# Computer algebra independent integration tests

## 3-Logarithms/3.2.1-f+g-x-^m-A+B-log-e-a+b-x-over-c+d-x-^n-^p

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3.134 $\int \frac{\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}{(ag+bgx)^3}\,dx$ . . . . . . . . . . 846

3.135 $\int \frac{\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}{(ag+bgx)^4}\,dx$ . . . . . . . . . . 854

3.136 $\int \frac{\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}{(ag+bgx)^5}\,dx$ . . . . . . . . . . 863

3.137 $\int \frac{(ag+bgx)^2}{A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}\,dx$ . . . . . . . . . . 873

3.138 $\int \frac{ag+bgx}{A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}\,dx$ . . . . . . . . . . 876

3.139 $\int \frac{1}{(ag+bgx)\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}\,dx$ . . . . . . . . . . 879

3.140 $\int \frac{1}{(ag+bgx)^2\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}\,dx$ . . . . . . . . . . 882

3.141 $\int \frac{1}{(ag+bgx)^3\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}\,dx$ . . . . . . . . . . 885

3.142 $\int \frac{(ag+bgx)^2}{\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}\,dx$ . . . . . . . . . . 888

3.143 $\int \frac{ag+bgx}{\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}\,dx$ . . . . . . . . . . 892

3.144 $\int \frac{1}{(ag+bgx)\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}\,dx$ . . . . . 896

3.145 $\int \frac{1}{(ag+bgx)^2\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}\,dx$ . . . . . 899

3.146 $\int \frac{1}{(ag+bgx)^3\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}\,dx$ . . . . . 903

3.147 $\int (a+bx)^4\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)\,dx$ . . . . . 907

3.148 $\int (a+bx)^3\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)\,dx$ . . . . . 913

3.149 $\int (a+bx)^2\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)\,dx$ . . . . . 918

3.150 $\int (a+bx)\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)\,dx$ . . . . . 923

3.151 $\int \frac{A+B\log(e(a+bx)^n(c+dx)^{-n})}{a+bx}\,dx$ . . . . . 927

3.152 $\int \frac{A+B\log(e(a+bx)^n(c+dx)^{-n})}{(a+bx)^2}\,dx$ . . . . . 932

3.153 $\int \frac{A+B\log(e(a+bx)^n(c+dx)^{-n})}{(a+bx)^3}\,dx$ . . . . . 936

3.154 $\int \frac{A+B\log(e(a+bx)^n(c+dx)^{-n})}{(a+bx)^4}\,dx$ . . . . . 941

3.155 $\int \frac{A+B\log(e(a+bx)^n(c+dx)^{-n})}{(a+bx)^5}\,dx$ . . . . . 946

3.156 $\int (a+bx)^3\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^2\,dx$ . . . . . 952

3.157 $\int (a+bx)^2\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^2\,dx$ . . . . . 960

3.158 $\int (a+bx)\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^2\,dx$ . . . . . 967

3.159 $\int \frac{\left(A+B\log(e(a+bx)^n(c+dx)^{-n})\right)^2}{a+bx}\,dx$ . . . . . 974

3.160 $\int \frac{\left(A+B\log(e(a+bx)^n(c+dx)^{-n})\right)^2}{(a+bx)^2}\,dx$ . . . . . 979

3.161 $\int \frac{\left(A+B\log(e(a+bx)^n(c+dx)^{-n})\right)^2}{(a+bx)^3}\,dx$ . . . . . 983

3.162 $\int \frac{\left(A+B\log(e(a+bx)^n(c+dx)^{-n})\right)^2}{(a+bx)^4}\,dx$ . . . . . 989

3.163 $\int \frac{\left(A+B\log(e(a+bx)^n(c+dx)^{-n})\right)^2}{(a+bx)^5}\,dx$ . . . . . 997

3.164 $\int (a+bx)^3\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^3\,dx$ . . . . . 1006

3.165 $\int (a+bx)^2\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^3\,dx$ . . . . . 1014

3.166 $\int (a+bx)\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^3\,dx$ . . . . . 1022

3.167 $\int \frac{\left(A+B\log(e(a+bx)^n(c+dx)^{-n})\right)^3}{a+bx}\,dx$ . . . . . 1030

3.168 $\int \frac{\left(A+B\log(e(a+bx)^n(c+dx)^{-n})\right)^3}{(a+bx)^2}\,dx$ . . . . . 1037

3.169 $\int \frac{\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^3}{(a+bx)^3}\,dx$ . . . . . 1042

3.170 $\int \frac{\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^3}{(a+bx)^4}\,dx$ . . . . . 1050

3.171 $\int \frac{\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^3}{(a+bx)^5}\,dx$ . . . . . 1063

3.172 $\int \frac{1}{(ag+bgx)^2\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)}\,dx$ . . . . . 1079

3.173 $\int (ag+bgx)^4\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)\,dx$ . . . . . 1082

3.174 $\int (ag+bgx)^3\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)\,dx$ . . . . . 1091

3.175 $\int (ag+bgx)^2\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)\,dx$ . . . . . 1098

3.176 $\int (ag+bgx)\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)\,dx$ . . . . . 1104

3.177 $\int \frac{A+B\log\left(\frac{e(c+dx)}{a+bx}\right)}{ag+bgx}\,dx$ . . . . . 1109

3.178 $\int \frac{A+B\log\left(\frac{e(c+dx)}{a+bx}\right)}{(ag+bgx)^2}\,dx$ . . . . . 1114

3.179 $\int \frac{A+B\log\left(\frac{e(c+dx)}{a+bx}\right)}{(ag+bgx)^3}\,dx$ . . . . . 1118

3.180 $\int \frac{A+B\log\left(\frac{e(c+dx)}{a+bx}\right)}{(ag+bgx)^4}\,dx$ . . . . . 1123

3.181 $\int \frac{A+B\log\left(\frac{e(c+dx)}{a+bx}\right)}{(ag+bgx)^5}\,dx$ . . . . . 1128

3.182 $\int (ag+bgx)^4\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2\,dx$ . . . . . 1134

3.183 $\int (ag+bgx)^3\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2\,dx$ . . . . . 1141

3.184 $\int (ag+bgx)^2\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2\,dx$ . . . . . 1148

3.185 $\int (ag+bgx)\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2\,dx$ . . . . . 1155

3.186 $\int \frac{\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2}{ag+bgx}\,dx$ . . . . . 1161

3.187 $\int \frac{\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2}{(ag+bgx)^2}\,dx$ . . . . . 1170

3.188 $\int \frac{\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2}{(ag+bgx)^3}\,dx$ . . . . . 1178

3.189 $\int \frac{\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2}{(ag+bgx)^4}\,dx$ . . . . . 1187

3.190 $\int \frac{\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2}{(ag+bgx)^5}\,dx$ . . . . . 1197

3.191 $\int \frac{(ag+bgx)^2}{A+B\log\left(\frac{e(c+dx)}{a+bx}\right)}\,dx$ . . . . . 1208

3.192 $\int \frac{ag+bgx}{A+B\log\left(\frac{e(c+dx)}{a+bx}\right)}\,dx$ . . . . . 1211

3.193 $\int \frac{1}{(ag+bgx)\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)}\,dx$ . . . . . 1214

3.194 $\int \frac{1}{(ag+bgx)^2\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)}\,dx$ . . . . . 1217

3.195 $\int \frac{1}{(ag+bgx)^3\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)}\,dx$ . . . . . 1220

3.196 $\int \frac{(ag+bgx)^2}{\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2}\,dx$ . . . . . 1223

3.197 $\int \frac{ag+bgx}{\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2}\,dx$ . . . . . 1226

3.198 $\int \frac{1}{(ag+bgx)\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2}\,dx$ . . . . . 1229

3.199 $\int \frac{1}{(ag+bgx)^2\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2}\,dx$ . . . . . 1232

3.200 $\int \frac{1}{(ag+bgx)^3\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2}\,dx$ . . . . . 1236

3.201 $\int (ag+bgx)^4\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)\,dx$ . . . . . 1240

3.202 $\int (ag+bgx)^3\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)\,dx$ . . . . . 1246

3.203 $\int (ag+bgx)^2\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)\,dx$ . . . . . 1251

3.204 $\int (ag+bgx)\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)\,dx$ . . . . . 1256

3.205 $\int \frac{A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)}{ag+bgx}\,dx$ . . . . . 1260

3.206 $\int \frac{A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)}{(ag+bgx)^2}\,dx$ . . . . . 1265

3.207 $\int \frac{A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)}{(ag+bgx)^3}\,dx$ . . . . . 1269

3.208 $\int \frac{A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)}{(ag+bgx)^4}\,dx$ . . . . . 1274

3.209 $\int \frac{A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)}{(ag+bgx)^5}\,dx$ . . . . . 1279

3.210 $\int (ag+bgx)^4 \left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2 dx$ . . . . . . . . . . . . . . . . . . . . . . . 1285

3.211 $\int (ag+bgx)^3 \left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2 dx$ . . . . . . . . . . . . . . . . . . . . . . . 1293

3.212 $\int (ag+bgx)^2 \left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2 dx$ . . . . . . . . . . . . . . . . . . . . . . . 1300

3.213 $\int (ag+bgx) \left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2 dx$ . . . . . . . . . . . . . . . . . . . . . . . 1307

3.214 $\int \frac{\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2}{ag+bgx} dx$ . . . . . . . . . . . . . . . . . . . . . . . 1314

3.215 $\int \frac{\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2}{(ag+bgx)^2} dx$ . . . . . . . . . . . . . . . . . . . . . . . 1323

3.216 $\int \frac{\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2}{(ag+bgx)^3} dx$ . . . . . . . . . . . . . . . . . . . . . . . 1331

3.217 $\int \frac{\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2}{(ag+bgx)^4} dx$ . . . . . . . . . . . . . . . . . . . . . . . 1339

3.218 $\int \frac{\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2}{(ag+bgx)^5} dx$ . . . . . . . . . . . . . . . . . . . . . . . 1348

3.219 $\int \frac{(ag+bgx)^2}{A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)} dx$ . . . . . . . . . . . . . . . . . . . . . . . 1358

3.220 $\int \frac{ag+bgx}{A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)} dx$ . . . . . . . . . . . . . . . . . . . . . . . 1361

3.221 $\int \frac{1}{(ag+bgx)\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)} dx$ . . . . . . . . . . . . . . . . . . . . . . . 1364

3.222 $\int \frac{1}{(ag+bgx)^2\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)} dx$ . . . . . . . . . . . . . . . . . . . . . . . 1367

3.223 $\int \frac{1}{(ag+bgx)^3\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)} dx$ . . . . . . . . . . . . . . . . . . . . . . . 1370

3.224 $\int \frac{(ag+bgx)^2}{\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2} dx$ . . . . . . . . . . . . . . . . . . . . . . . 1373

3.225 $\int \frac{ag+bgx}{\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2} dx$ . . . . . . . . . . . . . . . . . . . . . . . 1377

3.226 $\int \frac{1}{(ag+bgx)\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2} dx$ . . . . . . . . . . . . . . . . . . . . . . . 1381

3.227 $\int \frac{1}{(ag+bgx)^2\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2} dx$ . . . . . . . . . . . . . . . . . . . . . . . 1384

3.228 $\int \frac{1}{(ag+bgx)^3\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2}\,dx$ . . . . . . . . . . 1388

3.229 $\int \frac{1}{(ag+bgx)^2\left(A+B\log(e(a+bx)^n(c+dx)^{-n})\right)}\,dx$ . . . . . . . . . . 1392

3.230 $\int (f+gx)^4\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)dx$ . . . . . . . . . . 1395

3.231 $\int (f+gx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)dx$ . . . . . . . . . . 1401

3.232 $\int (f+gx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)dx$ . . . . . . . . . . 1411

3.233 $\int (f+gx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)dx$ . . . . . . . . . . 1420

3.234 $\int \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)dx$ . . . . . . . . . . 1426

3.235 $\int \frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{f+gx}\,dx$ . . . . . . . . . . 1430

3.236 $\int \frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{(f+gx)^2}\,dx$ . . . . . . . . . . 1435

3.237 $\int \frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{(f+gx)^3}\,dx$ . . . . . . . . . . 1440

3.238 $\int \frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{(f+gx)^4}\,dx$ . . . . . . . . . . 1448

3.239 $\int \frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{(f+gx)^5}\,dx$ . . . . . . . . . . 1453

3.240 $\int (f+gx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2dx$ . . . . . . . . . . 1459

3.241 $\int (f+gx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2dx$ . . . . . . . . . . 1467

3.242 $\int (f+gx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2dx$ . . . . . . . . . . 1474

3.243 $\int \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2dx$ . . . . . . . . . . 1481

3.244 $\int \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{f+gx}\,dx$ . . . . . . . . . . 1487

3.245 $\int \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(f+gx)^2}\,dx$ . . . . . . . . . . 1497

3.246 $\int \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(f+gx)^3}\,dx$ . . . . . . . . . . 1504

3.247 $\int \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(f+gx)^4}\,dx$ . . . . . . . . . . 1511

3.248 $\int \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(f+gx)^5}\,dx$ . . . . . . . . . . 1519

3.249 $\int \frac{\log\left(\frac{1+x}{-1+x}\right)}{x^2} dx$ . . . 1528

3.250 $\int \frac{(f+gx)^2}{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)} dx$ . . . 1532

3.251 $\int \frac{f+gx}{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)} dx$ . . . 1535

3.252 $\int \frac{1}{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)} dx$ . . . 1538

3.253 $\int \frac{1}{(f+gx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)} dx$ . . . 1541

3.254 $\int \frac{1}{(f+gx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)} dx$ . . . 1544

3.255 $\int \frac{1}{(f+gx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)} dx$ . . . 1547

3.256 $\int \frac{(f+gx)^2}{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2} dx$ . . . 1550

3.257 $\int \frac{f+gx}{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2} dx$ . . . 1554

3.258 $\int \frac{1}{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2} dx$ . . . 1557

3.259 $\int \frac{1}{(f+gx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2} dx$ . . . 1560

3.260 $\int \frac{1}{(f+gx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2} dx$ . . . 1563

3.261 $\int \frac{1}{(f+gx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2} dx$ . . . 1566

3.262 $\int (f+gx)^4\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right) dx$ . . . 1569

3.263 $\int (f+gx)^3\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right) dx$ . . . 1576

3.264 $\int (f+gx)^2\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right) dx$ . . . 1582

3.265 $\int (f+gx)\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right) dx$ . . . 1587

3.266 $\int \left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right) dx$ . . . 1591

3.267 $\int \frac{A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}{f+gx} dx$ . . . 1595

3.268 $\int \frac{A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}{(f+gx)^2} dx$ . . . 1600

3.269 $\int \frac{A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}{(f+gx)^3}\,dx$ . . . . . . . . . . 1604

3.270 $\int \frac{A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}{(f+gx)^4}\,dx$ . . . . . . . . . . 1609

3.271 $\int \frac{A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}{(f+gx)^5}\,dx$ . . . . . . . . . . 1616

3.272 $\int (f+gx)^3\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2 dx$ . . . . . . . . . . 1623

3.273 $\int (f+gx)^2\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2 dx$ . . . . . . . . . . 1632

3.274 $\int (f+gx)\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2 dx$ . . . . . . . . . . 1639

3.275 $\int \left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2 dx$ . . . . . . . . . . 1646

3.276 $\int \frac{\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}{f+gx}\,dx$ . . . . . . . . . . 1652

3.277 $\int \frac{\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}{(f+gx)^2}\,dx$ . . . . . . . . . . 1661

3.278 $\int \frac{\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}{(f+gx)^3}\,dx$ . . . . . . . . . . 1668

3.279 $\int \frac{\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}{(f+gx)^4}\,dx$ . . . . . . . . . . 1675

3.280 $\int \frac{\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}{(f+gx)^5}\,dx$ . . . . . . . . . . 1683

3.281 $\int \frac{(f+gx)^2}{A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}\,dx$ . . . . . . . . . . 1692

3.282 $\int \frac{f+gx}{A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}\,dx$ . . . . . . . . . . 1695

3.283 $\int \frac{1}{A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}\,dx$ . . . . . . . . . . 1698

3.284 $\int \frac{1}{(f+gx)\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}\,dx$ . . . . . . . . . . 1701

3.285 $\int \frac{1}{(f+gx)^2\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}\,dx$ . . . . . . . . . . 1704

3.286 $\int \frac{1}{(f+gx)^3\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}\,dx$ . . . . . . . . . . 1707

3.287 $\int \frac{(f+gx)^2}{\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}\,dx$ . . . . 1710
3.288 $\int \frac{f+gx}{\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}\,dx$ . . . . 1714
3.289 $\int \frac{1}{\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}\,dx$ . . . . 1718
3.290 $\int \frac{1}{(f+gx)\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}\,dx$ . . . . 1721
3.291 $\int \frac{1}{(f+gx)^2\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}\,dx$ . . . . 1724
3.292 $\int \frac{1}{(f+gx)^3\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}\,dx$ . . . . 1727
3.293 $\int (g+hx)^4\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)\,dx$ . . . . 1730
3.294 $\int (g+hx)^3\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)\,dx$ . . . . 1736
3.295 $\int (g+hx)^2\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)\,dx$ . . . . 1741
3.296 $\int (g+hx)\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)\,dx$ . . . . 1746
3.297 $\int \left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)\,dx$ . . . . 1750
3.298 $\int \frac{A+B\log(e(a+bx)^n(c+dx)^{-n})}{g+hx}\,dx$ . . . . 1753
3.299 $\int \frac{A+B\log(e(a+bx)^n(c+dx)^{-n})}{(g+hx)^2}\,dx$ . . . . 1757
3.300 $\int \frac{A+B\log(e(a+bx)^n(c+dx)^{-n})}{(g+hx)^3}\,dx$ . . . . 1762
3.301 $\int \frac{A+B\log(e(a+bx)^n(c+dx)^{-n})}{(g+hx)^4}\,dx$ . . . . 1769
3.302 $\int \frac{A+B\log(e(a+bx)^n(c+dx)^{-n})}{(g+hx)^5}\,dx$ . . . . 1774
3.303 $\int (g+hx)^2\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^2\,dx$ . . . . 1781
3.304 $\int (g+hx)\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^2\,dx$ . . . . 1788
3.305 $\int \left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^2\,dx$ . . . . 1795
3.306 $\int \frac{\left(A+B\log(e(a+bx)^n(c+dx)^{-n})\right)^2}{g+hx}\,dx$ . . . . 1802
3.307 $\int \frac{\left(A+B\log(e(a+bx)^n(c+dx)^{-n})\right)^2}{(g+hx)^2}\,dx$ . . . . 1808
3.308 $\int \frac{\left(A+B\log(e(a+bx)^n(c+dx)^{-n})\right)^2}{(g+hx)^3}\,dx$ . . . . 1815
3.309 $\int (g+hx)^2\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^3\,dx$ . . . . 1822
3.310 $\int (g+hx)\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^3\,dx$ . . . . 1830

3.311 $\int \left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^3\,dx$ . . . . . . . . . . . . . . . . . . . . . .1838

3.312 $\int \frac{\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^3}{g+hx}\,dx$ . . . . . . . . . . . . . . . . . . . . . . . . . . . . . .1843

3.313 $\int \frac{\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^3}{(g+hx)^2}\,dx$ . . . . . . . . . . . . . . . . . . . . . . . . . . . . . .1849

3.314 $\int \frac{\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^3}{(g+hx)^3}\,dx$ . . . . . . . . . . . . . . . . . . . . . . . . . . . . . .1855

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# Chapter 1

# Introduction

This report gives the result of running the computer algebra independent integration problems. The listing of the problems are maintained by and can be downloaded from `https://rulebasedintegration.org`

The number of integrals in this report is [ 314 ]. This is test number [ 59 ].

## 1.1 Listing of CAS systems tested

The following systems were tested at this time.

1. Mathematica 12.3 (64 bit) on windows 10.
2. Rubi 4.16.1 in Mathematica 12.1 on windows 10.
3. Maple 2021.1 (64 bit) on windows 10.
4. Maxima 5.44 on Linux. (via sagemath 9.3)
5. Fricas 1.3.7 on Linux (via sagemath 9.3)
6. Giac/Xcas 1.7 on Linux. (via sagemath 9.3)
7. Sympy 1.8 under Python 3.8.8 using Anaconda distribution on Ubuntu.
8. Mupad using Matlab 2021a with Symbolic Math Toolbox Version 8.7 under windows 10 (64 bit)

Maxima, Fricas and Giac/Xcas were called from inside SageMath. This was done using SageMath integrate command by changing the name of the algorithm to use the different CAS systems.

Sympy was called directly using Python.

## 1.2 Results

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or `Hypergeometric2F1` functions. `RootSum` and `RootOf` are not allowed.

If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

| System | solved | Failed |
|---|---|---|
| Rubi | % 91.72 ( 288 ) | % 8.28 ( 26 ) |
| Mathematica | % 94.90 ( 298 ) | % 5.10 ( 16 ) |
| Maple | % 59.55 ( 187 ) | % 40.45 ( 127 ) |
| Maxima | % 75.80 ( 238 ) | % 24.20 ( 76 ) |
| Fricas | % 66.88 ( 210 ) | % 33.12 ( 104 ) |
| Sympy | % 37.58 ( 118 ) | % 62.42 ( 196 ) |
| Giac | % 48.41 ( 152 ) | % 51.59 ( 162 ) |
| Mupad | % 63.69 ( 200 ) | % 36.31 ( 114 ) |

Table 1.1: Percentage solved for each CAS

The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.

| grade | description |
|---|---|
| A | Integral was solved and antiderivative is optimal in quality and leaf size. |
| B | Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size. |
| C | Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons<br>1. antiderivative contains a hypergeometric function and the optimal antiderivative does not.<br>2. antiderivative contains a special function and the optimal antiderivative does not.<br>3. antiderivative contains the imaginary unit and the optimal antiderivative does not. |
| F | Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised. |

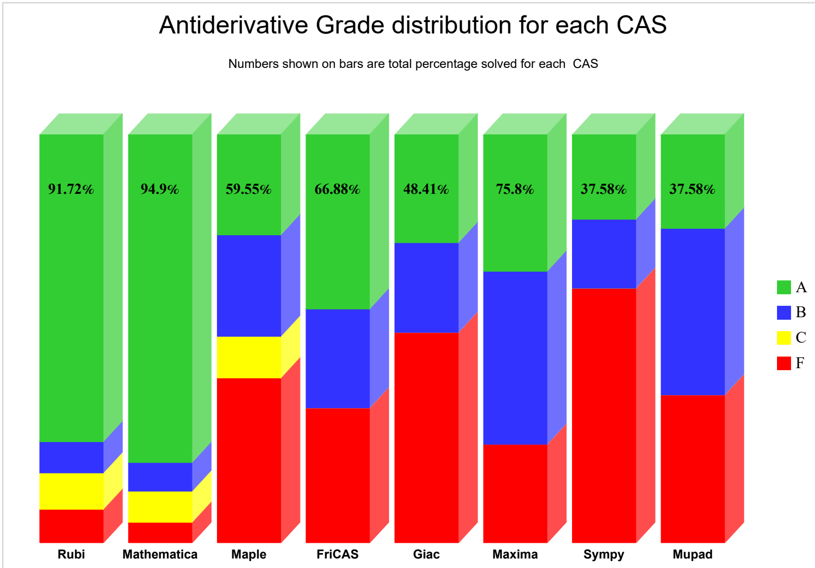
Table 1.2: Description of grading applied to integration result

Grading is implemented for all CAS systems. Based on the above, the following table summarizes the grading for this test suite.

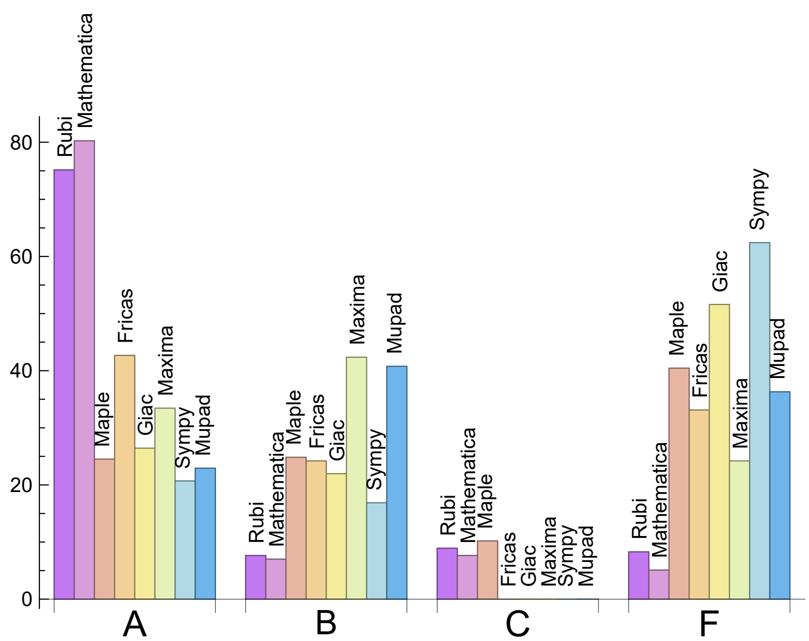
| System | % A grade | % B grade | % C grade | % F grade |
|---|---|---|---|---|
| Rubi | 75.16 | 7.64 | 8.92 | 8.28 |
| Mathematica | 80.25 | 7.01 | 7.64 | 5.10 |
| Maple | 24.52 | 24.84 | 10.19 | 40.45 |
| Maxima | 33.44 | 42.36 | 0.00 | 24.20 |
| Fricas | 42.68 | 24.20 | 0.00 | 33.12 |
| Sympy | 20.70 | 16.88 | 0.00 | 62.42 |
| Giac | 26.43 | 21.97 | 0.00 | 51.59 |
| Mupad | 22.93 | 40.76 | 0.00 | 36.31 |

Table 1.3: Antiderivative Grade distribution of each CAS

The following is a Bar chart illustration of the data in the above table.




The figure below compares the CAS systems for each grade level.




The following table shows the distribution of the different types of failure for each CAS. There are 3 types of reasons why it can fail. The first is when CAS returns back the input within the time limit, which means it could not solve it. This the typical normal failure **F** .

The second is due to time out. CAS could not solve the integral within the 3 minutes time limit which is assigned **F(-1)**.

The third is due to an exception generated. Assigned **F(-2)**. This most likely indicates an interface problem between sagemath and the CAS (applicable only to FriCAS, Maxima and Giac) or it could be an indication of an internal error in CAS. This type of error requires more investigations to determine the cause.

| System | Number failed | Percentage normal failure | Percentage time-out failure | Percentage exception failure |
|---|---|---|---|---|
| Rubi | 26 | 100.00 % | 0.00 % | 0.00 % |
| Mathematica | 16 | 100.00 % | 0.00 % | 0.00 % |
| Maple | 127 | 100.00 % | 0.00 % | 0.00 % |
| Maxima | 76 | 100.00 % | 0.00 % | 0.00 % |
| Fricas | 104 | 92.31 % | 7.69 % | 0.00 % |
| Sympy | 196 | 24.49 % | 60.20 % | 15.31 % |
| Giac | 162 | 40.74 % | 58.02 % | 1.23 % |
| Mupad | 114 | 100.00 % | 0.00 % | 0.00 % |

Table 1.4: Time and leaf size performance for each CAS

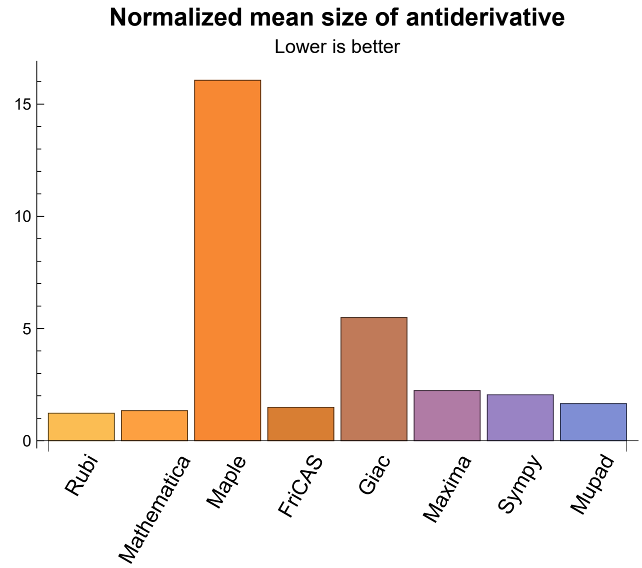
## 1.3 Performance

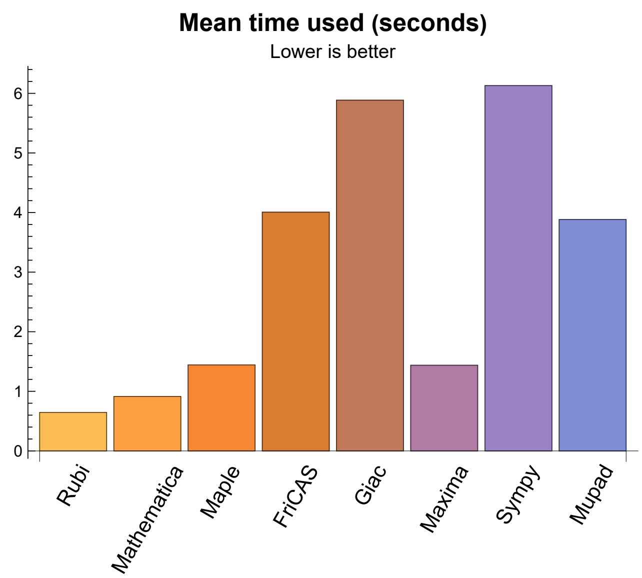
The table below summarizes the performance of each CAS system in terms of CPU time and leaf size of results.

| System | Mean time (sec) | Mean size | Normalized mean | Median size | Normalized median |
|---|---|---|---|---|---|
| Rubi | 0.65 | 340.11 | 1.23 | 179.00 | 1.00 |
| Mathematica | 0.91 | 374.14 | 1.34 | 142.50 | 0.93 |
| Maple | 1.44 | 5560.30 | 16.06 | 452.00 | 3.19 |
| Maxima | 1.44 | 633.40 | 2.24 | 368.50 | 2.37 |
| Fricas | 4.01 | 343.14 | 1.49 | 161.00 | 1.54 |
| Sympy | 6.13 | 351.95 | 2.04 | 131.00 | 2.20 |
| Giac | 5.89 | 942.50 | 5.49 | 288.50 | 1.82 |
| Mupad | 3.88 | 417.30 | 1.66 | 160.00 | 1.54 |

Table 1.5: Time and leaf size performance for each CAS

The following are bar charts for the normalized leafsize and time used columns from the above table.




## 1.4 list of integrals that has no closed form antiderivative

{19, 20, 21, 24, 25, 26, 47, 48, 49, 52, 53, 54, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 109, 110, 111, 114, 115, 116, 137, 138, 139, 142, 143, 144, 191, 192, 193, 196, 197, 198, 219, 220, 221, 224, 225, 226, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292}

## 1.5 list of integrals solved by CAS but has no known antiderivative

**Rubi** {}

**Mathematica** {}

**Maple** {}

**Maxima** {}

**Fricas** {}

**Sympy** {}

**Giac** {}

**Mupad** {}

## 1.6 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not mean necessarily that the anti-derivative is wrong, as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it easier to do further investigation to determine why it was not possible to verify the result produced.

**Rubi** {}

**Mathematica** {}

**Maple** Verification phase not implemented yet.

**Maxima** Verification phase not implemented yet.

**Fricas** Verification phase not implemented yet.

**Sympy** Verification phase not implemented yet.

**Giac** Verification phase not implemented yet.

**Mupad** Verification phase not implemented yet.

## 1.7 Timing

The command `AboluteTiming[]` was used in Mathematica to obtain the elapsed time for each integrate call. In Maple, the command `Usage` was used as in the following example

```
cpu_time := Usage(assign ('result_of _int',int(expr,x)),output='realtime'
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call has completed from the time before the call was made. This was done using Python's `time.time()` call.

All elapsed times shown are in seconds. A time limit of 3 minutes was used for each integral. If the integrate command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by failed integrals due to time out is not counted in the final statistics.

## 1.8 Verification

A verification phase was applied on the result of integration for Rubi and Mathematica. Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative produced was correct.

Verification phase has 3 minutes time out. An integral whose result was not verified could still be correct. Further investigation is needed on those integrals which failed verifications. Such integrals are marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

## 1.9 Important notes about some of the results

### 1.9.1 Important note about Maxima results

Since these integrals are run in a batch mode, using an automated script, and by using sagemath (SageMath uses Maxima), then any integral where Maxima needs an interactive response from the user to answer a question during evaluation of the integral in order to complete the integration, will fail and is counted as failed.

The exception raised is `ValueError`. Therefore Maxima result below is lower than what could result if Maxima was run directly and each question Maxima asks was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the Timofeev test file, there were about 14 such integrals out of total 705, or about 2 percent. This pecrentage can be higher or lower depending on the specific input test file.

Such integrals can be indentified by looking at the output of the integration in each section for Maxima. The exception message will indicate of the error is due to the interactive question being asked or not.

Maxima integrate was run using SageMath with the following settings set by default

```
'besselexpand : true'
'display2d : false'
'domain : complex'
'keepfloat : true'
'load(to_poly_solve)'
'load(simplify_sum)'
'load(abs_integrate)' 'load(diag)'
```

SageMath loading of Maxima `abs_integrate` was found to cause some problem. So the following code was added to disable this effect.

```
from sage.interfaces.maxima_lib import maxima_lib
maxima_lib.set('extra_definite_integration_methods', '[]')
maxima_lib.set('extra_integration_methods', '[]')
```

See https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/ for reference.

### 1.9.2 Important note about FriCAS and Giac/XCAS results

There are Few integrals which failed due to SageMath not able to translate the result back to SageMath syntax and not because these CAS system were not able to do the integrations.

These will fail With error `Exception raised: NotImplementedError`

The number of such cases seems to be very small. About 1 or 2 percent of all integrals.

Hopefully the next version of SageMath will have complete translation of FriCAS and XCAS syntax and I will re-run all the tests again when this happens.

### 1.9.3 Important note about finding leaf size of antiderivative

For Mathematica, Rubi and Maple, the buildin system function `LeafSize` is used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special buildin function for this purpose at this time. Therefore the leaf size for Fricas and Sympy and Giac antiderivatives is determined using the following function, thanks to user `slelievre` at https://

ask.sagemath.org/question/57123/could-we-have-a-leaf_count-function-in-base-sagemath/

```
def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)
```

For Sympy, which is called directly from Python, the following code is used to obtain the leafsize of its result

```
try:
  # 1.7 is a fudge factor since it is low side from actual leaf count
  leafCount = round(1.7*count_ops(anti))

  except Exception as ee:
         leafCount =1
```

### 1.9.4 Important note about Mupad results

Matlab's symbolic toolbox does not have a leaf count function to measure the size of the antiderivative, Maple was used to determine the leaf size of Mupad output by post processing.

Currently no grading of the antiderivative for Mupad is implemented. If it can integrate the problem, it was assigned a B grade automatically as a placeholder. In the future, when grading function is implemented for Mupad, the tests will be rerun again.

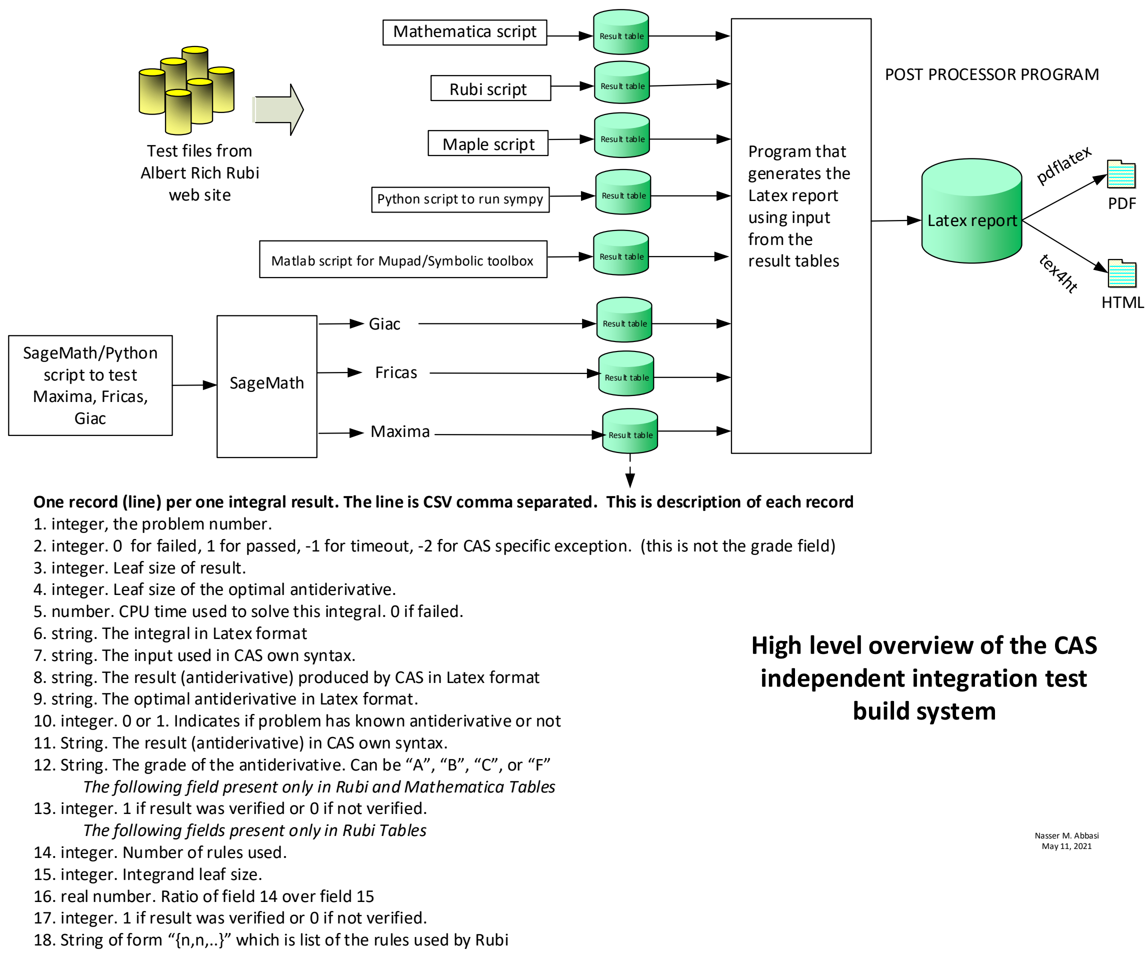
The following is an example of using Matlab's symbolic toolbox (Mupad) to solve an integral

```
integrand = evalin(symengine,'cos(x)*sin(x)')
the_variable = evalin(symengine,'x')
anti = int(integrand,the_variable)
```

Which gives `sin(x)^2/2`

## 1.10 Design of the test system

The following diagram gives a high level view of the current test build system.



# Chapter 2

# detailed summary tables of results

## 2.1 List of integrals sorted by grade for each CAS

### 2.1.1 Rubi

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 19, 20, 21, 24, 25, 26, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 47, 48, 49, 52, 53, 54, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 106, 107, 108, 109, 110, 111, 114, 115, 116, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 137, 138, 139, 142, 143, 144, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 164, 165, 166, 168, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 191, 192, 193, 196, 197, 198, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 219, 220, 221, 224, 225, 226, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 309 }

B grade: { 14, 42, 70, 71, 72, 73, 101, 132, 167, 169, 186, 214, 244, 245, 246, 276, 277, 278, 308, 310, 311, 312, 313, 314 }

C grade: { 15, 16, 17, 18, 43, 44, 45, 46, 102, 103, 104, 105, 133, 134, 135, 136, 162, 163, 170, 171, 187, 188, 189, 190, 215, 216, 217, 218 }

F grade: { 22, 23, 27, 28, 50, 51, 55, 56, 112, 113, 117, 118, 140, 141, 145, 146, 172, 194, 195, 199, 200, 222, 223, 227, 228, 229 }

### 2.1.2 Mathematica

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 137, 138, 139, 142, 143, 144, 148, 149, 150, 151, 152, 153, 154, 155, 160, 161, 162, 163, 169, 170, 171, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 219, 220, 221, 224, 225, 226, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 311 }

B grade: { 14, 42, 71, 72, 106, 107, 108, 147, 156, 157, 158, 159, 164, 165, 166, 167, 168, 245, 277, 306, 307, 308 }

C grade: { 15, 16, 17, 18, 43, 44, 45, 46, 102, 103, 104, 105, 133, 134, 135, 136, 187, 188, 189, 190, 215, 216, 217, 218 }

F grade: { 140, 141, 145, 146, 172, 222, 223, 227, 228, 229, 276, 309, 310, 312, 313, 314 }

### 2.1.3 Maple

A grade: { 19, 20, 21, 24, 25, 26, 47, 48, 49, 52, 53, 54, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 106, 107, 108, 109, 110, 111, 114, 115, 116, 137, 138, 139, 142, 143, 144, 191, 192, 193, 194, 196, 197, 198, 219, 220, 221, 224, 225, 226, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292 }

B grade: { 61, 88, 89, 90, 91, 92, 93, 94, 95, 96, 101, 102, 103, 104, 105, 119, 120, 121, 122, 123, 124, 125, 126, 127, 133, 134, 135, 136, 173, 174, 175, 176, 177, 178, 179, 180, 181, 186, 187, 188, 189, 190, 199, 201, 202, 203, 204, 205, 206, 207, 208, 209, 215, 216, 217, 218, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 244, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 297 }

C grade: { 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 160, 161, 162, 163, 168, 169, 170, 171, 293, 294, 295, 296, 298, 299, 300, 301, 302, 303, 304, 305 }

F grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 22, 23, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 50, 51, 55, 56, 57, 58, 59, 60, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 97, 98, 99, 100, 112, 113, 117, 118, 128, 129, 130, 131, 132, 140, 141, 145, 146, 159, 164, 165, 166, 167, 172, 182, 183, 184, 185, 195, 200, 210, 211, 212, 213, 214, 222, 223, 227, 228, 229, 240, 241, 242, 243, 245, 246, 247, 248, 272, 273, 274, 275, 276, 277, 278, 279, 280, 306, 307, 308, 309, 310, 311, 312, 313, 314 }

### 2.1.4 Maxima

A grade: { 4, 7, 19, 20, 21, 24, 25, 26, 32, 34, 35, 47, 48, 49, 52, 53, 54, 57, 58, 59, 60, 61, 63, 64, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 91, 94, 109, 110, 111, 114, 115, 116, 137, 138, 139, 142, 143, 144, 150, 152, 153, 176, 179, 191, 192, 193, 196, 197, 198, 206, 219, 220, 221, 224, 225, 226, 230, 231, 232, 233, 234, 236, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 266, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 295, 296, 297, 299 }

B grade: { 1, 2, 3, 6, 8, 9, 10, 11, 12, 13, 15, 16, 17, 18, 29, 30, 31, 36, 37, 38, 39, 40, 41, 43, 44, 45, 46, 65, 66, 67, 68, 69, 88, 89, 90, 93, 95, 96, 97, 98, 99, 100, 102, 103, 104, 105, 106, 107, 108, 119, 120, 121, 122, 124, 125, 126, 127, 128, 129, 130, 131, 133, 134, 135, 136, 147, 148, 149, 154, 155, 156, 157, 158, 160, 161, 162, 163, 168, 169, 170, 171, 173, 174, 175, 178, 180, 181, 182, 183, 184, 185, 187, 188, 189, 190, 201, 202, 203, 204, 207, 208, 209, 210, 211, 212, 213, 215, 216, 217, 218, 237, 238, 239, 240, 241, 242, 262, 263, 264, 265, 268, 269, 270, 271, 272, 273, 274, 294, 300, 301, 302, 303, 304 }

C grade: { }

F grade: { 5, 14, 22, 23, 27, 28, 33, 42, 50, 51, 55, 56, 62, 70, 71, 72, 73, 74, 75, 92, 101, 112, 113, 117, 118, 123, 132, 140, 141, 145, 146, 151, 159, 164, 165, 166, 167, 172, 177, 186, 194, 195, 199, 200, 205, 214, 222, 223, 227, 228, 229, 235, 243, 244, 245, 246, 247, 248, 267, 275, 276, 277, 278, 279, 280, 298, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314 }

### 2.1.5 FriCAS

A grade: { 4, 6, 7, 15, 19, 20, 21, 22, 23, 24, 25, 26, 27, 34, 35, 43, 47, 48, 49, 50, 51, 52, 53, 54, 55, 60, 61, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 91, 93, 94, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 122, 124, 125, 133, 134, 135, 136, 137, 138, 139, 142, 143, 144, 152, 172, 176, 178, 179, 187, 188, 189, 191, 192, 193, 194, 195, 196, 197, 198, 204, 206, 207, 215, 216, 217, 219, 220, 221, 224, 225, 226, 229, 230, 232, 233, 234, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 265, 266, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 296, 297 }

B grade: { 1, 2, 3, 8, 9, 16, 17, 18, 28, 29, 30, 31, 32, 36, 37, 44, 45, 46, 56, 57, 58, 59, 63, 64, 88, 89, 90, 95, 96, 118, 119, 120, 121, 126, 127, 147, 148, 149, 150, 153, 154, 155, 160, 161, 162, 163, 168, 169, 170, 171, 173, 174, 175, 180, 181, 190, 199, 200, 201, 202, 203, 208, 209, 218, 231, 236, 237, 263, 264, 268, 269, 293, 294, 295, 299, 300 }

C grade: { }

F grade: { 5, 10, 11, 12, 13, 14, 33, 38, 39, 40, 41, 42, 62, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 92, 97, 98, 99, 100, 101, 123, 128, 129, 130, 131, 132, 140, 141, 145, 146, 151, 156, 157, 158, 159, 164, 165, 166, 167, 177, 182, 183, 184, 185, 186, 205, 210, 211, 212, 213, 214, 222, 223, 227, 228, 235, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 267, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 298, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314 }

### 2.1.6 Sympy

A grade: { 3, 4, 19, 20, 21, 24, 25, 31, 32, 47, 48, 49, 52, 53, 59, 60, 61, 76, 77, 78, 79, 82, 83, 84, 109, 110, 111, 114, 115, 116, 137, 138, 139, 142, 143, 144, 191, 192, 193, 197, 198, 219, 220, 221, 224, 225, 226, 234, 249, 250, 251, 252, 253, 254, 256, 257, 258, 281, 282, 283, 284, 285, 287, 288, 289 }

B grade: { 88, 89, 90, 91, 93, 94, 95, 96, 102, 103, 104, 119, 120, 121, 122, 124, 125, 126, 127, 133, 134, 135, 173, 174, 175, 176, 178, 179, 180, 181, 187, 188, 189, 201, 202, 203, 204, 206, 207, 208, 209, 215, 216, 217, 230, 231, 232, 233, 262, 263, 264, 265, 266 }

C grade: { }

F grade: { 1, 2, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 22, 23, 26, 27, 28, 29, 30, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 50, 51, 54, 55, 56, 57, 58, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 80, 81, 85, 86, 87, 92, 97, 98, 99, 100, 101, 105, 106, 107, 108, 112, 113, 117, 118, 123, 128, 129, 130, 131, 132, 136, 140, 141, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 177, 182, 183, 184, 185, 186, 190, 194, 195, 196, 199, 200, 205, 210, 211, 212, 213, 214, 218, 222, 223, 227, 228, 229, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 255, 259, 260, 261, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 286, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314 }

### 2.1.7 Giac

A grade: { 6, 7, 15, 16, 17, 18, 24, 25, 26, 34, 35, 43, 44, 45, 52, 53, 54, 55, 56, 82, 83, 84, 85, 86, 87, 93, 94, 102, 103, 104, 105, 116, 122, 125, 136, 137, 138, 139, 142, 143, 144, 150, 152, 153, 178, 179, 187, 188, 189, 198, 199, 204, 206, 207, 218, 219, 220, 221, 224, 225, 226, 258, 259, 260, 261, 264, 265, 266, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 296, 297, 299 }

B grade: { 1, 2, 3, 4, 8, 9, 29, 30, 31, 32, 36, 37, 46, 57, 58, 59, 60, 61, 63, 64, 65, 88, 89, 90, 91, 95, 96, 107, 108, 119, 120, 121, 124, 126, 127, 133, 147, 148, 149, 154, 155, 173, 174, 175, 176, 180, 181, 190, 200, 201, 202, 203, 208, 209, 215, 231, 232, 233, 234, 236, 237, 249, 263, 269, 270, 295, 300, 301, 302 }

C grade: { }

F grade: { 5, 10, 11, 12, 13, 14, 19, 20, 21, 22, 23, 27, 28, 33, 38, 39, 40, 41, 42, 47, 48, 49, 50, 51, 62, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 92, 97, 98, 99, 100, 101, 106, 109, 110, 111, 112, 113, 114, 115, 117, 118, 123, 128, 129, 130, 131, 132, 134, 135, 140, 141, 145, 146, 151, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 177, 182, 183, 184, 185, 186, 191, 192, 193, 194, 195, 196, 197, 205, 210, 211, 212, 213, 214, 216, 217, 222, 223, 227, 228, 229, 230, 235, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 250, 251, 252, 253, 254, 255, 256, 257, 262, 267, 268, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 293, 294, 298, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314 }

### 2.1.8 Mupad

A grade: { 19, 20, 21, 24, 25, 26, 47, 48, 49, 52, 53, 54, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 109, 110, 111, 114, 115, 116, 137, 138, 139, 142, 143, 144, 191, 192, 193, 196, 197, 198, 219, 220, 221, 224, 225, 226, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292 }

B grade: { 1, 2, 3, 4, 6, 7, 8, 9, 15, 16, 17, 18, 29, 30, 31, 32, 34, 35, 36, 37, 43, 44, 45, 46, 57, 58, 59, 60, 61, 63, 64, 65, 66, 88, 89, 90, 91, 93, 94, 95, 96, 102, 103, 104, 105, 106, 107, 108, 119, 120, 121, 122, 124, 125, 126, 127, 133, 134, 135, 136, 147, 148, 149, 150, 152, 153, 154, 155, 160, 161, 162, 163, 168, 169, 170, 171, 173, 174, 175, 176, 178, 179, 180, 181, 187, 188, 189, 190, 201, 202, 203, 204, 206, 207, 208, 209, 215, 216, 217, 218, 230, 231, 232, 233, 234, 236, 237, 238, 239, 249, 262, 263, 264, 265, 266, 268, 269, 270, 271, 293, 294, 295, 296, 297, 299, 300, 301, 302 }

C grade: { }

F grade: { 5, 10, 11, 12, 13, 14, 22, 23, 27, 28, 33, 38, 39, 40, 41, 42, 50, 51, 55, 56, 62, 67, 68, 69, 70, 71, 72, 73, 74, 75, 92, 97, 98, 99, 100, 101, 112, 113, 117, 118, 123, 128, 129, 130, 131, 132, 140, 141, 145, 146, 151, 156, 157, 158, 159, 164, 165, 166, 167, 172, 177, 182, 183, 184, 185, 186, 194, 195, 199, 200, 205, 210, 211, 212, 213, 214, 222, 223, 227, 228, 229, 235, 240, 241, 242, 243, 244, 245, 246, 247, 248, 267, 272, 273, 274, 275, 276, 277, 278, 279, 280, 298, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314 }

## 2.2 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by table below. The elapsed time is in seconds. For failed result it is given as F(-1) if the failure was due to timeout. It is given as F(-2) if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given just an F.

In this table,the column **normalized size** is defined as $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$

| Problem 1 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | F | B | B | F(-1) | B | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 188 | 188 | 146 | 0 | 676 | 569 | 0 | 4392 | 1046 |
| normalized size | 1 | 1.00 | 0.78 | 0.00 | 3.60 | 3.03 | 0.00 | 23.36 | 5.56 |
| time (sec) | N/A | 0.143 | 0.119 | 0.333 | 1.473 | 1.141 | 0.000 | 6.201 | 4.548 |

| Problem 2 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | F | B | B | F(-1) | B | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 156 | 156 | 124 | 0 | 479 | 426 | 0 | 2986 | 588 |
| normalized size | 1 | 1.00 | 0.79 | 0.00 | 3.07 | 2.73 | 0.00 | 19.14 | 3.77 |
| time (sec) | N/A | 0.106 | 0.108 | 0.265 | 1.408 | 1.128 | 0.000 | 4.412 | 4.399 |

| Problem 3 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | F | B | B | A | B | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 124 | 124 | 103 | 0 | 309 | 296 | 673 | 1836 | 303 |
| normalized size | 1 | 1.00 | 0.83 | 0.00 | 2.49 | 2.39 | 5.43 | 14.81 | 2.44 |
| time (sec) | N/A | 0.090 | 0.058 | 0.267 | 1.386 | 0.770 | 60.494 | 2.292 | 4.284 |

| Problem 4 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | F | A | A | A | B | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 86 | 86 | 73 | 0 | 156 | 160 | 398 | 864 | 134 |
| normalized size | 1 | 1.00 | 0.85 | 0.00 | 1.81 | 1.86 | 4.63 | 10.05 | 1.56 |
| time (sec) | N/A | 0.061 | 0.038 | 0.175 | 1.347 | 0.927 | 40.673 | 1.241 | 4.073 |

| Problem 5 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | F | F | F | F | F(-1) | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 84 | 126 | 101 | 0 | 0 | 0 | 0 | 0 | -1 |
| normalized size | 1 | 1.50 | 1.20 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | -0.01 |
| time (sec) | N/A | 0.209 | 0.053 | 0.365 | 0.000 | 0.839 | 0.000 | 0.000 | 0.000 |

| Problem 6 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | F | B | A | F(-2) | A | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 67 | 108 | 115 | 0 | 137 | 103 | 0 | 85 | 112 |
| normalized size | 1 | 1.61 | 1.72 | 0.00 | 2.04 | 1.54 | 0.00 | 1.27 | 1.67 |
| time (sec) | N/A | 0.090 | 0.061 | 0.286 | 1.191 | 0.979 | 0.000 | 2.914 | 5.655 |

| Problem 7 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | F | A | A | F(-2) | A | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 151 | 151 | 114 | 0 | 259 | 265 | 0 | 220 | 222 |
| normalized size | 1 | 1.00 | 0.75 | 0.00 | 1.72 | 1.75 | 0.00 | 1.46 | 1.47 |
| time (sec) | N/A | 0.123 | 0.152 | 0.281 | 1.363 | 0.922 | 0.000 | 4.801 | 4.522 |

| Problem 8 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | F | B | B | F(-1) | B | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 183 | 183 | 145 | 0 | 432 | 482 | 0 | 375 | 349 |
| normalized size | 1 | 1.00 | 0.79 | 0.00 | 2.36 | 2.63 | 0.00 | 2.05 | 1.91 |
| time (sec) | N/A | 0.152 | 0.174 | 0.288 | 1.348 | 0.943 | 0.000 | 5.218 | 4.845 |

| Problem 9 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | F | B | B | F(-1) | B | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 215 | 215 | 162 | 0 | 651 | 733 | 0 | 533 | 603 |
| normalized size | 1 | 1.00 | 0.75 | 0.00 | 3.03 | 3.41 | 0.00 | 2.48 | 2.80 |
| time (sec) | N/A | 0.189 | 0.223 | 0.280 | 1.600 | 0.714 | 0.000 | 8.372 | 5.121 |

| Problem 10 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | F | B | F | F(-1) | F(-1) | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 396 | 602 | 535 | 0 | 2945 | 0 | 0 | 0 | -1 |
| normalized size | 1 | 1.52 | 1.35 | 0.00 | 7.44 | 0.00 | 0.00 | 0.00 | -0.00 |
| time (sec) | N/A | 0.871 | 0.515 | 0.273 | 8.285 | 0.899 | 0.000 | 0.000 | 0.000 |

| Problem 11 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | F | B | F | F(-1) | F(-1) | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 335 | 512 | 411 | 0 | 2175 | 0 | 0 | 0 | -1 |
| normalized size | 1 | 1.53 | 1.23 | 0.00 | 6.49 | 0.00 | 0.00 | 0.00 | -0.00 |
| time (sec) | N/A | 0.707 | 0.349 | 0.286 | 8.015 | 0.810 | 0.000 | 0.000 | 0.000 |

| Problem 12 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | F | B | F | F(-1) | F(-1) | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 274 | 420 | 303 | 0 | 1501 | 0 | 0 | 0 | -1 |
| normalized size | 1 | 1.53 | 1.11 | 0.00 | 5.48 | 0.00 | 0.00 | 0.00 | -0.00 |
| time (sec) | N/A | 0.583 | 0.233 | 0.275 | 11.466 | 0.800 | 0.000 | 0.000 | 0.000 |

| Problem 13 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | F | B | F | F | F(-1) | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 196 | 309 | 215 | 0 | 828 | 0 | 0 | 0 | -1 |
| normalized size | 1 | 1.58 | 1.10 | 0.00 | 4.22 | 0.00 | 0.00 | 0.00 | -0.01 |
| time (sec) | N/A | 0.443 | 0.195 | 0.115 | 7.481 | 0.937 | 0.000 | 0.000 | 0.000 |

| Problem 14 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | B | B | F | F | F | F | F(-1) | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 138 | 789 | 537 | 0 | 0 | 0 | 0 | 0 | -1 |
| normalized size | 1 | 5.72 | 3.89 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | -0.01 |
| time (sec) | N/A | 3.573 | 0.427 | 0.286 | 0.000 | 0.678 | 0.000 | 0.000 | 0.000 |

| Problem 15 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | C | C | F | B | A | F | A | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 136 | 512 | 330 | 0 | 430 | 258 | 0 | 163 | 238 |
| normalized size | 1 | 3.76 | 2.43 | 0.00 | 3.16 | 1.90 | 0.00 | 1.20 | 1.75 |
| time (sec) | N/A | 0.840 | 0.575 | 0.271 | 1.473 | 0.830 | 0.000 | 7.357 | 5.590 |

| Problem 16 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | C | C | F | B | B | F | A | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 288 | 626 | 463 | 0 | 861 | 651 | 0 | 458 | 506 |
| normalized size | 1 | 2.17 | 1.61 | 0.00 | 2.99 | 2.26 | 0.00 | 1.59 | 1.76 |
| time (sec) | N/A | 0.923 | 0.494 | 0.284 | 1.740 | 0.861 | 0.000 | 10.434 | 6.171 |

| Problem 17 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | C | C | F | B | B | F | A | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 448 | 736 | 609 | 0 | 1432 | 1164 | 0 | 810 | 1038 |
| normalized size | 1 | 1.64 | 1.36 | 0.00 | 3.20 | 2.60 | 0.00 | 1.81 | 2.32 |
| time (sec) | N/A | 1.087 | 0.704 | 0.281 | 2.344 | 0.871 | 0.000 | 13.074 | 7.693 |

| Problem 18 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | C | C | F | B | B | F(-1) | A | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 615 | 826 | 776 | 0 | 2136 | 1762 | 0 | 1166 | 1769 |
| normalized size | 1 | 1.34 | 1.26 | 0.00 | 3.47 | 2.87 | 0.00 | 1.90 | 2.88 |
| time (sec) | N/A | 1.314 | 1.025 | 0.279 | 2.978 | 1.077 | 0.000 | 17.895 | 9.221 |

| Problem 19 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | A | F(-1) | A |
| verified | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 38 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | -1 |
| normalized size | 1 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | -0.03 |
| time (sec) | N/A | 0.233 | 0.771 | 0.263 | 0.000 | 0.881 | 0.000 | 0.000 | 0.000 |

| Problem 20 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | A | F(-1) | A |
| verified | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 36 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | -1 |
| normalized size | 1 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | -0.03 |
| time (sec) | N/A | 0.115 | 0.297 | 0.154 | 0.000 | 0.917 | 0.000 | 0.000 | 0.000 |

| Problem 21 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | A | F(-1) | A |
| verified | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 38 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | -1 |
| normalized size | 1 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | -0.03 |
| time (sec) | N/A | 0.081 | 0.150 | 0.369 | 0.000 | 0.859 | 0.000 | 0.000 | 0.000 |

| Problem 22 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | F | A | F | F | A | F | F(-1) | F |
| verified | N/A | N/A | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 94 | 0 | 94 | 0 | 0 | 62 | 0 | 0 | -1 |
| normalized size | 1 | 0.00 | 1.00 | 0.00 | 0.00 | 0.66 | 0.00 | 0.00 | -0.01 |
| time (sec) | N/A | 0.099 | 0.166 | 0.290 | 0.000 | 0.905 | 0.000 | 0.000 | 0.000 |

| Problem 23 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | F | A | F | F | A | F(-1) | F(-1) | F |
| verified | N/A | N/A | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 197 | 0 | 172 | 0 | 0 | 149 | 0 | 0 | -1 |
| normalized size | 1 | 0.00 | 0.87 | 0.00 | 0.00 | 0.76 | 0.00 | 0.00 | -0.01 |
| time (sec) | N/A | 0.082 | 0.328 | 0.286 | 0.000 | 0.953 | 0.000 | 0.000 | 0.000 |

| Problem 24 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | A | A | A |
| verified | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 38 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | -1 |
| normalized size | 1 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | -0.03 |
| time (sec) | N/A | 0.238 | 1.050 | 0.261 | 0.000 | 0.763 | 0.000 | 0.000 | 0.000 |

| Problem 25 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | A | A | A |
| verified | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 36 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | -1 |
| normalized size | 1 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | -0.03 |
| time (sec) | N/A | 0.126 | 0.987 | 0.068 | 0.000 | 0.889 | 0.000 | 0.000 | 0.000 |

| Problem 26 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | F(-1) | A | A |
| verified | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 38 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | -1 |
| normalized size | 1 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | -0.03 |
| time (sec) | N/A | 0.091 | 0.557 | 0.281 | 0.000 | 0.625 | 0.000 | 0.000 | 0.000 |

| Problem 27 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | F | A | F | F | A | F(-1) | F | F |
| verified | N/A | N/A | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 153 | 0 | 146 | 0 | 0 | 274 | 0 | 0 | -1 |
| normalized size | 1 | 0.00 | 0.95 | 0.00 | 0.00 | 1.79 | 0.00 | 0.00 | -0.01 |
| time (sec) | N/A | 0.105 | 0.186 | 0.283 | 0.000 | 0.882 | 0.000 | 0.000 | 0.000 |

| Problem 28 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | F | A | F | F | B | F(-1) | F | F |
| verified | N/A | N/A | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 314 | 0 | 254 | 0 | 0 | 755 | 0 | 0 | -1 |
| normalized size | 1 | 0.00 | 0.81 | 0.00 | 0.00 | 2.40 | 0.00 | 0.00 | -0.00 |
| time (sec) | N/A | 0.093 | 0.616 | 0.286 | 0.000 | 0.930 | 0.000 | 0.000 | 0.000 |

| Problem 29 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | F | B | B | F(-1) | B | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 188 | 188 | 146 | 0 | 676 | 572 | 0 | 1862 | 1045 |
| normalized size | 1 | 1.00 | 0.78 | 0.00 | 3.60 | 3.04 | 0.00 | 9.90 | 5.56 |
| time (sec) | N/A | 0.128 | 0.101 | 0.297 | 1.378 | 1.037 | 0.000 | 5.994 | 4.484 |

| Problem 30 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | F | B | B | F(-1) | B | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 156 | 156 | 124 | 0 | 479 | 429 | 0 | 1390 | 588 |
| normalized size | 1 | 1.00 | 0.79 | 0.00 | 3.07 | 2.75 | 0.00 | 8.91 | 3.77 |
| time (sec) | N/A | 0.102 | 0.093 | 0.274 | 1.368 | 1.025 | 0.000 | 4.452 | 4.367 |

| Problem 31 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | F | B | B | A | B | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 124 | 124 | 101 | 0 | 309 | 297 | 779 | 980 | 303 |
| normalized size | 1 | 1.00 | 0.81 | 0.00 | 2.49 | 2.40 | 6.28 | 7.90 | 2.44 |
| time (sec) | N/A | 0.081 | 0.061 | 0.272 | 1.288 | 1.038 | 60.458 | 3.154 | 4.307 |

| Problem 32 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | F | A | B | A | B | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 86 | 86 | 74 | 0 | 156 | 162 | 444 | 572 | 134 |
| normalized size | 1 | 1.00 | 0.86 | 0.00 | 1.81 | 1.88 | 5.16 | 6.65 | 1.56 |
| time (sec) | N/A | 0.060 | 0.039 | 0.176 | 1.136 | 0.698 | 40.678 | 1.267 | 4.098 |

| Problem 33 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | F | F | F | F | F(-1) | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 80 | 128 | 101 | 0 | 0 | 0 | 0 | 0 | -1 |
| normalized size | 1 | 1.60 | 1.26 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | -0.01 |
| time (sec) | N/A | 0.201 | 0.044 | 0.372 | 0.000 | 0.993 | 0.000 | 0.000 | 0.000 |

| Problem 34 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | F | A | A | F(-2) | A | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 102 | 107 | 114 | 0 | 136 | 105 | 0 | 89 | 113 |
| normalized size | 1 | 1.05 | 1.12 | 0.00 | 1.33 | 1.03 | 0.00 | 0.87 | 1.11 |
| time (sec) | N/A | 0.087 | 0.059 | 0.292 | 1.133 | 0.843 | 0.000 | 3.766 | 4.017 |

| Problem 35 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | F | A | A | F(-2) | A | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 151 | 151 | 115 | 0 | 259 | 266 | 0 | 203 | 221 |
| normalized size | 1 | 1.00 | 0.76 | 0.00 | 1.72 | 1.76 | 0.00 | 1.34 | 1.46 |
| time (sec) | N/A | 0.114 | 0.151 | 0.285 | 1.355 | 0.969 | 0.000 | 6.453 | 4.551 |

| Problem 36 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | F | B | B | F(-1) | B | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 183 | 183 | 146 | 0 | 433 | 483 | 0 | 399 | 349 |
| normalized size | 1 | 1.00 | 0.80 | 0.00 | 2.37 | 2.64 | 0.00 | 2.18 | 1.91 |
| time (sec) | N/A | 0.143 | 0.170 | 0.279 | 1.365 | 0.931 | 0.000 | 7.700 | 4.723 |

| Problem 37 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | F | B | B | F(-1) | B | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 215 | 215 | 162 | 0 | 652 | 735 | 0 | 676 | 603 |
| normalized size | 1 | 1.00 | 0.75 | 0.00 | 3.03 | 3.42 | 0.00 | 3.14 | 2.80 |
| time (sec) | N/A | 0.176 | 0.231 | 0.298 | 1.058 | 1.086 | 0.000 | 11.422 | 4.985 |

| Problem 38 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | F | B | F | F(-1) | F(-1) | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 544 | 634 | 533 | 0 | 2880 | 0 | 0 | 0 | -1 |
| normalized size | 1 | 1.17 | 0.98 | 0.00 | 5.29 | 0.00 | 0.00 | 0.00 | -0.00 |
| time (sec) | N/A | 0.880 | 0.503 | 0.277 | 4.783 | 0.855 | 0.000 | 0.000 | 0.000 |

| Problem 39 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | F | B | F | F(-1) | F(-1) | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 454 | 544 | 409 | 0 | 2129 | 0 | 0 | 0 | -1 |
| normalized size | 1 | 1.20 | 0.90 | 0.00 | 4.69 | 0.00 | 0.00 | 0.00 | -0.00 |
| time (sec) | N/A | 0.684 | 0.335 | 0.272 | 4.846 | 1.038 | 0.000 | 0.000 | 0.000 |

| Problem 40 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | F | B | F | F(-1) | F(-1) | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 361 | 454 | 303 | 0 | 1473 | 0 | 0 | 0 | -1 |
| normalized size | 1 | 1.26 | 0.84 | 0.00 | 4.08 | 0.00 | 0.00 | 0.00 | -0.00 |
| time (sec) | N/A | 0.567 | 0.239 | 0.265 | 5.751 | 0.773 | 0.000 | 0.000 | 0.000 |

| Problem 41 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | F | B | F | F | F(-1) | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 220 | 307 | 216 | 0 | 825 | 0 | 0 | 0 | -1 |
| normalized size | 1 | 1.40 | 0.98 | 0.00 | 3.75 | 0.00 | 0.00 | 0.00 | -0.00 |
| time (sec) | N/A | 0.423 | 0.205 | 0.110 | 4.367 | 0.924 | 0.000 | 0.000 | 0.000 |

| Problem 42 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | B | B | F | F | F | F | F(-1) | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 137 | 782 | 537 | 0 | 0 | 0 | 0 | 0 | -1 |
| normalized size | 1 | 5.71 | 3.92 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | -0.01 |
| time (sec) | N/A | 3.302 | 0.407 | 0.300 | 0.000 | 0.957 | 0.000 | 0.000 | 0.000 |

| Problem 43 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | C | C | F | B | A | F | A | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 163 | 514 | 331 | 0 | 428 | 263 | 0 | 164 | 237 |
| normalized size | 1 | 3.15 | 2.03 | 0.00 | 2.63 | 1.61 | 0.00 | 1.01 | 1.45 |
| time (sec) | N/A | 0.774 | 0.431 | 0.282 | 0.861 | 0.912 | 0.000 | 7.263 | 5.649 |

| Problem 44 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | C | C | F | B | B | F | A | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 317 | 626 | 464 | 0 | 861 | 654 | 0 | 387 | 505 |
| normalized size | 1 | 1.97 | 1.46 | 0.00 | 2.72 | 2.06 | 0.00 | 1.22 | 1.59 |
| time (sec) | N/A | 0.915 | 0.444 | 0.280 | 1.226 | 1.084 | 0.000 | 9.303 | 5.467 |

| Problem 45 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | C | C | F | B | B | F | A | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 429 | 736 | 609 | 0 | 1435 | 1167 | 0 | 746 | 1040 |
| normalized size | 1 | 1.72 | 1.42 | 0.00 | 3.34 | 2.72 | 0.00 | 1.74 | 2.42 |
| time (sec) | N/A | 1.098 | 0.673 | 0.285 | 1.293 | 0.852 | 0.000 | 13.815 | 7.162 |

| Problem 46 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | C | C | F | B | B | F(-1) | B | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 536 | 826 | 776 | 0 | 2138 | 1768 | 0 | 1225 | 1765 |
| normalized size | 1 | 1.54 | 1.45 | 0.00 | 3.99 | 3.30 | 0.00 | 2.29 | 3.29 |
| time (sec) | N/A | 1.295 | 0.929 | 0.285 | 2.109 | 1.157 | 0.000 | 19.414 | 9.081 |

| Problem 47 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | A | F(-1) | A |
| verified | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 38 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | -1 |
| normalized size | 1 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | -0.03 |
| time (sec) | N/A | 0.214 | 0.439 | 0.265 | 0.000 | 0.875 | 0.000 | 0.000 | 0.000 |

| Problem 48 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | A | F(-1) | A |
| verified | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 36 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | -1 |
| normalized size | 1 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | -0.03 |
| time (sec) | N/A | 0.115 | 0.298 | 0.152 | 0.000 | 1.315 | 0.000 | 0.000 | 0.000 |

| Problem 49 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | A | F(-1) | A |
| verified | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 38 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | -1 |
| normalized size | 1 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | -0.03 |
| time (sec) | N/A | 0.083 | 0.178 | 0.385 | 0.000 | 0.803 | 0.000 | 0.000 | 0.000 |

| Problem 50 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | F | A | F | F | A | F | F(-1) | F |
| verified | N/A | N/A | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 96 | 0 | 96 | 0 | 0 | 62 | 0 | 0 | -1 |
| normalized size | 1 | 0.00 | 1.00 | 0.00 | 0.00 | 0.65 | 0.00 | 0.00 | -0.01 |
| time (sec) | N/A | 0.100 | 0.123 | 0.283 | 0.000 | 0.897 | 0.000 | 0.000 | 0.000 |

| Problem 51 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | F | A | F | F | A | F(-1) | F(-1) | F |
| verified | N/A | N/A | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 199 | 0 | 174 | 0 | 0 | 147 | 0 | 0 | -1 |
| normalized size | 1 | 0.00 | 0.87 | 0.00 | 0.00 | 0.74 | 0.00 | 0.00 | -0.01 |
| time (sec) | N/A | 0.083 | 0.285 | 0.284 | 0.000 | 0.904 | 0.000 | 0.000 | 0.000 |

| Problem 52 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | A | A | A |
| verified | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 38 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | -1 |
| normalized size | 1 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | -0.03 |
| time (sec) | N/A | 0.233 | 0.996 | 0.264 | 0.000 | 1.031 | 0.000 | 0.000 | 0.000 |

| Problem 53 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | A | A | A |
| verified | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 36 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | -1 |
| normalized size | 1 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | -0.03 |
| time (sec) | N/A | 0.124 | 1.004 | 0.063 | 0.000 | 0.898 | 0.000 | 0.000 | 0.000 |

| Problem 54 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | F(-1) | A | A |
| verified | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 38 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | -1 |
| normalized size | 1 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | -0.03 |
| time (sec) | N/A | 0.090 | 0.559 | 0.304 | 0.000 | 0.903 | 0.000 | 0.000 | 0.000 |

| Problem 55 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | F | A | F | F | A | F(-1) | A | F |
| verified | N/A | N/A | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 154 | 0 | 180 | 0 | 0 | 291 | 0 | 140 | -1 |
| normalized size | 1 | 0.00 | 1.17 | 0.00 | 0.00 | 1.89 | 0.00 | 0.91 | -0.01 |
| time (sec) | N/A | 0.104 | 0.171 | 0.289 | 0.000 | 0.805 | 0.000 | 1.160 | 0.000 |

| Problem 56 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | F | A | F | F | B | F(-1) | A | F |
| verified | N/A | N/A | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 256 | 0 | 288 | 0 | 0 | 770 | 0 | 312 | -1 |
| normalized size | 1 | 0.00 | 1.12 | 0.00 | 0.00 | 3.01 | 0.00 | 1.22 | -0.00 |
| time (sec) | N/A | 0.094 | 0.544 | 0.288 | 0.000 | 0.899 | 0.000 | 2.108 | 0.000 |

| Problem 57 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | F | A | B | F(-1) | B | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 364 | 348 | 285 | 0 | 631 | 736 | 0 | 11806 | 1433 |
| normalized size | 1 | 0.96 | 0.78 | 0.00 | 1.73 | 2.02 | 0.00 | 32.43 | 3.94 |
| time (sec) | N/A | 0.603 | 0.630 | 0.342 | 0.908 | 2.394 | 0.000 | 15.160 | 4.680 |

| Problem 58 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | F | A | B | F(-1) | B | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 235 | 235 | 219 | 0 | 443 | 521 | 0 | 6660 | 766 |
| normalized size | 1 | 1.00 | 0.93 | 0.00 | 1.89 | 2.22 | 0.00 | 28.34 | 3.26 |
| time (sec) | N/A | 0.359 | 0.283 | 0.262 | 0.927 | 1.607 | 0.000 | 9.224 | 4.742 |

| Problem 59 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | F | A | B | A | B | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 157 | 157 | 146 | 0 | 282 | 334 | 1027 | 3346 | 371 |
| normalized size | 1 | 1.00 | 0.93 | 0.00 | 1.80 | 2.13 | 6.54 | 21.31 | 2.36 |
| time (sec) | N/A | 0.180 | 0.146 | 0.269 | 0.888 | 0.961 | 70.520 | 5.580 | 4.179 |

| Problem 60 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | F | A | A | A | B | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 115 | 115 | 120 | 0 | 150 | 179 | 551 | 1189 | 153 |
| normalized size | 1 | 1.00 | 1.04 | 0.00 | 1.30 | 1.56 | 4.79 | 10.34 | 1.33 |
| time (sec) | N/A | 0.105 | 0.127 | 0.178 | 0.760 | 1.166 | 44.182 | 2.374 | 4.257 |

| Problem 61 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | A | A | A | B | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 56 | 56 | 56 | 122 | 52 | 63 | 158 | 237 | 52 |
| normalized size | 1 | 1.00 | 1.00 | 2.18 | 0.93 | 1.12 | 2.82 | 4.23 | 0.93 |
| time (sec) | N/A | 0.033 | 0.010 | 0.050 | 0.626 | 0.915 | 5.414 | 0.942 | 4.005 |

| Problem 62 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | F | F | F | F | F(-1) | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 147 | 147 | 122 | 0 | 0 | 0 | 0 | 0 | -1 |
| normalized size | 1 | 1.00 | 0.83 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | -0.01 |
| time (sec) | N/A | 0.226 | 0.060 | 0.391 | 0.000 | 1.002 | 0.000 | 0.000 | 0.000 |

| Problem 63 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | F | A | B | F(-2) | B | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 91 | 119 | 109 | 0 | 142 | 294 | 0 | 455 | 140 |
| normalized size | 1 | 1.31 | 1.20 | 0.00 | 1.56 | 3.23 | 0.00 | 5.00 | 1.54 |
| time (sec) | N/A | 0.125 | 0.147 | 0.288 | 0.814 | 11.245 | 0.000 | 4.104 | 4.640 |

| Problem 64 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | F | A | B | F(-1) | B | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 190 | 190 | 173 | 0 | 355 | 1175 | 0 | 2952 | 430 |
| normalized size | 1 | 1.00 | 0.91 | 0.00 | 1.87 | 6.18 | 0.00 | 15.54 | 2.26 |
| time (sec) | N/A | 0.236 | 0.544 | 0.280 | 0.997 | 159.269 | 0.000 | 6.805 | 6.199 |

| Problem 65 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | F | B | F(-1) | F(-1) | B | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 283 | 283 | 264 | 0 | 852 | 0 | 0 | 9570 | 1182 |
| normalized size | 1 | 1.00 | 0.93 | 0.00 | 3.01 | 0.00 | 0.00 | 33.82 | 4.18 |
| time (sec) | N/A | 0.458 | 0.893 | 0.285 | 1.409 | 0.000 | 0.000 | 9.250 | 9.225 |

| Problem 66 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | F | B | F(-1) | F(-1) | F(-1) | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 388 | 388 | 359 | 0 | 1761 | 0 | 0 | 0 | 2569 |
| normalized size | 1 | 1.00 | 0.93 | 0.00 | 4.54 | 0.00 | 0.00 | 0.00 | 6.62 |
| time (sec) | N/A | 0.713 | 1.070 | 0.292 | 1.798 | 0.000 | 0.000 | 0.000 | 13.771 |

| Problem 67 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | F | B | F | F(-1) | F(-1) | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 923 | 1060 | 757 | 0 | 2651 | 0 | 0 | 0 | -1 |
| normalized size | 1 | 1.15 | 0.82 | 0.00 | 2.87 | 0.00 | 0.00 | 0.00 | -0.00 |
| time (sec) | N/A | 1.835 | 1.040 | 0.277 | 5.353 | 1.334 | 0.000 | 0.000 | 0.000 |

| Problem 68 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | F | B | F | F(-1) | F(-1) | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 565 | 699 | 506 | 0 | 1659 | 0 | 0 | 0 | -1 |
| normalized size | 1 | 1.24 | 0.90 | 0.00 | 2.94 | 0.00 | 0.00 | 0.00 | -0.00 |
| time (sec) | N/A | 1.150 | 0.558 | 0.281 | 4.830 | 1.154 | 0.000 | 0.000 | 0.000 |

| Problem 69 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | F | B | F | F | F(-1) | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 290 | 481 | 362 | 0 | 899 | 0 | 0 | 0 | -1 |
| normalized size | 1 | 1.66 | 1.25 | 0.00 | 3.10 | 0.00 | 0.00 | 0.00 | -0.00 |
| time (sec) | N/A | 0.831 | 0.313 | 0.112 | 5.191 | 1.368 | 0.000 | 0.000 | 0.000 |

| Problem 70 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | B | A | F | F | F | F | F(-1) | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 135 | 275 | 226 | 0 | 0 | 0 | 0 | 0 | -1 |
| normalized size | 1 | 2.04 | 1.67 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | -0.01 |
| time (sec) | N/A | 0.617 | 0.173 | 0.146 | 0.000 | 1.097 | 0.000 | 0.000 | 0.000 |

| Problem 71 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | B | B | F | F | F | F | F(-1) | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 297 | 2233 | 1441 | 0 | 0 | 0 | 0 | 0 | -1 |
| normalized size | 1 | 7.52 | 4.85 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | -0.00 |
| time (sec) | N/A | 5.184 | 0.458 | 0.309 | 0.000 | 1.230 | 0.000 | 0.000 | 0.000 |

| Problem 72 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | B | B | F | F | F | F | F(-1) | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 206 | 657 | 418 | 0 | 0 | 0 | 0 | 0 | -1 |
| normalized size | 1 | 3.19 | 2.03 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | -0.00 |
| time (sec) | N/A | 1.134 | 0.521 | 0.286 | 0.000 | 1.413 | 0.000 | 0.000 | 0.000 |

| Problem 73 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | B | A | F | F | F | F(-1) | F(-1) | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 389 | 941 | 615 | 0 | 0 | 0 | 0 | 0 | -1 |
| normalized size | 1 | 2.42 | 1.58 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | -0.00 |
| time (sec) | N/A | 1.561 | 1.652 | 0.287 | 0.000 | 0.798 | 0.000 | 0.000 | 0.000 |

| Problem 74 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | F | F | F | F(-1) | F(-1) | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 747 | 1427 | 918 | 0 | 0 | 0 | 0 | 0 | -1 |
| normalized size | 1 | 1.91 | 1.23 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | -0.00 |
| time (sec) | N/A | 2.500 | 3.713 | 0.278 | 0.000 | 1.187 | 0.000 | 0.000 | 0.000 |

| Problem 75 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | F | F | F | F(-1) | F(-1) | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 1208 | 1968 | 1476 | 0 | 0 | 0 | 0 | 0 | -1 |
| normalized size | 1 | 1.63 | 1.22 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | -0.00 |
| time (sec) | N/A | 3.545 | 7.322 | 0.290 | 0.000 | 1.093 | 0.000 | 0.000 | 0.000 |

| Problem 76 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | A | F(-1) | A |
| verified | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 35 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | -1 |
| normalized size | 1 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | -0.03 |
| time (sec) | N/A | 0.189 | 0.428 | 0.261 | 0.000 | 0.742 | 0.000 | 0.000 | 0.000 |

| Problem 77 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | A | F(-1) | A |
| verified | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 33 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | -1 |
| normalized size | 1 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | -0.03 |
| time (sec) | N/A | 0.102 | 0.289 | 0.148 | 0.000 | 0.859 | 0.000 | 0.000 | 0.000 |

| Problem 78 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | A | F(-1) | A |
| verified | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 27 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | -1 |
| normalized size | 1 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | -0.04 |
| time (sec) | N/A | 0.015 | 0.016 | 0.121 | 0.000 | 1.068 | 0.000 | 0.000 | 0.000 |

| Problem 79 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | A | F(-1) | A |
| verified | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 35 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | -1 |
| normalized size | 1 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | -0.03 |
| time (sec) | N/A | 0.071 | 0.980 | 0.385 | 0.000 | 0.811 | 0.000 | 0.000 | 0.000 |

| Problem 80 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | F(-1) | F(-1) | A |
| verified | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 35 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | -1 |
| normalized size | 1 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | -0.03 |
| time (sec) | N/A | 0.075 | 1.104 | 0.287 | 0.000 | 0.900 | 0.000 | 0.000 | 0.000 |

| Problem 81 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | F(-1) | F(-1) | A |
| verified | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 35 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | -1 |
| normalized size | 1 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | -0.03 |
| time (sec) | N/A | 0.072 | 11.860 | 0.293 | 0.000 | 2.879 | 0.000 | 0.000 | 0.000 |

| Problem 82 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | A | A | A |
| verified | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 35 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | -1 |
| normalized size | 1 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | -0.03 |
| time (sec) | N/A | 0.207 | 0.942 | 0.268 | 0.000 | 0.810 | 0.000 | 0.000 | 0.000 |

| Problem 83 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | A | A | A |
| verified | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 33 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | -1 |
| normalized size | 1 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | -0.03 |
| time (sec) | N/A | 0.114 | 0.741 | 0.059 | 0.000 | 0.896 | 0.000 | 0.000 | 0.000 |

| Problem 84 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | A | A | A |
| verified | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 27 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | -1 |
| normalized size | 1 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | -0.04 |
| time (sec) | N/A | 0.015 | 0.665 | 0.141 | 0.000 | 0.875 | 0.000 | 0.000 | 0.000 |

| Problem 85 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | F(-1) | A | A |
| verified | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 35 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | -1 |
| normalized size | 1 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | -0.03 |
| time (sec) | N/A | 0.078 | 1.712 | 0.305 | 0.000 | 0.858 | 0.000 | 0.000 | 0.000 |

| Problem 86 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | F(-1) | A | A |
| verified | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 35 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | -1 |
| normalized size | 1 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | -0.03 |
| time (sec) | N/A | 0.081 | 3.823 | 0.285 | 0.000 | 1.216 | 0.000 | 0.000 | 0.000 |

| Problem 87 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | F(-1) | A | A |
| verified | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 35 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | -1 |
| normalized size | 1 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | -0.03 |
| time (sec) | N/A | 0.080 | 35.462 | 0.288 | 0.000 | 0.827 | 0.000 | 0.000 | 0.000 |

| Problem 88 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | B | B | B | B | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 180 | 180 | 142 | 8417 | 623 | 431 | 969 | 5428 | 1009 |
| normalized size | 1 | 1.00 | 0.79 | 46.76 | 3.46 | 2.39 | 5.38 | 30.16 | 5.61 |
| time (sec) | N/A | 0.124 | 0.105 | 0.185 | 1.317 | 1.060 | 6.400 | 2.558 | 4.780 |

| Problem 89 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | B | B | B | B | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 149 | 149 | 120 | 5556 | 439 | 318 | 706 | 3795 | 566 |
| normalized size | 1 | 1.00 | 0.81 | 37.29 | 2.95 | 2.13 | 4.74 | 25.47 | 3.80 |
| time (sec) | N/A | 0.096 | 0.097 | 0.164 | 1.330 | 1.014 | 4.298 | 1.807 | 4.645 |

| Problem 90 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | B | B | B | B | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 118 | 118 | 99 | 3283 | 280 | 222 | 491 | 2450 | 290 |
| normalized size | 1 | 1.00 | 0.84 | 27.82 | 2.37 | 1.88 | 4.16 | 20.76 | 2.46 |
| time (sec) | N/A | 0.079 | 0.055 | 0.149 | 1.238 | 0.567 | 2.920 | 1.290 | 4.480 |

| Problem 91 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | A | A | B | B | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 81 | 81 | 69 | 1544 | 144 | 125 | 253 | 1319 | 126 |
| normalized size | 1 | 1.00 | 0.85 | 19.06 | 1.78 | 1.54 | 3.12 | 16.28 | 1.56 |
| time (sec) | N/A | 0.053 | 0.035 | 0.134 | 1.459 | 0.787 | 1.951 | 0.966 | 4.303 |

| Problem 92 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | F | F | F | F(-1) | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 80 | 120 | 95 | 602 | 0 | 0 | 0 | 0 | -1 |
| normalized size | 1 | 1.50 | 1.19 | 7.52 | 0.00 | 0.00 | 0.00 | 0.00 | -0.01 |
| time (sec) | N/A | 0.217 | 0.045 | 0.086 | 0.000 | 0.982 | 0.000 | 0.000 | 0.000 |

| Problem 93 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | B | A | B | A | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 63 | 102 | 105 | 373 | 132 | 83 | 233 | 110 | 104 |
| normalized size | 1 | 1.62 | 1.67 | 5.92 | 2.10 | 1.32 | 3.70 | 1.75 | 1.65 |
| time (sec) | N/A | 0.079 | 0.058 | 0.046 | 1.089 | 1.366 | 1.579 | 1.300 | 5.020 |

| Problem 94 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | A | A | B | A | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 144 | 144 | 110 | 777 | 255 | 217 | 422 | 237 | 209 |
| normalized size | 1 | 1.00 | 0.76 | 5.40 | 1.77 | 1.51 | 2.93 | 1.65 | 1.45 |
| time (sec) | N/A | 0.100 | 0.129 | 0.049 | 1.256 | 0.611 | 2.710 | 1.686 | 5.039 |

| Problem 95 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | B | B | B | B | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 175 | 175 | 141 | 1191 | 428 | 406 | 656 | 382 | 339 |
| normalized size | 1 | 1.00 | 0.81 | 6.81 | 2.45 | 2.32 | 3.75 | 2.18 | 1.94 |
| time (sec) | N/A | 0.130 | 0.155 | 0.049 | 1.360 | 0.607 | 4.246 | 2.131 | 5.584 |

| Problem 96 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | B | B | B | B | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 206 | 206 | 158 | 1607 | 647 | 629 | 944 | 528 | 577 |
| normalized size | 1 | 1.00 | 0.77 | 7.80 | 3.14 | 3.05 | 4.58 | 2.56 | 2.80 |
| time (sec) | N/A | 0.157 | 0.209 | 0.049 | 1.540 | 3.016 | 5.926 | 2.043 | 6.166 |

| Problem 97 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | F | B | F | F(-1) | F(-1) | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 365 | 557 | 511 | 0 | 2389 | 0 | 0 | 0 | -1 |
| normalized size | 1 | 1.53 | 1.40 | 0.00 | 6.55 | 0.00 | 0.00 | 0.00 | -0.00 |
| time (sec) | N/A | 0.855 | 0.486 | 2.566 | 2.376 | 1.846 | 0.000 | 0.000 | 0.000 |

| Problem 98 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | F | B | F | F(-1) | F(-1) | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 309 | 474 | 391 | 0 | 1732 | 0 | 0 | 0 | -1 |
| normalized size | 1 | 1.53 | 1.27 | 0.00 | 5.61 | 0.00 | 0.00 | 0.00 | -0.00 |
| time (sec) | N/A | 0.646 | 0.373 | 2.177 | 2.369 | 1.895 | 0.000 | 0.000 | 0.000 |

| Problem 99 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | F | B | F | F(-1) | F(-1) | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 253 | 389 | 287 | 0 | 1165 | 0 | 0 | 0 | -1 |
| normalized size | 1 | 1.54 | 1.13 | 0.00 | 4.60 | 0.00 | 0.00 | 0.00 | -0.00 |
| time (sec) | N/A | 0.553 | 0.287 | 1.901 | 2.333 | 0.958 | 0.000 | 0.000 | 0.000 |

| Problem 100 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | F | B | F | F(-1) | F(-1) | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 180 | 285 | 203 | 0 | 611 | 0 | 0 | 0 | -1 |
| normalized size | 1 | 1.58 | 1.13 | 0.00 | 3.39 | 0.00 | 0.00 | 0.00 | -0.01 |
| time (sec) | N/A | 0.456 | 0.173 | 1.622 | 2.210 | 0.877 | 0.000 | 0.000 | 0.000 |

| Problem 101 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | B | A | B | F | F | F | F(-1) | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 128 | 728 | 250 | 1186 | 0 | 0 | 0 | 0 | -1 |
| normalized size | 1 | 5.69 | 1.95 | 9.27 | 0.00 | 0.00 | 0.00 | 0.00 | -0.01 |
| time (sec) | N/A | 3.416 | 0.613 | 0.065 | 0.000 | 1.639 | 0.000 | 0.000 | 0.000 |

| Problem 102 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | C | C | B | B | A | B | A | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 126 | 470 | 314 | 828 | 416 | 150 | 434 | 176 | 222 |
| normalized size | 1 | 3.73 | 2.49 | 6.57 | 3.30 | 1.19 | 3.44 | 1.40 | 1.76 |
| time (sec) | N/A | 0.772 | 0.454 | 0.046 | 1.405 | 1.096 | 3.591 | 2.076 | 5.260 |

| Problem 103 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | C | C | B | B | A | B | A | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 268 | 577 | 443 | 1715 | 848 | 367 | 894 | 424 | 507 |
| normalized size | 1 | 2.15 | 1.65 | 6.40 | 3.16 | 1.37 | 3.34 | 1.58 | 1.89 |
| time (sec) | N/A | 0.910 | 0.465 | 0.048 | 1.815 | 0.573 | 6.550 | 1.753 | 5.851 |

| Problem 104 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | C | C | B | B | A | B | A | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 418 | 680 | 585 | 2624 | 1419 | 672 | 1544 | 709 | 1064 |
| normalized size | 1 | 1.63 | 1.40 | 6.28 | 3.39 | 1.61 | 3.69 | 1.70 | 2.55 |
| time (sec) | N/A | 1.059 | 0.691 | 0.053 | 2.452 | 0.691 | 34.300 | 1.888 | 7.405 |

| Problem 105 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | C | C | B | B | A | F(-1) | A | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 575 | 763 | 748 | 3538 | 2123 | 1035 | 0 | 995 | 1881 |
| normalized size | 1 | 1.33 | 1.30 | 6.15 | 3.69 | 1.80 | 0.00 | 1.73 | 3.27 |
| time (sec) | N/A | 1.227 | 0.967 | 0.051 | 3.410 | 1.503 | 0.000 | 2.232 | 10.303 |

| Problem 106 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | B | A | B | A | F | F(-1) | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 28 | 29 | 114 | 30 | 158 | 30 | 0 | 0 | 25 |
| normalized size | 1 | 1.04 | 4.07 | 1.07 | 5.64 | 1.07 | 0.00 | 0.00 | 0.89 |
| time (sec) | N/A | 0.022 | 0.050 | 0.047 | 1.171 | 0.678 | 0.000 | 0.000 | 4.248 |

| Problem 107 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | B | A | B | A | F | B | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 15 | 15 | 140 | 15 | 61 | 22 | 0 | 320 | 15 |
| normalized size | 1 | 1.00 | 9.33 | 1.00 | 4.07 | 1.47 | 0.00 | 21.33 | 1.00 |
| time (sec) | N/A | 0.014 | 0.015 | 0.041 | 1.211 | 0.662 | 0.000 | 39.245 | 4.027 |

| Problem 108 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | B | A | B | A | F | B | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 13 | 13 | 133 | 17 | 59 | 22 | 0 | 322 | 13 |
| normalized size | 1 | 1.00 | 10.23 | 1.31 | 4.54 | 1.69 | 0.00 | 24.77 | 1.00 |
| time (sec) | N/A | 0.013 | 0.015 | 0.043 | 1.304 | 0.812 | 0.000 | 30.471 | 4.231 |

| Problem 109 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | A | F(-1) | A |
| verified | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 35 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | -1 |
| normalized size | 1 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | -0.03 |
| time (sec) | N/A | 0.200 | 0.616 | 1.148 | 0.000 | 0.789 | 0.000 | 0.000 | 0.000 |

| Problem 110 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | A | F(-1) | A |
| verified | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 33 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | -1 |
| normalized size | 1 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | -0.03 |
| time (sec) | N/A | 0.101 | 0.250 | 1.015 | 0.000 | 1.444 | 0.000 | 0.000 | 0.000 |

| Problem 111 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | A | F(-1) | A |
| verified | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 35 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | -1 |
| normalized size | 1 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | -0.03 |
| time (sec) | N/A | 0.072 | 0.228 | 1.180 | 0.000 | 1.639 | 0.000 | 0.000 | 0.000 |

| Problem 112 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | F | A | F | F | A | F | F(-1) | F |
| verified | N/A | N/A | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 50 | 0 | 52 | 0 | 0 | 47 | 0 | 0 | -1 |
| normalized size | 1 | 0.00 | 1.04 | 0.00 | 0.00 | 0.94 | 0.00 | 0.00 | -0.02 |
| time (sec) | N/A | 0.090 | 0.104 | 1.260 | 0.000 | 1.353 | 0.000 | 0.000 | 0.000 |

| Problem 113 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | F | A | F | F | A | F | F(-1) | F |
| verified | N/A | N/A | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 107 | 0 | 89 | 0 | 0 | 130 | 0 | 0 | -1 |
| normalized size | 1 | 0.00 | 0.83 | 0.00 | 0.00 | 1.21 | 0.00 | 0.00 | -0.01 |
| time (sec) | N/A | 0.073 | 0.171 | 1.321 | 0.000 | 0.657 | 0.000 | 0.000 | 0.000 |

| Problem 114 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | A | F(-1) | A |
| verified | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 35 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | -1 |
| normalized size | 1 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | -0.03 |
| time (sec) | N/A | 0.206 | 1.350 | 1.090 | 0.000 | 0.721 | 0.000 | 0.000 | 0.000 |

| Problem 115 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | A | F(-1) | A |
| verified | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 33 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | -1 |
| normalized size | 1 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | -0.03 |
| time (sec) | N/A | 0.107 | 0.930 | 1.085 | 0.000 | 0.950 | 0.000 | 0.000 | 0.000 |

| Problem 116 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | A | A | A |
| verified | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 35 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | -1 |
| normalized size | 1 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | -0.03 |
| time (sec) | N/A | 0.081 | 0.635 | 0.995 | 0.000 | 0.597 | 0.000 | 0.000 | 0.000 |

| Problem 117 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | F | A | F | F | A | F | F | F |
| verified | N/A | N/A | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 103 | 0 | 87 | 0 | 0 | 199 | 0 | 0 | -1 |
| normalized size | 1 | 0.00 | 0.84 | 0.00 | 0.00 | 1.93 | 0.00 | 0.00 | -0.01 |
| time (sec) | N/A | 0.095 | 0.178 | 1.338 | 0.000 | 0.744 | 0.000 | 0.000 | 0.000 |

| Problem 118 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | F | A | F | F | B | F(-1) | F | F |
| verified | N/A | N/A | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 212 | 0 | 136 | 0 | 0 | 570 | 0 | 0 | -1 |
| normalized size | 1 | 0.00 | 0.64 | 0.00 | 0.00 | 2.69 | 0.00 | 0.00 | -0.00 |
| time (sec) | N/A | 0.084 | 0.654 | 1.506 | 0.000 | 0.616 | 0.000 | 0.000 | 0.000 |

| Problem 119 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | B | B | B | B | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 182 | 182 | 144 | 1606 | 885 | 454 | 998 | 496 | 1025 |
| normalized size | 1 | 1.00 | 0.79 | 8.82 | 4.86 | 2.49 | 5.48 | 2.73 | 5.63 |
| time (sec) | N/A | 0.110 | 0.091 | 0.191 | 1.445 | 0.776 | 6.744 | 72.969 | 4.990 |

| Problem 120 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | B | B | B | B | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 151 | 151 | 122 | 1249 | 647 | 341 | 707 | 361 | 567 |
| normalized size | 1 | 1.00 | 0.81 | 8.27 | 4.28 | 2.26 | 4.68 | 2.39 | 3.75 |
| time (sec) | N/A | 0.097 | 0.095 | 0.085 | 1.602 | 1.093 | 4.888 | 17.615 | 4.740 |

| Problem 121 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | B | B | B | B | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 120 | 120 | 98 | 915 | 437 | 243 | 517 | 252 | 296 |
| normalized size | 1 | 1.00 | 0.82 | 7.62 | 3.64 | 2.02 | 4.31 | 2.10 | 2.47 |
| time (sec) | N/A | 0.074 | 0.065 | 0.078 | 1.414 | 0.754 | 3.513 | 3.434 | 4.589 |

| Problem 122 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | B | A | B | A | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 78 | 78 | 72 | 560 | 250 | 148 | 250 | 131 | 120 |
| normalized size | 1 | 1.00 | 0.92 | 7.18 | 3.21 | 1.90 | 3.21 | 1.68 | 1.54 |
| time (sec) | N/A | 0.057 | 0.046 | 0.074 | 1.324 | 0.585 | 2.019 | 0.843 | 4.389 |

| Problem 123 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | F | F | F | F | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 83 | 122 | 88 | 552 | 0 | 0 | 0 | 0 | -1 |
| normalized size | 1 | 1.47 | 1.06 | 6.65 | 0.00 | 0.00 | 0.00 | 0.00 | -0.01 |
| time (sec) | N/A | 0.292 | 0.048 | 0.134 | 0.000 | 2.740 | 0.000 | 0.000 | 0.000 |

| Problem 124 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | B | A | B | B | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 65 | 105 | 111 | 157 | 187 | 110 | 255 | 188 | 108 |
| normalized size | 1 | 1.62 | 1.71 | 2.42 | 2.88 | 1.69 | 3.92 | 2.89 | 1.66 |
| time (sec) | N/A | 0.077 | 0.063 | 0.076 | 1.222 | 0.676 | 1.723 | 0.455 | 5.251 |

| Problem 125 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | B | A | B | A | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 138 | 138 | 109 | 355 | 307 | 238 | 418 | 264 | 206 |
| normalized size | 1 | 1.00 | 0.79 | 2.57 | 2.22 | 1.72 | 3.03 | 1.91 | 1.49 |
| time (sec) | N/A | 0.093 | 0.136 | 0.115 | 1.237 | 0.605 | 2.741 | 0.300 | 5.140 |

| Problem 126 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | B | B | B | B | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 177 | 177 | 140 | 579 | 480 | 430 | 677 | 473 | 341 |
| normalized size | 1 | 1.00 | 0.79 | 3.27 | 2.71 | 2.43 | 3.82 | 2.67 | 1.93 |
| time (sec) | N/A | 0.115 | 0.171 | 0.159 | 1.387 | 0.586 | 4.244 | 0.326 | 5.800 |

| Problem 127 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | B | B | B | B | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 208 | 208 | 162 | 833 | 699 | 654 | 947 | 419 | 579 |
| normalized size | 1 | 1.00 | 0.78 | 4.00 | 3.36 | 3.14 | 4.55 | 2.01 | 2.78 |
| time (sec) | N/A | 0.144 | 0.222 | 0.218 | 1.618 | 0.951 | 5.814 | 0.768 | 6.511 |

| Problem 128 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | F | B | F | F(-1) | F | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 377 | 569 | 523 | 0 | 2650 | 0 | 0 | 0 | -1 |
| normalized size | 1 | 1.51 | 1.39 | 0.00 | 7.03 | 0.00 | 0.00 | 0.00 | -0.00 |
| time (sec) | N/A | 0.872 | 0.462 | 1.368 | 3.088 | 0.699 | 0.000 | 0.000 | 0.000 |

| Problem 129 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | F | B | F | F(-1) | F | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 319 | 470 | 402 | 0 | 1948 | 0 | 0 | 0 | -1 |
| normalized size | 1 | 1.47 | 1.26 | 0.00 | 6.11 | 0.00 | 0.00 | 0.00 | -0.00 |
| time (sec) | N/A | 0.760 | 0.318 | 1.204 | 2.984 | 0.673 | 0.000 | 0.000 | 0.000 |

| Problem 130 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | F | B | F | F(-1) | F | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 255 | 397 | 298 | 0 | 1326 | 0 | 0 | 0 | -1 |
| normalized size | 1 | 1.56 | 1.17 | 0.00 | 5.20 | 0.00 | 0.00 | 0.00 | -0.00 |
| time (sec) | N/A | 0.621 | 0.217 | 1.158 | 2.634 | 0.510 | 0.000 | 0.000 | 0.000 |

| Problem 131 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | F | B | F | F(-1) | F | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 188 | 291 | 207 | 0 | 727 | 0 | 0 | 0 | -1 |
| normalized size | 1 | 1.55 | 1.10 | 0.00 | 3.87 | 0.00 | 0.00 | 0.00 | -0.01 |
| time (sec) | N/A | 0.493 | 0.170 | 0.954 | 2.420 | 0.918 | 0.000 | 0.000 | 0.000 |

| Problem 132 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | B | A | F | F | F | F | F | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 132 | 749 | 257 | 0 | 0 | 0 | 0 | 0 | -1 |
| normalized size | 1 | 5.67 | 1.95 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | -0.01 |
| time (sec) | N/A | 4.141 | 0.321 | 0.985 | 0.000 | 2.025 | 0.000 | 0.000 | 0.000 |

| Problem 133 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | C | C | B | B | A | B | B | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 130 | 480 | 321 | 357 | 574 | 200 | 454 | 378 | 228 |
| normalized size | 1 | 3.69 | 2.47 | 2.75 | 4.42 | 1.54 | 3.49 | 2.91 | 1.75 |
| time (sec) | N/A | 0.888 | 0.429 | 0.097 | 1.595 | 0.759 | 3.660 | 1.736 | 5.971 |

| Problem 134 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | C | C | B | B | A | B | F | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 272 | 579 | 451 | 815 | 1001 | 410 | 879 | 0 | 503 |
| normalized size | 1 | 2.13 | 1.66 | 3.00 | 3.68 | 1.51 | 3.23 | 0.00 | 1.85 |
| time (sec) | N/A | 1.046 | 0.443 | 0.154 | 1.937 | 0.661 | 6.236 | 0.000 | 5.887 |

| Problem 135 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | C | C | B | B | A | B | F | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 429 | 692 | 598 | 1343 | 1575 | 719 | 1561 | 0 | 1069 |
| normalized size | 1 | 1.61 | 1.39 | 3.13 | 3.67 | 1.68 | 3.64 | 0.00 | 2.49 |
| time (sec) | N/A | 1.225 | 0.638 | 0.230 | 2.668 | 0.918 | 34.025 | 0.000 | 7.669 |

| Problem 136 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | C | C | B | B | A | F(-1) | A | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 587 | 757 | 762 | 1943 | 2279 | 1084 | 0 | 874 | 1883 |
| normalized size | 1 | 1.29 | 1.30 | 3.31 | 3.88 | 1.85 | 0.00 | 1.49 | 3.21 |
| time (sec) | N/A | 1.394 | 0.936 | 0.343 | 3.443 | 1.027 | 0.000 | 3.181 | 10.546 |

| Problem 137 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | A | A | A |
| verified | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 37 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | -1 |
| normalized size | 1 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | -0.03 |
| time (sec) | N/A | 0.197 | 0.162 | 0.796 | 0.000 | 0.570 | 0.000 | 0.000 | 0.000 |

| Problem 138 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | A | A | A |
| verified | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 35 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | -1 |
| normalized size | 1 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | -0.03 |
| time (sec) | N/A | 0.101 | 0.116 | 0.632 | 0.000 | 0.920 | 0.000 | 0.000 | 0.000 |

| Problem 139 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | A | A | A |
| verified | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 37 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | -1 |
| normalized size | 1 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | -0.03 |
| time (sec) | N/A | 0.073 | 0.071 | 0.822 | 0.000 | 0.744 | 0.000 | 0.000 | 0.000 |

| Problem 140 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | F | F | F | F | F | F | F | F |
| verified | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 94 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | -1 |
| normalized size | 1 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | -0.01 |
| time (sec) | N/A | 0.087 | 0.075 | 0.875 | 0.000 | 0.913 | 0.000 | 0.000 | 0.000 |

| Problem 141 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | F | F | F | F | F | F | F | F |
| verified | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 152 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | -1 |
| normalized size | 1 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | -0.01 |
| time (sec) | N/A | 0.074 | 0.081 | 0.959 | 0.000 | 0.823 | 0.000 | 0.000 | 0.000 |

| Problem 142 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | A | A | A |
| verified | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 37 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | -1 |
| normalized size | 1 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | -0.03 |
| time (sec) | N/A | 0.215 | 0.467 | 0.764 | 0.000 | 0.732 | 0.000 | 0.000 | 0.000 |

| Problem 143 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | A | A | A |
| verified | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 35 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | -1 |
| normalized size | 1 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | -0.03 |
| time (sec) | N/A | 0.114 | 0.345 | 0.754 | 0.000 | 0.835 | 0.000 | 0.000 | 0.000 |

| Problem 144 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | A | A | A |
| verified | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 37 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | -1 |
| normalized size | 1 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | -0.03 |
| time (sec) | N/A | 0.081 | 0.144 | 0.737 | 0.000 | 0.819 | 0.000 | 0.000 | 0.000 |

| Problem 145 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | F | F | F | F | F | F | F | F |
| verified | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 150 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | -1 |
| normalized size | 1 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | -0.01 |
| time (sec) | N/A | 0.094 | 0.179 | 1.046 | 0.000 | 1.435 | 0.000 | 0.000 | 0.000 |

| Problem 146 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | F | F | F | F | F | F(-1) | F | F |
| verified | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 266 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | -1 |
| normalized size | 1 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | -0.00 |
| time (sec) | N/A | 0.082 | 0.348 | 1.317 | 0.000 | 0.917 | 0.000 | 0.000 | 0.000 |

| Problem 147 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | B | C | B | B | F(-2) | B | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 171 | 183 | 364 | 2374 | 671 | 563 | 0 | 497 | 936 |
| normalized size | 1 | 1.07 | 2.13 | 13.88 | 3.92 | 3.29 | 0.00 | 2.91 | 5.47 |
| time (sec) | N/A | 0.183 | 0.806 | 0.779 | 1.468 | 0.773 | 0.000 | 11.526 | 4.564 |

| Problem 148 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | C | B | B | F(-2) | B | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 142 | 154 | 273 | 1840 | 467 | 417 | 0 | 355 | 520 |
| normalized size | 1 | 1.08 | 1.92 | 12.96 | 3.29 | 2.94 | 0.00 | 2.50 | 3.66 |
| time (sec) | N/A | 0.135 | 0.486 | 0.513 | 1.402 | 0.573 | 0.000 | 3.824 | 4.486 |

| Problem 149 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | C | B | B | F(-2) | B | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 113 | 125 | 194 | 1325 | 294 | 282 | 0 | 235 | 262 |
| normalized size | 1 | 1.11 | 1.72 | 11.73 | 2.60 | 2.50 | 0.00 | 2.08 | 2.32 |
| time (sec) | N/A | 0.121 | 0.287 | 0.467 | 1.268 | 0.956 | 0.000 | 1.382 | 4.239 |

| Problem 150 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | C | A | B | F(-2) | A | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 84 | 96 | 126 | 817 | 154 | 163 | 0 | 127 | 127 |
| normalized size | 1 | 1.14 | 1.50 | 9.73 | 1.83 | 1.94 | 0.00 | 1.51 | 1.51 |
| time (sec) | N/A | 0.091 | 0.149 | 0.427 | 1.233 | 1.112 | 0.000 | 0.584 | 4.283 |

| Problem 151 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | C | F | F | F | F | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 79 | 87 | 129 | 523 | 0 | 0 | 0 | 0 | -1 |
| normalized size | 1 | 1.10 | 1.63 | 6.62 | 0.00 | 0.00 | 0.00 | 0.00 | -0.01 |
| time (sec) | N/A | 0.268 | 0.098 | 1.244 | 0.000 | 0.990 | 0.000 | 0.000 | 0.000 |

| Problem 152 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | C | A | A | F(-2) | A | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 97 | 72 | 89 | 823 | 116 | 107 | 0 | 108 | 97 |
| normalized size | 1 | 0.74 | 0.92 | 8.48 | 1.20 | 1.10 | 0.00 | 1.11 | 1.00 |
| time (sec) | N/A | 0.084 | 0.085 | 0.402 | 1.195 | 2.045 | 0.000 | 0.196 | 4.909 |

| Problem 153 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | C | A | B | F(-1) | A | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 137 | 149 | 121 | 1379 | 230 | 296 | 0 | 239 | 192 |
| normalized size | 1 | 1.09 | 0.88 | 10.07 | 1.68 | 2.16 | 0.00 | 1.74 | 1.40 |
| time (sec) | N/A | 0.154 | 0.311 | 0.487 | 1.376 | 0.555 | 0.000 | 0.217 | 4.663 |

| Problem 154 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | C | B | B | F(-1) | B | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 166 | 178 | 143 | 1976 | 400 | 540 | 0 | 448 | 317 |
| normalized size | 1 | 1.07 | 0.86 | 11.90 | 2.41 | 3.25 | 0.00 | 2.70 | 1.91 |
| time (sec) | N/A | 0.170 | 0.377 | 0.585 | 1.289 | 0.606 | 0.000 | 0.230 | 4.912 |

| Problem 155 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | C | B | B | F(-1) | B | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 195 | 207 | 165 | 2583 | 618 | 820 | 0 | 710 | 555 |
| normalized size | 1 | 1.06 | 0.85 | 13.25 | 3.17 | 4.21 | 0.00 | 3.64 | 2.85 |
| time (sec) | N/A | 0.193 | 0.358 | 0.670 | 1.381 | 1.521 | 0.000 | 0.247 | 5.334 |

| Problem 156 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | B | C | B | F | F(-2) | F | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 322 | 542 | 1709 | 26948 | 1871 | 0 | 0 | 0 | -1 |
| normalized size | 1 | 1.68 | 5.31 | 83.69 | 5.81 | 0.00 | 0.00 | 0.00 | -0.00 |
| time (sec) | N/A | 0.772 | 1.799 | 3.550 | 7.070 | 1.035 | 0.000 | 0.000 | 0.000 |

| Problem 157 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | B | C | B | F | F(-2) | F | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 263 | 427 | 1149 | 19969 | 1284 | 0 | 0 | 0 | -1 |
| normalized size | 1 | 1.62 | 4.37 | 75.93 | 4.88 | 0.00 | 0.00 | 0.00 | -0.00 |
| time (sec) | N/A | 0.626 | 1.062 | 2.841 | 7.047 | 0.522 | 0.000 | 0.000 | 0.000 |

| Problem 158 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | B | C | B | F | F(-2) | F | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 195 | 308 | 656 | 10210 | 779 | 0 | 0 | 0 | -1 |
| normalized size | 1 | 1.58 | 3.36 | 52.36 | 3.99 | 0.00 | 0.00 | 0.00 | -0.01 |
| time (sec) | N/A | 0.486 | 0.731 | 2.093 | 6.912 | 0.571 | 0.000 | 0.000 | 0.000 |

| Problem 159 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | B | F | F | F | F | F | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 131 | 227 | 269 | 0 | 0 | 0 | 0 | 0 | -1 |
| normalized size | 1 | 1.73 | 2.05 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | -0.01 |
| time (sec) | N/A | 0.504 | 0.190 | 2.572 | 0.000 | 0.777 | 0.000 | 0.000 | 0.000 |

| Problem 160 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | C | B | B | F(-1) | F | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 129 | 189 | 236 | 10098 | 449 | 339 | 0 | 0 | 200 |
| normalized size | 1 | 1.47 | 1.83 | 78.28 | 3.48 | 2.63 | 0.00 | 0.00 | 1.55 |
| time (sec) | N/A | 0.182 | 0.372 | 2.253 | 1.531 | 0.842 | 0.000 | 0.000 | 5.269 |

| Problem 161 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | C | B | B | F(-1) | F | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 274 | 411 | 332 | 17300 | 899 | 919 | 0 | 0 | 444 |
| normalized size | 1 | 1.50 | 1.21 | 63.14 | 3.28 | 3.35 | 0.00 | 0.00 | 1.62 |
| time (sec) | N/A | 0.423 | 0.523 | 3.360 | 1.848 | 0.774 | 0.000 | 0.000 | 5.316 |

| Problem 162 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | C | A | C | B | B | F(-1) | F | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 427 | 730 | 432 | 25057 | 1500 | 1635 | 0 | 0 | 911 |
| normalized size | 1 | 1.71 | 1.01 | 58.68 | 3.51 | 3.83 | 0.00 | 0.00 | 2.13 |
| time (sec) | N/A | 1.212 | 0.726 | 4.703 | 2.435 | 1.158 | 0.000 | 0.000 | 6.842 |

| Problem 163 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | C | A | C | B | B | F(-1) | F | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 587 | 843 | 1011 | 33370 | 2238 | 2458 | 0 | 0 | 1579 |
| normalized size | 1 | 1.44 | 1.72 | 56.85 | 3.81 | 4.19 | 0.00 | 0.00 | 2.69 |
| time (sec) | N/A | 1.408 | 0.954 | 5.845 | 2.996 | 0.837 | 0.000 | 0.000 | 9.607 |

| Problem 164 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | B | F | F | F | F(-2) | F(-1) | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 809 | 1203 | 9054 | 0 | 0 | 0 | 0 | 0 | -1 |
| normalized size | 1 | 1.49 | 11.19 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | -0.00 |
| time (sec) | N/A | 2.404 | 10.008 | 8.023 | 0.000 | 0.845 | 0.000 | 0.000 | 0.000 |

| Problem 165 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | B | F | F | F | F(-2) | F | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 614 | 915 | 5668 | 0 | 0 | 0 | 0 | 0 | -1 |
| normalized size | 1 | 1.49 | 9.23 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | -0.00 |
| time (sec) | N/A | 1.737 | 4.216 | 7.008 | 0.000 | 0.930 | 0.000 | 0.000 | 0.000 |

| Problem 166 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | B | F | F | F | F(-2) | F | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 376 | 700 | 3813 | 0 | 0 | 0 | 0 | 0 | -1 |
| normalized size | 1 | 1.86 | 10.14 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | -0.00 |
| time (sec) | N/A | 1.183 | 3.088 | 10.047 | 0.000 | 0.815 | 0.000 | 0.000 | 0.000 |

| Problem 167 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | B | B | F | F | F | F(-1) | F | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 186 | 424 | 2513 | 0 | 0 | 0 | 0 | 0 | -1 |
| normalized size | 1 | 2.28 | 13.51 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | -0.01 |
| time (sec) | N/A | 0.851 | 0.986 | 3.339 | 0.000 | 0.731 | 0.000 | 0.000 | 0.000 |

| Problem 168 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | B | C | B | B | F(-1) | F | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 184 | 360 | 524 | 69354 | 1129 | 825 | 0 | 0 | 474 |
| normalized size | 1 | 1.96 | 2.85 | 376.92 | 6.14 | 4.48 | 0.00 | 0.00 | 2.58 |
| time (sec) | N/A | 0.315 | 0.788 | 20.955 | 2.161 | 0.921 | 0.000 | 0.000 | 6.060 |

| Problem 169 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | B | A | C | B | B | F(-1) | F | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 390 | 811 | 693 | 120138 | 2246 | 2244 | 0 | 0 | 966 |
| normalized size | 1 | 2.08 | 1.78 | 308.05 | 5.76 | 5.75 | 0.00 | 0.00 | 2.48 |
| time (sec) | N/A | 0.804 | 1.184 | 32.817 | 2.762 | 0.731 | 0.000 | 0.000 | 8.988 |

| Problem 170 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | C | A | C | B | B | F(-1) | F | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 611 | 1876 | 1003 | 175812 | 3630 | 4008 | 0 | 0 | 2069 |
| normalized size | 1 | 3.07 | 1.64 | 287.74 | 5.94 | 6.56 | 0.00 | 0.00 | 3.39 |
| time (sec) | N/A | 3.432 | 1.503 | 48.030 | 4.174 | 1.112 | 0.000 | 0.000 | 10.731 |

| Problem 171 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | C | A | C | B | B | F(-1) | F | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 830 | 2173 | 1370 | 236754 | 5280 | 6057 | 0 | 0 | 4257 |
| normalized size | 1 | 2.62 | 1.65 | 285.25 | 6.36 | 7.30 | 0.00 | 0.00 | 5.13 |
| time (sec) | N/A | 4.675 | 2.218 | 58.078 | 5.793 | 1.617 | 0.000 | 0.000 | 11.290 |

| Problem 172 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | F | F | F | F | A | F(-1) | F | F |
| verified | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 96 | 0 | 0 | 0 | 0 | 62 | 0 | 0 | -1 |
| normalized size | 1 | 0.00 | 0.00 | 0.00 | 0.00 | 0.65 | 0.00 | 0.00 | -0.01 |
| time (sec) | N/A | 0.100 | 0.087 | 4.518 | 0.000 | 0.606 | 0.000 | 0.000 | 0.000 |

| Problem 173 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | B | B | B | B | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 180 | 180 | 142 | 2930 | 619 | 433 | 969 | 5960 | 1008 |
| normalized size | 1 | 1.00 | 0.79 | 16.28 | 3.44 | 2.41 | 5.38 | 33.11 | 5.60 |
| time (sec) | N/A | 0.124 | 0.103 | 0.145 | 1.259 | 0.849 | 6.523 | 1.533 | 4.863 |

| Problem 174 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | B | B | B | B | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 149 | 149 | 120 | 2191 | 436 | 320 | 706 | 4137 | 566 |
| normalized size | 1 | 1.00 | 0.81 | 14.70 | 2.93 | 2.15 | 4.74 | 27.77 | 3.80 |
| time (sec) | N/A | 0.101 | 0.081 | 0.138 | 1.305 | 1.119 | 4.345 | 1.192 | 4.692 |

| Problem 175 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | B | B | B | B | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 118 | 118 | 99 | 1537 | 278 | 223 | 491 | 2640 | 290 |
| normalized size | 1 | 1.00 | 0.84 | 13.03 | 2.36 | 1.89 | 4.16 | 22.37 | 2.46 |
| time (sec) | N/A | 0.081 | 0.052 | 0.191 | 1.163 | 0.830 | 2.910 | 0.887 | 4.572 |

| Problem 176 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | A | A | B | B | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 81 | 81 | 69 | 951 | 143 | 127 | 253 | 1395 | 126 |
| normalized size | 1 | 1.00 | 0.85 | 11.74 | 1.77 | 1.57 | 3.12 | 17.22 | 1.56 |
| time (sec) | N/A | 0.055 | 0.038 | 0.139 | 1.119 | 0.956 | 1.917 | 0.670 | 4.310 |

| Problem 177 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | F | F | F | F(-1) | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 81 | 122 | 95 | 419 | 0 | 0 | 0 | 0 | -1 |
| normalized size | 1 | 1.51 | 1.17 | 5.17 | 0.00 | 0.00 | 0.00 | 0.00 | -0.01 |
| time (sec) | N/A | 0.211 | 0.043 | 0.056 | 0.000 | 0.775 | 0.000 | 0.000 | 0.000 |

| Problem 178 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | B | A | B | A | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 64 | 101 | 86 | 520 | 134 | 87 | 231 | 126 | 106 |
| normalized size | 1 | 1.58 | 1.34 | 8.12 | 2.09 | 1.36 | 3.61 | 1.97 | 1.66 |
| time (sec) | N/A | 0.077 | 0.054 | 0.046 | 1.071 | 1.729 | 1.539 | 0.902 | 5.006 |

| Problem 179 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | A | A | B | A | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 144 | 144 | 128 | 753 | 255 | 221 | 422 | 254 | 208 |
| normalized size | 1 | 1.00 | 0.89 | 5.23 | 1.77 | 1.53 | 2.93 | 1.76 | 1.44 |
| time (sec) | N/A | 0.104 | 0.095 | 0.051 | 1.280 | 1.697 | 2.684 | 0.861 | 5.195 |

| Problem 180 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | B | B | B | B | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 175 | 175 | 141 | 1012 | 428 | 412 | 656 | 382 | 339 |
| normalized size | 1 | 1.00 | 0.81 | 5.78 | 2.45 | 2.35 | 3.75 | 2.18 | 1.94 |
| time (sec) | N/A | 0.131 | 0.141 | 0.050 | 1.372 | 1.119 | 4.061 | 1.057 | 5.875 |

| Problem 181 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | B | B | B | B | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 206 | 206 | 166 | 1306 | 647 | 637 | 944 | 511 | 578 |
| normalized size | 1 | 1.00 | 0.81 | 6.34 | 3.14 | 3.09 | 4.58 | 2.48 | 2.81 |
| time (sec) | N/A | 0.149 | 0.192 | 0.055 | 1.515 | 0.922 | 5.534 | 1.388 | 6.623 |

| Problem 182 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | F | B | F | F(-1) | F(-1) | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 503 | 557 | 512 | 0 | 2395 | 0 | 0 | 0 | -1 |
| normalized size | 1 | 1.11 | 1.02 | 0.00 | 4.76 | 0.00 | 0.00 | 0.00 | -0.00 |
| time (sec) | N/A | 0.818 | 0.506 | 2.199 | 2.528 | 0.627 | 0.000 | 0.000 | 0.000 |

| Problem 183 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | F | B | F | F(-1) | F(-1) | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 420 | 474 | 392 | 0 | 1735 | 0 | 0 | 0 | -1 |
| normalized size | 1 | 1.13 | 0.93 | 0.00 | 4.13 | 0.00 | 0.00 | 0.00 | -0.00 |
| time (sec) | N/A | 0.648 | 0.346 | 1.980 | 2.160 | 0.706 | 0.000 | 0.000 | 0.000 |

| Problem 184 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | F | B | F | F(-1) | F(-1) | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 335 | 389 | 290 | 0 | 1172 | 0 | 0 | 0 | -1 |
| normalized size | 1 | 1.16 | 0.87 | 0.00 | 3.50 | 0.00 | 0.00 | 0.00 | -0.00 |
| time (sec) | N/A | 0.556 | 0.238 | 1.814 | 2.086 | 1.020 | 0.000 | 0.000 | 0.000 |

| Problem 185 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | F | B | F | F(-1) | F(-1) | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 202 | 284 | 203 | 0 | 619 | 0 | 0 | 0 | -1 |
| normalized size | 1 | 1.41 | 1.00 | 0.00 | 3.06 | 0.00 | 0.00 | 0.00 | -0.00 |
| time (sec) | N/A | 0.418 | 0.180 | 1.526 | 1.969 | 0.660 | 0.000 | 0.000 | 0.000 |

| Problem 186 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | B | A | B | F | F | F | F(-1) | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 128 | 719 | 251 | 906 | 0 | 0 | 0 | 0 | -1 |
| normalized size | 1 | 5.62 | 1.96 | 7.08 | 0.00 | 0.00 | 0.00 | 0.00 | -0.01 |
| time (sec) | N/A | 4.423 | 0.286 | 0.057 | 0.000 | 0.689 | 0.000 | 0.000 | 0.000 |

| Problem 187 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | C | C | B | B | A | B | A | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 153 | 470 | 314 | 1251 | 416 | 154 | 430 | 188 | 223 |
| normalized size | 1 | 3.07 | 2.05 | 8.18 | 2.72 | 1.01 | 2.81 | 1.23 | 1.46 |
| time (sec) | N/A | 0.762 | 0.476 | 0.050 | 1.254 | 0.670 | 3.749 | 1.555 | 6.379 |

| Problem 188 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | C | C | B | B | A | B | A | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 296 | 578 | 444 | 1934 | 847 | 373 | 892 | 493 | 507 |
| normalized size | 1 | 1.95 | 1.50 | 6.53 | 2.86 | 1.26 | 3.01 | 1.67 | 1.71 |
| time (sec) | N/A | 0.910 | 0.437 | 0.051 | 1.517 | 0.817 | 6.554 | 2.193 | 6.002 |

| Problem 189 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | C | C | B | B | A | B | A | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 399 | 680 | 585 | 2758 | 1420 | 680 | 1544 | 760 | 1064 |
| normalized size | 1 | 1.70 | 1.47 | 6.91 | 3.56 | 1.70 | 3.87 | 1.90 | 2.67 |
| time (sec) | N/A | 1.074 | 0.685 | 0.051 | 2.172 | 2.026 | 34.712 | 2.742 | 7.703 |

| Problem 190 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | C | C | B | B | B | F(-1) | B | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 498 | 763 | 748 | 3717 | 2122 | 1045 | 0 | 1029 | 1880 |
| normalized size | 1 | 1.53 | 1.50 | 7.46 | 4.26 | 2.10 | 0.00 | 2.07 | 3.78 |
| time (sec) | N/A | 1.264 | 0.920 | 0.052 | 2.908 | 0.604 | 0.000 | 2.337 | 10.940 |

| Problem 191 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | A | F(-1) | A |
| verified | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 35 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | -1 |
| normalized size | 1 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | -0.03 |
| time (sec) | N/A | 0.196 | 0.601 | 1.171 | 0.000 | 0.764 | 0.000 | 0.000 | 0.000 |

| Problem 192 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | A | F(-1) | A |
| verified | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 33 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | -1 |
| normalized size | 1 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | -0.03 |
| time (sec) | N/A | 0.101 | 0.247 | 0.932 | 0.000 | 2.548 | 0.000 | 0.000 | 0.000 |

| Problem 193 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | A | F(-1) | A |
| verified | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 35 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | -1 |
| normalized size | 1 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | -0.03 |
| time (sec) | N/A | 0.073 | 0.246 | 1.096 | 0.000 | 0.998 | 0.000 | 0.000 | 0.000 |

| Problem 194 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | F | A | A | F | A | F | F(-1) | F |
| verified | N/A | N/A | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 53 | 0 | 50 | 69 | 0 | 50 | 0 | 0 | -1 |
| normalized size | 1 | 0.00 | 0.94 | 1.30 | 0.00 | 0.94 | 0.00 | 0.00 | -0.02 |
| time (sec) | N/A | 0.086 | 0.068 | 0.533 | 0.000 | 0.916 | 0.000 | 0.000 | 0.000 |

| Problem 195 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | F | A | F | F | A | F | F(-1) | F |
| verified | N/A | N/A | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 109 | 0 | 89 | 0 | 0 | 129 | 0 | 0 | -1 |
| normalized size | 1 | 0.00 | 0.82 | 0.00 | 0.00 | 1.18 | 0.00 | 0.00 | -0.01 |
| time (sec) | N/A | 0.072 | 0.162 | 1.149 | 0.000 | 0.865 | 0.000 | 0.000 | 0.000 |

| Problem 196 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | F(-1) | F(-1) | A |
| verified | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 35 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | -1 |
| normalized size | 1 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | -0.03 |
| time (sec) | N/A | 0.206 | 1.398 | 1.066 | 0.000 | 2.135 | 0.000 | 0.000 | 0.000 |

| Problem 197 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | A | F(-1) | A |
| verified | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 33 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | -1 |
| normalized size | 1 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | -0.03 |
| time (sec) | N/A | 0.106 | 0.970 | 1.122 | 0.000 | 0.807 | 0.000 | 0.000 | 0.000 |

| Problem 198 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | A | A | A |
| verified | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 35 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | -1 |
| normalized size | 1 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | -0.03 |
| time (sec) | N/A | 0.079 | 0.523 | 0.976 | 0.000 | 0.637 | 0.000 | 0.000 | 0.000 |

| Problem 199 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | F | A | B | F | B | F | A | F |
| verified | N/A | N/A | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 104 | 0 | 88 | 258 | 0 | 208 | 0 | 152 | -1 |
| normalized size | 1 | 0.00 | 0.85 | 2.48 | 0.00 | 2.00 | 0.00 | 1.46 | -0.01 |
| time (sec) | N/A | 0.089 | 0.119 | 0.452 | 0.000 | 0.571 | 0.000 | 1.341 | 0.000 |

| Problem 200 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | F | A | F | F | B | F(-1) | B | F |
| verified | N/A | N/A | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 159 | 0 | 135 | 0 | 0 | 584 | 0 | 317 | -1 |
| normalized size | 1 | 0.00 | 0.85 | 0.00 | 0.00 | 3.67 | 0.00 | 1.99 | -0.01 |
| time (sec) | N/A | 0.080 | 0.415 | 1.493 | 0.000 | 0.535 | 0.000 | 1.827 | 0.000 |

| Problem 201 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | B | B | B | B | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 182 | 182 | 144 | 1030 | 882 | 457 | 998 | 493 | 1024 |
| normalized size | 1 | 1.00 | 0.79 | 5.66 | 4.85 | 2.51 | 5.48 | 2.71 | 5.63 |
| time (sec) | N/A | 0.118 | 0.101 | 0.132 | 1.397 | 0.705 | 6.699 | 77.997 | 4.789 |

| Problem 202 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | B | B | B | B | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 151 | 151 | 122 | 788 | 645 | 343 | 707 | 364 | 567 |
| normalized size | 1 | 1.00 | 0.81 | 5.22 | 4.27 | 2.27 | 4.68 | 2.41 | 3.75 |
| time (sec) | N/A | 0.098 | 0.075 | 0.066 | 1.306 | 0.530 | 4.354 | 18.855 | 4.853 |

| Problem 203 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | B | B | B | B | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 120 | 120 | 98 | 569 | 436 | 245 | 517 | 248 | 296 |
| normalized size | 1 | 1.00 | 0.82 | 4.74 | 3.63 | 2.04 | 4.31 | 2.07 | 2.47 |
| time (sec) | N/A | 0.078 | 0.051 | 0.066 | 1.511 | 0.604 | 3.125 | 3.510 | 4.646 |

| Problem 204 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | B | A | B | A | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 78 | 78 | 72 | 340 | 250 | 149 | 250 | 128 | 120 |
| normalized size | 1 | 1.00 | 0.92 | 4.36 | 3.21 | 1.91 | 3.21 | 1.64 | 1.54 |
| time (sec) | N/A | 0.052 | 0.038 | 0.064 | 1.442 | 0.639 | 1.924 | 0.900 | 4.380 |

| Problem 205 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | F | F | F | F | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 83 | 121 | 87 | 265 | 0 | 0 | 0 | 0 | -1 |
| normalized size | 1 | 1.46 | 1.05 | 3.19 | 0.00 | 0.00 | 0.00 | 0.00 | -0.01 |
| time (sec) | N/A | 0.292 | 0.038 | 0.064 | 0.000 | 2.562 | 0.000 | 0.000 | 0.000 |

| Problem 206 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | A | A | B | A | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 102 | 105 | 89 | 212 | 187 | 110 | 253 | 188 | 108 |
| normalized size | 1 | 1.03 | 0.87 | 2.08 | 1.83 | 1.08 | 2.48 | 1.84 | 1.06 |
| time (sec) | N/A | 0.076 | 0.052 | 0.054 | 1.078 | 0.492 | 1.641 | 0.400 | 5.943 |

| Problem 207 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | B | A | B | A | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 139 | 139 | 128 | 300 | 306 | 240 | 418 | 259 | 206 |
| normalized size | 1 | 1.00 | 0.92 | 2.16 | 2.20 | 1.73 | 3.01 | 1.86 | 1.48 |
| time (sec) | N/A | 0.101 | 0.092 | 0.056 | 1.155 | 0.647 | 2.600 | 0.344 | 5.928 |

| Problem 208 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | B | B | B | B | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 177 | 177 | 140 | 427 | 480 | 432 | 677 | 473 | 341 |
| normalized size | 1 | 1.00 | 0.79 | 2.41 | 2.71 | 2.44 | 3.82 | 2.67 | 1.93 |
| time (sec) | N/A | 0.121 | 0.104 | 0.055 | 1.223 | 0.800 | 4.056 | 0.328 | 6.733 |

| Problem 209 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | B | B | B | B | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 208 | 208 | 162 | 587 | 699 | 658 | 947 | 416 | 579 |
| normalized size | 1 | 1.00 | 0.78 | 2.82 | 3.36 | 3.16 | 4.55 | 2.00 | 2.78 |
| time (sec) | N/A | 0.143 | 0.169 | 0.059 | 1.377 | 1.511 | 5.685 | 0.687 | 7.897 |

| Problem 210 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | F | B | F | F(-1) | F | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 515 | 569 | 524 | 0 | 2660 | 0 | 0 | 0 | -1 |
| normalized size | 1 | 1.10 | 1.02 | 0.00 | 5.17 | 0.00 | 0.00 | 0.00 | -0.00 |
| time (sec) | N/A | 0.864 | 0.468 | 1.359 | 2.597 | 0.710 | 0.000 | 0.000 | 0.000 |

| Problem 211 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | F | B | F | F(-1) | F | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 422 | 469 | 402 | 0 | 1950 | 0 | 0 | 0 | -1 |
| normalized size | 1 | 1.11 | 0.95 | 0.00 | 4.62 | 0.00 | 0.00 | 0.00 | -0.00 |
| time (sec) | N/A | 0.741 | 0.323 | 1.237 | 2.597 | 0.645 | 0.000 | 0.000 | 0.000 |

| Problem 212 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | F | B | F | F(-1) | F | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 343 | 397 | 298 | 0 | 1333 | 0 | 0 | 0 | -1 |
| normalized size | 1 | 1.16 | 0.87 | 0.00 | 3.89 | 0.00 | 0.00 | 0.00 | -0.00 |
| time (sec) | N/A | 0.625 | 0.225 | 1.146 | 1.699 | 2.203 | 0.000 | 0.000 | 0.000 |

| Problem 213 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | F | B | F | F(-1) | F | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 211 | 291 | 195 | 0 | 730 | 0 | 0 | 0 | -1 |
| normalized size | 1 | 1.38 | 0.92 | 0.00 | 3.46 | 0.00 | 0.00 | 0.00 | -0.00 |
| time (sec) | N/A | 0.504 | 0.172 | 0.925 | 2.086 | 0.543 | 0.000 | 0.000 | 0.000 |

| Problem 214 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | B | A | F | F | F | F | F | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 132 | 740 | 257 | 0 | 0 | 0 | 0 | 0 | -1 |
| normalized size | 1 | 5.61 | 1.95 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | -0.01 |
| time (sec) | N/A | 4.080 | 0.335 | 1.000 | 0.000 | 1.434 | 0.000 | 0.000 | 0.000 |

| Problem 215 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | C | C | B | B | A | B | B | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 157 | 480 | 322 | 452 | 573 | 200 | 450 | 374 | 227 |
| normalized size | 1 | 3.06 | 2.05 | 2.88 | 3.65 | 1.27 | 2.87 | 2.38 | 1.45 |
| time (sec) | N/A | 0.924 | 0.461 | 0.063 | 1.160 | 0.744 | 3.803 | 1.279 | 6.489 |

| Problem 216 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | C | C | B | B | A | B | F | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 299 | 578 | 452 | 664 | 1001 | 413 | 877 | 0 | 504 |
| normalized size | 1 | 1.93 | 1.51 | 2.22 | 3.35 | 1.38 | 2.93 | 0.00 | 1.69 |
| time (sec) | N/A | 1.081 | 0.458 | 0.064 | 1.482 | 1.043 | 6.647 | 0.000 | 6.707 |

| Problem 217 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | C | C | B | B | A | B | F | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 407 | 692 | 598 | 947 | 1576 | 721 | 1561 | 0 | 1069 |
| normalized size | 1 | 1.70 | 1.47 | 2.33 | 3.87 | 1.77 | 3.84 | 0.00 | 2.63 |
| time (sec) | N/A | 1.228 | 0.760 | 0.067 | 1.895 | 0.665 | 35.552 | 0.000 | 9.083 |

| Problem 218 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | C | C | B | B | B | F(-1) | A | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 501 | 758 | 762 | 1285 | 2278 | 1088 | 0 | 868 | 1882 |
| normalized size | 1 | 1.51 | 1.52 | 2.56 | 4.55 | 2.17 | 0.00 | 1.73 | 3.76 |
| time (sec) | N/A | 1.429 | 0.873 | 0.073 | 2.508 | 0.818 | 0.000 | 2.225 | 12.108 |

| Problem 219 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | A | A | A |
| verified | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 37 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | -1 |
| normalized size | 1 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | -0.03 |
| time (sec) | N/A | 0.202 | 0.159 | 0.798 | 0.000 | 0.702 | 0.000 | 0.000 | 0.000 |

| Problem 220 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | A | A | A |
| verified | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 35 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | -1 |
| normalized size | 1 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | -0.03 |
| time (sec) | N/A | 0.101 | 0.118 | 0.653 | 0.000 | 0.742 | 0.000 | 0.000 | 0.000 |

| Problem 221 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | A | A | A |
| verified | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 37 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | -1 |
| normalized size | 1 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | -0.03 |
| time (sec) | N/A | 0.073 | 0.073 | 0.829 | 0.000 | 0.618 | 0.000 | 0.000 | 0.000 |

| Problem 222 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | F | F | F | F | F | F | F | F |
| verified | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 91 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | -1 |
| normalized size | 1 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | -0.01 |
| time (sec) | N/A | 0.087 | 0.078 | 0.872 | 0.000 | 0.649 | 0.000 | 0.000 | 0.000 |

| Problem 223 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | F | F | F | F | F | F | F | F |
| verified | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 151 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | -1 |
| normalized size | 1 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | -0.01 |
| time (sec) | N/A | 0.074 | 0.082 | 0.978 | 0.000 | 1.925 | 0.000 | 0.000 | 0.000 |

| Problem 224 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | A | A | A |
| verified | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 37 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | -1 |
| normalized size | 1 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | -0.03 |
| time (sec) | N/A | 0.212 | 0.469 | 0.763 | 0.000 | 0.766 | 0.000 | 0.000 | 0.000 |

| Problem 225 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | A | A | A |
| verified | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 35 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | -1 |
| normalized size | 1 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | -0.03 |
| time (sec) | N/A | 0.110 | 0.348 | 0.759 | 0.000 | 0.527 | 0.000 | 0.000 | 0.000 |

| Problem 226 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | A | A | A |
| verified | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 37 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | -1 |
| normalized size | 1 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | -0.03 |
| time (sec) | N/A | 0.082 | 0.146 | 0.757 | 0.000 | 0.811 | 0.000 | 0.000 | 0.000 |

| Problem 227 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | F | F | F | F | F | F | F | F |
| verified | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 147 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | -1 |
| normalized size | 1 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | -0.01 |
| time (sec) | N/A | 0.093 | 0.186 | 1.048 | 0.000 | 0.725 | 0.000 | 0.000 | 0.000 |

| Problem 228 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | F | F | F | F | F | F(-1) | F | F |
| verified | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 206 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | -1 |
| normalized size | 1 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | -0.00 |
| time (sec) | N/A | 0.085 | 0.354 | 1.322 | 0.000 | 0.712 | 0.000 | 0.000 | 0.000 |

| Problem 229 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | F | F | F | F | A | F(-1) | F | F |
| verified | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 96 | 0 | 0 | 0 | 0 | 62 | 0 | 0 | -1 |
| normalized size | 1 | 0.00 | 0.00 | 0.00 | 0.00 | 0.65 | 0.00 | 0.00 | -0.01 |
| time (sec) | N/A | 0.102 | 0.064 | 0.035 | 0.000 | 0.747 | 0.000 | 0.000 | 0.000 |

| Problem 230 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | A | A | B | F(-1) | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 355 | 339 | 279 | 14719 | 593 | 636 | 1436 | 0 | 1392 |
| normalized size | 1 | 0.95 | 0.79 | 41.46 | 1.67 | 1.79 | 4.05 | 0.00 | 3.92 |
| time (sec) | N/A | 0.557 | 0.588 | 0.287 | 0.823 | 1.597 | 26.354 | 0.000 | 5.342 |

| Problem 231 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | A | B | B | B | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 227 | 227 | 215 | 8605 | 415 | 445 | 998 | 11299 | 741 |
| normalized size | 1 | 1.00 | 0.95 | 37.91 | 1.83 | 1.96 | 4.40 | 49.78 | 3.26 |
| time (sec) | N/A | 0.341 | 0.266 | 0.174 | 0.976 | 0.947 | 13.268 | 3.229 | 4.689 |

| Problem 232 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | A | A | B | B | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 150 | 150 | 142 | 4406 | 262 | 280 | 658 | 5950 | 356 |
| normalized size | 1 | 1.00 | 0.95 | 29.37 | 1.75 | 1.87 | 4.39 | 39.67 | 2.37 |
| time (sec) | N/A | 0.169 | 0.131 | 0.161 | 0.636 | 0.884 | 6.544 | 1.958 | 4.730 |

| Problem 233 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | A | A | B | B | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 109 | 109 | 114 | 1809 | 140 | 150 | 318 | 2355 | 144 |
| normalized size | 1 | 1.00 | 1.05 | 16.60 | 1.28 | 1.38 | 2.92 | 21.61 | 1.32 |
| time (sec) | N/A | 0.098 | 0.110 | 0.143 | 0.906 | 0.691 | 3.006 | 1.063 | 4.241 |

| Problem 234 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | A | A | A | B | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 52 | 52 | 52 | 418 | 54 | 56 | 83 | 427 | 47 |
| normalized size | 1 | 1.00 | 1.00 | 8.04 | 1.04 | 1.08 | 1.60 | 8.21 | 0.90 |
| time (sec) | N/A | 0.027 | 0.008 | 0.129 | 0.617 | 1.323 | 0.999 | 0.521 | 4.118 |

| Problem 235 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | F | F | F | F(-1) | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 140 | 140 | 115 | 1400 | 0 | 0 | 0 | 0 | -1 |
| normalized size | 1 | 1.00 | 0.82 | 10.00 | 0.00 | 0.00 | 0.00 | 0.00 | -0.01 |
| time (sec) | N/A | 0.247 | 0.060 | 0.201 | 0.000 | 1.884 | 0.000 | 0.000 | 0.000 |

| Problem 236 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | A | B | F(-1) | B | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 87 | 113 | 105 | 926 | 138 | 255 | 0 | 1537 | 166 |
| normalized size | 1 | 1.30 | 1.21 | 10.64 | 1.59 | 2.93 | 0.00 | 17.67 | 1.91 |
| time (sec) | N/A | 0.106 | 0.130 | 0.131 | 0.767 | 10.904 | 0.000 | 1.106 | 5.169 |

| Problem 237 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | B | B | F(-1) | B | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 183 | 183 | 169 | 5274 | 351 | 1017 | 0 | 7600 | 417 |
| normalized size | 1 | 1.00 | 0.92 | 28.82 | 1.92 | 5.56 | 0.00 | 41.53 | 2.28 |
| time (sec) | N/A | 0.185 | 0.505 | 0.158 | 0.991 | 139.536 | 0.000 | 2.088 | 7.208 |

| Problem 238 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | B | F(-1) | F(-1) | F(-1) | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 275 | 275 | 260 | 18285 | 848 | 0 | 0 | 0 | 1154 |
| normalized size | 1 | 1.00 | 0.95 | 66.49 | 3.08 | 0.00 | 0.00 | 0.00 | 4.20 |
| time (sec) | N/A | 0.396 | 0.716 | 0.184 | 1.285 | 0.000 | 0.000 | 0.000 | 10.670 |

| Problem 239 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | B | F(-1) | F(-1) | F(-1) | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 379 | 379 | 355 | 44893 | 1757 | 0 | 0 | 0 | 2518 |
| normalized size | 1 | 1.00 | 0.94 | 118.45 | 4.64 | 0.00 | 0.00 | 0.00 | 6.64 |
| time (sec) | N/A | 0.618 | 0.931 | 0.386 | 1.989 | 0.000 | 0.000 | 0.000 | 16.223 |

| Problem 240 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | F | B | F | F(-1) | F(-1) | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 874 | 994 | 733 | 0 | 2140 | 0 | 0 | 0 | -1 |
| normalized size | 1 | 1.14 | 0.84 | 0.00 | 2.45 | 0.00 | 0.00 | 0.00 | -0.00 |
| time (sec) | N/A | 1.744 | 0.979 | 2.581 | 1.907 | 1.869 | 0.000 | 0.000 | 0.000 |

| Problem 241 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | F | B | F | F(-1) | F(-1) | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 532 | 649 | 486 | 0 | 1300 | 0 | 0 | 0 | -1 |
| normalized size | 1 | 1.22 | 0.91 | 0.00 | 2.44 | 0.00 | 0.00 | 0.00 | -0.00 |
| time (sec) | N/A | 1.094 | 0.531 | 2.096 | 2.044 | 0.825 | 0.000 | 0.000 | 0.000 |

| Problem 242 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | F | B | F | F(-1) | F(-1) | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 270 | 444 | 346 | 0 | 673 | 0 | 0 | 0 | -1 |
| normalized size | 1 | 1.64 | 1.28 | 0.00 | 2.49 | 0.00 | 0.00 | 0.00 | -0.00 |
| time (sec) | N/A | 0.820 | 0.352 | 1.668 | 1.915 | 1.120 | 0.000 | 0.000 | 0.000 |

| Problem 243 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | F | F | F | F(-1) | F(-1) | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 125 | 246 | 214 | 0 | 0 | 0 | 0 | 0 | -1 |
| normalized size | 1 | 1.97 | 1.71 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | -0.01 |
| time (sec) | N/A | 0.637 | 0.206 | 1.519 | 0.000 | 1.172 | 0.000 | 0.000 | 0.000 |

| Problem 244 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | B | A | B | F | F | F | F(-1) | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 277 | 1998 | 431 | 2428 | 0 | 0 | 0 | 0 | -1 |
| normalized size | 1 | 7.21 | 1.56 | 8.77 | 0.00 | 0.00 | 0.00 | 0.00 | -0.00 |
| time (sec) | N/A | 4.900 | 0.850 | 0.082 | 0.000 | 1.080 | 0.000 | 0.000 | 0.000 |

| Problem 245 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | B | B | F | F | F | F(-1) | F(-1) | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 196 | 612 | 402 | 0 | 0 | 0 | 0 | 0 | -1 |
| normalized size | 1 | 3.12 | 2.05 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | -0.01 |
| time (sec) | N/A | 1.119 | 0.579 | 1.795 | 0.000 | 0.811 | 0.000 | 0.000 | 0.000 |

| Problem 246 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | B | A | F | F | F | F(-1) | F(-1) | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 369 | 883 | 595 | 0 | 0 | 0 | 0 | 0 | -1 |
| normalized size | 1 | 2.39 | 1.61 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | -0.00 |
| time (sec) | N/A | 1.481 | 1.535 | 2.731 | 0.000 | 0.904 | 0.000 | 0.000 | 0.000 |

| Problem 247 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | F | F | F | F(-1) | F(-1) | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 714 | 1356 | 894 | 0 | 0 | 0 | 0 | 0 | -1 |
| normalized size | 1 | 1.90 | 1.25 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | -0.00 |
| time (sec) | N/A | 2.381 | 3.182 | 4.238 | 0.000 | 1.162 | 0.000 | 0.000 | 0.000 |

| Problem 248 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | F | F | F | F(-1) | F(-1) | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 1159 | 1881 | 1448 | 0 | 0 | 0 | 0 | 0 | -1 |
| normalized size | 1 | 1.62 | 1.25 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | -0.00 |
| time (sec) | N/A | 3.403 | 7.419 | 6.982 | 0.000 | 1.210 | 0.000 | 0.000 | 0.000 |

| Problem 249 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | B | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 35 | 34 | 30 | 46 | 32 | 29 | 20 | 103 | 28 |
| normalized size | 1 | 0.97 | 0.86 | 1.31 | 0.91 | 0.83 | 0.57 | 2.94 | 0.80 |
| time (sec) | N/A | 0.014 | 0.006 | 0.102 | 0.478 | 0.999 | 0.152 | 0.415 | 0.186 |

| Problem 250 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | A | F(-1) | A |
| verified | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 32 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | -1 |
| normalized size | 1 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | -0.03 |
| time (sec) | N/A | 0.168 | 0.429 | 1.149 | 0.000 | 0.750 | 0.000 | 0.000 | 0.000 |

| Problem 251 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | A | F(-1) | A |
| verified | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 30 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | -1 |
| normalized size | 1 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | -0.03 |
| time (sec) | N/A | 0.090 | 0.252 | 0.987 | 0.000 | 0.903 | 0.000 | 0.000 | 0.000 |

| Problem 252 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | A | F(-1) | A |
| verified | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 24 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | -1 |
| normalized size | 1 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | -0.04 |
| time (sec) | N/A | 0.014 | 0.023 | 0.921 | 0.000 | 0.975 | 0.000 | 0.000 | 0.000 |

| Problem 253 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | A | F(-1) | A |
| verified | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 32 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | -1 |
| normalized size | 1 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | -0.03 |
| time (sec) | N/A | 0.065 | 0.897 | 1.306 | 0.000 | 0.893 | 0.000 | 0.000 | 0.000 |

| Problem 254 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | A | F(-1) | A |
| verified | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 32 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | -1 |
| normalized size | 1 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | -0.03 |
| time (sec) | N/A | 0.065 | 0.880 | 1.190 | 0.000 | 0.754 | 0.000 | 0.000 | 0.000 |

| Problem 255 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | F(-1) | F(-1) | A |
| verified | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 32 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | -1 |
| normalized size | 1 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | -0.03 |
| time (sec) | N/A | 0.061 | 8.146 | 1.260 | 0.000 | 0.771 | 0.000 | 0.000 | 0.000 |

| Problem 256 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | A | F(-1) | A |
| verified | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 32 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | -1 |
| normalized size | 1 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | -0.03 |
| time (sec) | N/A | 0.184 | 1.366 | 1.111 | 0.000 | 0.888 | 0.000 | 0.000 | 0.000 |

| Problem 257 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | A | F(-1) | A |
| verified | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 30 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | -1 |
| normalized size | 1 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | -0.03 |
| time (sec) | N/A | 0.100 | 0.947 | 1.039 | 0.000 | 1.002 | 0.000 | 0.000 | 0.000 |

| Problem 258 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | A | A | A |
| verified | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 24 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | -1 |
| normalized size | 1 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | -0.04 |
| time (sec) | N/A | 0.014 | 0.587 | 1.081 | 0.000 | 0.576 | 0.000 | 0.000 | 0.000 |

| Problem 259 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | F(-1) | A | A |
| verified | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 32 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | -1 |
| normalized size | 1 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | -0.03 |
| time (sec) | N/A | 0.070 | 2.603 | 1.367 | 0.000 | 0.817 | 0.000 | 0.000 | 0.000 |

| Problem 260 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | F(-1) | A | A |
| verified | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 32 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | -1 |
| normalized size | 1 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | -0.03 |
| time (sec) | N/A | 0.075 | 3.931 | 1.496 | 0.000 | 0.862 | 0.000 | 0.000 | 0.000 |

| Problem 261 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | F(-1) | A | A |
| verified | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 32 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | -1 |
| normalized size | 1 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | -0.03 |
| time (sec) | N/A | 0.070 | 30.006 | 1.795 | 0.000 | 0.895 | 0.000 | 0.000 | 0.000 |

| Problem 262 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | B | A | B | F(-1) | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 357 | 341 | 282 | 2438 | 855 | 660 | 1477 | 0 | 1403 |
| normalized size | 1 | 0.96 | 0.79 | 6.83 | 2.39 | 1.85 | 4.14 | 0.00 | 3.93 |
| time (sec) | N/A | 0.501 | 0.598 | 0.146 | 1.173 | 2.350 | 26.589 | 0.000 | 5.333 |

| Problem 263 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | B | B | B | B | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 229 | 229 | 217 | 1783 | 623 | 468 | 998 | 447 | 743 |
| normalized size | 1 | 1.00 | 0.95 | 7.79 | 2.72 | 2.04 | 4.36 | 1.95 | 3.24 |
| time (sec) | N/A | 0.324 | 0.257 | 0.089 | 1.055 | 1.309 | 12.639 | 167.452 | 5.032 |

| Problem 264 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | B | B | B | A | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 152 | 152 | 142 | 1188 | 419 | 301 | 692 | 279 | 362 |
| normalized size | 1 | 1.00 | 0.93 | 7.82 | 2.76 | 1.98 | 4.55 | 1.84 | 2.38 |
| time (sec) | N/A | 0.161 | 0.138 | 0.081 | 0.795 | 0.993 | 6.812 | 12.579 | 4.786 |

| Problem 265 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | B | A | B | A | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 104 | 104 | 118 | 656 | 246 | 174 | 314 | 145 | 133 |
| normalized size | 1 | 1.00 | 1.13 | 6.31 | 2.37 | 1.67 | 3.02 | 1.39 | 1.28 |
| time (sec) | N/A | 0.087 | 0.107 | 0.075 | 0.758 | 0.868 | 2.791 | 1.106 | 4.500 |

| Problem 266 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | A | A | B | A | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 54 | 54 | 54 | 233 | 57 | 80 | 104 | 83 | 50 |
| normalized size | 1 | 1.00 | 1.00 | 4.31 | 1.06 | 1.48 | 1.93 | 1.54 | 0.93 |
| time (sec) | N/A | 0.027 | 0.024 | 0.063 | 0.634 | 0.867 | 1.074 | 0.259 | 4.289 |

| Problem 267 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | F | F | F | F | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 144 | 144 | 119 | 1143 | 0 | 0 | 0 | 0 | -1 |
| normalized size | 1 | 1.00 | 0.83 | 7.94 | 0.00 | 0.00 | 0.00 | 0.00 | -0.01 |
| time (sec) | N/A | 0.309 | 0.061 | 0.148 | 0.000 | 0.919 | 0.000 | 0.000 | 0.000 |

| Problem 268 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | B | B | F(-1) | F(-2) | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 90 | 117 | 108 | 388 | 192 | 279 | 0 | 0 | 191 |
| normalized size | 1 | 1.30 | 1.20 | 4.31 | 2.13 | 3.10 | 0.00 | 0.00 | 2.12 |
| time (sec) | N/A | 0.092 | 0.134 | 0.092 | 0.700 | 10.606 | 0.000 | 0.000 | 5.339 |

| Problem 269 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | B | B | F(-1) | B | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 175 | 175 | 172 | 1554 | 405 | 1036 | 0 | 495 | 412 |
| normalized size | 1 | 1.00 | 0.98 | 8.88 | 2.31 | 5.92 | 0.00 | 2.83 | 2.35 |
| time (sec) | N/A | 0.170 | 0.547 | 0.191 | 1.052 | 151.718 | 0.000 | 0.771 | 7.448 |

| Problem 270 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | B | F(-1) | F(-1) | B | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 277 | 277 | 263 | 4421 | 900 | 0 | 0 | 1391 | 1147 |
| normalized size | 1 | 1.00 | 0.95 | 15.96 | 3.25 | 0.00 | 0.00 | 5.02 | 4.14 |
| time (sec) | N/A | 0.327 | 0.738 | 0.295 | 1.725 | 0.000 | 0.000 | 3.668 | 11.577 |

| Problem 271 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | B | F(-1) | F(-1) | F(-2) | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 381 | 381 | 358 | 10401 | 1809 | 0 | 0 | 0 | 2520 |
| normalized size | 1 | 1.00 | 0.94 | 27.30 | 4.75 | 0.00 | 0.00 | 0.00 | 6.61 |
| time (sec) | N/A | 0.553 | 0.967 | 0.446 | 2.000 | 0.000 | 0.000 | 0.000 | 17.435 |

| Problem 272 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | F | B | F | F(-1) | F | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 869 | 973 | 746 | 0 | 2351 | 0 | 0 | 0 | -1 |
| normalized size | 1 | 1.12 | 0.86 | 0.00 | 2.71 | 0.00 | 0.00 | 0.00 | -0.00 |
| time (sec) | N/A | 1.808 | 0.998 | 1.348 | 1.959 | 1.076 | 0.000 | 0.000 | 0.000 |

| Problem 273 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | F | B | F | F(-1) | F | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 542 | 659 | 497 | 0 | 1458 | 0 | 0 | 0 | -1 |
| normalized size | 1 | 1.22 | 0.92 | 0.00 | 2.69 | 0.00 | 0.00 | 0.00 | -0.00 |
| time (sec) | N/A | 1.199 | 0.546 | 1.236 | 1.770 | 1.097 | 0.000 | 0.000 | 0.000 |

| Problem 274 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | F | B | F | F(-1) | F | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 281 | 450 | 351 | 0 | 786 | 0 | 0 | 0 | -1 |
| normalized size | 1 | 1.60 | 1.25 | 0.00 | 2.80 | 0.00 | 0.00 | 0.00 | -0.00 |
| time (sec) | N/A | 0.959 | 0.305 | 1.013 | 1.551 | 0.744 | 0.000 | 0.000 | 0.000 |

| Problem 275 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | F | F | F | F(-1) | F | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 129 | 252 | 220 | 0 | 0 | 0 | 0 | 0 | -1 |
| normalized size | 1 | 1.95 | 1.71 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | -0.01 |
| time (sec) | N/A | 0.774 | 0.173 | 0.885 | 0.000 | 0.765 | 0.000 | 0.000 | 0.000 |

| Problem 276 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | B | F | F | F | F | F | F | F |
| verified | N/A | Yes | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 285 | 2126 | 0 | 0 | 0 | 0 | 0 | 0 | -1 |
| normalized size | 1 | 7.46 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | -0.00 |
| time (sec) | N/A | 5.846 | 2.302 | 1.064 | 0.000 | 0.798 | 0.000 | 0.000 | 0.000 |

| Problem 277 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | B | B | F | F | F | F(-1) | F | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 200 | 620 | 409 | 0 | 0 | 0 | 0 | 0 | -1 |
| normalized size | 1 | 3.10 | 2.04 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | -0.00 |
| time (sec) | N/A | 1.301 | 0.588 | 1.260 | 0.000 | 1.464 | 0.000 | 0.000 | 0.000 |

| Problem 278 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | B | A | F | F | F | F(-1) | F | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 381 | 899 | 603 | 0 | 0 | 0 | 0 | 0 | -1 |
| normalized size | 1 | 2.36 | 1.58 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | -0.00 |
| time (sec) | N/A | 1.644 | 1.576 | 1.751 | 0.000 | 0.869 | 0.000 | 0.000 | 0.000 |

| Problem 279 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | F | F | F | F(-1) | F | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 724 | 1369 | 909 | 0 | 0 | 0 | 0 | 0 | -1 |
| normalized size | 1 | 1.89 | 1.26 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | -0.00 |
| time (sec) | N/A | 2.540 | 3.370 | 2.707 | 0.000 | 0.981 | 0.000 | 0.000 | 0.000 |

| Problem 280 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | F | F | F | F(-1) | F | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 1154 | 1854 | 1453 | 0 | 0 | 0 | 0 | 0 | -1 |
| normalized size | 1 | 1.61 | 1.26 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | -0.00 |
| time (sec) | N/A | 3.543 | 7.345 | 4.090 | 0.000 | 2.076 | 0.000 | 0.000 | 0.000 |

| Problem 281 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | A | A | A |
| verified | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 34 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | -1 |
| normalized size | 1 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | -0.03 |
| time (sec) | N/A | 0.174 | 0.180 | 0.820 | 0.000 | 0.852 | 0.000 | 0.000 | 0.000 |

| Problem 282 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | A | A | A |
| verified | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 32 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | -1 |
| normalized size | 1 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | -0.03 |
| time (sec) | N/A | 0.092 | 0.146 | 0.692 | 0.000 | 0.693 | 0.000 | 0.000 | 0.000 |

| Problem 283 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | A | A | A |
| verified | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 26 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | -1 |
| normalized size | 1 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | -0.04 |
| time (sec) | N/A | 0.014 | 0.042 | 0.665 | 0.000 | 0.872 | 0.000 | 0.000 | 0.000 |

| Problem 284 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | A | A | A |
| verified | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 34 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | -1 |
| normalized size | 1 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | -0.03 |
| time (sec) | N/A | 0.064 | 0.090 | 0.845 | 0.000 | 0.926 | 0.000 | 0.000 | 0.000 |

| Problem 285 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | A | A | A |
| verified | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 34 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | -1 |
| normalized size | 1 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | -0.03 |
| time (sec) | N/A | 0.068 | 0.094 | 0.897 | 0.000 | 1.840 | 0.000 | 0.000 | 0.000 |

| Problem 286 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | F(-1) | A | A |
| verified | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 34 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | -1 |
| normalized size | 1 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | -0.03 |
| time (sec) | N/A | 0.062 | 0.096 | 1.046 | 0.000 | 1.538 | 0.000 | 0.000 | 0.000 |

| Problem 287 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | A | A | A |
| verified | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 34 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | -1 |
| normalized size | 1 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | -0.03 |
| time (sec) | N/A | 0.191 | 0.715 | 0.775 | 0.000 | 1.693 | 0.000 | 0.000 | 0.000 |

| Problem 288 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | A | A | A |
| verified | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 32 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | -1 |
| normalized size | 1 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | -0.03 |
| time (sec) | N/A | 0.103 | 0.433 | 0.769 | 0.000 | 2.118 | 0.000 | 0.000 | 0.000 |

| Problem 289 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | A | A | A |
| verified | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 26 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | -1 |
| normalized size | 1 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | -0.04 |
| time (sec) | N/A | 0.014 | 0.232 | 0.755 | 0.000 | 1.041 | 0.000 | 0.000 | 0.000 |

| Problem 290 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | F(-1) | A | A |
| verified | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 34 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | -1 |
| normalized size | 1 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | -0.03 |
| time (sec) | N/A | 0.072 | 0.498 | 1.049 | 0.000 | 2.102 | 0.000 | 0.000 | 0.000 |

| Problem 291 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | F(-1) | A | A |
| verified | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 34 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | -1 |
| normalized size | 1 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | -0.03 |
| time (sec) | N/A | 0.076 | 0.606 | 1.329 | 0.000 | 0.799 | 0.000 | 0.000 | 0.000 |

| Problem 292 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | F(-1) | A | A |
| verified | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 34 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | -1 |
| normalized size | 1 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | -0.03 |
| time (sec) | N/A | 0.071 | 0.641 | 1.918 | 0.000 | 1.024 | 0.000 | 0.000 | 0.000 |

| Problem 293 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | C | A | B | F(-2) | F(-1) | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 365 | 377 | 463 | 2576 | 671 | 805 | 0 | 0 | 1434 |
| normalized size | 1 | 1.03 | 1.27 | 7.06 | 1.84 | 2.21 | 0.00 | 0.00 | 3.93 |
| time (sec) | N/A | 0.712 | 1.009 | 0.656 | 0.802 | 0.856 | 0.000 | 0.000 | 5.132 |

| Problem 294 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | C | B | B | F(-2) | F(-1) | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 236 | 248 | 314 | 1967 | 467 | 571 | 0 | 0 | 767 |
| normalized size | 1 | 1.05 | 1.33 | 8.33 | 1.98 | 2.42 | 0.00 | 0.00 | 3.25 |
| time (sec) | N/A | 0.456 | 0.610 | 0.572 | 0.659 | 0.861 | 0.000 | 0.000 | 4.761 |

| Problem 295 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | C | A | B | F(-2) | B | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 158 | 170 | 204 | 1389 | 294 | 365 | 0 | 298 | 372 |
| normalized size | 1 | 1.08 | 1.29 | 8.79 | 1.86 | 2.31 | 0.00 | 1.89 | 2.35 |
| time (sec) | N/A | 0.240 | 0.393 | 0.513 | 0.940 | 0.931 | 0.000 | 97.810 | 4.471 |

| Problem 296 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | C | A | A | F(-2) | A | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 116 | 128 | 124 | 839 | 154 | 192 | 0 | 149 | 154 |
| normalized size | 1 | 1.10 | 1.07 | 7.23 | 1.33 | 1.66 | 0.00 | 1.28 | 1.33 |
| time (sec) | N/A | 0.149 | 0.194 | 0.452 | 0.624 | 0.811 | 0.000 | 7.100 | 4.393 |

| Problem 297 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | A | A | F(-2) | A | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 57 | 57 | 57 | 123 | 59 | 59 | 0 | 55 | 53 |
| normalized size | 1 | 1.00 | 1.00 | 2.16 | 1.04 | 1.04 | 0.00 | 0.96 | 0.93 |
| time (sec) | N/A | 0.030 | 0.013 | 0.051 | 0.820 | 0.904 | 0.000 | 0.205 | 4.111 |

| Problem 298 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | C | F | F | F(-2) | F | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 148 | 156 | 150 | 597 | 0 | 0 | 0 | 0 | -1 |
| normalized size | 1 | 1.05 | 1.01 | 4.03 | 0.00 | 0.00 | 0.00 | 0.00 | -0.01 |
| time (sec) | N/A | 0.189 | 0.111 | 0.454 | 0.000 | 0.969 | 0.000 | 0.000 | 0.000 |

| Problem 299 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | C | A | B | F(-1) | A | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 120 | 132 | 117 | 1796 | 151 | 250 | 0 | 166 | 141 |
| normalized size | 1 | 1.10 | 0.98 | 14.97 | 1.26 | 2.08 | 0.00 | 1.38 | 1.18 |
| time (sec) | N/A | 0.120 | 0.203 | 0.526 | 0.762 | 11.672 | 0.000 | 0.376 | 4.717 |

| Problem 300 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | C | B | B | F(-1) | B | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 191 | 203 | 178 | 4925 | 382 | 1127 | 0 | 523 | 431 |
| normalized size | 1 | 1.06 | 0.93 | 25.79 | 2.00 | 5.90 | 0.00 | 2.74 | 2.26 |
| time (sec) | N/A | 0.301 | 0.617 | 0.843 | 0.914 | 149.493 | 0.000 | 0.803 | 6.349 |

| Problem 301 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | C | B | F(-1) | F(-1) | B | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 284 | 296 | 273 | 9645 | 920 | 0 | 0 | 1512 | 1183 |
| normalized size | 1 | 1.04 | 0.96 | 33.96 | 3.24 | 0.00 | 0.00 | 5.32 | 4.17 |
| time (sec) | N/A | 0.536 | 1.139 | 1.191 | 1.889 | 0.000 | 0.000 | 3.175 | 9.259 |

| Problem 302 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | C | B | F(-1) | F(-1) | B | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 389 | 401 | 366 | 16077 | 1912 | 0 | 0 | 3293 | 2570 |
| normalized size | 1 | 1.03 | 0.94 | 41.33 | 4.92 | 0.00 | 0.00 | 8.47 | 6.61 |
| time (sec) | N/A | 0.821 | 1.209 | 1.728 | 2.969 | 0.000 | 0.000 | 15.659 | 14.282 |

| Problem 303 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | C | B | F | F(-2) | F(-1) | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 570 | 697 | 906 | 22955 | 1671 | 0 | 0 | 0 | -1 |
| normalized size | 1 | 1.22 | 1.59 | 40.27 | 2.93 | 0.00 | 0.00 | 0.00 | -0.00 |
| time (sec) | N/A | 1.294 | 1.844 | 4.816 | 6.789 | 1.144 | 0.000 | 0.000 | 0.000 |

| Problem 304 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | C | B | F | F(-2) | F | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 294 | 449 | 472 | 11007 | 903 | 0 | 0 | 0 | -1 |
| normalized size | 1 | 1.53 | 1.61 | 37.44 | 3.07 | 0.00 | 0.00 | 0.00 | -0.00 |
| time (sec) | N/A | 0.955 | 0.975 | 2.711 | 6.534 | 0.936 | 0.000 | 0.000 | 0.000 |

| Problem 305 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | C | F | F | F(-2) | F | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 137 | 195 | 217 | 4749 | 0 | 0 | 0 | 0 | -1 |
| normalized size | 1 | 1.42 | 1.58 | 34.66 | 0.00 | 0.00 | 0.00 | 0.00 | -0.01 |
| time (sec) | N/A | 0.321 | 0.157 | 1.323 | 0.000 | 1.310 | 0.000 | 0.000 | 0.000 |

| Problem 306 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | B | F | F | F | F(-2) | F | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 301 | 473 | 1082 | 0 | 0 | 0 | 0 | 0 | -1 |
| normalized size | 1 | 1.57 | 3.59 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | -0.00 |
| time (sec) | N/A | 0.815 | 0.488 | 2.627 | 0.000 | 0.872 | 0.000 | 0.000 | 0.000 |

| Problem 307 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | B | F | F | F | F(-1) | F | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 208 | 343 | 3460 | 0 | 0 | 0 | 0 | 0 | -1 |
| normalized size | 1 | 1.65 | 16.63 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | -0.00 |
| time (sec) | N/A | 0.407 | 1.302 | 4.454 | 0.000 | 1.035 | 0.000 | 0.000 | 0.000 |

| Problem 308 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | B | B | F | F | F | F(-1) | F | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 393 | 968 | 15406 | 0 | 0 | 0 | 0 | 0 | -1 |
| normalized size | 1 | 2.46 | 39.20 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | -0.00 |
| time (sec) | N/A | 1.631 | 6.460 | 1.962 | 0.000 | 0.722 | 0.000 | 0.000 | 0.000 |

| Problem 309 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | F | F | F | F | F(-2) | F(-1) | F |
| verified | N/A | Yes | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 875 | 1640 | 0 | 0 | 0 | 0 | 0 | 0 | -1 |
| normalized size | 1 | 1.87 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | -0.00 |
| time (sec) | N/A | 3.481 | 6.022 | 8.289 | 0.000 | 0.793 | 0.000 | 0.000 | 0.000 |

| Problem 310 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | B | F | F | F | F | F(-2) | F(-1) | F |
| verified | N/A | Yes | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 466 | 1030 | 0 | 0 | 0 | 0 | 0 | 0 | -1 |
| normalized size | 1 | 2.21 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | -0.00 |
| time (sec) | N/A | 2.112 | 3.292 | 8.961 | 0.000 | 0.938 | 0.000 | 0.000 | 0.000 |

| Problem 311 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | B | A | F | F | F | F(-2) | F | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 203 | 408 | 378 | 0 | 0 | 0 | 0 | 0 | -1 |
| normalized size | 1 | 2.01 | 1.86 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | -0.00 |
| time (sec) | N/A | 0.590 | 0.310 | 4.517 | 0.000 | 2.937 | 0.000 | 0.000 | 0.000 |

| Problem 312 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | B | F | F | F | F | F(-2) | F | F |
| verified | N/A | Yes | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 425 | 921 | 0 | 0 | 0 | 0 | 0 | 0 | -1 |
| normalized size | 1 | 2.17 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | -0.00 |
| time (sec) | N/A | 1.642 | 1.528 | 3.758 | 0.000 | 1.133 | 0.000 | 0.000 | 0.000 |

| Problem 313 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | B | F | F | F | F | F(-1) | F | F |
| verified | N/A | Yes | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 302 | 650 | 0 | 0 | 0 | 0 | 0 | 0 | -1 |
| normalized size | 1 | 2.15 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | -0.00 |
| time (sec) | N/A | 0.807 | 3.128 | 3.364 | 0.000 | 0.817 | 0.000 | 0.000 | 0.000 |

| Problem 314 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | B | F | F | F | F | F(-1) | F | F |
| verified | N/A | Yes | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 629 | 2207 | 0 | 0 | 0 | 0 | 0 | 0 | -1 |
| normalized size | 1 | 3.51 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | -0.00 |
| time (sec) | N/A | 3.745 | 6.372 | 5.694 | 0.000 | 1.001 | 0.000 | 0.000 | 0.000 |

## 2.3 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi. It gives additional statistics for each integral. the column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio $\frac{\text{number of rules}}{\text{integrand size}}$ is given. The larger this ratio is, the harder the integral was to solve. In this test, problem number [186] had the largest ratio of [.7500]

Table 2.1: Rubi specific breakdown of results for each integral

| # | grade | number of steps used | number of unique rules | normalized antiderivative leaf size | integrand leaf size | $\frac{\text{number of rules}}{\text{integrand leaf size}}$ |
|---|---|---|---|---|---|---|
| 1 | A | 4 | 3 | 1.00 | 33 | 0.091 |
| 2 | A | 4 | 3 | 1.00 | 33 | 0.091 |
| 3 | A | 4 | 3 | 1.00 | 33 | 0.091 |
| 4 | A | 4 | 3 | 1.00 | 31 | 0.097 |
| 5 | A | 9 | 8 | 1.50 | 33 | 0.242 |
| 6 | A | 4 | 3 | 1.61 | 33 | 0.091 |
| 7 | A | 4 | 3 | 1.00 | 33 | 0.091 |
| 8 | A | 4 | 3 | 1.00 | 33 | 0.091 |
| 9 | A | 4 | 3 | 1.00 | 33 | 0.091 |
| 10 | A | 27 | 13 | 1.52 | 35 | 0.371 |

Continued on next page

Table 2.1 – continued from previous page

| # | grade | number of steps used | number of unique rules | normalized antiderivative leaf size | integrand leaf size | $\frac{\text{number of rules}}{\text{integrand leaf size}}$ |
|---|---|---|---|---|---|---|
| 11 | A | 23 | 13 | 1.53 | 35 | 0.371 |
| 12 | A | 19 | 13 | 1.53 | 35 | 0.371 |
| 13 | A | 15 | 12 | 1.58 | 33 | 0.364 |
| 14 | B | 45 | 23 | 5.72 | 35 | 0.657 |
| 15 | C | 24 | 11 | 3.76 | 35 | 0.314 |
| 16 | C | 28 | 11 | 2.17 | 35 | 0.314 |
| 17 | C | 32 | 11 | 1.64 | 35 | 0.314 |
| 18 | C | 36 | 11 | 1.34 | 35 | 0.314 |
| 19 | A | 0 | 0 | 0.00 | 0 | 0.000 |
| 20 | A | 0 | 0 | 0.00 | 0 | 0.000 |
| 21 | A | 0 | 0 | 0.00 | 0 | 0.000 |
| 22 | F | 0 | 0 | N/A | 0 | N/A |
| 23 | F | 0 | 0 | N/A | 0 | N/A |
| 24 | A | 0 | 0 | 0.00 | 0 | 0.000 |
| 25 | A | 0 | 0 | 0.00 | 0 | 0.000 |
| 26 | A | 0 | 0 | 0.00 | 0 | 0.000 |
| 27 | F | 0 | 0 | N/A | 0 | N/A |
| 28 | F | 0 | 0 | N/A | 0 | N/A |
| 29 | A | 4 | 3 | 1.00 | 33 | 0.091 |
| 30 | A | 4 | 3 | 1.00 | 33 | 0.091 |
| 31 | A | 4 | 3 | 1.00 | 33 | 0.091 |
| 32 | A | 4 | 3 | 1.00 | 31 | 0.097 |
| 33 | A | 9 | 8 | 1.60 | 33 | 0.242 |
| 34 | A | 4 | 3 | 1.05 | 33 | 0.091 |
| 35 | A | 4 | 3 | 1.00 | 33 | 0.091 |

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Table 2.1 – continued from previous page

| # | grade | number of steps used | number of unique rules | normalized antiderivative leaf size | integrand leaf size | $\frac{\text{number of rules}}{\text{integrand leaf size}}$ |
|---|---|---|---|---|---|---|
| 36 | A | 4 | 3 | 1.00 | 33 | 0.091 |
| 37 | A | 4 | 3 | 1.00 | 33 | 0.091 |
| 38 | A | 27 | 13 | 1.17 | 35 | 0.371 |
| 39 | A | 23 | 13 | 1.20 | 35 | 0.371 |
| 40 | A | 19 | 13 | 1.26 | 35 | 0.371 |
| 41 | A | 15 | 12 | 1.40 | 33 | 0.364 |
| 42 | B | 45 | 23 | 5.71 | 35 | 0.657 |
| 43 | C | 24 | 11 | 3.15 | 35 | 0.314 |
| 44 | C | 28 | 11 | 1.97 | 35 | 0.314 |
| 45 | C | 32 | 11 | 1.72 | 35 | 0.314 |
| 46 | C | 36 | 11 | 1.54 | 35 | 0.314 |
| 47 | A | 0 | 0 | 0.00 | 0 | 0.000 |
| 48 | A | 0 | 0 | 0.00 | 0 | 0.000 |
| 49 | A | 0 | 0 | 0.00 | 0 | 0.000 |
| 50 | F | 0 | 0 | N/A | 0 | N/A |
| 51 | F | 0 | 0 | N/A | 0 | N/A |
| 52 | A | 0 | 0 | 0.00 | 0 | 0.000 |
| 53 | A | 0 | 0 | 0.00 | 0 | 0.000 |
| 54 | A | 0 | 0 | 0.00 | 0 | 0.000 |
| 55 | F | 0 | 0 | N/A | 0 | N/A |
| 56 | F | 0 | 0 | N/A | 0 | N/A |
| 57 | A | 4 | 3 | 0.96 | 30 | 0.100 |
| 58 | A | 4 | 3 | 1.00 | 30 | 0.100 |
| 59 | A | 4 | 3 | 1.00 | 30 | 0.100 |
| 60 | A | 4 | 3 | 1.00 | 28 | 0.107 |

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Table 2.1 – continued from previous page

| # | grade | number of steps used | number of unique rules | normalized antiderivative leaf size | integrand leaf size | $\frac{\text{number of rules}}{\text{integrand leaf size}}$ |
|---|---|---|---|---|---|---|
| 61 | A | 3 | 2 | 1.00 | 22 | 0.091 |
| 62 | A | 9 | 5 | 1.00 | 30 | 0.167 |
| 63 | A | 4 | 3 | 1.31 | 30 | 0.100 |
| 64 | A | 4 | 3 | 1.00 | 30 | 0.100 |
| 65 | A | 4 | 3 | 1.00 | 30 | 0.100 |
| 66 | A | 4 | 3 | 1.00 | 30 | 0.100 |
| 67 | A | 31 | 13 | 1.15 | 32 | 0.406 |
| 68 | A | 27 | 13 | 1.24 | 32 | 0.406 |
| 69 | A | 23 | 12 | 1.66 | 30 | 0.400 |
| 70 | B | 20 | 10 | 2.04 | 24 | 0.417 |
| 71 | B | 43 | 21 | 7.52 | 32 | 0.656 |
| 72 | B | 29 | 10 | 3.19 | 32 | 0.312 |
| 73 | B | 33 | 11 | 2.42 | 32 | 0.344 |
| 74 | A | 37 | 11 | 1.91 | 32 | 0.344 |
| 75 | A | 41 | 11 | 1.63 | 32 | 0.344 |
| 76 | A | 0 | 0 | 0.00 | 0 | 0.000 |
| 77 | A | 0 | 0 | 0.00 | 0 | 0.000 |
| 78 | A | 0 | 0 | 0.00 | 0 | 0.000 |
| 79 | A | 0 | 0 | 0.00 | 0 | 0.000 |
| 80 | A | 0 | 0 | 0.00 | 0 | 0.000 |
| 81 | A | 0 | 0 | 0.00 | 0 | 0.000 |
| 82 | A | 0 | 0 | 0.00 | 0 | 0.000 |
| 83 | A | 0 | 0 | 0.00 | 0 | 0.000 |
| 84 | A | 0 | 0 | 0.00 | 0 | 0.000 |
| 85 | A | 0 | 0 | 0.00 | 0 | 0.000 |
| 86 | A | 0 | 0 | 0.00 | 0 | 0.000 |

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Table 2.1 – continued from previous page

| # | grade | number of steps used | number of unique rules | normalized antiderivative leaf size | integrand leaf size | $\frac{\text{number of rules}}{\text{integrand leaf size}}$ |
|---|---|---|---|---|---|---|
| 87 | A | 0 | 0 | 0.00 | 0 | 0.000 |
| 88 | A | 4 | 3 | 1.00 | 30 | 0.100 |
| 89 | A | 4 | 3 | 1.00 | 30 | 0.100 |
| 90 | A | 4 | 3 | 1.00 | 30 | 0.100 |
| 91 | A | 4 | 3 | 1.00 | 28 | 0.107 |
| 92 | A | 10 | 8 | 1.50 | 30 | 0.267 |
| 93 | A | 4 | 3 | 1.62 | 30 | 0.100 |
| 94 | A | 4 | 3 | 1.00 | 30 | 0.100 |
| 95 | A | 4 | 3 | 1.00 | 30 | 0.100 |
| 96 | A | 4 | 3 | 1.00 | 30 | 0.100 |
| 97 | A | 28 | 13 | 1.53 | 32 | 0.406 |
| 98 | A | 24 | 13 | 1.53 | 32 | 0.406 |
| 99 | A | 20 | 13 | 1.54 | 32 | 0.406 |
| 100 | A | 16 | 12 | 1.58 | 30 | 0.400 |
| 101 | B | 46 | 23 | 5.69 | 32 | 0.719 |
| 102 | C | 26 | 11 | 3.73 | 32 | 0.344 |
| 103 | C | 30 | 11 | 2.15 | 32 | 0.344 |
| 104 | C | 34 | 11 | 1.63 | 32 | 0.344 |
| 105 | C | 38 | 11 | 1.33 | 32 | 0.344 |
| 106 | A | 1 | 1 | 1.04 | 29 | 0.034 |
| 107 | A | 1 | 1 | 1.00 | 18 | 0.056 |
| 108 | A | 1 | 1 | 1.00 | 20 | 0.050 |
| 109 | A | 0 | 0 | 0.00 | 0 | 0.000 |
| 110 | A | 0 | 0 | 0.00 | 0 | 0.000 |
| 111 | A | 0 | 0 | 0.00 | 0 | 0.000 |
| 112 | F | 0 | 0 | N/A | 0 | N/A |

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Table 2.1 – continued from previous page

| # | grade | number of steps used | number of unique rules | normalized antiderivative leaf size | integrand leaf size | $\frac{\text{number of rules}}{\text{integrand leaf size}}$ |
|---|---|---|---|---|---|---|
| 113 | F | 0 | 0 | N/A | 0 | N/A |
| 114 | A | 0 | 0 | 0.00 | 0 | 0.000 |
| 115 | A | 0 | 0 | 0.00 | 0 | 0.000 |
| 116 | A | 0 | 0 | 0.00 | 0 | 0.000 |
| 117 | F | 0 | 0 | N/A | 0 | N/A |
| 118 | F | 0 | 0 | N/A | 0 | N/A |
| 119 | A | 4 | 3 | 1.00 | 32 | 0.094 |
| 120 | A | 4 | 3 | 1.00 | 32 | 0.094 |
| 121 | A | 4 | 3 | 1.00 | 32 | 0.094 |
| 122 | A | 4 | 3 | 1.00 | 30 | 0.100 |
| 123 | A | 10 | 8 | 1.47 | 32 | 0.250 |
| 124 | A | 4 | 3 | 1.62 | 32 | 0.094 |
| 125 | A | 4 | 3 | 1.00 | 32 | 0.094 |
| 126 | A | 4 | 3 | 1.00 | 32 | 0.094 |
| 127 | A | 4 | 3 | 1.00 | 32 | 0.094 |
| 128 | A | 28 | 13 | 1.51 | 34 | 0.382 |
| 129 | A | 24 | 13 | 1.47 | 34 | 0.382 |
| 130 | A | 20 | 13 | 1.56 | 34 | 0.382 |
| 131 | A | 16 | 12 | 1.55 | 32 | 0.375 |
| 132 | B | 46 | 23 | 5.67 | 34 | 0.676 |
| 133 | C | 26 | 11 | 3.69 | 34 | 0.324 |
| 134 | C | 30 | 11 | 2.13 | 34 | 0.324 |
| 135 | C | 34 | 11 | 1.61 | 34 | 0.324 |
| 136 | C | 38 | 11 | 1.29 | 34 | 0.324 |
| 137 | A | 0 | 0 | 0.00 | 0 | 0.000 |

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Table 2.1 – continued from previous page

| # | grade | number of steps used | number of unique rules | normalized antiderivative leaf size | integrand leaf size | number of rules / integrand leaf size |
|---|---|---|---|---|---|---|
| 138 | A | 0 | 0 | 0.00 | 0 | 0.000 |
| 139 | A | 0 | 0 | 0.00 | 0 | 0.000 |
| 140 | F | 0 | 0 | N/A | 0 | N/A |
| 141 | F | 0 | 0 | N/A | 0 | N/A |
| 142 | A | 0 | 0 | 0.00 | 0 | 0.000 |
| 143 | A | 0 | 0 | 0.00 | 0 | 0.000 |
| 144 | A | 0 | 0 | 0.00 | 0 | 0.000 |
| 145 | F | 0 | 0 | N/A | 0 | N/A |
| 146 | F | 0 | 0 | N/A | 0 | N/A |
| 147 | A | 5 | 3 | 1.07 | 31 | 0.097 |
| 148 | A | 5 | 3 | 1.08 | 31 | 0.097 |
| 149 | A | 5 | 3 | 1.11 | 31 | 0.097 |
| 150 | A | 5 | 3 | 1.14 | 29 | 0.103 |
| 151 | A | 7 | 6 | 1.10 | 31 | 0.194 |
| 152 | A | 4 | 3 | 0.74 | 31 | 0.097 |
| 153 | A | 5 | 3 | 1.09 | 31 | 0.097 |
| 154 | A | 5 | 3 | 1.07 | 31 | 0.097 |
| 155 | A | 5 | 3 | 1.06 | 31 | 0.097 |
| 156 | A | 21 | 11 | 1.68 | 33 | 0.333 |
| 157 | A | 18 | 11 | 1.62 | 33 | 0.333 |
| 158 | A | 15 | 11 | 1.58 | 31 | 0.355 |
| 159 | A | 10 | 8 | 1.73 | 33 | 0.242 |
| 160 | A | 7 | 3 | 1.47 | 33 | 0.091 |
| 161 | A | 12 | 8 | 1.50 | 33 | 0.242 |
| 162 | C | 26 | 11 | 1.71 | 33 | 0.333 |

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Table 2.1 – continued from previous page

| # | grade | number of steps used | number of unique rules | normalized antiderivative leaf size | integrand leaf size | $\frac{\text{number of rules}}{\text{integrand leaf size}}$ |
|---|---|---|---|---|---|---|
| 163 | C | 29 | 11 | 1.44 | 33 | 0.333 |
| 164 | A | 56 | 13 | 1.49 | 33 | 0.394 |
| 165 | A | 40 | 13 | 1.49 | 33 | 0.394 |
| 166 | A | 27 | 13 | 1.86 | 31 | 0.419 |
| 167 | B | 14 | 9 | 2.28 | 33 | 0.273 |
| 168 | A | 11 | 3 | 1.96 | 33 | 0.091 |
| 169 | B | 21 | 8 | 2.08 | 33 | 0.242 |
| 170 | C | 66 | 16 | 3.07 | 33 | 0.485 |
| 171 | C | 93 | 16 | 2.62 | 33 | 0.485 |
| 172 | F | 0 | 0 | N/A | 0 | N/A |
| 173 | A | 4 | 3 | 1.00 | 30 | 0.100 |
| 174 | A | 4 | 3 | 1.00 | 30 | 0.100 |
| 175 | A | 4 | 3 | 1.00 | 30 | 0.100 |
| 176 | A | 4 | 3 | 1.00 | 28 | 0.107 |
| 177 | A | 10 | 8 | 1.51 | 30 | 0.267 |
| 178 | A | 4 | 3 | 1.58 | 30 | 0.100 |
| 179 | A | 4 | 3 | 1.00 | 30 | 0.100 |
| 180 | A | 4 | 3 | 1.00 | 30 | 0.100 |
| 181 | A | 4 | 3 | 1.00 | 30 | 0.100 |
| 182 | A | 28 | 13 | 1.11 | 32 | 0.406 |
| 183 | A | 24 | 13 | 1.13 | 32 | 0.406 |
| 184 | A | 20 | 13 | 1.16 | 32 | 0.406 |
| 185 | A | 16 | 12 | 1.41 | 30 | 0.400 |
| 186 | B | 47 | 24 | 5.62 | 32 | 0.750 |
| 187 | C | 26 | 11 | 3.07 | 32 | 0.344 |
| 188 | C | 30 | 11 | 1.95 | 32 | 0.344 |

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Table 2.1 – continued from previous page

| # | grade | number of steps used | number of unique rules | normalized antiderivative leaf size | integrand leaf size | $\frac{\text{number of rules}}{\text{integrand leaf size}}$ |
|---|---|---|---|---|---|---|
| 189 | C | 34 | 11 | 1.70 | 32 | 0.344 |
| 190 | C | 38 | 11 | 1.53 | 32 | 0.344 |
| 191 | A | 0 | 0 | 0.00 | 0 | 0.000 |
| 192 | A | 0 | 0 | 0.00 | 0 | 0.000 |
| 193 | A | 0 | 0 | 0.00 | 0 | 0.000 |
| 194 | F | 0 | 0 | N/A | 0 | N/A |
| 195 | F | 0 | 0 | N/A | 0 | N/A |
| 196 | A | 0 | 0 | 0.00 | 0 | 0.000 |
| 197 | A | 0 | 0 | 0.00 | 0 | 0.000 |
| 198 | A | 0 | 0 | 0.00 | 0 | 0.000 |
| 199 | F | 0 | 0 | N/A | 0 | N/A |
| 200 | F | 0 | 0 | N/A | 0 | N/A |
| 201 | A | 4 | 3 | 1.00 | 32 | 0.094 |
| 202 | A | 4 | 3 | 1.00 | 32 | 0.094 |
| 203 | A | 4 | 3 | 1.00 | 32 | 0.094 |
| 204 | A | 4 | 3 | 1.00 | 30 | 0.100 |
| 205 | A | 10 | 8 | 1.46 | 32 | 0.250 |
| 206 | A | 4 | 3 | 1.03 | 32 | 0.094 |
| 207 | A | 4 | 3 | 1.00 | 32 | 0.094 |
| 208 | A | 4 | 3 | 1.00 | 32 | 0.094 |
| 209 | A | 4 | 3 | 1.00 | 32 | 0.094 |
| 210 | A | 28 | 13 | 1.10 | 34 | 0.382 |
| 211 | A | 24 | 13 | 1.11 | 34 | 0.382 |
| 212 | A | 20 | 13 | 1.16 | 34 | 0.382 |
| 213 | A | 16 | 12 | 1.38 | 32 | 0.375 |

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Table 2.1 – continued from previous page

| # | grade | number of steps used | number of unique rules | normalized antiderivative leaf size | integrand leaf size | $\frac{\text{number of rules}}{\text{integrand leaf size}}$ |
|---|---|---|---|---|---|---|
| 214 | B | 46 | 23 | 5.61 | 34 | 0.676 |
| 215 | C | 26 | 11 | 3.06 | 34 | 0.324 |
| 216 | C | 30 | 11 | 1.93 | 34 | 0.324 |
| 217 | C | 34 | 11 | 1.70 | 34 | 0.324 |
| 218 | C | 38 | 11 | 1.51 | 34 | 0.324 |
| 219 | A | 0 | 0 | 0.00 | 0 | 0.000 |
| 220 | A | 0 | 0 | 0.00 | 0 | 0.000 |
| 221 | A | 0 | 0 | 0.00 | 0 | 0.000 |
| 222 | F | 0 | 0 | N/A | 0 | N/A |
| 223 | F | 0 | 0 | N/A | 0 | N/A |
| 224 | A | 0 | 0 | 0.00 | 0 | 0.000 |
| 225 | A | 0 | 0 | 0.00 | 0 | 0.000 |
| 226 | A | 0 | 0 | 0.00 | 0 | 0.000 |
| 227 | F | 0 | 0 | N/A | 0 | N/A |
| 228 | F | 0 | 0 | N/A | 0 | N/A |
| 229 | F | 0 | 0 | N/A | 0 | N/A |
| 230 | A | 4 | 3 | 0.95 | 27 | 0.111 |
| 231 | A | 4 | 3 | 1.00 | 27 | 0.111 |
| 232 | A | 4 | 3 | 1.00 | 27 | 0.111 |
| 233 | A | 4 | 3 | 1.00 | 25 | 0.120 |
| 234 | A | 3 | 2 | 1.00 | 19 | 0.105 |
| 235 | A | 10 | 6 | 1.00 | 27 | 0.222 |
| 236 | A | 4 | 3 | 1.30 | 27 | 0.111 |
| 237 | A | 4 | 3 | 1.00 | 27 | 0.111 |
| 238 | A | 4 | 3 | 1.00 | 27 | 0.111 |

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Table 2.1 – continued from previous page

| # | grade | number of steps used | number of unique rules | normalized antiderivative leaf size | integrand leaf size | $\frac{\text{number of rules}}{\text{integrand leaf size}}$ |
|---|---|---|---|---|---|---|
| 239 | A | 4 | 3 | 1.00 | 27 | 0.111 |
| 240 | A | 33 | 13 | 1.14 | 29 | 0.448 |
| 241 | A | 29 | 13 | 1.22 | 29 | 0.448 |
| 242 | A | 25 | 12 | 1.64 | 27 | 0.444 |
| 243 | A | 22 | 10 | 1.97 | 21 | 0.476 |
| 244 | B | 41 | 21 | 7.21 | 29 | 0.724 |
| 245 | B | 32 | 10 | 3.12 | 29 | 0.345 |
| 246 | B | 36 | 11 | 2.39 | 29 | 0.379 |
| 247 | A | 40 | 11 | 1.90 | 29 | 0.379 |
| 248 | A | 44 | 11 | 1.62 | 29 | 0.379 |
| 249 | A | 4 | 4 | 0.97 | 14 | 0.286 |
| 250 | A | 0 | 0 | 0.00 | 0 | 0.000 |
| 251 | A | 0 | 0 | 0.00 | 0 | 0.000 |
| 252 | A | 0 | 0 | 0.00 | 0 | 0.000 |
| 253 | A | 0 | 0 | 0.00 | 0 | 0.000 |
| 254 | A | 0 | 0 | 0.00 | 0 | 0.000 |
| 255 | A | 0 | 0 | 0.00 | 0 | 0.000 |
| 256 | A | 0 | 0 | 0.00 | 0 | 0.000 |
| 257 | A | 0 | 0 | 0.00 | 0 | 0.000 |
| 258 | A | 0 | 0 | 0.00 | 0 | 0.000 |
| 259 | A | 0 | 0 | 0.00 | 0 | 0.000 |
| 260 | A | 0 | 0 | 0.00 | 0 | 0.000 |
| 261 | A | 0 | 0 | 0.00 | 0 | 0.000 |
| 262 | A | 4 | 3 | 0.96 | 29 | 0.103 |
| 263 | A | 4 | 3 | 1.00 | 29 | 0.103 |
| 264 | A | 4 | 3 | 1.00 | 29 | 0.103 |

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Table 2.1 – continued from previous page

| # | grade | number of steps used | number of unique rules | normalized antiderivative leaf size | integrand leaf size | $\frac{\text{number of rules}}{\text{integrand leaf size}}$ |
|---|---|---|---|---|---|---|
| 265 | A | 4 | 3 | 1.00 | 27 | 0.111 |
| 266 | A | 3 | 2 | 1.00 | 21 | 0.095 |
| 267 | A | 10 | 6 | 1.00 | 29 | 0.207 |
| 268 | A | 4 | 3 | 1.30 | 29 | 0.103 |
| 269 | A | 4 | 3 | 1.00 | 29 | 0.103 |
| 270 | A | 4 | 3 | 1.00 | 29 | 0.103 |
| 271 | A | 4 | 3 | 1.00 | 29 | 0.103 |
| 272 | A | 33 | 13 | 1.12 | 31 | 0.419 |
| 273 | A | 29 | 13 | 1.22 | 31 | 0.419 |
| 274 | A | 25 | 12 | 1.60 | 29 | 0.414 |
| 275 | A | 22 | 10 | 1.95 | 23 | 0.435 |
| 276 | B | 44 | 21 | 7.46 | 31 | 0.677 |
| 277 | B | 32 | 10 | 3.10 | 31 | 0.323 |
| 278 | B | 36 | 11 | 2.36 | 31 | 0.355 |
| 279 | A | 40 | 11 | 1.89 | 31 | 0.355 |
| 280 | A | 44 | 11 | 1.61 | 31 | 0.355 |
| 281 | A | 0 | 0 | 0.00 | 0 | 0.000 |
| 282 | A | 0 | 0 | 0.00 | 0 | 0.000 |
| 283 | A | 0 | 0 | 0.00 | 0 | 0.000 |
| 284 | A | 0 | 0 | 0.00 | 0 | 0.000 |
| 285 | A | 0 | 0 | 0.00 | 0 | 0.000 |
| 286 | A | 0 | 0 | 0.00 | 0 | 0.000 |
| 287 | A | 0 | 0 | 0.00 | 0 | 0.000 |
| 288 | A | 0 | 0 | 0.00 | 0 | 0.000 |
| 289 | A | 0 | 0 | 0.00 | 0 | 0.000 |
| 290 | A | 0 | 0 | 0.00 | 0 | 0.000 |

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Table 2.1 – continued from previous page

| # | grade | number of steps used | number of unique rules | normalized antiderivative leaf size | integrand leaf size | $\frac{\text{number of rules}}{\text{integrand leaf size}}$ |
|---|---|---|---|---|---|---|
| 291 | A | 0 | 0 | 0.00 | 0 | 0.000 |
| 292 | A | 0 | 0 | 0.00 | 0 | 0.000 |
| 293 | A | 5 | 3 | 1.03 | 31 | 0.097 |
| 294 | A | 5 | 3 | 1.05 | 31 | 0.097 |
| 295 | A | 5 | 3 | 1.08 | 31 | 0.097 |
| 296 | A | 5 | 3 | 1.10 | 29 | 0.103 |
| 297 | A | 3 | 2 | 1.00 | 23 | 0.087 |
| 298 | A | 9 | 5 | 1.05 | 31 | 0.161 |
| 299 | A | 6 | 4 | 1.10 | 31 | 0.129 |
| 300 | A | 5 | 3 | 1.06 | 31 | 0.097 |
| 301 | A | 5 | 3 | 1.04 | 31 | 0.097 |
| 302 | A | 5 | 3 | 1.03 | 31 | 0.097 |
| 303 | A | 23 | 11 | 1.22 | 33 | 0.333 |
| 304 | A | 20 | 11 | 1.53 | 31 | 0.355 |
| 305 | A | 10 | 8 | 1.42 | 25 | 0.320 |
| 306 | A | 16 | 10 | 1.57 | 33 | 0.303 |
| 307 | A | 10 | 7 | 1.65 | 33 | 0.212 |
| 308 | B | 29 | 16 | 2.46 | 33 | 0.485 |
| 309 | A | 53 | 13 | 1.87 | 33 | 0.394 |
| 310 | B | 35 | 13 | 2.21 | 31 | 0.419 |
| 311 | B | 14 | 10 | 2.01 | 25 | 0.400 |
| 312 | B | 25 | 11 | 2.17 | 33 | 0.333 |
| 313 | B | 14 | 9 | 2.15 | 33 | 0.273 |
| 314 | B | 49 | 21 | 3.51 | 33 | 0.636 |

# Chapter 3

# Listing of integrals

## 3.1 $\int (ag+bgx)^4 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\,dx$

**Optimal**. Leaf size=188

$$\frac{g^4(a+bx)^5\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{5b}-\frac{Bg^4n(bc-ad)^5\log(c+dx)}{5bd^5}+\frac{Bg^4nx(bc-ad)^4}{5d^4}-\frac{Bg^4n(a+bx)^2(bc-ad)^3}{10bd^3}+$$

```
[Out] 1/5*B*(-a*d+b*c)^4*g^4*n*x/d^4-1/10*B*(-a*d+b*c)^3*g^4*n*(b*x+a)^2/b/d^3+1/
15*B*(-a*d+b*c)^2*g^4*n*(b*x+a)^3/b/d^2-1/20*B*(-a*d+b*c)*g^4*n*(b*x+a)^4/b
/d+1/5*g^4*(b*x+a)^5*(A+B*ln(e*((b*x+a)/(d*x+c))^n))/b-1/5*B*(-a*d+b*c)^5*g
^4*n*ln(d*x+c)/b/d^5
```

**Rubi [A]** time = 0.14, antiderivative size = 188, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {2525, 12, 43}

$$\frac{g^4(a+bx)^5\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{5b}+\frac{Bg^4nx(bc-ad)^4}{5d^4}-\frac{Bg^4n(a+bx)^2(bc-ad)^3}{10bd^3}+\frac{Bg^4n(a+bx)^3(bc-ad)^2}{15bd^2}-Bg$$

Antiderivative was successfully verified.

```
[In] Int[(a*g + b*g*x)^4*(A + B*Log[e*((a + b*x)/(c + d*x))^n]),x]
```

```
[Out] (B*(b*c - a*d)^4*g^4*n*x)/(5*d^4) - (B*(b*c - a*d)^3*g^4*n*(a + b*x)^2)/(10
*b*d^3) + (B*(b*c - a*d)^2*g^4*n*(a + b*x)^3)/(15*b*d^2) - (B*(b*c - a*d)*g
^4*n*(a + b*x)^4)/(20*b*d) + (g^4*(a + b*x)^5*(A + B*Log[e*((a + b*x)/(c +
d*x))^n]))/(5*b) - (B*(b*c - a*d)^5*g^4*n*Log[c + d*x])/(5*b*d^5)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2525

```
Int[((a_.) + Log[(c_.)*(RFx_)^(p_.)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_))^(m_.
), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(a + b*Log[c*RFx^p])^n)/(e*(m + 1))
, x] - Dist[(b*n*p)/(e*(m + 1)), Int[SimplifyIntegrand[((d + e*x)^(m + 1)*(
a + b*Log[c*RFx^p])^(n - 1)*D[RFx, x])/RFx, x], x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && RationalFunctionQ[RFx, x] && IGtQ[n, 0] && (EqQ[n, 1] ||
IntegerQ[m]) && NeQ[m, -1]
```

Rubi steps

$$\int (ag+bgx)^4\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)dx = \frac{g^4(a+bx)^5\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{5b} - \frac{(Bn)\int \frac{(bc-ad)g^5(a+bx)^4}{c+dx}\,dx}{5bg}$$

$$= \frac{g^4(a+bx)^5\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{5b} - \frac{\left(B(bc-ad)g^4n\right)\int \frac{(a+bx)^4}{c+dx}\,d}{5b}$$

$$= \frac{g^4(a+bx)^5\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{5b} - \frac{\left(B(bc-ad)g^4n\right)\int\left(-\frac{b(bc-a}{d^4}\right.}{}$$

$$= \frac{B(bc-ad)^4g^4nx}{5d^4} - \frac{B(bc-ad)^3g^4n(a+bx)^2}{10bd^3} + \frac{B(bc-ad)^2g^4n(a+}{15bd^2}$$

**Mathematica [A]** time = 0.12, size = 146, normalized size = 0.78

$$\frac{g^4\left((a+bx)^5\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)-\frac{Bn(bc-ad)\left(4d^3(a+bx)^3(ad-bc)+6d^2(a+bx)^2(bc-ad)^2-12bdx(bc-ad)^3+12(bc-ad)^4\log(c+dx)+3d^4(\right.}{12d^5}}{5b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*g + b*g*x)^4*(A + B*Log[e*((a + b*x)/(c + d*x))^n]),x]
```

```
[Out] (g^4*((a + b*x)^5*(A + B*Log[e*((a + b*x)/(c + d*x))^n]) - (B*(b*c - a*d)*n
*(-12*b*d*(b*c - a*d)^3*x + 6*d^2*(b*c - a*d)^2*(a + b*x)^2 + 4*d^3*(-(b*c)
 + a*d)*(a + b*x)^3 + 3*d^4*(a + b*x)^4 + 12*(b*c - a*d)^4*Log[c + d*x]))/(
12*d^5)))/(5*b)
```

**fricas [B]** time = 1.14, size = 569, normalized size = 3.03

$$\frac{12\,Ab^5d^5g^4x^5+12\,Ba^5d^5g^4n\log\left(bx+a\right)-12\left(Bb^5c^5-5\,Bab^4c^4d+10\,Ba^2b^3c^3d^2-10\,Ba^3b^2c^2d^3+5\,Ba^4bcd^4\right)g}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*g*x+a*g)^4*(A+B*log(e*((b*x+a)/(d*x+c))^n)),x, algorithm="fric
as")
```

```
[Out] 1/60*(12*A*b^5*d^5*g^4*x^5 + 12*B*a^5*d^5*g^4*n*log(b*x + a) - 12*(B*b^5*c^
5 - 5*B*a*b^4*c^4*d + 10*B*a^2*b^3*c^3*d^2 - 10*B*a^3*b^2*c^2*d^3 + 5*B*a^4
*b*c*d^4)*g^4*n*log(d*x + c) + 3*(20*A*a*b^4*d^5*g^4 - (B*b^5*c*d^4 - B*a*b
^4*d^5)*g^4*n)*x^4 + 4*(30*A*a^2*b^3*d^5*g^4 + (B*b^5*c^2*d^3 - 5*B*a*b^4*c
*d^4 + 4*B*a^2*b^3*d^5)*g^4*n)*x^3 + 6*(20*A*a^3*b^2*d^5*g^4 - (B*b^5*c^3*d
^2 - 5*B*a*b^4*c^2*d^3 + 10*B*a^2*b^3*c*d^4 - 6*B*a^3*b^2*d^5)*g^4*n)*x^2 +
 12*(5*A*a^4*b*d^5*g^4 + (B*b^5*c^4*d - 5*B*a*b^4*c^3*d^2 + 10*B*a^2*b^3*c^
2*d^3 - 10*B*a^3*b^2*c*d^4 + 4*B*a^4*b*d^5)*g^4*n)*x + 12*(B*b^5*d^5*g^4*x^
5 + 5*B*a*b^4*d^5*g^4*x^4 + 10*B*a^2*b^3*d^5*g^4*x^3 + 10*B*a^3*b^2*d^5*g^4
*x^2 + 5*B*a^4*b*d^5*g^4*x)*log(e) + 12*(B*b^5*d^5*g^4*n*x^5 + 5*B*a*b^4*d^
5*g^4*n*x^4 + 10*B*a^2*b^3*d^5*g^4*n*x^3 + 10*B*a^3*b^2*d^5*g^4*n*x^2 + 5*B
*a^4*b*d^5*g^4*n*x)*log((b*x + a)/(d*x + c)))/(b*d^5)
```

**giac [B]** time = 6.20, size = 4392, normalized size = 23.36

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*g*x+a*g)^4*(A+B*log(e*((b*x+a)/(d*x+c))^n)),x, algorithm="giac
")
```

```
[Out] 1/60*(12*(B*b^10*c^6*g^4*n - 6*B*a*b^9*c^5*d*g^4*n - 5*(b*x + a)*B*b^9*c^6*
d*g^4*n/(d*x + c) + 15*B*a^2*b^8*c^4*d^2*g^4*n + 30*(b*x + a)*B*a*b^8*c^5*d
^2*g^4*n/(d*x + c) + 10*(b*x + a)^2*B*b^8*c^6*d^2*g^4*n/(d*x + c)^2 - 20*B*
a^3*b^7*c^3*d^3*g^4*n - 75*(b*x + a)*B*a^2*b^7*c^4*d^3*g^4*n/(d*x + c) - 60
*(b*x + a)^2*B*a*b^7*c^5*d^3*g^4*n/(d*x + c)^2 - 10*(b*x + a)^3*B*b^7*c^6*d
```

```
^3*g^4*n/(d*x + c)^3 + 15*B*a^4*b^6*c^2*d^4*g^4*n + 100*(b*x + a)*B*a^3*b^6
*c^3*d^4*g^4*n/(d*x + c) + 150*(b*x + a)^2*B*a^2*b^6*c^4*d^4*g^4*n/(d*x + c
)^2 + 60*(b*x + a)^3*B*a*b^6*c^5*d^4*g^4*n/(d*x + c)^3 + 5*(b*x + a)^4*B*b^
6*c^6*d^4*g^4*n/(d*x + c)^4 - 6*B*a^5*b^5*c*d^5*g^4*n - 75*(b*x + a)*B*a^4*
b^5*c^2*d^5*g^4*n/(d*x + c) - 200*(b*x + a)^2*B*a^3*b^5*c^3*d^5*g^4*n/(d*x
+ c)^2 - 150*(b*x + a)^3*B*a^2*b^5*c^4*d^5*g^4*n/(d*x + c)^3 - 30*(b*x + a)
^4*B*a*b^5*c^5*d^5*g^4*n/(d*x + c)^4 + B*a^6*b^4*d^6*g^4*n + 30*(b*x + a)*B
*a^5*b^4*c*d^6*g^4*n/(d*x + c) + 150*(b*x + a)^2*B*a^4*b^4*c^2*d^6*g^4*n/(d
*x + c)^2 + 200*(b*x + a)^3*B*a^3*b^4*c^3*d^6*g^4*n/(d*x + c)^3 + 75*(b*x +
 a)^4*B*a^2*b^4*c^4*d^6*g^4*n/(d*x + c)^4 - 5*(b*x + a)*B*a^6*b^3*d^7*g^4*n
/(d*x + c) - 60*(b*x + a)^2*B*a^5*b^3*c*d^7*g^4*n/(d*x + c)^2 - 150*(b*x +
a)^3*B*a^4*b^3*c^2*d^7*g^4*n/(d*x + c)^3 - 100*(b*x + a)^4*B*a^3*b^3*c^3*d^
7*g^4*n/(d*x + c)^4 + 10*(b*x + a)^2*B*a^6*b^2*d^8*g^4*n/(d*x + c)^2 + 60*(
b*x + a)^3*B*a^5*b^2*c*d^8*g^4*n/(d*x + c)^3 + 75*(b*x + a)^4*B*a^4*b^2*c^2
*d^8*g^4*n/(d*x + c)^4 - 10*(b*x + a)^3*B*a^6*b*d^9*g^4*n/(d*x + c)^3 - 30*
(b*x + a)^4*B*a^5*b*c*d^9*g^4*n/(d*x + c)^4 + 5*(b*x + a)^4*B*a^6*d^10*g^4*
n/(d*x + c)^4)*log((b*x + a)/(d*x + c))/(b^5*d^5 - 5*(b*x + a)*b^4*d^6/(d*x
 + c) + 10*(b*x + a)^2*b^3*d^7/(d*x + c)^2 - 10*(b*x + a)^3*b^2*d^8/(d*x +
c)^3 + 5*(b*x + a)^4*b*d^9/(d*x + c)^4 - (b*x + a)^5*d^10/(d*x + c)^5) + (2
5*B*b^10*c^6*g^4*n - 150*B*a*b^9*c^5*d*g^4*n - 113*(b*x + a)*B*b^9*c^6*d*g^
4*n/(d*x + c) + 375*B*a^2*b^8*c^4*d^2*g^4*n + 678*(b*x + a)*B*a*b^8*c^5*d^2
*g^4*n/(d*x + c) + 196*(b*x + a)^2*B*b^8*c^6*d^2*g^4*n/(d*x + c)^2 - 500*B*
a^3*b^7*c^3*d^3*g^4*n - 1695*(b*x + a)*B*a^2*b^7*c^4*d^3*g^4*n/(d*x + c) -
1176*(b*x + a)^2*B*a*b^7*c^5*d^3*g^4*n/(d*x + c)^2 - 156*(b*x + a)^3*B*b^7*
c^6*d^3*g^4*n/(d*x + c)^3 + 375*B*a^4*b^6*c^2*d^4*g^4*n + 2260*(b*x + a)*B*
a^3*b^6*c^3*d^4*g^4*n/(d*x + c) + 2940*(b*x + a)^2*B*a^2*b^6*c^4*d^4*g^4*n/
(d*x + c)^2 + 936*(b*x + a)^3*B*a*b^6*c^5*d^4*g^4*n/(d*x + c)^3 + 48*(b*x +
 a)^4*B*b^6*c^6*d^4*g^4*n/(d*x + c)^4 - 150*B*a^5*b^5*c*d^5*g^4*n - 1695*(b
*x + a)*B*a^4*b^5*c^2*d^5*g^4*n/(d*x + c) - 3920*(b*x + a)^2*B*a^3*b^5*c^3*
d^5*g^4*n/(d*x + c)^2 - 2340*(b*x + a)^3*B*a^2*b^5*c^4*d^5*g^4*n/(d*x + c)^
3 - 288*(b*x + a)^4*B*a*b^5*c^5*d^5*g^4*n/(d*x + c)^4 + 25*B*a^6*b^4*d^6*g^
4*n + 678*(b*x + a)*B*a^5*b^4*c*d^6*g^4*n/(d*x + c) + 2940*(b*x + a)^2*B*a^
4*b^4*c^2*d^6*g^4*n/(d*x + c)^2 + 3120*(b*x + a)^3*B*a^3*b^4*c^3*d^6*g^4*n/
(d*x + c)^3 + 720*(b*x + a)^4*B*a^2*b^4*c^4*d^6*g^4*n/(d*x + c)^4 - 113*(b*
x + a)*B*a^6*b^3*d^7*g^4*n/(d*x + c) - 1176*(b*x + a)^2*B*a^5*b^3*c*d^7*g^4
*n/(d*x + c)^2 - 2340*(b*x + a)^3*B*a^4*b^3*c^2*d^7*g^4*n/(d*x + c)^3 - 960
*(b*x + a)^4*B*a^3*b^3*c^3*d^7*g^4*n/(d*x + c)^4 + 196*(b*x + a)^2*B*a^6*b^
2*d^8*g^4*n/(d*x + c)^2 + 936*(b*x + a)^3*B*a^5*b^2*c*d^8*g^4*n/(d*x + c)^3
 + 720*(b*x + a)^4*B*a^4*b^2*c^2*d^8*g^4*n/(d*x + c)^4 - 156*(b*x + a)^3*B*
a^6*b*d^9*g^4*n/(d*x + c)^3 - 288*(b*x + a)^4*B*a^5*b*c*d^9*g^4*n/(d*x + c)
^4 + 48*(b*x + a)^4*B*a^6*d^10*g^4*n/(d*x + c)^4 + 12*A*b^10*c^6*g^4 + 12*B
*b^10*c^6*g^4 - 72*A*a*b^9*c^5*d*g^4 - 72*B*a*b^9*c^5*d*g^4 - 60*(b*x + a)*
A*b^9*c^6*d*g^4/(d*x + c) - 60*(b*x + a)*B*b^9*c^6*d*g^4/(d*x + c) + 180*A*
a^2*b^8*c^4*d^2*g^4 + 180*B*a^2*b^8*c^4*d^2*g^4 + 360*(b*x + a)*A*a*b^8*c^5
*d^2*g^4/(d*x + c) + 360*(b*x + a)*B*a*b^8*c^5*d^2*g^4/(d*x + c) + 120*(b*x
```

```
 + a)^2*A*b^8*c^6*d^2*g^4/(d*x + c)^2 + 120*(b*x + a)^2*B*b^8*c^6*d^2*g^4/(
d*x + c)^2 - 240*A*a^3*b^7*c^3*d^3*g^4 - 240*B*a^3*b^7*c^3*d^3*g^4 - 900*(b
*x + a)*A*a^2*b^7*c^4*d^3*g^4/(d*x + c) - 900*(b*x + a)*B*a^2*b^7*c^4*d^3*g
^4/(d*x + c) - 720*(b*x + a)^2*A*a*b^7*c^5*d^3*g^4/(d*x + c)^2 - 720*(b*x +
 a)^2*B*a*b^7*c^5*d^3*g^4/(d*x + c)^2 - 120*(b*x + a)^3*A*b^7*c^6*d^3*g^4/(
d*x + c)^3 - 120*(b*x + a)^3*B*b^7*c^6*d^3*g^4/(d*x + c)^3 + 180*A*a^4*b^6*
c^2*d^4*g^4 + 180*B*a^4*b^6*c^2*d^4*g^4 + 1200*(b*x + a)*A*a^3*b^6*c^3*d^4*
g^4/(d*x + c) + 1200*(b*x + a)*B*a^3*b^6*c^3*d^4*g^4/(d*x + c) + 1800*(b*x
+ a)^2*A*a^2*b^6*c^4*d^4*g^4/(d*x + c)^2 + 1800*(b*x + a)^2*B*a^2*b^6*c^4*d
^4*g^4/(d*x + c)^2 + 720*(b*x + a)^3*A*a*b^6*c^5*d^4*g^4/(d*x + c)^3 + 720*
(b*x + a)^3*B*a*b^6*c^5*d^4*g^4/(d*x + c)^3 + 60*(b*x + a)^4*A*b^6*c^6*d^4*
g^4/(d*x + c)^4 + 60*(b*x + a)^4*B*b^6*c^6*d^4*g^4/(d*x + c)^4 - 72*A*a^5*b
^5*c*d^5*g^4 - 72*B*a^5*b^5*c*d^5*g^4 - 900*(b*x + a)*A*a^4*b^5*c^2*d^5*g^4
/(d*x + c) - 900*(b*x + a)*B*a^4*b^5*c^2*d^5*g^4/(d*x + c) - 2400*(b*x + a)
^2*A*a^3*b^5*c^3*d^5*g^4/(d*x + c)^2 - 2400*(b*x + a)^2*B*a^3*b^5*c^3*d^5*g
^4/(d*x + c)^2 - 1800*(b*x + a)^3*A*a^2*b^5*c^4*d^5*g^4/(d*x + c)^3 - 1800*
(b*x + a)^3*B*a^2*b^5*c^4*d^5*g^4/(d*x + c)^3 - 360*(b*x + a)^4*A*a*b^5*c^5
*d^5*g^4/(d*x + c)^4 - 360*(b*x + a)^4*B*a*b^5*c^5*d^5*g^4/(d*x + c)^4 + 12
*A*a^6*b^4*d^6*g^4 + 12*B*a^6*b^4*d^6*g^4 + 360*(b*x + a)*A*a^5*b^4*c*d^6*g
^4/(d*x + c) + 360*(b*x + a)*B*a^5*b^4*c*d^6*g^4/(d*x + c) + 1800*(b*x + a)
^2*A*a^4*b^4*c^2*d^6*g^4/(d*x + c)^2 + 1800*(b*x + a)^2*B*a^4*b^4*c^2*d^6*g
^4/(d*x + c)^2 + 2400*(b*x + a)^3*A*a^3*b^4*c^3*d^6*g^4/(d*x + c)^3 + 2400*
(b*x + a)^3*B*a^3*b^4*c^3*d^6*g^4/(d*x + c)^3 + 900*(b*x + a)^4*A*a^2*b^4*c
^4*d^6*g^4/(d*x + c)^4 + 900*(b*x + a)^4*B*a^2*b^4*c^4*d^6*g^4/(d*x + c)^4
- 60*(b*x + a)*A*a^6*b^3*d^7*g^4/(d*x + c) - 60*(b*x + a)*B*a^6*b^3*d^7*g^4
/(d*x + c) - 720*(b*x + a)^2*A*a^5*b^3*c*d^7*g^4/(d*x + c)^2 - 720*(b*x + a
)^2*B*a^5*b^3*c*d^7*g^4/(d*x + c)^2 - 1800*(b*x + a)^3*A*a^4*b^3*c^2*d^7*g^
4/(d*x + c)^3 - 1800*(b*x + a)^3*B*a^4*b^3*c^2*d^7*g^4/(d*x + c)^3 - 1200*(
b*x + a)^4*A*a^3*b^3*c^3*d^7*g^4/(d*x + c)^4 - 1200*(b*x + a)^4*B*a^3*b^3*c
^3*d^7*g^4/(d*x + c)^4 + 120*(b*x + a)^2*A*a^6*b^2*d^8*g^4/(d*x + c)^2 + 12
0*(b*x + a)^2*B*a^6*b^2*d^8*g^4/(d*x + c)^2 + 720*(b*x + a)^3*A*a^5*b^2*c*d
^8*g^4/(d*x + c)^3 + 720*(b*x + a)^3*B*a^5*b^2*c*d^8*g^4/(d*x + c)^3 + 900*
(b*x + a)^4*A*a^4*b^2*c^2*d^8*g^4/(d*x + c)^4 + 900*(b*x + a)^4*B*a^4*b^2*c
^2*d^8*g^4/(d*x + c)^4 - 120*(b*x + a)^3*A*a^6*b*d^9*g^4/(d*x + c)^3 - 120*
(b*x + a)^3*B*a^6*b*d^9*g^4/(d*x + c)^3 - 360*(b*x + a)^4*A*a^5*b*c*d^9*g^4
/(d*x + c)^4 - 360*(b*x + a)^4*B*a^5*b*c*d^9*g^4/(d*x + c)^4 + 60*(b*x + a)
^4*A*a^6*d^10*g^4/(d*x + c)^4 + 60*(b*x + a)^4*B*a^6*d^10*g^4/(d*x + c)^4)/
(b^5*d^5 - 5*(b*x + a)*b^4*d^6/(d*x + c) + 10*(b*x + a)^2*b^3*d^7/(d*x + c)
^2 - 10*(b*x + a)^3*b^2*d^8/(d*x + c)^3 + 5*(b*x + a)^4*b*d^9/(d*x + c)^4 -
 (b*x + a)^5*d^10/(d*x + c)^5) + 12*(B*b^6*c^6*g^4*n - 6*B*a*b^5*c^5*d*g^4*
n + 15*B*a^2*b^4*c^4*d^2*g^4*n - 20*B*a^3*b^3*c^3*d^3*g^4*n + 15*B*a^4*b^2*
c^2*d^4*g^4*n - 6*B*a^5*b*c*d^5*g^4*n + B*a^6*d^6*g^4*n)*log(b - (b*x + a)*
d/(d*x + c))/(b*d^5) - 12*(B*b^6*c^6*g^4*n - 6*B*a*b^5*c^5*d*g^4*n + 15*B*a
^2*b^4*c^4*d^2*g^4*n - 20*B*a^3*b^3*c^3*d^3*g^4*n + 15*B*a^4*b^2*c^2*d^4*g^
4*n - 6*B*a^5*b*c*d^5*g^4*n + B*a^6*d^6*g^4*n)*log((b*x + a)/(d*x + c))/(b*
```

```
d^5))*(b*c/(b*c - a*d)^2 - a*d/(b*c - a*d)^2)
```

**maple [F]** time = 0.33, size = 0, normalized size = 0.00

$$\int \left(bgx+ag\right)^{4} \left(B\ln\left(e\left(\frac{bx+a}{dx+c}\right)^{n}\right)+A\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*g*x+a*g)^4*(A+B*ln(e*((b*x+a)/(d*x+c))^n)),x)
```

```
[Out] int((b*g*x+a*g)^4*(A+B*ln(e*((b*x+a)/(d*x+c))^n)),x)
```

**maxima [B]** time = 1.47, size = 676, normalized size = 3.60

$$\frac{1}{5}Bb^{4}g^{4}x^{5}\log\left(e\left(\frac{bx}{dx+c}+\frac{a}{dx+c}\right)^{n}\right)+\frac{1}{5}Ab^{4}g^{4}x^{5}+Bab^{3}g^{4}x^{4}\log\left(e\left(\frac{bx}{dx+c}+\frac{a}{dx+c}\right)^{n}\right)+Aab^{3}g^{4}x^{4}+2Ba^{2}b^{2}g^{4}x^{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*g*x+a*g)^4*(A+B*log(e*((b*x+a)/(d*x+c))^n)),x, algorithm="maxi
ma")
```

```
[Out] 1/5*B*b^4*g^4*x^5*log(e*(b*x/(d*x + c) + a/(d*x + c))^n) + 1/5*A*b^4*g^4*x^
5 + B*a*b^3*g^4*x^4*log(e*(b*x/(d*x + c) + a/(d*x + c))^n) + A*a*b^3*g^4*x^
4 + 2*B*a^2*b^2*g^4*x^3*log(e*(b*x/(d*x + c) + a/(d*x + c))^n) + 2*A*a^2*b^
2*g^4*x^3 + 2*B*a^3*b*g^4*x^2*log(e*(b*x/(d*x + c) + a/(d*x + c))^n) + 2*A*
a^3*b*g^4*x^2 + 1/60*B*b^4*g^4*n*(12*a^5*log(b*x + a)/b^5 - 12*c^5*log(d*x
+ c)/d^5 - (3*(b^4*c*d^3 - a*b^3*d^4)*x^4 - 4*(b^4*c^2*d^2 - a^2*b^2*d^4)*x
^3 + 6*(b^4*c^3*d - a^3*b*d^4)*x^2 - 12*(b^4*c^4 - a^4*d^4)*x)/(b^4*d^4)) -
 1/6*B*a*b^3*g^4*n*(6*a^4*log(b*x + a)/b^4 - 6*c^4*log(d*x + c)/d^4 + (2*(b
^3*c*d^2 - a*b^2*d^3)*x^3 - 3*(b^3*c^2*d - a^2*b*d^3)*x^2 + 6*(b^3*c^3 - a^
3*d^3)*x)/(b^3*d^3)) + B*a^2*b^2*g^4*n*(2*a^3*log(b*x + a)/b^3 - 2*c^3*log(
d*x + c)/d^3 - ((b^2*c*d - a*b*d^2)*x^2 - 2*(b^2*c^2 - a^2*d^2)*x)/(b^2*d^2
)) - 2*B*a^3*b*g^4*n*(a^2*log(b*x + a)/b^2 - c^2*log(d*x + c)/d^2 + (b*c -
a*d)*x/(b*d)) + B*a^4*g^4*n*(a*log(b*x + a)/b - c*log(d*x + c)/d) + B*a^4*g
^4*x*log(e*(b*x/(d*x + c) + a/(d*x + c))^n) + A*a^4*g^4*x
```

**mupad [B]** time = 4.55, size = 1046, normalized size = 5.56

$$x^2\left(\frac{(5\,a\,d+5\,b\,c)\left(\frac{\left(\frac{b^3\,g^4\,(25\,A\,a\,d+5\,A\,b\,c+B\,a\,d\,n-B\,b\,c\,n)}{5\,d}-\frac{A\,b^3\,g^4\,(5\,a\,d+5\,b\,c)}{5\,d}\right)(5\,a\,d+5\,b\,c)}{5\,b\,d}-\frac{a\,b^2\,g^4\,(10\,A\,a\,d+5\,A\,b\,c+B\,a\,d\,n-B\,b\,c\,n)}{d}+\frac{A\,a\,b^3\,c\,g}{d}\right.}{10\,b\,d}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*g + b*g*x)^4*(A + B*log(e*((a + b*x)/(c + d*x))^n)),x)
```

```
[Out] x^2*(((5*a*d + 5*b*c)*((((b^3*g^4*(25*A*a*d + 5*A*b*c + B*a*d*n - B*b*c*n))
/(5*d) - (A*b^3*g^4*(5*a*d + 5*b*c))/(5*d))*(5*a*d + 5*b*c))/(5*b*d) - (a*b
^2*g^4*(10*A*a*d + 5*A*b*c + B*a*d*n - B*b*c*n))/d + (A*a*b^3*c*g^4)/d))/(1
0*b*d) - (a*c*((b^3*g^4*(25*A*a*d + 5*A*b*c + B*a*d*n - B*b*c*n))/(5*d) - (
A*b^3*g^4*(5*a*d + 5*b*c))/(5*d)))/(2*b*d) + (a^2*b*g^4*(5*A*a*d + 5*A*b*c
+ B*a*d*n - B*b*c*n))/d) - x^3*((((b^3*g^4*(25*A*a*d + 5*A*b*c + B*a*d*n -
B*b*c*n))/(5*d) - (A*b^3*g^4*(5*a*d + 5*b*c))/(5*d))*(5*a*d + 5*b*c))/(15*b
*d) - (a*b^2*g^4*(10*A*a*d + 5*A*b*c + B*a*d*n - B*b*c*n))/(3*d) + (A*a*b^3
*c*g^4)/(3*d)) + log(e*((a + b*x)/(c + d*x))^n)*((B*b^4*g^4*x^5)/5 + B*a^4*
g^4*x + 2*B*a^3*b*g^4*x^2 + B*a*b^3*g^4*x^4 + 2*B*a^2*b^2*g^4*x^3) + x*((a^
3*g^4*(5*A*a*d + 10*A*b*c + 2*B*a*d*n - 2*B*b*c*n))/d - ((5*a*d + 5*b*c)*((
(5*a*d + 5*b*c)*((((b^3*g^4*(25*A*a*d + 5*A*b*c + B*a*d*n - B*b*c*n))/(5*d)
 - (A*b^3*g^4*(5*a*d + 5*b*c))/(5*d))*(5*a*d + 5*b*c))/(5*b*d) - (a*b^2*g^4
*(10*A*a*d + 5*A*b*c + B*a*d*n - B*b*c*n))/d + (A*a*b^3*c*g^4)/d))/(5*b*d)
- (a*c*((b^3*g^4*(25*A*a*d + 5*A*b*c + B*a*d*n - B*b*c*n))/(5*d) - (A*b^3*g
^4*(5*a*d + 5*b*c))/(5*d)))/(b*d) + (2*a^2*b*g^4*(5*A*a*d + 5*A*b*c + B*a*d
*n - B*b*c*n))/d))/(5*b*d) + (a*c*((((b^3*g^4*(25*A*a*d + 5*A*b*c + B*a*d*n
 - B*b*c*n))/(5*d) - (A*b^3*g^4*(5*a*d + 5*b*c))/(5*d))*(5*a*d + 5*b*c))/(5
*b*d) - (a*b^2*g^4*(10*A*a*d + 5*A*b*c + B*a*d*n - B*b*c*n))/d + (A*a*b^3*c
*g^4)/d))/(b*d)) + x^4*((b^3*g^4*(25*A*a*d + 5*A*b*c + B*a*d*n - B*b*c*n))/
(20*d) - (A*b^3*g^4*(5*a*d + 5*b*c))/(20*d)) - (log(c + d*x)*(B*b^4*c^5*g^4
*n + 5*B*a^4*c*d^4*g^4*n - 5*B*a*b^3*c^4*d*g^4*n - 10*B*a^3*b*c^2*d^3*g^4*n
 + 10*B*a^2*b^2*c^3*d^2*g^4*n))/(5*d^5) + (A*b^4*g^4*x^5)/5 + (B*a^5*g^4*n*
log(a + b*x))/(5*b)
```

**sympy [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*g*x+a*g)**4*(A+B*ln(e*((b*x+a)/(d*x+c))**n)),x)
```

```
[Out] Timed out
```

## 3.2 $\int (ag+bgx)^3 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\,dx$

**Optimal**. Leaf size=156

$$\frac{g^3(a+bx)^4\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{4b}+\frac{Bg^3n(bc-ad)^4\log(c+dx)}{4bd^4}-\frac{Bg^3nx(bc-ad)^3}{4d^3}+\frac{Bg^3n(a+bx)^2(bc-ad)^2}{8bd^2}-$$

```
[Out] -1/4*B*(-a*d+b*c)^3*g^3*n*x/d^3+1/8*B*(-a*d+b*c)^2*g^3*n*(b*x+a)^2/b/d^2-1/
12*B*(-a*d+b*c)*g^3*n*(b*x+a)^3/b/d+1/4*g^3*(b*x+a)^4*(A+B*ln(e*((b*x+a)/(d
*x+c))^n))/b+1/4*B*(-a*d+b*c)^4*g^3*n*ln(d*x+c)/b/d^4
```

**Rubi [A]** time = 0.11, antiderivative size = 156, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {2525, 12, 43}

$$\frac{g^3(a+bx)^4\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{4b}-\frac{Bg^3nx(bc-ad)^3}{4d^3}+\frac{Bg^3n(a+bx)^2(bc-ad)^2}{8bd^2}+\frac{Bg^3n(bc-ad)^4\log(c+dx)}{4bd^4}-$$

Antiderivative was successfully verified.

```
[In] Int[(a*g + b*g*x)^3*(A + B*Log[e*((a + b*x)/(c + d*x))^n]),x]
```

```
[Out] -(B*(b*c - a*d)^3*g^3*n*x)/(4*d^3) + (B*(b*c - a*d)^2*g^3*n*(a + b*x)^2)/(8
*b*d^2) - (B*(b*c - a*d)*g^3*n*(a + b*x)^3)/(12*b*d) + (g^3*(a + b*x)^4*(A
+ B*Log[e*((a + b*x)/(c + d*x))^n]))/(4*b) + (B*(b*c - a*d)^4*g^3*n*Log[c +
 d*x])/(4*b*d^4)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2525

```
Int[((a_.) + Log[(c_.)*(RFx_)^(p_.)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_))^(m_.
), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(a + b*Log[c*RFx^p])^n)/(e*(m + 1))
```

```
, x] - Dist[(b*n*p)/(e*(m + 1)), Int[SimplifyIntegrand[((d + e*x)^(m + 1)*(
a + b*Log[c*RFx^p])^(n - 1)*D[RFx, x])/RFx, x], x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && RationalFunctionQ[RFx, x] && IGtQ[n, 0] && (EqQ[n, 1] ||
IntegerQ[m]) && NeQ[m, -1]
```

Rubi steps

$$\int (ag+bgx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)dx=\frac{g^3(a+bx)^4\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{4b}-\frac{(Bn)\int\frac{(bc-ad)g^4(a+bx)^3}{c+dx}dx}{4bg}$$

$$=\frac{g^3(a+bx)^4\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{4b}-\frac{\left(B(bc-ad)g^3n\right)\int\frac{(a+bx)^3}{c+dx}d}{4b}$$

$$=\frac{g^3(a+bx)^4\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{4b}-\frac{\left(B(bc-ad)g^3n\right)\int\left(\frac{b(bc-ad)}{d^3}\right.}{}$$

$$=-\frac{B(bc-ad)^3g^3nx}{4d^3}+\frac{B(bc-ad)^2g^3n(a+bx)^2}{8bd^2}-\frac{B(bc-ad)g^3n(a+}{12bd}$$

**Mathematica [A]** time = 0.11, size = 124, normalized size = 0.79

$$\frac{g^3\left((a+bx)^4\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)-\frac{Bn(bc-ad)\left(3d^2(a+bx)^2(ad-bc)+6bdx(bc-ad)^2-6(bc-ad)^3\log(c+dx)+2d^3(a+bx)^3\right)}{6d^4}\right)}{4b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*g + b*g*x)^3*(A + B*Log[e*((a + b*x)/(c + d*x))^n]),x]
[Out] (g^3*((a + b*x)^4*(A + B*Log[e*((a + b*x)/(c + d*x))^n]) - (B*(b*c - a*d)*n
*(6*b*d*(b*c - a*d)^2*x + 3*d^2*(-(b*c) + a*d)*(a + b*x)^2 + 2*d^3*(a + b*x
)^3 - 6*(b*c - a*d)^3*Log[c + d*x]))/(6*d^4)))/(4*b)
```

**fricas [B]** time = 1.13, size = 426, normalized size = 2.73

$$\frac{6\,Ab^4d^4g^3x^4+6\,Ba^4d^4g^3n\log\left(bx+a\right)+6\left(Bb^4c^4-4\,Bab^3c^3d+6\,Ba^2b^2c^2d^2-4\,Ba^3bcd^3\right)g^3n\log\left(dx+c\right)+2\left(1\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*g*x+a*g)^3*(A+B*log(e*((b*x+a)/(d*x+c))^n)),x, algorithm="fric
as")
```

```
[Out] 1/24*(6*A*b^4*d^4*g^3*x^4 + 6*B*a^4*d^4*g^3*n*log(b*x + a) + 6*(B*b^4*c^4 -
 4*B*a*b^3*c^3*d + 6*B*a^2*b^2*c^2*d^2 - 4*B*a^3*b*c*d^3)*g^3*n*log(d*x + c
) + 2*(12*A*a*b^3*d^4*g^3 - (B*b^4*c*d^3 - B*a*b^3*d^4)*g^3*n)*x^3 + 3*(12*
A*a^2*b^2*d^4*g^3 + (B*b^4*c^2*d^2 - 4*B*a*b^3*c*d^3 + 3*B*a^2*b^2*d^4)*g^3
*n)*x^2 + 6*(4*A*a^3*b*d^4*g^3 - (B*b^4*c^3*d - 4*B*a*b^3*c^2*d^2 + 6*B*a^2
*b^2*c*d^3 - 3*B*a^3*b*d^4)*g^3*n)*x + 6*(B*b^4*d^4*g^3*x^4 + 4*B*a*b^3*d^4
*g^3*x^3 + 6*B*a^2*b^2*d^4*g^3*x^2 + 4*B*a^3*b*d^4*g^3*x)*log(e) + 6*(B*b^4
*d^4*g^3*n*x^4 + 4*B*a*b^3*d^4*g^3*n*x^3 + 6*B*a^2*b^2*d^4*g^3*n*x^2 + 4*B*
a^3*b*d^4*g^3*n*x)*log((b*x + a)/(d*x + c)))/(b*d^4)
```

**giac [B]** time = 4.41, size = 2986, normalized size = 19.14

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*g*x+a*g)^3*(A+B*log(e*((b*x+a)/(d*x+c))^n)),x, algorithm="giac
")
```

```
[Out] -1/24*(6*(B*b^8*c^5*g^3*n - 5*B*a*b^7*c^4*d*g^3*n - 4*(b*x + a)*B*b^7*c^5*d
*g^3*n/(d*x + c) + 10*B*a^2*b^6*c^3*d^2*g^3*n + 20*(b*x + a)*B*a*b^6*c^4*d^
2*g^3*n/(d*x + c) + 6*(b*x + a)^2*B*b^6*c^5*d^2*g^3*n/(d*x + c)^2 - 10*B*a^
3*b^5*c^2*d^3*g^3*n - 40*(b*x + a)*B*a^2*b^5*c^3*d^3*g^3*n/(d*x + c) - 30*(
b*x + a)^2*B*a*b^5*c^4*d^3*g^3*n/(d*x + c)^2 - 4*(b*x + a)^3*B*b^5*c^5*d^3*
g^3*n/(d*x + c)^3 + 5*B*a^4*b^4*c*d^4*g^3*n + 40*(b*x + a)*B*a^3*b^4*c^2*d^
4*g^3*n/(d*x + c) + 60*(b*x + a)^2*B*a^2*b^4*c^3*d^4*g^3*n/(d*x + c)^2 + 20
*(b*x + a)^3*B*a*b^4*c^4*d^4*g^3*n/(d*x + c)^3 - B*a^5*b^3*d^5*g^3*n - 20*(
b*x + a)*B*a^4*b^3*c*d^5*g^3*n/(d*x + c) - 60*(b*x + a)^2*B*a^3*b^3*c^2*d^5
*g^3*n/(d*x + c)^2 - 40*(b*x + a)^3*B*a^2*b^3*c^3*d^5*g^3*n/(d*x + c)^3 + 4
*(b*x + a)*B*a^5*b^2*d^6*g^3*n/(d*x + c) + 30*(b*x + a)^2*B*a^4*b^2*c*d^6*g
^3*n/(d*x + c)^2 + 40*(b*x + a)^3*B*a^3*b^2*c^2*d^6*g^3*n/(d*x + c)^3 - 6*(
b*x + a)^2*B*a^5*b*d^7*g^3*n/(d*x + c)^2 - 20*(b*x + a)^3*B*a^4*b*c*d^7*g^3
*n/(d*x + c)^3 + 4*(b*x + a)^3*B*a^5*d^8*g^3*n/(d*x + c)^3)*log((b*x + a)/(
d*x + c))/(b^4*d^4 - 4*(b*x + a)*b^3*d^5/(d*x + c) + 6*(b*x + a)^2*b^2*d^6/
(d*x + c)^2 - 4*(b*x + a)^3*b*d^7/(d*x + c)^3 + (b*x + a)^4*d^8/(d*x + c)^4
) + (11*B*b^8*c^5*g^3*n - 55*B*a*b^7*c^4*d*g^3*n - 38*(b*x + a)*B*b^7*c^5*d
*g^3*n/(d*x + c) + 110*B*a^2*b^6*c^3*d^2*g^3*n + 190*(b*x + a)*B*a*b^6*c^4*
d^2*g^3*n/(d*x + c) + 45*(b*x + a)^2*B*b^6*c^5*d^2*g^3*n/(d*x + c)^2 - 110*
B*a^3*b^5*c^2*d^3*g^3*n - 380*(b*x + a)*B*a^2*b^5*c^3*d^3*g^3*n/(d*x + c) -
 225*(b*x + a)^2*B*a*b^5*c^4*d^3*g^3*n/(d*x + c)^2 - 18*(b*x + a)^3*B*b^5*c
^5*d^3*g^3*n/(d*x + c)^3 + 55*B*a^4*b^4*c*d^4*g^3*n + 380*(b*x + a)*B*a^3*b
^4*c^2*d^4*g^3*n/(d*x + c) + 450*(b*x + a)^2*B*a^2*b^4*c^3*d^4*g^3*n/(d*x +
 c)^2 + 90*(b*x + a)^3*B*a*b^4*c^4*d^4*g^3*n/(d*x + c)^3 - 11*B*a^5*b^3*d^5
```

```
*g^3*n - 190*(b*x + a)*B*a^4*b^3*c*d^5*g^3*n/(d*x + c) - 450*(b*x + a)^2*B*
a^3*b^3*c^2*d^5*g^3*n/(d*x + c)^2 - 180*(b*x + a)^3*B*a^2*b^3*c^3*d^5*g^3*n
/(d*x + c)^3 + 38*(b*x + a)*B*a^5*b^2*d^6*g^3*n/(d*x + c) + 225*(b*x + a)^2
*B*a^4*b^2*c*d^6*g^3*n/(d*x + c)^2 + 180*(b*x + a)^3*B*a^3*b^2*c^2*d^6*g^3*
n/(d*x + c)^3 - 45*(b*x + a)^2*B*a^5*b*d^7*g^3*n/(d*x + c)^2 - 90*(b*x + a)
^3*B*a^4*b*c*d^7*g^3*n/(d*x + c)^3 + 18*(b*x + a)^3*B*a^5*d^8*g^3*n/(d*x +
c)^3 + 6*A*b^8*c^5*g^3 + 6*B*b^8*c^5*g^3 - 30*A*a*b^7*c^4*d*g^3 - 30*B*a*b^
7*c^4*d*g^3 - 24*(b*x + a)*A*b^7*c^5*d*g^3/(d*x + c) - 24*(b*x + a)*B*b^7*c
^5*d*g^3/(d*x + c) + 60*A*a^2*b^6*c^3*d^2*g^3 + 60*B*a^2*b^6*c^3*d^2*g^3 +
120*(b*x + a)*A*a*b^6*c^4*d^2*g^3/(d*x + c) + 120*(b*x + a)*B*a*b^6*c^4*d^2
*g^3/(d*x + c) + 36*(b*x + a)^2*A*b^6*c^5*d^2*g^3/(d*x + c)^2 + 36*(b*x + a
)^2*B*b^6*c^5*d^2*g^3/(d*x + c)^2 - 60*A*a^3*b^5*c^2*d^3*g^3 - 60*B*a^3*b^5
*c^2*d^3*g^3 - 240*(b*x + a)*A*a^2*b^5*c^3*d^3*g^3/(d*x + c) - 240*(b*x + a
)*B*a^2*b^5*c^3*d^3*g^3/(d*x + c) - 180*(b*x + a)^2*A*a*b^5*c^4*d^3*g^3/(d*
x + c)^2 - 180*(b*x + a)^2*B*a*b^5*c^4*d^3*g^3/(d*x + c)^2 - 24*(b*x + a)^3
*A*b^5*c^5*d^3*g^3/(d*x + c)^3 - 24*(b*x + a)^3*B*b^5*c^5*d^3*g^3/(d*x + c)
^3 + 30*A*a^4*b^4*c*d^4*g^3 + 30*B*a^4*b^4*c*d^4*g^3 + 240*(b*x + a)*A*a^3*
b^4*c^2*d^4*g^3/(d*x + c) + 240*(b*x + a)*B*a^3*b^4*c^2*d^4*g^3/(d*x + c) +
 360*(b*x + a)^2*A*a^2*b^4*c^3*d^4*g^3/(d*x + c)^2 + 360*(b*x + a)^2*B*a^2*
b^4*c^3*d^4*g^3/(d*x + c)^2 + 120*(b*x + a)^3*A*a*b^4*c^4*d^4*g^3/(d*x + c)
^3 + 120*(b*x + a)^3*B*a*b^4*c^4*d^4*g^3/(d*x + c)^3 - 6*A*a^5*b^3*d^5*g^3
- 6*B*a^5*b^3*d^5*g^3 - 120*(b*x + a)*A*a^4*b^3*c*d^5*g^3/(d*x + c) - 120*(
b*x + a)*B*a^4*b^3*c*d^5*g^3/(d*x + c) - 360*(b*x + a)^2*A*a^3*b^3*c^2*d^5*
g^3/(d*x + c)^2 - 360*(b*x + a)^2*B*a^3*b^3*c^2*d^5*g^3/(d*x + c)^2 - 240*(
b*x + a)^3*A*a^2*b^3*c^3*d^5*g^3/(d*x + c)^3 - 240*(b*x + a)^3*B*a^2*b^3*c^
3*d^5*g^3/(d*x + c)^3 + 24*(b*x + a)*A*a^5*b^2*d^6*g^3/(d*x + c) + 24*(b*x
+ a)*B*a^5*b^2*d^6*g^3/(d*x + c) + 180*(b*x + a)^2*A*a^4*b^2*c*d^6*g^3/(d*x
 + c)^2 + 180*(b*x + a)^2*B*a^4*b^2*c*d^6*g^3/(d*x + c)^2 + 240*(b*x + a)^3
*A*a^3*b^2*c^2*d^6*g^3/(d*x + c)^3 + 240*(b*x + a)^3*B*a^3*b^2*c^2*d^6*g^3/
(d*x + c)^3 - 36*(b*x + a)^2*A*a^5*b*d^7*g^3/(d*x + c)^2 - 36*(b*x + a)^2*B
*a^5*b*d^7*g^3/(d*x + c)^2 - 120*(b*x + a)^3*A*a^4*b*c*d^7*g^3/(d*x + c)^3
- 120*(b*x + a)^3*B*a^4*b*c*d^7*g^3/(d*x + c)^3 + 24*(b*x + a)^3*A*a^5*d^8*
g^3/(d*x + c)^3 + 24*(b*x + a)^3*B*a^5*d^8*g^3/(d*x + c)^3)/(b^4*d^4 - 4*(b
*x + a)*b^3*d^5/(d*x + c) + 6*(b*x + a)^2*b^2*d^6/(d*x + c)^2 - 4*(b*x + a)
^3*b*d^7/(d*x + c)^3 + (b*x + a)^4*d^8/(d*x + c)^4) + 6*(B*b^5*c^5*g^3*n -
5*B*a*b^4*c^4*d*g^3*n + 10*B*a^2*b^3*c^3*d^2*g^3*n - 10*B*a^3*b^2*c^2*d^3*g
^3*n + 5*B*a^4*b*c*d^4*g^3*n - B*a^5*d^5*g^3*n)*log(-b + (b*x + a)*d/(d*x +
 c))/(b*d^4) - 6*(B*b^5*c^5*g^3*n - 5*B*a*b^4*c^4*d*g^3*n + 10*B*a^2*b^3*c^
3*d^2*g^3*n - 10*B*a^3*b^2*c^2*d^3*g^3*n + 5*B*a^4*b*c*d^4*g^3*n - B*a^5*d^
5*g^3*n)*log((b*x + a)/(d*x + c))/(b*d^4))*(b*c/(b*c - a*d)^2 - a*d/(b*c -
a*d)^2)
```

**maple [F]** time = 0.26, size = 0, normalized size = 0.00

$$\int \left(bgx+ag\right)^{3} \left(B\ln\left(e\left(\frac{bx+a}{dx+c}\right)^{n}\right)+A\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*g*x+a*g)^3*(B*ln(e*((b*x+a)/(d*x+c))^n)+A),x)
```

```
[Out] int((b*g*x+a*g)^3*(B*ln(e*((b*x+a)/(d*x+c))^n)+A),x)
```

**maxima [B]** time = 1.41, size = 479, normalized size = 3.07

$$\frac{1}{4}Bb^{3}g^{3}x^{4}\log\left(e\left(\frac{bx}{dx+c}+\frac{a}{dx+c}\right)^{n}\right)+\frac{1}{4}Ab^{3}g^{3}x^{4}+Bab^{2}g^{3}x^{3}\log\left(e\left(\frac{bx}{dx+c}+\frac{a}{dx+c}\right)^{n}\right)+Aab^{2}g^{3}x^{3}+\frac{3}{2}Ba^{2}bg^{3}x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*g*x+a*g)^3*(A+B*log(e*((b*x+a)/(d*x+c))^n)),x, algorithm="maxi
ma")
```

```
[Out] 1/4*B*b^3*g^3*x^4*log(e*(b*x/(d*x + c) + a/(d*x + c))^n) + 1/4*A*b^3*g^3*x^
4 + B*a*b^2*g^3*x^3*log(e*(b*x/(d*x + c) + a/(d*x + c))^n) + A*a*b^2*g^3*x^
3 + 3/2*B*a^2*b*g^3*x^2*log(e*(b*x/(d*x + c) + a/(d*x + c))^n) + 3/2*A*a^2*
b*g^3*x^2 - 1/24*B*b^3*g^3*n*(6*a^4*log(b*x + a)/b^4 - 6*c^4*log(d*x + c)/d
^4 + (2*(b^3*c*d^2 - a*b^2*d^3)*x^3 - 3*(b^3*c^2*d - a^2*b*d^3)*x^2 + 6*(b^
3*c^3 - a^3*d^3)*x)/(b^3*d^3)) + 1/2*B*a*b^2*g^3*n*(2*a^3*log(b*x + a)/b^3
- 2*c^3*log(d*x + c)/d^3 - ((b^2*c*d - a*b*d^2)*x^2 - 2*(b^2*c^2 - a^2*d^2)
*x)/(b^2*d^2)) - 3/2*B*a^2*b*g^3*n*(a^2*log(b*x + a)/b^2 - c^2*log(d*x + c)
/d^2 + (b*c - a*d)*x/(b*d)) + B*a^3*g^3*n*(a*log(b*x + a)/b - c*log(d*x + c
)/d) + B*a^3*g^3*x*log(e*(b*x/(d*x + c) + a/(d*x + c))^n) + A*a^3*g^3*x
```

**mupad [B]** time = 4.40, size = 588, normalized size = 3.77

$$x^{3}\left(\frac{b^{2}g^{3}\left(16\,A\,a\,d+4\,A\,b\,c+B\,a\,d\,n-B\,b\,c\,n\right)}{12\,d}-\frac{A\,b^{2}\,g^{3}\,\left(4\,a\,d+4\,b\,c\right)}{12\,d}\right)-x^{2}\left(\frac{\left(\frac{b^{2}g^{3}\left(16\,A\,a\,d+4\,A\,b\,c+B\,a\,d\,n-B\,b\,c\,n\right)}{4\,d}-\right.}{8\,b}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*g + b*g*x)^3*(A + B*log(e*((a + b*x)/(c + d*x))^n)),x)
```

```
[Out] x^3*((b^2*g^3*(16*A*a*d + 4*A*b*c + B*a*d*n - B*b*c*n))/(12*d) - (A*b^2*g^3
*(4*a*d + 4*b*c))/(12*d)) - x^2*((((b^2*g^3*(16*A*a*d + 4*A*b*c + B*a*d*n -
 B*b*c*n))/(4*d) - (A*b^2*g^3*(4*a*d + 4*b*c))/(4*d))*(4*a*d + 4*b*c))/(8*b
*d) - (a*b*g^3*(6*A*a*d + 4*A*b*c + B*a*d*n - B*b*c*n))/(2*d) + (A*a*b^2*c*
g^3)/(2*d)) + log(e*((a + b*x)/(c + d*x))^n)*((B*b^3*g^3*x^4)/4 + B*a^3*g^3
*x + (3*B*a^2*b*g^3*x^2)/2 + B*a*b^2*g^3*x^3) + x*(((4*a*d + 4*b*c)*((((b^2
*g^3*(16*A*a*d + 4*A*b*c + B*a*d*n - B*b*c*n))/(4*d) - (A*b^2*g^3*(4*a*d +
4*b*c))/(4*d))*(4*a*d + 4*b*c))/(4*b*d) - (a*b*g^3*(6*A*a*d + 4*A*b*c + B*a
*d*n - B*b*c*n))/d + (A*a*b^2*c*g^3)/d))/(4*b*d) + (a^2*g^3*(8*A*a*d + 12*A
*b*c + 3*B*a*d*n - 3*B*b*c*n))/(2*d) - (a*c*((b^2*g^3*(16*A*a*d + 4*A*b*c +
 B*a*d*n - B*b*c*n))/(4*d) - (A*b^2*g^3*(4*a*d + 4*b*c))/(4*d)))/(b*d)) + (
log(c + d*x)*(B*b^3*c^4*g^3*n - 4*B*a^3*c*d^3*g^3*n - 4*B*a*b^2*c^3*d*g^3*n
 + 6*B*a^2*b*c^2*d^2*g^3*n))/(4*d^4) + (A*b^3*g^3*x^4)/4 + (B*a^4*g^3*n*log
(a + b*x))/(4*b)
```

**sympy [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*g*x+a*g)**3*(A+B*ln(e*((b*x+a)/(d*x+c))**n)),x)
```

```
[Out] Timed out
```

## 3.3 $\int (ag+bgx)^2 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right) dx$

**Optimal**. Leaf size=124

$$\frac{g^2(a+bx)^3\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{3b}-\frac{Bg^2n(bc-ad)^3\log(c+dx)}{3bd^3}+\frac{Bg^2nx(bc-ad)^2}{3d^2}-\frac{Bg^2n(a+bx)^2(bc-ad)}{6bd}$$

```
[Out] 1/3*B*(-a*d+b*c)^2*g^2*n*x/d^2-1/6*B*(-a*d+b*c)*g^2*n*(b*x+a)^2/b/d+1/3*g^2
*(b*x+a)^3*(A+B*ln(e*((b*x+a)/(d*x+c))^n))/b-1/3*B*(-a*d+b*c)^3*g^2*n*ln(d*
x+c)/b/d^3
```

**Rubi [A]** time = 0.09, antiderivative size = 124, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {2525, 12, 43}

$$\frac{g^2(a+bx)^3\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{3b}+\frac{Bg^2nx(bc-ad)^2}{3d^2}-\frac{Bg^2n(bc-ad)^3\log(c+dx)}{3bd^3}-\frac{Bg^2n(a+bx)^2(bc-ad)}{6bd}$$

Antiderivative was successfully verified.

```
[In] Int[(a*g + b*g*x)^2*(A + B*Log[e*((a + b*x)/(c + d*x))^n]),x]
```

```
[Out] (B*(b*c - a*d)^2*g^2*n*x)/(3*d^2) - (B*(b*c - a*d)*g^2*n*(a + b*x)^2)/(6*b*
d) + (g^2*(a + b*x)^3*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/(3*b) - (B*(b
*c - a*d)^3*g^2*n*Log[c + d*x])/(3*b*d^3)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2525

```
Int[((a_.) + Log[(c_.)*(RFx_)^(p_.)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_))^(m_.
), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(a + b*Log[c*RFx^p])^n)/(e*(m + 1))
, x] - Dist[(b*n*p)/(e*(m + 1)), Int[SimplifyIntegrand[((d + e*x)^(m + 1)*(
```

```
a + b*Log[c*RFx^p])^(n - 1)*D[RFx, x])/RFx, x], x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && RationalFunctionQ[RFx, x] && IGtQ[n, 0] && (EqQ[n, 1] ||
IntegerQ[m]) && NeQ[m, -1]
```

Rubi steps

$$\int (ag+bgx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)dx = \frac{g^2(a+bx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{3b} - \frac{(Bn)\int\frac{(bc-ad)g^3(a+bx)^2}{c+dx}dx}{3bg}$$

$$= \frac{g^2(a+bx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{3b} - \frac{\left(B(bc-ad)g^2n\right)\int\frac{(a+bx)^2}{c+dx}d}{3b}$$

$$= \frac{g^2(a+bx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{3b} - \frac{\left(B(bc-ad)g^2n\right)\int\left(-\frac{b(bc-a}{d^2}\right.}{3}$$

$$= \frac{B(bc-ad)^2g^2nx}{3d^2} - \frac{B(bc-ad)g^2n(a+bx)^2}{6bd} + \frac{g^2(a+bx)^3\left(A+B\right.}{3b}$$

**Mathematica [A]** time = 0.06, size = 103, normalized size = 0.83

$$\frac{g^2\left(\frac{Bn(ad-bc)\left(d\left(a^2d+4abdx+b^2x(dx-2c)\right)+2(bc-ad)^2\log(c+dx)\right)}{2d^3}+(a+bx)^3\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)\right)}{3b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*g + b*g*x)^2*(A + B*Log[e*((a + b*x)/(c + d*x))^n]),x]
```

```
[Out] (g^2*((a + b*x)^3*(A + B*Log[e*((a + b*x)/(c + d*x))^n]) + (B*(-(b*c) + a*d
)*n*(d*(a^2*d + 4*a*b*d*x + b^2*x*(-2*c + d*x)) + 2*(b*c - a*d)^2*Log[c + d
*x]))/(2*d^3)))/(3*b)
```

**fricas [B]** time = 0.77, size = 296, normalized size = 2.39

$$\frac{2\,Ab^3d^3g^2x^3+2\,Ba^3d^3g^2n\log(bx+a)-2\left(Bb^3c^3-3\,Bab^2c^2d+3\,Ba^2bcd^2\right)g^2n\log(dx+c)+\left(6\,Aab^2d^3g^2-(Bl\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*g*x+a*g)^2*(A+B*log(e*((b*x+a)/(d*x+c))^n)),x, algorithm="fric
as")
```

```
[Out] 1/6*(2*A*b^3*d^3*g^2*x^3 + 2*B*a^3*d^3*g^2*n*log(b*x + a) - 2*(B*b^3*c^3 -
3*B*a*b^2*c^2*d + 3*B*a^2*b*c*d^2)*g^2*n*log(d*x + c) + (6*A*a*b^2*d^3*g^2
- (B*b^3*c*d^2 - B*a*b^2*d^3)*g^2*n)*x^2 + 2*(3*A*a^2*b*d^3*g^2 + (B*b^3*c^
2*d - 3*B*a*b^2*c*d^2 + 2*B*a^2*b*d^3)*g^2*n)*x + 2*(B*b^3*d^3*g^2*x^3 + 3*
B*a*b^2*d^3*g^2*x^2 + 3*B*a^2*b*d^3*g^2*x)*log(e) + 2*(B*b^3*d^3*g^2*n*x^3
+ 3*B*a*b^2*d^3*g^2*n*x^2 + 3*B*a^2*b*d^3*g^2*n*x)*log((b*x + a)/(d*x + c))
)/(b*d^3)
```

**giac** [B] time = 2.29, size = 1836, normalized size = 14.81

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*g*x+a*g)^2*(A+B*log(e*((b*x+a)/(d*x+c))^n)),x, algorithm="giac
")
```

```
[Out] 1/6*(2*(B*b^6*c^4*g^2*n - 4*B*a*b^5*c^3*d*g^2*n - 3*(b*x + a)*B*b^5*c^4*d*g
^2*n/(d*x + c) + 6*B*a^2*b^4*c^2*d^2*g^2*n + 12*(b*x + a)*B*a*b^4*c^3*d^2*g
^2*n/(d*x + c) + 3*(b*x + a)^2*B*b^4*c^4*d^2*g^2*n/(d*x + c)^2 - 4*B*a^3*b^
3*c*d^3*g^2*n - 18*(b*x + a)*B*a^2*b^3*c^2*d^3*g^2*n/(d*x + c) - 12*(b*x +
a)^2*B*a*b^3*c^3*d^3*g^2*n/(d*x + c)^2 + B*a^4*b^2*d^4*g^2*n + 12*(b*x + a)
*B*a^3*b^2*c*d^4*g^2*n/(d*x + c) + 18*(b*x + a)^2*B*a^2*b^2*c^2*d^4*g^2*n/(
d*x + c)^2 - 3*(b*x + a)*B*a^4*b*d^5*g^2*n/(d*x + c) - 12*(b*x + a)^2*B*a^3
*b*c*d^5*g^2*n/(d*x + c)^2 + 3*(b*x + a)^2*B*a^4*d^6*g^2*n/(d*x + c)^2)*log
((b*x + a)/(d*x + c))/(b^3*d^3 - 3*(b*x + a)*b^2*d^4/(d*x + c) + 3*(b*x + a
)^2*b*d^5/(d*x + c)^2 - (b*x + a)^3*d^6/(d*x + c)^3) + (3*B*b^6*c^4*g^2*n -
 12*B*a*b^5*c^3*d*g^2*n - 7*(b*x + a)*B*b^5*c^4*d*g^2*n/(d*x + c) + 18*B*a^
2*b^4*c^2*d^2*g^2*n + 28*(b*x + a)*B*a*b^4*c^3*d^2*g^2*n/(d*x + c) + 4*(b*x
 + a)^2*B*b^4*c^4*d^2*g^2*n/(d*x + c)^2 - 12*B*a^3*b^3*c*d^3*g^2*n - 42*(b*
x + a)*B*a^2*b^3*c^2*d^3*g^2*n/(d*x + c) - 16*(b*x + a)^2*B*a*b^3*c^3*d^3*g
^2*n/(d*x + c)^2 + 3*B*a^4*b^2*d^4*g^2*n + 28*(b*x + a)*B*a^3*b^2*c*d^4*g^2
*n/(d*x + c) + 24*(b*x + a)^2*B*a^2*b^2*c^2*d^4*g^2*n/(d*x + c)^2 - 7*(b*x
+ a)*B*a^4*b*d^5*g^2*n/(d*x + c) - 16*(b*x + a)^2*B*a^3*b*c*d^5*g^2*n/(d*x
+ c)^2 + 4*(b*x + a)^2*B*a^4*d^6*g^2*n/(d*x + c)^2 + 2*A*b^6*c^4*g^2 + 2*B*
b^6*c^4*g^2 - 8*A*a*b^5*c^3*d*g^2 - 8*B*a*b^5*c^3*d*g^2 - 6*(b*x + a)*A*b^5
*c^4*d*g^2/(d*x + c) - 6*(b*x + a)*B*b^5*c^4*d*g^2/(d*x + c) + 12*A*a^2*b^4
*c^2*d^2*g^2 + 12*B*a^2*b^4*c^2*d^2*g^2 + 24*(b*x + a)*A*a*b^4*c^3*d^2*g^2/
(d*x + c) + 24*(b*x + a)*B*a*b^4*c^3*d^2*g^2/(d*x + c) + 6*(b*x + a)^2*A*b^
4*c^4*d^2*g^2/(d*x + c)^2 + 6*(b*x + a)^2*B*b^4*c^4*d^2*g^2/(d*x + c)^2 - 8
*A*a^3*b^3*c*d^3*g^2 - 8*B*a^3*b^3*c*d^3*g^2 - 36*(b*x + a)*A*a^2*b^3*c^2*d
^3*g^2/(d*x + c) - 36*(b*x + a)*B*a^2*b^3*c^2*d^3*g^2/(d*x + c) - 24*(b*x +
 a)^2*A*a*b^3*c^3*d^3*g^2/(d*x + c)^2 - 24*(b*x + a)^2*B*a*b^3*c^3*d^3*g^2/
(d*x + c)^2 + 2*A*a^4*b^2*d^4*g^2 + 2*B*a^4*b^2*d^4*g^2 + 24*(b*x + a)*A*a^
3*b^2*c*d^4*g^2/(d*x + c) + 24*(b*x + a)*B*a^3*b^2*c*d^4*g^2/(d*x + c) + 36
*(b*x + a)^2*A*a^2*b^2*c^2*d^4*g^2/(d*x + c)^2 + 36*(b*x + a)^2*B*a^2*b^2*c
```

```
^2*d^4*g^2/(d*x + c)^2 - 6*(b*x + a)*A*a^4*b*d^5*g^2/(d*x + c) - 6*(b*x + a
)*B*a^4*b*d^5*g^2/(d*x + c) - 24*(b*x + a)^2*A*a^3*b*c*d^5*g^2/(d*x + c)^2
- 24*(b*x + a)^2*B*a^3*b*c*d^5*g^2/(d*x + c)^2 + 6*(b*x + a)^2*A*a^4*d^6*g^
2/(d*x + c)^2 + 6*(b*x + a)^2*B*a^4*d^6*g^2/(d*x + c)^2)/(b^3*d^3 - 3*(b*x
+ a)*b^2*d^4/(d*x + c) + 3*(b*x + a)^2*b*d^5/(d*x + c)^2 - (b*x + a)^3*d^6/
(d*x + c)^3) + 2*(B*b^4*c^4*g^2*n - 4*B*a*b^3*c^3*d*g^2*n + 6*B*a^2*b^2*c^2
*d^2*g^2*n - 4*B*a^3*b*c*d^3*g^2*n + B*a^4*d^4*g^2*n)*log(b - (b*x + a)*d/(
d*x + c))/(b*d^3) - 2*(B*b^4*c^4*g^2*n - 4*B*a*b^3*c^3*d*g^2*n + 6*B*a^2*b^
2*c^2*d^2*g^2*n - 4*B*a^3*b*c*d^3*g^2*n + B*a^4*d^4*g^2*n)*log((b*x + a)/(d
*x + c))/(b*d^3))*(b*c/(b*c - a*d)^2 - a*d/(b*c - a*d)^2)
```

**maple [F]** time = 0.27, size = 0, normalized size = 0.00

$$\int \left(bgx+ag\right)^{2} \left(B\ln\left(e\left(\frac{bx+a}{dx+c}\right)^{n}\right)+A\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*g*x+a*g)^2*(B*ln(e*((b*x+a)/(d*x+c))^n)+A),x)
```

```
[Out] int((b*g*x+a*g)^2*(B*ln(e*((b*x+a)/(d*x+c))^n)+A),x)
```

**maxima [B]** time = 1.39, size = 309, normalized size = 2.49

$$\frac{1}{3}\,Bb^{2}g^{2}x^{3}\log\left(e\left(\frac{bx}{dx+c}+\frac{a}{dx+c}\right)^{n}\right)+\frac{1}{3}\,Ab^{2}g^{2}x^{3}+Babg^{2}x^{2}\log\left(e\left(\frac{bx}{dx+c}+\frac{a}{dx+c}\right)^{n}\right)+Aabg^{2}x^{2}+\frac{1}{6}\,Bb^{2}g^{2}n\Bigg(\frac{2\,a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*g*x+a*g)^2*(A+B*log(e*((b*x+a)/(d*x+c))^n)),x, algorithm="maxi
ma")
```

```
[Out] 1/3*B*b^2*g^2*x^3*log(e*(b*x/(d*x + c) + a/(d*x + c))^n) + 1/3*A*b^2*g^2*x^
3 + B*a*b*g^2*x^2*log(e*(b*x/(d*x + c) + a/(d*x + c))^n) + A*a*b*g^2*x^2 +
1/6*B*b^2*g^2*n*(2*a^3*log(b*x + a)/b^3 - 2*c^3*log(d*x + c)/d^3 - ((b^2*c*
d - a*b*d^2)*x^2 - 2*(b^2*c^2 - a^2*d^2)*x)/(b^2*d^2)) - B*a*b*g^2*n*(a^2*l
og(b*x + a)/b^2 - c^2*log(d*x + c)/d^2 + (b*c - a*d)*x/(b*d)) + B*a^2*g^2*n
*(a*log(b*x + a)/b - c*log(d*x + c)/d) + B*a^2*g^2*x*log(e*(b*x/(d*x + c) +
 a/(d*x + c))^n) + A*a^2*g^2*x
```

**mupad [B]** time = 4.28, size = 303, normalized size = 2.44

$$\ln\left(e\,{\left(\frac{a+b\,x}{c+d\,x}\right)}^n\right)\,\left(B\,a^2\,g^2\,x+B\,a\,b\,g^2\,x^2+\frac{B\,b^2\,g^2\,x^3}{3}\right)-x\,\left(\frac{\left(3\,a\,d+3\,b\,c\right)\,\left(\frac{b\,g^2\,\left(9\,A\,a\,d+3\,A\,b\,c+B\,a\,d\,n-B\,b\,c\,n\right)}{3\,d}-\frac{A\,b\,g^2\,\left(3\,a\,d\right.}{3\,d}\right.}{3\,b\,d}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*g + b*g*x)^2*(A + B*log(e*((a + b*x)/(c + d*x))^n)),x)
```

```
[Out] log(e*((a + b*x)/(c + d*x))^n)*((B*b^2*g^2*x^3)/3 + B*a^2*g^2*x + B*a*b*g^2
*x^2) - x*(((3*a*d + 3*b*c)*((b*g^2*(9*A*a*d + 3*A*b*c + B*a*d*n - B*b*c*n)
)/(3*d) - (A*b*g^2*(3*a*d + 3*b*c))/(3*d)))/(3*b*d) - (a*g^2*(3*A*a*d + 3*A
*b*c + B*a*d*n - B*b*c*n))/d + (A*a*b*c*g^2)/d) + x^2*((b*g^2*(9*A*a*d + 3*
A*b*c + B*a*d*n - B*b*c*n))/(6*d) - (A*b*g^2*(3*a*d + 3*b*c))/(6*d)) - (log
(c + d*x)*(B*b^2*c^3*g^2*n + 3*B*a^2*c*d^2*g^2*n - 3*B*a*b*c^2*d*g^2*n))/(3
*d^3) + (A*b^2*g^2*x^3)/3 + (B*a^3*g^2*n*log(a + b*x))/(3*b)
```

**sympy [A]** time = 60.49, size = 673, normalized size = 5.43

$$\begin{cases} a^2g^2x\left(A + B\log\left(e\left(\frac{a}{c}\right)^n\right)\right) \\ Aa^2g^2x + Aabg^2x^2 + \frac{Ab^2g^2x^3}{3} + \frac{Ba^3g^2n\log\left(\frac{a}{c}+\frac{bx}{c}\right)}{3b} + Ba^2g^2nx\log\left(\frac{a}{c} + \frac{bx}{c}\right) - \frac{Ba^2g^2nx}{3} + Ba^2g^2x\log(e) + Babg^2nx^2 \\ a^2g^2\left(Ax - \frac{Bcn\log(c+dx)}{d} + Bnx\log(a) - Bnx\log(c + dx) + Bnx + Bx\log(e)\right) \\ Aa^2g^2x + Aabg^2x^2 + \frac{Ab^2g^2x^3}{3} + \frac{Ba^3g^2n\log\left(\frac{a}{c+dx}+\frac{bx}{c+dx}\right)}{3b} + \frac{Ba^3g^2n\log\left(\frac{c}{d}+x\right)}{3b} - \frac{Ba^2cg^2n\log\left(\frac{c}{d}+x\right)}{d} + Ba^2g^2nx\log\left(\frac{a}{c+dx} + \frac{}{c}\right. \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*g*x+a*g)**2*(A+B*ln(e*((b*x+a)/(d*x+c))**n)),x)
```

```
[Out] Piecewise((a**2*g**2*x*(A + B*log(e*(a/c)**n)), Eq(b, 0) & Eq(d, 0)), (A*a*
*2*g**2*x + A*a*b*g**2*x**2 + A*b**2*g**2*x**3/3 + B*a**3*g**2*n*log(a/c +
b*x/c)/(3*b) + B*a**2*g**2*n*x*log(a/c + b*x/c) - B*a**2*g**2*n*x/3 + B*a**
2*g**2*x*log(e) + B*a*b*g**2*n*x**2*log(a/c + b*x/c) - B*a*b*g**2*n*x**2/3
+ B*a*b*g**2*x**2*log(e) + B*b**2*g**2*n*x**3*log(a/c + b*x/c)/3 - B*b**2*g
**2*n*x**3/9 + B*b**2*g**2*x**3*log(e)/3, Eq(d, 0)), (a**2*g**2*(A*x - B*c*
n*log(c + d*x)/d + B*n*x*log(a) - B*n*x*log(c + d*x) + B*n*x + B*x*log(e)),
 Eq(b, 0)), (A*a**2*g**2*x + A*a*b*g**2*x**2 + A*b**2*g**2*x**3/3 + B*a**3*
g**2*n*log(a/(c + d*x) + b*x/(c + d*x))/(3*b) + B*a**3*g**2*n*log(c/d + x)/
(3*b) - B*a**2*c*g**2*n*log(c/d + x)/d + B*a**2*g**2*n*x*log(a/(c + d*x) +
b*x/(c + d*x)) + 2*B*a**2*g**2*n*x/3 + B*a**2*g**2*x*log(e) + B*a*b*c**2*g*
*2*n*log(c/d + x)/d**2 - B*a*b*c*g**2*n*x/d + B*a*b*g**2*n*x**2*log(a/(c +
d*x) + b*x/(c + d*x)) + B*a*b*g**2*n*x**2/6 + B*a*b*g**2*x**2*log(e) - B*b*
*2*c**3*g**2*n*log(c/d + x)/(3*d**3) + B*b**2*c**2*g**2*n*x/(3*d**2) - B*b*
*2*c*g**2*n*x**2/(6*d) + B*b**2*g**2*n*x**3*log(a/(c + d*x) + b*x/(c + d*x)
)/3 + B*b**2*g**2*x**3*log(e)/3, True))
```

# 3.4 $\int (ag+bgx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)dx$

**Optimal**. Leaf size=86

$$\frac{g(a+bx)^2\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{2b}+\frac{Bgn(bc-ad)^2\log(c+dx)}{2bd^2}-\frac{Bgnx(bc-ad)}{2d}$$

```
[Out] -1/2*B*(-a*d+b*c)*g*n*x/d+1/2*g*(b*x+a)^2*(A+B*ln(e*((b*x+a)/(d*x+c))^n))/b
+1/2*B*(-a*d+b*c)^2*g*n*ln(d*x+c)/b/d^2
```

**Rubi [A]** time = 0.06, antiderivative size = 86, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.097, Rules used = {2525, 12, 43}

$$\frac{g(a+bx)^2\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{2b}+\frac{Bgn(bc-ad)^2\log(c+dx)}{2bd^2}-\frac{Bgnx(bc-ad)}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[(a*g + b*g*x)*(A + B*Log[e*((a + b*x)/(c + d*x))^n]),x]
```

```
[Out] -(B*(b*c - a*d)*g*n*x)/(2*d) + (g*(a + b*x)^2*(A + B*Log[e*((a + b*x)/(c +
d*x))^n]))/(2*b) + (B*(b*c - a*d)^2*g*n*Log[c + d*x])/(2*b*d^2)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2525

```
Int[((a_.) + Log[(c_.)*(RFx_)^(p_.)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_))^(m_.
), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(a + b*Log[c*RFx^p])^n)/(e*(m + 1))
, x] - Dist[(b*n*p)/(e*(m + 1)), Int[SimplifyIntegrand[((d + e*x)^(m + 1)*(
a + b*Log[c*RFx^p])^(n - 1)*D[RFx, x])/RFx, x], x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && RationalFunctionQ[RFx, x] && IGtQ[n, 0] && (EqQ[n, 1] ||
```

```
IntegerQ[m]) && NeQ[m, -1]
```

Rubi steps

$$\int (ag+bgx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)dx=\frac{g(a+bx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{2b}-\frac{(Bn)\int\frac{(bc-ad)g^2(a+bx)}{c+dx}\,dx}{2bg}$$
$$=\frac{g(a+bx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{2b}-\frac{(B(bc-ad)gn)\int\frac{a+bx}{c+dx}\,dx}{2b}$$
$$=\frac{g(a+bx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{2b}-\frac{(B(bc-ad)gn)\int\left(\frac{b}{d}+\frac{-bc+a}{d(c+d}\right.}{2b}$$
$$=-\frac{B(bc-ad)gnx}{2d}+\frac{g(a+bx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{2b}+\frac{B(bc-a}{}$$

**Mathematica [A]** time = 0.04, size = 73, normalized size = 0.85

$$\frac{g\left((a+bx)^2\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)+\frac{Bn(ad-bc)((ad-bc)\log(c+dx)+bdx)}{d^2}\right)}{2b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*g + b*g*x)*(A + B*Log[e*((a + b*x)/(c + d*x))^n]),x]
```

```
[Out] (g*((a + b*x)^2*(A + B*Log[e*((a + b*x)/(c + d*x))^n]) + (B*(-(b*c) + a*d)*
n*(b*d*x + (-(b*c) + a*d)*Log[c + d*x]))/d^2))/(2*b)
```

**fricas [A]** time = 0.93, size = 160, normalized size = 1.86

$$\frac{Ab^2d^2gx^2+Ba^2d^2gn\log(bx+a)+\left(Bb^2c^2-2\,Babcd\right)gn\log(dx+c)+\left(2\,Aabd^2g-\left(Bb^2cd-Babd^2\right)gn\right)x+\left(\right.}{2\,bd^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*g*x+a*g)*(A+B*log(e*((b*x+a)/(d*x+c))^n)),x, algorithm="fricas
")
```

```
[Out] 1/2*(A*b^2*d^2*g*x^2 + B*a^2*d^2*g*n*log(b*x + a) + (B*b^2*c^2 - 2*B*a*b*c*
d)*g*n*log(d*x + c) + (2*A*a*b*d^2*g - (B*b^2*c*d - B*a*b*d^2)*g*n)*x + (B*
```

```
b^2*d^2*g*x^2 + 2*B*a*b*d^2*g*x)*log(e) + (B*b^2*d^2*g*n*x^2 + 2*B*a*b*d^2*
g*n*x)*log((b*x + a)/(d*x + c)))/(b*d^2)
```

**giac [B]** time = 1.24, size = 864, normalized size = 10.05

$$-\frac{1}{2}\left(\frac{\left(Bb^4c^3gn - 3\,Bab^3c^2dgn - \frac{2\,(bx+a)Bb^3c^3dgn}{dx+c} + 3\,Ba^2b^2cd^2gn + \frac{6\,(bx+a)Bab^2c^2d^2gn}{dx+c} - Ba^3bd^3gn - \frac{6\,(bx+a)Ba^2bcd^3gn}{dx+c} + \right.}{b^2d^2 - \frac{2\,(bx+a)bd^3}{dx+c} + \frac{(bx+a)^2d^4}{(dx+c)^2}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*g*x+a*g)*(A+B*log(e*((b*x+a)/(d*x+c))^n)),x, algorithm="giac")
```

```
[Out] -1/2*((B*b^4*c^3*g*n - 3*B*a*b^3*c^2*d*g*n - 2*(b*x + a)*B*b^3*c^3*d*g*n/(d
*x + c) + 3*B*a^2*b^2*c*d^2*g*n + 6*(b*x + a)*B*a*b^2*c^2*d^2*g*n/(d*x + c)
 - B*a^3*b*d^3*g*n - 6*(b*x + a)*B*a^2*b*c*d^3*g*n/(d*x + c) + 2*(b*x + a)*
B*a^3*d^4*g*n/(d*x + c))*log((b*x + a)/(d*x + c))/(b^2*d^2 - 2*(b*x + a)*b*
d^3/(d*x + c) + (b*x + a)^2*d^4/(d*x + c)^2) + (B*b^4*c^3*g*n - 3*B*a*b^3*c
^2*d*g*n - (b*x + a)*B*b^3*c^3*d*g*n/(d*x + c) + 3*B*a^2*b^2*c*d^2*g*n + 3*
(b*x + a)*B*a*b^2*c^2*d^2*g*n/(d*x + c) - B*a^3*b*d^3*g*n - 3*(b*x + a)*B*a
^2*b*c*d^3*g*n/(d*x + c) + (b*x + a)*B*a^3*d^4*g*n/(d*x + c) + A*b^4*c^3*g
+ B*b^4*c^3*g - 3*A*a*b^3*c^2*d*g - 3*B*a*b^3*c^2*d*g - 2*(b*x + a)*A*b^3*c
^3*d*g/(d*x + c) - 2*(b*x + a)*B*b^3*c^3*d*g/(d*x + c) + 3*A*a^2*b^2*c*d^2*
g + 3*B*a^2*b^2*c*d^2*g + 6*(b*x + a)*A*a*b^2*c^2*d^2*g/(d*x + c) + 6*(b*x
+ a)*B*a*b^2*c^2*d^2*g/(d*x + c) - A*a^3*b*d^3*g - B*a^3*b*d^3*g - 6*(b*x +
 a)*A*a^2*b*c*d^3*g/(d*x + c) - 6*(b*x + a)*B*a^2*b*c*d^3*g/(d*x + c) + 2*(
b*x + a)*A*a^3*d^4*g/(d*x + c) + 2*(b*x + a)*B*a^3*d^4*g/(d*x + c))/(b^2*d^
2 - 2*(b*x + a)*b*d^3/(d*x + c) + (b*x + a)^2*d^4/(d*x + c)^2) + (B*b^3*c^3
*g*n - 3*B*a*b^2*c^2*d*g*n + 3*B*a^2*b*c*d^2*g*n - B*a^3*d^3*g*n)*log(-b +
(b*x + a)*d/(d*x + c))/(b*d^2) - (B*b^3*c^3*g*n - 3*B*a*b^2*c^2*d*g*n + 3*B
*a^2*b*c*d^2*g*n - B*a^3*d^3*g*n)*log((b*x + a)/(d*x + c))/(b*d^2))*(b*c/(b
*c - a*d)^2 - a*d/(b*c - a*d)^2)
```

**maple [F]** time = 0.18, size = 0, normalized size = 0.00

$$\int \left(bgx + ag\right)\left(B\ln\left(e\left(\frac{bx+a}{dx+c}\right)^{n}\right) + A\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*g*x+a*g)*(B*ln(e*((b*x+a)/(d*x+c))^n)+A),x)
```

```
[Out] int((b*g*x+a*g)*(B*ln(e*((b*x+a)/(d*x+c))^n)+A),x)
```

**maxima [A]** time = 1.35, size = 156, normalized size = 1.81

$$\frac{1}{2}\,Bbgx^2\log\left(e\left(\frac{bx}{dx+c} + \frac{a}{dx+c}\right)^{n}\right) + \frac{1}{2}\,Abgx^2 - \frac{1}{2}\,Bbgn\left(\frac{a^2\log(bx+a)}{b^2} - \frac{c^2\log(dx+c)}{d^2} + \frac{(bc-ad)x}{bd}\right) + Bagn\left(\frac{a}{\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*g*x+a*g)*(A+B*log(e*((b*x+a)/(d*x+c))^n)),x, algorithm="maxima
")
```

```
[Out] 1/2*B*b*g*x^2*log(e*(b*x/(d*x + c) + a/(d*x + c))^n) + 1/2*A*b*g*x^2 - 1/2*
B*b*g*n*(a^2*log(b*x + a)/b^2 - c^2*log(d*x + c)/d^2 + (b*c - a*d)*x/(b*d))
 + B*a*g*n*(a*log(b*x + a)/b - c*log(d*x + c)/d) + B*a*g*x*log(e*(b*x/(d*x
+ c) + a/(d*x + c))^n) + A*a*g*x
```

**mupad [B]** time = 4.07, size = 134, normalized size = 1.56

$$x\left(\frac{g\,(4\,A\,a\,d+2\,A\,b\,c+B\,a\,d\,n-B\,b\,c\,n)}{2\,d}-\frac{A\,g\,(2\,a\,d+2\,b\,c)}{2\,d}\right)+\ln\left(e\left(\frac{a+b\,x}{c+d\,x}\right)^n\right)\left(\frac{B\,b\,g\,x^2}{2}+B\,a\,g\,x\right)+\frac{\ln(c+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*g + b*g*x)*(A + B*log(e*((a + b*x)/(c + d*x))^n)),x)
```

```
[Out] x*((g*(4*A*a*d + 2*A*b*c + B*a*d*n - B*b*c*n))/(2*d) - (A*g*(2*a*d + 2*b*c)
)/(2*d)) + log(e*((a + b*x)/(c + d*x))^n)*((B*b*g*x^2)/2 + B*a*g*x) + (log(
c + d*x)*(B*b*c^2*g*n - 2*B*a*c*d*g*n))/(2*d^2) + (A*b*g*x^2)/2 + (B*a^2*g*
n*log(a + b*x))/(2*b)
```

**sympy [A]** time = 40.67, size = 398, normalized size = 4.63

$$\begin{cases} agx\left(A+B\log\left(e\left(\frac{a}{c}\right)^n\right)\right) \\ ag\left(Ax-\frac{Bcn\log(c+dx)}{d}+Bnx\log(a)-Bnx\log(c+dx)+Bnx+Bx\log(e)\right) \\ Aagx+\frac{Abgx^2}{2}+\frac{Ba^2gn\log\left(\frac{a}{c}+\frac{bx}{c}\right)}{2b}+Bagnx\log\left(\frac{a}{c}+\frac{bx}{c}\right)-\frac{Bagnx}{2}+Bagx\log(e)+\frac{Bbgnx^2\log\left(\frac{a}{c}+\frac{bx}{c}\right)}{2}-\frac{Bbgnx^2}{4}+\frac{Bbgx^2 \\ Aagx+\frac{Abgx^2}{2}+\frac{Ba^2gn\log\left(\frac{a}{c+dx}+\frac{bx}{c+dx}\right)}{2b}+\frac{Ba^2gn\log\left(\frac{c}{d}+x\right)}{2b}-\frac{Bacgn\log\left(\frac{c}{d}+x\right)}{d}+Bagnx\log\left(\frac{a}{c+dx}+\frac{bx}{c+dx}\right)+\frac{Bagnx}{2}+Bagx \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*g*x+a*g)*(A+B*ln(e*((b*x+a)/(d*x+c))**n)),x)
```

```
[Out] Piecewise((a*g*x*(A + B*log(e*(a/c)**n)), Eq(b, 0) & Eq(d, 0)), (a*g*(A*x -
 B*c*n*log(c + d*x)/d + B*n*x*log(a) - B*n*x*log(c + d*x) + B*n*x + B*x*log
(e)), Eq(b, 0)), (A*a*g*x + A*b*g*x**2/2 + B*a**2*g*n*log(a/c + b*x/c)/(2*b
) + B*a*g*n*x*log(a/c + b*x/c) - B*a*g*n*x/2 + B*a*g*x*log(e) + B*b*g*n*x**
2*log(a/c + b*x/c)/2 - B*b*g*n*x**2/4 + B*b*g*x**2*log(e)/2, Eq(d, 0)), (A*
```

```
a*g*x + A*b*g*x**2/2 + B*a**2*g*n*log(a/(c + d*x) + b*x/(c + d*x))/(2*b) +
B*a**2*g*n*log(c/d + x)/(2*b) - B*a*c*g*n*log(c/d + x)/d + B*a*g*n*x*log(a/
(c + d*x) + b*x/(c + d*x)) + B*a*g*n*x/2 + B*a*g*x*log(e) + B*b*c**2*g*n*lo
g(c/d + x)/(2*d**2) - B*b*c*g*n*x/(2*d) + B*b*g*n*x**2*log(a/(c + d*x) + b*
x/(c + d*x))/2 + B*b*g*x**2*log(e)/2, True))
```

## 3.5 $\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{ag+bgx}\,dx$

**Optimal**. Leaf size=84

$$\frac{Bn\mathrm{Li}_2\left(\frac{bc-ad}{d(a+bx)}+1\right)}{bg}-\frac{\log\left(-\frac{bc-ad}{d(a+bx)}\right)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{bg}$$

```
[Out] -ln((a*d-b*c)/d/(b*x+a))*(A+B*ln(e*((b*x+a)/(d*x+c))^n))/b/g+B*n*polylog(2,
1+(-a*d+b*c)/d/(b*x+a))/b/g
```

**Rubi [A]** time = 0.21, antiderivative size = 126, normalized size of antiderivative = 1.50, number of steps used = 9, number of rules used = 8, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.242, Rules used = {2524, 2418, 2390, 12, 2301, 2394, 2393, 2391}

$$\frac{Bn\mathrm{PolyLog}\left(2,-\frac{d(a+bx)}{bc-ad}\right)}{bg}+\frac{\log(ag+bgx)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{bg}+\frac{Bn\log(ag+bgx)\log\left(\frac{b(c+dx)}{bc-ad}\right)}{bg}-\frac{Bn\log^2(g(a$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Log[e*((a + b*x)/(c + d*x))^n])/(a*g + b*g*x),x]
```

```
[Out] -(B*n*Log[g*(a + b*x)]^2)/(2*b*g) + ((A + B*Log[e*((a + b*x)/(c + d*x))^n])
*Log[a*g + b*g*x])/(b*g) + (B*n*Log[(b*(c + d*x))/(b*c - a*d)]*Log[a*g + b*
g*x])/(b*g) + (B*n*PolyLog[2, -((d*(a + b*x))/(b*c - a*d))])/(b*g)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2301

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

Rule 2390

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_))^(q_.), x_Symbol] :> Dist[1/e, Subst[Int[((f*x)/d)^q*(a + b*Log[c*x^
n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && E
qQ[e*f - d*g, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2393

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + (c*e*x)/g])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2394

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d + e*x
)^n]))/g, x] - Dist[(b*e*n)/g, Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2418

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*(RFx_), x_Sy
mbol] :> With[{u = ExpandIntegrand[(a + b*Log[c*(d + e*x)^n])^p, RFx, x]},
Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d, e, n}, x] && RationalFunctionQ[
RFx, x] && IntegerQ[p]
```

Rule 2524

```
Int[((a_.) + Log[(c_.)*(RFx_)^(p_.)]*(b_.))^(n_.)/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[(Log[d + e*x]*(a + b*Log[c*RFx^p])^n)/e, x] - Dist[(b*n*p)/e
, Int[(Log[d + e*x]*(a + b*Log[c*RFx^p])^(n - 1)*D[RFx, x])/RFx, x], x] /;
FreeQ[{a, b, c, d, e, p}, x] && RationalFunctionQ[RFx, x] && IGtQ[n, 0]
```

Rubi steps

$$\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{ag+bgx}\,dx = \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\log(ag+bgx)}{bg} - \frac{(Bn)\int\frac{(c+dx)\left(-\frac{d(a+bx)}{(c+dx)^2}+\frac{b}{c+dx}\right)\log(ag+bgx)}{a+bx}\,dx}{bg}$$

$$= \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\log(ag+bgx)}{bg} - \frac{(Bn)\int\left(\frac{b\log(ag+bgx)}{a+bx}-\frac{d\log(ag+bgx)}{c+dx}\right)dx}{bg}$$

$$= \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\log(ag+bgx)}{bg} - \frac{(Bn)\int\frac{\log(ag+bgx)}{a+bx}\,dx}{g} + \frac{(Bdn)\int\frac{\log(ag+b}{c+dx}}{bg}$$

$$= \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\log(ag+bgx)}{bg} + \frac{Bn\log\left(\frac{b(c+dx)}{bc-ad}\right)\log(ag+bgx)}{bg} - (Bn)$$

$$= \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\log(ag+bgx)}{bg} + \frac{Bn\log\left(\frac{b(c+dx)}{bc-ad}\right)\log(ag+bgx)}{bg} - \frac{(Bn)}{}$$

$$= -\frac{Bn\log^2(g(a+bx))}{2bg} + \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\log(ag+bgx)}{bg} + \frac{Bn\log\left(\frac{b(c+dx)}{bc-ad}\right.}{b}$$

**Mathematica [A]** time = 0.05, size = 101, normalized size = 1.20

$$\frac{\log(g(a+bx))\left(2\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+Bn\log\left(\frac{b(c+dx)}{bc-ad}\right)+A\right)-Bn\log(g(a+bx))\right)+2Bn\text{Li}_2\left(\frac{d(a+bx)}{ad-bc}\right)}{2bg}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Log[e*((a + b*x)/(c + d*x))^n])/(a*g + b*g*x),x]
```

```
[Out] (Log[g*(a + b*x)]*(-(B*n*Log[g*(a + b*x)]) + 2*(A + B*Log[e*((a + b*x)/(c +
 d*x))^n] + B*n*Log[(b*(c + d*x))/(b*c - a*d)])) + 2*B*n*PolyLog[2, (d*(a +
 b*x))/(-(b*c) + a*d)])/(2*b*g)
```

**fricas [F]** time = 0.84, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A}{bgx+ag},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))/(b*g*x+a*g),x, algorithm="fricas
")
```

```
[Out] integral((B*log(e*((b*x + a)/(d*x + c))^n) + A)/(b*g*x + a*g), x)
```

**giac [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))/(b*g*x+a*g),x, algorithm="giac")
```

```
[Out] Timed out
```

**maple [F]** time = 0.36, size = 0, normalized size = 0.00

$$\int \frac{B \ln \left(e \left(\frac{bx+a}{dx+c}\right)^{n}\right) + A}{bgx + ag}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*ln(e*((b*x+a)/(d*x+c))^n)+A)/(b*g*x+a*g),x)
```

```
[Out] int((B*ln(e*((b*x+a)/(d*x+c))^n)+A)/(b*g*x+a*g),x)
```

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$B\left(\frac{\log\left(bx+a\right)\log\left(\left(bx+a\right)^{n}\right) - \log\left(bx+a\right)\log\left(\left(dx+c\right)^{n}\right)}{bg} + \int \frac{bdx\log(e) + bc\log(e) - (bcn - adn)\log\left(bx+\right.}{b^{2}dgx^{2} + abcg + \left(b^{2}cg + abdg\right)x}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))/(b*g*x+a*g),x, algorithm="maxima
")
```

```
[Out] B*((log(b*x + a)*log((b*x + a)^n) - log(b*x + a)*log((d*x + c)^n))/(b*g) +
integrate((b*d*x*log(e) + b*c*log(e) - (b*c*n - a*d*n)*log(b*x + a))/(b^2*d
*g*x^2 + a*b*c*g + (b^2*c*g + a*b*d*g)*x), x)) + A*log(b*g*x + a*g)/(b*g)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{A + B\,\ln\left(e\,{\left(\frac{a+b\,x}{c+d\,x}\right)}^{n}\right)}{a\,g + b\,g\,x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*log(e*((a + b*x)/(c + d*x))^n))/(a*g + b*g*x),x)
```

```
[Out] int((A + B*log(e*((a + b*x)/(c + d*x))^n))/(a*g + b*g*x), x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{A}{a+bx}\,dx + \int \frac{B\log\left(e\left(\frac{a}{c+dx}+\frac{bx}{c+dx}\right)^{n}\right)}{a+bx}\,dx}{g}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*ln(e*((b*x+a)/(d*x+c))**n))/(b*g*x+a*g),x)
```

```
[Out] (Integral(A/(a + b*x), x) + Integral(B*log(e*(a/(c + d*x) + b*x/(c + d*x))*
*n)/(a + b*x), x))/g
```

## 3.6 $\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(ag+bgx)^2}\,dx$

**Optimal**. Leaf size=67

$$-\frac{(c+dx)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{g^2(a+bx)(bc-ad)}-\frac{Bn}{bg^2(a+bx)}$$

```
[Out] -B*n/b/g^2/(b*x+a)-(d*x+c)*(A+B*ln(e*((b*x+a)/(d*x+c))^n))/(-a*d+b*c)/g^2/(
b*x+a)
```

**Rubi [A]** time = 0.09, antiderivative size = 108, normalized size of antiderivative = 1.61, number of steps used = 4, number of rules used = 3, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {2525, 12, 44}

$$-\frac{B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A}{bg^2(a+bx)}-\frac{Bdn\log(a+bx)}{bg^2(bc-ad)}+\frac{Bdn\log(c+dx)}{bg^2(bc-ad)}-\frac{Bn}{bg^2(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Log[e*((a + b*x)/(c + d*x))^n])/(a*g + b*g*x)^2,x]
```

```
[Out] -((B*n)/(b*g^2*(a + b*x))) - (B*d*n*Log[a + b*x])/(b*(b*c - a*d)*g^2) - (A
+ B*Log[e*((a + b*x)/(c + d*x))^n])/(b*g^2*(a + b*x)) + (B*d*n*Log[c + d*x]
)/(b*(b*c - a*d)*g^2)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 2525

```
Int[((a_.) + Log[(c_.)*(RFx_)^(p_.)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_))^(m_.
), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(a + b*Log[c*RFx^p])^n)/(e*(m + 1))
```

```
, x] - Dist[(b*n*p)/(e*(m + 1)), Int[SimplifyIntegrand[((d + e*x)^(m + 1)*(
a + b*Log[c*RFx^p])^(n - 1)*D[RFx, x])/RFx, x], x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && RationalFunctionQ[RFx, x] && IGtQ[n, 0] && (EqQ[n, 1] ||
IntegerQ[m]) && NeQ[m, -1]
```

Rubi steps

$$\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(ag+bgx)^2}\,dx = -\frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{bg^2(a+bx)} + \frac{(Bn)\int \frac{bc-ad}{g(a+bx)^2(c+dx)}\,dx}{bg}$$

$$= -\frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{bg^2(a+bx)} + \frac{(B(bc-ad)n)\int \frac{1}{(a+bx)^2(c+dx)}\,dx}{bg^2}$$

$$= -\frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{bg^2(a+bx)} + \frac{(B(bc-ad)n)\int \left(\frac{b}{(bc-ad)(a+bx)^2} - \frac{bd}{(bc-ad)^2(a+bx)} + \frac{d^2}{(bc-ad)^2(}\right.}{bg^2}$$

$$= -\frac{Bn}{bg^2(a+bx)} - \frac{Bdn\log(a+bx)}{b(bc-ad)g^2} - \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{bg^2(a+bx)} + \frac{Bdn\log(c+dx)}{b(bc-ad)g^2}$$

**Mathematica [A]** time = 0.06, size = 115, normalized size = 1.72

$$\frac{Bn(bc-ad)\left(-\frac{1}{(a+bx)(bc-ad)} - \frac{d\log(a+bx)}{(bc-ad)^2} + \frac{d\log(c+dx)}{(bc-ad)^2}\right)}{bg^2} - \frac{B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A}{bg(ag+bgx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Log[e*((a + b*x)/(c + d*x))^n])/(a*g + b*g*x)^2,x]
```

```
[Out] -((A + B*Log[e*((a + b*x)/(c + d*x))^n])/(b*g*(a*g + b*g*x))) + (B*(b*c - a
*d)*n*(-(1/((b*c - a*d)*(a + b*x))) - (d*Log[a + b*x])/(b*c - a*d)^2 + (d*L
og[c + d*x])/(b*c - a*d)^2))/(b*g^2)
```

**fricas [A]** time = 0.98, size = 103, normalized size = 1.54

$$-\frac{Abc - Aad + (Bbc - Bad)n + (Bbc - Bad)\log(e) + (Bbdnx + Bbcn)\log\left(\frac{bx+a}{dx+c}\right)}{\left(b^3c - ab^2d\right)g^2x + \left(ab^2c - a^2bd\right)g^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))/(b*g*x+a*g)^2,x, algorithm="fric
as")
```

```
[Out] -(A*b*c - A*a*d + (B*b*c - B*a*d)*n + (B*b*c - B*a*d)*log(e) + (B*b*d*n*x +
 B*b*c*n)*log((b*x + a)/(d*x + c)))/((b^3*c - a*b^2*d)*g^2*x + (a*b^2*c - a
^2*b*d)*g^2)
```

**giac [A]** time = 2.91, size = 85, normalized size = 1.27

$$-\left(\frac{(dx+c)Bn\log\left(\frac{bx+a}{dx+c}\right)}{(bx+a)g^2}+\frac{(Bn+A+B)(dx+c)}{(bx+a)g^2}\right)\left(\frac{bc}{(bc-ad)^2}-\frac{ad}{(bc-ad)^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))/(b*g*x+a*g)^2,x, algorithm="giac
")
```

```
[Out] -((d*x + c)*B*n*log((b*x + a)/(d*x + c))/((b*x + a)*g^2) + (B*n + A + B)*(d
*x + c)/((b*x + a)*g^2))*(b*c/(b*c - a*d)^2 - a*d/(b*c - a*d)^2)
```

**maple [F]** time = 0.29, size = 0, normalized size = 0.00

$$\int \frac{B\ln\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A}{\left(bgx+ag\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*ln(e*((b*x+a)/(d*x+c))^n)+A)/(b*g*x+a*g)^2,x)
```

```
[Out] int((B*ln(e*((b*x+a)/(d*x+c))^n)+A)/(b*g*x+a*g)^2,x)
```

**maxima [B]** time = 1.19, size = 137, normalized size = 2.04

$$-Bn\left(\frac{1}{b^2g^2x+abg^2}+\frac{d\log\left(bx+a\right)}{\left(b^2c-abd\right)g^2}-\frac{d\log\left(dx+c\right)}{\left(b^2c-abd\right)g^2}\right)-\frac{B\log\left(e\left(\frac{bx}{dx+c}+\frac{a}{dx+c}\right)^n\right)}{b^2g^2x+abg^2}-\frac{A}{b^2g^2x+abg^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))/(b*g*x+a*g)^2,x, algorithm="maxi
ma")
```

```
[Out] -B*n*(1/(b^2*g^2*x + a*b*g^2) + d*log(b*x + a)/((b^2*c - a*b*d)*g^2) - d*lo
g(d*x + c)/((b^2*c - a*b*d)*g^2)) - B*log(e*(b*x/(d*x + c) + a/(d*x + c))^n
)/(b^2*g^2*x + a*b*g^2) - A/(b^2*g^2*x + a*b*g^2)
```

**mupad [B]** time = 5.65, size = 112, normalized size = 1.67

$$-\frac{A+B\,n}{x\,b^2\,g^2+a\,b\,g^2}-\frac{B\,\ln\left(e\,{\left(\frac{a+b\,x}{c+d\,x}\right)}^n\right)}{b\,\left(a\,g^2+b\,g^2\,x\right)}-\frac{B\,d\,n\,\mathrm{atan}\left(\frac{b\,c\,2{}\mathrm{i}+b\,d\,x\,2{}\mathrm{i}}{a\,d-b\,c}+1{}\mathrm{i}\right)\,2{}\mathrm{i}}{b\,g^2\,\left(a\,d-b\,c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*log(e*((a + b*x)/(c + d*x))^n))/(a*g + b*g*x)^2,x)
```

```
[Out] - (A + B*n)/(b^2*g^2*x + a*b*g^2) - (B*log(e*((a + b*x)/(c + d*x))^n))/(b*(
a*g^2 + b*g^2*x)) - (B*d*n*atan((b*c*2i + b*d*x*2i)/(a*d - b*c) + 1i)*2i)/(
b*g^2*(a*d - b*c))
```

**sympy [F(-2)]** time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*ln(e*((b*x+a)/(d*x+c))**n))/(b*g*x+a*g)**2,x)
```

```
[Out] Exception raised: NotImplementedError
```

## 3.7 $\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(ag+bgx)^3}\,dx$

**Optimal**. Leaf size=151

$$-\frac{B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A}{2bg^3(a+bx)^2}+\frac{Bd^2n\log(a+bx)}{2bg^3(bc-ad)^2}-\frac{Bd^2n\log(c+dx)}{2bg^3(bc-ad)^2}+\frac{Bdn}{2bg^3(a+bx)(bc-ad)}-\frac{Bn}{4bg^3(a+bx)^2}$$

```
[Out] -1/4*B*n/b/g^3/(b*x+a)^2+1/2*B*d*n/b/(-a*d+b*c)/g^3/(b*x+a)+1/2*B*d^2*n*ln(
b*x+a)/b/(-a*d+b*c)^2/g^3+1/2*(-A-B*ln(e*((b*x+a)/(d*x+c))^n))/b/g^3/(b*x+a
)^2-1/2*B*d^2*n*ln(d*x+c)/b/(-a*d+b*c)^2/g^3
```

**Rubi [A]** time = 0.12, antiderivative size = 151, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {2525, 12, 44}

$$-\frac{B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A}{2bg^3(a+bx)^2}+\frac{Bd^2n\log(a+bx)}{2bg^3(bc-ad)^2}-\frac{Bd^2n\log(c+dx)}{2bg^3(bc-ad)^2}+\frac{Bdn}{2bg^3(a+bx)(bc-ad)}-\frac{Bn}{4bg^3(a+bx)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Log[e*((a + b*x)/(c + d*x))^n])/(a*g + b*g*x)^3,x]
```

```
[Out] -(B*n)/(4*b*g^3*(a + b*x)^2) + (B*d*n)/(2*b*(b*c - a*d)*g^3*(a + b*x)) + (B
*d^2*n*Log[a + b*x])/(2*b*(b*c - a*d)^2*g^3) - (A + B*Log[e*((a + b*x)/(c +
 d*x))^n])/(2*b*g^3*(a + b*x)^2) - (B*d^2*n*Log[c + d*x])/(2*b*(b*c - a*d)^
2*g^3)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 2525

```
Int[((a_.) + Log[(c_.)*(RFx_)^(p_.)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_))^(m_.
), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(a + b*Log[c*RFx^p])^n)/(e*(m + 1))
, x] - Dist[(b*n*p)/(e*(m + 1)), Int[SimplifyIntegrand[((d + e*x)^(m + 1)*(
a + b*Log[c*RFx^p])^(n - 1)*D[RFx, x])/RFx, x], x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && RationalFunctionQ[RFx, x] && IGtQ[n, 0] && (EqQ[n, 1] ||
IntegerQ[m]) && NeQ[m, -1]
```

Rubi steps

$$\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(ag+bgx)^3}\,dx = -\frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{2bg^3(a+bx)^2}+\frac{(Bn)\int \frac{bc-ad}{g^2(a+bx)^3(c+dx)}\,dx}{2bg}$$
$$= -\frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{2bg^3(a+bx)^2}+\frac{(B(bc-ad)n)\int \frac{1}{(a+bx)^3(c+dx)}\,dx}{2bg^3}$$
$$= -\frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{2bg^3(a+bx)^2}+\frac{(B(bc-ad)n)\int \left(\frac{b}{(bc-ad)(a+bx)^3}-\frac{bd}{(bc-ad)^2(a+bx)^2}+\frac{ba}{(bc-ad)^3}\right.}{2bg^3}$$
$$= -\frac{Bn}{4bg^3(a+bx)^2}+\frac{Bdn}{2b(bc-ad)g^3(a+bx)}+\frac{Bd^2n\log(a+bx)}{2b(bc-ad)^2g^3}-\frac{A+B\log\left(e\left(\frac{a+}{c+}\right.\right.}{2bg^3(a+bx)}$$

**Mathematica [A]** time = 0.15, size = 114, normalized size = 0.75

$$-\frac{2\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)+\frac{Bn\left(2d^2(a+bx)^2\log(c+dx)+(bc-ad)(b(c-2dx)-3ad)-2d^2(a+bx)^2\log(a+bx)\right)}{(bc-ad)^2}}{4bg^3(a+bx)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Log[e*((a + b*x)/(c + d*x))^n])/(a*g + b*g*x)^3,x]
```

```
[Out] -1/4*(2*(A + B*Log[e*((a + b*x)/(c + d*x))^n]) + (B*n*((b*c - a*d)*(-3*a*d
+ b*(c - 2*d*x)) - 2*d^2*(a + b*x)^2*Log[a + b*x] + 2*d^2*(a + b*x)^2*Log[c
 + d*x]))/(b*c - a*d)^2)/(b*g^3*(a + b*x)^2)
```

**fricas [A]** time = 0.92, size = 265, normalized size = 1.75

$$-\frac{2\,Ab^2c^2-4\,Aabcd+2\,Aa^2d^2-2\left(Bb^2cd-Babd^2\right)nx+\left(Bb^2c^2-4\,Babcd+3\,Ba^2d^2\right)n+2\left(Bb^2c^2-2\,Babcd+\right.}{4\left(\left(b^5c^2-2\,ab^4cd+a^2b^3d^2\right)g^3x^2+2\left(ab^4c^2-2\,a^2b^3cd+a^3b^2d^2\right)g^3x\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))/(b*g*x+a*g)^3,x, algorithm="fric
as")
```

```
[Out] -1/4*(2*A*b^2*c^2 - 4*A*a*b*c*d + 2*A*a^2*d^2 - 2*(B*b^2*c*d - B*a*b*d^2)*n
*x + (B*b^2*c^2 - 4*B*a*b*c*d + 3*B*a^2*d^2)*n + 2*(B*b^2*c^2 - 2*B*a*b*c*d
 + B*a^2*d^2)*log(e) - 2*(B*b^2*d^2*n*x^2 + 2*B*a*b*d^2*n*x - (B*b^2*c^2 -
2*B*a*b*c*d)*n)*log((b*x + a)/(d*x + c)))/((b^5*c^2 - 2*a*b^4*c*d + a^2*b^3
*d^2)*g^3*x^2 + 2*(a*b^4*c^2 - 2*a^2*b^3*c*d + a^3*b^2*d^2)*g^3*x + (a^2*b^
3*c^2 - 2*a^3*b^2*c*d + a^4*b*d^2)*g^3)
```

**giac [A]** time = 4.80, size = 220, normalized size = 1.46

$$-\frac{1}{4}\left(\frac{2\left(Bbn-\frac{2\,(bx+a)Bdn}{dx+c}\right)\log\left(\frac{bx+a}{dx+c}\right)}{\frac{(bx+a)^2bcg^3}{(dx+c)^2}-\frac{(bx+a)^2adg^3}{(dx+c)^2}}+\frac{Bbn-\frac{4\,(bx+a)Bdn}{dx+c}+2\,Ab+2\,Bb-\frac{4\,(bx+a)Ad}{dx+c}-\frac{4\,(bx+a)Bd}{dx+c}}{\frac{(bx+a)^2bcg^3}{(dx+c)^2}-\frac{(bx+a)^2adg^3}{(dx+c)^2}}\right)\left(\frac{bc}{(bc-ad)^2}-\frac{}{(bc}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))/(b*g*x+a*g)^3,x, algorithm="giac
")
```

```
[Out] -1/4*(2*(B*b*n - 2*(b*x + a)*B*d*n/(d*x + c))*log((b*x + a)/(d*x + c))/((b*
x + a)^2*b*c*g^3/(d*x + c)^2 - (b*x + a)^2*a*d*g^3/(d*x + c)^2) + (B*b*n -
4*(b*x + a)*B*d*n/(d*x + c) + 2*A*b + 2*B*b - 4*(b*x + a)*A*d/(d*x + c) - 4
*(b*x + a)*B*d/(d*x + c))/((b*x + a)^2*b*c*g^3/(d*x + c)^2 - (b*x + a)^2*a*
d*g^3/(d*x + c)^2))*(b*c/(b*c - a*d)^2 - a*d/(b*c - a*d)^2)
```

**maple [F]** time = 0.28, size = 0, normalized size = 0.00

$$\int\frac{B\ln\left(e\left(\frac{bx+a}{dx+c}\right)^{n}\right)+A}{\left(bgx+ag\right)^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*ln(e*((b*x+a)/(d*x+c))^n)+A)/(b*g*x+a*g)^3,x)
```

```
[Out] int((B*ln(e*((b*x+a)/(d*x+c))^n)+A)/(b*g*x+a*g)^3,x)
```

**maxima [A]** time = 1.36, size = 259, normalized size = 1.72

$$\frac{1}{4}\,Bn\left(\frac{2\,bdx-bc+3\,ad}{\left(b^4c-ab^3d\right)g^3x^2+2\left(ab^3c-a^2b^2d\right)g^3x+\left(a^2b^2c-a^3bd\right)g^3}+\frac{2\,d^2\log\left(bx+a\right)}{\left(b^3c^2-2\,ab^2cd+a^2bd^2\right)g^3}-\frac{2\,d^2\log(}{\left(b^3c^2-2\,ab^2c\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))/(b*g*x+a*g)^3,x, algorithm="maxi
ma")
```

```
[Out] 1/4*B*n*((2*b*d*x - b*c + 3*a*d)/((b^4*c - a*b^3*d)*g^3*x^2 + 2*(a*b^3*c -
a^2*b^2*d)*g^3*x + (a^2*b^2*c - a^3*b*d)*g^3) + 2*d^2*log(b*x + a)/((b^3*c^
2 - 2*a*b^2*c*d + a^2*b*d^2)*g^3) - 2*d^2*log(d*x + c)/((b^3*c^2 - 2*a*b^2*
c*d + a^2*b*d^2)*g^3)) - 1/2*B*log(e*(b*x/(d*x + c) + a/(d*x + c))^n)/(b^3*
g^3*x^2 + 2*a*b^2*g^3*x + a^2*b*g^3) - 1/2*A/(b^3*g^3*x^2 + 2*a*b^2*g^3*x +
 a^2*b*g^3)
```

**mupad** [B] time = 4.52, size = 222, normalized size = 1.47

$$-\frac{\frac{2\,A\,a\,d-2\,A\,b\,c+3\,B\,a\,d\,n-B\,b\,c\,n}{2\,\left(a\,d-b\,c\right)}+\frac{B\,b\,d\,n\,x}{a\,d-b\,c}}{2\,a^2\,b\,g^3+4\,a\,b^2\,g^3\,x+2\,b^3\,g^3\,x^2}-\frac{B\,\ln\left(e\,{\left(\frac{a+b\,x}{c+d\,x}\right)}^n\right)}{2\,b\,\left(a^2\,g^3+2\,a\,b\,g^3\,x+b^2\,g^3\,x^2\right)}-\frac{B\,d^2\,n\,\mathrm{atanh}\left(\frac{2\,b^3\,c^2\,g^3-2\,a^2\,b\,d^2\,g^3}{2\,b\,g^3\,{\left(a\,d-b\,c\right)}^2}-\frac{2\,b\,d\,x}{a\,d-b\,c}\right)}{b\,g^3\,{\left(a\,d-b\,c\right)}^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*log(e*((a + b*x)/(c + d*x))^n))/(a*g + b*g*x)^3,x)
```

```
[Out] - ((2*A*a*d - 2*A*b*c + 3*B*a*d*n - B*b*c*n)/(2*(a*d - b*c)) + (B*b*d*n*x)/
(a*d - b*c))/(2*a^2*b*g^3 + 2*b^3*g^3*x^2 + 4*a*b^2*g^3*x) - (B*log(e*((a +
 b*x)/(c + d*x))^n))/(2*b*(a^2*g^3 + b^2*g^3*x^2 + 2*a*b*g^3*x)) - (B*d^2*n
*atanh((2*b^3*c^2*g^3 - 2*a^2*b*d^2*g^3)/(2*b*g^3*(a*d - b*c)^2) - (2*b*d*x
)/(a*d - b*c)))/(b*g^3*(a*d - b*c)^2)
```

**sympy** [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*ln(e*((b*x+a)/(d*x+c))**n))/(b*g*x+a*g)**3,x)
```

```
[Out] Exception raised: NotImplementedError
```

## 3.8 $\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(ag+bgx)^4}\,dx$

**Optimal**. Leaf size=183

$$-\frac{B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A}{3bg^4(a+bx)^3}-\frac{Bd^3n\log(a+bx)}{3bg^4(bc-ad)^3}+\frac{Bd^3n\log(c+dx)}{3bg^4(bc-ad)^3}-\frac{Bd^2n}{3bg^4(a+bx)(bc-ad)^2}+\frac{Bdn}{6bg^4(a+bx)^2(bc-ad)}-$$

```
[Out] -1/9*B*n/b/g^4/(b*x+a)^3+1/6*B*d*n/b/(-a*d+b*c)/g^4/(b*x+a)^2-1/3*B*d^2*n/b
/(-a*d+b*c)^2/g^4/(b*x+a)-1/3*B*d^3*n*ln(b*x+a)/b/(-a*d+b*c)^3/g^4+1/3*(-A-
B*ln(e*((b*x+a)/(d*x+c))^n))/b/g^4/(b*x+a)^3+1/3*B*d^3*n*ln(d*x+c)/b/(-a*d+
b*c)^3/g^4
```

**Rubi [A]** time = 0.15, antiderivative size = 183, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {2525, 12, 44}

$$-\frac{B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A}{3bg^4(a+bx)^3}-\frac{Bd^2n}{3bg^4(a+bx)(bc-ad)^2}-\frac{Bd^3n\log(a+bx)}{3bg^4(bc-ad)^3}+\frac{Bd^3n\log(c+dx)}{3bg^4(bc-ad)^3}+\frac{Bdn}{6bg^4(a+bx)^2(bc-ad)}-$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Log[e*((a + b*x)/(c + d*x))^n])/(a*g + b*g*x)^4,x]
```

```
[Out] -(B*n)/(9*b*g^4*(a + b*x)^3) + (B*d*n)/(6*b*(b*c - a*d)*g^4*(a + b*x)^2) -
(B*d^2*n)/(3*b*(b*c - a*d)^2*g^4*(a + b*x)) - (B*d^3*n*Log[a + b*x])/(3*b*(
b*c - a*d)^3*g^4) - (A + B*Log[e*((a + b*x)/(c + d*x))^n])/(3*b*g^4*(a + b*
x)^3) + (B*d^3*n*Log[c + d*x])/(3*b*(b*c - a*d)^3*g^4)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
 + n + 2, 0])
```

Rule 2525

```
Int[((a_.) + Log[(c_.)*(RFx_)^(p_.)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_))^(m_.
), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(a + b*Log[c*RFx^p])^n)/(e*(m + 1))
, x] - Dist[(b*n*p)/(e*(m + 1)), Int[SimplifyIntegrand[((d + e*x)^(m + 1)*(
a + b*Log[c*RFx^p])^(n - 1)*D[RFx, x])/RFx, x], x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && RationalFunctionQ[RFx, x] && IGtQ[n, 0] && (EqQ[n, 1] ||
IntegerQ[m]) && NeQ[m, -1]
```

Rubi steps

$$\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(ag+bgx)^4}\,dx = -\frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{3bg^4(a+bx)^3}+\frac{(Bn)\int \frac{bc-ad}{g^3(a+bx)^4(c+dx)}\,dx}{3bg}$$

$$= -\frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{3bg^4(a+bx)^3}+\frac{(B(bc-ad)n)\int \frac{1}{(a+bx)^4(c+dx)}\,dx}{3bg^4}$$

$$= -\frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{3bg^4(a+bx)^3}+\frac{(B(bc-ad)n)\int \left(\frac{b}{(bc-ad)(a+bx)^4}-\frac{bd}{(bc-ad)^2(a+bx)^3}+\frac{b\cdots}{(bc-ad)^{\cdots}}\right.}{3bg^4}$$

$$= -\frac{Bn}{9bg^4(a+bx)^3}+\frac{Bdn}{6b(bc-ad)g^4(a+bx)^2}-\frac{Bd^2n}{3b(bc-ad)^2g^4(a+bx)}-\frac{Bd^3n\log(\cdots}{3b(bc-a\cdots}$$

**Mathematica [A]** time = 0.17, size = 145, normalized size = 0.79

$$-\frac{\frac{Bn\left((bc-ad)\left(11a^2d^2+abd(15dx-7c)+b^2\left(2c^2-3cdx+6d^2x^2\right)\right)-6d^3(a+bx)^3\log(c+dx)+6d^3(a+bx)^3\log(a+bx)\right)}{(bc-ad)^3}+6\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{18bg^4(a+bx)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Log[e*((a + b*x)/(c + d*x))^n])/(a*g + b*g*x)^4,x]
```

```
[Out] -1/18*(6*(A + B*Log[e*((a + b*x)/(c + d*x))^n]) + (B*n*((b*c - a*d)*(11*a^2
*d^2 + a*b*d*(-7*c + 15*d*x) + b^2*(2*c^2 - 3*c*d*x + 6*d^2*x^2)) + 6*d^3*(
a + b*x)^3*Log[a + b*x] - 6*d^3*(a + b*x)^3*Log[c + d*x]))/(b*c - a*d)^3)/(
b*g^4*(a + b*x)^3)
```

**fricas [B]** time = 0.94, size = 482, normalized size = 2.63

$$-\frac{6\,Ab^3c^3-18\,Aab^2c^2d+18\,Aa^2bcd^2-6\,Aa^3d^3+6\left(Bb^3cd^2-Bab^2d^3\right)nx^2-3\left(Bb^3c^2d-6\,Bab^2cd^2+5\,Ba^2bd^3\right)\cdots}{18\left(\left(b^7c^3-3\,ab^6c^2d+3\,a^2b^5cd^2-a^3b^4d^3\right)g^4x^3+3\cdots\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))/(b*g*x+a*g)^4,x, algorithm="fric
as")
```

```
[Out] -1/18*(6*A*b^3*c^3 - 18*A*a*b^2*c^2*d + 18*A*a^2*b*c*d^2 - 6*A*a^3*d^3 + 6*
(B*b^3*c*d^2 - B*a*b^2*d^3)*n*x^2 - 3*(B*b^3*c^2*d - 6*B*a*b^2*c*d^2 + 5*B*
a^2*b*d^3)*n*x + (2*B*b^3*c^3 - 9*B*a*b^2*c^2*d + 18*B*a^2*b*c*d^2 - 11*B*a
^3*d^3)*n + 6*(B*b^3*c^3 - 3*B*a*b^2*c^2*d + 3*B*a^2*b*c*d^2 - B*a^3*d^3)*l
og(e) + 6*(B*b^3*d^3*n*x^3 + 3*B*a*b^2*d^3*n*x^2 + 3*B*a^2*b*d^3*n*x + (B*b
^3*c^3 - 3*B*a*b^2*c^2*d + 3*B*a^2*b*c*d^2)*n)*log((b*x + a)/(d*x + c)))/((
b^7*c^3 - 3*a*b^6*c^2*d + 3*a^2*b^5*c*d^2 - a^3*b^4*d^3)*g^4*x^3 + 3*(a*b^6
*c^3 - 3*a^2*b^5*c^2*d + 3*a^3*b^4*c*d^2 - a^4*b^3*d^3)*g^4*x^2 + 3*(a^2*b^
5*c^3 - 3*a^3*b^4*c^2*d + 3*a^4*b^3*c*d^2 - a^5*b^2*d^3)*g^4*x + (a^3*b^4*c
^3 - 3*a^4*b^3*c^2*d + 3*a^5*b^2*c*d^2 - a^6*b*d^3)*g^4)
```

**giac [B]** time = 5.22, size = 375, normalized size = 2.05

$$-\frac{1}{18}\left(\frac{6\left(Bb^2n - \frac{3\,(bx+a)Bbdn}{dx+c} + \frac{3\,(bx+a)^2Bd^2n}{(dx+c)^2}\right)\log\left(\frac{bx+a}{dx+c}\right)}{\frac{(bx+a)^3b^2c^2g^4}{(dx+c)^3} - \frac{2\,(bx+a)^3abcdg^4}{(dx+c)^3} + \frac{(bx+a)^3a^2d^2g^4}{(dx+c)^3}} + \frac{2\,Bb^2n - \frac{9\,(bx+a)Bbdn}{dx+c} + \frac{18\,(bx+a)^2Bd^2n}{(dx+c)^2} + 6\,Ab^2 + 6\,Bb^2 - \frac{18}{}}{\frac{(bx+a)^3b^2c^2g^4}{(dx+c)^3} - \frac{2\,(bx+a)^3abc}{(dx+c)^3}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))/(b*g*x+a*g)^4,x, algorithm="giac
")
```

```
[Out] -1/18*(6*(B*b^2*n - 3*(b*x + a)*B*b*d*n/(d*x + c) + 3*(b*x + a)^2*B*d^2*n/(
d*x + c)^2)*log((b*x + a)/(d*x + c))/((b*x + a)^3*b^2*c^2*g^4/(d*x + c)^3 -
 2*(b*x + a)^3*a*b*c*d*g^4/(d*x + c)^3 + (b*x + a)^3*a^2*d^2*g^4/(d*x + c)^
3) + (2*B*b^2*n - 9*(b*x + a)*B*b*d*n/(d*x + c) + 18*(b*x + a)^2*B*d^2*n/(d
*x + c)^2 + 6*A*b^2 + 6*B*b^2 - 18*(b*x + a)*A*b*d/(d*x + c) - 18*(b*x + a)
*B*b*d/(d*x + c) + 18*(b*x + a)^2*A*d^2/(d*x + c)^2 + 18*(b*x + a)^2*B*d^2/
(d*x + c)^2)/((b*x + a)^3*b^2*c^2*g^4/(d*x + c)^3 - 2*(b*x + a)^3*a*b*c*d*g
^4/(d*x + c)^3 + (b*x + a)^3*a^2*d^2*g^4/(d*x + c)^3))*(b*c/(b*c - a*d)^2 -
 a*d/(b*c - a*d)^2)
```

**maple [F]** time = 0.29, size = 0, normalized size = 0.00

$$\int \frac{B\ln\left(e\left(\frac{bx+a}{dx+c}\right)^{n}\right)+A}{\left(bgx+ag\right)^{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*ln(e*((b*x+a)/(d*x+c))^n)+A)/(b*g*x+a*g)^4,x)
```

```
[Out] int((B*ln(e*((b*x+a)/(d*x+c))^n)+A)/(b*g*x+a*g)^4,x)
```

**maxima [B]** time = 1.35, size = 432, normalized size = 2.36

$$-\frac{1}{18}\,Bn\left(\frac{6\,b^2d^2x^2+2\,b^2c^2-7\,abcd+11\,a^2d^2-3\left(b^2cd-5\,abd^2\right)x}{\left(b^6c^2-2\,ab^5cd+a^2b^4d^2\right)g^4x^3+3\left(ab^5c^2-2\,a^2b^4cd+a^3b^3d^2\right)g^4x^2+3\left(a^2b^4c^2-2\,a^3b^3cd+a^4b^2d^2\right)g^4}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))/(b*g*x+a*g)^4,x, algorithm="maxi
ma")
```

```
[Out] -1/18*B*n*((6*b^2*d^2*x^2 + 2*b^2*c^2 - 7*a*b*c*d + 11*a^2*d^2 - 3*(b^2*c*d
 - 5*a*b*d^2)*x)/((b^6*c^2 - 2*a*b^5*c*d + a^2*b^4*d^2)*g^4*x^3 + 3*(a*b^5*
c^2 - 2*a^2*b^4*c*d + a^3*b^3*d^2)*g^4*x^2 + 3*(a^2*b^4*c^2 - 2*a^3*b^3*c*d
 + a^4*b^2*d^2)*g^4*x + (a^3*b^3*c^2 - 2*a^4*b^2*c*d + a^5*b*d^2)*g^4) + 6*
d^3*log(b*x + a)/((b^4*c^3 - 3*a*b^3*c^2*d + 3*a^2*b^2*c*d^2 - a^3*b*d^3)*g
^4) - 6*d^3*log(d*x + c)/((b^4*c^3 - 3*a*b^3*c^2*d + 3*a^2*b^2*c*d^2 - a^3*
b*d^3)*g^4)) - 1/3*B*log(e*(b*x/(d*x + c) + a/(d*x + c))^n)/(b^4*g^4*x^3 +
3*a*b^3*g^4*x^2 + 3*a^2*b^2*g^4*x + a^3*b*g^4) - 1/3*A/(b^4*g^4*x^3 + 3*a*b
^3*g^4*x^2 + 3*a^2*b^2*g^4*x + a^3*b*g^4)
```

**mupad [B]** time = 4.85, size = 349, normalized size = 1.91

$$\frac{2\,A\,a\,c\,d}{3\,g^4\,(a\,d-b\,c)^2\,(a+b\,x)^3}-\frac{A\,b\,c^2}{3\,g^4\,(a\,d-b\,c)^2\,(a+b\,x)^3}-\frac{A\,a^2\,d^2}{3\,b\,g^4\,(a\,d-b\,c)^2\,(a+b\,x)^3}-\frac{B\,b\,c^2\,n}{9\,g^4\,(a\,d-b\,c)^2\,(a+b\,x)^3}-\frac{B}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*log(e*((a + b*x)/(c + d*x))^n))/(a*g + b*g*x)^4,x)
```

```
[Out] (2*A*a*c*d)/(3*g^4*(a*d - b*c)^2*(a + b*x)^3) - (A*b*c^2)/(3*g^4*(a*d - b*c
)^2*(a + b*x)^3) - (A*a^2*d^2)/(3*b*g^4*(a*d - b*c)^2*(a + b*x)^3) - (B*b*c
^2*n)/(9*g^4*(a*d - b*c)^2*(a + b*x)^3) - (B*d^3*n*atan((a*d*1i + b*c*1i +
b*d*x*2i)/(a*d - b*c))*2i)/(3*b*g^4*(a*d - b*c)^3) - (B*log(e*((a + b*x)/(c
 + d*x))^n))/(3*b*g^4*(a + b*x)^3) - (B*b*d^2*n*x^2)/(3*g^4*(a*d - b*c)^2*(
a + b*x)^3) + (7*B*a*c*d*n)/(18*g^4*(a*d - b*c)^2*(a + b*x)^3) - (11*B*a^2*
d^2*n)/(18*b*g^4*(a*d - b*c)^2*(a + b*x)^3) - (5*B*a*d^2*n*x)/(6*g^4*(a*d -
 b*c)^2*(a + b*x)^3) + (B*b*c*d*n*x)/(6*g^4*(a*d - b*c)^2*(a + b*x)^3)
```

**sympy [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*ln(e*((b*x+a)/(d*x+c))**n))/(b*g*x+a*g)**4,x)
```

```
[Out] Timed out
```

## 3.9 $\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(ag+bgx)^5}\,dx$

**Optimal**. Leaf size=215

$$-\frac{B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A}{4bg^5(a+bx)^4}+\frac{Bd^4n\log(a+bx)}{4bg^5(bc-ad)^4}-\frac{Bd^4n\log(c+dx)}{4bg^5(bc-ad)^4}+\frac{Bd^3n}{4bg^5(a+bx)(bc-ad)^3}-\frac{Bd^2n}{8bg^5(a+bx)^2(bc-ad)^2}$$

```
[Out] -1/16*B*n/b/g^5/(b*x+a)^4+1/12*B*d*n/b/(-a*d+b*c)/g^5/(b*x+a)^3-1/8*B*d^2*n
/b/(-a*d+b*c)^2/g^5/(b*x+a)^2+1/4*B*d^3*n/b/(-a*d+b*c)^3/g^5/(b*x+a)+1/4*B*
d^4*n*ln(b*x+a)/b/(-a*d+b*c)^4/g^5+1/4*(-A-B*ln(e*((b*x+a)/(d*x+c))^n))/b/g
^5/(b*x+a)^4-1/4*B*d^4*n*ln(d*x+c)/b/(-a*d+b*c)^4/g^5
```

**Rubi [A]** time = 0.19, antiderivative size = 215, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {2525, 12, 44}

$$-\frac{B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A}{4bg^5(a+bx)^4}+\frac{Bd^3n}{4bg^5(a+bx)(bc-ad)^3}-\frac{Bd^2n}{8bg^5(a+bx)^2(bc-ad)^2}+\frac{Bd^4n\log(a+bx)}{4bg^5(bc-ad)^4}-\frac{Bd^4n\log(c+dx)}{4bg^5(bc-ad)^4}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Log[e*((a + b*x)/(c + d*x))^n])/(a*g + b*g*x)^5,x]
```

```
[Out] -(B*n)/(16*b*g^5*(a + b*x)^4) + (B*d*n)/(12*b*(b*c - a*d)*g^5*(a + b*x)^3)
- (B*d^2*n)/(8*b*(b*c - a*d)^2*g^5*(a + b*x)^2) + (B*d^3*n)/(4*b*(b*c - a*d
)^3*g^5*(a + b*x)) + (B*d^4*n*Log[a + b*x])/(4*b*(b*c - a*d)^4*g^5) - (A +
B*Log[e*((a + b*x)/(c + d*x))^n])/(4*b*g^5*(a + b*x)^4) - (B*d^4*n*Log[c +
d*x])/(4*b*(b*c - a*d)^4*g^5)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 2525

```
Int[((a_.) + Log[(c_.)*(RFx_)^(p_.)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_))^(m_.
), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(a + b*Log[c*RFx^p])^n)/(e*(m + 1))
, x] - Dist[(b*n*p)/(e*(m + 1)), Int[SimplifyIntegrand[((d + e*x)^(m + 1)*(
a + b*Log[c*RFx^p])^(n - 1)*D[RFx, x])/RFx, x], x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && RationalFunctionQ[RFx, x] && IGtQ[n, 0] && (EqQ[n, 1] ||
IntegerQ[m]) && NeQ[m, -1]
```

Rubi steps

$$\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(ag+bgx)^5}\,dx = -\frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{4bg^5(a+bx)^4}+\frac{(Bn)\int\frac{bc-ad}{g^4(a+bx)^5(c+dx)}\,dx}{4bg}$$

$$= -\frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{4bg^5(a+bx)^4}+\frac{(B(bc-ad)n)\int\frac{1}{(a+bx)^5(c+dx)}\,dx}{4bg^5}$$

$$= -\frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{4bg^5(a+bx)^4}+\frac{(B(bc-ad)n)\int\left(\frac{b}{(bc-ad)(a+bx)^5}-\frac{bd}{(bc-ad)^2(a+bx)^4}+\frac{bd^2}{(bc-ad)^3(a}\right.}{4b}$$

$$= -\frac{Bn}{16bg^5(a+bx)^4}+\frac{Bdn}{12b(bc-ad)g^5(a+bx)^3}-\frac{Bd^2n}{8b(bc-ad)^2g^5(a+bx)^2}+\frac{}{4b(bc-}$$

**Mathematica [A]** time = 0.22, size = 162, normalized size = 0.75

$$\frac{\frac{Bn\left(\frac{12d^3(bc-ad)}{a+bx}-\frac{6d^2(bc-ad)^2}{(a+bx)^2}+\frac{4d(bc-ad)^3}{(a+bx)^3}-\frac{3(bc-ad)^4}{(a+bx)^4}+12d^4\log(a+bx)-12d^4\log(c+dx)\right)}{12(bc-ad)^4}-\frac{B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A}{(a+bx)^4}}{4bg^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Log[e*((a + b*x)/(c + d*x))^n])/(a*g + b*g*x)^5,x]
```

```
[Out] (-((A + B*Log[e*((a + b*x)/(c + d*x))^n])/(a + b*x)^4) + (B*n*((-3*(b*c - a
*d)^4)/(a + b*x)^4 + (4*d*(b*c - a*d)^3)/(a + b*x)^3 - (6*d^2*(b*c - a*d)^2
)/(a + b*x)^2 + (12*d^3*(b*c - a*d))/(a + b*x) + 12*d^4*Log[a + b*x] - 12*d
^4*Log[c + d*x]))/(12*(b*c - a*d)^4))/(4*b*g^5)
```

**fricas [B]** time = 0.71, size = 733, normalized size = 3.41

$$-\frac{12\,Ab^4c^4 - 48\,Aab^3c^3d + 72\,Aa^2b^2c^2d^2 - 48\,Aa^3bcd^3 + 12\,Aa^4d^4 - 12\left(Bb^4cd^3 - Bab^3d^4\right)nx^3 + 6\left(Bb^4c^2d^2 - 8\right.}{48\left(\left(b^9c^4 - 4\,ab^8c^3d + 6\,a^2b^7c^2d^2 - 4\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))/(b*g*x+a*g)^5,x, algorithm="fric
as")
```

```
[Out] -1/48*(12*A*b^4*c^4 - 48*A*a*b^3*c^3*d + 72*A*a^2*b^2*c^2*d^2 - 48*A*a^3*b*
c*d^3 + 12*A*a^4*d^4 - 12*(B*b^4*c*d^3 - B*a*b^3*d^4)*n*x^3 + 6*(B*b^4*c^2*
d^2 - 8*B*a*b^3*c*d^3 + 7*B*a^2*b^2*d^4)*n*x^2 - 4*(B*b^4*c^3*d - 6*B*a*b^3
*c^2*d^2 + 18*B*a^2*b^2*c*d^3 - 13*B*a^3*b*d^4)*n*x + (3*B*b^4*c^4 - 16*B*a
*b^3*c^3*d + 36*B*a^2*b^2*c^2*d^2 - 48*B*a^3*b*c*d^3 + 25*B*a^4*d^4)*n + 12
*(B*b^4*c^4 - 4*B*a*b^3*c^3*d + 6*B*a^2*b^2*c^2*d^2 - 4*B*a^3*b*c*d^3 + B*a
^4*d^4)*log(e) - 12*(B*b^4*d^4*n*x^4 + 4*B*a*b^3*d^4*n*x^3 + 6*B*a^2*b^2*d^
4*n*x^2 + 4*B*a^3*b*d^4*n*x - (B*b^4*c^4 - 4*B*a*b^3*c^3*d + 6*B*a^2*b^2*c^
2*d^2 - 4*B*a^3*b*c*d^3)*n)*log((b*x + a)/(d*x + c)))/((b^9*c^4 - 4*a*b^8*c
^3*d + 6*a^2*b^7*c^2*d^2 - 4*a^3*b^6*c*d^3 + a^4*b^5*d^4)*g^5*x^4 + 4*(a*b^
8*c^4 - 4*a^2*b^7*c^3*d + 6*a^3*b^6*c^2*d^2 - 4*a^4*b^5*c*d^3 + a^5*b^4*d^4
)*g^5*x^3 + 6*(a^2*b^7*c^4 - 4*a^3*b^6*c^3*d + 6*a^4*b^5*c^2*d^2 - 4*a^5*b^
4*c*d^3 + a^6*b^3*d^4)*g^5*x^2 + 4*(a^3*b^6*c^4 - 4*a^4*b^5*c^3*d + 6*a^5*b
^4*c^2*d^2 - 4*a^6*b^3*c*d^3 + a^7*b^2*d^4)*g^5*x + (a^4*b^5*c^4 - 4*a^5*b^
4*c^3*d + 6*a^6*b^3*c^2*d^2 - 4*a^7*b^2*c*d^3 + a^8*b*d^4)*g^5)
```

**giac [B]** time = 8.37, size = 533, normalized size = 2.48

$$-\frac{1}{48}\left(\frac{12\left(Bb^3n - \frac{4\,(bx+a)Bb^2dn}{dx+c} + \frac{6\,(bx+a)^2Bbd^2n}{(dx+c)^2} - \frac{4\,(bx+a)^3Bd^3n}{(dx+c)^3}\right)\log\left(\frac{bx+a}{dx+c}\right)}{\frac{(bx+a)^4b^3c^3g^5}{(dx+c)^4} - \frac{3\,(bx+a)^4ab^2c^2dg^5}{(dx+c)^4} + \frac{3\,(bx+a)^4a^2bcd^2g^5}{(dx+c)^4} - \frac{(bx+a)^4a^3d^3g^5}{(dx+c)^4}} + \frac{3\,Bb^3n - \frac{16\,(bx+a)Bb^2dn}{dx+c} + \frac{36\,(bx+a)^2Bbd^2n}{(dx+c)^2}}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))/(b*g*x+a*g)^5,x, algorithm="giac
")
```

```
[Out] -1/48*(12*(B*b^3*n - 4*(b*x + a)*B*b^2*d*n/(d*x + c) + 6*(b*x + a)^2*B*b*d^
2*n/(d*x + c)^2 - 4*(b*x + a)^3*B*d^3*n/(d*x + c)^3)*log((b*x + a)/(d*x + c
))/((b*x + a)^4*b^3*c^3*g^5/(d*x + c)^4 - 3*(b*x + a)^4*a*b^2*c^2*d*g^5/(d*
x + c)^4 + 3*(b*x + a)^4*a^2*b*c*d^2*g^5/(d*x + c)^4 - (b*x + a)^4*a^3*d^3*
g^5/(d*x + c)^4) + (3*B*b^3*n - 16*(b*x + a)*B*b^2*d*n/(d*x + c) + 36*(b*x
+ a)^2*B*b*d^2*n/(d*x + c)^2 - 48*(b*x + a)^3*B*d^3*n/(d*x + c)^3 + 12*A*b^
3 + 12*B*b^3 - 48*(b*x + a)*A*b^2*d/(d*x + c) - 48*(b*x + a)*B*b^2*d/(d*x +
```

```
c) + 72*(b*x + a)^2*A*b*d^2/(d*x + c)^2 + 72*(b*x + a)^2*B*b*d^2/(d*x + c)
^2 - 48*(b*x + a)^3*A*d^3/(d*x + c)^3 - 48*(b*x + a)^3*B*d^3/(d*x + c)^3)/(
(b*x + a)^4*b^3*c^3*g^5/(d*x + c)^4 - 3*(b*x + a)^4*a*b^2*c^2*d*g^5/(d*x +
c)^4 + 3*(b*x + a)^4*a^2*b*c*d^2*g^5/(d*x + c)^4 - (b*x + a)^4*a^3*d^3*g^5/
(d*x + c)^4))*(b*c/(b*c - a*d)^2 - a*d/(b*c - a*d)^2)
```

**maple [F]** time = 0.28, size = 0, normalized size = 0.00

$$\int \frac{B \ln\left(e\left(\frac{bx+a}{dx+c}\right)^{n}\right) + A}{\left(bgx + ag\right)^{5}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*ln(e*((b*x+a)/(d*x+c))^n)+A)/(b*g*x+a*g)^5,x)
```

```
[Out] int((B*ln(e*((b*x+a)/(d*x+c))^n)+A)/(b*g*x+a*g)^5,x)
```

**maxima [B]** time = 1.60, size = 651, normalized size = 3.03

$$\frac{1}{48}\, Bn\left(\frac{12\, b^{3}d^{3}x^{3} - 3\, b^{3}c^{3} + 13\, ab^{2}c^{2}d - 23\, a^{2}bcd^{2}}{\left(b^{8}c^{3} - 3\, ab^{7}c^{2}d + 3\, a^{2}b^{6}cd^{2} - a^{3}b^{5}d^{3}\right)g^{5}x^{4} + 4\left(ab^{7}c^{3} - 3\, a^{2}b^{6}c^{2}d + 3\, a^{3}b^{5}cd^{2} - a^{4}b^{4}d^{3}\right)g^{5}x^{3} + 6\left(a^{2}b^{6}c^{3}\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))/(b*g*x+a*g)^5,x, algorithm="maxi
ma")
```

```
[Out] 1/48*B*n*((12*b^3*d^3*x^3 - 3*b^3*c^3 + 13*a*b^2*c^2*d - 23*a^2*b*c*d^2 + 2
5*a^3*d^3 - 6*(b^3*c*d^2 - 7*a*b^2*d^3)*x^2 + 4*(b^3*c^2*d - 5*a*b^2*c*d^2
+ 13*a^2*b*d^3)*x)/((b^8*c^3 - 3*a*b^7*c^2*d + 3*a^2*b^6*c*d^2 - a^3*b^5*d^
3)*g^5*x^4 + 4*(a*b^7*c^3 - 3*a^2*b^6*c^2*d + 3*a^3*b^5*c*d^2 - a^4*b^4*d^3
)*g^5*x^3 + 6*(a^2*b^6*c^3 - 3*a^3*b^5*c^2*d + 3*a^4*b^4*c*d^2 - a^5*b^3*d^
3)*g^5*x^2 + 4*(a^3*b^5*c^3 - 3*a^4*b^4*c^2*d + 3*a^5*b^3*c*d^2 - a^6*b^2*d
^3)*g^5*x + (a^4*b^4*c^3 - 3*a^5*b^3*c^2*d + 3*a^6*b^2*c*d^2 - a^7*b*d^3)*g
^5) + 12*d^4*log(b*x + a)/((b^5*c^4 - 4*a*b^4*c^3*d + 6*a^2*b^3*c^2*d^2 - 4
*a^3*b^2*c*d^3 + a^4*b*d^4)*g^5) - 12*d^4*log(d*x + c)/((b^5*c^4 - 4*a*b^4*
c^3*d + 6*a^2*b^3*c^2*d^2 - 4*a^3*b^2*c*d^3 + a^4*b*d^4)*g^5)) - 1/4*B*log(
e*(b*x/(d*x + c) + a/(d*x + c))^n)/(b^5*g^5*x^4 + 4*a*b^4*g^5*x^3 + 6*a^2*b
^3*g^5*x^2 + 4*a^3*b^2*g^5*x + a^4*b*g^5) - 1/4*A/(b^5*g^5*x^4 + 4*a*b^4*g^
5*x^3 + 6*a^2*b^3*g^5*x^2 + 4*a^3*b^2*g^5*x + a^4*b*g^5)
```

**mupad [B]** time = 5.12, size = 603, normalized size = 2.80

$$-\frac{\frac{12\,A\,a^3\,d^3-12\,A\,b^3\,c^3+25\,B\,a^3\,d^3\,n-3\,B\,b^3\,c^3\,n+36\,A\,a\,b^2\,c^2\,d-36\,A\,a^2\,b\,c\,d^2+13\,B\,a\,b^2\,c^2\,d\,n-23\,B\,a^2\,b\,c\,d^2\,n}{12\,\left(a^3\,d^3-3\,a^2\,b\,c\,d^2+3\,a\,b^2\,c^2\,d-b^3\,c^3\right)}+\frac{d\,x\,\left(13\,B\,n\,a^2\,b\,d^2-5\,B\,n\,a\,b^2\,c\,d+B\,n\right.}{3\,\left(a^3\,d^3-3\,a^2\,b\,c\,d^2+3\,a\,b^2\,c^2\,d-b\right.}}{4\,a^4\,b\,g^5+16\,a^3\,b^2\,g^5\,x+24\,a^2\,b^3\,g^5\,x^2+16\,a\,b^4\,g^5\,x^3+}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*log(e*((a + b*x)/(c + d*x))^n))/(a*g + b*g*x)^5,x)
```

```
[Out] - ((12*A*a^3*d^3 - 12*A*b^3*c^3 + 25*B*a^3*d^3*n - 3*B*b^3*c^3*n + 36*A*a*b
^2*c^2*d - 36*A*a^2*b*c*d^2 + 13*B*a*b^2*c^2*d*n - 23*B*a^2*b*c*d^2*n)/(12*
(a^3*d^3 - b^3*c^3 + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2)) + (d*x*(B*b^3*c^2*n +
13*B*a^2*b*d^2*n - 5*B*a*b^2*c*d*n))/(3*(a^3*d^3 - b^3*c^3 + 3*a*b^2*c^2*d
- 3*a^2*b*c*d^2)) - (d^2*x^2*(B*b^3*c*n - 7*B*a*b^2*d*n))/(2*(a^3*d^3 - b^3
*c^3 + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2)) + (B*b^3*d^3*n*x^3)/(a^3*d^3 - b^3*c
^3 + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2))/(4*a^4*b*g^5 + 4*b^5*g^5*x^4 + 16*a^3*
b^2*g^5*x + 16*a*b^4*g^5*x^3 + 24*a^2*b^3*g^5*x^2) - (B*log(e*((a + b*x)/(c
 + d*x))^n))/(4*b*(a^4*g^5 + b^4*g^5*x^4 + 4*a*b^3*g^5*x^3 + 6*a^2*b^2*g^5*
x^2 + 4*a^3*b*g^5*x)) - (B*d^4*n*atanh((4*b^5*c^4*g^5 - 4*a^4*b*d^4*g^5 - 8
*a*b^4*c^3*d*g^5 + 8*a^3*b^2*c*d^3*g^5)/(4*b*g^5*(a*d - b*c)^4) - (2*b*d*x*
(a^3*d^3 - b^3*c^3 + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2))/(a*d - b*c)^4))/(2*b*g
^5*(a*d - b*c)^4)
```

**sympy [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*ln(e*((b*x+a)/(d*x+c))**n))/(b*g*x+a*g)**5,x)
```

```
[Out] Timed out
```

## 3.10 $\int (ag+bgx)^4 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2 dx$

**Optimal**. Leaf size=396

$$\frac{Bg^4n(bc-ad)^5\log\left(\frac{bc-ad}{b(c+dx)}\right)\left(12B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+12A+25Bn\right)}{30bd^5}+\frac{Bg^4n(a+bx)(bc-ad)^4\left(12B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+\right.}{30bd^4}$$

```
[Out] -1/10*B*(-a*d+b*c)*g^4*n*(b*x+a)^4*(A+B*ln(e*((b*x+a)/(d*x+c))^n))/b/d+1/5*
g^4*(b*x+a)^5*(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2/b+1/30*B*(-a*d+b*c)^2*g^4*n
*(b*x+a)^3*(4*A+B*n+4*B*ln(e*((b*x+a)/(d*x+c))^n))/b/d^2-1/60*B*(-a*d+b*c)^
3*g^4*n*(b*x+a)^2*(12*A+7*B*n+12*B*ln(e*((b*x+a)/(d*x+c))^n))/b/d^3+1/30*B*
(-a*d+b*c)^4*g^4*n*(b*x+a)*(12*A+13*B*n+12*B*ln(e*((b*x+a)/(d*x+c))^n))/b/d
^4+1/30*B*(-a*d+b*c)^5*g^4*n*(12*A+25*B*n+12*B*ln(e*((b*x+a)/(d*x+c))^n))*l
n((-a*d+b*c)/b/(d*x+c))/b/d^5+2/5*B^2*(-a*d+b*c)^5*g^4*n^2*polylog(2,d*(b*x
+a)/b/(d*x+c))/b/d^5
```

**Rubi [A]** time = 0.87, antiderivative size = 602, normalized size of antiderivative = 1.52, number of steps used = 27, number of rules used = 13, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.371, Rules used = {2525, 12, 2528, 2486, 31, 43, 2524, 2418, 2394, 2393, 2391, 2390, 2301}

$$\frac{2B^2g^4n^2(bc-ad)^5\text{PolyLog}\left(2,\frac{b(c+dx)}{bc-ad}\right)}{5bd^5}-\frac{2Bg^4n(bc-ad)^5\log(c+dx)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{5bd^5}-\frac{Bg^4n(a+bx)^2(bc}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a*g + b*g*x)^4*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2,x]
```

```
[Out] (2*A*B*(b*c - a*d)^4*g^4*n*x)/(5*d^4) + (13*B^2*(b*c - a*d)^4*g^4*n^2*x)/(3
0*d^4) - (7*B^2*(b*c - a*d)^3*g^4*n^2*(a + b*x)^2)/(60*b*d^3) + (B^2*(b*c -
 a*d)^2*g^4*n^2*(a + b*x)^3)/(30*b*d^2) + (2*B^2*(b*c - a*d)^4*g^4*n*(a + b
*x)*Log[e*((a + b*x)/(c + d*x))^n])/(5*b*d^4) - (B*(b*c - a*d)^3*g^4*n*(a +
 b*x)^2*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/(5*b*d^3) + (2*B*(b*c - a*d
)^2*g^4*n*(a + b*x)^3*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/(15*b*d^2) -
(B*(b*c - a*d)*g^4*n*(a + b*x)^4*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/(1
0*b*d) + (g^4*(a + b*x)^5*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2)/(5*b) -
 (5*B^2*(b*c - a*d)^5*g^4*n^2*Log[c + d*x])/(6*b*d^5) + (2*B^2*(b*c - a*d)^
5*g^4*n^2*Log[-((d*(a + b*x))/(b*c - a*d))]*Log[c + d*x])/(5*b*d^5) - (2*B*
(b*c - a*d)^5*g^4*n*(A + B*Log[e*((a + b*x)/(c + d*x))^n])*Log[c + d*x])/(5
*b*d^5) - (B^2*(b*c - a*d)^5*g^4*n^2*Log[c + d*x]^2)/(5*b*d^5) + (2*B^2*(b*
c - a*d)^5*g^4*n^2*PolyLog[2, (b*(c + d*x))/(b*c - a*d)])/(5*b*d^5)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2301

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

Rule 2390

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_))^(q_.), x_Symbol] :> Dist[1/e, Subst[Int[((f*x)/d)^q*(a + b*Log[c*x^
n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && E
qQ[e*f - d*g, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2393

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + (c*e*x)/g])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2394

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d + e*x
```

```
)^n]))/g, x] - Dist[(b*e*n)/g, Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2418

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*(RFx_), x_Sy
mbol] :> With[{u = ExpandIntegrand[(a + b*Log[c*(d + e*x)^n])^p, RFx, x]},
Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d, e, n}, x] && RationalFunctionQ[
RFx, x] && IntegerQ[p]
```

Rule 2486

```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))
^(r_.)]^(s_.), x_Symbol] :> Simp[((a + b*x)*Log[e*(f*(a + b*x)^p*(c + d*x)^
q)^r]^s)/b, x] + Dist[(q*r*s*(b*c - a*d))/b, Int[Log[e*(f*(a + b*x)^p*(c +
d*x)^q)^r]^(s - 1)/(c + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, p, q, r, s
}, x] && NeQ[b*c - a*d, 0] && EqQ[p + q, 0] && IGtQ[s, 0]
```

Rule 2524

```
Int[((a_.) + Log[(c_.)*(RFx_)^(p_.)]*(b_.))^(n_.)/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[(Log[d + e*x]*(a + b*Log[c*RFx^p])^n)/e, x] - Dist[(b*n*p)/e
, Int[(Log[d + e*x]*(a + b*Log[c*RFx^p])^(n - 1)*D[RFx, x])/RFx, x], x] /;
FreeQ[{a, b, c, d, e, p}, x] && RationalFunctionQ[RFx, x] && IGtQ[n, 0]
```

Rule 2525

```
Int[((a_.) + Log[(c_.)*(RFx_)^(p_.)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_))^(m_.
), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(a + b*Log[c*RFx^p])^n)/(e*(m + 1))
, x] - Dist[(b*n*p)/(e*(m + 1)), Int[SimplifyIntegrand[((d + e*x)^(m + 1)*(
a + b*Log[c*RFx^p])^(n - 1)*D[RFx, x])/RFx, x], x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && RationalFunctionQ[RFx, x] && IGtQ[n, 0] && (EqQ[n, 1] ||
IntegerQ[m]) && NeQ[m, -1]
```

Rule 2528

```
Int[((a_.) + Log[(c_.)*(RFx_)^(p_.)]*(b_.))^(n_.)*(RGx_), x_Symbol] :> With
[{u = ExpandIntegrand[(a + b*Log[c*RFx^p])^n, RGx, x]}, Int[u, x] /; SumQ[u
]] /; FreeQ[{a, b, c, p}, x] && RationalFunctionQ[RFx, x] && RationalFuncti
onQ[RGx, x] && IGtQ[n, 0]
```

Rubi steps

$$\int (ag+bgx)^4 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2 dx = \frac{g^4(a+bx)^5\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{5b} - \frac{(2Bn)\int \frac{(bc-ad)g^5(a+bx)^4\left(\ldots\right.}{c+\ldots}}{5bg}$$

$$= \frac{g^4(a+bx)^5\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{5b} - \frac{\left(2B(bc-ad)g^4n\right)\int \frac{(a+\ldots}{\ldots}}{5\ldots}$$

$$= \frac{g^4(a+bx)^5\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{5b} - \frac{\left(2B(bc-ad)g^4n\right)\int\left(-\ldots\right.}{\ldots}$$

$$= \frac{g^4(a+bx)^5\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{5b} - \frac{\left(2B(bc-ad)g^4n\right)\int(a+\ldots}{\ldots}$$

$$= \frac{2AB(bc-ad)^4g^4nx}{5d^4} - \frac{B(bc-ad)^3g^4n(a+bx)^2\left(A+B\log\left(e\left(\ldots\right.\right.\right.}{5bd^3}$$

$$= \frac{2AB(bc-ad)^4g^4nx}{5d^4} + \frac{2B^2(bc-ad)^4g^4n(a+bx)\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\ldots\right.}{5bd^4}$$

$$= \frac{2AB(bc-ad)^4g^4nx}{5d^4} + \frac{2B^2(bc-ad)^4g^4n(a+bx)\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\ldots\right.}{5bd^4}$$

$$= \frac{2AB(bc-ad)^4g^4nx}{5d^4} + \frac{13B^2(bc-ad)^4g^4n^2x}{30d^4} - \frac{7B^2(bc-ad)^3g^4\ldots}{60bd^3\ldots}$$

$$= \frac{2AB(bc-ad)^4g^4nx}{5d^4} + \frac{13B^2(bc-ad)^4g^4n^2x}{30d^4} - \frac{7B^2(bc-ad)^3g^4\ldots}{60bd^3\ldots}$$

$$= \frac{2AB(bc-ad)^4g^4nx}{5d^4} + \frac{13B^2(bc-ad)^4g^4n^2x}{30d^4} - \frac{7B^2(bc-ad)^3g^4\ldots}{60bd^3\ldots}$$

**Mathematica [A]** time = 0.52, size = 535, normalized size = 1.35

$$g^4\left(\frac{Bn(bc-ad)\left(-6d^4(a+bx)^4\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)+8d^3(a+bx)^3(bc-ad)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)-12d^2(a+bx)^2(bc-ad)^2\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)-24(bc-ad)\right.}{}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*g + b*g*x)^4*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2,x]
```

```
[Out] (g^4*((a + b*x)^5*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2 + (B*(b*c - a*d)
*n*(24*A*b*d*(b*c - a*d)^3*x + 24*B*d*(b*c - a*d)^3*(a + b*x)*Log[e*((a + b
*x)/(c + d*x))^n] - 12*d^2*(b*c - a*d)^2*(a + b*x)^2*(A + B*Log[e*((a + b*x
)/(c + d*x))^n]) + 8*d^3*(b*c - a*d)*(a + b*x)^3*(A + B*Log[e*((a + b*x)/(c
 + d*x))^n]) - 6*d^4*(a + b*x)^4*(A + B*Log[e*((a + b*x)/(c + d*x))^n]) - 2
4*B*(b*c - a*d)^4*n*Log[c + d*x] - 24*(b*c - a*d)^4*(A + B*Log[e*((a + b*x)
/(c + d*x))^n])*Log[c + d*x] + 4*B*(b*c - a*d)^2*n*(2*b*d*(b*c - a*d)*x - d
^2*(a + b*x)^2 - 2*(b*c - a*d)^2*Log[c + d*x]) + B*(b*c - a*d)*n*(6*b*d*(b*
c - a*d)^2*x + 3*d^2*(-(b*c) + a*d)*(a + b*x)^2 + 2*d^3*(a + b*x)^3 - 6*(b*
c - a*d)^3*Log[c + d*x]) + 12*B*(b*c - a*d)^3*n*(b*d*x + (-(b*c) + a*d)*Log
[c + d*x]) + 12*B*(b*c - a*d)^4*n*((2*Log[(d*(a + b*x))/(-(b*c) + a*d)] - L
og[c + d*x])*Log[c + d*x] + 2*PolyLog[2, (b*(c + d*x))/(b*c - a*d)])))/(12*
d^5)))/(5*b)
```

**fricas [F]** time = 0.90, size = 0, normalized size = 0.00

$$\text{integral}\left(A^2b^4g^4x^4 + 4\,A^2ab^3g^4x^3 + 6\,A^2a^2b^2g^4x^2 + 4\,A^2a^3bg^4x + A^2a^4g^4 + \left(B^2b^4g^4x^4 + 4\,B^2ab^3g^4x^3 + 6\,B^2a^2b\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*g*x+a*g)^4*(A+B*log(e*((b*x+a)/(d*x+c))^n))^2,x, algorithm="fr
icas")
```

```
[Out] integral(A^2*b^4*g^4*x^4 + 4*A^2*a*b^3*g^4*x^3 + 6*A^2*a^2*b^2*g^4*x^2 + 4*
A^2*a^3*b*g^4*x + A^2*a^4*g^4 + (B^2*b^4*g^4*x^4 + 4*B^2*a*b^3*g^4*x^3 + 6*
B^2*a^2*b^2*g^4*x^2 + 4*B^2*a^3*b*g^4*x + B^2*a^4*g^4)*log(e*((b*x + a)/(d*
x + c))^n)^2 + 2*(A*B*b^4*g^4*x^4 + 4*A*B*a*b^3*g^4*x^3 + 6*A*B*a^2*b^2*g^4
*x^2 + 4*A*B*a^3*b*g^4*x + A*B*a^4*g^4)*log(e*((b*x + a)/(d*x + c))^n), x)
```

**giac [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*g*x+a*g)^4*(A+B*log(e*((b*x+a)/(d*x+c))^n))^2,x, algorithm="gi
ac")
```

```
[Out] Timed out
```

**maple [F]** time = 0.27, size = 0, normalized size = 0.00

$$\int \left(bgx+ag\right)^{4} \left(B\ln\left(e\left(\frac{bx+a}{dx+c}\right)^{n}\right)+A\right)^{2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*g*x+a*g)^4*(B*ln(e*((b*x+a)/(d*x+c))^n)+A)^2,x)
```

```
[Out] int((b*g*x+a*g)^4*(B*ln(e*((b*x+a)/(d*x+c))^n)+A)^2,x)
```

**maxima [B]** time = 8.29, size = 2945, normalized size = 7.44

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*g*x+a*g)^4*(A+B*log(e*((b*x+a)/(d*x+c))^n))^2,x, algorithm="ma
xima")
```

```
[Out] 2/5*A*B*b^4*g^4*x^5*log(e*(b*x/(d*x + c) + a/(d*x + c))^n) + 1/5*A^2*b^4*g^
4*x^5 + 2*A*B*a*b^3*g^4*x^4*log(e*(b*x/(d*x + c) + a/(d*x + c))^n) + A^2*a*
b^3*g^4*x^4 + 4*A*B*a^2*b^2*g^4*x^3*log(e*(b*x/(d*x + c) + a/(d*x + c))^n)
+ 2*A^2*a^2*b^2*g^4*x^3 + 4*A*B*a^3*b*g^4*x^2*log(e*(b*x/(d*x + c) + a/(d*x
 + c))^n) + 2*A^2*a^3*b*g^4*x^2 + 1/30*A*B*b^4*g^4*n*(12*a^5*log(b*x + a)/b
^5 - 12*c^5*log(d*x + c)/d^5 - (3*(b^4*c*d^3 - a*b^3*d^4)*x^4 - 4*(b^4*c^2*
d^2 - a^2*b^2*d^4)*x^3 + 6*(b^4*c^3*d - a^3*b*d^4)*x^2 - 12*(b^4*c^4 - a^4*
d^4)*x)/(b^4*d^4)) - 1/3*A*B*a*b^3*g^4*n*(6*a^4*log(b*x + a)/b^4 - 6*c^4*lo
g(d*x + c)/d^4 + (2*(b^3*c*d^2 - a*b^2*d^3)*x^3 - 3*(b^3*c^2*d - a^2*b*d^3)
*x^2 + 6*(b^3*c^3 - a^3*d^3)*x)/(b^3*d^3)) + 2*A*B*a^2*b^2*g^4*n*(2*a^3*log
(b*x + a)/b^3 - 2*c^3*log(d*x + c)/d^3 - ((b^2*c*d - a*b*d^2)*x^2 - 2*(b^2*
c^2 - a^2*d^2)*x)/(b^2*d^2)) - 4*A*B*a^3*b*g^4*n*(a^2*log(b*x + a)/b^2 - c^
2*log(d*x + c)/d^2 + (b*c - a*d)*x/(b*d)) + 2*A*B*a^4*g^4*n*(a*log(b*x + a)
/b - c*log(d*x + c)/d) + 2*A*B*a^4*g^4*x*log(e*(b*x/(d*x + c) + a/(d*x + c)
)^n) + A^2*a^4*g^4*x - 1/30*((25*g^4*n^2 + 12*g^4*n*log(e))*b^4*c^5 - (113*
g^4*n^2 + 60*g^4*n*log(e))*a*b^3*c^4*d + 4*(49*g^4*n^2 + 30*g^4*n*log(e))*a
^2*b^2*c^3*d^2 - 12*(13*g^4*n^2 + 10*g^4*n*log(e))*a^3*b*c^2*d^3 + 12*(4*g^
4*n^2 + 5*g^4*n*log(e))*a^4*c*d^4)*B^2*log(d*x + c)/d^5 - 2/5*(b^5*c^5*g^4*
n^2 - 5*a*b^4*c^4*d*g^4*n^2 + 10*a^2*b^3*c^3*d^2*g^4*n^2 - 10*a^3*b^2*c^2*d
^3*g^4*n^2 + 5*a^4*b*c*d^4*g^4*n^2 - a^5*d^5*g^4*n^2)*(log(b*x + a)*log((b*
d*x + a*d)/(b*c - a*d) + 1) + dilog(-(b*d*x + a*d)/(b*c - a*d)))*B^2/(b*d^5
) + 1/60*(12*B^2*b^5*d^5*g^4*x^5*log(e)^2 - 12*B^2*a^5*d^5*g^4*n^2*log(b*x
```

```
+ a)^2 - 6*(b^5*c*d^4*g^4*n*log(e) - (g^4*n*log(e) + 10*g^4*log(e)^2)*a*b^4
*d^5)*B^2*x^4 + 2*((g^4*n^2 + 4*g^4*n*log(e))*b^5*c^2*d^3 - 2*(g^4*n^2 + 10
*g^4*n*log(e))*a*b^4*c*d^4 + (g^4*n^2 + 16*g^4*n*log(e) + 60*g^4*log(e)^2)*
a^2*b^3*d^5)*B^2*x^3 - ((7*g^4*n^2 + 12*g^4*n*log(e))*b^5*c^3*d^2 - 3*(9*g^
4*n^2 + 20*g^4*n*log(e))*a*b^4*c^2*d^3 + 3*(11*g^4*n^2 + 40*g^4*n*log(e))*a
^2*b^3*c*d^4 - (13*g^4*n^2 + 72*g^4*n*log(e) + 120*g^4*log(e)^2)*a^3*b^2*d^
5)*B^2*x^2 + 24*(b^5*c^5*g^4*n^2 - 5*a*b^4*c^4*d*g^4*n^2 + 10*a^2*b^3*c^3*d
^2*g^4*n^2 - 10*a^3*b^2*c^2*d^3*g^4*n^2 + 5*a^4*b*c*d^4*g^4*n^2)*B^2*log(b*
x + a)*log(d*x + c) - 12*(b^5*c^5*g^4*n^2 - 5*a*b^4*c^4*d*g^4*n^2 + 10*a^2*
b^3*c^3*d^2*g^4*n^2 - 10*a^3*b^2*c^2*d^3*g^4*n^2 + 5*a^4*b*c*d^4*g^4*n^2)*B
^2*log(d*x + c)^2 + 2*((13*g^4*n^2 + 12*g^4*n*log(e))*b^5*c^4*d - (59*g^4*n
^2 + 60*g^4*n*log(e))*a*b^4*c^3*d^2 + 6*(17*g^4*n^2 + 20*g^4*n*log(e))*a^2*
b^3*c^2*d^3 - (79*g^4*n^2 + 120*g^4*n*log(e))*a^3*b^2*c*d^4 + (23*g^4*n^2 +
 48*g^4*n*log(e) + 30*g^4*log(e)^2)*a^4*b*d^5)*B^2*x + 2*(12*a*b^4*c^4*d*g^
4*n^2 - 54*a^2*b^3*c^3*d^2*g^4*n^2 + 94*a^3*b^2*c^2*d^3*g^4*n^2 - 77*a^4*b*
c*d^4*g^4*n^2 + (25*g^4*n^2 + 12*g^4*n*log(e))*a^5*d^5)*B^2*log(b*x + a) +
12*(B^2*b^5*d^5*g^4*x^5 + 5*B^2*a*b^4*d^5*g^4*x^4 + 10*B^2*a^2*b^3*d^5*g^4*
x^3 + 10*B^2*a^3*b^2*d^5*g^4*x^2 + 5*B^2*a^4*b*d^5*g^4*x)*log((b*x + a)^n)^
2 + 12*(B^2*b^5*d^5*g^4*x^5 + 5*B^2*a*b^4*d^5*g^4*x^4 + 10*B^2*a^2*b^3*d^5*
g^4*x^3 + 10*B^2*a^3*b^2*d^5*g^4*x^2 + 5*B^2*a^4*b*d^5*g^4*x)*log((d*x + c)
^n)^2 + 2*(12*B^2*b^5*d^5*g^4*x^5*log(e) + 12*B^2*a^5*d^5*g^4*n*log(b*x + a
) - 3*(b^5*c*d^4*g^4*n - (g^4*n + 20*g^4*log(e))*a*b^4*d^5)*B^2*x^4 + 4*(b^
5*c^2*d^3*g^4*n - 5*a*b^4*c*d^4*g^4*n + 2*(2*g^4*n + 15*g^4*log(e))*a^2*b^3
*d^5)*B^2*x^3 - 6*(b^5*c^3*d^2*g^4*n - 5*a*b^4*c^2*d^3*g^4*n + 10*a^2*b^3*c
*d^4*g^4*n - 2*(3*g^4*n + 10*g^4*log(e))*a^3*b^2*d^5)*B^2*x^2 + 12*(b^5*c^4
*d*g^4*n - 5*a*b^4*c^3*d^2*g^4*n + 10*a^2*b^3*c^2*d^3*g^4*n - 10*a^3*b^2*c*
d^4*g^4*n + (4*g^4*n + 5*g^4*log(e))*a^4*b*d^5)*B^2*x - 12*(b^5*c^5*g^4*n -
 5*a*b^4*c^4*d*g^4*n + 10*a^2*b^3*c^3*d^2*g^4*n - 10*a^3*b^2*c^2*d^3*g^4*n
+ 5*a^4*b*c*d^4*g^4*n)*B^2*log(d*x + c))*log((b*x + a)^n) - 2*(12*B^2*b^5*d
^5*g^4*x^5*log(e) + 12*B^2*a^5*d^5*g^4*n*log(b*x + a) - 3*(b^5*c*d^4*g^4*n
- (g^4*n + 20*g^4*log(e))*a*b^4*d^5)*B^2*x^4 + 4*(b^5*c^2*d^3*g^4*n - 5*a*b
^4*c*d^4*g^4*n + 2*(2*g^4*n + 15*g^4*log(e))*a^2*b^3*d^5)*B^2*x^3 - 6*(b^5*
c^3*d^2*g^4*n - 5*a*b^4*c^2*d^3*g^4*n + 10*a^2*b^3*c*d^4*g^4*n - 2*(3*g^4*n
 + 10*g^4*log(e))*a^3*b^2*d^5)*B^2*x^2 + 12*(b^5*c^4*d*g^4*n - 5*a*b^4*c^3*
d^2*g^4*n + 10*a^2*b^3*c^2*d^3*g^4*n - 10*a^3*b^2*c*d^4*g^4*n + (4*g^4*n +
5*g^4*log(e))*a^4*b*d^5)*B^2*x - 12*(b^5*c^5*g^4*n - 5*a*b^4*c^4*d*g^4*n +
10*a^2*b^3*c^3*d^2*g^4*n - 10*a^3*b^2*c^2*d^3*g^4*n + 5*a^4*b*c*d^4*g^4*n)*
B^2*log(d*x + c) + 12*(B^2*b^5*d^5*g^4*x^5 + 5*B^2*a*b^4*d^5*g^4*x^4 + 10*B
^2*a^2*b^3*d^5*g^4*x^3 + 10*B^2*a^3*b^2*d^5*g^4*x^2 + 5*B^2*a^4*b*d^5*g^4*x
)*log((b*x + a)^n))*log((d*x + c)^n))/(b*d^5)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.00

$$\int \left(a g + b g x\right)^4 \left(A + B \ln\left(e \left(\frac{a + b x}{c + d x}\right)^n\right)\right)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*g + b*g*x)^4*(A + B*log(e*((a + b*x)/(c + d*x))^n))^2,x)
```

```
[Out] int((a*g + b*g*x)^4*(A + B*log(e*((a + b*x)/(c + d*x))^n))^2, x)
```

**sympy [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*g*x+a*g)**4*(A+B*ln(e*((b*x+a)/(d*x+c))**n))**2,x)
```

```
[Out] Timed out
```

# 3.11 $\int (ag+bgx)^3 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2 dx$

**Optimal**. Leaf size=335

$$-\frac{Bg^3n(bc-ad)^4\log\left(\frac{bc-ad}{b(c+dx)}\right)\left(6B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+6A+11Bn\right)}{12bd^4}-\frac{Bg^3n(a+bx)(bc-ad)^3\left(6B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+6\right.}{12bd^3}$$

```
[Out] -1/6*B*(-a*d+b*c)*g^3*n*(b*x+a)^3*(A+B*ln(e*((b*x+a)/(d*x+c))^n))/b/d+1/4*g
^3*(b*x+a)^4*(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2/b+1/12*B*(-a*d+b*c)^2*g^3*n*
(b*x+a)^2*(3*A+B*n+3*B*ln(e*((b*x+a)/(d*x+c))^n))/b/d^2-1/12*B*(-a*d+b*c)^3
*g^3*n*(b*x+a)*(6*A+5*B*n+6*B*ln(e*((b*x+a)/(d*x+c))^n))/b/d^3-1/12*B*(-a*d
+b*c)^4*g^3*n*(6*A+11*B*n+6*B*ln(e*((b*x+a)/(d*x+c))^n))*ln((-a*d+b*c)/b/(d
*x+c))/b/d^4-1/2*B^2*(-a*d+b*c)^4*g^3*n^2*polylog(2,d*(b*x+a)/b/(d*x+c))/b/
d^4
```

**Rubi [A]** time = 0.71, antiderivative size = 512, normalized size of antiderivative = 1.53, number of steps used = 23, number of rules used = 13, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.371$, Rules used = {2525, 12, 2528, 2486, 31, 43, 2524, 2418, 2394, 2393, 2391, 2390, 2301}

$$-\frac{B^2g^3n^2(bc-ad)^4\text{PolyLog}\left(2,\frac{b(c+dx)}{bc-ad}\right)}{2bd^4}+\frac{Bg^3n(bc-ad)^4\log(c+dx)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{2bd^4}+\frac{Bg^3n(a+bx)^2(bc-}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a*g + b*g*x)^3*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2,x]
```

```
[Out] -(A*B*(b*c - a*d)^3*g^3*n*x)/(2*d^3) - (5*B^2*(b*c - a*d)^3*g^3*n^2*x)/(12*
d^3) + (B^2*(b*c - a*d)^2*g^3*n^2*(a + b*x)^2)/(12*b*d^2) - (B^2*(b*c - a*d
)^3*g^3*n*(a + b*x)*Log[e*((a + b*x)/(c + d*x))^n])/(2*b*d^3) + (B*(b*c - a
*d)^2*g^3*n*(a + b*x)^2*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/(4*b*d^2) -
 (B*(b*c - a*d)*g^3*n*(a + b*x)^3*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/(
6*b*d) + (g^3*(a + b*x)^4*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2)/(4*b) +
 (11*B^2*(b*c - a*d)^4*g^3*n^2*Log[c + d*x])/(12*b*d^4) - (B^2*(b*c - a*d)^
4*g^3*n^2*Log[-((d*(a + b*x))/(b*c - a*d))]*Log[c + d*x])/(2*b*d^4) + (B*(b
*c - a*d)^4*g^3*n*(A + B*Log[e*((a + b*x)/(c + d*x))^n])*Log[c + d*x])/(2*b
*d^4) + (B^2*(b*c - a*d)^4*g^3*n^2*Log[c + d*x]^2)/(4*b*d^4) - (B^2*(b*c -
a*d)^4*g^3*n^2*PolyLog[2, (b*(c + d*x))/(b*c - a*d)])/(2*b*d^4)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2301

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

Rule 2390

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_))^(q_.), x_Symbol] :> Dist[1/e, Subst[Int[((f*x)/d)^q*(a + b*Log[c*x^
n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && E
qQ[e*f - d*g, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2393

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + (c*e*x)/g])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2394

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d + e*x
)^n]))/g, x] - Dist[(b*e*n)/g, Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
```

```
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2418

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*(RFx_), x_Sy
mbol] :> With[{u = ExpandIntegrand[(a + b*Log[c*(d + e*x)^n])^p, RFx, x]},
Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d, e, n}, x] && RationalFunctionQ[
RFx, x] && IntegerQ[p]
```

Rule 2486

```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))
^(r_.)]^(s_.), x_Symbol] :> Simp[((a + b*x)*Log[e*(f*(a + b*x)^p*(c + d*x)^
q)^r]^s)/b, x] + Dist[(q*r*s*(b*c - a*d))/b, Int[Log[e*(f*(a + b*x)^p*(c +
d*x)^q)^r]^(s - 1)/(c + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, p, q, r, s
}, x] && NeQ[b*c - a*d, 0] && EqQ[p + q, 0] && IGtQ[s, 0]
```

Rule 2524

```
Int[((a_.) + Log[(c_.)*(RFx_)^(p_.)]*(b_.))^(n_.)/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[(Log[d + e*x]*(a + b*Log[c*RFx^p])^n)/e, x] - Dist[(b*n*p)/e
, Int[(Log[d + e*x]*(a + b*Log[c*RFx^p])^(n - 1)*D[RFx, x])/RFx, x], x] /;
FreeQ[{a, b, c, d, e, p}, x] && RationalFunctionQ[RFx, x] && IGtQ[n, 0]
```

Rule 2525

```
Int[((a_.) + Log[(c_.)*(RFx_)^(p_.)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_))^(m_.
), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(a + b*Log[c*RFx^p])^n)/(e*(m + 1))
, x] - Dist[(b*n*p)/(e*(m + 1)), Int[SimplifyIntegrand[((d + e*x)^(m + 1)*(
a + b*Log[c*RFx^p])^(n - 1)*D[RFx, x])/RFx, x], x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && RationalFunctionQ[RFx, x] && IGtQ[n, 0] && (EqQ[n, 1] ||
IntegerQ[m]) && NeQ[m, -1]
```

Rule 2528

```
Int[((a_.) + Log[(c_.)*(RFx_)^(p_.)]*(b_.))^(n_.)*(RGx_), x_Symbol] :> With
[{u = ExpandIntegrand[(a + b*Log[c*RFx^p])^n, RGx, x]}, Int[u, x] /; SumQ[u
]] /; FreeQ[{a, b, c, p}, x] && RationalFunctionQ[RFx, x] && RationalFuncti
onQ[RGx, x] && IGtQ[n, 0]
```

Rubi steps

$$\int (ag+bgx)^3 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2 dx = \frac{g^3(a+bx)^4\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{4b} - \frac{(Bn)\int \frac{(bc-ad)g^4(a+bx)^3\left(A\right.}{c+d}}{2bg}$$

$$= \frac{g^3(a+bx)^4\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{4b} - \frac{\left(B(bc-ad)g^3n\right)\int \frac{(a+bx}{}}{2b}$$

$$= \frac{g^3(a+bx)^4\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{4b} - \frac{\left(B(bc-ad)g^3n\right)\int\left(\frac{b(bc}{}\right.}{}$$

$$= \frac{g^3(a+bx)^4\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{4b} - \frac{\left(B(bc-ad)g^3n\right)\int(a+}{}$$

$$= -\frac{AB(bc-ad)^3g^3nx}{2d^3} + \frac{B(bc-ad)^2g^3n(a+bx)^2\left(A+B\log\left(e\left(\right.\right.\right.}{4bd^2}$$

$$= -\frac{AB(bc-ad)^3g^3nx}{2d^3} - \frac{B^2(bc-ad)^3g^3n(a+bx)\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{2bd^3}$$

$$= -\frac{AB(bc-ad)^3g^3nx}{2d^3} - \frac{B^2(bc-ad)^3g^3n(a+bx)\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{2bd^3}$$

$$= -\frac{AB(bc-ad)^3g^3nx}{2d^3} - \frac{5B^2(bc-ad)^3g^3n^2x}{12d^3} + \frac{B^2(bc-ad)^2g^3n^2}{12bd^2}$$

$$= -\frac{AB(bc-ad)^3g^3nx}{2d^3} - \frac{5B^2(bc-ad)^3g^3n^2x}{12d^3} + \frac{B^2(bc-ad)^2g^3n^2}{12bd^2}$$

$$= -\frac{AB(bc-ad)^3g^3nx}{2d^3} - \frac{5B^2(bc-ad)^3g^3n^2x}{12d^3} + \frac{B^2(bc-ad)^2g^3n^2}{12bd^2}$$

**Mathematica [A]** time = 0.35, size = 411, normalized size = 1.23

$$g^3\left((a+bx)^4\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2-\frac{Bn(bc-ad)\left(2d^3(a+bx)^3\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)+3d^2(a+bx)^2(ad-bc)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)-6(bc-a\right.}{}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*g + b*g*x)^3*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2,x]
```

```
[Out] (g^3*((a + b*x)^4*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2 - (B*(b*c - a*d)
*n*(6*A*b*d*(b*c - a*d)^2*x + 6*B*d*(b*c - a*d)^2*(a + b*x)*Log[e*((a + b*x
)/(c + d*x))^n] + 3*d^2*(-(b*c) + a*d)*(a + b*x)^2*(A + B*Log[e*((a + b*x)/
(c + d*x))^n]) + 2*d^3*(a + b*x)^3*(A + B*Log[e*((a + b*x)/(c + d*x))^n]) -
 6*B*(b*c - a*d)^3*n*Log[c + d*x] - 6*(b*c - a*d)^3*(A + B*Log[e*((a + b*x)
/(c + d*x))^n])*Log[c + d*x] + B*(b*c - a*d)*n*(2*b*d*(b*c - a*d)*x - d^2*(
a + b*x)^2 - 2*(b*c - a*d)^2*Log[c + d*x]) + 3*B*(b*c - a*d)^2*n*(b*d*x + (
-(b*c) + a*d)*Log[c + d*x]) + 3*B*(b*c - a*d)^3*n*((2*Log[(d*(a + b*x))/(-(
b*c) + a*d)] - Log[c + d*x])*Log[c + d*x] + 2*PolyLog[2, (b*(c + d*x))/(b*c
 - a*d)]))))/(3*d^4)))/(4*b)
```

**fricas [F]** time = 0.81, size = 0, normalized size = 0.00

$$\text{integral}\left(A^2b^3g^3x^3+3\,A^2ab^2g^3x^2+3\,A^2a^2bg^3x+A^2a^3g^3+\left(B^2b^3g^3x^3+3\,B^2ab^2g^3x^2+3\,B^2a^2bg^3x+B^2a^3g^3\right)\mathrm{l}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*g*x+a*g)^3*(A+B*log(e*((b*x+a)/(d*x+c))^n))^2,x, algorithm="fr
icas")
```

```
[Out] integral(A^2*b^3*g^3*x^3 + 3*A^2*a*b^2*g^3*x^2 + 3*A^2*a^2*b*g^3*x + A^2*a^
3*g^3 + (B^2*b^3*g^3*x^3 + 3*B^2*a*b^2*g^3*x^2 + 3*B^2*a^2*b*g^3*x + B^2*a^
3*g^3)*log(e*((b*x + a)/(d*x + c))^n)^2 + 2*(A*B*b^3*g^3*x^3 + 3*A*B*a*b^2*
g^3*x^2 + 3*A*B*a^2*b*g^3*x + A*B*a^3*g^3)*log(e*((b*x + a)/(d*x + c))^n),
x)
```

**giac [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*g*x+a*g)^3*(A+B*log(e*((b*x+a)/(d*x+c))^n))^2,x, algorithm="gi
ac")
```

```
[Out] Timed out
```

**maple [F]** time = 0.29, size = 0, normalized size = 0.00

$$\int \left(bgx+ag\right)^{3}\left(B\ln\left(e\left(\frac{bx+a}{dx+c}\right)^{n}\right)+A\right)^{2}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*g*x+a*g)^3*(B*ln(e*((b*x+a)/(d*x+c))^n)+A)^2,x)
```

```
[Out] int((b*g*x+a*g)^3*(B*ln(e*((b*x+a)/(d*x+c))^n)+A)^2,x)
```

**maxima [B]** time = 8.02, size = 2175, normalized size = 6.49

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*g*x+a*g)^3*(A+B*log(e*((b*x+a)/(d*x+c))^n))^2,x, algorithm="ma
xima")
```

```
[Out] 1/2*A*B*b^3*g^3*x^4*log(e*(b*x/(d*x + c) + a/(d*x + c))^n) + 1/4*A^2*b^3*g^
3*x^4 + 2*A*B*a*b^2*g^3*x^3*log(e*(b*x/(d*x + c) + a/(d*x + c))^n) + A^2*a*
b^2*g^3*x^3 + 3*A*B*a^2*b*g^3*x^2*log(e*(b*x/(d*x + c) + a/(d*x + c))^n) +
3/2*A^2*a^2*b*g^3*x^2 - 1/12*A*B*b^3*g^3*n*(6*a^4*log(b*x + a)/b^4 - 6*c^4*
log(d*x + c)/d^4 + (2*(b^3*c*d^2 - a*b^2*d^3)*x^3 - 3*(b^3*c^2*d - a^2*b*d^
3)*x^2 + 6*(b^3*c^3 - a^3*d^3)*x)/(b^3*d^3)) + A*B*a*b^2*g^3*n*(2*a^3*log(b
*x + a)/b^3 - 2*c^3*log(d*x + c)/d^3 - ((b^2*c*d - a*b*d^2)*x^2 - 2*(b^2*c^
2 - a^2*d^2)*x)/(b^2*d^2)) - 3*A*B*a^2*b*g^3*n*(a^2*log(b*x + a)/b^2 - c^2*
log(d*x + c)/d^2 + (b*c - a*d)*x/(b*d)) + 2*A*B*a^3*g^3*n*(a*log(b*x + a)/b
 - c*log(d*x + c)/d) + 2*A*B*a^3*g^3*x*log(e*(b*x/(d*x + c) + a/(d*x + c))^
n) + A^2*a^3*g^3*x + 1/12*((11*g^3*n^2 + 6*g^3*n*log(e))*b^3*c^4 - 2*(19*g^
3*n^2 + 12*g^3*n*log(e))*a*b^2*c^3*d + 9*(5*g^3*n^2 + 4*g^3*n*log(e))*a^2*b
*c^2*d^2 - 6*(3*g^3*n^2 + 4*g^3*n*log(e))*a^3*c*d^3)*B^2*log(d*x + c)/d^4 +
 1/2*(b^4*c^4*g^3*n^2 - 4*a*b^3*c^3*d*g^3*n^2 + 6*a^2*b^2*c^2*d^2*g^3*n^2 -
 4*a^3*b*c*d^3*g^3*n^2 + a^4*d^4*g^3*n^2)*(log(b*x + a)*log((b*d*x + a*d)/(
b*c - a*d) + 1) + dilog(-(b*d*x + a*d)/(b*c - a*d)))*B^2/(b*d^4) + 1/12*(3*
B^2*b^4*d^4*g^3*x^4*log(e)^2 - 3*B^2*a^4*d^4*g^3*n^2*log(b*x + a)^2 - 2*(b^
4*c*d^3*g^3*n*log(e) - (g^3*n*log(e) + 6*g^3*log(e)^2)*a*b^3*d^4)*B^2*x^3 +
 ((g^3*n^2 + 3*g^3*n*log(e))*b^4*c^2*d^2 - 2*(g^3*n^2 + 6*g^3*n*log(e))*a*b
^3*c*d^3 + (g^3*n^2 + 9*g^3*n*log(e) + 18*g^3*log(e)^2)*a^2*b^2*d^4)*B^2*x^
2 - 6*(b^4*c^4*g^3*n^2 - 4*a*b^3*c^3*d*g^3*n^2 + 6*a^2*b^2*c^2*d^2*g^3*n^2
- 4*a^3*b*c*d^3*g^3*n^2)*B^2*log(b*x + a)*log(d*x + c) + 3*(b^4*c^4*g^3*n^2
 - 4*a*b^3*c^3*d*g^3*n^2 + 6*a^2*b^2*c^2*d^2*g^3*n^2 - 4*a^3*b*c*d^3*g^3*n^
2)*B^2*log(d*x + c)^2 - ((5*g^3*n^2 + 6*g^3*n*log(e))*b^4*c^3*d - (17*g^3*n
```

```
^2 + 24*g^3*n*log(e))*a*b^3*c^2*d^2 + (19*g^3*n^2 + 36*g^3*n*log(e))*a^2*b^
2*c*d^3 - (7*g^3*n^2 + 18*g^3*n*log(e) + 12*g^3*log(e)^2)*a^3*b*d^4)*B^2*x
- (6*a*b^3*c^3*d*g^3*n^2 - 21*a^2*b^2*c^2*d^2*g^3*n^2 + 26*a^3*b*c*d^3*g^3*
n^2 - (11*g^3*n^2 + 6*g^3*n*log(e))*a^4*d^4)*B^2*log(b*x + a) + 3*(B^2*b^4*
d^4*g^3*x^4 + 4*B^2*a*b^3*d^4*g^3*x^3 + 6*B^2*a^2*b^2*d^4*g^3*x^2 + 4*B^2*a
^3*b*d^4*g^3*x)*log((b*x + a)^n)^2 + 3*(B^2*b^4*d^4*g^3*x^4 + 4*B^2*a*b^3*d
^4*g^3*x^3 + 6*B^2*a^2*b^2*d^4*g^3*x^2 + 4*B^2*a^3*b*d^4*g^3*x)*log((d*x +
c)^n)^2 + (6*B^2*b^4*d^4*g^3*x^4*log(e) + 6*B^2*a^4*d^4*g^3*n*log(b*x + a)
- 2*(b^4*c*d^3*g^3*n - (g^3*n + 12*g^3*log(e))*a*b^3*d^4)*B^2*x^3 + 3*(b^4*
c^2*d^2*g^3*n - 4*a*b^3*c*d^3*g^3*n + 3*(g^3*n + 4*g^3*log(e))*a^2*b^2*d^4)
*B^2*x^2 - 6*(b^4*c^3*d*g^3*n - 4*a*b^3*c^2*d^2*g^3*n + 6*a^2*b^2*c*d^3*g^3
*n - (3*g^3*n + 4*g^3*log(e))*a^3*b*d^4)*B^2*x + 6*(b^4*c^4*g^3*n - 4*a*b^3
*c^3*d*g^3*n + 6*a^2*b^2*c^2*d^2*g^3*n - 4*a^3*b*c*d^3*g^3*n)*B^2*log(d*x +
 c))*log((b*x + a)^n) - (6*B^2*b^4*d^4*g^3*x^4*log(e) + 6*B^2*a^4*d^4*g^3*n
*log(b*x + a) - 2*(b^4*c*d^3*g^3*n - (g^3*n + 12*g^3*log(e))*a*b^3*d^4)*B^2
*x^3 + 3*(b^4*c^2*d^2*g^3*n - 4*a*b^3*c*d^3*g^3*n + 3*(g^3*n + 4*g^3*log(e)
)*a^2*b^2*d^4)*B^2*x^2 - 6*(b^4*c^3*d*g^3*n - 4*a*b^3*c^2*d^2*g^3*n + 6*a^2
*b^2*c*d^3*g^3*n - (3*g^3*n + 4*g^3*log(e))*a^3*b*d^4)*B^2*x + 6*(b^4*c^4*g
^3*n - 4*a*b^3*c^3*d*g^3*n + 6*a^2*b^2*c^2*d^2*g^3*n - 4*a^3*b*c*d^3*g^3*n)
*B^2*log(d*x + c) + 6*(B^2*b^4*d^4*g^3*x^4 + 4*B^2*a*b^3*d^4*g^3*x^3 + 6*B^
2*a^2*b^2*d^4*g^3*x^2 + 4*B^2*a^3*b*d^4*g^3*x)*log((b*x + a)^n))*log((d*x +
 c)^n))/(b*d^4)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.00

$$\int \left(a\,g+b\,g\,x\right)^3\,\left(A+B\,\ln\left(e\,\left(\frac{a+b\,x}{c+d\,x}\right)^n\right)\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*g + b*g*x)^3*(A + B*log(e*((a + b*x)/(c + d*x))^n))^2,x)
```

```
[Out] int((a*g + b*g*x)^3*(A + B*log(e*((a + b*x)/(c + d*x))^n))^2, x)
```

**sympy [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*g*x+a*g)**3*(A+B*ln(e*((b*x+a)/(d*x+c))**n))**2,x)
```

```
[Out] Timed out
```

# 3.12 $\int (ag+bgx)^2 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2 dx$

**Optimal**. Leaf size=274

$$\frac{Bg^2n(bc-ad)^3\log\left(\frac{bc-ad}{b(c+dx)}\right)\left(2B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+2A+3Bn\right)}{3bd^3}+\frac{Bg^2n(a+bx)(bc-ad)^2\left(2B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+2A\right.}{3bd^2}$$

```
[Out] -1/3*B*(-a*d+b*c)*g^2*n*(b*x+a)^2*(A+B*ln(e*((b*x+a)/(d*x+c))^n))/b/d+1/3*g
^2*(b*x+a)^3*(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2/b+1/3*B*(-a*d+b*c)^2*g^2*n*(
b*x+a)*(2*A+B*n+2*B*ln(e*((b*x+a)/(d*x+c))^n))/b/d^2+1/3*B*(-a*d+b*c)^3*g^2
*n*(2*A+3*B*n+2*B*ln(e*((b*x+a)/(d*x+c))^n))*ln((-a*d+b*c)/b/(d*x+c))/b/d^3
+2/3*B^2*(-a*d+b*c)^3*g^2*n^2*polylog(2,d*(b*x+a)/b/(d*x+c))/b/d^3
```

**Rubi [A]** time = 0.58, antiderivative size = 420, normalized size of antiderivative = 1.53, number of steps used = 19, number of rules used = 13, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.371, Rules used = {2525, 12, 2528, 2486, 31, 43, 2524, 2418, 2394, 2393, 2391, 2390, 2301}

$$\frac{2B^2g^2n^2(bc-ad)^3\text{PolyLog}\left(2,\frac{b(c+dx)}{bc-ad}\right)}{3bd^3}-\frac{2Bg^2n(bc-ad)^3\log(c+dx)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{3bd^3}+\frac{2ABg^2nx(bc-a}{3d^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a*g + b*g*x)^2*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2,x]
```

```
[Out] (2*A*B*(b*c - a*d)^2*g^2*n*x)/(3*d^2) + (B^2*(b*c - a*d)^2*g^2*n^2*x)/(3*d^
2) + (2*B^2*(b*c - a*d)^2*g^2*n*(a + b*x)*Log[e*((a + b*x)/(c + d*x))^n])/(
3*b*d^2) - (B*(b*c - a*d)*g^2*n*(a + b*x)^2*(A + B*Log[e*((a + b*x)/(c + d*
x))^n]))/(3*b*d) + (g^2*(a + b*x)^3*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^
2)/(3*b) - (B^2*(b*c - a*d)^3*g^2*n^2*Log[c + d*x])/(b*d^3) + (2*B^2*(b*c -
 a*d)^3*g^2*n^2*Log[-((d*(a + b*x))/(b*c - a*d))]*Log[c + d*x])/(3*b*d^3) -
 (2*B*(b*c - a*d)^3*g^2*n*(A + B*Log[e*((a + b*x)/(c + d*x))^n])*Log[c + d*
x])/(3*b*d^3) - (B^2*(b*c - a*d)^3*g^2*n^2*Log[c + d*x]^2)/(3*b*d^3) + (2*B
^2*(b*c - a*d)^3*g^2*n^2*PolyLog[2, (b*(c + d*x))/(b*c - a*d)])/(3*b*d^3)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2301

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

Rule 2390

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_))^(q_.), x_Symbol] :> Dist[1/e, Subst[Int[((f*x)/d)^q*(a + b*Log[c*x^
n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && E
qQ[e*f - d*g, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2393

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + (c*e*x)/g])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2394

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d + e*x
)^n]))/g, x] - Dist[(b*e*n)/g, Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2418

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*(RFx_), x_Sy
mbol] :> With[{u = ExpandIntegrand[(a + b*Log[c*(d + e*x)^n])^p, RFx, x]},
```

```
Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d, e, n}, x] && RationalFunctionQ[
RFx, x] && IntegerQ[p]
```

Rule 2486

```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))
^(r_.)]^(s_.), x_Symbol] :> Simp[((a + b*x)*Log[e*(f*(a + b*x)^p*(c + d*x)^
q)^r]^s)/b, x] + Dist[(q*r*s*(b*c - a*d))/b, Int[Log[e*(f*(a + b*x)^p*(c +
d*x)^q)^r]^(s - 1)/(c + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, p, q, r, s
}, x] && NeQ[b*c - a*d, 0] && EqQ[p + q, 0] && IGtQ[s, 0]
```

Rule 2524

```
Int[((a_.) + Log[(c_.)*(RFx_)^(p_.)]*(b_.))^(n_.)/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[(Log[d + e*x]*(a + b*Log[c*RFx^p])^n)/e, x] - Dist[(b*n*p)/e
, Int[(Log[d + e*x]*(a + b*Log[c*RFx^p])^(n - 1)*D[RFx, x])/RFx, x], x] /;
FreeQ[{a, b, c, d, e, p}, x] && RationalFunctionQ[RFx, x] && IGtQ[n, 0]
```

Rule 2525

```
Int[((a_.) + Log[(c_.)*(RFx_)^(p_.)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_))^(m_.
), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(a + b*Log[c*RFx^p])^n)/(e*(m + 1))
, x] - Dist[(b*n*p)/(e*(m + 1)), Int[SimplifyIntegrand[((d + e*x)^(m + 1)*(
a + b*Log[c*RFx^p])^(n - 1)*D[RFx, x])/RFx, x], x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && RationalFunctionQ[RFx, x] && IGtQ[n, 0] && (EqQ[n, 1] ||
IntegerQ[m]) && NeQ[m, -1]
```

Rule 2528

```
Int[((a_.) + Log[(c_.)*(RFx_)^(p_.)]*(b_.))^(n_.)*(RGx_), x_Symbol] :> With
[{u = ExpandIntegrand[(a + b*Log[c*RFx^p])^n, RGx, x]}, Int[u, x] /; SumQ[u
]] /; FreeQ[{a, b, c, p}, x] && RationalFunctionQ[RFx, x] && RationalFuncti
onQ[RGx, x] && IGtQ[n, 0]
```

Rubi steps

$$\int (ag+bgx)^2 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2 dx = \frac{g^2(a+bx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{3b} - \frac{(2Bn)\int \frac{(bc-ad)g^3(a+bx)^2\left(A\cdots\right.}{c+d\cdots}}{3bg}$$

$$= \frac{g^2(a+bx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{3b} - \frac{\left(2B(bc-ad)g^2n\right)\int \frac{(a+bx\cdots}{\cdots}}{3b\cdots}$$

$$= \frac{g^2(a+bx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{3b} - \frac{\left(2B(bc-ad)g^2n\right)\int\left(-\frac{b(\cdots}{\cdots}\right.}{\cdots}$$

$$= \frac{g^2(a+bx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{3b} - \frac{\left(2B(bc-ad)g^2n\right)\int(a+\cdots}{\cdots}$$

$$= \frac{2AB(bc-ad)^2g^2nx}{3d^2} - \frac{B(bc-ad)g^2n(a+bx)^2\left(A+B\log\left(e\left(\frac{a+b\cdots}{c+d\cdots}\right.\right.\right.}{3bd}$$

$$= \frac{2AB(bc-ad)^2g^2nx}{3d^2} + \frac{2B^2(bc-ad)^2g^2n(a+bx)\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{3bd^2}\cdots$$

$$= \frac{2AB(bc-ad)^2g^2nx}{3d^2} + \frac{2B^2(bc-ad)^2g^2n(a+bx)\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{3bd^2}\cdots$$

$$= \frac{2AB(bc-ad)^2g^2nx}{3d^2} + \frac{B^2(bc-ad)^2g^2n^2x}{3d^2} + \frac{2B^2(bc-ad)^2g^2n(a\cdots}{3b\cdots}$$

$$= \frac{2AB(bc-ad)^2g^2nx}{3d^2} + \frac{B^2(bc-ad)^2g^2n^2x}{3d^2} + \frac{2B^2(bc-ad)^2g^2n(a\cdots}{3b\cdots}$$

$$= \frac{2AB(bc-ad)^2g^2nx}{3d^2} + \frac{B^2(bc-ad)^2g^2n^2x}{3d^2} + \frac{2B^2(bc-ad)^2g^2n(a\cdots}{3b\cdots}$$

**Mathematica [A]** time = 0.23, size = 303, normalized size = 1.11

$$g^2\left(\frac{Bn(bc-ad)\left(-d^2(a+bx)^2\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)-2(bc-ad)^2\log(c+dx)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)+2Abdx(bc-ad)+2Bd(a+bx)(bc-ad)\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+B\right.}{d^3}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*g + b*g*x)^2*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2,x]
```

```
[Out] (g^2*((a + b*x)^3*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2 + (B*(b*c - a*d)
*n*(2*A*b*d*(b*c - a*d)*x + 2*B*d*(b*c - a*d)*(a + b*x)*Log[e*((a + b*x)/(c
 + d*x))^n] - d^2*(a + b*x)^2*(A + B*Log[e*((a + b*x)/(c + d*x))^n]) - 2*B*
(b*c - a*d)^2*n*Log[c + d*x] - 2*(b*c - a*d)^2*(A + B*Log[e*((a + b*x)/(c +
 d*x))^n])*Log[c + d*x] + B*(b*c - a*d)*n*(b*d*x + (-(b*c) + a*d)*Log[c + d
*x]) + B*(b*c - a*d)^2*n*((2*Log[(d*(a + b*x))/(-(b*c) + a*d)] - Log[c + d*
x])*Log[c + d*x] + 2*PolyLog[2, (b*(c + d*x))/(b*c - a*d)])))/d^3))/(3*b)
```

**fricas [F]** time = 0.80, size = 0, normalized size = 0.00

$$\text{integral}\left(A^2b^2g^2x^2 + 2\,A^2abg^2x + A^2a^2g^2 + \left(B^2b^2g^2x^2 + 2\,B^2abg^2x + B^2a^2g^2\right)\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)^2 + 2\left(ABb^2g^2\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*g*x+a*g)^2*(A+B*log(e*((b*x+a)/(d*x+c))^n))^2,x, algorithm="fr
icas")
```

```
[Out] integral(A^2*b^2*g^2*x^2 + 2*A^2*a*b*g^2*x + A^2*a^2*g^2 + (B^2*b^2*g^2*x^2
 + 2*B^2*a*b*g^2*x + B^2*a^2*g^2)*log(e*((b*x + a)/(d*x + c))^n)^2 + 2*(A*B
*b^2*g^2*x^2 + 2*A*B*a*b*g^2*x + A*B*a^2*g^2)*log(e*((b*x + a)/(d*x + c))^n
), x)
```

**giac [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*g*x+a*g)^2*(A+B*log(e*((b*x+a)/(d*x+c))^n))^2,x, algorithm="gi
ac")
```

```
[Out] Timed out
```

**maple [F]** time = 0.28, size = 0, normalized size = 0.00

$$\int \left(bgx + ag\right)^{2} \left(B\ln\left(e\left(\frac{bx+a}{dx+c}\right)^{n}\right) + A\right)^{2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*g*x+a*g)^2*(B*ln(e*((b*x+a)/(d*x+c))^n)+A)^2,x)
```

```
[Out] int((b*g*x+a*g)^2*(B*ln(e*((b*x+a)/(d*x+c))^n)+A)^2,x)
```

**maxima [B]** time = 11.47, size = 1501, normalized size = 5.48

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*g*x+a*g)^2*(A+B*log(e*((b*x+a)/(d*x+c))^n))^2,x, algorithm="ma
xima")
```

```
[Out] 2/3*A*B*b^2*g^2*x^3*log(e*(b*x/(d*x + c) + a/(d*x + c))^n) + 1/3*A^2*b^2*g^
2*x^3 + 2*A*B*a*b*g^2*x^2*log(e*(b*x/(d*x + c) + a/(d*x + c))^n) + A^2*a*b*
g^2*x^2 + 1/3*A*B*b^2*g^2*n*(2*a^3*log(b*x + a)/b^3 - 2*c^3*log(d*x + c)/d^
3 - ((b^2*c*d - a*b*d^2)*x^2 - 2*(b^2*c^2 - a^2*d^2)*x)/(b^2*d^2)) - 2*A*B*
a*b*g^2*n*(a^2*log(b*x + a)/b^2 - c^2*log(d*x + c)/d^2 + (b*c - a*d)*x/(b*d
)) + 2*A*B*a^2*g^2*n*(a*log(b*x + a)/b - c*log(d*x + c)/d) + 2*A*B*a^2*g^2*
x*log(e*(b*x/(d*x + c) + a/(d*x + c))^n) + A^2*a^2*g^2*x - 1/3*((3*g^2*n^2
+ 2*g^2*n*log(e))*b^2*c^3 - (7*g^2*n^2 + 6*g^2*n*log(e))*a*b*c^2*d + 2*(2*g
^2*n^2 + 3*g^2*n*log(e))*a^2*c*d^2)*B^2*log(d*x + c)/d^3 - 2/3*(b^3*c^3*g^2
*n^2 - 3*a*b^2*c^2*d*g^2*n^2 + 3*a^2*b*c*d^2*g^2*n^2 - a^3*d^3*g^2*n^2)*(lo
g(b*x + a)*log((b*d*x + a*d)/(b*c - a*d) + 1) + dilog(-(b*d*x + a*d)/(b*c -
 a*d)))*B^2/(b*d^3) + 1/3*(B^2*b^3*d^3*g^2*x^3*log(e)^2 - B^2*a^3*d^3*g^2*n
^2*log(b*x + a)^2 - (b^3*c*d^2*g^2*n*log(e) - (g^2*n*log(e) + 3*g^2*log(e)^
2)*a*b^2*d^3)*B^2*x^2 + 2*(b^3*c^3*g^2*n^2 - 3*a*b^2*c^2*d*g^2*n^2 + 3*a^2*
b*c*d^2*g^2*n^2)*B^2*log(b*x + a)*log(d*x + c) - (b^3*c^3*g^2*n^2 - 3*a*b^2
*c^2*d*g^2*n^2 + 3*a^2*b*c*d^2*g^2*n^2)*B^2*log(d*x + c)^2 + ((g^2*n^2 + 2*
g^2*n*log(e))*b^3*c^2*d - 2*(g^2*n^2 + 3*g^2*n*log(e))*a*b^2*c*d^2 + (g^2*n
^2 + 4*g^2*n*log(e) + 3*g^2*log(e)^2)*a^2*b*d^3)*B^2*x + (2*a*b^2*c^2*d*g^2
*n^2 - 5*a^2*b*c*d^2*g^2*n^2 + (3*g^2*n^2 + 2*g^2*n*log(e))*a^3*d^3)*B^2*lo
g(b*x + a) + (B^2*b^3*d^3*g^2*x^3 + 3*B^2*a*b^2*d^3*g^2*x^2 + 3*B^2*a^2*b*d
^3*g^2*x)*log((b*x + a)^n)^2 + (B^2*b^3*d^3*g^2*x^3 + 3*B^2*a*b^2*d^3*g^2*x
^2 + 3*B^2*a^2*b*d^3*g^2*x)*log((d*x + c)^n)^2 + (2*B^2*b^3*d^3*g^2*x^3*log
(e) + 2*B^2*a^3*d^3*g^2*n*log(b*x + a) - (b^3*c*d^2*g^2*n - (g^2*n + 6*g^2*
log(e))*a*b^2*d^3)*B^2*x^2 + 2*(b^3*c^2*d*g^2*n - 3*a*b^2*c*d^2*g^2*n + (2*
g^2*n + 3*g^2*log(e))*a^2*b*d^3)*B^2*x - 2*(b^3*c^3*g^2*n - 3*a*b^2*c^2*d*g
^2*n + 3*a^2*b*c*d^2*g^2*n)*B^2*log(d*x + c))*log((b*x + a)^n) - (2*B^2*b^3
```

```
*d^3*g^2*x^3*log(e) + 2*B^2*a^3*d^3*g^2*n*log(b*x + a) - (b^3*c*d^2*g^2*n -
 (g^2*n + 6*g^2*log(e))*a*b^2*d^3)*B^2*x^2 + 2*(b^3*c^2*d*g^2*n - 3*a*b^2*c
*d^2*g^2*n + (2*g^2*n + 3*g^2*log(e))*a^2*b*d^3)*B^2*x - 2*(b^3*c^3*g^2*n -
 3*a*b^2*c^2*d*g^2*n + 3*a^2*b*c*d^2*g^2*n)*B^2*log(d*x + c) + 2*(B^2*b^3*d
^3*g^2*x^3 + 3*B^2*a*b^2*d^3*g^2*x^2 + 3*B^2*a^2*b*d^3*g^2*x)*log((b*x + a)
^n))*log((d*x + c)^n))/(b*d^3)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.00

$$\int \left(a\,g+b\,g\,x\right)^2\,{\left(A+B\,\ln\left(e\,{\left(\frac{a+b\,x}{c+d\,x}\right)}^n\right)\right)}^2 \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*g + b*g*x)^2*(A + B*log(e*((a + b*x)/(c + d*x))^n))^2,x)
```

```
[Out] int((a*g + b*g*x)^2*(A + B*log(e*((a + b*x)/(c + d*x))^n))^2, x)
```

**sympy [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*g*x+a*g)**2*(A+B*ln(e*((b*x+a)/(d*x+c))**n))**2,x)
```

```
[Out] Timed out
```

## 3.13 $\int (ag+bgx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2\,dx$

**Optimal**. Leaf size=196

$$-\frac{Bgn(bc-ad)^2\log\left(\frac{bc-ad}{b(c+dx)}\right)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A+Bn\right)}{bd^2}-\frac{Bgn(a+bx)(bc-ad)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{bd}+\frac{g(a+}{}$$

```
[Out] -B*(-a*d+b*c)*g*n*(b*x+a)*(A+B*ln(e*((b*x+a)/(d*x+c))^n))/b/d+1/2*g*(b*x+a)
^2*(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2/b-B*(-a*d+b*c)^2*g*n*(A+B*n+B*ln(e*((b
*x+a)/(d*x+c))^n))*ln((-a*d+b*c)/b/(d*x+c))/b/d^2-B^2*(-a*d+b*c)^2*g*n^2*po
lylog(2,d*(b*x+a)/b/(d*x+c))/b/d^2
```

**Rubi [A]** time = 0.44, antiderivative size = 309, normalized size of antiderivative = 1.58, number of steps used = 15, number of rules used = 12, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {2525, 12, 2528, 2486, 31, 2524, 2418, 2394, 2393, 2391, 2390, 2301}

$$-\frac{B^2gn^2(bc-ad)^2\text{PolyLog}\left(2,\frac{b(c+dx)}{bc-ad}\right)}{bd^2}+\frac{Bgn(bc-ad)^2\log(c+dx)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{bd^2}+\frac{g(a+bx)^2\left(B\log\left(e\right.\right.}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[(a*g + b*g*x)*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2,x]
```

```
[Out] -((A*B*(b*c - a*d)*g*n*x)/d) - (B^2*(b*c - a*d)*g*n*(a + b*x)*Log[e*((a + b
*x)/(c + d*x))^n])/(b*d) + (g*(a + b*x)^2*(A + B*Log[e*((a + b*x)/(c + d*x)
)^n])^2)/(2*b) + (B^2*(b*c - a*d)^2*g*n^2*Log[c + d*x])/(b*d^2) - (B^2*(b*c
 - a*d)^2*g*n^2*Log[-((d*(a + b*x))/(b*c - a*d))]*Log[c + d*x])/(b*d^2) + (
B*(b*c - a*d)^2*g*n*(A + B*Log[e*((a + b*x)/(c + d*x))^n])*Log[c + d*x])/(b
*d^2) + (B^2*(b*c - a*d)^2*g*n^2*Log[c + d*x]^2)/(2*b*d^2) - (B^2*(b*c - a*
d)^2*g*n^2*PolyLog[2, (b*(c + d*x))/(b*c - a*d)])/(b*d^2)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 2301

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

Rule 2390

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_))^(q_.), x_Symbol] :> Dist[1/e, Subst[Int[((f*x)/d)^q*(a + b*Log[c*x^
n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && E
qQ[e*f - d*g, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2393

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + (c*e*x)/g])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2394

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d + e*x
)^n]))/g, x] - Dist[(b*e*n)/g, Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2418

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*(RFx_), x_Sy
mbol] :> With[{u = ExpandIntegrand[(a + b*Log[c*(d + e*x)^n])^p, RFx, x]},
Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d, e, n}, x] && RationalFunctionQ[
RFx, x] && IntegerQ[p]
```

Rule 2486

```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))
^(r_.)]^(s_.), x_Symbol] :> Simp[((a + b*x)*Log[e*(f*(a + b*x)^p*(c + d*x)^
q)^r]^s)/b, x] + Dist[(q*r*s*(b*c - a*d))/b, Int[Log[e*(f*(a + b*x)^p*(c +
d*x)^q)^r]^(s - 1)/(c + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, p, q, r, s
```

```
}, x] && NeQ[b*c - a*d, 0] && EqQ[p + q, 0] && IGtQ[s, 0]
```

Rule 2524

```
Int[((a_.) + Log[(c_.)*(RFx_)^(p_.)]*(b_.))^(n_.)/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[(Log[d + e*x]*(a + b*Log[c*RFx^p])^n)/e, x] - Dist[(b*n*p)/e
, Int[(Log[d + e*x]*(a + b*Log[c*RFx^p])^(n - 1)*D[RFx, x])/RFx, x], x] /;
FreeQ[{a, b, c, d, e, p}, x] && RationalFunctionQ[RFx, x] && IGtQ[n, 0]
```

Rule 2525

```
Int[((a_.) + Log[(c_.)*(RFx_)^(p_.)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_))^(m_.
), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(a + b*Log[c*RFx^p])^n)/(e*(m + 1))
, x] - Dist[(b*n*p)/(e*(m + 1)), Int[SimplifyIntegrand[((d + e*x)^(m + 1)*(
a + b*Log[c*RFx^p])^(n - 1)*D[RFx, x])/RFx, x], x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && RationalFunctionQ[RFx, x] && IGtQ[n, 0] && (EqQ[n, 1] ||
IntegerQ[m]) && NeQ[m, -1]
```

Rule 2528

```
Int[((a_.) + Log[(c_.)*(RFx_)^(p_.)]*(b_.))^(n_.)*(RGx_), x_Symbol] :> With
[{u = ExpandIntegrand[(a + b*Log[c*RFx^p])^n, RGx, x]}, Int[u, x] /; SumQ[u
]] /; FreeQ[{a, b, c, p}, x] && RationalFunctionQ[RFx, x] && RationalFuncti
onQ[RGx, x] && IGtQ[n, 0]
```

Rubi steps

$$\int (ag+bgx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2 dx = \frac{g(a+bx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{2b} - \frac{(Bn)\int \frac{(bc-ad)g^2(a+bx)\left(A+B\,\mathrm{l}\right.}{c+dx}}{bg}$$

$$= \frac{g(a+bx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{2b} - \frac{(B(bc-ad)gn)\int \frac{(a+bx)\left(A\right.}{}}{b}$$

$$= \frac{g(a+bx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{2b} - \frac{(B(bc-ad)gn)\int\left(\frac{b\left(A+B\,\mathrm{lo}\right.}{}\right.}{}$$

$$= \frac{g(a+bx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{2b} - \frac{(B(bc-ad)gn)\int\left(A+B\right.}{d}$$

$$= -\frac{AB(bc-ad)gnx}{d} + \frac{g(a+bx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{2b} + \frac{B(bc}{}$$

$$= -\frac{AB(bc-ad)gnx}{d} - \frac{B^2(bc-ad)gn(a+bx)\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{bd} + \frac{g(}{}$$

$$= -\frac{AB(bc-ad)gnx}{d} - \frac{B^2(bc-ad)gn(a+bx)\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{bd} + \frac{g(}{}$$

$$= -\frac{AB(bc-ad)gnx}{d} - \frac{B^2(bc-ad)gn(a+bx)\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{bd} + \frac{g(}{}$$

$$= -\frac{AB(bc-ad)gnx}{d} - \frac{B^2(bc-ad)gn(a+bx)\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{bd} + \frac{g(}{}$$

$$= -\frac{AB(bc-ad)gnx}{d} - \frac{B^2(bc-ad)gn(a+bx)\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{bd} + \frac{g(}{}$$

**Mathematica [A]** time = 0.20, size = 215, normalized size = 1.10

$$\frac{g\left((a+bx)^2\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2-\frac{Bn(bc-ad)\left(-2(bc-ad)\log(c+dx)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)+2Bd(a+bx)\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+Bn(bc-ad)\left(2\mathrm{L}\right.\right.}{d^2}\right.}{2b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*g + b*g*x)*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2,x]
```

```
[Out] (g*((a + b*x)^2*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2 - (B*(b*c - a*d)*n
*(2*A*b*d*x + 2*B*d*(a + b*x)*Log[e*((a + b*x)/(c + d*x))^n] - 2*B*(b*c - a
*d)*n*Log[c + d*x] - 2*(b*c - a*d)*(A + B*Log[e*((a + b*x)/(c + d*x))^n])*L
og[c + d*x] + B*(b*c - a*d)*n*((2*Log[(d*(a + b*x))/(-(b*c) + a*d)] - Log[c
 + d*x])*Log[c + d*x] + 2*PolyLog[2, (b*(c + d*x))/(b*c - a*d)])))/d^2))/(2
*b)
```

**fricas [F]** time = 0.94, size = 0, normalized size = 0.00

$$\text{integral}\left(A^2bgx+A^2ag+\left(B^2bgx+B^2ag\right)\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)^2+2\left(ABbgx+ABag\right)\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*g*x+a*g)*(A+B*log(e*((b*x+a)/(d*x+c))^n))^2,x, algorithm="fric
as")
```

```
[Out] integral(A^2*b*g*x + A^2*a*g + (B^2*b*g*x + B^2*a*g)*log(e*((b*x + a)/(d*x
+ c))^n)^2 + 2*(A*B*b*g*x + A*B*a*g)*log(e*((b*x + a)/(d*x + c))^n), x)
```

**giac [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*g*x+a*g)*(A+B*log(e*((b*x+a)/(d*x+c))^n))^2,x, algorithm="giac
")
```

```
[Out] Timed out
```

**maple [F]** time = 0.12, size = 0, normalized size = 0.00

$$\int\left(bgx+ag\right)\left(B\ln\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*g*x+a*g)*(B*ln(e*((b*x+a)/(d*x+c))^n)+A)^2,x)
```

```
[Out] int((b*g*x+a*g)*(B*ln(e*((b*x+a)/(d*x+c))^n)+A)^2,x)
```

**maxima [B]** time = 7.48, size = 828, normalized size = 4.22

$$ABbgx^2\log\left(e\left(\frac{bx}{dx+c}+\frac{a}{dx+c}\right)^n\right)+\frac{1}{2}A^2bgx^2-ABbgn\left(\frac{a^2\log(bx+a)}{b^2}-\frac{c^2\log(dx+c)}{d^2}+\frac{(bc-ad)x}{bd}\right)+2\,ABa$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*g*x+a*g)*(A+B*log(e*((b*x+a)/(d*x+c))^n))^2,x, algorithm="maxi
ma")
```

```
[Out] A*B*b*g*x^2*log(e*(b*x/(d*x + c) + a/(d*x + c))^n) + 1/2*A^2*b*g*x^2 - A*B*
b*g*n*(a^2*log(b*x + a)/b^2 - c^2*log(d*x + c)/d^2 + (b*c - a*d)*x/(b*d)) +
 2*A*B*a*g*n*(a*log(b*x + a)/b - c*log(d*x + c)/d) + 2*A*B*a*g*x*log(e*(b*x
/(d*x + c) + a/(d*x + c))^n) + A^2*a*g*x + ((g*n^2 + g*n*log(e))*b*c^2 - (g
*n^2 + 2*g*n*log(e))*a*c*d)*B^2*log(d*x + c)/d^2 + (b^2*c^2*g*n^2 - 2*a*b*c
*d*g*n^2 + a^2*d^2*g*n^2)*(log(b*x + a)*log((b*d*x + a*d)/(b*c - a*d) + 1)
+ dilog(-(b*d*x + a*d)/(b*c - a*d)))*B^2/(b*d^2) - 1/2*(B^2*a^2*d^2*g*n^2*l
og(b*x + a)^2 - B^2*b^2*d^2*g*x^2*log(e)^2 + 2*(b^2*c^2*g*n^2 - 2*a*b*c*d*g
*n^2)*B^2*log(b*x + a)*log(d*x + c) - (b^2*c^2*g*n^2 - 2*a*b*c*d*g*n^2)*B^2
*log(d*x + c)^2 + 2*(b^2*c*d*g*n*log(e) - (g*n*log(e) + g*log(e)^2)*a*b*d^2
)*B^2*x + 2*(a*b*c*d*g*n^2 - (g*n^2 + g*n*log(e))*a^2*d^2)*B^2*log(b*x + a)
 - (B^2*b^2*d^2*g*x^2 + 2*B^2*a*b*d^2*g*x)*log((b*x + a)^n)^2 - (B^2*b^2*d^
2*g*x^2 + 2*B^2*a*b*d^2*g*x)*log((d*x + c)^n)^2 - 2*(B^2*b^2*d^2*g*x^2*log(
e) + B^2*a^2*d^2*g*n*log(b*x + a) - (b^2*c*d*g*n - (g*n + 2*g*log(e))*a*b*d
^2)*B^2*x + (b^2*c^2*g*n - 2*a*b*c*d*g*n)*B^2*log(d*x + c))*log((b*x + a)^n
) + 2*(B^2*b^2*d^2*g*x^2*log(e) + B^2*a^2*d^2*g*n*log(b*x + a) - (b^2*c*d*g
*n - (g*n + 2*g*log(e))*a*b*d^2)*B^2*x + (b^2*c^2*g*n - 2*a*b*c*d*g*n)*B^2*
log(d*x + c) + (B^2*b^2*d^2*g*x^2 + 2*B^2*a*b*d^2*g*x)*log((b*x + a)^n))*lo
g((d*x + c)^n))/(b*d^2)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.01

$$\int \left(a\,g+b\,g\,x\right)\,{\left(A+B\,\ln\left(e\,{\left(\frac{a+b\,x}{c+d\,x}\right)}^n\right)\right)}^2 \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*g + b*g*x)*(A + B*log(e*((a + b*x)/(c + d*x))^n))^2,x)
```

```
[Out] int((a*g + b*g*x)*(A + B*log(e*((a + b*x)/(c + d*x))^n))^2, x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$g\left(\int A^2a\,dx+\int A^2bx\,dx+\int B^2a\log\left(e\left(\frac{a}{c+dx}+\frac{bx}{c+dx}\right)^n\right)^2dx+\int 2ABa\log\left(e\left(\frac{a}{c+dx}+\frac{bx}{c+dx}\right)^n\right)dx+\ldots\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*g*x+a*g)*(A+B*ln(e*((b*x+a)/(d*x+c))**n))**2,x)
```

```
[Out] g*(Integral(A**2*a, x) + Integral(A**2*b*x, x) + Integral(B**2*a*log(e*(a/(
c + d*x) + b*x/(c + d*x))**n)**2, x) + Integral(2*A*B*a*log(e*(a/(c + d*x)
+ b*x/(c + d*x))**n), x) + Integral(B**2*b*x*log(e*(a/(c + d*x) + b*x/(c +
d*x))**n)**2, x) + Integral(2*A*B*b*x*log(e*(a/(c + d*x) + b*x/(c + d*x))**
n), x))
```

# 3.14 $\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{ag+bgx}\,dx$

**Optimal**. Leaf size=138

$$\frac{2Bn\text{Li}_2\left(\frac{b(c+dx)}{d(a+bx)}\right)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{bg}-\frac{\log\left(1-\frac{b(c+dx)}{d(a+bx)}\right)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{bg}+\frac{2B^2n^2\text{Li}_3\left(\frac{b(c+dx)}{d(a+bx)}\right)}{bg}$$

```
[Out] -(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2*ln(1-b*(d*x+c)/d/(b*x+a))/b/g+2*B*n*(A+B
*ln(e*((b*x+a)/(d*x+c))^n))*polylog(2,b*(d*x+c)/d/(b*x+a))/b/g+2*B^2*n^2*po
lylog(3,b*(d*x+c)/d/(b*x+a))/b/g
```

**Rubi [B]** time = 3.57, antiderivative size = 789, normalized size of antiderivative = 5.72, number of steps used = 45, number of rules used = 23, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.657, Rules used = {2524, 2528, 2418, 2390, 2301, 2394, 2393, 2391, 6688, 12, 6742, 2500, 2440, 2434, 2433, 2375, 2317, 2374, 6589, 2499, 2302, 30, 2396}

$$\frac{2ABn\text{PolyLog}\left(2,-\frac{d(a+bx)}{bc-ad}\right)}{bg}-\frac{2B^2n\text{PolyLog}\left(2,-\frac{d(a+bx)}{bc-ad}\right)\left(-\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+\log\left((a+bx)^n\right)+\log\left((c+dx)^{-n}\right)\right.}{bg}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2/(a*g + b*g*x),x]
```

```
[Out] -((A*B*n*Log[g*(a + b*x)]^2)/(b*g)) + (B^2*n^2*Log[g*(a + b*x)]^3)/(3*b*g)
- (B^2*n^2*Log[g*(a + b*x)]^2*Log[-c - d*x])/(b*g) + (2*B^2*n*Log[g*(a + b*
x)]*Log[(a + b*x)^n]*Log[-c - d*x])/(b*g) - (B^2*Log[(a + b*x)^n]^2*Log[-c
- d*x])/(b*g) + (B^2*n^2*Log[g*(a + b*x)]^2*Log[(b*(c + d*x))/(b*c - a*d)])
/(b*g) + (B^2*Log[(a + b*x)^n]^2*Log[(b*(c + d*x))/(b*c - a*d)])/(b*g) + (B
^2*Log[-((d*(a + b*x))/(b*c - a*d))]*Log[(c + d*x)^(-n)]^2)/(b*g) - (B^2*Lo
g[g*(a + b*x)]*Log[(c + d*x)^(-n)]^2)/(b*g) + ((A + B*Log[e*((a + b*x)/(c +
 d*x))^n])^2*Log[a*g + b*g*x])/(b*g) + (2*A*B*n*Log[(b*(c + d*x))/(b*c - a*
d)]*Log[a*g + b*g*x])/(b*g) - (2*B^2*n*Log[(b*(c + d*x))/(b*c - a*d)]*(Log[
(a + b*x)^n] - Log[e*((a + b*x)/(c + d*x))^n] + Log[(c + d*x)^(-n)])*Log[a*
g + b*g*x])/(b*g) - (B^2*n*Log[e*((a + b*x)/(c + d*x))^n]*Log[a*g + b*g*x]^
2)/(b*g) - (B^2*n^2*Log[(b*(c + d*x))/(b*c - a*d)]*Log[a*g + b*g*x]^2)/(b*g
) + (2*A*B*n*PolyLog[2, -((d*(a + b*x))/(b*c - a*d))])/(b*g) + (2*B^2*n*Log
[(a + b*x)^n]*PolyLog[2, -((d*(a + b*x))/(b*c - a*d))])/(b*g) - (2*B^2*n*(L
og[(a + b*x)^n] - Log[e*((a + b*x)/(c + d*x))^n] + Log[(c + d*x)^(-n)])*Pol
yLog[2, -((d*(a + b*x))/(b*c - a*d))])/(b*g) - (2*B^2*n*Log[(c + d*x)^(-n)]
```

```
*PolyLog[2, (b*(c + d*x))/(b*c - a*d)])/(b*g) - (2*B^2*n^2*PolyLog[3, -((d*
(a + b*x))/(b*c - a*d))])/(b*g) - (2*B^2*n^2*PolyLog[3, (b*(c + d*x))/(b*c
- a*d)])/(b*g)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2301

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

Rule 2302

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/(x_), x_Symbol] :> Dist[1/(
b*n), Subst[Int[x^p, x], x, a + b*Log[c*x^n]], x] /; FreeQ[{a, b, c, n, p},
 x]
```

Rule 2317

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symb
ol] :> Simp[(Log[1 + (e*x)/d]*(a + b*Log[c*x^n])^p)/e, x] - Dist[(b*n*p)/e,
 Int[(Log[1 + (e*x)/d]*(a + b*Log[c*x^n])^(p - 1))/x, x], x] /; FreeQ[{a, b
, c, d, e, n}, x] && IGtQ[p, 0]
```

Rule 2374

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> -Simp[(PolyLog[2, -(d*f*x^m)]*(a + b*Log[c*x
^n])^p)/m, x] + Dist[(b*n*p)/m, Int[(PolyLog[2, -(d*f*x^m)]*(a + b*Log[c*x^
n])^(p - 1))/x, x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p, 0]
&& EqQ[d*e, 1]
```

Rule 2375

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))^(r_.)]*((a_.) + Log[(c_.)*(x_)^(n_
.)]*(b_.))^(p_.))/(x_), x_Symbol] :> Simp[(Log[d*(e + f*x^m)^r]*(a + b*Log[
c*x^n])^(p + 1))/(b*n*(p + 1)), x] - Dist[(f*m*r)/(b*n*(p + 1)), Int[(x^(m
- 1)*(a + b*Log[c*x^n])^(p + 1))/(e + f*x^m), x], x] /; FreeQ[{a, b, c, d,
```

```
e, f, r, m, n}, x] && IGtQ[p, 0] && NeQ[d*e, 1]
```

Rule 2390

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_))^(q_.), x_Symbol] :> Dist[1/e, Subst[Int[((f*x)/d)^q*(a + b*Log[c*x^
n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && E
qQ[e*f - d*g, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2393

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + (c*e*x)/g])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2394

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d + e*x
)^n]))/g, x] - Dist[(b*e*n)/g, Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2396

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)/((f_.) + (g_.
)*(x_)), x_Symbol] :> Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d
+ e*x)^n])^p)/g, x] - Dist[(b*e*n*p)/g, Int[(Log[(e*(f + g*x))/(e*f - d*g)]
*(a + b*Log[c*(d + e*x)^n])^(p - 1))/(d + e*x), x], x] /; FreeQ[{a, b, c, d
, e, f, g, n, p}, x] && NeQ[e*f - d*g, 0] && IGtQ[p, 1]
```

Rule 2418

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*(RFx_), x_Sy
mbol] :> With[{u = ExpandIntegrand[(a + b*Log[c*(d + e*x)^n])^p, RFx, x]},
Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d, e, n}, x] && RationalFunctionQ[
RFx, x] && IntegerQ[p]
```

Rule 2433

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + Log
[(h_.)*((i_.) + (j_.)*(x_))^(m_.)]*(g_.))*((k_.) + (l_.)*(x_))^(r_.), x_Sym
bol] :> Dist[1/e, Subst[Int[((k*x)/d)^r*(a + b*Log[c*x^n])^p*(f + g*Log[h*(
(e*i - d*j)/e + (j*x)/e)^m]), x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e,
f, g, h, i, j, k, l, n, p, r}, x] && EqQ[e*k - d*l, 0]
```

Rule 2434

```
Int[(((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + Log[(h_.
)*((i_.) + (j_.)*(x_))^(m_.)]*(g_.)))/(x_), x_Symbol] :> Simp[Log[x]*(a + b
*Log[c*(d + e*x)^n])*(f + g*Log[h*(i + j*x)^m]), x] + (-Dist[e*g*m, Int[(Lo
g[x]*(a + b*Log[c*(d + e*x)^n]))/(d + e*x), x], x] - Dist[b*j*n, Int[(Log[x
]*(f + g*Log[h*(i + j*x)^m]))/(i + j*x), x], x]) /; FreeQ[{a, b, c, d, e, f
, g, h, i, j, m, n}, x] && EqQ[e*i - d*j, 0]
```

Rule 2440

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + Log[(h_.)
*((i_.) + (j_.)*(x_))^(m_.)]*(g_.))*((k_) + (l_.)*(x_))^(r_.), x_Symbol] :>
 Dist[1/l, Subst[Int[x^r*(a + b*Log[c*(-((e*k - d*l)/l) + (e*x)/l)^n])*(f +
 g*Log[h*(-((j*k - i*l)/l) + (j*x)/l)^m]), x], x, k + l*x], x] /; FreeQ[{a,
 b, c, d, e, f, g, h, i, j, k, l, m, n}, x] && IntegerQ[r]
```

Rule 2499

```
Int[(Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.)
)^(r_.)]*((s_.) + Log[(i_.)*((g_.) + (h_.)*(x_))^(n_.)]*(t_.))^(m_.))/((j_.
) + (k_.)*(x_)), x_Symbol] :> Simp[((s + t*Log[i*(g + h*x)^n])^(m + 1)*Log[
e*(f*(a + b*x)^p*(c + d*x)^q)^r])/(k*n*t*(m + 1)), x] + (-Dist[(b*p*r)/(k*n
*t*(m + 1)), Int[(s + t*Log[i*(g + h*x)^n])^(m + 1)/(a + b*x), x], x] - Dis
t[(d*q*r)/(k*n*t*(m + 1)), Int[(s + t*Log[i*(g + h*x)^n])^(m + 1)/(c + d*x)
, x], x]) /; FreeQ[{a, b, c, d, e, f, g, h, i, j, k, s, t, m, n, p, q, r},
x] && NeQ[b*c - a*d, 0] && EqQ[h*j - g*k, 0] && IGtQ[m, 0]
```

Rule 2500

```
Int[(Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.)
)^(r_.)]*((s_.) + Log[(i_.)*((g_.) + (h_.)*(x_))^(n_.)]*(t_.)))/((j_.) + (k
_.)*(x_)), x_Symbol] :> Dist[Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r] - Log[(a
+ b*x)^(p*r)] - Log[(c + d*x)^(q*r)], Int[(s + t*Log[i*(g + h*x)^n])/(j + k
*x), x], x] + (Int[(Log[(a + b*x)^(p*r)]*(s + t*Log[i*(g + h*x)^n]))/(j + k
*x), x] + Int[(Log[(c + d*x)^(q*r)]*(s + t*Log[i*(g + h*x)^n]))/(j + k*x),
x]) /; FreeQ[{a, b, c, d, e, f, g, h, i, j, k, s, t, n, p, q, r}, x] && NeQ
[b*c - a*d, 0]
```

Rule 2524

```
Int[((a_.) + Log[(c_.)*(RFx_)^(p_.)]*(b_.))^(n_.)/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[(Log[d + e*x]*(a + b*Log[c*RFx^p])^n)/e, x] - Dist[(b*n*p)/e
, Int[(Log[d + e*x]*(a + b*Log[c*RFx^p])^(n - 1)*D[RFx, x])/RFx, x], x] /;
FreeQ[{a, b, c, d, e, p}, x] && RationalFunctionQ[RFx, x] && IGtQ[n, 0]
```

Rule 2528

```
Int[((a_.) + Log[(c_.)*(RFx_)^(p_.)]*(b_.))^(n_.)*(RGx_), x_Symbol] :> With
[{u = ExpandIntegrand[(a + b*Log[c*RFx^p])^n, RGx, x]}, Int[u, x] /; SumQ[u
]] /; FreeQ[{a, b, c, p}, x] && RationalFunctionQ[RFx, x] && RationalFuncti
onQ[RGx, x] && IGtQ[n, 0]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6688

```
Int[u_, x_Symbol] :> With[{v = SimplifyIntegrand[u, x]}, Int[v, x] /; Simpl
erIntegrandQ[v, u, x]]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{ag+bgx}\,dx = \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2\log(ag+bgx)}{bg} - \frac{(2Bn)\int \frac{(c+dx)\left(-\frac{d(a+bx)}{(c+dx)^2}+\frac{b}{c+dx}\right)\left(A+B\log\left(e\left(\cdots\right.\right.\right.}{a+bx}}{bg}$$

$$= \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2\log(ag+bgx)}{bg} - \frac{(2Bn)\int \frac{(bc-ad)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\log(ag\cdots}{(a+bx)(c+dx)}}{bg}$$

$$= \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2\log(ag+bgx)}{bg} - \frac{(2B(bc-ad)n)\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\log\cdots}{(a+bx)(c+dx)}}{bg}$$

$$= \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2\log(ag+bgx)}{bg} - \frac{(2B(bc-ad)n)\int \left(\frac{d\left(-A-B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right.\right.\cdots}{(bc-ad)(c+\cdots}\right.}{\cdots}$$

$$= \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2\log(ag+bgx)}{bg} - \frac{(2Bn)\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\log(ag+bgx)}{a+bx}\,d\cdots}{g}$$

$$= \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2\log(ag+bgx)}{bg} - \frac{(2Bn)\int \left(\frac{A\log(ag+bgx)}{a+bx} + \frac{B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right.\cdots}{a+b\cdots}\right.}{g}$$

$$= \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2\log(ag+bgx)}{bg} - \frac{(2ABn)\int \frac{\log(ag+bgx)}{a+bx}\,dx}{g} - \frac{\left(2B^2n\right)\int\cdots}{\cdots}$$

$$= \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2\log(ag+bgx)}{bg} + \frac{2ABn\log\left(\frac{b(c+dx)}{bc-ad}\right)\log(ag+bgx)}{bg} - \cdots$$

$$= \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2\log(ag+bgx)}{bg} + \frac{2ABn\log\left(\frac{b(c+dx)}{bc-ad}\right)\log(ag+bgx)}{bg} - \cdots$$

$$= -\frac{ABn\log^2(g(a+bx))}{bg} + \frac{2B^2n\log(g(a+bx))\log\left((a+bx)^n\right)\log(-c-dx)}{bg} - \frac{B\cdots}{\cdots}$$

$$= -\frac{ABn\log^2(g(a+bx))}{bg} + \frac{2B^2n\log(g(a+bx))\log\left((a+bx)^n\right)\log(-c-dx)}{bg} + \frac{B\cdots}{\cdots}$$

**Mathematica [B]** time = 0.43, size = 537, normalized size = 3.89

$$3Bn\left(-2\left(\mathrm{Li}_2\left(\frac{b(c+dx)}{bc-ad}\right)+\log\left(\frac{c}{d}+x\right)\log\left(\frac{d(a+bx)}{ad-bc}\right)\right)-2\log(a+bx)\left(-\log\left(\frac{a+bx}{c+dx}\right)+\log\left(\frac{a}{b}+x\right)-\log\left(\frac{c}{d}+x\right)\right)+$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2/(a*g + b*g*x),x]
```

```
[Out] (3*Log[a + b*x]*(A + B*Log[e*((a + b*x)/(c + d*x))^n] - B*n*Log[(a + b*x)/(
c + d*x)])^2 + 3*B*n*(A + B*Log[e*((a + b*x)/(c + d*x))^n] - B*n*Log[(a + b
*x)/(c + d*x)])*(Log[a/b + x]^2 - 2*Log[a + b*x]*(Log[a/b + x] - Log[c/d +
x] - Log[(a + b*x)/(c + d*x)]) - 2*(Log[c/d + x]*Log[(d*(a + b*x))/(-(b*c)
+ a*d)] + PolyLog[2, (b*(c + d*x))/(b*c - a*d)])) + B^2*n^2*(Log[a/b + x]^3
 + 3*Log[c/d + x]^2*Log[(d*(a + b*x))/(-(b*c) + a*d)] + 3*Log[a + b*x]*(-Lo
g[a/b + x] + Log[c/d + x] + Log[(a + b*x)/(c + d*x)])^2 + 3*Log[a/b + x]^2*
(-Log[c/d + x] + Log[(b*(c + d*x))/(b*c - a*d)]) + 6*Log[a/b + x]*PolyLog[2
, (d*(a + b*x))/(-(b*c) + a*d)] + 6*Log[c/d + x]*PolyLog[2, (b*(c + d*x))/(
b*c - a*d)] - 3*(Log[a/b + x] - Log[c/d + x] - Log[(a + b*x)/(c + d*x)])*(L
og[a/b + x]^2 - 2*(Log[c/d + x]*Log[(d*(a + b*x))/(-(b*c) + a*d)] + PolyLog
[2, (b*(c + d*x))/(b*c - a*d)])) - 6*PolyLog[3, (d*(a + b*x))/(-(b*c) + a*d
)] - 6*PolyLog[3, (b*(c + d*x))/(b*c - a*d)]))/(3*b*g)
```

**fricas [F]** time = 0.68, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{B^2\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)^2+2\,AB\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A^2}{bgx+ag},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))^2/(b*g*x+a*g),x, algorithm="fric
as")
```

```
[Out] integral((B^2*log(e*((b*x + a)/(d*x + c))^n)^2 + 2*A*B*log(e*((b*x + a)/(d*
x + c))^n) + A^2)/(b*g*x + a*g), x)
```

**giac [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))^2/(b*g*x+a*g),x, algorithm="giac
")
```

```
[Out] Timed out
```

**maple** [F] time = 0.29, size = 0, normalized size = 0.00

$$\int \frac{\left(B\ln\left(e\left(\frac{bx+a}{dx+c}\right)^{n}\right)+A\right)^{2}}{bgx+ag}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*ln(e*((b*x+a)/(d*x+c))^n)+A)^2/(b*g*x+a*g),x)
```

```
[Out] int((B*ln(e*((b*x+a)/(d*x+c))^n)+A)^2/(b*g*x+a*g),x)
```

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{B^{2}\log\left(bx+a\right)\log\left(\left(dx+c\right)^{n}\right)^{2}}{bg}+\frac{A^{2}\log\left(bgx+ag\right)}{bg}-\int -\frac{B^{2}bc\log(e)^{2}+2\,ABbc\log(e)+\left(B^{2}bdx+B^{2}bc\right)\log\left((b\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))^2/(b*g*x+a*g),x, algorithm="maxi
ma")
```

```
[Out] B^2*log(b*x + a)*log((d*x + c)^n)^2/(b*g) + A^2*log(b*g*x + a*g)/(b*g) - in
tegrate(-(B^2*b*c*log(e)^2 + 2*A*B*b*c*log(e) + (B^2*b*d*x + B^2*b*c)*log((
b*x + a)^n)^2 + (B^2*b*d*log(e)^2 + 2*A*B*b*d*log(e))*x + 2*(B^2*b*c*log(e)
 + A*B*b*c + (B^2*b*d*log(e) + A*B*b*d)*x)*log((b*x + a)^n) - 2*(B^2*b*c*lo
g(e) + A*B*b*c + (B^2*b*d*log(e) + A*B*b*d)*x + (B^2*b*d*n*x + B^2*a*d*n)*l
og(b*x + a) + (B^2*b*d*x + B^2*b*c)*log((b*x + a)^n))*log((d*x + c)^n))/(b^
2*d*g*x^2 + a*b*c*g + (b^2*c*g + a*b*d*g)*x), x)
```

**mupad** [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\left(A+B\,\ln\left(e\,{\left(\frac{a+b\,x}{c+d\,x}\right)}^{n}\right)\right)}^{2}}{a\,g+b\,g\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*log(e*((a + b*x)/(c + d*x))^n))^2/(a*g + b*g*x),x)
```

```
[Out] int((A + B*log(e*((a + b*x)/(c + d*x))^n))^2/(a*g + b*g*x), x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{A^2}{a+bx}\,dx + \int \frac{B^2 \log\left(e\left(\frac{a}{c+dx}+\frac{bx}{c+dx}\right)^n\right)^2}{a+bx}\,dx + \int \frac{2AB\log\left(e\left(\frac{a}{c+dx}+\frac{bx}{c+dx}\right)^n\right)}{a+bx}\,dx}{g}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*ln(e*((b*x+a)/(d*x+c))**n))**2/(b*g*x+a*g),x)
```

```
[Out] (Integral(A**2/(a + b*x), x) + Integral(B**2*log(e*(a/(c + d*x) + b*x/(c +
d*x))**n)**2/(a + b*x), x) + Integral(2*A*B*log(e*(a/(c + d*x) + b*x/(c + d
*x))**n)/(a + b*x), x))/g
```

## 3.15 $\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag+bgx)^2}\,dx$

**Optimal**. Leaf size=136

$$-\frac{2Bn(c+dx)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{g^2(a+bx)(bc-ad)}-\frac{(c+dx)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{g^2(a+bx)(bc-ad)}-\frac{2B^2n^2(c+dx)}{g^2(a+bx)(bc-ad)}$$

```
[Out] -2*B^2*n^2*(d*x+c)/(-a*d+b*c)/g^2/(b*x+a)-2*B*n*(d*x+c)*(A+B*ln(e*((b*x+a)/
(d*x+c))^n))/(-a*d+b*c)/g^2/(b*x+a)-(d*x+c)*(A+B*ln(e*((b*x+a)/(d*x+c))^n))
^2/(-a*d+b*c)/g^2/(b*x+a)
```

**Rubi [C]** time = 0.84, antiderivative size = 512, normalized size of antiderivative = 3.76, number of steps used = 24, number of rules used = 11, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.314, Rules used = {2525, 12, 2528, 44, 2524, 2418, 2390, 2301, 2394, 2393, 2391}

$$-\frac{2B^2dn^2\text{PolyLog}\left(2,-\frac{d(a+bx)}{bc-ad}\right)}{bg^2(bc-ad)}-\frac{2B^2dn^2\text{PolyLog}\left(2,\frac{b(c+dx)}{bc-ad}\right)}{bg^2(bc-ad)}-\frac{2Bdn\log(a+bx)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{bg^2(bc-ad)}-\frac{2Bn\left(B\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2/(a*g + b*g*x)^2,x]
```

```
[Out] (-2*B^2*n^2)/(b*g^2*(a + b*x)) - (2*B^2*d*n^2*Log[a + b*x])/(b*(b*c - a*d)*
g^2) + (B^2*d*n^2*Log[a + b*x]^2)/(b*(b*c - a*d)*g^2) - (2*B*n*(A + B*Log[e
*((a + b*x)/(c + d*x))^n]))/(b*g^2*(a + b*x)) - (2*B*d*n*Log[a + b*x]*(A +
B*Log[e*((a + b*x)/(c + d*x))^n]))/(b*(b*c - a*d)*g^2) - (A + B*Log[e*((a +
 b*x)/(c + d*x))^n])^2/(b*g^2*(a + b*x)) + (2*B^2*d*n^2*Log[c + d*x])/(b*(b
*c - a*d)*g^2) - (2*B^2*d*n^2*Log[-((d*(a + b*x))/(b*c - a*d))]*Log[c + d*x
])/(b*(b*c - a*d)*g^2) + (2*B*d*n*(A + B*Log[e*((a + b*x)/(c + d*x))^n])*Lo
g[c + d*x])/(b*(b*c - a*d)*g^2) + (B^2*d*n^2*Log[c + d*x]^2)/(b*(b*c - a*d)
*g^2) - (2*B^2*d*n^2*Log[a + b*x]*Log[(b*(c + d*x))/(b*c - a*d)])/(b*(b*c -
 a*d)*g^2) - (2*B^2*d*n^2*PolyLog[2, -((d*(a + b*x))/(b*c - a*d))])/(b*(b*c
 - a*d)*g^2) - (2*B^2*d*n^2*PolyLog[2, (b*(c + d*x))/(b*c - a*d)])/(b*(b*c
- a*d)*g^2)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 2301

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

Rule 2390

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_))^(q_.), x_Symbol] :> Dist[1/e, Subst[Int[((f*x)/d)^q*(a + b*Log[c*x^
n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && E
qQ[e*f - d*g, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2393

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + (c*e*x)/g])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2394

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d + e*x
)^n]))/g, x] - Dist[(b*e*n)/g, Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2418

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*(RFx_), x_Sy
mbol] :> With[{u = ExpandIntegrand[(a + b*Log[c*(d + e*x)^n])^p, RFx, x]},
Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d, e, n}, x] && RationalFunctionQ[
RFx, x] && IntegerQ[p]
```

Rule 2524

```
Int[((a_.) + Log[(c_.)*(RFx_)^(p_.)]*(b_.))^(n_.)/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[(Log[d + e*x]*(a + b*Log[c*RFx^p])^n)/e, x] - Dist[(b*n*p)/e
, Int[(Log[d + e*x]*(a + b*Log[c*RFx^p])^(n - 1)*D[RFx, x])/RFx, x], x] /;
FreeQ[{a, b, c, d, e, p}, x] && RationalFunctionQ[RFx, x] && IGtQ[n, 0]
```

Rule 2525

```
Int[((a_.) + Log[(c_.)*(RFx_)^(p_.)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_))^(m_.
), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(a + b*Log[c*RFx^p])^n)/(e*(m + 1))
, x] - Dist[(b*n*p)/(e*(m + 1)), Int[SimplifyIntegrand[((d + e*x)^(m + 1)*(
a + b*Log[c*RFx^p])^(n - 1)*D[RFx, x])/RFx, x], x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && RationalFunctionQ[RFx, x] && IGtQ[n, 0] && (EqQ[n, 1] ||
IntegerQ[m]) && NeQ[m, -1]
```

Rule 2528

```
Int[((a_.) + Log[(c_.)*(RFx_)^(p_.)]*(b_.))^(n_.)*(RGx_), x_Symbol] :> With
[{u = ExpandIntegrand[(a + b*Log[c*RFx^p])^n, RGx, x]}, Int[u, x] /; SumQ[u
]] /; FreeQ[{a, b, c, p}, x] && RationalFunctionQ[RFx, x] && RationalFuncti
onQ[RGx, x] && IGtQ[n, 0]
```

Rubi steps

$$\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag+bgx)^2}\,dx = -\frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{bg^2(a+bx)} + \frac{(2Bn)\int \frac{(bc-ad)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{g(a+bx)^2(c+dx)}\,dx}{bg}$$

$$= -\frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{bg^2(a+bx)} + \frac{(2B(bc-ad)n)\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(a+bx)^2(c+dx)}\,dx}{bg^2}$$

$$= -\frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{bg^2(a+bx)} + \frac{(2B(bc-ad)n)\int \left(\frac{b\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(bc-ad)(a+bx)^2} - \frac{bd\left(A+B\log\right.}{(bc-ad}\right.}{bg^2}$$

$$= -\frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{bg^2(a+bx)} + \frac{(2Bn)\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(a+bx)^2}\,dx}{g^2} - \frac{(2Bdn)\int \frac{A+B\log\left(\right.}{a+}}{(bc-ad)}$$

$$= -\frac{2Bn\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{bg^2(a+bx)} - \frac{2Bdn\log(a+bx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{b(bc-ad)g^2} -$$

$$= -\frac{2Bn\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{bg^2(a+bx)} - \frac{2Bdn\log(a+bx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{b(bc-ad)g^2} -$$

$$= -\frac{2Bn\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{bg^2(a+bx)} - \frac{2Bdn\log(a+bx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{b(bc-ad)g^2} -$$

$$= -\frac{2B^2n^2}{bg^2(a+bx)} - \frac{2B^2dn^2\log(a+bx)}{b(bc-ad)g^2} - \frac{2Bn\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{bg^2(a+bx)} - \frac{2Bdn\,\mathrm{l}}{}$$

$$= -\frac{2B^2n^2}{bg^2(a+bx)} - \frac{2B^2dn^2\log(a+bx)}{b(bc-ad)g^2} + \frac{B^2dn^2\log^2(a+bx)}{b(bc-ad)g^2} - \frac{2Bn\left(A+B\log\right.}{bg^2(a+}$$

$$= -\frac{2B^2n^2}{bg^2(a+bx)} - \frac{2B^2dn^2\log(a+bx)}{b(bc-ad)g^2} + \frac{B^2dn^2\log^2(a+bx)}{b(bc-ad)g^2} - \frac{2Bn\left(A+B\log\right.}{bg^2(a+}$$

**Mathematica [C]** time = 0.57, size = 330, normalized size = 2.43

$$-\frac{Bn\left(2(bc-ad)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)+2d(a+bx)\log(a+bx)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)-2d(a+bx)\log(c+dx)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)-Bdn(a+bx)\left(\log(a+bx)\left(\log \cdots\right.\right.\right.}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2/(a*g + b*g*x)^2,x]
```

```
[Out] -(((A + B*Log[e*((a + b*x)/(c + d*x))^n])^2 + (B*n*(2*(b*c - a*d)*(A + B*Lo
g[e*((a + b*x)/(c + d*x))^n]) + 2*d*(a + b*x)*Log[a + b*x]*(A + B*Log[e*((a
 + b*x)/(c + d*x))^n]) - 2*d*(a + b*x)*(A + B*Log[e*((a + b*x)/(c + d*x))^n
])*Log[c + d*x] + 2*B*n*(b*c - a*d + d*(a + b*x)*Log[a + b*x] - d*(a + b*x)
*Log[c + d*x]) - B*d*n*(a + b*x)*(Log[a + b*x]*(Log[a + b*x] - 2*Log[(b*(c
+ d*x))/(b*c - a*d)]) - 2*PolyLog[2, (d*(a + b*x))/(-(b*c) + a*d)]) + B*d*n
*(a + b*x)*((2*Log[(d*(a + b*x))/(-(b*c) + a*d)] - Log[c + d*x])*Log[c + d*
x] + 2*PolyLog[2, (b*(c + d*x))/(b*c - a*d)])))/(b*c - a*d))/(b*g^2*(a + b*
x)))
```

**fricas [A]** time = 0.83, size = 258, normalized size = 1.90

$$-\frac{A^2bc - A^2ad + 2\left(B^2bc - B^2ad\right)n^2 + \left(B^2bc - B^2ad\right)\log(e)^2 + \left(B^2bdn^2x + B^2bcn^2\right)\log\left(\frac{bx+a}{dx+c}\right)^2 + 2\left(ABbc - ABa \cdots\right.}{\left(b^3 \cdots\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))^2/(b*g*x+a*g)^2,x, algorithm="fr
icas")
```

```
[Out] -(A^2*b*c - A^2*a*d + 2*(B^2*b*c - B^2*a*d)*n^2 + (B^2*b*c - B^2*a*d)*log(e
)^2 + (B^2*b*d*n^2*x + B^2*b*c*n^2)*log((b*x + a)/(d*x + c))^2 + 2*(A*B*b*c
 - A*B*a*d)*n + 2*(A*B*b*c - A*B*a*d + (B^2*b*c - B^2*a*d)*n + (B^2*b*d*n*x
 + B^2*b*c*n)*log((b*x + a)/(d*x + c)))*log(e) + 2*(B^2*b*c*n^2 + A*B*b*c*n
 + (B^2*b*d*n^2 + A*B*b*d*n)*x)*log((b*x + a)/(d*x + c)))/((b^3*c - a*b^2*d
)*g^2*x + (a*b^2*c - a^2*b*d)*g^2)
```

**giac [A]** time = 7.36, size = 163, normalized size = 1.20

$$-\left(\frac{(dx+c)B^2n^2\log\left(\frac{bx+a}{dx+c}\right)^2}{(bx+a)g^2} + \frac{2\left(B^2n^2 + ABn + B^2n\right)(dx+c)\log\left(\frac{bx+a}{dx+c}\right)}{(bx+a)g^2} + \frac{\left(2B^2n^2 + 2ABn + 2B^2n + A^2 + 2A \cdots\right.}{(bx+a)g^2} \cdots\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))^2/(b*g*x+a*g)^2,x, algorithm="gi
ac")
```

```
[Out] -((d*x + c)*B^2*n^2*log((b*x + a)/(d*x + c))^2/((b*x + a)*g^2) + 2*(B^2*n^2
 + A*B*n + B^2*n)*(d*x + c)*log((b*x + a)/(d*x + c))/((b*x + a)*g^2) + (2*B
^2*n^2 + 2*A*B*n + 2*B^2*n + A^2 + 2*A*B + B^2)*(d*x + c)/((b*x + a)*g^2))*
(b*c/(b*c - a*d)^2 - a*d/(b*c - a*d)^2)
```

**maple [F]** time = 0.27, size = 0, normalized size = 0.00

$$\int \frac{\left(B \ln \left(e \left(\frac{bx+a}{dx+c}\right)^{n}\right)+A\right)^{2}}{\left(bgx+ag\right)^{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*ln(e*((b*x+a)/(d*x+c))^n)+A)^2/(b*g*x+a*g)^2,x)
```

```
[Out] int((B*ln(e*((b*x+a)/(d*x+c))^n)+A)^2/(b*g*x+a*g)^2,x)
```

**maxima [B]** time = 1.47, size = 430, normalized size = 3.16

$$-2\,ABn\left(\frac{1}{b^{2}g^{2}x+abg^{2}}+\frac{d\log\left(bx+a\right)}{\left(b^{2}c-abd\right)g^{2}}-\frac{d\log\left(dx+c\right)}{\left(b^{2}c-abd\right)g^{2}}\right)-\left(2\,n\left(\frac{1}{b^{2}g^{2}x+abg^{2}}+\frac{d\log\left(bx+a\right)}{\left(b^{2}c-abd\right)g^{2}}-\frac{d\log\left(dx+c\right)}{\left(b^{2}c-abd\right)g^{2}}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))^2/(b*g*x+a*g)^2,x, algorithm="ma
xima")
```

```
[Out] -2*A*B*n*(1/(b^2*g^2*x + a*b*g^2) + d*log(b*x + a)/((b^2*c - a*b*d)*g^2) -
d*log(d*x + c)/((b^2*c - a*b*d)*g^2)) - (2*n*(1/(b^2*g^2*x + a*b*g^2) + d*l
og(b*x + a)/((b^2*c - a*b*d)*g^2) - d*log(d*x + c)/((b^2*c - a*b*d)*g^2))*l
og(e*(b*x/(d*x + c) + a/(d*x + c))^n) - ((b*d*x + a*d)*log(b*x + a)^2 + (b*
d*x + a*d)*log(d*x + c)^2 - 2*b*c + 2*a*d - 2*(b*d*x + a*d)*log(b*x + a) +
2*(b*d*x + a*d - (b*d*x + a*d)*log(b*x + a))*log(d*x + c))*n^2/(a*b^2*c*g^2
 - a^2*b*d*g^2 + (b^3*c*g^2 - a*b^2*d*g^2)*x))*B^2 - B^2*log(e*(b*x/(d*x +
c) + a/(d*x + c))^n)^2/(b^2*g^2*x + a*b*g^2) - 2*A*B*log(e*(b*x/(d*x + c) +
 a/(d*x + c))^n)/(b^2*g^2*x + a*b*g^2) - A^2/(b^2*g^2*x + a*b*g^2)
```

**mupad [B]** time = 5.59, size = 238, normalized size = 1.75

$$-\frac{A^{2}+2\,A\,B\,n+2\,B^{2}\,n^{2}}{x\,b^{2}\,g^{2}+a\,b\,g^{2}}-\ln\left(e\left(\frac{a+b\,x}{c+d\,x}\right)^{n}\right)^{2}\left(\frac{B^{2}}{b\,\left(a\,g^{2}+b\,g^{2}\,x\right)}-\frac{B^{2}\,d}{b\,g^{2}\,\left(a\,d-b\,c\right)}\right)-\ln\left(e\left(\frac{a+b\,x}{c+d\,x}\right)^{n}\right)\left(\frac{2\,B^{2}\,n}{x\,b^{2}\,g^{2}+a\,b\,g^{2}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*log(e*((a + b*x)/(c + d*x))^n))^2/(a*g + b*g*x)^2,x)
```

```
[Out] - (A^2 + 2*B^2*n^2 + 2*A*B*n)/(b^2*g^2*x + a*b*g^2) - log(e*((a + b*x)/(c +
 d*x))^n)^2*(B^2/(b*(a*g^2 + b*g^2*x)) - (B^2*d)/(b*g^2*(a*d - b*c))) - log
(e*((a + b*x)/(c + d*x))^n)*((2*B^2*n)/(b^2*g^2*x + a*b*g^2) + (2*A*B)/(b^2
*g^2*x + a*b*g^2)) - (B*d*n*atan(((2*b*d*x + (b^2*c*g^2 + a*b*d*g^2)/(b*g^2
))*1i)/(a*d - b*c))*(A + B*n)*4i)/(b*g^2*(a*d - b*c))
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{A^2}{a^2+2abx+b^2x^2}\,dx + \int \frac{B^2 \log\left(e\left(\frac{a}{c+dx}+\frac{bx}{c+dx}\right)^n\right)^2}{a^2+2abx+b^2x^2}\,dx + \int \frac{2AB\log\left(e\left(\frac{a}{c+dx}+\frac{bx}{c+dx}\right)^n\right)}{a^2+2abx+b^2x^2}\,dx}{g^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*ln(e*((b*x+a)/(d*x+c))**n))**2/(b*g*x+a*g)**2,x)
```

```
[Out] (Integral(A**2/(a**2 + 2*a*b*x + b**2*x**2), x) + Integral(B**2*log(e*(a/(c
 + d*x) + b*x/(c + d*x))**n)**2/(a**2 + 2*a*b*x + b**2*x**2), x) + Integral
(2*A*B*log(e*(a/(c + d*x) + b*x/(c + d*x))**n)/(a**2 + 2*a*b*x + b**2*x**2)
, x))/g**2
```

## 3.16 $\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag+bgx)^3}\,dx$

**Optimal**. Leaf size=288

$$-\frac{bBn(c+dx)^2\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{2g^3(a+bx)^2(bc-ad)^2}+\frac{2Bdn(c+dx)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{g^3(a+bx)(bc-ad)^2}-\frac{b(c+dx)^2\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+\right.}{2g^3(a+bx)^2(bc-ad)^2}$$

```
[Out] 2*B^2*d*n^2*(d*x+c)/(-a*d+b*c)^2/g^3/(b*x+a)-1/4*b*B^2*n^2*(d*x+c)^2/(-a*d+
b*c)^2/g^3/(b*x+a)^2+2*B*d*n*(d*x+c)*(A+B*ln(e*((b*x+a)/(d*x+c))^n))/(-a*d+
b*c)^2/g^3/(b*x+a)-1/2*b*B*n*(d*x+c)^2*(A+B*ln(e*((b*x+a)/(d*x+c))^n))/(-a*
d+b*c)^2/g^3/(b*x+a)^2+d*(d*x+c)*(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2/(-a*d+b*
c)^2/g^3/(b*x+a)-1/2*b*(d*x+c)^2*(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2/(-a*d+b*
c)^2/g^3/(b*x+a)^2
```

**Rubi [C]** time = 0.92, antiderivative size = 626, normalized size of antiderivative = 2.17, number of steps used = 28, number of rules used = 11, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.314, Rules used = {2525, 12, 2528, 44, 2524, 2418, 2390, 2301, 2394, 2393, 2391}

$$\frac{B^2d^2n^2\text{PolyLog}\left(2,-\frac{d(a+bx)}{bc-ad}\right)}{bg^3(bc-ad)^2}+\frac{B^2d^2n^2\text{PolyLog}\left(2,\frac{b(c+dx)}{bc-ad}\right)}{bg^3(bc-ad)^2}+\frac{Bd^2n\log(a+bx)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{bg^3(bc-ad)^2}-\frac{Bd^2n\,\mathrm{lo}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2/(a*g + b*g*x)^3,x]
```

```
[Out] -(B^2*n^2)/(4*b*g^3*(a + b*x)^2) + (3*B^2*d*n^2)/(2*b*(b*c - a*d)*g^3*(a +
b*x)) + (3*B^2*d^2*n^2*Log[a + b*x])/(2*b*(b*c - a*d)^2*g^3) - (B^2*d^2*n^2
*Log[a + b*x]^2)/(2*b*(b*c - a*d)^2*g^3) - (B*n*(A + B*Log[e*((a + b*x)/(c
+ d*x))^n]))/(2*b*g^3*(a + b*x)^2) + (B*d*n*(A + B*Log[e*((a + b*x)/(c + d*
x))^n]))/(b*(b*c - a*d)*g^3*(a + b*x)) + (B*d^2*n*Log[a + b*x]*(A + B*Log[e
*((a + b*x)/(c + d*x))^n]))/(b*(b*c - a*d)^2*g^3) - (A + B*Log[e*((a + b*x)
/(c + d*x))^n])^2/(2*b*g^3*(a + b*x)^2) - (3*B^2*d^2*n^2*Log[c + d*x])/(2*b
*(b*c - a*d)^2*g^3) + (B^2*d^2*n^2*Log[-((d*(a + b*x))/(b*c - a*d))]*Log[c
+ d*x])/(b*(b*c - a*d)^2*g^3) - (B*d^2*n*(A + B*Log[e*((a + b*x)/(c + d*x))
^n])*Log[c + d*x])/(b*(b*c - a*d)^2*g^3) - (B^2*d^2*n^2*Log[c + d*x]^2)/(2*
b*(b*c - a*d)^2*g^3) + (B^2*d^2*n^2*Log[a + b*x]*Log[(b*(c + d*x))/(b*c - a
*d)])/(b*(b*c - a*d)^2*g^3) + (B^2*d^2*n^2*PolyLog[2, -((d*(a + b*x))/(b*c
- a*d))])/(b*(b*c - a*d)^2*g^3) + (B^2*d^2*n^2*PolyLog[2, (b*(c + d*x))/(b*
c - a*d)])/(b*(b*c - a*d)^2*g^3)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 2301

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

Rule 2390

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_))^(q_.), x_Symbol] :> Dist[1/e, Subst[Int[((f*x)/d)^q*(a + b*Log[c*x^
n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && E
qQ[e*f - d*g, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2393

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + (c*e*x)/g])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2394

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d + e*x
)^n]))/g, x] - Dist[(b*e*n)/g, Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2418

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*(RFx_), x_Sy
mbol] :> With[{u = ExpandIntegrand[(a + b*Log[c*(d + e*x)^n])^p, RFx, x]},
Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d, e, n}, x] && RationalFunctionQ[
RFx, x] && IntegerQ[p]
```

Rule 2524

```
Int[((a_.) + Log[(c_.)*(RFx_)^(p_.)]*(b_.))^(n_.)/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[(Log[d + e*x]*(a + b*Log[c*RFx^p])^n)/e, x] - Dist[(b*n*p)/e
, Int[(Log[d + e*x]*(a + b*Log[c*RFx^p])^(n - 1)*D[RFx, x])/RFx, x], x] /;
FreeQ[{a, b, c, d, e, p}, x] && RationalFunctionQ[RFx, x] && IGtQ[n, 0]
```

Rule 2525

```
Int[((a_.) + Log[(c_.)*(RFx_)^(p_.)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_))^(m_.
), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(a + b*Log[c*RFx^p])^n)/(e*(m + 1))
, x] - Dist[(b*n*p)/(e*(m + 1)), Int[SimplifyIntegrand[((d + e*x)^(m + 1)*(
a + b*Log[c*RFx^p])^(n - 1)*D[RFx, x])/RFx, x], x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && RationalFunctionQ[RFx, x] && IGtQ[n, 0] && (EqQ[n, 1] ||
IntegerQ[m]) && NeQ[m, -1]
```

Rule 2528

```
Int[((a_.) + Log[(c_.)*(RFx_)^(p_.)]*(b_.))^(n_.)*(RGx_), x_Symbol] :> With
[{u = ExpandIntegrand[(a + b*Log[c*RFx^p])^n, RGx, x]}, Int[u, x] /; SumQ[u
]] /; FreeQ[{a, b, c, p}, x] && RationalFunctionQ[RFx, x] && RationalFuncti
onQ[RGx, x] && IGtQ[n, 0]
```

Rubi steps

$$\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag+bgx)^3}\,dx = -\frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{2bg^3(a+bx)^2} + \frac{(Bn)\int \frac{(bc-ad)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{g^2(a+bx)^3(c+dx)}\,dx}{bg}$$

$$= -\frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{2bg^3(a+bx)^2} + \frac{(B(bc-ad)n)\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(a+bx)^3(c+dx)}\,dx}{bg^3}$$

$$= -\frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{2bg^3(a+bx)^2} + \frac{(B(bc-ad)n)\int \left(\frac{b\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(bc-ad)(a+bx)^3} - \frac{bd\left(A+B\log\left(e\right.\right.}{(bc-ad)^2(a}\right.}{}$$

$$= -\frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{2bg^3(a+bx)^2} + \frac{(Bn)\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(a+bx)^3}\,dx}{g^3} + \frac{\left(Bd^2n\right)\int \frac{A+B\log\left(e\left(\right.\right.}{a+bx}}{(bc-ad)^2g^3}$$

$$= -\frac{Bn\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{2bg^3(a+bx)^2} + \frac{Bdn\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{b(bc-ad)g^3(a+bx)} + \frac{Bd^2n\log(a+b}{b}$$

$$= -\frac{Bn\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{2bg^3(a+bx)^2} + \frac{Bdn\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{b(bc-ad)g^3(a+bx)} + \frac{Bd^2n\log(a+b}{b}$$

$$= -\frac{Bn\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{2bg^3(a+bx)^2} + \frac{Bdn\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{b(bc-ad)g^3(a+bx)} + \frac{Bd^2n\log(a+b}{b}$$

$$= -\frac{B^2n^2}{4bg^3(a+bx)^2} + \frac{3B^2dn^2}{2b(bc-ad)g^3(a+bx)} + \frac{3B^2d^2n^2\log(a+bx)}{2b(bc-ad)^2g^3} - \frac{Bn\left(A+B\right.}{2bg^3}$$

$$= -\frac{B^2n^2}{4bg^3(a+bx)^2} + \frac{3B^2dn^2}{2b(bc-ad)g^3(a+bx)} + \frac{3B^2d^2n^2\log(a+bx)}{2b(bc-ad)^2g^3} - \frac{B^2d^2n^2\log^2}{2b(bc-a}$$

$$= -\frac{B^2n^2}{4bg^3(a+bx)^2} + \frac{3B^2dn^2}{2b(bc-ad)g^3(a+bx)} + \frac{3B^2d^2n^2\log(a+bx)}{2b(bc-ad)^2g^3} - \frac{B^2d^2n^2\log^2}{2b(bc-a}$$

**Mathematica [C]** time = 0.49, size = 463, normalized size = 1.61

$$-\frac{Bn\left(-4d^2(a+bx)^2\log(a+bx)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)+4d^2(a+bx)^2\log(c+dx)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)+2(bc-ad)^2\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)+4d(a+bx)(ad-bc\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2/(a*g + b*g*x)^3,x]
```

```
[Out] -1/4*(2*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2 + (B*n*(2*(b*c - a*d)^2*(A
 + B*Log[e*((a + b*x)/(c + d*x))^n]) + 4*d*(-(b*c) + a*d)*(a + b*x)*(A + B*
Log[e*((a + b*x)/(c + d*x))^n]) - 4*d^2*(a + b*x)^2*Log[a + b*x]*(A + B*Log
[e*((a + b*x)/(c + d*x))^n]) + 4*d^2*(a + b*x)^2*(A + B*Log[e*((a + b*x)/(c
 + d*x))^n])*Log[c + d*x] - 4*B*d*n*(a + b*x)*(b*c - a*d + d*(a + b*x)*Log[
a + b*x] - d*(a + b*x)*Log[c + d*x]) + B*n*((b*c - a*d)^2 + 2*d*(-(b*c) + a
*d)*(a + b*x) - 2*d^2*(a + b*x)^2*Log[a + b*x] + 2*d^2*(a + b*x)^2*Log[c +
d*x]) + 2*B*d^2*n*(a + b*x)^2*(Log[a + b*x]*(Log[a + b*x] - 2*Log[(b*(c + d
*x))/(b*c - a*d)]) - 2*PolyLog[2, (d*(a + b*x))/(-(b*c) + a*d)]) - 2*B*d^2*
n*(a + b*x)^2*((2*Log[(d*(a + b*x))/(-(b*c) + a*d)] - Log[c + d*x])*Log[c +
 d*x] + 2*PolyLog[2, (b*(c + d*x))/(b*c - a*d)])))/(b*c - a*d)^2)/(b*g^3*(a
 + b*x)^2)
```

**fricas [B]** time = 0.86, size = 651, normalized size = 2.26

$$-\frac{2\,A^2b^2c^2-4\,A^2abcd+2\,A^2a^2d^2+\left(B^2b^2c^2-8\,B^2abcd+7\,B^2a^2d^2\right)n^2+2\left(B^2b^2c^2-2\,B^2abcd+B^2a^2d^2\right)\log(e)}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))^2/(b*g*x+a*g)^3,x, algorithm="fr
icas")
```

```
[Out] -1/4*(2*A^2*b^2*c^2 - 4*A^2*a*b*c*d + 2*A^2*a^2*d^2 + (B^2*b^2*c^2 - 8*B^2*
a*b*c*d + 7*B^2*a^2*d^2)*n^2 + 2*(B^2*b^2*c^2 - 2*B^2*a*b*c*d + B^2*a^2*d^2
)*log(e)^2 - 2*(B^2*b^2*d^2*n^2*x^2 + 2*B^2*a*b*d^2*n^2*x - (B^2*b^2*c^2 -
2*B^2*a*b*c*d)*n^2)*log((b*x + a)/(d*x + c))^2 + 2*(A*B*b^2*c^2 - 4*A*B*a*b
*c*d + 3*A*B*a^2*d^2)*n - 2*(3*(B^2*b^2*c*d - B^2*a*b*d^2)*n^2 + 2*(A*B*b^2
*c*d - A*B*a*b*d^2)*n)*x + 2*(2*A*B*b^2*c^2 - 4*A*B*a*b*c*d + 2*A*B*a^2*d^2
 - 2*(B^2*b^2*c*d - B^2*a*b*d^2)*n*x + (B^2*b^2*c^2 - 4*B^2*a*b*c*d + 3*B^2
*a^2*d^2)*n - 2*(B^2*b^2*d^2*n*x^2 + 2*B^2*a*b*d^2*n*x - (B^2*b^2*c^2 - 2*B
^2*a*b*c*d)*n)*log((b*x + a)/(d*x + c)))*log(e) + 2*((B^2*b^2*c^2 - 4*B^2*a
*b*c*d)*n^2 - (3*B^2*b^2*d^2*n^2 + 2*A*B*b^2*d^2*n)*x^2 + 2*(A*B*b^2*c^2 -
2*A*B*a*b*c*d)*n - 2*(2*A*B*a*b*d^2*n + (B^2*b^2*c*d + 2*B^2*a*b*d^2)*n^2)*
```

```
x)*log((b*x + a)/(d*x + c)))/((b^5*c^2 - 2*a*b^4*c*d + a^2*b^3*d^2)*g^3*x^2
 + 2*(a*b^4*c^2 - 2*a^2*b^3*c*d + a^3*b^2*d^2)*g^3*x + (a^2*b^3*c^2 - 2*a^3
*b^2*c*d + a^4*b*d^2)*g^3)
```

**giac [A]** time = 10.43, size = 458, normalized size = 1.59

$$-\frac{1}{4}\left(\frac{2\left(B^2bn^2-\frac{2(bx+a)B^2dn^2}{dx+c}\right)\log\left(\frac{bx+a}{dx+c}\right)^2}{\frac{(bx+a)^2bcg^3}{(dx+c)^2}-\frac{(bx+a)^2adg^3}{(dx+c)^2}}+\frac{2\left(B^2bn^2-\frac{4(bx+a)B^2dn^2}{dx+c}+2\,ABbn+2\,B^2bn-\frac{4(bx+a)ABdn}{dx+c}-\frac{4(bx+a)B^2d}{dx+c}\right.}{\frac{(bx+a)^2bcg^3}{(dx+c)^2}-\frac{(bx+a)^2adg^3}{(dx+c)^2}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))^2/(b*g*x+a*g)^3,x, algorithm="gi
ac")
```

```
[Out] -1/4*(2*(B^2*b*n^2 - 2*(b*x + a)*B^2*d*n^2/(d*x + c))*log((b*x + a)/(d*x +
c))^2/((b*x + a)^2*b*c*g^3/(d*x + c)^2 - (b*x + a)^2*a*d*g^3/(d*x + c)^2) +
 2*(B^2*b*n^2 - 4*(b*x + a)*B^2*d*n^2/(d*x + c) + 2*A*B*b*n + 2*B^2*b*n - 4
*(b*x + a)*A*B*d*n/(d*x + c) - 4*(b*x + a)*B^2*d*n/(d*x + c))*log((b*x + a)
/(d*x + c))/((b*x + a)^2*b*c*g^3/(d*x + c)^2 - (b*x + a)^2*a*d*g^3/(d*x + c
)^2) + (B^2*b*n^2 - 8*(b*x + a)*B^2*d*n^2/(d*x + c) + 2*A*B*b*n + 2*B^2*b*n
 - 8*(b*x + a)*A*B*d*n/(d*x + c) - 8*(b*x + a)*B^2*d*n/(d*x + c) + 2*A^2*b
+ 4*A*B*b + 2*B^2*b - 4*(b*x + a)*A^2*d/(d*x + c) - 8*(b*x + a)*A*B*d/(d*x
+ c) - 4*(b*x + a)*B^2*d/(d*x + c))/((b*x + a)^2*b*c*g^3/(d*x + c)^2 - (b*x
 + a)^2*a*d*g^3/(d*x + c)^2))*(b*c/(b*c - a*d)^2 - a*d/(b*c - a*d)^2)
```

**maple [F]** time = 0.28, size = 0, normalized size = 0.00

$$\int\frac{\left(B\ln\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A\right)^2}{\left(bgx+ag\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*ln(e*((b*x+a)/(d*x+c))^n)+A)^2/(b*g*x+a*g)^3,x)
```

```
[Out] int((B*ln(e*((b*x+a)/(d*x+c))^n)+A)^2/(b*g*x+a*g)^3,x)
```

**maxima [B]** time = 1.74, size = 861, normalized size = 2.99

$$\frac{1}{2}\,ABn\left(\frac{2\,bdx-bc+3\,ad}{\left(b^4c-ab^3d\right)g^3x^2+2\left(ab^3c-a^2b^2d\right)g^3x+\left(a^2b^2c-a^3bd\right)g^3}+\frac{2\,d^2\log\left(bx+a\right)}{\left(b^3c^2-2\,ab^2cd+a^2bd^2\right)g^3}-\frac{2\,d^2\log}{\left(b^3c^2-2\,ab^2\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))^2/(b*g*x+a*g)^3,x, algorithm="ma
xima")
```

```
[Out] 1/2*A*B*n*((2*b*d*x - b*c + 3*a*d)/((b^4*c - a*b^3*d)*g^3*x^2 + 2*(a*b^3*c
- a^2*b^2*d)*g^3*x + (a^2*b^2*c - a^3*b*d)*g^3) + 2*d^2*log(b*x + a)/((b^3*
c^2 - 2*a*b^2*c*d + a^2*b*d^2)*g^3) - 2*d^2*log(d*x + c)/((b^3*c^2 - 2*a*b^
2*c*d + a^2*b*d^2)*g^3)) + 1/4*(2*n*((2*b*d*x - b*c + 3*a*d)/((b^4*c - a*b^
3*d)*g^3*x^2 + 2*(a*b^3*c - a^2*b^2*d)*g^3*x + (a^2*b^2*c - a^3*b*d)*g^3) +
 2*d^2*log(b*x + a)/((b^3*c^2 - 2*a*b^2*c*d + a^2*b*d^2)*g^3) - 2*d^2*log(d
*x + c)/((b^3*c^2 - 2*a*b^2*c*d + a^2*b*d^2)*g^3))*log(e*(b*x/(d*x + c) + a
/(d*x + c))^n) - (b^2*c^2 - 8*a*b*c*d + 7*a^2*d^2 + 2*(b^2*d^2*x^2 + 2*a*b*
d^2*x + a^2*d^2)*log(b*x + a)^2 + 2*(b^2*d^2*x^2 + 2*a*b*d^2*x + a^2*d^2)*l
og(d*x + c)^2 - 6*(b^2*c*d - a*b*d^2)*x - 6*(b^2*d^2*x^2 + 2*a*b*d^2*x + a^
2*d^2)*log(b*x + a) + 2*(3*b^2*d^2*x^2 + 6*a*b*d^2*x + 3*a^2*d^2 - 2*(b^2*d
^2*x^2 + 2*a*b*d^2*x + a^2*d^2)*log(b*x + a))*log(d*x + c))*n^2/(a^2*b^3*c^
2*g^3 - 2*a^3*b^2*c*d*g^3 + a^4*b*d^2*g^3 + (b^5*c^2*g^3 - 2*a*b^4*c*d*g^3
+ a^2*b^3*d^2*g^3)*x^2 + 2*(a*b^4*c^2*g^3 - 2*a^2*b^3*c*d*g^3 + a^3*b^2*d^2
*g^3)*x))*B^2 - 1/2*B^2*log(e*(b*x/(d*x + c) + a/(d*x + c))^n)^2/(b^3*g^3*x
^2 + 2*a*b^2*g^3*x + a^2*b*g^3) - A*B*log(e*(b*x/(d*x + c) + a/(d*x + c))^n
)/(b^3*g^3*x^2 + 2*a*b^2*g^3*x + a^2*b*g^3) - 1/2*A^2/(b^3*g^3*x^2 + 2*a*b^
2*g^3*x + a^2*b*g^3)
```

**mupad [B]** time = 6.17, size = 506, normalized size = 1.76

$$-\ln\left(e\left(\frac{a+b\,x}{c+d\,x}\right)^n\right)^2\left(\frac{B^2}{2\,b\,\left(a^2\,g^3+2\,a\,b\,g^3\,x+b^2\,g^3\,x^2\right)}-\frac{B^2\,d^2}{2\,b\,g^3\,\left(a^2\,d^2-2\,a\,b\,c\,d+b^2\,c^2\right)}\right)-\frac{\frac{2\,A^2\,a\,d-2\,A^2\,b\,c+7\,B^2\,a\,d\,n}{2}}{2\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*log(e*((a + b*x)/(c + d*x))^n))^2/(a*g + b*g*x)^3,x)
```

```
[Out] - log(e*((a + b*x)/(c + d*x))^n)^2*(B^2/(2*b*(a^2*g^3 + b^2*g^3*x^2 + 2*a*b
*g^3*x)) - (B^2*d^2)/(2*b*g^3*(a^2*d^2 + b^2*c^2 - 2*a*b*c*d))) - ((2*A^2*a
*d - 2*A^2*b*c + 7*B^2*a*d*n^2 - B^2*b*c*n^2 + 6*A*B*a*d*n - 2*A*B*b*c*n)/(
2*(a*d - b*c)) + (d*x*(3*B^2*b*n^2 + 2*A*B*b*n))/(a*d - b*c))/(2*a^2*b*g^3
+ 2*b^3*g^3*x^2 + 4*a*b^2*g^3*x) - log(e*((a + b*x)/(c + d*x))^n)*((A*B)/(a
^2*b*g^3 + b^3*g^3*x^2 + 2*a*b^2*g^3*x) + (B^2*d^2*((b*g^3*n*(a*d - b*c)*(2
*a*d - b*c))/(2*d^2) + (b^2*g^3*n*x*(a*d - b*c))/d + (a*b*g^3*n*(a*d - b*c)
)/(2*d)))/(b*g^3*(a^2*d^2 + b^2*c^2 - 2*a*b*c*d)*(a^2*b*g^3 + b^3*g^3*x^2 +
 2*a*b^2*g^3*x))) - (B*d^2*n*atan(((2*b*d*x - (2*b^3*c^2*g^3 - 2*a^2*b*d^2*
g^3)/(2*b*g^3*(a*d - b*c)))*1i)/(a*d - b*c))*(2*A + 3*B*n)*1i)/(b*g^3*(a*d
- b*c)^2)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{A^2}{a^3+3a^2bx+3ab^2x^2+b^3x^3}\,dx + \int \frac{B^2\log\left(e\left(\frac{a}{c+dx}+\frac{bx}{c+dx}\right)^n\right)^2}{a^3+3a^2bx+3ab^2x^2+b^3x^3}\,dx + \int \frac{2AB\log\left(e\left(\frac{a}{c+dx}+\frac{bx}{c+dx}\right)^n\right)}{a^3+3a^2bx+3ab^2x^2+b^3x^3}\,dx}{g^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*ln(e*((b*x+a)/(d*x+c))**n))**2/(b*g*x+a*g)**3,x)
```

```
[Out] (Integral(A**2/(a**3 + 3*a**2*b*x + 3*a*b**2*x**2 + b**3*x**3), x) + Integr
al(B**2*log(e*(a/(c + d*x) + b*x/(c + d*x))**n)**2/(a**3 + 3*a**2*b*x + 3*a
*b**2*x**2 + b**3*x**3), x) + Integral(2*A*B*log(e*(a/(c + d*x) + b*x/(c +
d*x))**n)/(a**3 + 3*a**2*b*x + 3*a*b**2*x**2 + b**3*x**3), x))/g**3
```

## 3.17 $\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag+bgx)^4}\,dx$

**Optimal**. Leaf size=448

$$-\frac{b^2(c+dx)^3\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{3g^4(a+bx)^3(bc-ad)^3}-\frac{2b^2Bn(c+dx)^3\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{9g^4(a+bx)^3(bc-ad)^3}-\frac{d^2(c+dx)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right.}{g^4(a+bx)(bc-ad)^3}$$

```
[Out] -2*B^2*d^2*n^2*(d*x+c)/(-a*d+b*c)^3/g^4/(b*x+a)+1/2*b*B^2*d*n^2*(d*x+c)^2/(
-a*d+b*c)^3/g^4/(b*x+a)^2-2/27*b^2*B^2*n^2*(d*x+c)^3/(-a*d+b*c)^3/g^4/(b*x+
a)^3-2*B*d^2*n*(d*x+c)*(A+B*ln(e*((b*x+a)/(d*x+c))^n))/(-a*d+b*c)^3/g^4/(b*
x+a)+b*B*d*n*(d*x+c)^2*(A+B*ln(e*((b*x+a)/(d*x+c))^n))/(-a*d+b*c)^3/g^4/(b*
x+a)^2-2/9*b^2*B*n*(d*x+c)^3*(A+B*ln(e*((b*x+a)/(d*x+c))^n))/(-a*d+b*c)^3/g
^4/(b*x+a)^3-d^2*(d*x+c)*(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2/(-a*d+b*c)^3/g^4
/(b*x+a)+b*d*(d*x+c)^2*(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2/(-a*d+b*c)^3/g^4/(
b*x+a)^2-1/3*b^2*(d*x+c)^3*(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2/(-a*d+b*c)^3/g
^4/(b*x+a)^3
```

**Rubi [C]** time = 1.09, antiderivative size = 736, normalized size of antiderivative = 1.64, number of steps used = 32, number of rules used = 11, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.314, Rules used = {2525, 12, 2528, 44, 2524, 2418, 2390, 2301, 2394, 2393, 2391}

$$-\frac{2B^2d^3n^2\text{PolyLog}\left(2,-\frac{d(a+bx)}{bc-ad}\right)}{3bg^4(bc-ad)^3}-\frac{2B^2d^3n^2\text{PolyLog}\left(2,\frac{b(c+dx)}{bc-ad}\right)}{3bg^4(bc-ad)^3}-\frac{2Bd^3n\log(a+bx)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{3bg^4(bc-ad)^3}+\cdots$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2/(a*g + b*g*x)^4,x]
```

```
[Out] (-2*B^2*n^2)/(27*b*g^4*(a + b*x)^3) + (5*B^2*d*n^2)/(18*b*(b*c - a*d)*g^4*(
a + b*x)^2) - (11*B^2*d^2*n^2)/(9*b*(b*c - a*d)^2*g^4*(a + b*x)) - (11*B^2*
d^3*n^2*Log[a + b*x])/(9*b*(b*c - a*d)^3*g^4) + (B^2*d^3*n^2*Log[a + b*x]^2
)/(3*b*(b*c - a*d)^3*g^4) - (2*B*n*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/
(9*b*g^4*(a + b*x)^3) + (B*d*n*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/(3*b
*(b*c - a*d)*g^4*(a + b*x)^2) - (2*B*d^2*n*(A + B*Log[e*((a + b*x)/(c + d*x
))^n]))/(3*b*(b*c - a*d)^2*g^4*(a + b*x)) - (2*B*d^3*n*Log[a + b*x]*(A + B*
Log[e*((a + b*x)/(c + d*x))^n]))/(3*b*(b*c - a*d)^3*g^4) - (A + B*Log[e*((a
 + b*x)/(c + d*x))^n])^2/(3*b*g^4*(a + b*x)^3) + (11*B^2*d^3*n^2*Log[c + d*
x])/(9*b*(b*c - a*d)^3*g^4) - (2*B^2*d^3*n^2*Log[-((d*(a + b*x))/(b*c - a*d
))]*Log[c + d*x])/(3*b*(b*c - a*d)^3*g^4) + (2*B*d^3*n*(A + B*Log[e*((a + b
```

```
*x)/(c + d*x))^n])*Log[c + d*x])/(3*b*(b*c - a*d)^3*g^4) + (B^2*d^3*n^2*Log
[c + d*x]^2)/(3*b*(b*c - a*d)^3*g^4) - (2*B^2*d^3*n^2*Log[a + b*x]*Log[(b*(
c + d*x))/(b*c - a*d)])/(3*b*(b*c - a*d)^3*g^4) - (2*B^2*d^3*n^2*PolyLog[2,
 -((d*(a + b*x))/(b*c - a*d))])/(3*b*(b*c - a*d)^3*g^4) - (2*B^2*d^3*n^2*Po
lyLog[2, (b*(c + d*x))/(b*c - a*d)])/(3*b*(b*c - a*d)^3*g^4)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 2301

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

Rule 2390

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_))^(q_.), x_Symbol] :> Dist[1/e, Subst[Int[((f*x)/d)^q*(a + b*Log[c*x^
n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && E
qQ[e*f - d*g, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2393

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + (c*e*x)/g])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2394

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d + e*x
```

```
)^n]))/g, x] - Dist[(b*e*n)/g, Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2418

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*(RFx_), x_Sy
mbol] :> With[{u = ExpandIntegrand[(a + b*Log[c*(d + e*x)^n])^p, RFx, x]},
Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d, e, n}, x] && RationalFunctionQ[
RFx, x] && IntegerQ[p]
```

Rule 2524

```
Int[((a_.) + Log[(c_.)*(RFx_)^(p_.)]*(b_.))^(n_.)/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[(Log[d + e*x]*(a + b*Log[c*RFx^p])^n)/e, x] - Dist[(b*n*p)/e
, Int[(Log[d + e*x]*(a + b*Log[c*RFx^p])^(n - 1)*D[RFx, x])/RFx, x], x] /;
FreeQ[{a, b, c, d, e, p}, x] && RationalFunctionQ[RFx, x] && IGtQ[n, 0]
```

Rule 2525

```
Int[((a_.) + Log[(c_.)*(RFx_)^(p_.)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_))^(m_.
), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(a + b*Log[c*RFx^p])^n)/(e*(m + 1))
, x] - Dist[(b*n*p)/(e*(m + 1)), Int[SimplifyIntegrand[((d + e*x)^(m + 1)*(
a + b*Log[c*RFx^p])^(n - 1)*D[RFx, x])/RFx, x], x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && RationalFunctionQ[RFx, x] && IGtQ[n, 0] && (EqQ[n, 1] ||
IntegerQ[m]) && NeQ[m, -1]
```

Rule 2528

```
Int[((a_.) + Log[(c_.)*(RFx_)^(p_.)]*(b_.))^(n_.)*(RGx_), x_Symbol] :> With
[{u = ExpandIntegrand[(a + b*Log[c*RFx^p])^n, RGx, x]}, Int[u, x] /; SumQ[u
]] /; FreeQ[{a, b, c, p}, x] && RationalFunctionQ[RFx, x] && RationalFuncti
onQ[RGx, x] && IGtQ[n, 0]
```

Rubi steps

$$\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag+bgx)^4}\,dx = -\frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{3bg^4(a+bx)^3} + \frac{(2Bn)\int \frac{(bc-ad)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{g^3(a+bx)^4(c+dx)}\,dx}{3bg}$$

$$= -\frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{3bg^4(a+bx)^3} + \frac{(2B(bc-ad)n)\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(a+bx)^4(c+dx)}\,dx}{3bg^4}$$

$$= -\frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{3bg^4(a+bx)^3} + \frac{(2B(bc-ad)n)\int \left(\frac{b\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(bc-ad)(a+bx)^4} - \frac{bd\left(A+B\log\left(\cdots\right.\right.}{(bc-ad)^2\cdots}\cdots\right.}{\cdots}$$

$$= -\frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{3bg^4(a+bx)^3} + \frac{(2Bn)\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(a+bx)^4}\,dx}{3g^4} - \frac{\left(2Bd^3n\right)\int \frac{A+B\log\left(\cdots\right.}{a+\cdots}}{3(bc-ad)^3\cdots}$$

$$= -\frac{2Bn\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{9bg^4(a+bx)^3} + \frac{Bdn\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{3b(bc-ad)g^4(a+bx)^2} - \frac{2Bd^2n\left(A+B\cdots\right.}{3b(bc-a\cdots}$$

$$= -\frac{2Bn\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{9bg^4(a+bx)^3} + \frac{Bdn\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{3b(bc-ad)g^4(a+bx)^2} - \frac{2Bd^2n\left(A+B\cdots\right.}{3b(bc-a\cdots}$$

$$= -\frac{2Bn\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{9bg^4(a+bx)^3} + \frac{Bdn\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{3b(bc-ad)g^4(a+bx)^2} - \frac{2Bd^2n\left(A+B\cdots\right.}{3b(bc-a\cdots}$$

$$= -\frac{2B^2n^2}{27bg^4(a+bx)^3} + \frac{5B^2dn^2}{18b(bc-ad)g^4(a+bx)^2} - \frac{11B^2d^2n^2}{9b(bc-ad)^2g^4(a+bx)} - \frac{11B^2d\cdots}{9b\cdots}$$

$$= -\frac{2B^2n^2}{27bg^4(a+bx)^3} + \frac{5B^2dn^2}{18b(bc-ad)g^4(a+bx)^2} - \frac{11B^2d^2n^2}{9b(bc-ad)^2g^4(a+bx)} - \frac{11B^2d\cdots}{9b\cdots}$$

$$= -\frac{2B^2n^2}{27bg^4(a+bx)^3} + \frac{5B^2dn^2}{18b(bc-ad)g^4(a+bx)^2} - \frac{11B^2d^2n^2}{9b(bc-ad)^2g^4(a+bx)} - \frac{11B^2d\cdots}{9b\cdots}$$

**Mathematica [C]** time = 0.70, size = 609, normalized size = 1.36

$$-\frac{Bn\left(36d^3(a+bx)^3\log(a+bx)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)-36d^3(a+bx)^3\log(c+dx)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)+36d^2(a+bx)^2(bc-ad)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)+12(b\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2/(a*g + b*g*x)^4,x]
```

```
[Out] -1/54*(18*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2 + (B*n*(12*(b*c - a*d)^3
*(A + B*Log[e*((a + b*x)/(c + d*x))^n]) - 18*d*(b*c - a*d)^2*(a + b*x)*(A +
 B*Log[e*((a + b*x)/(c + d*x))^n]) + 36*d^2*(b*c - a*d)*(a + b*x)^2*(A + B*
Log[e*((a + b*x)/(c + d*x))^n]) + 36*d^3*(a + b*x)^3*Log[a + b*x]*(A + B*Lo
g[e*((a + b*x)/(c + d*x))^n]) - 36*d^3*(a + b*x)^3*(A + B*Log[e*((a + b*x)/
(c + d*x))^n])*Log[c + d*x] + 36*B*d^2*n*(a + b*x)^2*(b*c - a*d + d*(a + b*
x)*Log[a + b*x] - d*(a + b*x)*Log[c + d*x]) - 9*B*d*n*(a + b*x)*((b*c - a*d
)^2 + 2*d*(-(b*c) + a*d)*(a + b*x) - 2*d^2*(a + b*x)^2*Log[a + b*x] + 2*d^2
*(a + b*x)^2*Log[c + d*x]) + 2*B*n*(2*(b*c - a*d)^3 - 3*d*(b*c - a*d)^2*(a
+ b*x) + 6*d^2*(b*c - a*d)*(a + b*x)^2 + 6*d^3*(a + b*x)^3*Log[a + b*x] - 6
*d^3*(a + b*x)^3*Log[c + d*x]) - 18*B*d^3*n*(a + b*x)^3*(Log[a + b*x]*(Log[
a + b*x] - 2*Log[(b*(c + d*x))/(b*c - a*d)]) - 2*PolyLog[2, (d*(a + b*x))/(
-(b*c) + a*d)]) + 18*B*d^3*n*(a + b*x)^3*((2*Log[(d*(a + b*x))/(-(b*c) + a*
d)] - Log[c + d*x])*Log[c + d*x] + 2*PolyLog[2, (b*(c + d*x))/(b*c - a*d)])
))/(b*c - a*d)^3)/(b*g^4*(a + b*x)^3)
```

**fricas [B]** time = 0.87, size = 1164, normalized size = 2.60

$$-\frac{18\,A^2b^3c^3-54\,A^2ab^2c^2d+54\,A^2a^2bcd^2-18\,A^2a^3d^3+\left(4\,B^2b^3c^3-27\,B^2ab^2c^2d+108\,B^2a^2bcd^2-85\,B^2a^3d^3\right)n}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))^2/(b*g*x+a*g)^4,x, algorithm="fr
icas")
```

```
[Out] -1/54*(18*A^2*b^3*c^3 - 54*A^2*a*b^2*c^2*d + 54*A^2*a^2*b*c*d^2 - 18*A^2*a^
3*d^3 + (4*B^2*b^3*c^3 - 27*B^2*a*b^2*c^2*d + 108*B^2*a^2*b*c*d^2 - 85*B^2*
a^3*d^3)*n^2 + 6*(11*(B^2*b^3*c*d^2 - B^2*a*b^2*d^3)*n^2 + 6*(A*B*b^3*c*d^2
 - A*B*a*b^2*d^3)*n)*x^2 + 18*(B^2*b^3*c^3 - 3*B^2*a*b^2*c^2*d + 3*B^2*a^2*
b*c*d^2 - B^2*a^3*d^3)*log(e)^2 + 18*(B^2*b^3*d^3*n^2*x^3 + 3*B^2*a*b^2*d^3
*n^2*x^2 + 3*B^2*a^2*b*d^3*n^2*x + (B^2*b^3*c^3 - 3*B^2*a*b^2*c^2*d + 3*B^2
*a^2*b*c*d^2)*n^2)*log((b*x + a)/(d*x + c))^2 + 6*(2*A*B*b^3*c^3 - 9*A*B*a*
b^2*c^2*d + 18*A*B*a^2*b*c*d^2 - 11*A*B*a^3*d^3)*n - 3*((5*B^2*b^3*c^2*d -
```

```
54*B^2*a*b^2*c*d^2 + 49*B^2*a^2*b*d^3)*n^2 + 6*(A*B*b^3*c^2*d - 6*A*B*a*b^2
*c*d^2 + 5*A*B*a^2*b*d^3)*n)*x + 6*(6*A*B*b^3*c^3 - 18*A*B*a*b^2*c^2*d + 18
*A*B*a^2*b*c*d^2 - 6*A*B*a^3*d^3 + 6*(B^2*b^3*c*d^2 - B^2*a*b^2*d^3)*n*x^2
- 3*(B^2*b^3*c^2*d - 6*B^2*a*b^2*c*d^2 + 5*B^2*a^2*b*d^3)*n*x + (2*B^2*b^3*
c^3 - 9*B^2*a*b^2*c^2*d + 18*B^2*a^2*b*c*d^2 - 11*B^2*a^3*d^3)*n + 6*(B^2*b
^3*d^3*n*x^3 + 3*B^2*a*b^2*d^3*n*x^2 + 3*B^2*a^2*b*d^3*n*x + (B^2*b^3*c^3 -
 3*B^2*a*b^2*c^2*d + 3*B^2*a^2*b*c*d^2)*n)*log((b*x + a)/(d*x + c)))*log(e)
 + 6*((11*B^2*b^3*d^3*n^2 + 6*A*B*b^3*d^3*n)*x^3 + (2*B^2*b^3*c^3 - 9*B^2*a
*b^2*c^2*d + 18*B^2*a^2*b*c*d^2)*n^2 + 3*(6*A*B*a*b^2*d^3*n + (2*B^2*b^3*c*
d^2 + 9*B^2*a*b^2*d^3)*n^2)*x^2 + 6*(A*B*b^3*c^3 - 3*A*B*a*b^2*c^2*d + 3*A*
B*a^2*b*c*d^2)*n + 3*(6*A*B*a^2*b*d^3*n - (B^2*b^3*c^2*d - 6*B^2*a*b^2*c*d^
2 - 6*B^2*a^2*b*d^3)*n^2)*x)*log((b*x + a)/(d*x + c)))/((b^7*c^3 - 3*a*b^6*
c^2*d + 3*a^2*b^5*c*d^2 - a^3*b^4*d^3)*g^4*x^3 + 3*(a*b^6*c^3 - 3*a^2*b^5*c
^2*d + 3*a^3*b^4*c*d^2 - a^4*b^3*d^3)*g^4*x^2 + 3*(a^2*b^5*c^3 - 3*a^3*b^4*
c^2*d + 3*a^4*b^3*c*d^2 - a^5*b^2*d^3)*g^4*x + (a^3*b^4*c^3 - 3*a^4*b^3*c^2
*d + 3*a^5*b^2*c*d^2 - a^6*b*d^3)*g^4)
```

**giac [A]** time = 13.07, size = 810, normalized size = 1.81

$$-\frac{1}{54}\left(\frac{18\left(B^2b^2n^2-\frac{3(bx+a)B^2bdn^2}{dx+c}+\frac{3(bx+a)^2B^2d^2n^2}{(dx+c)^2}\right)\log\left(\frac{bx+a}{dx+c}\right)^2}{\frac{(bx+a)^3b^2c^2g^4}{(dx+c)^3}-\frac{2(bx+a)^3abcdg^4}{(dx+c)^3}+\frac{(bx+a)^3a^2d^2g^4}{(dx+c)^3}}+\frac{6\left(2B^2b^2n^2-\frac{9(bx+a)B^2bdn^2}{dx+c}+\frac{18(bx+a)^2B^2d^2n^2}{(dx+c)^2}+6A\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))^2/(b*g*x+a*g)^4,x, algorithm="gi
ac")
```

```
[Out] -1/54*(18*(B^2*b^2*n^2 - 3*(b*x + a)*B^2*b*d*n^2/(d*x + c) + 3*(b*x + a)^2*
B^2*d^2*n^2/(d*x + c)^2)*log((b*x + a)/(d*x + c))^2/((b*x + a)^3*b^2*c^2*g^
4/(d*x + c)^3 - 2*(b*x + a)^3*a*b*c*d*g^4/(d*x + c)^3 + (b*x + a)^3*a^2*d^2
*g^4/(d*x + c)^3) + 6*(2*B^2*b^2*n^2 - 9*(b*x + a)*B^2*b*d*n^2/(d*x + c) +
18*(b*x + a)^2*B^2*d^2*n^2/(d*x + c)^2 + 6*A*B*b^2*n + 6*B^2*b^2*n - 18*(b*
x + a)*A*B*b*d*n/(d*x + c) - 18*(b*x + a)*B^2*b*d*n/(d*x + c) + 18*(b*x + a
)^2*A*B*d^2*n/(d*x + c)^2 + 18*(b*x + a)^2*B^2*d^2*n/(d*x + c)^2)*log((b*x
+ a)/(d*x + c))/((b*x + a)^3*b^2*c^2*g^4/(d*x + c)^3 - 2*(b*x + a)^3*a*b*c*
d*g^4/(d*x + c)^3 + (b*x + a)^3*a^2*d^2*g^4/(d*x + c)^3) + (4*B^2*b^2*n^2 -
 27*(b*x + a)*B^2*b*d*n^2/(d*x + c) + 108*(b*x + a)^2*B^2*d^2*n^2/(d*x + c)
^2 + 12*A*B*b^2*n + 12*B^2*b^2*n - 54*(b*x + a)*A*B*b*d*n/(d*x + c) - 54*(b
*x + a)*B^2*b*d*n/(d*x + c) + 108*(b*x + a)^2*A*B*d^2*n/(d*x + c)^2 + 108*(
b*x + a)^2*B^2*d^2*n/(d*x + c)^2 + 18*A^2*b^2 + 36*A*B*b^2 + 18*B^2*b^2 - 5
4*(b*x + a)*A^2*b*d/(d*x + c) - 108*(b*x + a)*A*B*b*d/(d*x + c) - 54*(b*x +
 a)*B^2*b*d/(d*x + c) + 54*(b*x + a)^2*A^2*d^2/(d*x + c)^2 + 108*(b*x + a)^
2*A*B*d^2/(d*x + c)^2 + 54*(b*x + a)^2*B^2*d^2/(d*x + c)^2)/((b*x + a)^3*b^
```

```
2*c^2*g^4/(d*x + c)^3 - 2*(b*x + a)^3*a*b*c*d*g^4/(d*x + c)^3 + (b*x + a)^3
*a^2*d^2*g^4/(d*x + c)^3))*(b*c/(b*c - a*d)^2 - a*d/(b*c - a*d)^2)
```

**maple [F]** time = 0.28, size = 0, normalized size = 0.00

$$\int \frac{\left(B\ln\left(e\left(\frac{bx+a}{dx+c}\right)^{n}\right)+A\right)^{2}}{\left(bgx+ag\right)^{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*ln(e*((b*x+a)/(d*x+c))^n)+A)^2/(b*g*x+a*g)^4,x)
```

```
[Out] int((B*ln(e*((b*x+a)/(d*x+c))^n)+A)^2/(b*g*x+a*g)^4,x)
```

**maxima [B]** time = 2.34, size = 1432, normalized size = 3.20

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))^2/(b*g*x+a*g)^4,x, algorithm="ma
xima")
```

```
[Out] -1/9*A*B*n*((6*b^2*d^2*x^2 + 2*b^2*c^2 - 7*a*b*c*d + 11*a^2*d^2 - 3*(b^2*c*
d - 5*a*b*d^2)*x)/((b^6*c^2 - 2*a*b^5*c*d + a^2*b^4*d^2)*g^4*x^3 + 3*(a*b^5
*c^2 - 2*a^2*b^4*c*d + a^3*b^3*d^2)*g^4*x^2 + 3*(a^2*b^4*c^2 - 2*a^3*b^3*c*
d + a^4*b^2*d^2)*g^4*x + (a^3*b^3*c^2 - 2*a^4*b^2*c*d + a^5*b*d^2)*g^4) + 6
*d^3*log(b*x + a)/((b^4*c^3 - 3*a*b^3*c^2*d + 3*a^2*b^2*c*d^2 - a^3*b*d^3)*
g^4) - 6*d^3*log(d*x + c)/((b^4*c^3 - 3*a*b^3*c^2*d + 3*a^2*b^2*c*d^2 - a^3
*b*d^3)*g^4)) - 1/54*(6*n*((6*b^2*d^2*x^2 + 2*b^2*c^2 - 7*a*b*c*d + 11*a^2*
d^2 - 3*(b^2*c*d - 5*a*b*d^2)*x)/((b^6*c^2 - 2*a*b^5*c*d + a^2*b^4*d^2)*g^4
*x^3 + 3*(a*b^5*c^2 - 2*a^2*b^4*c*d + a^3*b^3*d^2)*g^4*x^2 + 3*(a^2*b^4*c^2
 - 2*a^3*b^3*c*d + a^4*b^2*d^2)*g^4*x + (a^3*b^3*c^2 - 2*a^4*b^2*c*d + a^5*
b*d^2)*g^4) + 6*d^3*log(b*x + a)/((b^4*c^3 - 3*a*b^3*c^2*d + 3*a^2*b^2*c*d^
2 - a^3*b*d^3)*g^4) - 6*d^3*log(d*x + c)/((b^4*c^3 - 3*a*b^3*c^2*d + 3*a^2*
b^2*c*d^2 - a^3*b*d^3)*g^4))*log(e*(b*x/(d*x + c) + a/(d*x + c))^n) + (4*b^
3*c^3 - 27*a*b^2*c^2*d + 108*a^2*b*c*d^2 - 85*a^3*d^3 + 66*(b^3*c*d^2 - a*b
^2*d^3)*x^2 - 18*(b^3*d^3*x^3 + 3*a*b^2*d^3*x^2 + 3*a^2*b*d^3*x + a^3*d^3)*
log(b*x + a)^2 - 18*(b^3*d^3*x^3 + 3*a*b^2*d^3*x^2 + 3*a^2*b*d^3*x + a^3*d^
3)*log(d*x + c)^2 - 3*(5*b^3*c^2*d - 54*a*b^2*c*d^2 + 49*a^2*b*d^3)*x + 66*
(b^3*d^3*x^3 + 3*a*b^2*d^3*x^2 + 3*a^2*b*d^3*x + a^3*d^3)*log(b*x + a) - 6*
(11*b^3*d^3*x^3 + 33*a*b^2*d^3*x^2 + 33*a^2*b*d^3*x + 11*a^3*d^3 - 6*(b^3*d
^3*x^3 + 3*a*b^2*d^3*x^2 + 3*a^2*b*d^3*x + a^3*d^3)*log(b*x + a))*log(d*x +
 c))*n^2/(a^3*b^4*c^3*g^4 - 3*a^4*b^3*c^2*d*g^4 + 3*a^5*b^2*c*d^2*g^4 - a^6
*b*d^3*g^4 + (b^7*c^3*g^4 - 3*a*b^6*c^2*d*g^4 + 3*a^2*b^5*c*d^2*g^4 - a^3*b
```

```
^4*d^3*g^4)*x^3 + 3*(a*b^6*c^3*g^4 - 3*a^2*b^5*c^2*d*g^4 + 3*a^3*b^4*c*d^2*
g^4 - a^4*b^3*d^3*g^4)*x^2 + 3*(a^2*b^5*c^3*g^4 - 3*a^3*b^4*c^2*d*g^4 + 3*a
^4*b^3*c*d^2*g^4 - a^5*b^2*d^3*g^4)*x))*B^2 - 1/3*B^2*log(e*(b*x/(d*x + c)
+ a/(d*x + c))^n)^2/(b^4*g^4*x^3 + 3*a*b^3*g^4*x^2 + 3*a^2*b^2*g^4*x + a^3*
b*g^4) - 2/3*A*B*log(e*(b*x/(d*x + c) + a/(d*x + c))^n)/(b^4*g^4*x^3 + 3*a*
b^3*g^4*x^2 + 3*a^2*b^2*g^4*x + a^3*b*g^4) - 1/3*A^2/(b^4*g^4*x^3 + 3*a*b^3
*g^4*x^2 + 3*a^2*b^2*g^4*x + a^3*b*g^4)
```

**mupad [B]** time = 7.69, size = 1038, normalized size = 2.32

$$\frac{\frac{18\,A^2\,a^2\,d^2-36\,A^2\,a\,b\,c\,d+18\,A^2\,b^2\,c^2+66\,A\,B\,a^2\,d^2\,n-42\,A\,B\,a\,b\,c\,d\,n+12\,A\,B\,b^2\,c^2\,n+85\,B^2\,a^2\,d^2\,n^2-23\,B^2\,a\,b\,c\,d\,n^2+4\,B^2\,b^2\,c^2\,n^2}{6\,(a\,d-b\,c)}+\frac{x\,\left(-5\,c\,B^2\,b^2\,d\,n^2\cdots\right.}{\cdots}}{x\,\left(27\,a^2\,b^3\,c\,g^4-27\,a^3\,b^2\,d\,g^4\right)-x^2\,\left(27\,a^2\,b^3\,d\,g^4-27\,a\,b^4\,c\,g^4\right)+x^3\,\left(9\,b^5\,c\,g^4-9\,a\cdots\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*log(e*((a + b*x)/(c + d*x))^n))^2/(a*g + b*g*x)^4,x)
```

```
[Out] ((18*A^2*a^2*d^2 + 18*A^2*b^2*c^2 + 85*B^2*a^2*d^2*n^2 + 4*B^2*b^2*c^2*n^2
- 36*A^2*a*b*c*d + 66*A*B*a^2*d^2*n + 12*A*B*b^2*c^2*n - 23*B^2*a*b*c*d*n^2
 - 42*A*B*a*b*c*d*n)/(6*(a*d - b*c)) + (x*(49*B^2*a*b*d^2*n^2 - 5*B^2*b^2*c
*d*n^2 + 30*A*B*a*b*d^2*n - 6*A*B*b^2*c*d*n))/(2*(a*d - b*c)) + (d*x^2*(11*
B^2*b^2*d*n^2 + 6*A*B*b^2*d*n))/(a*d - b*c))/(x*(27*a^2*b^3*c*g^4 - 27*a^3*
b^2*d*g^4) - x^2*(27*a^2*b^3*d*g^4 - 27*a*b^4*c*g^4) + x^3*(9*b^5*c*g^4 - 9
*a*b^4*d*g^4) + 9*a^3*b^2*c*g^4 - 9*a^4*b*d*g^4) - log(e*((a + b*x)/(c + d*
x))^n)*((2*A*B)/(3*a^3*b*g^4 + 3*b^4*g^4*x^3 + 9*a^2*b^2*g^4*x + 9*a*b^3*g^
4*x^2) + (2*B^2*d^3*(x*(b*((b*g^4*n*(a*d - b*c)*(3*a*d - b*c))/(2*d^2) + (a
*b*g^4*n*(a*d - b*c))/d) + (2*a*b^2*g^4*n*(a*d - b*c))/d + (b^2*g^4*n*(a*d
- b*c)*(3*a*d - b*c))/d^2) + a*((b*g^4*n*(a*d - b*c)*(3*a*d - b*c))/(2*d^2)
 + (a*b*g^4*n*(a*d - b*c))/d) + (b*g^4*n*(a*d - b*c)*(3*a^2*d^2 + b^2*c^2 -
 3*a*b*c*d))/d^3 + (3*b^3*g^4*n*x^2*(a*d - b*c))/d))/(3*b*g^4*(a^3*d^3 - b^
3*c^3 + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2)*(3*a^3*b*g^4 + 3*b^4*g^4*x^3 + 9*a^2
*b^2*g^4*x + 9*a*b^3*g^4*x^2))) - log(e*((a + b*x)/(c + d*x))^n)^2*(B^2/(3*
b*(a^3*g^4 + b^3*g^4*x^3 + 3*a*b^2*g^4*x^2 + 3*a^2*b*g^4*x)) - (B^2*d^3)/(3
*b*g^4*(a^3*d^3 - b^3*c^3 + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2))) - (B*d^3*n*ata
n((B*d^3*n*(6*A + 11*B*n)*((b^4*c^3*g^4 + a^3*b*d^3*g^4 - a*b^3*c^2*d*g^4 -
 a^2*b^2*c*d^2*g^4)/(b^3*c^2*g^4 + a^2*b*d^2*g^4 - 2*a*b^2*c*d*g^4) + 2*b*d
*x)*(b^3*c^2*g^4 + a^2*b*d^2*g^4 - 2*a*b^2*c*d*g^4)*1i)/(b*g^4*(11*B^2*d^3*
n^2 + 6*A*B*d^3*n)*(a*d - b*c)^3))*(6*A + 11*B*n)*2i)/(9*b*g^4*(a*d - b*c)^
3)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{A^2}{a^4+4a^3bx+6a^2b^2x^2+4ab^3x^3+b^4x^4}\,dx + \int \frac{B^2\log\left(e\left(\frac{a}{c+dx}+\frac{bx}{c+dx}\right)^n\right)^2}{a^4+4a^3bx+6a^2b^2x^2+4ab^3x^3+b^4x^4}\,dx + \int \frac{2AB\log\left(e\left(\frac{a}{c+dx}+\frac{bx}{c+dx}\right)^n\right)}{a^4+4a^3bx+6a^2b^2x^2+4ab^3x^3+b^4x^4}\,dx}{g^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*ln(e*((b*x+a)/(d*x+c))**n))**2/(b*g*x+a*g)**4,x)
```

```
[Out] (Integral(A**2/(a**4 + 4*a**3*b*x + 6*a**2*b**2*x**2 + 4*a*b**3*x**3 + b**4
*x**4), x) + Integral(B**2*log(e*(a/(c + d*x) + b*x/(c + d*x))**n)**2/(a**4
 + 4*a**3*b*x + 6*a**2*b**2*x**2 + 4*a*b**3*x**3 + b**4*x**4), x) + Integra
l(2*A*B*log(e*(a/(c + d*x) + b*x/(c + d*x))**n)/(a**4 + 4*a**3*b*x + 6*a**2
*b**2*x**2 + 4*a*b**3*x**3 + b**4*x**4), x))/g**4
```

## 3.18 $\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag+bgx)^5}\,dx$

**Optimal**. Leaf size=615

$$-\frac{b^3(c+dx)^4\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{4g^5(a+bx)^4(bc-ad)^4}-\frac{b^3Bn(c+dx)^4\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{8g^5(a+bx)^4(bc-ad)^4}+\frac{b^2d(c+dx)^3\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+\right.}{g^5(a+bx)^3(bc-ad)^4}$$

```
[Out] 2*B^2*d^3*n^2*(d*x+c)/(-a*d+b*c)^4/g^5/(b*x+a)-3/4*b*B^2*d^2*n^2*(d*x+c)^2/
(-a*d+b*c)^4/g^5/(b*x+a)^2+2/9*b^2*B^2*d*n^2*(d*x+c)^3/(-a*d+b*c)^4/g^5/(b*
x+a)^3-1/32*b^3*B^2*n^2*(d*x+c)^4/(-a*d+b*c)^4/g^5/(b*x+a)^4+2*B*d^3*n*(d*x
+c)*(A+B*ln(e*((b*x+a)/(d*x+c))^n))/(-a*d+b*c)^4/g^5/(b*x+a)-3/2*b*B*d^2*n*
(d*x+c)^2*(A+B*ln(e*((b*x+a)/(d*x+c))^n))/(-a*d+b*c)^4/g^5/(b*x+a)^2+2/3*b^
2*B*d*n*(d*x+c)^3*(A+B*ln(e*((b*x+a)/(d*x+c))^n))/(-a*d+b*c)^4/g^5/(b*x+a)^
3-1/8*b^3*B*n*(d*x+c)^4*(A+B*ln(e*((b*x+a)/(d*x+c))^n))/(-a*d+b*c)^4/g^5/(b
*x+a)^4+d^3*(d*x+c)*(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2/(-a*d+b*c)^4/g^5/(b*x
+a)-3/2*b*d^2*(d*x+c)^2*(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2/(-a*d+b*c)^4/g^5/
(b*x+a)^2+b^2*d*(d*x+c)^3*(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2/(-a*d+b*c)^4/g^
5/(b*x+a)^3-1/4*b^3*(d*x+c)^4*(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2/(-a*d+b*c)^
4/g^5/(b*x+a)^4
```

**Rubi [C]** time = 1.31, antiderivative size = 826, normalized size of antiderivative = 1.34, number of steps used = 36, number of rules used = 11, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.314, Rules used = {2525, 12, 2528, 44, 2524, 2418, 2390, 2301, 2394, 2393, 2391}

$$-\frac{B^2n^2\log^2(a+bx)d^4}{4b(bc-ad)^4g^5}-\frac{B^2n^2\log^2(c+dx)d^4}{4b(bc-ad)^4g^5}+\frac{25B^2n^2\log(a+bx)d^4}{24b(bc-ad)^4g^5}+\frac{Bn\log(a+bx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)d^4}{2b(bc-ad)^4g^5}-$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2/(a*g + b*g*x)^5,x]
```

```
[Out] -(B^2*n^2)/(32*b*g^5*(a + b*x)^4) + (7*B^2*d*n^2)/(72*b*(b*c - a*d)*g^5*(a
+ b*x)^3) - (13*B^2*d^2*n^2)/(48*b*(b*c - a*d)^2*g^5*(a + b*x)^2) + (25*B^2
*d^3*n^2)/(24*b*(b*c - a*d)^3*g^5*(a + b*x)) + (25*B^2*d^4*n^2*Log[a + b*x]
)/(24*b*(b*c - a*d)^4*g^5) - (B^2*d^4*n^2*Log[a + b*x]^2)/(4*b*(b*c - a*d)^
4*g^5) - (B*n*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/(8*b*g^5*(a + b*x)^4)
 + (B*d*n*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/(6*b*(b*c - a*d)*g^5*(a +
 b*x)^3) - (B*d^2*n*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/(4*b*(b*c - a*d
)^2*g^5*(a + b*x)^2) + (B*d^3*n*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/(2*
```

```
b*(b*c - a*d)^3*g^5*(a + b*x)) + (B*d^4*n*Log[a + b*x]*(A + B*Log[e*((a + b
*x)/(c + d*x))^n]))/(2*b*(b*c - a*d)^4*g^5) - (A + B*Log[e*((a + b*x)/(c +
d*x))^n])^2/(4*b*g^5*(a + b*x)^4) - (25*B^2*d^4*n^2*Log[c + d*x])/(24*b*(b*
c - a*d)^4*g^5) + (B^2*d^4*n^2*Log[-((d*(a + b*x))/(b*c - a*d))]*Log[c + d*
x])/(2*b*(b*c - a*d)^4*g^5) - (B*d^4*n*(A + B*Log[e*((a + b*x)/(c + d*x))^n
])*Log[c + d*x])/(2*b*(b*c - a*d)^4*g^5) - (B^2*d^4*n^2*Log[c + d*x]^2)/(4*
b*(b*c - a*d)^4*g^5) + (B^2*d^4*n^2*Log[a + b*x]*Log[(b*(c + d*x))/(b*c - a
*d)])/(2*b*(b*c - a*d)^4*g^5) + (B^2*d^4*n^2*PolyLog[2, -((d*(a + b*x))/(b*
c - a*d))])/(2*b*(b*c - a*d)^4*g^5) + (B^2*d^4*n^2*PolyLog[2, (b*(c + d*x))
/(b*c - a*d)])/(2*b*(b*c - a*d)^4*g^5)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 2301

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

Rule 2390

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_))^(q_.), x_Symbol] :> Dist[1/e, Subst[Int[((f*x)/d)^q*(a + b*Log[c*x^
n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && E
qQ[e*f - d*g, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2393

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + (c*e*x)/g])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
```

```
(e*f - d*g), 0]
```

Rule 2394

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d + e*x
)^n]))/g, x] - Dist[(b*e*n)/g, Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2418

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*(RFx_), x_Sy
mbol] :> With[{u = ExpandIntegrand[(a + b*Log[c*(d + e*x)^n])^p, RFx, x]},
Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d, e, n}, x] && RationalFunctionQ[
RFx, x] && IntegerQ[p]
```

Rule 2524

```
Int[((a_.) + Log[(c_.)*(RFx_)^(p_.)]*(b_.))^(n_.)/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[(Log[d + e*x]*(a + b*Log[c*RFx^p])^n)/e, x] - Dist[(b*n*p)/e
, Int[(Log[d + e*x]*(a + b*Log[c*RFx^p])^(n - 1)*D[RFx, x])/RFx, x], x] /;
FreeQ[{a, b, c, d, e, p}, x] && RationalFunctionQ[RFx, x] && IGtQ[n, 0]
```

Rule 2525

```
Int[((a_.) + Log[(c_.)*(RFx_)^(p_.)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_))^(m_.
), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(a + b*Log[c*RFx^p])^n)/(e*(m + 1))
, x] - Dist[(b*n*p)/(e*(m + 1)), Int[SimplifyIntegrand[((d + e*x)^(m + 1)*(
a + b*Log[c*RFx^p])^(n - 1)*D[RFx, x])/RFx, x], x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && RationalFunctionQ[RFx, x] && IGtQ[n, 0] && (EqQ[n, 1] ||
IntegerQ[m]) && NeQ[m, -1]
```

Rule 2528

```
Int[((a_.) + Log[(c_.)*(RFx_)^(p_.)]*(b_.))^(n_.)*(RGx_), x_Symbol] :> With
[{u = ExpandIntegrand[(a + b*Log[c*RFx^p])^n, RGx, x]}, Int[u, x] /; SumQ[u
]] /; FreeQ[{a, b, c, p}, x] && RationalFunctionQ[RFx, x] && RationalFuncti
onQ[RGx, x] && IGtQ[n, 0]
```

Rubi steps

$$\int \frac{\left(A + B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag+bgx)^5}\,dx = -\frac{\left(A + B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{4bg^5(a+bx)^4} + \frac{(Bn)\int \frac{(bc-ad)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{g^4(a+bx)^5(c+dx)}\,dx}{2bg}$$

$$= -\frac{\left(A + B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{4bg^5(a+bx)^4} + \frac{(B(bc-ad)n)\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(a+bx)^5(c+dx)}\,dx}{2bg^5}$$

$$= -\frac{\left(A + B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{4bg^5(a+bx)^4} + \frac{(B(bc-ad)n)\int \left(\frac{b\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(bc-ad)(a+bx)^5} - \frac{bd\left(A+B\log\right.}{(bc-ad)^2 \cdots}\right.}{\cdots}$$

$$= -\frac{\left(A + B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{4bg^5(a+bx)^4} + \frac{(Bn)\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(a+bx)^5}\,dx}{2g^5} + \frac{\left(Bd^4n\right)\int \frac{A+B\log\left(e\left(\cdots\right.\right.}{a+b\cdots}}{2(bc-ad)^4 \cdots}$$

$$= -\frac{Bn\left(A + B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{8bg^5(a+bx)^4} + \frac{Bdn\left(A + B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{6b(bc-ad)g^5(a+bx)^3} - \frac{Bd^2n\left(A+B\cdots\right.}{4b(bc-ad\cdots}$$

$$= -\frac{Bn\left(A + B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{8bg^5(a+bx)^4} + \frac{Bdn\left(A + B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{6b(bc-ad)g^5(a+bx)^3} - \frac{Bd^2n\left(A+B\cdots\right.}{4b(bc-ad\cdots}$$

$$= -\frac{Bn\left(A + B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{8bg^5(a+bx)^4} + \frac{Bdn\left(A + B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{6b(bc-ad)g^5(a+bx)^3} - \frac{Bd^2n\left(A+B\cdots\right.}{4b(bc-ad\cdots}$$

$$= -\frac{B^2n^2}{32bg^5(a+bx)^4} + \frac{7B^2dn^2}{72b(bc-ad)g^5(a+bx)^3} - \frac{13B^2d^2n^2}{48b(bc-ad)^2g^5(a+bx)^2} + \frac{\cdots}{24\cdots}$$

$$= -\frac{B^2n^2}{32bg^5(a+bx)^4} + \frac{7B^2dn^2}{72b(bc-ad)g^5(a+bx)^3} - \frac{13B^2d^2n^2}{48b(bc-ad)^2g^5(a+bx)^2} + \frac{\cdots}{24\cdots}$$

$$= -\frac{B^2n^2}{32bg^5(a+bx)^4} + \frac{7B^2dn^2}{72b(bc-ad)g^5(a+bx)^3} - \frac{13B^2d^2n^2}{48b(bc-ad)^2g^5(a+bx)^2} + \frac{\cdots}{24\cdots}$$

**Mathematica [C]** time = 1.02, size = 776, normalized size = 1.26

$$-\frac{Bn\left(-144d^4(a+bx)^4\log(a+bx)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)+144d^4(a+bx)^4\log(c+dx)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)+144d^3(a+bx)^3(ad-bc)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)+7\cdots}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2/(a*g + b*g*x)^5,x]
```

```
[Out] -1/288*(72*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2 + (B*n*(36*(b*c - a*d)^
4*(A + B*Log[e*((a + b*x)/(c + d*x))^n]) + 48*d*(-(b*c) + a*d)^3*(a + b*x)*
(A + B*Log[e*((a + b*x)/(c + d*x))^n]) + 72*d^2*(b*c - a*d)^2*(a + b*x)^2*(
A + B*Log[e*((a + b*x)/(c + d*x))^n]) + 144*d^3*(-(b*c) + a*d)*(a + b*x)^3*
(A + B*Log[e*((a + b*x)/(c + d*x))^n]) - 144*d^4*(a + b*x)^4*Log[a + b*x]*(
A + B*Log[e*((a + b*x)/(c + d*x))^n]) + 144*d^4*(a + b*x)^4*(A + B*Log[e*((
a + b*x)/(c + d*x))^n])*Log[c + d*x] - 144*B*d^3*n*(a + b*x)^3*(b*c - a*d +
 d*(a + b*x)*Log[a + b*x] - d*(a + b*x)*Log[c + d*x]) + 36*B*d^2*n*(a + b*x
)^2*((b*c - a*d)^2 + 2*d*(-(b*c) + a*d)*(a + b*x) - 2*d^2*(a + b*x)^2*Log[a
 + b*x] + 2*d^2*(a + b*x)^2*Log[c + d*x]) - 8*B*d*n*(a + b*x)*(2*(b*c - a*d
)^3 - 3*d*(b*c - a*d)^2*(a + b*x) + 6*d^2*(b*c - a*d)*(a + b*x)^2 + 6*d^3*(
a + b*x)^3*Log[a + b*x] - 6*d^3*(a + b*x)^3*Log[c + d*x]) + 3*B*n*(3*(b*c -
 a*d)^4 + 4*d*(-(b*c) + a*d)^3*(a + b*x) + 6*d^2*(b*c - a*d)^2*(a + b*x)^2
+ 12*d^3*(-(b*c) + a*d)*(a + b*x)^3 - 12*d^4*(a + b*x)^4*Log[a + b*x] + 12*
d^4*(a + b*x)^4*Log[c + d*x]) + 72*B*d^4*n*(a + b*x)^4*(Log[a + b*x]*(Log[a
 + b*x] - 2*Log[(b*(c + d*x))/(b*c - a*d)]) - 2*PolyLog[2, (d*(a + b*x))/(-
(b*c) + a*d)]) - 72*B*d^4*n*(a + b*x)^4*((2*Log[(d*(a + b*x))/(-(b*c) + a*d
)] - Log[c + d*x])*Log[c + d*x] + 2*PolyLog[2, (b*(c + d*x))/(b*c - a*d)]))
)/(b*c - a*d)^4)/(b*g^5*(a + b*x)^4)
```

**fricas [B]** time = 1.08, size = 1762, normalized size = 2.87

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))^2/(b*g*x+a*g)^5,x, algorithm="fr
icas")
```

```
[Out] -1/288*(72*A^2*b^4*c^4 - 288*A^2*a*b^3*c^3*d + 432*A^2*a^2*b^2*c^2*d^2 - 28
8*A^2*a^3*b*c*d^3 + 72*A^2*a^4*d^4 - 12*(25*(B^2*b^4*c*d^3 - B^2*a*b^3*d^4)
*n^2 + 12*(A*B*b^4*c*d^3 - A*B*a*b^3*d^4)*n)*x^3 + (9*B^2*b^4*c^4 - 64*B^2*
a*b^3*c^3*d + 216*B^2*a^2*b^2*c^2*d^2 - 576*B^2*a^3*b*c*d^3 + 415*B^2*a^4*d
^4)*n^2 + 6*((13*B^2*b^4*c^2*d^2 - 176*B^2*a*b^3*c*d^3 + 163*B^2*a^2*b^2*d^
4)*n^2 + 12*(A*B*b^4*c^2*d^2 - 8*A*B*a*b^3*c*d^3 + 7*A*B*a^2*b^2*d^4)*n)*x^
```

```
2 + 72*(B^2*b^4*c^4 - 4*B^2*a*b^3*c^3*d + 6*B^2*a^2*b^2*c^2*d^2 - 4*B^2*a^3
*b*c*d^3 + B^2*a^4*d^4)*log(e)^2 - 72*(B^2*b^4*d^4*n^2*x^4 + 4*B^2*a*b^3*d^
4*n^2*x^3 + 6*B^2*a^2*b^2*d^4*n^2*x^2 + 4*B^2*a^3*b*d^4*n^2*x - (B^2*b^4*c^
4 - 4*B^2*a*b^3*c^3*d + 6*B^2*a^2*b^2*c^2*d^2 - 4*B^2*a^3*b*c*d^3)*n^2)*log
((b*x + a)/(d*x + c))^2 + 12*(3*A*B*b^4*c^4 - 16*A*B*a*b^3*c^3*d + 36*A*B*a
^2*b^2*c^2*d^2 - 48*A*B*a^3*b*c*d^3 + 25*A*B*a^4*d^4)*n - 4*((7*B^2*b^4*c^3
*d - 60*B^2*a*b^3*c^2*d^2 + 324*B^2*a^2*b^2*c*d^3 - 271*B^2*a^3*b*d^4)*n^2
+ 12*(A*B*b^4*c^3*d - 6*A*B*a*b^3*c^2*d^2 + 18*A*B*a^2*b^2*c*d^3 - 13*A*B*a
^3*b*d^4)*n)*x + 12*(12*A*B*b^4*c^4 - 48*A*B*a*b^3*c^3*d + 72*A*B*a^2*b^2*c
^2*d^2 - 48*A*B*a^3*b*c*d^3 + 12*A*B*a^4*d^4 - 12*(B^2*b^4*c*d^3 - B^2*a*b^
3*d^4)*n*x^3 + 6*(B^2*b^4*c^2*d^2 - 8*B^2*a*b^3*c*d^3 + 7*B^2*a^2*b^2*d^4)*
n*x^2 - 4*(B^2*b^4*c^3*d - 6*B^2*a*b^3*c^2*d^2 + 18*B^2*a^2*b^2*c*d^3 - 13*
B^2*a^3*b*d^4)*n*x + (3*B^2*b^4*c^4 - 16*B^2*a*b^3*c^3*d + 36*B^2*a^2*b^2*c
^2*d^2 - 48*B^2*a^3*b*c*d^3 + 25*B^2*a^4*d^4)*n - 12*(B^2*b^4*d^4*n*x^4 + 4
*B^2*a*b^3*d^4*n*x^3 + 6*B^2*a^2*b^2*d^4*n*x^2 + 4*B^2*a^3*b*d^4*n*x - (B^2
*b^4*c^4 - 4*B^2*a*b^3*c^3*d + 6*B^2*a^2*b^2*c^2*d^2 - 4*B^2*a^3*b*c*d^3)*n
)*log((b*x + a)/(d*x + c)))*log(e) - 12*((25*B^2*b^4*d^4*n^2 + 12*A*B*b^4*d
^4*n)*x^4 + 4*(12*A*B*a*b^3*d^4*n + (3*B^2*b^4*c*d^3 + 22*B^2*a*b^3*d^4)*n^
2)*x^3 - (3*B^2*b^4*c^4 - 16*B^2*a*b^3*c^3*d + 36*B^2*a^2*b^2*c^2*d^2 - 48*
B^2*a^3*b*c*d^3)*n^2 + 6*(12*A*B*a^2*b^2*d^4*n - (B^2*b^4*c^2*d^2 - 8*B^2*a
*b^3*c*d^3 - 18*B^2*a^2*b^2*d^4)*n^2)*x^2 - 12*(A*B*b^4*c^4 - 4*A*B*a*b^3*c
^3*d + 6*A*B*a^2*b^2*c^2*d^2 - 4*A*B*a^3*b*c*d^3)*n + 4*(12*A*B*a^3*b*d^4*n
 + (B^2*b^4*c^3*d - 6*B^2*a*b^3*c^2*d^2 + 18*B^2*a^2*b^2*c*d^3 + 12*B^2*a^3
*b*d^4)*n^2)*x)*log((b*x + a)/(d*x + c)))/((b^9*c^4 - 4*a*b^8*c^3*d + 6*a^2
*b^7*c^2*d^2 - 4*a^3*b^6*c*d^3 + a^4*b^5*d^4)*g^5*x^4 + 4*(a*b^8*c^4 - 4*a^
2*b^7*c^3*d + 6*a^3*b^6*c^2*d^2 - 4*a^4*b^5*c*d^3 + a^5*b^4*d^4)*g^5*x^3 +
6*(a^2*b^7*c^4 - 4*a^3*b^6*c^3*d + 6*a^4*b^5*c^2*d^2 - 4*a^5*b^4*c*d^3 + a^
6*b^3*d^4)*g^5*x^2 + 4*(a^3*b^6*c^4 - 4*a^4*b^5*c^3*d + 6*a^5*b^4*c^2*d^2 -
 4*a^6*b^3*c*d^3 + a^7*b^2*d^4)*g^5*x + (a^4*b^5*c^4 - 4*a^5*b^4*c^3*d + 6*
a^6*b^3*c^2*d^2 - 4*a^7*b^2*c*d^3 + a^8*b*d^4)*g^5)
```

**giac [A]** time = 17.90, size = 1166, normalized size = 1.90

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))^2/(b*g*x+a*g)^5,x, algorithm="gi
ac")
```

```
[Out] -1/288*(72*(B^2*b^3*n^2 - 4*(b*x + a)*B^2*b^2*d*n^2/(d*x + c) + 6*(b*x + a)
^2*B^2*b*d^2*n^2/(d*x + c)^2 - 4*(b*x + a)^3*B^2*d^3*n^2/(d*x + c)^3)*log((
b*x + a)/(d*x + c))^2/((b*x + a)^4*b^3*c^3*g^5/(d*x + c)^4 - 3*(b*x + a)^4*
a*b^2*c^2*d*g^5/(d*x + c)^4 + 3*(b*x + a)^4*a^2*b*c*d^2*g^5/(d*x + c)^4 - (
b*x + a)^4*a^3*d^3*g^5/(d*x + c)^4) + 12*(3*B^2*b^3*n^2 - 16*(b*x + a)*B^2*
b^2*d*n^2/(d*x + c) + 36*(b*x + a)^2*B^2*b*d^2*n^2/(d*x + c)^2 - 48*(b*x +
```

```
a)^3*B^2*d^3*n^2/(d*x + c)^3 + 12*A*B*b^3*n + 12*B^2*b^3*n - 48*(b*x + a)*A
*B*b^2*d*n/(d*x + c) - 48*(b*x + a)*B^2*b^2*d*n/(d*x + c) + 72*(b*x + a)^2*
A*B*b*d^2*n/(d*x + c)^2 + 72*(b*x + a)^2*B^2*b*d^2*n/(d*x + c)^2 - 48*(b*x
+ a)^3*A*B*d^3*n/(d*x + c)^3 - 48*(b*x + a)^3*B^2*d^3*n/(d*x + c)^3)*log((b
*x + a)/(d*x + c))/((b*x + a)^4*b^3*c^3*g^5/(d*x + c)^4 - 3*(b*x + a)^4*a*b
^2*c^2*d*g^5/(d*x + c)^4 + 3*(b*x + a)^4*a^2*b*c*d^2*g^5/(d*x + c)^4 - (b*x
 + a)^4*a^3*d^3*g^5/(d*x + c)^4) + (9*B^2*b^3*n^2 - 64*(b*x + a)*B^2*b^2*d*
n^2/(d*x + c) + 216*(b*x + a)^2*B^2*b*d^2*n^2/(d*x + c)^2 - 576*(b*x + a)^3
*B^2*d^3*n^2/(d*x + c)^3 + 36*A*B*b^3*n + 36*B^2*b^3*n - 192*(b*x + a)*A*B*
b^2*d*n/(d*x + c) - 192*(b*x + a)*B^2*b^2*d*n/(d*x + c) + 432*(b*x + a)^2*A
*B*b*d^2*n/(d*x + c)^2 + 432*(b*x + a)^2*B^2*b*d^2*n/(d*x + c)^2 - 576*(b*x
 + a)^3*A*B*d^3*n/(d*x + c)^3 - 576*(b*x + a)^3*B^2*d^3*n/(d*x + c)^3 + 72*
A^2*b^3 + 144*A*B*b^3 + 72*B^2*b^3 - 288*(b*x + a)*A^2*b^2*d/(d*x + c) - 57
6*(b*x + a)*A*B*b^2*d/(d*x + c) - 288*(b*x + a)*B^2*b^2*d/(d*x + c) + 432*(
b*x + a)^2*A^2*b*d^2/(d*x + c)^2 + 864*(b*x + a)^2*A*B*b*d^2/(d*x + c)^2 +
432*(b*x + a)^2*B^2*b*d^2/(d*x + c)^2 - 288*(b*x + a)^3*A^2*d^3/(d*x + c)^3
 - 576*(b*x + a)^3*A*B*d^3/(d*x + c)^3 - 288*(b*x + a)^3*B^2*d^3/(d*x + c)^
3)/((b*x + a)^4*b^3*c^3*g^5/(d*x + c)^4 - 3*(b*x + a)^4*a*b^2*c^2*d*g^5/(d*
x + c)^4 + 3*(b*x + a)^4*a^2*b*c*d^2*g^5/(d*x + c)^4 - (b*x + a)^4*a^3*d^3*
g^5/(d*x + c)^4))*(b*c/(b*c - a*d)^2 - a*d/(b*c - a*d)^2)
```

**maple [F]** time = 0.28, size = 0, normalized size = 0.00

$$\int \frac{\left(B \ln \left(e \left(\frac{bx+a}{dx+c}\right)^{n}\right)+A\right)^{2}}{\left(bgx+ag\right)^{5}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*ln(e*((b*x+a)/(d*x+c))^n)+A)^2/(b*g*x+a*g)^5,x)
```

```
[Out] int((B*ln(e*((b*x+a)/(d*x+c))^n)+A)^2/(b*g*x+a*g)^5,x)
```

**maxima [B]** time = 2.98, size = 2136, normalized size = 3.47

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))^2/(b*g*x+a*g)^5,x, algorithm="ma
xima")
```

```
[Out] 1/24*A*B*n*((12*b^3*d^3*x^3 - 3*b^3*c^3 + 13*a*b^2*c^2*d - 23*a^2*b*c*d^2 +
 25*a^3*d^3 - 6*(b^3*c*d^2 - 7*a*b^2*d^3)*x^2 + 4*(b^3*c^2*d - 5*a*b^2*c*d^
2 + 13*a^2*b*d^3)*x)/((b^8*c^3 - 3*a*b^7*c^2*d + 3*a^2*b^6*c*d^2 - a^3*b^5*
d^3)*g^5*x^4 + 4*(a*b^7*c^3 - 3*a^2*b^6*c^2*d + 3*a^3*b^5*c*d^2 - a^4*b^4*d
```

```
^3)*g^5*x^3 + 6*(a^2*b^6*c^3 - 3*a^3*b^5*c^2*d + 3*a^4*b^4*c*d^2 - a^5*b^3*
d^3)*g^5*x^2 + 4*(a^3*b^5*c^3 - 3*a^4*b^4*c^2*d + 3*a^5*b^3*c*d^2 - a^6*b^2
*d^3)*g^5*x + (a^4*b^4*c^3 - 3*a^5*b^3*c^2*d + 3*a^6*b^2*c*d^2 - a^7*b*d^3)
*g^5) + 12*d^4*log(b*x + a)/((b^5*c^4 - 4*a*b^4*c^3*d + 6*a^2*b^3*c^2*d^2 -
 4*a^3*b^2*c*d^3 + a^4*b*d^4)*g^5) - 12*d^4*log(d*x + c)/((b^5*c^4 - 4*a*b^
4*c^3*d + 6*a^2*b^3*c^2*d^2 - 4*a^3*b^2*c*d^3 + a^4*b*d^4)*g^5)) + 1/288*(1
2*n*((12*b^3*d^3*x^3 - 3*b^3*c^3 + 13*a*b^2*c^2*d - 23*a^2*b*c*d^2 + 25*a^3
*d^3 - 6*(b^3*c*d^2 - 7*a*b^2*d^3)*x^2 + 4*(b^3*c^2*d - 5*a*b^2*c*d^2 + 13*
a^2*b*d^3)*x)/((b^8*c^3 - 3*a*b^7*c^2*d + 3*a^2*b^6*c*d^2 - a^3*b^5*d^3)*g^
5*x^4 + 4*(a*b^7*c^3 - 3*a^2*b^6*c^2*d + 3*a^3*b^5*c*d^2 - a^4*b^4*d^3)*g^5
*x^3 + 6*(a^2*b^6*c^3 - 3*a^3*b^5*c^2*d + 3*a^4*b^4*c*d^2 - a^5*b^3*d^3)*g^
5*x^2 + 4*(a^3*b^5*c^3 - 3*a^4*b^4*c^2*d + 3*a^5*b^3*c*d^2 - a^6*b^2*d^3)*g
^5*x + (a^4*b^4*c^3 - 3*a^5*b^3*c^2*d + 3*a^6*b^2*c*d^2 - a^7*b*d^3)*g^5) +
 12*d^4*log(b*x + a)/((b^5*c^4 - 4*a*b^4*c^3*d + 6*a^2*b^3*c^2*d^2 - 4*a^3*
b^2*c*d^3 + a^4*b*d^4)*g^5) - 12*d^4*log(d*x + c)/((b^5*c^4 - 4*a*b^4*c^3*d
 + 6*a^2*b^3*c^2*d^2 - 4*a^3*b^2*c*d^3 + a^4*b*d^4)*g^5))*log(e*(b*x/(d*x +
 c) + a/(d*x + c))^n) - (9*b^4*c^4 - 64*a*b^3*c^3*d + 216*a^2*b^2*c^2*d^2 -
 576*a^3*b*c*d^3 + 415*a^4*d^4 - 300*(b^4*c*d^3 - a*b^3*d^4)*x^3 + 6*(13*b^
4*c^2*d^2 - 176*a*b^3*c*d^3 + 163*a^2*b^2*d^4)*x^2 + 72*(b^4*d^4*x^4 + 4*a*
b^3*d^4*x^3 + 6*a^2*b^2*d^4*x^2 + 4*a^3*b*d^4*x + a^4*d^4)*log(b*x + a)^2 +
 72*(b^4*d^4*x^4 + 4*a*b^3*d^4*x^3 + 6*a^2*b^2*d^4*x^2 + 4*a^3*b*d^4*x + a^
4*d^4)*log(d*x + c)^2 - 4*(7*b^4*c^3*d - 60*a*b^3*c^2*d^2 + 324*a^2*b^2*c*d
^3 - 271*a^3*b*d^4)*x - 300*(b^4*d^4*x^4 + 4*a*b^3*d^4*x^3 + 6*a^2*b^2*d^4*
x^2 + 4*a^3*b*d^4*x + a^4*d^4)*log(b*x + a) + 12*(25*b^4*d^4*x^4 + 100*a*b^
3*d^4*x^3 + 150*a^2*b^2*d^4*x^2 + 100*a^3*b*d^4*x + 25*a^4*d^4 - 12*(b^4*d^
4*x^4 + 4*a*b^3*d^4*x^3 + 6*a^2*b^2*d^4*x^2 + 4*a^3*b*d^4*x + a^4*d^4)*log(
b*x + a))*log(d*x + c))*n^2/(a^4*b^5*c^4*g^5 - 4*a^5*b^4*c^3*d*g^5 + 6*a^6*
b^3*c^2*d^2*g^5 - 4*a^7*b^2*c*d^3*g^5 + a^8*b*d^4*g^5 + (b^9*c^4*g^5 - 4*a*
b^8*c^3*d*g^5 + 6*a^2*b^7*c^2*d^2*g^5 - 4*a^3*b^6*c*d^3*g^5 + a^4*b^5*d^4*g
^5)*x^4 + 4*(a*b^8*c^4*g^5 - 4*a^2*b^7*c^3*d*g^5 + 6*a^3*b^6*c^2*d^2*g^5 -
4*a^4*b^5*c*d^3*g^5 + a^5*b^4*d^4*g^5)*x^3 + 6*(a^2*b^7*c^4*g^5 - 4*a^3*b^6
*c^3*d*g^5 + 6*a^4*b^5*c^2*d^2*g^5 - 4*a^5*b^4*c*d^3*g^5 + a^6*b^3*d^4*g^5)
*x^2 + 4*(a^3*b^6*c^4*g^5 - 4*a^4*b^5*c^3*d*g^5 + 6*a^5*b^4*c^2*d^2*g^5 - 4
*a^6*b^3*c*d^3*g^5 + a^7*b^2*d^4*g^5)*x))*B^2 - 1/4*B^2*log(e*(b*x/(d*x + c
) + a/(d*x + c))^n)^2/(b^5*g^5*x^4 + 4*a*b^4*g^5*x^3 + 6*a^2*b^3*g^5*x^2 +
4*a^3*b^2*g^5*x + a^4*b*g^5) - 1/2*A*B*log(e*(b*x/(d*x + c) + a/(d*x + c))^
n)/(b^5*g^5*x^4 + 4*a*b^4*g^5*x^3 + 6*a^2*b^3*g^5*x^2 + 4*a^3*b^2*g^5*x + a
^4*b*g^5) - 1/4*A^2/(b^5*g^5*x^4 + 4*a*b^4*g^5*x^3 + 6*a^2*b^3*g^5*x^2 + 4*
a^3*b^2*g^5*x + a^4*b*g^5)
```

**mupad** [**B**] time = 9.22, size = 1769, normalized size = 2.88

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*log(e*((a + b*x)/(c + d*x))^n))^2/(a*g + b*g*x)^5,x)
```

```
[Out] (B*d^4*n*atan((B*d^4*n*(12*A + 25*B*n)*(24*b^5*c^4*g^5 - 24*a^4*b*d^4*g^5 -
 48*a*b^4*c^3*d*g^5 + 48*a^3*b^2*c*d^3*g^5)*1i)/(24*b*g^5*(25*B^2*d^4*n^2 +
 12*A*B*d^4*n)*(a*d - b*c)^4) + (B*d^5*n*x*(12*A + 25*B*n)*(b^4*c^3*g^5 - a
^3*b*d^3*g^5 - 3*a*b^3*c^2*d*g^5 + 3*a^2*b^2*c*d^2*g^5)*2i)/(g^5*(25*B^2*d^
4*n^2 + 12*A*B*d^4*n)*(a*d - b*c)^4))*(12*A + 25*B*n)*1i)/(12*b*g^5*(a*d -
b*c)^4) - ((72*A^2*a^3*d^3 - 72*A^2*b^3*c^3 + 415*B^2*a^3*d^3*n^2 - 9*B^2*b
^3*c^3*n^2 + 216*A^2*a*b^2*c^2*d - 216*A^2*a^2*b*c*d^2 + 300*A*B*a^3*d^3*n
- 36*A*B*b^3*c^3*n + 55*B^2*a*b^2*c^2*d*n^2 - 161*B^2*a^2*b*c*d^2*n^2 + 156
*A*B*a*b^2*c^2*d*n - 276*A*B*a^2*b*c*d^2*n)/(12*(a*d - b*c)) + (x^2*(163*B^
2*a*b^2*d^3*n^2 - 13*B^2*b^3*c*d^2*n^2 + 84*A*B*a*b^2*d^3*n - 12*A*B*b^3*c*
d^2*n))/(2*(a*d - b*c)) + (x*(271*B^2*a^2*b*d^3*n^2 + 7*B^2*b^3*c^2*d*n^2 -
 53*B^2*a*b^2*c*d^2*n^2 + 156*A*B*a^2*b*d^3*n + 12*A*B*b^3*c^2*d*n - 60*A*B
*a*b^2*c*d^2*n))/(3*(a*d - b*c)) + (d*x^3*(25*B^2*b^3*d^2*n^2 + 12*A*B*b^3*
d^2*n))/(a*d - b*c))/(x*(96*a^3*b^4*c^2*g^5 + 96*a^5*b^2*d^2*g^5 - 192*a^4*
b^3*c*d*g^5) + x^3*(96*a*b^6*c^2*g^5 + 96*a^3*b^4*d^2*g^5 - 192*a^2*b^5*c*d
*g^5) + x^4*(24*b^7*c^2*g^5 + 24*a^2*b^5*d^2*g^5 - 48*a*b^6*c*d*g^5) + x^2*
(144*a^2*b^5*c^2*g^5 + 144*a^4*b^3*d^2*g^5 - 288*a^3*b^4*c*d*g^5) + 24*a^6*
b*d^2*g^5 + 24*a^4*b^3*c^2*g^5 - 48*a^5*b^2*c*d*g^5) - log(e*((a + b*x)/(c
+ d*x))^n)^2*(B^2/(4*b*(a^4*g^5 + b^4*g^5*x^4 + 4*a*b^3*g^5*x^3 + 6*a^2*b^2
*g^5*x^2 + 4*a^3*b*g^5*x)) - (B^2*d^4)/(4*b*g^5*(a^4*d^4 + b^4*c^4 + 6*a^2*
b^2*c^2*d^2 - 4*a*b^3*c^3*d - 4*a^3*b*c*d^3))) - log(e*((a + b*x)/(c + d*x)
)^n)*((A*B)/(2*a^4*b*g^5 + 2*b^5*g^5*x^4 + 8*a^3*b^2*g^5*x + 8*a*b^4*g^5*x^
3 + 12*a^2*b^3*g^5*x^2) + (B^2*d^4*(x*(b*(a*((b*g^5*n*(a*d - b*c)*(4*a*d -
b*c))/(6*d^2) + (a*b*g^5*n*(a*d - b*c))/(2*d)) + (b*g^5*n*(a*d - b*c)*(6*a^
2*d^2 + b^2*c^2 - 4*a*b*c*d))/(6*d^3)) + a*(b*((b*g^5*n*(a*d - b*c)*(4*a*d
- b*c))/(6*d^2) + (a*b*g^5*n*(a*d - b*c))/(2*d)) + (a*b^2*g^5*n*(a*d - b*c)
)/d + (b^2*g^5*n*(a*d - b*c)*(4*a*d - b*c))/(3*d^2)) + (b^2*g^5*n*(a*d - b*
c)*(6*a^2*d^2 + b^2*c^2 - 4*a*b*c*d))/(2*d^3)) + a*(a*((b*g^5*n*(a*d - b*c)
*(4*a*d - b*c))/(6*d^2) + (a*b*g^5*n*(a*d - b*c))/(2*d)) + (b*g^5*n*(a*d -
b*c)*(6*a^2*d^2 + b^2*c^2 - 4*a*b*c*d))/(6*d^3)) + x^2*(b*(b*((b*g^5*n*(a*d
 - b*c)*(4*a*d - b*c))/(6*d^2) + (a*b*g^5*n*(a*d - b*c))/(2*d)) + (a*b^2*g^
5*n*(a*d - b*c))/d + (b^2*g^5*n*(a*d - b*c)*(4*a*d - b*c))/(3*d^2)) + (3*a*
b^3*g^5*n*(a*d - b*c))/(2*d) + (b^3*g^5*n*(a*d - b*c)*(4*a*d - b*c))/(2*d^2
)) + (2*b^4*g^5*n*x^3*(a*d - b*c))/d + (b*g^5*n*(a*d - b*c)*(4*a^3*d^3 - b^
3*c^3 + 4*a*b^2*c^2*d - 6*a^2*b*c*d^2))/(2*d^4)))/(2*b*g^5*(2*a^4*b*g^5 + 2
*b^5*g^5*x^4 + 8*a^3*b^2*g^5*x + 8*a*b^4*g^5*x^3 + 12*a^2*b^3*g^5*x^2)*(a^4
*d^4 + b^4*c^4 + 6*a^2*b^2*c^2*d^2 - 4*a*b^3*c^3*d - 4*a^3*b*c*d^3)))
```

**sympy** [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*ln(e*((b*x+a)/(d*x+c))**n))**2/(b*g*x+a*g)**5,x)
```

```
[Out] Timed out
```

## 3.19 $\int \frac{(ag+bgx)^2}{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}\,dx$

**Optimal**. Leaf size=38

$$\text{Int}\left(\frac{(ag+bgx)^2}{B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A},x\right)$$

```
[Out] Unintegrable((b*g*x+a*g)^2/(A+B*ln(e*((b*x+a)/(d*x+c))^n)),x)
```

**Rubi [A]** time = 0.23, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{(ag+bgx)^2}{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(a*g + b*g*x)^2/(A + B*Log[e*((a + b*x)/(c + d*x))^n]),x]
```

```
[Out] a^2*g^2*Defer[Int][(A + B*Log[e*((a + b*x)/(c + d*x))^n])^(-1), x] + 2*a*b*
g^2*Defer[Int][x/(A + B*Log[e*((a + b*x)/(c + d*x))^n]), x] + b^2*g^2*Defer
[Int][x^2/(A + B*Log[e*((a + b*x)/(c + d*x))^n]), x]
```

Rubi steps

$$\begin{aligned}\int \frac{(ag+bgx)^2}{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}\,dx &= \int \left(\frac{a^2g^2}{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}+\frac{2abg^2x}{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}+\frac{b^2g^2x^2}{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}\right)dx\\ &= \left(a^2g^2\right)\int \frac{1}{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}\,dx+\left(2abg^2\right)\int \frac{x}{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}\,dx+\left(b^2g^2\right.\end{aligned}$$

**Mathematica [A]** time = 0.77, size = 0, normalized size = 0.00

$$\int \frac{(ag+bgx)^2}{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(a*g + b*g*x)^2/(A + B*Log[e*((a + b*x)/(c + d*x))^n]),x]
```

```
[Out] Integrate[(a*g + b*g*x)^2/(A + B*Log[e*((a + b*x)/(c + d*x))^n]), x]
```

**fricas [A]** time = 0.88, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{b^2g^2x^2 + 2\,abg^2x + a^2g^2}{B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right) + A}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*g*x+a*g)^2/(A+B*log(e*((b*x+a)/(d*x+c))^n)),x, algorithm="fric
as")
```

```
[Out] integral((b^2*g^2*x^2 + 2*a*b*g^2*x + a^2*g^2)/(B*log(e*((b*x + a)/(d*x + c
))^n) + A), x)
```

**giac [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*g*x+a*g)^2/(A+B*log(e*((b*x+a)/(d*x+c))^n)),x, algorithm="giac
")
```

```
[Out] Timed out
```

**maple [A]** time = 0.26, size = 0, normalized size = 0.00

$$\int \frac{\left(bgx + ag\right)^2}{B\ln\left(e\left(\frac{bx+a}{dx+c}\right)^n\right) + A}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*g*x+a*g)^2/(B*ln(e*((b*x+a)/(d*x+c))^n)+A),x)
```

```
[Out] int((b*g*x+a*g)^2/(B*ln(e*((b*x+a)/(d*x+c))^n)+A),x)
```

**maxima [A]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(bgx + ag\right)^2}{B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right) + A}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*g*x+a*g)^2/(A+B*log(e*((b*x+a)/(d*x+c))^n)),x, algorithm="maxi
ma")
```

```
[Out] integrate((b*g*x + a*g)^2/(B*log(e*((b*x + a)/(d*x + c))^n) + A), x)
```

**mupad [A]** time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{\left(a\,g+b\,g\,x\right)^2}{A+B\,\ln\left(e\,{\left(\frac{a+b\,x}{c+d\,x}\right)}^n\right)} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*g + b*g*x)^2/(A + B*log(e*((a + b*x)/(c + d*x))^n)),x)
```

```
[Out] int((a*g + b*g*x)^2/(A + B*log(e*((a + b*x)/(c + d*x))^n)), x)
```

**sympy [A]** time = 0.00, size = 0, normalized size = 0.00

$$g^{2} \left(\int \frac{a^{2}}{A + B \log{\left(e \left(\frac{a}{c + d x} + \frac{b x}{c + d x}\right)^{n} \right)}}\, dx + \int \frac{b^{2} x^{2}}{A + B \log{\left(e \left(\frac{a}{c + d x} + \frac{b x}{c + d x}\right)^{n} \right)}}\, dx + \int \frac{2 a b x}{A + B \log{\left(e \left(\frac{a}{c + d x} + \frac{b x}{c + d x}\right)^{n} \right)}}\, d$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*g*x+a*g)**2/(A+B*ln(e*((b*x+a)/(d*x+c))**n)),x)
```

```
[Out] g**2*(Integral(a**2/(A + B*log(e*(a/(c + d*x) + b*x/(c + d*x))**n)), x) + I
ntegral(b**2*x**2/(A + B*log(e*(a/(c + d*x) + b*x/(c + d*x))**n)), x) + Int
egral(2*a*b*x/(A + B*log(e*(a/(c + d*x) + b*x/(c + d*x))**n)), x))
```

# 3.20 $\int \frac{ag+bgx}{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}\,dx$

**Optimal**. Leaf size=36

$$\text{Int}\left(\frac{ag+bgx}{B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A},x\right)$$

```
[Out] Unintegrable((b*g*x+a*g)/(A+B*ln(e*((b*x+a)/(d*x+c))^n)),x)
```

**Rubi [A]** time = 0.11, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{ag+bgx}{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(a*g + b*g*x)/(A + B*Log[e*((a + b*x)/(c + d*x))^n]),x]
```

```
[Out] a*g*Defer[Int][(A + B*Log[e*((a + b*x)/(c + d*x))^n])^(-1), x] + b*g*Defer[
Int][x/(A + B*Log[e*((a + b*x)/(c + d*x))^n]), x]
```

Rubi steps

$$\begin{aligned}\int \frac{ag+bgx}{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}\,dx &= \int\left(\frac{ag}{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}+\frac{bgx}{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}\right)dx\\ &= (ag)\int \frac{1}{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}\,dx+(bg)\int \frac{x}{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}\,dx\end{aligned}$$

**Mathematica [A]** time = 0.30, size = 0, normalized size = 0.00

$$\int \frac{ag+bgx}{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(a*g + b*g*x)/(A + B*Log[e*((a + b*x)/(c + d*x))^n]),x]
```

```
[Out] Integrate[(a*g + b*g*x)/(A + B*Log[e*((a + b*x)/(c + d*x))^n]), x]
```

**fricas [A]** time = 0.92, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{bgx+ag}{B\log\left(e\left(\frac{bx+a}{dx+c}\right)^{n}\right)+A},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*g*x+a*g)/(A+B*log(e*((b*x+a)/(d*x+c))^n)),x, algorithm="fricas
")
```

```
[Out] integral((b*g*x + a*g)/(B*log(e*((b*x + a)/(d*x + c))^n) + A), x)
```

**giac [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*g*x+a*g)/(A+B*log(e*((b*x+a)/(d*x+c))^n)),x, algorithm="giac")
```

```
[Out] Timed out
```

**maple [A]** time = 0.15, size = 0, normalized size = 0.00

$$\int \frac{bgx+ag}{B\ln\left(e\left(\frac{bx+a}{dx+c}\right)^{n}\right)+A}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*g*x+a*g)/(B*ln(e*((b*x+a)/(d*x+c))^n)+A),x)
```

```
[Out] int((b*g*x+a*g)/(B*ln(e*((b*x+a)/(d*x+c))^n)+A),x)
```

**maxima [A]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{bgx+ag}{B\log\left(e\left(\frac{bx+a}{dx+c}\right)^{n}\right)+A}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*g*x+a*g)/(A+B*log(e*((b*x+a)/(d*x+c))^n)),x, algorithm="maxima
")
```

```
[Out] integrate((b*g*x + a*g)/(B*log(e*((b*x + a)/(d*x + c))^n) + A), x)
```

**mupad [A]** time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{a\,g+b\,g\,x}{A+B\,\ln\left(e\,{\left(\frac{a+b\,x}{c+d\,x}\right)}^n\right)} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*g + b*g*x)/(A + B*log(e*((a + b*x)/(c + d*x))^n)),x)
```

```
[Out] int((a*g + b*g*x)/(A + B*log(e*((a + b*x)/(c + d*x))^n)), x)
```

**sympy [A]** time = 0.00, size = 0, normalized size = 0.00

$$g \left(\int \frac{a}{A + B \log{\left(e \left(\frac{a}{c + d x} + \frac{b x}{c + d x}\right)^{n} \right)}}\, dx + \int \frac{b x}{A + B \log{\left(e \left(\frac{a}{c + d x} + \frac{b x}{c + d x}\right)^{n} \right)}}\, dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*g*x+a*g)/(A+B*ln(e*((b*x+a)/(d*x+c))**n)),x)
```

```
[Out] g*(Integral(a/(A + B*log(e*(a/(c + d*x) + b*x/(c + d*x))**n)), x) + Integra
l(b*x/(A + B*log(e*(a/(c + d*x) + b*x/(c + d*x))**n)), x))
```

# 3.21 $\int \frac{1}{(ag+bgx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}\,dx$

**Optimal**. Leaf size=38

$$\text{Int}\left(\frac{1}{(ag+bgx)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)},x\right)$$

```
[Out] Unintegrable(1/(b*g*x+a*g)/(A+B*ln(e*((b*x+a)/(d*x+c))^n)),x)
```

**Rubi [A]** time = 0.08, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{1}{(ag+bgx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[1/((a*g + b*g*x)*(A + B*Log[e*((a + b*x)/(c + d*x))^n])),x]
[Out] Defer[Int][1/((a*g + b*g*x)*(A + B*Log[e*((a + b*x)/(c + d*x))^n])), x]
```

Rubi steps

$$\int \frac{1}{(ag+bgx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}\,dx=\int \frac{1}{(ag+bgx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}\,dx$$

**Mathematica [A]** time = 0.15, size = 0, normalized size = 0.00

$$\int \frac{1}{(ag+bgx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[1/((a*g + b*g*x)*(A + B*Log[e*((a + b*x)/(c + d*x))^n])),x]
```

```
[Out] Integrate[1/((a*g + b*g*x)*(A + B*Log[e*((a + b*x)/(c + d*x))^n])), x]
```

**fricas [A]** time = 0.86, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{1}{Abgx + Aag + \left(Bbgx + Bag\right)\log\left(e\left(\frac{bx+a}{dx+c}\right)^{n}\right)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*g*x+a*g)/(A+B*log(e*((b*x+a)/(d*x+c))^n)),x, algorithm="fric
as")
```

```
[Out] integral(1/(A*b*g*x + A*a*g + (B*b*g*x + B*a*g)*log(e*((b*x + a)/(d*x + c))
^n)), x)
```

**giac [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*g*x+a*g)/(A+B*log(e*((b*x+a)/(d*x+c))^n)),x, algorithm="giac
")
```

```
[Out] Timed out
```

**maple [A]** time = 0.37, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(bgx + ag\right)\left(B\ln\left(e\left(\frac{bx+a}{dx+c}\right)^{n}\right) + A\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*g*x+a*g)/(B*ln(e*((b*x+a)/(d*x+c))^n)+A),x)
```

```
[Out] int(1/(b*g*x+a*g)/(B*ln(e*((b*x+a)/(d*x+c))^n)+A),x)
```

**maxima [A]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(bgx + ag\right)\left(B\log\left(e\left(\frac{bx+a}{dx+c}\right)^{n}\right) + A\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*g*x+a*g)/(A+B*log(e*((b*x+a)/(d*x+c))^n)),x, algorithm="maxi
ma")
```

```
[Out] integrate(1/((b*g*x + a*g)*(B*log(e*((b*x + a)/(d*x + c))^n) + A)), x)
```

**mupad [A]** time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{1}{\left(a\,g+b\,g\,x\right)\,\left(A+B\,\ln\left(e\,{\left(\frac{a+b\,x}{c+d\,x}\right)}^n\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*g + b*g*x)*(A + B*log(e*((a + b*x)/(c + d*x))^n))),x)
```

```
[Out] int(1/((a*g + b*g*x)*(A + B*log(e*((a + b*x)/(c + d*x))^n))), x)
```

**sympy [A]** time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{1}{Aa+Abx+Ba\log\left(e\left(\frac{a}{c+dx}+\frac{bx}{c+dx}\right)^{n}\right)+Bbx\log\left(e\left(\frac{a}{c+dx}+\frac{bx}{c+dx}\right)^{n}\right)}\, dx}{g}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*g*x+a*g)/(A+B*ln(e*((b*x+a)/(d*x+c))**n)),x)
```

```
[Out] Integral(1/(A*a + A*b*x + B*a*log(e*(a/(c + d*x) + b*x/(c + d*x))**n) + B*b
*x*log(e*(a/(c + d*x) + b*x/(c + d*x))**n)), x)/g
```

## 3.22 $\int \frac{1}{(ag+bgx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}\,dx$

**Optimal**. Leaf size=94

$$\frac{e^{\frac{A}{Bn}}(c+dx)\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)^{\frac{1}{n}}\operatorname{Ei}\left(-\frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{Bn}\right)}{Bg^2n(a+bx)(bc-ad)}$$

[Out] exp(A/B/n)*(e*((b*x+a)/(d*x+c))^n)^(1/n)*(d*x+c)*Ei((-A-B*ln(e*((b*x+a)/(d*x+c))^n))/B/n)/B/(-a*d+b*c)/g^2/n/(b*x+a)

**Rubi [F]** time = 0.10, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{1}{(ag+bgx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}\,dx$$

Verification is Not applicable to the result.

[In] Int[1/((a*g + b*g*x)^2*(A + B*Log[e*((a + b*x)/(c + d*x))^n])),x]

[Out] Defer[Int][1/((a*g + b*g*x)^2*(A + B*Log[e*((a + b*x)/(c + d*x))^n])), x]

Rubi steps

$$\int \frac{1}{(ag+bgx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}\,dx=\int \frac{1}{(ag+bgx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}\,dx$$

**Mathematica [A]** time = 0.17, size = 94, normalized size = 1.00

$$\frac{e^{\frac{A}{Bn}}(c+dx)\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)^{\frac{1}{n}}\operatorname{Ei}\left(-\frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{Bn}\right)}{Bg^2n(a+bx)(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a*g + b*g*x)^2*(A + B*Log[e*((a + b*x)/(c + d*x))^n])),x]
```

```
[Out] (E^(A/(B*n))*(e*((a + b*x)/(c + d*x))^n)^n^(-1)*(c + d*x)*ExpIntegralEi[-((
A + B*Log[e*((a + b*x)/(c + d*x))^n])/(B*n))])/(B*(b*c - a*d)*g^2*n*(a + b*
x))
```

**fricas [A]** time = 0.90, size = 62, normalized size = 0.66

$$\frac{e^{\left(\frac{B\log(e)+A}{Bn}\right)}\operatorname{log\_integral}\left(\frac{(dx+c)e^{\left(-\frac{B\log(e)+A}{Bn}\right)}}{bx+a}\right)}{(Bbc-Bad)g^2n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*g*x+a*g)^2/(A+B*log(e*((b*x+a)/(d*x+c))^n)),x, algorithm="fr
icas")
```

```
[Out] e^((B*log(e) + A)/(B*n))*log_integral((d*x + c)*e^(-(B*log(e) + A)/(B*n))/(
b*x + a))/((B*b*c - B*a*d)*g^2*n)
```

**giac [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*g*x+a*g)^2/(A+B*log(e*((b*x+a)/(d*x+c))^n)),x, algorithm="gi
ac")
```

```
[Out] Timed out
```

**maple [F]** time = 0.29, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(bgx+ag\right)^2\left(B\ln\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*g*x+a*g)^2/(B*ln(e*((b*x+a)/(d*x+c))^n)+A),x)
```

```
[Out] int(1/(b*g*x+a*g)^2/(B*ln(e*((b*x+a)/(d*x+c))^n)+A),x)
```

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(bgx+ag\right)^2\left(B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*g*x+a*g)^2/(A+B*log(e*((b*x+a)/(d*x+c))^n)),x, algorithm="ma
xima")
```

```
[Out] integrate(1/((b*g*x + a*g)^2*(B*log(e*((b*x + a)/(d*x + c))^n) + A)), x)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{{\left(a\,g+b\,g\,x\right)}^2\,\left(A+B\,\ln\left(e\,{\left(\frac{a+b\,x}{c+d\,x}\right)}^n\right)\right)} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*g + b*g*x)^2*(A + B*log(e*((a + b*x)/(c + d*x))^n))),x)
```

```
[Out] int(1/((a*g + b*g*x)^2*(A + B*log(e*((a + b*x)/(c + d*x))^n))), x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{1}{Aa^{2} + 2Aabx + Ab^{2}x^{2} + Ba^{2}\log{\left(e\left(\frac{a}{c+dx} + \frac{bx}{c+dx}\right)^{n} \right)} + 2Babx\log{\left(e\left(\frac{a}{c+dx} + \frac{bx}{c+dx}\right)^{n} \right)} + Bb^{2}x^{2}\log{\left(e\left(\frac{a}{c+dx} + \frac{bx}{c+dx}\right)^{n} \right)}}\, dx}{g^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*g*x+a*g)**2/(A+B*ln(e*((b*x+a)/(d*x+c))**n)),x)
```

```
[Out] Integral(1/(A*a**2 + 2*A*a*b*x + A*b**2*x**2 + B*a**2*log(e*(a/(c + d*x) +
b*x/(c + d*x))**n) + 2*B*a*b*x*log(e*(a/(c + d*x) + b*x/(c + d*x))**n) + B*
b**2*x**2*log(e*(a/(c + d*x) + b*x/(c + d*x))**n)), x)/g**2
```

## 3.23 $\int \frac{1}{(ag+bgx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}\,dx$

**Optimal**. Leaf size=197

$$\frac{be^{\frac{2A}{Bn}}(c+dx)^2\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)^{2/n}\text{Ei}\left(-\frac{2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{Bn}\right)}{Bg^3n(a+bx)^2(bc-ad)^2}-\frac{de^{\frac{A}{Bn}}(c+dx)\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)^{\frac{1}{n}}\text{Ei}\left(-\frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{Bn}\right)}{Bg^3n(a+bx)(bc-ad)^2}$$

```
[Out] b*exp(2*A/B/n)*(e*((b*x+a)/(d*x+c))^n)^(2/n)*(d*x+c)^2*Ei(-2*(A+B*ln(e*((b*
x+a)/(d*x+c))^n))/B/n)/B/(-a*d+b*c)^2/g^3/n/(b*x+a)^2-d*exp(A/B/n)*(e*((b*x
+a)/(d*x+c))^n)^(1/n)*(d*x+c)*Ei((-A-B*ln(e*((b*x+a)/(d*x+c))^n))/B/n)/B/(-
a*d+b*c)^2/g^3/n/(b*x+a)
```

**Rubi [F]** time = 0.08, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{1}{(ag+bgx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[1/((a*g + b*g*x)^3*(A + B*Log[e*((a + b*x)/(c + d*x))^n])),x]
```

```
[Out] Defer[Int][1/((a*g + b*g*x)^3*(A + B*Log[e*((a + b*x)/(c + d*x))^n])), x]
```

Rubi steps

$$\int \frac{1}{(ag+bgx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}\,dx=\int \frac{1}{(ag+bgx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}\,dx$$

**Mathematica [A]** time = 0.33, size = 172, normalized size = 0.87

$$\frac{e^{\frac{A}{Bn}}(c+dx)\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)^{\frac{1}{n}}\left(be^{\frac{A}{Bn}}(c+dx)\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)^{\frac{1}{n}}\text{Ei}\left(-\frac{2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{Bn}\right)-d(a+bx)\text{Ei}\left(-\frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{Bn}\right)\right)}{Bg^3n(a+bx)^2(bc-ad)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a*g + b*g*x)^3*(A + B*Log[e*((a + b*x)/(c + d*x))^n])),x]
```

```
[Out] (E^(A/(B*n))*(e*((a + b*x)/(c + d*x))^n)^n^(-1)*(c + d*x)*(b*E^(A/(B*n))*(e
*((a + b*x)/(c + d*x))^n)^n^(-1)*(c + d*x)*ExpIntegralEi[(-2*(A + B*Log[e*(
(a + b*x)/(c + d*x))^n]))/(B*n)] - d*(a + b*x)*ExpIntegralEi[-((A + B*Log[e
*((a + b*x)/(c + d*x))^n])/(B*n))]))/(B*(b*c - a*d)^2*g^3*n*(a + b*x)^2)
```

**fricas [A]** time = 0.95, size = 149, normalized size = 0.76

$$-\frac{de^{\left(\frac{B\log(e)+A}{Bn}\right)}\text{log\_integral}\left(\frac{(dx+c)e^{\left(-\frac{B\log(e)+A}{Bn}\right)}}{bx+a}\right)-be^{\left(\frac{2\,(B\log(e)+A)}{Bn}\right)}\text{log\_integral}\left(\frac{(d^2x^2+2\,cdx+c^2)e^{\left(-\frac{2\,(B\log(e)+A)}{Bn}\right)}}{b^2x^2+2\,abx+a^2}\right)}{\left(Bb^2c^2-2\,Babcd+Ba^2d^2\right)g^3n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*g*x+a*g)^3/(A+B*log(e*((b*x+a)/(d*x+c))^n)),x, algorithm="fr
icas")
```

```
[Out] -(d*e^((B*log(e) + A)/(B*n))*log_integral((d*x + c)*e^(-(B*log(e) + A)/(B*n
))/(b*x + a)) - b*e^(2*(B*log(e) + A)/(B*n))*log_integral((d^2*x^2 + 2*c*d*
x + c^2)*e^(-2*(B*log(e) + A)/(B*n))/(b^2*x^2 + 2*a*b*x + a^2)))/((B*b^2*c^
2 - 2*B*a*b*c*d + B*a^2*d^2)*g^3*n)
```

**giac [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*g*x+a*g)^3/(A+B*log(e*((b*x+a)/(d*x+c))^n)),x, algorithm="gi
ac")
```

```
[Out] Timed out
```

**maple [F]** time = 0.29, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(bgx+ag\right)^{3}\left(B\ln\left(e\left(\frac{bx+a}{dx+c}\right)^{n}\right)+A\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*g*x+a*g)^3/(B*ln(e*((b*x+a)/(d*x+c))^n)+A),x)
```

```
[Out] int(1/(b*g*x+a*g)^3/(B*ln(e*((b*x+a)/(d*x+c))^n)+A),x)
```

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(bgx+ag)^3 \left(B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*g*x+a*g)^3/(A+B*log(e*((b*x+a)/(d*x+c))^n)),x, algorithm="ma
xima")
```

```
[Out] integrate(1/((b*g*x + a*g)^3*(B*log(e*((b*x + a)/(d*x + c))^n) + A)), x)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{{\left(a\,g+b\,g\,x\right)}^3\,\left(A+B\,\ln\left(e\,{\left(\frac{a+b\,x}{c+d\,x}\right)}^n\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*g + b*g*x)^3*(A + B*log(e*((a + b*x)/(c + d*x))^n))),x)
```

```
[Out] int(1/((a*g + b*g*x)^3*(A + B*log(e*((a + b*x)/(c + d*x))^n))), x)
```

**sympy [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*g*x+a*g)**3/(A+B*ln(e*((b*x+a)/(d*x+c))**n)),x)
```

```
[Out] Timed out
```

# 3.24 $\int \frac{(ag+bgx)^2}{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}\,dx$

**Optimal**. Leaf size=38

$$\operatorname{Int}\left(\frac{(ag+bgx)^2}{\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2},x\right)$$

```
[Out] Unintegrable((b*g*x+a*g)^2/(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2,x)
```

**Rubi [A]** time = 0.24, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{(ag+bgx)^2}{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(a*g + b*g*x)^2/(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2,x]
```

```
[Out] a^2*g^2*Defer[Int][(A + B*Log[e*((a + b*x)/(c + d*x))^n])^(-2), x] + 2*a*b*
g^2*Defer[Int][x/(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2, x] + b^2*g^2*Def
er[Int][x^2/(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2, x]
```

Rubi steps

$$\int \frac{(ag+bgx)^2}{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}\,dx=\int\left(\frac{a^2g^2}{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}+\frac{2abg^2x}{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}+\frac{b^2g^2x^2}{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}\right)dx$$

$$=\left(a^2g^2\right)\int\frac{1}{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}\,dx+\left(2abg^2\right)\int\frac{x}{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}$$

**Mathematica** [A] time = 1.05, size = 0, normalized size = 0.00

$$\int \frac{(ag+bgx)^2}{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(a*g + b*g*x)^2/(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2,x]
```

```
[Out] Integrate[(a*g + b*g*x)^2/(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2, x]
```

**fricas** [A] time = 0.76, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{b^2g^2x^2+2\,abg^2x+a^2g^2}{B^2\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)^2+2\,AB\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*g*x+a*g)^2/(A+B*log(e*((b*x+a)/(d*x+c))^n))^2,x, algorithm="fr
icas")
```

```
[Out] integral((b^2*g^2*x^2 + 2*a*b*g^2*x + a^2*g^2)/(B^2*log(e*((b*x + a)/(d*x +
 c))^n)^2 + 2*A*B*log(e*((b*x + a)/(d*x + c))^n) + A^2), x)
```

**giac** [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(bgx+ag)^2}{\left(B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*g*x+a*g)^2/(A+B*log(e*((b*x+a)/(d*x+c))^n))^2,x, algorithm="gi
ac")
```

```
[Out] integrate((b*g*x + a*g)^2/(B*log(e*((b*x + a)/(d*x + c))^n) + A)^2, x)
```

**maple** [A] time = 0.26, size = 0, normalized size = 0.00

$$\int \frac{(bgx+ag)^2}{\left(B\ln\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*g*x+a*g)^2/(B*ln(e*((b*x+a)/(d*x+c))^n)+A)^2,x)
```

```
[Out] int((b*g*x+a*g)^2/(B*ln(e*((b*x+a)/(d*x+c))^n)+A)^2,x)
```

**maxima [A]** time = 0.00, size = 0, normalized size = 0.00

$$-\frac{b^3dg^2x^4 + a^3cg^2 + \left(b^3cg^2 + 3\,ab^2dg^2\right)x^3 + 3\left(ab^2cg^2 + a^2bdg^2\right)x^2 + \left(3\,a^2bcg^2 + a^3dg^2\right)x}{(bcn - adn)B^2\log\left((bx+a)^n\right) - (bcn - adn)B^2\log\left((dx+c)^n\right) + (bcn - adn)AB + \left(bcn\log(e) - adn\log(e)\right)B^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*g*x+a*g)^2/(A+B*log(e*((b*x+a)/(d*x+c))^n))^2,x, algorithm="ma
xima")
```

```
[Out] -(b^3*d*g^2*x^4 + a^3*c*g^2 + (b^3*c*g^2 + 3*a*b^2*d*g^2)*x^3 + 3*(a*b^2*c*
g^2 + a^2*b*d*g^2)*x^2 + (3*a^2*b*c*g^2 + a^3*d*g^2)*x)/((b*c*n - a*d*n)*B^
2*log((b*x + a)^n) - (b*c*n - a*d*n)*B^2*log((d*x + c)^n) + (b*c*n - a*d*n)
*A*B + (b*c*n*log(e) - a*d*n*log(e))*B^2) + integrate((4*b^3*d*g^2*x^3 + 3*
a^2*b*c*g^2 + a^3*d*g^2 + 3*(b^3*c*g^2 + 3*a*b^2*d*g^2)*x^2 + 6*(a*b^2*c*g^
2 + a^2*b*d*g^2)*x)/((b*c*n - a*d*n)*B^2*log((b*x + a)^n) - (b*c*n - a*d*n)
*B^2*log((d*x + c)^n) + (b*c*n - a*d*n)*A*B + (b*c*n*log(e) - a*d*n*log(e))
*B^2), x)
```

**mupad [A]** time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{\left(a\,g+b\,g\,x\right)^2}{\left(A+B\,\ln\left(e\,\left(\frac{a+b\,x}{c+d\,x}\right)^n\right)\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*g + b*g*x)^2/(A + B*log(e*((a + b*x)/(c + d*x))^n))^2,x)
```

```
[Out] int((a*g + b*g*x)^2/(A + B*log(e*((a + b*x)/(c + d*x))^n))^2, x)
```

**sympy [A]** time = 0.00, size = 0, normalized size = 0.00

$$g^2\left(\int \frac{a^2}{A^2 + 2AB\log\left(e\left(\frac{a}{c+dx} + \frac{bx}{c+dx}\right)^n\right) + B^2\log\left(e\left(\frac{a}{c+dx} + \frac{bx}{c+dx}\right)^n\right)^2}\,dx + \int \frac{b^2x^2}{A^2 + 2AB\log\left(e\left(\frac{a}{c+dx} + \frac{bx}{c+dx}\right)^n\right) + \cdots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*g*x+a*g)**2/(A+B*ln(e*((b*x+a)/(d*x+c))**n))**2,x)
```

```
[Out] g**2*(Integral(a**2/(A**2 + 2*A*B*log(e*(a/(c + d*x) + b*x/(c + d*x))**n) +
 B**2*log(e*(a/(c + d*x) + b*x/(c + d*x))**n)**2), x) + Integral(b**2*x**2/
(A**2 + 2*A*B*log(e*(a/(c + d*x) + b*x/(c + d*x))**n) + B**2*log(e*(a/(c +
d*x) + b*x/(c + d*x))**n)**2), x) + Integral(2*a*b*x/(A**2 + 2*A*B*log(e*(a
/(c + d*x) + b*x/(c + d*x))**n) + B**2*log(e*(a/(c + d*x) + b*x/(c + d*x))*
*n)**2), x))
```

# 3.25 $\int \frac{ag+bgx}{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}\,dx$

**Optimal**. Leaf size=36

$$\text{Int}\left(\frac{ag+bgx}{\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2},x\right)$$

```
[Out] Unintegrable((b*g*x+a*g)/(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2,x)
```

**Rubi [A]** time = 0.13, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{ag+bgx}{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(a*g + b*g*x)/(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2,x]
```

```
[Out] a*g*Defer[Int][(A + B*Log[e*((a + b*x)/(c + d*x))^n])^(-2), x] + b*g*Defer[
Int][x/(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2, x]
```

Rubi steps

$$\begin{aligned}\int \frac{ag+bgx}{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}\,dx &= \int \left(\frac{ag}{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}+\frac{bgx}{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}\right)dx\\ &=(ag)\int \frac{1}{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}\,dx+(bg)\int \frac{x}{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}\,dx\end{aligned}$$

**Mathematica [A]** time = 0.99, size = 0, normalized size = 0.00

$$\int \frac{ag+bgx}{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(a*g + b*g*x)/(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2,x]
```

```
[Out] Integrate[(a*g + b*g*x)/(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2, x]
```

**fricas [A]** time = 0.89, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{bgx + ag}{B^2 \log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)^2 + 2\,AB\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right) + A^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*g*x+a*g)/(A+B*log(e*((b*x+a)/(d*x+c))^n))^2,x, algorithm="fric
as")
```

```
[Out] integral((b*g*x + a*g)/(B^2*log(e*((b*x + a)/(d*x + c))^n)^2 + 2*A*B*log(e*
((b*x + a)/(d*x + c))^n) + A^2), x)
```

**giac [A]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{bgx + ag}{\left(B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right) + A\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*g*x+a*g)/(A+B*log(e*((b*x+a)/(d*x+c))^n))^2,x, algorithm="giac
")
```

```
[Out] integrate((b*g*x + a*g)/(B*log(e*((b*x + a)/(d*x + c))^n) + A)^2, x)
```

**maple [A]** time = 0.07, size = 0, normalized size = 0.00

$$\int \frac{bgx + ag}{\left(B\ln\left(e\left(\frac{bx+a}{dx+c}\right)^n\right) + A\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*g*x+a*g)/(B*ln(e*((b*x+a)/(d*x+c))^n)+A)^2,x)
```

```
[Out] int((b*g*x+a*g)/(B*ln(e*((b*x+a)/(d*x+c))^n)+A)^2,x)
```

**maxima** [A] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{b^2dgx^3+a^2cg+\left(b^2cg+2\,abdg\right)x^2+\left(2\,abcg+a^2dg\right)x}{(bcn-adn)B^2\log\left((bx+a)^n\right)-(bcn-adn)B^2\log\left((dx+c)^n\right)+(bcn-adn)AB+\left(bcn\log(e)-adn\log(e)\right)B^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*g*x+a*g)/(A+B*log(e*((b*x+a)/(d*x+c))^n))^2,x, algorithm="maxi
ma")
```

```
[Out] -(b^2*d*g*x^3 + a^2*c*g + (b^2*c*g + 2*a*b*d*g)*x^2 + (2*a*b*c*g + a^2*d*g)
*x)/((b*c*n - a*d*n)*B^2*log((b*x + a)^n) - (b*c*n - a*d*n)*B^2*log((d*x +
c)^n) + (b*c*n - a*d*n)*A*B + (b*c*n*log(e) - a*d*n*log(e))*B^2) + integrat
e((3*b^2*d*g*x^2 + 2*a*b*c*g + a^2*d*g + 2*(b^2*c*g + 2*a*b*d*g)*x)/((b*c*n
 - a*d*n)*B^2*log((b*x + a)^n) - (b*c*n - a*d*n)*B^2*log((d*x + c)^n) + (b*
c*n - a*d*n)*A*B + (b*c*n*log(e) - a*d*n*log(e))*B^2), x)
```

**mupad** [A] time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{a\,g+b\,g\,x}{{\left(A+B\,\ln\left(e\,{\left(\frac{a+b\,x}{c+d\,x}\right)}^n\right)\right)}^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*g + b*g*x)/(A + B*log(e*((a + b*x)/(c + d*x))^n))^2,x)
```

```
[Out] int((a*g + b*g*x)/(A + B*log(e*((a + b*x)/(c + d*x))^n))^2, x)
```

**sympy** [A] time = 0.00, size = 0, normalized size = 0.00

$$g\left(\int \frac{a}{A^2+2AB\log\left(e\left(\frac{a}{c+dx}+\frac{bx}{c+dx}\right)^n\right)+B^2\log\left(e\left(\frac{a}{c+dx}+\frac{bx}{c+dx}\right)^n\right)^2}\,dx+\int \frac{bx}{A^2+2AB\log\left(e\left(\frac{a}{c+dx}+\frac{bx}{c+dx}\right)^n\right)+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*g*x+a*g)/(A+B*ln(e*((b*x+a)/(d*x+c))**n))**2,x)
```

```
[Out] g*(Integral(a/(A**2 + 2*A*B*log(e*(a/(c + d*x) + b*x/(c + d*x))**n) + B**2*
log(e*(a/(c + d*x) + b*x/(c + d*x))**n)**2), x) + Integral(b*x/(A**2 + 2*A*
B*log(e*(a/(c + d*x) + b*x/(c + d*x))**n) + B**2*log(e*(a/(c + d*x) + b*x/(
c + d*x))**n)**2), x))
```

## 3.26 $\int \frac{1}{(ag+bgx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}\,dx$

**Optimal**. Leaf size=38

$$\text{Int}\left(\frac{1}{(ag+bgx)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2},x\right)$$

```
[Out] Unintegrable(1/(b*g*x+a*g)/(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2,x)
```

**Rubi** [A] time = 0.09, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{1}{(ag+bgx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[1/((a*g + b*g*x)*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2),x]
[Out] Defer[Int][1/((a*g + b*g*x)*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2), x]
```

Rubi steps

$$\int \frac{1}{(ag+bgx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}\,dx=\int \frac{1}{(ag+bgx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}\,dx$$

**Mathematica** [A] time = 0.56, size = 0, normalized size = 0.00

$$\int \frac{1}{(ag+bgx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[1/((a*g + b*g*x)*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2),x]
```

```
[Out] Integrate[1/((a*g + b*g*x)*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2), x]
```

**fricas [A]** time = 0.63, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{1}{A^2bgx + A^2ag + \left(B^2bgx + B^2ag\right)\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)^2 + 2\left(ABbgx + ABag\right)\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*g*x+a*g)/(A+B*log(e*((b*x+a)/(d*x+c))^n))^2,x, algorithm="fr
icas")
```

```
[Out] integral(1/(A^2*b*g*x + A^2*a*g + (B^2*b*g*x + B^2*a*g)*log(e*((b*x + a)/(d
*x + c))^n)^2 + 2*(A*B*b*g*x + A*B*a*g)*log(e*((b*x + a)/(d*x + c))^n)), x)
```

**giac [A]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(bgx + ag\right)\left(B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right) + A\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*g*x+a*g)/(A+B*log(e*((b*x+a)/(d*x+c))^n))^2,x, algorithm="gi
ac")
```

```
[Out] integrate(1/((b*g*x + a*g)*(B*log(e*((b*x + a)/(d*x + c))^n) + A)^2), x)
```

**maple [A]** time = 0.28, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(bgx + ag\right)\left(B\ln\left(e\left(\frac{bx+a}{dx+c}\right)^n\right) + A\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*g*x+a*g)/(B*ln(e*((b*x+a)/(d*x+c))^n)+A)^2,x)
```

```
[Out] int(1/(b*g*x+a*g)/(B*ln(e*((b*x+a)/(d*x+c))^n)+A)^2,x)
```

**maxima [A]** time = 0.00, size = 0, normalized size = 0.00

$$d\int \frac{1}{\left(bcgn - adgn\right)B^2\log\left((bx+a)^n\right) - \left(bcgn - adgn\right)B^2\log\left((dx+c)^n\right) + \left(bcgn - adgn\right)AB + \left(bcgn\log(e) - a\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*g*x+a*g)/(A+B*log(e*((b*x+a)/(d*x+c))^n))^2,x, algorithm="ma
xima")
```

```
[Out] d*integrate(1/((b*c*g*n - a*d*g*n)*B^2*log((b*x + a)^n) - (b*c*g*n - a*d*g*
n)*B^2*log((d*x + c)^n) + (b*c*g*n - a*d*g*n)*A*B + (b*c*g*n*log(e) - a*d*g
*n*log(e))*B^2), x) - (d*x + c)/((b*c*g*n - a*d*g*n)*B^2*log((b*x + a)^n) -
 (b*c*g*n - a*d*g*n)*B^2*log((d*x + c)^n) + (b*c*g*n - a*d*g*n)*A*B + (b*c*
g*n*log(e) - a*d*g*n*log(e))*B^2)
```

**mupad [A]** time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{1}{\left(a\,g+b\,g\,x\right)\,{\left(A+B\,\ln\left(e\,{\left(\frac{a+b\,x}{c+d\,x}\right)}^n\right)\right)}^2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*g + b*g*x)*(A + B*log(e*((a + b*x)/(c + d*x))^n))^2),x)
```

```
[Out] int(1/((a*g + b*g*x)*(A + B*log(e*((a + b*x)/(c + d*x))^n))^2), x)
```

**sympy [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*g*x+a*g)/(A+B*ln(e*((b*x+a)/(d*x+c))**n))**2,x)
```

```
[Out] Timed out
```

## 3.27 $\int \frac{1}{(ag+bgx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}\,dx$

**Optimal**. Leaf size=153

$$-\frac{e^{\frac{A}{Bn}}(c+dx)\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)^{\frac{1}{n}}\operatorname{Ei}\left(-\frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{Bn}\right)}{B^2g^2n^2(a+bx)(bc-ad)}-\frac{c+dx}{Bg^2n(a+bx)(bc-ad)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}$$

```
[Out] -exp(A/B/n)*(e*((b*x+a)/(d*x+c))^n)^(1/n)*(d*x+c)*Ei((-A-B*ln(e*((b*x+a)/(d
*x+c))^n))/B/n)/B^2/(-a*d+b*c)/g^2/n^2/(b*x+a)+(-d*x-c)/B/(-a*d+b*c)/g^2/n/
(b*x+a)/(A+B*ln(e*((b*x+a)/(d*x+c))^n))
```

**Rubi [F]** time = 0.10, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{1}{(ag+bgx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[1/((a*g + b*g*x)^2*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2),x]
```

```
[Out] Defer[Int][1/((a*g + b*g*x)^2*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2), x]
```

Rubi steps

$$\int \frac{1}{(ag+bgx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}\,dx=\int \frac{1}{(ag+bgx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}\,dx$$

**Mathematica [A]** time = 0.19, size = 146, normalized size = 0.95

$$-\frac{(c+dx)\left(e^{\frac{A}{Bn}}\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)^{\frac{1}{n}}\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)\operatorname{Ei}\left(-\frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{Bn}\right)+Bn\right)}{B^2g^2n^2(a+bx)(bc-ad)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a*g + b*g*x)^2*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2),x]
```

```
[Out] -(((c + d*x)*(B*n + E^(A/(B*n))*(e*((a + b*x)/(c + d*x))^n)^n^(-1)*ExpInteg
ralEi[-((A + B*Log[e*((a + b*x)/(c + d*x))^n])/(B*n))]*(A + B*Log[e*((a + b
*x)/(c + d*x))^n])))/(B^2*(b*c - a*d)*g^2*n^2*(a + b*x)*(A + B*Log[e*((a +
b*x)/(c + d*x))^n])))
```

**fricas [A]** time = 0.88, size = 274, normalized size = 1.79

$$-\frac{Bdnx + Bcn + \left(Abx + Aa + (Bbx + Ba)\log(e) + (Bbnx + Ban)\log\left(\frac{bx+a}{dx+c}\right)\right)e^{\left(\frac{B\log(e)+A}{Bn}\right.}}{\left(AB^2b^2c - AB^2abd\right)g^2n^2x + \left(AB^2abc - AB^2a^2d\right)g^2n^2 + \left(\left(B^3b^2c - B^3abd\right)g^2n^2x + \left(B^3abc - B^3a^2d\right)g^2n^2\right)\log(e)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*g*x+a*g)^2/(A+B*log(e*((b*x+a)/(d*x+c))^n))^2,x, algorithm="
fricas")
```

```
[Out] -(B*d*n*x + B*c*n + (A*b*x + A*a + (B*b*x + B*a)*log(e) + (B*b*n*x + B*a*n)
*log((b*x + a)/(d*x + c)))*e^((B*log(e) + A)/(B*n))*log_integral((d*x + c)*
e^(-(B*log(e) + A)/(B*n))/(b*x + a)))/((A*B^2*b^2*c - A*B^2*a*b*d)*g^2*n^2*
x + (A*B^2*a*b*c - A*B^2*a^2*d)*g^2*n^2 + ((B^3*b^2*c - B^3*a*b*d)*g^2*n^2*
x + (B^3*a*b*c - B^3*a^2*d)*g^2*n^2)*log(e) + ((B^3*b^2*c - B^3*a*b*d)*g^2*
n^3*x + (B^3*a*b*c - B^3*a^2*d)*g^2*n^3)*log((b*x + a)/(d*x + c)))
```

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(bgx + ag\right)^2\left(B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right) + A\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*g*x+a*g)^2/(A+B*log(e*((b*x+a)/(d*x+c))^n))^2,x, algorithm="
giac")
```

```
[Out] integrate(1/((b*g*x + a*g)^2*(B*log(e*((b*x + a)/(d*x + c))^n) + A)^2), x)
```

**maple [F]** time = 0.28, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(bgx + ag\right)^2\left(B\ln\left(e\left(\frac{bx+a}{dx+c}\right)^n\right) + A\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*g*x+a*g)^2/(B*ln(e*((b*x+a)/(d*x+c))^n)+A)^2,x)
```

```
[Out] int(1/(b*g*x+a*g)^2/(B*ln(e*((b*x+a)/(d*x+c))^n)+A)^2,x)
```

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$-\frac{}{\left(abcg^2n - a^2dg^2n\right)AB + \left(abcg^2n\log(e) - a^2dg^2n\log(e)\right)B^2 + \left(\left(b^2cg^2n - abdg^2n\right)AB + \left(b^2cg^2n\log(e) - abdg\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*g*x+a*g)^2/(A+B*log(e*((b*x+a)/(d*x+c))^n))^2,x, algorithm="
maxima")
```

```
[Out] -(d*x + c)/((a*b*c*g^2*n - a^2*d*g^2*n)*A*B + (a*b*c*g^2*n*log(e) - a^2*d*g
^2*n*log(e))*B^2 + ((b^2*c*g^2*n - a*b*d*g^2*n)*A*B + (b^2*c*g^2*n*log(e) -
 a*b*d*g^2*n*log(e))*B^2)*x + ((b^2*c*g^2*n - a*b*d*g^2*n)*B^2*x + (a*b*c*g
^2*n - a^2*d*g^2*n)*B^2)*log((b*x + a)^n) - ((b^2*c*g^2*n - a*b*d*g^2*n)*B^
2*x + (a*b*c*g^2*n - a^2*d*g^2*n)*B^2)*log((d*x + c)^n)) + integrate(-1/(B^
2*a^2*g^2*n*log(e) + A*B*a^2*g^2*n + (B^2*b^2*g^2*n*log(e) + A*B*b^2*g^2*n)
*x^2 + 2*(B^2*a*b*g^2*n*log(e) + A*B*a*b*g^2*n)*x + (B^2*b^2*g^2*n*x^2 + 2*
B^2*a*b*g^2*n*x + B^2*a^2*g^2*n)*log((b*x + a)^n) - (B^2*b^2*g^2*n*x^2 + 2*
B^2*a*b*g^2*n*x + B^2*a^2*g^2*n)*log((d*x + c)^n)), x)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{{\left(a\,g+b\,g\,x\right)}^2\,{\left(A+B\,\ln\left(e\,{\left(\frac{a+b\,x}{c+d\,x}\right)}^n\right)\right)}^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*g + b*g*x)^2*(A + B*log(e*((a + b*x)/(c + d*x))^n))^2),x)
```

```
[Out] int(1/((a*g + b*g*x)^2*(A + B*log(e*((a + b*x)/(c + d*x))^n))^2), x)
```

**sympy [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*g*x+a*g)**2/(A+B*ln(e*((b*x+a)/(d*x+c))**n))**2,x)
```

```
[Out] Timed out
```

## 3.28 $\int \frac{1}{(ag+bgx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}\,dx$

**Optimal**. Leaf size=314

$$-\frac{2be^{\frac{2A}{Bn}}(c+dx)^2\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)^{2/n}\operatorname{Ei}\left(-\frac{2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{Bn}\right)}{B^2g^3n^2(a+bx)^2(bc-ad)^2}+\frac{de^{\frac{A}{Bn}}(c+dx)\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)^{\frac{1}{n}}\operatorname{Ei}\left(-\frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{Bn}\right)}{B^2g^3n^2(a+bx)(bc-ad)^2}-\frac{}{Bg^3n(}$$

```
[Out] -2*b*exp(2*A/B/n)*(e*((b*x+a)/(d*x+c))^n)^(2/n)*(d*x+c)^2*Ei(-2*(A+B*ln(e*(
(b*x+a)/(d*x+c))^n))/B/n)/B^2/(-a*d+b*c)^2/g^3/n^2/(b*x+a)^2+d*exp(A/B/n)*(
e*((b*x+a)/(d*x+c))^n)^(1/n)*(d*x+c)*Ei((-A-B*ln(e*((b*x+a)/(d*x+c))^n))/B/
n)/B^2/(-a*d+b*c)^2/g^3/n^2/(b*x+a)+d*(d*x+c)/B/(-a*d+b*c)^2/g^3/n/(b*x+a)/
(A+B*ln(e*((b*x+a)/(d*x+c))^n))-b*(d*x+c)^2/B/(-a*d+b*c)^2/g^3/n/(b*x+a)^2/
(A+B*ln(e*((b*x+a)/(d*x+c))^n))
```

**Rubi [F]** time = 0.09, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{1}{(ag+bgx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[1/((a*g + b*g*x)^3*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2),x]
```

```
[Out] Defer[Int][1/((a*g + b*g*x)^3*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2), x]
```

Rubi steps

$$\int \frac{1}{(ag+bgx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}\,dx=\int \frac{1}{(ag+bgx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}\,dx$$

**Mathematica [A]** time = 0.62, size = 254, normalized size = 0.81

$$\frac{(c+dx)\left(-2be^{\frac{2A}{Bn}}(c+dx)\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)^{2/n}\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)\text{Ei}\left(-\frac{2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{Bn}\right)+d(a+bx)e^{\frac{A}{Bn}}\left(e\left(\frac{a+bx}{c+dx}\right.\right.\right.}{B^2g^3n^2(a+bx)^2(bc-ad)^2\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+\right.}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a*g + b*g*x)^3*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2),x]
```

```
[Out] ((c + d*x)*(B*(-(b*c) + a*d)*n - 2*b*E^((2*A)/(B*n))*(e*((a + b*x)/(c + d*x
))^n)^(2/n)*(c + d*x)*ExpIntegralEi[(-2*(A + B*Log[e*((a + b*x)/(c + d*x))^
n]))/(B*n)]*(A + B*Log[e*((a + b*x)/(c + d*x))^n]) + d*E^(A/(B*n))*(a + b*x
)*(e*((a + b*x)/(c + d*x))^n)^n^(-1)*ExpIntegralEi[-((A + B*Log[e*((a + b*x
)/(c + d*x))^n])/(B*n))]*(A + B*Log[e*((a + b*x)/(c + d*x))^n])))/(B^2*(b*c
 - a*d)^2*g^3*n^2*(a + b*x)^2*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))
```

**fricas [B]** time = 0.93, size = 755, normalized size = 2.40

$$-\frac{\left(Bbcd-Bad^2\right)nx-\left(Ab^2dx^2+2\,Aabdx+Aa^2d+\left(Bb^2dx^2+2\,Babdx+Ba^2d\right)\right.}{\left(AB^2b^4c^2-2\,AB^2ab^3cd+AB^2a^2b^2d^2\right)g^3n^2x^2+2\left(AB^2ab^3c^2-2\,AB^2a^2b^2cd+AB^2a^3bd^2\right)g^3n^2x+\left(AB^2a^2b^2c\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*g*x+a*g)^3/(A+B*log(e*((b*x+a)/(d*x+c))^n))^2,x, algorithm="
fricas")
```

```
[Out] -((B*b*c*d - B*a*d^2)*n*x - (A*b^2*d*x^2 + 2*A*a*b*d*x + A*a^2*d + (B*b^2*d
*x^2 + 2*B*a*b*d*x + B*a^2*d)*log(e) + (B*b^2*d*n*x^2 + 2*B*a*b*d*n*x + B*a
^2*d*n)*log((b*x + a)/(d*x + c)))*e^((B*log(e) + A)/(B*n))*log_integral((d*
x + c)*e^(-(B*log(e) + A)/(B*n))/(b*x + a)) + 2*(A*b^3*x^2 + 2*A*a*b^2*x +
A*a^2*b + (B*b^3*x^2 + 2*B*a*b^2*x + B*a^2*b)*log(e) + (B*b^3*n*x^2 + 2*B*a
*b^2*n*x + B*a^2*b*n)*log((b*x + a)/(d*x + c)))*e^(2*(B*log(e) + A)/(B*n))*
log_integral((d^2*x^2 + 2*c*d*x + c^2)*e^(-2*(B*log(e) + A)/(B*n))/(b^2*x^2
 + 2*a*b*x + a^2)) + (B*b*c^2 - B*a*c*d)*n)/((A*B^2*b^4*c^2 - 2*A*B^2*a*b^3
*c*d + A*B^2*a^2*b^2*d^2)*g^3*n^2*x^2 + 2*(A*B^2*a*b^3*c^2 - 2*A*B^2*a^2*b^
2*c*d + A*B^2*a^3*b*d^2)*g^3*n^2*x + (A*B^2*a^2*b^2*c^2 - 2*A*B^2*a^3*b*c*d
 + A*B^2*a^4*d^2)*g^3*n^2 + ((B^3*b^4*c^2 - 2*B^3*a*b^3*c*d + B^3*a^2*b^2*d
^2)*g^3*n^2*x^2 + 2*(B^3*a*b^3*c^2 - 2*B^3*a^2*b^2*c*d + B^3*a^3*b*d^2)*g^3
*n^2*x + (B^3*a^2*b^2*c^2 - 2*B^3*a^3*b*c*d + B^3*a^4*d^2)*g^3*n^2)*log(e)
```

```
+ ((B^3*b^4*c^2 - 2*B^3*a*b^3*c*d + B^3*a^2*b^2*d^2)*g^3*n^3*x^2 + 2*(B^3*a
*b^3*c^2 - 2*B^3*a^2*b^2*c*d + B^3*a^3*b*d^2)*g^3*n^3*x + (B^3*a^2*b^2*c^2
- 2*B^3*a^3*b*c*d + B^3*a^4*d^2)*g^3*n^3)*log((b*x + a)/(d*x + c)))
```

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(bgx+ag)^3\left(B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*g*x+a*g)^3/(A+B*log(e*((b*x+a)/(d*x+c))^n))^2,x, algorithm="
giac")
```

```
[Out] integrate(1/((b*g*x + a*g)^3*(B*log(e*((b*x + a)/(d*x + c))^n) + A)^2), x)
```

**maple [F]** time = 0.29, size = 0, normalized size = 0.00

$$\int \frac{1}{(bgx+ag)^3\left(B\ln\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*g*x+a*g)^3/(B*ln(e*((b*x+a)/(d*x+c))^n)+A)^2,x)
```

```
[Out] int(1/(b*g*x+a*g)^3/(B*ln(e*((b*x+a)/(d*x+c))^n)+A)^2,x)
```

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$-\frac{}{(a^2bcg^3n-a^3dg^3n)AB+(a^2bcg^3n\log(e)-a^3dg^3n\log(e))B^2+((b^3cg^3n-ab^2dg^3n)AB+(b^3cg^3n\log(e)-ab^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*g*x+a*g)^3/(A+B*log(e*((b*x+a)/(d*x+c))^n))^2,x, algorithm="
maxima")
```

```
[Out] -(d*x + c)/((a^2*b*c*g^3*n - a^3*d*g^3*n)*A*B + (a^2*b*c*g^3*n*log(e) - a^3
*d*g^3*n*log(e))*B^2 + ((b^3*c*g^3*n - a*b^2*d*g^3*n)*A*B + (b^3*c*g^3*n*lo
g(e) - a*b^2*d*g^3*n*log(e))*B^2)*x^2 + 2*((a*b^2*c*g^3*n - a^2*b*d*g^3*n)*
A*B + (a*b^2*c*g^3*n*log(e) - a^2*b*d*g^3*n*log(e))*B^2)*x + ((b^3*c*g^3*n
- a*b^2*d*g^3*n)*B^2*x^2 + 2*(a*b^2*c*g^3*n - a^2*b*d*g^3*n)*B^2*x + (a^2*b
*c*g^3*n - a^3*d*g^3*n)*B^2)*log((b*x + a)^n) - ((b^3*c*g^3*n - a*b^2*d*g^3
*n)*B^2*x^2 + 2*(a*b^2*c*g^3*n - a^2*b*d*g^3*n)*B^2*x + (a^2*b*c*g^3*n - a^
```

```
3*d*g^3*n)*B^2)*log((d*x + c)^n)) - integrate((b*d*x + 2*b*c - a*d)/(((b^4*
c*g^3*n - a*b^3*d*g^3*n)*A*B + (b^4*c*g^3*n*log(e) - a*b^3*d*g^3*n*log(e))*
B^2)*x^3 + (a^3*b*c*g^3*n - a^4*d*g^3*n)*A*B + (a^3*b*c*g^3*n*log(e) - a^4*
d*g^3*n*log(e))*B^2 + 3*((a*b^3*c*g^3*n - a^2*b^2*d*g^3*n)*A*B + (a*b^3*c*g
^3*n*log(e) - a^2*b^2*d*g^3*n*log(e))*B^2)*x^2 + 3*((a^2*b^2*c*g^3*n - a^3*
b*d*g^3*n)*A*B + (a^2*b^2*c*g^3*n*log(e) - a^3*b*d*g^3*n*log(e))*B^2)*x + (
(b^4*c*g^3*n - a*b^3*d*g^3*n)*B^2*x^3 + 3*(a*b^3*c*g^3*n - a^2*b^2*d*g^3*n)
*B^2*x^2 + 3*(a^2*b^2*c*g^3*n - a^3*b*d*g^3*n)*B^2*x + (a^3*b*c*g^3*n - a^4
*d*g^3*n)*B^2)*log((b*x + a)^n) - ((b^4*c*g^3*n - a*b^3*d*g^3*n)*B^2*x^3 +
3*(a*b^3*c*g^3*n - a^2*b^2*d*g^3*n)*B^2*x^2 + 3*(a^2*b^2*c*g^3*n - a^3*b*d*
g^3*n)*B^2*x + (a^3*b*c*g^3*n - a^4*d*g^3*n)*B^2)*log((d*x + c)^n)), x)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\left(a\,g+b\,g\,x\right)^3\,\left(A+B\,\ln\left(e\,\left(\frac{a+b\,x}{c+d\,x}\right)^n\right)\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*g + b*g*x)^3*(A + B*log(e*((a + b*x)/(c + d*x))^n))^2),x)
[Out] int(1/((a*g + b*g*x)^3*(A + B*log(e*((a + b*x)/(c + d*x))^n))^2), x)
```

**sympy [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*g*x+a*g)**3/(A+B*ln(e*((b*x+a)/(d*x+c))**n))**2,x)
[Out] Timed out
```

# 3.29 $\int (cg+dgx)^4 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right) dx$

**Optimal**. Leaf size=188

$$\frac{g^4(c+dx)^5\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{5d}-\frac{Bg^4n(bc-ad)^5\log(a+bx)}{5b^5d}-\frac{Bg^4nx(bc-ad)^4}{5b^4}-\frac{Bg^4n(c+dx)^2(bc-ad)^3}{10b^3d}-\frac{Bg}{}$$

```
[Out] -1/5*B*(-a*d+b*c)^4*g^4*n*x/b^4-1/10*B*(-a*d+b*c)^3*g^4*n*(d*x+c)^2/b^3/d-1
/15*B*(-a*d+b*c)^2*g^4*n*(d*x+c)^3/b^2/d-1/20*B*(-a*d+b*c)*g^4*n*(d*x+c)^4/
b/d-1/5*B*(-a*d+b*c)^5*g^4*n*ln(b*x+a)/b^5/d+1/5*g^4*(d*x+c)^5*(A+B*ln(e*((
b*x+a)/(d*x+c))^n))/d
```

**Rubi [A]** time = 0.13, antiderivative size = 188, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {2525, 12, 43}

$$\frac{g^4(c+dx)^5\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{5d}-\frac{Bg^4nx(bc-ad)^4}{5b^4}-\frac{Bg^4n(c+dx)^2(bc-ad)^3}{10b^3d}-\frac{Bg^4n(c+dx)^3(bc-ad)^2}{15b^2d}-\frac{Bg^4n}{}$$

Antiderivative was successfully verified.

```
[In] Int[(c*g + d*g*x)^4*(A + B*Log[e*((a + b*x)/(c + d*x))^n]),x]
```

```
[Out] -(B*(b*c - a*d)^4*g^4*n*x)/(5*b^4) - (B*(b*c - a*d)^3*g^4*n*(c + d*x)^2)/(1
0*b^3*d) - (B*(b*c - a*d)^2*g^4*n*(c + d*x)^3)/(15*b^2*d) - (B*(b*c - a*d)*
g^4*n*(c + d*x)^4)/(20*b*d) - (B*(b*c - a*d)^5*g^4*n*Log[a + b*x])/(5*b^5*d
) + (g^4*(c + d*x)^5*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/(5*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2525

```
Int[((a_.) + Log[(c_.)*(RFx_)^(p_.)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_))^(m_.
), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(a + b*Log[c*RFx^p])^n)/(e*(m + 1))
, x] - Dist[(b*n*p)/(e*(m + 1)), Int[SimplifyIntegrand[((d + e*x)^(m + 1)*(
a + b*Log[c*RFx^p])^(n - 1)*D[RFx, x])/RFx, x], x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && RationalFunctionQ[RFx, x] && IGtQ[n, 0] && (EqQ[n, 1] ||
IntegerQ[m]) && NeQ[m, -1]
```

Rubi steps

$$\int (cg+dgx)^4\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)dx = \frac{g^4(c+dx)^5\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{5d} - \frac{(Bn)\int\frac{(bc-ad)g^5(c+dx)^4}{a+bx}\,dx}{5dg}$$

$$= \frac{g^4(c+dx)^5\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{5d} - \frac{\left(B(bc-ad)g^4n\right)\int\frac{(c+dx)^4}{a+bx}}{5d}$$

$$= \frac{g^4(c+dx)^5\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{5d} - \frac{\left(B(bc-ad)g^4n\right)\int\left(\frac{d(bc-a}{b^4}\right.}{}$$

$$= -\frac{B(bc-ad)^4g^4nx}{5b^4} - \frac{B(bc-ad)^3g^4n(c+dx)^2}{10b^3d} - \frac{B(bc-ad)^2g^4n(}{15b^2d}$$

**Mathematica [A]** time = 0.10, size = 146, normalized size = 0.78

$$\frac{g^4\left((c+dx)^5\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)-\frac{Bn(bc-ad)\left(4b^3(c+dx)^3(bc-ad)+6b^2(c+dx)^2(bc-ad)^2+12bdx(bc-ad)^3+12(bc-ad)^4\log(a+bx)+3b\right.}{12b^5}}{5d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*g + d*g*x)^4*(A + B*Log[e*((a + b*x)/(c + d*x))^n]),x]
```

```
[Out] (g^4*(-1/12*(B*(b*c - a*d)*n*(12*b*d*(b*c - a*d)^3*x + 6*b^2*(b*c - a*d)^2*
(c + d*x)^2 + 4*b^3*(b*c - a*d)*(c + d*x)^3 + 3*b^4*(c + d*x)^4 + 12*(b*c -
 a*d)^4*Log[a + b*x]))/b^5 + (c + d*x)^5*(A + B*Log[e*((a + b*x)/(c + d*x))
^n])))/(5*d)
```

**fricas [B]** time = 1.04, size = 572, normalized size = 3.04

$$\frac{12\,Ab^5d^5g^4x^5 - 12\,Bb^5c^5g^4n\log(dx+c) + 12\left(5\,Bab^4c^4d - 10\,Ba^2b^3c^3d^2 + 10\,Ba^3b^2c^2d^3 - 5\,Ba^4bcd^4 + Ba^5d^5\right)g}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*g*x+c*g)^4*(A+B*log(e*((b*x+a)/(d*x+c))^n)),x, algorithm="fric
as")
```

```
[Out] 1/60*(12*A*b^5*d^5*g^4*x^5 - 12*B*b^5*c^5*g^4*n*log(d*x + c) + 12*(5*B*a*b^
4*c^4*d - 10*B*a^2*b^3*c^3*d^2 + 10*B*a^3*b^2*c^2*d^3 - 5*B*a^4*b*c*d^4 + B
*a^5*d^5)*g^4*n*log(b*x + a) + 3*(20*A*b^5*c*d^4*g^4 - (B*b^5*c*d^4 - B*a*b
^4*d^5)*g^4*n)*x^4 + 4*(30*A*b^5*c^2*d^3*g^4 - (4*B*b^5*c^2*d^3 - 5*B*a*b^4
*c*d^4 + B*a^2*b^3*d^5)*g^4*n)*x^3 + 6*(20*A*b^5*c^3*d^2*g^4 - (6*B*b^5*c^3
*d^2 - 10*B*a*b^4*c^2*d^3 + 5*B*a^2*b^3*c*d^4 - B*a^3*b^2*d^5)*g^4*n)*x^2 +
 12*(5*A*b^5*c^4*d*g^4 - (4*B*b^5*c^4*d - 10*B*a*b^4*c^3*d^2 + 10*B*a^2*b^3
*c^2*d^3 - 5*B*a^3*b^2*c*d^4 + B*a^4*b*d^5)*g^4*n)*x + 12*(B*b^5*d^5*g^4*x^
5 + 5*B*b^5*c*d^4*g^4*x^4 + 10*B*b^5*c^2*d^3*g^4*x^3 + 10*B*b^5*c^3*d^2*g^4
*x^2 + 5*B*b^5*c^4*d*g^4*x)*log(e) + 12*(B*b^5*d^5*g^4*n*x^5 + 5*B*b^5*c*d^
4*g^4*n*x^4 + 10*B*b^5*c^2*d^3*g^4*n*x^3 + 10*B*b^5*c^3*d^2*g^4*n*x^2 + 5*B
*b^5*c^4*d*g^4*n*x)*log((b*x + a)/(d*x + c)))/(b^5*d)
```

**giac [B]** time = 5.99, size = 1862, normalized size = 9.90

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*g*x+c*g)^4*(A+B*log(e*((b*x+a)/(d*x+c))^n)),x, algorithm="giac
")
```

```
[Out] 1/60*(12*(B*b^6*c^6*g^4*n - 6*B*a*b^5*c^5*d*g^4*n + 15*B*a^2*b^4*c^4*d^2*g^
4*n - 20*B*a^3*b^3*c^3*d^3*g^4*n + 15*B*a^4*b^2*c^2*d^4*g^4*n - 6*B*a^5*b*c
*d^5*g^4*n + B*a^6*d^6*g^4*n)*log((b*x + a)/(d*x + c))/(b^5*d - 5*(b*x + a)
*b^4*d^2/(d*x + c) + 10*(b*x + a)^2*b^3*d^3/(d*x + c)^2 - 10*(b*x + a)^3*b^
2*d^4/(d*x + c)^3 + 5*(b*x + a)^4*b*d^5/(d*x + c)^4 - (b*x + a)^5*d^6/(d*x
+ c)^5) - (25*B*b^10*c^6*g^4*n - 150*B*a*b^9*c^5*d*g^4*n - 77*(b*x + a)*B*b
^9*c^6*d*g^4*n/(d*x + c) + 375*B*a^2*b^8*c^4*d^2*g^4*n + 462*(b*x + a)*B*a*
b^8*c^5*d^2*g^4*n/(d*x + c) + 94*(b*x + a)^2*B*b^8*c^6*d^2*g^4*n/(d*x + c)^
2 - 500*B*a^3*b^7*c^3*d^3*g^4*n - 1155*(b*x + a)*B*a^2*b^7*c^4*d^3*g^4*n/(d
*x + c) - 564*(b*x + a)^2*B*a*b^7*c^5*d^3*g^4*n/(d*x + c)^2 - 54*(b*x + a)^
3*B*b^7*c^6*d^3*g^4*n/(d*x + c)^3 + 375*B*a^4*b^6*c^2*d^4*g^4*n + 1540*(b*x
 + a)*B*a^3*b^6*c^3*d^4*g^4*n/(d*x + c) + 1410*(b*x + a)^2*B*a^2*b^6*c^4*d^
4*g^4*n/(d*x + c)^2 + 324*(b*x + a)^3*B*a*b^6*c^5*d^4*g^4*n/(d*x + c)^3 + 1
2*(b*x + a)^4*B*b^6*c^6*d^4*g^4*n/(d*x + c)^4 - 150*B*a^5*b^5*c*d^5*g^4*n -
 1155*(b*x + a)*B*a^4*b^5*c^2*d^5*g^4*n/(d*x + c) - 1880*(b*x + a)^2*B*a^3*
b^5*c^3*d^5*g^4*n/(d*x + c)^2 - 810*(b*x + a)^3*B*a^2*b^5*c^4*d^5*g^4*n/(d*
x + c)^3 - 72*(b*x + a)^4*B*a*b^5*c^5*d^5*g^4*n/(d*x + c)^4 + 25*B*a^6*b^4*
d^6*g^4*n + 462*(b*x + a)*B*a^5*b^4*c*d^6*g^4*n/(d*x + c) + 1410*(b*x + a)^
2*B*a^4*b^4*c^2*d^6*g^4*n/(d*x + c)^2 + 1080*(b*x + a)^3*B*a^3*b^4*c^3*d^6*
```

```
g^4*n/(d*x + c)^3 + 180*(b*x + a)^4*B*a^2*b^4*c^4*d^6*g^4*n/(d*x + c)^4 - 7
7*(b*x + a)*B*a^6*b^3*d^7*g^4*n/(d*x + c) - 564*(b*x + a)^2*B*a^5*b^3*c*d^7
*g^4*n/(d*x + c)^2 - 810*(b*x + a)^3*B*a^4*b^3*c^2*d^7*g^4*n/(d*x + c)^3 -
240*(b*x + a)^4*B*a^3*b^3*c^3*d^7*g^4*n/(d*x + c)^4 + 94*(b*x + a)^2*B*a^6*
b^2*d^8*g^4*n/(d*x + c)^2 + 324*(b*x + a)^3*B*a^5*b^2*c*d^8*g^4*n/(d*x + c)
^3 + 180*(b*x + a)^4*B*a^4*b^2*c^2*d^8*g^4*n/(d*x + c)^4 - 54*(b*x + a)^3*B
*a^6*b*d^9*g^4*n/(d*x + c)^3 - 72*(b*x + a)^4*B*a^5*b*c*d^9*g^4*n/(d*x + c)
^4 + 12*(b*x + a)^4*B*a^6*d^10*g^4*n/(d*x + c)^4 - 12*A*b^10*c^6*g^4 - 12*B
*b^10*c^6*g^4 + 72*A*a*b^9*c^5*d*g^4 + 72*B*a*b^9*c^5*d*g^4 - 180*A*a^2*b^8
*c^4*d^2*g^4 - 180*B*a^2*b^8*c^4*d^2*g^4 + 240*A*a^3*b^7*c^3*d^3*g^4 + 240*
B*a^3*b^7*c^3*d^3*g^4 - 180*A*a^4*b^6*c^2*d^4*g^4 - 180*B*a^4*b^6*c^2*d^4*g
^4 + 72*A*a^5*b^5*c*d^5*g^4 + 72*B*a^5*b^5*c*d^5*g^4 - 12*A*a^6*b^4*d^6*g^4
 - 12*B*a^6*b^4*d^6*g^4)/(b^9*d - 5*(b*x + a)*b^8*d^2/(d*x + c) + 10*(b*x +
 a)^2*b^7*d^3/(d*x + c)^2 - 10*(b*x + a)^3*b^6*d^4/(d*x + c)^3 + 5*(b*x + a
)^4*b^5*d^5/(d*x + c)^4 - (b*x + a)^5*b^4*d^6/(d*x + c)^5) + 12*(B*b^6*c^6*
g^4*n - 6*B*a*b^5*c^5*d*g^4*n + 15*B*a^2*b^4*c^4*d^2*g^4*n - 20*B*a^3*b^3*c
^3*d^3*g^4*n + 15*B*a^4*b^2*c^2*d^4*g^4*n - 6*B*a^5*b*c*d^5*g^4*n + B*a^6*d
^6*g^4*n)*log(b - (b*x + a)*d/(d*x + c))/(b^5*d) - 12*(B*b^6*c^6*g^4*n - 6*
B*a*b^5*c^5*d*g^4*n + 15*B*a^2*b^4*c^4*d^2*g^4*n - 20*B*a^3*b^3*c^3*d^3*g^4
*n + 15*B*a^4*b^2*c^2*d^4*g^4*n - 6*B*a^5*b*c*d^5*g^4*n + B*a^6*d^6*g^4*n)*
log((b*x + a)/(d*x + c))/(b^5*d))*(b*c/(b*c - a*d)^2 - a*d/(b*c - a*d)^2)
```

**maple [F]** time = 0.30, size = 0, normalized size = 0.00

$$\int \left(dgx + cg\right)^{4} \left(B \ln \left(e \left(\frac{bx + a}{dx + c}\right)^{n}\right) + A\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*g*x+c*g)^4*(B*ln(e*((b*x+a)/(d*x+c))^n)+A),x)
```

```
[Out] int((d*g*x+c*g)^4*(B*ln(e*((b*x+a)/(d*x+c))^n)+A),x)
```

**maxima [B]** time = 1.38, size = 676, normalized size = 3.60

$$\frac{1}{5}\,Bd^{4}g^{4}x^{5}\log\left(e\left(\frac{bx}{dx+c}+\frac{a}{dx+c}\right)^{n}\right)+\frac{1}{5}\,Ad^{4}g^{4}x^{5}+Bcd^{3}g^{4}x^{4}\log\left(e\left(\frac{bx}{dx+c}+\frac{a}{dx+c}\right)^{n}\right)+Acd^{3}g^{4}x^{4}+2\,Bc^{2}d^{2}g^{4}x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*g*x+c*g)^4*(A+B*log(e*((b*x+a)/(d*x+c))^n)),x, algorithm="maxi
ma")
```

```
[Out] 1/5*B*d^4*g^4*x^5*log(e*(b*x/(d*x + c) + a/(d*x + c))^n) + 1/5*A*d^4*g^4*x^
5 + B*c*d^3*g^4*x^4*log(e*(b*x/(d*x + c) + a/(d*x + c))^n) + A*c*d^3*g^4*x^
4 + 2*B*c^2*d^2*g^4*x^3*log(e*(b*x/(d*x + c) + a/(d*x + c))^n) + 2*A*c^2*d^
```

```
2*g^4*x^3 + 2*B*c^3*d*g^4*x^2*log(e*(b*x/(d*x + c) + a/(d*x + c))^n) + 2*A*
c^3*d*g^4*x^2 + 1/60*B*d^4*g^4*n*(12*a^5*log(b*x + a)/b^5 - 12*c^5*log(d*x
+ c)/d^5 - (3*(b^4*c*d^3 - a*b^3*d^4)*x^4 - 4*(b^4*c^2*d^2 - a^2*b^2*d^4)*x
^3 + 6*(b^4*c^3*d - a^3*b*d^4)*x^2 - 12*(b^4*c^4 - a^4*d^4)*x)/(b^4*d^4)) -
 1/6*B*c*d^3*g^4*n*(6*a^4*log(b*x + a)/b^4 - 6*c^4*log(d*x + c)/d^4 + (2*(b
^3*c*d^2 - a*b^2*d^3)*x^3 - 3*(b^3*c^2*d - a^2*b*d^3)*x^2 + 6*(b^3*c^3 - a^
3*d^3)*x)/(b^3*d^3)) + B*c^2*d^2*g^4*n*(2*a^3*log(b*x + a)/b^3 - 2*c^3*log(
d*x + c)/d^3 - ((b^2*c*d - a*b*d^2)*x^2 - 2*(b^2*c^2 - a^2*d^2)*x)/(b^2*d^2
)) - 2*B*c^3*d*g^4*n*(a^2*log(b*x + a)/b^2 - c^2*log(d*x + c)/d^2 + (b*c -
a*d)*x/(b*d)) + B*c^4*g^4*n*(a*log(b*x + a)/b - c*log(d*x + c)/d) + B*c^4*g
^4*x*log(e*(b*x/(d*x + c) + a/(d*x + c))^n) + A*c^4*g^4*x
```

**mupad [B]** time = 4.48, size = 1045, normalized size = 5.56

$$x^2 \left( \frac{(5\,a\,d+5\,b\,c)\left(\frac{\left(\frac{d^3\,g^4\,(5\,A\,a\,d+25\,A\,b\,c+B\,a\,d\,n-B\,b\,c\,n)}{5\,b}-\frac{A\,d^3\,g^4\,(5\,a\,d+5\,b\,c)}{5\,b}\right)(5\,a\,d+5\,b\,c)}{5\,b\,d}-\frac{c\,d^2\,g^4\,(5\,A\,a\,d+10\,A\,b\,c+B\,a\,d\,n-B\,b\,c\,n)}{b}+\frac{A\,a\,c\,d^3\,g^4}{b}\right.}{10\,b\,d} \right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*g + d*g*x)^4*(A + B*log(e*((a + b*x)/(c + d*x))^n)),x)
```

```
[Out] x^2*(((5*a*d + 5*b*c)*((((d^3*g^4*(5*A*a*d + 25*A*b*c + B*a*d*n - B*b*c*n))
/(5*b) - (A*d^3*g^4*(5*a*d + 5*b*c))/(5*b))*(5*a*d + 5*b*c))/(5*b*d) - (c*d
^2*g^4*(5*A*a*d + 10*A*b*c + B*a*d*n - B*b*c*n))/b + (A*a*c*d^3*g^4)/b))/(1
0*b*d) - (a*c*((d^3*g^4*(5*A*a*d + 25*A*b*c + B*a*d*n - B*b*c*n))/(5*b) - (
A*d^3*g^4*(5*a*d + 5*b*c))/(5*b)))/(2*b*d) + (c^2*d*g^4*(5*A*a*d + 5*A*b*c
+ B*a*d*n - B*b*c*n))/b) - x^3*((((d^3*g^4*(5*A*a*d + 25*A*b*c + B*a*d*n -
B*b*c*n))/(5*b) - (A*d^3*g^4*(5*a*d + 5*b*c))/(5*b))*(5*a*d + 5*b*c))/(15*b
*d) - (c*d^2*g^4*(5*A*a*d + 10*A*b*c + B*a*d*n - B*b*c*n))/(3*b) + (A*a*c*d
^3*g^4)/(3*b)) + x^4*((d^3*g^4*(5*A*a*d + 25*A*b*c + B*a*d*n - B*b*c*n))/(2
0*b) - (A*d^3*g^4*(5*a*d + 5*b*c))/(20*b)) + log(e*((a + b*x)/(c + d*x))^n)
*((B*d^4*g^4*x^5)/5 + B*c^4*g^4*x + 2*B*c^3*d*g^4*x^2 + B*c*d^3*g^4*x^4 + 2
*B*c^2*d^2*g^4*x^3) + x*((c^3*g^4*(10*A*a*d + 5*A*b*c + 2*B*a*d*n - 2*B*b*c
*n))/b - ((5*a*d + 5*b*c)*(((5*a*d + 5*b*c)*((((d^3*g^4*(5*A*a*d + 25*A*b*c
 + B*a*d*n - B*b*c*n))/(5*b) - (A*d^3*g^4*(5*a*d + 5*b*c))/(5*b))*(5*a*d +
5*b*c))/(5*b*d) - (c*d^2*g^4*(5*A*a*d + 10*A*b*c + B*a*d*n - B*b*c*n))/b +
```

```
(A*a*c*d^3*g^4)/b))/(5*b*d) - (a*c*((d^3*g^4*(5*A*a*d + 25*A*b*c + B*a*d*n
- B*b*c*n))/(5*b) - (A*d^3*g^4*(5*a*d + 5*b*c))/(5*b)))/(b*d) + (2*c^2*d*g^
4*(5*A*a*d + 5*A*b*c + B*a*d*n - B*b*c*n))/b))/(5*b*d) + (a*c*((((d^3*g^4*(
5*A*a*d + 25*A*b*c + B*a*d*n - B*b*c*n))/(5*b) - (A*d^3*g^4*(5*a*d + 5*b*c)
)/(5*b))*(5*a*d + 5*b*c))/(5*b*d) - (c*d^2*g^4*(5*A*a*d + 10*A*b*c + B*a*d*
n - B*b*c*n))/b + (A*a*c*d^3*g^4)/b))/(b*d)) + (log(a + b*x)*((B*a^5*d^4*g^
4*n)/5 + B*a*b^4*c^4*g^4*n - B*a^4*b*c*d^3*g^4*n - 2*B*a^2*b^3*c^3*d*g^4*n
+ 2*B*a^3*b^2*c^2*d^2*g^4*n))/b^5 + (A*d^4*g^4*x^5)/5 - (B*c^5*g^4*n*log(c
+ d*x))/(5*d)
```

**sympy [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*g*x+c*g)**4*(A+B*ln(e*((b*x+a)/(d*x+c))**n)),x)
```

```
[Out] Timed out
```

# 3.30 $\int (cg+dgx)^3 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right) dx$

**Optimal**. Leaf size=156

$$\frac{g^3(c+dx)^4\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{4d}-\frac{Bg^3n(bc-ad)^4\log(a+bx)}{4b^4d}-\frac{Bg^3nx(bc-ad)^3}{4b^3}-\frac{Bg^3n(c+dx)^2(bc-ad)^2}{8b^2d}-Bg$$

```
[Out] -1/4*B*(-a*d+b*c)^3*g^3*n*x/b^3-1/8*B*(-a*d+b*c)^2*g^3*n*(d*x+c)^2/b^2/d-1/
12*B*(-a*d+b*c)*g^3*n*(d*x+c)^3/b/d-1/4*B*(-a*d+b*c)^4*g^3*n*ln(b*x+a)/b^4/
d+1/4*g^3*(d*x+c)^4*(A+B*ln(e*((b*x+a)/(d*x+c))^n))/d
```

**Rubi [A]** time = 0.10, antiderivative size = 156, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {2525, 12, 43}

$$\frac{g^3(c+dx)^4\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{4d}-\frac{Bg^3nx(bc-ad)^3}{4b^3}-\frac{Bg^3n(c+dx)^2(bc-ad)^2}{8b^2d}-\frac{Bg^3n(bc-ad)^4\log(a+bx)}{4b^4d}-Bg$$

Antiderivative was successfully verified.

```
[In] Int[(c*g + d*g*x)^3*(A + B*Log[e*((a + b*x)/(c + d*x))^n]),x]
```

```
[Out] -(B*(b*c - a*d)^3*g^3*n*x)/(4*b^3) - (B*(b*c - a*d)^2*g^3*n*(c + d*x)^2)/(8
*b^2*d) - (B*(b*c - a*d)*g^3*n*(c + d*x)^3)/(12*b*d) - (B*(b*c - a*d)^4*g^3
*n*Log[a + b*x])/(4*b^4*d) + (g^3*(c + d*x)^4*(A + B*Log[e*((a + b*x)/(c +
d*x))^n]))/(4*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2525

```
Int[((a_.) + Log[(c_.)*(RFx_)^(p_.)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_))^(m_.
), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(a + b*Log[c*RFx^p])^n)/(e*(m + 1))
```

```
, x] - Dist[(b*n*p)/(e*(m + 1)), Int[SimplifyIntegrand[((d + e*x)^(m + 1)*(
a + b*Log[c*RFx^p])^(n - 1)*D[RFx, x])/RFx, x], x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && RationalFunctionQ[RFx, x] && IGtQ[n, 0] && (EqQ[n, 1] ||
IntegerQ[m]) && NeQ[m, -1]
```

Rubi steps

$$\int (cg+dgx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)dx = \frac{g^3(c+dx)^4\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{4d}-\frac{(Bn)\int\frac{(bc-ad)g^4(c+dx)^3}{a+bx}\,dx}{4dg}$$

$$=\frac{g^3(c+dx)^4\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{4d}-\frac{\left(B(bc-ad)g^3n\right)\int\frac{(c+dx)^3}{a+bx}}{4d}$$

$$=\frac{g^3(c+dx)^4\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{4d}-\frac{\left(B(bc-ad)g^3n\right)\int\left(\frac{d(bc-ad}{b^3}\right.}{}$$

$$=-\frac{B(bc-ad)^3g^3nx}{4b^3}-\frac{B(bc-ad)^2g^3n(c+dx)^2}{8b^2d}-\frac{B(bc-ad)g^3n(c}{12bd}$$

**Mathematica [A]** time = 0.09, size = 124, normalized size = 0.79

$$\frac{g^3\left((c+dx)^4\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)-\frac{Bn(bc-ad)\left(3b^2(c+dx)^2(bc-ad)+6bdx(bc-ad)^2+6(bc-ad)^3\log(a+bx)+2b^3(c+dx)^3\right)}{6b^4}\right)}{4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*g + d*g*x)^3*(A + B*Log[e*((a + b*x)/(c + d*x))^n]),x]
```

```
[Out] (g^3*(-1/6*(B*(b*c - a*d)*n*(6*b*d*(b*c - a*d)^2*x + 3*b^2*(b*c - a*d)*(c +
 d*x)^2 + 2*b^3*(c + d*x)^3 + 6*(b*c - a*d)^3*Log[a + b*x]))/b^4 + (c + d*x
)^4*(A + B*Log[e*((a + b*x)/(c + d*x))^n])))/(4*d)
```

**fricas [B]** time = 1.03, size = 429, normalized size = 2.75

$$\frac{6\,Ab^4d^4g^3x^4-6\,Bb^4c^4g^3n\log(dx+c)+6\left(4\,Bab^3c^3d-6\,Ba^2b^2c^2d^2+4\,Ba^3bcd^3-Ba^4d^4\right)g^3n\log(bx+a)+2\,(}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*g*x+c*g)^3*(A+B*log(e*((b*x+a)/(d*x+c))^n)),x, algorithm="fric
as")
```

```
[Out] 1/24*(6*A*b^4*d^4*g^3*x^4 - 6*B*b^4*c^4*g^3*n*log(d*x + c) + 6*(4*B*a*b^3*c
^3*d - 6*B*a^2*b^2*c^2*d^2 + 4*B*a^3*b*c*d^3 - B*a^4*d^4)*g^3*n*log(b*x + a
) + 2*(12*A*b^4*c*d^3*g^3 - (B*b^4*c*d^3 - B*a*b^3*d^4)*g^3*n)*x^3 + 3*(12*
A*b^4*c^2*d^2*g^3 - (3*B*b^4*c^2*d^2 - 4*B*a*b^3*c*d^3 + B*a^2*b^2*d^4)*g^3
*n)*x^2 + 6*(4*A*b^4*c^3*d*g^3 - (3*B*b^4*c^3*d - 6*B*a*b^3*c^2*d^2 + 4*B*a
^2*b^2*c*d^3 - B*a^3*b*d^4)*g^3*n)*x + 6*(B*b^4*d^4*g^3*x^4 + 4*B*b^4*c*d^3
*g^3*x^3 + 6*B*b^4*c^2*d^2*g^3*x^2 + 4*B*b^4*c^3*d*g^3*x)*log(e) + 6*(B*b^4
*d^4*g^3*n*x^4 + 4*B*b^4*c*d^3*g^3*n*x^3 + 6*B*b^4*c^2*d^2*g^3*n*x^2 + 4*B*
b^4*c^3*d*g^3*n*x)*log((b*x + a)/(d*x + c)))/(b^4*d)
```

**giac [B]** time = 4.45, size = 1390, normalized size = 8.91

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*g*x+c*g)^3*(A+B*log(e*((b*x+a)/(d*x+c))^n)),x, algorithm="giac
")
```

```
[Out] 1/24*(6*(B*b^5*c^5*g^3*n - 5*B*a*b^4*c^4*d*g^3*n + 10*B*a^2*b^3*c^3*d^2*g^3
*n - 10*B*a^3*b^2*c^2*d^3*g^3*n + 5*B*a^4*b*c*d^4*g^3*n - B*a^5*d^5*g^3*n)*
log((b*x + a)/(d*x + c))/(b^4*d - 4*(b*x + a)*b^3*d^2/(d*x + c) + 6*(b*x +
a)^2*b^2*d^3/(d*x + c)^2 - 4*(b*x + a)^3*b*d^4/(d*x + c)^3 + (b*x + a)^4*d^
5/(d*x + c)^4) - (11*B*b^8*c^5*g^3*n - 55*B*a*b^7*c^4*d*g^3*n - 26*(b*x + a
)*B*b^7*c^5*d*g^3*n/(d*x + c) + 110*B*a^2*b^6*c^3*d^2*g^3*n + 130*(b*x + a)
*B*a*b^6*c^4*d^2*g^3*n/(d*x + c) + 21*(b*x + a)^2*B*b^6*c^5*d^2*g^3*n/(d*x
+ c)^2 - 110*B*a^3*b^5*c^2*d^3*g^3*n - 260*(b*x + a)*B*a^2*b^5*c^3*d^3*g^3*
n/(d*x + c) - 105*(b*x + a)^2*B*a*b^5*c^4*d^3*g^3*n/(d*x + c)^2 - 6*(b*x +
a)^3*B*b^5*c^5*d^3*g^3*n/(d*x + c)^3 + 55*B*a^4*b^4*c*d^4*g^3*n + 260*(b*x
+ a)*B*a^3*b^4*c^2*d^4*g^3*n/(d*x + c) + 210*(b*x + a)^2*B*a^2*b^4*c^3*d^4*
g^3*n/(d*x + c)^2 + 30*(b*x + a)^3*B*a*b^4*c^4*d^4*g^3*n/(d*x + c)^3 - 11*B
*a^5*b^3*d^5*g^3*n - 130*(b*x + a)*B*a^4*b^3*c*d^5*g^3*n/(d*x + c) - 210*(b
*x + a)^2*B*a^3*b^3*c^2*d^5*g^3*n/(d*x + c)^2 - 60*(b*x + a)^3*B*a^2*b^3*c^
3*d^5*g^3*n/(d*x + c)^3 + 26*(b*x + a)*B*a^5*b^2*d^6*g^3*n/(d*x + c) + 105*
(b*x + a)^2*B*a^4*b^2*c*d^6*g^3*n/(d*x + c)^2 + 60*(b*x + a)^3*B*a^3*b^2*c^
2*d^6*g^3*n/(d*x + c)^3 - 21*(b*x + a)^2*B*a^5*b*d^7*g^3*n/(d*x + c)^2 - 30
*(b*x + a)^3*B*a^4*b*c*d^7*g^3*n/(d*x + c)^3 + 6*(b*x + a)^3*B*a^5*d^8*g^3*
n/(d*x + c)^3 - 6*A*b^8*c^5*g^3 - 6*B*b^8*c^5*g^3 + 30*A*a*b^7*c^4*d*g^3 +
30*B*a*b^7*c^4*d*g^3 - 60*A*a^2*b^6*c^3*d^2*g^3 - 60*B*a^2*b^6*c^3*d^2*g^3
+ 60*A*a^3*b^5*c^2*d^3*g^3 + 60*B*a^3*b^5*c^2*d^3*g^3 - 30*A*a^4*b^4*c*d^4*
g^3 - 30*B*a^4*b^4*c*d^4*g^3 + 6*A*a^5*b^3*d^5*g^3 + 6*B*a^5*b^3*d^5*g^3)/(
b^7*d - 4*(b*x + a)*b^6*d^2/(d*x + c) + 6*(b*x + a)^2*b^5*d^3/(d*x + c)^2 -
 4*(b*x + a)^3*b^4*d^4/(d*x + c)^3 + (b*x + a)^4*b^3*d^5/(d*x + c)^4) + 6*(
```

```
B*b^5*c^5*g^3*n - 5*B*a*b^4*c^4*d*g^3*n + 10*B*a^2*b^3*c^3*d^2*g^3*n - 10*B
*a^3*b^2*c^2*d^3*g^3*n + 5*B*a^4*b*c*d^4*g^3*n - B*a^5*d^5*g^3*n)*log(-b +
(b*x + a)*d/(d*x + c))/(b^4*d) - 6*(B*b^5*c^5*g^3*n - 5*B*a*b^4*c^4*d*g^3*n
 + 10*B*a^2*b^3*c^3*d^2*g^3*n - 10*B*a^3*b^2*c^2*d^3*g^3*n + 5*B*a^4*b*c*d^
4*g^3*n - B*a^5*d^5*g^3*n)*log((b*x + a)/(d*x + c))/(b^4*d))*(b*c/(b*c - a*
d)^2 - a*d/(b*c - a*d)^2)
```

**maple [F]** time = 0.27, size = 0, normalized size = 0.00

$$\int \left(dgx+cg\right)^{3}\left(B\ln\left(e\left(\frac{bx+a}{dx+c}\right)^{n}\right)+A\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*g*x+c*g)^3*(B*ln(e*((b*x+a)/(d*x+c))^n)+A),x)
```

```
[Out] int((d*g*x+c*g)^3*(B*ln(e*((b*x+a)/(d*x+c))^n)+A),x)
```

**maxima [B]** time = 1.37, size = 479, normalized size = 3.07

$$\frac{1}{4}Bd^{3}g^{3}x^{4}\log\left(e\left(\frac{bx}{dx+c}+\frac{a}{dx+c}\right)^{n}\right)+\frac{1}{4}Ad^{3}g^{3}x^{4}+Bcd^{2}g^{3}x^{3}\log\left(e\left(\frac{bx}{dx+c}+\frac{a}{dx+c}\right)^{n}\right)+Acd^{2}g^{3}x^{3}+\frac{3}{2}Bc^{2}dg^{3}x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*g*x+c*g)^3*(A+B*log(e*((b*x+a)/(d*x+c))^n)),x, algorithm="maxi
ma")
```

```
[Out] 1/4*B*d^3*g^3*x^4*log(e*(b*x/(d*x + c) + a/(d*x + c))^n) + 1/4*A*d^3*g^3*x^
4 + B*c*d^2*g^3*x^3*log(e*(b*x/(d*x + c) + a/(d*x + c))^n) + A*c*d^2*g^3*x^
3 + 3/2*B*c^2*d*g^3*x^2*log(e*(b*x/(d*x + c) + a/(d*x + c))^n) + 3/2*A*c^2*
d*g^3*x^2 - 1/24*B*d^3*g^3*n*(6*a^4*log(b*x + a)/b^4 - 6*c^4*log(d*x + c)/d
^4 + (2*(b^3*c*d^2 - a*b^2*d^3)*x^3 - 3*(b^3*c^2*d - a^2*b*d^3)*x^2 + 6*(b^
3*c^3 - a^3*d^3)*x)/(b^3*d^3)) + 1/2*B*c*d^2*g^3*n*(2*a^3*log(b*x + a)/b^3
- 2*c^3*log(d*x + c)/d^3 - ((b^2*c*d - a*b*d^2)*x^2 - 2*(b^2*c^2 - a^2*d^2)
*x)/(b^2*d^2)) - 3/2*B*c^2*d*g^3*n*(a^2*log(b*x + a)/b^2 - c^2*log(d*x + c)
/d^2 + (b*c - a*d)*x/(b*d)) + B*c^3*g^3*n*(a*log(b*x + a)/b - c*log(d*x + c
)/d) + B*c^3*g^3*x*log(e*(b*x/(d*x + c) + a/(d*x + c))^n) + A*c^3*g^3*x
```

**mupad [B]** time = 4.37, size = 588, normalized size = 3.77

$$x^{3}\left(\frac{d^{2}\,g^{3}\,\left(4\,A\,a\,d+16\,A\,b\,c+B\,a\,d\,n-B\,b\,c\,n\right)}{12\,b}-\frac{A\,d^{2}\,g^{3}\,\left(4\,a\,d+4\,b\,c\right)}{12\,b}\right)-x^{2}\left(\frac{\left(\frac{d^{2}\,g^{3}\,\left(4\,A\,a\,d+16\,A\,b\,c+B\,a\,d\,n-B\,b\,c\,n\right)}{4\,b}-\right.}{8\,b}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*g + d*g*x)^3*(A + B*log(e*((a + b*x)/(c + d*x))^n)),x)
```

```
[Out] x^3*((d^2*g^3*(4*A*a*d + 16*A*b*c + B*a*d*n - B*b*c*n))/(12*b) - (A*d^2*g^3
*(4*a*d + 4*b*c))/(12*b)) - x^2*((((d^2*g^3*(4*A*a*d + 16*A*b*c + B*a*d*n -
 B*b*c*n))/(4*b) - (A*d^2*g^3*(4*a*d + 4*b*c))/(4*b))*(4*a*d + 4*b*c))/(8*b
*d) - (c*d*g^3*(4*A*a*d + 6*A*b*c + B*a*d*n - B*b*c*n))/(2*b) + (A*a*c*d^2*
g^3)/(2*b)) + log(e*((a + b*x)/(c + d*x))^n)*((B*d^3*g^3*x^4)/4 + B*c^3*g^3
*x + (3*B*c^2*d*g^3*x^2)/2 + B*c*d^2*g^3*x^3) + x*(((4*a*d + 4*b*c)*((((d^2
*g^3*(4*A*a*d + 16*A*b*c + B*a*d*n - B*b*c*n))/(4*b) - (A*d^2*g^3*(4*a*d +
4*b*c))/(4*b))*(4*a*d + 4*b*c))/(4*b*d) - (c*d*g^3*(4*A*a*d + 6*A*b*c + B*a
*d*n - B*b*c*n))/b + (A*a*c*d^2*g^3)/b))/(4*b*d) + (c^2*g^3*(12*A*a*d + 8*A
*b*c + 3*B*a*d*n - 3*B*b*c*n))/(2*b) - (a*c*((d^2*g^3*(4*A*a*d + 16*A*b*c +
 B*a*d*n - B*b*c*n))/(4*b) - (A*d^2*g^3*(4*a*d + 4*b*c))/(4*b)))/(b*d)) - (
log(a + b*x)*(B*a^4*d^3*g^3*n - 4*B*a*b^3*c^3*g^3*n - 4*B*a^3*b*c*d^2*g^3*n
 + 6*B*a^2*b^2*c^2*d*g^3*n))/(4*b^4) + (A*d^3*g^3*x^4)/4 - (B*c^4*g^3*n*log
(c + d*x))/(4*d)
```

**sympy [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*g*x+c*g)**3*(A+B*ln(e*((b*x+a)/(d*x+c))**n)),x)
```

```
[Out] Timed out
```

# 3.31 $\int (cg+dgx)^2 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right) dx$

**Optimal**. Leaf size=124

$$\frac{g^2(c+dx)^3\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{3d}-\frac{Bg^2n(bc-ad)^3\log(a+bx)}{3b^3d}-\frac{Bg^2nx(bc-ad)^2}{3b^2}-\frac{Bg^2n(c+dx)^2(bc-ad)}{6bd}$$

```
[Out] -1/3*B*(-a*d+b*c)^2*g^2*n*x/b^2-1/6*B*(-a*d+b*c)*g^2*n*(d*x+c)^2/b/d-1/3*B*
(-a*d+b*c)^3*g^2*n*ln(b*x+a)/b^3/d+1/3*g^2*(d*x+c)^3*(A+B*ln(e*((b*x+a)/(d*
x+c))^n))/d
```

**Rubi [A]** time = 0.08, antiderivative size = 124, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {2525, 12, 43}

$$\frac{g^2(c+dx)^3\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{3d}-\frac{Bg^2nx(bc-ad)^2}{3b^2}-\frac{Bg^2n(bc-ad)^3\log(a+bx)}{3b^3d}-\frac{Bg^2n(c+dx)^2(bc-ad)}{6bd}$$

Antiderivative was successfully verified.

```
[In] Int[(c*g + d*g*x)^2*(A + B*Log[e*((a + b*x)/(c + d*x))^n]),x]
```

```
[Out] -(B*(b*c - a*d)^2*g^2*n*x)/(3*b^2) - (B*(b*c - a*d)*g^2*n*(c + d*x)^2)/(6*b
*d) - (B*(b*c - a*d)^3*g^2*n*Log[a + b*x])/(3*b^3*d) + (g^2*(c + d*x)^3*(A
+ B*Log[e*((a + b*x)/(c + d*x))^n]))/(3*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2525

```
Int[((a_.) + Log[(c_.)*(RFx_)^(p_.)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_))^(m_.
), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(a + b*Log[c*RFx^p])^n)/(e*(m + 1))
, x] - Dist[(b*n*p)/(e*(m + 1)), Int[SimplifyIntegrand[((d + e*x)^(m + 1)*(
```

```
a + b*Log[c*RFx^p])^(n - 1)*D[RFx, x])/RFx, x], x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && RationalFunctionQ[RFx, x] && IGtQ[n, 0] && (EqQ[n, 1] ||
IntegerQ[m]) && NeQ[m, -1]
```

Rubi steps

$$\int (cg+dgx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)dx = \frac{g^2(c+dx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{3d} - \frac{(Bn)\int\frac{(bc-ad)g^3(c+dx)^2}{a+bx}dx}{3dg}$$

$$= \frac{g^2(c+dx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{3d} - \frac{\left(B(bc-ad)g^2n\right)\int\frac{(c+dx)^2}{a+bx}d}{3d}$$

$$= \frac{g^2(c+dx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{3d} - \frac{\left(B(bc-ad)g^2n\right)\int\left(\frac{d(bc-ad)}{b^2}\right.}{3d}$$

$$= -\frac{B(bc-ad)^2g^2nx}{3b^2} - \frac{B(bc-ad)g^2n(c+dx)^2}{6bd} - \frac{B(bc-ad)^3g^2n\log}{3b^3d}$$

**Mathematica [A]** time = 0.06, size = 101, normalized size = 0.81

$$\frac{g^2\left((c+dx)^3\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)-\frac{Bn(bc-ad)\left(2bdx(bc-ad)+2(bc-ad)^2\log(a+bx)+b^2(c+dx)^2\right)}{2b^3}\right)}{3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*g + d*g*x)^2*(A + B*Log[e*((a + b*x)/(c + d*x))^n]),x]
```

```
[Out] (g^2*(-1/2*(B*(b*c - a*d)*n*(2*b*d*(b*c - a*d)*x + b^2*(c + d*x)^2 + 2*(b*c
 - a*d)^2*Log[a + b*x]))/b^3 + (c + d*x)^3*(A + B*Log[e*((a + b*x)/(c + d*x
))^n])))/(3*d)
```

**fricas [B]** time = 1.04, size = 297, normalized size = 2.40

$$2\,Ab^3d^3g^2x^3 - 2\,Bb^3c^3g^2n\log(dx+c) + 2\left(3\,Bab^2c^2d - 3\,Ba^2bcd^2 + Ba^3d^3\right)g^2n\log(bx+a) + \left(6\,Ab^3cd^2g^2 - \left(Bb\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*g*x+c*g)^2*(A+B*log(e*((b*x+a)/(d*x+c))^n)),x, algorithm="fric
as")
```

```
[Out] 1/6*(2*A*b^3*d^3*g^2*x^3 - 2*B*b^3*c^3*g^2*n*log(d*x + c) + 2*(3*B*a*b^2*c^
2*d - 3*B*a^2*b*c*d^2 + B*a^3*d^3)*g^2*n*log(b*x + a) + (6*A*b^3*c*d^2*g^2
- (B*b^3*c*d^2 - B*a*b^2*d^3)*g^2*n)*x^2 + 2*(3*A*b^3*c^2*d*g^2 - (2*B*b^3*
c^2*d - 3*B*a*b^2*c*d^2 + B*a^2*b*d^3)*g^2*n)*x + 2*(B*b^3*d^3*g^2*x^3 + 3*
B*b^3*c*d^2*g^2*x^2 + 3*B*b^3*c^2*d*g^2*x)*log(e) + 2*(B*b^3*d^3*g^2*n*x^3
+ 3*B*b^3*c*d^2*g^2*n*x^2 + 3*B*b^3*c^2*d*g^2*n*x)*log((b*x + a)/(d*x + c))
)/(b^3*d)
```

**giac [B]** time = 3.15, size = 980, normalized size = 7.90

$$\frac{1}{6}\left(\frac{2\left(Bb^4c^4g^2n - 4\,Bab^3c^3dg^2n + 6\,Ba^2b^2c^2d^2g^2n - 4\,Ba^3bcd^3g^2n + Ba^4d^4g^2n\right)\log\left(\frac{bx+a}{dx+c}\right)}{b^3d - \frac{3\,(bx+a)b^2d^2}{dx+c} + \frac{3\,(bx+a)^2bd^3}{(dx+c)^2} - \frac{(bx+a)^3d^4}{(dx+c)^3}} - \frac{3\,Bb^6c^4g^2n - 12\,B\ldots}{\ldots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*g*x+c*g)^2*(A+B*log(e*((b*x+a)/(d*x+c))^n)),x, algorithm="giac
")
```

```
[Out] 1/6*(2*(B*b^4*c^4*g^2*n - 4*B*a*b^3*c^3*d*g^2*n + 6*B*a^2*b^2*c^2*d^2*g^2*n
 - 4*B*a^3*b*c*d^3*g^2*n + B*a^4*d^4*g^2*n)*log((b*x + a)/(d*x + c))/(b^3*d
 - 3*(b*x + a)*b^2*d^2/(d*x + c) + 3*(b*x + a)^2*b*d^3/(d*x + c)^2 - (b*x +
 a)^3*d^4/(d*x + c)^3) - (3*B*b^6*c^4*g^2*n - 12*B*a*b^5*c^3*d*g^2*n - 5*(b
*x + a)*B*b^5*c^4*d*g^2*n/(d*x + c) + 18*B*a^2*b^4*c^2*d^2*g^2*n + 20*(b*x
+ a)*B*a*b^4*c^3*d^2*g^2*n/(d*x + c) + 2*(b*x + a)^2*B*b^4*c^4*d^2*g^2*n/(d
*x + c)^2 - 12*B*a^3*b^3*c*d^3*g^2*n - 30*(b*x + a)*B*a^2*b^3*c^2*d^3*g^2*n
/(d*x + c) - 8*(b*x + a)^2*B*a*b^3*c^3*d^3*g^2*n/(d*x + c)^2 + 3*B*a^4*b^2*
d^4*g^2*n + 20*(b*x + a)*B*a^3*b^2*c*d^4*g^2*n/(d*x + c) + 12*(b*x + a)^2*B
*a^2*b^2*c^2*d^4*g^2*n/(d*x + c)^2 - 5*(b*x + a)*B*a^4*b*d^5*g^2*n/(d*x + c
) - 8*(b*x + a)^2*B*a^3*b*c*d^5*g^2*n/(d*x + c)^2 + 2*(b*x + a)^2*B*a^4*d^6
*g^2*n/(d*x + c)^2 - 2*A*b^6*c^4*g^2 - 2*B*b^6*c^4*g^2 + 8*A*a*b^5*c^3*d*g^
2 + 8*B*a*b^5*c^3*d*g^2 - 12*A*a^2*b^4*c^2*d^2*g^2 - 12*B*a^2*b^4*c^2*d^2*g
^2 + 8*A*a^3*b^3*c*d^3*g^2 + 8*B*a^3*b^3*c*d^3*g^2 - 2*A*a^4*b^2*d^4*g^2 -
2*B*a^4*b^2*d^4*g^2)/(b^5*d - 3*(b*x + a)*b^4*d^2/(d*x + c) + 3*(b*x + a)^2
*b^3*d^3/(d*x + c)^2 - (b*x + a)^3*b^2*d^4/(d*x + c)^3) + 2*(B*b^4*c^4*g^2*
n - 4*B*a*b^3*c^3*d*g^2*n + 6*B*a^2*b^2*c^2*d^2*g^2*n - 4*B*a^3*b*c*d^3*g^2
*n + B*a^4*d^4*g^2*n)*log(b - (b*x + a)*d/(d*x + c))/(b^3*d) - 2*(B*b^4*c^4
*g^2*n - 4*B*a*b^3*c^3*d*g^2*n + 6*B*a^2*b^2*c^2*d^2*g^2*n - 4*B*a^3*b*c*d^
3*g^2*n + B*a^4*d^4*g^2*n)*log((b*x + a)/(d*x + c))/(b^3*d))*(b*c/(b*c - a*
d)^2 - a*d/(b*c - a*d)^2)
```

**maple [F]** time = 0.27, size = 0, normalized size = 0.00

$$\int \left(dgx + cg\right)^2 \left(B\ln\left(e\left(\frac{bx + a}{dx + c}\right)^n\right) + A\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*g*x+c*g)^2*(B*ln(e*((b*x+a)/(d*x+c))^n)+A),x)
```

```
[Out] int((d*g*x+c*g)^2*(B*ln(e*((b*x+a)/(d*x+c))^n)+A),x)
```

**maxima [B]** time = 1.29, size = 309, normalized size = 2.49

$$\frac{1}{3}Bd^2g^2x^3\log\left(e\left(\frac{bx}{dx+c}+\frac{a}{dx+c}\right)^n\right)+\frac{1}{3}Ad^2g^2x^3+Bcdg^2x^2\log\left(e\left(\frac{bx}{dx+c}+\frac{a}{dx+c}\right)^n\right)+Acdg^2x^2+\frac{1}{6}Bd^2g^2n\left(\frac{2a\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*g*x+c*g)^2*(A+B*log(e*((b*x+a)/(d*x+c))^n)),x, algorithm="maxi
ma")
```

```
[Out] 1/3*B*d^2*g^2*x^3*log(e*(b*x/(d*x + c) + a/(d*x + c))^n) + 1/3*A*d^2*g^2*x^
3 + B*c*d*g^2*x^2*log(e*(b*x/(d*x + c) + a/(d*x + c))^n) + A*c*d*g^2*x^2 +
1/6*B*d^2*g^2*n*(2*a^3*log(b*x + a)/b^3 - 2*c^3*log(d*x + c)/d^3 - ((b^2*c*
d - a*b*d^2)*x^2 - 2*(b^2*c^2 - a^2*d^2)*x)/(b^2*d^2)) - B*c*d*g^2*n*(a^2*l
og(b*x + a)/b^2 - c^2*log(d*x + c)/d^2 + (b*c - a*d)*x/(b*d)) + B*c^2*g^2*n
*(a*log(b*x + a)/b - c*log(d*x + c)/d) + B*c^2*g^2*x*log(e*(b*x/(d*x + c) +
 a/(d*x + c))^n) + A*c^2*g^2*x
```

**mupad [B]** time = 4.31, size = 303, normalized size = 2.44

$$\ln\left(e\left(\frac{a+b\,x}{c+d\,x}\right)^n\right)\left(B\,c^2\,g^2\,x+B\,c\,d\,g^2\,x^2+\frac{B\,d^2\,g^2\,x^3}{3}\right)-x\left(\frac{(3\,a\,d+3\,b\,c)\,\left(\frac{d\,g^2\,(3\,A\,a\,d+9\,A\,b\,c+B\,a\,d\,n-B\,b\,c\,n)}{3\,b}-\frac{A\,d\,g^2\,(3\,a\,d}{3\,b}\right.}{3\,b\,d}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*g + d*g*x)^2*(A + B*log(e*((a + b*x)/(c + d*x))^n)),x)
```

```
[Out] log(e*((a + b*x)/(c + d*x))^n)*((B*d^2*g^2*x^3)/3 + B*c^2*g^2*x + B*c*d*g^2
*x^2) - x*(((3*a*d + 3*b*c)*((d*g^2*(3*A*a*d + 9*A*b*c + B*a*d*n - B*b*c*n)
)/(3*b) - (A*d*g^2*(3*a*d + 3*b*c))/(3*b)))/(3*b*d) - (c*g^2*(3*A*a*d + 3*A
*b*c + B*a*d*n - B*b*c*n))/b + (A*a*c*d*g^2)/b) + x^2*((d*g^2*(3*A*a*d + 9*
A*b*c + B*a*d*n - B*b*c*n))/(6*b) - (A*d*g^2*(3*a*d + 3*b*c))/(6*b)) + (log
(a + b*x)*(B*a^3*d^2*g^2*n + 3*B*a*b^2*c^2*g^2*n - 3*B*a^2*b*c*d*g^2*n))/(3
*b^3) + (A*d^2*g^2*x^3)/3 - (B*c^3*g^2*n*log(c + d*x))/(3*d)
```

**sympy [A]** time = 60.46, size = 779, normalized size = 6.28

$$\begin{cases} c^2g^2x\left(A+B\log\left(e\left(\frac{a}{c}\right)^n\right)\right) \\ Ac^2g^2x+Acdg^2x^2+\frac{Ad^2g^2x^3}{3}-\frac{Bc^3g^2n\log(c+dx)}{3d}+Bc^2g^2nx\log(a)-Bc^2g^2nx\log(c+dx)+\frac{Bc^2g^2nx}{3}+Bc^2g^2x\log \\ c^2g^2\left(Ax+\frac{Ban\log\left(\frac{a}{c}+\frac{bx}{c}\right)}{b}+Bnx\log\left(\frac{a}{c}+\frac{bx}{c}\right)-Bnx+Bx\log(e)\right) \\ Ac^2g^2x+Acdg^2x^2+\frac{Ad^2g^2x^3}{3}+\frac{Ba^3d^2g^2n\log\left(\frac{a}{c+dx}+\frac{bx}{c+dx}\right)}{3b^3}+\frac{Ba^3d^2g^2n\log\left(\frac{c}{d}+x\right)}{3b^3}-\frac{Ba^2cdg^2n\log\left(\frac{a}{c+dx}+\frac{bx}{c+dx}\right)}{b^2}-\frac{Ba^2cdg^2n\log\left(\frac{c}{d}\right.}{b^2} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*g*x+c*g)**2*(A+B*ln(e*((b*x+a)/(d*x+c))**n)),x)
```

```
[Out] Piecewise((c**2*g**2*x*(A + B*log(e*(a/c)**n)), Eq(b, 0) & Eq(d, 0)), (A*c*
*2*g**2*x + A*c*d*g**2*x**2 + A*d**2*g**2*x**3/3 - B*c**3*g**2*n*log(c + d*
x)/(3*d) + B*c**2*g**2*n*x*log(a) - B*c**2*g**2*n*x*log(c + d*x) + B*c**2*g
**2*n*x/3 + B*c**2*g**2*x*log(e) + B*c*d*g**2*n*x**2*log(a) - B*c*d*g**2*n*
x**2*log(c + d*x) + B*c*d*g**2*n*x**2/3 + B*c*d*g**2*x**2*log(e) + B*d**2*g
**2*n*x**3*log(a)/3 - B*d**2*g**2*n*x**3*log(c + d*x)/3 + B*d**2*g**2*n*x**
3/9 + B*d**2*g**2*x**3*log(e)/3, Eq(b, 0)), (c**2*g**2*(A*x + B*a*n*log(a/c
 + b*x/c)/b + B*n*x*log(a/c + b*x/c) - B*n*x + B*x*log(e)), Eq(d, 0)), (A*c
**2*g**2*x + A*c*d*g**2*x**2 + A*d**2*g**2*x**3/3 + B*a**3*d**2*g**2*n*log(
a/(c + d*x) + b*x/(c + d*x))/(3*b**3) + B*a**3*d**2*g**2*n*log(c/d + x)/(3*
b**3) - B*a**2*c*d*g**2*n*log(a/(c + d*x) + b*x/(c + d*x))/b**2 - B*a**2*c*
d*g**2*n*log(c/d + x)/b**2 - B*a**2*d**2*g**2*n*x/(3*b**2) + B*a*c**2*g**2*
n*log(a/(c + d*x) + b*x/(c + d*x))/b + B*a*c**2*g**2*n*log(c/d + x)/b + B*a
*c*d*g**2*n*x/b + B*a*d**2*g**2*n*x**2/(6*b) - B*c**3*g**2*n*log(c/d + x)/(
3*d) + B*c**2*g**2*n*x*log(a/(c + d*x) + b*x/(c + d*x)) - 2*B*c**2*g**2*n*x
/3 + B*c**2*g**2*x*log(e) + B*c*d*g**2*n*x**2*log(a/(c + d*x) + b*x/(c + d*
x)) - B*c*d*g**2*n*x**2/6 + B*c*d*g**2*x**2*log(e) + B*d**2*g**2*n*x**3*log
(a/(c + d*x) + b*x/(c + d*x))/3 + B*d**2*g**2*x**3*log(e)/3, True))
```

## 3.32 $\int (cg+dgx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)dx$

**Optimal**. Leaf size=86

$$\frac{g(c+dx)^2\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{2d}-\frac{Bgn(bc-ad)^2\log(a+bx)}{2b^2d}-\frac{Bgnx(bc-ad)}{2b}$$

```
[Out] -1/2*B*(-a*d+b*c)*g*n*x/b-1/2*B*(-a*d+b*c)^2*g*n*ln(b*x+a)/b^2/d+1/2*g*(d*x
+c)^2*(A+B*ln(e*((b*x+a)/(d*x+c))^n))/d
```

**Rubi [A]** time = 0.06, antiderivative size = 86, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.097, Rules used = {2525, 12, 43}

$$\frac{g(c+dx)^2\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{2d}-\frac{Bgn(bc-ad)^2\log(a+bx)}{2b^2d}-\frac{Bgnx(bc-ad)}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[(c*g + d*g*x)*(A + B*Log[e*((a + b*x)/(c + d*x))^n]),x]
```

```
[Out] -(B*(b*c - a*d)*g*n*x)/(2*b) - (B*(b*c - a*d)^2*g*n*Log[a + b*x])/(2*b^2*d)
 + (g*(c + d*x)^2*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/(2*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2525

```
Int[((a_.) + Log[(c_.)*(RFx_)^(p_.)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_))^(m_.
), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(a + b*Log[c*RFx^p])^n)/(e*(m + 1))
, x] - Dist[(b*n*p)/(e*(m + 1)), Int[SimplifyIntegrand[((d + e*x)^(m + 1)*(
a + b*Log[c*RFx^p])^(n - 1)*D[RFx, x])/RFx, x], x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && RationalFunctionQ[RFx, x] && IGtQ[n, 0] && (EqQ[n, 1] ||
```

```
IntegerQ[m]) && NeQ[m, -1]
```

Rubi steps

$$\int (cg+dgx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)dx=\frac{g(c+dx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{2d}-\frac{(Bn)\int\frac{(bc-ad)g^2(c+dx)}{a+bx}dx}{2dg}$$
$$=\frac{g(c+dx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{2d}-\frac{(B(bc-ad)gn)\int\frac{c+dx}{a+bx}dx}{2d}$$
$$=\frac{g(c+dx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{2d}-\frac{(B(bc-ad)gn)\int\left(\frac{d}{b}+\frac{bc-a}{b(a+b}\right.}{2d}$$
$$=-\frac{B(bc-ad)gnx}{2b}-\frac{B(bc-ad)^2gn\log(a+bx)}{2b^2d}+\frac{g(c+dx)^2\left(A+\right.}{2}$$

**Mathematica [A]** time = 0.04, size = 74, normalized size = 0.86

$$\frac{g\left((c+dx)^2\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)-\frac{Bn(bc-ad)((bc-ad)\log(a+bx)+bdx)}{b^2}\right)}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*g + d*g*x)*(A + B*Log[e*((a + b*x)/(c + d*x))^n]),x]
```

```
[Out] (g*(-((B*(b*c - a*d)*n*(b*d*x + (b*c - a*d)*Log[a + b*x]))/b^2) + (c + d*x)
^2*(A + B*Log[e*((a + b*x)/(c + d*x))^n])))/(2*d)
```

**fricas [B]** time = 0.70, size = 162, normalized size = 1.88

$$\frac{Ab^2d^2gx^2-Bb^2c^2gn\log(dx+c)+\left(2\,Babcd-Ba^2d^2\right)gn\log(bx+a)+\left(2\,Ab^2cdg-\left(Bb^2cd-Babd^2\right)gn\right)x+\left(B\right.}{2\,b^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*g*x+c*g)*(A+B*log(e*((b*x+a)/(d*x+c))^n)),x, algorithm="fricas
")
```

```
[Out] 1/2*(A*b^2*d^2*g*x^2 - B*b^2*c^2*g*n*log(d*x + c) + (2*B*a*b*c*d - B*a^2*d^
2)*g*n*log(b*x + a) + (2*A*b^2*c*d*g - (B*b^2*c*d - B*a*b*d^2)*g*n)*x + (B*
```

```
b^2*d^2*g*x^2 + 2*B*b^2*c*d*g*x)*log(e) + (B*b^2*d^2*g*n*x^2 + 2*B*b^2*c*d*
g*n*x)*log((b*x + a)/(d*x + c)))/(b^2*d)
```

**giac [B]** time = 1.27, size = 572, normalized size = 6.65

$$\frac{1}{2}\left(\frac{\left(Bb^3c^3gn - 3\,Bab^2c^2dgn + 3\,Ba^2bcd^2gn - Ba^3d^3gn\right)\log\left(\frac{bx+a}{dx+c}\right)}{b^2d - \frac{2\,(bx+a)bd^2}{dx+c} + \frac{(bx+a)^2d^3}{(dx+c)^2}} - \frac{Bb^4c^3gn - 3\,Bab^3c^2dgn - \frac{(bx+a)Bb^3c^3dgn}{dx+c} + 3\,B\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*g*x+c*g)*(A+B*log(e*((b*x+a)/(d*x+c))^n)),x, algorithm="giac")
```

```
[Out] 1/2*((B*b^3*c^3*g*n - 3*B*a*b^2*c^2*d*g*n + 3*B*a^2*b*c*d^2*g*n - B*a^3*d^3
*g*n)*log((b*x + a)/(d*x + c))/(b^2*d - 2*(b*x + a)*b*d^2/(d*x + c) + (b*x
+ a)^2*d^3/(d*x + c)^2) - (B*b^4*c^3*g*n - 3*B*a*b^3*c^2*d*g*n - (b*x + a)*
B*b^3*c^3*d*g*n/(d*x + c) + 3*B*a^2*b^2*c*d^2*g*n + 3*(b*x + a)*B*a*b^2*c^2
*d^2*g*n/(d*x + c) - B*a^3*b*d^3*g*n - 3*(b*x + a)*B*a^2*b*c*d^3*g*n/(d*x +
 c) + (b*x + a)*B*a^3*d^4*g*n/(d*x + c) - A*b^4*c^3*g - B*b^4*c^3*g + 3*A*a
*b^3*c^2*d*g + 3*B*a*b^3*c^2*d*g - 3*A*a^2*b^2*c*d^2*g - 3*B*a^2*b^2*c*d^2*
g + A*a^3*b*d^3*g + B*a^3*b*d^3*g)/(b^3*d - 2*(b*x + a)*b^2*d^2/(d*x + c) +
 (b*x + a)^2*b*d^3/(d*x + c)^2) + (B*b^3*c^3*g*n - 3*B*a*b^2*c^2*d*g*n + 3*
B*a^2*b*c*d^2*g*n - B*a^3*d^3*g*n)*log(-b + (b*x + a)*d/(d*x + c))/(b^2*d)
- (B*b^3*c^3*g*n - 3*B*a*b^2*c^2*d*g*n + 3*B*a^2*b*c*d^2*g*n - B*a^3*d^3*g*
n)*log((b*x + a)/(d*x + c))/(b^2*d))*(b*c/(b*c - a*d)^2 - a*d/(b*c - a*d)^2
)
```

**maple [F]** time = 0.18, size = 0, normalized size = 0.00

$$\int \left(dgx + cg\right)\left(B\ln\left(e\left(\frac{bx+a}{dx+c}\right)^n\right) + A\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*g*x+c*g)*(B*ln(e*((b*x+a)/(d*x+c))^n)+A),x)
```

```
[Out] int((d*g*x+c*g)*(B*ln(e*((b*x+a)/(d*x+c))^n)+A),x)
```

**maxima [A]** time = 1.14, size = 156, normalized size = 1.81

$$\frac{1}{2}\,Bdgx^2\log\left(e\left(\frac{bx}{dx+c} + \frac{a}{dx+c}\right)^n\right) + \frac{1}{2}\,Adgx^2 - \frac{1}{2}\,Bdgn\left(\frac{a^2\log(bx+a)}{b^2} - \frac{c^2\log(dx+c)}{d^2} + \frac{(bc-ad)x}{bd}\right) + Bcgn\left(\frac{a}{\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*g*x+c*g)*(A+B*log(e*((b*x+a)/(d*x+c))^n)),x, algorithm="maxima
")
```

```
[Out] 1/2*B*d*g*x^2*log(e*(b*x/(d*x + c) + a/(d*x + c))^n) + 1/2*A*d*g*x^2 - 1/2*
B*d*g*n*(a^2*log(b*x + a)/b^2 - c^2*log(d*x + c)/d^2 + (b*c - a*d)*x/(b*d))
 + B*c*g*n*(a*log(b*x + a)/b - c*log(d*x + c)/d) + B*c*g*x*log(e*(b*x/(d*x
+ c) + a/(d*x + c))^n) + A*c*g*x
```

**mupad [B]** time = 4.10, size = 134, normalized size = 1.56

$$x\left(\frac{g\,(2\,A\,a\,d+4\,A\,b\,c+B\,a\,d\,n-B\,b\,c\,n)}{2\,b}-\frac{A\,g\,(2\,a\,d+2\,b\,c)}{2\,b}\right)+\ln\left(e\left(\frac{a+b\,x}{c+d\,x}\right)^n\right)\left(\frac{B\,d\,g\,x^2}{2}+B\,c\,g\,x\right)-\frac{\ln(a+}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*g + d*g*x)*(A + B*log(e*((a + b*x)/(c + d*x))^n)),x)
```

```
[Out] x*((g*(2*A*a*d + 4*A*b*c + B*a*d*n - B*b*c*n))/(2*b) - (A*g*(2*a*d + 2*b*c)
)/(2*b)) + log(e*((a + b*x)/(c + d*x))^n)*((B*d*g*x^2)/2 + B*c*g*x) - (log(
a + b*x)*(B*a^2*d*g*n - 2*B*a*b*c*g*n))/(2*b^2) + (A*d*g*x^2)/2 - (B*c^2*g*
n*log(c + d*x))/(2*d)
```

**sympy [A]** time = 40.68, size = 444, normalized size = 5.16

$$\begin{cases} cgx\left(A+B\log\left(e\left(\frac{a}{c}\right)^n\right)\right) \\ Acgx+\frac{Adgx^2}{2}-\frac{Bc^2gn\log(c+dx)}{2d}+Bcgnx\log(a)-Bcgnx\log(c+dx)+\frac{Bcgnx}{2}+Bcgx\log(e)+\frac{Bdgnx^2\log(a)}{2}-\frac{Bdg}{} \\ cg\left(Ax+\frac{Ban\log\left(\frac{a}{c}+\frac{bx}{c}\right)}{b}+Bnx\log\left(\frac{a}{c}+\frac{bx}{c}\right)-Bnx+Bx\log(e)\right) \\ Acgx+\frac{Adgx^2}{2}-\frac{Ba^2dgn\log\left(\frac{a}{c+dx}+\frac{bx}{c+dx}\right)}{2b^2}-\frac{Ba^2dgn\log\left(\frac{c}{d}+x\right)}{2b^2}+\frac{Bacgn\log\left(\frac{a}{c+dx}+\frac{bx}{c+dx}\right)}{b}+\frac{Bacgn\log\left(\frac{c}{d}+x\right)}{b}+\frac{Badgnx}{2b}-\frac{Bc^2gn\log\left(\right.}{2d} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*g*x+c*g)*(A+B*ln(e*((b*x+a)/(d*x+c))**n)),x)
```

```
[Out] Piecewise((c*g*x*(A + B*log(e*(a/c)**n)), Eq(b, 0) & Eq(d, 0)), (A*c*g*x +
A*d*g*x**2/2 - B*c**2*g*n*log(c + d*x)/(2*d) + B*c*g*n*x*log(a) - B*c*g*n*x
*log(c + d*x) + B*c*g*n*x/2 + B*c*g*x*log(e) + B*d*g*n*x**2*log(a)/2 - B*d*
g*n*x**2*log(c + d*x)/2 + B*d*g*n*x**2/4 + B*d*g*x**2*log(e)/2, Eq(b, 0)),
(c*g*(A*x + B*a*n*log(a/c + b*x/c)/b + B*n*x*log(a/c + b*x/c) - B*n*x + B*x
*log(e)), Eq(d, 0)), (A*c*g*x + A*d*g*x**2/2 - B*a**2*d*g*n*log(a/(c + d*x)
 + b*x/(c + d*x))/(2*b**2) - B*a**2*d*g*n*log(c/d + x)/(2*b**2) + B*a*c*g*n
```

```
*log(a/(c + d*x) + b*x/(c + d*x))/b + B*a*c*g*n*log(c/d + x)/b + B*a*d*g*n*
x/(2*b) - B*c**2*g*n*log(c/d + x)/(2*d) + B*c*g*n*x*log(a/(c + d*x) + b*x/(
c + d*x)) - B*c*g*n*x/2 + B*c*g*x*log(e) + B*d*g*n*x**2*log(a/(c + d*x) + b
*x/(c + d*x))/2 + B*d*g*x**2*log(e)/2, True))
```

# 3.33 $\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{cg+dgx}\,dx$

**Optimal**. Leaf size=80

$$-\frac{\log\left(\frac{bc-ad}{b(c+dx)}\right)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{dg}-\frac{Bn\,\mathrm{Li}_2\left(\frac{d(a+bx)}{b(c+dx)}\right)}{dg}$$

```
[Out] -(A+B*ln(e*((b*x+a)/(d*x+c))^n))*ln((-a*d+b*c)/b/(d*x+c))/d/g-B*n*polylog(2
,d*(b*x+a)/b/(d*x+c))/d/g
```

**Rubi [A]** time = 0.20, antiderivative size = 128, normalized size of antiderivative = 1.60, number of steps used = 9, number of rules used = 8, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.242, Rules used = {2524, 2418, 2394, 2393, 2391, 2390, 12, 2301}

$$-\frac{Bn\,\mathrm{PolyLog}\left(2,\frac{b(c+dx)}{bc-ad}\right)}{dg}+\frac{\log(cg+dgx)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{dg}-\frac{Bn\log(cg+dgx)\log\left(-\frac{d(a+bx)}{bc-ad}\right)}{dg}+\frac{Bn\log^2(g(\ldots}{2d\ldots}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Log[e*((a + b*x)/(c + d*x))^n])/(c*g + d*g*x),x]
```

```
[Out] (B*n*Log[g*(c + d*x)]^2)/(2*d*g) - (B*n*Log[-((d*(a + b*x))/(b*c - a*d))]*L
og[c*g + d*g*x])/(d*g) + ((A + B*Log[e*((a + b*x)/(c + d*x))^n])*Log[c*g +
d*g*x])/(d*g) - (B*n*PolyLog[2, (b*(c + d*x))/(b*c - a*d)])/(d*g)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2301

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

Rule 2390

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_))^(q_.), x_Symbol] :> Dist[1/e, Subst[Int[((f*x)/d)^q*(a + b*Log[c*x^
n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && E
qQ[e*f - d*g, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2393

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + (c*e*x)/g])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2394

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d + e*x
)^n]))/g, x] - Dist[(b*e*n)/g, Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2418

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*(RFx_), x_Sy
mbol] :> With[{u = ExpandIntegrand[(a + b*Log[c*(d + e*x)^n])^p, RFx, x]},
Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d, e, n}, x] && RationalFunctionQ[
RFx, x] && IntegerQ[p]
```

Rule 2524

```
Int[((a_.) + Log[(c_.)*(RFx_)^(p_.)]*(b_.))^(n_.)/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[(Log[d + e*x]*(a + b*Log[c*RFx^p])^n)/e, x] - Dist[(b*n*p)/e
, Int[(Log[d + e*x]*(a + b*Log[c*RFx^p])^(n - 1)*D[RFx, x])/RFx, x], x] /;
FreeQ[{a, b, c, d, e, p}, x] && RationalFunctionQ[RFx, x] && IGtQ[n, 0]
```

Rubi steps

$$\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{cg+dgx}\,dx = \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\log(cg+dgx)}{dg} - \frac{(Bn)\int\frac{(c+dx)\left(-\frac{d(a+bx)}{(c+dx)^2}+\frac{b}{c+dx}\right)\log(cg+dgx)}{a+bx}\,dx}{dg}$$

$$= \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\log(cg+dgx)}{dg} - \frac{(Bn)\int\left(\frac{b\log(cg+dgx)}{a+bx}-\frac{d\log(cg+dgx)}{c+dx}\right)dx}{dg}$$

$$= \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\log(cg+dgx)}{dg} + \frac{(Bn)\int\frac{\log(cg+dgx)}{c+dx}\,dx}{g} - \frac{(bBn)\int\frac{\log(cg+d}{a+bx}}{dg}$$

$$= -\frac{Bn\log\left(-\frac{d(a+bx)}{bc-ad}\right)\log(cg+dgx)}{dg} + \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\log(cg+dgx)}{dg} + (B$$

$$= -\frac{Bn\log\left(-\frac{d(a+bx)}{bc-ad}\right)\log(cg+dgx)}{dg} + \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\log(cg+dgx)}{dg} + \frac{(B}{}$$

$$= \frac{Bn\log^2(g(c+dx))}{2dg} - \frac{Bn\log\left(-\frac{d(a+bx)}{bc-ad}\right)\log(cg+dgx)}{dg} + \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{dg}$$

**Mathematica [A]** time = 0.04, size = 101, normalized size = 1.26

$$\frac{\log(g(c+dx))\left(2B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)-2Bn\log\left(\frac{d(a+bx)}{ad-bc}\right)+2A+Bn\log(g(c+dx))\right)-2Bn\,\mathrm{Li}_2\left(\frac{b(c+dx)}{bc-ad}\right)}{2dg}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Log[e*((a + b*x)/(c + d*x))^n])/(c*g + d*g*x),x]
```

```
[Out] (Log[g*(c + d*x)]*(2*A - 2*B*n*Log[(d*(a + b*x))/(-(b*c) + a*d)] + 2*B*Log[
e*((a + b*x)/(c + d*x))^n] + B*n*Log[g*(c + d*x)]) - 2*B*n*PolyLog[2, (b*(c
 + d*x))/(b*c - a*d)])/(2*d*g)
```

**fricas [F]** time = 0.99, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A}{dgx+cg},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))/(d*g*x+c*g),x, algorithm="fricas
")
```

```
[Out] integral((B*log(e*((b*x + a)/(d*x + c))^n) + A)/(d*g*x + c*g), x)
```

**giac [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))/(d*g*x+c*g),x, algorithm="giac")
```

```
[Out] Timed out
```

**maple [F]** time = 0.37, size = 0, normalized size = 0.00

$$\int \frac{B \ln\left(e \left(\frac{bx+a}{dx+c}\right)^{n}\right) + A}{dgx + cg}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*ln(e*((b*x+a)/(d*x+c))^n)+A)/(d*g*x+c*g),x)
```

```
[Out] int((B*ln(e*((b*x+a)/(d*x+c))^n)+A)/(d*g*x+c*g),x)
```

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$-\frac{1}{2}\, B\left(\frac{2\, n \log\left(bx + a\right) \log\left(dx + c\right) - n \log\left(dx + c\right)^{2} - 2\, \log\left(dx + c\right) \log\left(\left(bx + a\right)^{n}\right) + 2\, \log\left(dx + c\right) \log\left(\left(dx + c\right.\right.}{dg}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))/(d*g*x+c*g),x, algorithm="maxima
")
```

```
[Out] -1/2*B*((2*n*log(b*x + a)*log(d*x + c) - n*log(d*x + c)^2 - 2*log(d*x + c)*
log((b*x + a)^n) + 2*log(d*x + c)*log((d*x + c)^n))/(d*g) - 2*integrate((n*
log(b*x + a) + log(e))/(d*g*x + c*g), x)) + A*log(d*g*x + c*g)/(d*g)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{A + B\, \ln\left(e \left(\frac{a+b\,x}{c+d\,x}\right)^{n}\right)}{c\,g + d\,g\,x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*log(e*((a + b*x)/(c + d*x))^n))/(c*g + d*g*x),x)
```

```
[Out] int((A + B*log(e*((a + b*x)/(c + d*x))^n))/(c*g + d*g*x), x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{A}{c+dx}\,dx + \int \frac{B\log\left(e\left(\frac{a}{c+dx}+\frac{bx}{c+dx}\right)^{n}\right)}{c+dx}\,dx}{g}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*ln(e*((b*x+a)/(d*x+c))**n))/(d*g*x+c*g),x)
```

```
[Out] (Integral(A/(c + d*x), x) + Integral(B*log(e*(a/(c + d*x) + b*x/(c + d*x))*
*n)/(c + d*x), x))/g
```

## 3.34 $\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(cg+dgx)^2}\,dx$

**Optimal**. Leaf size=102

$$\frac{A(a+bx)}{g^2(c+dx)(bc-ad)}+\frac{B(a+bx)\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{g^2(c+dx)(bc-ad)}-\frac{Bn(a+bx)}{g^2(c+dx)(bc-ad)}$$

```
[Out] A*(b*x+a)/(-a*d+b*c)/g^2/(d*x+c)-B*n*(b*x+a)/(-a*d+b*c)/g^2/(d*x+c)+B*(b*x+
a)*ln(e*((b*x+a)/(d*x+c))^n)/(-a*d+b*c)/g^2/(d*x+c)
```

**Rubi [A]** time = 0.09, antiderivative size = 107, normalized size of antiderivative = 1.05, number of steps used = 4, number of rules used = 3, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {2525, 12, 44}

$$-\frac{B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A}{dg^2(c+dx)}+\frac{bBn\log(a+bx)}{dg^2(bc-ad)}-\frac{bBn\log(c+dx)}{dg^2(bc-ad)}+\frac{Bn}{dg^2(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Log[e*((a + b*x)/(c + d*x))^n])/(c*g + d*g*x)^2,x]
```

```
[Out] (B*n)/(d*g^2*(c + d*x)) + (b*B*n*Log[a + b*x])/(d*(b*c - a*d)*g^2) - (A + B
*Log[e*((a + b*x)/(c + d*x))^n])/(d*g^2*(c + d*x)) - (b*B*n*Log[c + d*x])/(
d*(b*c - a*d)*g^2)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
 + n + 2, 0])
```

Rule 2525

```
Int[((a_.) + Log[(c_.)*(RFx_)^(p_.)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_))^(m_.
), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(a + b*Log[c*RFx^p])^n)/(e*(m + 1))
```

```
, x] - Dist[(b*n*p)/(e*(m + 1)), Int[SimplifyIntegrand[((d + e*x)^(m + 1)*(
a + b*Log[c*RFx^p])^(n - 1)*D[RFx, x])/RFx, x], x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && RationalFunctionQ[RFx, x] && IGtQ[n, 0] && (EqQ[n, 1] ||
IntegerQ[m]) && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(cg+dgx)^2}\,dx &= -\frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{dg^2(c+dx)}+\frac{(Bn)\int\frac{bc-ad}{g(a+bx)(c+dx)^2}\,dx}{dg}\\ &= -\frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{dg^2(c+dx)}+\frac{(B(bc-ad)n)\int\frac{1}{(a+bx)(c+dx)^2}\,dx}{dg^2}\\ &= -\frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{dg^2(c+dx)}+\frac{(B(bc-ad)n)\int\left(\frac{b^2}{(bc-ad)^2(a+bx)}-\frac{d}{(bc-ad)(c+dx)^2}-\frac{bd}{(bc-ad)^2(}\right.}{dg^2}\\ &= \frac{Bn}{dg^2(c+dx)}+\frac{bBn\log(a+bx)}{d(bc-ad)g^2}-\frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{dg^2(c+dx)}-\frac{bBn\log(c+dx)}{d(bc-ad)g^2}\end{aligned}$$

**Mathematica [A]** time = 0.06, size = 114, normalized size = 1.12

$$\frac{Bn(bc-ad)\left(\frac{1}{(c+dx)(bc-ad)}+\frac{b\log(a+bx)}{(bc-ad)^2}-\frac{b\log(c+dx)}{(bc-ad)^2}\right)}{dg^2}-\frac{B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A}{dg(cg+dgx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Log[e*((a + b*x)/(c + d*x))^n])/(c*g + d*g*x)^2,x]
```

```
[Out] -((A + B*Log[e*((a + b*x)/(c + d*x))^n])/(d*g*(c*g + d*g*x))) + (B*(b*c - a
*d)*n*(1/((b*c - a*d)*(c + d*x)) + (b*Log[a + b*x])/(b*c - a*d)^2 - (b*Log[
c + d*x])/(b*c - a*d)^2))/(d*g^2)
```

**fricas [A]** time = 0.84, size = 105, normalized size = 1.03

$$-\frac{Abc-Aad-(Bbc-Bad)n+(Bbc-Bad)\log(e)-(Bbdnx+Badn)\log\left(\frac{bx+a}{dx+c}\right)}{\left(bcd^2-ad^3\right)g^2x+\left(bc^2d-acd^2\right)g^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))/(d*g*x+c*g)^2,x, algorithm="fric
as")
```

```
[Out] -(A*b*c - A*a*d - (B*b*c - B*a*d)*n + (B*b*c - B*a*d)*log(e) - (B*b*d*n*x +
 B*a*d*n)*log((b*x + a)/(d*x + c)))/((b*c*d^2 - a*d^3)*g^2*x + (b*c^2*d - a
*c*d^2)*g^2)
```

**giac [A]** time = 3.77, size = 89, normalized size = 0.87

$$\left(\frac{(bx+a)Bn\log\left(\frac{bx+a}{dx+c}\right)}{(dx+c)g^2}-\frac{(Bn-A-B)(bx+a)}{(dx+c)g^2}\right)\left(\frac{bc}{(bc-ad)^2}-\frac{ad}{(bc-ad)^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))/(d*g*x+c*g)^2,x, algorithm="giac
")
```

```
[Out] ((b*x + a)*B*n*log((b*x + a)/(d*x + c))/((d*x + c)*g^2) - (B*n - A - B)*(b*
x + a)/((d*x + c)*g^2))*(b*c/(b*c - a*d)^2 - a*d/(b*c - a*d)^2)
```

**maple [F]** time = 0.29, size = 0, normalized size = 0.00

$$\int \frac{B\ln\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A}{\left(dgx+cg\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*ln(e*((b*x+a)/(d*x+c))^n)+A)/(d*g*x+c*g)^2,x)
```

```
[Out] int((B*ln(e*((b*x+a)/(d*x+c))^n)+A)/(d*g*x+c*g)^2,x)
```

**maxima [A]** time = 1.13, size = 136, normalized size = 1.33

$$Bn\left(\frac{1}{d^2g^2x+cdg^2}+\frac{b\log(bx+a)}{\left(bcd-ad^2\right)g^2}-\frac{b\log(dx+c)}{\left(bcd-ad^2\right)g^2}\right)-\frac{B\log\left(e\left(\frac{bx}{dx+c}+\frac{a}{dx+c}\right)^n\right)}{d^2g^2x+cdg^2}-\frac{A}{d^2g^2x+cdg^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))/(d*g*x+c*g)^2,x, algorithm="maxi
ma")
```

```
[Out] B*n*(1/(d^2*g^2*x + c*d*g^2) + b*log(b*x + a)/((b*c*d - a*d^2)*g^2) - b*log
(d*x + c)/((b*c*d - a*d^2)*g^2)) - B*log(e*(b*x/(d*x + c) + a/(d*x + c))^n)
/(d^2*g^2*x + c*d*g^2) - A/(d^2*g^2*x + c*d*g^2)
```

**mupad [B]** time = 4.02, size = 113, normalized size = 1.11

$$-\frac{A-B\,n}{x\,d^2\,g^2+c\,d\,g^2}-\frac{B\,\ln\left(e\,{\left(\frac{a+b\,x}{c+d\,x}\right)}^n\right)}{d\,\left(c\,g^2+d\,g^2\,x\right)}+\frac{B\,b\,n\,\mathrm{atan}\left(\frac{b\,c\,2{}\mathrm{i}+b\,d\,x\,2{}\mathrm{i}}{a\,d-b\,c}+1{}\mathrm{i}\right)\,2{}\mathrm{i}}{d\,g^2\,\left(a\,d-b\,c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*log(e*((a + b*x)/(c + d*x))^n))/(c*g + d*g*x)^2,x)
```

```
[Out] (B*b*n*atan((b*c*2i + b*d*x*2i)/(a*d - b*c) + 1i)*2i)/(d*g^2*(a*d - b*c)) -
 (B*log(e*((a + b*x)/(c + d*x))^n))/(d*(c*g^2 + d*g^2*x)) - (A - B*n)/(d^2*
g^2*x + c*d*g^2)
```

**sympy [F(-2)]** time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*ln(e*((b*x+a)/(d*x+c))**n))/(d*g*x+c*g)**2,x)
```

```
[Out] Exception raised: NotImplementedError
```

## 3.35 $\int \frac{A+B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(cg+dgx)^3}\,dx$

**Optimal**. Leaf size=151

$$-\frac{B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A}{2dg^3(c+dx)^2}+\frac{b^2Bn\log(a+bx)}{2dg^3(bc-ad)^2}-\frac{b^2Bn\log(c+dx)}{2dg^3(bc-ad)^2}+\frac{bBn}{2dg^3(c+dx)(bc-ad)}+\frac{Bn}{4dg^3(c+dx)^2}$$

```
[Out] 1/4*B*n/d/g^3/(d*x+c)^2+1/2*b*B*n/d/(-a*d+b*c)/g^3/(d*x+c)+1/2*b^2*B*n*ln(b
*x+a)/d/(-a*d+b*c)^2/g^3+1/2*(-A-B*ln(e*((b*x+a)/(d*x+c))^n))/d/g^3/(d*x+c)
^2-1/2*b^2*B*n*ln(d*x+c)/d/(-a*d+b*c)^2/g^3
```

**Rubi [A]** time = 0.11, antiderivative size = 151, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {2525, 12, 44}

$$-\frac{B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A}{2dg^3(c+dx)^2}+\frac{b^2Bn\log(a+bx)}{2dg^3(bc-ad)^2}-\frac{b^2Bn\log(c+dx)}{2dg^3(bc-ad)^2}+\frac{bBn}{2dg^3(c+dx)(bc-ad)}+\frac{Bn}{4dg^3(c+dx)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Log[e*((a + b*x)/(c + d*x))^n])/(c*g + d*g*x)^3,x]
```

```
[Out] (B*n)/(4*d*g^3*(c + d*x)^2) + (b*B*n)/(2*d*(b*c - a*d)*g^3*(c + d*x)) + (b^
2*B*n*Log[a + b*x])/(2*d*(b*c - a*d)^2*g^3) - (A + B*Log[e*((a + b*x)/(c +
d*x))^n])/(2*d*g^3*(c + d*x)^2) - (b^2*B*n*Log[c + d*x])/(2*d*(b*c - a*d)^2
*g^3)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 2525

```
Int[((a_.) + Log[(c_.)*(RFx_)^(p_.)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_))^(m_.
), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(a + b*Log[c*RFx^p])^n)/(e*(m + 1))
, x] - Dist[(b*n*p)/(e*(m + 1)), Int[SimplifyIntegrand[((d + e*x)^(m + 1)*(
a + b*Log[c*RFx^p])^(n - 1)*D[RFx, x])/RFx, x], x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && RationalFunctionQ[RFx, x] && IGtQ[n, 0] && (EqQ[n, 1] ||
IntegerQ[m]) && NeQ[m, -1]
```

Rubi steps

$$\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(cg+dgx)^3}\,dx = -\frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{2dg^3(c+dx)^2} + \frac{(Bn)\int \frac{bc-ad}{g^2(a+bx)(c+dx)^3}\,dx}{2dg}$$

$$= -\frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{2dg^3(c+dx)^2} + \frac{(B(bc-ad)n)\int \frac{1}{(a+bx)(c+dx)^3}\,dx}{2dg^3}$$

$$= -\frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{2dg^3(c+dx)^2} + \frac{(B(bc-ad)n)\int \left(\frac{b^3}{(bc-ad)^3(a+bx)} - \frac{d}{(bc-ad)(c+dx)^3} - \frac{bd}{(bc-ad)^2(}\right.}{2dg^3}$$

$$= \frac{Bn}{4dg^3(c+dx)^2} + \frac{bBn}{2d(bc-ad)g^3(c+dx)} + \frac{b^2Bn\log(a+bx)}{2d(bc-ad)^2g^3} - \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right.\right.}{2dg^3(c+dx)^2}$$

**Mathematica [A]** time = 0.15, size = 115, normalized size = 0.76

$$\frac{\frac{Bn\left(2b^2(c+dx)^2\log(a+bx)+(bc-ad)(-ad+3bc+2bdx)-2b^2(c+dx)^2\log(c+dx)\right)}{(bc-ad)^2} - 2\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{4dg^3(c+dx)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Log[e*((a + b*x)/(c + d*x))^n])/(c*g + d*g*x)^3,x]
```

```
[Out] (-2*(A + B*Log[e*((a + b*x)/(c + d*x))^n]) + (B*n*((b*c - a*d)*(3*b*c - a*d
 + 2*b*d*x) + 2*b^2*(c + d*x)^2*Log[a + b*x] - 2*b^2*(c + d*x)^2*Log[c + d*
x]))/(b*c - a*d)^2)/(4*d*g^3*(c + d*x)^2)
```

**fricas [A]** time = 0.97, size = 266, normalized size = 1.76

$$-\frac{2\,Ab^2c^2 - 4\,Aabcd + 2\,Aa^2d^2 - 2\left(Bb^2cd - Babd^2\right)nx - \left(3\,Bb^2c^2 - 4\,Babcd + Ba^2d^2\right)n + 2\left(Bb^2c^2 - 2\,Babcd +\right.}{4\left(\left(b^2c^2d^3 - 2\,abcd^4 + a^2d^5\right)g^3x^2 + 2\left(b^2c^3d^2 - 2\,abc^2d^3 + a^2cd^4\right)g^3x\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))/(d*g*x+c*g)^3,x, algorithm="fric
as")
```

```
[Out] -1/4*(2*A*b^2*c^2 - 4*A*a*b*c*d + 2*A*a^2*d^2 - 2*(B*b^2*c*d - B*a*b*d^2)*n
*x - (3*B*b^2*c^2 - 4*B*a*b*c*d + B*a^2*d^2)*n + 2*(B*b^2*c^2 - 2*B*a*b*c*d
 + B*a^2*d^2)*log(e) - 2*(B*b^2*d^2*n*x^2 + 2*B*b^2*c*d*n*x + (2*B*a*b*c*d
- B*a^2*d^2)*n)*log((b*x + a)/(d*x + c)))/((b^2*c^2*d^3 - 2*a*b*c*d^4 + a^2
*d^5)*g^3*x^2 + 2*(b^2*c^3*d^2 - 2*a*b*c^2*d^3 + a^2*c*d^4)*g^3*x + (b^2*c^
4*d - 2*a*b*c^3*d^2 + a^2*c^2*d^3)*g^3)
```

**giac [A]** time = 6.45, size = 203, normalized size = 1.34

$$\frac{1}{4}\left(2\left(\frac{2\,(bx+a)Bbn}{\left(bcg^3-adg^3\right)(dx+c)}-\frac{(bx+a)^2Bdn}{\left(bcg^3-adg^3\right)(dx+c)^2}\right)\log\left(\frac{bx+a}{dx+c}\right)+\frac{(Bdn-2\,Ad-2\,Bd)(bx+a)^2}{\left(bcg^3-adg^3\right)(dx+c)^2}-\frac{4\,(Bbn-}{\left(bcg^3-\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))/(d*g*x+c*g)^3,x, algorithm="giac
")
```

```
[Out] 1/4*(2*(2*(b*x + a)*B*b*n/((b*c*g^3 - a*d*g^3)*(d*x + c)) - (b*x + a)^2*B*d
*n/((b*c*g^3 - a*d*g^3)*(d*x + c)^2))*log((b*x + a)/(d*x + c)) + (B*d*n - 2
*A*d - 2*B*d)*(b*x + a)^2/((b*c*g^3 - a*d*g^3)*(d*x + c)^2) - 4*(B*b*n - A*
b - B*b)*(b*x + a)/((b*c*g^3 - a*d*g^3)*(d*x + c)))*(b*c/(b*c - a*d)^2 - a*
d/(b*c - a*d)^2)
```

**maple [F]** time = 0.28, size = 0, normalized size = 0.00

$$\int \frac{B\ln\left(e\left(\frac{bx+a}{dx+c}\right)^{n}\right)+A}{\left(dgx+cg\right)^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*ln(e*((b*x+a)/(d*x+c))^n)+A)/(d*g*x+c*g)^3,x)
```

```
[Out] int((B*ln(e*((b*x+a)/(d*x+c))^n)+A)/(d*g*x+c*g)^3,x)
```

**maxima [A]** time = 1.36, size = 259, normalized size = 1.72

$$\frac{1}{4}\,Bn\left(\frac{2\,bdx+3\,bc-ad}{\left(bcd^3-ad^4\right)g^3x^2+2\left(bc^2d^2-acd^3\right)g^3x+\left(bc^3d-ac^2d^2\right)g^3}+\frac{2\,b^2\log\left(bx+a\right)}{\left(b^2c^2d-2\,abcd^2+a^2d^3\right)g^3}-\frac{2\,b^2\log\left(\right.}{\left(b^2c^2d-2\,abc\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))/(d*g*x+c*g)^3,x, algorithm="maxi
ma")
```

```
[Out] 1/4*B*n*((2*b*d*x + 3*b*c - a*d)/((b*c*d^3 - a*d^4)*g^3*x^2 + 2*(b*c^2*d^2
- a*c*d^3)*g^3*x + (b*c^3*d - a*c^2*d^2)*g^3) + 2*b^2*log(b*x + a)/((b^2*c^
2*d - 2*a*b*c*d^2 + a^2*d^3)*g^3) - 2*b^2*log(d*x + c)/((b^2*c^2*d - 2*a*b*
c*d^2 + a^2*d^3)*g^3)) - 1/2*B*log(e*(b*x/(d*x + c) + a/(d*x + c))^n)/(d^3*
g^3*x^2 + 2*c*d^2*g^3*x + c^2*d*g^3) - 1/2*A/(d^3*g^3*x^2 + 2*c*d^2*g^3*x +
 c^2*d*g^3)
```

**mupad [B]** time = 4.55, size = 221, normalized size = 1.46

$$\frac{B\,b^2\,n\,\mathrm{atanh}\left(\frac{2\,a^2\,d^3\,g^3-2\,b^2\,c^2\,d\,g^3}{2\,d\,g^3\,{\left(a\,d-b\,c\right)}^2}+\frac{2\,b\,d\,x}{a\,d-b\,c}\right)}{d\,g^3\,{\left(a\,d-b\,c\right)}^2}-\frac{B\,\ln\left(e\,{\left(\frac{a+b\,x}{c+d\,x}\right)}^n\right)}{2\,d\,\left(c^2\,g^3+2\,c\,d\,g^3\,x+d^2\,g^3\,x^2\right)}-\frac{\frac{2\,A\,a\,d-2\,A\,b\,c-B\,a\,d\,n+3\,B\,b\,c\,n}{2\,\left(a\,d-b\,c\right)}+\frac{B\,b\,d\,n\,x}{a\,d-b\,c}}{2\,c^2\,d\,g^3+4\,c\,d^2\,g^3\,x+2\,d^3\,g^3\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*log(e*((a + b*x)/(c + d*x))^n))/(c*g + d*g*x)^3,x)
```

```
[Out] (B*b^2*n*atanh((2*a^2*d^3*g^3 - 2*b^2*c^2*d*g^3)/(2*d*g^3*(a*d - b*c)^2) +
(2*b*d*x)/(a*d - b*c)))/(d*g^3*(a*d - b*c)^2) - (B*log(e*((a + b*x)/(c + d*
x))^n))/(2*d*(c^2*g^3 + d^2*g^3*x^2 + 2*c*d*g^3*x)) - ((2*A*a*d - 2*A*b*c -
 B*a*d*n + 3*B*b*c*n)/(2*(a*d - b*c)) + (B*b*d*n*x)/(a*d - b*c))/(2*c^2*d*g
^3 + 2*d^3*g^3*x^2 + 4*c*d^2*g^3*x)
```

**sympy [F(-2)]** time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*ln(e*((b*x+a)/(d*x+c))**n))/(d*g*x+c*g)**3,x)
```

```
[Out] Exception raised: NotImplementedError
```

# 3.36 $\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(cg+dgx)^4}\,dx$

**Optimal**. Leaf size=183

$$-\frac{B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A}{3dg^4(c+dx)^3}+\frac{b^3Bn\log(a+bx)}{3dg^4(bc-ad)^3}-\frac{b^3Bn\log(c+dx)}{3dg^4(bc-ad)^3}+\frac{b^2Bn}{3dg^4(c+dx)(bc-ad)^2}+\frac{bBn}{6dg^4(c+dx)^2(bc-ad)}+$$

```
[Out] 1/9*B*n/d/g^4/(d*x+c)^3+1/6*b*B*n/d/(-a*d+b*c)/g^4/(d*x+c)^2+1/3*b^2*B*n/d/
(-a*d+b*c)^2/g^4/(d*x+c)+1/3*b^3*B*n*ln(b*x+a)/d/(-a*d+b*c)^3/g^4+1/3*(-A-B
*ln(e*((b*x+a)/(d*x+c))^n))/d/g^4/(d*x+c)^3-1/3*b^3*B*n*ln(d*x+c)/d/(-a*d+b
*c)^3/g^4
```

**Rubi [A]** time = 0.14, antiderivative size = 183, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {2525, 12, 44}

$$-\frac{B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A}{3dg^4(c+dx)^3}+\frac{b^2Bn}{3dg^4(c+dx)(bc-ad)^2}+\frac{b^3Bn\log(a+bx)}{3dg^4(bc-ad)^3}-\frac{b^3Bn\log(c+dx)}{3dg^4(bc-ad)^3}+\frac{bBn}{6dg^4(c+dx)^2(bc-ad)}+$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Log[e*((a + b*x)/(c + d*x))^n])/(c*g + d*g*x)^4,x]
```

```
[Out] (B*n)/(9*d*g^4*(c + d*x)^3) + (b*B*n)/(6*d*(b*c - a*d)*g^4*(c + d*x)^2) + (
b^2*B*n)/(3*d*(b*c - a*d)^2*g^4*(c + d*x)) + (b^3*B*n*Log[a + b*x])/(3*d*(b
*c - a*d)^3*g^4) - (A + B*Log[e*((a + b*x)/(c + d*x))^n])/(3*d*g^4*(c + d*x
)^3) - (b^3*B*n*Log[c + d*x])/(3*d*(b*c - a*d)^3*g^4)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 2525

```
Int[((a_.) + Log[(c_.)*(RFx_)^(p_.)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_))^(m_.
), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(a + b*Log[c*RFx^p])^n)/(e*(m + 1))
, x] - Dist[(b*n*p)/(e*(m + 1)), Int[SimplifyIntegrand[((d + e*x)^(m + 1)*(
a + b*Log[c*RFx^p])^(n - 1)*D[RFx, x])/RFx, x], x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && RationalFunctionQ[RFx, x] && IGtQ[n, 0] && (EqQ[n, 1] ||
IntegerQ[m]) && NeQ[m, -1]
```

Rubi steps

$$\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(cg+dgx)^4}\,dx = -\frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{3dg^4(c+dx)^3} + \frac{(Bn)\int \frac{bc-ad}{g^3(a+bx)(c+dx)^4}\,dx}{3dg}$$

$$= -\frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{3dg^4(c+dx)^3} + \frac{(B(bc-ad)n)\int \frac{1}{(a+bx)(c+dx)^4}\,dx}{3dg^4}$$

$$= -\frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{3dg^4(c+dx)^3} + \frac{(B(bc-ad)n)\int \left(\frac{b^4}{(bc-ad)^4(a+bx)} - \frac{d}{(bc-ad)(c+dx)^4} - \frac{bd}{(bc-ad)^2(}\right.}{3dg^4}$$

$$= \frac{Bn}{9dg^4(c+dx)^3} + \frac{bBn}{6d(bc-ad)g^4(c+dx)^2} + \frac{b^2Bn}{3d(bc-ad)^2g^4(c+dx)} + \frac{b^3Bn\log(a}{3d(bc-a}$$

**Mathematica [A]** time = 0.17, size = 146, normalized size = 0.80

$$\frac{\frac{Bn\left((bc-ad)\left(2a^2d^2-abd(7c+3dx)+b^2\left(11c^2+15cdx+6d^2x^2\right)\right)+6b^3(c+dx)^3\log(a+bx)-6b^3(c+dx)^3\log(c+dx)\right)}{(bc-ad)^3} - 6\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{18dg^4(c+dx)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Log[e*((a + b*x)/(c + d*x))^n])/(c*g + d*g*x)^4,x]
```

```
[Out] (-6*(A + B*Log[e*((a + b*x)/(c + d*x))^n]) + (B*n*((b*c - a*d)*(2*a^2*d^2 -
 a*b*d*(7*c + 3*d*x) + b^2*(11*c^2 + 15*c*d*x + 6*d^2*x^2)) + 6*b^3*(c + d*
x)^3*Log[a + b*x] - 6*b^3*(c + d*x)^3*Log[c + d*x]))/(b*c - a*d)^3)/(18*d*g
^4*(c + d*x)^3)
```

**fricas [B]** time = 0.93, size = 483, normalized size = 2.64

$$-\frac{6\,Ab^3c^3 - 18\,Aab^2c^2d + 18\,Aa^2bcd^2 - 6\,Aa^3d^3 - 6\left(Bb^3cd^2 - Bab^2d^3\right)nx^2 - 3\left(5\,Bb^3c^2d - 6\,Bab^2cd^2 + Ba^2bd^3\right)}{18\left(\left(b^3c^3d^4 - 3\,ab^2c^2d^5 + 3\,a^2bcd^6 - a^3d^7\right)g^4x^3 + 3\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))/(d*g*x+c*g)^4,x, algorithm="fric
as")
```

```
[Out] -1/18*(6*A*b^3*c^3 - 18*A*a*b^2*c^2*d + 18*A*a^2*b*c*d^2 - 6*A*a^3*d^3 - 6*
(B*b^3*c*d^2 - B*a*b^2*d^3)*n*x^2 - 3*(5*B*b^3*c^2*d - 6*B*a*b^2*c*d^2 + B*
a^2*b*d^3)*n*x - (11*B*b^3*c^3 - 18*B*a*b^2*c^2*d + 9*B*a^2*b*c*d^2 - 2*B*a
^3*d^3)*n + 6*(B*b^3*c^3 - 3*B*a*b^2*c^2*d + 3*B*a^2*b*c*d^2 - B*a^3*d^3)*l
og(e) - 6*(B*b^3*d^3*n*x^3 + 3*B*b^3*c*d^2*n*x^2 + 3*B*b^3*c^2*d*n*x + (3*B
*a*b^2*c^2*d - 3*B*a^2*b*c*d^2 + B*a^3*d^3)*n)*log((b*x + a)/(d*x + c)))/((
b^3*c^3*d^4 - 3*a*b^2*c^2*d^5 + 3*a^2*b*c*d^6 - a^3*d^7)*g^4*x^3 + 3*(b^3*c
^4*d^3 - 3*a*b^2*c^3*d^4 + 3*a^2*b*c^2*d^5 - a^3*c*d^6)*g^4*x^2 + 3*(b^3*c^
5*d^2 - 3*a*b^2*c^4*d^3 + 3*a^2*b*c^3*d^4 - a^3*c^2*d^5)*g^4*x + (b^3*c^6*d
 - 3*a*b^2*c^5*d^2 + 3*a^2*b*c^4*d^3 - a^3*c^3*d^4)*g^4)
```

**giac [B]** time = 7.70, size = 399, normalized size = 2.18

$$\frac{1}{18}\left(6\left(\frac{3\,(bx+a)Bb^2n}{\left(b^2c^2g^4-2\,abcdg^4+a^2d^2g^4\right)(dx+c)}-\frac{3\,(bx+a)^2Bbdn}{\left(b^2c^2g^4-2\,abcdg^4+a^2d^2g^4\right)(dx+c)^2}+\frac{(bx+a)^3Bd^2}{\left(b^2c^2g^4-2\,abcdg^4+a^2d\right.}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))/(d*g*x+c*g)^4,x, algorithm="giac
")
```

```
[Out] 1/18*(6*(3*(b*x + a)*B*b^2*n/((b^2*c^2*g^4 - 2*a*b*c*d*g^4 + a^2*d^2*g^4)*(
d*x + c)) - 3*(b*x + a)^2*B*b*d*n/((b^2*c^2*g^4 - 2*a*b*c*d*g^4 + a^2*d^2*g
^4)*(d*x + c)^2) + (b*x + a)^3*B*d^2*n/((b^2*c^2*g^4 - 2*a*b*c*d*g^4 + a^2*
d^2*g^4)*(d*x + c)^3))*log((b*x + a)/(d*x + c)) - 2*(B*d^2*n - 3*A*d^2 - 3*
B*d^2)*(b*x + a)^3/((b^2*c^2*g^4 - 2*a*b*c*d*g^4 + a^2*d^2*g^4)*(d*x + c)^3
) + 9*(B*b*d*n - 2*A*b*d - 2*B*b*d)*(b*x + a)^2/((b^2*c^2*g^4 - 2*a*b*c*d*g
^4 + a^2*d^2*g^4)*(d*x + c)^2) - 18*(B*b^2*n - A*b^2 - B*b^2)*(b*x + a)/((b
^2*c^2*g^4 - 2*a*b*c*d*g^4 + a^2*d^2*g^4)*(d*x + c)))*(b*c/(b*c - a*d)^2 -
a*d/(b*c - a*d)^2)
```

**maple [F]** time = 0.28, size = 0, normalized size = 0.00

$$\int \frac{B\ln\left(e\left(\frac{bx+a}{dx+c}\right)^{n}\right)+A}{\left(dgx+cg\right)^{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*ln(e*((b*x+a)/(d*x+c))^n)+A)/(d*g*x+c*g)^4,x)
```

```
[Out] int((B*ln(e*((b*x+a)/(d*x+c))^n)+A)/(d*g*x+c*g)^4,x)
```

**maxima [B]** time = 1.37, size = 433, normalized size = 2.37

$$\frac{1}{18}Bn\left(\frac{6\,b^2d^2x^2+11\,b^2c^2-7\,abcd+2\,a^2d^2+3\left(5\,b^2cd-abd^2\right)x}{\left(b^2c^2d^4-2\,abcd^5+a^2d^6\right)g^4x^3+3\left(b^2c^3d^3-2\,abc^2d^4+a^2cd^5\right)g^4x^2+3\left(b^2c^4d^2-2\,abc^3d^3+a^2c^2d^4\right)g^4x\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))/(d*g*x+c*g)^4,x, algorithm="maxi
ma")
```

```
[Out] 1/18*B*n*((6*b^2*d^2*x^2 + 11*b^2*c^2 - 7*a*b*c*d + 2*a^2*d^2 + 3*(5*b^2*c*
d - a*b*d^2)*x)/((b^2*c^2*d^4 - 2*a*b*c*d^5 + a^2*d^6)*g^4*x^3 + 3*(b^2*c^3
*d^3 - 2*a*b*c^2*d^4 + a^2*c*d^5)*g^4*x^2 + 3*(b^2*c^4*d^2 - 2*a*b*c^3*d^3
+ a^2*c^2*d^4)*g^4*x + (b^2*c^5*d - 2*a*b*c^4*d^2 + a^2*c^3*d^3)*g^4) + 6*b
^3*log(b*x + a)/((b^3*c^3*d - 3*a*b^2*c^2*d^2 + 3*a^2*b*c*d^3 - a^3*d^4)*g^
4) - 6*b^3*log(d*x + c)/((b^3*c^3*d - 3*a*b^2*c^2*d^2 + 3*a^2*b*c*d^3 - a^3
*d^4)*g^4)) - 1/3*B*log(e*(b*x/(d*x + c) + a/(d*x + c))^n)/(d^4*g^4*x^3 + 3
*c*d^3*g^4*x^2 + 3*c^2*d^2*g^4*x + c^3*d*g^4) - 1/3*A/(d^4*g^4*x^3 + 3*c*d^
3*g^4*x^2 + 3*c^2*d^2*g^4*x + c^3*d*g^4)
```

**mupad [B]** time = 4.72, size = 349, normalized size = 1.91

$$\frac{B\,a^2\,d\,n}{9\,g^4\,(a\,d-b\,c)^2\,(c+d\,x)^3}-\frac{A\,a^2\,d}{3\,g^4\,(a\,d-b\,c)^2\,(c+d\,x)^3}-\frac{A\,b^2\,c^2}{3\,d\,g^4\,(a\,d-b\,c)^2\,(c+d\,x)^3}-\frac{B\,\ln\left(e\left(\frac{a+b\,x}{c+d\,x}\right)^n\right)}{3\,d\,g^4\,(c+d\,x)^3}+\frac{2\,}{3\,g^4\,(a\,d-}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*log(e*((a + b*x)/(c + d*x))^n))/(c*g + d*g*x)^4,x)
```

```
[Out] (B*a^2*d*n)/(9*g^4*(a*d - b*c)^2*(c + d*x)^3) - (A*a^2*d)/(3*g^4*(a*d - b*c
)^2*(c + d*x)^3) - (A*b^2*c^2)/(3*d*g^4*(a*d - b*c)^2*(c + d*x)^3) - (B*log
(e*((a + b*x)/(c + d*x))^n))/(3*d*g^4*(c + d*x)^3) + (B*b^3*n*atan((a*d*1i
+ b*c*1i + b*d*x*2i)/(a*d - b*c))*2i)/(3*d*g^4*(a*d - b*c)^3) + (2*A*a*b*c)
/(3*g^4*(a*d - b*c)^2*(c + d*x)^3) + (B*b^2*d*n*x^2)/(3*g^4*(a*d - b*c)^2*(
c + d*x)^3) - (7*B*a*b*c*n)/(18*g^4*(a*d - b*c)^2*(c + d*x)^3) + (11*B*b^2*
c^2*n)/(18*d*g^4*(a*d - b*c)^2*(c + d*x)^3) + (5*B*b^2*c*n*x)/(6*g^4*(a*d -
 b*c)^2*(c + d*x)^3) - (B*a*b*d*n*x)/(6*g^4*(a*d - b*c)^2*(c + d*x)^3)
```

**sympy [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*ln(e*((b*x+a)/(d*x+c))**n))/(d*g*x+c*g)**4,x)
```

```
[Out] Timed out
```

# 3.37 $\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(cg+dgx)^5}\,dx$

**Optimal**. Leaf size=215

$$-\frac{B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A}{4dg^5(c+dx)^4}+\frac{b^4Bn\log(a+bx)}{4dg^5(bc-ad)^4}-\frac{b^4Bn\log(c+dx)}{4dg^5(bc-ad)^4}+\frac{b^3Bn}{4dg^5(c+dx)(bc-ad)^3}+\frac{b^2Bn}{8dg^5(c+dx)^2(bc-ad)^2}$$

```
[Out] 1/16*B*n/d/g^5/(d*x+c)^4+1/12*b*B*n/d/(-a*d+b*c)/g^5/(d*x+c)^3+1/8*b^2*B*n/
d/(-a*d+b*c)^2/g^5/(d*x+c)^2+1/4*b^3*B*n/d/(-a*d+b*c)^3/g^5/(d*x+c)+1/4*b^4
*B*n*ln(b*x+a)/d/(-a*d+b*c)^4/g^5+1/4*(-A-B*ln(e*((b*x+a)/(d*x+c))^n))/d/g^
5/(d*x+c)^4-1/4*b^4*B*n*ln(d*x+c)/d/(-a*d+b*c)^4/g^5
```

**Rubi [A]** time = 0.18, antiderivative size = 215, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {2525, 12, 44}

$$-\frac{B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A}{4dg^5(c+dx)^4}+\frac{b^3Bn}{4dg^5(c+dx)(bc-ad)^3}+\frac{b^2Bn}{8dg^5(c+dx)^2(bc-ad)^2}+\frac{b^4Bn\log(a+bx)}{4dg^5(bc-ad)^4}-\frac{b^4Bn\log(c+dx)}{4dg^5(bc-ad)^4}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Log[e*((a + b*x)/(c + d*x))^n])/(c*g + d*g*x)^5,x]
```

```
[Out] (B*n)/(16*d*g^5*(c + d*x)^4) + (b*B*n)/(12*d*(b*c - a*d)*g^5*(c + d*x)^3) +
 (b^2*B*n)/(8*d*(b*c - a*d)^2*g^5*(c + d*x)^2) + (b^3*B*n)/(4*d*(b*c - a*d)
^3*g^5*(c + d*x)) + (b^4*B*n*Log[a + b*x])/(4*d*(b*c - a*d)^4*g^5) - (A + B
*Log[e*((a + b*x)/(c + d*x))^n])/(4*d*g^5*(c + d*x)^4) - (b^4*B*n*Log[c + d
*x])/(4*d*(b*c - a*d)^4*g^5)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 2525

```
Int[((a_.) + Log[(c_.)*(RFx_)^(p_.)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_))^(m_.
), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(a + b*Log[c*RFx^p])^n)/(e*(m + 1))
, x] - Dist[(b*n*p)/(e*(m + 1)), Int[SimplifyIntegrand[((d + e*x)^(m + 1)*(
a + b*Log[c*RFx^p])^(n - 1)*D[RFx, x])/RFx, x], x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && RationalFunctionQ[RFx, x] && IGtQ[n, 0] && (EqQ[n, 1] ||
IntegerQ[m]) && NeQ[m, -1]
```

Rubi steps

$$\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(cg+dgx)^5}\,dx = -\frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{4dg^5(c+dx)^4} + \frac{(Bn)\int \frac{bc-ad}{g^4(a+bx)(c+dx)^5}\,dx}{4dg}$$

$$= -\frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{4dg^5(c+dx)^4} + \frac{(B(bc-ad)n)\int \frac{1}{(a+bx)(c+dx)^5}\,dx}{4dg^5}$$

$$= -\frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{4dg^5(c+dx)^4} + \frac{(B(bc-ad)n)\int \left(\frac{b^5}{(bc-ad)^5(a+bx)} - \frac{d}{(bc-ad)(c+dx)^5} - \frac{bd}{(bc-ad)^2(c+}\right.}{4dg}$$

$$= \frac{Bn}{16dg^5(c+dx)^4} + \frac{bBn}{12d(bc-ad)g^5(c+dx)^3} + \frac{b^2Bn}{8d(bc-ad)^2g^5(c+dx)^2} + \frac{b}{4d(bc-a}$$

**Mathematica [A]** time = 0.23, size = 162, normalized size = 0.75

$$\frac{\frac{Bn\left(12b^4\log(a+bx)+\frac{12b^3(bc-ad)}{c+dx}+\frac{6b^2(bc-ad)^2}{(c+dx)^2}+\frac{4b(bc-ad)^3}{(c+dx)^3}+\frac{3(bc-ad)^4}{(c+dx)^4}-12b^4\log(c+dx)\right)}{12(bc-ad)^4}-\frac{B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A}{(c+dx)^4}}{4dg^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Log[e*((a + b*x)/(c + d*x))^n])/(c*g + d*g*x)^5,x]
```

```
[Out] (-((A + B*Log[e*((a + b*x)/(c + d*x))^n])/(c + d*x)^4) + (B*n*((3*(b*c - a*
d)^4)/(c + d*x)^4 + (4*b*(b*c - a*d)^3)/(c + d*x)^3 + (6*b^2*(b*c - a*d)^2)
/(c + d*x)^2 + (12*b^3*(b*c - a*d))/(c + d*x) + 12*b^4*Log[a + b*x] - 12*b^
4*Log[c + d*x]))/(12*(b*c - a*d)^4))/(4*d*g^5)
```

**fricas [B]** time = 1.09, size = 735, normalized size = 3.42

$$-\frac{12\,Ab^4c^4 - 48\,Aab^3c^3d + 72\,Aa^2b^2c^2d^2 - 48\,Aa^3bcd^3 + 12\,Aa^4d^4 - 12\left(Bb^4cd^3 - Bab^3d^4\right)nx^3 - 6\left(7\,Bb^4c^2d^2 -\right.}{48\left(\left(b^4c^4d^5 - 4\,ab^3c^3d^6 + 6\,a^2b^2c^2d^7\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))/(d*g*x+c*g)^5,x, algorithm="fric
as")
```

```
[Out] -1/48*(12*A*b^4*c^4 - 48*A*a*b^3*c^3*d + 72*A*a^2*b^2*c^2*d^2 - 48*A*a^3*b*
c*d^3 + 12*A*a^4*d^4 - 12*(B*b^4*c*d^3 - B*a*b^3*d^4)*n*x^3 - 6*(7*B*b^4*c^
2*d^2 - 8*B*a*b^3*c*d^3 + B*a^2*b^2*d^4)*n*x^2 - 4*(13*B*b^4*c^3*d - 18*B*a
*b^3*c^2*d^2 + 6*B*a^2*b^2*c*d^3 - B*a^3*b*d^4)*n*x - (25*B*b^4*c^4 - 48*B*
a*b^3*c^3*d + 36*B*a^2*b^2*c^2*d^2 - 16*B*a^3*b*c*d^3 + 3*B*a^4*d^4)*n + 12
*(B*b^4*c^4 - 4*B*a*b^3*c^3*d + 6*B*a^2*b^2*c^2*d^2 - 4*B*a^3*b*c*d^3 + B*a
^4*d^4)*log(e) - 12*(B*b^4*d^4*n*x^4 + 4*B*b^4*c*d^3*n*x^3 + 6*B*b^4*c^2*d^
2*n*x^2 + 4*B*b^4*c^3*d*n*x + (4*B*a*b^3*c^3*d - 6*B*a^2*b^2*c^2*d^2 + 4*B*
a^3*b*c*d^3 - B*a^4*d^4)*n)*log((b*x + a)/(d*x + c)))/((b^4*c^4*d^5 - 4*a*b
^3*c^3*d^6 + 6*a^2*b^2*c^2*d^7 - 4*a^3*b*c*d^8 + a^4*d^9)*g^5*x^4 + 4*(b^4*
c^5*d^4 - 4*a*b^3*c^4*d^5 + 6*a^2*b^2*c^3*d^6 - 4*a^3*b*c^2*d^7 + a^4*c*d^8
)*g^5*x^3 + 6*(b^4*c^6*d^3 - 4*a*b^3*c^5*d^4 + 6*a^2*b^2*c^4*d^5 - 4*a^3*b*
c^3*d^6 + a^4*c^2*d^7)*g^5*x^2 + 4*(b^4*c^7*d^2 - 4*a*b^3*c^6*d^3 + 6*a^2*b
^2*c^5*d^4 - 4*a^3*b*c^4*d^5 + a^4*c^3*d^6)*g^5*x + (b^4*c^8*d - 4*a*b^3*c^
7*d^2 + 6*a^2*b^2*c^6*d^3 - 4*a^3*b*c^5*d^4 + a^4*c^4*d^5)*g^5)
```

**giac [B]** time = 11.42, size = 676, normalized size = 3.14

$$\frac{1}{48}\left(12\left(\frac{4\,(bx+a)Bb^3n}{\left(b^3c^3g^5 - 3\,ab^2c^2dg^5 + 3\,a^2bcd^2g^5 - a^3d^3g^5\right)(dx+c)} - \frac{6\,(bx+a)^2Bb^2dn}{\left(b^3c^3g^5 - 3\,ab^2c^2dg^5 + 3\,a^2bcd^2g^5 - a^3d^3g^5\right)(dx\right.}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))/(d*g*x+c*g)^5,x, algorithm="giac
")
```

```
[Out] 1/48*(12*(4*(b*x + a)*B*b^3*n/((b^3*c^3*g^5 - 3*a*b^2*c^2*d*g^5 + 3*a^2*b*c
*d^2*g^5 - a^3*d^3*g^5)*(d*x + c)) - 6*(b*x + a)^2*B*b^2*d*n/((b^3*c^3*g^5
- 3*a*b^2*c^2*d*g^5 + 3*a^2*b*c*d^2*g^5 - a^3*d^3*g^5)*(d*x + c)^2) + 4*(b*
x + a)^3*B*b*d^2*n/((b^3*c^3*g^5 - 3*a*b^2*c^2*d*g^5 + 3*a^2*b*c*d^2*g^5 -
a^3*d^3*g^5)*(d*x + c)^3) - (b*x + a)^4*B*d^3*n/((b^3*c^3*g^5 - 3*a*b^2*c^2
*d*g^5 + 3*a^2*b*c*d^2*g^5 - a^3*d^3*g^5)*(d*x + c)^4))*log((b*x + a)/(d*x
+ c)) + 3*(B*d^3*n - 4*A*d^3 - 4*B*d^3)*(b*x + a)^4/((b^3*c^3*g^5 - 3*a*b^2
*c^2*d*g^5 + 3*a^2*b*c*d^2*g^5 - a^3*d^3*g^5)*(d*x + c)^4) - 16*(B*b*d^2*n
```

```
- 3*A*b*d^2 - 3*B*b*d^2)*(b*x + a)^3/((b^3*c^3*g^5 - 3*a*b^2*c^2*d*g^5 + 3*
a^2*b*c*d^2*g^5 - a^3*d^3*g^5)*(d*x + c)^3) + 36*(B*b^2*d*n - 2*A*b^2*d - 2
*B*b^2*d)*(b*x + a)^2/((b^3*c^3*g^5 - 3*a*b^2*c^2*d*g^5 + 3*a^2*b*c*d^2*g^5
 - a^3*d^3*g^5)*(d*x + c)^2) - 48*(B*b^3*n - A*b^3 - B*b^3)*(b*x + a)/((b^3
*c^3*g^5 - 3*a*b^2*c^2*d*g^5 + 3*a^2*b*c*d^2*g^5 - a^3*d^3*g^5)*(d*x + c)))
*(b*c/(b*c - a*d)^2 - a*d/(b*c - a*d)^2)
```

**maple [F]** time = 0.30, size = 0, normalized size = 0.00

$$\int \frac{B \ln\left(e \left(\frac{bx+a}{dx+c}\right)^{n}\right) + A}{\left(dgx + cg\right)^{5}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*ln(e*((b*x+a)/(d*x+c))^n)+A)/(d*g*x+c*g)^5,x)
```

```
[Out] int((B*ln(e*((b*x+a)/(d*x+c))^n)+A)/(d*g*x+c*g)^5,x)
```

**maxima [B]** time = 1.06, size = 652, normalized size = 3.03

$$\frac{1}{48}\,Bn\left(\frac{12\,b^3d^3x^3 + 25\,b^3c^3 - 23\,ab^2c^2d + 13\,a^2bcd}{\left(b^3c^3d^5 - 3\,ab^2c^2d^6 + 3\,a^2bcd^7 - a^3d^8\right)g^5x^4 + 4\left(b^3c^4d^4 - 3\,ab^2c^3d^5 + 3\,a^2bc^2d^6 - a^3cd^7\right)g^5x^3 + 6\left(b^3c^5d^3\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))/(d*g*x+c*g)^5,x, algorithm="maxi
ma")
```

```
[Out] 1/48*B*n*((12*b^3*d^3*x^3 + 25*b^3*c^3 - 23*a*b^2*c^2*d + 13*a^2*b*c*d^2 -
3*a^3*d^3 + 6*(7*b^3*c*d^2 - a*b^2*d^3)*x^2 + 4*(13*b^3*c^2*d - 5*a*b^2*c*d
^2 + a^2*b*d^3)*x)/((b^3*c^3*d^5 - 3*a*b^2*c^2*d^6 + 3*a^2*b*c*d^7 - a^3*d^
8)*g^5*x^4 + 4*(b^3*c^4*d^4 - 3*a*b^2*c^3*d^5 + 3*a^2*b*c^2*d^6 - a^3*c*d^7
)*g^5*x^3 + 6*(b^3*c^5*d^3 - 3*a*b^2*c^4*d^4 + 3*a^2*b*c^3*d^5 - a^3*c^2*d^
6)*g^5*x^2 + 4*(b^3*c^6*d^2 - 3*a*b^2*c^5*d^3 + 3*a^2*b*c^4*d^4 - a^3*c^3*d
^5)*g^5*x + (b^3*c^7*d - 3*a*b^2*c^6*d^2 + 3*a^2*b*c^5*d^3 - a^3*c^4*d^4)*g
^5) + 12*b^4*log(b*x + a)/((b^4*c^4*d - 4*a*b^3*c^3*d^2 + 6*a^2*b^2*c^2*d^3
 - 4*a^3*b*c*d^4 + a^4*d^5)*g^5) - 12*b^4*log(d*x + c)/((b^4*c^4*d - 4*a*b^
3*c^3*d^2 + 6*a^2*b^2*c^2*d^3 - 4*a^3*b*c*d^4 + a^4*d^5)*g^5)) - 1/4*B*log(
e*(b*x/(d*x + c) + a/(d*x + c))^n)/(d^5*g^5*x^4 + 4*c*d^4*g^5*x^3 + 6*c^2*d
^3*g^5*x^2 + 4*c^3*d^2*g^5*x + c^4*d*g^5) - 1/4*A/(d^5*g^5*x^4 + 4*c*d^4*g^
5*x^3 + 6*c^2*d^3*g^5*x^2 + 4*c^3*d^2*g^5*x + c^4*d*g^5)
```

**mupad [B]** time = 4.99, size = 603, normalized size = 2.80

$$\frac{B\,b^4\,n\,\mathrm{atanh}\left(\frac{4\,a^4\,d^5\,g^5-8\,a^3\,b\,c\,d^4\,g^5+8\,a\,b^3\,c^3\,d^2\,g^5-4\,b^4\,c^4\,d\,g^5}{4\,d\,g^5\,(a\,d-b\,c)^4}+\frac{2\,b\,d\,x\,\left(a^3\,d^3-3\,a^2\,b\,c\,d^2+3\,a\,b^2\,c^2\,d-b^3\,c^3\right)}{(a\,d-b\,c)^4}\right)}{2\,d\,g^5\,(a\,d-b\,c)^4}-\frac{\cdots}{4\,d\,\left(c^4\,g^5+4\,c^3\,d\,g^5\,x\cdots\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*log(e*((a + b*x)/(c + d*x))^n))/(c*g + d*g*x)^5,x)
```

```
[Out] (B*b^4*n*atanh((4*a^4*d^5*g^5 - 4*b^4*c^4*d*g^5 - 8*a^3*b*c*d^4*g^5 + 8*a*b
^3*c^3*d^2*g^5)/(4*d*g^5*(a*d - b*c)^4) + (2*b*d*x*(a^3*d^3 - b^3*c^3 + 3*a
*b^2*c^2*d - 3*a^2*b*c*d^2))/(a*d - b*c)^4))/(2*d*g^5*(a*d - b*c)^4) - (B*l
og(e*((a + b*x)/(c + d*x))^n))/(4*d*(c^4*g^5 + d^4*g^5*x^4 + 4*c*d^3*g^5*x^
3 + 6*c^2*d^2*g^5*x^2 + 4*c^3*d*g^5*x)) - ((12*A*a^3*d^3 - 12*A*b^3*c^3 - 3
*B*a^3*d^3*n + 25*B*b^3*c^3*n + 36*A*a*b^2*c^2*d - 36*A*a^2*b*c*d^2 - 23*B*
a*b^2*c^2*d*n + 13*B*a^2*b*c*d^2*n)/(12*(a^3*d^3 - b^3*c^3 + 3*a*b^2*c^2*d
- 3*a^2*b*c*d^2)) + (b*x*(B*a^2*d^3*n + 13*B*b^2*c^2*d*n - 5*B*a*b*c*d^2*n)
)/(3*(a^3*d^3 - b^3*c^3 + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2)) - (b^2*x^2*(B*a*d
^3*n - 7*B*b*c*d^2*n))/(2*(a^3*d^3 - b^3*c^3 + 3*a*b^2*c^2*d - 3*a^2*b*c*d^
2)) + (B*b^3*d^3*n*x^3)/(a^3*d^3 - b^3*c^3 + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2)
)/(4*c^4*d*g^5 + 4*d^5*g^5*x^4 + 16*c^3*d^2*g^5*x + 16*c*d^4*g^5*x^3 + 24*c
^2*d^3*g^5*x^2)
```

**sympy [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*ln(e*((b*x+a)/(d*x+c))**n))/(d*g*x+c*g)**5,x)
```

```
[Out] Timed out
```

# 3.38 $\int (cg+dgx)^4 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2 dx$

**Optimal**. Leaf size=544

$$\frac{2Bg^4n(bc-ad)^5\log\left(1-\frac{b(c+dx)}{d(a+bx)}\right)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{5b^5d}-\frac{2Bg^4n(a+bx)(bc-ad)^4\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{5b^5}-\frac{Bg}{}$$

```
[Out] 13/30*B^2*(-a*d+b*c)^4*g^4*n^2*x/b^4+7/60*B^2*(-a*d+b*c)^3*g^4*n^2*(d*x+c)^
2/b^3/d+1/30*B^2*(-a*d+b*c)^2*g^4*n^2*(d*x+c)^3/b^2/d-2/5*B*(-a*d+b*c)^4*g^
4*n*(b*x+a)*(A+B*ln(e*((b*x+a)/(d*x+c))^n))/b^5-1/5*B*(-a*d+b*c)^3*g^4*n*(d
*x+c)^2*(A+B*ln(e*((b*x+a)/(d*x+c))^n))/b^3/d-2/15*B*(-a*d+b*c)^2*g^4*n*(d*
x+c)^3*(A+B*ln(e*((b*x+a)/(d*x+c))^n))/b^2/d-1/10*B*(-a*d+b*c)*g^4*n*(d*x+c
)^4*(A+B*ln(e*((b*x+a)/(d*x+c))^n))/b/d+1/5*g^4*(d*x+c)^5*(A+B*ln(e*((b*x+a
)/(d*x+c))^n))^2/d+13/30*B^2*(-a*d+b*c)^5*g^4*n^2*ln((b*x+a)/(d*x+c))/b^5/d
+5/6*B^2*(-a*d+b*c)^5*g^4*n^2*ln(d*x+c)/b^5/d+2/5*B*(-a*d+b*c)^5*g^4*n*(A+B
*ln(e*((b*x+a)/(d*x+c))^n))*ln(1-b*(d*x+c)/d/(b*x+a))/b^5/d-2/5*B^2*(-a*d+b
*c)^5*g^4*n^2*polylog(2,b*(d*x+c)/d/(b*x+a))/b^5/d
```

**Rubi [A]** time = 0.88, antiderivative size = 634, normalized size of antiderivative = 1.17, number of steps used = 27, number of rules used = 13, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.371, Rules used = {2525, 12, 2528, 2486, 31, 2524, 2418, 2390, 2301, 2394, 2393, 2391, 43}

$$-\frac{2B^2g^4n^2(bc-ad)^5\text{PolyLog}\left(2,-\frac{d(a+bx)}{bc-ad}\right)}{5b^5d}-\frac{2Bg^4n(bc-ad)^5\log(a+bx)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{5b^5d}-\frac{Bg^4n(c+dx)^2}{}$$

Antiderivative was successfully verified.

```
[In] Int[(c*g + d*g*x)^4*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2,x]
```

```
[Out] (-2*A*B*(b*c - a*d)^4*g^4*n*x)/(5*b^4) + (13*B^2*(b*c - a*d)^4*g^4*n^2*x)/(
30*b^4) + (7*B^2*(b*c - a*d)^3*g^4*n^2*(c + d*x)^2)/(60*b^3*d) + (B^2*(b*c
- a*d)^2*g^4*n^2*(c + d*x)^3)/(30*b^2*d) + (13*B^2*(b*c - a*d)^5*g^4*n^2*Lo
g[a + b*x])/(30*b^5*d) + (B^2*(b*c - a*d)^5*g^4*n^2*Log[a + b*x]^2)/(5*b^5*
d) - (2*B^2*(b*c - a*d)^4*g^4*n*(a + b*x)*Log[e*((a + b*x)/(c + d*x))^n])/(
5*b^5) - (B*(b*c - a*d)^3*g^4*n*(c + d*x)^2*(A + B*Log[e*((a + b*x)/(c + d*
x))^n]))/(5*b^3*d) - (2*B*(b*c - a*d)^2*g^4*n*(c + d*x)^3*(A + B*Log[e*((a
+ b*x)/(c + d*x))^n]))/(15*b^2*d) - (B*(b*c - a*d)*g^4*n*(c + d*x)^4*(A + B
*Log[e*((a + b*x)/(c + d*x))^n]))/(10*b*d) - (2*B*(b*c - a*d)^5*g^4*n*Log[a
 + b*x]*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/(5*b^5*d) + (g^4*(c + d*x)^
5*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2)/(5*d) + (2*B^2*(b*c - a*d)^5*g^
```

```
4*n^2*Log[c + d*x])/(5*b^5*d) - (2*B^2*(b*c - a*d)^5*g^4*n^2*Log[a + b*x]*L
og[(b*(c + d*x))/(b*c - a*d)])/(5*b^5*d) - (2*B^2*(b*c - a*d)^5*g^4*n^2*Pol
yLog[2, -((d*(a + b*x))/(b*c - a*d))])/(5*b^5*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2301

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

Rule 2390

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_))^(q_.), x_Symbol] :> Dist[1/e, Subst[Int[((f*x)/d)^q*(a + b*Log[c*x^
n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && E
qQ[e*f - d*g, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2393

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + (c*e*x)/g])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2394

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d + e*x
)^n]))/g, x] - Dist[(b*e*n)/g, Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2418

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*(RFx_), x_Sy
mbol] :> With[{u = ExpandIntegrand[(a + b*Log[c*(d + e*x)^n])^p, RFx, x]},
Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d, e, n}, x] && RationalFunctionQ[
RFx, x] && IntegerQ[p]
```

Rule 2486

```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))
^(r_.)]^(s_.), x_Symbol] :> Simp[((a + b*x)*Log[e*(f*(a + b*x)^p*(c + d*x)^
q)^r]^s)/b, x] + Dist[(q*r*s*(b*c - a*d))/b, Int[Log[e*(f*(a + b*x)^p*(c +
d*x)^q)^r]^(s - 1)/(c + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, p, q, r, s
}, x] && NeQ[b*c - a*d, 0] && EqQ[p + q, 0] && IGtQ[s, 0]
```

Rule 2524

```
Int[((a_.) + Log[(c_.)*(RFx_)^(p_.)]*(b_.))^(n_.)/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[(Log[d + e*x]*(a + b*Log[c*RFx^p])^n)/e, x] - Dist[(b*n*p)/e
, Int[(Log[d + e*x]*(a + b*Log[c*RFx^p])^(n - 1)*D[RFx, x])/RFx, x], x] /;
FreeQ[{a, b, c, d, e, p}, x] && RationalFunctionQ[RFx, x] && IGtQ[n, 0]
```

Rule 2525

```
Int[((a_.) + Log[(c_.)*(RFx_)^(p_.)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_))^(m_.
), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(a + b*Log[c*RFx^p])^n)/(e*(m + 1))
, x] - Dist[(b*n*p)/(e*(m + 1)), Int[SimplifyIntegrand[((d + e*x)^(m + 1)*(
a + b*Log[c*RFx^p])^(n - 1)*D[RFx, x])/RFx, x], x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && RationalFunctionQ[RFx, x] && IGtQ[n, 0] && (EqQ[n, 1] ||
IntegerQ[m]) && NeQ[m, -1]
```

Rule 2528

```
Int[((a_.) + Log[(c_.)*(RFx_)^(p_.)]*(b_.))^(n_.)*(RGx_), x_Symbol] :> With
[{u = ExpandIntegrand[(a + b*Log[c*RFx^p])^n, RGx, x]}, Int[u, x] /; SumQ[u
]] /; FreeQ[{a, b, c, p}, x] && RationalFunctionQ[RFx, x] && RationalFuncti
onQ[RGx, x] && IGtQ[n, 0]
```

Rubi steps

$$\int (cg+dgx)^4 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2 dx = \frac{g^4(c+dx)^5\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{5d} - \frac{(2Bn)\int \frac{(bc-ad)g^5(c+dx)^4\left(A\right.}{a+}}{5dg}$$

$$= \frac{g^4(c+dx)^5\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{5d} - \frac{\left(2B(bc-ad)g^4n\right)\int \frac{(c+}{}}{5}$$

$$= \frac{g^4(c+dx)^5\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{5d} - \frac{\left(2B(bc-ad)g^4n\right)\int\left(\frac{d(b}{}\right.}{}$$

$$= \frac{g^4(c+dx)^5\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{5d} - \frac{\left(2B(bc-ad)g^4n\right)\int(c+}{}$$

$$= -\frac{2AB(bc-ad)^4g^4nx}{5b^4} - \frac{B(bc-ad)^3g^4n(c+dx)^2\left(A+B\log\left(e\right.\right.}{5b^3d}$$

$$= -\frac{2AB(bc-ad)^4g^4nx}{5b^4} - \frac{2B^2(bc-ad)^4g^4n(a+bx)\log\left(e\left(\frac{a+bx}{c+dx}\right)\right.}{5b^5}$$

$$= -\frac{2AB(bc-ad)^4g^4nx}{5b^4} - \frac{2B^2(bc-ad)^4g^4n(a+bx)\log\left(e\left(\frac{a+bx}{c+dx}\right)\right.}{5b^5}$$

$$= -\frac{2AB(bc-ad)^4g^4nx}{5b^4} + \frac{13B^2(bc-ad)^4g^4n^2x}{30b^4} + \frac{7B^2(bc-ad)^3g}{60b^3}$$

$$= -\frac{2AB(bc-ad)^4g^4nx}{5b^4} + \frac{13B^2(bc-ad)^4g^4n^2x}{30b^4} + \frac{7B^2(bc-ad)^3g}{60b^3}$$

$$= -\frac{2AB(bc-ad)^4g^4nx}{5b^4} + \frac{13B^2(bc-ad)^4g^4n^2x}{30b^4} + \frac{7B^2(bc-ad)^3g}{60b^3}$$

**Mathematica [A]** time = 0.50, size = 533, normalized size = 0.98

$$\frac{g^4\left((c+dx)^5\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2-\frac{Bn(bc-ad)\left(6b^4(c+dx)^4\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)+8b^3(c+dx)^3(bc-ad)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)+12b^2(c\cdots}{\cdots}\right)}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*g + d*g*x)^4*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2,x]
```

```
[Out] (g^4*((c + d*x)^5*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2 - (B*(b*c - a*d)
*n*(24*A*b*d*(b*c - a*d)^3*x - 12*B*(b*c - a*d)^3*n*(b*d*x + (b*c - a*d)*Lo
g[a + b*x]) - 4*B*(b*c - a*d)^2*n*(2*b*d*(b*c - a*d)*x + b^2*(c + d*x)^2 +
2*(b*c - a*d)^2*Log[a + b*x]) - B*(b*c - a*d)*n*(6*b*d*(b*c - a*d)^2*x + 3*
b^2*(b*c - a*d)*(c + d*x)^2 + 2*b^3*(c + d*x)^3 + 6*(b*c - a*d)^3*Log[a + b
*x]) + 24*B*d*(b*c - a*d)^3*(a + b*x)*Log[e*((a + b*x)/(c + d*x))^n] + 12*b
^2*(b*c - a*d)^2*(c + d*x)^2*(A + B*Log[e*((a + b*x)/(c + d*x))^n]) + 8*b^3
*(b*c - a*d)*(c + d*x)^3*(A + B*Log[e*((a + b*x)/(c + d*x))^n]) + 6*b^4*(c
+ d*x)^4*(A + B*Log[e*((a + b*x)/(c + d*x))^n]) + 24*(b*c - a*d)^4*Log[a +
b*x]*(A + B*Log[e*((a + b*x)/(c + d*x))^n]) - 24*B*(b*c - a*d)^4*n*Log[c +
d*x] - 12*B*(b*c - a*d)^4*n*(Log[a + b*x]*(Log[a + b*x] - 2*Log[(b*(c + d*x
))/(b*c - a*d)]) - 2*PolyLog[2, (d*(a + b*x))/(-(b*c) + a*d)])))/(12*b^5)))
/(5*d)
```

**fricas [F]** time = 0.85, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(A^2d^4g^4x^4+4\,A^2cd^3g^4x^3+6\,A^2c^2d^2g^4x^2+4\,A^2c^3dg^4x+A^2c^4g^4+\left(B^2d^4g^4x^4+4\,B^2cd^3g^4x^3+6\,B^2c^2d\cdots\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*g*x+c*g)^4*(A+B*log(e*((b*x+a)/(d*x+c))^n))^2,x, algorithm="fr
icas")
```

```
[Out] integral(A^2*d^4*g^4*x^4 + 4*A^2*c*d^3*g^4*x^3 + 6*A^2*c^2*d^2*g^4*x^2 + 4*
A^2*c^3*d*g^4*x + A^2*c^4*g^4 + (B^2*d^4*g^4*x^4 + 4*B^2*c*d^3*g^4*x^3 + 6*
B^2*c^2*d^2*g^4*x^2 + 4*B^2*c^3*d*g^4*x + B^2*c^4*g^4)*log(e*((b*x + a)/(d*
x + c))^n)^2 + 2*(A*B*d^4*g^4*x^4 + 4*A*B*c*d^3*g^4*x^3 + 6*A*B*c^2*d^2*g^4
*x^2 + 4*A*B*c^3*d*g^4*x + A*B*c^4*g^4)*log(e*((b*x + a)/(d*x + c))^n), x)
```

**giac [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*g*x+c*g)^4*(A+B*log(e*((b*x+a)/(d*x+c))^n))^2,x, algorithm="gi
ac")
```

```
[Out] Timed out
```

**maple [F]** time = 0.28, size = 0, normalized size = 0.00

$$\int (dgx+cg)^4 \left(B\ln\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A\right)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*g*x+c*g)^4*(B*ln(e*((b*x+a)/(d*x+c))^n)+A)^2,x)
```

```
[Out] int((d*g*x+c*g)^4*(B*ln(e*((b*x+a)/(d*x+c))^n)+A)^2,x)
```

**maxima [B]** time = 4.78, size = 2880, normalized size = 5.29

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*g*x+c*g)^4*(A+B*log(e*((b*x+a)/(d*x+c))^n))^2,x, algorithm="ma
xima")
```

```
[Out] 2/5*A*B*d^4*g^4*x^5*log(e*(b*x/(d*x + c) + a/(d*x + c))^n) + 1/5*A^2*d^4*g^
4*x^5 + 2*A*B*c*d^3*g^4*x^4*log(e*(b*x/(d*x + c) + a/(d*x + c))^n) + A^2*c*
d^3*g^4*x^4 + 4*A*B*c^2*d^2*g^4*x^3*log(e*(b*x/(d*x + c) + a/(d*x + c))^n)
+ 2*A^2*c^2*d^2*g^4*x^3 + 4*A*B*c^3*d*g^4*x^2*log(e*(b*x/(d*x + c) + a/(d*x
 + c))^n) + 2*A^2*c^3*d*g^4*x^2 + 1/30*A*B*d^4*g^4*n*(12*a^5*log(b*x + a)/b
^5 - 12*c^5*log(d*x + c)/d^5 - (3*(b^4*c*d^3 - a*b^3*d^4)*x^4 - 4*(b^4*c^2*
d^2 - a^2*b^2*d^4)*x^3 + 6*(b^4*c^3*d - a^3*b*d^4)*x^2 - 12*(b^4*c^4 - a^4*
d^4)*x)/(b^4*d^4)) - 1/3*A*B*c*d^3*g^4*n*(6*a^4*log(b*x + a)/b^4 - 6*c^4*lo
g(d*x + c)/d^4 + (2*(b^3*c*d^2 - a*b^2*d^3)*x^3 - 3*(b^3*c^2*d - a^2*b*d^3)
*x^2 + 6*(b^3*c^3 - a^3*d^3)*x)/(b^3*d^3)) + 2*A*B*c^2*d^2*g^4*n*(2*a^3*log
(b*x + a)/b^3 - 2*c^3*log(d*x + c)/d^3 - ((b^2*c*d - a*b*d^2)*x^2 - 2*(b^2*
c^2 - a^2*d^2)*x)/(b^2*d^2)) - 4*A*B*c^3*d*g^4*n*(a^2*log(b*x + a)/b^2 - c^
2*log(d*x + c)/d^2 + (b*c - a*d)*x/(b*d)) + 2*A*B*c^4*g^4*n*(a*log(b*x + a)
/b - c*log(d*x + c)/d) + 2*A*B*c^4*g^4*x*log(e*(b*x/(d*x + c) + a/(d*x + c)
)^n) + A^2*c^4*g^4*x - 1/30*(77*a*b^3*c^4*d*g^4*n^2 - 94*a^2*b^2*c^3*d^2*g^
4*n^2 + 54*a^3*b*c^2*d^3*g^4*n^2 - 12*a^4*c*d^4*g^4*n^2 - (25*g^4*n^2 - 12*
g^4*n*log(e))*b^4*c^5)*B^2*log(d*x + c)/(b^4*d) - 2/5*(b^5*c^5*g^4*n^2 - 5*
a*b^4*c^4*d*g^4*n^2 + 10*a^2*b^3*c^3*d^2*g^4*n^2 - 10*a^3*b^2*c^2*d^3*g^4*n
^2 + 5*a^4*b*c*d^4*g^4*n^2 - a^5*d^5*g^4*n^2)*(log(b*x + a)*log((b*d*x + a*
d)/(b*c - a*d) + 1) + dilog(-(b*d*x + a*d)/(b*c - a*d)))*B^2/(b^5*d) + 1/60
*(12*B^2*b^5*d^5*g^4*x^5*log(e)^2 + 24*B^2*b^5*c^5*g^4*n^2*log(b*x + a)*log
(d*x + c) - 12*B^2*b^5*c^5*g^4*n^2*log(d*x + c)^2 + 6*(a*b^4*d^5*g^4*n*log(
```

```
e) - (g^4*n*log(e) - 10*g^4*log(e)^2)*b^5*c*d^4)*B^2*x^4 + 2*((g^4*n^2 - 16
*g^4*n*log(e) + 60*g^4*log(e)^2)*b^5*c^2*d^3 - 2*(g^4*n^2 - 10*g^4*n*log(e)
)*a*b^4*c*d^4 + (g^4*n^2 - 4*g^4*n*log(e))*a^2*b^3*d^5)*B^2*x^3 + ((13*g^4*
n^2 - 72*g^4*n*log(e) + 120*g^4*log(e)^2)*b^5*c^3*d^2 - 3*(11*g^4*n^2 - 40*
g^4*n*log(e))*a*b^4*c^2*d^3 + 3*(9*g^4*n^2 - 20*g^4*n*log(e))*a^2*b^3*c*d^4
 - (7*g^4*n^2 - 12*g^4*n*log(e))*a^3*b^2*d^5)*B^2*x^2 - 12*(5*a*b^4*c^4*d*g
^4*n^2 - 10*a^2*b^3*c^3*d^2*g^4*n^2 + 10*a^3*b^2*c^2*d^3*g^4*n^2 - 5*a^4*b*
c*d^4*g^4*n^2 + a^5*d^5*g^4*n^2)*B^2*log(b*x + a)^2 + 2*((23*g^4*n^2 - 48*g
^4*n*log(e) + 30*g^4*log(e)^2)*b^5*c^4*d - (79*g^4*n^2 - 120*g^4*n*log(e))*
a*b^4*c^3*d^2 + 6*(17*g^4*n^2 - 20*g^4*n*log(e))*a^2*b^3*c^2*d^3 - (59*g^4*
n^2 - 60*g^4*n*log(e))*a^3*b^2*c*d^4 + (13*g^4*n^2 - 12*g^4*n*log(e))*a^4*b
*d^5)*B^2*x - 2*(12*(4*g^4*n^2 - 5*g^4*n*log(e))*a*b^4*c^4*d - 12*(13*g^4*n
^2 - 10*g^4*n*log(e))*a^2*b^3*c^3*d^2 + 4*(49*g^4*n^2 - 30*g^4*n*log(e))*a^
3*b^2*c^2*d^3 - (113*g^4*n^2 - 60*g^4*n*log(e))*a^4*b*c*d^4 + (25*g^4*n^2 -
 12*g^4*n*log(e))*a^5*d^5)*B^2*log(b*x + a) + 12*(B^2*b^5*d^5*g^4*x^5 + 5*B
^2*b^5*c*d^4*g^4*x^4 + 10*B^2*b^5*c^2*d^3*g^4*x^3 + 10*B^2*b^5*c^3*d^2*g^4*
x^2 + 5*B^2*b^5*c^4*d*g^4*x)*log((b*x + a)^n)^2 + 12*(B^2*b^5*d^5*g^4*x^5 +
 5*B^2*b^5*c*d^4*g^4*x^4 + 10*B^2*b^5*c^2*d^3*g^4*x^3 + 10*B^2*b^5*c^3*d^2*
g^4*x^2 + 5*B^2*b^5*c^4*d*g^4*x)*log((d*x + c)^n)^2 + 2*(12*B^2*b^5*d^5*g^4
*x^5*log(e) - 12*B^2*b^5*c^5*g^4*n*log(d*x + c) + 3*(a*b^4*d^5*g^4*n - (g^4
*n - 20*g^4*log(e))*b^5*c*d^4)*B^2*x^4 + 4*(5*a*b^4*c*d^4*g^4*n - a^2*b^3*d
^5*g^4*n - 2*(2*g^4*n - 15*g^4*log(e))*b^5*c^2*d^3)*B^2*x^3 + 6*(10*a*b^4*c
^2*d^3*g^4*n - 5*a^2*b^3*c*d^4*g^4*n + a^3*b^2*d^5*g^4*n - 2*(3*g^4*n - 10*
g^4*log(e))*b^5*c^3*d^2)*B^2*x^2 + 12*(10*a*b^4*c^3*d^2*g^4*n - 10*a^2*b^3*
c^2*d^3*g^4*n + 5*a^3*b^2*c*d^4*g^4*n - a^4*b*d^5*g^4*n - (4*g^4*n - 5*g^4*
log(e))*b^5*c^4*d)*B^2*x + 12*(5*a*b^4*c^4*d*g^4*n - 10*a^2*b^3*c^3*d^2*g^4
*n + 10*a^3*b^2*c^2*d^3*g^4*n - 5*a^4*b*c*d^4*g^4*n + a^5*d^5*g^4*n)*B^2*lo
g(b*x + a))*log((b*x + a)^n) - 2*(12*B^2*b^5*d^5*g^4*x^5*log(e) - 12*B^2*b^
5*c^5*g^4*n*log(d*x + c) + 3*(a*b^4*d^5*g^4*n - (g^4*n - 20*g^4*log(e))*b^5
*c*d^4)*B^2*x^4 + 4*(5*a*b^4*c*d^4*g^4*n - a^2*b^3*d^5*g^4*n - 2*(2*g^4*n -
 15*g^4*log(e))*b^5*c^2*d^3)*B^2*x^3 + 6*(10*a*b^4*c^2*d^3*g^4*n - 5*a^2*b^
3*c*d^4*g^4*n + a^3*b^2*d^5*g^4*n - 2*(3*g^4*n - 10*g^4*log(e))*b^5*c^3*d^2
)*B^2*x^2 + 12*(10*a*b^4*c^3*d^2*g^4*n - 10*a^2*b^3*c^2*d^3*g^4*n + 5*a^3*b
^2*c*d^4*g^4*n - a^4*b*d^5*g^4*n - (4*g^4*n - 5*g^4*log(e))*b^5*c^4*d)*B^2*
x + 12*(5*a*b^4*c^4*d*g^4*n - 10*a^2*b^3*c^3*d^2*g^4*n + 10*a^3*b^2*c^2*d^3
*g^4*n - 5*a^4*b*c*d^4*g^4*n + a^5*d^5*g^4*n)*B^2*log(b*x + a) + 12*(B^2*b^
5*d^5*g^4*x^5 + 5*B^2*b^5*c*d^4*g^4*x^4 + 10*B^2*b^5*c^2*d^3*g^4*x^3 + 10*B
^2*b^5*c^3*d^2*g^4*x^2 + 5*B^2*b^5*c^4*d*g^4*x)*log((b*x + a)^n))*log((d*x
+ c)^n))/(b^5*d)
```

**mupad** **[F]** time = 0.00, size = -1, normalized size = -0.00

$$\int \left(c\,g+d\,g\,x\right)^4\,{\left(A+B\,\ln\left(e\,{\left(\frac{a+b\,x}{c+d\,x}\right)}^n\right)\right)}^2 \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*g + d*g*x)^4*(A + B*log(e*((a + b*x)/(c + d*x))^n))^2,x)
```

```
[Out] int((c*g + d*g*x)^4*(A + B*log(e*((a + b*x)/(c + d*x))^n))^2, x)
```

**sympy [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*g*x+c*g)**4*(A+B*ln(e*((b*x+a)/(d*x+c))**n))**2,x)
```

```
[Out] Timed out
```

## 3.39 $\int (cg+dgx)^3 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2 dx$

**Optimal**. Leaf size=454

$$\frac{Bg^3n(bc-ad)^4\log\left(1-\frac{b(c+dx)}{d(a+bx)}\right)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{2b^4d}-\frac{Bg^3n(a+bx)(bc-ad)^3\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{2b^4}-\frac{Bg^3n}{}$$

```
[Out] 5/12*B^2*(-a*d+b*c)^3*g^3*n^2*x/b^3+1/12*B^2*(-a*d+b*c)^2*g^3*n^2*(d*x+c)^2
/b^2/d-1/2*B*(-a*d+b*c)^3*g^3*n*(b*x+a)*(A+B*ln(e*((b*x+a)/(d*x+c))^n))/b^4
-1/4*B*(-a*d+b*c)^2*g^3*n*(d*x+c)^2*(A+B*ln(e*((b*x+a)/(d*x+c))^n))/b^2/d-1
/6*B*(-a*d+b*c)*g^3*n*(d*x+c)^3*(A+B*ln(e*((b*x+a)/(d*x+c))^n))/b/d+1/4*g^3
*(d*x+c)^4*(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2/d+5/12*B^2*(-a*d+b*c)^4*g^3*n^
2*ln((b*x+a)/(d*x+c))/b^4/d+11/12*B^2*(-a*d+b*c)^4*g^3*n^2*ln(d*x+c)/b^4/d+
1/2*B*(-a*d+b*c)^4*g^3*n*(A+B*ln(e*((b*x+a)/(d*x+c))^n))*ln(1-b*(d*x+c)/d/(
b*x+a))/b^4/d-1/2*B^2*(-a*d+b*c)^4*g^3*n^2*polylog(2,b*(d*x+c)/d/(b*x+a))/b
^4/d
```

**Rubi [A]** time = 0.68, antiderivative size = 544, normalized size of antiderivative = 1.20, number of steps used = 23, number of rules used = 13, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.371, Rules used = {2525, 12, 2528, 2486, 31, 2524, 2418, 2390, 2301, 2394, 2393, 2391, 43}

$$-\frac{B^2g^3n^2(bc-ad)^4\text{PolyLog}\left(2,-\frac{d(a+bx)}{bc-ad}\right)}{2b^4d}-\frac{Bg^3n(bc-ad)^4\log(a+bx)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{2b^4d}-\frac{Bg^3n(c+dx)^2(bc}{}$$

Antiderivative was successfully verified.

```
[In] Int[(c*g + d*g*x)^3*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2,x]
```

```
[Out] -(A*B*(b*c - a*d)^3*g^3*n*x)/(2*b^3) + (5*B^2*(b*c - a*d)^3*g^3*n^2*x)/(12*
b^3) + (B^2*(b*c - a*d)^2*g^3*n^2*(c + d*x)^2)/(12*b^2*d) + (5*B^2*(b*c - a
*d)^4*g^3*n^2*Log[a + b*x])/(12*b^4*d) + (B^2*(b*c - a*d)^4*g^3*n^2*Log[a +
 b*x]^2)/(4*b^4*d) - (B^2*(b*c - a*d)^3*g^3*n*(a + b*x)*Log[e*((a + b*x)/(c
 + d*x))^n])/(2*b^4) - (B*(b*c - a*d)^2*g^3*n*(c + d*x)^2*(A + B*Log[e*((a
+ b*x)/(c + d*x))^n]))/(4*b^2*d) - (B*(b*c - a*d)*g^3*n*(c + d*x)^3*(A + B*
Log[e*((a + b*x)/(c + d*x))^n]))/(6*b*d) - (B*(b*c - a*d)^4*g^3*n*Log[a + b
*x]*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/(2*b^4*d) + (g^3*(c + d*x)^4*(A
 + B*Log[e*((a + b*x)/(c + d*x))^n])^2)/(4*d) + (B^2*(b*c - a*d)^4*g^3*n^2*
Log[c + d*x])/(2*b^4*d) - (B^2*(b*c - a*d)^4*g^3*n^2*Log[a + b*x]*Log[(b*(c
 + d*x))/(b*c - a*d)])/(2*b^4*d) - (B^2*(b*c - a*d)^4*g^3*n^2*PolyLog[2, -(
(d*(a + b*x))/(b*c - a*d))])/(2*b^4*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2301

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

Rule 2390

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_))^(q_.), x_Symbol] :> Dist[1/e, Subst[Int[((f*x)/d)^q*(a + b*Log[c*x^
n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && E
qQ[e*f - d*g, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2393

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + (c*e*x)/g])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2394

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d + e*x
```

```
)^n]))/g, x] - Dist[(b*e*n)/g, Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2418

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*(RFx_), x_Sy
mbol] :> With[{u = ExpandIntegrand[(a + b*Log[c*(d + e*x)^n])^p, RFx, x]},
Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d, e, n}, x] && RationalFunctionQ[
RFx, x] && IntegerQ[p]
```

Rule 2486

```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))
^(r_.)]^(s_.), x_Symbol] :> Simp[((a + b*x)*Log[e*(f*(a + b*x)^p*(c + d*x)^
q)^r]^s)/b, x] + Dist[(q*r*s*(b*c - a*d))/b, Int[Log[e*(f*(a + b*x)^p*(c +
d*x)^q)^r]^(s - 1)/(c + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, p, q, r, s
}, x] && NeQ[b*c - a*d, 0] && EqQ[p + q, 0] && IGtQ[s, 0]
```

Rule 2524

```
Int[((a_.) + Log[(c_.)*(RFx_)^(p_.)]*(b_.))^(n_.)/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[(Log[d + e*x]*(a + b*Log[c*RFx^p])^n)/e, x] - Dist[(b*n*p)/e
, Int[(Log[d + e*x]*(a + b*Log[c*RFx^p])^(n - 1)*D[RFx, x])/RFx, x], x] /;
FreeQ[{a, b, c, d, e, p}, x] && RationalFunctionQ[RFx, x] && IGtQ[n, 0]
```

Rule 2525

```
Int[((a_.) + Log[(c_.)*(RFx_)^(p_.)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_))^(m_.
), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(a + b*Log[c*RFx^p])^n)/(e*(m + 1))
, x] - Dist[(b*n*p)/(e*(m + 1)), Int[SimplifyIntegrand[((d + e*x)^(m + 1)*(
a + b*Log[c*RFx^p])^(n - 1)*D[RFx, x])/RFx, x], x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && RationalFunctionQ[RFx, x] && IGtQ[n, 0] && (EqQ[n, 1] ||
IntegerQ[m]) && NeQ[m, -1]
```

Rule 2528

```
Int[((a_.) + Log[(c_.)*(RFx_)^(p_.)]*(b_.))^(n_.)*(RGx_), x_Symbol] :> With
[{u = ExpandIntegrand[(a + b*Log[c*RFx^p])^n, RGx, x]}, Int[u, x] /; SumQ[u
]] /; FreeQ[{a, b, c, p}, x] && RationalFunctionQ[RFx, x] && RationalFuncti
onQ[RGx, x] && IGtQ[n, 0]
```

Rubi steps

$$\int (cg+dgx)^3 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2 dx = \frac{g^3(c+dx)^4\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{4d} - \frac{(Bn)\int \frac{(bc-ad)g^4(c+dx)^3\left(A\cdots\right.}{a+b\cdots}}{2dg}$$

$$= \frac{g^3(c+dx)^4\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{4d} - \frac{\left(B(bc-ad)g^3n\right)\int \frac{(c+dx\cdots}{\cdots}}{2d\cdots}$$

$$= \frac{g^3(c+dx)^4\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{4d} - \frac{\left(B(bc-ad)g^3n\right)\int\left(\frac{d(bc\cdots}{\cdots}\right.}{\cdots}$$

$$= \frac{g^3(c+dx)^4\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{4d} - \frac{\left(B(bc-ad)g^3n\right)\int(c+\cdots}{\cdots}$$

$$= -\frac{AB(bc-ad)^3g^3nx}{2b^3} - \frac{B(bc-ad)^2g^3n(c+dx)^2\left(A+B\log\left(e\left(\cdots\right.\right.\right.}{4b^2d}$$

$$= -\frac{AB(bc-ad)^3g^3nx}{2b^3} - \frac{B^2(bc-ad)^3g^3n(a+bx)\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\cdots}{2b^4}$$

$$= -\frac{AB(bc-ad)^3g^3nx}{2b^3} - \frac{B^2(bc-ad)^3g^3n(a+bx)\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\cdots}{2b^4}$$

$$= -\frac{AB(bc-ad)^3g^3nx}{2b^3} + \frac{5B^2(bc-ad)^3g^3n^2x}{12b^3} + \frac{B^2(bc-ad)^2g^3n^2\cdots}{12b^2d}$$

$$= -\frac{AB(bc-ad)^3g^3nx}{2b^3} + \frac{5B^2(bc-ad)^3g^3n^2x}{12b^3} + \frac{B^2(bc-ad)^2g^3n^2\cdots}{12b^2d}$$

$$= -\frac{AB(bc-ad)^3g^3nx}{2b^3} + \frac{5B^2(bc-ad)^3g^3n^2x}{12b^3} + \frac{B^2(bc-ad)^2g^3n^2\cdots}{12b^2d}$$

**Mathematica [A]** time = 0.34, size = 409, normalized size = 0.90

$$g^3\left((c+dx)^4\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2-\frac{Bn(bc-ad)\left(2b^3(c+dx)^3\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)+3b^2(c+dx)^2(bc-ad)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)+6(bc-a\cdots}{\cdots}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*g + d*g*x)^3*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2,x]
```

```
[Out] (g^3*((c + d*x)^4*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2 - (B*(b*c - a*d)
*n*(6*A*b*d*(b*c - a*d)^2*x - 3*B*(b*c - a*d)^2*n*(b*d*x + (b*c - a*d)*Log[
a + b*x]) - B*(b*c - a*d)*n*(2*b*d*(b*c - a*d)*x + b^2*(c + d*x)^2 + 2*(b*c
 - a*d)^2*Log[a + b*x]) + 6*B*d*(b*c - a*d)^2*(a + b*x)*Log[e*((a + b*x)/(c
 + d*x))^n] + 3*b^2*(b*c - a*d)*(c + d*x)^2*(A + B*Log[e*((a + b*x)/(c + d*
x))^n]) + 2*b^3*(c + d*x)^3*(A + B*Log[e*((a + b*x)/(c + d*x))^n]) + 6*(b*c
 - a*d)^3*Log[a + b*x]*(A + B*Log[e*((a + b*x)/(c + d*x))^n]) - 6*B*(b*c -
a*d)^3*n*Log[c + d*x] - 3*B*(b*c - a*d)^3*n*(Log[a + b*x]*(Log[a + b*x] - 2
*Log[(b*(c + d*x))/(b*c - a*d)]) - 2*PolyLog[2, (d*(a + b*x))/(-(b*c) + a*d
)])))/(3*b^4)))/(4*d)
```

**fricas [F]** time = 1.04, size = 0, normalized size = 0.00

$$\text{integral}\left(A^2d^3g^3x^3+3\,A^2cd^2g^3x^2+3\,A^2c^2dg^3x+A^2c^3g^3+\left(B^2d^3g^3x^3+3\,B^2cd^2g^3x^2+3\,B^2c^2dg^3x+B^2c^3g^3\right)\text{lo}\cdots\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*g*x+c*g)^3*(A+B*log(e*((b*x+a)/(d*x+c))^n))^2,x, algorithm="fr
icas")
```

```
[Out] integral(A^2*d^3*g^3*x^3 + 3*A^2*c*d^2*g^3*x^2 + 3*A^2*c^2*d*g^3*x + A^2*c^
3*g^3 + (B^2*d^3*g^3*x^3 + 3*B^2*c*d^2*g^3*x^2 + 3*B^2*c^2*d*g^3*x + B^2*c^
3*g^3)*log(e*((b*x + a)/(d*x + c))^n)^2 + 2*(A*B*d^3*g^3*x^3 + 3*A*B*c*d^2*
g^3*x^2 + 3*A*B*c^2*d*g^3*x + A*B*c^3*g^3)*log(e*((b*x + a)/(d*x + c))^n),
x)
```

**giac [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*g*x+c*g)^3*(A+B*log(e*((b*x+a)/(d*x+c))^n))^2,x, algorithm="gi
ac")
```

```
[Out] Timed out
```

**maple [F]** time = 0.27, size = 0, normalized size = 0.00

$$\int \left(dgx + cg\right)^{3} \left(B \ln \left(e \left(\frac{bx + a}{dx + c}\right)^{n}\right) + A\right)^{2} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*g*x+c*g)^3*(B*ln(e*((b*x+a)/(d*x+c))^n)+A)^2,x)
```

```
[Out] int((d*g*x+c*g)^3*(B*ln(e*((b*x+a)/(d*x+c))^n)+A)^2,x)
```

**maxima [B]** time = 4.85, size = 2129, normalized size = 4.69

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*g*x+c*g)^3*(A+B*log(e*((b*x+a)/(d*x+c))^n))^2,x, algorithm="ma
xima")
```

```
[Out] 1/2*A*B*d^3*g^3*x^4*log(e*(b*x/(d*x + c) + a/(d*x + c))^n) + 1/4*A^2*d^3*g^
3*x^4 + 2*A*B*c*d^2*g^3*x^3*log(e*(b*x/(d*x + c) + a/(d*x + c))^n) + A^2*c*
d^2*g^3*x^3 + 3*A*B*c^2*d*g^3*x^2*log(e*(b*x/(d*x + c) + a/(d*x + c))^n) +
3/2*A^2*c^2*d*g^3*x^2 - 1/12*A*B*d^3*g^3*n*(6*a^4*log(b*x + a)/b^4 - 6*c^4*
log(d*x + c)/d^4 + (2*(b^3*c*d^2 - a*b^2*d^3)*x^3 - 3*(b^3*c^2*d - a^2*b*d^
3)*x^2 + 6*(b^3*c^3 - a^3*d^3)*x)/(b^3*d^3)) + A*B*c*d^2*g^3*n*(2*a^3*log(b
*x + a)/b^3 - 2*c^3*log(d*x + c)/d^3 - ((b^2*c*d - a*b*d^2)*x^2 - 2*(b^2*c^
2 - a^2*d^2)*x)/(b^2*d^2)) - 3*A*B*c^2*d*g^3*n*(a^2*log(b*x + a)/b^2 - c^2*
log(d*x + c)/d^2 + (b*c - a*d)*x/(b*d)) + 2*A*B*c^3*g^3*n*(a*log(b*x + a)/b
 - c*log(d*x + c)/d) + 2*A*B*c^3*g^3*x*log(e*(b*x/(d*x + c) + a/(d*x + c))^
n) + A^2*c^3*g^3*x - 1/12*(26*a*b^2*c^3*d*g^3*n^2 - 21*a^2*b*c^2*d^2*g^3*n^
2 + 6*a^3*c*d^3*g^3*n^2 - (11*g^3*n^2 - 6*g^3*n*log(e))*b^3*c^4)*B^2*log(d*
x + c)/(b^3*d) - 1/2*(b^4*c^4*g^3*n^2 - 4*a*b^3*c^3*d*g^3*n^2 + 6*a^2*b^2*c
^2*d^2*g^3*n^2 - 4*a^3*b*c*d^3*g^3*n^2 + a^4*d^4*g^3*n^2)*(log(b*x + a)*log
((b*d*x + a*d)/(b*c - a*d) + 1) + dilog(-(b*d*x + a*d)/(b*c - a*d)))*B^2/(b
^4*d) + 1/12*(3*B^2*b^4*d^4*g^3*x^4*log(e)^2 + 6*B^2*b^4*c^4*g^3*n^2*log(b*
x + a)*log(d*x + c) - 3*B^2*b^4*c^4*g^3*n^2*log(d*x + c)^2 + 2*(a*b^3*d^4*g
^3*n*log(e) - (g^3*n*log(e) - 6*g^3*log(e)^2)*b^4*c*d^3)*B^2*x^3 + ((g^3*n^
2 - 9*g^3*n*log(e) + 18*g^3*log(e)^2)*b^4*c^2*d^2 - 2*(g^3*n^2 - 6*g^3*n*lo
g(e))*a*b^3*c*d^3 + (g^3*n^2 - 3*g^3*n*log(e))*a^2*b^2*d^4)*B^2*x^2 - 3*(4*
a*b^3*c^3*d*g^3*n^2 - 6*a^2*b^2*c^2*d^2*g^3*n^2 + 4*a^3*b*c*d^3*g^3*n^2 - a
^4*d^4*g^3*n^2)*B^2*log(b*x + a)^2 + ((7*g^3*n^2 - 18*g^3*n*log(e) + 12*g^3
*log(e)^2)*b^4*c^3*d - (19*g^3*n^2 - 36*g^3*n*log(e))*a*b^3*c^2*d^2 + (17*g
^3*n^2 - 24*g^3*n*log(e))*a^2*b^2*c*d^3 - (5*g^3*n^2 - 6*g^3*n*log(e))*a^3*
```

```
b*d^4)*B^2*x - (6*(3*g^3*n^2 - 4*g^3*n*log(e))*a*b^3*c^3*d - 9*(5*g^3*n^2 -
 4*g^3*n*log(e))*a^2*b^2*c^2*d^2 + 2*(19*g^3*n^2 - 12*g^3*n*log(e))*a^3*b*c
*d^3 - (11*g^3*n^2 - 6*g^3*n*log(e))*a^4*d^4)*B^2*log(b*x + a) + 3*(B^2*b^4
*d^4*g^3*x^4 + 4*B^2*b^4*c*d^3*g^3*x^3 + 6*B^2*b^4*c^2*d^2*g^3*x^2 + 4*B^2*
b^4*c^3*d*g^3*x)*log((b*x + a)^n)^2 + 3*(B^2*b^4*d^4*g^3*x^4 + 4*B^2*b^4*c*
d^3*g^3*x^3 + 6*B^2*b^4*c^2*d^2*g^3*x^2 + 4*B^2*b^4*c^3*d*g^3*x)*log((d*x +
 c)^n)^2 + (6*B^2*b^4*d^4*g^3*x^4*log(e) - 6*B^2*b^4*c^4*g^3*n*log(d*x + c)
 + 2*(a*b^3*d^4*g^3*n - (g^3*n - 12*g^3*log(e))*b^4*c*d^3)*B^2*x^3 + 3*(4*a
*b^3*c*d^3*g^3*n - a^2*b^2*d^4*g^3*n - 3*(g^3*n - 4*g^3*log(e))*b^4*c^2*d^2
)*B^2*x^2 + 6*(6*a*b^3*c^2*d^2*g^3*n - 4*a^2*b^2*c*d^3*g^3*n + a^3*b*d^4*g^
3*n - (3*g^3*n - 4*g^3*log(e))*b^4*c^3*d)*B^2*x + 6*(4*a*b^3*c^3*d*g^3*n -
6*a^2*b^2*c^2*d^2*g^3*n + 4*a^3*b*c*d^3*g^3*n - a^4*d^4*g^3*n)*B^2*log(b*x
+ a))*log((b*x + a)^n) - (6*B^2*b^4*d^4*g^3*x^4*log(e) - 6*B^2*b^4*c^4*g^3*
n*log(d*x + c) + 2*(a*b^3*d^4*g^3*n - (g^3*n - 12*g^3*log(e))*b^4*c*d^3)*B^
2*x^3 + 3*(4*a*b^3*c*d^3*g^3*n - a^2*b^2*d^4*g^3*n - 3*(g^3*n - 4*g^3*log(e
))*b^4*c^2*d^2)*B^2*x^2 + 6*(6*a*b^3*c^2*d^2*g^3*n - 4*a^2*b^2*c*d^3*g^3*n
+ a^3*b*d^4*g^3*n - (3*g^3*n - 4*g^3*log(e))*b^4*c^3*d)*B^2*x + 6*(4*a*b^3*
c^3*d*g^3*n - 6*a^2*b^2*c^2*d^2*g^3*n + 4*a^3*b*c*d^3*g^3*n - a^4*d^4*g^3*n
)*B^2*log(b*x + a) + 6*(B^2*b^4*d^4*g^3*x^4 + 4*B^2*b^4*c*d^3*g^3*x^3 + 6*B
^2*b^4*c^2*d^2*g^3*x^2 + 4*B^2*b^4*c^3*d*g^3*x)*log((b*x + a)^n))*log((d*x
+ c)^n))/(b^4*d)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.00

$$\int (c\,g+d\,g\,x)^3 \left(A+B\,\ln\left(e\left(\frac{a+b\,x}{c+d\,x}\right)^n\right)\right)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*g + d*g*x)^3*(A + B*log(e*((a + b*x)/(c + d*x))^n))^2,x)
[Out] int((c*g + d*g*x)^3*(A + B*log(e*((a + b*x)/(c + d*x))^n))^2, x)
```

**sympy [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*g*x+c*g)**3*(A+B*ln(e*((b*x+a)/(d*x+c))**n))**2,x)
[Out] Timed out
```

# 3.40 $\int (cg+dgx)^2 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2 \, dx$

**Optimal**. Leaf size=361

$$\frac{2Bg^2n(bc-ad)^3\log\left(1-\frac{b(c+dx)}{d(a+bx)}\right)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{3b^3d}-\frac{2Bg^2n(a+bx)(bc-ad)^2\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{3b^3}-\frac{B}{}$$

```
[Out] 1/3*B^2*(-a*d+b*c)^2*g^2*n^2*x/b^2-2/3*B*(-a*d+b*c)^2*g^2*n*(b*x+a)*(A+B*ln
(e*((b*x+a)/(d*x+c))^n))/b^3-1/3*B*(-a*d+b*c)*g^2*n*(d*x+c)^2*(A+B*ln(e*((b
*x+a)/(d*x+c))^n))/b/d+1/3*g^2*(d*x+c)^3*(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2/
d+1/3*B^2*(-a*d+b*c)^3*g^2*n^2*ln((b*x+a)/(d*x+c))/b^3/d+B^2*(-a*d+b*c)^3*g
^2*n^2*ln(d*x+c)/b^3/d+2/3*B*(-a*d+b*c)^3*g^2*n*(A+B*ln(e*((b*x+a)/(d*x+c))
^n))*ln(1-b*(d*x+c)/d/(b*x+a))/b^3/d-2/3*B^2*(-a*d+b*c)^3*g^2*n^2*polylog(2
,b*(d*x+c)/d/(b*x+a))/b^3/d
```

**Rubi [A]** time = 0.57, antiderivative size = 454, normalized size of antiderivative = 1.26, number of steps used = 19, number of rules used = 13, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.371, Rules used = {2525, 12, 2528, 2486, 31, 2524, 2418, 2390, 2301, 2394, 2393, 2391, 43}

$$-\frac{2B^2g^2n^2(bc-ad)^3\text{PolyLog}\left(2,-\frac{d(a+bx)}{bc-ad}\right)}{3b^3d}-\frac{2Bg^2n(bc-ad)^3\log(a+bx)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{3b^3d}-\frac{2ABg^2nx(bc}{3b^2}$$

Antiderivative was successfully verified.

```
[In] Int[(c*g + d*g*x)^2*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2,x]
```

```
[Out] (-2*A*B*(b*c - a*d)^2*g^2*n*x)/(3*b^2) + (B^2*(b*c - a*d)^2*g^2*n^2*x)/(3*b
^2) + (B^2*(b*c - a*d)^3*g^2*n^2*Log[a + b*x])/(3*b^3*d) + (B^2*(b*c - a*d)
^3*g^2*n^2*Log[a + b*x]^2)/(3*b^3*d) - (2*B^2*(b*c - a*d)^2*g^2*n*(a + b*x)
*Log[e*((a + b*x)/(c + d*x))^n])/(3*b^3) - (B*(b*c - a*d)*g^2*n*(c + d*x)^2
*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/(3*b*d) - (2*B*(b*c - a*d)^3*g^2*n
*Log[a + b*x]*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/(3*b^3*d) + (g^2*(c +
 d*x)^3*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2)/(3*d) + (2*B^2*(b*c - a*d
)^3*g^2*n^2*Log[c + d*x])/(3*b^3*d) - (2*B^2*(b*c - a*d)^3*g^2*n^2*Log[a +
b*x]*Log[(b*(c + d*x))/(b*c - a*d)])/(3*b^3*d) - (2*B^2*(b*c - a*d)^3*g^2*n
^2*PolyLog[2, -((d*(a + b*x))/(b*c - a*d))])/(3*b^3*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2301

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

Rule 2390

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_))^(q_.), x_Symbol] :> Dist[1/e, Subst[Int[((f*x)/d)^q*(a + b*Log[c*x^
n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && E
qQ[e*f - d*g, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2393

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + (c*e*x)/g])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2394

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d + e*x
)^n]))/g, x] - Dist[(b*e*n)/g, Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2418

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*(RFx_), x_Sy
mbol] :> With[{u = ExpandIntegrand[(a + b*Log[c*(d + e*x)^n])^p, RFx, x]},
Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d, e, n}, x] && RationalFunctionQ[
RFx, x] && IntegerQ[p]
```

Rule 2486

```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))
^(r_.)]^(s_.), x_Symbol] :> Simp[((a + b*x)*Log[e*(f*(a + b*x)^p*(c + d*x)^
q)^r]^s)/b, x] + Dist[(q*r*s*(b*c - a*d))/b, Int[Log[e*(f*(a + b*x)^p*(c +
d*x)^q)^r]^(s - 1)/(c + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, p, q, r, s
}, x] && NeQ[b*c - a*d, 0] && EqQ[p + q, 0] && IGtQ[s, 0]
```

Rule 2524

```
Int[((a_.) + Log[(c_.)*(RFx_)^(p_.)]*(b_.))^(n_.)/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[(Log[d + e*x]*(a + b*Log[c*RFx^p])^n)/e, x] - Dist[(b*n*p)/e
, Int[(Log[d + e*x]*(a + b*Log[c*RFx^p])^(n - 1)*D[RFx, x])/RFx, x], x] /;
FreeQ[{a, b, c, d, e, p}, x] && RationalFunctionQ[RFx, x] && IGtQ[n, 0]
```

Rule 2525

```
Int[((a_.) + Log[(c_.)*(RFx_)^(p_.)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_))^(m_.
), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(a + b*Log[c*RFx^p])^n)/(e*(m + 1))
, x] - Dist[(b*n*p)/(e*(m + 1)), Int[SimplifyIntegrand[((d + e*x)^(m + 1)*(
a + b*Log[c*RFx^p])^(n - 1)*D[RFx, x])/RFx, x], x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && RationalFunctionQ[RFx, x] && IGtQ[n, 0] && (EqQ[n, 1] ||
IntegerQ[m]) && NeQ[m, -1]
```

Rule 2528

```
Int[((a_.) + Log[(c_.)*(RFx_)^(p_.)]*(b_.))^(n_.)*(RGx_), x_Symbol] :> With
[{u = ExpandIntegrand[(a + b*Log[c*RFx^p])^n, RGx, x]}, Int[u, x] /; SumQ[u
]] /; FreeQ[{a, b, c, p}, x] && RationalFunctionQ[RFx, x] && RationalFuncti
onQ[RGx, x] && IGtQ[n, 0]
```

Rubi steps

$$\int (cg+dgx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2 dx = \frac{g^2(c+dx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{3d} - \frac{(2Bn)\int \frac{(bc-ad)g^3(c+dx)^2\left(A+\right.}{a+b}}{3dg}$$

$$= \frac{g^2(c+dx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{3d} - \frac{\left(2B(bc-ad)g^2n\right)\int \frac{(c+dx)}{}}{3d}$$

$$= \frac{g^2(c+dx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{3d} - \frac{\left(2B(bc-ad)g^2n\right)\int\left(\frac{d(bc-}{}\right.}{}$$

$$= \frac{g^2(c+dx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{3d} - \frac{\left(2B(bc-ad)g^2n\right)\int (c+}{}$$

$$= -\frac{2AB(bc-ad)^2g^2nx}{3b^2} - \frac{B(bc-ad)g^2n(c+dx)^2\left(A+B\log\left(e\left(\frac{a+}{c+}\right.\right.\right.}{3bd}$$

$$= -\frac{2AB(bc-ad)^2g^2nx}{3b^2} - \frac{2B^2(bc-ad)^2g^2n(a+bx)\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right.}{3b^3}$$

$$= -\frac{2AB(bc-ad)^2g^2nx}{3b^2} - \frac{2B^2(bc-ad)^2g^2n(a+bx)\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right.}{3b^3}$$

$$= -\frac{2AB(bc-ad)^2g^2nx}{3b^2} + \frac{B^2(bc-ad)^2g^2n^2x}{3b^2} + \frac{B^2(bc-ad)^3g^2n^2\,\mathrm{l}}{3b^3d}$$

$$= -\frac{2AB(bc-ad)^2g^2nx}{3b^2} + \frac{B^2(bc-ad)^2g^2n^2x}{3b^2} + \frac{B^2(bc-ad)^3g^2n^2\,\mathrm{l}}{3b^3d}$$

$$= -\frac{2AB(bc-ad)^2g^2nx}{3b^2} + \frac{B^2(bc-ad)^2g^2n^2x}{3b^2} + \frac{B^2(bc-ad)^3g^2n^2\,\mathrm{l}}{3b^3d}$$

**Mathematica [A]** time = 0.24, size = 303, normalized size = 0.84

$$\frac{g^2\left((c+dx)^3\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2-\frac{Bn(bc-ad)\left(b^2(c+dx)^2\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)+2(bc-ad)^2\log(a+bx)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)+2Ab\cdots\right.}{\cdots}\right.}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*g + d*g*x)^2*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2,x]
```

```
[Out] (g^2*((c + d*x)^3*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2 - (B*(b*c - a*d)
*n*(2*A*b*d*(b*c - a*d)*x - B*(b*c - a*d)*n*(b*d*x + (b*c - a*d)*Log[a + b*
x]) + 2*B*d*(b*c - a*d)*(a + b*x)*Log[e*((a + b*x)/(c + d*x))^n] + b^2*(c +
 d*x)^2*(A + B*Log[e*((a + b*x)/(c + d*x))^n]) + 2*(b*c - a*d)^2*Log[a + b*
x]*(A + B*Log[e*((a + b*x)/(c + d*x))^n]) - 2*B*(b*c - a*d)^2*n*Log[c + d*x
] - B*(b*c - a*d)^2*n*(Log[a + b*x]*(Log[a + b*x] - 2*Log[(b*(c + d*x))/(b*
c - a*d)]) - 2*PolyLog[2, (d*(a + b*x))/(-(b*c) + a*d)])))/b^3))/(3*d)
```

**fricas [F]** time = 0.77, size = 0, normalized size = 0.00

$$\text{integral}\left(A^2d^2g^2x^2+2\,A^2cdg^2x+A^2c^2g^2+\left(B^2d^2g^2x^2+2\,B^2cdg^2x+B^2c^2g^2\right)\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)^2+2\left(ABd^2g^2\cdots\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*g*x+c*g)^2*(A+B*log(e*((b*x+a)/(d*x+c))^n))^2,x, algorithm="fr
icas")
```

```
[Out] integral(A^2*d^2*g^2*x^2 + 2*A^2*c*d*g^2*x + A^2*c^2*g^2 + (B^2*d^2*g^2*x^2
 + 2*B^2*c*d*g^2*x + B^2*c^2*g^2)*log(e*((b*x + a)/(d*x + c))^n)^2 + 2*(A*B
*d^2*g^2*x^2 + 2*A*B*c*d*g^2*x + A*B*c^2*g^2)*log(e*((b*x + a)/(d*x + c))^n
), x)
```

**giac [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*g*x+c*g)^2*(A+B*log(e*((b*x+a)/(d*x+c))^n))^2,x, algorithm="gi
ac")
```

```
[Out] Timed out
```

**maple [F]** time = 0.26, size = 0, normalized size = 0.00

$$\int \left(dgx + cg\right)^{2} \left(B \ln \left(e \left(\frac{bx + a}{dx + c}\right)^{n}\right) + A\right)^{2} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*g*x+c*g)^2*(B*ln(e*((b*x+a)/(d*x+c))^n)+A)^2,x)
```

```
[Out] int((d*g*x+c*g)^2*(B*ln(e*((b*x+a)/(d*x+c))^n)+A)^2,x)
```

**maxima [B]** time = 5.75, size = 1473, normalized size = 4.08

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*g*x+c*g)^2*(A+B*log(e*((b*x+a)/(d*x+c))^n))^2,x, algorithm="ma
xima")
```

```
[Out] 2/3*A*B*d^2*g^2*x^3*log(e*(b*x/(d*x + c) + a/(d*x + c))^n) + 1/3*A^2*d^2*g^
2*x^3 + 2*A*B*c*d*g^2*x^2*log(e*(b*x/(d*x + c) + a/(d*x + c))^n) + A^2*c*d*
g^2*x^2 + 1/3*A*B*d^2*g^2*n*(2*a^3*log(b*x + a)/b^3 - 2*c^3*log(d*x + c)/d^
3 - ((b^2*c*d - a*b*d^2)*x^2 - 2*(b^2*c^2 - a^2*d^2)*x)/(b^2*d^2)) - 2*A*B*
c*d*g^2*n*(a^2*log(b*x + a)/b^2 - c^2*log(d*x + c)/d^2 + (b*c - a*d)*x/(b*d
)) + 2*A*B*c^2*g^2*n*(a*log(b*x + a)/b - c*log(d*x + c)/d) + 2*A*B*c^2*g^2*
x*log(e*(b*x/(d*x + c) + a/(d*x + c))^n) + A^2*c^2*g^2*x - 1/3*(5*a*b*c^2*d
*g^2*n^2 - 2*a^2*c*d^2*g^2*n^2 - (3*g^2*n^2 - 2*g^2*n*log(e))*b^2*c^3)*B^2*
log(d*x + c)/(b^2*d) - 2/3*(b^3*c^3*g^2*n^2 - 3*a*b^2*c^2*d*g^2*n^2 + 3*a^2
*b*c*d^2*g^2*n^2 - a^3*d^3*g^2*n^2)*(log(b*x + a)*log((b*d*x + a*d)/(b*c -
a*d) + 1) + dilog(-(b*d*x + a*d)/(b*c - a*d)))*B^2/(b^3*d) + 1/3*(B^2*b^3*d
^3*g^2*x^3*log(e)^2 + 2*B^2*b^3*c^3*g^2*n^2*log(b*x + a)*log(d*x + c) - B^2
*b^3*c^3*g^2*n^2*log(d*x + c)^2 + (a*b^2*d^3*g^2*n*log(e) - (g^2*n*log(e) -
 3*g^2*log(e)^2)*b^3*c*d^2)*B^2*x^2 - (3*a*b^2*c^2*d*g^2*n^2 - 3*a^2*b*c*d^
2*g^2*n^2 + a^3*d^3*g^2*n^2)*B^2*log(b*x + a)^2 + ((g^2*n^2 - 4*g^2*n*log(e
) + 3*g^2*log(e)^2)*b^3*c^2*d - 2*(g^2*n^2 - 3*g^2*n*log(e))*a*b^2*c*d^2 +
(g^2*n^2 - 2*g^2*n*log(e))*a^2*b*d^3)*B^2*x - (2*(2*g^2*n^2 - 3*g^2*n*log(e
))*a*b^2*c^2*d - (7*g^2*n^2 - 6*g^2*n*log(e))*a^2*b*c*d^2 + (3*g^2*n^2 - 2*
g^2*n*log(e))*a^3*d^3)*B^2*log(b*x + a) + (B^2*b^3*d^3*g^2*x^3 + 3*B^2*b^3*
c*d^2*g^2*x^2 + 3*B^2*b^3*c^2*d*g^2*x)*log((b*x + a)^n)^2 + (B^2*b^3*d^3*g^
2*x^3 + 3*B^2*b^3*c*d^2*g^2*x^2 + 3*B^2*b^3*c^2*d*g^2*x)*log((d*x + c)^n)^2
 + (2*B^2*b^3*d^3*g^2*x^3*log(e) - 2*B^2*b^3*c^3*g^2*n*log(d*x + c) + (a*b^
2*d^3*g^2*n - (g^2*n - 6*g^2*log(e))*b^3*c*d^2)*B^2*x^2 + 2*(3*a*b^2*c*d^2*
g^2*n - a^2*b*d^3*g^2*n - (2*g^2*n - 3*g^2*log(e))*b^3*c^2*d)*B^2*x + 2*(3*
a*b^2*c^2*d*g^2*n - 3*a^2*b*c*d^2*g^2*n + a^3*d^3*g^2*n)*B^2*log(b*x + a))*
log((b*x + a)^n) - (2*B^2*b^3*d^3*g^2*x^3*log(e) - 2*B^2*b^3*c^3*g^2*n*log(
```

```
d*x + c) + (a*b^2*d^3*g^2*n - (g^2*n - 6*g^2*log(e))*b^3*c*d^2)*B^2*x^2 + 2
*(3*a*b^2*c*d^2*g^2*n - a^2*b*d^3*g^2*n - (2*g^2*n - 3*g^2*log(e))*b^3*c^2*
d)*B^2*x + 2*(3*a*b^2*c^2*d*g^2*n - 3*a^2*b*c*d^2*g^2*n + a^3*d^3*g^2*n)*B^
2*log(b*x + a) + 2*(B^2*b^3*d^3*g^2*x^3 + 3*B^2*b^3*c*d^2*g^2*x^2 + 3*B^2*b
^3*c^2*d*g^2*x)*log((b*x + a)^n))*log((d*x + c)^n))/(b^3*d)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.00

$$\int \left(c\,g+d\,g\,x\right)^2\,{\left(A+B\,\ln\left(e\,{\left(\frac{a+b\,x}{c+d\,x}\right)}^n\right)\right)}^2 \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*g + d*g*x)^2*(A + B*log(e*((a + b*x)/(c + d*x))^n))^2,x)
[Out] int((c*g + d*g*x)^2*(A + B*log(e*((a + b*x)/(c + d*x))^n))^2, x)
```

**sympy [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*g*x+c*g)**2*(A+B*ln(e*((b*x+a)/(d*x+c))**n))**2,x)
[Out] Timed out
```

# 3.41 $\int (cg+dgx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2\,dx$

**Optimal**. Leaf size=220

$$\frac{Bgn(bc-ad)^2\log\left(1-\frac{b(c+dx)}{d(a+bx)}\right)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{b^2d}-\frac{Bgn(a+bx)(bc-ad)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{b^2}+\frac{g(c+dx}{}$$

```
[Out] -B*(-a*d+b*c)*g*n*(b*x+a)*(A+B*ln(e*((b*x+a)/(d*x+c))^n))/b^2+1/2*g*(d*x+c)
^2*(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2/d+B^2*(-a*d+b*c)^2*g*n^2*ln(d*x+c)/b^2
/d+B*(-a*d+b*c)^2*g*n*(A+B*ln(e*((b*x+a)/(d*x+c))^n))*ln(1-b*(d*x+c)/d/(b*x
+a))/b^2/d-B^2*(-a*d+b*c)^2*g*n^2*polylog(2,b*(d*x+c)/d/(b*x+a))/b^2/d
```

**Rubi [A]** time = 0.42, antiderivative size = 307, normalized size of antiderivative = 1.40, number of steps used = 15, number of rules used = 12, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {2525, 12, 2528, 2486, 31, 2524, 2418, 2390, 2301, 2394, 2393, 2391}

$$-\frac{B^2gn^2(bc-ad)^2\text{PolyLog}\left(2,-\frac{d(a+bx)}{bc-ad}\right)}{b^2d}-\frac{Bgn(bc-ad)^2\log(a+bx)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{b^2d}+\frac{g(c+dx)^2\left(B\log\left(\right.\right.}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[(c*g + d*g*x)*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2,x]
```

```
[Out] -((A*B*(b*c - a*d)*g*n*x)/b) + (B^2*(b*c - a*d)^2*g*n^2*Log[a + b*x]^2)/(2*
b^2*d) - (B^2*(b*c - a*d)*g*n*(a + b*x)*Log[e*((a + b*x)/(c + d*x))^n])/b^2
 - (B*(b*c - a*d)^2*g*n*Log[a + b*x]*(A + B*Log[e*((a + b*x)/(c + d*x))^n])
)/(b^2*d) + (g*(c + d*x)^2*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2)/(2*d)
+ (B^2*(b*c - a*d)^2*g*n^2*Log[c + d*x])/(b^2*d) - (B^2*(b*c - a*d)^2*g*n^2
*Log[a + b*x]*Log[(b*(c + d*x))/(b*c - a*d)])/(b^2*d) - (B^2*(b*c - a*d)^2*
g*n^2*PolyLog[2, -((d*(a + b*x))/(b*c - a*d))])/(b^2*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 2301

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

Rule 2390

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_))^(q_.), x_Symbol] :> Dist[1/e, Subst[Int[((f*x)/d)^q*(a + b*Log[c*x^
n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && E
qQ[e*f - d*g, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2393

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + (c*e*x)/g])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2394

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d + e*x
)^n]))/g, x] - Dist[(b*e*n)/g, Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2418

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*(RFx_), x_Sy
mbol] :> With[{u = ExpandIntegrand[(a + b*Log[c*(d + e*x)^n])^p, RFx, x]},
Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d, e, n}, x] && RationalFunctionQ[
RFx, x] && IntegerQ[p]
```

Rule 2486

```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))
^(r_.)]^(s_.), x_Symbol] :> Simp[((a + b*x)*Log[e*(f*(a + b*x)^p*(c + d*x)^
q)^r]^s)/b, x] + Dist[(q*r*s*(b*c - a*d))/b, Int[Log[e*(f*(a + b*x)^p*(c +
d*x)^q)^r]^(s - 1)/(c + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, p, q, r, s
```

```
}, x] && NeQ[b*c - a*d, 0] && EqQ[p + q, 0] && IGtQ[s, 0]
```

Rule 2524

```
Int[((a_.) + Log[(c_.)*(RFx_)^(p_.)]*(b_.))^(n_.)/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[(Log[d + e*x]*(a + b*Log[c*RFx^p])^n)/e, x] - Dist[(b*n*p)/e
, Int[(Log[d + e*x]*(a + b*Log[c*RFx^p])^(n - 1)*D[RFx, x])/RFx, x], x] /;
FreeQ[{a, b, c, d, e, p}, x] && RationalFunctionQ[RFx, x] && IGtQ[n, 0]
```

Rule 2525

```
Int[((a_.) + Log[(c_.)*(RFx_)^(p_.)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_))^(m_.
), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(a + b*Log[c*RFx^p])^n)/(e*(m + 1))
, x] - Dist[(b*n*p)/(e*(m + 1)), Int[SimplifyIntegrand[((d + e*x)^(m + 1)*(
a + b*Log[c*RFx^p])^(n - 1)*D[RFx, x])/RFx, x], x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && RationalFunctionQ[RFx, x] && IGtQ[n, 0] && (EqQ[n, 1] ||
IntegerQ[m]) && NeQ[m, -1]
```

Rule 2528

```
Int[((a_.) + Log[(c_.)*(RFx_)^(p_.)]*(b_.))^(n_.)*(RGx_), x_Symbol] :> With
[{u = ExpandIntegrand[(a + b*Log[c*RFx^p])^n, RGx, x]}, Int[u, x] /; SumQ[u
]] /; FreeQ[{a, b, c, p}, x] && RationalFunctionQ[RFx, x] && RationalFuncti
onQ[RGx, x] && IGtQ[n, 0]
```

Rubi steps

$$\int (cg+dgx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2 dx = \frac{g(c+dx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{2d} - \frac{(Bn)\int \frac{(bc-ad)g^2(c+dx)\left(A+B\,\mathrm{l}\right.}{a+bx}}{dg}$$

$$= \frac{g(c+dx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{2d} - \frac{(B(bc-ad)gn)\int \frac{(c+dx)\left(A\right.}{}}{d}$$

$$= \frac{g(c+dx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{2d} - \frac{(B(bc-ad)gn)\int\left(\frac{d\left(A+B\,\mathrm{l}\right.}{}\right.}{}$$

$$= \frac{g(c+dx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{2d} - \frac{(B(bc-ad)gn)\int\left(A+B\right.}{b}$$

$$= -\frac{AB(bc-ad)gnx}{b} - \frac{B(bc-ad)^2gn\log(a+bx)\left(A+B\log\left(e\left(\frac{a}{c}\right.\right.\right.}{b^2d}$$

$$= -\frac{AB(bc-ad)gnx}{b} - \frac{B^2(bc-ad)gn(a+bx)\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{b^2} - \frac{B(}{}$$

$$= -\frac{AB(bc-ad)gnx}{b} - \frac{B^2(bc-ad)gn(a+bx)\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{b^2} - \frac{B(}{}$$

$$= -\frac{AB(bc-ad)gnx}{b} - \frac{B^2(bc-ad)gn(a+bx)\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{b^2} - \frac{B(}{}$$

$$= -\frac{AB(bc-ad)gnx}{b} + \frac{B^2(bc-ad)^2gn^2\log^2(a+bx)}{2b^2d} - \frac{B^2(bc-ad}{}$$

$$= -\frac{AB(bc-ad)gnx}{b} + \frac{B^2(bc-ad)^2gn^2\log^2(a+bx)}{2b^2d} - \frac{B^2(bc-ad}{}$$

**Mathematica [A]** time = 0.21, size = 216, normalized size = 0.98

$$\frac{g\left(\frac{Bn(bc-ad)\left(-2(bc-ad)\log(a+bx)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+Bn\log\left(\frac{b(c+dx)}{bc-ad}\right)+A\right)-2\left(Bd(a+bx)\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+Bn(ad-bc)\log(c+dx)+Abdx\right)+2Bn(ad-bc)\mathrm{Li}}{b^2}\right.}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*g + d*g*x)*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2,x]
```

```
[Out] (g*((c + d*x)^2*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2 + (B*(b*c - a*d)*n
*(B*(b*c - a*d)*n*Log[a + b*x]^2 - 2*(A*b*d*x + B*d*(a + b*x)*Log[e*((a + b
*x)/(c + d*x))^n] + B*(-(b*c) + a*d)*n*Log[c + d*x]) - 2*(b*c - a*d)*Log[a
+ b*x]*(A + B*Log[e*((a + b*x)/(c + d*x))^n] + B*n*Log[(b*(c + d*x))/(b*c -
 a*d)]) + 2*B*(-(b*c) + a*d)*n*PolyLog[2, (d*(a + b*x))/(-(b*c) + a*d)]))/b
^2))/(2*d)
```

**fricas [F]** time = 0.92, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(A^2dgx + A^2cg + \left(B^2dgx + B^2cg\right)\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)^2 + 2\left(ABdgx + ABcg\right)\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right), x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*g*x+c*g)*(A+B*log(e*((b*x+a)/(d*x+c))^n))^2,x, algorithm="fric
as")
```

```
[Out] integral(A^2*d*g*x + A^2*c*g + (B^2*d*g*x + B^2*c*g)*log(e*((b*x + a)/(d*x
+ c))^n)^2 + 2*(A*B*d*g*x + A*B*c*g)*log(e*((b*x + a)/(d*x + c))^n), x)
```

**giac [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*g*x+c*g)*(A+B*log(e*((b*x+a)/(d*x+c))^n))^2,x, algorithm="giac
")
```

```
[Out] Timed out
```

**maple [F]** time = 0.11, size = 0, normalized size = 0.00

$$\int \left(dgx+cg\right)\left(B\ln\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*g*x+c*g)*(B*ln(e*((b*x+a)/(d*x+c))^n)+A)^2,x)
```

```
[Out] int((d*g*x+c*g)*(B*ln(e*((b*x+a)/(d*x+c))^n)+A)^2,x)
```

**maxima [B]** time = 4.37, size = 825, normalized size = 3.75

$$ABdgx^2\log\left(e\left(\frac{bx}{dx+c}+\frac{a}{dx+c}\right)^n\right)+\frac{1}{2}A^2dgx^2-ABdgn\left(\frac{a^2\log(bx+a)}{b^2}-\frac{c^2\log(dx+c)}{d^2}+\frac{(bc-ad)x}{bd}\right)+2\,ABc$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*g*x+c*g)*(A+B*log(e*((b*x+a)/(d*x+c))^n))^2,x, algorithm="maxi
ma")
```

```
[Out] A*B*d*g*x^2*log(e*(b*x/(d*x + c) + a/(d*x + c))^n) + 1/2*A^2*d*g*x^2 - A*B*
d*g*n*(a^2*log(b*x + a)/b^2 - c^2*log(d*x + c)/d^2 + (b*c - a*d)*x/(b*d)) +
 2*A*B*c*g*n*(a*log(b*x + a)/b - c*log(d*x + c)/d) + 2*A*B*c*g*x*log(e*(b*x
/(d*x + c) + a/(d*x + c))^n) + A^2*c*g*x - (a*c*d*g*n^2 - (g*n^2 - g*n*log(
e))*b*c^2)*B^2*log(d*x + c)/(b*d) - (b^2*c^2*g*n^2 - 2*a*b*c*d*g*n^2 + a^2*
d^2*g*n^2)*(log(b*x + a)*log((b*d*x + a*d)/(b*c - a*d) + 1) + dilog(-(b*d*x
 + a*d)/(b*c - a*d)))*B^2/(b^2*d) + 1/2*(2*B^2*b^2*c^2*g*n^2*log(b*x + a)*l
og(d*x + c) - B^2*b^2*c^2*g*n^2*log(d*x + c)^2 + B^2*b^2*d^2*g*x^2*log(e)^2
 - (2*a*b*c*d*g*n^2 - a^2*d^2*g*n^2)*B^2*log(b*x + a)^2 + 2*(a*b*d^2*g*n*lo
g(e) - (g*n*log(e) - g*log(e)^2)*b^2*c*d)*B^2*x - 2*((g*n^2 - 2*g*n*log(e))
*a*b*c*d - (g*n^2 - g*n*log(e))*a^2*d^2)*B^2*log(b*x + a) + (B^2*b^2*d^2*g*
x^2 + 2*B^2*b^2*c*d*g*x)*log((b*x + a)^n)^2 + (B^2*b^2*d^2*g*x^2 + 2*B^2*b^
2*c*d*g*x)*log((d*x + c)^n)^2 + 2*(B^2*b^2*d^2*g*x^2*log(e) - B^2*b^2*c^2*g
*n*log(d*x + c) + (a*b*d^2*g*n - (g*n - 2*g*log(e))*b^2*c*d)*B^2*x + (2*a*b
*c*d*g*n - a^2*d^2*g*n)*B^2*log(b*x + a))*log((b*x + a)^n) - 2*(B^2*b^2*d^2
*g*x^2*log(e) - B^2*b^2*c^2*g*n*log(d*x + c) + (a*b*d^2*g*n - (g*n - 2*g*lo
g(e))*b^2*c*d)*B^2*x + (2*a*b*c*d*g*n - a^2*d^2*g*n)*B^2*log(b*x + a) + (B^
2*b^2*d^2*g*x^2 + 2*B^2*b^2*c*d*g*x)*log((b*x + a)^n))*log((d*x + c)^n))/(b
^2*d)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.00

$$\int (c\,g+d\,g\,x)\,\left(A+B\,\ln\left(e\,\left(\frac{a+b\,x}{c+d\,x}\right)^n\right)\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*g + d*g*x)*(A + B*log(e*((a + b*x)/(c + d*x))^n))^2,x)
```

```
[Out] int((c*g + d*g*x)*(A + B*log(e*((a + b*x)/(c + d*x))^n))^2, x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$g\left(\int A^2c\,dx+\int A^2dx\,dx+\int B^2c\log\left(e\left(\frac{a}{c+dx}+\frac{bx}{c+dx}\right)^n\right)^2dx+\int 2ABc\log\left(e\left(\frac{a}{c+dx}+\frac{bx}{c+dx}\right)^n\right)dx+\int\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*g*x+c*g)*(A+B*ln(e*((b*x+a)/(d*x+c))**n))**2,x)
```

```
[Out] g*(Integral(A**2*c, x) + Integral(A**2*d*x, x) + Integral(B**2*c*log(e*(a/(
c + d*x) + b*x/(c + d*x))**n)**2, x) + Integral(2*A*B*c*log(e*(a/(c + d*x)
+ b*x/(c + d*x))**n), x) + Integral(B**2*d*x*log(e*(a/(c + d*x) + b*x/(c +
d*x))**n)**2, x) + Integral(2*A*B*d*x*log(e*(a/(c + d*x) + b*x/(c + d*x))**
n), x))
```

# 3.42 $\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{cg+dgx}\,dx$

**Optimal**. Leaf size=137

$$-\frac{2Bn\text{Li}_2\left(\frac{d(a+bx)}{b(c+dx)}\right)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{dg}-\frac{\log\left(\frac{bc-ad}{b(c+dx)}\right)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{dg}+\frac{2B^2n^2\text{Li}_3\left(\frac{d(a+bx)}{b(c+dx)}\right)}{dg}$$

```
[Out] -(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2*ln((-a*d+b*c)/b/(d*x+c))/d/g-2*B*n*(A+B*
ln(e*((b*x+a)/(d*x+c))^n))*polylog(2,d*(b*x+a)/b/(d*x+c))/d/g+2*B^2*n^2*pol
ylog(3,d*(b*x+a)/b/(d*x+c))/d/g
```

**Rubi [B]** time = 3.30, antiderivative size = 782, normalized size of antiderivative = 5.71, number of steps used = 45, number of rules used = 23, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.657, Rules used = {2524, 2528, 2418, 2390, 2301, 2394, 2393, 2391, 6688, 12, 6742, 2499, 2396, 2433, 2374, 6589, 2302, 30, 2500, 2375, 2317, 2440, 2434}

$$-\frac{2ABn\text{PolyLog}\left(2,\frac{b(c+dx)}{bc-ad}\right)}{dg}+\frac{2B^2n\text{PolyLog}\left(2,\frac{b(c+dx)}{bc-ad}\right)\left(-\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+\log\left((a+bx)^n\right)+\log\left((c+dx)^{-n}\right)\right)}{dg}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2/(c*g + d*g*x),x]
```

```
[Out] (B^2*Log[(a + b*x)^n]^2*Log[(b*(c + d*x))/(b*c - a*d)])/(d*g) - (B^2*Log[(a
 + b*x)^n]^2*Log[g*(c + d*x)])/(d*g) + (A*B*n*Log[g*(c + d*x)]^2)/(d*g) - (
B^2*n^2*Log[a + b*x]*Log[g*(c + d*x)]^2)/(d*g) + (B^2*n^2*Log[-((d*(a + b*x
))/(b*c - a*d))]*Log[g*(c + d*x)]^2)/(d*g) + (B^2*n^2*Log[g*(c + d*x)]^3)/(
3*d*g) - (2*B^2*n*Log[a + b*x]*Log[g*(c + d*x)]*Log[(c + d*x)^(-n)])/(d*g)
- (B^2*Log[a + b*x]*Log[(c + d*x)^(-n)]^2)/(d*g) + (B^2*Log[-((d*(a + b*x))
/(b*c - a*d))]*Log[(c + d*x)^(-n)]^2)/(d*g) - (2*A*B*n*Log[-((d*(a + b*x))/
(b*c - a*d))]*Log[c*g + d*g*x])/(d*g) + ((A + B*Log[e*((a + b*x)/(c + d*x))
^n])^2*Log[c*g + d*g*x])/(d*g) + (2*B^2*n*Log[-((d*(a + b*x))/(b*c - a*d))]
*(Log[(a + b*x)^n] - Log[e*((a + b*x)/(c + d*x))^n] + Log[(c + d*x)^(-n)])*
Log[c*g + d*g*x])/(d*g) - (B^2*n^2*Log[-((d*(a + b*x))/(b*c - a*d))]*Log[c*
g + d*g*x]^2)/(d*g) + (B^2*n*Log[e*((a + b*x)/(c + d*x))^n]*Log[c*g + d*g*x
]^2)/(d*g) + (2*B^2*n*Log[(a + b*x)^n]*PolyLog[2, -((d*(a + b*x))/(b*c - a*
d))])/(d*g) - (2*A*B*n*PolyLog[2, (b*(c + d*x))/(b*c - a*d)])/(d*g) - (2*B^
2*n*Log[(c + d*x)^(-n)]*PolyLog[2, (b*(c + d*x))/(b*c - a*d)])/(d*g) + (2*B
^2*n*(Log[(a + b*x)^n] - Log[e*((a + b*x)/(c + d*x))^n] + Log[(c + d*x)^(-n
```

```
)])*PolyLog[2, (b*(c + d*x))/(b*c - a*d)])/(d*g) - (2*B^2*n^2*PolyLog[3, -(
(d*(a + b*x))/(b*c - a*d))])/(d*g) - (2*B^2*n^2*PolyLog[3, (b*(c + d*x))/(b
*c - a*d)])/(d*g)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2301

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

Rule 2302

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/(x_), x_Symbol] :> Dist[1/(
b*n), Subst[Int[x^p, x], x, a + b*Log[c*x^n]], x] /; FreeQ[{a, b, c, n, p},
 x]
```

Rule 2317

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symb
ol] :> Simp[(Log[1 + (e*x)/d]*(a + b*Log[c*x^n])^p)/e, x] - Dist[(b*n*p)/e,
 Int[(Log[1 + (e*x)/d]*(a + b*Log[c*x^n])^(p - 1))/x, x], x] /; FreeQ[{a, b
, c, d, e, n}, x] && IGtQ[p, 0]
```

Rule 2374

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> -Simp[(PolyLog[2, -(d*f*x^m)]*(a + b*Log[c*x
^n])^p)/m, x] + Dist[(b*n*p)/m, Int[(PolyLog[2, -(d*f*x^m)]*(a + b*Log[c*x^
n])^(p - 1))/x, x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p, 0]
&& EqQ[d*e, 1]
```

Rule 2375

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))^(r_.)]*((a_.) + Log[(c_.)*(x_)^(n_
.)]*(b_.))^(p_.))/(x_), x_Symbol] :> Simp[(Log[d*(e + f*x^m)^r]*(a + b*Log[
c*x^n])^(p + 1))/(b*n*(p + 1)), x] - Dist[(f*m*r)/(b*n*(p + 1)), Int[(x^(m
- 1)*(a + b*Log[c*x^n])^(p + 1))/(e + f*x^m), x], x] /; FreeQ[{a, b, c, d,
```

```
e, f, r, m, n}, x] && IGtQ[p, 0] && NeQ[d*e, 1]
```

Rule 2390

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_))^(q_.), x_Symbol] :> Dist[1/e, Subst[Int[((f*x)/d)^q*(a + b*Log[c*x^
n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && E
qQ[e*f - d*g, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2393

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + (c*e*x)/g])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2394

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d + e*x
)^n]))/g, x] - Dist[(b*e*n)/g, Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2396

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)/((f_.) + (g_.
)*(x_)), x_Symbol] :> Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d
+ e*x)^n])^p)/g, x] - Dist[(b*e*n*p)/g, Int[(Log[(e*(f + g*x))/(e*f - d*g)]
*(a + b*Log[c*(d + e*x)^n])^(p - 1))/(d + e*x), x], x] /; FreeQ[{a, b, c, d
, e, f, g, n, p}, x] && NeQ[e*f - d*g, 0] && IGtQ[p, 1]
```

Rule 2418

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*(RFx_), x_Sy
mbol] :> With[{u = ExpandIntegrand[(a + b*Log[c*(d + e*x)^n])^p, RFx, x]},
Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d, e, n}, x] && RationalFunctionQ[
RFx, x] && IntegerQ[p]
```

Rule 2433

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + Log
[(h_.)*((i_.) + (j_.)*(x_))^(m_.)]*(g_.))*((k_.) + (l_.)*(x_))^(r_.), x_Sym
bol] :> Dist[1/e, Subst[Int[((k*x)/d)^r*(a + b*Log[c*x^n])^p*(f + g*Log[h*(
(e*i - d*j)/e + (j*x)/e)^m]), x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e,
f, g, h, i, j, k, l, n, p, r}, x] && EqQ[e*k - d*l, 0]
```

Rule 2434

```
Int[(((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + Log[(h_.
)*((i_.) + (j_.)*(x_))^(m_.)]*(g_.)))/(x_), x_Symbol] :> Simp[Log[x]*(a + b
*Log[c*(d + e*x)^n])*(f + g*Log[h*(i + j*x)^m]), x] + (-Dist[e*g*m, Int[(Lo
g[x]*(a + b*Log[c*(d + e*x)^n]))/(d + e*x), x], x] - Dist[b*j*n, Int[(Log[x
]*(f + g*Log[h*(i + j*x)^m]))/(i + j*x), x], x]) /; FreeQ[{a, b, c, d, e, f
, g, h, i, j, m, n}, x] && EqQ[e*i - d*j, 0]
```

Rule 2440

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + Log[(h_.)
*((i_.) + (j_.)*(x_))^(m_.)]*(g_.))*((k_) + (l_.)*(x_))^(r_.), x_Symbol] :>
 Dist[1/l, Subst[Int[x^r*(a + b*Log[c*(-((e*k - d*l)/l) + (e*x)/l)^n])*(f +
 g*Log[h*(-((j*k - i*l)/l) + (j*x)/l)^m]), x], x, k + l*x], x] /; FreeQ[{a,
 b, c, d, e, f, g, h, i, j, k, l, m, n}, x] && IntegerQ[r]
```

Rule 2499

```
Int[(Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.)
)^(r_.)]*((s_.) + Log[(i_.)*((g_.) + (h_.)*(x_))^(n_.)]*(t_.))^(m_.))/((j_.
) + (k_.)*(x_)), x_Symbol] :> Simp[((s + t*Log[i*(g + h*x)^n])^(m + 1)*Log[
e*(f*(a + b*x)^p*(c + d*x)^q)^r])/(k*n*t*(m + 1)), x] + (-Dist[(b*p*r)/(k*n
*t*(m + 1)), Int[(s + t*Log[i*(g + h*x)^n])^(m + 1)/(a + b*x), x], x] - Dis
t[(d*q*r)/(k*n*t*(m + 1)), Int[(s + t*Log[i*(g + h*x)^n])^(m + 1)/(c + d*x)
, x], x]) /; FreeQ[{a, b, c, d, e, f, g, h, i, j, k, s, t, m, n, p, q, r},
x] && NeQ[b*c - a*d, 0] && EqQ[h*j - g*k, 0] && IGtQ[m, 0]
```

Rule 2500

```
Int[(Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.)
)^(r_.)]*((s_.) + Log[(i_.)*((g_.) + (h_.)*(x_))^(n_.)]*(t_.)))/((j_.) + (k
_.)*(x_)), x_Symbol] :> Dist[Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r] - Log[(a
+ b*x)^(p*r)] - Log[(c + d*x)^(q*r)], Int[(s + t*Log[i*(g + h*x)^n])/(j + k
*x), x], x] + (Int[(Log[(a + b*x)^(p*r)]*(s + t*Log[i*(g + h*x)^n]))/(j + k
*x), x] + Int[(Log[(c + d*x)^(q*r)]*(s + t*Log[i*(g + h*x)^n]))/(j + k*x),
x]) /; FreeQ[{a, b, c, d, e, f, g, h, i, j, k, s, t, n, p, q, r}, x] && NeQ
[b*c - a*d, 0]
```

Rule 2524

```
Int[((a_.) + Log[(c_.)*(RFx_)^(p_.)]*(b_.))^(n_.)/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[(Log[d + e*x]*(a + b*Log[c*RFx^p])^n)/e, x] - Dist[(b*n*p)/e
, Int[(Log[d + e*x]*(a + b*Log[c*RFx^p])^(n - 1)*D[RFx, x])/RFx, x], x] /;
FreeQ[{a, b, c, d, e, p}, x] && RationalFunctionQ[RFx, x] && IGtQ[n, 0]
```

Rule 2528

```
Int[((a_.) + Log[(c_.)*(RFx_)^(p_.)]*(b_.))^(n_.)*(RGx_), x_Symbol] :> With
[{u = ExpandIntegrand[(a + b*Log[c*RFx^p])^n, RGx, x]}, Int[u, x] /; SumQ[u
]] /; FreeQ[{a, b, c, p}, x] && RationalFunctionQ[RFx, x] && RationalFuncti
onQ[RGx, x] && IGtQ[n, 0]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6688

```
Int[u_, x_Symbol] :> With[{v = SimplifyIntegrand[u, x]}, Int[v, x] /; Simpl
erIntegrandQ[v, u, x]]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{cg+dgx}\,dx = \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2\log(cg+dgx)}{dg} - \frac{(2Bn)\int \frac{(c+dx)\left(-\frac{d(a+bx)}{(c+dx)^2}+\frac{b}{c+dx}\right)\left(A+B\log\left(e\left(\cdots\right.\right.\right.}{a+bx}}{dg}$$

$$= \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2\log(cg+dgx)}{dg} - \frac{(2Bn)\int \frac{(bc-ad)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\log(cg\cdots}{(a+bx)(c+dx)}}{dg}$$

$$= \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2\log(cg+dgx)}{dg} - \frac{(2B(bc-ad)n)\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\log\cdots}{(a+bx)(c+dx)}}{dg}$$

$$= \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2\log(cg+dgx)}{dg} - \frac{(2B(bc-ad)n)\int \left(\frac{d\left(-A-B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\cdots\right.\right.\right.}{(bc-ad)(c+\cdots}\right.}{\cdots}$$

$$= \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2\log(cg+dgx)}{dg} - \frac{(2Bn)\int \frac{\left(-A-B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\log(cg+dgx)}{c+dx}\cdots}{g}$$

$$= \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2\log(cg+dgx)}{dg} - \frac{(2Bn)\int \left(\frac{A\log(cg+dgx)}{-c-dx}+\frac{B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\cdots\right.\right.}{-c-\cdots}\right.}{g}$$

$$= \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2\log(cg+dgx)}{dg} - \frac{(2ABn)\int \frac{\log(cg+dgx)}{-c-dx}\,dx}{g} - \frac{\left(2B^2n\right)\int\cdots}{\cdots}$$

$$= -\frac{2ABn\log\left(-\frac{d(a+bx)}{bc-ad}\right)\log(cg+dgx)}{dg} + \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2\log(cg+dgx\cdots}{dg}$$

$$= -\frac{2ABn\log\left(-\frac{d(a+bx)}{bc-ad}\right)\log(cg+dgx)}{dg} + \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2\log(cg+dgx\cdots}{dg}$$

$$= -\frac{B^2\log^2\left((a+bx)^n\right)\log(g(c+dx))}{dg} + \frac{ABn\log^2(g(c+dx))}{dg} - \frac{2B^2n\log(a+bx\cdots}{\cdots}$$

$$= \frac{B^2\log^2\left((a+bx)^n\right)\log\left(\frac{b(c+dx)}{bc-ad}\right)}{dg} - \frac{B^2\log^2\left((a+bx)^n\right)\log(g(c+dx))}{dg} + \frac{ABn\log\cdots}{\cdots}$$

**Mathematica [B]** time = 0.41, size = 537, normalized size = 3.92

$$-3Bn\left(-2\left(\text{Li}_2\left(\frac{d(a+bx)}{ad-bc}\right)+\log\left(\frac{a}{b}+x\right)\log\left(\frac{b(c+dx)}{bc-ad}\right)\right)+2\log(c+dx)\left(-\log\left(\frac{a+bx}{c+dx}\right)+\log\left(\frac{a}{b}+x\right)-\log\left(\frac{c}{d}+x\right)\right)\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2/(c*g + d*g*x),x]
```

```
[Out] (3*(A + B*Log[e*((a + b*x)/(c + d*x))^n] - B*n*Log[(a + b*x)/(c + d*x)])^2*
Log[c + d*x] - 3*B*n*(A + B*Log[e*((a + b*x)/(c + d*x))^n] - B*n*Log[(a + b
*x)/(c + d*x)])*(Log[c/d + x]^2 + 2*(Log[a/b + x] - Log[c/d + x] - Log[(a +
 b*x)/(c + d*x)])*Log[c + d*x] - 2*(Log[a/b + x]*Log[(b*(c + d*x))/(b*c - a
*d)] + PolyLog[2, (d*(a + b*x))/(-(b*c) + a*d)])) + B^2*n^2*(Log[c/d + x]^3
 + 3*Log[c/d + x]^2*(-Log[a/b + x] + Log[(d*(a + b*x))/(-(b*c) + a*d)]) + 3
*(-Log[a/b + x] + Log[c/d + x] + Log[(a + b*x)/(c + d*x)])^2*Log[c + d*x] +
 3*Log[a/b + x]^2*Log[(b*(c + d*x))/(b*c - a*d)] + 6*Log[a/b + x]*PolyLog[2
, (d*(a + b*x))/(-(b*c) + a*d)] + 3*(Log[a/b + x] - Log[c/d + x] - Log[(a +
 b*x)/(c + d*x)])*(Log[c/d + x]^2 - 2*(Log[a/b + x]*Log[(b*(c + d*x))/(b*c
- a*d)] + PolyLog[2, (d*(a + b*x))/(-(b*c) + a*d)])) + 6*Log[c/d + x]*PolyL
og[2, (b*(c + d*x))/(b*c - a*d)] - 6*PolyLog[3, (d*(a + b*x))/(-(b*c) + a*d
)] - 6*PolyLog[3, (b*(c + d*x))/(b*c - a*d)]))/(3*d*g)
```

**fricas [F]** time = 0.96, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{B^2\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)^2+2\,AB\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A^2}{dgx+cg},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))^2/(d*g*x+c*g),x, algorithm="fric
as")
```

```
[Out] integral((B^2*log(e*((b*x + a)/(d*x + c))^n)^2 + 2*A*B*log(e*((b*x + a)/(d*
x + c))^n) + A^2)/(d*g*x + c*g), x)
```

**giac [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))^2/(d*g*x+c*g),x, algorithm="giac
")
```

```
[Out] Timed out
```

**maple [F]** time = 0.30, size = 0, normalized size = 0.00

$$\int \frac{\left(B \ln\left(e \left(\frac{bx+a}{dx+c}\right)^{n}\right)+A\right)^{2}}{dgx+cg}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*ln(e*((b*x+a)/(d*x+c))^n)+A)^2/(d*g*x+c*g),x)
```

```
[Out] int((B*ln(e*((b*x+a)/(d*x+c))^n)+A)^2/(d*g*x+c*g),x)
```

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$\frac{B^{2} \log\left(dx+c\right) \log\left(\left(dx+c\right)^{n}\right)^{2}}{dg}+\frac{A^{2} \log\left(dgx+cg\right)}{dg}-\int -\frac{B^{2} \log\left(\left(bx+a\right)^{n}\right)^{2}+B^{2} \log(e)^{2}+2\,AB \log(e)+2\left(B^{2}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))^2/(d*g*x+c*g),x, algorithm="maxi
ma")
```

```
[Out] B^2*log(d*x + c)*log((d*x + c)^n)^2/(d*g) + A^2*log(d*g*x + c*g)/(d*g) - in
tegrate(-(B^2*log((b*x + a)^n)^2 + B^2*log(e)^2 + 2*A*B*log(e) + 2*(B^2*log
(e) + A*B)*log((b*x + a)^n) - 2*(B^2*n*log(d*x + c) + B^2*log((b*x + a)^n)
+ B^2*log(e) + A*B)*log((d*x + c)^n))/(d*g*x + c*g), x)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(A+B\,\ln\left(e\,\left(\frac{a+b\,x}{c+d\,x}\right)^{n}\right)\right)^{2}}{c\,g+d\,g\,x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*log(e*((a + b*x)/(c + d*x))^n))^2/(c*g + d*g*x),x)
```

```
[Out] int((A + B*log(e*((a + b*x)/(c + d*x))^n))^2/(c*g + d*g*x), x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{A^{2}}{c+dx}\, dx+\int \frac{B^{2} \log\left(e\left(\frac{a}{c+dx}+\frac{bx}{c+dx}\right)^{n}\right)^{2}}{c+dx}\, dx+\int \frac{2AB \log\left(e\left(\frac{a}{c+dx}+\frac{bx}{c+dx}\right)^{n}\right)}{c+dx}\, dx}{g}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*ln(e*((b*x+a)/(d*x+c))**n))**2/(d*g*x+c*g),x)
```

```
[Out] (Integral(A**2/(c + d*x), x) + Integral(B**2*log(e*(a/(c + d*x) + b*x/(c +
d*x))**n)**2/(c + d*x), x) + Integral(2*A*B*log(e*(a/(c + d*x) + b*x/(c + d
*x))**n)/(c + d*x), x))/g
```

## 3.43 $\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(cg+dgx)^2}\,dx$

**Optimal**. Leaf size=163

$$\frac{(a+bx)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{g^2(c+dx)(bc-ad)}-\frac{2ABn(a+bx)}{g^2(c+dx)(bc-ad)}-\frac{2B^2n(a+bx)\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{g^2(c+dx)(bc-ad)}+\frac{2B^2n^2(a+bx)}{g^2(c+dx)(bc-ad)}$$

```
[Out] -2*A*B*n*(b*x+a)/(-a*d+b*c)/g^2/(d*x+c)+2*B^2*n^2*(b*x+a)/(-a*d+b*c)/g^2/(d
*x+c)-2*B^2*n*(b*x+a)*ln(e*((b*x+a)/(d*x+c))^n)/(-a*d+b*c)/g^2/(d*x+c)+(b*x
+a)*(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2/(-a*d+b*c)/g^2/(d*x+c)
```

**Rubi [C]** time = 0.77, antiderivative size = 514, normalized size of antiderivative = 3.15, number of steps used = 24, number of rules used = 11, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.314, Rules used = {2525, 12, 2528, 2524, 2418, 2390, 2301, 2394, 2393, 2391, 44}

$$\frac{2bB^2n^2\text{PolyLog}\left(2,-\frac{d(a+bx)}{bc-ad}\right)}{dg^2(bc-ad)}+\frac{2bB^2n^2\text{PolyLog}\left(2,\frac{b(c+dx)}{bc-ad}\right)}{dg^2(bc-ad)}+\frac{2bBn\log(a+bx)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{dg^2(bc-ad)}+\frac{2Bn\left(B\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2/(c*g + d*g*x)^2,x]
```

```
[Out] (-2*B^2*n^2)/(d*g^2*(c + d*x)) - (2*b*B^2*n^2*Log[a + b*x])/(d*(b*c - a*d)*
g^2) - (b*B^2*n^2*Log[a + b*x]^2)/(d*(b*c - a*d)*g^2) + (2*B*n*(A + B*Log[e
*((a + b*x)/(c + d*x))^n]))/(d*g^2*(c + d*x)) + (2*b*B*n*Log[a + b*x]*(A +
B*Log[e*((a + b*x)/(c + d*x))^n]))/(d*(b*c - a*d)*g^2) - (A + B*Log[e*((a +
 b*x)/(c + d*x))^n])^2/(d*g^2*(c + d*x)) + (2*b*B^2*n^2*Log[c + d*x])/(d*(b
*c - a*d)*g^2) + (2*b*B^2*n^2*Log[-((d*(a + b*x))/(b*c - a*d))]*Log[c + d*x
])/(d*(b*c - a*d)*g^2) - (2*b*B*n*(A + B*Log[e*((a + b*x)/(c + d*x))^n])*Lo
g[c + d*x])/(d*(b*c - a*d)*g^2) - (b*B^2*n^2*Log[c + d*x]^2)/(d*(b*c - a*d)
*g^2) + (2*b*B^2*n^2*Log[a + b*x]*Log[(b*(c + d*x))/(b*c - a*d)])/(d*(b*c -
 a*d)*g^2) + (2*b*B^2*n^2*PolyLog[2, -((d*(a + b*x))/(b*c - a*d))])/(d*(b*c
 - a*d)*g^2) + (2*b*B^2*n^2*PolyLog[2, (b*(c + d*x))/(b*c - a*d)])/(d*(b*c
- a*d)*g^2)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 2301

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

Rule 2390

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_))^(q_.), x_Symbol] :> Dist[1/e, Subst[Int[((f*x)/d)^q*(a + b*Log[c*x^
n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && E
qQ[e*f - d*g, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2393

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + (c*e*x)/g])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2394

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d + e*x
)^n]))/g, x] - Dist[(b*e*n)/g, Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2418

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*(RFx_), x_Sy
mbol] :> With[{u = ExpandIntegrand[(a + b*Log[c*(d + e*x)^n])^p, RFx, x]},
Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d, e, n}, x] && RationalFunctionQ[
RFx, x] && IntegerQ[p]
```

Rule 2524

```
Int[((a_.) + Log[(c_.)*(RFx_)^(p_.)]*(b_.))^(n_.)/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[(Log[d + e*x]*(a + b*Log[c*RFx^p])^n)/e, x] - Dist[(b*n*p)/e
, Int[(Log[d + e*x]*(a + b*Log[c*RFx^p])^(n - 1)*D[RFx, x])/RFx, x], x] /;
FreeQ[{a, b, c, d, e, p}, x] && RationalFunctionQ[RFx, x] && IGtQ[n, 0]
```

Rule 2525

```
Int[((a_.) + Log[(c_.)*(RFx_)^(p_.)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_))^(m_.
), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(a + b*Log[c*RFx^p])^n)/(e*(m + 1))
, x] - Dist[(b*n*p)/(e*(m + 1)), Int[SimplifyIntegrand[((d + e*x)^(m + 1)*(
a + b*Log[c*RFx^p])^(n - 1)*D[RFx, x])/RFx, x], x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && RationalFunctionQ[RFx, x] && IGtQ[n, 0] && (EqQ[n, 1] ||
IntegerQ[m]) && NeQ[m, -1]
```

Rule 2528

```
Int[((a_.) + Log[(c_.)*(RFx_)^(p_.)]*(b_.))^(n_.)*(RGx_), x_Symbol] :> With
[{u = ExpandIntegrand[(a + b*Log[c*RFx^p])^n, RGx, x]}, Int[u, x] /; SumQ[u
]] /; FreeQ[{a, b, c, p}, x] && RationalFunctionQ[RFx, x] && RationalFuncti
onQ[RGx, x] && IGtQ[n, 0]
```

Rubi steps

$$\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(cg+dgx)^2}\,dx = -\frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{dg^2(c+dx)} + \frac{(2Bn)\int \frac{(bc-ad)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{g(a+bx)(c+dx)^2}\,dx}{dg}$$

$$= -\frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{dg^2(c+dx)} + \frac{(2B(bc-ad)n)\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(a+bx)(c+dx)^2}\,dx}{dg^2}$$

$$= -\frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{dg^2(c+dx)} + \frac{(2B(bc-ad)n)\int \left(\frac{b^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(bc-ad)^2(a+bx)} - \frac{d\left(A+B\log \cdots\right.}{(bc-a \cdots} \cdots\right.}{dg^2 \cdots}$$

$$= -\frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{dg^2(c+dx)} - \frac{(2Bn)\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(c+dx)^2}\,dx}{g^2} - \frac{(2bBn)\int \frac{A+B\log\left(\cdots\right.}{c+\cdots}}{(bc-ad)\cdots}$$

$$= \frac{2Bn\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{dg^2(c+dx)} + \frac{2bBn\log(a+bx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{d(bc-ad)g^2} - \cdots$$

$$= \frac{2Bn\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{dg^2(c+dx)} + \frac{2bBn\log(a+bx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{d(bc-ad)g^2} - \cdots$$

$$= \frac{2Bn\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{dg^2(c+dx)} + \frac{2bBn\log(a+bx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{d(bc-ad)g^2} - \cdots$$

$$= -\frac{2B^2n^2}{dg^2(c+dx)} - \frac{2bB^2n^2\log(a+bx)}{d(bc-ad)g^2} + \frac{2Bn\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{dg^2(c+dx)} + \frac{2bBn\,\mathrm{l}\cdots}{\cdots}$$

$$= -\frac{2B^2n^2}{dg^2(c+dx)} - \frac{2bB^2n^2\log(a+bx)}{d(bc-ad)g^2} - \frac{bB^2n^2\log^2(a+bx)}{d(bc-ad)g^2} + \frac{2Bn\left(A+B\log \cdots\right.}{dg^2(c+\cdots}$$

$$= -\frac{2B^2n^2}{dg^2(c+dx)} - \frac{2bB^2n^2\log(a+bx)}{d(bc-ad)g^2} - \frac{bB^2n^2\log^2(a+bx)}{d(bc-ad)g^2} + \frac{2Bn\left(A+B\log \cdots\right.}{dg^2(c+\cdots}$$

**Mathematica [C]** time = 0.43, size = 331, normalized size = 2.03

$$\frac{Bn\left(2(bc-ad)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)+2b(c+dx)\log(a+bx)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)-2b(c+dx)\log(c+dx)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)-bBn(c+dx)\left(\log(a+bx)\left(\log(\right.\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2/(c*g + d*g*x)^2,x]
```

```
[Out] (-(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2 + (B*n*(2*(b*c - a*d)*(A + B*Log
[e*((a + b*x)/(c + d*x))^n]) + 2*b*(c + d*x)*Log[a + b*x]*(A + B*Log[e*((a
+ b*x)/(c + d*x))^n]) - 2*b*(c + d*x)*(A + B*Log[e*((a + b*x)/(c + d*x))^n]
)*Log[c + d*x] - 2*B*n*(b*c - a*d + b*(c + d*x)*Log[a + b*x] - b*(c + d*x)*
Log[c + d*x]) - b*B*n*(c + d*x)*(Log[a + b*x]*(Log[a + b*x] - 2*Log[(b*(c +
 d*x))/(b*c - a*d)]) - 2*PolyLog[2, (d*(a + b*x))/(-(b*c) + a*d)]) + b*B*n*
(c + d*x)*((2*Log[(d*(a + b*x))/(-(b*c) + a*d)] - Log[c + d*x])*Log[c + d*x
] + 2*PolyLog[2, (b*(c + d*x))/(b*c - a*d)])))/(b*c - a*d))/(d*g^2*(c + d*x
))
```

**fricas [A]** time = 0.91, size = 263, normalized size = 1.61

$$-\frac{A^2bc-A^2ad+2\left(B^2bc-B^2ad\right)n^2+\left(B^2bc-B^2ad\right)\log(e)^2-\left(B^2bdn^2x+B^2adn^2\right)\log\left(\frac{bx+a}{dx+c}\right)^2-2\left(ABbc-ABa\right.}{\left(bc\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))^2/(d*g*x+c*g)^2,x, algorithm="fr
icas")
```

```
[Out] -(A^2*b*c - A^2*a*d + 2*(B^2*b*c - B^2*a*d)*n^2 + (B^2*b*c - B^2*a*d)*log(e
)^2 - (B^2*b*d*n^2*x + B^2*a*d*n^2)*log((b*x + a)/(d*x + c))^2 - 2*(A*B*b*c
 - A*B*a*d)*n + 2*(A*B*b*c - A*B*a*d - (B^2*b*c - B^2*a*d)*n - (B^2*b*d*n*x
 + B^2*a*d*n)*log((b*x + a)/(d*x + c)))*log(e) + 2*(B^2*a*d*n^2 - A*B*a*d*n
 + (B^2*b*d*n^2 - A*B*b*d*n)*x)*log((b*x + a)/(d*x + c)))/((b*c*d^2 - a*d^3
)*g^2*x + (b*c^2*d - a*c*d^2)*g^2)
```

**giac [A]** time = 7.26, size = 164, normalized size = 1.01

$$\left(\frac{(bx+a)B^2n^2\log\left(\frac{bx+a}{dx+c}\right)^2}{(dx+c)g^2}-\frac{2\left(B^2n^2-ABn-B^2n\right)(bx+a)\log\left(\frac{bx+a}{dx+c}\right)}{(dx+c)g^2}+\frac{\left(2B^2n^2-2ABn-2B^2n+A^2+2AB\right.}{(dx+c)g^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))^2/(d*g*x+c*g)^2,x, algorithm="gi
ac")
```

```
[Out] ((b*x + a)*B^2*n^2*log((b*x + a)/(d*x + c))^2/((d*x + c)*g^2) - 2*(B^2*n^2
- A*B*n - B^2*n)*(b*x + a)*log((b*x + a)/(d*x + c))/((d*x + c)*g^2) + (2*B^
2*n^2 - 2*A*B*n - 2*B^2*n + A^2 + 2*A*B + B^2)*(b*x + a)/((d*x + c)*g^2))*(
b*c/(b*c - a*d)^2 - a*d/(b*c - a*d)^2)
```

**maple [F]** time = 0.28, size = 0, normalized size = 0.00

$$\int \frac{\left(B\ln\left(e\left(\frac{bx+a}{dx+c}\right)^{n}\right)+A\right)^{2}}{\left(dgx+cg\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*ln(e*((b*x+a)/(d*x+c))^n)+A)^2/(d*g*x+c*g)^2,x)
```

```
[Out] int((B*ln(e*((b*x+a)/(d*x+c))^n)+A)^2/(d*g*x+c*g)^2,x)
```

**maxima [B]** time = 0.86, size = 428, normalized size = 2.63

$$2\,ABn\left(\frac{1}{d^2g^2x+cdg^2}+\frac{b\log\left(bx+a\right)}{\left(bcd-ad^2\right)g^2}-\frac{b\log\left(dx+c\right)}{\left(bcd-ad^2\right)g^2}\right)+\left(2\,n\left(\frac{1}{d^2g^2x+cdg^2}+\frac{b\log\left(bx+a\right)}{\left(bcd-ad^2\right)g^2}-\frac{b\log\left(dx+c\right)}{\left(bcd-ad^2\right)g^2}\right)\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))^2/(d*g*x+c*g)^2,x, algorithm="ma
xima")
```

```
[Out] 2*A*B*n*(1/(d^2*g^2*x + c*d*g^2) + b*log(b*x + a)/((b*c*d - a*d^2)*g^2) - b
*log(d*x + c)/((b*c*d - a*d^2)*g^2)) + (2*n*(1/(d^2*g^2*x + c*d*g^2) + b*lo
g(b*x + a)/((b*c*d - a*d^2)*g^2) - b*log(d*x + c)/((b*c*d - a*d^2)*g^2))*lo
g(e*(b*x/(d*x + c) + a/(d*x + c))^n) - ((b*d*x + b*c)*log(b*x + a)^2 + (b*d
*x + b*c)*log(d*x + c)^2 + 2*b*c - 2*a*d + 2*(b*d*x + b*c)*log(b*x + a) - 2
*(b*d*x + b*c + (b*d*x + b*c)*log(b*x + a))*log(d*x + c))*n^2/(b*c^2*d*g^2
- a*c*d^2*g^2 + (b*c*d^2*g^2 - a*d^3*g^2)*x))*B^2 - B^2*log(e*(b*x/(d*x + c
) + a/(d*x + c))^n)^2/(d^2*g^2*x + c*d*g^2) - 2*A*B*log(e*(b*x/(d*x + c) +
a/(d*x + c))^n)/(d^2*g^2*x + c*d*g^2) - A^2/(d^2*g^2*x + c*d*g^2)
```

**mupad [B]** time = 5.65, size = 237, normalized size = 1.45

$$\ln\left(e\left(\frac{a+b\,x}{c+d\,x}\right)^n\right)\left(\frac{2\,B^2\,n}{x\,d^2\,g^2+c\,d\,g^2}-\frac{2\,A\,B}{x\,d^2\,g^2+c\,d\,g^2}\right)-\ln\left(e\left(\frac{a+b\,x}{c+d\,x}\right)^n\right)^2\left(\frac{B^2}{d\,\left(c\,g^2+d\,g^2\,x\right)}+\frac{B^2\,b}{d\,g^2\,\left(a\,d-b\,c\right)}\right)-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*log(e*((a + b*x)/(c + d*x))^n))^2/(c*g + d*g*x)^2,x)
```

```
[Out] log(e*((a + b*x)/(c + d*x))^n)*((2*B^2*n)/(d^2*g^2*x + c*d*g^2) - (2*A*B)/(
d^2*g^2*x + c*d*g^2)) - log(e*((a + b*x)/(c + d*x))^n)^2*(B^2/(d*(c*g^2 + d
*g^2*x)) + (B^2*b)/(d*g^2*(a*d - b*c))) - (A^2 + 2*B^2*n^2 - 2*A*B*n)/(d^2*
g^2*x + c*d*g^2) + (B*b*n*atan(((2*b*d*x + (a*d^2*g^2 + b*c*d*g^2)/(d*g^2))
*1i)/(a*d - b*c))*(A - B*n)*4i)/(d*g^2*(a*d - b*c))
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{A^2}{c^2+2cdx+d^2x^2}\,dx + \int \frac{B^2\log\left(e\left(\frac{a}{c+dx}+\frac{bx}{c+dx}\right)^n\right)^2}{c^2+2cdx+d^2x^2}\,dx + \int \frac{2AB\log\left(e\left(\frac{a}{c+dx}+\frac{bx}{c+dx}\right)^n\right)}{c^2+2cdx+d^2x^2}\,dx}{g^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*ln(e*((b*x+a)/(d*x+c))**n))**2/(d*g*x+c*g)**2,x)
```

```
[Out] (Integral(A**2/(c**2 + 2*c*d*x + d**2*x**2), x) + Integral(B**2*log(e*(a/(c
 + d*x) + b*x/(c + d*x))**n)**2/(c**2 + 2*c*d*x + d**2*x**2), x) + Integral
(2*A*B*log(e*(a/(c + d*x) + b*x/(c + d*x))**n)/(c**2 + 2*c*d*x + d**2*x**2)
, x))/g**2
```

# 3.44 $\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(cg+dgx)^3}\,dx$

**Optimal**. Leaf size=317

$$\frac{Bdn(a+bx)^2\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{2g^3(c+dx)^2(bc-ad)^2}+\frac{b(a+bx)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{g^3(c+dx)(bc-ad)^2}-\frac{d(a+bx)^2\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{2g^3(c+dx)^2(bc-ad)^2}$$

```
[Out] -1/4*B^2*d*n^2*(b*x+a)^2/(-a*d+b*c)^2/g^3/(d*x+c)^2-2*A*b*B*n*(b*x+a)/(-a*d
+b*c)^2/g^3/(d*x+c)+2*b*B^2*n^2*(b*x+a)/(-a*d+b*c)^2/g^3/(d*x+c)-2*b*B^2*n*
(b*x+a)*ln(e*((b*x+a)/(d*x+c))^n)/(-a*d+b*c)^2/g^3/(d*x+c)+1/2*B*d*n*(b*x+a
)^2*(A+B*ln(e*((b*x+a)/(d*x+c))^n))/(-a*d+b*c)^2/g^3/(d*x+c)^2-1/2*d*(b*x+a
)^2*(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2/(-a*d+b*c)^2/g^3/(d*x+c)^2+b*(b*x+a)*
(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2/(-a*d+b*c)^2/g^3/(d*x+c)
```

**Rubi [C]** time = 0.92, antiderivative size = 626, normalized size of antiderivative = 1.97, number of steps used = 28, number of rules used = 11, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.314, Rules used = {2525, 12, 2528, 2524, 2418, 2390, 2301, 2394, 2393, 2391, 44}

$$\frac{b^2B^2n^2\text{PolyLog}\left(2,-\frac{d(a+bx)}{bc-ad}\right)}{dg^3(bc-ad)^2}+\frac{b^2B^2n^2\text{PolyLog}\left(2,\frac{b(c+dx)}{bc-ad}\right)}{dg^3(bc-ad)^2}+\frac{b^2Bn\log(a+bx)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{dg^3(bc-ad)^2}-\frac{b^2Bn\log\ldots}{\ldots}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2/(c*g + d*g*x)^3,x]
```

```
[Out] -(B^2*n^2)/(4*d*g^3*(c + d*x)^2) - (3*b*B^2*n^2)/(2*d*(b*c - a*d)*g^3*(c +
d*x)) - (3*b^2*B^2*n^2*Log[a + b*x])/(2*d*(b*c - a*d)^2*g^3) - (b^2*B^2*n^2
*Log[a + b*x]^2)/(2*d*(b*c - a*d)^2*g^3) + (B*n*(A + B*Log[e*((a + b*x)/(c
+ d*x))^n]))/(2*d*g^3*(c + d*x)^2) + (b*B*n*(A + B*Log[e*((a + b*x)/(c + d*
x))^n]))/(d*(b*c - a*d)*g^3*(c + d*x)) + (b^2*B*n*Log[a + b*x]*(A + B*Log[e
*((a + b*x)/(c + d*x))^n]))/(d*(b*c - a*d)^2*g^3) - (A + B*Log[e*((a + b*x)
/(c + d*x))^n])^2/(2*d*g^3*(c + d*x)^2) + (3*b^2*B^2*n^2*Log[c + d*x])/(2*d
*(b*c - a*d)^2*g^3) + (b^2*B^2*n^2*Log[-((d*(a + b*x))/(b*c - a*d))]*Log[c
+ d*x])/(d*(b*c - a*d)^2*g^3) - (b^2*B*n*(A + B*Log[e*((a + b*x)/(c + d*x))
^n])*Log[c + d*x])/(d*(b*c - a*d)^2*g^3) - (b^2*B^2*n^2*Log[c + d*x]^2)/(2*
d*(b*c - a*d)^2*g^3) + (b^2*B^2*n^2*Log[a + b*x]*Log[(b*(c + d*x))/(b*c - a
*d)])/(d*(b*c - a*d)^2*g^3) + (b^2*B^2*n^2*PolyLog[2, -((d*(a + b*x))/(b*c
- a*d))])/(d*(b*c - a*d)^2*g^3) + (b^2*B^2*n^2*PolyLog[2, (b*(c + d*x))/(b*
c - a*d)])/(d*(b*c - a*d)^2*g^3)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 2301

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

Rule 2390

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_))^(q_.), x_Symbol] :> Dist[1/e, Subst[Int[((f*x)/d)^q*(a + b*Log[c*x^
n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && E
qQ[e*f - d*g, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2393

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + (c*e*x)/g])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2394

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d + e*x
)^n]))/g, x] - Dist[(b*e*n)/g, Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2418

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*(RFx_), x_Sy
mbol] :> With[{u = ExpandIntegrand[(a + b*Log[c*(d + e*x)^n])^p, RFx, x]},
Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d, e, n}, x] && RationalFunctionQ[
RFx, x] && IntegerQ[p]
```

Rule 2524

```
Int[((a_.) + Log[(c_.)*(RFx_)^(p_.)]*(b_.))^(n_.)/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[(Log[d + e*x]*(a + b*Log[c*RFx^p])^n)/e, x] - Dist[(b*n*p)/e
, Int[(Log[d + e*x]*(a + b*Log[c*RFx^p])^(n - 1)*D[RFx, x])/RFx, x], x] /;
FreeQ[{a, b, c, d, e, p}, x] && RationalFunctionQ[RFx, x] && IGtQ[n, 0]
```

Rule 2525

```
Int[((a_.) + Log[(c_.)*(RFx_)^(p_.)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_))^(m_.
), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(a + b*Log[c*RFx^p])^n)/(e*(m + 1))
, x] - Dist[(b*n*p)/(e*(m + 1)), Int[SimplifyIntegrand[((d + e*x)^(m + 1)*(
a + b*Log[c*RFx^p])^(n - 1)*D[RFx, x])/RFx, x], x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && RationalFunctionQ[RFx, x] && IGtQ[n, 0] && (EqQ[n, 1] ||
IntegerQ[m]) && NeQ[m, -1]
```

Rule 2528

```
Int[((a_.) + Log[(c_.)*(RFx_)^(p_.)]*(b_.))^(n_.)*(RGx_), x_Symbol] :> With
[{u = ExpandIntegrand[(a + b*Log[c*RFx^p])^n, RGx, x]}, Int[u, x] /; SumQ[u
]] /; FreeQ[{a, b, c, p}, x] && RationalFunctionQ[RFx, x] && RationalFuncti
onQ[RGx, x] && IGtQ[n, 0]
```

Rubi steps

$$\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(cg+dgx)^3}\,dx = -\frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{2dg^3(c+dx)^2} + \frac{(Bn)\int \frac{(bc-ad)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{g^2(a+bx)(c+dx)^3}\,dx}{dg}$$

$$= -\frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{2dg^3(c+dx)^2} + \frac{(B(bc-ad)n)\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(a+bx)(c+dx)^3}\,dx}{dg^3}$$

$$= -\frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{2dg^3(c+dx)^2} + \frac{(B(bc-ad)n)\int \left(\frac{b^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(bc-ad)^3(a+bx)} - \frac{d\left(A+B\log\left(e\right.\right.}{(bc-ad)(c}\right.}{}$$

$$= -\frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{2dg^3(c+dx)^2} - \frac{(Bn)\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(c+dx)^3}\,dx}{g^3} - \frac{\left(b^2Bn\right)\int \frac{A+B\log\left(e\left(\frac{a+}{c+}\right.\right.}{c+dx}}{(bc-ad)^2g^3}$$

$$= \frac{Bn\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{2dg^3(c+dx)^2} + \frac{bBn\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{d(bc-ad)g^3(c+dx)} + \frac{b^2Bn\log(a+bx}{d(}$$

$$= \frac{Bn\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{2dg^3(c+dx)^2} + \frac{bBn\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{d(bc-ad)g^3(c+dx)} + \frac{b^2Bn\log(a+bx}{d(}$$

$$= \frac{Bn\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{2dg^3(c+dx)^2} + \frac{bBn\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{d(bc-ad)g^3(c+dx)} + \frac{b^2Bn\log(a+bx}{d(}$$

$$= -\frac{B^2n^2}{4dg^3(c+dx)^2} - \frac{3bB^2n^2}{2d(bc-ad)g^3(c+dx)} - \frac{3b^2B^2n^2\log(a+bx)}{2d(bc-ad)^2g^3} + \frac{Bn\left(A+B\mathrm{l}\right.}{2dg^3(}$$

$$= -\frac{B^2n^2}{4dg^3(c+dx)^2} - \frac{3bB^2n^2}{2d(bc-ad)g^3(c+dx)} - \frac{3b^2B^2n^2\log(a+bx)}{2d(bc-ad)^2g^3} - \frac{b^2B^2n^2\log^2}{2d(bc-a}$$

$$= -\frac{B^2n^2}{4dg^3(c+dx)^2} - \frac{3bB^2n^2}{2d(bc-ad)g^3(c+dx)} - \frac{3b^2B^2n^2\log(a+bx)}{2d(bc-ad)^2g^3} - \frac{b^2B^2n^2\log^2}{2d(bc-a}$$

**Mathematica [C]** time = 0.44, size = 464, normalized size = 1.46

$$\frac{Bn\left(4b^2(c+dx)^2\log(a+bx)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)-4b^2(c+dx)^2\log(c+dx)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)+2(bc-ad)^2\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)+4b(c+dx)(bc-ad)\left(B\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2/(c*g + d*g*x)^3,x]
```

```
[Out] (-2*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2 + (B*n*(2*(b*c - a*d)^2*(A + B
*Log[e*((a + b*x)/(c + d*x))^n]) + 4*b*(b*c - a*d)*(c + d*x)*(A + B*Log[e*(
(a + b*x)/(c + d*x))^n]) + 4*b^2*(c + d*x)^2*Log[a + b*x]*(A + B*Log[e*((a
+ b*x)/(c + d*x))^n]) - 4*b^2*(c + d*x)^2*(A + B*Log[e*((a + b*x)/(c + d*x)
)^n])*Log[c + d*x] - 4*b*B*n*(c + d*x)*(b*c - a*d + b*(c + d*x)*Log[a + b*x
] - b*(c + d*x)*Log[c + d*x]) - B*n*((b*c - a*d)^2 + 2*b*(b*c - a*d)*(c + d
*x) + 2*b^2*(c + d*x)^2*Log[a + b*x] - 2*b^2*(c + d*x)^2*Log[c + d*x]) - 2*
b^2*B*n*(c + d*x)^2*(Log[a + b*x]*(Log[a + b*x] - 2*Log[(b*(c + d*x))/(b*c
- a*d)]) - 2*PolyLog[2, (d*(a + b*x))/(-(b*c) + a*d)]) + 2*b^2*B*n*(c + d*x
)^2*((2*Log[(d*(a + b*x))/(-(b*c) + a*d)] - Log[c + d*x])*Log[c + d*x] + 2*
PolyLog[2, (b*(c + d*x))/(b*c - a*d)])))/(b*c - a*d)^2)/(4*d*g^3*(c + d*x)^
2)
```

**fricas [B]** time = 1.08, size = 654, normalized size = 2.06

$$-\frac{2\,A^2b^2c^2 - 4\,A^2abcd + 2\,A^2a^2d^2 + \left(7\,B^2b^2c^2 - 8\,B^2abcd + B^2a^2d^2\right)n^2 + 2\left(B^2b^2c^2 - 2\,B^2abcd + B^2a^2d^2\right)\log(e)}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))^2/(d*g*x+c*g)^3,x, algorithm="fr
icas")
```

```
[Out] -1/4*(2*A^2*b^2*c^2 - 4*A^2*a*b*c*d + 2*A^2*a^2*d^2 + (7*B^2*b^2*c^2 - 8*B^
2*a*b*c*d + B^2*a^2*d^2)*n^2 + 2*(B^2*b^2*c^2 - 2*B^2*a*b*c*d + B^2*a^2*d^2
)*log(e)^2 - 2*(B^2*b^2*d^2*n^2*x^2 + 2*B^2*b^2*c*d*n^2*x + (2*B^2*a*b*c*d
- B^2*a^2*d^2)*n^2)*log((b*x + a)/(d*x + c))^2 - 2*(3*A*B*b^2*c^2 - 4*A*B*a
*b*c*d + A*B*a^2*d^2)*n + 2*(3*(B^2*b^2*c*d - B^2*a*b*d^2)*n^2 - 2*(A*B*b^2
*c*d - A*B*a*b*d^2)*n)*x + 2*(2*A*B*b^2*c^2 - 4*A*B*a*b*c*d + 2*A*B*a^2*d^2
 - 2*(B^2*b^2*c*d - B^2*a*b*d^2)*n*x - (3*B^2*b^2*c^2 - 4*B^2*a*b*c*d + B^2
*a^2*d^2)*n - 2*(B^2*b^2*d^2*n*x^2 + 2*B^2*b^2*c*d*n*x + (2*B^2*a*b*c*d - B
^2*a^2*d^2)*n)*log((b*x + a)/(d*x + c)))*log(e) + 2*((4*B^2*a*b*c*d - B^2*a
^2*d^2)*n^2 + (3*B^2*b^2*d^2*n^2 - 2*A*B*b^2*d^2*n)*x^2 - 2*(2*A*B*a*b*c*d
- A*B*a^2*d^2)*n - 2*(2*A*B*b^2*c*d*n - (2*B^2*b^2*c*d + B^2*a*b*d^2)*n^2)*
```

```
x)*log((b*x + a)/(d*x + c)))/((b^2*c^2*d^3 - 2*a*b*c*d^4 + a^2*d^5)*g^3*x^2
 + 2*(b^2*c^3*d^2 - 2*a*b*c^2*d^3 + a^2*c*d^4)*g^3*x + (b^2*c^4*d - 2*a*b*c
^3*d^2 + a^2*c^2*d^3)*g^3)
```

**giac [A]** time = 9.30, size = 387, normalized size = 1.22

$$\frac{1}{4}\left(2\left(\frac{2(bx+a)B^2bn^2}{(bcg^3-adg^3)(dx+c)}-\frac{(bx+a)^2B^2dn^2}{(bcg^3-adg^3)(dx+c)^2}\right)\log\left(\frac{bx+a}{dx+c}\right)^2+2\left(\frac{(B^2dn^2-2ABdn-2B^2dn)(bx+a)^2}{(bcg^3-adg^3)(dx+c)^2}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))^2/(d*g*x+c*g)^3,x, algorithm="gi
ac")
```

```
[Out] 1/4*(2*(2*(b*x + a)*B^2*b*n^2/((b*c*g^3 - a*d*g^3)*(d*x + c)) - (b*x + a)^2
*B^2*d*n^2/((b*c*g^3 - a*d*g^3)*(d*x + c)^2))*log((b*x + a)/(d*x + c))^2 +
2*((B^2*d*n^2 - 2*A*B*d*n - 2*B^2*d*n)*(b*x + a)^2/((b*c*g^3 - a*d*g^3)*(d*
x + c)^2) - 4*(B^2*b*n^2 - A*B*b*n - B^2*b*n)*(b*x + a)/((b*c*g^3 - a*d*g^3
)*(d*x + c)))*log((b*x + a)/(d*x + c)) - (B^2*d*n^2 - 2*A*B*d*n - 2*B^2*d*n
 + 2*A^2*d + 4*A*B*d + 2*B^2*d)*(b*x + a)^2/((b*c*g^3 - a*d*g^3)*(d*x + c)^
2) + 4*(2*B^2*b*n^2 - 2*A*B*b*n - 2*B^2*b*n + A^2*b + 2*A*B*b + B^2*b)*(b*x
 + a)/((b*c*g^3 - a*d*g^3)*(d*x + c)))*(b*c/(b*c - a*d)^2 - a*d/(b*c - a*d)
^2)
```

**maple [F]** time = 0.28, size = 0, normalized size = 0.00

$$\int \frac{\left(B\ln\left(e\left(\frac{bx+a}{dx+c}\right)^{n}\right)+A\right)^{2}}{\left(dgx+cg\right)^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*ln(e*((b*x+a)/(d*x+c))^n)+A)^2/(d*g*x+c*g)^3,x)
```

```
[Out] int((B*ln(e*((b*x+a)/(d*x+c))^n)+A)^2/(d*g*x+c*g)^3,x)
```

**maxima [B]** time = 1.23, size = 861, normalized size = 2.72

$$\frac{1}{2}ABn\left(\frac{2bdx+3bc-ad}{(bcd^3-ad^4)g^3x^2+2(bc^2d^2-acd^3)g^3x+(bc^3d-ac^2d^2)g^3}+\frac{2b^2\log(bx+a)}{(b^2c^2d-2abcd^2+a^2d^3)g^3}-\frac{2b^2\log}{(b^2c^2d-2ab}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))^2/(d*g*x+c*g)^3,x, algorithm="ma
xima")
```

```
[Out] 1/2*A*B*n*((2*b*d*x + 3*b*c - a*d)/((b*c*d^3 - a*d^4)*g^3*x^2 + 2*(b*c^2*d^
2 - a*c*d^3)*g^3*x + (b*c^3*d - a*c^2*d^2)*g^3) + 2*b^2*log(b*x + a)/((b^2*
c^2*d - 2*a*b*c*d^2 + a^2*d^3)*g^3) - 2*b^2*log(d*x + c)/((b^2*c^2*d - 2*a*
b*c*d^2 + a^2*d^3)*g^3)) + 1/4*(2*n*((2*b*d*x + 3*b*c - a*d)/((b*c*d^3 - a*
d^4)*g^3*x^2 + 2*(b*c^2*d^2 - a*c*d^3)*g^3*x + (b*c^3*d - a*c^2*d^2)*g^3) +
 2*b^2*log(b*x + a)/((b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)*g^3) - 2*b^2*log(d
*x + c)/((b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)*g^3))*log(e*(b*x/(d*x + c) + a
/(d*x + c))^n) - (7*b^2*c^2 - 8*a*b*c*d + a^2*d^2 + 2*(b^2*d^2*x^2 + 2*b^2*
c*d*x + b^2*c^2)*log(b*x + a)^2 + 2*(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*l
og(d*x + c)^2 + 6*(b^2*c*d - a*b*d^2)*x + 6*(b^2*d^2*x^2 + 2*b^2*c*d*x + b^
2*c^2)*log(b*x + a) - 2*(3*b^2*d^2*x^2 + 6*b^2*c*d*x + 3*b^2*c^2 + 2*(b^2*d
^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*log(b*x + a))*log(d*x + c))*n^2/(b^2*c^4*d*
g^3 - 2*a*b*c^3*d^2*g^3 + a^2*c^2*d^3*g^3 + (b^2*c^2*d^3*g^3 - 2*a*b*c*d^4*
g^3 + a^2*d^5*g^3)*x^2 + 2*(b^2*c^3*d^2*g^3 - 2*a*b*c^2*d^3*g^3 + a^2*c*d^4
*g^3)*x))*B^2 - 1/2*B^2*log(e*(b*x/(d*x + c) + a/(d*x + c))^n)^2/(d^3*g^3*x
^2 + 2*c*d^2*g^3*x + c^2*d*g^3) - A*B*log(e*(b*x/(d*x + c) + a/(d*x + c))^n
)/(d^3*g^3*x^2 + 2*c*d^2*g^3*x + c^2*d*g^3) - 1/2*A^2/(d^3*g^3*x^2 + 2*c*d^
2*g^3*x + c^2*d*g^3)
```

**mupad [B]** time = 5.47, size = 505, normalized size = 1.59

$$-\ln\left(e\left(\frac{a+b\,x}{c+d\,x}\right)^n\right)^2\left(\frac{B^2}{2\,d\,\left(c^2\,g^3+2\,c\,d\,g^3\,x+d^2\,g^3\,x^2\right)}-\frac{B^2\,b^2}{2\,d\,g^3\,\left(a^2\,d^2-2\,a\,b\,c\,d+b^2\,c^2\right)}\right)-\frac{\frac{2\,A^2\,a\,d-2\,A^2\,b\,c+B^2\,a\,d\,n^2\cdots}{2}}{2\,c\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*log(e*((a + b*x)/(c + d*x))^n))^2/(c*g + d*g*x)^3,x)
```

```
[Out] - log(e*((a + b*x)/(c + d*x))^n)^2*(B^2/(2*d*(c^2*g^3 + d^2*g^3*x^2 + 2*c*d
*g^3*x)) - (B^2*b^2)/(2*d*g^3*(a^2*d^2 + b^2*c^2 - 2*a*b*c*d))) - ((2*A^2*a
*d - 2*A^2*b*c + B^2*a*d*n^2 - 7*B^2*b*c*n^2 - 2*A*B*a*d*n + 6*A*B*b*c*n)/(
2*(a*d - b*c)) - (b*x*(3*B^2*d*n^2 - 2*A*B*d*n))/(a*d - b*c))/(2*c^2*d*g^3
+ 2*d^3*g^3*x^2 + 4*c*d^2*g^3*x) - log(e*((a + b*x)/(c + d*x))^n)*((A*B)/(c
^2*d*g^3 + d^3*g^3*x^2 + 2*c*d^2*g^3*x) + (B^2*b^2*((d^2*g^3*n*x*(a*d - b*c
))/b - (d*g^3*n*(a*d - b*c)*(a*d - 2*b*c))/(2*b^2) + (c*d*g^3*n*(a*d - b*c)
)/(2*b)))/(d*g^3*(a^2*d^2 + b^2*c^2 - 2*a*b*c*d)*(c^2*d*g^3 + d^3*g^3*x^2 +
 2*c*d^2*g^3*x))) - (B*b^2*n*atan(((2*b*d*x + (2*a^2*d^3*g^3 - 2*b^2*c^2*d*
g^3)/(2*d*g^3*(a*d - b*c)))*1i)/(a*d - b*c))*(2*A - 3*B*n)*1i)/(d*g^3*(a*d
- b*c)^2)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{A^2}{c^3+3c^2dx+3cd^2x^2+d^3x^3}\,dx + \int \frac{B^2\log\left(e\left(\frac{a}{c+dx}+\frac{bx}{c+dx}\right)^n\right)^2}{c^3+3c^2dx+3cd^2x^2+d^3x^3}\,dx + \int \frac{2AB\log\left(e\left(\frac{a}{c+dx}+\frac{bx}{c+dx}\right)^n\right)}{c^3+3c^2dx+3cd^2x^2+d^3x^3}\,dx}{g^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*ln(e*((b*x+a)/(d*x+c))**n))**2/(d*g*x+c*g)**3,x)
```

```
[Out] (Integral(A**2/(c**3 + 3*c**2*d*x + 3*c*d**2*x**2 + d**3*x**3), x) + Integr
al(B**2*log(e*(a/(c + d*x) + b*x/(c + d*x))**n)**2/(c**3 + 3*c**2*d*x + 3*c
*d**2*x**2 + d**3*x**3), x) + Integral(2*A*B*log(e*(a/(c + d*x) + b*x/(c +
d*x))**n)/(c**3 + 3*c**2*d*x + 3*c*d**2*x**2 + d**3*x**3), x))/g**3
```

## 3.45 $\displaystyle \int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(cg+dgx)^4}\,dx$

**Optimal**. Leaf size=429

$$\frac{2b^3Bn\log\left(\frac{a+bx}{c+dx}\right)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{3dg^4(bc-ad)^3}-\frac{2b^2Bn(a+bx)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{g^4(c+dx)(bc-ad)^3}-\frac{2Bd^2n(a+bx)^3\left(B\log\left(e\left(\cdots\right.\right.\right.}{9g^4(c+dx)^3(bc-\cdots}$$

```
[Out] 2/27*B^2*d^2*n^2*(b*x+a)^3/(-a*d+b*c)^3/g^4/(d*x+c)^3-1/2*b*B^2*d*n^2*(b*x+
a)^2/(-a*d+b*c)^3/g^4/(d*x+c)^2+2*b^2*B^2*n^2*(b*x+a)/(-a*d+b*c)^3/g^4/(d*x
+c)-2/9*B*d^2*n*(b*x+a)^3*(A+B*ln(e*((b*x+a)/(d*x+c))^n))/(-a*d+b*c)^3/g^4/
(d*x+c)^3+b*B*d*n*(b*x+a)^2*(A+B*ln(e*((b*x+a)/(d*x+c))^n))/(-a*d+b*c)^3/g^
4/(d*x+c)^2-2*b^2*B*n*(b*x+a)*(A+B*ln(e*((b*x+a)/(d*x+c))^n))/(-a*d+b*c)^3/
g^4/(d*x+c)-1/3*(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2/d/g^4/(d*x+c)^3+2/3*b^3*B
*n*(A+B*ln(e*((b*x+a)/(d*x+c))^n))*ln((b*x+a)/(d*x+c))/d/(-a*d+b*c)^3/g^4-1
/3*b^3*B^2*n^2*ln((b*x+a)/(d*x+c))^2/d/(-a*d+b*c)^3/g^4
```

**Rubi [C]** time = 1.10, antiderivative size = 736, normalized size of antiderivative = 1.72, number of steps used = 32, number of rules used = 11, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.314, Rules used = {2525, 12, 2528, 2524, 2418, 2390, 2301, 2394, 2393, 2391, 44}

$$\frac{2b^3B^2n^2\text{PolyLog}\left(2,-\frac{d(a+bx)}{bc-ad}\right)}{3dg^4(bc-ad)^3}+\frac{2b^3B^2n^2\text{PolyLog}\left(2,\frac{b(c+dx)}{bc-ad}\right)}{3dg^4(bc-ad)^3}+\frac{2b^3Bn\log(a+bx)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{3dg^4(bc-ad)^3}-\frac{2b^3\cdots}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2/(c*g + d*g*x)^4,x]
```

```
[Out] (-2*B^2*n^2)/(27*d*g^4*(c + d*x)^3) - (5*b*B^2*n^2)/(18*d*(b*c - a*d)*g^4*(
c + d*x)^2) - (11*b^2*B^2*n^2)/(9*d*(b*c - a*d)^2*g^4*(c + d*x)) - (11*b^3*
B^2*n^2*Log[a + b*x])/(9*d*(b*c - a*d)^3*g^4) - (b^3*B^2*n^2*Log[a + b*x]^2
)/(3*d*(b*c - a*d)^3*g^4) + (2*B*n*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/
(9*d*g^4*(c + d*x)^3) + (b*B*n*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/(3*d
*(b*c - a*d)*g^4*(c + d*x)^2) + (2*b^2*B*n*(A + B*Log[e*((a + b*x)/(c + d*x
))^n]))/(3*d*(b*c - a*d)^2*g^4*(c + d*x)) + (2*b^3*B*n*Log[a + b*x]*(A + B*
Log[e*((a + b*x)/(c + d*x))^n]))/(3*d*(b*c - a*d)^3*g^4) - (A + B*Log[e*((a
 + b*x)/(c + d*x))^n])^2/(3*d*g^4*(c + d*x)^3) + (11*b^3*B^2*n^2*Log[c + d*
x])/(9*d*(b*c - a*d)^3*g^4) + (2*b^3*B^2*n^2*Log[-((d*(a + b*x))/(b*c - a*d
))]*Log[c + d*x])/(3*d*(b*c - a*d)^3*g^4) - (2*b^3*B*n*(A + B*Log[e*((a + b
*x)/(c + d*x))^n])*Log[c + d*x])/(3*d*(b*c - a*d)^3*g^4) - (b^3*B^2*n^2*Log
```

```
[c + d*x]^2)/(3*d*(b*c - a*d)^3*g^4) + (2*b^3*B^2*n^2*Log[a + b*x]*Log[(b*(
c + d*x))/(b*c - a*d)])/(3*d*(b*c - a*d)^3*g^4) + (2*b^3*B^2*n^2*PolyLog[2,
 -((d*(a + b*x))/(b*c - a*d))])/(3*d*(b*c - a*d)^3*g^4) + (2*b^3*B^2*n^2*Po
lyLog[2, (b*(c + d*x))/(b*c - a*d)])/(3*d*(b*c - a*d)^3*g^4)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 2301

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

Rule 2390

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_))^(q_.), x_Symbol] :> Dist[1/e, Subst[Int[((f*x)/d)^q*(a + b*Log[c*x^
n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && E
qQ[e*f - d*g, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2393

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + (c*e*x)/g])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2394

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d + e*x
)^n]))/g, x] - Dist[(b*e*n)/g, Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
```

```
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2418

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*(RFx_), x_Sy
mbol] :> With[{u = ExpandIntegrand[(a + b*Log[c*(d + e*x)^n])^p, RFx, x]},
Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d, e, n}, x] && RationalFunctionQ[
RFx, x] && IntegerQ[p]
```

Rule 2524

```
Int[((a_.) + Log[(c_.)*(RFx_)^(p_.)]*(b_.))^(n_.)/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[(Log[d + e*x]*(a + b*Log[c*RFx^p])^n)/e, x] - Dist[(b*n*p)/e
, Int[(Log[d + e*x]*(a + b*Log[c*RFx^p])^(n - 1)*D[RFx, x])/RFx, x], x] /;
FreeQ[{a, b, c, d, e, p}, x] && RationalFunctionQ[RFx, x] && IGtQ[n, 0]
```

Rule 2525

```
Int[((a_.) + Log[(c_.)*(RFx_)^(p_.)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_))^(m_.
), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(a + b*Log[c*RFx^p])^n)/(e*(m + 1))
, x] - Dist[(b*n*p)/(e*(m + 1)), Int[SimplifyIntegrand[((d + e*x)^(m + 1)*(
a + b*Log[c*RFx^p])^(n - 1)*D[RFx, x])/RFx, x], x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && RationalFunctionQ[RFx, x] && IGtQ[n, 0] && (EqQ[n, 1] ||
IntegerQ[m]) && NeQ[m, -1]
```

Rule 2528

```
Int[((a_.) + Log[(c_.)*(RFx_)^(p_.)]*(b_.))^(n_.)*(RGx_), x_Symbol] :> With
[{u = ExpandIntegrand[(a + b*Log[c*RFx^p])^n, RGx, x]}, Int[u, x] /; SumQ[u
]] /; FreeQ[{a, b, c, p}, x] && RationalFunctionQ[RFx, x] && RationalFuncti
onQ[RGx, x] && IGtQ[n, 0]
```

Rubi steps

$$\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(cg+dgx)^4}\,dx = -\frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{3dg^4(c+dx)^3} + \frac{(2Bn)\int \frac{(bc-ad)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{g^3(a+bx)(c+dx)^4}\,dx}{3dg}$$

$$= -\frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{3dg^4(c+dx)^3} + \frac{(2B(bc-ad)n)\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(a+bx)(c+dx)^4}\,dx}{3dg^4}$$

$$= -\frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{3dg^4(c+dx)^3} + \frac{(2B(bc-ad)n)\int\left(\frac{b^4\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(bc-ad)^4(a+bx)} - \frac{d\left(A+B\log\left(\cdots\right.\right.}{(bc-ad)\cdots}\right.}{\cdots}$$

$$= -\frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{3dg^4(c+dx)^3} - \frac{(2Bn)\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(c+dx)^4}\,dx}{3g^4} - \frac{\left(2b^3Bn\right)\int \frac{A+B\log\left(e\cdots\right.}{c+\cdots}}{3(bc-ad)^3}$$

$$= \frac{2Bn\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{9dg^4(c+dx)^3} + \frac{bBn\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{3d(bc-ad)g^4(c+dx)^2} + \frac{2b^2Bn\left(A+B\,\mathrm{l}\cdots\right.}{3d(bc-ad\cdots}$$

$$= \frac{2Bn\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{9dg^4(c+dx)^3} + \frac{bBn\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{3d(bc-ad)g^4(c+dx)^2} + \frac{2b^2Bn\left(A+B\,\mathrm{l}\cdots\right.}{3d(bc-ad\cdots}$$

$$= \frac{2Bn\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{9dg^4(c+dx)^3} + \frac{bBn\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{3d(bc-ad)g^4(c+dx)^2} + \frac{2b^2Bn\left(A+B\,\mathrm{l}\cdots\right.}{3d(bc-ad\cdots}$$

$$= -\frac{2B^2n^2}{27dg^4(c+dx)^3} - \frac{5bB^2n^2}{18d(bc-ad)g^4(c+dx)^2} - \frac{11b^2B^2n^2}{9d(bc-ad)^2g^4(c+dx)} - \frac{11b^3B\cdots}{9d\cdots}$$

$$= -\frac{2B^2n^2}{27dg^4(c+dx)^3} - \frac{5bB^2n^2}{18d(bc-ad)g^4(c+dx)^2} - \frac{11b^2B^2n^2}{9d(bc-ad)^2g^4(c+dx)} - \frac{11b^3B\cdots}{9d\cdots}$$

$$= -\frac{2B^2n^2}{27dg^4(c+dx)^3} - \frac{5bB^2n^2}{18d(bc-ad)g^4(c+dx)^2} - \frac{11b^2B^2n^2}{9d(bc-ad)^2g^4(c+dx)} - \frac{11b^3B\cdots}{9d\cdots}$$

**Mathematica [C]** time = 0.67, size = 609, normalized size = 1.42

$$\frac{Bn\left(36b^3(c+dx)^3\log(a+bx)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)-36b^3(c+dx)^3\log(c+dx)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)+36b^2(c+dx)^2(bc-ad)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)+12(bc-\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2/(c*g + d*g*x)^4,x]
```

```
[Out] (-18*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2 + (B*n*(12*(b*c - a*d)^3*(A +
 B*Log[e*((a + b*x)/(c + d*x))^n]) + 18*b*(b*c - a*d)^2*(c + d*x)*(A + B*Lo
g[e*((a + b*x)/(c + d*x))^n]) + 36*b^2*(b*c - a*d)*(c + d*x)^2*(A + B*Log[e
*((a + b*x)/(c + d*x))^n]) + 36*b^3*(c + d*x)^3*Log[a + b*x]*(A + B*Log[e*(
(a + b*x)/(c + d*x))^n]) - 36*b^3*(c + d*x)^3*(A + B*Log[e*((a + b*x)/(c +
d*x))^n])*Log[c + d*x] - 36*b^2*B*n*(c + d*x)^2*(b*c - a*d + b*(c + d*x)*Lo
g[a + b*x] - b*(c + d*x)*Log[c + d*x]) - 9*b*B*n*(c + d*x)*((b*c - a*d)^2 +
 2*b*(b*c - a*d)*(c + d*x) + 2*b^2*(c + d*x)^2*Log[a + b*x] - 2*b^2*(c + d*
x)^2*Log[c + d*x]) - 2*B*n*(2*(b*c - a*d)^3 + 3*b*(b*c - a*d)^2*(c + d*x) +
 6*b^2*(b*c - a*d)*(c + d*x)^2 + 6*b^3*(c + d*x)^3*Log[a + b*x] - 6*b^3*(c
+ d*x)^3*Log[c + d*x]) - 18*b^3*B*n*(c + d*x)^3*(Log[a + b*x]*(Log[a + b*x]
 - 2*Log[(b*(c + d*x))/(b*c - a*d)]) - 2*PolyLog[2, (d*(a + b*x))/(-(b*c) +
 a*d)]) + 18*b^3*B*n*(c + d*x)^3*((2*Log[(d*(a + b*x))/(-(b*c) + a*d)] - Lo
g[c + d*x])*Log[c + d*x] + 2*PolyLog[2, (b*(c + d*x))/(b*c - a*d)])))/(b*c
- a*d)^3)/(54*d*g^4*(c + d*x)^3)
```

**fricas [B]** time = 0.85, size = 1167, normalized size = 2.72

$$-\frac{18\,A^2b^3c^3-54\,A^2ab^2c^2d+54\,A^2a^2bcd^2-18\,A^2a^3d^3+\left(85\,B^2b^3c^3-108\,B^2ab^2c^2d+27\,B^2a^2bcd^2-4\,B^2a^3d^3\right)\ldots}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))^2/(d*g*x+c*g)^4,x, algorithm="fr
icas")
```

```
[Out] -1/54*(18*A^2*b^3*c^3 - 54*A^2*a*b^2*c^2*d + 54*A^2*a^2*b*c*d^2 - 18*A^2*a^
3*d^3 + (85*B^2*b^3*c^3 - 108*B^2*a*b^2*c^2*d + 27*B^2*a^2*b*c*d^2 - 4*B^2*
a^3*d^3)*n^2 + 6*(11*(B^2*b^3*c*d^2 - B^2*a*b^2*d^3)*n^2 - 6*(A*B*b^3*c*d^2
 - A*B*a*b^2*d^3)*n)*x^2 + 18*(B^2*b^3*c^3 - 3*B^2*a*b^2*c^2*d + 3*B^2*a^2*
b*c*d^2 - B^2*a^3*d^3)*log(e)^2 - 18*(B^2*b^3*d^3*n^2*x^3 + 3*B^2*b^3*c*d^2
*n^2*x^2 + 3*B^2*b^3*c^2*d*n^2*x + (3*B^2*a*b^2*c^2*d - 3*B^2*a^2*b*c*d^2 +
 B^2*a^3*d^3)*n^2)*log((b*x + a)/(d*x + c))^2 - 6*(11*A*B*b^3*c^3 - 18*A*B*
a*b^2*c^2*d + 9*A*B*a^2*b*c*d^2 - 2*A*B*a^3*d^3)*n + 3*((49*B^2*b^3*c^2*d -
```

```
54*B^2*a*b^2*c*d^2 + 5*B^2*a^2*b*d^3)*n^2 - 6*(5*A*B*b^3*c^2*d - 6*A*B*a*b
^2*c*d^2 + A*B*a^2*b*d^3)*n)*x + 6*(6*A*B*b^3*c^3 - 18*A*B*a*b^2*c^2*d + 18
*A*B*a^2*b*c*d^2 - 6*A*B*a^3*d^3 - 6*(B^2*b^3*c*d^2 - B^2*a*b^2*d^3)*n*x^2
- 3*(5*B^2*b^3*c^2*d - 6*B^2*a*b^2*c*d^2 + B^2*a^2*b*d^3)*n*x - (11*B^2*b^3
*c^3 - 18*B^2*a*b^2*c^2*d + 9*B^2*a^2*b*c*d^2 - 2*B^2*a^3*d^3)*n - 6*(B^2*b
^3*d^3*n*x^3 + 3*B^2*b^3*c*d^2*n*x^2 + 3*B^2*b^3*c^2*d*n*x + (3*B^2*a*b^2*c
^2*d - 3*B^2*a^2*b*c*d^2 + B^2*a^3*d^3)*n)*log((b*x + a)/(d*x + c)))*log(e)
 + 6*((11*B^2*b^3*d^3*n^2 - 6*A*B*b^3*d^3*n)*x^3 + (18*B^2*a*b^2*c^2*d - 9*
B^2*a^2*b*c*d^2 + 2*B^2*a^3*d^3)*n^2 - 3*(6*A*B*b^3*c*d^2*n - (9*B^2*b^3*c*
d^2 + 2*B^2*a*b^2*d^3)*n^2)*x^2 - 6*(3*A*B*a*b^2*c^2*d - 3*A*B*a^2*b*c*d^2
+ A*B*a^3*d^3)*n - 3*(6*A*B*b^3*c^2*d*n - (6*B^2*b^3*c^2*d + 6*B^2*a*b^2*c*
d^2 - B^2*a^2*b*d^3)*n^2)*x)*log((b*x + a)/(d*x + c)))/((b^3*c^3*d^4 - 3*a*
b^2*c^2*d^5 + 3*a^2*b*c*d^6 - a^3*d^7)*g^4*x^3 + 3*(b^3*c^4*d^3 - 3*a*b^2*c
^3*d^4 + 3*a^2*b*c^2*d^5 - a^3*c*d^6)*g^4*x^2 + 3*(b^3*c^5*d^2 - 3*a*b^2*c^
4*d^3 + 3*a^2*b*c^3*d^4 - a^3*c^2*d^5)*g^4*x + (b^3*c^6*d - 3*a*b^2*c^5*d^2
 + 3*a^2*b*c^4*d^3 - a^3*c^3*d^4)*g^4)
```

**giac [A]** time = 13.81, size = 746, normalized size = 1.74

$$\frac{1}{54}\left(18\left(\frac{3\,(bx+a)B^2b^2n^2}{\left(b^2c^2g^4-2\,abcdg^4+a^2d^2g^4\right)(dx+c)}-\frac{3\,(bx+a)^2B^2bdn^2}{\left(b^2c^2g^4-2\,abcdg^4+a^2d^2g^4\right)(dx+c)^2}+\frac{(bx+a)^3B^2d}{\left(b^2c^2g^4-2\,abcdg^4+a^2\right.}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))^2/(d*g*x+c*g)^4,x, algorithm="gi
ac")
```

```
[Out] 1/54*(18*(3*(b*x + a)*B^2*b^2*n^2/((b^2*c^2*g^4 - 2*a*b*c*d*g^4 + a^2*d^2*g
^4)*(d*x + c)) - 3*(b*x + a)^2*B^2*b*d*n^2/((b^2*c^2*g^4 - 2*a*b*c*d*g^4 +
a^2*d^2*g^4)*(d*x + c)^2) + (b*x + a)^3*B^2*d^2*n^2/((b^2*c^2*g^4 - 2*a*b*c
*d*g^4 + a^2*d^2*g^4)*(d*x + c)^3))*log((b*x + a)/(d*x + c))^2 - 6*(2*(B^2*
d^2*n^2 - 3*A*B*d^2*n - 3*B^2*d^2*n)*(b*x + a)^3/((b^2*c^2*g^4 - 2*a*b*c*d*
g^4 + a^2*d^2*g^4)*(d*x + c)^3) - 9*(B^2*b*d*n^2 - 2*A*B*b*d*n - 2*B^2*b*d*
n)*(b*x + a)^2/((b^2*c^2*g^4 - 2*a*b*c*d*g^4 + a^2*d^2*g^4)*(d*x + c)^2) +
18*(B^2*b^2*n^2 - A*B*b^2*n - B^2*b^2*n)*(b*x + a)/((b^2*c^2*g^4 - 2*a*b*c*
d*g^4 + a^2*d^2*g^4)*(d*x + c)))*log((b*x + a)/(d*x + c)) + 2*(2*B^2*d^2*n^
2 - 6*A*B*d^2*n - 6*B^2*d^2*n + 9*A^2*d^2 + 18*A*B*d^2 + 9*B^2*d^2)*(b*x +
a)^3/((b^2*c^2*g^4 - 2*a*b*c*d*g^4 + a^2*d^2*g^4)*(d*x + c)^3) - 27*(B^2*b*
d*n^2 - 2*A*B*b*d*n - 2*B^2*b*d*n + 2*A^2*b*d + 4*A*B*b*d + 2*B^2*b*d)*(b*x
 + a)^2/((b^2*c^2*g^4 - 2*a*b*c*d*g^4 + a^2*d^2*g^4)*(d*x + c)^2) + 54*(2*B
^2*b^2*n^2 - 2*A*B*b^2*n - 2*B^2*b^2*n + A^2*b^2 + 2*A*B*b^2 + B^2*b^2)*(b*
x + a)/((b^2*c^2*g^4 - 2*a*b*c*d*g^4 + a^2*d^2*g^4)*(d*x + c)))*(b*c/(b*c -
 a*d)^2 - a*d/(b*c - a*d)^2)
```

**maple [F]** time = 0.28, size = 0, normalized size = 0.00

$$\int \frac{\left(B\ln\left(e\left(\frac{bx+a}{dx+c}\right)^{n}\right)+A\right)^{2}}{\left(dgx+cg\right)^{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*ln(e*((b*x+a)/(d*x+c))^n)+A)^2/(d*g*x+c*g)^4,x)
```

```
[Out] int((B*ln(e*((b*x+a)/(d*x+c))^n)+A)^2/(d*g*x+c*g)^4,x)
```

**maxima [B]** time = 1.29, size = 1435, normalized size = 3.34

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))^2/(d*g*x+c*g)^4,x, algorithm="ma
xima")
```

```
[Out] 1/9*A*B*n*((6*b^2*d^2*x^2 + 11*b^2*c^2 - 7*a*b*c*d + 2*a^2*d^2 + 3*(5*b^2*c
*d - a*b*d^2)*x)/((b^2*c^2*d^4 - 2*a*b*c*d^5 + a^2*d^6)*g^4*x^3 + 3*(b^2*c^
3*d^3 - 2*a*b*c^2*d^4 + a^2*c*d^5)*g^4*x^2 + 3*(b^2*c^4*d^2 - 2*a*b*c^3*d^3
 + a^2*c^2*d^4)*g^4*x + (b^2*c^5*d - 2*a*b*c^4*d^2 + a^2*c^3*d^3)*g^4) + 6*
b^3*log(b*x + a)/((b^3*c^3*d - 3*a*b^2*c^2*d^2 + 3*a^2*b*c*d^3 - a^3*d^4)*g
^4) - 6*b^3*log(d*x + c)/((b^3*c^3*d - 3*a*b^2*c^2*d^2 + 3*a^2*b*c*d^3 - a^
3*d^4)*g^4)) + 1/54*(6*n*((6*b^2*d^2*x^2 + 11*b^2*c^2 - 7*a*b*c*d + 2*a^2*d
^2 + 3*(5*b^2*c*d - a*b*d^2)*x)/((b^2*c^2*d^4 - 2*a*b*c*d^5 + a^2*d^6)*g^4*
x^3 + 3*(b^2*c^3*d^3 - 2*a*b*c^2*d^4 + a^2*c*d^5)*g^4*x^2 + 3*(b^2*c^4*d^2
- 2*a*b*c^3*d^3 + a^2*c^2*d^4)*g^4*x + (b^2*c^5*d - 2*a*b*c^4*d^2 + a^2*c^3
*d^3)*g^4) + 6*b^3*log(b*x + a)/((b^3*c^3*d - 3*a*b^2*c^2*d^2 + 3*a^2*b*c*d
^3 - a^3*d^4)*g^4) - 6*b^3*log(d*x + c)/((b^3*c^3*d - 3*a*b^2*c^2*d^2 + 3*a
^2*b*c*d^3 - a^3*d^4)*g^4))*log(e*(b*x/(d*x + c) + a/(d*x + c))^n) - (85*b^
3*c^3 - 108*a*b^2*c^2*d + 27*a^2*b*c*d^2 - 4*a^3*d^3 + 66*(b^3*c*d^2 - a*b^
2*d^3)*x^2 + 18*(b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*b^3*c^2*d*x + b^3*c^3)*l
og(b*x + a)^2 + 18*(b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*b^3*c^2*d*x + b^3*c^3
)*log(d*x + c)^2 + 3*(49*b^3*c^2*d - 54*a*b^2*c*d^2 + 5*a^2*b*d^3)*x + 66*(
b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*b^3*c^2*d*x + b^3*c^3)*log(b*x + a) - 6*(
11*b^3*d^3*x^3 + 33*b^3*c*d^2*x^2 + 33*b^3*c^2*d*x + 11*b^3*c^3 + 6*(b^3*d^
3*x^3 + 3*b^3*c*d^2*x^2 + 3*b^3*c^2*d*x + b^3*c^3)*log(b*x + a))*log(d*x +
c))*n^2/(b^3*c^6*d*g^4 - 3*a*b^2*c^5*d^2*g^4 + 3*a^2*b*c^4*d^3*g^4 - a^3*c^
3*d^4*g^4 + (b^3*c^3*d^4*g^4 - 3*a*b^2*c^2*d^5*g^4 + 3*a^2*b*c*d^6*g^4 - a^
3*d^7*g^4)*x^3 + 3*(b^3*c^4*d^3*g^4 - 3*a*b^2*c^3*d^4*g^4 + 3*a^2*b*c^2*d^5
*g^4 - a^3*c*d^6*g^4)*x^2 + 3*(b^3*c^5*d^2*g^4 - 3*a*b^2*c^4*d^3*g^4 + 3*a^
```

```
2*b*c^3*d^4*g^4 - a^3*c^2*d^5*g^4)*x))*B^2 - 1/3*B^2*log(e*(b*x/(d*x + c) +
 a/(d*x + c))^n)^2/(d^4*g^4*x^3 + 3*c*d^3*g^4*x^2 + 3*c^2*d^2*g^4*x + c^3*d
*g^4) - 2/3*A*B*log(e*(b*x/(d*x + c) + a/(d*x + c))^n)/(d^4*g^4*x^3 + 3*c*d
^3*g^4*x^2 + 3*c^2*d^2*g^4*x + c^3*d*g^4) - 1/3*A^2/(d^4*g^4*x^3 + 3*c*d^3*
g^4*x^2 + 3*c^2*d^2*g^4*x + c^3*d*g^4)
```

**mupad [B]** time = 7.16, size = 1040, normalized size = 2.42

$$-\ln\left(e\left(\frac{a+b\,x}{c+d\,x}\right)^n\right)^2\left(\frac{B^2}{3\,d\,\left(c^3\,g^4+3\,c^2\,d\,g^4\,x+3\,c\,d^2\,g^4\,x^2+d^3\,g^4\,x^3\right)}+\frac{B^2\,b^3}{3\,d\,g^4\,\left(a^3\,d^3-3\,a^2\,b\,c\,d^2+3\,a\,b^2\,c^2\,d-b^3\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*log(e*((a + b*x)/(c + d*x))^n))^2/(c*g + d*g*x)^4,x)
```

```
[Out] - log(e*((a + b*x)/(c + d*x))^n)^2*(B^2/(3*d*(c^3*g^4 + d^3*g^4*x^3 + 3*c*d
^2*g^4*x^2 + 3*c^2*d*g^4*x)) + (B^2*b^3)/(3*d*g^4*(a^3*d^3 - b^3*c^3 + 3*a*
b^2*c^2*d - 3*a^2*b*c*d^2))) - ((18*A^2*a^2*d^2 + 18*A^2*b^2*c^2 + 4*B^2*a^
2*d^2*n^2 + 85*B^2*b^2*c^2*n^2 - 36*A^2*a*b*c*d - 12*A*B*a^2*d^2*n - 66*A*B
*b^2*c^2*n - 23*B^2*a*b*c*d*n^2 + 42*A*B*a*b*c*d*n)/(6*(a*d - b*c)) - (x*(5
*B^2*a*b*d^2*n^2 - 49*B^2*b^2*c*d*n^2 - 6*A*B*a*b*d^2*n + 30*A*B*b^2*c*d*n)
)/(2*(a*d - b*c)) + (b*x^2*(11*B^2*b*d^2*n^2 - 6*A*B*b*d^2*n))/(a*d - b*c))
/(x*(27*a*c^2*d^3*g^4 - 27*b*c^3*d^2*g^4) - x^2*(27*b*c^2*d^3*g^4 - 27*a*c*
d^4*g^4) + x^3*(9*a*d^5*g^4 - 9*b*c*d^4*g^4) + 9*a*c^3*d^2*g^4 - 9*b*c^4*d*
g^4) - log(e*((a + b*x)/(c + d*x))^n)*((2*A*B)/(3*c^3*d*g^4 + 3*d^4*g^4*x^3
 + 9*c^2*d^2*g^4*x + 9*c*d^3*g^4*x^2) + (2*B^2*b^3*(x*(d*((d*g^4*n*(a*d - b
*c)*(a*d - 3*b*c))/(2*b^2) - (c*d*g^4*n*(a*d - b*c))/b) - (2*c*d^2*g^4*n*(a
*d - b*c))/b + (d^2*g^4*n*(a*d - b*c)*(a*d - 3*b*c))/b^2) + c*((d*g^4*n*(a*
d - b*c)*(a*d - 3*b*c))/(2*b^2) - (c*d*g^4*n*(a*d - b*c))/b) - (d*g^4*n*(a*
d - b*c)*(a^2*d^2 + 3*b^2*c^2 - 3*a*b*c*d))/b^3 - (3*d^3*g^4*n*x^2*(a*d - b
*c))/b))/(3*d*g^4*(a^3*d^3 - b^3*c^3 + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2)*(3*c^
3*d*g^4 + 3*d^4*g^4*x^3 + 9*c^2*d^2*g^4*x + 9*c*d^3*g^4*x^2))) - (B*b^3*n*a
tan((B*b^3*n*(6*A - 11*B*n)*((a^3*d^4*g^4 + b^3*c^3*d*g^4 - a^2*b*c*d^3*g^4
 - a*b^2*c^2*d^2*g^4)/(a^2*d^3*g^4 + b^2*c^2*d*g^4 - 2*a*b*c*d^2*g^4) + 2*b
*d*x)*(a^2*d^3*g^4 + b^2*c^2*d*g^4 - 2*a*b*c*d^2*g^4)*1i)/(d*g^4*(11*B^2*b^
3*n^2 - 6*A*B*b^3*n)*(a*d - b*c)^3))*(6*A - 11*B*n)*2i)/(9*d*g^4*(a*d - b*c
)^3)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{A^2}{c^4+4c^3dx+6c^2d^2x^2+4cd^3x^3+d^4x^4}\,dx+\int \frac{B^2\log\left(e\left(\frac{a}{c+dx}+\frac{bx}{c+dx}\right)^n\right)^2}{c^4+4c^3dx+6c^2d^2x^2+4cd^3x^3+d^4x^4}\,dx+\int \frac{2AB\log\left(e\left(\frac{a}{c+dx}+\frac{bx}{c+dx}\right)^n\right)}{c^4+4c^3dx+6c^2d^2x^2+4cd^3x^3+d^4x^4}\,dx}{g^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*ln(e*((b*x+a)/(d*x+c))**n))**2/(d*g*x+c*g)**4,x)
```

```
[Out] (Integral(A**2/(c**4 + 4*c**3*d*x + 6*c**2*d**2*x**2 + 4*c*d**3*x**3 + d**4
*x**4), x) + Integral(B**2*log(e*(a/(c + d*x) + b*x/(c + d*x))**n)**2/(c**4
 + 4*c**3*d*x + 6*c**2*d**2*x**2 + 4*c*d**3*x**3 + d**4*x**4), x) + Integra
l(2*A*B*log(e*(a/(c + d*x) + b*x/(c + d*x))**n)/(c**4 + 4*c**3*d*x + 6*c**2
*d**2*x**2 + 4*c*d**3*x**3 + d**4*x**4), x))/g**4
```

# 3.46 $\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(cg+dgx)^5}\,dx$

**Optimal**. Leaf size=536

$$\frac{b^4Bn\log\left(\frac{a+bx}{c+dx}\right)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{2dg^5(bc-ad)^4}-\frac{2b^3Bn(a+bx)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{g^5(c+dx)(bc-ad)^4}+\frac{3b^2Bdn(a+bx)^2\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right.}{2g^5(c+dx)^2(bc-ad}$$

```
[Out] -1/32*B^2*d^3*n^2*(b*x+a)^4/(-a*d+b*c)^4/g^5/(d*x+c)^4+2/9*b*B^2*d^2*n^2*(b
*x+a)^3/(-a*d+b*c)^4/g^5/(d*x+c)^3-3/4*b^2*B^2*d*n^2*(b*x+a)^2/(-a*d+b*c)^4
/g^5/(d*x+c)^2+2*b^3*B^2*n^2*(b*x+a)/(-a*d+b*c)^4/g^5/(d*x+c)+1/8*B*d^3*n*(
b*x+a)^4*(A+B*ln(e*((b*x+a)/(d*x+c))^n))/(-a*d+b*c)^4/g^5/(d*x+c)^4-2/3*b*B
*d^2*n*(b*x+a)^3*(A+B*ln(e*((b*x+a)/(d*x+c))^n))/(-a*d+b*c)^4/g^5/(d*x+c)^3
+3/2*b^2*B*d*n*(b*x+a)^2*(A+B*ln(e*((b*x+a)/(d*x+c))^n))/(-a*d+b*c)^4/g^5/(
d*x+c)^2-2*b^3*B*n*(b*x+a)*(A+B*ln(e*((b*x+a)/(d*x+c))^n))/(-a*d+b*c)^4/g^5
/(d*x+c)-1/4*(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2/d/g^5/(d*x+c)^4+1/2*b^4*B*n*
(A+B*ln(e*((b*x+a)/(d*x+c))^n))*ln((b*x+a)/(d*x+c))/d/(-a*d+b*c)^4/g^5-1/4*
b^4*B^2*n^2*ln((b*x+a)/(d*x+c))^2/d/(-a*d+b*c)^4/g^5
```

**Rubi [C]** time = 1.30, antiderivative size = 826, normalized size of antiderivative = 1.54, number of steps used = 36, number of rules used = 11, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.314, Rules used = {2525, 12, 2528, 2524, 2418, 2390, 2301, 2394, 2393, 2391, 44}

$$-\frac{B^2n^2\log^2(a+bx)b^4}{4d(bc-ad)^4g^5}-\frac{B^2n^2\log^2(c+dx)b^4}{4d(bc-ad)^4g^5}-\frac{25B^2n^2\log(a+bx)b^4}{24d(bc-ad)^4g^5}+\frac{Bn\log(a+bx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)b^4}{2d(bc-ad)^4g^5}+$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2/(c*g + d*g*x)^5,x]
```

```
[Out] -(B^2*n^2)/(32*d*g^5*(c + d*x)^4) - (7*b*B^2*n^2)/(72*d*(b*c - a*d)*g^5*(c
 + d*x)^3) - (13*b^2*B^2*n^2)/(48*d*(b*c - a*d)^2*g^5*(c + d*x)^2) - (25*b^3
*B^2*n^2)/(24*d*(b*c - a*d)^3*g^5*(c + d*x)) - (25*b^4*B^2*n^2*Log[a + b*x]
)/(24*d*(b*c - a*d)^4*g^5) - (b^4*B^2*n^2*Log[a + b*x]^2)/(4*d*(b*c - a*d)^
4*g^5) + (B*n*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/(8*d*g^5*(c + d*x)^4)
 + (b*B*n*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/(6*d*(b*c - a*d)*g^5*(c +
 d*x)^3) + (b^2*B*n*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/(4*d*(b*c - a*d
)^2*g^5*(c + d*x)^2) + (b^3*B*n*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/(2*
d*(b*c - a*d)^3*g^5*(c + d*x)) + (b^4*B*n*Log[a + b*x]*(A + B*Log[e*((a + b
*x)/(c + d*x))^n]))/(2*d*(b*c - a*d)^4*g^5) - (A + B*Log[e*((a + b*x)/(c +
```

```
d*x))^n])^2/(4*d*g^5*(c + d*x)^4) + (25*b^4*B^2*n^2*Log[c + d*x])/(24*d*(b*
c - a*d)^4*g^5) + (b^4*B^2*n^2*Log[-((d*(a + b*x))/(b*c - a*d))]*Log[c + d*
x])/(2*d*(b*c - a*d)^4*g^5) - (b^4*B*n*(A + B*Log[e*((a + b*x)/(c + d*x))^n
])*Log[c + d*x])/(2*d*(b*c - a*d)^4*g^5) - (b^4*B^2*n^2*Log[c + d*x]^2)/(4*
d*(b*c - a*d)^4*g^5) + (b^4*B^2*n^2*Log[a + b*x]*Log[(b*(c + d*x))/(b*c - a
*d)])/(2*d*(b*c - a*d)^4*g^5) + (b^4*B^2*n^2*PolyLog[2, -((d*(a + b*x))/(b*
c - a*d))])/(2*d*(b*c - a*d)^4*g^5) + (b^4*B^2*n^2*PolyLog[2, (b*(c + d*x))
/(b*c - a*d)])/(2*d*(b*c - a*d)^4*g^5)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 2301

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

Rule 2390

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_))^(q_.), x_Symbol] :> Dist[1/e, Subst[Int[((f*x)/d)^q*(a + b*Log[c*x^
n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && E
qQ[e*f - d*g, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2393

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + (c*e*x)/g])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2394

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d + e*x
)^n]))/g, x] - Dist[(b*e*n)/g, Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2418

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*(RFx_), x_Sy
mbol] :> With[{u = ExpandIntegrand[(a + b*Log[c*(d + e*x)^n])^p, RFx, x]},
Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d, e, n}, x] && RationalFunctionQ[
RFx, x] && IntegerQ[p]
```

Rule 2524

```
Int[((a_.) + Log[(c_.)*(RFx_)^(p_.)]*(b_.))^(n_.)/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[(Log[d + e*x]*(a + b*Log[c*RFx^p])^n)/e, x] - Dist[(b*n*p)/e
, Int[(Log[d + e*x]*(a + b*Log[c*RFx^p])^(n - 1)*D[RFx, x])/RFx, x], x] /;
FreeQ[{a, b, c, d, e, p}, x] && RationalFunctionQ[RFx, x] && IGtQ[n, 0]
```

Rule 2525

```
Int[((a_.) + Log[(c_.)*(RFx_)^(p_.)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_))^(m_.
), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(a + b*Log[c*RFx^p])^n)/(e*(m + 1))
, x] - Dist[(b*n*p)/(e*(m + 1)), Int[SimplifyIntegrand[((d + e*x)^(m + 1)*(
a + b*Log[c*RFx^p])^(n - 1)*D[RFx, x])/RFx, x], x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && RationalFunctionQ[RFx, x] && IGtQ[n, 0] && (EqQ[n, 1] ||
IntegerQ[m]) && NeQ[m, -1]
```

Rule 2528

```
Int[((a_.) + Log[(c_.)*(RFx_)^(p_.)]*(b_.))^(n_.)*(RGx_), x_Symbol] :> With
[{u = ExpandIntegrand[(a + b*Log[c*RFx^p])^n, RGx, x]}, Int[u, x] /; SumQ[u
]] /; FreeQ[{a, b, c, p}, x] && RationalFunctionQ[RFx, x] && RationalFuncti
onQ[RGx, x] && IGtQ[n, 0]
```

Rubi steps

$$\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(cg+dgx)^5}\,dx = -\frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{4dg^5(c+dx)^4} + \frac{(Bn)\int \frac{(bc-ad)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{g^4(a+bx)(c+dx)^5}\,dx}{2dg}$$

$$= -\frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{4dg^5(c+dx)^4} + \frac{(B(bc-ad)n)\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(a+bx)(c+dx)^5}\,dx}{2dg^5}$$

$$= -\frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{4dg^5(c+dx)^4} + \frac{(B(bc-ad)n)\int \left(\frac{b^5\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(bc-ad)^5(a+bx)} - \frac{d\left(A+B\log\cdots\right.}{(bc-ad)\cdots}\cdots\right.}{\cdots}$$

$$= -\frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{4dg^5(c+dx)^4} - \frac{(Bn)\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(c+dx)^5}\,dx}{2g^5} - \frac{\left(b^4Bn\right)\int \frac{A+B\log\left(e\left(\cdots\right.\right.}{c+dx\cdots}}{2(bc-ad)^4\cdots}\cdots$$

$$= \frac{Bn\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{8dg^5(c+dx)^4} + \frac{bBn\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{6d(bc-ad)g^5(c+dx)^3} + \frac{b^2Bn\left(A+B\,\mathrm{lo}\cdots\right.}{4d(bc-ad)^2\cdots}\cdots$$

$$= \frac{Bn\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{8dg^5(c+dx)^4} + \frac{bBn\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{6d(bc-ad)g^5(c+dx)^3} + \frac{b^2Bn\left(A+B\,\mathrm{lo}\cdots\right.}{4d(bc-ad)^2\cdots}\cdots$$

$$= \frac{Bn\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{8dg^5(c+dx)^4} + \frac{bBn\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{6d(bc-ad)g^5(c+dx)^3} + \frac{b^2Bn\left(A+B\,\mathrm{lo}\cdots\right.}{4d(bc-ad)^2\cdots}\cdots$$

$$= -\frac{B^2n^2}{32dg^5(c+dx)^4} - \frac{7bB^2n^2}{72d(bc-ad)g^5(c+dx)^3} - \frac{13b^2B^2n^2}{48d(bc-ad)^2g^5(c+dx)^2} - \frac{\cdots}{24\cdots}\cdots$$

$$= -\frac{B^2n^2}{32dg^5(c+dx)^4} - \frac{7bB^2n^2}{72d(bc-ad)g^5(c+dx)^3} - \frac{13b^2B^2n^2}{48d(bc-ad)^2g^5(c+dx)^2} - \frac{\cdots}{24\cdots}\cdots$$

$$= -\frac{B^2n^2}{32dg^5(c+dx)^4} - \frac{7bB^2n^2}{72d(bc-ad)g^5(c+dx)^3} - \frac{13b^2B^2n^2}{48d(bc-ad)^2g^5(c+dx)^2} - \frac{\cdots}{24\cdots}\cdots$$

**Mathematica [C]** time = 0.93, size = 776, normalized size = 1.45

$$\frac{Bn\left(144b^4(c+dx)^4\log(a+bx)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)-144b^4(c+dx)^4\log(c+dx)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)+144b^3(c+dx)^3(bc-ad)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)+72b^2(\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2/(c*g + d*g*x)^5,x]
```

```
[Out] (-72*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2 + (B*n*(36*(b*c - a*d)^4*(A +
 B*Log[e*((a + b*x)/(c + d*x))^n]) + 48*b*(b*c - a*d)^3*(c + d*x)*(A + B*Lo
g[e*((a + b*x)/(c + d*x))^n]) + 72*b^2*(b*c - a*d)^2*(c + d*x)^2*(A + B*Log
[e*((a + b*x)/(c + d*x))^n]) + 144*b^3*(b*c - a*d)*(c + d*x)^3*(A + B*Log[e
*((a + b*x)/(c + d*x))^n]) + 144*b^4*(c + d*x)^4*Log[a + b*x]*(A + B*Log[e*
((a + b*x)/(c + d*x))^n]) - 144*b^4*(c + d*x)^4*(A + B*Log[e*((a + b*x)/(c
+ d*x))^n])*Log[c + d*x] - 144*b^3*B*n*(c + d*x)^3*(b*c - a*d + b*(c + d*x)
*Log[a + b*x] - b*(c + d*x)*Log[c + d*x]) - 36*b^2*B*n*(c + d*x)^2*((b*c -
a*d)^2 + 2*b*(b*c - a*d)*(c + d*x) + 2*b^2*(c + d*x)^2*Log[a + b*x] - 2*b^2
*(c + d*x)^2*Log[c + d*x]) - 8*b*B*n*(c + d*x)*(2*(b*c - a*d)^3 + 3*b*(b*c
- a*d)^2*(c + d*x) + 6*b^2*(b*c - a*d)*(c + d*x)^2 + 6*b^3*(c + d*x)^3*Log[
a + b*x] - 6*b^3*(c + d*x)^3*Log[c + d*x]) - 3*B*n*(3*(b*c - a*d)^4 + 4*b*(
b*c - a*d)^3*(c + d*x) + 6*b^2*(b*c - a*d)^2*(c + d*x)^2 + 12*b^3*(b*c - a*
d)*(c + d*x)^3 + 12*b^4*(c + d*x)^4*Log[a + b*x] - 12*b^4*(c + d*x)^4*Log[c
 + d*x]) - 72*b^4*B*n*(c + d*x)^4*(Log[a + b*x]*(Log[a + b*x] - 2*Log[(b*(c
 + d*x))/(b*c - a*d)]) - 2*PolyLog[2, (d*(a + b*x))/(-(b*c) + a*d)]) + 72*b
^4*B*n*(c + d*x)^4*((2*Log[(d*(a + b*x))/(-(b*c) + a*d)] - Log[c + d*x])*Lo
g[c + d*x] + 2*PolyLog[2, (b*(c + d*x))/(b*c - a*d)])))/(b*c - a*d)^4)/(288
*d*g^5*(c + d*x)^4)
```

**fricas [B]** time = 1.16, size = 1768, normalized size = 3.30

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))^2/(d*g*x+c*g)^5,x, algorithm="fr
icas")
```

```
[Out] -1/288*(72*A^2*b^4*c^4 - 288*A^2*a*b^3*c^3*d + 432*A^2*a^2*b^2*c^2*d^2 - 28
8*A^2*a^3*b*c*d^3 + 72*A^2*a^4*d^4 + 12*(25*(B^2*b^4*c*d^3 - B^2*a*b^3*d^4)
*n^2 - 12*(A*B*b^4*c*d^3 - A*B*a*b^3*d^4)*n)*x^3 + (415*B^2*b^4*c^4 - 576*B
^2*a*b^3*c^3*d + 216*B^2*a^2*b^2*c^2*d^2 - 64*B^2*a^3*b*c*d^3 + 9*B^2*a^4*d
^4)*n^2 + 6*((163*B^2*b^4*c^2*d^2 - 176*B^2*a*b^3*c*d^3 + 13*B^2*a^2*b^2*d^
4)*n^2 - 12*(7*A*B*b^4*c^2*d^2 - 8*A*B*a*b^3*c*d^3 + A*B*a^2*b^2*d^4)*n)*x^
```

```
2 + 72*(B^2*b^4*c^4 - 4*B^2*a*b^3*c^3*d + 6*B^2*a^2*b^2*c^2*d^2 - 4*B^2*a^3
*b*c*d^3 + B^2*a^4*d^4)*log(e)^2 - 72*(B^2*b^4*d^4*n^2*x^4 + 4*B^2*b^4*c*d^
3*n^2*x^3 + 6*B^2*b^4*c^2*d^2*n^2*x^2 + 4*B^2*b^4*c^3*d*n^2*x + (4*B^2*a*b^
3*c^3*d - 6*B^2*a^2*b^2*c^2*d^2 + 4*B^2*a^3*b*c*d^3 - B^2*a^4*d^4)*n^2)*log
((b*x + a)/(d*x + c))^2 - 12*(25*A*B*b^4*c^4 - 48*A*B*a*b^3*c^3*d + 36*A*B*
a^2*b^2*c^2*d^2 - 16*A*B*a^3*b*c*d^3 + 3*A*B*a^4*d^4)*n + 4*((271*B^2*b^4*c
^3*d - 324*B^2*a*b^3*c^2*d^2 + 60*B^2*a^2*b^2*c*d^3 - 7*B^2*a^3*b*d^4)*n^2
- 12*(13*A*B*b^4*c^3*d - 18*A*B*a*b^3*c^2*d^2 + 6*A*B*a^2*b^2*c*d^3 - A*B*a
^3*b*d^4)*n)*x + 12*(12*A*B*b^4*c^4 - 48*A*B*a*b^3*c^3*d + 72*A*B*a^2*b^2*c
^2*d^2 - 48*A*B*a^3*b*c*d^3 + 12*A*B*a^4*d^4 - 12*(B^2*b^4*c*d^3 - B^2*a*b^
3*d^4)*n*x^3 - 6*(7*B^2*b^4*c^2*d^2 - 8*B^2*a*b^3*c*d^3 + B^2*a^2*b^2*d^4)*
n*x^2 - 4*(13*B^2*b^4*c^3*d - 18*B^2*a*b^3*c^2*d^2 + 6*B^2*a^2*b^2*c*d^3 -
B^2*a^3*b*d^4)*n*x - (25*B^2*b^4*c^4 - 48*B^2*a*b^3*c^3*d + 36*B^2*a^2*b^2*
c^2*d^2 - 16*B^2*a^3*b*c*d^3 + 3*B^2*a^4*d^4)*n - 12*(B^2*b^4*d^4*n*x^4 + 4
*B^2*b^4*c*d^3*n*x^3 + 6*B^2*b^4*c^2*d^2*n*x^2 + 4*B^2*b^4*c^3*d*n*x + (4*B
^2*a*b^3*c^3*d - 6*B^2*a^2*b^2*c^2*d^2 + 4*B^2*a^3*b*c*d^3 - B^2*a^4*d^4)*n
)*log((b*x + a)/(d*x + c)))*log(e) + 12*((25*B^2*b^4*d^4*n^2 - 12*A*B*b^4*d
^4*n)*x^4 - 4*(12*A*B*b^4*c*d^3*n - (22*B^2*b^4*c*d^3 + 3*B^2*a*b^3*d^4)*n^
2)*x^3 + (48*B^2*a*b^3*c^3*d - 36*B^2*a^2*b^2*c^2*d^2 + 16*B^2*a^3*b*c*d^3
- 3*B^2*a^4*d^4)*n^2 - 6*(12*A*B*b^4*c^2*d^2*n - (18*B^2*b^4*c^2*d^2 + 8*B^
2*a*b^3*c*d^3 - B^2*a^2*b^2*d^4)*n^2)*x^2 - 12*(4*A*B*a*b^3*c^3*d - 6*A*B*a
^2*b^2*c^2*d^2 + 4*A*B*a^3*b*c*d^3 - A*B*a^4*d^4)*n - 4*(12*A*B*b^4*c^3*d*n
 - (12*B^2*b^4*c^3*d + 18*B^2*a*b^3*c^2*d^2 - 6*B^2*a^2*b^2*c*d^3 + B^2*a^3
*b*d^4)*n^2)*x)*log((b*x + a)/(d*x + c)))/((b^4*c^4*d^5 - 4*a*b^3*c^3*d^6 +
 6*a^2*b^2*c^2*d^7 - 4*a^3*b*c*d^8 + a^4*d^9)*g^5*x^4 + 4*(b^4*c^5*d^4 - 4*
a*b^3*c^4*d^5 + 6*a^2*b^2*c^3*d^6 - 4*a^3*b*c^2*d^7 + a^4*c*d^8)*g^5*x^3 +
6*(b^4*c^6*d^3 - 4*a*b^3*c^5*d^4 + 6*a^2*b^2*c^4*d^5 - 4*a^3*b*c^3*d^6 + a^
4*c^2*d^7)*g^5*x^2 + 4*(b^4*c^7*d^2 - 4*a*b^3*c^6*d^3 + 6*a^2*b^2*c^5*d^4 -
 4*a^3*b*c^4*d^5 + a^4*c^3*d^6)*g^5*x + (b^4*c^8*d - 4*a*b^3*c^7*d^2 + 6*a^
2*b^2*c^6*d^3 - 4*a^3*b*c^5*d^4 + a^4*c^4*d^5)*g^5)
```

**giac [B]** time = 19.41, size = 1225, normalized size = 2.29

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))^2/(d*g*x+c*g)^5,x, algorithm="gi
ac")
```

```
[Out] 1/288*(72*(4*(b*x + a)*B^2*b^3*n^2/((b^3*c^3*g^5 - 3*a*b^2*c^2*d*g^5 + 3*a^
2*b*c*d^2*g^5 - a^3*d^3*g^5)*(d*x + c)) - 6*(b*x + a)^2*B^2*b^2*d*n^2/((b^3
*c^3*g^5 - 3*a*b^2*c^2*d*g^5 + 3*a^2*b*c*d^2*g^5 - a^3*d^3*g^5)*(d*x + c)^2
) + 4*(b*x + a)^3*B^2*b*d^2*n^2/((b^3*c^3*g^5 - 3*a*b^2*c^2*d*g^5 + 3*a^2*b
*c*d^2*g^5 - a^3*d^3*g^5)*(d*x + c)^3) - (b*x + a)^4*B^2*d^3*n^2/((b^3*c^3*
g^5 - 3*a*b^2*c^2*d*g^5 + 3*a^2*b*c*d^2*g^5 - a^3*d^3*g^5)*(d*x + c)^4))*lo
```

```
g((b*x + a)/(d*x + c))^2 + 12*(3*(B^2*d^3*n^2 - 4*A*B*d^3*n - 4*B^2*d^3*n)*
(b*x + a)^4/((b^3*c^3*g^5 - 3*a*b^2*c^2*d*g^5 + 3*a^2*b*c*d^2*g^5 - a^3*d^3
*g^5)*(d*x + c)^4) - 16*(B^2*b*d^2*n^2 - 3*A*B*b*d^2*n - 3*B^2*b*d^2*n)*(b*
x + a)^3/((b^3*c^3*g^5 - 3*a*b^2*c^2*d*g^5 + 3*a^2*b*c*d^2*g^5 - a^3*d^3*g^
5)*(d*x + c)^3) + 36*(B^2*b^2*d*n^2 - 2*A*B*b^2*d*n - 2*B^2*b^2*d*n)*(b*x +
 a)^2/((b^3*c^3*g^5 - 3*a*b^2*c^2*d*g^5 + 3*a^2*b*c*d^2*g^5 - a^3*d^3*g^5)*
(d*x + c)^2) - 48*(B^2*b^3*n^2 - A*B*b^3*n - B^2*b^3*n)*(b*x + a)/((b^3*c^3
*g^5 - 3*a*b^2*c^2*d*g^5 + 3*a^2*b*c*d^2*g^5 - a^3*d^3*g^5)*(d*x + c)))*log
((b*x + a)/(d*x + c)) - 9*(B^2*d^3*n^2 - 4*A*B*d^3*n - 4*B^2*d^3*n + 8*A^2*
d^3 + 16*A*B*d^3 + 8*B^2*d^3)*(b*x + a)^4/((b^3*c^3*g^5 - 3*a*b^2*c^2*d*g^5
 + 3*a^2*b*c*d^2*g^5 - a^3*d^3*g^5)*(d*x + c)^4) + 32*(2*B^2*b*d^2*n^2 - 6*
A*B*b*d^2*n - 6*B^2*b*d^2*n + 9*A^2*b*d^2 + 18*A*B*b*d^2 + 9*B^2*b*d^2)*(b*
x + a)^3/((b^3*c^3*g^5 - 3*a*b^2*c^2*d*g^5 + 3*a^2*b*c*d^2*g^5 - a^3*d^3*g^
5)*(d*x + c)^3) - 216*(B^2*b^2*d*n^2 - 2*A*B*b^2*d*n - 2*B^2*b^2*d*n + 2*A^
2*b^2*d + 4*A*B*b^2*d + 2*B^2*b^2*d)*(b*x + a)^2/((b^3*c^3*g^5 - 3*a*b^2*c^
2*d*g^5 + 3*a^2*b*c*d^2*g^5 - a^3*d^3*g^5)*(d*x + c)^2) + 288*(2*B^2*b^3*n^
2 - 2*A*B*b^3*n - 2*B^2*b^3*n + A^2*b^3 + 2*A*B*b^3 + B^2*b^3)*(b*x + a)/((
b^3*c^3*g^5 - 3*a*b^2*c^2*d*g^5 + 3*a^2*b*c*d^2*g^5 - a^3*d^3*g^5)*(d*x + c
)))*(b*c/(b*c - a*d)^2 - a*d/(b*c - a*d)^2)
```

**maple [F]** time = 0.28, size = 0, normalized size = 0.00

$$\int \frac{\left(B \ln \left(e \left(\frac{bx+a}{dx+c}\right)^{n}\right)+A\right)^{2}}{\left(dgx+cg\right)^{5}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*ln(e*((b*x+a)/(d*x+c))^n)+A)^2/(d*g*x+c*g)^5,x)
[Out] int((B*ln(e*((b*x+a)/(d*x+c))^n)+A)^2/(d*g*x+c*g)^5,x)
```

**maxima [B]** time = 2.11, size = 2138, normalized size = 3.99

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))^2/(d*g*x+c*g)^5,x, algorithm="ma
xima")
[Out] 1/24*A*B*n*((12*b^3*d^3*x^3 + 25*b^3*c^3 - 23*a*b^2*c^2*d + 13*a^2*b*c*d^2
- 3*a^3*d^3 + 6*(7*b^3*c*d^2 - a*b^2*d^3)*x^2 + 4*(13*b^3*c^2*d - 5*a*b^2*c
*d^2 + a^2*b*d^3)*x)/((b^3*c^3*d^5 - 3*a*b^2*c^2*d^6 + 3*a^2*b*c*d^7 - a^3*
d^8)*g^5*x^4 + 4*(b^3*c^4*d^4 - 3*a*b^2*c^3*d^5 + 3*a^2*b*c^2*d^6 - a^3*c*d
^7)*g^5*x^3 + 6*(b^3*c^5*d^3 - 3*a*b^2*c^4*d^4 + 3*a^2*b*c^3*d^5 - a^3*c^2*
```

```
d^6)*g^5*x^2 + 4*(b^3*c^6*d^2 - 3*a*b^2*c^5*d^3 + 3*a^2*b*c^4*d^4 - a^3*c^3
*d^5)*g^5*x + (b^3*c^7*d - 3*a*b^2*c^6*d^2 + 3*a^2*b*c^5*d^3 - a^3*c^4*d^4)
*g^5) + 12*b^4*log(b*x + a)/((b^4*c^4*d - 4*a*b^3*c^3*d^2 + 6*a^2*b^2*c^2*d
^3 - 4*a^3*b*c*d^4 + a^4*d^5)*g^5) - 12*b^4*log(d*x + c)/((b^4*c^4*d - 4*a*
b^3*c^3*d^2 + 6*a^2*b^2*c^2*d^3 - 4*a^3*b*c*d^4 + a^4*d^5)*g^5)) + 1/288*(1
2*n*((12*b^3*d^3*x^3 + 25*b^3*c^3 - 23*a*b^2*c^2*d + 13*a^2*b*c*d^2 - 3*a^3
*d^3 + 6*(7*b^3*c*d^2 - a*b^2*d^3)*x^2 + 4*(13*b^3*c^2*d - 5*a*b^2*c*d^2 +
a^2*b*d^3)*x)/((b^3*c^3*d^5 - 3*a*b^2*c^2*d^6 + 3*a^2*b*c*d^7 - a^3*d^8)*g^
5*x^4 + 4*(b^3*c^4*d^4 - 3*a*b^2*c^3*d^5 + 3*a^2*b*c^2*d^6 - a^3*c*d^7)*g^5
*x^3 + 6*(b^3*c^5*d^3 - 3*a*b^2*c^4*d^4 + 3*a^2*b*c^3*d^5 - a^3*c^2*d^6)*g^
5*x^2 + 4*(b^3*c^6*d^2 - 3*a*b^2*c^5*d^3 + 3*a^2*b*c^4*d^4 - a^3*c^3*d^5)*g
^5*x + (b^3*c^7*d - 3*a*b^2*c^6*d^2 + 3*a^2*b*c^5*d^3 - a^3*c^4*d^4)*g^5) +
 12*b^4*log(b*x + a)/((b^4*c^4*d - 4*a*b^3*c^3*d^2 + 6*a^2*b^2*c^2*d^3 - 4*
a^3*b*c*d^4 + a^4*d^5)*g^5) - 12*b^4*log(d*x + c)/((b^4*c^4*d - 4*a*b^3*c^3
*d^2 + 6*a^2*b^2*c^2*d^3 - 4*a^3*b*c*d^4 + a^4*d^5)*g^5))*log(e*(b*x/(d*x +
 c) + a/(d*x + c))^n) - (415*b^4*c^4 - 576*a*b^3*c^3*d + 216*a^2*b^2*c^2*d^
2 - 64*a^3*b*c*d^3 + 9*a^4*d^4 + 300*(b^4*c*d^3 - a*b^3*d^4)*x^3 + 6*(163*b
^4*c^2*d^2 - 176*a*b^3*c*d^3 + 13*a^2*b^2*d^4)*x^2 + 72*(b^4*d^4*x^4 + 4*b^
4*c*d^3*x^3 + 6*b^4*c^2*d^2*x^2 + 4*b^4*c^3*d*x + b^4*c^4)*log(b*x + a)^2 +
 72*(b^4*d^4*x^4 + 4*b^4*c*d^3*x^3 + 6*b^4*c^2*d^2*x^2 + 4*b^4*c^3*d*x + b^
4*c^4)*log(d*x + c)^2 + 4*(271*b^4*c^3*d - 324*a*b^3*c^2*d^2 + 60*a^2*b^2*c
*d^3 - 7*a^3*b*d^4)*x + 300*(b^4*d^4*x^4 + 4*b^4*c*d^3*x^3 + 6*b^4*c^2*d^2*
x^2 + 4*b^4*c^3*d*x + b^4*c^4)*log(b*x + a) - 12*(25*b^4*d^4*x^4 + 100*b^4*
c*d^3*x^3 + 150*b^4*c^2*d^2*x^2 + 100*b^4*c^3*d*x + 25*b^4*c^4 + 12*(b^4*d^
4*x^4 + 4*b^4*c*d^3*x^3 + 6*b^4*c^2*d^2*x^2 + 4*b^4*c^3*d*x + b^4*c^4)*log(
b*x + a))*log(d*x + c))*n^2/(b^4*c^8*d*g^5 - 4*a*b^3*c^7*d^2*g^5 + 6*a^2*b^
2*c^6*d^3*g^5 - 4*a^3*b*c^5*d^4*g^5 + a^4*c^4*d^5*g^5 + (b^4*c^4*d^5*g^5 -
4*a*b^3*c^3*d^6*g^5 + 6*a^2*b^2*c^2*d^7*g^5 - 4*a^3*b*c*d^8*g^5 + a^4*d^9*g
^5)*x^4 + 4*(b^4*c^5*d^4*g^5 - 4*a*b^3*c^4*d^5*g^5 + 6*a^2*b^2*c^3*d^6*g^5
- 4*a^3*b*c^2*d^7*g^5 + a^4*c*d^8*g^5)*x^3 + 6*(b^4*c^6*d^3*g^5 - 4*a*b^3*c
^5*d^4*g^5 + 6*a^2*b^2*c^4*d^5*g^5 - 4*a^3*b*c^3*d^6*g^5 + a^4*c^2*d^7*g^5)
*x^2 + 4*(b^4*c^7*d^2*g^5 - 4*a*b^3*c^6*d^3*g^5 + 6*a^2*b^2*c^5*d^4*g^5 - 4
*a^3*b*c^4*d^5*g^5 + a^4*c^3*d^6*g^5)*x))*B^2 - 1/4*B^2*log(e*(b*x/(d*x + c
) + a/(d*x + c))^n)^2/(d^5*g^5*x^4 + 4*c*d^4*g^5*x^3 + 6*c^2*d^3*g^5*x^2 +
4*c^3*d^2*g^5*x + c^4*d*g^5) - 1/2*A*B*log(e*(b*x/(d*x + c) + a/(d*x + c))^
n)/(d^5*g^5*x^4 + 4*c*d^4*g^5*x^3 + 6*c^2*d^3*g^5*x^2 + 4*c^3*d^2*g^5*x + c
^4*d*g^5) - 1/4*A^2/(d^5*g^5*x^4 + 4*c*d^4*g^5*x^3 + 6*c^2*d^3*g^5*x^2 + 4*
c^3*d^2*g^5*x + c^4*d*g^5)
```

**mupad [B]** time = 9.08, size = 1765, normalized size = 3.29

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*log(e*((a + b*x)/(c + d*x))^n))^2/(c*g + d*g*x)^5,x)
```

```
[Out] (B*b^4*n*atan((B*b^4*n*(12*A - 25*B*n)*(24*a^4*d^5*g^5 - 24*b^4*c^4*d*g^5 -
 48*a^3*b*c*d^4*g^5 + 48*a*b^3*c^3*d^2*g^5)*1i)/(24*d*g^5*(25*B^2*b^4*n^2 -
 12*A*B*b^4*n)*(a*d - b*c)^4) + (B*b^5*n*x*(12*A - 25*B*n)*(a^3*d^4*g^5 - b
^3*c^3*d*g^5 - 3*a^2*b*c*d^3*g^5 + 3*a*b^2*c^2*d^2*g^5)*2i)/(g^5*(25*B^2*b^
4*n^2 - 12*A*B*b^4*n)*(a*d - b*c)^4))*(12*A - 25*B*n)*1i)/(12*d*g^5*(a*d -
b*c)^4) - ((72*A^2*a^3*d^3 - 72*A^2*b^3*c^3 + 9*B^2*a^3*d^3*n^2 - 415*B^2*b
^3*c^3*n^2 + 216*A^2*a*b^2*c^2*d - 216*A^2*a^2*b*c*d^2 - 36*A*B*a^3*d^3*n +
 300*A*B*b^3*c^3*n + 161*B^2*a*b^2*c^2*d*n^2 - 55*B^2*a^2*b*c*d^2*n^2 - 276
*A*B*a*b^2*c^2*d*n + 156*A*B*a^2*b*c*d^2*n)/(12*(a*d - b*c)) + (x^2*(13*B^2
*a*b^2*d^3*n^2 - 163*B^2*b^3*c*d^2*n^2 - 12*A*B*a*b^2*d^3*n + 84*A*B*b^3*c*
d^2*n))/(2*(a*d - b*c)) - (x*(7*B^2*a^2*b*d^3*n^2 + 271*B^2*b^3*c^2*d*n^2 -
 53*B^2*a*b^2*c*d^2*n^2 - 12*A*B*a^2*b*d^3*n - 156*A*B*b^3*c^2*d*n + 60*A*B
*a*b^2*c*d^2*n))/(3*(a*d - b*c)) - (b*x^3*(25*B^2*b^2*d^3*n^2 - 12*A*B*b^2*
d^3*n))/(a*d - b*c))/(x*(96*a^2*c^3*d^4*g^5 + 96*b^2*c^5*d^2*g^5 - 192*a*b*
c^4*d^3*g^5) + x^3*(96*a^2*c*d^6*g^5 + 96*b^2*c^3*d^4*g^5 - 192*a*b*c^2*d^5
*g^5) + x^4*(24*a^2*d^7*g^5 + 24*b^2*c^2*d^5*g^5 - 48*a*b*c*d^6*g^5) + x^2*
(144*a^2*c^2*d^5*g^5 + 144*b^2*c^4*d^3*g^5 - 288*a*b*c^3*d^4*g^5) + 24*b^2*
c^6*d*g^5 + 24*a^2*c^4*d^3*g^5 - 48*a*b*c^5*d^2*g^5) - log(e*((a + b*x)/(c
+ d*x))^n)^2*(B^2/(4*d*(c^4*g^5 + d^4*g^5*x^4 + 4*c*d^3*g^5*x^3 + 6*c^2*d^2
*g^5*x^2 + 4*c^3*d*g^5*x)) - (B^2*b^4)/(4*d*g^5*(a^4*d^4 + b^4*c^4 + 6*a^2*
b^2*c^2*d^2 - 4*a*b^3*c^3*d - 4*a^3*b*c*d^3))) - log(e*((a + b*x)/(c + d*x)
)^n)*((A*B)/(2*c^4*d*g^5 + 2*d^5*g^5*x^4 + 8*c^3*d^2*g^5*x + 8*c*d^4*g^5*x^
3 + 12*c^2*d^3*g^5*x^2) - (B^2*b^4*(x*(d*(c*((d*g^5*n*(a*d - b*c)*(a*d - 4*
b*c))/(6*b^2) - (c*d*g^5*n*(a*d - b*c))/(2*b)) - (d*g^5*n*(a*d - b*c)*(a^2*
d^2 + 6*b^2*c^2 - 4*a*b*c*d))/(6*b^3)) + c*(d*((d*g^5*n*(a*d - b*c)*(a*d -
4*b*c))/(6*b^2) - (c*d*g^5*n*(a*d - b*c))/(2*b)) - (c*d^2*g^5*n*(a*d - b*c)
)/b + (d^2*g^5*n*(a*d - b*c)*(a*d - 4*b*c))/(3*b^2)) - (d^2*g^5*n*(a*d - b*
c)*(a^2*d^2 + 6*b^2*c^2 - 4*a*b*c*d))/(2*b^3)) + c*(c*((d*g^5*n*(a*d - b*c)
*(a*d - 4*b*c))/(6*b^2) - (c*d*g^5*n*(a*d - b*c))/(2*b)) - (d*g^5*n*(a*d -
b*c)*(a^2*d^2 + 6*b^2*c^2 - 4*a*b*c*d))/(6*b^3)) + x^2*(d*(d*((d*g^5*n*(a*d
 - b*c)*(a*d - 4*b*c))/(6*b^2) - (c*d*g^5*n*(a*d - b*c))/(2*b)) - (c*d^2*g^
5*n*(a*d - b*c))/b + (d^2*g^5*n*(a*d - b*c)*(a*d - 4*b*c))/(3*b^2)) - (3*c*
d^3*g^5*n*(a*d - b*c))/(2*b) + (d^3*g^5*n*(a*d - b*c)*(a*d - 4*b*c))/(2*b^2
)) - (2*d^4*g^5*n*x^3*(a*d - b*c))/b + (d*g^5*n*(a*d - b*c)*(a^3*d^3 - 4*b^
3*c^3 + 6*a*b^2*c^2*d - 4*a^2*b*c*d^2))/(2*b^4)))/(2*d*g^5*(2*c^4*d*g^5 + 2
*d^5*g^5*x^4 + 8*c^3*d^2*g^5*x + 8*c*d^4*g^5*x^3 + 12*c^2*d^3*g^5*x^2)*(a^4
*d^4 + b^4*c^4 + 6*a^2*b^2*c^2*d^2 - 4*a*b^3*c^3*d - 4*a^3*b*c*d^3)))
```

**sympy [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*ln(e*((b*x+a)/(d*x+c))**n))**2/(d*g*x+c*g)**5,x)
```

```
[Out] Timed out
```

## 3.47 $\int \frac{(cg+dgx)^2}{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}\,dx$

**Optimal**. Leaf size=38

$$\text{Int}\left(\frac{(cg+dgx)^2}{B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A},x\right)$$

```
[Out] Unintegrable((d*g*x+c*g)^2/(A+B*ln(e*((b*x+a)/(d*x+c))^n)),x)
```

**Rubi [A]** time = 0.21, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{(cg+dgx)^2}{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(c*g + d*g*x)^2/(A + B*Log[e*((a + b*x)/(c + d*x))^n]),x]
```

```
[Out] c^2*g^2*Defer[Int][(A + B*Log[e*((a + b*x)/(c + d*x))^n])^(-1), x] + 2*c*d*
g^2*Defer[Int][x/(A + B*Log[e*((a + b*x)/(c + d*x))^n]), x] + d^2*g^2*Defer
[Int][x^2/(A + B*Log[e*((a + b*x)/(c + d*x))^n]), x]
```

Rubi steps

$$\int \frac{(cg+dgx)^2}{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}\,dx = \int \left(\frac{c^2g^2}{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}+\frac{2cdg^2x}{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}+\frac{d^2g^2x^2}{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}\right)dx$$
$$=\left(c^2g^2\right)\int \frac{1}{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}\,dx+\left(2cdg^2\right)\int \frac{x}{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}\,dx+\left(d^2g^2\right.$$

**Mathematica [A]** time = 0.44, size = 0, normalized size = 0.00

$$\int \frac{(cg+dgx)^2}{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(c*g + d*g*x)^2/(A + B*Log[e*((a + b*x)/(c + d*x))^n]),x]
```

```
[Out] Integrate[(c*g + d*g*x)^2/(A + B*Log[e*((a + b*x)/(c + d*x))^n]), x]
```

**fricas [A]** time = 0.87, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{d^2g^2x^2 + 2\,cdg^2x + c^2g^2}{B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right) + A}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*g*x+c*g)^2/(A+B*log(e*((b*x+a)/(d*x+c))^n)),x, algorithm="fric
as")
```

```
[Out] integral((d^2*g^2*x^2 + 2*c*d*g^2*x + c^2*g^2)/(B*log(e*((b*x + a)/(d*x + c
))^n) + A), x)
```

**giac [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*g*x+c*g)^2/(A+B*log(e*((b*x+a)/(d*x+c))^n)),x, algorithm="giac
")
```

```
[Out] Timed out
```

**maple [A]** time = 0.26, size = 0, normalized size = 0.00

$$\int \frac{\left(dgx + cg\right)^2}{B\ln\left(e\left(\frac{bx+a}{dx+c}\right)^n\right) + A}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*g*x+c*g)^2/(B*ln(e*((b*x+a)/(d*x+c))^n)+A),x)
```

```
[Out] int((d*g*x+c*g)^2/(B*ln(e*((b*x+a)/(d*x+c))^n)+A),x)
```

**maxima [A]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(dgx + cg\right)^2}{B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right) + A}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*g*x+c*g)^2/(A+B*log(e*((b*x+a)/(d*x+c))^n)),x, algorithm="maxi
ma")
```

```
[Out] integrate((d*g*x + c*g)^2/(B*log(e*((b*x + a)/(d*x + c))^n) + A), x)
```

**mupad** **[A]** time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{{\left(c\,g+d\,g\,x\right)}^2}{A+B\,\ln\left(e\,{\left(\frac{a+b\,x}{c+d\,x}\right)}^n\right)} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*g + d*g*x)^2/(A + B*log(e*((a + b*x)/(c + d*x))^n)),x)
```

```
[Out] int((c*g + d*g*x)^2/(A + B*log(e*((a + b*x)/(c + d*x))^n)), x)
```

**sympy** **[A]** time = 0.00, size = 0, normalized size = 0.00

$$g^{2} \left(\int \frac{c^{2}}{A + B \log{\left(e \left(\frac{a}{c + d x} + \frac{b x}{c + d x}\right)^{n} \right)}}\, dx + \int \frac{d^{2} x^{2}}{A + B \log{\left(e \left(\frac{a}{c + d x} + \frac{b x}{c + d x}\right)^{n} \right)}}\, dx + \int \frac{2 c d x}{A + B \log{\left(e \left(\frac{a}{c + d x} + \frac{b x}{c + d x}\right)^{n} \right)}}\, d$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*g*x+c*g)**2/(A+B*ln(e*((b*x+a)/(d*x+c))**n)),x)
```

```
[Out] g**2*(Integral(c**2/(A + B*log(e*(a/(c + d*x) + b*x/(c + d*x))**n)), x) + I
ntegral(d**2*x**2/(A + B*log(e*(a/(c + d*x) + b*x/(c + d*x))**n)), x) + Int
egral(2*c*d*x/(A + B*log(e*(a/(c + d*x) + b*x/(c + d*x))**n)), x))
```

## 3.48 $\int \frac{cg+dgx}{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}\,dx$

**Optimal**. Leaf size=36

$$\text{Int}\left(\frac{cg+dgx}{B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A},x\right)$$

```
[Out] Unintegrable((d*g*x+c*g)/(A+B*ln(e*((b*x+a)/(d*x+c))^n)),x)
```

**Rubi [A]** time = 0.11, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{cg+dgx}{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(c*g + d*g*x)/(A + B*Log[e*((a + b*x)/(c + d*x))^n]),x]
```

```
[Out] c*g*Defer[Int][(A + B*Log[e*((a + b*x)/(c + d*x))^n])^(-1), x] + d*g*Defer[
Int][x/(A + B*Log[e*((a + b*x)/(c + d*x))^n]), x]
```

Rubi steps

$$\begin{aligned}\int \frac{cg+dgx}{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}\,dx &= \int \left(\frac{cg}{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}+\frac{dgx}{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}\right)dx\\ &= (cg)\int \frac{1}{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}\,dx+(dg)\int \frac{x}{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}\,dx\end{aligned}$$

**Mathematica [A]** time = 0.30, size = 0, normalized size = 0.00

$$\int \frac{cg+dgx}{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(c*g + d*g*x)/(A + B*Log[e*((a + b*x)/(c + d*x))^n]),x]
[Out] Integrate[(c*g + d*g*x)/(A + B*Log[e*((a + b*x)/(c + d*x))^n]), x]
```

**fricas [A]** time = 1.32, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{dgx+cg}{B\log\left(e\left(\frac{bx+a}{dx+c}\right)^{n}\right)+A}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*g*x+c*g)/(A+B*log(e*((b*x+a)/(d*x+c))^n)),x, algorithm="fricas
")
[Out] integral((d*g*x + c*g)/(B*log(e*((b*x + a)/(d*x + c))^n) + A), x)
```

**giac [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*g*x+c*g)/(A+B*log(e*((b*x+a)/(d*x+c))^n)),x, algorithm="giac")
[Out] Timed out
```

**maple [A]** time = 0.15, size = 0, normalized size = 0.00

$$\int \frac{dgx+cg}{B\ln\left(e\left(\frac{bx+a}{dx+c}\right)^{n}\right)+A}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*g*x+c*g)/(B*ln(e*((b*x+a)/(d*x+c))^n)+A),x)
[Out] int((d*g*x+c*g)/(B*ln(e*((b*x+a)/(d*x+c))^n)+A),x)
```

**maxima [A]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{dgx+cg}{B\log\left(e\left(\frac{bx+a}{dx+c}\right)^{n}\right)+A}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*g*x+c*g)/(A+B*log(e*((b*x+a)/(d*x+c))^n)),x, algorithm="maxima
")
```

```
[Out] integrate((d*g*x + c*g)/(B*log(e*((b*x + a)/(d*x + c))^n) + A), x)
```

**mupad** [A] time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{c\,g+d\,g\,x}{A+B\,\ln\left(e\,{\left(\frac{a+b\,x}{c+d\,x}\right)}^n\right)} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*g + d*g*x)/(A + B*log(e*((a + b*x)/(c + d*x))^n)),x)
```

```
[Out] int((c*g + d*g*x)/(A + B*log(e*((a + b*x)/(c + d*x))^n)), x)
```

**sympy** [A] time = 0.00, size = 0, normalized size = 0.00

$$g \left(\int \frac{c}{A + B \log{\left(e \left(\frac{a}{c + d x} + \frac{b x}{c + d x}\right)^{n} \right)}}\, dx + \int \frac{d x}{A + B \log{\left(e \left(\frac{a}{c + d x} + \frac{b x}{c + d x}\right)^{n} \right)}}\, dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*g*x+c*g)/(A+B*ln(e*((b*x+a)/(d*x+c))**n)),x)
```

```
[Out] g*(Integral(c/(A + B*log(e*(a/(c + d*x) + b*x/(c + d*x))**n)), x) + Integra
l(d*x/(A + B*log(e*(a/(c + d*x) + b*x/(c + d*x))**n)), x))
```

## 3.49 $\int \frac{1}{(cg+dgx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}\,dx$

**Optimal**. Leaf size=38

$$\mathrm{Int}\left(\frac{1}{(cg+dgx)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)},x\right)$$

[Out] Unintegrable(1/(d*g*x+c*g)/(A+B*ln(e*((b*x+a)/(d*x+c))^n)),x)

**Rubi [A]** time = 0.08, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{1}{(cg+dgx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}\,dx$$

Verification is Not applicable to the result.

[In] Int[1/((c*g + d*g*x)*(A + B*Log[e*((a + b*x)/(c + d*x))^n])),x]

[Out] Defer[Int][1/((c*g + d*g*x)*(A + B*Log[e*((a + b*x)/(c + d*x))^n])), x]

Rubi steps

$$\int \frac{1}{(cg+dgx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}\,dx=\int \frac{1}{(cg+dgx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}\,dx$$

**Mathematica [A]** time = 0.18, size = 0, normalized size = 0.00

$$\int \frac{1}{(cg+dgx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}\,dx$$

Verification is Not applicable to the result.

[In] Integrate[1/((c*g + d*g*x)*(A + B*Log[e*((a + b*x)/(c + d*x))^n])),x]

```
[Out] Integrate[1/((c*g + d*g*x)*(A + B*Log[e*((a + b*x)/(c + d*x))^n])), x]
```

**fricas [A]** time = 0.80, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{1}{Adgx + Acg + (Bdgx + Bcg)\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*g*x+c*g)/(A+B*log(e*((b*x+a)/(d*x+c))^n)),x, algorithm="fric
as")
```

```
[Out] integral(1/(A*d*g*x + A*c*g + (B*d*g*x + B*c*g)*log(e*((b*x + a)/(d*x + c))
^n)), x)
```

**giac [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*g*x+c*g)/(A+B*log(e*((b*x+a)/(d*x+c))^n)),x, algorithm="giac
")
```

```
[Out] Timed out
```

**maple [A]** time = 0.38, size = 0, normalized size = 0.00

$$\int \frac{1}{(dgx + cg)\left(B\ln\left(e\left(\frac{bx+a}{dx+c}\right)^n\right) + A\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(d*g*x+c*g)/(B*ln(e*((b*x+a)/(d*x+c))^n)+A),x)
```

```
[Out] int(1/(d*g*x+c*g)/(B*ln(e*((b*x+a)/(d*x+c))^n)+A),x)
```

**maxima [A]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(dgx + cg)\left(B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right) + A\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*g*x+c*g)/(A+B*log(e*((b*x+a)/(d*x+c))^n)),x, algorithm="maxi
ma")
```

```
[Out] integrate(1/((d*g*x + c*g)*(B*log(e*((b*x + a)/(d*x + c))^n) + A)), x)
```

**mupad [A]** time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{1}{\left(c\,g+d\,g\,x\right)\,\left(A+B\,\ln\left(e\,{\left(\frac{a+b\,x}{c+d\,x}\right)}^n\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((c*g + d*g*x)*(A + B*log(e*((a + b*x)/(c + d*x))^n))),x)
```

```
[Out] int(1/((c*g + d*g*x)*(A + B*log(e*((a + b*x)/(c + d*x))^n))), x)
```

**sympy [A]** time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{1}{Ac+Adx+Bc\log\left(e\left(\frac{a}{c+dx}+\frac{bx}{c+dx}\right)^n\right)+Bdx\log\left(e\left(\frac{a}{c+dx}+\frac{bx}{c+dx}\right)^n\right)}\,dx}{g}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*g*x+c*g)/(A+B*ln(e*((b*x+a)/(d*x+c))**n)),x)
```

```
[Out] Integral(1/(A*c + A*d*x + B*c*log(e*(a/(c + d*x) + b*x/(c + d*x))**n) + B*d
*x*log(e*(a/(c + d*x) + b*x/(c + d*x))**n)), x)/g
```

## 3.50 $\int \frac{1}{(cg+dgx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}\,dx$

**Optimal**. Leaf size=96

$$\frac{(a+bx)e^{-\frac{A}{Bn}}\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)^{-1/n}\operatorname{Ei}\left(\frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{Bn}\right)}{Bg^2n(c+dx)(bc-ad)}$$

```
[Out] (b*x+a)*Ei((A+B*ln(e*((b*x+a)/(d*x+c))^n))/B/n)/B/(-a*d+b*c)/exp(A/B/n)/g^2
/n/((e*((b*x+a)/(d*x+c))^n)^(1/n))/(d*x+c)
```

**Rubi [F]** time = 0.10, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{1}{(cg+dgx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[1/((c*g + d*g*x)^2*(A + B*Log[e*((a + b*x)/(c + d*x))^n])),x]
```

```
[Out] Defer[Int][1/((c*g + d*g*x)^2*(A + B*Log[e*((a + b*x)/(c + d*x))^n])), x]
```

Rubi steps

$$\int \frac{1}{(cg+dgx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}\,dx=\int \frac{1}{(cg+dgx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}\,dx$$

**Mathematica [A]** time = 0.12, size = 96, normalized size = 1.00

$$\frac{(a+bx)e^{-\frac{A}{Bn}}\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)^{-1/n}\operatorname{Ei}\left(\frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{Bn}\right)}{Bg^2n(c+dx)(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((c*g + d*g*x)^2*(A + B*Log[e*((a + b*x)/(c + d*x))^n])),x]
```

```
[Out] ((a + b*x)*ExpIntegralEi[(A + B*Log[e*((a + b*x)/(c + d*x))^n])/(B*n)])/(B*
(b*c - a*d)*E^(A/(B*n))*g^2*n*(e*((a + b*x)/(c + d*x))^n)^n^(-1)*(c + d*x))
```

**fricas [A]** time = 0.90, size = 62, normalized size = 0.65

$$\frac{e^{\left(-\frac{B\log(e)+A}{Bn}\right)}\operatorname{log\_integral}\left(\frac{(bx+a)e^{\left(\frac{B\log(e)+A}{Bn}\right)}}{dx+c}\right)}{(Bbc-Bad)g^2n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*g*x+c*g)^2/(A+B*log(e*((b*x+a)/(d*x+c))^n)),x, algorithm="fr
icas")
```

```
[Out] e^(-(B*log(e) + A)/(B*n))*log_integral((b*x + a)*e^((B*log(e) + A)/(B*n))/(
d*x + c))/((B*b*c - B*a*d)*g^2*n)
```

**giac [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*g*x+c*g)^2/(A+B*log(e*((b*x+a)/(d*x+c))^n)),x, algorithm="gi
ac")
```

```
[Out] Timed out
```

**maple [F]** time = 0.28, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(dgx+cg\right)^2\left(B\ln\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(d*g*x+c*g)^2/(B*ln(e*((b*x+a)/(d*x+c))^n)+A),x)
```

```
[Out] int(1/(d*g*x+c*g)^2/(B*ln(e*((b*x+a)/(d*x+c))^n)+A),x)
```

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(dgx+cg\right)^2\left(B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*g*x+c*g)^2/(A+B*log(e*((b*x+a)/(d*x+c))^n)),x, algorithm="ma
xima")
```

```
[Out] integrate(1/((d*g*x + c*g)^2*(B*log(e*((b*x + a)/(d*x + c))^n) + A)), x)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{{\left(c\,g+d\,g\,x\right)}^2\,\left(A+B\,\ln\left(e\,{\left(\frac{a+b\,x}{c+d\,x}\right)}^n\right)\right)} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((c*g + d*g*x)^2*(A + B*log(e*((a + b*x)/(c + d*x))^n))),x)
```

```
[Out] int(1/((c*g + d*g*x)^2*(A + B*log(e*((a + b*x)/(c + d*x))^n))), x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{1}{Ac^{2}+2Acdx+Ad^{2}x^{2}+Bc^{2}\log\left(e\left(\frac{a}{c+dx}+\frac{bx}{c+dx}\right)^{n}\right)+2Bcdx\log\left(e\left(\frac{a}{c+dx}+\frac{bx}{c+dx}\right)^{n}\right)+Bd^{2}x^{2}\log\left(e\left(\frac{a}{c+dx}+\frac{bx}{c+dx}\right)^{n}\right)}\, dx}{g^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*g*x+c*g)**2/(A+B*ln(e*((b*x+a)/(d*x+c))**n)),x)
```

```
[Out] Integral(1/(A*c**2 + 2*A*c*d*x + A*d**2*x**2 + B*c**2*log(e*(a/(c + d*x) +
b*x/(c + d*x))**n) + 2*B*c*d*x*log(e*(a/(c + d*x) + b*x/(c + d*x))**n) + B*
d**2*x**2*log(e*(a/(c + d*x) + b*x/(c + d*x))**n)), x)/g**2
```

# 3.51 $\int \frac{1}{(cg+dgx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}\,dx$

**Optimal**. Leaf size=199

$$\frac{b(a+bx)e^{-\frac{A}{Bn}}\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)^{-1/n}\operatorname{Ei}\left(\frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{Bn}\right)}{Bg^3n(c+dx)(bc-ad)^2}-\frac{d(a+bx)^2e^{-\frac{2A}{Bn}}\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)^{-2/n}\operatorname{Ei}\left(\frac{2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{Bn}\right)}{Bg^3n(c+dx)^2(bc-ad)^2}$$

```
[Out] b*(b*x+a)*Ei((A+B*ln(e*((b*x+a)/(d*x+c))^n))/B/n)/B/(-a*d+b*c)^2/exp(A/B/n)
/g^3/n/((e*((b*x+a)/(d*x+c))^n)^(1/n))/(d*x+c)-d*(b*x+a)^2*Ei(2*(A+B*ln(e*(
(b*x+a)/(d*x+c))^n))/B/n)/B/(-a*d+b*c)^2/exp(2*A/B/n)/g^3/n/((e*((b*x+a)/(d
*x+c))^n)^(2/n))/(d*x+c)^2
```

**Rubi [F]** time = 0.08, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{1}{(cg+dgx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[1/((c*g + d*g*x)^3*(A + B*Log[e*((a + b*x)/(c + d*x))^n])),x]
```

```
[Out] Defer[Int][1/((c*g + d*g*x)^3*(A + B*Log[e*((a + b*x)/(c + d*x))^n])), x]
```

Rubi steps

$$\int \frac{1}{(cg+dgx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}\,dx=\int \frac{1}{(cg+dgx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}\,dx$$

**Mathematica [A]** time = 0.29, size = 174, normalized size = 0.87

$$\frac{(a+bx)e^{-\frac{2A}{Bn}}\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)^{-2/n}\left(be^{\frac{A}{Bn}}(c+dx)\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)^{\frac{1}{n}}\operatorname{Ei}\left(\frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{Bn}\right)-d(a+bx)\operatorname{Ei}\left(\frac{2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{Bn}\right)\right)}{Bg^3n(c+dx)^2(bc-ad)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((c*g + d*g*x)^3*(A + B*Log[e*((a + b*x)/(c + d*x))^n])),x]
```

```
[Out] ((a + b*x)*(b*E^(A/(B*n))*(e*((a + b*x)/(c + d*x))^n)^n^(-1)*(c + d*x)*ExpI
ntegralEi[(A + B*Log[e*((a + b*x)/(c + d*x))^n])/(B*n)] - d*(a + b*x)*ExpIn
tegralEi[(2*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/(B*n)]))/(B*(b*c - a*d)
^2*E^((2*A)/(B*n))*g^3*n*(e*((a + b*x)/(c + d*x))^n)^(2/n)*(c + d*x)^2)
```

**fricas [A]** time = 0.90, size = 147, normalized size = 0.74

$$\frac{\left(be^{\left(\frac{B\log(e)+A}{Bn}\right)}\operatorname{log\_integral}\left(\frac{(bx+a)e^{\left(\frac{B\log(e)+A}{Bn}\right)}}{dx+c}\right)-d\operatorname{log\_integral}\left(\frac{\left(b^2x^2+2\,abx+a^2\right)e^{\left(\frac{2\left(B\log(e)+A\right)}{Bn}\right)}}{d^2x^2+2\,cdx+c^2}\right)\right)e^{\left(-\frac{2\left(B\log(e)+A\right)}{Bn}\right)}}{\left(Bb^2c^2-2\,Babcd+Ba^2d^2\right)g^3n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*g*x+c*g)^3/(A+B*log(e*((b*x+a)/(d*x+c))^n)),x, algorithm="fr
icas")
```

```
[Out] (b*e^((B*log(e) + A)/(B*n))*log_integral((b*x + a)*e^((B*log(e) + A)/(B*n))
/(d*x + c)) - d*log_integral((b^2*x^2 + 2*a*b*x + a^2)*e^(2*(B*log(e) + A)/
(B*n))/(d^2*x^2 + 2*c*d*x + c^2)))*e^(-2*(B*log(e) + A)/(B*n))/((B*b^2*c^2
- 2*B*a*b*c*d + B*a^2*d^2)*g^3*n)
```

**giac [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*g*x+c*g)^3/(A+B*log(e*((b*x+a)/(d*x+c))^n)),x, algorithm="gi
ac")
```

```
[Out] Timed out
```

**maple [F]** time = 0.28, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(dgx+cg\right)^{3}\left(B\ln\left(e\left(\frac{bx+a}{dx+c}\right)^{n}\right)+A\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(d*g*x+c*g)^3/(B*ln(e*((b*x+a)/(d*x+c))^n)+A),x)
```

```
[Out] int(1/(d*g*x+c*g)^3/(B*ln(e*((b*x+a)/(d*x+c))^n)+A),x)
```

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(dgx+cg)^3\left(B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*g*x+c*g)^3/(A+B*log(e*((b*x+a)/(d*x+c))^n)),x, algorithm="ma
xima")
```

```
[Out] integrate(1/((d*g*x + c*g)^3*(B*log(e*((b*x + a)/(d*x + c))^n) + A)), x)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{(c\,g+d\,g\,x)^3\,\left(A+B\,\ln\left(e\,\left(\frac{a+b\,x}{c+d\,x}\right)^n\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((c*g + d*g*x)^3*(A + B*log(e*((a + b*x)/(c + d*x))^n))),x)
```

```
[Out] int(1/((c*g + d*g*x)^3*(A + B*log(e*((a + b*x)/(c + d*x))^n))), x)
```

**sympy [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*g*x+c*g)**3/(A+B*ln(e*((b*x+a)/(d*x+c))**n)),x)
```

```
[Out] Timed out
```

# 3.52 $\int \frac{(cg+dgx)^2}{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}\,dx$

**Optimal**. Leaf size=38

$$\text{Int}\left(\frac{(cg+dgx)^2}{\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2},x\right)$$

```
[Out] Unintegrable((d*g*x+c*g)^2/(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2,x)
```

**Rubi [A]** time = 0.23, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{(cg+dgx)^2}{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(c*g + d*g*x)^2/(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2,x]
```

```
[Out] c^2*g^2*Defer[Int][(A + B*Log[e*((a + b*x)/(c + d*x))^n])^(-2), x] + 2*c*d*
g^2*Defer[Int][x/(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2, x] + d^2*g^2*Def
er[Int][x^2/(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2, x]
```

Rubi steps

$$\int \frac{(cg+dgx)^2}{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}\,dx = \int \left(\frac{c^2g^2}{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}+\frac{2cdg^2x}{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}+\frac{d^2g^2x^2}{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}\right)dx$$

$$=\left(c^2g^2\right)\int \frac{1}{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}\,dx+\left(2cdg^2\right)\int \frac{x}{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}$$

**Mathematica [A]** time = 1.00, size = 0, normalized size = 0.00

$$\int \frac{(cg+dgx)^2}{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(c*g + d*g*x)^2/(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2,x]
```

```
[Out] Integrate[(c*g + d*g*x)^2/(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2, x]
```

**fricas [A]** time = 1.03, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{d^2g^2x^2+2\,cdg^2x+c^2g^2}{B^2\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)^2+2\,AB\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*g*x+c*g)^2/(A+B*log(e*((b*x+a)/(d*x+c))^n))^2,x, algorithm="fr
icas")
```

```
[Out] integral((d^2*g^2*x^2 + 2*c*d*g^2*x + c^2*g^2)/(B^2*log(e*((b*x + a)/(d*x +
 c))^n)^2 + 2*A*B*log(e*((b*x + a)/(d*x + c))^n) + A^2), x)
```

**giac [A]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(dgx+cg\right)^2}{\left(B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*g*x+c*g)^2/(A+B*log(e*((b*x+a)/(d*x+c))^n))^2,x, algorithm="gi
ac")
```

```
[Out] integrate((d*g*x + c*g)^2/(B*log(e*((b*x + a)/(d*x + c))^n) + A)^2, x)
```

**maple [A]** time = 0.26, size = 0, normalized size = 0.00

$$\int \frac{\left(dgx+cg\right)^2}{\left(B\ln\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*g*x+c*g)^2/(B*ln(e*((b*x+a)/(d*x+c))^n)+A)^2,x)
```

```
[Out] int((d*g*x+c*g)^2/(B*ln(e*((b*x+a)/(d*x+c))^n)+A)^2,x)
```

**maxima [A]** time = 0.00, size = 0, normalized size = 0.00

$$-\frac{bd^3g^2x^4 + ac^3g^2 + \left(3\,bcd^2g^2 + ad^3g^2\right)x^3 + 3\left(bc^2dg^2 + acd^2g^2\right)x^2 + \left(bc^3g^2 + 3\,ac^2dg^2\right)x}{(bcn - adn)B^2\log\left((bx + a)^n\right) - (bcn - adn)B^2\log\left((dx + c)^n\right) + (bcn - adn)AB + \left(bcn\log(e) - adn\log(e)\right)B^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*g*x+c*g)^2/(A+B*log(e*((b*x+a)/(d*x+c))^n))^2,x, algorithm="ma
xima")
```

```
[Out] -(b*d^3*g^2*x^4 + a*c^3*g^2 + (3*b*c*d^2*g^2 + a*d^3*g^2)*x^3 + 3*(b*c^2*d*
g^2 + a*c*d^2*g^2)*x^2 + (b*c^3*g^2 + 3*a*c^2*d*g^2)*x)/((b*c*n - a*d*n)*B^
2*log((b*x + a)^n) - (b*c*n - a*d*n)*B^2*log((d*x + c)^n) + (b*c*n - a*d*n)
*A*B + (b*c*n*log(e) - a*d*n*log(e))*B^2) + integrate((4*b*d^3*g^2*x^3 + b*
c^3*g^2 + 3*a*c^2*d*g^2 + 3*(3*b*c*d^2*g^2 + a*d^3*g^2)*x^2 + 6*(b*c^2*d*g^
2 + a*c*d^2*g^2)*x)/((b*c*n - a*d*n)*B^2*log((b*x + a)^n) - (b*c*n - a*d*n)
*B^2*log((d*x + c)^n) + (b*c*n - a*d*n)*A*B + (b*c*n*log(e) - a*d*n*log(e))
*B^2), x)
```

**mupad [A]** time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{{\left(c\,g+d\,g\,x\right)}^2}{{\left(A+B\,\ln\left(e\,{\left(\frac{a+b\,x}{c+d\,x}\right)}^n\right)\right)}^2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*g + d*g*x)^2/(A + B*log(e*((a + b*x)/(c + d*x))^n))^2,x)
```

```
[Out] int((c*g + d*g*x)^2/(A + B*log(e*((a + b*x)/(c + d*x))^n))^2, x)
```

**sympy [A]** time = 0.00, size = 0, normalized size = 0.00

$$g^2\left(\int \frac{c^2}{A^2 + 2AB\log\left(e\left(\frac{a}{c+dx} + \frac{bx}{c+dx}\right)^n\right) + B^2\log\left(e\left(\frac{a}{c+dx} + \frac{bx}{c+dx}\right)^n\right)^2}\,dx + \int \frac{d^2x^2}{A^2 + 2AB\log\left(e\left(\frac{a}{c+dx} + \frac{bx}{c+dx}\right)^n\right) + \cdots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*g*x+c*g)**2/(A+B*ln(e*((b*x+a)/(d*x+c))**n))**2,x)
```

```
[Out] g**2*(Integral(c**2/(A**2 + 2*A*B*log(e*(a/(c + d*x) + b*x/(c + d*x))**n) +
 B**2*log(e*(a/(c + d*x) + b*x/(c + d*x))**n)**2), x) + Integral(d**2*x**2/
(A**2 + 2*A*B*log(e*(a/(c + d*x) + b*x/(c + d*x))**n) + B**2*log(e*(a/(c +
d*x) + b*x/(c + d*x))**n)**2), x) + Integral(2*c*d*x/(A**2 + 2*A*B*log(e*(a
/(c + d*x) + b*x/(c + d*x))**n) + B**2*log(e*(a/(c + d*x) + b*x/(c + d*x))*
*n)**2), x))
```

# 3.53 $\int \frac{cg+dgx}{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}\,dx$

**Optimal**. Leaf size=36

$$\mathrm{Int}\left(\frac{cg+dgx}{\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2},x\right)$$

```
[Out] Unintegrable((d*g*x+c*g)/(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2,x)
```

**Rubi** [A] time = 0.12, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{cg+dgx}{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(c*g + d*g*x)/(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2,x]
```

```
[Out] c*g*Defer[Int][(A + B*Log[e*((a + b*x)/(c + d*x))^n])^(-2), x] + d*g*Defer[
Int][x/(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2, x]
```

Rubi steps

$$\begin{aligned}\int \frac{cg+dgx}{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}\,dx &= \int \left(\frac{cg}{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}+\frac{dgx}{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}\right)dx\\ &= (cg)\int \frac{1}{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}\,dx+(dg)\int \frac{x}{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}\,dx\end{aligned}$$

**Mathematica** [A] time = 1.00, size = 0, normalized size = 0.00

$$\int \frac{cg+dgx}{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(c*g + d*g*x)/(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2,x]
```

```
[Out] Integrate[(c*g + d*g*x)/(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2, x]
```

**fricas [A]** time = 0.90, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{dgx + cg}{B^2 \log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)^2 + 2\,AB\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right) + A^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*g*x+c*g)/(A+B*log(e*((b*x+a)/(d*x+c))^n))^2,x, algorithm="fric
as")
```

```
[Out] integral((d*g*x + c*g)/(B^2*log(e*((b*x + a)/(d*x + c))^n)^2 + 2*A*B*log(e*
((b*x + a)/(d*x + c))^n) + A^2), x)
```

**giac [A]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{dgx + cg}{\left(B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right) + A\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*g*x+c*g)/(A+B*log(e*((b*x+a)/(d*x+c))^n))^2,x, algorithm="giac
")
```

```
[Out] integrate((d*g*x + c*g)/(B*log(e*((b*x + a)/(d*x + c))^n) + A)^2, x)
```

**maple [A]** time = 0.06, size = 0, normalized size = 0.00

$$\int \frac{dgx + cg}{\left(B\ln\left(e\left(\frac{bx+a}{dx+c}\right)^n\right) + A\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*g*x+c*g)/(B*ln(e*((b*x+a)/(d*x+c))^n)+A)^2,x)
```

```
[Out] int((d*g*x+c*g)/(B*ln(e*((b*x+a)/(d*x+c))^n)+A)^2,x)
```

**maxima [A]** time = 0.00, size = 0, normalized size = 0.00

$$-\frac{bd^2gx^3 + ac^2g + \left(2\,bcdg + ad^2g\right)x^2 + \left(bc^2g + 2\,acdg\right)x}{(bcn - adn)B^2\log\left((bx+a)^n\right) - (bcn - adn)B^2\log\left((dx+c)^n\right) + (bcn - adn)AB + \left(bcn\log(e) - adn\log(e)\right)B^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*g*x+c*g)/(A+B*log(e*((b*x+a)/(d*x+c))^n))^2,x, algorithm="maxi
ma")
```

```
[Out] -(b*d^2*g*x^3 + a*c^2*g + (2*b*c*d*g + a*d^2*g)*x^2 + (b*c^2*g + 2*a*c*d*g)
*x)/((b*c*n - a*d*n)*B^2*log((b*x + a)^n) - (b*c*n - a*d*n)*B^2*log((d*x +
c)^n) + (b*c*n - a*d*n)*A*B + (b*c*n*log(e) - a*d*n*log(e))*B^2) + integrat
e((3*b*d^2*g*x^2 + b*c^2*g + 2*a*c*d*g + 2*(2*b*c*d*g + a*d^2*g)*x)/((b*c*n
 - a*d*n)*B^2*log((b*x + a)^n) - (b*c*n - a*d*n)*B^2*log((d*x + c)^n) + (b*
c*n - a*d*n)*A*B + (b*c*n*log(e) - a*d*n*log(e))*B^2), x)
```

**mupad [A]** time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{c\,g+d\,g\,x}{{\left(A+B\,\ln\left(e\,{\left(\frac{a+b\,x}{c+d\,x}\right)}^n\right)\right)}^2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*g + d*g*x)/(A + B*log(e*((a + b*x)/(c + d*x))^n))^2,x)
```

```
[Out] int((c*g + d*g*x)/(A + B*log(e*((a + b*x)/(c + d*x))^n))^2, x)
```

**sympy [A]** time = 0.00, size = 0, normalized size = 0.00

$$g\left(\int \frac{c}{A^2 + 2AB\log\left(e\left(\frac{a}{c+dx} + \frac{bx}{c+dx}\right)^n\right) + B^2\log\left(e\left(\frac{a}{c+dx} + \frac{bx}{c+dx}\right)^n\right)^2}\, dx + \int \frac{dx}{A^2 + 2AB\log\left(e\left(\frac{a}{c+dx} + \frac{bx}{c+dx}\right)^n\right) + B^2\log\left(e\left(\frac{a}{c+dx} + \frac{bx}{c+dx}\right)^n\right)^2}\, dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*g*x+c*g)/(A+B*ln(e*((b*x+a)/(d*x+c))**n))**2,x)
```

```
[Out] g*(Integral(c/(A**2 + 2*A*B*log(e*(a/(c + d*x) + b*x/(c + d*x))**n) + B**2*
log(e*(a/(c + d*x) + b*x/(c + d*x))**n)**2), x) + Integral(d*x/(A**2 + 2*A*
B*log(e*(a/(c + d*x) + b*x/(c + d*x))**n) + B**2*log(e*(a/(c + d*x) + b*x/(
c + d*x))**n)**2), x))
```

# 3.54 $\int \frac{1}{(cg+dgx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}\,dx$

**Optimal**. Leaf size=38

$$\text{Int}\left(\frac{1}{(cg+dgx)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2},x\right)$$

```
[Out] Unintegrable(1/(d*g*x+c*g)/(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2,x)
```

**Rubi [A]** time = 0.09, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{1}{(cg+dgx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[1/((c*g + d*g*x)*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2),x]
[Out] Defer[Int][1/((c*g + d*g*x)*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2), x]
```

Rubi steps

$$\int \frac{1}{(cg+dgx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}\,dx=\int \frac{1}{(cg+dgx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}\,dx$$

**Mathematica [A]** time = 0.56, size = 0, normalized size = 0.00

$$\int \frac{1}{(cg+dgx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[1/((c*g + d*g*x)*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2),x]
```

```
[Out] Integrate[1/((c*g + d*g*x)*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2), x]
```

**fricas [A]** time = 0.90, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{1}{A^2dgx + A^2cg + \left(B^2dgx + B^2cg\right)\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)^2 + 2\left(ABdgx + ABcg\right)\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*g*x+c*g)/(A+B*log(e*((b*x+a)/(d*x+c))^n))^2,x, algorithm="fr
icas")
```

```
[Out] integral(1/(A^2*d*g*x + A^2*c*g + (B^2*d*g*x + B^2*c*g)*log(e*((b*x + a)/(d
*x + c))^n)^2 + 2*(A*B*d*g*x + A*B*c*g)*log(e*((b*x + a)/(d*x + c))^n)), x)
```

**giac [A]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(dgx + cg\right)\left(B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right) + A\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*g*x+c*g)/(A+B*log(e*((b*x+a)/(d*x+c))^n))^2,x, algorithm="gi
ac")
```

```
[Out] integrate(1/((d*g*x + c*g)*(B*log(e*((b*x + a)/(d*x + c))^n) + A)^2), x)
```

**maple [A]** time = 0.30, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(dgx + cg\right)\left(B\ln\left(e\left(\frac{bx+a}{dx+c}\right)^n\right) + A\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(d*g*x+c*g)/(B*ln(e*((b*x+a)/(d*x+c))^n)+A)^2,x)
```

```
[Out] int(1/(d*g*x+c*g)/(B*ln(e*((b*x+a)/(d*x+c))^n)+A)^2,x)
```

**maxima [A]** time = 0.00, size = 0, normalized size = 0.00

$$b\int \frac{1}{\left(bcgn - adgn\right)B^2\log\left((bx+a)^n\right) - \left(bcgn - adgn\right)B^2\log\left((dx+c)^n\right) + \left(bcgn - adgn\right)AB + \left(bcgn\log(e) - a\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*g*x+c*g)/(A+B*log(e*((b*x+a)/(d*x+c))^n))^2,x, algorithm="ma
xima")
```

```
[Out] b*integrate(1/((b*c*g*n - a*d*g*n)*B^2*log((b*x + a)^n) - (b*c*g*n - a*d*g*
n)*B^2*log((d*x + c)^n) + (b*c*g*n - a*d*g*n)*A*B + (b*c*g*n*log(e) - a*d*g
*n*log(e))*B^2), x) - (b*x + a)/((b*c*g*n - a*d*g*n)*B^2*log((b*x + a)^n) -
 (b*c*g*n - a*d*g*n)*B^2*log((d*x + c)^n) + (b*c*g*n - a*d*g*n)*A*B + (b*c*
g*n*log(e) - a*d*g*n*log(e))*B^2)
```

**mupad** [**A**] time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{1}{\left(c\,g+d\,g\,x\right)\,{\left(A+B\,\ln\left(e\,{\left(\frac{a+b\,x}{c+d\,x}\right)}^n\right)\right)}^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((c*g + d*g*x)*(A + B*log(e*((a + b*x)/(c + d*x))^n))^2),x)
```

```
[Out] int(1/((c*g + d*g*x)*(A + B*log(e*((a + b*x)/(c + d*x))^n))^2), x)
```

**sympy** [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*g*x+c*g)/(A+B*ln(e*((b*x+a)/(d*x+c))**n))**2,x)
```

```
[Out] Timed out
```

# 3.55 $\int \frac{1}{(cg+dgx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}\,dx$

**Optimal**. Leaf size=154

$$\frac{(a+bx)e^{-\frac{A}{Bn}}\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)^{-1/n}\text{Ei}\left(\frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{Bn}\right)}{B^2g^2n^2(c+dx)(bc-ad)}-\frac{a+bx}{Bg^2n(c+dx)(bc-ad)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}$$

```
[Out] (b*x+a)*Ei((A+B*ln(e*((b*x+a)/(d*x+c))^n))/B/n)/B^2/(-a*d+b*c)/exp(A/B/n)/g
^2/n^2/((e*((b*x+a)/(d*x+c))^n)^(1/n))/(d*x+c)+(-b*x-a)/B/(-a*d+b*c)/g^2/n/
(d*x+c)/(A+B*ln(e*((b*x+a)/(d*x+c))^n))
```

**Rubi [F]** time = 0.10, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{1}{(cg+dgx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[1/((c*g + d*g*x)^2*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2),x]
```

```
[Out] Defer[Int][1/((c*g + d*g*x)^2*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2), x]
```

Rubi steps

$$\int \frac{1}{(cg+dgx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}\,dx=\int \frac{1}{(cg+dgx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}\,dx$$

**Mathematica [A]** time = 0.17, size = 180, normalized size = 1.17

$$-\frac{(a+bx)e^{-\frac{A}{Bn}}\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)^{-1/n}\left(Bne^{\frac{A}{Bn}}\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)^{\frac{1}{n}}-\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)\text{Ei}\left(\frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{Bn}\right)\right)}{B^2g^2n^2(c+dx)(bc-ad)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((c*g + d*g*x)^2*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2),x]
```

```
[Out] -(((a + b*x)*(B*E^(A/(B*n))*n*(e*((a + b*x)/(c + d*x))^n)^n^(-1) - ExpInteg
ralEi[(A + B*Log[e*((a + b*x)/(c + d*x))^n])/(B*n)]*(A + B*Log[e*((a + b*x)
/(c + d*x))^n])))/(B^2*(b*c - a*d)*E^(A/(B*n))*g^2*n^2*(e*((a + b*x)/(c + d
*x))^n)^n^(-1)*(c + d*x)*(A + B*Log[e*((a + b*x)/(c + d*x))^n])))
```

**fricas [A]** time = 0.81, size = 291, normalized size = 1.89

$$-\frac{\left((Bbnx + Ban)e^{\left(\frac{B\log(e)+A}{Bn}\right)} - \left(Adx + Ac + (Bdx + Bc)\log(e) + (Bdnx + Bcn)\log\left(\frac{bx+a}{dx+c}\right)\right)\log\right.}{\left(AB^2bcd - AB^2ad^2\right)g^2n^2x + \left(AB^2bc^2 - AB^2acd\right)g^2n^2 + \left(\left(B^3bcd - B^3ad^2\right)g^2n^2x + \left(B^3bc^2 - B^3acd\right)g^2n^2\right)\log(e)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*g*x+c*g)^2/(A+B*log(e*((b*x+a)/(d*x+c))^n))^2,x, algorithm="
fricas")
```

```
[Out] -((B*b*n*x + B*a*n)*e^((B*log(e) + A)/(B*n)) - (A*d*x + A*c + (B*d*x + B*c)
*log(e) + (B*d*n*x + B*c*n)*log((b*x + a)/(d*x + c)))*log_integral((b*x + a
)*e^((B*log(e) + A)/(B*n))/(d*x + c)))*e^(-(B*log(e) + A)/(B*n))/((A*B^2*b*
c*d - A*B^2*a*d^2)*g^2*n^2*x + (A*B^2*b*c^2 - A*B^2*a*c*d)*g^2*n^2 + ((B^3*
b*c*d - B^3*a*d^2)*g^2*n^2*x + (B^3*b*c^2 - B^3*a*c*d)*g^2*n^2)*log(e) + ((
B^3*b*c*d - B^3*a*d^2)*g^2*n^3*x + (B^3*b*c^2 - B^3*a*c*d)*g^2*n^3)*log((b*
x + a)/(d*x + c)))
```

**giac [A]** time = 1.16, size = 140, normalized size = 0.91

$$-\left(\frac{bc}{(bc-ad)^2} - \frac{ad}{(bc-ad)^2}\right)\left(\frac{bx+a}{\left(B^2g^2n^2\log\left(\frac{bx+a}{dx+c}\right) + ABg^2n + B^2g^2n\right)(dx+c)} - \frac{\mathrm{Ei}\left(\frac{A}{Bn} + \frac{1}{n} + \log\left(\frac{bx+a}{dx+c}\right)\right)e^{\left(-\frac{A}{Bn}-\frac{1}{n}\right)}}{B^2g^2n^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*g*x+c*g)^2/(A+B*log(e*((b*x+a)/(d*x+c))^n))^2,x, algorithm="
giac")
```

```
[Out] -(b*c/(b*c - a*d)^2 - a*d/(b*c - a*d)^2)*((b*x + a)/((B^2*g^2*n^2*log((b*x
+ a)/(d*x + c)) + A*B*g^2*n + B^2*g^2*n)*(d*x + c)) - Ei(A/(B*n) + 1/n + lo
g((b*x + a)/(d*x + c)))*e^(-A/(B*n) - 1/n)/(B^2*g^2*n^2))
```

**maple [F]** time = 0.29, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(dgx+cg\right)^{2}\left(B\ln\left(e\left(\frac{bx+a}{dx+c}\right)^{n}\right)+A\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(d*g*x+c*g)^2/(B*ln(e*((b*x+a)/(d*x+c))^n)+A)^2,x)
```

```
[Out] int(1/(d*g*x+c*g)^2/(B*ln(e*((b*x+a)/(d*x+c))^n)+A)^2,x)
```

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$-\frac{}{\left(bc^{2}g^{2}n-acdg^{2}n\right)AB+\left(bc^{2}g^{2}n\log(e)-acdg^{2}n\log(e)\right)B^{2}+\left(\left(bcdg^{2}n-ad^{2}g^{2}n\right)AB+\left(bcdg^{2}n\log(e)-ad^{2}g\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*g*x+c*g)^2/(A+B*log(e*((b*x+a)/(d*x+c))^n))^2,x, algorithm="
maxima")
```

```
[Out] -(b*x + a)/((b*c^2*g^2*n - a*c*d*g^2*n)*A*B + (b*c^2*g^2*n*log(e) - a*c*d*g
^2*n*log(e))*B^2 + ((b*c*d*g^2*n - a*d^2*g^2*n)*A*B + (b*c*d*g^2*n*log(e) -
 a*d^2*g^2*n*log(e))*B^2)*x + ((b*c*d*g^2*n - a*d^2*g^2*n)*B^2*x + (b*c^2*g
^2*n - a*c*d*g^2*n)*B^2)*log((b*x + a)^n) - ((b*c*d*g^2*n - a*d^2*g^2*n)*B^
2*x + (b*c^2*g^2*n - a*c*d*g^2*n)*B^2)*log((d*x + c)^n)) - integrate(-1/(B^
2*c^2*g^2*n*log(e) + A*B*c^2*g^2*n + (B^2*d^2*g^2*n*log(e) + A*B*d^2*g^2*n)
*x^2 + 2*(B^2*c*d*g^2*n*log(e) + A*B*c*d*g^2*n)*x + (B^2*d^2*g^2*n*x^2 + 2*
B^2*c*d*g^2*n*x + B^2*c^2*g^2*n)*log((b*x + a)^n) - (B^2*d^2*g^2*n*x^2 + 2*
B^2*c*d*g^2*n*x + B^2*c^2*g^2*n)*log((d*x + c)^n)), x)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{{\left(c\,g+d\,g\,x\right)}^2\,{\left(A+B\,\ln\left(e\,{\left(\frac{a+b\,x}{c+d\,x}\right)}^n\right)\right)}^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((c*g + d*g*x)^2*(A + B*log(e*((a + b*x)/(c + d*x))^n))^2),x)
```

```
[Out] int(1/((c*g + d*g*x)^2*(A + B*log(e*((a + b*x)/(c + d*x))^n))^2), x)
```

**sympy [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*g*x+c*g)**2/(A+B*ln(e*((b*x+a)/(d*x+c))**n))**2,x)
```

```
[Out] Timed out
```

# 3.56 $\displaystyle\int \frac{1}{(cg+dgx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}\,dx$

**Optimal**. Leaf size=256

$$-\frac{2d(a+bx)^2e^{-\frac{2A}{Bn}}\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)^{-2/n}\text{Ei}\left(\frac{2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{Bn}\right)}{B^2g^3n^2(c+dx)^2(bc-ad)^2}+\frac{b(a+bx)e^{-\frac{A}{Bn}}\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)^{-1/n}\text{Ei}\left(\frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{Bn}\right)}{B^2g^3n^2(c+dx)(bc-ad)^2}-- \quad B$$

```
[Out] b*(b*x+a)*Ei((A+B*ln(e*((b*x+a)/(d*x+c))^n))/B/n)/B^2/(-a*d+b*c)^2/exp(A/B/
n)/g^3/n^2/((e*((b*x+a)/(d*x+c))^n)^(1/n))/(d*x+c)-2*d*(b*x+a)^2*Ei(2*(A+B*
ln(e*((b*x+a)/(d*x+c))^n))/B/n)/B^2/(-a*d+b*c)^2/exp(2*A/B/n)/g^3/n^2/((e*(
(b*x+a)/(d*x+c))^n)^(2/n))/(d*x+c)^2+(-b*x-a)/B/(-a*d+b*c)/g^3/n/(d*x+c)^2/
(A+B*ln(e*((b*x+a)/(d*x+c))^n))
```

**Rubi** [F] time = 0.09, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{1}{(cg+dgx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[1/((c*g + d*g*x)^3*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2),x]
```

```
[Out] Defer[Int][1/((c*g + d*g*x)^3*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2), x]
```

Rubi steps

$$\int \frac{1}{(cg+dgx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}\,dx=\int \frac{1}{(cg+dgx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}\,dx$$

**Mathematica [A]** time = 0.54, size = 288, normalized size = 1.12

$$\frac{(a+bx)e^{-\frac{2A}{Bn}}\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)^{-2/n}\left(be^{\frac{A}{Bn}}(c+dx)\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)^{\frac{1}{n}}\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)\text{Ei}\left(\frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{Bn}\right)-2d(a+bx)}{B^2g^3n^2(c+dx)^2(bc-ad)^2\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right.\right.\right.}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((c*g + d*g*x)^3*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2),x]
```

```
[Out] ((a + b*x)*(-(B*(b*c - a*d)*E^((2*A)/(B*n))*n*(e*((a + b*x)/(c + d*x))^n)^(
2/n)) + b*E^(A/(B*n))*(e*((a + b*x)/(c + d*x))^n)^n^(-1)*(c + d*x)*ExpInteg
ralEi[(A + B*Log[e*((a + b*x)/(c + d*x))^n])/(B*n)]*(A + B*Log[e*((a + b*x)
/(c + d*x))^n]) - 2*d*(a + b*x)*ExpIntegralEi[(2*(A + B*Log[e*((a + b*x)/(c
 + d*x))^n]))/(B*n)]*(A + B*Log[e*((a + b*x)/(c + d*x))^n])))/(B^2*(b*c - a
*d)^2*E^((2*A)/(B*n))*g^3*n^2*(e*((a + b*x)/(c + d*x))^n)^(2/n)*(c + d*x)^2
*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))
```

**fricas [B]** time = 0.90, size = 770, normalized size = 3.01

$$\frac{\left(\left(Abd^2x^2+2\,Abcdx+Abc^2+\left(Bbd^2x^2+2\,Bbcdx+Bbc^2\right)\log(e)+\left(Bbd^2nx^2+2\,Bbcdnx\right.\right.\right.}{\left(AB^2b^2c^2d^2-2\,AB^2abcd^3+AB^2a^2d^4\right)g^3n^2x^2+2\left(AB^2b^2c^3d-2\,AB^2abc^2d^2+AB^2a^2cd^3\right)g^3n^2x+\left(AB^2b^2c^4-2\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*g*x+c*g)^3/(A+B*log(e*((b*x+a)/(d*x+c))^n))^2,x, algorithm="
fricas")
```

```
[Out] ((A*b*d^2*x^2 + 2*A*b*c*d*x + A*b*c^2 + (B*b*d^2*x^2 + 2*B*b*c*d*x + B*b*c^
2)*log(e) + (B*b*d^2*n*x^2 + 2*B*b*c*d*n*x + B*b*c^2*n)*log((b*x + a)/(d*x
+ c)))*e^((B*log(e) + A)/(B*n))*log_integral((b*x + a)*e^((B*log(e) + A)/(B
*n))/(d*x + c)) - ((B*b^2*c - B*a*b*d)*n*x + (B*a*b*c - B*a^2*d)*n)*e^(2*(B
*log(e) + A)/(B*n)) - 2*(A*d^3*x^2 + 2*A*c*d^2*x + A*c^2*d + (B*d^3*x^2 + 2
*B*c*d^2*x + B*c^2*d)*log(e) + (B*d^3*n*x^2 + 2*B*c*d^2*n*x + B*c^2*d*n)*lo
g((b*x + a)/(d*x + c)))*log_integral((b^2*x^2 + 2*a*b*x + a^2)*e^(2*(B*log(
e) + A)/(B*n))/(d^2*x^2 + 2*c*d*x + c^2)))*e^(-2*(B*log(e) + A)/(B*n))/((A*
B^2*b^2*c^2*d^2 - 2*A*B^2*a*b*c*d^3 + A*B^2*a^2*d^4)*g^3*n^2*x^2 + 2*(A*B^2
*b^2*c^3*d - 2*A*B^2*a*b*c^2*d^2 + A*B^2*a^2*c*d^3)*g^3*n^2*x + (A*B^2*b^2*
c^4 - 2*A*B^2*a*b*c^3*d + A*B^2*a^2*c^2*d^2)*g^3*n^2 + ((B^3*b^2*c^2*d^2 -
2*B^3*a*b*c*d^3 + B^3*a^2*d^4)*g^3*n^2*x^2 + 2*(B^3*b^2*c^3*d - 2*B^3*a*b*c
```

```
^2*d^2 + B^3*a^2*c*d^3)*g^3*n^2*x + (B^3*b^2*c^4 - 2*B^3*a*b*c^3*d + B^3*a^
2*c^2*d^2)*g^3*n^2)*log(e) + ((B^3*b^2*c^2*d^2 - 2*B^3*a*b*c*d^3 + B^3*a^2*
d^4)*g^3*n^3*x^2 + 2*(B^3*b^2*c^3*d - 2*B^3*a*b*c^2*d^2 + B^3*a^2*c*d^3)*g^
3*n^3*x + (B^3*b^2*c^4 - 2*B^3*a*b*c^3*d + B^3*a^2*c^2*d^2)*g^3*n^3)*log((b
*x + a)/(d*x + c)))
```

**giac [A]** time = 2.11, size = 312, normalized size = 1.22

$$\left(\frac{b\text{Ei}\left(\frac{A}{Bn}+\frac{1}{n}+\log\left(\frac{bx+a}{dx+c}\right)\right)e^{\left(-\frac{A}{Bn}-\frac{1}{n}\right)}}{B^2bcg^3n^2-B^2adg^3n^2}-\frac{2\,d\text{Ei}\left(\frac{2\,A}{Bn}+\frac{2}{n}+2\,\log\left(\frac{bx+a}{dx+c}\right)\right)e^{\left(-\frac{2\,A}{Bn}-\frac{2}{n}\right)}}{B^2bcg^3n^2-B^2adg^3n^2}-\frac{}{B^2bcg^3n^2\log\left(\frac{bx+a}{dx+c}\right)-B^2adg^3}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*g*x+c*g)^3/(A+B*log(e*((b*x+a)/(d*x+c))^n))^2,x, algorithm="
giac")
```

```
[Out] (b*Ei(A/(B*n) + 1/n + log((b*x + a)/(d*x + c)))*e^(-A/(B*n) - 1/n)/(B^2*b*c
*g^3*n^2 - B^2*a*d*g^3*n^2) - 2*d*Ei(2*A/(B*n) + 2/n + 2*log((b*x + a)/(d*x
 + c)))*e^(-2*A/(B*n) - 2/n)/(B^2*b*c*g^3*n^2 - B^2*a*d*g^3*n^2) - ((b*x +
a)*b/(d*x + c) - (b*x + a)^2*d/(d*x + c)^2)/(B^2*b*c*g^3*n^2*log((b*x + a)/
(d*x + c)) - B^2*a*d*g^3*n^2*log((b*x + a)/(d*x + c)) + A*B*b*c*g^3*n + B^2
*b*c*g^3*n - A*B*a*d*g^3*n - B^2*a*d*g^3*n))*(b*c/(b*c - a*d)^2 - a*d/(b*c
- a*d)^2)
```

**maple [F]** time = 0.29, size = 0, normalized size = 0.00

$$\int\frac{1}{\left(dgx+cg\right)^{3}\left(B\ln\left(e\left(\frac{bx+a}{dx+c}\right)^{n}\right)+A\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(d*g*x+c*g)^3/(B*ln(e*((b*x+a)/(d*x+c))^n)+A)^2,x)
```

```
[Out] int(1/(d*g*x+c*g)^3/(B*ln(e*((b*x+a)/(d*x+c))^n)+A)^2,x)
```

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$-\frac{}{\left(bc^3g^3n-ac^2dg^3n\right)AB+\left(bc^3g^3n\log(e)-ac^2dg^3n\log(e)\right)B^2+\left(\left(bcd^2g^3n-ad^3g^3n\right)AB+\left(bcd^2g^3n\log(e)-a\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*g*x+c*g)^3/(A+B*log(e*((b*x+a)/(d*x+c))^n))^2,x, algorithm="
maxima")
```

```
[Out] -(b*x + a)/((b*c^3*g^3*n - a*c^2*d*g^3*n)*A*B + (b*c^3*g^3*n*log(e) - a*c^2
*d*g^3*n*log(e))*B^2 + ((b*c*d^2*g^3*n - a*d^3*g^3*n)*A*B + (b*c*d^2*g^3*n*
log(e) - a*d^3*g^3*n*log(e))*B^2)*x^2 + 2*((b*c^2*d*g^3*n - a*c*d^2*g^3*n)*
A*B + (b*c^2*d*g^3*n*log(e) - a*c*d^2*g^3*n*log(e))*B^2)*x + ((b*c*d^2*g^3*
n - a*d^3*g^3*n)*B^2*x^2 + 2*(b*c^2*d*g^3*n - a*c*d^2*g^3*n)*B^2*x + (b*c^3
*g^3*n - a*c^2*d*g^3*n)*B^2)*log((b*x + a)^n) - ((b*c*d^2*g^3*n - a*d^3*g^3
*n)*B^2*x^2 + 2*(b*c^2*d*g^3*n - a*c*d^2*g^3*n)*B^2*x + (b*c^3*g^3*n - a*c^
2*d*g^3*n)*B^2)*log((d*x + c)^n)) - integrate((b*d*x - b*c + 2*a*d)/(((b*c*
d^3*g^3*n - a*d^4*g^3*n)*A*B + (b*c*d^3*g^3*n*log(e) - a*d^4*g^3*n*log(e))*
B^2)*x^3 + (b*c^4*g^3*n - a*c^3*d*g^3*n)*A*B + (b*c^4*g^3*n*log(e) - a*c^3*
d*g^3*n*log(e))*B^2 + 3*((b*c^2*d^2*g^3*n - a*c*d^3*g^3*n)*A*B + (b*c^2*d^2
*g^3*n*log(e) - a*c*d^3*g^3*n*log(e))*B^2)*x^2 + 3*((b*c^3*d*g^3*n - a*c^2*
d^2*g^3*n)*A*B + (b*c^3*d*g^3*n*log(e) - a*c^2*d^2*g^3*n*log(e))*B^2)*x + (
(b*c*d^3*g^3*n - a*d^4*g^3*n)*B^2*x^3 + 3*(b*c^2*d^2*g^3*n - a*c*d^3*g^3*n)
*B^2*x^2 + 3*(b*c^3*d*g^3*n - a*c^2*d^2*g^3*n)*B^2*x + (b*c^4*g^3*n - a*c^3
*d*g^3*n)*B^2)*log((b*x + a)^n) - ((b*c*d^3*g^3*n - a*d^4*g^3*n)*B^2*x^3 +
3*(b*c^2*d^2*g^3*n - a*c*d^3*g^3*n)*B^2*x^2 + 3*(b*c^3*d*g^3*n - a*c^2*d^2*
g^3*n)*B^2*x + (b*c^4*g^3*n - a*c^3*d*g^3*n)*B^2)*log((d*x + c)^n)), x)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\left(c\,g+d\,g\,x\right)^3\,\left(A+B\,\ln\left(e\,\left(\frac{a+b\,x}{c+d\,x}\right)^n\right)\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((c*g + d*g*x)^3*(A + B*log(e*((a + b*x)/(c + d*x))^n))^2),x)
```

```
[Out] int(1/((c*g + d*g*x)^3*(A + B*log(e*((a + b*x)/(c + d*x))^n))^2), x)
```

**sympy [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*g*x+c*g)**3/(A+B*ln(e*((b*x+a)/(d*x+c))**n))**2,x)
```

```
[Out] Timed out
```

# 3.57 $\int (f+gx)^4 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\,dx$

**Optimal**. Leaf size=364

$$-\frac{Bg^2nx^2(bc-ad)\left(a^2d^2g^2-abdg(5df-cg)+b^2\left(c^2g^2-5cdfg+10d^2f^2\right)\right)}{10b^3d^3}+\frac{Bgnx(bc-ad)\left(a^3d^3g^3-a^2bd^2g^2(\right.}{}$$

```
[Out] 1/5*B*(-a*d+b*c)*g*(a^3*d^3*g^3-a^2*b*d^2*g^2*(-c*g+5*d*f)+a*b^2*d*g*(c^2*g
^2-5*c*d*f*g+10*d^2*f^2)-b^3*(-c^3*g^3+5*c^2*d*f*g^2-10*c*d^2*f^2*g+10*d^3*
f^3))*n*x/b^4/d^4-1/10*B*(-a*d+b*c)*g^2*(a^2*d^2*g^2-a*b*d*g*(-c*g+5*d*f)+b
^2*(c^2*g^2-5*c*d*f*g+10*d^2*f^2))*n*x^2/b^3/d^3-1/15*B*(-a*d+b*c)*g^3*(-a*
d*g-b*c*g+5*b*d*f)*n*x^3/b^2/d^2-1/20*B*(-a*d+b*c)*g^4*n*x^4/b/d-1/5*B*(-a*
g+b*f)^5*n*ln(b*x+a)/b^5/g+1/5*(g*x+f)^5*(A+B*ln(e*((b*x+a)/(d*x+c))^n))/g+
1/5*B*(-c*g+d*f)^5*n*ln(d*x+c)/d^5/g
```

**Rubi [A]** time = 0.60, antiderivative size = 348, normalized size of antiderivative = 0.96, number of steps used = 4, number of rules used = 3, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {2525, 12, 72}

$$-\frac{Bg^2nx^2(bc-ad)\left(a^2d^2g^2-abdg(5df-cg)+b^2\left(c^2g^2-5cdfg+10d^2f^2\right)\right)}{10b^3d^3}+\frac{Bgnx\left(-10a^2b^2d^4f^2g+5a^3bd^4fg^2\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(f + g*x)^4*(A + B*Log[e*((a + b*x)/(c + d*x))^n]),x]
```

```
[Out] (B*g*(10*a*b^3*d^4*f^3 - 10*a^2*b^2*d^4*f^2*g + 5*a^3*b*d^4*f*g^2 - a^4*d^4
*g^3 - b^4*c*(10*d^3*f^3 - 10*c*d^2*f^2*g + 5*c^2*d*f*g^2 - c^3*g^3))*n*x)/
(5*b^4*d^4) - (B*(b*c - a*d)*g^2*(a^2*d^2*g^2 - a*b*d*g*(5*d*f - c*g) + b^2
*(10*d^2*f^2 - 5*c*d*f*g + c^2*g^2))*n*x^2)/(10*b^3*d^3) - (B*(b*c - a*d)*g
^3*(5*b*d*f - b*c*g - a*d*g)*n*x^3)/(15*b^2*d^2) - (B*(b*c - a*d)*g^4*n*x^4
)/(20*b*d) - (B*(b*f - a*g)^5*n*Log[a + b*x])/(5*b^5*g) + ((f + g*x)^5*(A +
 B*Log[e*((a + b*x)/(c + d*x))^n]))/(5*g) + (B*(d*f - c*g)^5*n*Log[c + d*x]
)/(5*d^5*g)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 72

```
Int[((e_.) + (f_.)*(x_))^(p_.)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
 x_Symbol] :> Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x]
 /; FreeQ[{a, b, c, d, e, f}, x] && IntegerQ[p]
```

Rule 2525

```
Int[((a_.) + Log[(c_.)*(RFx_)^(p_.)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_))^(m_.
), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(a + b*Log[c*RFx^p])^n)/(e*(m + 1))
, x] - Dist[(b*n*p)/(e*(m + 1)), Int[SimplifyIntegrand[((d + e*x)^(m + 1)*(
a + b*Log[c*RFx^p])^(n - 1)*D[RFx, x])/RFx, x], x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && RationalFunctionQ[RFx, x] && IGtQ[n, 0] && (EqQ[n, 1] ||
IntegerQ[m]) && NeQ[m, -1]
```

Rubi steps

$$\int (f+gx)^4\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)dx=\frac{(f+gx)^5\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{5g}-\frac{(Bn)\int\frac{(bc-ad)(f+gx)^5}{(a+bx)(c+dx)}\,dx}{5g}$$

$$=\frac{(f+gx)^5\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{5g}-\frac{(B(bc-ad)n)\int\frac{(f+gx)^5}{(a+bx)(c+dx)}\,dx}{5g}$$

$$=\frac{(f+gx)^5\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{5g}-\frac{(B(bc-ad)n)\int\left(\frac{g^2\left(-a^3d^3g^3+a^2bd\right.}{}\right.}{}$$

$$=\frac{B(bc-ad)g\left(a^3d^3g^3-a^2bd^2g^2(5df-cg)+ab^2dg\left(10d^2f^2-5cdfg\right.\right.}{5b^4d^4}$$

**Mathematica [A]** time = 0.63, size = 285, normalized size = 0.78

$$\frac{Bg^2nx(ad-bc)\left(-12a^3d^3g^3+6a^2bd^2g^2(-2cg+10df+dgx)-2ab^2dg\left(6c^2g^2-3cdg(10f+gx)+d^2\left(60f^2+15fgx+2g^2x^2\right)\right)+b^3\left(-12c^3g^3+6c^2dg^2(10f+gx)-2cd^2\right.\right.}{12b^4d^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(f + g*x)^4*(A + B*Log[e*((a + b*x)/(c + d*x))^n]),x]
```

```
[Out] ((B*(-(b*c) + a*d)*g^2*n*x*(-12*a^3*d^3*g^3 + 6*a^2*b*d^2*g^2*(10*d*f - 2*c
*g + d*g*x) - 2*a*b^2*d*g*(6*c^2*g^2 - 3*c*d*g*(10*f + g*x) + d^2*(60*f^2 +
```

```
 15*f*g*x + 2*g^2*x^2)) + b^3*(-12*c^3*g^3 + 6*c^2*d*g^2*(10*f + g*x) - 2*c
*d^2*g*(60*f^2 + 15*f*g*x + 2*g^2*x^2) + d^3*(120*f^3 + 60*f^2*g*x + 20*f*g
^2*x^2 + 3*g^3*x^3))))/(12*b^4*d^4) - (B*(b*f - a*g)^5*n*Log[a + b*x])/b^5
+ (f + g*x)^5*(A + B*Log[e*((a + b*x)/(c + d*x))^n]) + (B*(d*f - c*g)^5*n*L
og[c + d*x])/d^5)/(5*g)
```

**fricas [B]** time = 2.39, size = 736, normalized size = 2.02

$$\frac{12\,Ab^5d^5g^4x^5 + 3\left(20\,Ab^5d^5fg^3 - \left(Bb^5cd^4 - Bab^4d^5\right)g^4n\right)x^4 + 4\left(30\,Ab^5d^5f^2g^2 - \left(5\left(Bb^5cd^4 - Bab^4d^5\right)fg^3 - \left(\right.\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^4*(A+B*log(e*((b*x+a)/(d*x+c))^n)),x, algorithm="fricas")
```

```
[Out] 1/60*(12*A*b^5*d^5*g^4*x^5 + 3*(20*A*b^5*d^5*f*g^3 - (B*b^5*c*d^4 - B*a*b^4
*d^5)*g^4*n)*x^4 + 4*(30*A*b^5*d^5*f^2*g^2 - (5*(B*b^5*c*d^4 - B*a*b^4*d^5)
*f*g^3 - (B*b^5*c^2*d^3 - B*a^2*b^3*d^5)*g^4)*n)*x^3 + 6*(20*A*b^5*d^5*f^3*
g - (10*(B*b^5*c*d^4 - B*a*b^4*d^5)*f^2*g^2 - 5*(B*b^5*c^2*d^3 - B*a^2*b^3*
d^5)*f*g^3 + (B*b^5*c^3*d^2 - B*a^3*b^2*d^5)*g^4)*n)*x^2 + 12*(5*B*a*b^4*d^
5*f^4 - 10*B*a^2*b^3*d^5*f^3*g + 10*B*a^3*b^2*d^5*f^2*g^2 - 5*B*a^4*b*d^5*f
*g^3 + B*a^5*d^5*g^4)*n*log(b*x + a) - 12*(5*B*b^5*c*d^4*f^4 - 10*B*b^5*c^2
*d^3*f^3*g + 10*B*b^5*c^3*d^2*f^2*g^2 - 5*B*b^5*c^4*d*f*g^3 + B*b^5*c^5*g^4
)*n*log(d*x + c) + 12*(5*A*b^5*d^5*f^4 - (10*(B*b^5*c*d^4 - B*a*b^4*d^5)*f^
3*g - 10*(B*b^5*c^2*d^3 - B*a^2*b^3*d^5)*f^2*g^2 + 5*(B*b^5*c^3*d^2 - B*a^3
*b^2*d^5)*f*g^3 - (B*b^5*c^4*d - B*a^4*b*d^5)*g^4)*n)*x + 12*(B*b^5*d^5*g^4
*x^5 + 5*B*b^5*d^5*f*g^3*x^4 + 10*B*b^5*d^5*f^2*g^2*x^3 + 10*B*b^5*d^5*f^3*
g*x^2 + 5*B*b^5*d^5*f^4*x)*log(e) + 12*(B*b^5*d^5*g^4*n*x^5 + 5*B*b^5*d^5*f
*g^3*n*x^4 + 10*B*b^5*d^5*f^2*g^2*n*x^3 + 10*B*b^5*d^5*f^3*g*n*x^2 + 5*B*b^
5*d^5*f^4*n*x)*log((b*x + a)/(d*x + c)))/(b^5*d^5)
```

**giac [B]** time = 15.16, size = 11806, normalized size = 32.43

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^4*(A+B*log(e*((b*x+a)/(d*x+c))^n)),x, algorithm="giac")
```

```
[Out] 1/60*(12*(5*B*b^6*c^2*d^4*f^4*n - 10*B*a*b^5*c*d^5*f^4*n - 20*(b*x + a)*B*b
^5*c^2*d^5*f^4*n/(d*x + c) + 5*B*a^2*b^4*d^6*f^4*n + 40*(b*x + a)*B*a*b^4*c
*d^6*f^4*n/(d*x + c) + 30*(b*x + a)^2*B*b^4*c^2*d^6*f^4*n/(d*x + c)^2 - 20*
(b*x + a)*B*a^2*b^3*d^7*f^4*n/(d*x + c) - 60*(b*x + a)^2*B*a*b^3*c*d^7*f^4*
n/(d*x + c)^2 - 20*(b*x + a)^3*B*b^3*c^2*d^7*f^4*n/(d*x + c)^3 + 30*(b*x +
a)^2*B*a^2*b^2*d^8*f^4*n/(d*x + c)^2 + 40*(b*x + a)^3*B*a*b^2*c*d^8*f^4*n/(
d*x + c)^3 + 5*(b*x + a)^4*B*b^2*c^2*d^8*f^4*n/(d*x + c)^4 - 20*(b*x + a)^3
```

*B*a^2*b*d^9*f^4*n/(d*x + c)^3 - 10*(b*x + a)^4*B*a*b*c*d^9*f^4*n/(d*x + c)
^4 + 5*(b*x + a)^4*B*a^2*d^10*f^4*n/(d*x + c)^4 - 10*B*b^6*c^3*d^3*f^3*g*n
+ 10*B*a*b^5*c^2*d^4*f^3*g*n + 50*(b*x + a)*B*b^5*c^3*d^4*f^3*g*n/(d*x + c)
 + 10*B*a^2*b^4*c*d^5*f^3*g*n - 70*(b*x + a)*B*a*b^4*c^2*d^5*f^3*g*n/(d*x +
 c) - 90*(b*x + a)^2*B*b^4*c^3*d^5*f^3*g*n/(d*x + c)^2 - 10*B*a^3*b^3*d^6*f
^3*g*n - 10*(b*x + a)*B*a^2*b^3*c*d^6*f^3*g*n/(d*x + c) + 150*(b*x + a)^2*B
*a*b^3*c^2*d^6*f^3*g*n/(d*x + c)^2 + 70*(b*x + a)^3*B*b^3*c^3*d^6*f^3*g*n/(
d*x + c)^3 + 30*(b*x + a)*B*a^3*b^2*d^7*f^3*g*n/(d*x + c) - 30*(b*x + a)^2*
B*a^2*b^2*c*d^7*f^3*g*n/(d*x + c)^2 - 130*(b*x + a)^3*B*a*b^2*c^2*d^7*f^3*g
*n/(d*x + c)^3 - 20*(b*x + a)^4*B*b^2*c^3*d^7*f^3*g*n/(d*x + c)^4 - 30*(b*x
 + a)^2*B*a^3*b*d^8*f^3*g*n/(d*x + c)^2 + 50*(b*x + a)^3*B*a^2*b*c*d^8*f^3*
g*n/(d*x + c)^3 + 40*(b*x + a)^4*B*a*b*c^2*d^8*f^3*g*n/(d*x + c)^4 + 10*(b*
x + a)^3*B*a^3*d^9*f^3*g*n/(d*x + c)^3 - 20*(b*x + a)^4*B*a^2*c*d^9*f^3*g*n
/(d*x + c)^4 + 10*B*b^6*c^4*d^2*f^2*g^2*n - 10*B*a*b^5*c^3*d^3*f^2*g^2*n -
50*(b*x + a)*B*b^5*c^4*d^3*f^2*g^2*n/(d*x + c) + 50*(b*x + a)*B*a*b^4*c^3*d
^4*f^2*g^2*n/(d*x + c) + 100*(b*x + a)^2*B*b^4*c^4*d^4*f^2*g^2*n/(d*x + c)^
2 - 10*B*a^3*b^3*c*d^5*f^2*g^2*n + 30*(b*x + a)*B*a^2*b^3*c^2*d^5*f^2*g^2*n
/(d*x + c) - 130*(b*x + a)^2*B*a*b^3*c^3*d^5*f^2*g^2*n/(d*x + c)^2 - 90*(b*
x + a)^3*B*b^3*c^4*d^5*f^2*g^2*n/(d*x + c)^3 + 10*B*a^4*b^2*d^6*f^2*g^2*n -
 10*(b*x + a)*B*a^3*b^2*c*d^6*f^2*g^2*n/(d*x + c) - 30*(b*x + a)^2*B*a^2*b^
2*c^2*d^6*f^2*g^2*n/(d*x + c)^2 + 150*(b*x + a)^3*B*a*b^2*c^3*d^6*f^2*g^2*n
/(d*x + c)^3 + 30*(b*x + a)^4*B*b^2*c^4*d^6*f^2*g^2*n/(d*x + c)^4 - 20*(b*x
 + a)*B*a^4*b*d^7*f^2*g^2*n/(d*x + c) + 50*(b*x + a)^2*B*a^3*b*c*d^7*f^2*g^
2*n/(d*x + c)^2 - 30*(b*x + a)^3*B*a^2*b*c^2*d^7*f^2*g^2*n/(d*x + c)^3 - 60
*(b*x + a)^4*B*a*b*c^3*d^7*f^2*g^2*n/(d*x + c)^4 + 10*(b*x + a)^2*B*a^4*d^8
*f^2*g^2*n/(d*x + c)^2 - 30*(b*x + a)^3*B*a^3*c*d^8*f^2*g^2*n/(d*x + c)^3 +
 30*(b*x + a)^4*B*a^2*c^2*d^8*f^2*g^2*n/(d*x + c)^4 - 5*B*b^6*c^5*d*f*g^3*n
 + 5*B*a*b^5*c^4*d^2*f*g^3*n + 25*(b*x + a)*B*b^5*c^5*d^2*f*g^3*n/(d*x + c)
 - 25*(b*x + a)*B*a*b^4*c^4*d^3*f*g^3*n/(d*x + c) - 50*(b*x + a)^2*B*b^4*c^
5*d^3*f*g^3*n/(d*x + c)^2 + 50*(b*x + a)^2*B*a*b^3*c^4*d^4*f*g^3*n/(d*x + c
)^2 + 50*(b*x + a)^3*B*b^3*c^5*d^4*f*g^3*n/(d*x + c)^3 + 5*B*a^4*b^2*c*d^5*
f*g^3*n - 20*(b*x + a)*B*a^3*b^2*c^2*d^5*f*g^3*n/(d*x + c) + 30*(b*x + a)^2
*B*a^2*b^2*c^3*d^5*f*g^3*n/(d*x + c)^2 - 70*(b*x + a)^3*B*a*b^2*c^4*d^5*f*g
^3*n/(d*x + c)^3 - 20*(b*x + a)^4*B*b^2*c^5*d^5*f*g^3*n/(d*x + c)^4 - 5*B*a
^5*b*d^6*f*g^3*n + 15*(b*x + a)*B*a^4*b*c*d^6*f*g^3*n/(d*x + c) - 10*(b*x +
 a)^2*B*a^3*b*c^2*d^6*f*g^3*n/(d*x + c)^2 - 10*(b*x + a)^3*B*a^2*b*c^3*d^6*
f*g^3*n/(d*x + c)^3 + 40*(b*x + a)^4*B*a*b*c^4*d^6*f*g^3*n/(d*x + c)^4 + 5*
(b*x + a)*B*a^5*d^7*f*g^3*n/(d*x + c) - 20*(b*x + a)^2*B*a^4*c*d^7*f*g^3*n/
(d*x + c)^2 + 30*(b*x + a)^3*B*a^3*c^2*d^7*f*g^3*n/(d*x + c)^3 - 20*(b*x +
a)^4*B*a^2*c^3*d^7*f*g^3*n/(d*x + c)^4 + B*b^6*c^6*g^4*n - B*a*b^5*c^5*d*g^
4*n - 5*(b*x + a)*B*b^5*c^6*d*g^4*n/(d*x + c) + 5*(b*x + a)*B*a*b^4*c^5*d^2
*g^4*n/(d*x + c) + 10*(b*x + a)^2*B*b^4*c^6*d^2*g^4*n/(d*x + c)^2 - 10*(b*x
 + a)^2*B*a*b^3*c^5*d^3*g^4*n/(d*x + c)^2 - 10*(b*x + a)^3*B*b^3*c^6*d^3*g^
4*n/(d*x + c)^3 + 10*(b*x + a)^3*B*a*b^2*c^5*d^4*g^4*n/(d*x + c)^3 + 5*(b*x
 + a)^4*B*b^2*c^6*d^4*g^4*n/(d*x + c)^4 - B*a^5*b*c*d^5*g^4*n + 5*(b*x + a)

*B*a^4*b*c^2*d^5*g^4*n/(d*x + c) - 10*(b*x + a)^2*B*a^3*b*c^3*d^5*g^4*n/(d*
x + c)^2 + 10*(b*x + a)^3*B*a^2*b*c^4*d^5*g^4*n/(d*x + c)^3 - 10*(b*x + a)^
4*B*a*b*c^5*d^5*g^4*n/(d*x + c)^4 + B*a^6*d^6*g^4*n - 5*(b*x + a)*B*a^5*c*d
^6*g^4*n/(d*x + c) + 10*(b*x + a)^2*B*a^4*c^2*d^6*g^4*n/(d*x + c)^2 - 10*(b
*x + a)^3*B*a^3*c^3*d^6*g^4*n/(d*x + c)^3 + 5*(b*x + a)^4*B*a^2*c^4*d^6*g^4
*n/(d*x + c)^4)*log((b*x + a)/(d*x + c))/(b^5*d^5 - 5*(b*x + a)*b^4*d^6/(d*
x + c) + 10*(b*x + a)^2*b^3*d^7/(d*x + c)^2 - 10*(b*x + a)^3*b^2*d^8/(d*x +
 c)^3 + 5*(b*x + a)^4*b*d^9/(d*x + c)^4 - (b*x + a)^5*d^10/(d*x + c)^5) - (
120*B*b^10*c^3*d^3*f^3*g*n - 360*B*a*b^9*c^2*d^4*f^3*g*n - 480*(b*x + a)*B*
b^9*c^3*d^4*f^3*g*n/(d*x + c) + 360*B*a^2*b^8*c*d^5*f^3*g*n + 1440*(b*x + a
)*B*a*b^8*c^2*d^5*f^3*g*n/(d*x + c) + 720*(b*x + a)^2*B*b^8*c^3*d^5*f^3*g*n
/(d*x + c)^2 - 120*B*a^3*b^7*d^6*f^3*g*n - 1440*(b*x + a)*B*a^2*b^7*c*d^6*f
^3*g*n/(d*x + c) - 2160*(b*x + a)^2*B*a*b^7*c^2*d^6*f^3*g*n/(d*x + c)^2 - 4
80*(b*x + a)^3*B*b^7*c^3*d^6*f^3*g*n/(d*x + c)^3 + 480*(b*x + a)*B*a^3*b^6*
d^7*f^3*g*n/(d*x + c) + 2160*(b*x + a)^2*B*a^2*b^6*c*d^7*f^3*g*n/(d*x + c)^
2 + 1440*(b*x + a)^3*B*a*b^6*c^2*d^7*f^3*g*n/(d*x + c)^3 + 120*(b*x + a)^4*
B*b^6*c^3*d^7*f^3*g*n/(d*x + c)^4 - 720*(b*x + a)^2*B*a^3*b^5*d^8*f^3*g*n/(
d*x + c)^2 - 1440*(b*x + a)^3*B*a^2*b^5*c*d^8*f^3*g*n/(d*x + c)^3 - 360*(b*
x + a)^4*B*a*b^5*c^2*d^8*f^3*g*n/(d*x + c)^4 + 480*(b*x + a)^3*B*a^3*b^4*d^
9*f^3*g*n/(d*x + c)^3 + 360*(b*x + a)^4*B*a^2*b^4*c*d^9*f^3*g*n/(d*x + c)^4
 - 120*(b*x + a)^4*B*a^3*b^3*d^10*f^3*g*n/(d*x + c)^4 - 180*B*b^10*c^4*d^2*
f^2*g^2*n + 360*B*a*b^9*c^3*d^3*f^2*g^2*n + 780*(b*x + a)*B*b^9*c^4*d^3*f^2
*g^2*n/(d*x + c) - 1680*(b*x + a)*B*a*b^8*c^3*d^4*f^2*g^2*n/(d*x + c) - 126
0*(b*x + a)^2*B*b^8*c^4*d^4*f^2*g^2*n/(d*x + c)^2 - 360*B*a^3*b^7*c*d^5*f^2
*g^2*n + 360*(b*x + a)*B*a^2*b^7*c^2*d^5*f^2*g^2*n/(d*x + c) + 2880*(b*x +
a)^2*B*a*b^7*c^3*d^5*f^2*g^2*n/(d*x + c)^2 + 900*(b*x + a)^3*B*b^7*c^4*d^5*
f^2*g^2*n/(d*x + c)^3 + 180*B*a^4*b^6*d^6*f^2*g^2*n + 1200*(b*x + a)*B*a^3*
b^6*c*d^6*f^2*g^2*n/(d*x + c) - 1080*(b*x + a)^2*B*a^2*b^6*c^2*d^6*f^2*g^2*
n/(d*x + c)^2 - 2160*(b*x + a)^3*B*a*b^6*c^3*d^6*f^2*g^2*n/(d*x + c)^3 - 24
0*(b*x + a)^4*B*b^6*c^4*d^6*f^2*g^2*n/(d*x + c)^4 - 660*(b*x + a)*B*a^4*b^5
*d^7*f^2*g^2*n/(d*x + c) - 1440*(b*x + a)^2*B*a^3*b^5*c*d^7*f^2*g^2*n/(d*x
+ c)^2 + 1080*(b*x + a)^3*B*a^2*b^5*c^2*d^7*f^2*g^2*n/(d*x + c)^3 + 600*(b*
x + a)^4*B*a*b^5*c^3*d^7*f^2*g^2*n/(d*x + c)^4 + 900*(b*x + a)^2*B*a^4*b^4*
d^8*f^2*g^2*n/(d*x + c)^2 + 720*(b*x + a)^3*B*a^3*b^4*c*d^8*f^2*g^2*n/(d*x
+ c)^3 - 360*(b*x + a)^4*B*a^2*b^4*c^2*d^8*f^2*g^2*n/(d*x + c)^4 - 540*(b*x
 + a)^3*B*a^4*b^3*d^9*f^2*g^2*n/(d*x + c)^3 - 120*(b*x + a)^4*B*a^3*b^3*c*d
^9*f^2*g^2*n/(d*x + c)^4 + 120*(b*x + a)^4*B*a^4*b^2*d^10*f^2*g^2*n/(d*x +
c)^4 + 110*B*b^10*c^5*d*f*g^3*n - 190*B*a*b^9*c^4*d^2*f*g^3*n - 490*(b*x +
a)*B*b^9*c^5*d^2*f*g^3*n/(d*x + c) + 20*B*a^2*b^8*c^3*d^3*f*g^3*n + 890*(b*
x + a)*B*a*b^8*c^4*d^3*f*g^3*n/(d*x + c) + 830*(b*x + a)^2*B*b^8*c^5*d^3*f*
g^3*n/(d*x + c)^2 - 20*B*a^3*b^7*c^2*d^4*f*g^3*n - 100*(b*x + a)*B*a^2*b^7*
c^3*d^4*f*g^3*n/(d*x + c) - 1630*(b*x + a)^2*B*a*b^7*c^4*d^4*f*g^3*n/(d*x +
 c)^2 - 630*(b*x + a)^3*B*b^7*c^5*d^4*f*g^3*n/(d*x + c)^3 + 190*B*a^4*b^6*c
*d^5*f*g^3*n - 140*(b*x + a)*B*a^3*b^6*c^2*d^5*f*g^3*n/(d*x + c) + 380*(b*x
 + a)^2*B*a^2*b^6*c^3*d^5*f*g^3*n/(d*x + c)^2 + 1350*(b*x + a)^3*B*a*b^6*c^

4*d^5*f*g^3*n/(d*x + c)^3 + 180*(b*x + a)^4*B*b^6*c^5*d^5*f*g^3*n/(d*x + c)
^4 - 110*B*a^5*b^5*d^6*f*g^3*n - 530*(b*x + a)*B*a^4*b^5*c*d^6*f*g^3*n/(d*x
 + c) + 340*(b*x + a)^2*B*a^3*b^5*c^2*d^6*f*g^3*n/(d*x + c)^2 - 540*(b*x +
a)^3*B*a^2*b^5*c^3*d^6*f*g^3*n/(d*x + c)^3 - 420*(b*x + a)^4*B*a*b^5*c^4*d^
6*f*g^3*n/(d*x + c)^4 + 370*(b*x + a)*B*a^5*b^4*d^7*f*g^3*n/(d*x + c) + 550
*(b*x + a)^2*B*a^4*b^4*c*d^7*f*g^3*n/(d*x + c)^2 - 180*(b*x + a)^3*B*a^3*b^
4*c^2*d^7*f*g^3*n/(d*x + c)^3 + 240*(b*x + a)^4*B*a^2*b^4*c^3*d^7*f*g^3*n/(
d*x + c)^4 - 470*(b*x + a)^2*B*a^5*b^3*d^8*f*g^3*n/(d*x + c)^2 - 270*(b*x +
 a)^3*B*a^4*b^3*c*d^8*f*g^3*n/(d*x + c)^3 + 270*(b*x + a)^3*B*a^5*b^2*d^9*f
*g^3*n/(d*x + c)^3 + 60*(b*x + a)^4*B*a^4*b^2*c*d^9*f*g^3*n/(d*x + c)^4 - 6
0*(b*x + a)^4*B*a^5*b*d^10*f*g^3*n/(d*x + c)^4 - 25*B*b^10*c^6*g^4*n + 40*B
*a*b^9*c^5*d*g^4*n + 113*(b*x + a)*B*b^9*c^6*d*g^4*n/(d*x + c) - 5*B*a^2*b^
8*c^4*d^2*g^4*n - 188*(b*x + a)*B*a*b^8*c^5*d^2*g^4*n/(d*x + c) - 196*(b*x
+ a)^2*B*b^8*c^6*d^2*g^4*n/(d*x + c)^2 + 25*(b*x + a)*B*a^2*b^7*c^4*d^3*g^4
*n/(d*x + c) + 346*(b*x + a)^2*B*a*b^7*c^5*d^3*g^4*n/(d*x + c)^2 + 156*(b*x
 + a)^3*B*b^7*c^6*d^3*g^4*n/(d*x + c)^3 + 5*B*a^4*b^6*c^2*d^4*g^4*n - 50*(b
*x + a)^2*B*a^2*b^6*c^4*d^4*g^4*n/(d*x + c)^2 - 306*(b*x + a)^3*B*a*b^6*c^5
*d^4*g^4*n/(d*x + c)^3 - 48*(b*x + a)^4*B*b^6*c^6*d^4*g^4*n/(d*x + c)^4 - 4
0*B*a^5*b^5*c*d^5*g^4*n + 35*(b*x + a)*B*a^4*b^5*c^2*d^5*g^4*n/(d*x + c) -
60*(b*x + a)^2*B*a^3*b^5*c^3*d^5*g^4*n/(d*x + c)^2 + 90*(b*x + a)^3*B*a^2*b
^5*c^4*d^5*g^4*n/(d*x + c)^3 + 108*(b*x + a)^4*B*a*b^5*c^5*d^5*g^4*n/(d*x +
 c)^4 + 25*B*a^6*b^4*d^6*g^4*n + 92*(b*x + a)*B*a^5*b^4*c*d^6*g^4*n/(d*x +
c) - 40*(b*x + a)^2*B*a^4*b^4*c^2*d^6*g^4*n/(d*x + c)^2 + 60*(b*x + a)^3*B*
a^3*b^4*c^3*d^6*g^4*n/(d*x + c)^3 - 60*(b*x + a)^4*B*a^2*b^4*c^4*d^6*g^4*n/
(d*x + c)^4 - 77*(b*x + a)*B*a^6*b^3*d^7*g^4*n/(d*x + c) - 94*(b*x + a)^2*B
*a^5*b^3*c*d^7*g^4*n/(d*x + c)^2 + 94*(b*x + a)^2*B*a^6*b^2*d^8*g^4*n/(d*x
+ c)^2 + 54*(b*x + a)^3*B*a^5*b^2*c*d^8*g^4*n/(d*x + c)^3 - 54*(b*x + a)^3*
B*a^6*b*d^9*g^4*n/(d*x + c)^3 - 12*(b*x + a)^4*B*a^5*b*c*d^9*g^4*n/(d*x + c
)^4 + 12*(b*x + a)^4*B*a^6*d^10*g^4*n/(d*x + c)^4 - 60*A*b^10*c^2*d^4*f^4 -
 60*B*b^10*c^2*d^4*f^4 + 120*A*a*b^9*c*d^5*f^4 + 120*B*a*b^9*c*d^5*f^4 + 24
0*(b*x + a)*A*b^9*c^2*d^5*f^4/(d*x + c) + 240*(b*x + a)*B*b^9*c^2*d^5*f^4/(
d*x + c) - 60*A*a^2*b^8*d^6*f^4 - 60*B*a^2*b^8*d^6*f^4 - 480*(b*x + a)*A*a*
b^8*c*d^6*f^4/(d*x + c) - 480*(b*x + a)*B*a*b^8*c*d^6*f^4/(d*x + c) - 360*(
b*x + a)^2*A*b^8*c^2*d^6*f^4/(d*x + c)^2 - 360*(b*x + a)^2*B*b^8*c^2*d^6*f^
4/(d*x + c)^2 + 240*(b*x + a)*A*a^2*b^7*d^7*f^4/(d*x + c) + 240*(b*x + a)*B
*a^2*b^7*d^7*f^4/(d*x + c) + 720*(b*x + a)^2*A*a*b^7*c*d^7*f^4/(d*x + c)^2
+ 720*(b*x + a)^2*B*a*b^7*c*d^7*f^4/(d*x + c)^2 + 240*(b*x + a)^3*A*b^7*c^2
*d^7*f^4/(d*x + c)^3 + 240*(b*x + a)^3*B*b^7*c^2*d^7*f^4/(d*x + c)^3 - 360*
(b*x + a)^2*A*a^2*b^6*d^8*f^4/(d*x + c)^2 - 360*(b*x + a)^2*B*a^2*b^6*d^8*f
^4/(d*x + c)^2 - 480*(b*x + a)^3*A*a*b^6*c*d^8*f^4/(d*x + c)^3 - 480*(b*x +
 a)^3*B*a*b^6*c*d^8*f^4/(d*x + c)^3 - 60*(b*x + a)^4*A*b^6*c^2*d^8*f^4/(d*x
 + c)^4 - 60*(b*x + a)^4*B*b^6*c^2*d^8*f^4/(d*x + c)^4 + 240*(b*x + a)^3*A*
a^2*b^5*d^9*f^4/(d*x + c)^3 + 240*(b*x + a)^3*B*a^2*b^5*d^9*f^4/(d*x + c)^3
 + 120*(b*x + a)^4*A*a*b^5*c*d^9*f^4/(d*x + c)^4 + 120*(b*x + a)^4*B*a*b^5*
c*d^9*f^4/(d*x + c)^4 - 60*(b*x + a)^4*A*a^2*b^4*d^10*f^4/(d*x + c)^4 - 60*

(b*x + a)^4*B*a^2*b^4*d^10*f^4/(d*x + c)^4 + 120*A*b^10*c^3*d^3*f^3*g + 120
*B*b^10*c^3*d^3*f^3*g - 120*A*a*b^9*c^2*d^4*f^3*g - 120*B*a*b^9*c^2*d^4*f^3
*g - 600*(b*x + a)*A*b^9*c^3*d^4*f^3*g/(d*x + c) - 600*(b*x + a)*B*b^9*c^3*
d^4*f^3*g/(d*x + c) - 120*A*a^2*b^8*c*d^5*f^3*g - 120*B*a^2*b^8*c*d^5*f^3*g
 + 840*(b*x + a)*A*a*b^8*c^2*d^5*f^3*g/(d*x + c) + 840*(b*x + a)*B*a*b^8*c^
2*d^5*f^3*g/(d*x + c) + 1080*(b*x + a)^2*A*b^8*c^3*d^5*f^3*g/(d*x + c)^2 +
1080*(b*x + a)^2*B*b^8*c^3*d^5*f^3*g/(d*x + c)^2 + 120*A*a^3*b^7*d^6*f^3*g
+ 120*B*a^3*b^7*d^6*f^3*g + 120*(b*x + a)*A*a^2*b^7*c*d^6*f^3*g/(d*x + c) +
 120*(b*x + a)*B*a^2*b^7*c*d^6*f^3*g/(d*x + c) - 1800*(b*x + a)^2*A*a*b^7*c
^2*d^6*f^3*g/(d*x + c)^2 - 1800*(b*x + a)^2*B*a*b^7*c^2*d^6*f^3*g/(d*x + c)
^2 - 840*(b*x + a)^3*A*b^7*c^3*d^6*f^3*g/(d*x + c)^3 - 840*(b*x + a)^3*B*b^
7*c^3*d^6*f^3*g/(d*x + c)^3 - 360*(b*x + a)*A*a^3*b^6*d^7*f^3*g/(d*x + c) -
 360*(b*x + a)*B*a^3*b^6*d^7*f^3*g/(d*x + c) + 360*(b*x + a)^2*A*a^2*b^6*c*
d^7*f^3*g/(d*x + c)^2 + 360*(b*x + a)^2*B*a^2*b^6*c*d^7*f^3*g/(d*x + c)^2 +
 1560*(b*x + a)^3*A*a*b^6*c^2*d^7*f^3*g/(d*x + c)^3 + 1560*(b*x + a)^3*B*a*
b^6*c^2*d^7*f^3*g/(d*x + c)^3 + 240*(b*x + a)^4*A*b^6*c^3*d^7*f^3*g/(d*x +
c)^4 + 240*(b*x + a)^4*B*b^6*c^3*d^7*f^3*g/(d*x + c)^4 + 360*(b*x + a)^2*A*
a^3*b^5*d^8*f^3*g/(d*x + c)^2 + 360*(b*x + a)^2*B*a^3*b^5*d^8*f^3*g/(d*x +
c)^2 - 600*(b*x + a)^3*A*a^2*b^5*c*d^8*f^3*g/(d*x + c)^3 - 600*(b*x + a)^3*
B*a^2*b^5*c*d^8*f^3*g/(d*x + c)^3 - 480*(b*x + a)^4*A*a*b^5*c^2*d^8*f^3*g/(
d*x + c)^4 - 480*(b*x + a)^4*B*a*b^5*c^2*d^8*f^3*g/(d*x + c)^4 - 120*(b*x +
 a)^3*A*a^3*b^4*d^9*f^3*g/(d*x + c)^3 - 120*(b*x + a)^3*B*a^3*b^4*d^9*f^3*g
/(d*x + c)^3 + 240*(b*x + a)^4*A*a^2*b^4*c*d^9*f^3*g/(d*x + c)^4 + 240*(b*x
 + a)^4*B*a^2*b^4*c*d^9*f^3*g/(d*x + c)^4 - 120*A*b^10*c^4*d^2*f^2*g^2 - 12
0*B*b^10*c^4*d^2*f^2*g^2 + 120*A*a*b^9*c^3*d^3*f^2*g^2 + 120*B*a*b^9*c^3*d^
3*f^2*g^2 + 600*(b*x + a)*A*b^9*c^4*d^3*f^2*g^2/(d*x + c) + 600*(b*x + a)*B
*b^9*c^4*d^3*f^2*g^2/(d*x + c) - 600*(b*x + a)*A*a*b^8*c^3*d^4*f^2*g^2/(d*x
 + c) - 600*(b*x + a)*B*a*b^8*c^3*d^4*f^2*g^2/(d*x + c) - 1200*(b*x + a)^2*
A*b^8*c^4*d^4*f^2*g^2/(d*x + c)^2 - 1200*(b*x + a)^2*B*b^8*c^4*d^4*f^2*g^2/
(d*x + c)^2 + 120*A*a^3*b^7*c*d^5*f^2*g^2 + 120*B*a^3*b^7*c*d^5*f^2*g^2 - 3
60*(b*x + a)*A*a^2*b^7*c^2*d^5*f^2*g^2/(d*x + c) - 360*(b*x + a)*B*a^2*b^7*
c^2*d^5*f^2*g^2/(d*x + c) + 1560*(b*x + a)^2*A*a*b^7*c^3*d^5*f^2*g^2/(d*x +
 c)^2 + 1560*(b*x + a)^2*B*a*b^7*c^3*d^5*f^2*g^2/(d*x + c)^2 + 1080*(b*x +
a)^3*A*b^7*c^4*d^5*f^2*g^2/(d*x + c)^3 + 1080*(b*x + a)^3*B*b^7*c^4*d^5*f^2
*g^2/(d*x + c)^3 - 120*A*a^4*b^6*d^6*f^2*g^2 - 120*B*a^4*b^6*d^6*f^2*g^2 +
120*(b*x + a)*A*a^3*b^6*c*d^6*f^2*g^2/(d*x + c) + 120*(b*x + a)*B*a^3*b^6*c
*d^6*f^2*g^2/(d*x + c) + 360*(b*x + a)^2*A*a^2*b^6*c^2*d^6*f^2*g^2/(d*x + c
)^2 + 360*(b*x + a)^2*B*a^2*b^6*c^2*d^6*f^2*g^2/(d*x + c)^2 - 1800*(b*x + a
)^3*A*a*b^6*c^3*d^6*f^2*g^2/(d*x + c)^3 - 1800*(b*x + a)^3*B*a*b^6*c^3*d^6*
f^2*g^2/(d*x + c)^3 - 360*(b*x + a)^4*A*b^6*c^4*d^6*f^2*g^2/(d*x + c)^4 - 3
60*(b*x + a)^4*B*b^6*c^4*d^6*f^2*g^2/(d*x + c)^4 + 240*(b*x + a)*A*a^4*b^5*
d^7*f^2*g^2/(d*x + c) + 240*(b*x + a)*B*a^4*b^5*d^7*f^2*g^2/(d*x + c) - 600
*(b*x + a)^2*A*a^3*b^5*c*d^7*f^2*g^2/(d*x + c)^2 - 600*(b*x + a)^2*B*a^3*b^
5*c*d^7*f^2*g^2/(d*x + c)^2 + 360*(b*x + a)^3*A*a^2*b^5*c^2*d^7*f^2*g^2/(d*
x + c)^3 + 360*(b*x + a)^3*B*a^2*b^5*c^2*d^7*f^2*g^2/(d*x + c)^3 + 720*(b*x

+ a)^4*A*a*b^5*c^3*d^7*f^2*g^2/(d*x + c)^4 + 720*(b*x + a)^4*B*a*b^5*c^3*d
^7*f^2*g^2/(d*x + c)^4 - 120*(b*x + a)^2*A*a^4*b^4*d^8*f^2*g^2/(d*x + c)^2
- 120*(b*x + a)^2*B*a^4*b^4*d^8*f^2*g^2/(d*x + c)^2 + 360*(b*x + a)^3*A*a^3
*b^4*c*d^8*f^2*g^2/(d*x + c)^3 + 360*(b*x + a)^3*B*a^3*b^4*c*d^8*f^2*g^2/(d
*x + c)^3 - 360*(b*x + a)^4*A*a^2*b^4*c^2*d^8*f^2*g^2/(d*x + c)^4 - 360*(b*
x + a)^4*B*a^2*b^4*c^2*d^8*f^2*g^2/(d*x + c)^4 + 60*A*b^10*c^5*d*f*g^3 + 60
*B*b^10*c^5*d*f*g^3 - 60*A*a*b^9*c^4*d^2*f*g^3 - 60*B*a*b^9*c^4*d^2*f*g^3 -
300*(b*x + a)*A*b^9*c^5*d^2*f*g^3/(d*x + c) - 300*(b*x + a)*B*b^9*c^5*d^2*
f*g^3/(d*x + c) + 300*(b*x + a)*A*a*b^8*c^4*d^3*f*g^3/(d*x + c) + 300*(b*x
+ a)*B*a*b^8*c^4*d^3*f*g^3/(d*x + c) + 600*(b*x + a)^2*A*b^8*c^5*d^3*f*g^3/
(d*x + c)^2 + 600*(b*x + a)^2*B*b^8*c^5*d^3*f*g^3/(d*x + c)^2 - 600*(b*x +
a)^2*A*a*b^7*c^4*d^4*f*g^3/(d*x + c)^2 - 600*(b*x + a)^2*B*a*b^7*c^4*d^4*f*
g^3/(d*x + c)^2 - 600*(b*x + a)^3*A*b^7*c^5*d^4*f*g^3/(d*x + c)^3 - 600*(b*
x + a)^3*B*b^7*c^5*d^4*f*g^3/(d*x + c)^3 - 60*A*a^4*b^6*c*d^5*f*g^3 - 60*B*
a^4*b^6*c*d^5*f*g^3 + 240*(b*x + a)*A*a^3*b^6*c^2*d^5*f*g^3/(d*x + c) + 240
*(b*x + a)*B*a^3*b^6*c^2*d^5*f*g^3/(d*x + c) - 360*(b*x + a)^2*A*a^2*b^6*c^
3*d^5*f*g^3/(d*x + c)^2 - 360*(b*x + a)^2*B*a^2*b^6*c^3*d^5*f*g^3/(d*x + c)
^2 + 840*(b*x + a)^3*A*a*b^6*c^4*d^5*f*g^3/(d*x + c)^3 + 840*(b*x + a)^3*B*
a*b^6*c^4*d^5*f*g^3/(d*x + c)^3 + 240*(b*x + a)^4*A*b^6*c^5*d^5*f*g^3/(d*x
+ c)^4 + 240*(b*x + a)^4*B*b^6*c^5*d^5*f*g^3/(d*x + c)^4 + 60*A*a^5*b^5*d^6
*f*g^3 + 60*B*a^5*b^5*d^6*f*g^3 - 180*(b*x + a)*A*a^4*b^5*c*d^6*f*g^3/(d*x
+ c) - 180*(b*x + a)*B*a^4*b^5*c*d^6*f*g^3/(d*x + c) + 120*(b*x + a)^2*A*a^
3*b^5*c^2*d^6*f*g^3/(d*x + c)^2 + 120*(b*x + a)^2*B*a^3*b^5*c^2*d^6*f*g^3/(
d*x + c)^2 + 120*(b*x + a)^3*A*a^2*b^5*c^3*d^6*f*g^3/(d*x + c)^3 + 120*(b*x
+ a)^3*B*a^2*b^5*c^3*d^6*f*g^3/(d*x + c)^3 - 480*(b*x + a)^4*A*a*b^5*c^4*d
^6*f*g^3/(d*x + c)^4 - 480*(b*x + a)^4*B*a*b^5*c^4*d^6*f*g^3/(d*x + c)^4 -
60*(b*x + a)*A*a^5*b^4*d^7*f*g^3/(d*x + c) - 60*(b*x + a)*B*a^5*b^4*d^7*f*g
^3/(d*x + c) + 240*(b*x + a)^2*A*a^4*b^4*c*d^7*f*g^3/(d*x + c)^2 + 240*(b*x
+ a)^2*B*a^4*b^4*c*d^7*f*g^3/(d*x + c)^2 - 360*(b*x + a)^3*A*a^3*b^4*c^2*d
^7*f*g^3/(d*x + c)^3 - 360*(b*x + a)^3*B*a^3*b^4*c^2*d^7*f*g^3/(d*x + c)^3
+ 240*(b*x + a)^4*A*a^2*b^4*c^3*d^7*f*g^3/(d*x + c)^4 + 240*(b*x + a)^4*B*a
^2*b^4*c^3*d^7*f*g^3/(d*x + c)^4 - 12*A*b^10*c^6*g^4 - 12*B*b^10*c^6*g^4 +
12*A*a*b^9*c^5*d*g^4 + 12*B*a*b^9*c^5*d*g^4 + 60*(b*x + a)*A*b^9*c^6*d*g^4/
(d*x + c) + 60*(b*x + a)*B*b^9*c^6*d*g^4/(d*x + c) - 60*(b*x + a)*A*a*b^8*c
^5*d^2*g^4/(d*x + c) - 60*(b*x + a)*B*a*b^8*c^5*d^2*g^4/(d*x + c) - 120*(b*
x + a)^2*A*b^8*c^6*d^2*g^4/(d*x + c)^2 - 120*(b*x + a)^2*B*b^8*c^6*d^2*g^4/
(d*x + c)^2 + 120*(b*x + a)^2*A*a*b^7*c^5*d^3*g^4/(d*x + c)^2 + 120*(b*x +
a)^2*B*a*b^7*c^5*d^3*g^4/(d*x + c)^2 + 120*(b*x + a)^3*A*b^7*c^6*d^3*g^4/(d
*x + c)^3 + 120*(b*x + a)^3*B*b^7*c^6*d^3*g^4/(d*x + c)^3 - 120*(b*x + a)^3
*A*a*b^6*c^5*d^4*g^4/(d*x + c)^3 - 120*(b*x + a)^3*B*a*b^6*c^5*d^4*g^4/(d*x
+ c)^3 - 60*(b*x + a)^4*A*b^6*c^6*d^4*g^4/(d*x + c)^4 - 60*(b*x + a)^4*B*b
^6*c^6*d^4*g^4/(d*x + c)^4 + 12*A*a^5*b^5*c*d^5*g^4 + 12*B*a^5*b^5*c*d^5*g^
4 - 60*(b*x + a)*A*a^4*b^5*c^2*d^5*g^4/(d*x + c) - 60*(b*x + a)*B*a^4*b^5*c
^2*d^5*g^4/(d*x + c) + 120*(b*x + a)^2*A*a^3*b^5*c^3*d^5*g^4/(d*x + c)^2 +
120*(b*x + a)^2*B*a^3*b^5*c^3*d^5*g^4/(d*x + c)^2 - 120*(b*x + a)^3*A*a^2*b

```
^5*c^4*d^5*g^4/(d*x + c)^3 - 120*(b*x + a)^3*B*a^2*b^5*c^4*d^5*g^4/(d*x + c
)^3 + 120*(b*x + a)^4*A*a*b^5*c^5*d^5*g^4/(d*x + c)^4 + 120*(b*x + a)^4*B*a
*b^5*c^5*d^5*g^4/(d*x + c)^4 - 12*A*a^6*b^4*d^6*g^4 - 12*B*a^6*b^4*d^6*g^4
+ 60*(b*x + a)*A*a^5*b^4*c*d^6*g^4/(d*x + c) + 60*(b*x + a)*B*a^5*b^4*c*d^6
*g^4/(d*x + c) - 120*(b*x + a)^2*A*a^4*b^4*c^2*d^6*g^4/(d*x + c)^2 - 120*(b
*x + a)^2*B*a^4*b^4*c^2*d^6*g^4/(d*x + c)^2 + 120*(b*x + a)^3*A*a^3*b^4*c^3
*d^6*g^4/(d*x + c)^3 + 120*(b*x + a)^3*B*a^3*b^4*c^3*d^6*g^4/(d*x + c)^3 -
60*(b*x + a)^4*A*a^2*b^4*c^4*d^6*g^4/(d*x + c)^4 - 60*(b*x + a)^4*B*a^2*b^4
*c^4*d^6*g^4/(d*x + c)^4)/(b^9*d^5 - 5*(b*x + a)*b^8*d^6/(d*x + c) + 10*(b*
x + a)^2*b^7*d^7/(d*x + c)^2 - 10*(b*x + a)^3*b^6*d^8/(d*x + c)^3 + 5*(b*x
+ a)^4*b^5*d^9/(d*x + c)^4 - (b*x + a)^5*b^4*d^10/(d*x + c)^5) + 12*(5*B*b^
6*c^2*d^4*f^4*n - 10*B*a*b^5*c*d^5*f^4*n + 5*B*a^2*b^4*d^6*f^4*n - 10*B*b^6
*c^3*d^3*f^3*g*n + 10*B*a*b^5*c^2*d^4*f^3*g*n + 10*B*a^2*b^4*c*d^5*f^3*g*n
- 10*B*a^3*b^3*d^6*f^3*g*n + 10*B*b^6*c^4*d^2*f^2*g^2*n - 10*B*a*b^5*c^3*d^
3*f^2*g^2*n - 10*B*a^3*b^3*c*d^5*f^2*g^2*n + 10*B*a^4*b^2*d^6*f^2*g^2*n - 5
*B*b^6*c^5*d*f*g^3*n + 5*B*a*b^5*c^4*d^2*f*g^3*n + 5*B*a^4*b^2*c*d^5*f*g^3*
n - 5*B*a^5*b*d^6*f*g^3*n + B*b^6*c^6*g^4*n - B*a*b^5*c^5*d*g^4*n - B*a^5*b
*c*d^5*g^4*n + B*a^6*d^6*g^4*n)*log(b - (b*x + a)*d/(d*x + c))/(b^5*d^5) -
12*(5*B*b^6*c^2*d^4*f^4*n - 10*B*a*b^5*c*d^5*f^4*n + 5*B*a^2*b^4*d^6*f^4*n
- 10*B*b^6*c^3*d^3*f^3*g*n + 10*B*a*b^5*c^2*d^4*f^3*g*n + 10*B*a^2*b^4*c*d^
5*f^3*g*n - 10*B*a^3*b^3*d^6*f^3*g*n + 10*B*b^6*c^4*d^2*f^2*g^2*n - 10*B*a*
b^5*c^3*d^3*f^2*g^2*n - 10*B*a^3*b^3*c*d^5*f^2*g^2*n + 10*B*a^4*b^2*d^6*f^2
*g^2*n - 5*B*b^6*c^5*d*f*g^3*n + 5*B*a*b^5*c^4*d^2*f*g^3*n + 5*B*a^4*b^2*c*
d^5*f*g^3*n - 5*B*a^5*b*d^6*f*g^3*n + B*b^6*c^6*g^4*n - B*a*b^5*c^5*d*g^4*n
 - B*a^5*b*c*d^5*g^4*n + B*a^6*d^6*g^4*n)*log((b*x + a)/(d*x + c))/(b^5*d^5
))*(b*c/(b*c - a*d)^2 - a*d/(b*c - a*d)^2)
```

**maple [F]** time = 0.34, size = 0, normalized size = 0.00

$$\int \left(gx+f\right)^{4} \left(B\ln\left(e\left(\frac{bx+a}{dx+c}\right)^{n}\right)+A\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x+f)^4*(B*ln(e*((b*x+a)/(d*x+c))^n)+A),x)
```

```
[Out] int((g*x+f)^4*(B*ln(e*((b*x+a)/(d*x+c))^n)+A),x)
```

**maxima [A]** time = 0.91, size = 631, normalized size = 1.73

$$\frac{1}{5}Bg^4x^5\log\left(e\left(\frac{bx}{dx+c}+\frac{a}{dx+c}\right)^n\right)+\frac{1}{5}Ag^4x^5+Bfg^3x^4\log\left(e\left(\frac{bx}{dx+c}+\frac{a}{dx+c}\right)^n\right)+Afg^3x^4+2Bf^2g^2x^3\log\left(e\left(\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^4*(A+B*log(e*((b*x+a)/(d*x+c))^n)),x, algorithm="maxima")
```

```
[Out] 1/5*B*g^4*x^5*log(e*(b*x/(d*x + c) + a/(d*x + c))^n) + 1/5*A*g^4*x^5 + B*f*
g^3*x^4*log(e*(b*x/(d*x + c) + a/(d*x + c))^n) + A*f*g^3*x^4 + 2*B*f^2*g^2*
x^3*log(e*(b*x/(d*x + c) + a/(d*x + c))^n) + 2*A*f^2*g^2*x^3 + 2*B*f^3*g*x^
2*log(e*(b*x/(d*x + c) + a/(d*x + c))^n) + 2*A*f^3*g*x^2 + 1/60*B*g^4*n*(12
*a^5*log(b*x + a)/b^5 - 12*c^5*log(d*x + c)/d^5 - (3*(b^4*c*d^3 - a*b^3*d^4
)*x^4 - 4*(b^4*c^2*d^2 - a^2*b^2*d^4)*x^3 + 6*(b^4*c^3*d - a^3*b*d^4)*x^2 -
 12*(b^4*c^4 - a^4*d^4)*x)/(b^4*d^4)) - 1/6*B*f*g^3*n*(6*a^4*log(b*x + a)/b
^4 - 6*c^4*log(d*x + c)/d^4 + (2*(b^3*c*d^2 - a*b^2*d^3)*x^3 - 3*(b^3*c^2*d
 - a^2*b*d^3)*x^2 + 6*(b^3*c^3 - a^3*d^3)*x)/(b^3*d^3)) + B*f^2*g^2*n*(2*a^
3*log(b*x + a)/b^3 - 2*c^3*log(d*x + c)/d^3 - ((b^2*c*d - a*b*d^2)*x^2 - 2*
(b^2*c^2 - a^2*d^2)*x)/(b^2*d^2)) - 2*B*f^3*g*n*(a^2*log(b*x + a)/b^2 - c^2
*log(d*x + c)/d^2 + (b*c - a*d)*x/(b*d)) + B*f^4*n*(a*log(b*x + a)/b - c*lo
g(d*x + c)/d) + B*f^4*x*log(e*(b*x/(d*x + c) + a/(d*x + c))^n) + A*f^4*x
```

**mupad [B]** time = 4.68, size = 1433, normalized size = 3.94

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f + g*x)^4*(A + B*log(e*((a + b*x)/(c + d*x))^n)),x)
```

```
[Out] x^4*((5*A*a*d*g^4 + 5*A*b*c*g^4 + 20*A*b*d*f*g^3 + B*a*d*g^4*n - B*b*c*g^4*
n)/(20*b*d) - (A*g^4*(5*a*d + 5*b*c))/(20*b*d)) + x^2*((20*A*a*c*f*g^3 + 20
*A*b*d*f^3*g + 30*A*a*d*f^2*g^2 + 30*A*b*c*f^2*g^2 + 10*B*a*d*f^2*g^2*n - 1
0*B*b*c*f^2*g^2*n)/(10*b*d) + ((5*a*d + 5*b*c)*((((5*A*a*d*g^4 + 5*A*b*c*g^
4 + 20*A*b*d*f*g^3 + B*a*d*g^4*n - B*b*c*g^4*n)/(5*b*d) - (A*g^4*(5*a*d + 5
*b*c))/(5*b*d))*(5*a*d + 5*b*c))/(5*b*d) - (5*A*a*c*g^4 + 20*A*a*d*f*g^3 +
20*A*b*c*f*g^3 + 30*A*b*d*f^2*g^2 + 5*B*a*d*f*g^3*n - 5*B*b*c*f*g^3*n)/(5*b
*d) + (A*a*c*g^4)/(b*d)))/(10*b*d) - (a*c*((5*A*a*d*g^4 + 5*A*b*c*g^4 + 20*
A*b*d*f*g^3 + B*a*d*g^4*n - B*b*c*g^4*n)/(5*b*d) - (A*g^4*(5*a*d + 5*b*c))/
(5*b*d)))/(2*b*d)) - x^3*((((5*A*a*d*g^4 + 5*A*b*c*g^4 + 20*A*b*d*f*g^3 + B
*a*d*g^4*n - B*b*c*g^4*n)/(5*b*d) - (A*g^4*(5*a*d + 5*b*c))/(5*b*d))*(5*a*d
 + 5*b*c))/(15*b*d) - (5*A*a*c*g^4 + 20*A*a*d*f*g^3 + 20*A*b*c*f*g^3 + 30*A
*b*d*f^2*g^2 + 5*B*a*d*f*g^3*n - 5*B*b*c*f*g^3*n)/(15*b*d) + (A*a*c*g^4)/(3
*b*d)) + x*((5*A*b*d*f^4 + 20*A*a*d*f^3*g + 20*A*b*c*f^3*g + 30*A*a*c*f^2*g
^2 + 10*B*a*d*f^3*g*n - 10*B*b*c*f^3*g*n)/(5*b*d) - ((5*a*d + 5*b*c)*((20*A
*a*c*f*g^3 + 20*A*b*d*f^3*g + 30*A*a*d*f^2*g^2 + 30*A*b*c*f^2*g^2 + 10*B*a*
d*f^2*g^2*n - 10*B*b*c*f^2*g^2*n)/(5*b*d) + ((5*a*d + 5*b*c)*((((5*A*a*d*g^
4 + 5*A*b*c*g^4 + 20*A*b*d*f*g^3 + B*a*d*g^4*n - B*b*c*g^4*n)/(5*b*d) - (A*
g^4*(5*a*d + 5*b*c))/(5*b*d))*(5*a*d + 5*b*c))/(5*b*d) - (5*A*a*c*g^4 + 20*
A*a*d*f*g^3 + 20*A*b*c*f*g^3 + 30*A*b*d*f^2*g^2 + 5*B*a*d*f*g^3*n - 5*B*b*c
*f*g^3*n)/(5*b*d) + (A*a*c*g^4)/(b*d)))/(5*b*d) - (a*c*((5*A*a*d*g^4 + 5*A*
b*c*g^4 + 20*A*b*d*f*g^3 + B*a*d*g^4*n - B*b*c*g^4*n)/(5*b*d) - (A*g^4*(5*a
*d + 5*b*c))/(5*b*d)))/(b*d)))/(5*b*d) + (a*c*((((5*A*a*d*g^4 + 5*A*b*c*g^4
 + 20*A*b*d*f*g^3 + B*a*d*g^4*n - B*b*c*g^4*n)/(5*b*d) - (A*g^4*(5*a*d + 5*
```

```
b*c))/(5*b*d))*(5*a*d + 5*b*c))/(5*b*d) - (5*A*a*c*g^4 + 20*A*a*d*f*g^3 + 2
0*A*b*c*f*g^3 + 30*A*b*d*f^2*g^2 + 5*B*a*d*f*g^3*n - 5*B*b*c*f*g^3*n)/(5*b*
d) + (A*a*c*g^4)/(b*d)))/(b*d)) + log(e*((a + b*x)/(c + d*x))^n)*((B*g^4*x^
5)/5 + B*f^4*x + 2*B*f^2*g^2*x^3 + 2*B*f^3*g*x^2 + B*f*g^3*x^4) + (A*g^4*x^
5)/5 + (log(a + b*x)*((B*a^5*g^4*n)/5 + B*a*b^4*f^4*n + 2*B*a^3*b^2*f^2*g^2
*n - B*a^4*b*f*g^3*n - 2*B*a^2*b^3*f^3*g*n))/b^5 - (log(c + d*x)*(B*c^5*g^4
*n + 5*B*c*d^4*f^4*n + 10*B*c^3*d^2*f^2*g^2*n - 5*B*c^4*d*f*g^3*n - 10*B*c^
2*d^3*f^3*g*n))/(5*d^5)
```

**sympy [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)**4*(A+B*ln(e*((b*x+a)/(d*x+c))**n)),x)
[Out] Timed out
```

# 3.58 $\int (f+gx)^3 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right) dx$

**Optimal**. Leaf size=235

$$-\frac{Bgnx(bc-ad)\left(a^2d^2g^2-abdg(4df-cg)+b^2\left(c^2g^2-4cdfg+6d^2f^2\right)\right)}{4b^3d^3}+\frac{(f+gx)^4\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{4g}-Bn$$

```
[Out] -1/4*B*(-a*d+b*c)*g*(a^2*d^2*g^2-a*b*d*g*(-c*g+4*d*f)+b^2*(c^2*g^2-4*c*d*f*
g+6*d^2*f^2))*n*x/b^3/d^3-1/8*B*(-a*d+b*c)*g^2*(-a*d*g-b*c*g+4*b*d*f)*n*x^2
/b^2/d^2-1/12*B*(-a*d+b*c)*g^3*n*x^3/b/d-1/4*B*(-a*g+b*f)^4*n*ln(b*x+a)/b^4
/g+1/4*(g*x+f)^4*(A+B*ln(e*((b*x+a)/(d*x+c))^n))/g+1/4*B*(-c*g+d*f)^4*n*ln(
d*x+c)/d^4/g
```

**Rubi [A]** time = 0.36, antiderivative size = 235, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {2525, 12, 72}

$$-\frac{Bgnx(bc-ad)\left(a^2d^2g^2-abdg(4df-cg)+b^2\left(c^2g^2-4cdfg+6d^2f^2\right)\right)}{4b^3d^3}+\frac{(f+gx)^4\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{4g}-Bg$$

Antiderivative was successfully verified.

```
[In] Int[(f + g*x)^3*(A + B*Log[e*((a + b*x)/(c + d*x))^n]),x]
```

```
[Out] -(B*(b*c - a*d)*g*(a^2*d^2*g^2 - a*b*d*g*(4*d*f - c*g) + b^2*(6*d^2*f^2 - 4
*c*d*f*g + c^2*g^2))*n*x)/(4*b^3*d^3) - (B*(b*c - a*d)*g^2*(4*b*d*f - b*c*g
 - a*d*g)*n*x^2)/(8*b^2*d^2) - (B*(b*c - a*d)*g^3*n*x^3)/(12*b*d) - (B*(b*f
 - a*g)^4*n*Log[a + b*x])/(4*b^4*g) + ((f + g*x)^4*(A + B*Log[e*((a + b*x)/
(c + d*x))^n]))/(4*g) + (B*(d*f - c*g)^4*n*Log[c + d*x])/(4*d^4*g)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 72

```
Int[((e_.) + (f_.)*(x_))^(p_.)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
 x_Symbol] :> Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x]
 /; FreeQ[{a, b, c, d, e, f}, x] && IntegerQ[p]
```

Rule 2525

```
Int[((a_.) + Log[(c_.)*(RFx_)^(p_.)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_))^(m_.
), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(a + b*Log[c*RFx^p])^n)/(e*(m + 1))
, x] - Dist[(b*n*p)/(e*(m + 1)), Int[SimplifyIntegrand[((d + e*x)^(m + 1)*(
a + b*Log[c*RFx^p])^(n - 1)*D[RFx, x])/RFx, x], x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && RationalFunctionQ[RFx, x] && IGtQ[n, 0] && (EqQ[n, 1] ||
IntegerQ[m]) && NeQ[m, -1]
```

Rubi steps

$$\int (f+gx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)dx = \frac{(f+gx)^4\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{4g} - \frac{(Bn)\int\frac{(bc-ad)(f+gx)^4}{(a+bx)(c+dx)}dx}{4g}$$

$$= \frac{(f+gx)^4\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{4g} - \frac{(B(bc-ad)n)\int\frac{(f+gx)^4}{(a+bx)(c+dx)}dx}{4g}$$

$$= \frac{(f+gx)^4\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{4g} - \frac{(B(bc-ad)n)\int\left(\frac{g^2\left(a^2d^2g^2-abdg\right.}{}\right.}{}$$

$$= -\frac{B(bc-ad)g\left(a^2d^2g^2-abdg(4df-cg)+b^2\left(6d^2f^2-4cdfg+c^2g\right.\right.}{4b^3d^3}$$

**Mathematica [A]** time = 0.28, size = 219, normalized size = 0.93

$$\frac{(f+gx)^4\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)-\frac{Bn\left(6bdg^2x(bc-ad)\left(a^2d^2g^2+abdg(cg-4df)+b^2\left(c^2g^2-4cdfg+6d^2f^2\right)\right)+2b^3d^3g^4x^3(bc-ad)+3b^2d^2g^3x^2\right.}{6b^4d^4}}{4g}$$

Antiderivative was successfully verified.

```
[In] Integrate[(f + g*x)^3*(A + B*Log[e*((a + b*x)/(c + d*x))^n]),x]
```

```
[Out] ((f + g*x)^4*(A + B*Log[e*((a + b*x)/(c + d*x))^n]) - (B*n*(6*b*d*(b*c - a*
d)*g^2*(a^2*d^2*g^2 + a*b*d*g*(-4*d*f + c*g) + b^2*(6*d^2*f^2 - 4*c*d*f*g +
 c^2*g^2))*x + 3*b^2*d^2*(b*c - a*d)*g^3*(4*b*d*f - b*c*g - a*d*g)*x^2 + 2*
b^3*d^3*(b*c - a*d)*g^4*x^3 + 6*d^4*(b*f - a*g)^4*Log[a + b*x] - 6*b^4*(d*f
 - c*g)^4*Log[c + d*x]))/(6*b^4*d^4))/(4*g)
```

**fricas [B]** time = 1.61, size = 521, normalized size = 2.22

$$\frac{6\,Ab^4d^4g^3x^4+2\left(12\,Ab^4d^4fg^2-\left(Bb^4cd^3-Bab^3d^4\right)g^3n\right)x^3+3\left(12\,Ab^4d^4f^2g-\left(4\left(Bb^4cd^3-Bab^3d^4\right)fg^2-\left(Bb\right.\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^3*(A+B*log(e*((b*x+a)/(d*x+c))^n)),x, algorithm="fricas")
```

```
[Out] 1/24*(6*A*b^4*d^4*g^3*x^4 + 2*(12*A*b^4*d^4*f*g^2 - (B*b^4*c*d^3 - B*a*b^3*
d^4)*g^3*n)*x^3 + 3*(12*A*b^4*d^4*f^2*g - (4*(B*b^4*c*d^3 - B*a*b^3*d^4)*f*
g^2 - (B*b^4*c^2*d^2 - B*a^2*b^2*d^4)*g^3)*n)*x^2 + 6*(4*B*a*b^3*d^4*f^3 -
6*B*a^2*b^2*d^4*f^2*g + 4*B*a^3*b*d^4*f*g^2 - B*a^4*d^4*g^3)*n*log(b*x + a)
 - 6*(4*B*b^4*c*d^3*f^3 - 6*B*b^4*c^2*d^2*f^2*g + 4*B*b^4*c^3*d*f*g^2 - B*b
^4*c^4*g^3)*n*log(d*x + c) + 6*(4*A*b^4*d^4*f^3 - (6*(B*b^4*c*d^3 - B*a*b^3
*d^4)*f^2*g - 4*(B*b^4*c^2*d^2 - B*a^2*b^2*d^4)*f*g^2 + (B*b^4*c^3*d - B*a^
3*b*d^4)*g^3)*n)*x + 6*(B*b^4*d^4*g^3*x^4 + 4*B*b^4*d^4*f*g^2*x^3 + 6*B*b^4
*d^4*f^2*g*x^2 + 4*B*b^4*d^4*f^3*x)*log(e) + 6*(B*b^4*d^4*g^3*n*x^4 + 4*B*b
^4*d^4*f*g^2*n*x^3 + 6*B*b^4*d^4*f^2*g*n*x^2 + 4*B*b^4*d^4*f^3*n*x)*log((b*
x + a)/(d*x + c)))/(b^4*d^4)
```

**giac [B]** time = 9.22, size = 6660, normalized size = 28.34

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^3*(A+B*log(e*((b*x+a)/(d*x+c))^n)),x, algorithm="giac")
```

```
[Out] 1/24*(6*(4*B*b^5*c^2*d^3*f^3*n - 8*B*a*b^4*c*d^4*f^3*n - 12*(b*x + a)*B*b^4
*c^2*d^4*f^3*n/(d*x + c) + 4*B*a^2*b^3*d^5*f^3*n + 24*(b*x + a)*B*a*b^3*c*d
^5*f^3*n/(d*x + c) + 12*(b*x + a)^2*B*b^3*c^2*d^5*f^3*n/(d*x + c)^2 - 12*(b
*x + a)*B*a^2*b^2*d^6*f^3*n/(d*x + c) - 24*(b*x + a)^2*B*a*b^2*c*d^6*f^3*n/
(d*x + c)^2 - 4*(b*x + a)^3*B*b^2*c^2*d^6*f^3*n/(d*x + c)^3 + 12*(b*x + a)^
2*B*a^2*b*d^7*f^3*n/(d*x + c)^2 + 8*(b*x + a)^3*B*a*b*c*d^7*f^3*n/(d*x + c)
^3 - 4*(b*x + a)^3*B*a^2*d^8*f^3*n/(d*x + c)^3 - 6*B*b^5*c^3*d^2*f^2*g*n +
6*B*a*b^4*c^2*d^3*f^2*g*n + 24*(b*x + a)*B*b^4*c^3*d^3*f^2*g*n/(d*x + c) +
6*B*a^2*b^3*c*d^4*f^2*g*n - 36*(b*x + a)*B*a*b^3*c^2*d^4*f^2*g*n/(d*x + c)
- 30*(b*x + a)^2*B*b^3*c^3*d^4*f^2*g*n/(d*x + c)^2 - 6*B*a^3*b^2*d^5*f^2*g*
n + 54*(b*x + a)^2*B*a*b^2*c^2*d^5*f^2*g*n/(d*x + c)^2 + 12*(b*x + a)^3*B*b
^2*c^3*d^5*f^2*g*n/(d*x + c)^3 + 12*(b*x + a)*B*a^3*b*d^6*f^2*g*n/(d*x + c)
 - 18*(b*x + a)^2*B*a^2*b*c*d^6*f^2*g*n/(d*x + c)^2 - 24*(b*x + a)^3*B*a*b*
c^2*d^6*f^2*g*n/(d*x + c)^3 - 6*(b*x + a)^2*B*a^3*d^7*f^2*g*n/(d*x + c)^2 +
 12*(b*x + a)^3*B*a^2*c*d^7*f^2*g*n/(d*x + c)^3 + 4*B*b^5*c^4*d*f*g^2*n - 4
*B*a*b^4*c^3*d^2*f*g^2*n - 16*(b*x + a)*B*b^4*c^4*d^2*f*g^2*n/(d*x + c) + 1
6*(b*x + a)*B*a*b^3*c^3*d^3*f*g^2*n/(d*x + c) + 24*(b*x + a)^2*B*b^3*c^4*d^
3*f*g^2*n/(d*x + c)^2 - 4*B*a^3*b^2*c*d^4*f*g^2*n + 12*(b*x + a)*B*a^2*b^2*
c^2*d^4*f*g^2*n/(d*x + c) - 36*(b*x + a)^2*B*a*b^2*c^3*d^4*f*g^2*n/(d*x + c
)^2 - 12*(b*x + a)^3*B*b^2*c^4*d^4*f*g^2*n/(d*x + c)^3 + 4*B*a^4*b*d^5*f*g^
2*n - 8*(b*x + a)*B*a^3*b*c*d^5*f*g^2*n/(d*x + c) + 24*(b*x + a)^3*B*a*b*c^
3*d^5*f*g^2*n/(d*x + c)^3 - 4*(b*x + a)*B*a^4*d^6*f*g^2*n/(d*x + c) + 12*(b
```

*x + a)^2*B*a^3*c*d^6*f*g^2*n/(d*x + c)^2 - 12*(b*x + a)^3*B*a^2*c^2*d^6*f*
g^2*n/(d*x + c)^3 - B*b^5*c^5*g^3*n + B*a*b^4*c^4*d*g^3*n + 4*(b*x + a)*B*b
^4*c^5*d*g^3*n/(d*x + c) - 4*(b*x + a)*B*a*b^3*c^4*d^2*g^3*n/(d*x + c) - 6*
(b*x + a)^2*B*b^3*c^5*d^2*g^3*n/(d*x + c)^2 + 6*(b*x + a)^2*B*a*b^2*c^4*d^3
*g^3*n/(d*x + c)^2 + 4*(b*x + a)^3*B*b^2*c^5*d^3*g^3*n/(d*x + c)^3 + B*a^4*
b*c*d^4*g^3*n - 4*(b*x + a)*B*a^3*b*c^2*d^4*g^3*n/(d*x + c) + 6*(b*x + a)^2
*B*a^2*b*c^3*d^4*g^3*n/(d*x + c)^2 - 8*(b*x + a)^3*B*a*b*c^4*d^4*g^3*n/(d*x
 + c)^3 - B*a^5*d^5*g^3*n + 4*(b*x + a)*B*a^4*c*d^5*g^3*n/(d*x + c) - 6*(b*
x + a)^2*B*a^3*c^2*d^5*g^3*n/(d*x + c)^2 + 4*(b*x + a)^3*B*a^2*c^3*d^5*g^3*
n/(d*x + c)^3)*log((b*x + a)/(d*x + c))/(b^4*d^4 - 4*(b*x + a)*b^3*d^5/(d*x
 + c) + 6*(b*x + a)^2*b^2*d^6/(d*x + c)^2 - 4*(b*x + a)^3*b*d^7/(d*x + c)^3
 + (b*x + a)^4*d^8/(d*x + c)^4) - (36*B*b^8*c^3*d^2*f^2*g*n - 108*B*a*b^7*c
^2*d^3*f^2*g*n - 108*(b*x + a)*B*b^7*c^3*d^3*f^2*g*n/(d*x + c) + 108*B*a^2*
b^6*c*d^4*f^2*g*n + 324*(b*x + a)*B*a*b^6*c^2*d^4*f^2*g*n/(d*x + c) + 108*(
b*x + a)^2*B*b^6*c^3*d^4*f^2*g*n/(d*x + c)^2 - 36*B*a^3*b^5*d^5*f^2*g*n - 3
24*(b*x + a)*B*a^2*b^5*c*d^5*f^2*g*n/(d*x + c) - 324*(b*x + a)^2*B*a*b^5*c^
2*d^5*f^2*g*n/(d*x + c)^2 - 36*(b*x + a)^3*B*b^5*c^3*d^5*f^2*g*n/(d*x + c)^
3 + 108*(b*x + a)*B*a^3*b^4*d^6*f^2*g*n/(d*x + c) + 324*(b*x + a)^2*B*a^2*b
^4*c*d^6*f^2*g*n/(d*x + c)^2 + 108*(b*x + a)^3*B*a*b^4*c^2*d^6*f^2*g*n/(d*x
 + c)^3 - 108*(b*x + a)^2*B*a^3*b^3*d^7*f^2*g*n/(d*x + c)^2 - 108*(b*x + a)
^3*B*a^2*b^3*c*d^7*f^2*g*n/(d*x + c)^3 + 36*(b*x + a)^3*B*a^3*b^2*d^8*f^2*g
*n/(d*x + c)^3 - 36*B*b^8*c^4*d*f*g^2*n + 72*B*a*b^7*c^3*d^2*f*g^2*n + 120*
(b*x + a)*B*b^7*c^4*d^2*f*g^2*n/(d*x + c) - 264*(b*x + a)*B*a*b^6*c^3*d^3*f
*g^2*n/(d*x + c) - 132*(b*x + a)^2*B*b^6*c^4*d^3*f*g^2*n/(d*x + c)^2 - 72*B
*a^3*b^5*c*d^4*f*g^2*n + 72*(b*x + a)*B*a^2*b^5*c^2*d^4*f*g^2*n/(d*x + c) +
 312*(b*x + a)^2*B*a*b^5*c^3*d^4*f*g^2*n/(d*x + c)^2 + 48*(b*x + a)^3*B*b^5
*c^4*d^4*f*g^2*n/(d*x + c)^3 + 36*B*a^4*b^4*d^5*f*g^2*n + 168*(b*x + a)*B*a
^3*b^4*c*d^5*f*g^2*n/(d*x + c) - 144*(b*x + a)^2*B*a^2*b^4*c^2*d^5*f*g^2*n/
(d*x + c)^2 - 120*(b*x + a)^3*B*a*b^4*c^3*d^5*f*g^2*n/(d*x + c)^3 - 96*(b*x
 + a)*B*a^4*b^3*d^6*f*g^2*n/(d*x + c) - 120*(b*x + a)^2*B*a^3*b^3*c*d^6*f*g
^2*n/(d*x + c)^2 + 72*(b*x + a)^3*B*a^2*b^3*c^2*d^6*f*g^2*n/(d*x + c)^3 + 8
4*(b*x + a)^2*B*a^4*b^2*d^7*f*g^2*n/(d*x + c)^2 + 24*(b*x + a)^3*B*a^3*b^2*
c*d^7*f*g^2*n/(d*x + c)^3 - 24*(b*x + a)^3*B*a^4*b*d^8*f*g^2*n/(d*x + c)^3
+ 11*B*b^8*c^5*g^3*n - 19*B*a*b^7*c^4*d*g^3*n - 38*(b*x + a)*B*b^7*c^5*d*g^
3*n/(d*x + c) + 2*B*a^2*b^6*c^3*d^2*g^3*n + 70*(b*x + a)*B*a*b^6*c^4*d^2*g^
3*n/(d*x + c) + 45*(b*x + a)^2*B*b^6*c^5*d^2*g^3*n/(d*x + c)^2 - 2*B*a^3*b^
5*c^2*d^3*g^3*n - 8*(b*x + a)*B*a^2*b^5*c^3*d^3*g^3*n/(d*x + c) - 93*(b*x +
 a)^2*B*a*b^5*c^4*d^3*g^3*n/(d*x + c)^2 - 18*(b*x + a)^3*B*b^5*c^5*d^3*g^3*
n/(d*x + c)^3 + 19*B*a^4*b^4*c*d^4*g^3*n - 16*(b*x + a)*B*a^3*b^4*c^2*d^4*g
^3*n/(d*x + c) + 30*(b*x + a)^2*B*a^2*b^4*c^3*d^4*g^3*n/(d*x + c)^2 + 42*(b
*x + a)^3*B*a*b^4*c^4*d^4*g^3*n/(d*x + c)^3 - 11*B*a^5*b^3*d^5*g^3*n - 34*(
b*x + a)*B*a^4*b^3*c*d^5*g^3*n/(d*x + c) + 18*(b*x + a)^2*B*a^3*b^3*c^2*d^5
*g^3*n/(d*x + c)^2 - 24*(b*x + a)^3*B*a^2*b^3*c^3*d^5*g^3*n/(d*x + c)^3 + 2
6*(b*x + a)*B*a^5*b^2*d^6*g^3*n/(d*x + c) + 21*(b*x + a)^2*B*a^4*b^2*c*d^6*
g^3*n/(d*x + c)^2 - 21*(b*x + a)^2*B*a^5*b*d^7*g^3*n/(d*x + c)^2 - 6*(b*x +

a)^3*B*a^4*b*c*d^7*g^3*n/(d*x + c)^3 + 6*(b*x + a)^3*B*a^5*d^8*g^3*n/(d*x
+ c)^3 - 24*A*b^8*c^2*d^3*f^3 - 24*B*b^8*c^2*d^3*f^3 + 48*A*a*b^7*c*d^4*f^3
+ 48*B*a*b^7*c*d^4*f^3 + 72*(b*x + a)*A*b^7*c^2*d^4*f^3/(d*x + c) + 72*(b*
x + a)*B*b^7*c^2*d^4*f^3/(d*x + c) - 24*A*a^2*b^6*d^5*f^3 - 24*B*a^2*b^6*d^
5*f^3 - 144*(b*x + a)*A*a*b^6*c*d^5*f^3/(d*x + c) - 144*(b*x + a)*B*a*b^6*c
*d^5*f^3/(d*x + c) - 72*(b*x + a)^2*A*b^6*c^2*d^5*f^3/(d*x + c)^2 - 72*(b*x
+ a)^2*B*b^6*c^2*d^5*f^3/(d*x + c)^2 + 72*(b*x + a)*A*a^2*b^5*d^6*f^3/(d*x
+ c) + 72*(b*x + a)*B*a^2*b^5*d^6*f^3/(d*x + c) + 144*(b*x + a)^2*A*a*b^5*
c*d^6*f^3/(d*x + c)^2 + 144*(b*x + a)^2*B*a*b^5*c*d^6*f^3/(d*x + c)^2 + 24*
(b*x + a)^3*A*b^5*c^2*d^6*f^3/(d*x + c)^3 + 24*(b*x + a)^3*B*b^5*c^2*d^6*f^
3/(d*x + c)^3 - 72*(b*x + a)^2*A*a^2*b^4*d^7*f^3/(d*x + c)^2 - 72*(b*x + a)
^2*B*a^2*b^4*d^7*f^3/(d*x + c)^2 - 48*(b*x + a)^3*A*a*b^4*c*d^7*f^3/(d*x +
c)^3 - 48*(b*x + a)^3*B*a*b^4*c*d^7*f^3/(d*x + c)^3 + 24*(b*x + a)^3*A*a^2*
b^3*d^8*f^3/(d*x + c)^3 + 24*(b*x + a)^3*B*a^2*b^3*d^8*f^3/(d*x + c)^3 + 36
*A*b^8*c^3*d^2*f^2*g + 36*B*b^8*c^3*d^2*f^2*g - 36*A*a*b^7*c^2*d^3*f^2*g -
36*B*a*b^7*c^2*d^3*f^2*g - 144*(b*x + a)*A*b^7*c^3*d^3*f^2*g/(d*x + c) - 14
4*(b*x + a)*B*b^7*c^3*d^3*f^2*g/(d*x + c) - 36*A*a^2*b^6*c*d^4*f^2*g - 36*B
*a^2*b^6*c*d^4*f^2*g + 216*(b*x + a)*A*a*b^6*c^2*d^4*f^2*g/(d*x + c) + 216*
(b*x + a)*B*a*b^6*c^2*d^4*f^2*g/(d*x + c) + 180*(b*x + a)^2*A*b^6*c^3*d^4*f
^2*g/(d*x + c)^2 + 180*(b*x + a)^2*B*b^6*c^3*d^4*f^2*g/(d*x + c)^2 + 36*A*a
^3*b^5*d^5*f^2*g + 36*B*a^3*b^5*d^5*f^2*g - 324*(b*x + a)^2*A*a*b^5*c^2*d^5
*f^2*g/(d*x + c)^2 - 324*(b*x + a)^2*B*a*b^5*c^2*d^5*f^2*g/(d*x + c)^2 - 72
*(b*x + a)^3*A*b^5*c^3*d^5*f^2*g/(d*x + c)^3 - 72*(b*x + a)^3*B*b^5*c^3*d^5
*f^2*g/(d*x + c)^3 - 72*(b*x + a)*A*a^3*b^4*d^6*f^2*g/(d*x + c) - 72*(b*x +
a)*B*a^3*b^4*d^6*f^2*g/(d*x + c) + 108*(b*x + a)^2*A*a^2*b^4*c*d^6*f^2*g/(
d*x + c)^2 + 108*(b*x + a)^2*B*a^2*b^4*c*d^6*f^2*g/(d*x + c)^2 + 144*(b*x +
a)^3*A*a*b^4*c^2*d^6*f^2*g/(d*x + c)^3 + 144*(b*x + a)^3*B*a*b^4*c^2*d^6*f
^2*g/(d*x + c)^3 + 36*(b*x + a)^2*A*a^3*b^3*d^7*f^2*g/(d*x + c)^2 + 36*(b*x
+ a)^2*B*a^3*b^3*d^7*f^2*g/(d*x + c)^2 - 72*(b*x + a)^3*A*a^2*b^3*c*d^7*f^
2*g/(d*x + c)^3 - 72*(b*x + a)^3*B*a^2*b^3*c*d^7*f^2*g/(d*x + c)^3 - 24*A*b
^8*c^4*d*f*g^2 - 24*B*b^8*c^4*d*f*g^2 + 24*A*a*b^7*c^3*d^2*f*g^2 + 24*B*a*b
^7*c^3*d^2*f*g^2 + 96*(b*x + a)*A*b^7*c^4*d^2*f*g^2/(d*x + c) + 96*(b*x + a
)*B*b^7*c^4*d^2*f*g^2/(d*x + c) - 96*(b*x + a)*A*a*b^6*c^3*d^3*f*g^2/(d*x +
c) - 96*(b*x + a)*B*a*b^6*c^3*d^3*f*g^2/(d*x + c) - 144*(b*x + a)^2*A*b^6*
c^4*d^3*f*g^2/(d*x + c)^2 - 144*(b*x + a)^2*B*b^6*c^4*d^3*f*g^2/(d*x + c)^2
+ 24*A*a^3*b^5*c*d^4*f*g^2 + 24*B*a^3*b^5*c*d^4*f*g^2 - 72*(b*x + a)*A*a^2
*b^5*c^2*d^4*f*g^2/(d*x + c) - 72*(b*x + a)*B*a^2*b^5*c^2*d^4*f*g^2/(d*x +
c) + 216*(b*x + a)^2*A*a*b^5*c^3*d^4*f*g^2/(d*x + c)^2 + 216*(b*x + a)^2*B*
a*b^5*c^3*d^4*f*g^2/(d*x + c)^2 + 72*(b*x + a)^3*A*b^5*c^4*d^4*f*g^2/(d*x +
c)^3 + 72*(b*x + a)^3*B*b^5*c^4*d^4*f*g^2/(d*x + c)^3 - 24*A*a^4*b^4*d^5*f
*g^2 - 24*B*a^4*b^4*d^5*f*g^2 + 48*(b*x + a)*A*a^3*b^4*c*d^5*f*g^2/(d*x + c
) + 48*(b*x + a)*B*a^3*b^4*c*d^5*f*g^2/(d*x + c) - 144*(b*x + a)^3*A*a*b^4*
c^3*d^5*f*g^2/(d*x + c)^3 - 144*(b*x + a)^3*B*a*b^4*c^3*d^5*f*g^2/(d*x + c)
^3 + 24*(b*x + a)*A*a^4*b^3*d^6*f*g^2/(d*x + c) + 24*(b*x + a)*B*a^4*b^3*d^
6*f*g^2/(d*x + c) - 72*(b*x + a)^2*A*a^3*b^3*c*d^6*f*g^2/(d*x + c)^2 - 72*(

```
b*x + a)^2*B*a^3*b^3*c*d^6*f*g^2/(d*x + c)^2 + 72*(b*x + a)^3*A*a^2*b^3*c^2
*d^6*f*g^2/(d*x + c)^3 + 72*(b*x + a)^3*B*a^2*b^3*c^2*d^6*f*g^2/(d*x + c)^3
 + 6*A*b^8*c^5*g^3 + 6*B*b^8*c^5*g^3 - 6*A*a*b^7*c^4*d*g^3 - 6*B*a*b^7*c^4*
d*g^3 - 24*(b*x + a)*A*b^7*c^5*d*g^3/(d*x + c) - 24*(b*x + a)*B*b^7*c^5*d*g
^3/(d*x + c) + 24*(b*x + a)*A*a*b^6*c^4*d^2*g^3/(d*x + c) + 24*(b*x + a)*B*
a*b^6*c^4*d^2*g^3/(d*x + c) + 36*(b*x + a)^2*A*b^6*c^5*d^2*g^3/(d*x + c)^2
+ 36*(b*x + a)^2*B*b^6*c^5*d^2*g^3/(d*x + c)^2 - 36*(b*x + a)^2*A*a*b^5*c^4
*d^3*g^3/(d*x + c)^2 - 36*(b*x + a)^2*B*a*b^5*c^4*d^3*g^3/(d*x + c)^2 - 24*
(b*x + a)^3*A*b^5*c^5*d^3*g^3/(d*x + c)^3 - 24*(b*x + a)^3*B*b^5*c^5*d^3*g^
3/(d*x + c)^3 - 6*A*a^4*b^4*c*d^4*g^3 - 6*B*a^4*b^4*c*d^4*g^3 + 24*(b*x + a
)*A*a^3*b^4*c^2*d^4*g^3/(d*x + c) + 24*(b*x + a)*B*a^3*b^4*c^2*d^4*g^3/(d*x
 + c) - 36*(b*x + a)^2*A*a^2*b^4*c^3*d^4*g^3/(d*x + c)^2 - 36*(b*x + a)^2*B
*a^2*b^4*c^3*d^4*g^3/(d*x + c)^2 + 48*(b*x + a)^3*A*a*b^4*c^4*d^4*g^3/(d*x
+ c)^3 + 48*(b*x + a)^3*B*a*b^4*c^4*d^4*g^3/(d*x + c)^3 + 6*A*a^5*b^3*d^5*g
^3 + 6*B*a^5*b^3*d^5*g^3 - 24*(b*x + a)*A*a^4*b^3*c*d^5*g^3/(d*x + c) - 24*
(b*x + a)*B*a^4*b^3*c*d^5*g^3/(d*x + c) + 36*(b*x + a)^2*A*a^3*b^3*c^2*d^5*
g^3/(d*x + c)^2 + 36*(b*x + a)^2*B*a^3*b^3*c^2*d^5*g^3/(d*x + c)^2 - 24*(b*
x + a)^3*A*a^2*b^3*c^3*d^5*g^3/(d*x + c)^3 - 24*(b*x + a)^3*B*a^2*b^3*c^3*d
^5*g^3/(d*x + c)^3)/(b^7*d^4 - 4*(b*x + a)*b^6*d^5/(d*x + c) + 6*(b*x + a)^
2*b^5*d^6/(d*x + c)^2 - 4*(b*x + a)^3*b^4*d^7/(d*x + c)^3 + (b*x + a)^4*b^3
*d^8/(d*x + c)^4) + 6*(4*B*b^5*c^2*d^3*f^3*n - 8*B*a*b^4*c*d^4*f^3*n + 4*B*
a^2*b^3*d^5*f^3*n - 6*B*b^5*c^3*d^2*f^2*g*n + 6*B*a*b^4*c^2*d^3*f^2*g*n + 6
*B*a^2*b^3*c*d^4*f^2*g*n - 6*B*a^3*b^2*d^5*f^2*g*n + 4*B*b^5*c^4*d*f*g^2*n
- 4*B*a*b^4*c^3*d^2*f*g^2*n - 4*B*a^3*b^2*c*d^4*f*g^2*n + 4*B*a^4*b*d^5*f*g
^2*n - B*b^5*c^5*g^3*n + B*a*b^4*c^4*d*g^3*n + B*a^4*b*c*d^4*g^3*n - B*a^5*
d^5*g^3*n)*log(-b + (b*x + a)*d/(d*x + c))/(b^4*d^4) - 6*(4*B*b^5*c^2*d^3*f
^3*n - 8*B*a*b^4*c*d^4*f^3*n + 4*B*a^2*b^3*d^5*f^3*n - 6*B*b^5*c^3*d^2*f^2*
g*n + 6*B*a*b^4*c^2*d^3*f^2*g*n + 6*B*a^2*b^3*c*d^4*f^2*g*n - 6*B*a^3*b^2*d
^5*f^2*g*n + 4*B*b^5*c^4*d*f*g^2*n - 4*B*a*b^4*c^3*d^2*f*g^2*n - 4*B*a^3*b^
2*c*d^4*f*g^2*n + 4*B*a^4*b*d^5*f*g^2*n - B*b^5*c^5*g^3*n + B*a*b^4*c^4*d*g
^3*n + B*a^4*b*c*d^4*g^3*n - B*a^5*d^5*g^3*n)*log((b*x + a)/(d*x + c))/(b^4
*d^4))*(b*c/(b*c - a*d)^2 - a*d/(b*c - a*d)^2)
```

**maple [F]** time = 0.26, size = 0, normalized size = 0.00

$$\int \left(gx+f\right)^{3} \left(B\ln\left(e\left(\frac{bx+a}{dx+c}\right)^{n}\right)+A\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x+f)^3*(B*ln(e*((b*x+a)/(d*x+c))^n)+A),x)
```

```
[Out] int((g*x+f)^3*(B*ln(e*((b*x+a)/(d*x+c))^n)+A),x)
```

**maxima [A]** time = 0.93, size = 443, normalized size = 1.89

$$\frac{1}{4}Bg^3x^4\log\left(e\left(\frac{bx}{dx+c}+\frac{a}{dx+c}\right)^n\right)+\frac{1}{4}Ag^3x^4+Bfg^2x^3\log\left(e\left(\frac{bx}{dx+c}+\frac{a}{dx+c}\right)^n\right)+Afg^2x^3+\frac{3}{2}Bf^2gx^2\log\left(e\left(\frac{b}{dx}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^3*(A+B*log(e*((b*x+a)/(d*x+c))^n)),x, algorithm="maxima")
```

```
[Out] 1/4*B*g^3*x^4*log(e*(b*x/(d*x + c) + a/(d*x + c))^n) + 1/4*A*g^3*x^4 + B*f*
g^2*x^3*log(e*(b*x/(d*x + c) + a/(d*x + c))^n) + A*f*g^2*x^3 + 3/2*B*f^2*g*
x^2*log(e*(b*x/(d*x + c) + a/(d*x + c))^n) + 3/2*A*f^2*g*x^2 - 1/24*B*g^3*n
*(6*a^4*log(b*x + a)/b^4 - 6*c^4*log(d*x + c)/d^4 + (2*(b^3*c*d^2 - a*b^2*d
^3)*x^3 - 3*(b^3*c^2*d - a^2*b*d^3)*x^2 + 6*(b^3*c^3 - a^3*d^3)*x)/(b^3*d^3
)) + 1/2*B*f*g^2*n*(2*a^3*log(b*x + a)/b^3 - 2*c^3*log(d*x + c)/d^3 - ((b^2
*c*d - a*b*d^2)*x^2 - 2*(b^2*c^2 - a^2*d^2)*x)/(b^2*d^2)) - 3/2*B*f^2*g*n*(
a^2*log(b*x + a)/b^2 - c^2*log(d*x + c)/d^2 + (b*c - a*d)*x/(b*d)) + B*f^3*
n*(a*log(b*x + a)/b - c*log(d*x + c)/d) + B*f^3*x*log(e*(b*x/(d*x + c) + a/
(d*x + c))^n) + A*f^3*x
```

**mupad [B]** time = 4.74, size = 766, normalized size = 3.26

$$x\left(\frac{4\,A\,b\,d\,f^3+12\,A\,a\,c\,f\,g^2+12\,A\,a\,d\,f^2\,g+12\,A\,b\,c\,f^2\,g+6\,B\,a\,d\,f^2\,g\,n-6\,B\,b\,c\,f^2\,g\,n}{4\,b\,d}+\frac{(4\,a\,d+4\,b\,c)\left(\left(\frac{4\,A\,a\,d}{\cdots}\right.\right.}{\cdots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f + g*x)^3*(A + B*log(e*((a + b*x)/(c + d*x))^n)),x)
```

```
[Out] x*((4*A*b*d*f^3 + 12*A*a*c*f*g^2 + 12*A*a*d*f^2*g + 12*A*b*c*f^2*g + 6*B*a*
d*f^2*g*n - 6*B*b*c*f^2*g*n)/(4*b*d) + ((4*a*d + 4*b*c)*((((4*A*a*d*g^3 + 4
*A*b*c*g^3 + 12*A*b*d*f*g^2 + B*a*d*g^3*n - B*b*c*g^3*n)/(4*b*d) - (A*g^3*(
4*a*d + 4*b*c))/(4*b*d))*(4*a*d + 4*b*c))/(4*b*d) - (4*A*a*c*g^3 + 12*A*a*d
*f*g^2 + 12*A*b*c*f*g^2 + 12*A*b*d*f^2*g + 4*B*a*d*f*g^2*n - 4*B*b*c*f*g^2*
n)/(4*b*d) + (A*a*c*g^3)/(b*d)))/(4*b*d) - (a*c*((4*A*a*d*g^3 + 4*A*b*c*g^3
 + 12*A*b*d*f*g^2 + B*a*d*g^3*n - B*b*c*g^3*n)/(4*b*d) - (A*g^3*(4*a*d + 4*
b*c))/(4*b*d)))/(b*d)) - x^2*((((4*A*a*d*g^3 + 4*A*b*c*g^3 + 12*A*b*d*f*g^2
 + B*a*d*g^3*n - B*b*c*g^3*n)/(4*b*d) - (A*g^3*(4*a*d + 4*b*c))/(4*b*d))*(4
*a*d + 4*b*c))/(8*b*d) - (4*A*a*c*g^3 + 12*A*a*d*f*g^2 + 12*A*b*c*f*g^2 + 1
2*A*b*d*f^2*g + 4*B*a*d*f*g^2*n - 4*B*b*c*f*g^2*n)/(8*b*d) + (A*a*c*g^3)/(2
*b*d)) + x^3*((4*A*a*d*g^3 + 4*A*b*c*g^3 + 12*A*b*d*f*g^2 + B*a*d*g^3*n - B
```

```
*b*c*g^3*n)/(12*b*d) - (A*g^3*(4*a*d + 4*b*c))/(12*b*d)) + log(e*((a + b*x)
/(c + d*x))^n)*((B*g^3*x^4)/4 + B*f^3*x + (3*B*f^2*g*x^2)/2 + B*f*g^2*x^3)
+ (A*g^3*x^4)/4 - (log(a + b*x)*(B*a^4*g^3*n - 4*B*a*b^3*f^3*n - 4*B*a^3*b*
f*g^2*n + 6*B*a^2*b^2*f^2*g*n))/(4*b^4) + (log(c + d*x)*(B*c^4*g^3*n - 4*B*
c*d^3*f^3*n - 4*B*c^3*d*f*g^2*n + 6*B*c^2*d^2*f^2*g*n))/(4*d^4)
```

**sympy** [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)**3*(A+B*ln(e*((b*x+a)/(d*x+c))**n)),x)
```

```
[Out] Timed out
```

# 3.59 $\int (f+gx)^2 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right) dx$

**Optimal**. Leaf size=157

$$\frac{(f+gx)^3\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{3g}-\frac{Bn(bf-ag)^3\log(a+bx)}{3b^3g}-\frac{Bgnx(bc-ad)(-adg-bcg+3bdf)}{3b^2d^2}-\frac{Bg^2nx^2(bc-}{6bd}$$

[Out] -1/3*B*(-a*d+b*c)*g*(-a*d*g-b*c*g+3*b*d*f)*n*x/b^2/d^2-1/6*B*(-a*d+b*c)*g^2 *n*x^2/b/d-1/3*B*(-a*g+b*f)^3*n*ln(b*x+a)/b^3/g+1/3*(g*x+f)^3*(A+B*ln(e*((b *x+a)/(d*x+c))^n))/g+1/3*B*(-c*g+d*f)^3*n*ln(d*x+c)/d^3/g

**Rubi [A]** time = 0.18, antiderivative size = 157, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {2525, 12, 72}

$$\frac{(f+gx)^3\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{3g}-\frac{Bgnx(bc-ad)(-adg-bcg+3bdf)}{3b^2d^2}-\frac{Bn(bf-ag)^3\log(a+bx)}{3b^3g}-\frac{Bg^2nx^2(bc-}{6bd}$$

Antiderivative was successfully verified.

[In] Int[(f + g*x)^2*(A + B*Log[e*((a + b*x)/(c + d*x))^n]),x]

[Out] -(B*(b*c - a*d)*g*(3*b*d*f - b*c*g - a*d*g)*n*x)/(3*b^2*d^2) - (B*(b*c - a*d)*g^2*n*x^2)/(6*b*d) - (B*(b*f - a*g)^3*n*Log[a + b*x])/(3*b^3*g) + ((f + g*x)^3*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/(3*g) + (B*(d*f - c*g)^3*n*Log[c + d*x])/(3*d^3*g)

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 72

```
Int[((e_.) + (f_.)*(x_))^(p_.)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
 x_Symbol] :> Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x]
 /; FreeQ[{a, b, c, d, e, f}, x] && IntegerQ[p]
```

Rule 2525

```
Int[((a_.) + Log[(c_.)*(RFx_)^(p_.)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_))^(m_.
), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(a + b*Log[c*RFx^p])^n)/(e*(m + 1))
```

```
, x] - Dist[(b*n*p)/(e*(m + 1)), Int[SimplifyIntegrand[((d + e*x)^(m + 1)*(
a + b*Log[c*RFx^p])^(n - 1)*D[RFx, x])/RFx, x], x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && RationalFunctionQ[RFx, x] && IGtQ[n, 0] && (EqQ[n, 1] ||
IntegerQ[m]) && NeQ[m, -1]
```

Rubi steps

$$\int (f+gx)^2 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right) dx = \frac{(f+gx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{3g} - \frac{(Bn)\int \frac{(bc-ad)(f+gx)^3}{(a+bx)(c+dx)}\,dx}{3g}$$

$$= \frac{(f+gx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{3g} - \frac{(B(bc-ad)n)\int \frac{(f+gx)^3}{(a+bx)(c+dx)}\,dx}{3g}$$

$$= \frac{(f+gx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{3g} - \frac{(B(bc-ad)n)\int \left(\frac{g^2(3bdf-bcg-a}{b^2d^2}\right.}{}$$

$$= -\frac{B(bc-ad)g(3bdf-bcg-adg)nx}{3b^2d^2} - \frac{B(bc-ad)g^2nx^2}{6bd} - \frac{B(bf-a}{}$$

**Mathematica [A]** time = 0.15, size = 146, normalized size = 0.93

$$\frac{(f+gx)^3\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right) - \frac{Bn\left(b^2d^2g^3x^2(bc-ad)+2bdg^2x(bc-ad)(-adg-bcg+3bdf)+2d^3(bf-ag)^3\log(a+bx)-2b^3(df-cg)^3\log(c}{2b^3d^3}}{3g}$$

Antiderivative was successfully verified.

```
[In] Integrate[(f + g*x)^2*(A + B*Log[e*((a + b*x)/(c + d*x))^n]),x]
```

```
[Out] ((f + g*x)^3*(A + B*Log[e*((a + b*x)/(c + d*x))^n]) - (B*n*(2*b*d*(b*c - a*
d)*g^2*(3*b*d*f - b*c*g - a*d*g)*x + b^2*d^2*(b*c - a*d)*g^3*x^2 + 2*d^3*(b
*f - a*g)^3*Log[a + b*x] - 2*b^3*(d*f - c*g)^3*Log[c + d*x]))/(2*b^3*d^3))/
(3*g)
```

**fricas [B]** time = 0.96, size = 334, normalized size = 2.13

$$\frac{2\,Ab^3d^3g^2x^3 + \left(6\,Ab^3d^3fg - \left(Bb^3cd^2 - Bab^2d^3\right)g^2n\right)x^2 + 2\left(3\,Bab^2d^3f^2 - 3\,Ba^2bd^3fg + Ba^3d^3g^2\right)n\log(bx+a)}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^2*(A+B*log(e*((b*x+a)/(d*x+c))^n)),x, algorithm="fricas")
```

```
[Out] 1/6*(2*A*b^3*d^3*g^2*x^3 + (6*A*b^3*d^3*f*g - (B*b^3*c*d^2 - B*a*b^2*d^3)*g
^2*n)*x^2 + 2*(3*B*a*b^2*d^3*f^2 - 3*B*a^2*b*d^3*f*g + B*a^3*d^3*g^2)*n*log
(b*x + a) - 2*(3*B*b^3*c*d^2*f^2 - 3*B*b^3*c^2*d*f*g + B*b^3*c^3*g^2)*n*log
(d*x + c) + 2*(3*A*b^3*d^3*f^2 - (3*(B*b^3*c*d^2 - B*a*b^2*d^3)*f*g - (B*b^
3*c^2*d - B*a^2*b*d^3)*g^2)*n)*x + 2*(B*b^3*d^3*g^2*x^3 + 3*B*b^3*d^3*f*g*x
^2 + 3*B*b^3*d^3*f^2*x)*log(e) + 2*(B*b^3*d^3*g^2*n*x^3 + 3*B*b^3*d^3*f*g*n
*x^2 + 3*B*b^3*d^3*f^2*n*x)*log((b*x + a)/(d*x + c)))/(b^3*d^3)
```

**giac [B]** time = 5.58, size = 3346, normalized size = 21.31

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^2*(A+B*log(e*((b*x+a)/(d*x+c))^n)),x, algorithm="giac")
```

```
[Out] 1/6*(2*(3*B*b^4*c^2*d^2*f^2*n - 6*B*a*b^3*c*d^3*f^2*n - 6*(b*x + a)*B*b^3*c
^2*d^3*f^2*n/(d*x + c) + 3*B*a^2*b^2*d^4*f^2*n + 12*(b*x + a)*B*a*b^2*c*d^4
*f^2*n/(d*x + c) + 3*(b*x + a)^2*B*b^2*c^2*d^4*f^2*n/(d*x + c)^2 - 6*(b*x +
 a)*B*a^2*b*d^5*f^2*n/(d*x + c) - 6*(b*x + a)^2*B*a*b*c*d^5*f^2*n/(d*x + c)
^2 + 3*(b*x + a)^2*B*a^2*d^6*f^2*n/(d*x + c)^2 - 3*B*b^4*c^3*d*f*g*n + 3*B*
a*b^3*c^2*d^2*f*g*n + 9*(b*x + a)*B*b^3*c^3*d^2*f*g*n/(d*x + c) + 3*B*a^2*b
^2*c*d^3*f*g*n - 15*(b*x + a)*B*a*b^2*c^2*d^3*f*g*n/(d*x + c) - 6*(b*x + a)
^2*B*b^2*c^3*d^3*f*g*n/(d*x + c)^2 - 3*B*a^3*b*d^4*f*g*n + 3*(b*x + a)*B*a^
2*b*c*d^4*f*g*n/(d*x + c) + 12*(b*x + a)^2*B*a*b*c^2*d^4*f*g*n/(d*x + c)^2
+ 3*(b*x + a)*B*a^3*d^5*f*g*n/(d*x + c) - 6*(b*x + a)^2*B*a^2*c*d^5*f*g*n/(
d*x + c)^2 + B*b^4*c^4*g^2*n - B*a*b^3*c^3*d*g^2*n - 3*(b*x + a)*B*b^3*c^4*
d*g^2*n/(d*x + c) + 3*(b*x + a)*B*a*b^2*c^3*d^2*g^2*n/(d*x + c) + 3*(b*x +
a)^2*B*b^2*c^4*d^2*g^2*n/(d*x + c)^2 - B*a^3*b*c*d^3*g^2*n + 3*(b*x + a)*B*
a^2*b*c^2*d^3*g^2*n/(d*x + c) - 6*(b*x + a)^2*B*a*b*c^3*d^3*g^2*n/(d*x + c)
^2 + B*a^4*d^4*g^2*n - 3*(b*x + a)*B*a^3*c*d^4*g^2*n/(d*x + c) + 3*(b*x + a
)^2*B*a^2*c^2*d^4*g^2*n/(d*x + c)^2)*log((b*x + a)/(d*x + c))/(b^3*d^3 - 3*
(b*x + a)*b^2*d^4/(d*x + c) + 3*(b*x + a)^2*b*d^5/(d*x + c)^2 - (b*x + a)^3
*d^6/(d*x + c)^3) - (6*B*b^6*c^3*d*f*g*n - 18*B*a*b^5*c^2*d^2*f*g*n - 12*(b
*x + a)*B*b^5*c^3*d^2*f*g*n/(d*x + c) + 18*B*a^2*b^4*c*d^3*f*g*n + 36*(b*x
+ a)*B*a*b^4*c^2*d^3*f*g*n/(d*x + c) + 6*(b*x + a)^2*B*b^4*c^3*d^3*f*g*n/(d
*x + c)^2 - 6*B*a^3*b^3*d^4*f*g*n - 36*(b*x + a)*B*a^2*b^3*c*d^4*f*g*n/(d*x
 + c) - 18*(b*x + a)^2*B*a*b^3*c^2*d^4*f*g*n/(d*x + c)^2 + 12*(b*x + a)*B*a
^3*b^2*d^5*f*g*n/(d*x + c) + 18*(b*x + a)^2*B*a^2*b^2*c*d^5*f*g*n/(d*x + c)
^2 - 6*(b*x + a)^2*B*a^3*b*d^6*f*g*n/(d*x + c)^2 - 3*B*b^6*c^4*g^2*n + 6*B*
a*b^5*c^3*d*g^2*n + 7*(b*x + a)*B*b^5*c^4*d*g^2*n/(d*x + c) - 16*(b*x + a)*
B*a*b^4*c^3*d^2*g^2*n/(d*x + c) - 4*(b*x + a)^2*B*b^4*c^4*d^2*g^2*n/(d*x +
c)^2 - 6*B*a^3*b^3*c*d^3*g^2*n + 6*(b*x + a)*B*a^2*b^3*c^2*d^3*g^2*n/(d*x +
 c) + 10*(b*x + a)^2*B*a*b^3*c^3*d^3*g^2*n/(d*x + c)^2 + 3*B*a^4*b^2*d^4*g^
```

```
2*n + 8*(b*x + a)*B*a^3*b^2*c*d^4*g^2*n/(d*x + c) - 6*(b*x + a)^2*B*a^2*b^2
*c^2*d^4*g^2*n/(d*x + c)^2 - 5*(b*x + a)*B*a^4*b*d^5*g^2*n/(d*x + c) - 2*(b
*x + a)^2*B*a^3*b*c*d^5*g^2*n/(d*x + c)^2 + 2*(b*x + a)^2*B*a^4*d^6*g^2*n/(
d*x + c)^2 - 6*A*b^6*c^2*d^2*f^2 - 6*B*b^6*c^2*d^2*f^2 + 12*A*a*b^5*c*d^3*f
^2 + 12*B*a*b^5*c*d^3*f^2 + 12*(b*x + a)*A*b^5*c^2*d^3*f^2/(d*x + c) + 12*(
b*x + a)*B*b^5*c^2*d^3*f^2/(d*x + c) - 6*A*a^2*b^4*d^4*f^2 - 6*B*a^2*b^4*d^
4*f^2 - 24*(b*x + a)*A*a*b^4*c*d^4*f^2/(d*x + c) - 24*(b*x + a)*B*a*b^4*c*d
^4*f^2/(d*x + c) - 6*(b*x + a)^2*A*b^4*c^2*d^4*f^2/(d*x + c)^2 - 6*(b*x + a
)^2*B*b^4*c^2*d^4*f^2/(d*x + c)^2 + 12*(b*x + a)*A*a^2*b^3*d^5*f^2/(d*x + c
) + 12*(b*x + a)*B*a^2*b^3*d^5*f^2/(d*x + c) + 12*(b*x + a)^2*A*a*b^3*c*d^5
*f^2/(d*x + c)^2 + 12*(b*x + a)^2*B*a*b^3*c*d^5*f^2/(d*x + c)^2 - 6*(b*x +
a)^2*A*a^2*b^2*d^6*f^2/(d*x + c)^2 - 6*(b*x + a)^2*B*a^2*b^2*d^6*f^2/(d*x +
 c)^2 + 6*A*b^6*c^3*d*f*g + 6*B*b^6*c^3*d*f*g - 6*A*a*b^5*c^2*d^2*f*g - 6*B
*a*b^5*c^2*d^2*f*g - 18*(b*x + a)*A*b^5*c^3*d^2*f*g/(d*x + c) - 18*(b*x + a
)*B*b^5*c^3*d^2*f*g/(d*x + c) - 6*A*a^2*b^4*c*d^3*f*g - 6*B*a^2*b^4*c*d^3*f
*g + 30*(b*x + a)*A*a*b^4*c^2*d^3*f*g/(d*x + c) + 30*(b*x + a)*B*a*b^4*c^2*
d^3*f*g/(d*x + c) + 12*(b*x + a)^2*A*b^4*c^3*d^3*f*g/(d*x + c)^2 + 12*(b*x
+ a)^2*B*b^4*c^3*d^3*f*g/(d*x + c)^2 + 6*A*a^3*b^3*d^4*f*g + 6*B*a^3*b^3*d^
4*f*g - 6*(b*x + a)*A*a^2*b^3*c*d^4*f*g/(d*x + c) - 6*(b*x + a)*B*a^2*b^3*c
*d^4*f*g/(d*x + c) - 24*(b*x + a)^2*A*a*b^3*c^2*d^4*f*g/(d*x + c)^2 - 24*(b
*x + a)^2*B*a*b^3*c^2*d^4*f*g/(d*x + c)^2 - 6*(b*x + a)*A*a^3*b^2*d^5*f*g/(
d*x + c) - 6*(b*x + a)*B*a^3*b^2*d^5*f*g/(d*x + c) + 12*(b*x + a)^2*A*a^2*b
^2*c*d^5*f*g/(d*x + c)^2 + 12*(b*x + a)^2*B*a^2*b^2*c*d^5*f*g/(d*x + c)^2 -
 2*A*b^6*c^4*g^2 - 2*B*b^6*c^4*g^2 + 2*A*a*b^5*c^3*d*g^2 + 2*B*a*b^5*c^3*d*
g^2 + 6*(b*x + a)*A*b^5*c^4*d*g^2/(d*x + c) + 6*(b*x + a)*B*b^5*c^4*d*g^2/(
d*x + c) - 6*(b*x + a)*A*a*b^4*c^3*d^2*g^2/(d*x + c) - 6*(b*x + a)*B*a*b^4*
c^3*d^2*g^2/(d*x + c) - 6*(b*x + a)^2*A*b^4*c^4*d^2*g^2/(d*x + c)^2 - 6*(b*
x + a)^2*B*b^4*c^4*d^2*g^2/(d*x + c)^2 + 2*A*a^3*b^3*c*d^3*g^2 + 2*B*a^3*b^
3*c*d^3*g^2 - 6*(b*x + a)*A*a^2*b^3*c^2*d^3*g^2/(d*x + c) - 6*(b*x + a)*B*a
^2*b^3*c^2*d^3*g^2/(d*x + c) + 12*(b*x + a)^2*A*a*b^3*c^3*d^3*g^2/(d*x + c)
^2 + 12*(b*x + a)^2*B*a*b^3*c^3*d^3*g^2/(d*x + c)^2 - 2*A*a^4*b^2*d^4*g^2 -
 2*B*a^4*b^2*d^4*g^2 + 6*(b*x + a)*A*a^3*b^2*c*d^4*g^2/(d*x + c) + 6*(b*x +
 a)*B*a^3*b^2*c*d^4*g^2/(d*x + c) - 6*(b*x + a)^2*A*a^2*b^2*c^2*d^4*g^2/(d*
x + c)^2 - 6*(b*x + a)^2*B*a^2*b^2*c^2*d^4*g^2/(d*x + c)^2)/(b^5*d^3 - 3*(b
*x + a)*b^4*d^4/(d*x + c) + 3*(b*x + a)^2*b^3*d^5/(d*x + c)^2 - (b*x + a)^3
*b^2*d^6/(d*x + c)^3) + 2*(3*B*b^4*c^2*d^2*f^2*n - 6*B*a*b^3*c*d^3*f^2*n +
3*B*a^2*b^2*d^4*f^2*n - 3*B*b^4*c^3*d*f*g*n + 3*B*a*b^3*c^2*d^2*f*g*n + 3*B
*a^2*b^2*c*d^3*f*g*n - 3*B*a^3*b*d^4*f*g*n + B*b^4*c^4*g^2*n - B*a*b^3*c^3*
d*g^2*n - B*a^3*b*c*d^3*g^2*n + B*a^4*d^4*g^2*n)*log(b - (b*x + a)*d/(d*x +
 c))/(b^3*d^3) - 2*(3*B*b^4*c^2*d^2*f^2*n - 6*B*a*b^3*c*d^3*f^2*n + 3*B*a^2
*b^2*d^4*f^2*n - 3*B*b^4*c^3*d*f*g*n + 3*B*a*b^3*c^2*d^2*f*g*n + 3*B*a^2*b^
2*c*d^3*f*g*n - 3*B*a^3*b*d^4*f*g*n + B*b^4*c^4*g^2*n - B*a*b^3*c^3*d*g^2*n
 - B*a^3*b*c*d^3*g^2*n + B*a^4*d^4*g^2*n)*log((b*x + a)/(d*x + c))/(b^3*d^3
))*(b*c/(b*c - a*d)^2 - a*d/(b*c - a*d)^2)
```

**maple [F]** time = 0.27, size = 0, normalized size = 0.00

$$\int (gx+f)^2 \left(B\ln\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x+f)^2*(B*ln(e*((b*x+a)/(d*x+c))^n)+A),x)
```

```
[Out] int((g*x+f)^2*(B*ln(e*((b*x+a)/(d*x+c))^n)+A),x)
```

**maxima [A]** time = 0.89, size = 282, normalized size = 1.80

$$\frac{1}{3}Bg^2x^3\log\left(e\left(\frac{bx}{dx+c}+\frac{a}{dx+c}\right)^n\right)+\frac{1}{3}Ag^2x^3+Bfgx^2\log\left(e\left(\frac{bx}{dx+c}+\frac{a}{dx+c}\right)^n\right)+Afgx^2+\frac{1}{6}Bg^2n\left(\frac{2a^3\log(bx+}{b^3}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^2*(A+B*log(e*((b*x+a)/(d*x+c))^n)),x, algorithm="maxima")
```

```
[Out] 1/3*B*g^2*x^3*log(e*(b*x/(d*x + c) + a/(d*x + c))^n) + 1/3*A*g^2*x^3 + B*f*
g*x^2*log(e*(b*x/(d*x + c) + a/(d*x + c))^n) + A*f*g*x^2 + 1/6*B*g^2*n*(2*a
^3*log(b*x + a)/b^3 - 2*c^3*log(d*x + c)/d^3 - ((b^2*c*d - a*b*d^2)*x^2 - 2
*(b^2*c^2 - a^2*d^2)*x)/(b^2*d^2)) - B*f*g*n*(a^2*log(b*x + a)/b^2 - c^2*lo
g(d*x + c)/d^2 + (b*c - a*d)*x/(b*d)) + B*f^2*n*(a*log(b*x + a)/b - c*log(d
*x + c)/d) + B*f^2*x*log(e*(b*x/(d*x + c) + a/(d*x + c))^n) + A*f^2*x
```

**mupad [B]** time = 4.18, size = 371, normalized size = 2.36

$$x^2\left(\frac{3\,A\,a\,d\,g^2+3\,A\,b\,c\,g^2+6\,A\,b\,d\,f\,g+B\,a\,d\,g^2\,n-B\,b\,c\,g^2\,n}{6\,b\,d}-\frac{A\,g^2\,(3\,a\,d+3\,b\,c)}{6\,b\,d}\right)-x\left(\frac{(3\,a\,d+3\,b\,c)\left(\frac{3\,A\,a\,d\,g^2}{}\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f + g*x)^2*(A + B*log(e*((a + b*x)/(c + d*x))^n)),x)
```

```
[Out] x^2*((3*A*a*d*g^2 + 3*A*b*c*g^2 + 6*A*b*d*f*g + B*a*d*g^2*n - B*b*c*g^2*n)/
(6*b*d) - (A*g^2*(3*a*d + 3*b*c))/(6*b*d)) - x*(((3*a*d + 3*b*c)*((3*A*a*d*
g^2 + 3*A*b*c*g^2 + 6*A*b*d*f*g + B*a*d*g^2*n - B*b*c*g^2*n)/(3*b*d) - (A*g
^2*(3*a*d + 3*b*c))/(3*b*d)))/(3*b*d) - (3*A*a*c*g^2 + 3*A*b*d*f^2 + 6*A*a*
d*f*g + 6*A*b*c*f*g + 3*B*a*d*f*g*n - 3*B*b*c*f*g*n)/(3*b*d) + (A*a*c*g^2)/
(b*d)) + log(e*((a + b*x)/(c + d*x))^n)*((B*g^2*x^3)/3 + B*f^2*x + B*f*g*x^
2) + (A*g^2*x^3)/3 + (log(a + b*x)*(B*a^3*g^2*n + 3*B*a*b^2*f^2*n - 3*B*a^2
*b*f*g*n))/(3*b^3) - (log(c + d*x)*(B*c^3*g^2*n + 3*B*c*d^2*f^2*n - 3*B*c^2
*d*f*g*n))/(3*d^3)
```

**sympy [A]** time = 70.52, size = 1027, normalized size = 6.54

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)**2*(A+B*ln(e*((b*x+a)/(d*x+c))**n)),x)
```

```
[Out] Piecewise(((A + B*log(e*(a/c)**n))*(f**2*x + f*g*x**2 + g**2*x**3/3), Eq(b,
 0) & Eq(d, 0)), (A*f**2*x + A*f*g*x**2 + A*g**2*x**3/3 + B*a**3*g**2*n*log
(a/c + b*x/c)/(3*b**3) - B*a**2*f*g*n*log(a/c + b*x/c)/b**2 - B*a**2*g**2*n
*x/(3*b**2) + B*a*f**2*n*log(a/c + b*x/c)/b + B*a*f*g*n*x/b + B*a*g**2*n*x*
*2/(6*b) + B*f**2*n*x*log(a/c + b*x/c) - B*f**2*n*x + B*f**2*x*log(e) + B*f
*g*n*x**2*log(a/c + b*x/c) - B*f*g*n*x**2/2 + B*f*g*x**2*log(e) + B*g**2*n*
x**3*log(a/c + b*x/c)/3 - B*g**2*n*x**3/9 + B*g**2*x**3*log(e)/3, Eq(d, 0))
, (A*f**2*x + A*f*g*x**2 + A*g**2*x**3/3 - B*c**3*g**2*n*log(c + d*x)/(3*d*
*3) + B*c**2*f*g*n*log(c + d*x)/d**2 + B*c**2*g**2*n*x/(3*d**2) - B*c*f**2*
n*log(c + d*x)/d - B*c*f*g*n*x/d - B*c*g**2*n*x**2/(6*d) + B*f**2*n*x*log(a
) - B*f**2*n*x*log(c + d*x) + B*f**2*n*x + B*f**2*x*log(e) + B*f*g*n*x**2*l
og(a) - B*f*g*n*x**2*log(c + d*x) + B*f*g*n*x**2/2 + B*f*g*x**2*log(e) + B*
g**2*n*x**3*log(a)/3 - B*g**2*n*x**3*log(c + d*x)/3 + B*g**2*n*x**3/9 + B*g
**2*x**3*log(e)/3, Eq(b, 0)), (A*f**2*x + A*f*g*x**2 + A*g**2*x**3/3 + B*a*
*3*g**2*n*log(a/(c + d*x) + b*x/(c + d*x))/(3*b**3) + B*a**3*g**2*n*log(c/d
 + x)/(3*b**3) - B*a**2*f*g*n*log(a/(c + d*x) + b*x/(c + d*x))/b**2 - B*a**
2*f*g*n*log(c/d + x)/b**2 - B*a**2*g**2*n*x/(3*b**2) + B*a*f**2*n*log(a/(c
+ d*x) + b*x/(c + d*x))/b + B*a*f**2*n*log(c/d + x)/b + B*a*f*g*n*x/b + B*a
*g**2*n*x**2/(6*b) - B*c**3*g**2*n*log(c/d + x)/(3*d**3) + B*c**2*f*g*n*log
(c/d + x)/d**2 + B*c**2*g**2*n*x/(3*d**2) - B*c*f**2*n*log(c/d + x)/d - B*c
*f*g*n*x/d - B*c*g**2*n*x**2/(6*d) + B*f**2*n*x*log(a/(c + d*x) + b*x/(c +
d*x)) + B*f**2*x*log(e) + B*f*g*n*x**2*log(a/(c + d*x) + b*x/(c + d*x)) + B
*f*g*x**2*log(e) + B*g**2*n*x**3*log(a/(c + d*x) + b*x/(c + d*x))/3 + B*g**
2*x**3*log(e)/3, True))
```

## 3.60 $\int (f+gx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\,dx$

**Optimal**. Leaf size=115

$$\frac{(f+gx)^2\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{2g}-\frac{Bn(bf-ag)^2\log(a+bx)}{2b^2g}-\frac{Bgnx(bc-ad)}{2bd}+\frac{Bn(df-cg)^2\log(c+dx)}{2d^2g}$$

```
[Out] -1/2*B*(-a*d+b*c)*g*n*x/b/d-1/2*B*(-a*g+b*f)^2*n*ln(b*x+a)/b^2/g+1/2*(g*x+f
)^2*(A+B*ln(e*((b*x+a)/(d*x+c))^n))/g+1/2*B*(-c*g+d*f)^2*n*ln(d*x+c)/d^2/g
```

**Rubi [A]** time = 0.10, antiderivative size = 115, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.107, Rules used = {2525, 12, 72}

$$\frac{(f+gx)^2\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{2g}-\frac{Bn(bf-ag)^2\log(a+bx)}{2b^2g}-\frac{Bgnx(bc-ad)}{2bd}+\frac{Bn(df-cg)^2\log(c+dx)}{2d^2g}$$

Antiderivative was successfully verified.

```
[In] Int[(f + g*x)*(A + B*Log[e*((a + b*x)/(c + d*x))^n]),x]
```

```
[Out] -(B*(b*c - a*d)*g*n*x)/(2*b*d) - (B*(b*f - a*g)^2*n*Log[a + b*x])/(2*b^2*g)
 + ((f + g*x)^2*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/(2*g) + (B*(d*f - c
*g)^2*n*Log[c + d*x])/(2*d^2*g)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 72

```
Int[((e_.) + (f_.)*(x_))^(p_.)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
 x_Symbol] :> Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x]
 /; FreeQ[{a, b, c, d, e, f}, x] && IntegerQ[p]
```

Rule 2525

```
Int[((a_.) + Log[(c_.)*(RFx_)^(p_.)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_))^(m_.
), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(a + b*Log[c*RFx^p])^n)/(e*(m + 1))
, x] - Dist[(b*n*p)/(e*(m + 1)), Int[SimplifyIntegrand[((d + e*x)^(m + 1)*(
a + b*Log[c*RFx^p])^(n - 1)*D[RFx, x])/RFx, x], x], x] /; FreeQ[{a, b, c, d
```

```
, e, m, p}, x] && RationalFunctionQ[RFx, x] && IGtQ[n, 0] && (EqQ[n, 1] ||
IntegerQ[m]) && NeQ[m, -1]
```

Rubi steps

$$\int (f+gx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)dx = \frac{(f+gx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{2g} - \frac{(Bn)\int\frac{(bc-ad)(f+gx)^2}{(a+bx)(c+dx)}dx}{2g}$$

$$= \frac{(f+gx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{2g} - \frac{(B(bc-ad)n)\int\frac{(f+gx)^2}{(a+bx)(c+dx)}dx}{2g}$$

$$= \frac{(f+gx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{2g} - \frac{(B(bc-ad)n)\int\left(\frac{g^2}{bd}+\frac{(bf-ag}{b(bc-ad)(a}\right.}{2g}$$

$$= -\frac{B(bc-ad)gnx}{2bd} - \frac{B(bf-ag)^2n\log(a+bx)}{2b^2g} + \frac{(f+gx)^2\left(A+B\log\right.}{2g}$$

**Mathematica [A]** time = 0.13, size = 120, normalized size = 1.04

$$\frac{b\left(d\left(Bg^2nx(ad-bc)+Abd(f+gx)^2\right)+bBd^2(f+gx)^2\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+bBn(df-cg)^2\log(c+dx)\right)-Bd^2n(b}{2b^2d^2g}$$

Antiderivative was successfully verified.

```
[In] Integrate[(f + g*x)*(A + B*Log[e*((a + b*x)/(c + d*x))^n]),x]
```

```
[Out] (-(B*d^2*(b*f - a*g)^2*n*Log[a + b*x]) + b*(d*(B*(-(b*c) + a*d)*g^2*n*x + A
*b*d*(f + g*x)^2) + b*B*d^2*(f + g*x)^2*Log[e*((a + b*x)/(c + d*x))^n] + b*
B*(d*f - c*g)^2*n*Log[c + d*x]))/(2*b^2*d^2*g)
```

**fricas [A]** time = 1.17, size = 179, normalized size = 1.56

$$\frac{Ab^2d^2gx^2+\left(2\,Babd^2f-Ba^2d^2g\right)n\log\left(bx+a\right)-\left(2\,Bb^2cdf-Bb^2c^2g\right)n\log\left(dx+c\right)+\left(2\,Ab^2d^2f-\left(Bb^2cd-B\right.\right.}{2\,b^2d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)*(A+B*log(e*((b*x+a)/(d*x+c))^n)),x, algorithm="fricas")
```

```
[Out] 1/2*(A*b^2*d^2*g*x^2 + (2*B*a*b*d^2*f - B*a^2*d^2*g)*n*log(b*x + a) - (2*B*
b^2*c*d*f - B*b^2*c^2*g)*n*log(d*x + c) + (2*A*b^2*d^2*f - (B*b^2*c*d - B*a
*b*d^2)*g*n)*x + (B*b^2*d^2*g*x^2 + 2*B*b^2*d^2*f*x)*log(e) + (B*b^2*d^2*g*
n*x^2 + 2*B*b^2*d^2*f*n*x)*log((b*x + a)/(d*x + c)))/(b^2*d^2)
```

**giac [B]** time = 2.37, size = 1189, normalized size = 10.34

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)*(A+B*log(e*((b*x+a)/(d*x+c))^n)),x, algorithm="giac")
```

```
[Out] 1/2*((2*B*b^3*c^2*d*f*n - 4*B*a*b^2*c*d^2*f*n - 2*(b*x + a)*B*b^2*c^2*d^2*f
*n/(d*x + c) + 2*B*a^2*b*d^3*f*n + 4*(b*x + a)*B*a*b*c*d^3*f*n/(d*x + c) -
2*(b*x + a)*B*a^2*d^4*f*n/(d*x + c) - B*b^3*c^3*g*n + B*a*b^2*c^2*d*g*n + 2
*(b*x + a)*B*b^2*c^3*d*g*n/(d*x + c) + B*a^2*b*c*d^2*g*n - 4*(b*x + a)*B*a*
b*c^2*d^2*g*n/(d*x + c) - B*a^3*d^3*g*n + 2*(b*x + a)*B*a^2*c*d^3*g*n/(d*x
+ c))*log((b*x + a)/(d*x + c))/(b^2*d^2 - 2*(b*x + a)*b*d^3/(d*x + c) + (b*
x + a)^2*d^4/(d*x + c)^2) - (B*b^4*c^3*g*n - 3*B*a*b^3*c^2*d*g*n - (b*x + a
)*B*b^3*c^3*d*g*n/(d*x + c) + 3*B*a^2*b^2*c*d^2*g*n + 3*(b*x + a)*B*a*b^2*c
^2*d^2*g*n/(d*x + c) - B*a^3*b*d^3*g*n - 3*(b*x + a)*B*a^2*b*c*d^3*g*n/(d*x
 + c) + (b*x + a)*B*a^3*d^4*g*n/(d*x + c) - 2*A*b^4*c^2*d*f - 2*B*b^4*c^2*d
*f + 4*A*a*b^3*c*d^2*f + 4*B*a*b^3*c*d^2*f + 2*(b*x + a)*A*b^3*c^2*d^2*f/(d
*x + c) + 2*(b*x + a)*B*b^3*c^2*d^2*f/(d*x + c) - 2*A*a^2*b^2*d^3*f - 2*B*a
^2*b^2*d^3*f - 4*(b*x + a)*A*a*b^2*c*d^3*f/(d*x + c) - 4*(b*x + a)*B*a*b^2*
c*d^3*f/(d*x + c) + 2*(b*x + a)*A*a^2*b*d^4*f/(d*x + c) + 2*(b*x + a)*B*a^2
*b*d^4*f/(d*x + c) + A*b^4*c^3*g + B*b^4*c^3*g - A*a*b^3*c^2*d*g - B*a*b^3*
c^2*d*g - 2*(b*x + a)*A*b^3*c^3*d*g/(d*x + c) - 2*(b*x + a)*B*b^3*c^3*d*g/(
d*x + c) - A*a^2*b^2*c*d^2*g - B*a^2*b^2*c*d^2*g + 4*(b*x + a)*A*a*b^2*c^2*
d^2*g/(d*x + c) + 4*(b*x + a)*B*a*b^2*c^2*d^2*g/(d*x + c) + A*a^3*b*d^3*g +
 B*a^3*b*d^3*g - 2*(b*x + a)*A*a^2*b*c*d^3*g/(d*x + c) - 2*(b*x + a)*B*a^2*
b*c*d^3*g/(d*x + c))/(b^3*d^2 - 2*(b*x + a)*b^2*d^3/(d*x + c) + (b*x + a)^2
*b*d^4/(d*x + c)^2) + (2*B*b^3*c^2*d*f*n - 4*B*a*b^2*c*d^2*f*n + 2*B*a^2*b*
d^3*f*n - B*b^3*c^3*g*n + B*a*b^2*c^2*d*g*n + B*a^2*b*c*d^2*g*n - B*a^3*d^3
*g*n)*log(-b + (b*x + a)*d/(d*x + c))/(b^2*d^2) - (2*B*b^3*c^2*d*f*n - 4*B*
a*b^2*c*d^2*f*n + 2*B*a^2*b*d^3*f*n - B*b^3*c^3*g*n + B*a*b^2*c^2*d*g*n + B
*a^2*b*c*d^2*g*n - B*a^3*d^3*g*n)*log((b*x + a)/(d*x + c))/(b^2*d^2))*(b*c/
(b*c - a*d)^2 - a*d/(b*c - a*d)^2)
```

**maple [F]** time = 0.18, size = 0, normalized size = 0.00

$$\int \left(gx+f\right)\left(B\ln\left(e\left(\frac{bx+a}{dx+c}\right)^{n}\right)+A\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x+f)*(B*ln(e*((b*x+a)/(d*x+c))^n)+A),x)
```

```
[Out] int((g*x+f)*(B*ln(e*((b*x+a)/(d*x+c))^n)+A),x)
```

**maxima [A]** time = 0.76, size = 150, normalized size = 1.30

$$\frac{1}{2}Bgx^2\log\left(e\left(\frac{bx}{dx+c}+\frac{a}{dx+c}\right)^n\right)+\frac{1}{2}Agx^2-\frac{1}{2}Bgn\left(\frac{a^2\log(bx+a)}{b^2}-\frac{c^2\log(dx+c)}{d^2}+\frac{(bc-ad)x}{bd}\right)+Bfn\left(\frac{a\log}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)*(A+B*log(e*((b*x+a)/(d*x+c))^n)),x, algorithm="maxima")
```

```
[Out] 1/2*B*g*x^2*log(e*(b*x/(d*x + c) + a/(d*x + c))^n) + 1/2*A*g*x^2 - 1/2*B*g*
n*(a^2*log(b*x + a)/b^2 - c^2*log(d*x + c)/d^2 + (b*c - a*d)*x/(b*d)) + B*f
*n*(a*log(b*x + a)/b - c*log(d*x + c)/d) + B*f*x*log(e*(b*x/(d*x + c) + a/(
d*x + c))^n) + A*f*x
```

**mupad [B]** time = 4.26, size = 153, normalized size = 1.33

$$x\left(\frac{2\,A\,a\,d\,g+2\,A\,b\,c\,g+2\,A\,b\,d\,f+B\,a\,d\,g\,n-B\,b\,c\,g\,n}{2\,b\,d}-\frac{A\,g\,(2\,a\,d+2\,b\,c)}{2\,b\,d}\right)+\ln\left(e\left(\frac{a+b\,x}{c+d\,x}\right)^n\right)\left(\frac{B\,g\,x^2}{2}+B\,f\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f + g*x)*(A + B*log(e*((a + b*x)/(c + d*x))^n)),x)
```

```
[Out] x*((2*A*a*d*g + 2*A*b*c*g + 2*A*b*d*f + B*a*d*g*n - B*b*c*g*n)/(2*b*d) - (A
*g*(2*a*d + 2*b*c))/(2*b*d)) + log(e*((a + b*x)/(c + d*x))^n)*(B*f*x + (B*g
*x^2)/2) - (log(a + b*x)*(B*a^2*g*n - 2*B*a*b*f*n))/(2*b^2) + (log(c + d*x)
*(B*c^2*g*n - 2*B*c*d*f*n))/(2*d^2) + (A*g*x^2)/2
```

**sympy [A]** time = 44.18, size = 551, normalized size = 4.79

$$\begin{cases}\left(A+B\log\left(e\left(\frac{a}{c}\right)^n\right)\right)\left(fx+\frac{gx^2}{2}\right)\\ Afx+\frac{Agx^2}{2}+\frac{Bc^2gn\log(c+dx)}{2d^2}-\frac{Bcfn\log(c+dx)}{d}-\frac{Bcgnx}{2d}+Bfnx\log(a)-Bfnx\log(c+dx)+Bfnx+Bfx\log(e)\\ Afx+\frac{Agx^2}{2}-\frac{Ba^2gn\log\left(\frac{a}{c}+\frac{bx}{c}\right)}{2b^2}+\frac{Bafn\log\left(\frac{a}{c}+\frac{bx}{c}\right)}{b}+\frac{Bagnx}{2b}+Bfnx\log\left(\frac{a}{c}+\frac{bx}{c}\right)-Bfnx+Bfx\log(e)+\frac{Bgnx^2\log\left(\frac{a}{c}\right.}{2}\\ Afx+\frac{Agx^2}{2}-\frac{Ba^2gn\log\left(\frac{a}{c+dx}+\frac{bx}{c+dx}\right)}{2b^2}-\frac{Ba^2gn\log\left(\frac{c}{d}+x\right)}{2b^2}+\frac{Bafn\log\left(\frac{a}{c+dx}+\frac{bx}{c+dx}\right)}{b}+\frac{Bafn\log\left(\frac{c}{d}+x\right)}{b}+\frac{Bagnx}{2b}+\frac{Bc^2gn\log\left(\frac{c}{d}+x\right)}{2d^2}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)*(A+B*ln(e*((b*x+a)/(d*x+c))**n)),x)
```

```
[Out] Piecewise(((A + B*log(e*(a/c)**n))*(f*x + g*x**2/2), Eq(b, 0) & Eq(d, 0)),
(A*f*x + A*g*x**2/2 + B*c**2*g*n*log(c + d*x)/(2*d**2) - B*c*f*n*log(c + d*
x)/d - B*c*g*n*x/(2*d) + B*f*n*x*log(a) - B*f*n*x*log(c + d*x) + B*f*n*x +
B*f*x*log(e) + B*g*n*x**2*log(a)/2 - B*g*n*x**2*log(c + d*x)/2 + B*g*n*x**2
/4 + B*g*x**2*log(e)/2, Eq(b, 0)), (A*f*x + A*g*x**2/2 - B*a**2*g*n*log(a/c
 + b*x/c)/(2*b**2) + B*a*f*n*log(a/c + b*x/c)/b + B*a*g*n*x/(2*b) + B*f*n*x
*log(a/c + b*x/c) - B*f*n*x + B*f*x*log(e) + B*g*n*x**2*log(a/c + b*x/c)/2
- B*g*n*x**2/4 + B*g*x**2*log(e)/2, Eq(d, 0)), (A*f*x + A*g*x**2/2 - B*a**2
*g*n*log(a/(c + d*x) + b*x/(c + d*x))/(2*b**2) - B*a**2*g*n*log(c/d + x)/(2
*b**2) + B*a*f*n*log(a/(c + d*x) + b*x/(c + d*x))/b + B*a*f*n*log(c/d + x)/
b + B*a*g*n*x/(2*b) + B*c**2*g*n*log(c/d + x)/(2*d**2) - B*c*f*n*log(c/d +
x)/d - B*c*g*n*x/(2*d) + B*f*n*x*log(a/(c + d*x) + b*x/(c + d*x)) + B*f*x*l
og(e) + B*g*n*x**2*log(a/(c + d*x) + b*x/(c + d*x))/2 + B*g*x**2*log(e)/2,
True))
```

# 3.61 $\int \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)dx$

**Optimal**. Leaf size=56

$$\frac{B(a+bx)\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{b}-\frac{Bn(bc-ad)\log(c+dx)}{bd}+Ax$$

```
[Out] A*x+B*(b*x+a)*ln(e*((b*x+a)/(d*x+c))^n)/b-B*(-a*d+b*c)*n*ln(d*x+c)/b/d
```

**Rubi [A]** time = 0.03, antiderivative size = 56, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {2486, 31}

$$\frac{B(a+bx)\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{b}-\frac{Bn(bc-ad)\log(c+dx)}{bd}+Ax$$

Antiderivative was successfully verified.

```
[In] Int[A + B*Log[e*((a + b*x)/(c + d*x))^n],x]
```

```
[Out] A*x + (B*(a + b*x)*Log[e*((a + b*x)/(c + d*x))^n])/b - (B*(b*c - a*d)*n*Log
[c + d*x])/(b*d)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 2486

```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))
^(r_.)]^(s_.), x_Symbol] :> Simp[((a + b*x)*Log[e*(f*(a + b*x)^p*(c + d*x)^
q)^r]^s)/b, x] + Dist[(q*r*s*(b*c - a*d))/b, Int[Log[e*(f*(a + b*x)^p*(c +
d*x)^q)^r]^(s - 1)/(c + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, p, q, r, s
}, x] && NeQ[b*c - a*d, 0] && EqQ[p + q, 0] && IGtQ[s, 0]
```

Rubi steps

$$\int \left(A + B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)dx = Ax + B\int \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)dx$$
$$= Ax + \frac{B(a+bx)\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{b} - \frac{(B(bc-ad)n)\int \frac{1}{c+dx}\,dx}{b}$$
$$= Ax + \frac{B(a+bx)\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{b} - \frac{B(bc-ad)n\log(c+dx)}{bd}$$

**Mathematica [A]** time = 0.01, size = 56, normalized size = 1.00

$$\frac{B(a+bx)\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{b} - \frac{Bn(bc-ad)\log(c+dx)}{bd} + Ax$$

Antiderivative was successfully verified.

```
[In] Integrate[A + B*Log[e*((a + b*x)/(c + d*x))^n],x]
```

```
[Out] A*x + (B*(a + b*x)*Log[e*((a + b*x)/(c + d*x))^n])/b - (B*(b*c - a*d)*n*Log
[c + d*x])/(b*d)
```

**fricas [A]** time = 0.92, size = 63, normalized size = 1.12

$$\frac{Bbdnx\log\left(\frac{bx+a}{dx+c}\right) + Badn\log(bx+a) - Bbcn\log(dx+c) + Bbdx\log(e) + Abdx}{bd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(A+B*log(e*((b*x+a)/(d*x+c))^n),x, algorithm="fricas")
```

```
[Out] (B*b*d*n*x*log((b*x + a)/(d*x + c)) + B*a*d*n*log(b*x + a) - B*b*c*n*log(d*
x + c) + B*b*d*x*log(e) + A*b*d*x)/(b*d)
```

**giac [B]** time = 0.94, size = 237, normalized size = 4.23

$$B\left(\frac{\left(b^2c^2n - 2\,abcdn + a^2d^2n\right)\log\left(\frac{bx+a}{dx+c}\right)}{bd - \frac{(bx+a)d^2}{dx+c}} + \frac{b^2c^2 - 2\,abcd + a^2d^2}{bd - \frac{(bx+a)d^2}{dx+c}} + \frac{\left(b^2c^2n - 2\,abcdn + a^2d^2n\right)\log\left(b - \frac{(bx+a)d}{dx+c}\right)}{bd} - \right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(A+B*log(e*((b*x+a)/(d*x+c))^n),x, algorithm="giac")
```

```
[Out] B*((b^2*c^2*n - 2*a*b*c*d*n + a^2*d^2*n)*log((b*x + a)/(d*x + c))/(b*d - (b
*x + a)*d^2/(d*x + c)) + (b^2*c^2 - 2*a*b*c*d + a^2*d^2)/(b*d - (b*x + a)*d
^2/(d*x + c)) + (b^2*c^2*n - 2*a*b*c*d*n + a^2*d^2*n)*log(b - (b*x + a)*d/(
d*x + c))/(b*d) - (b^2*c^2*n - 2*a*b*c*d*n + a^2*d^2*n)*log((b*x + a)/(d*x
+ c))/(b*d))*(b*c/(b*c - a*d)^2 - a*d/(b*c - a*d)^2) + A*x
```

**maple [B]** time = 0.05, size = 122, normalized size = 2.18

$$\frac{B\,a^{2} d n \ln \left(b x +a \right)}{\left(a d -b c \right) b}-\frac{B a c n \ln \left(b x +a \right)}{a d -b c}-\frac{B a c n \ln \left(d x +c \right)}{a d -b c}+\frac{B b\,c^{2} n \ln \left(d x +c \right)}{\left(a d -b c \right) d}+B x \ln \left(e \left(\frac{b x +a}{d x +c}\right)^{n}\right)+A x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(B*ln(e*((b*x+a)/(d*x+c))^n)+A,x)
```

```
[Out] A*x+B*x*ln(e*((b*x+a)/(d*x+c))^n)-B*n*c/(a*d-b*c)*ln(d*x+c)*a+B*n*c^2/(a*d-
b*c)/d*ln(d*x+c)*b+B*n*a^2/(a*d-b*c)/b*ln(b*x+a)*d-B*n*a/(a*d-b*c)*ln(b*x+a
)*c
```

**maxima [A]** time = 0.63, size = 52, normalized size = 0.93

$$B n \left(\frac{a \log \left(b x + a\right)}{b} - \frac{c \log \left(d x + c\right)}{d}\right) + B x \log \left(e \left(\frac{b x + a}{d x + c}\right)^{n}\right) + A x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(A+B*log(e*((b*x+a)/(d*x+c))^n),x, algorithm="maxima")
```

```
[Out] B*n*(a*log(b*x + a)/b - c*log(d*x + c)/d) + B*x*log(e*((b*x + a)/(d*x + c))
^n) + A*x
```

**mupad [B]** time = 4.01, size = 52, normalized size = 0.93

$$A\,x+B\,x\,\ln\left(e\,{\left(\frac{a+b\,x}{c+d\,x}\right)}^n\right)+\frac{B\,a\,n\,\ln\left(a+b\,x\right)}{b}-\frac{B\,c\,n\,\ln\left(c+d\,x\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(A + B*log(e*((a + b*x)/(c + d*x))^n),x)
```

```
[Out] A*x + B*x*log(e*((a + b*x)/(c + d*x))^n) + (B*a*n*log(a + b*x))/b - (B*c*n*
log(c + d*x))/d
```

**sympy [A]** time = 5.41, size = 158, normalized size = 2.82

$$Ax+B\left(\begin{cases} x\log\left(e\left(\frac{a}{c}\right)^n\right) & \text{for } b=0 \wedge d=0 \\ -\frac{cn\log(c+dx)}{d}+nx\log(a)-nx\log(c+dx)+nx+x\log(e) & \text{for } b=0 \\ \frac{an\log\left(\frac{a}{c}+\frac{bx}{c}\right)}{b}+nx\log\left(\frac{a}{c}+\frac{bx}{c}\right)-nx+x\log(e) & \text{for } d=0 \\ \frac{an\log(c+dx)}{b}+\frac{an\log\left(\frac{a}{c+dx}+\frac{bx}{c+dx}\right)}{b}-\frac{cn\log(c+dx)}{d}+nx\log\left(\frac{a}{c+dx}+\frac{bx}{c+dx}\right)+x\log(e) & \text{otherwise} \end{cases}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(A+B*ln(e*((b*x+a)/(d*x+c))**n),x)
```

```
[Out] A*x + B*Piecewise((x*log(e*(a/c)**n), Eq(b, 0) & Eq(d, 0)), (-c*n*log(c + d
*x)/d + n*x*log(a) - n*x*log(c + d*x) + n*x + x*log(e), Eq(b, 0)), (a*n*log
(a/c + b*x/c)/b + n*x*log(a/c + b*x/c) - n*x + x*log(e), Eq(d, 0)), (a*n*lo
g(c + d*x)/b + a*n*log(a/(c + d*x) + b*x/(c + d*x))/b - c*n*log(c + d*x)/d
+ n*x*log(a/(c + d*x) + b*x/(c + d*x)) + x*log(e), True))
```

## 3.62 $\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{f+gx}\,dx$

**Optimal**. Leaf size=147

$$\frac{\log(f+gx)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{g}-\frac{Bn\text{Li}_2\left(\frac{b(f+gx)}{bf-ag}\right)}{g}-\frac{Bn\log(f+gx)\log\left(-\frac{g(a+bx)}{bf-ag}\right)}{g}+\frac{Bn\text{Li}_2\left(\frac{d(f+gx)}{df-cg}\right)}{g}+\frac{Bn\log}{}$$

```
[Out] -B*n*ln(-g*(b*x+a)/(-a*g+b*f))*ln(g*x+f)/g+(A+B*ln(e*((b*x+a)/(d*x+c))^n))*
ln(g*x+f)/g+B*n*ln(-g*(d*x+c)/(-c*g+d*f))*ln(g*x+f)/g-B*n*polylog(2,b*(g*x+
f)/(-a*g+b*f))/g+B*n*polylog(2,d*(g*x+f)/(-c*g+d*f))/g
```

**Rubi [A]** time = 0.23, antiderivative size = 147, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 5, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {2524, 2418, 2394, 2393, 2391}

$$-\frac{Bn\text{PolyLog}\left(2,\frac{b(f+gx)}{bf-ag}\right)}{g}+\frac{Bn\text{PolyLog}\left(2,\frac{d(f+gx)}{df-cg}\right)}{g}+\frac{\log(f+gx)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{g}-\frac{Bn\log(f+gx)\log\left(\right.}{g}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Log[e*((a + b*x)/(c + d*x))^n])/(f + g*x),x]
```

```
[Out] -((B*n*Log[-((g*(a + b*x))/(b*f - a*g))]*Log[f + g*x])/g) + ((A + B*Log[e*(
(a + b*x)/(c + d*x))^n])*Log[f + g*x])/g + (B*n*Log[-((g*(c + d*x))/(d*f -
c*g))]*Log[f + g*x])/g - (B*n*PolyLog[2, (b*(f + g*x))/(b*f - a*g)])/g + (B
*n*PolyLog[2, (d*(f + g*x))/(d*f - c*g)])/g
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2393

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + (c*e*x)/g])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2394

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d + e*x
)^n]))/g, x] - Dist[(b*e*n)/g, Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2418

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*(RFx_), x_Sy
mbol] :> With[{u = ExpandIntegrand[(a + b*Log[c*(d + e*x)^n])^p, RFx, x]},
Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d, e, n}, x] && RationalFunctionQ[
RFx, x] && IntegerQ[p]
```

Rule 2524

```
Int[((a_.) + Log[(c_.)*(RFx_)^(p_.)]*(b_.))^(n_.)/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[(Log[d + e*x]*(a + b*Log[c*RFx^p])^n)/e, x] - Dist[(b*n*p)/e
, Int[(Log[d + e*x]*(a + b*Log[c*RFx^p])^(n - 1)*D[RFx, x])/RFx, x], x] /;
FreeQ[{a, b, c, d, e, p}, x] && RationalFunctionQ[RFx, x] && IGtQ[n, 0]
```

Rubi steps

$$\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{f+gx}\,dx = \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\log(f+gx)}{g} - \frac{(Bn)\int \frac{(c+dx)\left(-\frac{d(a+bx)}{(c+dx)^2}+\frac{b}{c+dx}\right)\log(f+gx)}{a+bx}\,dx}{g}$$

$$= \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\log(f+gx)}{g} - \frac{(Bn)\int \left(\frac{b\log(f+gx)}{a+bx}-\frac{d\log(f+gx)}{c+dx}\right)dx}{g}$$

$$= \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\log(f+gx)}{g} - \frac{(bBn)\int \frac{\log(f+gx)}{a+bx}\,dx}{g} + \frac{(Bdn)\int \frac{\log(f+gx)}{c+dx}\,dx}{g}$$

$$= -\frac{Bn\log\left(-\frac{g(a+bx)}{bf-ag}\right)\log(f+gx)}{g} + \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\log(f+gx)}{g} + \frac{Bn\log(\cdots}{\cdots}$$

$$= -\frac{Bn\log\left(-\frac{g(a+bx)}{bf-ag}\right)\log(f+gx)}{g} + \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\log(f+gx)}{g} + \frac{Bn\log(\cdots}{\cdots}$$

$$= -\frac{Bn\log\left(-\frac{g(a+bx)}{bf-ag}\right)\log(f+gx)}{g} + \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\log(f+gx)}{g} + \frac{Bn\log(\cdots}{\cdots}$$

**Mathematica [A]** time = 0.06, size = 122, normalized size = 0.83

$$\frac{\log(f+gx)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)-Bn\log\left(\frac{g(a+bx)}{ag-bf}\right)+A+Bn\log\left(\frac{g(c+dx)}{cg-df}\right)\right)-Bn\text{Li}_2\left(\frac{b(f+gx)}{bf-ag}\right)+Bn\text{Li}_2\left(\frac{d(f+gx)}{df-cg}\right)}{g}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Log[e*((a + b*x)/(c + d*x))^n])/(f + g*x),x]
```

```
[Out] ((A - B*n*Log[(g*(a + b*x))/(-(b*f) + a*g)] + B*Log[e*((a + b*x)/(c + d*x))
^n] + B*n*Log[(g*(c + d*x))/(-(d*f) + c*g)])*Log[f + g*x] - B*n*PolyLog[2,
(b*(f + g*x))/(b*f - a*g)] + B*n*PolyLog[2, (d*(f + g*x))/(d*f - c*g)])/g
```

**fricas [F]** time = 1.00, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A}{gx+f},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))/(g*x+f),x, algorithm="fricas")
```

```
[Out] integral((B*log(e*((b*x + a)/(d*x + c))^n) + A)/(g*x + f), x)
```

**giac [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))/(g*x+f),x, algorithm="giac")
```

```
[Out] Timed out
```

**maple [F]** time = 0.39, size = 0, normalized size = 0.00

$$\int\frac{B\ln\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A}{gx+f}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*ln(e*((b*x+a)/(d*x+c))^n)+A)/(g*x+f),x)
```

```
[Out] int((B*ln(e*((b*x+a)/(d*x+c))^n)+A)/(g*x+f),x)
```

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$-B\int -\frac{\log\left((bx+a)^n\right)-\log\left((dx+c)^n\right)+\log(e)}{gx+f}\,dx+\frac{A\log\left(gx+f\right)}{g}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))/(g*x+f),x, algorithm="maxima")
```

```
[Out] -B*integrate(-(log((b*x + a)^n) - log((d*x + c)^n) + log(e))/(g*x + f), x)
+ A*log(g*x + f)/g
```

**mupad** [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{A+B\,\ln\left(e\left(\frac{a+b\,x}{c+d\,x}\right)^n\right)}{f+g\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*log(e*((a + b*x)/(c + d*x))^n))/(f + g*x),x)
```

```
[Out] int((A + B*log(e*((a + b*x)/(c + d*x))^n))/(f + g*x), x)
```

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{A+B\log\left(e\left(\frac{a}{c+dx}+\frac{bx}{c+dx}\right)^n\right)}{f+gx}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*ln(e*((b*x+a)/(d*x+c))**n))/(g*x+f),x)
```

```
[Out] Integral((A + B*log(e*(a/(c + d*x) + b*x/(c + d*x))**n))/(f + g*x), x)
```

## 3.63 $\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(f+gx)^2}\,dx$

**Optimal**. Leaf size=91

$$\frac{(a+bx)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{(f+gx)(bf-ag)}+\frac{Bn(bc-ad)\log\left(\frac{f+gx}{c+dx}\right)}{(bf-ag)(df-cg)}$$

```
[Out] (b*x+a)*(A+B*ln(e*((b*x+a)/(d*x+c))^n))/(-a*g+b*f)/(g*x+f)+B*(-a*d+b*c)*n*l
n((g*x+f)/(d*x+c))/(-a*g+b*f)/(-c*g+d*f)
```

**Rubi [A]** time = 0.12, antiderivative size = 119, normalized size of antiderivative = 1.31, number of steps used = 4, number of rules used = 3, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {2525, 12, 72}

$$-\frac{B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A}{g(f+gx)}+\frac{Bn(bc-ad)\log(f+gx)}{(bf-ag)(df-cg)}+\frac{bBn\log(a+bx)}{g(bf-ag)}-\frac{Bdn\log(c+dx)}{g(df-cg)}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Log[e*((a + b*x)/(c + d*x))^n])/(f + g*x)^2,x]
```

```
[Out] (b*B*n*Log[a + b*x])/(g*(b*f - a*g)) - (A + B*Log[e*((a + b*x)/(c + d*x))^n
])/(g*(f + g*x)) - (B*d*n*Log[c + d*x])/(g*(d*f - c*g)) + (B*(b*c - a*d)*n*
Log[f + g*x])/((b*f - a*g)*(d*f - c*g))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 72

```
Int[((e_.) + (f_.)*(x_))^(p_.)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
 x_Symbol] :> Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x]
 /; FreeQ[{a, b, c, d, e, f}, x] && IntegerQ[p]
```

Rule 2525

```
Int[((a_.) + Log[(c_.)*(RFx_)^(p_.)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_))^(m_.
), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(a + b*Log[c*RFx^p])^n)/(e*(m + 1))
, x] - Dist[(b*n*p)/(e*(m + 1)), Int[SimplifyIntegrand[((d + e*x)^(m + 1)*(
```

```
a + b*Log[c*RFx^p])^(n - 1)*D[RFx, x])/RFx, x], x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && RationalFunctionQ[RFx, x] && IGtQ[n, 0] && (EqQ[n, 1] ||
IntegerQ[m]) && NeQ[m, -1]
```

Rubi steps

$$\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(f+gx)^2}\,dx = -\frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{g(f+gx)}+\frac{(Bn)\int \frac{bc-ad}{(a+bx)(c+dx)(f+gx)}\,dx}{g}$$

$$= -\frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{g(f+gx)}+\frac{(B(bc-ad)n)\int \frac{1}{(a+bx)(c+dx)(f+gx)}\,dx}{g}$$

$$= -\frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{g(f+gx)}+\frac{(B(bc-ad)n)\int \left(\frac{b^2}{(bc-ad)(bf-ag)(a+bx)}+\frac{d^2}{(bc-ad)(-df+cg)(c+dx)}\right.}{g}$$

$$= \frac{bBn\log(a+bx)}{g(bf-ag)}-\frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{g(f+gx)}-\frac{Bdn\log(c+dx)}{g(df-cg)}+\frac{B(bc-ad)n\log(f}{(bf-ag)(df-}$$

**Mathematica [A]** time = 0.15, size = 109, normalized size = 1.20

$$\frac{\frac{Bn(b\log(a+bx)(df-cg)+\log(c+dx)(adg-bdf)+g(bc-ad)\log(f+gx))}{(bf-ag)(df-cg)}-\frac{B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A}{f+gx}}{g}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Log[e*((a + b*x)/(c + d*x))^n])/(f + g*x)^2,x]
```

```
[Out] (-((A + B*Log[e*((a + b*x)/(c + d*x))^n])/(f + g*x)) + (B*n*(b*(d*f - c*g)*
Log[a + b*x] + (-(b*d*f) + a*d*g)*Log[c + d*x] + (b*c - a*d)*g*Log[f + g*x]
))/((b*f - a*g)*(d*f - c*g)))/g
```

**fricas [B]** time = 11.25, size = 294, normalized size = 3.23

$$-\frac{Abdf^2+Aacg^2-(Abc+Aad)fg+\left(Bbdf^2+Bacg^2-(Bbc+Bad)fg\right)n\log\left(\frac{bx+a}{dx+c}\right)-\left(\left(Bbdfg-Bbcg^2\right)nx+\left(\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))/(g*x+f)^2,x, algorithm="fricas")
```

```
[Out] -(A*b*d*f^2 + A*a*c*g^2 - (A*b*c + A*a*d)*f*g + (B*b*d*f^2 + B*a*c*g^2 - (B
*b*c + B*a*d)*f*g)*n*log((b*x + a)/(d*x + c)) - ((B*b*d*f*g - B*b*c*g^2)*n*
x + (B*b*d*f^2 - B*b*c*f*g)*n)*log(b*x + a) + ((B*b*d*f*g - B*a*d*g^2)*n*x
+ (B*b*d*f^2 - B*a*d*f*g)*n)*log(d*x + c) - ((B*b*c - B*a*d)*g^2*n*x + (B*b
*c - B*a*d)*f*g*n)*log(g*x + f) + (B*b*d*f^2 + B*a*c*g^2 - (B*b*c + B*a*d)*
f*g)*log(e))/(b*d*f^3*g + a*c*f*g^3 - (b*c + a*d)*f^2*g^2 + (b*d*f^2*g^2 +
a*c*g^4 - (b*c + a*d)*f*g^3)*x)
```

**giac [B]** time = 4.10, size = 455, normalized size = 5.00

$$\left(\frac{\left(Bb^2c^2n - 2\,Babcdn + Ba^2d^2n\right)\log\left(-bf + \frac{(bx+a)df}{dx+c} + ag - \frac{(bx+a)cg}{dx+c}\right)}{bdf^2 - bcfg - adfg + acg^2} + \frac{\left(Bb^2c^2n - 2\,Babcdn + Ba^2d^2\right.}{bdf^2 - \frac{(bx+a)d^2f^2}{dx+c} - bcfg - adfg + \frac{2\,(bx+}{dx}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))/(g*x+f)^2,x, algorithm="giac")
```

```
[Out] ((B*b^2*c^2*n - 2*B*a*b*c*d*n + B*a^2*d^2*n)*log(-b*f + (b*x + a)*d*f/(d*x
+ c) + a*g - (b*x + a)*c*g/(d*x + c))/(b*d*f^2 - b*c*f*g - a*d*f*g + a*c*g^
2) + (B*b^2*c^2*n - 2*B*a*b*c*d*n + B*a^2*d^2*n)*log((b*x + a)/(d*x + c))/(
b*d*f^2 - (b*x + a)*d^2*f^2/(d*x + c) - b*c*f*g - a*d*f*g + 2*(b*x + a)*c*d
*f*g/(d*x + c) + a*c*g^2 - (b*x + a)*c^2*g^2/(d*x + c)) - (B*b^2*c^2*n - 2*
B*a*b*c*d*n + B*a^2*d^2*n)*log((b*x + a)/(d*x + c))/(b*d*f^2 - b*c*f*g - a*
d*f*g + a*c*g^2) + (A*b^2*c^2 + B*b^2*c^2 - 2*A*a*b*c*d - 2*B*a*b*c*d + A*a
^2*d^2 + B*a^2*d^2)/(b*d*f^2 - (b*x + a)*d^2*f^2/(d*x + c) - b*c*f*g - a*d*
f*g + 2*(b*x + a)*c*d*f*g/(d*x + c) + a*c*g^2 - (b*x + a)*c^2*g^2/(d*x + c)
))*(b*c/(b*c - a*d)^2 - a*d/(b*c - a*d)^2)
```

**maple [F]** time = 0.29, size = 0, normalized size = 0.00

$$\int \frac{B\ln\left(e\left(\frac{bx+a}{dx+c}\right)^{n}\right) + A}{\left(gx+f\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*ln(e*((b*x+a)/(d*x+c))^n)+A)/(g*x+f)^2,x)
```

```
[Out] int((B*ln(e*((b*x+a)/(d*x+c))^n)+A)/(g*x+f)^2,x)
```

**maxima [A]** time = 0.81, size = 142, normalized size = 1.56

$$Bn\left(\frac{b\log\left(bx+a\right)}{bfg-ag^2} - \frac{d\log\left(dx+c\right)}{dfg-cg^2} + \frac{(bc-ad)\log\left(gx+f\right)}{bdf^2+acg^2-(bc+ad)fg}\right) - \frac{B\log\left(e\left(\frac{bx}{dx+c}+\frac{a}{dx+c}\right)^{n}\right)}{g^2x+fg} - \frac{A}{g^2x+fg}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))/(g*x+f)^2,x, algorithm="maxima")
```

```
[Out] B*n*(b*log(b*x + a)/(b*f*g - a*g^2) - d*log(d*x + c)/(d*f*g - c*g^2) + (b*c
 - a*d)*log(g*x + f)/(b*d*f^2 + a*c*g^2 - (b*c + a*d)*f*g)) - B*log(e*(b*x/
(d*x + c) + a/(d*x + c))^n)/(g^2*x + f*g) - A/(g^2*x + f*g)
```

**mupad [B]** time = 4.64, size = 140, normalized size = 1.54

$$\frac{B\,d\,n\,\ln\left(c+d\,x\right)}{c\,g^2-d\,f\,g}-\frac{\ln\left(f+g\,x\right)\,\left(B\,a\,d\,n-B\,b\,c\,n\right)}{a\,c\,g^2+b\,d\,f^2-a\,d\,f\,g-b\,c\,f\,g}-\frac{B\,\ln\left(e\,{\left(\frac{a+b\,x}{c+d\,x}\right)}^n\right)}{g\,\left(f+g\,x\right)}-\frac{B\,b\,n\,\ln\left(a+b\,x\right)}{a\,g^2-b\,f\,g}-\frac{A}{x\,g^2+f\,g}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*log(e*((a + b*x)/(c + d*x))^n))/(f + g*x)^2,x)
```

```
[Out] (B*d*n*log(c + d*x))/(c*g^2 - d*f*g) - (log(f + g*x)*(B*a*d*n - B*b*c*n))/(
a*c*g^2 + b*d*f^2 - a*d*f*g - b*c*f*g) - (B*log(e*((a + b*x)/(c + d*x))^n))
/(g*(f + g*x)) - (B*b*n*log(a + b*x))/(a*g^2 - b*f*g) - A/(f*g + g^2*x)
```

**sympy [F(-2)]** time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*ln(e*((b*x+a)/(d*x+c))**n))/(g*x+f)**2,x)
```

```
[Out] Exception raised: NotImplementedError
```

## 3.64 $\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(f+gx)^3}\,dx$

**Optimal**. Leaf size=190

$$-\frac{B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A}{2g(f+gx)^2}+\frac{b^2Bn\log(a+bx)}{2g(bf-ag)^2}-\frac{Bn(bc-ad)}{2(f+gx)(bf-ag)(df-cg)}+\frac{Bn(bc-ad)\log(f+gx)(-adg-bcg}{2(bf-ag)^2(df-cg)^2}$$

```
[Out] -1/2*B*(-a*d+b*c)*n/(-a*g+b*f)/(-c*g+d*f)/(g*x+f)+1/2*b^2*B*n*ln(b*x+a)/g/(
-a*g+b*f)^2+1/2*(-A-B*ln(e*((b*x+a)/(d*x+c))^n))/g/(g*x+f)^2-1/2*B*d^2*n*ln
(d*x+c)/g/(-c*g+d*f)^2+1/2*B*(-a*d+b*c)*(-a*d*g-b*c*g+2*b*d*f)*n*ln(g*x+f)/
(-a*g+b*f)^2/(-c*g+d*f)^2
```

**Rubi [A]** time = 0.24, antiderivative size = 190, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {2525, 12, 72}

$$-\frac{B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A}{2g(f+gx)^2}+\frac{b^2Bn\log(a+bx)}{2g(bf-ag)^2}-\frac{Bn(bc-ad)}{2(f+gx)(bf-ag)(df-cg)}+\frac{Bn(bc-ad)\log(f+gx)(-adg-bcg}{2(bf-ag)^2(df-cg)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Log[e*((a + b*x)/(c + d*x))^n])/(f + g*x)^3,x]
```

```
[Out] -(B*(b*c - a*d)*n)/(2*(b*f - a*g)*(d*f - c*g)*(f + g*x)) + (b^2*B*n*Log[a +
 b*x])/(2*g*(b*f - a*g)^2) - (A + B*Log[e*((a + b*x)/(c + d*x))^n])/(2*g*(f
 + g*x)^2) - (B*d^2*n*Log[c + d*x])/(2*g*(d*f - c*g)^2) + (B*(b*c - a*d)*(2
*b*d*f - b*c*g - a*d*g)*n*Log[f + g*x])/(2*(b*f - a*g)^2*(d*f - c*g)^2)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 72

```
Int[((e_.) + (f_.)*(x_))^(p_.)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
 x_Symbol] :> Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x]
 /; FreeQ[{a, b, c, d, e, f}, x] && IntegerQ[p]
```

Rule 2525

```
Int[((a_.) + Log[(c_.)*(RFx_)^(p_.)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_))^(m_.
), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(a + b*Log[c*RFx^p])^n)/(e*(m + 1))
, x] - Dist[(b*n*p)/(e*(m + 1)), Int[SimplifyIntegrand[((d + e*x)^(m + 1)*(
a + b*Log[c*RFx^p])^(n - 1)*D[RFx, x])/RFx, x], x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && RationalFunctionQ[RFx, x] && IGtQ[n, 0] && (EqQ[n, 1] ||
IntegerQ[m]) && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(f+gx)^3}\,dx &= -\frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{2g(f+gx)^2}+\frac{(Bn)\int\frac{bc-ad}{(a+bx)(c+dx)(f+gx)^2}\,dx}{2g}\\ &= -\frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{2g(f+gx)^2}+\frac{(B(bc-ad)n)\int\frac{1}{(a+bx)(c+dx)(f+gx)^2}\,dx}{2g}\\ &= -\frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{2g(f+gx)^2}+\frac{(B(bc-ad)n)\int\left(\frac{b^3}{(bc-ad)(bf-ag)^2(a+bx)}-\frac{d^3}{(bc-ad)(-df+cg)^2(c+d}\right.}{2g}\\ &= -\frac{B(bc-ad)n}{2(bf-ag)(df-cg)(f+gx)}+\frac{b^2Bn\log(a+bx)}{2g(bf-ag)^2}-\frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{2g(f+gx)^2}-\frac{Bd^2}{2g}\end{aligned}$$

**Mathematica [A]** time = 0.54, size = 173, normalized size = 0.91

$$\frac{Bn(bc-ad)\left(\frac{b^2\log(a+bx)}{(bc-ad)(bf-ag)^2}+\frac{\frac{d^2\log(c+dx)}{ad-bc}+\frac{g(cg-df)}{(f+gx)(bf-ag)}-\frac{g\log(f+gx)(adg+bcg-2bdf)}{(bf-ag)^2}}{(df-cg)^2}\right)-\frac{B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A}{(f+gx)^2}}{2g}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Log[e*((a + b*x)/(c + d*x))^n])/(f + g*x)^3,x]
```

```
[Out] (-((A + B*Log[e*((a + b*x)/(c + d*x))^n])/(f + g*x)^2) + B*(b*c - a*d)*n*((
b^2*Log[a + b*x])/((b*c - a*d)*(b*f - a*g)^2) + ((g*(-(d*f) + c*g))/((b*f -
 a*g)*(f + g*x)) + (d^2*Log[c + d*x])/(-(b*c) + a*d) - (g*(-2*b*d*f + b*c*g
 + a*d*g)*Log[f + g*x])/(b*f - a*g)^2)/(d*f - c*g)^2))/(2*g)
```

**fricas [B]** time = 159.27, size = 1175, normalized size = 6.18

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))/(g*x+f)^3,x, algorithm="fricas")
```

```
[Out] -1/2*(A*b^2*d^2*f^4 + A*a^2*c^2*g^4 - 2*(A*b^2*c*d + A*a*b*d^2)*f^3*g + (A*
b^2*c^2 + 4*A*a*b*c*d + A*a^2*d^2)*f^2*g^2 - 2*(A*a*b*c^2 + A*a^2*c*d)*f*g^
3 + ((B*b^2*c*d - B*a*b*d^2)*f^2*g^2 - (B*b^2*c^2 - B*a^2*d^2)*f*g^3 + (B*a
*b*c^2 - B*a^2*c*d)*g^4)*n*x + (B*b^2*d^2*f^4 + B*a^2*c^2*g^4 - 2*(B*b^2*c*
d + B*a*b*d^2)*f^3*g + (B*b^2*c^2 + 4*B*a*b*c*d + B*a^2*d^2)*f^2*g^2 - 2*(B
*a*b*c^2 + B*a^2*c*d)*f*g^3)*n*log((b*x + a)/(d*x + c)) + ((B*b^2*c*d - B*a
*b*d^2)*f^3*g - (B*b^2*c^2 - B*a^2*d^2)*f^2*g^2 + (B*a*b*c^2 - B*a^2*c*d)*f
*g^3)*n - ((B*b^2*d^2*f^2*g^2 - 2*B*b^2*c*d*f*g^3 + B*b^2*c^2*g^4)*n*x^2 +
2*(B*b^2*d^2*f^3*g - 2*B*b^2*c*d*f^2*g^2 + B*b^2*c^2*f*g^3)*n*x + (B*b^2*d^
2*f^4 - 2*B*b^2*c*d*f^3*g + B*b^2*c^2*f^2*g^2)*n)*log(b*x + a) + ((B*b^2*d^
2*f^2*g^2 - 2*B*a*b*d^2*f*g^3 + B*a^2*d^2*g^4)*n*x^2 + 2*(B*b^2*d^2*f^3*g -
 2*B*a*b*d^2*f^2*g^2 + B*a^2*d^2*f*g^3)*n*x + (B*b^2*d^2*f^4 - 2*B*a*b*d^2*
f^3*g + B*a^2*d^2*f^2*g^2)*n)*log(d*x + c) - ((2*(B*b^2*c*d - B*a*b*d^2)*f*
g^3 - (B*b^2*c^2 - B*a^2*d^2)*g^4)*n*x^2 + 2*(2*(B*b^2*c*d - B*a*b*d^2)*f^2
*g^2 - (B*b^2*c^2 - B*a^2*d^2)*f*g^3)*n*x + (2*(B*b^2*c*d - B*a*b*d^2)*f^3*
g - (B*b^2*c^2 - B*a^2*d^2)*f^2*g^2)*n)*log(g*x + f) + (B*b^2*d^2*f^4 + B*a
^2*c^2*g^4 - 2*(B*b^2*c*d + B*a*b*d^2)*f^3*g + (B*b^2*c^2 + 4*B*a*b*c*d + B
*a^2*d^2)*f^2*g^2 - 2*(B*a*b*c^2 + B*a^2*c*d)*f*g^3)*log(e))/(b^2*d^2*f^6*g
 + a^2*c^2*f^2*g^5 - 2*(b^2*c*d + a*b*d^2)*f^5*g^2 + (b^2*c^2 + 4*a*b*c*d +
 a^2*d^2)*f^4*g^3 - 2*(a*b*c^2 + a^2*c*d)*f^3*g^4 + (b^2*d^2*f^4*g^3 + a^2*
c^2*g^7 - 2*(b^2*c*d + a*b*d^2)*f^3*g^4 + (b^2*c^2 + 4*a*b*c*d + a^2*d^2)*f
^2*g^5 - 2*(a*b*c^2 + a^2*c*d)*f*g^6)*x^2 + 2*(b^2*d^2*f^5*g^2 + a^2*c^2*f*
g^6 - 2*(b^2*c*d + a*b*d^2)*f^4*g^3 + (b^2*c^2 + 4*a*b*c*d + a^2*d^2)*f^3*g
^4 - 2*(a*b*c^2 + a^2*c*d)*f^2*g^5)*x)
```

**giac [B]** time = 6.80, size = 2952, normalized size = 15.54

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))/(g*x+f)^3,x, algorithm="giac")
```

```
[Out] 1/2*((2*B*b^3*c^2*d*f*n - 4*B*a*b^2*c*d^2*f*n + 2*B*a^2*b*d^3*f*n - B*b^3*c
^3*g*n + B*a*b^2*c^2*d*g*n + B*a^2*b*c*d^2*g*n - B*a^3*d^3*g*n)*log(-b*f +
(b*x + a)*d*f/(d*x + c) + a*g - (b*x + a)*c*g/(d*x + c))/(b^2*d^2*f^4 - 2*b
^2*c*d*f^3*g - 2*a*b*d^2*f^3*g + b^2*c^2*f^2*g^2 + 4*a*b*c*d*f^2*g^2 + a^2*
d^2*f^2*g^2 - 2*a*b*c^2*f*g^3 - 2*a^2*c*d*f*g^3 + a^2*c^2*g^4) + (2*B*b^3*c
^2*d*f*n - 4*B*a*b^2*c*d^2*f*n - 2*(b*x + a)*B*b^2*c^2*d^2*f*n/(d*x + c) +
2*B*a^2*b*d^3*f*n + 4*(b*x + a)*B*a*b*c*d^3*f*n/(d*x + c) - 2*(b*x + a)*B*a
^2*d^4*f*n/(d*x + c) - B*b^3*c^3*g*n + B*a*b^2*c^2*d*g*n + 2*(b*x + a)*B*b^
2*c^3*d*g*n/(d*x + c) + B*a^2*b*c*d^2*g*n - 4*(b*x + a)*B*a*b*c^2*d^2*g*n/(
```

d*x + c) - B*a^3*d^3*g*n + 2*(b*x + a)*B*a^2*c*d^3*g*n/(d*x + c))*log((b*x + a)/(d*x + c))/(b^2*d^2*f^4 - 2*(b*x + a)*b*d^3*f^4/(d*x + c) + (b*x + a)^2*d^4*f^4/(d*x + c)^2 - 2*b^2*c*d*f^3*g - 2*a*b*d^2*f^3*g + 6*(b*x + a)*b*c*d^2*f^3*g/(d*x + c) + 2*(b*x + a)*a*d^3*f^3*g/(d*x + c) - 4*(b*x + a)^2*c*d^3*f^3*g/(d*x + c)^2 + b^2*c^2*f^2*g^2 + 4*a*b*c*d*f^2*g^2 - 6*(b*x + a)*b*c^2*d*f^2*g^2/(d*x + c) + a^2*d^2*f^2*g^2 - 6*(b*x + a)*a*c*d^2*f^2*g^2/(d*x + c) + 6*(b*x + a)^2*c^2*d^2*f^2*g^2/(d*x + c)^2 - 2*a*b*c^2*f*g^3 + 2*(b*x + a)*b*c^3*f*g^3/(d*x + c) - 2*a^2*c*d*f*g^3 + 6*(b*x + a)*a*c^2*d*f*g^3/(d*x + c) - 4*(b*x + a)^2*c^3*d*f*g^3/(d*x + c)^2 + a^2*c^2*g^4 - 2*(b*x + a)*a*c^3*g^4/(d*x + c) + (b*x + a)^2*c^4*g^4/(d*x + c)^2) - (2*B*b^3*c^2*d*f*n - 4*B*a*b^2*c*d^2*f*n + 2*B*a^2*b*d^3*f*n - B*b^3*c^3*g*n + B*a*b^2*c^2*d*g*n + B*a^2*b*c*d^2*g*n - B*a^3*d^3*g*n)*log((b*x + a)/(d*x + c))/(b^2*d^2*f^4 - 2*b^2*c*d*f^3*g - 2*a*b*d^2*f^3*g + b^2*c^2*f^2*g^2 + 4*a*b*c*d*f^2*g^2 + a^2*d^2*f^2*g^2 - 2*a*b*c^2*f*g^3 - 2*a^2*c*d*f*g^3 + a^2*c^2*g^4) + (B*b^4*c^3*f*g*n - 3*B*a*b^3*c^2*d*f*g*n - (b*x + a)*B*b^3*c^3*d*f*g*n/(d*x + c) + 3*B*a^2*b^2*c*d^2*f*g*n + 3*(b*x + a)*B*a*b^2*c^2*d^2*f*g*n/(d*x + c) - B*a^3*b*d^3*f*g*n - 3*(b*x + a)*B*a^2*b*c*d^3*f*g*n/(d*x + c) + (b*x + a)*B*a^3*d^4*f*g*n/(d*x + c) - B*a*b^3*c^3*g^2*n + (b*x + a)*B*b^3*c^4*g^2*n/(d*x + c) + 3*B*a^2*b^2*c^2*d*g^2*n - 3*(b*x + a)*B*a*b^2*c^3*d*g^2*n/(d*x + c) - 3*B*a^3*b*c*d^2*g^2*n + 3*(b*x + a)*B*a^2*b*c^2*d^2*g^2*n/(d*x + c) + B*a^4*d^3*g^2*n - (b*x + a)*B*a^3*c*d^3*g^2*n/(d*x + c) + 2*A*b^4*c^2*d*f^2 + 2*B*b^4*c^2*d*f^2 - 4*A*a*b^3*c*d^2*f^2 - 4*B*a*b^3*c*d^2*f^2 - 2*(b*x + a)*A*b^3*c^2*d^2*f^2/(d*x + c) - 2*(b*x + a)*B*b^3*c^2*d^2*f^2/(d*x + c) + 2*A*a^2*b^2*d^3*f^2 + 2*B*a^2*b^2*d^3*f^2 + 4*(b*x + a)*A*a*b^2*c*d^3*f^2/(d*x + c) + 4*(b*x + a)*B*a*b^2*c*d^3*f^2/(d*x + c) - 2*(b*x + a)*A*a^2*b*d^4*f^2/(d*x + c) - 2*(b*x + a)*B*a^2*b*d^4*f^2/(d*x + c) - A*b^4*c^3*f*g - B*b^4*c^3*f*g - A*a*b^3*c^2*d*f*g - B*a*b^3*c^2*d*f*g + 2*(b*x + a)*A*b^3*c^3*d*f*g/(d*x + c) + 2*(b*x + a)*B*b^3*c^3*d*f*g/(d*x + c) + 5*A*a^2*b^2*c*d^2*f*g + 5*B*a^2*b^2*c*d^2*f*g - 2*(b*x + a)*A*a*b^2*c^2*d^2*f*g/(d*x + c) - 2*(b*x + a)*B*a*b^2*c^2*d^2*f*g/(d*x + c) - 3*A*a^3*b*d^3*f*g - 3*B*a^3*b*d^3*f*g - 2*(b*x + a)*A*a^2*b*c*d^3*f*g/(d*x + c) - 2*(b*x + a)*B*a^2*b*c*d^3*f*g/(d*x + c) + 2*(b*x + a)*A*a^3*d^4*f*g/(d*x + c) + 2*(b*x + a)*B*a^3*d^4*f*g/(d*x + c) + A*a*b^3*c^3*g^2 + B*a*b^3*c^3*g^2 - A*a^2*b^2*c^2*d*g^2 - B*a^2*b^2*c^2*d*g^2 - 2*(b*x + a)*A*a*b^2*c^3*d*g^2/(d*x + c) - 2*(b*x + a)*B*a*b^2*c^3*d*g^2/(d*x + c) - A*a^3*b*c*d^2*g^2 - B*a^3*b*c*d^2*g^2 + 4*(b*x + a)*A*a^2*b*c^2*d^2*g^2/(d*x + c) + 4*(b*x + a)*B*a^2*b*c^2*d^2*g^2/(d*x + c) + A*a^4*d^3*g^2 + B*a^4*d^3*g^2 - 2*(b*x + a)*A*a^3*c*d^3*g^2/(d*x + c) - 2*(b*x + a)*B*a^3*c*d^3*g^2/(d*x + c))/(b^3*d^2*f^5 - 2*(b*x + a)*b^2*d^3*f^5/(d*x + c) + (b*x + a)^2*b*d^4*f^5/(d*x + c)^2 - 2*b^3*c*d*f^4*g - 3*a*b^2*d^2*f^4*g + 6*(b*x + a)*b^2*c*d^2*f^4*g/(d*x + c) + 4*(b*x + a)*a*b*d^3*f^4*g/(d*x + c) - 4*(b*x + a)^2*b*c*d^3*f^4*g/(d*x + c)^2 - (b*x + a)^2*a*d^4*f^4*g/(d*x + c)^2 + b^3*c^2*f^3*g^2 + 6*a*b^2*c*d*f^3*g^2 - 6*(b*x + a)*b^2*c^2*d*f^3*g^2/(d*x + c) + 3*a^2*b*d^2*f^3*g^2 - 12*(b*x + a)*a*b*c*d^2*f^3*g^2/(d*x + c) + 6*(b*x + a)^2*b*c^2*d^2*f^3*g^2/(d*x + c)^2 - 2*(b*x + a)*a^2*d^3*f^3*g^2/(d*x + c) + 4*(b*x + a)^2*a*c*d^3*f^3

```
*g^2/(d*x + c)^2 - 3*a*b^2*c^2*f^2*g^3 + 2*(b*x + a)*b^2*c^3*f^2*g^3/(d*x +
 c) - 6*a^2*b*c*d*f^2*g^3 + 12*(b*x + a)*a*b*c^2*d*f^2*g^3/(d*x + c) - 4*(b
*x + a)^2*b*c^3*d*f^2*g^3/(d*x + c)^2 - a^3*d^2*f^2*g^3 + 6*(b*x + a)*a^2*c
*d^2*f^2*g^3/(d*x + c) - 6*(b*x + a)^2*a*c^2*d^2*f^2*g^3/(d*x + c)^2 + 3*a^
2*b*c^2*f*g^4 - 4*(b*x + a)*a*b*c^3*f*g^4/(d*x + c) + (b*x + a)^2*b*c^4*f*g
^4/(d*x + c)^2 + 2*a^3*c*d*f*g^4 - 6*(b*x + a)*a^2*c^2*d*f*g^4/(d*x + c) +
4*(b*x + a)^2*a*c^3*d*f*g^4/(d*x + c)^2 - a^3*c^2*g^5 + 2*(b*x + a)*a^2*c^3
*g^5/(d*x + c) - (b*x + a)^2*a*c^4*g^5/(d*x + c)^2))*(b*c/(b*c - a*d)^2 - a
*d/(b*c - a*d)^2)
```

**maple [F]** time = 0.28, size = 0, normalized size = 0.00

$$\int \frac{B\ln\left(e\left(\frac{bx+a}{dx+c}\right)^{n}\right)+A}{\left(gx+f\right)^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*ln(e*((b*x+a)/(d*x+c))^n)+A)/(g*x+f)^3,x)
```

```
[Out] int((B*ln(e*((b*x+a)/(d*x+c))^n)+A)/(g*x+f)^3,x)
```

**maxima [A]** time = 1.00, size = 355, normalized size = 1.87

$$\frac{1}{2}\left(\frac{b^{2}\log\left(bx+a\right)}{b^{2}f^{2}g-2\,abfg^{2}+a^{2}g^{3}}-\frac{d^{2}\log\left(dx+c\right)}{d^{2}f^{2}g-2\,cdfg^{2}+c^{2}g^{3}}+\frac{\left(2\left(b^{2}cd-abd^{2}\right)f-\left(b^{2}c^{2}-a^{2}d^{2}\right)g}{b^{2}d^{2}f^{4}+a^{2}c^{2}g^{4}-2\left(b^{2}cd+abd^{2}\right)f^{3}g+\left(b^{2}c^{2}+4\,abcd\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))/(g*x+f)^3,x, algorithm="maxima")
```

```
[Out] 1/2*(b^2*log(b*x + a)/(b^2*f^2*g - 2*a*b*f*g^2 + a^2*g^3) - d^2*log(d*x + c
)/(d^2*f^2*g - 2*c*d*f*g^2 + c^2*g^3) + (2*(b^2*c*d - a*b*d^2)*f - (b^2*c^2
 - a^2*d^2)*g)*log(g*x + f)/(b^2*d^2*f^4 + a^2*c^2*g^4 - 2*(b^2*c*d + a*b*d
^2)*f^3*g + (b^2*c^2 + 4*a*b*c*d + a^2*d^2)*f^2*g^2 - 2*(a*b*c^2 + a^2*c*d)
*f*g^3) - (b*c - a*d)/(b*d*f^3 + a*c*f*g^2 - (b*c + a*d)*f^2*g + (b*d*f^2*g
 + a*c*g^3 - (b*c + a*d)*f*g^2)*x))*B*n - 1/2*B*log(e*(b*x/(d*x + c) + a/(d
*x + c))^n)/(g^3*x^2 + 2*f*g^2*x + f^2*g) - 1/2*A/(g^3*x^2 + 2*f*g^2*x + f^
2*g)
```

**mupad [B]** time = 6.20, size = 430, normalized size = 2.26

$$\frac{\ln\left(f+g\,x\right)\,\left(g\,\left(B\,a^2\,d^2\,n-B\,b^2\,c^2\,n\right)-2\,B\,a\,b\,d^2\,f\,n+2\,B\,b^2\,c\,d\,f\,n\right)}{2\,a^2\,c^2\,g^4-4\,a^2\,c\,d\,f\,g^3+2\,a^2\,d^2\,f^2\,g^2-4\,a\,b\,c^2\,f\,g^3+8\,a\,b\,c\,d\,f^2\,g^2-4\,a\,b\,d^2\,f^3\,g+2\,b^2\,c^2\,f^2\,g^2-4\,b^2\,c\,d\,f^3\,g}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*log(e*((a + b*x)/(c + d*x))^n))/(f + g*x)^3,x)
```

```
[Out] (log(f + g*x)*(g*(B*a^2*d^2*n - B*b^2*c^2*n) - 2*B*a*b*d^2*f*n + 2*B*b^2*c*
d*f*n))/(2*a^2*c^2*g^4 + 2*b^2*d^2*f^4 + 2*a^2*d^2*f^2*g^2 + 2*b^2*c^2*f^2*
g^2 - 4*a*b*c^2*f*g^3 - 4*a*b*d^2*f^3*g - 4*a^2*c*d*f*g^3 - 4*b^2*c*d*f^3*g
 + 8*a*b*c*d*f^2*g^2) - ((A*a*c*g^2 + A*b*d*f^2 - A*a*d*f*g - A*b*c*f*g - B
*a*d*f*g*n + B*b*c*f*g*n)/(a*c*g^2 + b*d*f^2 - a*d*f*g - b*c*f*g) - (x*(B*a
*d*g^2*n - B*b*c*g^2*n))/(a*c*g^2 + b*d*f^2 - a*d*f*g - b*c*f*g))/(2*f^2*g
+ 2*g^3*x^2 + 4*f*g^2*x) - (B*log(e*((a + b*x)/(c + d*x))^n))/(2*g*(f^2 + g
^2*x^2 + 2*f*g*x)) + (B*b^2*n*log(a + b*x))/(2*a^2*g^3 + 2*b^2*f^2*g - 4*a*
b*f*g^2) - (B*d^2*n*log(c + d*x))/(2*c^2*g^3 + 2*d^2*f^2*g - 4*c*d*f*g^2)
```

**sympy [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*ln(e*((b*x+a)/(d*x+c))**n))/(g*x+f)**3,x)
```

```
[Out] Timed out
```

## 3.65 $\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(f+gx)^4}\,dx$

**Optimal**. Leaf size=283

$$\frac{Bn(bc-ad)\log(f+gx)\left(a^2d^2g^2-abdg(3df-cg)+b^2\left(c^2g^2-3cdfg+3d^2f^2\right)\right)}{3(bf-ag)^3(df-cg)^3}-\frac{B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A}{3g(f+gx)^3}+\frac{b^3Bn}{3g(}$$

[Out] -1/6*B*(-a*d+b*c)*n/(-a*g+b*f)/(-c*g+d*f)/(g*x+f)^2-1/3*B*(-a*d+b*c)*(-a*d*g-b*c*g+2*b*d*f)*n/(-a*g+b*f)^2/(-c*g+d*f)^2/(g*x+f)+1/3*b^3*B*n*ln(b*x+a)/g/(-a*g+b*f)^3+1/3*(-A-B*ln(e*((b*x+a)/(d*x+c))^n))/g/(g*x+f)^3-1/3*B*d^3*n*ln(d*x+c)/g/(-c*g+d*f)^3+1/3*B*(-a*d+b*c)*(a^2*d^2*g^2-a*b*d*g*(-c*g+3*d*f)+b^2*(c^2*g^2-3*c*d*f*g+3*d^2*f^2))*n*ln(g*x+f)/(-a*g+b*f)^3/(-c*g+d*f)^3

**Rubi [A]** time = 0.46, antiderivative size = 283, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {2525, 12, 72}

$$\frac{Bn(bc-ad)\log(f+gx)\left(a^2d^2g^2-abdg(3df-cg)+b^2\left(c^2g^2-3cdfg+3d^2f^2\right)\right)}{3(bf-ag)^3(df-cg)^3}-\frac{B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A}{3g(f+gx)^3}+\frac{b^3Bn}{3g(}$$

Antiderivative was successfully verified.

[In] Int[(A + B*Log[e*((a + b*x)/(c + d*x))^n])/(f + g*x)^4,x]

[Out] -(B*(b*c - a*d)*n)/(6*(b*f - a*g)*(d*f - c*g)*(f + g*x)^2) - (B*(b*c - a*d)*(2*b*d*f - b*c*g - a*d*g)*n)/(3*(b*f - a*g)^2*(d*f - c*g)^2*(f + g*x)) + (b^3*B*n*Log[a + b*x])/(3*g*(b*f - a*g)^3) - (A + B*Log[e*((a + b*x)/(c + d*x))^n])/(3*g*(f + g*x)^3) - (B*d^3*n*Log[c + d*x])/(3*g*(d*f - c*g)^3) + (B*(b*c - a*d)*(a^2*d^2*g^2 - a*b*d*g*(3*d*f - c*g) + b^2*(3*d^2*f^2 - 3*c*d*f*g + c^2*g^2))*n*Log[f + g*x])/(3*(b*f - a*g)^3*(d*f - c*g)^3)

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !MatchQ[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 72

```
Int[((e_.) + (f_.)*(x_))^(p_.)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IntegerQ[p]
```

Rule 2525

```
Int[((a_.) + Log[(c_.)*(RFx_)^(p_.)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_))^(m_.
), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(a + b*Log[c*RFx^p])^n)/(e*(m + 1))
, x] - Dist[(b*n*p)/(e*(m + 1)), Int[SimplifyIntegrand[((d + e*x)^(m + 1)*(
a + b*Log[c*RFx^p])^(n - 1)*D[RFx, x])/RFx, x], x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && RationalFunctionQ[RFx, x] && IGtQ[n, 0] && (EqQ[n, 1] ||
IntegerQ[m]) && NeQ[m, -1]
```

Rubi steps

$$\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(f+gx)^4}\,dx = -\frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{3g(f+gx)^3}+\frac{(Bn)\int\frac{bc-ad}{(a+bx)(c+dx)(f+gx)^3}\,dx}{3g}$$

$$= -\frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{3g(f+gx)^3}+\frac{(B(bc-ad)n)\int\frac{1}{(a+bx)(c+dx)(f+gx)^3}\,dx}{3g}$$

$$= -\frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{3g(f+gx)^3}+\frac{(B(bc-ad)n)\int\left(\frac{b^4}{(bc-ad)(bf-ag)^3(a+bx)}+\frac{d^4}{(bc-ad)(-df+cg)^3(c+d}\right.}{}$$

$$= -\frac{B(bc-ad)n}{6(bf-ag)(df-cg)(f+gx)^2}-\frac{B(bc-ad)(2bdf-bcg-adg)n}{3(bf-ag)^2(df-cg)^2(f+gx)}+\frac{b^3Bn\log(a+}{3g(bf-ag)}$$

**Mathematica [A]** time = 0.89, size = 264, normalized size = 0.93

$$\frac{Bn(bc-ad)\left(\frac{g\log(f+gx)\left(a^2d^2g^2+abdg(cg-3df)+b^2\left(c^2g^2-3cdfg+3d^2f^2\right)\right)}{(bf-ag)^3(df-cg)^3}+\frac{b^3\log(a+bx)}{(bc-ad)(bf-ag)^3}+\frac{d^3\log(c+dx)}{(bc-ad)(cg-df)^3}+\frac{g(adg+bcg-2bdf)}{(f+gx)(bf-ag)^2(df-cg)}\right.}{3g}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Log[e*((a + b*x)/(c + d*x))^n])/(f + g*x)^4,x]
```

```
[Out] (-((A + B*Log[e*((a + b*x)/(c + d*x))^n])/(f + g*x)^3) + B*(b*c - a*d)*n*(-
1/2*g/((b*f - a*g)*(d*f - c*g)*(f + g*x)^2) + (g*(-2*b*d*f + b*c*g + a*d*g)
)/((b*f - a*g)^2*(d*f - c*g)^2*(f + g*x)) + (b^3*Log[a + b*x])/((b*c - a*d)
*(b*f - a*g)^3) + (d^3*Log[c + d*x])/((b*c - a*d)*(-(d*f) + c*g)^3) + (g*(a
^2*d^2*g^2 + a*b*d*g*(-3*d*f + c*g) + b^2*(3*d^2*f^2 - 3*c*d*f*g + c^2*g^2)
)*Log[f + g*x])/((b*f - a*g)^3*(d*f - c*g)^3)))/(3*g)
```

**fricas [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))/(g*x+f)^4,x, algorithm="fricas")
```

```
[Out] Timed out
```

**giac [B]** time = 9.25, size = 9570, normalized size = 33.82

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))/(g*x+f)^4,x, algorithm="giac")
```

```
[Out] 1/6*(2*(3*B*b^4*c^2*d^2*f^2*n - 6*B*a*b^3*c*d^3*f^2*n + 3*B*a^2*b^2*d^4*f^2
*n - 3*B*b^4*c^3*d*f*g*n + 3*B*a*b^3*c^2*d^2*f*g*n + 3*B*a^2*b^2*c*d^3*f*g*
n - 3*B*a^3*b*d^4*f*g*n + B*b^4*c^4*g^2*n - B*a*b^3*c^3*d*g^2*n - B*a^3*b*c
*d^3*g^2*n + B*a^4*d^4*g^2*n)*log(-b*f + (b*x + a)*d*f/(d*x + c) + a*g - (b
*x + a)*c*g/(d*x + c))/(b^3*d^3*f^6 - 3*b^3*c*d^2*f^5*g - 3*a*b^2*d^3*f^5*g
 + 3*b^3*c^2*d*f^4*g^2 + 9*a*b^2*c*d^2*f^4*g^2 + 3*a^2*b*d^3*f^4*g^2 - b^3*
c^3*f^3*g^3 - 9*a*b^2*c^2*d*f^3*g^3 - 9*a^2*b*c*d^2*f^3*g^3 - a^3*d^3*f^3*g
^3 + 3*a*b^2*c^3*f^2*g^4 + 9*a^2*b*c^2*d*f^2*g^4 + 3*a^3*c*d^2*f^2*g^4 - 3*
a^2*b*c^3*f*g^5 - 3*a^3*c^2*d*f*g^5 + a^3*c^3*g^6) + 2*(3*B*b^4*c^2*d^2*f^2
*n - 6*B*a*b^3*c*d^3*f^2*n - 6*(b*x + a)*B*b^3*c^2*d^3*f^2*n/(d*x + c) + 3*
B*a^2*b^2*d^4*f^2*n + 12*(b*x + a)*B*a*b^2*c*d^4*f^2*n/(d*x + c) + 3*(b*x +
 a)^2*B*b^2*c^2*d^4*f^2*n/(d*x + c)^2 - 6*(b*x + a)*B*a^2*b*d^5*f^2*n/(d*x
+ c) - 6*(b*x + a)^2*B*a*b*c*d^5*f^2*n/(d*x + c)^2 + 3*(b*x + a)^2*B*a^2*d^
6*f^2*n/(d*x + c)^2 - 3*B*b^4*c^3*d*f*g*n + 3*B*a*b^3*c^2*d^2*f*g*n + 9*(b*
x + a)*B*b^3*c^3*d^2*f*g*n/(d*x + c) + 3*B*a^2*b^2*c*d^3*f*g*n - 15*(b*x +
a)*B*a*b^2*c^2*d^3*f*g*n/(d*x + c) - 6*(b*x + a)^2*B*b^2*c^3*d^3*f*g*n/(d*x
 + c)^2 - 3*B*a^3*b*d^4*f*g*n + 3*(b*x + a)*B*a^2*b*c*d^4*f*g*n/(d*x + c) +
 12*(b*x + a)^2*B*a*b*c^2*d^4*f*g*n/(d*x + c)^2 + 3*(b*x + a)*B*a^3*d^5*f*g
*n/(d*x + c) - 6*(b*x + a)^2*B*a^2*c*d^5*f*g*n/(d*x + c)^2 + B*b^4*c^4*g^2*
n - B*a*b^3*c^3*d*g^2*n - 3*(b*x + a)*B*b^3*c^4*d*g^2*n/(d*x + c) + 3*(b*x
+ a)*B*a*b^2*c^3*d^2*g^2*n/(d*x + c) + 3*(b*x + a)^2*B*b^2*c^4*d^2*g^2*n/(d
*x + c)^2 - B*a^3*b*c*d^3*g^2*n + 3*(b*x + a)*B*a^2*b*c^2*d^3*g^2*n/(d*x +
c) - 6*(b*x + a)^2*B*a*b*c^3*d^3*g^2*n/(d*x + c)^2 + B*a^4*d^4*g^2*n - 3*(b
*x + a)*B*a^3*c*d^4*g^2*n/(d*x + c) + 3*(b*x + a)^2*B*a^2*c^2*d^4*g^2*n/(d*
x + c)^2)*log((b*x + a)/(d*x + c))/(b^3*d^3*f^6 - 3*(b*x + a)*b^2*d^4*f^6/(
d*x + c) + 3*(b*x + a)^2*b*d^5*f^6/(d*x + c)^2 - (b*x + a)^3*d^6*f^6/(d*x +
 c)^3 - 3*b^3*c*d^2*f^5*g - 3*a*b^2*d^3*f^5*g + 12*(b*x + a)*b^2*c*d^3*f^5*
g/(d*x + c) + 6*(b*x + a)*a*b*d^4*f^5*g/(d*x + c) - 15*(b*x + a)^2*b*c*d^4*
```

f^5*g/(d*x + c)^2 - 3*(b*x + a)^2*a*d^5*f^5*g/(d*x + c)^2 + 6*(b*x + a)^3*c
*d^5*f^5*g/(d*x + c)^3 + 3*b^3*c^2*d*f^4*g^2 + 9*a*b^2*c*d^2*f^4*g^2 - 18*(
b*x + a)*b^2*c^2*d^2*f^4*g^2/(d*x + c) + 3*a^2*b*d^3*f^4*g^2 - 24*(b*x + a)
*a*b*c*d^3*f^4*g^2/(d*x + c) + 30*(b*x + a)^2*b*c^2*d^3*f^4*g^2/(d*x + c)^2
 - 3*(b*x + a)*a^2*d^4*f^4*g^2/(d*x + c) + 15*(b*x + a)^2*a*c*d^4*f^4*g^2/(
d*x + c)^2 - 15*(b*x + a)^3*c^2*d^4*f^4*g^2/(d*x + c)^3 - b^3*c^3*f^3*g^3 -
 9*a*b^2*c^2*d*f^3*g^3 + 12*(b*x + a)*b^2*c^3*d*f^3*g^3/(d*x + c) - 9*a^2*b
*c*d^2*f^3*g^3 + 36*(b*x + a)*a*b*c^2*d^2*f^3*g^3/(d*x + c) - 30*(b*x + a)^
2*b*c^3*d^2*f^3*g^3/(d*x + c)^2 - a^3*d^3*f^3*g^3 + 12*(b*x + a)*a^2*c*d^3*
f^3*g^3/(d*x + c) - 30*(b*x + a)^2*a*c^2*d^3*f^3*g^3/(d*x + c)^2 + 20*(b*x
+ a)^3*c^3*d^3*f^3*g^3/(d*x + c)^3 + 3*a*b^2*c^3*f^2*g^4 - 3*(b*x + a)*b^2*
c^4*f^2*g^4/(d*x + c) + 9*a^2*b*c^2*d*f^2*g^4 - 24*(b*x + a)*a*b*c^3*d*f^2*
g^4/(d*x + c) + 15*(b*x + a)^2*b*c^4*d*f^2*g^4/(d*x + c)^2 + 3*a^3*c*d^2*f^
2*g^4 - 18*(b*x + a)*a^2*c^2*d^2*f^2*g^4/(d*x + c) + 30*(b*x + a)^2*a*c^3*d
^2*f^2*g^4/(d*x + c)^2 - 15*(b*x + a)^3*c^4*d^2*f^2*g^4/(d*x + c)^3 - 3*a^2
*b*c^3*f*g^5 + 6*(b*x + a)*a*b*c^4*f*g^5/(d*x + c) - 3*(b*x + a)^2*b*c^5*f*
g^5/(d*x + c)^2 - 3*a^3*c^2*d*f*g^5 + 12*(b*x + a)*a^2*c^3*d*f*g^5/(d*x + c
) - 15*(b*x + a)^2*a*c^4*d*f*g^5/(d*x + c)^2 + 6*(b*x + a)^3*c^5*d*f*g^5/(d
*x + c)^3 + a^3*c^3*g^6 - 3*(b*x + a)*a^2*c^4*g^6/(d*x + c) + 3*(b*x + a)^2
*a*c^5*g^6/(d*x + c)^2 - (b*x + a)^3*c^6*g^6/(d*x + c)^3) - 2*(3*B*b^4*c^2*
d^2*f^2*n - 6*B*a*b^3*c*d^3*f^2*n + 3*B*a^2*b^2*d^4*f^2*n - 3*B*b^4*c^3*d*f
*g*n + 3*B*a*b^3*c^2*d^2*f*g*n + 3*B*a^2*b^2*c*d^3*f*g*n - 3*B*a^3*b*d^4*f*
g*n + B*b^4*c^4*g^2*n - B*a*b^3*c^3*d*g^2*n - B*a^3*b*c*d^3*g^2*n + B*a^4*d
^4*g^2*n)*log((b*x + a)/(d*x + c))/(b^3*d^3*f^6 - 3*b^3*c*d^2*f^5*g - 3*a*b
^2*d^3*f^5*g + 3*b^3*c^2*d*f^4*g^2 + 9*a*b^2*c*d^2*f^4*g^2 + 3*a^2*b*d^3*f^
4*g^2 - b^3*c^3*f^3*g^3 - 9*a*b^2*c^2*d*f^3*g^3 - 9*a^2*b*c*d^2*f^3*g^3 - a
^3*d^3*f^3*g^3 + 3*a*b^2*c^3*f^2*g^4 + 9*a^2*b*c^2*d*f^2*g^4 + 3*a^3*c*d^2*
f^2*g^4 - 3*a^2*b*c^3*f*g^5 - 3*a^3*c^2*d*f*g^5 + a^3*c^3*g^6) + (6*B*b^6*c
^3*d*f^3*g*n - 18*B*a*b^5*c^2*d^2*f^3*g*n - 12*(b*x + a)*B*b^5*c^3*d^2*f^3*
g*n/(d*x + c) + 18*B*a^2*b^4*c*d^3*f^3*g*n + 36*(b*x + a)*B*a*b^4*c^2*d^3*f
^3*g*n/(d*x + c) + 6*(b*x + a)^2*B*b^4*c^3*d^3*f^3*g*n/(d*x + c)^2 - 6*B*a^
3*b^3*d^4*f^3*g*n - 36*(b*x + a)*B*a^2*b^3*c*d^4*f^3*g*n/(d*x + c) - 18*(b*
x + a)^2*B*a*b^3*c^2*d^4*f^3*g*n/(d*x + c)^2 + 12*(b*x + a)*B*a^3*b^2*d^5*f
^3*g*n/(d*x + c) + 18*(b*x + a)^2*B*a^2*b^2*c*d^5*f^3*g*n/(d*x + c)^2 - 6*(
b*x + a)^2*B*a^3*b*d^6*f^3*g*n/(d*x + c)^2 - 3*B*b^6*c^4*f^2*g^2*n - 6*B*a*
b^5*c^3*d*f^2*g^2*n + 17*(b*x + a)*B*b^5*c^4*d*f^2*g^2*n/(d*x + c) + 36*B*a
^2*b^4*c^2*d^2*f^2*g^2*n - 32*(b*x + a)*B*a*b^4*c^3*d^2*f^2*g^2*n/(d*x + c)
 - 14*(b*x + a)^2*B*b^4*c^4*d^2*f^2*g^2*n/(d*x + c)^2 - 42*B*a^3*b^3*c*d^3*
f^2*g^2*n - 6*(b*x + a)*B*a^2*b^3*c^2*d^3*f^2*g^2*n/(d*x + c) + 38*(b*x + a
)^2*B*a*b^3*c^3*d^3*f^2*g^2*n/(d*x + c)^2 + 15*B*a^4*b^2*d^4*f^2*g^2*n + 40
*(b*x + a)*B*a^3*b^2*c*d^4*f^2*g^2*n/(d*x + c) - 30*(b*x + a)^2*B*a^2*b^2*c
^2*d^4*f^2*g^2*n/(d*x + c)^2 - 19*(b*x + a)*B*a^4*b*d^5*f^2*g^2*n/(d*x + c)
 + 2*(b*x + a)^2*B*a^3*b*c*d^5*f^2*g^2*n/(d*x + c)^2 + 4*(b*x + a)^2*B*a^4*
d^6*f^2*g^2*n/(d*x + c)^2 + 6*B*a*b^5*c^4*f*g^3*n - 5*(b*x + a)*B*b^5*c^5*f
*g^3*n/(d*x + c) - 6*B*a^2*b^4*c^3*d*f*g^3*n - 9*(b*x + a)*B*a*b^4*c^4*d*f*

g^3*n/(d*x + c) + 10*(b*x + a)^2*B*b^4*c^5*d*f*g^3*n/(d*x + c)^2 - 18*B*a^3
*b^3*c^2*d^2*f*g^3*n + 50*(b*x + a)*B*a^2*b^3*c^3*d^2*f*g^3*n/(d*x + c) - 2
2*(b*x + a)^2*B*a*b^3*c^4*d^2*f*g^3*n/(d*x + c)^2 + 30*B*a^4*b^2*c*d^3*f*g^
3*n - 46*(b*x + a)*B*a^3*b^2*c^2*d^3*f*g^3*n/(d*x + c) + 6*(b*x + a)^2*B*a^
2*b^2*c^3*d^3*f*g^3*n/(d*x + c)^2 - 12*B*a^5*b*d^4*f*g^3*n + 3*(b*x + a)*B*
a^4*b*c*d^4*f*g^3*n/(d*x + c) + 14*(b*x + a)^2*B*a^3*b*c^2*d^4*f*g^3*n/(d*x
 + c)^2 + 7*(b*x + a)*B*a^5*d^5*f*g^3*n/(d*x + c) - 8*(b*x + a)^2*B*a^4*c*d
^5*f*g^3*n/(d*x + c)^2 - 3*B*a^2*b^4*c^4*g^4*n + 5*(b*x + a)*B*a*b^4*c^5*g^
4*n/(d*x + c) - 2*(b*x + a)^2*B*b^4*c^6*g^4*n/(d*x + c)^2 + 6*B*a^3*b^3*c^3
*d*g^4*n - 8*(b*x + a)*B*a^2*b^3*c^4*d*g^4*n/(d*x + c) + 2*(b*x + a)^2*B*a*
b^3*c^5*d*g^4*n/(d*x + c)^2 - 6*(b*x + a)*B*a^3*b^2*c^3*d^2*g^4*n/(d*x + c)
 + 6*(b*x + a)^2*B*a^2*b^2*c^4*d^2*g^4*n/(d*x + c)^2 - 6*B*a^5*b*c*d^3*g^4*
n + 16*(b*x + a)*B*a^4*b*c^2*d^3*g^4*n/(d*x + c) - 10*(b*x + a)^2*B*a^3*b*c
^3*d^3*g^4*n/(d*x + c)^2 + 3*B*a^6*d^4*g^4*n - 7*(b*x + a)*B*a^5*c*d^4*g^4*
n/(d*x + c) + 4*(b*x + a)^2*B*a^4*c^2*d^4*g^4*n/(d*x + c)^2 + 6*A*b^6*c^2*d
^2*f^4 + 6*B*b^6*c^2*d^2*f^4 - 12*A*a*b^5*c*d^3*f^4 - 12*B*a*b^5*c*d^3*f^4
- 12*(b*x + a)*A*b^5*c^2*d^3*f^4/(d*x + c) - 12*(b*x + a)*B*b^5*c^2*d^3*f^4
/(d*x + c) + 6*A*a^2*b^4*d^4*f^4 + 6*B*a^2*b^4*d^4*f^4 + 24*(b*x + a)*A*a*b
^4*c*d^4*f^4/(d*x + c) + 24*(b*x + a)*B*a*b^4*c*d^4*f^4/(d*x + c) + 6*(b*x
+ a)^2*A*b^4*c^2*d^4*f^4/(d*x + c)^2 + 6*(b*x + a)^2*B*b^4*c^2*d^4*f^4/(d*x
 + c)^2 - 12*(b*x + a)*A*a^2*b^3*d^5*f^4/(d*x + c) - 12*(b*x + a)*B*a^2*b^3
*d^5*f^4/(d*x + c) - 12*(b*x + a)^2*A*a*b^3*c*d^5*f^4/(d*x + c)^2 - 12*(b*x
 + a)^2*B*a*b^3*c*d^5*f^4/(d*x + c)^2 + 6*(b*x + a)^2*A*a^2*b^2*d^6*f^4/(d*
x + c)^2 + 6*(b*x + a)^2*B*a^2*b^2*d^6*f^4/(d*x + c)^2 - 6*A*b^6*c^3*d*f^3*
g - 6*B*b^6*c^3*d*f^3*g - 6*A*a*b^5*c^2*d^2*f^3*g - 6*B*a*b^5*c^2*d^2*f^3*g
 + 18*(b*x + a)*A*b^5*c^3*d^2*f^3*g/(d*x + c) + 18*(b*x + a)*B*b^5*c^3*d^2*
f^3*g/(d*x + c) + 30*A*a^2*b^4*c*d^3*f^3*g + 30*B*a^2*b^4*c*d^3*f^3*g - 6*(
b*x + a)*A*a*b^4*c^2*d^3*f^3*g/(d*x + c) - 6*(b*x + a)*B*a*b^4*c^2*d^3*f^3*
g/(d*x + c) - 12*(b*x + a)^2*A*b^4*c^3*d^3*f^3*g/(d*x + c)^2 - 12*(b*x + a)
^2*B*b^4*c^3*d^3*f^3*g/(d*x + c)^2 - 18*A*a^3*b^3*d^4*f^3*g - 18*B*a^3*b^3*
d^4*f^3*g - 42*(b*x + a)*A*a^2*b^3*c*d^4*f^3*g/(d*x + c) - 42*(b*x + a)*B*a
^2*b^3*c*d^4*f^3*g/(d*x + c) + 12*(b*x + a)^2*A*a*b^3*c^2*d^4*f^3*g/(d*x +
c)^2 + 12*(b*x + a)^2*B*a*b^3*c^2*d^4*f^3*g/(d*x + c)^2 + 30*(b*x + a)*A*a^
3*b^2*d^5*f^3*g/(d*x + c) + 30*(b*x + a)*B*a^3*b^2*d^5*f^3*g/(d*x + c) + 12
*(b*x + a)^2*A*a^2*b^2*c*d^5*f^3*g/(d*x + c)^2 + 12*(b*x + a)^2*B*a^2*b^2*c
*d^5*f^3*g/(d*x + c)^2 - 12*(b*x + a)^2*A*a^3*b*d^6*f^3*g/(d*x + c)^2 - 12*
(b*x + a)^2*B*a^3*b*d^6*f^3*g/(d*x + c)^2 + 2*A*b^6*c^4*f^2*g^2 + 2*B*b^6*c
^4*f^2*g^2 + 10*A*a*b^5*c^3*d*f^2*g^2 + 10*B*a*b^5*c^3*d*f^2*g^2 - 6*(b*x +
 a)*A*b^5*c^4*d*f^2*g^2/(d*x + c) - 6*(b*x + a)*B*b^5*c^4*d*f^2*g^2/(d*x +
c) - 6*A*a^2*b^4*c^2*d^2*f^2*g^2 - 6*B*a^2*b^4*c^2*d^2*f^2*g^2 - 30*(b*x +
a)*A*a*b^4*c^3*d^2*f^2*g^2/(d*x + c) - 30*(b*x + a)*B*a*b^4*c^3*d^2*f^2*g^2
/(d*x + c) + 6*(b*x + a)^2*A*b^4*c^4*d^2*f^2*g^2/(d*x + c)^2 + 6*(b*x + a)^
2*B*b^4*c^4*d^2*f^2*g^2/(d*x + c)^2 - 26*A*a^3*b^3*c*d^3*f^2*g^2 - 26*B*a^3
*b^3*c*d^3*f^2*g^2 + 54*(b*x + a)*A*a^2*b^3*c^2*d^3*f^2*g^2/(d*x + c) + 54*
(b*x + a)*B*a^2*b^3*c^2*d^3*f^2*g^2/(d*x + c) + 12*(b*x + a)^2*A*a*b^3*c^3*

d^3*f^2*g^2/(d*x + c)^2 + 12*(b*x + a)^2*B*a*b^3*c^3*d^3*f^2*g^2/(d*x + c)^
2 + 20*A*a^4*b^2*d^4*f^2*g^2 + 20*B*a^4*b^2*d^4*f^2*g^2 + 6*(b*x + a)*A*a^3
*b^2*c*d^4*f^2*g^2/(d*x + c) + 6*(b*x + a)*B*a^3*b^2*c*d^4*f^2*g^2/(d*x + c
) - 36*(b*x + a)^2*A*a^2*b^2*c^2*d^4*f^2*g^2/(d*x + c)^2 - 36*(b*x + a)^2*B
*a^2*b^2*c^2*d^4*f^2*g^2/(d*x + c)^2 - 24*(b*x + a)*A*a^4*b*d^5*f^2*g^2/(d*
x + c) - 24*(b*x + a)*B*a^4*b*d^5*f^2*g^2/(d*x + c) + 12*(b*x + a)^2*A*a^3*
b*c*d^5*f^2*g^2/(d*x + c)^2 + 12*(b*x + a)^2*B*a^3*b*c*d^5*f^2*g^2/(d*x + c
)^2 + 6*(b*x + a)^2*A*a^4*d^6*f^2*g^2/(d*x + c)^2 + 6*(b*x + a)^2*B*a^4*d^6
*f^2*g^2/(d*x + c)^2 - 4*A*a*b^5*c^4*f*g^3 - 4*B*a*b^5*c^4*f*g^3 - 2*A*a^2*
b^4*c^3*d*f*g^3 - 2*B*a^2*b^4*c^3*d*f*g^3 + 12*(b*x + a)*A*a*b^4*c^4*d*f*g^
3/(d*x + c) + 12*(b*x + a)*B*a*b^4*c^4*d*f*g^3/(d*x + c) + 6*A*a^3*b^3*c^2*
d^2*f*g^3 + 6*B*a^3*b^3*c^2*d^2*f*g^3 + 6*(b*x + a)*A*a^2*b^3*c^3*d^2*f*g^3
/(d*x + c) + 6*(b*x + a)*B*a^2*b^3*c^3*d^2*f*g^3/(d*x + c) - 12*(b*x + a)^2
*A*a*b^3*c^4*d^2*f*g^3/(d*x + c)^2 - 12*(b*x + a)^2*B*a*b^3*c^4*d^2*f*g^3/(
d*x + c)^2 + 10*A*a^4*b^2*c*d^3*f*g^3 + 10*B*a^4*b^2*c*d^3*f*g^3 - 42*(b*x
+ a)*A*a^3*b^2*c^2*d^3*f*g^3/(d*x + c) - 42*(b*x + a)*B*a^3*b^2*c^2*d^3*f*g
^3/(d*x + c) + 12*(b*x + a)^2*A*a^2*b^2*c^3*d^3*f*g^3/(d*x + c)^2 + 12*(b*x
 + a)^2*B*a^2*b^2*c^3*d^3*f*g^3/(d*x + c)^2 - 10*A*a^5*b*d^4*f*g^3 - 10*B*a
^5*b*d^4*f*g^3 + 18*(b*x + a)*A*a^4*b*c*d^4*f*g^3/(d*x + c) + 18*(b*x + a)*
B*a^4*b*c*d^4*f*g^3/(d*x + c) + 12*(b*x + a)^2*A*a^3*b*c^2*d^4*f*g^3/(d*x +
 c)^2 + 12*(b*x + a)^2*B*a^3*b*c^2*d^4*f*g^3/(d*x + c)^2 + 6*(b*x + a)*A*a^
5*d^5*f*g^3/(d*x + c) + 6*(b*x + a)*B*a^5*d^5*f*g^3/(d*x + c) - 12*(b*x + a
)^2*A*a^4*c*d^5*f*g^3/(d*x + c)^2 - 12*(b*x + a)^2*B*a^4*c*d^5*f*g^3/(d*x +
 c)^2 + 2*A*a^2*b^4*c^4*g^4 + 2*B*a^2*b^4*c^4*g^4 - 2*A*a^3*b^3*c^3*d*g^4 -
 2*B*a^3*b^3*c^3*d*g^4 - 6*(b*x + a)*A*a^2*b^3*c^4*d*g^4/(d*x + c) - 6*(b*x
 + a)*B*a^2*b^3*c^4*d*g^4/(d*x + c) + 6*(b*x + a)*A*a^3*b^2*c^3*d^2*g^4/(d*
x + c) + 6*(b*x + a)*B*a^3*b^2*c^3*d^2*g^4/(d*x + c) + 6*(b*x + a)^2*A*a^2*
b^2*c^4*d^2*g^4/(d*x + c)^2 + 6*(b*x + a)^2*B*a^2*b^2*c^4*d^2*g^4/(d*x + c)
^2 - 2*A*a^5*b*c*d^3*g^4 - 2*B*a^5*b*c*d^3*g^4 + 6*(b*x + a)*A*a^4*b*c^2*d^
3*g^4/(d*x + c) + 6*(b*x + a)*B*a^4*b*c^2*d^3*g^4/(d*x + c) - 12*(b*x + a)^
2*A*a^3*b*c^3*d^3*g^4/(d*x + c)^2 - 12*(b*x + a)^2*B*a^3*b*c^3*d^3*g^4/(d*x
 + c)^2 + 2*A*a^6*d^4*g^4 + 2*B*a^6*d^4*g^4 - 6*(b*x + a)*A*a^5*c*d^4*g^4/(
d*x + c) - 6*(b*x + a)*B*a^5*c*d^4*g^4/(d*x + c) + 6*(b*x + a)^2*A*a^4*c^2*
d^4*g^4/(d*x + c)^2 + 6*(b*x + a)^2*B*a^4*c^2*d^4*g^4/(d*x + c)^2)/(b^5*d^3
*f^8 - 3*(b*x + a)*b^4*d^4*f^8/(d*x + c) + 3*(b*x + a)^2*b^3*d^5*f^8/(d*x +
 c)^2 - (b*x + a)^3*b^2*d^6*f^8/(d*x + c)^3 - 3*b^5*c*d^2*f^7*g - 5*a*b^4*d
^3*f^7*g + 12*(b*x + a)*b^4*c*d^3*f^7*g/(d*x + c) + 12*(b*x + a)*a*b^3*d^4*
f^7*g/(d*x + c) - 15*(b*x + a)^2*b^3*c*d^4*f^7*g/(d*x + c)^2 - 9*(b*x + a)^
2*a*b^2*d^5*f^7*g/(d*x + c)^2 + 6*(b*x + a)^3*b^2*c*d^5*f^7*g/(d*x + c)^3 +
 2*(b*x + a)^3*a*b*d^6*f^7*g/(d*x + c)^3 + 3*b^5*c^2*d*f^6*g^2 + 15*a*b^4*c
*d^2*f^6*g^2 - 18*(b*x + a)*b^4*c^2*d^2*f^6*g^2/(d*x + c) + 10*a^2*b^3*d^3*
f^6*g^2 - 48*(b*x + a)*a*b^3*c*d^3*f^6*g^2/(d*x + c) + 30*(b*x + a)^2*b^3*c
^2*d^3*f^6*g^2/(d*x + c)^2 - 18*(b*x + a)*a^2*b^2*d^4*f^6*g^2/(d*x + c) + 4
5*(b*x + a)^2*a*b^2*c*d^4*f^6*g^2/(d*x + c)^2 - 15*(b*x + a)^3*b^2*c^2*d^4*
f^6*g^2/(d*x + c)^3 + 9*(b*x + a)^2*a^2*b*d^5*f^6*g^2/(d*x + c)^2 - 12*(b*x

```
 + a)^3*a*b*c*d^5*f^6*g^2/(d*x + c)^3 - (b*x + a)^3*a^2*d^6*f^6*g^2/(d*x +
c)^3 - b^5*c^3*f^5*g^3 - 15*a*b^4*c^2*d*f^5*g^3 + 12*(b*x + a)*b^4*c^3*d*f^
5*g^3/(d*x + c) - 30*a^2*b^3*c*d^2*f^5*g^3 + 72*(b*x + a)*a*b^3*c^2*d^2*f^5
*g^3/(d*x + c) - 30*(b*x + a)^2*b^3*c^3*d^2*f^5*g^3/(d*x + c)^2 - 10*a^3*b^
2*d^3*f^5*g^3 + 72*(b*x + a)*a^2*b^2*c*d^3*f^5*g^3/(d*x + c) - 90*(b*x + a)
^2*a*b^2*c^2*d^3*f^5*g^3/(d*x + c)^2 + 20*(b*x + a)^3*b^2*c^3*d^3*f^5*g^3/(
d*x + c)^3 + 12*(b*x + a)*a^3*b*d^4*f^5*g^3/(d*x + c) - 45*(b*x + a)^2*a^2*
b*c*d^4*f^5*g^3/(d*x + c)^2 + 30*(b*x + a)^3*a*b*c^2*d^4*f^5*g^3/(d*x + c)^
3 - 3*(b*x + a)^2*a^3*d^5*f^5*g^3/(d*x + c)^2 + 6*(b*x + a)^3*a^2*c*d^5*f^5
*g^3/(d*x + c)^3 + 5*a*b^4*c^3*f^4*g^4 - 3*(b*x + a)*b^4*c^4*f^4*g^4/(d*x +
 c) + 30*a^2*b^3*c^2*d*f^4*g^4 - 48*(b*x + a)*a*b^3*c^3*d*f^4*g^4/(d*x + c)
 + 15*(b*x + a)^2*b^3*c^4*d*f^4*g^4/(d*x + c)^2 + 30*a^3*b^2*c*d^2*f^4*g^4
- 108*(b*x + a)*a^2*b^2*c^2*d^2*f^4*g^4/(d*x + c) + 90*(b*x + a)^2*a*b^2*c^
3*d^2*f^4*g^4/(d*x + c)^2 - 15*(b*x + a)^3*b^2*c^4*d^2*f^4*g^4/(d*x + c)^3
+ 5*a^4*b*d^3*f^4*g^4 - 48*(b*x + a)*a^3*b*c*d^3*f^4*g^4/(d*x + c) + 90*(b*
x + a)^2*a^2*b*c^2*d^3*f^4*g^4/(d*x + c)^2 - 40*(b*x + a)^3*a*b*c^3*d^3*f^4
*g^4/(d*x + c)^3 - 3*(b*x + a)*a^4*d^4*f^4*g^4/(d*x + c) + 15*(b*x + a)^2*a
^3*c*d^4*f^4*g^4/(d*x + c)^2 - 15*(b*x + a)^3*a^2*c^2*d^4*f^4*g^4/(d*x + c)
^3 - 10*a^2*b^3*c^3*f^3*g^5 + 12*(b*x + a)*a*b^3*c^4*f^3*g^5/(d*x + c) - 3*
(b*x + a)^2*b^3*c^5*f^3*g^5/(d*x + c)^2 - 30*a^3*b^2*c^2*d*f^3*g^5 + 72*(b*
x + a)*a^2*b^2*c^3*d*f^3*g^5/(d*x + c) - 45*(b*x + a)^2*a*b^2*c^4*d*f^3*g^5
/(d*x + c)^2 + 6*(b*x + a)^3*b^2*c^5*d*f^3*g^5/(d*x + c)^3 - 15*a^4*b*c*d^2
*f^3*g^5 + 72*(b*x + a)*a^3*b*c^2*d^2*f^3*g^5/(d*x + c) - 90*(b*x + a)^2*a^
2*b*c^3*d^2*f^3*g^5/(d*x + c)^2 + 30*(b*x + a)^3*a*b*c^4*d^2*f^3*g^5/(d*x +
 c)^3 - a^5*d^3*f^3*g^5 + 12*(b*x + a)*a^4*c*d^3*f^3*g^5/(d*x + c) - 30*(b*
x + a)^2*a^3*c^2*d^3*f^3*g^5/(d*x + c)^2 + 20*(b*x + a)^3*a^2*c^3*d^3*f^3*g
^5/(d*x + c)^3 + 10*a^3*b^2*c^3*f^2*g^6 - 18*(b*x + a)*a^2*b^2*c^4*f^2*g^6/
(d*x + c) + 9*(b*x + a)^2*a*b^2*c^5*f^2*g^6/(d*x + c)^2 - (b*x + a)^3*b^2*c
^6*f^2*g^6/(d*x + c)^3 + 15*a^4*b*c^2*d*f^2*g^6 - 48*(b*x + a)*a^3*b*c^3*d*
f^2*g^6/(d*x + c) + 45*(b*x + a)^2*a^2*b*c^4*d*f^2*g^6/(d*x + c)^2 - 12*(b*
x + a)^3*a*b*c^5*d*f^2*g^6/(d*x + c)^3 + 3*a^5*c*d^2*f^2*g^6 - 18*(b*x + a)
*a^4*c^2*d^2*f^2*g^6/(d*x + c) + 30*(b*x + a)^2*a^3*c^3*d^2*f^2*g^6/(d*x +
c)^2 - 15*(b*x + a)^3*a^2*c^4*d^2*f^2*g^6/(d*x + c)^3 - 5*a^4*b*c^3*f*g^7 +
 12*(b*x + a)*a^3*b*c^4*f*g^7/(d*x + c) - 9*(b*x + a)^2*a^2*b*c^5*f*g^7/(d*
x + c)^2 + 2*(b*x + a)^3*a*b*c^6*f*g^7/(d*x + c)^3 - 3*a^5*c^2*d*f*g^7 + 12
*(b*x + a)*a^4*c^3*d*f*g^7/(d*x + c) - 15*(b*x + a)^2*a^3*c^4*d*f*g^7/(d*x
+ c)^2 + 6*(b*x + a)^3*a^2*c^5*d*f*g^7/(d*x + c)^3 + a^5*c^3*g^8 - 3*(b*x +
 a)*a^4*c^4*g^8/(d*x + c) + 3*(b*x + a)^2*a^3*c^5*g^8/(d*x + c)^2 - (b*x +
a)^3*a^2*c^6*g^8/(d*x + c)^3))*(b*c/(b*c - a*d)^2 - a*d/(b*c - a*d)^2)
```

**maple [F]** time = 0.28, size = 0, normalized size = 0.00

$$\int \frac{B \ln\left(e \left(\frac{bx+a}{dx+c}\right)^{n}\right)+A}{\left(gx+f\right)^{4}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*ln(e*((b*x+a)/(d*x+c))^n)+A)/(g*x+f)^4,x)
```

```
[Out] int((B*ln(e*((b*x+a)/(d*x+c))^n)+A)/(g*x+f)^4,x)
```

**maxima [B]** time = 1.41, size = 852, normalized size = 3.01

$$\frac{1}{6}\left(\frac{2\,b^3\log\left(bx+a\right)}{b^3f^3g-3\,ab^2f^2g^2+3\,a^2bfg^3-a^3g^4}-\frac{2\,d^3\log\left(dx+c\right)}{d^3f^3g-3\,cd^2f^2g^2+3\,c^2dfg^3-c^3g^4}+\frac{}{b^3d^3f^6+a^3c^3g^6-3\left(b^3cd^2+\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))/(g*x+f)^4,x, algorithm="maxima")
```

```
[Out] 1/6*(2*b^3*log(b*x + a)/(b^3*f^3*g - 3*a*b^2*f^2*g^2 + 3*a^2*b*f*g^3 - a^3*
g^4) - 2*d^3*log(d*x + c)/(d^3*f^3*g - 3*c*d^2*f^2*g^2 + 3*c^2*d*f*g^3 - c^
3*g^4) + 2*(3*(b^3*c*d^2 - a*b^2*d^3)*f^2 - 3*(b^3*c^2*d - a^2*b*d^3)*f*g +
 (b^3*c^3 - a^3*d^3)*g^2)*log(g*x + f)/(b^3*d^3*f^6 + a^3*c^3*g^6 - 3*(b^3*
c*d^2 + a*b^2*d^3)*f^5*g + 3*(b^3*c^2*d + 3*a*b^2*c*d^2 + a^2*b*d^3)*f^4*g^
2 - (b^3*c^3 + 9*a*b^2*c^2*d + 9*a^2*b*c*d^2 + a^3*d^3)*f^3*g^3 + 3*(a*b^2*
c^3 + 3*a^2*b*c^2*d + a^3*c*d^2)*f^2*g^4 - 3*(a^2*b*c^3 + a^3*c^2*d)*f*g^5)
 - (5*(b^2*c*d - a*b*d^2)*f^2 - 3*(b^2*c^2 - a^2*d^2)*f*g + (a*b*c^2 - a^2*
c*d)*g^2 + 2*(2*(b^2*c*d - a*b*d^2)*f*g - (b^2*c^2 - a^2*d^2)*g^2)*x)/(b^2*
d^2*f^6 + a^2*c^2*f^2*g^4 - 2*(b^2*c*d + a*b*d^2)*f^5*g + (b^2*c^2 + 4*a*b*
c*d + a^2*d^2)*f^4*g^2 - 2*(a*b*c^2 + a^2*c*d)*f^3*g^3 + (b^2*d^2*f^4*g^2 +
 a^2*c^2*g^6 - 2*(b^2*c*d + a*b*d^2)*f^3*g^3 + (b^2*c^2 + 4*a*b*c*d + a^2*d
^2)*f^2*g^4 - 2*(a*b*c^2 + a^2*c*d)*f*g^5)*x^2 + 2*(b^2*d^2*f^5*g + a^2*c^2
*f*g^5 - 2*(b^2*c*d + a*b*d^2)*f^4*g^2 + (b^2*c^2 + 4*a*b*c*d + a^2*d^2)*f^
3*g^3 - 2*(a*b*c^2 + a^2*c*d)*f^2*g^4)*x))*B*n - 1/3*B*log(e*(b*x/(d*x + c)
 + a/(d*x + c))^n)/(g^4*x^3 + 3*f*g^3*x^2 + 3*f^2*g^2*x + f^3*g) - 1/3*A/(g
^4*x^3 + 3*f*g^3*x^2 + 3*f^2*g^2*x + f^3*g)
```

**mupad [B]** time = 9.23, size = 1182, normalized size = 4.18

$$\frac{B\,d^3\,n\,\ln\left(c+d\,x\right)}{3\,c^3\,g^4-9\,c^2\,d\,f\,g^3+9\,c\,d^2\,f^2\,g^2-3\,d^3\,f^3\,g}-\frac{}{3\,a^3\,c^3\,g^6-9\,a^3\,c^2\,d\,f\,g^5+9\,a^3\,c\,d^2\,f^2\,g^4-3\,a^3\,d^3\,f^3\,g^3-9\,a^2\,b\,c^3\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*log(e*((a + b*x)/(c + d*x))^n))/(f + g*x)^4,x)
```

```
[Out] (B*d^3*n*log(c + d*x))/(3*c^3*g^4 - 3*d^3*f^3*g + 9*c*d^2*f^2*g^2 - 9*c^2*d
*f*g^3) - (log(f + g*x)*(g^2*(B*a^3*d^3*n - B*b^3*c^3*n) - g*(3*B*a^2*b*d^3
```

```
*f*n - 3*B*b^3*c^2*d*f*n) + 3*B*a*b^2*d^3*f^2*n - 3*B*b^3*c*d^2*f^2*n))/(3*
a^3*c^3*g^6 + 3*b^3*d^3*f^6 - 3*a^3*d^3*f^3*g^3 - 3*b^3*c^3*f^3*g^3 - 9*a^2
*b*c^3*f*g^5 - 9*a*b^2*d^3*f^5*g - 9*a^3*c^2*d*f*g^5 - 9*b^3*c*d^2*f^5*g +
9*a*b^2*c^3*f^2*g^4 + 9*a^2*b*d^3*f^4*g^2 + 9*a^3*c*d^2*f^2*g^4 + 9*b^3*c^2
*d*f^4*g^2 + 27*a*b^2*c*d^2*f^4*g^2 - 27*a*b^2*c^2*d*f^3*g^3 - 27*a^2*b*c*d
^2*f^3*g^3 + 27*a^2*b*c^2*d*f^2*g^4) - (B*log(e*((a + b*x)/(c + d*x))^n))/(
3*g*(f^3 + g^3*x^3 + 3*f^2*g*x + 3*f*g^2*x^2)) - (B*b^3*n*log(a + b*x))/(3*
a^3*g^4 - 3*b^3*f^3*g + 9*a*b^2*f^2*g^2 - 9*a^2*b*f*g^3) - ((2*A*a^2*c^2*g^
4 + 2*A*b^2*d^2*f^4 + 2*A*a^2*d^2*f^2*g^2 + 2*A*b^2*c^2*f^2*g^2 + 3*B*a^2*d
^2*f^2*g^2*n - 3*B*b^2*c^2*f^2*g^2*n - 4*A*a*b*c^2*f*g^3 - 4*A*a*b*d^2*f^3*
g - 4*A*a^2*c*d*f*g^3 - 4*A*b^2*c*d*f^3*g + 8*A*a*b*c*d*f^2*g^2 + B*a*b*c^2
*f*g^3*n - 5*B*a*b*d^2*f^3*g*n - B*a^2*c*d*f*g^3*n + 5*B*b^2*c*d*f^3*g*n)/(
2*(a^2*c^2*g^4 + b^2*d^2*f^4 + a^2*d^2*f^2*g^2 + b^2*c^2*f^2*g^2 - 2*a*b*c^
2*f*g^3 - 2*a*b*d^2*f^3*g - 2*a^2*c*d*f*g^3 - 2*b^2*c*d*f^3*g + 4*a*b*c*d*f
^2*g^2)) + (x*(B*a*b*c^2*g^4*n - B*a^2*c*d*g^4*n + 5*B*a^2*d^2*f*g^3*n - 5*
B*b^2*c^2*f*g^3*n - 9*B*a*b*d^2*f^2*g^2*n + 9*B*b^2*c*d*f^2*g^2*n))/(2*(a^2
*c^2*g^4 + b^2*d^2*f^4 + a^2*d^2*f^2*g^2 + b^2*c^2*f^2*g^2 - 2*a*b*c^2*f*g^
3 - 2*a*b*d^2*f^3*g - 2*a^2*c*d*f*g^3 - 2*b^2*c*d*f^3*g + 4*a*b*c*d*f^2*g^2
)) + (x^2*(B*a^2*d^2*g^4*n - B*b^2*c^2*g^4*n - 2*B*a*b*d^2*f*g^3*n + 2*B*b^
2*c*d*f*g^3*n))/(a^2*c^2*g^4 + b^2*d^2*f^4 + a^2*d^2*f^2*g^2 + b^2*c^2*f^2*
g^2 - 2*a*b*c^2*f*g^3 - 2*a*b*d^2*f^3*g - 2*a^2*c*d*f*g^3 - 2*b^2*c*d*f^3*g
 + 4*a*b*c*d*f^2*g^2))/(3*f^3*g + 3*g^4*x^3 + 9*f^2*g^2*x + 9*f*g^3*x^2)
```

**sympy [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*ln(e*((b*x+a)/(d*x+c))**n))/(g*x+f)**4,x)
[Out] Timed out
```

## 3.66 $\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(f+gx)^5}\,dx$

**Optimal**. Leaf size=388

$$-\frac{Bn(bc-ad)\left(a^2d^2g^2-abdg(3df-cg)+b^2\left(c^2g^2-3cdfg+3d^2f^2\right)\right)}{4(f+gx)(bf-ag)^3(df-cg)^3}-\frac{Bn(bc-ad)\log(f+gx)(-adg-bcg+2b}{4(b}$$

```
[Out] -1/12*B*(-a*d+b*c)*n/(-a*g+b*f)/(-c*g+d*f)/(g*x+f)^3-1/8*B*(-a*d+b*c)*(-a*d
*g-b*c*g+2*b*d*f)*n/(-a*g+b*f)^2/(-c*g+d*f)^2/(g*x+f)^2-1/4*B*(-a*d+b*c)*(a
^2*d^2*g^2-a*b*d*g*(-c*g+3*d*f)+b^2*(c^2*g^2-3*c*d*f*g+3*d^2*f^2))*n/(-a*g+
b*f)^3/(-c*g+d*f)^3/(g*x+f)+1/4*b^4*B*n*ln(b*x+a)/g/(-a*g+b*f)^4+1/4*(-A-B*
ln(e*((b*x+a)/(d*x+c))^n))/g/(g*x+f)^4-1/4*B*d^4*n*ln(d*x+c)/g/(-c*g+d*f)^4
-1/4*B*(-a*d+b*c)*(-a*d*g-b*c*g+2*b*d*f)*(2*a*b*d^2*f*g-a^2*d^2*g^2-b^2*(c^
2*g^2-2*c*d*f*g+2*d^2*f^2))*n*ln(g*x+f)/(-a*g+b*f)^4/(-c*g+d*f)^4
```

**Rubi [A]** time = 0.71, antiderivative size = 388, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {2525, 12, 72}

$$-\frac{Bn(bc-ad)\left(a^2d^2g^2-abdg(3df-cg)+b^2\left(c^2g^2-3cdfg+3d^2f^2\right)\right)}{4(f+gx)(bf-ag)^3(df-cg)^3}-\frac{Bn(bc-ad)\log(f+gx)(-adg-bcg+2b}{4(}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Log[e*((a + b*x)/(c + d*x))^n])/(f + g*x)^5,x]
```

```
[Out] -(B*(b*c - a*d)*n)/(12*(b*f - a*g)*(d*f - c*g)*(f + g*x)^3) - (B*(b*c - a*d
)*(2*b*d*f - b*c*g - a*d*g)*n)/(8*(b*f - a*g)^2*(d*f - c*g)^2*(f + g*x)^2)
- (B*(b*c - a*d)*(a^2*d^2*g^2 - a*b*d*g*(3*d*f - c*g) + b^2*(3*d^2*f^2 - 3*
c*d*f*g + c^2*g^2))*n)/(4*(b*f - a*g)^3*(d*f - c*g)^3*(f + g*x)) + (b^4*B*n
*Log[a + b*x])/(4*g*(b*f - a*g)^4) - (A + B*Log[e*((a + b*x)/(c + d*x))^n])
/(4*g*(f + g*x)^4) - (B*d^4*n*Log[c + d*x])/(4*g*(d*f - c*g)^4) - (B*(b*c -
 a*d)*(2*b*d*f - b*c*g - a*d*g)*(2*a*b*d^2*f*g - a^2*d^2*g^2 - b^2*(2*d^2*f
^2 - 2*c*d*f*g + c^2*g^2))*n*Log[f + g*x])/(4*(b*f - a*g)^4*(d*f - c*g)^4)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 72

```
Int[((e_.) + (f_.)*(x_))^(p_.)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
 x_Symbol] :> Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x]
 /; FreeQ[{a, b, c, d, e, f}, x] && IntegerQ[p]
```

Rule 2525

```
Int[((a_.) + Log[(c_.)*(RFx_)^(p_.)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_))^(m_.
), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(a + b*Log[c*RFx^p])^n)/(e*(m + 1))
, x] - Dist[(b*n*p)/(e*(m + 1)), Int[SimplifyIntegrand[((d + e*x)^(m + 1)*(
a + b*Log[c*RFx^p])^(n - 1)*D[RFx, x])/RFx, x], x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && RationalFunctionQ[RFx, x] && IGtQ[n, 0] && (EqQ[n, 1] ||
IntegerQ[m]) && NeQ[m, -1]
```

Rubi steps

$$\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(f+gx)^5}\,dx = -\frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{4g(f+gx)^4} + \frac{(Bn)\int\frac{bc-ad}{(a+bx)(c+dx)(f+gx)^4}\,dx}{4g}$$

$$= -\frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{4g(f+gx)^4} + \frac{(B(bc-ad)n)\int\frac{1}{(a+bx)(c+dx)(f+gx)^4}\,dx}{4g}$$

$$= -\frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{4g(f+gx)^4} + \frac{(B(bc-ad)n)\int\left(\frac{b^5}{(bc-ad)(bf-ag)^4(a+bx)} - \frac{d^5}{(bc-ad)(-df+cg)^4(c+$$

$$= -\frac{B(bc-ad)n}{12(bf-ag)(df-cg)(f+gx)^3} - \frac{B(bc-ad)(2bdf-bcg-adg)n}{8(bf-ag)^2(df-cg)^2(f+gx)^2} - \frac{B(bc-ad)}{}$$

**Mathematica [A]** time = 1.07, size = 359, normalized size = 0.93

$$\frac{Bn(bc-ad)\left(-\frac{g\left(a^2d^2g^2+abdg(cg-3df)+b^2\left(c^2g^2-3cdfg+3d^2f^2\right)\right)}{(f+gx)(bf-ag)^3(df-cg)^3} - \frac{g\log(f+gx)(adg+bcg-2bdf)\left(a^2d^2g^2-2abd^2fg+b^2\left(c^2g^2-2cdfg+2d^2f^2\right)\right)}{(bf-ag)^4(df-cg)^4}}{4g}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Log[e*((a + b*x)/(c + d*x))^n])/(f + g*x)^5,x]
```

```
[Out] (-((A + B*Log[e*((a + b*x)/(c + d*x))^n])/(f + g*x)^4) + B*(b*c - a*d)*n*(-
1/3*g/((b*f - a*g)*(d*f - c*g)*(f + g*x)^3) + (g*(-2*b*d*f + b*c*g + a*d*g)
```

```
)/(2*(b*f - a*g)^2*(d*f - c*g)^2*(f + g*x)^2) - (g*(a^2*d^2*g^2 + a*b*d*g*(
-3*d*f + c*g) + b^2*(3*d^2*f^2 - 3*c*d*f*g + c^2*g^2)))/((b*f - a*g)^3*(d*f
 - c*g)^3*(f + g*x)) + (b^4*Log[a + b*x])/((b*c - a*d)*(b*f - a*g)^4) - (d^
4*Log[c + d*x])/((b*c - a*d)*(d*f - c*g)^4) - (g*(-2*b*d*f + b*c*g + a*d*g)
*(-2*a*b*d^2*f*g + a^2*d^2*g^2 + b^2*(2*d^2*f^2 - 2*c*d*f*g + c^2*g^2))*Log
[f + g*x])/((b*f - a*g)^4*(d*f - c*g)^4)))/(4*g)
```

**fricas [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))/(g*x+f)^5,x, algorithm="fricas")
[Out] Timed out
```

**giac [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))/(g*x+f)^5,x, algorithm="giac")
[Out] Timed out
```

**maple [F]** time = 0.29, size = 0, normalized size = 0.00

$$\int \frac{B\ln\left(e\left(\frac{bx+a}{dx+c}\right)^{n}\right)+A}{\left(gx+f\right)^{5}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*ln(e*((b*x+a)/(d*x+c))^n)+A)/(g*x+f)^5,x)
[Out] int((B*ln(e*((b*x+a)/(d*x+c))^n)+A)/(g*x+f)^5,x)
```

**maxima [B]** time = 1.80, size = 1761, normalized size = 4.54

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))/(g*x+f)^5,x, algorithm="maxima")
```

```
[Out] 1/24*(6*b^4*log(b*x + a)/(b^4*f^4*g - 4*a*b^3*f^3*g^2 + 6*a^2*b^2*f^2*g^3 -
 4*a^3*b*f*g^4 + a^4*g^5) - 6*d^4*log(d*x + c)/(d^4*f^4*g - 4*c*d^3*f^3*g^2
 + 6*c^2*d^2*f^2*g^3 - 4*c^3*d*f*g^4 + c^4*g^5) + 6*(4*(b^4*c*d^3 - a*b^3*d
^4)*f^3 - 6*(b^4*c^2*d^2 - a^2*b^2*d^4)*f^2*g + 4*(b^4*c^3*d - a^3*b*d^4)*f
*g^2 - (b^4*c^4 - a^4*d^4)*g^3)*log(g*x + f)/(b^4*d^4*f^8 + a^4*c^4*g^8 - 4
*(b^4*c*d^3 + a*b^3*d^4)*f^7*g + 2*(3*b^4*c^2*d^2 + 8*a*b^3*c*d^3 + 3*a^2*b
^2*d^4)*f^6*g^2 - 4*(b^4*c^3*d + 6*a*b^3*c^2*d^2 + 6*a^2*b^2*c*d^3 + a^3*b*
d^4)*f^5*g^3 + (b^4*c^4 + 16*a*b^3*c^3*d + 36*a^2*b^2*c^2*d^2 + 16*a^3*b*c*
d^3 + a^4*d^4)*f^4*g^4 - 4*(a*b^3*c^4 + 6*a^2*b^2*c^3*d + 6*a^3*b*c^2*d^2 +
 a^4*c*d^3)*f^3*g^5 + 2*(3*a^2*b^2*c^4 + 8*a^3*b*c^3*d + 3*a^4*c^2*d^2)*f^2
*g^6 - 4*(a^3*b*c^4 + a^4*c^3*d)*f*g^7) - (26*(b^3*c*d^2 - a*b^2*d^3)*f^4 -
 31*(b^3*c^2*d - a^2*b*d^3)*f^3*g + (11*b^3*c^3 + 15*a*b^2*c^2*d - 15*a^2*b
*c*d^2 - 11*a^3*d^3)*f^2*g^2 - 7*(a*b^2*c^3 - a^3*c*d^2)*f*g^3 + 2*(a^2*b*c
^3 - a^3*c^2*d)*g^4 + 6*(3*(b^3*c*d^2 - a*b^2*d^3)*f^2*g^2 - 3*(b^3*c^2*d -
 a^2*b*d^3)*f*g^3 + (b^3*c^3 - a^3*d^3)*g^4)*x^2 + 3*(14*(b^3*c*d^2 - a*b^2
*d^3)*f^3*g - 15*(b^3*c^2*d - a^2*b*d^3)*f^2*g^2 + (5*b^3*c^3 + 3*a*b^2*c^2
*d - 3*a^2*b*c*d^2 - 5*a^3*d^3)*f*g^3 - (a*b^2*c^3 - a^3*c*d^2)*g^4)*x)/(b^
3*d^3*f^9 + a^3*c^3*f^3*g^6 - 3*(b^3*c*d^2 + a*b^2*d^3)*f^8*g + 3*(b^3*c^2*
d + 3*a*b^2*c*d^2 + a^2*b*d^3)*f^7*g^2 - (b^3*c^3 + 9*a*b^2*c^2*d + 9*a^2*b
*c*d^2 + a^3*d^3)*f^6*g^3 + 3*(a*b^2*c^3 + 3*a^2*b*c^2*d + a^3*c*d^2)*f^5*g
^4 - 3*(a^2*b*c^3 + a^3*c^2*d)*f^4*g^5 + (b^3*d^3*f^6*g^3 + a^3*c^3*g^9 - 3
*(b^3*c*d^2 + a*b^2*d^3)*f^5*g^4 + 3*(b^3*c^2*d + 3*a*b^2*c*d^2 + a^2*b*d^3
)*f^4*g^5 - (b^3*c^3 + 9*a*b^2*c^2*d + 9*a^2*b*c*d^2 + a^3*d^3)*f^3*g^6 + 3
*(a*b^2*c^3 + 3*a^2*b*c^2*d + a^3*c*d^2)*f^2*g^7 - 3*(a^2*b*c^3 + a^3*c^2*d
)*f*g^8)*x^3 + 3*(b^3*d^3*f^7*g^2 + a^3*c^3*f*g^8 - 3*(b^3*c*d^2 + a*b^2*d^
3)*f^6*g^3 + 3*(b^3*c^2*d + 3*a*b^2*c*d^2 + a^2*b*d^3)*f^5*g^4 - (b^3*c^3 +
 9*a*b^2*c^2*d + 9*a^2*b*c*d^2 + a^3*d^3)*f^4*g^5 + 3*(a*b^2*c^3 + 3*a^2*b*
c^2*d + a^3*c*d^2)*f^3*g^6 - 3*(a^2*b*c^3 + a^3*c^2*d)*f^2*g^7)*x^2 + 3*(b^
3*d^3*f^8*g + a^3*c^3*f^2*g^7 - 3*(b^3*c*d^2 + a*b^2*d^3)*f^7*g^2 + 3*(b^3*
c^2*d + 3*a*b^2*c*d^2 + a^2*b*d^3)*f^6*g^3 - (b^3*c^3 + 9*a*b^2*c^2*d + 9*a
^2*b*c*d^2 + a^3*d^3)*f^5*g^4 + 3*(a*b^2*c^3 + 3*a^2*b*c^2*d + a^3*c*d^2)*f
^4*g^5 - 3*(a^2*b*c^3 + a^3*c^2*d)*f^3*g^6)*x))*B*n - 1/4*B*log(e*(b*x/(d*x
 + c) + a/(d*x + c))^n)/(g^5*x^4 + 4*f*g^4*x^3 + 6*f^2*g^3*x^2 + 4*f^3*g^2*
x + f^4*g) - 1/4*A/(g^5*x^4 + 4*f*g^4*x^3 + 6*f^2*g^3*x^2 + 4*f^3*g^2*x + f
^4*g)
```

**mupad [B]** time = 13.77, size = 2569, normalized size = 6.62

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*log(e*((a + b*x)/(c + d*x))^n))/(f + g*x)^5,x)
```

```
[Out] ((x^3*(B*a^3*d^3*g^6*n - B*b^3*c^3*g^6*n - 3*B*a^2*b*d^3*f*g^5*n + 3*B*b^3*
c^2*d*f*g^5*n + 3*B*a*b^2*d^3*f^2*g^4*n - 3*B*b^3*c*d^2*f^2*g^4*n))/(a^3*c^
```

3*g^6 + b^3*d^3*f^6 - a^3*d^3*f^3*g^3 - b^3*c^3*f^3*g^3 - 3*a^2*b*c^3*f*g^5 - 3*a*b^2*d^3*f^5*g - 3*a^3*c^2*d*f*g^5 - 3*b^3*c*d^2*f^5*g + 3*a*b^2*c^3*f^2*g^4 + 3*a^2*b*d^3*f^4*g^2 + 3*a^3*c*d^2*f^2*g^4 + 3*b^3*c^2*d*f^4*g^2 + 9*a*b^2*c*d^2*f^4*g^2 - 9*a*b^2*c^2*d*f^3*g^3 - 9*a^2*b*c*d^2*f^3*g^3 + 9*a^2*b*c^2*d*f^2*g^4) - (6*A*a^3*c^3*g^6 + 6*A*b^3*d^3*f^6 - 6*A*a^3*d^3*f^3*g^3 - 6*A*b^3*c^3*f^3*g^3 + 18*A*a*b^2*c^3*f^2*g^4 + 18*A*a^2*b*d^3*f^4*g^2 + 18*A*a^3*c*d^2*f^2*g^4 + 18*A*b^3*c^2*d*f^4*g^2 - 11*B*a^3*d^3*f^3*g^3*n + 11*B*b^3*c^3*f^3*g^3*n - 18*A*a^2*b*c^3*f*g^5 - 18*A*a*b^2*d^3*f^5*g - 18*A*a^3*c^2*d*f*g^5 - 18*A*b^3*c*d^2*f^5*g + 2*B*a^2*b*c^3*f*g^5*n - 26*B*a*b^2*d^3*f^5*g*n - 2*B*a^3*c^2*d*f*g^5*n + 26*B*b^3*c*d^2*f^5*g*n + 54*A*a*b^2*c*d^2*f^4*g^2 - 54*A*a*b^2*c^2*d*f^3*g^3 - 54*A*a^2*b*c*d^2*f^3*g^3 + 54*A*a^2*b*c^2*d*f^2*g^4 - 7*B*a*b^2*c^3*f^2*g^4*n + 31*B*a^2*b*d^3*f^4*g^2*n + 7*B*a^3*c*d^2*f^2*g^4*n - 31*B*b^3*c^2*d*f^4*g^2*n + 15*B*a*b^2*c^2*d*f^3*g^3*n - 15*B*a^2*b*c*d^2*f^3*g^3*n)/(6*(a^3*c^3*g^6 + b^3*d^3*f^6 - a^3*d^3*f^3*g^3 - b^3*c^3*f^3*g^3 - 3*a^2*b*c^3*f*g^5 - 3*a*b^2*d^3*f^5*g - 3*a^3*c^2*d*f*g^5 - 3*b^3*c*d^2*f^5*g + 3*a*b^2*c^3*f^2*g^4 + 3*a^2*b*d^3*f^4*g^2 + 3*a^3*c*d^2*f^2*g^4 + 3*b^3*c^2*d*f^4*g^2 + 9*a*b^2*c*d^2*f^4*g^2 - 9*a*b^2*c^2*d*f^3*g^3 - 9*a^2*b*c*d^2*f^3*g^3 + 9*a^2*b*c^2*d*f^2*g^4)) + (x^2*(B*a*b^2*c^3*g^6*n - B*a^3*c*d^2*g^6*n + 7*B*a^3*d^3*f*g^5*n - 7*B*b^3*c^3*f*g^5*n + 20*B*a*b^2*d^3*f^3*g^3*n - 21*B*a^2*b*d^3*f^2*g^4*n - 20*B*b^3*c*d^2*f^3*g^3*n + 21*B*b^3*c^2*d*f^2*g^4*n - 3*B*a*b^2*c^2*d*f*g^5*n + 3*B*a^2*b*c*d^2*f*g^5*n))/(2*(a^3*c^3*g^6 + b^3*d^3*f^6 - a^3*d^3*f^3*g^3 - b^3*c^3*f^3*g^3 - 3*a^2*b*c^3*f*g^5 - 3*a*b^2*d^3*f^5*g - 3*a^3*c^2*d*f*g^5 - 3*b^3*c*d^2*f^5*g + 3*a*b^2*c^3*f^2*g^4 + 3*a^2*b*d^3*f^4*g^2 + 3*a^3*c*d^2*f^2*g^4 + 3*b^3*c^2*d*f^4*g^2 + 9*a*b^2*c*d^2*f^4*g^2 - 9*a*b^2*c^2*d*f^3*g^3 - 9*a^2*b*c*d^2*f^3*g^3 + 9*a^2*b*c^2*d*f^2*g^4)) + (x*(13*B*a^3*d^3*f^2*g^4*n - 13*B*b^3*c^3*f^2*g^4*n - B*a^2*b*c^3*g^6*n + B*a^3*c^2*d*g^6*n + 5*B*a*b^2*c^3*f*g^5*n - 5*B*a^3*c*d^2*f*g^5*n + 34*B*a*b^2*d^3*f^4*g^2*n - 38*B*a^2*b*d^3*f^3*g^3*n - 34*B*b^3*c*d^2*f^4*g^2*n + 38*B*b^3*c^2*d*f^3*g^3*n - 12*B*a*b^2*c^2*d*f^2*g^4*n + 12*B*a^2*b*c*d^2*f^2*g^4*n))/(3*(a^3*c^3*g^6 + b^3*d^3*f^6 - a^3*d^3*f^3*g^3 - b^3*c^3*f^3*g^3 - 3*a^2*b*c^3*f*g^5 - 3*a*b^2*d^3*f^5*g - 3*a^3*c^2*d*f*g^5 - 3*b^3*c*d^2*f^5*g + 3*a*b^2*c^3*f^2*g^4 + 3*a^2*b*d^3*f^4*g^2 + 3*a^3*c*d^2*f^2*g^4 + 3*b^3*c^2*d*f^4*g^2 + 9*a*b^2*c*d^2*f^4*g^2 - 9*a*b^2*c^2*d*f^3*g^3 - 9*a^2*b*c*d^2*f^3*g^3 + 9*a^2*b*c^2*d*f^2*g^4)))/(4*f^4*g + 4*g^5*x^4 + 16*f^3*g^2*x + 16*f*g^4*x^3 + 24*f^2*g^3*x^2) + (log(f + g*x)*(g*(6*B*a^2*b^2*d^4*f^2*n - 6*B*b^4*c^2*d^2*f^2*n) - g^2*(4*B*a^3*b*d^4*f*n - 4*B*b^4*c^3*d*f*n) + g^3*(B*a^4*d^4*n - B*b^4*c^4*n) - 4*B*a*b^3*d^4*f^3*n + 4*B*b^4*c*d^3*f^3*n))/(4*a^4*c^4*g^8 + 4*b^4*d^4*f^8 + 4*a^4*d^4*f^4*g^4 + 4*b^4*c^4*f^4*g^4 + 24*a^2*b^2*c^4*f^2*g^6 + 24*a^2*b^2*d^4*f^6*g^2 + 24*a^4*c^2*d^2*f^2*g^6 + 24*b^4*c^2*d^2*f^6*g^2 - 16*a^3*b*c^4*f*g^7 - 16*a*b^3*d^4*f^7*g - 16*a^4*c^3*d*f*g^7 - 16*b^4*c*d^3*f^7*g - 16*a*b^3*c^4*f^3*g^5 - 16*a^3*b*d^4*f^5*g^3 - 16*a^4*c*d^3*f^3*g^5 - 16*b^4*c^3*d*f^5*g^3 + 64*a*b^3*c*d^3*f^6*g^2 + 64*a*b^3*c^3*d*f^4*g^4 + 64*a^3*b*c*d^3*f^4*g^4 + 64*a^3*b*c^3*d*f^2*g^6 - 96*a*b^3*c^2*d^2*f^5*g^3 - 96*a^2*b^2*c*d^3*f^5*g^3 - 96*a^2*b^2*c^3*d*f^3*g^5 - 96*a^3*b*

```
c^2*d^2*f^3*g^5 + 144*a^2*b^2*c^2*d^2*f^4*g^4) - (B*log(e*((a + b*x)/(c + d
*x))^n))/(4*g*(f^4 + g^4*x^4 + 4*f^3*g*x + 4*f*g^3*x^3 + 6*f^2*g^2*x^2)) +
(B*b^4*n*log(a + b*x))/(4*a^4*g^5 + 4*b^4*f^4*g - 16*a*b^3*f^3*g^2 + 24*a^2
*b^2*f^2*g^3 - 16*a^3*b*f*g^4) - (B*d^4*n*log(c + d*x))/(4*c^4*g^5 + 4*d^4*
f^4*g - 16*c*d^3*f^3*g^2 + 24*c^2*d^2*f^2*g^3 - 16*c^3*d*f*g^4)
```

**sympy [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*ln(e*((b*x+a)/(d*x+c))**n))/(g*x+f)**5,x)
```

```
[Out] Timed out
```

# 3.67 $\int (f+gx)^3 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2 dx$

**Optimal**. Leaf size=923

$$\frac{B^2g^3n^2\log\left(\frac{a+bx}{c+dx}\right)(bc-ad)^4}{6b^4d^4}+\frac{B^2g^3n^2\log(c+dx)(bc-ad)^4}{6b^4d^4}+\frac{B^2g^3n^2x(bc-ad)^3}{6b^3d^3}+\frac{B^2g^2(4bdf-3bcg-adg)n^2\log}{4b^4d^4}$$

```
[Out] 1/6*B^2*(-a*d+b*c)^3*g^3*n^2*x/b^3/d^3+1/4*B^2*(-a*d+b*c)^2*g^2*(-a*d*g-3*b
*c*g+4*b*d*f)*n^2*x/b^3/d^3+1/12*B^2*(-a*d+b*c)^2*g^3*n^2*(d*x+c)^2/b^2/d^4
-1/2*B*(-a*d+b*c)*g*(a^2*d^2*g^2-2*a*b*d*g*(-c*g+2*d*f)+b^2*(3*c^2*g^2-8*c*
d*f*g+6*d^2*f^2))*n*(b*x+a)*(A+B*ln(e*((b*x+a)/(d*x+c))^n))/b^4/d^3-1/4*B*(
-a*d+b*c)*g^2*(-a*d*g-3*b*c*g+4*b*d*f)*n*(d*x+c)^2*(A+B*ln(e*((b*x+a)/(d*x+
c))^n))/b^2/d^4-1/6*B*(-a*d+b*c)*g^3*n*(d*x+c)^3*(A+B*ln(e*((b*x+a)/(d*x+c)
)^n))/b/d^4-1/4*(-a*g+b*f)^4*(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2/b^4/g+1/4*(g
*x+f)^4*(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2/g-1/2*B*(-a*d+b*c)*(-a*d*g-b*c*g+
2*b*d*f)*(2*a*b*d^2*f*g-a^2*d^2*g^2-b^2*(c^2*g^2-2*c*d*f*g+2*d^2*f^2))*n*(A
+B*ln(e*((b*x+a)/(d*x+c))^n))*ln((-a*d+b*c)/b/(d*x+c))/b^4/d^4+1/6*B^2*(-a*
d+b*c)^4*g^3*n^2*ln((b*x+a)/(d*x+c))/b^4/d^4+1/4*B^2*(-a*d+b*c)^3*g^2*(-a*d
*g-3*b*c*g+4*b*d*f)*n^2*ln((b*x+a)/(d*x+c))/b^4/d^4+1/6*B^2*(-a*d+b*c)^4*g^
3*n^2*ln(d*x+c)/b^4/d^4+1/4*B^2*(-a*d+b*c)^3*g^2*(-a*d*g-3*b*c*g+4*b*d*f)*n
^2*ln(d*x+c)/b^4/d^4+1/2*B^2*(-a*d+b*c)^2*g*(a^2*d^2*g^2-2*a*b*d*g*(-c*g+2*
d*f)+b^2*(3*c^2*g^2-8*c*d*f*g+6*d^2*f^2))*n^2*ln(d*x+c)/b^4/d^4-1/2*B^2*(-a
*d+b*c)*(-a*d*g-b*c*g+2*b*d*f)*(2*a*b*d^2*f*g-a^2*d^2*g^2-b^2*(c^2*g^2-2*c*
d*f*g+2*d^2*f^2))*n^2*polylog(2,d*(b*x+a)/b/(d*x+c))/b^4/d^4
```

**Rubi** [A] time = 1.84, antiderivative size = 1060, normalized size of antiderivative = 1.15, number of steps used = 31, number of rules used = 13, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.406, Rules used = {2525, 12, 2528, 2486, 31, 72, 2524, 2418, 2390, 2301, 2394, 2393, 2391}

$$\frac{B^2n^2\log^2(a+bx)(bf-ag)^4}{4b^4g}-\frac{Bn\log(a+bx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)(bf-ag)^4}{2b^4g}-\frac{B^2n^2\log(a+bx)\log\left(\frac{b(c+dx)}{bc-ad}\right)(}{2b^4g}$$

Antiderivative was successfully verified.

```
[In] Int[(f + g*x)^3*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2,x]
```

```
[Out] -(A*B*(b*c - a*d)*g*(a^2*d^2*g^2 - a*b*d*g*(4*d*f - c*g) + b^2*(6*d^2*f^2 -
 4*c*d*f*g + c^2*g^2))*n*x)/(2*b^3*d^3) - (B^2*(b*c - a*d)^2*(b*c + a*d)*g^
3*n^2*x)/(6*b^3*d^3) + (B^2*(b*c - a*d)^2*g^2*(4*b*d*f - b*c*g - a*d*g)*n^2
*x)/(4*b^3*d^3) + (B^2*(b*c - a*d)^2*g^3*n^2*x^2)/(12*b^2*d^2) - (a^3*B^2*(
```

```
b*c - a*d)*g^3*n^2*Log[a + b*x])/(6*b^4*d) + (a^2*B^2*(b*c - a*d)*g^2*(4*b*
d*f - b*c*g - a*d*g)*n^2*Log[a + b*x])/(4*b^4*d^2) + (B^2*(b*f - a*g)^4*n^2
*Log[a + b*x]^2)/(4*b^4*g) - (B^2*(b*c - a*d)*g*(a^2*d^2*g^2 - a*b*d*g*(4*d
*f - c*g) + b^2*(6*d^2*f^2 - 4*c*d*f*g + c^2*g^2))*n*(a + b*x)*Log[e*((a +
b*x)/(c + d*x))^n])/(2*b^4*d^3) - (B*(b*c - a*d)*g^2*(4*b*d*f - b*c*g - a*d
*g)*n*x^2*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/(4*b^2*d^2) - (B*(b*c - a
*d)*g^3*n*x^3*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/(6*b*d) - (B*(b*f - a
*g)^4*n*Log[a + b*x]*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/(2*b^4*g) + ((
f + g*x)^4*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2)/(4*g) + (B^2*c^3*(b*c
- a*d)*g^3*n^2*Log[c + d*x])/(6*b*d^4) - (B^2*c^2*(b*c - a*d)*g^2*(4*b*d*f
- b*c*g - a*d*g)*n^2*Log[c + d*x])/(4*b^2*d^4) + (B^2*(b*c - a*d)^2*g*(a^2*
d^2*g^2 - a*b*d*g*(4*d*f - c*g) + b^2*(6*d^2*f^2 - 4*c*d*f*g + c^2*g^2))*n^
2*Log[c + d*x])/(2*b^4*d^4) - (B^2*(d*f - c*g)^4*n^2*Log[-((d*(a + b*x))/(b
*c - a*d))]*Log[c + d*x])/(2*d^4*g) + (B*(d*f - c*g)^4*n*(A + B*Log[e*((a +
 b*x)/(c + d*x))^n])*Log[c + d*x])/(2*d^4*g) + (B^2*(d*f - c*g)^4*n^2*Log[c
 + d*x]^2)/(4*d^4*g) - (B^2*(b*f - a*g)^4*n^2*Log[a + b*x]*Log[(b*(c + d*x)
)/(b*c - a*d)])/(2*b^4*g) - (B^2*(b*f - a*g)^4*n^2*PolyLog[2, -((d*(a + b*x
))/(b*c - a*d))])/(2*b^4*g) - (B^2*(d*f - c*g)^4*n^2*PolyLog[2, (b*(c + d*x
))/(b*c - a*d)])/(2*d^4*g)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 72

```
Int[((e_.) + (f_.)*(x_))^(p_.)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
 x_Symbol] :> Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x]
 /; FreeQ[{a, b, c, d, e, f}, x] && IntegerQ[p]
```

Rule 2301

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

Rule 2390

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_))^(q_.), x_Symbol] :> Dist[1/e, Subst[Int[((f*x)/d)^q*(a + b*Log[c*x^
n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && E
```

```
qQ[e*f - d*g, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2393

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + (c*e*x)/g])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2394

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d + e*x
)^n]))/g, x] - Dist[(b*e*n)/g, Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2418

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*(RFx_), x_Sy
mbol] :> With[{u = ExpandIntegrand[(a + b*Log[c*(d + e*x)^n])^p, RFx, x]},
Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d, e, n}, x] && RationalFunctionQ[
RFx, x] && IntegerQ[p]
```

Rule 2486

```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))
^(r_.)]^(s_.), x_Symbol] :> Simp[((a + b*x)*Log[e*(f*(a + b*x)^p*(c + d*x)^
q)^r]^s)/b, x] + Dist[(q*r*s*(b*c - a*d))/b, Int[Log[e*(f*(a + b*x)^p*(c +
d*x)^q)^r]^(s - 1)/(c + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, p, q, r, s
}, x] && NeQ[b*c - a*d, 0] && EqQ[p + q, 0] && IGtQ[s, 0]
```

Rule 2524

```
Int[((a_.) + Log[(c_.)*(RFx_)^(p_.)]*(b_.))^(n_.)/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[(Log[d + e*x]*(a + b*Log[c*RFx^p])^n)/e, x] - Dist[(b*n*p)/e
, Int[(Log[d + e*x]*(a + b*Log[c*RFx^p])^(n - 1)*D[RFx, x])/RFx, x], x] /;
FreeQ[{a, b, c, d, e, p}, x] && RationalFunctionQ[RFx, x] && IGtQ[n, 0]
```

Rule 2525

```
Int[((a_.) + Log[(c_.)*(RFx_)^(p_.)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_))^(m_.
), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(a + b*Log[c*RFx^p])^n)/(e*(m + 1))
, x] - Dist[(b*n*p)/(e*(m + 1)), Int[SimplifyIntegrand[((d + e*x)^(m + 1)*(
a + b*Log[c*RFx^p])^(n - 1)*D[RFx, x])/RFx, x], x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && RationalFunctionQ[RFx, x] && IGtQ[n, 0] && (EqQ[n, 1] ||
IntegerQ[m]) && NeQ[m, -1]
```

Rule 2528

```
Int[((a_.) + Log[(c_.)*(RFx_)^(p_.)]*(b_.))^(n_.)*(RGx_), x_Symbol] :> With
[{u = ExpandIntegrand[(a + b*Log[c*RFx^p])^n, RGx, x]}, Int[u, x] /; SumQ[u
]] /; FreeQ[{a, b, c, p}, x] && RationalFunctionQ[RFx, x] && RationalFuncti
onQ[RGx, x] && IGtQ[n, 0]
```

Rubi steps

$$\int (f+gx)^3 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2 dx = \frac{(f+gx)^4\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{4g} - \frac{(Bn)\int \frac{(bc-ad)(f+gx)^4\left(A+B\log\left(e\right.\right.}{(a+bx)(c+dx)}}{2g}$$

$$= \frac{(f+gx)^4\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{4g} - \frac{(B(bc-ad)n)\int \frac{(f+gx)^4\left(A+B\log\right.}{(a+bx)(c}}{2g}$$

$$= \frac{(f+gx)^4\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{4g} - \frac{(B(bc-ad)n)\int\left(\frac{g^2\left(a^2d^2g^2-abd\right.}{}\right.}{}$$

$$= \frac{(f+gx)^4\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{4g} - \frac{\left(B(bc-ad)g^3n\right)\int x^2\left(A+B\right.}{2bd}$$

$$= -\frac{AB(bc-ad)g\left(a^2d^2g^2-abdg(4df-cg)+b^2\left(6d^2f^2-4cdfg+c^2\right.\right.}{2b^3d^3}$$

$$= -\frac{AB(bc-ad)g\left(a^2d^2g^2-abdg(4df-cg)+b^2\left(6d^2f^2-4cdfg+c^2\right.\right.}{2b^3d^3}$$

$$= -\frac{AB(bc-ad)g\left(a^2d^2g^2-abdg(4df-cg)+b^2\left(6d^2f^2-4cdfg+c^2\right.\right.}{2b^3d^3}$$

$$= -\frac{AB(bc-ad)g\left(a^2d^2g^2-abdg(4df-cg)+b^2\left(6d^2f^2-4cdfg+c^2\right.\right.}{2b^3d^3}$$

$$= -\frac{AB(bc-ad)g\left(a^2d^2g^2-abdg(4df-cg)+b^2\left(6d^2f^2-4cdfg+c^2\right.\right.}{2b^3d^3}$$

$$= -\frac{AB(bc-ad)g\left(a^2d^2g^2-abdg(4df-cg)+b^2\left(6d^2f^2-4cdfg+c^2\right.\right.}{2b^3d^3}$$

**Mathematica [A]** time = 1.04, size = 757, normalized size = 0.82

$$\frac{(f+gx)^4\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2-\frac{Bn\left(Bg^4n(bc-ad)\left(2a^3d^3\log(a+bx)+bdx(bc-ad)(2ad+2bc-bdx)-2b^3c^3\log(c+dx)\right)+6Abdg^2x(bc-ad)\right.}{}}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(f + g*x)^3*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2,x]
```

```
[Out] ((f + g*x)^4*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2 - (B*n*(6*A*b*d*(b*c
- a*d)*g^2*(a^2*d^2*g^2 + a*b*d*g*(-4*d*f + c*g) + b^2*(6*d^2*f^2 - 4*c*d*f
*g + c^2*g^2))*x + 6*B*d*(b*c - a*d)*g^2*(a^2*d^2*g^2 + a*b*d*g*(-4*d*f + c
*g) + b^2*(6*d^2*f^2 - 4*c*d*f*g + c^2*g^2))*(a + b*x)*Log[e*((a + b*x)/(c
+ d*x))^n] + 3*b^2*d^2*(b*c - a*d)*g^3*(4*b*d*f - b*c*g - a*d*g)*x^2*(A + B
*Log[e*((a + b*x)/(c + d*x))^n]) + 2*b^3*d^3*(b*c - a*d)*g^4*x^3*(A + B*Log
[e*((a + b*x)/(c + d*x))^n]) + 6*d^4*(b*f - a*g)^4*Log[a + b*x]*(A + B*Log[
e*((a + b*x)/(c + d*x))^n]) - 6*B*(b*c - a*d)^2*g^2*(a^2*d^2*g^2 + a*b*d*g*
(-4*d*f + c*g) + b^2*(6*d^2*f^2 - 4*c*d*f*g + c^2*g^2))*n*Log[c + d*x] - 6*
b^4*(d*f - c*g)^4*(A + B*Log[e*((a + b*x)/(c + d*x))^n])*Log[c + d*x] + B*(
b*c - a*d)*g^4*n*(b*d*(b*c - a*d)*x*(2*b*c + 2*a*d - b*d*x) + 2*a^3*d^3*Log
[a + b*x] - 2*b^3*c^3*Log[c + d*x]) - 3*B*(b*c - a*d)*g^3*(-4*b*d*f + b*c*g
 + a*d*g)*n*(-(a^2*d^2*Log[a + b*x]) + b*(d*(-(b*c) + a*d)*x + b*c^2*Log[c
+ d*x])) - 3*B*d^4*(b*f - a*g)^4*n*(Log[a + b*x]*(Log[a + b*x] - 2*Log[(b*(
c + d*x))/(b*c - a*d)]) - 2*PolyLog[2, (d*(a + b*x))/(-(b*c) + a*d)]) + 3*b
^4*B*(d*f - c*g)^4*n*((2*Log[(d*(a + b*x))/(-(b*c) + a*d)] - Log[c + d*x])*
Log[c + d*x] + 2*PolyLog[2, (b*(c + d*x))/(b*c - a*d)])))/(3*b^4*d^4))/(4*g
)
```

**fricas [F]** time = 1.33, size = 0, normalized size = 0.00

$$\text{integral}\left(A^2g^3x^3+3\,A^2fg^2x^2+3\,A^2f^2gx+A^2f^3+\left(B^2g^3x^3+3\,B^2fg^2x^2+3\,B^2f^2gx+B^2f^3\right)\log\left(e\left(\frac{bx+a}{dx+c}\right)\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^3*(A+B*log(e*((b*x+a)/(d*x+c))^n))^2,x, algorithm="fricas
")
```

```
[Out] integral(A^2*g^3*x^3 + 3*A^2*f*g^2*x^2 + 3*A^2*f^2*g*x + A^2*f^3 + (B^2*g^3
*x^3 + 3*B^2*f*g^2*x^2 + 3*B^2*f^2*g*x + B^2*f^3)*log(e*((b*x + a)/(d*x + c
))^n)^2 + 2*(A*B*g^3*x^3 + 3*A*B*f*g^2*x^2 + 3*A*B*f^2*g*x + A*B*f^3)*log(e
*((b*x + a)/(d*x + c))^n), x)
```

**giac [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^3*(A+B*log(e*((b*x+a)/(d*x+c))^n))^2,x, algorithm="giac")
```

```
[Out] Timed out
```

**maple [F]** time = 0.28, size = 0, normalized size = 0.00

$$\int (gx+f)^3 \left(B \ln \left(e \left(\frac{bx+a}{dx+c}\right)^n\right)+A\right)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x+f)^3*(B*ln(e*((b*x+a)/(d*x+c))^n)+A)^2,x)
```

```
[Out] int((g*x+f)^3*(B*ln(e*((b*x+a)/(d*x+c))^n)+A)^2,x)
```

**maxima [B]** time = 5.35, size = 2651, normalized size = 2.87

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^3*(A+B*log(e*((b*x+a)/(d*x+c))^n))^2,x, algorithm="maxima
")
```

```
[Out] 1/2*A*B*g^3*x^4*log(e*(b*x/(d*x + c) + a/(d*x + c))^n) + 1/4*A^2*g^3*x^4 +
2*A*B*f*g^2*x^3*log(e*(b*x/(d*x + c) + a/(d*x + c))^n) + A^2*f*g^2*x^3 + 3*
A*B*f^2*g*x^2*log(e*(b*x/(d*x + c) + a/(d*x + c))^n) + 3/2*A^2*f^2*g*x^2 -
1/12*A*B*g^3*n*(6*a^4*log(b*x + a)/b^4 - 6*c^4*log(d*x + c)/d^4 + (2*(b^3*c
*d^2 - a*b^2*d^3)*x^3 - 3*(b^3*c^2*d - a^2*b*d^3)*x^2 + 6*(b^3*c^3 - a^3*d^
3)*x)/(b^3*d^3)) + A*B*f*g^2*n*(2*a^3*log(b*x + a)/b^3 - 2*c^3*log(d*x + c)
/d^3 - ((b^2*c*d - a*b*d^2)*x^2 - 2*(b^2*c^2 - a^2*d^2)*x)/(b^2*d^2)) - 3*A
*B*f^2*g*n*(a^2*log(b*x + a)/b^2 - c^2*log(d*x + c)/d^2 + (b*c - a*d)*x/(b*
d)) + 2*A*B*f^3*n*(a*log(b*x + a)/b - c*log(d*x + c)/d) + 2*A*B*f^3*x*log(e
*(b*x/(d*x + c) + a/(d*x + c))^n) + A^2*f^3*x - 1/12*(6*a^3*c*d^3*g^3*n^2 -
 3*(8*c*d^3*f*g^2*n^2 - c^2*d^2*g^3*n^2)*a^2*b + 2*(18*c*d^3*f^2*g*n^2 - 6*
c^2*d^2*f*g^2*n^2 + c^3*d*g^3*n^2)*a*b^2 + (24*c*d^3*f^3*n*log(e) - (11*g^3
*n^2 + 6*g^3*n*log(e))*c^4 + 12*(3*f*g^2*n^2 + 2*f*g^2*n*log(e))*c^3*d - 36
*(f^2*g*n^2 + f^2*g*n*log(e))*c^2*d^2)*b^3)*B^2*log(d*x + c)/(b^3*d^4) + 1/
2*(4*a*b^3*d^4*f^3*n^2 - 6*a^2*b^2*d^4*f^2*g*n^2 + 4*a^3*b*d^4*f*g^2*n^2 -
a^4*d^4*g^3*n^2 - (4*c*d^3*f^3*n^2 - 6*c^2*d^2*f^2*g*n^2 + 4*c^3*d*f*g^2*n^
2 - c^4*g^3*n^2)*b^4)*(log(b*x + a)*log((b*d*x + a*d)/(b*c - a*d) + 1) + di
log(-(b*d*x + a*d)/(b*c - a*d)))*B^2/(b^4*d^4) + 1/12*(3*B^2*b^4*d^4*g^3*x^
4*log(e)^2 + 6*(4*c*d^3*f^3*n^2 - 6*c^2*d^2*f^2*g*n^2 + 4*c^3*d*f*g^2*n^2 -
 c^4*g^3*n^2)*B^2*b^4*log(b*x + a)*log(d*x + c) - 3*(4*c*d^3*f^3*n^2 - 6*c^
2*d^2*f^2*g*n^2 + 4*c^3*d*f*g^2*n^2 - c^4*g^3*n^2)*B^2*b^4*log(d*x + c)^2 +
```

```
 2*(a*b^3*d^4*g^3*n*log(e) - (c*d^3*g^3*n*log(e) - 6*d^4*f*g^2*log(e)^2)*b^
4)*B^2*x^3 + ((g^3*n^2 - 3*g^3*n*log(e))*a^2*b^2*d^4 - 2*(c*d^3*g^3*n^2 - 6
*d^4*f*g^2*n*log(e))*a*b^3 - (12*c*d^3*f*g^2*n*log(e) - 18*d^4*f^2*g*log(e)
^2 - (g^3*n^2 + 3*g^3*n*log(e))*c^2*d^2)*b^4)*B^2*x^2 - 3*(4*a*b^3*d^4*f^3*
n^2 - 6*a^2*b^2*d^4*f^2*g*n^2 + 4*a^3*b*d^4*f*g^2*n^2 - a^4*d^4*g^3*n^2)*B^
2*log(b*x + a)^2 - ((5*g^3*n^2 - 6*g^3*n*log(e))*a^3*b*d^4 - (5*c*d^3*g^3*n
^2 + 12*(f*g^2*n^2 - 2*f*g^2*n*log(e))*d^4)*a^2*b^2 + (24*c*d^3*f*g^2*n^2 -
 5*c^2*d^2*g^3*n^2 - 36*d^4*f^2*g*n*log(e))*a*b^3 + (36*c*d^3*f^2*g*n*log(e
) - 12*d^4*f^3*log(e)^2 + (5*g^3*n^2 + 6*g^3*n*log(e))*c^3*d - 12*(f*g^2*n^
2 + 2*f*g^2*n*log(e))*c^2*d^2)*b^4)*B^2*x + ((11*g^3*n^2 - 6*g^3*n*log(e))*
a^4*d^4 - 2*(c*d^3*g^3*n^2 + 6*(3*f*g^2*n^2 - 2*f*g^2*n*log(e))*d^4)*a^3*b
+ 3*(4*c*d^3*f*g^2*n^2 - c^2*d^2*g^3*n^2 + 12*(f^2*g*n^2 - f^2*g*n*log(e))*
d^4)*a^2*b^2 - 6*(6*c*d^3*f^2*g*n^2 - 4*c^2*d^2*f*g^2*n^2 + c^3*d*g^3*n^2 -
 4*d^4*f^3*n*log(e))*a*b^3)*B^2*log(b*x + a) + 3*(B^2*b^4*d^4*g^3*x^4 + 4*B
^2*b^4*d^4*f*g^2*x^3 + 6*B^2*b^4*d^4*f^2*g*x^2 + 4*B^2*b^4*d^4*f^3*x)*log((
b*x + a)^n)^2 + 3*(B^2*b^4*d^4*g^3*x^4 + 4*B^2*b^4*d^4*f*g^2*x^3 + 6*B^2*b^
4*d^4*f^2*g*x^2 + 4*B^2*b^4*d^4*f^3*x)*log((d*x + c)^n)^2 + (6*B^2*b^4*d^4*
g^3*x^4*log(e) - 6*(4*c*d^3*f^3*n - 6*c^2*d^2*f^2*g*n + 4*c^3*d*f*g^2*n - c
^4*g^3*n)*B^2*b^4*log(d*x + c) + 2*(a*b^3*d^4*g^3*n - (c*d^3*g^3*n - 12*d^4
*f*g^2*log(e))*b^4)*B^2*x^3 + 3*(4*a*b^3*d^4*f*g^2*n - a^2*b^2*d^4*g^3*n -
(4*c*d^3*f*g^2*n - c^2*d^2*g^3*n - 12*d^4*f^2*g*log(e))*b^4)*B^2*x^2 + 6*(6
*a*b^3*d^4*f^2*g*n - 4*a^2*b^2*d^4*f*g^2*n + a^3*b*d^4*g^3*n - (6*c*d^3*f^2
*g*n - 4*c^2*d^2*f*g^2*n + c^3*d*g^3*n - 4*d^4*f^3*log(e))*b^4)*B^2*x + 6*(
4*a*b^3*d^4*f^3*n - 6*a^2*b^2*d^4*f^2*g*n + 4*a^3*b*d^4*f*g^2*n - a^4*d^4*g
^3*n)*B^2*log(b*x + a))*log((b*x + a)^n) - (6*B^2*b^4*d^4*g^3*x^4*log(e) -
6*(4*c*d^3*f^3*n - 6*c^2*d^2*f^2*g*n + 4*c^3*d*f*g^2*n - c^4*g^3*n)*B^2*b^4
*log(d*x + c) + 2*(a*b^3*d^4*g^3*n - (c*d^3*g^3*n - 12*d^4*f*g^2*log(e))*b^
4)*B^2*x^3 + 3*(4*a*b^3*d^4*f*g^2*n - a^2*b^2*d^4*g^3*n - (4*c*d^3*f*g^2*n
- c^2*d^2*g^3*n - 12*d^4*f^2*g*log(e))*b^4)*B^2*x^2 + 6*(6*a*b^3*d^4*f^2*g*
n - 4*a^2*b^2*d^4*f*g^2*n + a^3*b*d^4*g^3*n - (6*c*d^3*f^2*g*n - 4*c^2*d^2*
f*g^2*n + c^3*d*g^3*n - 4*d^4*f^3*log(e))*b^4)*B^2*x + 6*(4*a*b^3*d^4*f^3*n
 - 6*a^2*b^2*d^4*f^2*g*n + 4*a^3*b*d^4*f*g^2*n - a^4*d^4*g^3*n)*B^2*log(b*x
 + a) + 6*(B^2*b^4*d^4*g^3*x^4 + 4*B^2*b^4*d^4*f*g^2*x^3 + 6*B^2*b^4*d^4*f^
2*g*x^2 + 4*B^2*b^4*d^4*f^3*x)*log((b*x + a)^n))*log((d*x + c)^n))/(b^4*d^4
)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.00

$$\int \left(f+g x\right)^3 \left(A+B \ln\left(e \left(\frac{a+b x}{c+d x}\right)^n\right)\right)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f + g*x)^3*(A + B*log(e*((a + b*x)/(c + d*x))^n))^2,x)
```

```
[Out] int((f + g*x)^3*(A + B*log(e*((a + b*x)/(c + d*x))^n))^2, x)
```

<u>**sympy**</u> **[F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)**3*(A+B*ln(e*((b*x+a)/(d*x+c))**n))**2,x)
```

```
[Out] Timed out
```

## 3.68 $\int (f+gx)^2 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2 dx$

**Optimal**. Leaf size=565

$$\frac{2Bn(bc-ad)\left(a^2d^2g^2-abdg(3df-cg)+b^2\left(c^2g^2-3cdfg+3d^2f^2\right)\right)\log\left(\frac{bc-ad}{b(c+dx)}\right)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{3b^3d^3}+\frac{2B}{}$$

```
[Out] 1/3*B^2*(-a*d+b*c)^2*g^2*n^2*x/b^2/d^2-2/3*B*(-a*d+b*c)*g*(-a*d*g-2*b*c*g+3
*b*d*f)*n*(b*x+a)*(A+B*ln(e*((b*x+a)/(d*x+c))^n))/b^3/d^2-1/3*B*(-a*d+b*c)*
g^2*n*(d*x+c)^2*(A+B*ln(e*((b*x+a)/(d*x+c))^n))/b/d^3-1/3*(-a*g+b*f)^3*(A+B
*ln(e*((b*x+a)/(d*x+c))^n))^2/b^3/g+1/3*(g*x+f)^3*(A+B*ln(e*((b*x+a)/(d*x+c
))^n))^2/g+2/3*B*(-a*d+b*c)*(a^2*d^2*g^2-a*b*d*g*(-c*g+3*d*f)+b^2*(c^2*g^2-
3*c*d*f*g+3*d^2*f^2))*n*(A+B*ln(e*((b*x+a)/(d*x+c))^n))*ln((-a*d+b*c)/b/(d*
x+c))/b^3/d^3+1/3*B^2*(-a*d+b*c)^3*g^2*n^2*ln((b*x+a)/(d*x+c))/b^3/d^3+1/3*
B^2*(-a*d+b*c)^3*g^2*n^2*ln(d*x+c)/b^3/d^3+2/3*B^2*(-a*d+b*c)^2*g*(-a*d*g-2
*b*c*g+3*b*d*f)*n^2*ln(d*x+c)/b^3/d^3+2/3*B^2*(-a*d+b*c)*(a^2*d^2*g^2-a*b*d
*g*(-c*g+3*d*f)+b^2*(c^2*g^2-3*c*d*f*g+3*d^2*f^2))*n^2*polylog(2,d*(b*x+a)/
b/(d*x+c))/b^3/d^3
```

**Rubi [A]** time = 1.15, antiderivative size = 699, normalized size of antiderivative = 1.24, number of steps used = 27, number of rules used = 13, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}} = 0.406$, Rules used = {2525, 12, 2528, 2486, 31, 72, 2524, 2418, 2390, 2301, 2394, 2393, 2391}

$$-\frac{2B^2n^2(bf-ag)^3\text{PolyLog}\left(2,-\frac{d(a+bx)}{bc-ad}\right)}{3b^3g}-\frac{2B^2n^2(df-cg)^3\text{PolyLog}\left(2,\frac{b(c+dx)}{bc-ad}\right)}{3d^3g}+\frac{a^2B^2g^2n^2(bc-ad)\log(a+bx)}{3b^3d}$$

Antiderivative was successfully verified.

```
[In] Int[(f + g*x)^2*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2,x]
```

```
[Out] (-2*A*B*(b*c - a*d)*g*(3*b*d*f - b*c*g - a*d*g)*n*x)/(3*b^2*d^2) + (B^2*(b*
c - a*d)^2*g^2*n^2*x)/(3*b^2*d^2) + (a^2*B^2*(b*c - a*d)*g^2*n^2*Log[a + b*
x])/(3*b^3*d) + (B^2*(b*f - a*g)^3*n^2*Log[a + b*x]^2)/(3*b^3*g) - (2*B^2*(
b*c - a*d)*g*(3*b*d*f - b*c*g - a*d*g)*n*(a + b*x)*Log[e*((a + b*x)/(c + d*
x))^n])/(3*b^3*d^2) - (B*(b*c - a*d)*g^2*n*x^2*(A + B*Log[e*((a + b*x)/(c +
 d*x))^n]))/(3*b*d) - (2*B*(b*f - a*g)^3*n*Log[a + b*x]*(A + B*Log[e*((a +
b*x)/(c + d*x))^n]))/(3*b^3*g) + ((f + g*x)^3*(A + B*Log[e*((a + b*x)/(c +
d*x))^n])^2)/(3*g) - (B^2*c^2*(b*c - a*d)*g^2*n^2*Log[c + d*x])/(3*b*d^3) +
 (2*B^2*(b*c - a*d)^2*g*(3*b*d*f - b*c*g - a*d*g)*n^2*Log[c + d*x])/(3*b^3*
d^3) - (2*B^2*(d*f - c*g)^3*n^2*Log[-((d*(a + b*x))/(b*c - a*d))]*Log[c + d
```

```
*x])/(3*d^3*g) + (2*B*(d*f - c*g)^3*n*(A + B*Log[e*((a + b*x)/(c + d*x))^n]
)*Log[c + d*x])/(3*d^3*g) + (B^2*(d*f - c*g)^3*n^2*Log[c + d*x]^2)/(3*d^3*g
) - (2*B^2*(b*f - a*g)^3*n^2*Log[a + b*x]*Log[(b*(c + d*x))/(b*c - a*d)])/(
3*b^3*g) - (2*B^2*(b*f - a*g)^3*n^2*PolyLog[2, -((d*(a + b*x))/(b*c - a*d))
])/(3*b^3*g) - (2*B^2*(d*f - c*g)^3*n^2*PolyLog[2, (b*(c + d*x))/(b*c - a*d
)])/(3*d^3*g)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 72

```
Int[((e_.) + (f_.)*(x_))^(p_.)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
 x_Symbol] :> Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x]
 /; FreeQ[{a, b, c, d, e, f}, x] && IntegerQ[p]
```

Rule 2301

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

Rule 2390

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_))^(q_.), x_Symbol] :> Dist[1/e, Subst[Int[((f*x)/d)^q*(a + b*Log[c*x^
n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && E
qQ[e*f - d*g, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2393

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + (c*e*x)/g])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
```

```
(e*f - d*g), 0]
```

Rule 2394

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d + e*x
)^n]))/g, x] - Dist[(b*e*n)/g, Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2418

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*(RFx_), x_Sy
mbol] :> With[{u = ExpandIntegrand[(a + b*Log[c*(d + e*x)^n])^p, RFx, x]},
Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d, e, n}, x] && RationalFunctionQ[
RFx, x] && IntegerQ[p]
```

Rule 2486

```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))
^(r_.)]^(s_.), x_Symbol] :> Simp[((a + b*x)*Log[e*(f*(a + b*x)^p*(c + d*x)^
q)^r]^s)/b, x] + Dist[(q*r*s*(b*c - a*d))/b, Int[Log[e*(f*(a + b*x)^p*(c +
d*x)^q)^r]^(s - 1)/(c + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, p, q, r, s
}, x] && NeQ[b*c - a*d, 0] && EqQ[p + q, 0] && IGtQ[s, 0]
```

Rule 2524

```
Int[((a_.) + Log[(c_.)*(RFx_)^(p_.)]*(b_.))^(n_.)/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[(Log[d + e*x]*(a + b*Log[c*RFx^p])^n)/e, x] - Dist[(b*n*p)/e
, Int[(Log[d + e*x]*(a + b*Log[c*RFx^p])^(n - 1)*D[RFx, x])/RFx, x], x] /;
FreeQ[{a, b, c, d, e, p}, x] && RationalFunctionQ[RFx, x] && IGtQ[n, 0]
```

Rule 2525

```
Int[((a_.) + Log[(c_.)*(RFx_)^(p_.)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_))^(m_.
), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(a + b*Log[c*RFx^p])^n)/(e*(m + 1))
, x] - Dist[(b*n*p)/(e*(m + 1)), Int[SimplifyIntegrand[((d + e*x)^(m + 1)*(
a + b*Log[c*RFx^p])^(n - 1)*D[RFx, x])/RFx, x], x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && RationalFunctionQ[RFx, x] && IGtQ[n, 0] && (EqQ[n, 1] ||
IntegerQ[m]) && NeQ[m, -1]
```

Rule 2528

```
Int[((a_.) + Log[(c_.)*(RFx_)^(p_.)]*(b_.))^(n_.)*(RGx_), x_Symbol] :> With
[{u = ExpandIntegrand[(a + b*Log[c*RFx^p])^n, RGx, x]}, Int[u, x] /; SumQ[u
]] /; FreeQ[{a, b, c, p}, x] && RationalFunctionQ[RFx, x] && RationalFuncti
```

```
onQ[RGx, x] && IGtQ[n, 0]
```

Rubi steps

$$\int (f+gx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2 dx = \frac{(f+gx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{3g} - \frac{(2Bn)\int \frac{(bc-ad)(f+gx)^3\left(A+B\log \cdots\right.}{(a+bx)(c+dx \cdots}}{3g}$$

$$= \frac{(f+gx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{3g} - \frac{(2B(bc-ad)n)\int \frac{(f+gx)^3\left(A+ \cdots\right.}{(a+b \cdots}}{3g}$$

$$= \frac{(f+gx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{3g} - \frac{(2B(bc-ad)n)\int\left(\frac{g^2(3bdf-bc \cdots}{\cdots}\right.}{\cdots}$$

$$= \frac{(f+gx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{3g} - \frac{\left(2B(bc-ad)g^2n\right)\int x\left(A+ \cdots\right.}{3bd}$$

$$= -\frac{2AB(bc-ad)g(3bdf-bcg-adg)nx}{3b^2d^2} - \frac{B(bc-ad)g^2nx^2\left(A+B \cdots\right.}{3bd}$$

$$= -\frac{2AB(bc-ad)g(3bdf-bcg-adg)nx}{3b^2d^2} - \frac{2B^2(bc-ad)g(3bdf-b \cdots}{\cdots}$$

$$= -\frac{2AB(bc-ad)g(3bdf-bcg-adg)nx}{3b^2d^2} - \frac{2B^2(bc-ad)g(3bdf-b \cdots}{\cdots}$$

$$= -\frac{2AB(bc-ad)g(3bdf-bcg-adg)nx}{3b^2d^2} + \frac{B^2(bc-ad)^2g^2n^2x}{3b^2d^2} + \frac{a^2 \cdots}{\cdots}$$

$$= -\frac{2AB(bc-ad)g(3bdf-bcg-adg)nx}{3b^2d^2} + \frac{B^2(bc-ad)^2g^2n^2x}{3b^2d^2} + \frac{a^2 \cdots}{\cdots}$$

$$= -\frac{2AB(bc-ad)g(3bdf-bcg-adg)nx}{3b^2d^2} + \frac{B^2(bc-ad)^2g^2n^2x}{3b^2d^2} + \frac{a^2 \cdots}{\cdots}$$

**Mathematica [A]** time = 0.56, size = 506, normalized size = 0.90

$$\frac{(f+gx)^3\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2-\frac{Bn\left(-Bg^3n(bc-ad)\left(a^2d^2\log(a+bx)-b\left(dx(ad-bc)+bc^2\log(c+dx)\right)\right)-2b^3(df-cg)^3\log(c+dx)\left(B\log\left(e\left(\frac{a}{c}\right.\right.\right.\right.}{}}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(f + g*x)^2*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2,x]
```

```
[Out] ((f + g*x)^3*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2 - (B*n*(2*A*b*d*(b*c
- a*d)*g^2*(3*b*d*f - b*c*g - a*d*g)*x + 2*B*d*(b*c - a*d)*g^2*(3*b*d*f - b
*c*g - a*d*g)*(a + b*x)*Log[e*((a + b*x)/(c + d*x))^n] + b^2*d^2*(b*c - a*d
)*g^3*x^2*(A + B*Log[e*((a + b*x)/(c + d*x))^n]) + 2*d^3*(b*f - a*g)^3*Log[
a + b*x]*(A + B*Log[e*((a + b*x)/(c + d*x))^n]) + 2*B*(b*c - a*d)^2*g^2*(-3
*b*d*f + b*c*g + a*d*g)*n*Log[c + d*x] - 2*b^3*(d*f - c*g)^3*(A + B*Log[e*(
(a + b*x)/(c + d*x))^n])*Log[c + d*x] - B*(b*c - a*d)*g^3*n*(a^2*d^2*Log[a
+ b*x] - b*(d*(-(b*c) + a*d)*x + b*c^2*Log[c + d*x])) - B*d^3*(b*f - a*g)^3
*n*(Log[a + b*x]*(Log[a + b*x] - 2*Log[(b*(c + d*x))/(b*c - a*d)]) - 2*Poly
Log[2, (d*(a + b*x))/(-(b*c) + a*d)]) + b^3*B*(d*f - c*g)^3*n*((2*Log[(d*(a
 + b*x))/(-(b*c) + a*d)] - Log[c + d*x])*Log[c + d*x] + 2*PolyLog[2, (b*(c
+ d*x))/(b*c - a*d)])))/(b^3*d^3))/(3*g)
```

**fricas [F]** time = 1.15, size = 0, normalized size = 0.00

$$\text{integral}\left(A^2g^2x^2+2\,A^2fgx+A^2f^2+\left(B^2g^2x^2+2\,B^2fgx+B^2f^2\right)\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)^2+2\left(ABg^2x^2+2\,ABfgx+\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^2*(A+B*log(e*((b*x+a)/(d*x+c))^n))^2,x, algorithm="fricas
")
```

```
[Out] integral(A^2*g^2*x^2 + 2*A^2*f*g*x + A^2*f^2 + (B^2*g^2*x^2 + 2*B^2*f*g*x +
 B^2*f^2)*log(e*((b*x + a)/(d*x + c))^n)^2 + 2*(A*B*g^2*x^2 + 2*A*B*f*g*x +
 A*B*f^2)*log(e*((b*x + a)/(d*x + c))^n), x)
```

**giac [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^2*(A+B*log(e*((b*x+a)/(d*x+c))^n))^2,x, algorithm="giac")
```

```
[Out] Timed out
```

**maple [F]** time = 0.28, size = 0, normalized size = 0.00

$$\int \left(gx+f\right)^{2} \left(B\ln\left(e\left(\frac{bx+a}{dx+c}\right)^{n}\right)+A\right)^{2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x+f)^2*(B*ln(e*((b*x+a)/(d*x+c))^n)+A)^2,x)
```

```
[Out] int((g*x+f)^2*(B*ln(e*((b*x+a)/(d*x+c))^n)+A)^2,x)
```

**maxima [B]** time = 4.83, size = 1659, normalized size = 2.94

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^2*(A+B*log(e*((b*x+a)/(d*x+c))^n))^2,x, algorithm="maxima
")
```

```
[Out] 2/3*A*B*g^2*x^3*log(e*(b*x/(d*x + c) + a/(d*x + c))^n) + 1/3*A^2*g^2*x^3 +
2*A*B*f*g*x^2*log(e*(b*x/(d*x + c) + a/(d*x + c))^n) + A^2*f*g*x^2 + 1/3*A*
B*g^2*n*(2*a^3*log(b*x + a)/b^3 - 2*c^3*log(d*x + c)/d^3 - ((b^2*c*d - a*b*
d^2)*x^2 - 2*(b^2*c^2 - a^2*d^2)*x)/(b^2*d^2)) - 2*A*B*f*g*n*(a^2*log(b*x +
 a)/b^2 - c^2*log(d*x + c)/d^2 + (b*c - a*d)*x/(b*d)) + 2*A*B*f^2*n*(a*log(
b*x + a)/b - c*log(d*x + c)/d) + 2*A*B*f^2*x*log(e*(b*x/(d*x + c) + a/(d*x
+ c))^n) + A^2*f^2*x + 1/3*(2*a^2*c*d^2*g^2*n^2 - (6*c*d^2*f*g*n^2 - c^2*d*
g^2*n^2)*a*b - (6*c*d^2*f^2*n*log(e) + (3*g^2*n^2 + 2*g^2*n*log(e))*c^3 - 6
*(f*g*n^2 + f*g*n*log(e))*c^2*d)*b^2)*B^2*log(d*x + c)/(b^2*d^3) + 2/3*(3*a
*b^2*d^3*f^2*n^2 - 3*a^2*b*d^3*f*g*n^2 + a^3*d^3*g^2*n^2 - (3*c*d^2*f^2*n^2
 - 3*c^2*d*f*g*n^2 + c^3*g^2*n^2)*b^3)*(log(b*x + a)*log((b*d*x + a*d)/(b*c
 - a*d) + 1) + dilog(-(b*d*x + a*d)/(b*c - a*d)))*B^2/(b^3*d^3) + 1/3*(B^2*
b^3*d^3*g^2*x^3*log(e)^2 + 2*(3*c*d^2*f^2*n^2 - 3*c^2*d*f*g*n^2 + c^3*g^2*n
^2)*B^2*b^3*log(b*x + a)*log(d*x + c) - (3*c*d^2*f^2*n^2 - 3*c^2*d*f*g*n^2
+ c^3*g^2*n^2)*B^2*b^3*log(d*x + c)^2 + (a*b^2*d^3*g^2*n*log(e) - (c*d^2*g^
2*n*log(e) - 3*d^3*f*g*log(e)^2)*b^3)*B^2*x^2 - (3*a*b^2*d^3*f^2*n^2 - 3*a^
2*b*d^3*f*g*n^2 + a^3*d^3*g^2*n^2)*B^2*log(b*x + a)^2 + ((g^2*n^2 - 2*g^2*n
*log(e))*a^2*b*d^3 - 2*(c*d^2*g^2*n^2 - 3*d^3*f*g*n*log(e))*a*b^2 - (6*c*d^
2*f*g*n*log(e) - 3*d^3*f^2*log(e)^2 - (g^2*n^2 + 2*g^2*n*log(e))*c^2*d)*b^3
)*B^2*x - ((3*g^2*n^2 - 2*g^2*n*log(e))*a^3*d^3 - (c*d^2*g^2*n^2 + 6*(f*g*n
^2 - f*g*n*log(e))*d^3)*a^2*b + 2*(3*c*d^2*f*g*n^2 - c^2*d*g^2*n^2 - 3*d^3*
f^2*n*log(e))*a*b^2)*B^2*log(b*x + a) + (B^2*b^3*d^3*g^2*x^3 + 3*B^2*b^3*d^
3*f*g*x^2 + 3*B^2*b^3*d^3*f^2*x)*log((b*x + a)^n)^2 + (B^2*b^3*d^3*g^2*x^3
+ 3*B^2*b^3*d^3*f*g*x^2 + 3*B^2*b^3*d^3*f^2*x)*log((d*x + c)^n)^2 + (2*B^2*
b^3*d^3*g^2*x^3*log(e) - 2*(3*c*d^2*f^2*n - 3*c^2*d*f*g*n + c^3*g^2*n)*B^2*
b^3*log(d*x + c) + (a*b^2*d^3*g^2*n - (c*d^2*g^2*n - 6*d^3*f*g*log(e))*b^3)
```

```
*B^2*x^2 + 2*(3*a*b^2*d^3*f*g*n - a^2*b*d^3*g^2*n - (3*c*d^2*f*g*n - c^2*d*
g^2*n - 3*d^3*f^2*log(e))*b^3)*B^2*x + 2*(3*a*b^2*d^3*f^2*n - 3*a^2*b*d^3*f
*g*n + a^3*d^3*g^2*n)*B^2*log(b*x + a))*log((b*x + a)^n) - (2*B^2*b^3*d^3*g
^2*x^3*log(e) - 2*(3*c*d^2*f^2*n - 3*c^2*d*f*g*n + c^3*g^2*n)*B^2*b^3*log(d
*x + c) + (a*b^2*d^3*g^2*n - (c*d^2*g^2*n - 6*d^3*f*g*log(e))*b^3)*B^2*x^2
+ 2*(3*a*b^2*d^3*f*g*n - a^2*b*d^3*g^2*n - (3*c*d^2*f*g*n - c^2*d*g^2*n - 3
*d^3*f^2*log(e))*b^3)*B^2*x + 2*(3*a*b^2*d^3*f^2*n - 3*a^2*b*d^3*f*g*n + a^
3*d^3*g^2*n)*B^2*log(b*x + a) + 2*(B^2*b^3*d^3*g^2*x^3 + 3*B^2*b^3*d^3*f*g*
x^2 + 3*B^2*b^3*d^3*f^2*x)*log((b*x + a)^n))*log((d*x + c)^n))/(b^3*d^3)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.00

$$\int (f+g\,x)^2\,\left(A+B\,\ln\left(e\,{\left(\frac{a+b\,x}{c+d\,x}\right)}^n\right)\right)^2 \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f + g*x)^2*(A + B*log(e*((a + b*x)/(c + d*x))^n))^2,x)
[Out] int((f + g*x)^2*(A + B*log(e*((a + b*x)/(c + d*x))^n))^2, x)
```

**sympy [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)**2*(A+B*ln(e*((b*x+a)/(d*x+c))**n))**2,x)
[Out] Timed out
```

# 3.69 $\int (f+gx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2\,dx$

**Optimal**. Leaf size=290

$$\frac{Bn(bc-ad)(-adg-bcg+2bdf)\log\left(\frac{bc-ad}{b(c+dx)}\right)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{b^2d^2}-\frac{(bf-ag)^2\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{2b^2g}-\frac{Bg\cdots}{\cdots}$$

```
[Out] -B*(-a*d+b*c)*g*n*(b*x+a)*(A+B*ln(e*((b*x+a)/(d*x+c))^n))/b^2/d-1/2*(-a*g+b
*f)^2*(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2/b^2/g+1/2*(g*x+f)^2*(A+B*ln(e*((b*x
+a)/(d*x+c))^n))^2/g+B*(-a*d+b*c)*(-a*d*g-b*c*g+2*b*d*f)*n*(A+B*ln(e*((b*x+
a)/(d*x+c))^n))*ln((-a*d+b*c)/b/(d*x+c))/b^2/d^2+B^2*(-a*d+b*c)^2*g*n^2*ln(
d*x+c)/b^2/d^2+B^2*(-a*d+b*c)*(-a*d*g-b*c*g+2*b*d*f)*n^2*polylog(2,d*(b*x+a
)/b/(d*x+c))/b^2/d^2
```

**Rubi [A]** time = 0.83, antiderivative size = 481, normalized size of antiderivative = 1.66, number of steps used = 23, number of rules used = 12, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {2525, 12, 2528, 2486, 31, 2524, 2418, 2390, 2301, 2394, 2393, 2391}

$$-\frac{B^2n^2(bf-ag)^2\text{PolyLog}\left(2,-\frac{d(a+bx)}{bc-ad}\right)}{b^2g}-\frac{B^2n^2(df-cg)^2\text{PolyLog}\left(2,\frac{b(c+dx)}{bc-ad}\right)}{d^2g}-\frac{Bn(bf-ag)^2\log(a+bx)\left(B\log\cdots\right.}{b^2g\cdots}$$

Antiderivative was successfully verified.

```
[In] Int[(f + g*x)*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2,x]
```

```
[Out] -((A*B*(b*c - a*d)*g*n*x)/(b*d)) + (B^2*(b*f - a*g)^2*n^2*Log[a + b*x]^2)/(
2*b^2*g) - (B^2*(b*c - a*d)*g*n*(a + b*x)*Log[e*((a + b*x)/(c + d*x))^n])/(
b^2*d) - (B*(b*f - a*g)^2*n*Log[a + b*x]*(A + B*Log[e*((a + b*x)/(c + d*x))
^n]))/(b^2*g) + ((f + g*x)^2*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2)/(2*g
) + (B^2*(b*c - a*d)^2*g*n^2*Log[c + d*x])/(b^2*d^2) - (B^2*(d*f - c*g)^2*n
^2*Log[-((d*(a + b*x))/(b*c - a*d))]*Log[c + d*x])/(d^2*g) + (B*(d*f - c*g)
^2*n*(A + B*Log[e*((a + b*x)/(c + d*x))^n])*Log[c + d*x])/(d^2*g) + (B^2*(d
*f - c*g)^2*n^2*Log[c + d*x]^2)/(2*d^2*g) - (B^2*(b*f - a*g)^2*n^2*Log[a +
b*x]*Log[(b*(c + d*x))/(b*c - a*d)])/(b^2*g) - (B^2*(b*f - a*g)^2*n^2*PolyL
og[2, -((d*(a + b*x))/(b*c - a*d))])/(b^2*g) - (B^2*(d*f - c*g)^2*n^2*PolyL
og[2, (b*(c + d*x))/(b*c - a*d)])/(d^2*g)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 2301

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

Rule 2390

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_))^(q_.), x_Symbol] :> Dist[1/e, Subst[Int[((f*x)/d)^q*(a + b*Log[c*x^
n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && E
qQ[e*f - d*g, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2393

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + (c*e*x)/g])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2394

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d + e*x
)^n]))/g, x] - Dist[(b*e*n)/g, Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2418

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*(RFx_), x_Sy
mbol] :> With[{u = ExpandIntegrand[(a + b*Log[c*(d + e*x)^n])^p, RFx, x]},
Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d, e, n}, x] && RationalFunctionQ[
RFx, x] && IntegerQ[p]
```

Rule 2486

```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))
^(r_.)]^(s_.), x_Symbol] :> Simp[((a + b*x)*Log[e*(f*(a + b*x)^p*(c + d*x)^
q)^r]^s)/b, x] + Dist[(q*r*s*(b*c - a*d))/b, Int[Log[e*(f*(a + b*x)^p*(c +
d*x)^q)^r]^(s - 1)/(c + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, p, q, r, s
}, x] && NeQ[b*c - a*d, 0] && EqQ[p + q, 0] && IGtQ[s, 0]
```

Rule 2524

```
Int[((a_.) + Log[(c_.)*(RFx_)^(p_.)]*(b_.))^(n_.)/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[(Log[d + e*x]*(a + b*Log[c*RFx^p])^n)/e, x] - Dist[(b*n*p)/e
, Int[(Log[d + e*x]*(a + b*Log[c*RFx^p])^(n - 1)*D[RFx, x])/RFx, x], x] /;
FreeQ[{a, b, c, d, e, p}, x] && RationalFunctionQ[RFx, x] && IGtQ[n, 0]
```

Rule 2525

```
Int[((a_.) + Log[(c_.)*(RFx_)^(p_.)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_))^(m_.
), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(a + b*Log[c*RFx^p])^n)/(e*(m + 1))
, x] - Dist[(b*n*p)/(e*(m + 1)), Int[SimplifyIntegrand[((d + e*x)^(m + 1)*(
a + b*Log[c*RFx^p])^(n - 1)*D[RFx, x])/RFx, x], x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && RationalFunctionQ[RFx, x] && IGtQ[n, 0] && (EqQ[n, 1] ||
IntegerQ[m]) && NeQ[m, -1]
```

Rule 2528

```
Int[((a_.) + Log[(c_.)*(RFx_)^(p_.)]*(b_.))^(n_.)*(RGx_), x_Symbol] :> With
[{u = ExpandIntegrand[(a + b*Log[c*RFx^p])^n, RGx, x]}, Int[u, x] /; SumQ[u
]] /; FreeQ[{a, b, c, p}, x] && RationalFunctionQ[RFx, x] && RationalFuncti
onQ[RGx, x] && IGtQ[n, 0]
```

Rubi steps

$$\int (f+gx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2 dx = \frac{(f+gx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{2g} - \frac{(Bn)\int \frac{(bc-ad)(f+gx)^2\left(A+B\log\left(e\left(\right.\right.\right.}{(a+bx)(c+dx)}}{g}$$

$$= \frac{(f+gx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{2g} - \frac{(B(bc-ad)n)\int \frac{(f+gx)^2\left(A+B\log\right.}{(a+bx)(c+}}{g}$$

$$= \frac{(f+gx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{2g} - \frac{(B(bc-ad)n)\int \left(\frac{g^2\left(A+B\log\left(e\left(\right.\right.\right.}{bd}\right.}{}$$

$$= \frac{(f+gx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{2g} - \frac{(B(bc-ad)gn)\int \left(A+B\log\left(\right.\right.}{bd}$$

$$= -\frac{AB(bc-ad)gnx}{bd} - \frac{B(bf-ag)^2 n\log(a+bx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)\right.\right.}{b^2 g}$$

$$= -\frac{AB(bc-ad)gnx}{bd} - \frac{B^2(bc-ad)gn(a+bx)\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{b^2 d} - \frac{B(bf}{}$$

$$= -\frac{AB(bc-ad)gnx}{bd} - \frac{B^2(bc-ad)gn(a+bx)\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{b^2 d} - \frac{B(bf}{}$$

$$= -\frac{AB(bc-ad)gnx}{bd} - \frac{B^2(bc-ad)gn(a+bx)\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{b^2 d} - \frac{B(bf}{}$$

$$= -\frac{AB(bc-ad)gnx}{bd} + \frac{B^2(bf-ag)^2 n^2\log^2(a+bx)}{2b^2 g} - \frac{B^2(bc-ad)gn(}{}$$

$$= -\frac{AB(bc-ad)gnx}{bd} + \frac{B^2(bf-ag)^2 n^2\log^2(a+bx)}{2b^2 g} - \frac{B^2(bc-ad)gn(}{}$$

**Mathematica [A]** time = 0.31, size = 362, normalized size = 1.25

$$(f+gx)^2\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2 - \frac{Bn\left(-2b^2(df-cg)^2\log(c+dx)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)+2d^2(bf-ag)^2\log(a+bx)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)+\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(f + g*x)*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2,x]
```

```
[Out] ((f + g*x)^2*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2 - (B*n*(2*A*b*d*(b*c
- a*d)*g^2*x + 2*B*d*(b*c - a*d)*g^2*(a + b*x)*Log[e*((a + b*x)/(c + d*x))^
n] + 2*d^2*(b*f - a*g)^2*Log[a + b*x]*(A + B*Log[e*((a + b*x)/(c + d*x))^n]
) - 2*B*(b*c - a*d)^2*g^2*n*Log[c + d*x] - 2*b^2*(d*f - c*g)^2*(A + B*Log[e
*((a + b*x)/(c + d*x))^n])*Log[c + d*x] - B*d^2*(b*f - a*g)^2*n*(Log[a + b*
x]*(Log[a + b*x] - 2*Log[(b*(c + d*x))/(b*c - a*d)]) - 2*PolyLog[2, (d*(a +
 b*x))/(-(b*c) + a*d)]) + b^2*B*(d*f - c*g)^2*n*((2*Log[(d*(a + b*x))/(-(b*
c) + a*d)] - Log[c + d*x])*Log[c + d*x] + 2*PolyLog[2, (b*(c + d*x))/(b*c -
 a*d)])))/(b^2*d^2))/(2*g)
```

**fricas [F]** time = 1.37, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(A^2gx + A^2f + \left(B^2gx + B^2f\right)\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)^2 + 2\left(ABgx + ABf\right)\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right), x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)*(A+B*log(e*((b*x+a)/(d*x+c))^n))^2,x, algorithm="fricas")
```

```
[Out] integral(A^2*g*x + A^2*f + (B^2*g*x + B^2*f)*log(e*((b*x + a)/(d*x + c))^n)
^2 + 2*(A*B*g*x + A*B*f)*log(e*((b*x + a)/(d*x + c))^n), x)
```

**giac [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)*(A+B*log(e*((b*x+a)/(d*x+c))^n))^2,x, algorithm="giac")
```

```
[Out] Timed out
```

**maple [F]** time = 0.11, size = 0, normalized size = 0.00

$$\int \left(gx+f\right)\left(B\ln\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x+f)*(B*ln(e*((b*x+a)/(d*x+c))^n)+A)^2,x)
```

```
[Out] int((g*x+f)*(B*ln(e*((b*x+a)/(d*x+c))^n)+A)^2,x)
```

**maxima [B]** time = 5.19, size = 899, normalized size = 3.10

$$ABgx^2\log\left(e\left(\frac{bx}{dx+c}+\frac{a}{dx+c}\right)^n\right)+\frac{1}{2}A^2gx^2-ABgn\left(\frac{a^2\log(bx+a)}{b^2}-\frac{c^2\log(dx+c)}{d^2}+\frac{(bc-ad)x}{bd}\right)+2ABfn\left(\frac{a\,l}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)*(A+B*log(e*((b*x+a)/(d*x+c))^n))^2,x, algorithm="maxima")
```

```
[Out] A*B*g*x^2*log(e*(b*x/(d*x + c) + a/(d*x + c))^n) + 1/2*A^2*g*x^2 - A*B*g*n*
(a^2*log(b*x + a)/b^2 - c^2*log(d*x + c)/d^2 + (b*c - a*d)*x/(b*d)) + 2*A*B
*f*n*(a*log(b*x + a)/b - c*log(d*x + c)/d) + 2*A*B*f*x*log(e*(b*x/(d*x + c)
 + a/(d*x + c))^n) + A^2*f*x - (a*c*d*g*n^2 + (2*c*d*f*n*log(e) - (g*n^2 +
g*n*log(e))*c^2)*b)*B^2*log(d*x + c)/(b*d^2) + (2*a*b*d^2*f*n^2 - a^2*d^2*g
*n^2 - (2*c*d*f*n^2 - c^2*g*n^2)*b^2)*(log(b*x + a)*log((b*d*x + a*d)/(b*c
- a*d) + 1) + dilog(-(b*d*x + a*d)/(b*c - a*d)))*B^2/(b^2*d^2) + 1/2*(B^2*b
^2*d^2*g*x^2*log(e)^2 + 2*(2*c*d*f*n^2 - c^2*g*n^2)*B^2*b^2*log(b*x + a)*lo
g(d*x + c) - (2*c*d*f*n^2 - c^2*g*n^2)*B^2*b^2*log(d*x + c)^2 - (2*a*b*d^2*
f*n^2 - a^2*d^2*g*n^2)*B^2*log(b*x + a)^2 + 2*(a*b*d^2*g*n*log(e) - (c*d*g*
n*log(e) - d^2*f*log(e)^2)*b^2)*B^2*x + 2*((g*n^2 - g*n*log(e))*a^2*d^2 - (
c*d*g*n^2 - 2*d^2*f*n*log(e))*a*b)*B^2*log(b*x + a) + (B^2*b^2*d^2*g*x^2 +
2*B^2*b^2*d^2*f*x)*log((b*x + a)^n)^2 + (B^2*b^2*d^2*g*x^2 + 2*B^2*b^2*d^2*
f*x)*log((d*x + c)^n)^2 + 2*(B^2*b^2*d^2*g*x^2*log(e) - (2*c*d*f*n - c^2*g*
n)*B^2*b^2*log(d*x + c) + (a*b*d^2*g*n - (c*d*g*n - 2*d^2*f*log(e))*b^2)*B^
2*x + (2*a*b*d^2*f*n - a^2*d^2*g*n)*B^2*log(b*x + a))*log((b*x + a)^n) - 2*
(B^2*b^2*d^2*g*x^2*log(e) - (2*c*d*f*n - c^2*g*n)*B^2*b^2*log(d*x + c) + (a
*b*d^2*g*n - (c*d*g*n - 2*d^2*f*log(e))*b^2)*B^2*x + (2*a*b*d^2*f*n - a^2*d
^2*g*n)*B^2*log(b*x + a) + (B^2*b^2*d^2*g*x^2 + 2*B^2*b^2*d^2*f*x)*log((b*x
 + a)^n))*log((d*x + c)^n))/(b^2*d^2)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.00

$$\int (f+g\,x)\,\left(A+B\,\ln\left(e\,{\left(\frac{a+b\,x}{c+d\,x}\right)}^n\right)\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f + g*x)*(A + B*log(e*((a + b*x)/(c + d*x))^n))^2,x)
```

```
[Out] int((f + g*x)*(A + B*log(e*((a + b*x)/(c + d*x))^n))^2, x)
```

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(A+B\log\left(e\left(\frac{a}{c+dx}+\frac{bx}{c+dx}\right)^{n}\right)\right)^{2}\left(f+gx\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)*(A+B*ln(e*((b*x+a)/(d*x+c))**n))**2,x)
```

```
[Out] Integral((A + B*log(e*(a/(c + d*x) + b*x/(c + d*x))**n))**2*(f + g*x), x)
```

# 3.70 $\int \left(A+B\log \left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2 \, dx$

**Optimal**. Leaf size=135

$$\frac{2Bn(bc-ad)\log \left(\frac{bc-ad}{b(c+dx)}\right)\left(B\log \left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{bd}+\frac{(a+bx)\left(B\log \left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{b}+\frac{2B^2n^2(bc-ad)\text{Li}_2\left(\frac{d(a+bx)}{b(c+dx)}\right)}{bd}$$

```
[Out] (b*x+a)*(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2/b+2*B*(-a*d+b*c)*n*(A+B*ln(e*((b*
x+a)/(d*x+c))^n))*ln((-a*d+b*c)/b/(d*x+c))/b/d+2*B^2*(-a*d+b*c)*n^2*polylog
(2,d*(b*x+a)/b/(d*x+c))/b/d
```

**Rubi [B]** time = 0.62, antiderivative size = 275, normalized size of antiderivative = 2.04, number of steps used = 20, number of rules used = 10, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.417, Rules used = {2523, 12, 2528, 2524, 2418, 2390, 2301, 2394, 2393, 2391}

$$\frac{2aB^2n^2\text{PolyLog}\left(2,-\frac{d(a+bx)}{bc-ad}\right)}{b}+\frac{2B^2cn^2\text{PolyLog}\left(2,\frac{b(c+dx)}{bc-ad}\right)}{d}+\frac{2aBn\log(a+bx)\left(B\log \left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{b}-\frac{2Bcn\log \ldots}{\ldots}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2,x]
```

```
[Out] -((a*B^2*n^2*Log[a + b*x]^2)/b) + (2*a*B*n*Log[a + b*x]*(A + B*Log[e*((a +
b*x)/(c + d*x))^n]))/b + x*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2 + (2*B^
2*c*n^2*Log[-((d*(a + b*x))/(b*c - a*d))]*Log[c + d*x])/d - (2*B*c*n*(A + B
*Log[e*((a + b*x)/(c + d*x))^n])*Log[c + d*x])/d - (B^2*c*n^2*Log[c + d*x]^
2)/d + (2*a*B^2*n^2*Log[a + b*x]*Log[(b*(c + d*x))/(b*c - a*d)])/b + (2*a*B
^2*n^2*PolyLog[2, -((d*(a + b*x))/(b*c - a*d))])/b + (2*B^2*c*n^2*PolyLog[2
, (b*(c + d*x))/(b*c - a*d)])/d
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2301

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

Rule 2390

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_))^(q_.), x_Symbol] :> Dist[1/e, Subst[Int[((f*x)/d)^q*(a + b*Log[c*x^
n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && E
qQ[e*f - d*g, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2393

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + (c*e*x)/g])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2394

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d + e*x
)^n]))/g, x] - Dist[(b*e*n)/g, Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2418

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*(RFx_), x_Sy
mbol] :> With[{u = ExpandIntegrand[(a + b*Log[c*(d + e*x)^n])^p, RFx, x]},
Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d, e, n}, x] && RationalFunctionQ[
RFx, x] && IntegerQ[p]
```

Rule 2523

```
Int[((a_.) + Log[(c_.)*(RFx_)^(p_.)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a +
 b*Log[c*RFx^p])^n, x] - Dist[b*n*p, Int[SimplifyIntegrand[(x*(a + b*Log[c*
RFx^p])^(n - 1)*D[RFx, x])/RFx, x], x], x] /; FreeQ[{a, b, c, p}, x] && Rat
ionalFunctionQ[RFx, x] && IGtQ[n, 0]
```

Rule 2524

```
Int[((a_.) + Log[(c_.)*(RFx_)^(p_.)]*(b_.))^(n_.)/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[(Log[d + e*x]*(a + b*Log[c*RFx^p])^n)/e, x] - Dist[(b*n*p)/e
, Int[(Log[d + e*x]*(a + b*Log[c*RFx^p])^(n - 1)*D[RFx, x])/RFx, x], x] /;
FreeQ[{a, b, c, d, e, p}, x] && RationalFunctionQ[RFx, x] && IGtQ[n, 0]
```

Rule 2528

```
Int[((a_.) + Log[(c_.)*(RFx_)^(p_.)]*(b_.))^(n_.)*(RGx_), x_Symbol] :> With
[{u = ExpandIntegrand[(a + b*Log[c*RFx^p])^n, RGx, x]}, Int[u, x] /; SumQ[u
]] /; FreeQ[{a, b, c, p}, x] && RationalFunctionQ[RFx, x] && RationalFuncti
onQ[RGx, x] && IGtQ[n, 0]
```

Rubi steps

$$\int \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2 dx = x\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2 - (2Bn)\int \frac{(bc-ad)x\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right.}{(a+bx)(c+dx)}$$

$$= x\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2 - (2B(bc-ad)n)\int \frac{x\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right.}{(a+bx)(c+dx)}$$

$$= x\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2 - (2B(bc-ad)n)\int \left(-\frac{a\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)\right.\right.}{(bc-ad)(a+bx)}\right.$$

$$= x\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2 + (2aBn)\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{a+bx}\,dx - (2Bc$$

$$= \frac{2aBn\log(a+bx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{b} + x\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2$$

$$= \frac{2aBn\log(a+bx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{b} + x\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2$$

$$= \frac{2aBn\log(a+bx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{b} + x\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2$$

$$= \frac{2aBn\log(a+bx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{b} + x\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2$$

$$= -\frac{aB^2n^2\log^2(a+bx)}{b} + \frac{2aBn\log(a+bx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{b} + x\left(A\right.$$

$$= -\frac{aB^2n^2\log^2(a+bx)}{b} + \frac{2aBn\log(a+bx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{b} + x\left(A\right.$$

**Mathematica** [A] time = 0.17, size = 226, normalized size = 1.67

$$\frac{Bn\left(2ad\log(a+bx)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)-2bc\log(c+dx)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)-aBdn\left(\log(a+bx)\left(\log\right.\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2,x]
```

```
[Out] x*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2 + (B*n*(2*a*d*Log[a + b*x]*(A +
B*Log[e*((a + b*x)/(c + d*x))^n]) - 2*b*c*(A + B*Log[e*((a + b*x)/(c + d*x)
)^n])*Log[c + d*x] - a*B*d*n*(Log[a + b*x]*(Log[a + b*x] - 2*Log[(b*(c + d*
x))/(b*c - a*d)]) - 2*PolyLog[2, (d*(a + b*x))/(-(b*c) + a*d)]) + b*B*c*n*(
(2*Log[(d*(a + b*x))/(-(b*c) + a*d)] - Log[c + d*x])*Log[c + d*x] + 2*PolyL
og[2, (b*(c + d*x))/(b*c - a*d)])))/(b*d)
```

**fricas [F]** time = 1.10, size = 0, normalized size = 0.00

$$\text{integral}\left(B^2\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)^2 + 2\,AB\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right) + A^2, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))^2,x, algorithm="fricas")
```

```
[Out] integral(B^2*log(e*((b*x + a)/(d*x + c))^n)^2 + 2*A*B*log(e*((b*x + a)/(d*x
 + c))^n) + A^2, x)
```

**giac [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))^2,x, algorithm="giac")
```

```
[Out] Timed out
```

**maple [F]** time = 0.15, size = 0, normalized size = 0.00

$$\int\left(B\ln\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*ln(e*((b*x+a)/(d*x+c))^n)+A)^2,x)
```

```
[Out] int((B*ln(e*((b*x+a)/(d*x+c))^n)+A)^2,x)
```

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$2\,ABn\left(\frac{a\log(bx+a)}{b} - \frac{c\log(dx+c)}{d}\right)+2\,ABx\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A^2x+B^2\left(\frac{2\,bcn^2\log(bx+a)\log(dx+c) - bcr}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))^2,x, algorithm="maxima")
```

```
[Out] 2*A*B*n*(a*log(b*x + a)/b - c*log(d*x + c)/d) + 2*A*B*x*log(e*((b*x + a)/(d
*x + c))^n) + A^2*x + B^2*((2*b*c*n^2*log(b*x + a)*log(d*x + c) - b*c*n^2*l
og(d*x + c)^2 + b*d*x*log((b*x + a)^n)^2 + b*d*x*log((d*x + c)^n)^2 + 2*(a*
d*n*log(b*x + a) - b*c*n*log(d*x + c) + b*d*x*log(e))*log((b*x + a)^n) - 2*
(a*d*n*log(b*x + a) - b*c*n*log(d*x + c) + b*d*x*log((b*x + a)^n) + b*d*x*l
og(e))*log((d*x + c)^n))/(b*d) - integrate(-(b^2*d*x^2*log(e)^2 + a*b*c*log
(e)^2 - ((2*n*log(e) - log(e)^2)*b^2*c - (2*n*log(e) + log(e)^2)*a*b*d)*x -
 2*(b^2*c*n^2*x + 2*a*b*c*n^2 - a^2*d*n^2)*log(b*x + a))/(b^2*d*x^2 + a*b*c
 + (b^2*c + a*b*d)*x), x))
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.01

$$\int \left(A+B\,\ln\left(e\,{\left(\frac{a+b\,x}{c+d\,x}\right)}^n\right)\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*log(e*((a + b*x)/(c + d*x))^n))^2,x)
```

```
[Out] int((A + B*log(e*((a + b*x)/(c + d*x))^n))^2, x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \left(A + B \log{\left(e \left(\frac{a + bx}{c + dx}\right)^{n} \right)}\right)^{2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*ln(e*((b*x+a)/(d*x+c))**n))**2,x)
```

```
[Out] Integral((A + B*log(e*((a + b*x)/(c + d*x))**n))**2, x)
```

## 3.71 $\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{f+gx}\,dx$

**Optimal**. Leaf size=297

$$\frac{2Bn\mathrm{Li}_2\left(\frac{(df-cg)(a+bx)}{(bf-ag)(c+dx)}\right)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{g}+\frac{\log\left(1-\frac{(a+bx)(df-cg)}{(c+dx)(bf-ag)}\right)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{g}-\frac{2Bn\mathrm{Li}_2\left(\frac{d(a+bx)}{b(c+dx)}\right)}{}$$

```
[Out] -(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2*ln((-a*d+b*c)/b/(d*x+c))/g+(A+B*ln(e*((b
*x+a)/(d*x+c))^n))^2*ln(1-(-c*g+d*f)*(b*x+a)/(-a*g+b*f)/(d*x+c))/g-2*B*n*(A
+B*ln(e*((b*x+a)/(d*x+c))^n))*polylog(2,d*(b*x+a)/b/(d*x+c))/g+2*B*n*(A+B*l
n(e*((b*x+a)/(d*x+c))^n))*polylog(2,(-c*g+d*f)*(b*x+a)/(-a*g+b*f)/(d*x+c))/
g+2*B^2*n^2*polylog(3,d*(b*x+a)/b/(d*x+c))/g-2*B^2*n^2*polylog(3,(-c*g+d*f)
*(b*x+a)/(-a*g+b*f)/(d*x+c))/g
```

**Rubi [B]** time = 5.18, antiderivative size = 2233, normalized size of antiderivative = 7.52, number of steps used = 43, number of rules used = 21, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.656, Rules used = {2524, 2528, 2418, 2390, 2301, 2394, 2393, 2391, 6688, 12, 6742, 2500, 2433, 2375, 2317, 2374, 6589, 2440, 2437, 2435, 2315}

result too large to display

Antiderivative was successfully verified.

```
[In] Int[(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2/(f + g*x),x]
```

```
[Out] (-2*A*B*n*Log[-((g*(a + b*x))/(b*f - a*g))]*Log[f + g*x])/g - (B^2*Log[(a +
 b*x)^n]^2*Log[f + g*x])/g + ((A + B*Log[e*((a + b*x)/(c + d*x))^n])^2*Log[
f + g*x])/g + (2*B^2*n^2*Log[-((d*(a + b*x))/(b*c - a*d))]*Log[c + d*x]*Log
[f + g*x])/g + (2*B^2*n^2*Log[a + b*x]*Log[(b*(c + d*x))/(b*c - a*d)]*Log[f
 + g*x])/g + (2*A*B*n*Log[-((g*(c + d*x))/(d*f - c*g))]*Log[f + g*x])/g - (
2*B^2*n*(n*Log[a + b*x] - Log[(a + b*x)^n])*Log[-((g*(c + d*x))/(d*f - c*g)
)]*Log[f + g*x])/g - (B^2*Log[(c + d*x)^(-n)]^2*Log[f + g*x])/g + (2*B^2*n*
Log[-((g*(a + b*x))/(b*f - a*g))]*(Log[(a + b*x)^n] - Log[e*((a + b*x)/(c +
 d*x))^n] + Log[(c + d*x)^(-n)])*Log[f + g*x])/g - (2*B^2*n*Log[-((g*(c + d
*x))/(d*f - c*g))]*(Log[(a + b*x)^n] - Log[e*((a + b*x)/(c + d*x))^n] + Log
[(c + d*x)^(-n)])*Log[f + g*x])/g - (2*B^2*n*Log[-((g*(a + b*x))/(b*f - a*g
))]*(n*Log[c + d*x] + Log[(c + d*x)^(-n)])*Log[f + g*x])/g + (B^2*Log[(a +
b*x)^n]^2*Log[(b*(f + g*x))/(b*f - a*g)])/g + (B^2*Log[(c + d*x)^(-n)]^2*Lo
g[(d*(f + g*x))/(d*f - c*g)])/g + (B^2*n^2*(Log[(b*(c + d*x))/(b*c - a*d)]
+ Log[(b*f - a*g)/(b*(f + g*x))] - Log[((b*f - a*g)*(c + d*x))/((b*c - a*d)
*(f + g*x))])*Log[-(((b*c - a*d)*(f + g*x))/((d*f - c*g)*(a + b*x)))]^2)/g
```

```
- (B^2*n^2*(Log[(b*(c + d*x))/(b*c - a*d)] - Log[-((g*(c + d*x))/(d*f - c*g
))])*(Log[a + b*x] + Log[-(((b*c - a*d)*(f + g*x))/((d*f - c*g)*(a + b*x)))
])^2)/g + (B^2*n^2*(Log[-((d*(a + b*x))/(b*c - a*d))] + Log[(d*f - c*g)/(d*
(f + g*x))] - Log[-(((d*f - c*g)*(a + b*x))/((b*c - a*d)*(f + g*x)))])*Log[
((b*c - a*d)*(f + g*x))/((b*f - a*g)*(c + d*x))]^2)/g - (B^2*n^2*(Log[-((d*
(a + b*x))/(b*c - a*d))] - Log[-((g*(a + b*x))/(b*f - a*g))])*(Log[c + d*x]
 + Log[((b*c - a*d)*(f + g*x))/((b*f - a*g)*(c + d*x))])^2)/g + (2*B^2*n^2*
(Log[f + g*x] - Log[-(((b*c - a*d)*(f + g*x))/((d*f - c*g)*(a + b*x)))])*Po
lyLog[2, -((d*(a + b*x))/(b*c - a*d))])/g + (2*B^2*n*Log[(a + b*x)^n]*PolyL
og[2, -((g*(a + b*x))/(b*f - a*g))])/g + (2*B^2*n^2*(Log[f + g*x] - Log[((b
*c - a*d)*(f + g*x))/((b*f - a*g)*(c + d*x))])*PolyLog[2, (b*(c + d*x))/(b*
c - a*d)])/g - (2*B^2*n*Log[(c + d*x)^(-n)]*PolyLog[2, -((g*(c + d*x))/(d*f
 - c*g))])/g - (2*B^2*n^2*Log[-(((b*c - a*d)*(f + g*x))/((d*f - c*g)*(a + b
*x)))]*PolyLog[2, (g*(a + b*x))/(b*(f + g*x))])/g + (2*B^2*n^2*Log[-(((b*c
- a*d)*(f + g*x))/((d*f - c*g)*(a + b*x)))]*PolyLog[2, -(((d*f - c*g)*(a +
b*x))/((b*c - a*d)*(f + g*x)))])/g - (2*B^2*n^2*Log[((b*c - a*d)*(f + g*x))
/((b*f - a*g)*(c + d*x))]*PolyLog[2, (g*(c + d*x))/(d*(f + g*x))])/g + (2*B
^2*n^2*Log[((b*c - a*d)*(f + g*x))/((b*f - a*g)*(c + d*x))]*PolyLog[2, ((b*
f - a*g)*(c + d*x))/((b*c - a*d)*(f + g*x))])/g - (2*A*B*n*PolyLog[2, (b*(f
 + g*x))/(b*f - a*g)])/g + (2*B^2*n*(Log[(a + b*x)^n] - Log[e*((a + b*x)/(c
 + d*x))^n] + Log[(c + d*x)^(-n)])*PolyLog[2, (b*(f + g*x))/(b*f - a*g)])/g
 - (2*B^2*n*(n*Log[c + d*x] + Log[(c + d*x)^(-n)])*PolyLog[2, (b*(f + g*x))
/(b*f - a*g)])/g + (2*B^2*n^2*(Log[c + d*x] + Log[((b*c - a*d)*(f + g*x))/(
(b*f - a*g)*(c + d*x))])*PolyLog[2, (b*(f + g*x))/(b*f - a*g)])/g + (2*A*B*
n*PolyLog[2, (d*(f + g*x))/(d*f - c*g)])/g - (2*B^2*n*(n*Log[a + b*x] - Log
[(a + b*x)^n])*PolyLog[2, (d*(f + g*x))/(d*f - c*g)])/g - (2*B^2*n*(Log[(a
+ b*x)^n] - Log[e*((a + b*x)/(c + d*x))^n] + Log[(c + d*x)^(-n)])*PolyLog[2
, (d*(f + g*x))/(d*f - c*g)])/g + (2*B^2*n^2*(Log[a + b*x] + Log[-(((b*c -
a*d)*(f + g*x))/((d*f - c*g)*(a + b*x)))])*PolyLog[2, (d*(f + g*x))/(d*f -
c*g)])/g - (2*B^2*n^2*PolyLog[3, -((d*(a + b*x))/(b*c - a*d))])/g - (2*B^2*
n^2*PolyLog[3, -((g*(a + b*x))/(b*f - a*g))])/g - (2*B^2*n^2*PolyLog[3, (b*
(c + d*x))/(b*c - a*d)])/g - (2*B^2*n^2*PolyLog[3, -((g*(c + d*x))/(d*f - c
*g))])/g - (2*B^2*n^2*PolyLog[3, (g*(a + b*x))/(b*(f + g*x))])/g + (2*B^2*n
^2*PolyLog[3, -(((d*f - c*g)*(a + b*x))/((b*c - a*d)*(f + g*x)))])/g - (2*B
^2*n^2*PolyLog[3, (g*(c + d*x))/(d*(f + g*x))])/g + (2*B^2*n^2*PolyLog[3, (
(b*f - a*g)*(c + d*x))/((b*c - a*d)*(f + g*x))])/g - (2*B^2*n^2*PolyLog[3,
(b*(f + g*x))/(b*f - a*g)])/g - (2*B^2*n^2*PolyLog[3, (d*(f + g*x))/(d*f -
c*g)])/g
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2301

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2317

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symb
ol] :> Simp[(Log[1 + (e*x)/d]*(a + b*Log[c*x^n])^p)/e, x] - Dist[(b*n*p)/e,
 Int[(Log[1 + (e*x)/d]*(a + b*Log[c*x^n])^(p - 1))/x, x], x] /; FreeQ[{a, b
, c, d, e, n}, x] && IGtQ[p, 0]
```

Rule 2374

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> -Simp[(PolyLog[2, -(d*f*x^m)]*(a + b*Log[c*x
^n])^p)/m, x] + Dist[(b*n*p)/m, Int[(PolyLog[2, -(d*f*x^m)]*(a + b*Log[c*x^
n])^(p - 1))/x, x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p, 0]
&& EqQ[d*e, 1]
```

Rule 2375

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))^(r_.)]*((a_.) + Log[(c_.)*(x_)^(n_
.)]*(b_.))^(p_.))/(x_), x_Symbol] :> Simp[(Log[d*(e + f*x^m)^r]*(a + b*Log[
c*x^n])^(p + 1))/(b*n*(p + 1)), x] - Dist[(f*m*r)/(b*n*(p + 1)), Int[(x^(m
- 1)*(a + b*Log[c*x^n])^(p + 1))/(e + f*x^m), x], x] /; FreeQ[{a, b, c, d,
e, f, r, m, n}, x] && IGtQ[p, 0] && NeQ[d*e, 1]
```

Rule 2390

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_))^(q_.), x_Symbol] :> Dist[1/e, Subst[Int[((f*x)/d)^q*(a + b*Log[c*x^
n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && E
qQ[e*f - d*g, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2393

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + (c*e*x)/g])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2394

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d + e*x
)^n]))/g, x] - Dist[(b*e*n)/g, Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2418

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*(RFx_), x_Sy
mbol] :> With[{u = ExpandIntegrand[(a + b*Log[c*(d + e*x)^n])^p, RFx, x]},
Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d, e, n}, x] && RationalFunctionQ[
RFx, x] && IntegerQ[p]
```

Rule 2433

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + Log
[(h_.)*((i_.) + (j_.)*(x_))^(m_.)]*(g_.))*((k_.) + (l_.)*(x_))^(r_.), x_Sym
bol] :> Dist[1/e, Subst[Int[((k*x)/d)^r*(a + b*Log[c*x^n])^p*(f + g*Log[h*(
(e*i - d*j)/e + (j*x)/e)^m]), x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e,
f, g, h, i, j, k, l, n, p, r}, x] && EqQ[e*k - d*l, 0]
```

Rule 2435

```
Int[(Log[(a_) + (b_.)*(x_)]*Log[(c_) + (d_.)*(x_)])/(x_), x_Symbol] :> Simp
[Log[-((b*x)/a)]*Log[a + b*x]*Log[c + d*x], x] + (Simp[(1*(Log[-((b*x)/a)]
- Log[-(((b*c - a*d)*x)/(a*(c + d*x)))] + Log[(b*c - a*d)/(b*(c + d*x))])*L
og[(a*(c + d*x))/(c*(a + b*x))]^2)/2, x] - Simp[(1*(Log[-((b*x)/a)] - Log[-
((d*x)/c)])*(Log[a + b*x] + Log[(a*(c + d*x))/(c*(a + b*x))])^2)/2, x] + Si
mp[(Log[c + d*x] - Log[(a*(c + d*x))/(c*(a + b*x))])*PolyLog[2, 1 + (b*x)/a
], x] + Simp[(Log[a + b*x] + Log[(a*(c + d*x))/(c*(a + b*x))])*PolyLog[2, 1
 + (d*x)/c], x] + Simp[Log[(a*(c + d*x))/(c*(a + b*x))]*PolyLog[2, (c*(a +
b*x))/(a*(c + d*x))], x] - Simp[Log[(a*(c + d*x))/(c*(a + b*x))]*PolyLog[2,
 (d*(a + b*x))/(b*(c + d*x))], x] - Simp[PolyLog[3, 1 + (b*x)/a], x] - Simp
[PolyLog[3, 1 + (d*x)/c], x] + Simp[PolyLog[3, (c*(a + b*x))/(a*(c + d*x))]
, x] - Simp[PolyLog[3, (d*(a + b*x))/(b*(c + d*x))], x]) /; FreeQ[{a, b, c,
 d}, x] && NeQ[b*c - a*d, 0]
```

Rule 2437

```
Int[(Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*Log[(h_.)*((i_.) + (j_.)*(x_))^(m
_.)])/(x_), x_Symbol] :> Dist[m, Int[(Log[i + j*x]*Log[c*(d + e*x)^n])/x, x
], x] - Dist[m*Log[i + j*x] - Log[h*(i + j*x)^m], Int[Log[c*(d + e*x)^n]/x,
 x], x] /; FreeQ[{c, d, e, h, i, j, m, n}, x] && NeQ[e*i - d*j, 0] && NeQ[i
 + j*x, h*(i + j*x)^m]
```

Rule 2440

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + Log[(h_.)
*((i_.) + (j_.)*(x_))^(m_.)]*(g_.))*((k_) + (l_.)*(x_))^(r_.), x_Symbol] :>
 Dist[1/l, Subst[Int[x^r*(a + b*Log[c*(-((e*k - d*l)/l) + (e*x)/l)^n])*(f +
 g*Log[h*(-((j*k - i*l)/l) + (j*x)/l)^m]), x], x, k + l*x], x] /; FreeQ[{a,
 b, c, d, e, f, g, h, i, j, k, l, m, n}, x] && IntegerQ[r]
```

Rule 2500

```
Int[(Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.)
)^(r_.)]*((s_.) + Log[(i_.)*((g_.) + (h_.)*(x_))^(n_.)]*(t_.)))/((j_.) + (k
_.)*(x_)), x_Symbol] :> Dist[Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r] - Log[(a
+ b*x)^(p*r)] - Log[(c + d*x)^(q*r)], Int[(s + t*Log[i*(g + h*x)^n])/(j + k
*x), x], x] + (Int[(Log[(a + b*x)^(p*r)]*(s + t*Log[i*(g + h*x)^n]))/(j + k
*x), x] + Int[(Log[(c + d*x)^(q*r)]*(s + t*Log[i*(g + h*x)^n]))/(j + k*x),
x]) /; FreeQ[{a, b, c, d, e, f, g, h, i, j, k, s, t, n, p, q, r}, x] && NeQ
[b*c - a*d, 0]
```

Rule 2524

```
Int[((a_.) + Log[(c_.)*(RFx_)^(p_.)]*(b_.))^(n_.)/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[(Log[d + e*x]*(a + b*Log[c*RFx^p])^n)/e, x] - Dist[(b*n*p)/e
, Int[(Log[d + e*x]*(a + b*Log[c*RFx^p])^(n - 1)*D[RFx, x])/RFx, x], x] /;
FreeQ[{a, b, c, d, e, p}, x] && RationalFunctionQ[RFx, x] && IGtQ[n, 0]
```

Rule 2528

```
Int[((a_.) + Log[(c_.)*(RFx_)^(p_.)]*(b_.))^(n_.)*(RGx_), x_Symbol] :> With
[{u = ExpandIntegrand[(a + b*Log[c*RFx^p])^n, RGx, x]}, Int[u, x] /; SumQ[u
]] /; FreeQ[{a, b, c, p}, x] && RationalFunctionQ[RFx, x] && RationalFuncti
onQ[RGx, x] && IGtQ[n, 0]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6688

```
Int[u_, x_Symbol] :> With[{v = SimplifyIntegrand[u, x]}, Int[v, x] /; Simpl
erIntegrandQ[v, u, x]]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{f+gx}\,dx = \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2\log(f+gx)}{g} - \frac{(2Bn)\int \frac{(c+dx)\left(-\frac{d(a+bx)}{(c+dx)^2}+\frac{b}{c+dx}\right)\left(A+B\log\left(e\left(\frac{a+}{c+}\right.\right.\right.}{a+bx}}{g}$$

$$= \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2\log(f+gx)}{g} - \frac{(2Bn)\int \frac{(bc-ad)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\log(f+gx}{(a+bx)(c+dx)}}{g}$$

$$= \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2\log(f+gx)}{g} - \frac{(2B(bc-ad)n)\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\log(f}{(a+bx)(c+dx)}}{g}$$

$$= \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2\log(f+gx)}{g} - \frac{(2B(bc-ad)n)\int\left(\frac{b\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\log}{(bc-ad)(a+bx)}\right.}{}$$

$$= \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2\log(f+gx)}{g} - \frac{(2bBn)\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\log(f+gx)}{a+bx}\,dx}{g}$$

$$= \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2\log(f+gx)}{g} - \frac{(2bBn)\int\left(\frac{A\log(f+gx)}{a+bx}+\frac{B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\log}{a+bx}\right.}{g}$$

$$= \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2\log(f+gx)}{g} - \frac{(2AbBn)\int\frac{\log(f+gx)}{a+bx}\,dx}{g} - \frac{\left(2bB^2n\right)\int -\frac{\mathrm{lo}}{}}{}$$

$$= -\frac{2ABn\log\left(-\frac{g(a+bx)}{bf-ag}\right)\log(f+gx)}{g} + \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2\log(f+gx)}{g} + \frac{2}{}$$

$$= -\frac{2ABn\log\left(-\frac{g(a+bx)}{bf-ag}\right)\log(f+gx)}{g} + \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2\log(f+gx)}{g} + \frac{2}{}$$

$$= -\frac{2ABn\log\left(-\frac{g(a+bx)}{bf-ag}\right)\log(f+gx)}{g} - \frac{B^2\log^2\left((a+bx)^n\right)\log(f+gx)}{g} + \frac{\left(A+B\right.}{}$$

$$= -\frac{2ABn\log\left(-\frac{g(a+bx)}{bf-ag}\right)\log(f+gx)}{g} - \frac{B^2\log^2\left((a+bx)^n\right)\log(f+gx)}{g} + \frac{\left(A+B\right.}{}$$

$$= -\frac{2ABn\log\left(-\frac{g(a+bx)}{bf-ag}\right)\log(f+gx)}{g} - \frac{B^2\log^2\left((a+bx)^n\right)\log(f+gx)}{g} + \frac{\left(A+B\right.}{}$$

**Mathematica [B]** time = 0.46, size = 1441, normalized size = 4.85

result too large to display

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2/(f + g*x),x]
```

```
[Out] (-(B^2*n^2*Log[(-(b*c) + a*d)/(d*(a + b*x))]*Log[((b*f - a*g)*(c + d*x))/((
d*f - c*g)*(a + b*x))]^2) + A^2*Log[f + g*x] - 2*A*B*n*Log[a/b + x]*Log[f +
 g*x] + B^2*n^2*Log[a/b + x]^2*Log[f + g*x] + 2*A*B*n*Log[c/d + x]*Log[f +
g*x] - 2*B^2*n^2*Log[a/b + x]*Log[c/d + x]*Log[f + g*x] + B^2*n^2*Log[c/d +
 x]^2*Log[f + g*x] + 2*A*B*Log[e*((a + b*x)/(c + d*x))^n]*Log[f + g*x] - 2*
B^2*n*Log[a/b + x]*Log[e*((a + b*x)/(c + d*x))^n]*Log[f + g*x] + 2*B^2*n*Lo
g[c/d + x]*Log[e*((a + b*x)/(c + d*x))^n]*Log[f + g*x] + B^2*Log[e*((a + b*
x)/(c + d*x))^n]^2*Log[f + g*x] + 2*A*B*n*Log[a/b + x]*Log[(b*(f + g*x))/(b
*f - a*g)] - B^2*n^2*Log[a/b + x]^2*Log[(b*(f + g*x))/(b*f - a*g)] + 2*B^2*
n*Log[a/b + x]*Log[e*((a + b*x)/(c + d*x))^n]*Log[(b*(f + g*x))/(b*f - a*g)
] + 2*B^2*n^2*Log[a/b + x]*Log[(g*(c + d*x))/(-(d*f) + c*g)]*Log[(b*(f + g*
x))/(b*f - a*g)] - B^2*n^2*Log[(g*(c + d*x))/(-(d*f) + c*g)]^2*Log[(b*(f +
g*x))/(b*f - a*g)] + 2*B^2*n^2*Log[(g*(c + d*x))/(-(d*f) + c*g)]*Log[((b*f
- a*g)*(c + d*x))/((d*f - c*g)*(a + b*x))]*Log[(b*(f + g*x))/(b*f - a*g)] -
 B^2*n^2*Log[((b*f - a*g)*(c + d*x))/((d*f - c*g)*(a + b*x))]^2*Log[(b*(f +
 g*x))/(b*f - a*g)] - 2*A*B*n*Log[c/d + x]*Log[(d*(f + g*x))/(d*f - c*g)] +
 2*B^2*n^2*Log[a/b + x]*Log[c/d + x]*Log[(d*(f + g*x))/(d*f - c*g)] - B^2*n
^2*Log[c/d + x]^2*Log[(d*(f + g*x))/(d*f - c*g)] - 2*B^2*n*Log[c/d + x]*Log
[e*((a + b*x)/(c + d*x))^n]*Log[(d*(f + g*x))/(d*f - c*g)] - 2*B^2*n^2*Log[
a/b + x]*Log[(g*(c + d*x))/(-(d*f) + c*g)]*Log[(d*(f + g*x))/(d*f - c*g)] +
 B^2*n^2*Log[(g*(c + d*x))/(-(d*f) + c*g)]^2*Log[(d*(f + g*x))/(d*f - c*g)]
 - 2*B^2*n^2*Log[(g*(c + d*x))/(-(d*f) + c*g)]*Log[((b*f - a*g)*(c + d*x))/
((d*f - c*g)*(a + b*x))]*Log[(d*(f + g*x))/(d*f - c*g)] + B^2*n^2*Log[((b*f
 - a*g)*(c + d*x))/((d*f - c*g)*(a + b*x))]^2*Log[((-(b*c) + a*d)*(f + g*x)
)/((d*f - c*g)*(a + b*x))] + 2*B*n*(A + B*Log[e*((a + b*x)/(c + d*x))^n] +
B*n*Log[((b*f - a*g)*(c + d*x))/((d*f - c*g)*(a + b*x))])*PolyLog[2, (g*(a
+ b*x))/(-(b*f) + a*g)] - 2*B*n*(A + B*Log[e*((a + b*x)/(c + d*x))^n] + B*n
*Log[((b*f - a*g)*(c + d*x))/((d*f - c*g)*(a + b*x))])*PolyLog[2, (g*(c + d
*x))/(-(d*f) + c*g)] - 2*B^2*n^2*Log[((b*f - a*g)*(c + d*x))/((d*f - c*g)*(
a + b*x))]*PolyLog[2, (b*(c + d*x))/(d*(a + b*x))] + 2*B^2*n^2*Log[((b*f -
a*g)*(c + d*x))/((d*f - c*g)*(a + b*x))]*PolyLog[2, ((b*f - a*g)*(c + d*x))
/((d*f - c*g)*(a + b*x))] + 2*B^2*n^2*PolyLog[3, (b*(c + d*x))/(d*(a + b*x)
)] - 2*B^2*n^2*PolyLog[3, ((b*f - a*g)*(c + d*x))/((d*f - c*g)*(a + b*x))])
/g
```

**fricas [F]** time = 1.23, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{B^2\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)^2+2\,AB\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A^2}{gx+f},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))^2/(g*x+f),x, algorithm="fricas")
```

```
[Out] integral((B^2*log(e*((b*x + a)/(d*x + c))^n)^2 + 2*A*B*log(e*((b*x + a)/(d*
x + c))^n) + A^2)/(g*x + f), x)
```

**giac [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))^2/(g*x+f),x, algorithm="giac")
```

```
[Out] Timed out
```

**maple [F]** time = 0.31, size = 0, normalized size = 0.00

$$\int\frac{\left(B\ln\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A\right)^2}{gx+f}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*ln(e*((b*x+a)/(d*x+c))^n)+A)^2/(g*x+f),x)
```

```
[Out] int((B*ln(e*((b*x+a)/(d*x+c))^n)+A)^2/(g*x+f),x)
```

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$\frac{A^2\log\left(gx+f\right)}{g}+\int\frac{B^2\log\left((bx+a)^n\right)^2+B^2\log\left((dx+c)^n\right)^2+B^2\log(e)^2+2\,AB\log(e)+2\left(B^2\log(e)+AB\right)\text{lo}}{gx+f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))^2/(g*x+f),x, algorithm="maxima")
```

```
[Out] A^2*log(g*x + f)/g + integrate((B^2*log((b*x + a)^n)^2 + B^2*log((d*x + c)^
n)^2 + B^2*log(e)^2 + 2*A*B*log(e) + 2*(B^2*log(e) + A*B)*log((b*x + a)^n)
- 2*(B^2*log((b*x + a)^n) + B^2*log(e) + A*B)*log((d*x + c)^n))/(g*x + f),
x)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(A+B\,\ln\left(e\,{\left(\frac{a+b\,x}{c+d\,x}\right)}^n\right)\right)^2}{f+g\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*log(e*((a + b*x)/(c + d*x))^n))^2/(f + g*x),x)
```

```
[Out] int((A + B*log(e*((a + b*x)/(c + d*x))^n))^2/(f + g*x), x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(A+B\log\left(e\left(\frac{a}{c+dx}+\frac{bx}{c+dx}\right)^n\right)\right)^2}{f+gx}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*ln(e*((b*x+a)/(d*x+c))**n))**2/(g*x+f),x)
```

```
[Out] Integral((A + B*log(e*(a/(c + d*x) + b*x/(c + d*x))**n))**2/(f + g*x), x)
```

## 3.72 $\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(f+gx)^2}\,dx$

**Optimal**. Leaf size=206

$$\frac{2Bn(bc-ad)\log\left(1-\frac{(a+bx)(df-cg)}{(c+dx)(bf-ag)}\right)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{(bf-ag)(df-cg)}+\frac{(a+bx)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{(f+gx)(bf-ag)}+\frac{2B^2n^2(bc-ad)\mathrm{L}}{(bf-ag)}$$

```
[Out] (b*x+a)*(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2/(-a*g+b*f)/(g*x+f)+2*B*(-a*d+b*c)
*n*(A+B*ln(e*((b*x+a)/(d*x+c))^n))*ln(1-(-c*g+d*f)*(b*x+a)/(-a*g+b*f)/(d*x+
c))/(-a*g+b*f)/(-c*g+d*f)+2*B^2*(-a*d+b*c)*n^2*polylog(2,(-c*g+d*f)*(b*x+a)
/(-a*g+b*f)/(d*x+c))/(-a*g+b*f)/(-c*g+d*f)
```

**Rubi [B]** time = 1.13, antiderivative size = 657, normalized size of antiderivative = 3.19, number of steps used = 29, number of rules used = 10, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.312, Rules used = {2525, 12, 2528, 2524, 2418, 2390, 2301, 2394, 2393, 2391}

$$\frac{2bB^2n^2\text{PolyLog}\left(2,-\frac{d(a+bx)}{bc-ad}\right)}{g(bf-ag)}+\frac{2B^2dn^2\text{PolyLog}\left(2,\frac{b(c+dx)}{bc-ad}\right)}{g(df-cg)}-\frac{2B^2n^2(bc-ad)\text{PolyLog}\left(2,\frac{b(f+gx)}{bf-ag}\right)}{(bf-ag)(df-cg)}+\frac{2B^2n^2(bc-ad}{(bf-}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2/(f + g*x)^2,x]
```

```
[Out] -((b*B^2*n^2*Log[a + b*x]^2)/(g*(b*f - a*g))) + (2*b*B*n*Log[a + b*x]*(A +
B*Log[e*((a + b*x)/(c + d*x))^n]))/(g*(b*f - a*g)) - (A + B*Log[e*((a + b*x
)/(c + d*x))^n])^2/(g*(f + g*x)) + (2*B^2*d*n^2*Log[-((d*(a + b*x))/(b*c -
a*d))]*Log[c + d*x])/(g*(d*f - c*g)) - (2*B*d*n*(A + B*Log[e*((a + b*x)/(c
+ d*x))^n])*Log[c + d*x])/(g*(d*f - c*g)) - (B^2*d*n^2*Log[c + d*x]^2)/(g*(
d*f - c*g)) + (2*b*B^2*n^2*Log[a + b*x]*Log[(b*(c + d*x))/(b*c - a*d)])/(g*
(b*f - a*g)) - (2*B^2*(b*c - a*d)*n^2*Log[-((g*(a + b*x))/(b*f - a*g))]*Log
[f + g*x])/((b*f - a*g)*(d*f - c*g)) + (2*B*(b*c - a*d)*n*(A + B*Log[e*((a
+ b*x)/(c + d*x))^n])*Log[f + g*x])/((b*f - a*g)*(d*f - c*g)) + (2*B^2*(b*c
 - a*d)*n^2*Log[-((g*(c + d*x))/(d*f - c*g))]*Log[f + g*x])/((b*f - a*g)*(d
*f - c*g)) + (2*b*B^2*n^2*PolyLog[2, -((d*(a + b*x))/(b*c - a*d))])/(g*(b*f
 - a*g)) + (2*B^2*d*n^2*PolyLog[2, (b*(c + d*x))/(b*c - a*d)])/(g*(d*f - c*
g)) - (2*B^2*(b*c - a*d)*n^2*PolyLog[2, (b*(f + g*x))/(b*f - a*g)])/((b*f -
 a*g)*(d*f - c*g)) + (2*B^2*(b*c - a*d)*n^2*PolyLog[2, (d*(f + g*x))/(d*f -
 c*g)])/((b*f - a*g)*(d*f - c*g))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2301

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

Rule 2390

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_))^(q_.), x_Symbol] :> Dist[1/e, Subst[Int[((f*x)/d)^q*(a + b*Log[c*x^
n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && E
qQ[e*f - d*g, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2393

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + (c*e*x)/g])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2394

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d + e*x
)^n]))/g, x] - Dist[(b*e*n)/g, Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2418

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*(RFx_), x_Sy
mbol] :> With[{u = ExpandIntegrand[(a + b*Log[c*(d + e*x)^n])^p, RFx, x]},
Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d, e, n}, x] && RationalFunctionQ[
RFx, x] && IntegerQ[p]
```

Rule 2524

```
Int[((a_.) + Log[(c_.)*(RFx_)^(p_.)]*(b_.))^(n_.)/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[(Log[d + e*x]*(a + b*Log[c*RFx^p])^n)/e, x] - Dist[(b*n*p)/e
```

```
, Int[(Log[d + e*x]*(a + b*Log[c*RFx^p])^(n - 1)*D[RFx, x])/RFx, x], x] /;
FreeQ[{a, b, c, d, e, p}, x] && RationalFunctionQ[RFx, x] && IGtQ[n, 0]
```

Rule 2525

```
Int[((a_.) + Log[(c_.)*(RFx_)^(p_.)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_))^(m_.
), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(a + b*Log[c*RFx^p])^n)/(e*(m + 1))
, x] - Dist[(b*n*p)/(e*(m + 1)), Int[SimplifyIntegrand[((d + e*x)^(m + 1)*(
a + b*Log[c*RFx^p])^(n - 1)*D[RFx, x])/RFx, x], x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && RationalFunctionQ[RFx, x] && IGtQ[n, 0] && (EqQ[n, 1] ||
IntegerQ[m]) && NeQ[m, -1]
```

Rule 2528

```
Int[((a_.) + Log[(c_.)*(RFx_)^(p_.)]*(b_.))^(n_.)*(RGx_), x_Symbol] :> With
[{u = ExpandIntegrand[(a + b*Log[c*RFx^p])^n, RGx, x]}, Int[u, x] /; SumQ[u
]] /; FreeQ[{a, b, c, p}, x] && RationalFunctionQ[RFx, x] && RationalFuncti
onQ[RGx, x] && IGtQ[n, 0]
```

Rubi steps

$$\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(f+gx)^2}\,dx = -\frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{g(f+gx)} + \frac{(2Bn)\int \frac{(bc-ad)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(a+bx)(c+dx)(f+gx)}\,dx}{g}$$

$$= -\frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{g(f+gx)} + \frac{(2B(bc-ad)n)\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(a+bx)(c+dx)(f+gx)}\,dx}{g}$$

$$= -\frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{g(f+gx)} + \frac{(2B(bc-ad)n)\int \left(\frac{b^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(bc-ad)(bf-ag)(a+bx)} + \frac{d^2\left(A+B\,\mathrm{l}\right.}{(bc-ad)(\cdots}\right.}{g}$$

$$= -\frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{g(f+gx)} + \frac{\left(2b^2Bn\right)\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{a+bx}\,dx}{g(bf-ag)} - \frac{\left(2Bd^2n\right)\int \frac{A+B}{\cdots}}{g(df}$$

$$= \frac{2bBn\log(a+bx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{g(bf-ag)} - \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{g(f+gx)} - \frac{2Bd}{\cdots}$$

$$= \frac{2bBn\log(a+bx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{g(bf-ag)} - \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{g(f+gx)} - \frac{2Bd}{\cdots}$$

$$= \frac{2bBn\log(a+bx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{g(bf-ag)} - \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{g(f+gx)} - \frac{2Bd}{\cdots}$$

$$= \frac{2bBn\log(a+bx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{g(bf-ag)} - \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{g(f+gx)} + \frac{2B^2}{\cdots}$$

$$= -\frac{bB^2n^2\log^2(a+bx)}{g(bf-ag)} + \frac{2bBn\log(a+bx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{g(bf-ag)} - \frac{\left(A+B\,\mathrm{l}\right.}{g}$$

$$= -\frac{bB^2n^2\log^2(a+bx)}{g(bf-ag)} + \frac{2bBn\log(a+bx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{g(bf-ag)} - \frac{\left(A+B\,\mathrm{l}\right.}{g}$$

**Mathematica [B]** time = 0.52, size = 418, normalized size = 2.03

$$\frac{Bn\left(2b\log(a+bx)(df-cg)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^{n}\right)+A\right)-2d(bf-ag)\log(c+dx)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^{n}\right)+A\right)+2g(bc-ad)\log(f+gx)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^{n}\right)+A\right)-bBn(df-cg)\left(\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2/(f + g*x)^2,x]
```

```
[Out] (-((A + B*Log[e*((a + b*x)/(c + d*x))^n])^2/(f + g*x)) + (B*n*(2*b*(d*f - c
*g)*Log[a + b*x]*(A + B*Log[e*((a + b*x)/(c + d*x))^n]) - 2*d*(b*f - a*g)*(
A + B*Log[e*((a + b*x)/(c + d*x))^n])*Log[c + d*x] + 2*(b*c - a*d)*g*(A + B
*Log[e*((a + b*x)/(c + d*x))^n])*Log[f + g*x] - b*B*(d*f - c*g)*n*(Log[a +
b*x]*(Log[a + b*x] - 2*Log[(b*(c + d*x))/(b*c - a*d)]) - 2*PolyLog[2, (d*(a
 + b*x))/(-(b*c) + a*d)]) + B*d*(b*f - a*g)*n*((2*Log[(d*(a + b*x))/(-(b*c)
 + a*d)] - Log[c + d*x])*Log[c + d*x] + 2*PolyLog[2, (b*(c + d*x))/(b*c - a
*d)]) - 2*B*(b*c - a*d)*g*n*((Log[(g*(a + b*x))/(-(b*f) + a*g)] - Log[(g*(c
 + d*x))/(-(d*f) + c*g)])*Log[f + g*x] + PolyLog[2, (b*(f + g*x))/(b*f - a*
g)] - PolyLog[2, (d*(f + g*x))/(d*f - c*g)])))/((b*f - a*g)*(d*f - c*g)))/g
```

**fricas [F]** time = 1.41, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{B^{2}\log\left(e\left(\frac{bx+a}{dx+c}\right)^{n}\right)^{2}+2\,AB\log\left(e\left(\frac{bx+a}{dx+c}\right)^{n}\right)+A^{2}}{g^{2}x^{2}+2\,fgx+f^{2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))^2/(g*x+f)^2,x, algorithm="fricas
")
```

```
[Out] integral((B^2*log(e*((b*x + a)/(d*x + c))^n)^2 + 2*A*B*log(e*((b*x + a)/(d*
x + c))^n) + A^2)/(g^2*x^2 + 2*f*g*x + f^2), x)
```

**giac [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))^2/(g*x+f)^2,x, algorithm="giac")
```

```
[Out] Timed out
```

**maple [F]** time = 0.29, size = 0, normalized size = 0.00

$$\int \frac{\left(B\ln\left(e\left(\frac{bx+a}{dx+c}\right)^{n}\right)+A\right)^{2}}{\left(gx+f\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*ln(e*((b*x+a)/(d*x+c))^n)+A)^2/(g*x+f)^2,x)
[Out] int((B*ln(e*((b*x+a)/(d*x+c))^n)+A)^2/(g*x+f)^2,x)
```

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$2\,ABn\left(\frac{b\log\left(bx+a\right)}{bfg-ag^{2}}-\frac{d\log\left(dx+c\right)}{dfg-cg^{2}}+\frac{\left(bc-ad\right)\log\left(gx+f\right)}{bdf^{2}+acg^{2}-\left(bc+ad\right)fg}\right)-B^{2}\left(\frac{\log\left(\left(dx+c\right)^{n}\right)^{2}}{g^{2}x+fg}+\int-\frac{dgx\log(e)^{2}+}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))^2/(g*x+f)^2,x, algorithm="maxima
")
[Out] 2*A*B*n*(b*log(b*x + a)/(b*f*g - a*g^2) - d*log(d*x + c)/(d*f*g - c*g^2) +
(b*c - a*d)*log(g*x + f)/(b*d*f^2 + a*c*g^2 - (b*c + a*d)*f*g)) - B^2*(log(
(d*x + c)^n)^2/(g^2*x + f*g) + integrate(-(d*g*x*log(e)^2 + c*g*log(e)^2 +
(d*g*x + c*g)*log((b*x + a)^n)^2 + 2*(d*g*x*log(e) + c*g*log(e))*log((b*x +
 a)^n) + 2*(d*f*n + (g*n - g*log(e))*d*x - c*g*log(e) - (d*g*x + c*g)*log((
b*x + a)^n))*log((d*x + c)^n))/(d*g^3*x^3 + c*f^2*g + (2*d*f*g^2 + c*g^3)*x
^2 + (d*f^2*g + 2*c*f*g^2)*x), x)) - 2*A*B*log(e*(b*x/(d*x + c) + a/(d*x +
c))^n)/(g^2*x + f*g) - A^2/(g^2*x + f*g)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(A+B\,\ln\left(e\,\left(\frac{a+b\,x}{c+d\,x}\right)^{n}\right)\right)^{2}}{\left(f+g\,x\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*log(e*((a + b*x)/(c + d*x))^n))^2/(f + g*x)^2,x)
[Out] int((A + B*log(e*((a + b*x)/(c + d*x))^n))^2/(f + g*x)^2, x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(A+B\log\left(e\left(\frac{a}{c+dx}+\frac{bx}{c+dx}\right)^{n}\right)\right)^{2}}{\left(f+gx\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*ln(e*((b*x+a)/(d*x+c))**n))**2/(g*x+f)**2,x)
```

```
[Out] Integral((A + B*log(e*(a/(c + d*x) + b*x/(c + d*x))**n))**2/(f + g*x)**2, x
)
```

## 3.73 $\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(f+gx)^3}\,dx$

**Optimal**. Leaf size=389

$$\frac{b^2\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{2g(bf-ag)^2}+\frac{Bgn(a+bx)(bc-ad)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{(f+gx)(bf-ag)^2(df-cg)}+\frac{Bn(bc-ad)(-adg-bcg+2bdf)\,\mathrm{l}}{(bf-}$$

[Out] B*(-a*d+b*c)*g*n*(b*x+a)*(A+B*ln(e*((b*x+a)/(d*x+c))^n))/(-a*g+b*f)^2/(-c*g
+d*f)/(g*x+f)+1/2*b^2*(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2/g/(-a*g+b*f)^2-1/2*
(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2/g/(g*x+f)^2+B^2*(-a*d+b*c)^2*g*n^2*ln((g*
x+f)/(d*x+c))/(-a*g+b*f)^2/(-c*g+d*f)^2+B*(-a*d+b*c)*(-a*d*g-b*c*g+2*b*d*f)
*n*(A+B*ln(e*((b*x+a)/(d*x+c))^n))*ln(1-(-c*g+d*f)*(b*x+a)/(-a*g+b*f)/(d*x+
c))/(-a*g+b*f)^2/(-c*g+d*f)^2+B^2*(-a*d+b*c)*(-a*d*g-b*c*g+2*b*d*f)*n^2*pol
ylog(2,(-c*g+d*f)*(b*x+a)/(-a*g+b*f)/(d*x+c))/(-a*g+b*f)^2/(-c*g+d*f)^2

**Rubi [B]** time = 1.56, antiderivative size = 941, normalized size of antiderivative = 2.42, number of steps used = 33, number of rules used = 11, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.344, Rules used = {2525, 12, 2528, 2524, 2418, 2390, 2301, 2394, 2393, 2391, 72}

$$-\frac{B^2n^2\log^2(a+bx)b^2}{2g(bf-ag)^2}+\frac{Bn\log(a+bx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)b^2}{g(bf-ag)^2}+\frac{B^2n^2\log(a+bx)\log\left(\frac{b(c+dx)}{bc-ad}\right)b^2}{g(bf-ag)^2}+\frac{B^2n^2\mathrm{Poly}}{g}$$

Antiderivative was successfully verified.

[In] Int[(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2/(f + g*x)^3,x]

[Out] (b*B^2*(b*c - a*d)*n^2*Log[a + b*x])/((b*f - a*g)^2*(d*f - c*g)) - (b^2*B^2
*n^2*Log[a + b*x]^2)/(2*g*(b*f - a*g)^2) - (B*(b*c - a*d)*n*(A + B*Log[e*((
a + b*x)/(c + d*x))^n]))/((b*f - a*g)*(d*f - c*g)*(f + g*x)) + (b^2*B*n*Log
[a + b*x]*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/(g*(b*f - a*g)^2) - (A +
B*Log[e*((a + b*x)/(c + d*x))^n])^2/(2*g*(f + g*x)^2) - (B^2*d*(b*c - a*d)*
n^2*Log[c + d*x])/((b*f - a*g)*(d*f - c*g)^2) + (B^2*d^2*n^2*Log[-((d*(a +
b*x))/(b*c - a*d))]*Log[c + d*x])/(g*(d*f - c*g)^2) - (B*d^2*n*(A + B*Log[e
*((a + b*x)/(c + d*x))^n])*Log[c + d*x])/(g*(d*f - c*g)^2) - (B^2*d^2*n^2*L
og[c + d*x]^2)/(2*g*(d*f - c*g)^2) + (b^2*B^2*n^2*Log[a + b*x]*Log[(b*(c +
d*x))/(b*c - a*d)])/(g*(b*f - a*g)^2) + (B^2*(b*c - a*d)^2*g*n^2*Log[f + g*
x])/((b*f - a*g)^2*(d*f - c*g)^2) - (B^2*(b*c - a*d)*(2*b*d*f - b*c*g - a*d
*g)*n^2*Log[-((g*(a + b*x))/(b*f - a*g))]*Log[f + g*x])/((b*f - a*g)^2*(d*f
 - c*g)^2) + (B*(b*c - a*d)*(2*b*d*f - b*c*g - a*d*g)*n*(A + B*Log[e*((a +

```
b*x)/(c + d*x))^n])*Log[f + g*x])/((b*f - a*g)^2*(d*f - c*g)^2) + (B^2*(b*c
 - a*d)*(2*b*d*f - b*c*g - a*d*g)*n^2*Log[-((g*(c + d*x))/(d*f - c*g))]*Log
[f + g*x])/((b*f - a*g)^2*(d*f - c*g)^2) + (b^2*B^2*n^2*PolyLog[2, -((d*(a
+ b*x))/(b*c - a*d))])/(g*(b*f - a*g)^2) + (B^2*d^2*n^2*PolyLog[2, (b*(c +
d*x))/(b*c - a*d)])/(g*(d*f - c*g)^2) - (B^2*(b*c - a*d)*(2*b*d*f - b*c*g -
 a*d*g)*n^2*PolyLog[2, (b*(f + g*x))/(b*f - a*g)])/((b*f - a*g)^2*(d*f - c*
g)^2) + (B^2*(b*c - a*d)*(2*b*d*f - b*c*g - a*d*g)*n^2*PolyLog[2, (d*(f + g
*x))/(d*f - c*g)])/((b*f - a*g)^2*(d*f - c*g)^2)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 72

```
Int[((e_.) + (f_.)*(x_))^(p_.)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
 x_Symbol] :> Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x]
 /; FreeQ[{a, b, c, d, e, f}, x] && IntegerQ[p]
```

Rule 2301

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

Rule 2390

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_))^(q_.), x_Symbol] :> Dist[1/e, Subst[Int[((f*x)/d)^q*(a + b*Log[c*x^
n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && E
qQ[e*f - d*g, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2393

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + (c*e*x)/g])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2394

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d + e*x
)^n]))/g, x] - Dist[(b*e*n)/g, Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2418

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*(RFx_), x_Sy
mbol] :> With[{u = ExpandIntegrand[(a + b*Log[c*(d + e*x)^n])^p, RFx, x]},
Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d, e, n}, x] && RationalFunctionQ[
RFx, x] && IntegerQ[p]
```

Rule 2524

```
Int[((a_.) + Log[(c_.)*(RFx_)^(p_.)]*(b_.))^(n_.)/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[(Log[d + e*x]*(a + b*Log[c*RFx^p])^n)/e, x] - Dist[(b*n*p)/e
, Int[(Log[d + e*x]*(a + b*Log[c*RFx^p])^(n - 1)*D[RFx, x])/RFx, x], x] /;
FreeQ[{a, b, c, d, e, p}, x] && RationalFunctionQ[RFx, x] && IGtQ[n, 0]
```

Rule 2525

```
Int[((a_.) + Log[(c_.)*(RFx_)^(p_.)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_))^(m_.
), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(a + b*Log[c*RFx^p])^n)/(e*(m + 1))
, x] - Dist[(b*n*p)/(e*(m + 1)), Int[SimplifyIntegrand[((d + e*x)^(m + 1)*(
a + b*Log[c*RFx^p])^(n - 1)*D[RFx, x])/RFx, x], x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && RationalFunctionQ[RFx, x] && IGtQ[n, 0] && (EqQ[n, 1] ||
IntegerQ[m]) && NeQ[m, -1]
```

Rule 2528

```
Int[((a_.) + Log[(c_.)*(RFx_)^(p_.)]*(b_.))^(n_.)*(RGx_), x_Symbol] :> With
[{u = ExpandIntegrand[(a + b*Log[c*RFx^p])^n, RGx, x]}, Int[u, x] /; SumQ[u
]] /; FreeQ[{a, b, c, p}, x] && RationalFunctionQ[RFx, x] && RationalFuncti
onQ[RGx, x] && IGtQ[n, 0]
```

Rubi steps

$$\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(f+gx)^3}\,dx = -\frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{2g(f+gx)^2} + \frac{(Bn)\int \frac{(bc-ad)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(a+bx)(c+dx)(f+gx)^2}\,dx}{g}$$

$$= -\frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{2g(f+gx)^2} + \frac{(B(bc-ad)n)\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(a+bx)(c+dx)(f+gx)^2}\,dx}{g}$$

$$= -\frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{2g(f+gx)^2} + \frac{(B(bc-ad)n)\int \left(\frac{b^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(bc-ad)(bf-ag)^2(a+bx)} - \frac{d^3\left(A+B\log\left(\ldots\right.\right.}{(bc-ad)(-df+\ldots}\right.}{\ldots}$$

$$= -\frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{2g(f+gx)^2} + \frac{\left(b^3Bn\right)\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{a+bx}\,dx}{g(bf-ag)^2} - \frac{\left(Bd^3n\right)\int \frac{A+B\log\left(\ldots\right.}{c+\ldots}}{g(df-c\ldots}$$

$$= -\frac{B(bc-ad)n\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(bf-ag)(df-cg)(f+gx)} + \frac{b^2Bn\log(a+bx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right.\right.\right.}{g(bf-ag)^2}\ldots$$

$$= -\frac{B(bc-ad)n\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(bf-ag)(df-cg)(f+gx)} + \frac{b^2Bn\log(a+bx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right.\right.\right.}{g(bf-ag)^2}\ldots$$

$$= -\frac{B(bc-ad)n\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(bf-ag)(df-cg)(f+gx)} + \frac{b^2Bn\log(a+bx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right.\right.\right.}{g(bf-ag)^2}\ldots$$

$$= \frac{bB^2(bc-ad)n^2\log(a+bx)}{(bf-ag)^2(df-cg)} - \frac{B(bc-ad)n\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(bf-ag)(df-cg)(f+gx)} + \frac{b^2Bn\log\ldots}{\ldots}$$

$$= \frac{bB^2(bc-ad)n^2\log(a+bx)}{(bf-ag)^2(df-cg)} - \frac{b^2B^2n^2\log^2(a+bx)}{2g(bf-ag)^2} - \frac{B(bc-ad)n\left(A+B\log\left(\ldots\right.\right.}{(bf-ag)(df-cg)(f\ldots}$$

$$= \frac{bB^2(bc-ad)n^2\log(a+bx)}{(bf-ag)^2(df-cg)} - \frac{b^2B^2n^2\log^2(a+bx)}{2g(bf-ag)^2} - \frac{B(bc-ad)n\left(A+B\log\left(\ldots\right.\right.}{(bf-ag)(df-cg)(f\ldots}$$

**Mathematica [A]** time = 1.65, size = 615, normalized size = 1.58

$$-\frac{Bn(f+gx)\left(-2b^2(f+gx)\log(a+bx)(df-cg)^2\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)+2d^2(f+gx)(bf-ag)^2\log(c+dx)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)+2g(bc-ad)(bf-ag)(df-cg)\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2/(f + g*x)^3,x]
```

```
[Out] -1/2*((A + B*Log[e*((a + b*x)/(c + d*x))^n])^2 + (B*n*(f + g*x)*(2*(b*c - a
*d)*g*(b*f - a*g)*(d*f - c*g)*(A + B*Log[e*((a + b*x)/(c + d*x))^n]) - 2*b^
2*(d*f - c*g)^2*(f + g*x)*Log[a + b*x]*(A + B*Log[e*((a + b*x)/(c + d*x))^n
]) + 2*d^2*(b*f - a*g)^2*(f + g*x)*(A + B*Log[e*((a + b*x)/(c + d*x))^n])*L
og[c + d*x] + 2*(b*c - a*d)*g*(-2*b*d*f + b*c*g + a*d*g)*(f + g*x)*(A + B*L
og[e*((a + b*x)/(c + d*x))^n])*Log[f + g*x] - 2*B*(b*c - a*d)*g*n*(f + g*x)
*(b*(d*f - c*g)*Log[a + b*x] + (-(b*d*f) + a*d*g)*Log[c + d*x] + (b*c - a*d
)*g*Log[f + g*x]) + b^2*B*(d*f - c*g)^2*n*(f + g*x)*(Log[a + b*x]*(Log[a +
b*x] - 2*Log[(b*(c + d*x))/(b*c - a*d)]) - 2*PolyLog[2, (d*(a + b*x))/(-(b*
c) + a*d)]) - B*d^2*(b*f - a*g)^2*n*(f + g*x)*((2*Log[(d*(a + b*x))/(-(b*c)
 + a*d)] - Log[c + d*x])*Log[c + d*x] + 2*PolyLog[2, (b*(c + d*x))/(b*c - a
*d)]) - 2*B*(b*c - a*d)*g*(-2*b*d*f + b*c*g + a*d*g)*n*(f + g*x)*((Log[(g*(
a + b*x))/(-(b*f) + a*g)] - Log[(g*(c + d*x))/(-(d*f) + c*g)])*Log[f + g*x]
 + PolyLog[2, (b*(f + g*x))/(b*f - a*g)] - PolyLog[2, (d*(f + g*x))/(d*f -
c*g)])))/((b*f - a*g)^2*(d*f - c*g)^2))/(g*(f + g*x)^2)
```

**fricas [F]** time = 0.80, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{B^2\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)^2+2\,AB\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A^2}{g^3x^3+3\,fg^2x^2+3\,f^2gx+f^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))^2/(g*x+f)^3,x, algorithm="fricas
")
```

```
[Out] integral((B^2*log(e*((b*x + a)/(d*x + c))^n)^2 + 2*A*B*log(e*((b*x + a)/(d*
x + c))^n) + A^2)/(g^3*x^3 + 3*f*g^2*x^2 + 3*f^2*g*x + f^3), x)
```

**giac [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))^2/(g*x+f)^3,x, algorithm="giac")
```

```
[Out] Timed out
```

**maple [F]** time = 0.29, size = 0, normalized size = 0.00

$$\int \frac{\left(B \ln \left(e \left(\frac{bx+a}{dx+c}\right)^{n}\right)+A\right)^{2}}{\left(gx+f\right)^{3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*ln(e*((b*x+a)/(d*x+c))^n)+A)^2/(g*x+f)^3,x)
```

```
[Out] int((B*ln(e*((b*x+a)/(d*x+c))^n)+A)^2/(g*x+f)^3,x)
```

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$\left(\frac{b^2 \log\left(bx+a\right)}{b^2 f^2 g - 2\,abfg^2 + a^2 g^3} - \frac{d^2 \log\left(dx+c\right)}{d^2 f^2 g - 2\,cdfg^2 + c^2 g^3} + \frac{\left(2\left(b^2cd - abd^2\right)f - \left(b^2c^2 - a^2d^2\right)g\right) \text{lo}}{b^2d^2f^4 + a^2c^2g^4 - 2\left(b^2cd + abd^2\right)f^3g + \left(b^2c^2 + 4\,abcd + a\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))^2/(g*x+f)^3,x, algorithm="maxima
")
```

```
[Out] (b^2*log(b*x + a)/(b^2*f^2*g - 2*a*b*f*g^2 + a^2*g^3) - d^2*log(d*x + c)/(d
^2*f^2*g - 2*c*d*f*g^2 + c^2*g^3) + (2*(b^2*c*d - a*b*d^2)*f - (b^2*c^2 - a
^2*d^2)*g)*log(g*x + f)/(b^2*d^2*f^4 + a^2*c^2*g^4 - 2*(b^2*c*d + a*b*d^2)*
f^3*g + (b^2*c^2 + 4*a*b*c*d + a^2*d^2)*f^2*g^2 - 2*(a*b*c^2 + a^2*c*d)*f*g
^3) - (b*c - a*d)/(b*d*f^3 + a*c*f*g^2 - (b*c + a*d)*f^2*g + (b*d*f^2*g + a
*c*g^3 - (b*c + a*d)*f*g^2)*x))*A*B*n - 1/2*B^2*(log((d*x + c)^n)^2/(g^3*x^
2 + 2*f*g^2*x + f^2*g) + 2*integrate(-(d*g*x*log(e)^2 + c*g*log(e)^2 + (d*g
*x + c*g)*log((b*x + a)^n)^2 + 2*(d*g*x*log(e) + c*g*log(e))*log((b*x + a)^
n) + (d*f*n + (g*n - 2*g*log(e))*d*x - 2*c*g*log(e) - 2*(d*g*x + c*g)*log((
b*x + a)^n))*log((d*x + c)^n))/(d*g^4*x^4 + c*f^3*g + (3*d*f*g^3 + c*g^4)*x
^3 + 3*(d*f^2*g^2 + c*f*g^3)*x^2 + (d*f^3*g + 3*c*f^2*g^2)*x), x)) - A*B*lo
g(e*(b*x/(d*x + c) + a/(d*x + c))^n)/(g^3*x^2 + 2*f*g^2*x + f^2*g) - 1/2*A^
2/(g^3*x^2 + 2*f*g^2*x + f^2*g)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(A+B\,\ln\left(e\,{\left(\frac{a+b\,x}{c+d\,x}\right)}^n\right)\right)^2}{{\left(f+g\,x\right)}^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*log(e*((a + b*x)/(c + d*x))^n))^2/(f + g*x)^3,x)
[Out] int((A + B*log(e*((a + b*x)/(c + d*x))^n))^2/(f + g*x)^3, x)
```

**sympy [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*ln(e*((b*x+a)/(d*x+c))**n))**2/(g*x+f)**3,x)
[Out] Timed out
```

# 3.74 $\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(f+gx)^4}\,dx$

**Optimal**. Leaf size=747

$$\frac{2Bn(bc-ad)\left(a^2d^2g^2-abdg(3df-cg)+b^2\left(c^2g^2-3cdfg+3d^2f^2\right)\right)\log\left(1-\frac{(a+bx)(df-cg)}{(c+dx)(bf-ag)}\right)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+\right.}{3(bf-ag)^3(df-cg)^3}$$

```
[Out] 1/3*B^2*(-a*d+b*c)^2*g^2*n^2*(d*x+c)/(-a*g+b*f)^2/(-c*g+d*f)^3/(g*x+f)-1/3*
B*(-a*d+b*c)*g^2*n*(d*x+c)^2*(A+B*ln(e*((b*x+a)/(d*x+c))^n))/(-a*g+b*f)/(-c
*g+d*f)^3/(g*x+f)^2+2/3*B*(-a*d+b*c)*g*(-2*a*d*g-b*c*g+3*b*d*f)*n*(b*x+a)*(
A+B*ln(e*((b*x+a)/(d*x+c))^n))/(-a*g+b*f)^3/(-c*g+d*f)^2/(g*x+f)+1/3*b^3*(A
+B*ln(e*((b*x+a)/(d*x+c))^n))^2/g/(-a*g+b*f)^3-1/3*(A+B*ln(e*((b*x+a)/(d*x+
c))^n))^2/g/(g*x+f)^3+1/3*B^2*(-a*d+b*c)^3*g^2*n^2*ln((b*x+a)/(d*x+c))/(-a*
g+b*f)^3/(-c*g+d*f)^3-1/3*B^2*(-a*d+b*c)^3*g^2*n^2*ln((g*x+f)/(d*x+c))/(-a*
g+b*f)^3/(-c*g+d*f)^3+2/3*B^2*(-a*d+b*c)^2*g*(-2*a*d*g-b*c*g+3*b*d*f)*n^2*l
n((g*x+f)/(d*x+c))/(-a*g+b*f)^3/(-c*g+d*f)^3+2/3*B*(-a*d+b*c)*(a^2*d^2*g^2-
a*b*d*g*(-c*g+3*d*f)+b^2*(c^2*g^2-3*c*d*f*g+3*d^2*f^2))*n*(A+B*ln(e*((b*x+a
)/(d*x+c))^n))*ln(1-(-c*g+d*f)*(b*x+a)/(-a*g+b*f)/(d*x+c))/(-a*g+b*f)^3/(-c
*g+d*f)^3+2/3*B^2*(-a*d+b*c)*(a^2*d^2*g^2-a*b*d*g*(-c*g+3*d*f)+b^2*(c^2*g^2
-3*c*d*f*g+3*d^2*f^2))*n^2*polylog(2,(-c*g+d*f)*(b*x+a)/(-a*g+b*f)/(d*x+c))
/(-a*g+b*f)^3/(-c*g+d*f)^3
```

**Rubi [A]** time = 2.50, antiderivative size = 1427, normalized size of antiderivative = 1.91, number of steps used = 37, number of rules used = 11, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.344, Rules used = {2525, 12, 2528, 2524, 2418, 2390, 2301, 2394, 2393, 2391, 72}

$$-\frac{B^2n^2\log^2(a+bx)b^3}{3g(bf-ag)^3}+\frac{2Bn\log(a+bx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)b^3}{3g(bf-ag)^3}+\frac{2B^2n^2\log(a+bx)\log\left(\frac{b(c+dx)}{bc-ad}\right)b^3}{3g(bf-ag)^3}+\frac{2B^2n^2\mathrm{Po}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2/(f + g*x)^4,x]
```

```
[Out] -(B^2*(b*c - a*d)^2*g*n^2)/(3*(b*f - a*g)^2*(d*f - c*g)^2*(f + g*x)) + (b^2
*B^2*(b*c - a*d)*n^2*Log[a + b*x])/(3*(b*f - a*g)^3*(d*f - c*g)) + (2*b*B^2
*(b*c - a*d)*(2*b*d*f - b*c*g - a*d*g)*n^2*Log[a + b*x])/(3*(b*f - a*g)^3*(
d*f - c*g)^2) - (b^3*B^2*n^2*Log[a + b*x]^2)/(3*g*(b*f - a*g)^3) - (B*(b*c
- a*d)*n*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/(3*(b*f - a*g)*(d*f - c*g)
```

```
*(f + g*x)^2) - (2*B*(b*c - a*d)*(2*b*d*f - b*c*g - a*d*g)*n*(A + B*Log[e*(
(a + b*x)/(c + d*x))^n]))/(3*(b*f - a*g)^2*(d*f - c*g)^2*(f + g*x)) + (2*b^
3*B*n*Log[a + b*x]*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/(3*g*(b*f - a*g)
^3) - (A + B*Log[e*((a + b*x)/(c + d*x))^n])^2/(3*g*(f + g*x)^3) - (B^2*d^2
*(b*c - a*d)*n^2*Log[c + d*x])/(3*(b*f - a*g)*(d*f - c*g)^3) - (2*B^2*d*(b*
c - a*d)*(2*b*d*f - b*c*g - a*d*g)*n^2*Log[c + d*x])/(3*(b*f - a*g)^2*(d*f
- c*g)^3) + (2*B^2*d^3*n^2*Log[-((d*(a + b*x))/(b*c - a*d))]*Log[c + d*x])/
(3*g*(d*f - c*g)^3) - (2*B*d^3*n*(A + B*Log[e*((a + b*x)/(c + d*x))^n])*Log
[c + d*x])/(3*g*(d*f - c*g)^3) - (B^2*d^3*n^2*Log[c + d*x]^2)/(3*g*(d*f - c
*g)^3) + (2*b^3*B^2*n^2*Log[a + b*x]*Log[(b*(c + d*x))/(b*c - a*d)])/(3*g*(
b*f - a*g)^3) + (B^2*(b*c - a*d)^2*g*(2*b*d*f - b*c*g - a*d*g)*n^2*Log[f +
g*x])/((b*f - a*g)^3*(d*f - c*g)^3) - (2*B^2*(b*c - a*d)*(a^2*d^2*g^2 - a*b
*d*g*(3*d*f - c*g) + b^2*(3*d^2*f^2 - 3*c*d*f*g + c^2*g^2))*n^2*Log[-((g*(a
 + b*x))/(b*f - a*g))]*Log[f + g*x])/(3*(b*f - a*g)^3*(d*f - c*g)^3) + (2*B
*(b*c - a*d)*(a^2*d^2*g^2 - a*b*d*g*(3*d*f - c*g) + b^2*(3*d^2*f^2 - 3*c*d*
f*g + c^2*g^2))*n*(A + B*Log[e*((a + b*x)/(c + d*x))^n])*Log[f + g*x])/(3*(
b*f - a*g)^3*(d*f - c*g)^3) + (2*B^2*(b*c - a*d)*(a^2*d^2*g^2 - a*b*d*g*(3*
d*f - c*g) + b^2*(3*d^2*f^2 - 3*c*d*f*g + c^2*g^2))*n^2*Log[-((g*(c + d*x))
/(d*f - c*g))]*Log[f + g*x])/(3*(b*f - a*g)^3*(d*f - c*g)^3) + (2*b^3*B^2*n
^2*PolyLog[2, -((d*(a + b*x))/(b*c - a*d))])/(3*g*(b*f - a*g)^3) + (2*B^2*d
^3*n^2*PolyLog[2, (b*(c + d*x))/(b*c - a*d)])/(3*g*(d*f - c*g)^3) - (2*B^2*
(b*c - a*d)*(a^2*d^2*g^2 - a*b*d*g*(3*d*f - c*g) + b^2*(3*d^2*f^2 - 3*c*d*f
*g + c^2*g^2))*n^2*PolyLog[2, (b*(f + g*x))/(b*f - a*g)])/(3*(b*f - a*g)^3*
(d*f - c*g)^3) + (2*B^2*(b*c - a*d)*(a^2*d^2*g^2 - a*b*d*g*(3*d*f - c*g) +
b^2*(3*d^2*f^2 - 3*c*d*f*g + c^2*g^2))*n^2*PolyLog[2, (d*(f + g*x))/(d*f -
c*g)])/(3*(b*f - a*g)^3*(d*f - c*g)^3)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 72

```
Int[((e_.) + (f_.)*(x_))^(p_.)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
 x_Symbol] :> Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x]
 /; FreeQ[{a, b, c, d, e, f}, x] && IntegerQ[p]
```

Rule 2301

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

Rule 2390

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
```

```
)*(x_))^(q_.), x_Symbol] :> Dist[1/e, Subst[Int[((f*x)/d)^q*(a + b*Log[c*x^
n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && E
qQ[e*f - d*g, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2393

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + (c*e*x)/g])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2394

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d + e*x
)^n]))/g, x] - Dist[(b*e*n)/g, Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2418

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*(RFx_), x_Sy
mbol] :> With[{u = ExpandIntegrand[(a + b*Log[c*(d + e*x)^n])^p, RFx, x]},
Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d, e, n}, x] && RationalFunctionQ[
RFx, x] && IntegerQ[p]
```

Rule 2524

```
Int[((a_.) + Log[(c_.)*(RFx_)^(p_.)]*(b_.))^(n_.)/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[(Log[d + e*x]*(a + b*Log[c*RFx^p])^n)/e, x] - Dist[(b*n*p)/e
, Int[(Log[d + e*x]*(a + b*Log[c*RFx^p])^(n - 1)*D[RFx, x])/RFx, x], x] /;
FreeQ[{a, b, c, d, e, p}, x] && RationalFunctionQ[RFx, x] && IGtQ[n, 0]
```

Rule 2525

```
Int[((a_.) + Log[(c_.)*(RFx_)^(p_.)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_))^(m_.
), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(a + b*Log[c*RFx^p])^n)/(e*(m + 1))
, x] - Dist[(b*n*p)/(e*(m + 1)), Int[SimplifyIntegrand[((d + e*x)^(m + 1)*(
a + b*Log[c*RFx^p])^(n - 1)*D[RFx, x])/RFx, x], x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && RationalFunctionQ[RFx, x] && IGtQ[n, 0] && (EqQ[n, 1] ||
```

```
IntegerQ[m]) && NeQ[m, -1]
```

Rule 2528

```
Int[((a_.) + Log[(c_.)*(RFx_)^(p_.)]*(b_.))^(n_.)*(RGx_), x_Symbol] :> With
[{u = ExpandIntegrand[(a + b*Log[c*RFx^p])^n, RGx, x]}, Int[u, x] /; SumQ[u
]] /; FreeQ[{a, b, c, p}, x] && RationalFunctionQ[RFx, x] && RationalFuncti
onQ[RGx, x] && IGtQ[n, 0]
```

Rubi steps

$$\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(f+gx)^4}\,dx = -\frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{3g(f+gx)^3} + \frac{(2Bn)\int \frac{(bc-ad)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(a+bx)(c+dx)(f+gx)^3}\,dx}{3g}$$

$$= -\frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{3g(f+gx)^3} + \frac{(2B(bc-ad)n)\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(a+bx)(c+dx)(f+gx)^3}\,dx}{3g}$$

$$= -\frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{3g(f+gx)^3} + \frac{(2B(bc-ad)n)\int \left(\frac{b^4\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(bc-ad)(bf-ag)^3(a+bx)} + \frac{d^4\left(A+B\log\right.}{(bc-ad)(-d}\right.}{}$$

$$= -\frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{3g(f+gx)^3} + \frac{\left(2b^4Bn\right)\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{a+bx}\,dx}{3g(bf-ag)^3} - \frac{\left(2Bd^4n\right)\int \frac{A+B\log}{}}{3g(df-}$$

$$= -\frac{B(bc-ad)n\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{3(bf-ag)(df-cg)(f+gx)^2} - \frac{2B(bc-ad)(2bdf-bcg-adg)n\left(A+\right.}{3(bf-ag)^2(df-cg)^2(f}$$

$$= -\frac{B(bc-ad)n\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{3(bf-ag)(df-cg)(f+gx)^2} - \frac{2B(bc-ad)(2bdf-bcg-adg)n\left(A+\right.}{3(bf-ag)^2(df-cg)^2(f}$$

$$= -\frac{B(bc-ad)n\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{3(bf-ag)(df-cg)(f+gx)^2} - \frac{2B(bc-ad)(2bdf-bcg-adg)n\left(A+\right.}{3(bf-ag)^2(df-cg)^2(f}$$

$$= -\frac{B^2(bc-ad)^2gn^2}{3(bf-ag)^2(df-cg)^2(f+gx)} + \frac{b^2B^2(bc-ad)n^2\log(a+bx)}{3(bf-ag)^3(df-cg)} + \frac{2bB^2(bc-a}{}$$

$$= -\frac{B^2(bc-ad)^2gn^2}{3(bf-ag)^2(df-cg)^2(f+gx)} + \frac{b^2B^2(bc-ad)n^2\log(a+bx)}{3(bf-ag)^3(df-cg)} + \frac{2bB^2(bc-a}{}$$

$$= -\frac{B^2(bc-ad)^2gn^2}{3(bf-ag)^2(df-cg)^2(f+gx)} + \frac{b^2B^2(bc-ad)n^2\log(a+bx)}{3(bf-ag)^3(df-cg)} + \frac{2bB^2(bc-a}{}$$

**Mathematica [A]** time = 3.71, size = 918, normalized size = 1.23

$$-\frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2+\frac{Bn(f+gx)\left(2d^3(f+gx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\log(c+dx)(bf-ag)^3-Bd^3n(f+gx)^2\left(\left(2\log\left(\frac{d(a+bx)}{ad-bc}\right)-\log(c+dx)\right)\log\right.}{}}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2/(f + g*x)^4,x]
```

```
[Out] -1/3*((A + B*Log[e*((a + b*x)/(c + d*x))^n])^2 + (B*n*(f + g*x)*((b*c - a*d
)*g*(b*f - a*g)^2*(d*f - c*g)^2*(A + B*Log[e*((a + b*x)/(c + d*x))^n]) + 2*
(b*c - a*d)*g*(b*f - a*g)*(-(d*f) + c*g)*(-2*b*d*f + b*c*g + a*d*g)*(f + g*
x)*(A + B*Log[e*((a + b*x)/(c + d*x))^n]) - 2*b^3*(d*f - c*g)^3*(f + g*x)^2
*Log[a + b*x]*(A + B*Log[e*((a + b*x)/(c + d*x))^n]) + 2*d^3*(b*f - a*g)^3*
(f + g*x)^2*(A + B*Log[e*((a + b*x)/(c + d*x))^n])*Log[c + d*x] - 2*(b*c -
a*d)*g*(a^2*d^2*g^2 + a*b*d*g*(-3*d*f + c*g) + b^2*(3*d^2*f^2 - 3*c*d*f*g +
 c^2*g^2))*(f + g*x)^2*(A + B*Log[e*((a + b*x)/(c + d*x))^n])*Log[f + g*x]
- 2*B*(b*c - a*d)*g*(2*b*d*f - b*c*g - a*d*g)*n*(f + g*x)^2*(b*(d*f - c*g)*
Log[a + b*x] + (-(b*d*f) + a*d*g)*Log[c + d*x] + (b*c - a*d)*g*Log[f + g*x]
) + B*(b*c - a*d)*g*n*(f + g*x)*((b*c - a*d)*g*(b*f - a*g)*(d*f - c*g) - b^
2*(d*f - c*g)^2*(f + g*x)*Log[a + b*x] + d^2*(b*f - a*g)^2*(f + g*x)*Log[c
+ d*x] + (b*c - a*d)*g*(-2*b*d*f + b*c*g + a*d*g)*(f + g*x)*Log[f + g*x]) +
 b^3*B*(d*f - c*g)^3*n*(f + g*x)^2*(Log[a + b*x]*(Log[a + b*x] - 2*Log[(b*(
c + d*x))/(b*c - a*d)]) - 2*PolyLog[2, (d*(a + b*x))/(-(b*c) + a*d)]) - B*d
^3*(b*f - a*g)^3*n*(f + g*x)^2*((2*Log[(d*(a + b*x))/(-(b*c) + a*d)] - Log[
c + d*x])*Log[c + d*x] + 2*PolyLog[2, (b*(c + d*x))/(b*c - a*d)]) + 2*B*(b*
c - a*d)*g*(a^2*d^2*g^2 + a*b*d*g*(-3*d*f + c*g) + b^2*(3*d^2*f^2 - 3*c*d*f
*g + c^2*g^2))*n*(f + g*x)^2*((Log[(g*(a + b*x))/(-(b*f) + a*g)] - Log[(g*(
c + d*x))/(-(d*f) + c*g)])*Log[f + g*x] + PolyLog[2, (b*(f + g*x))/(b*f - a
*g)] - PolyLog[2, (d*(f + g*x))/(d*f - c*g)])))/((b*f - a*g)^3*(d*f - c*g)^
3))/(g*(f + g*x)^3)
```

**fricas [F]** time = 1.19, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{B^2\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)^2+2\,AB\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A^2}{g^4x^4+4\,fg^3x^3+6\,f^2g^2x^2+4\,f^3gx+f^4},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))^2/(g*x+f)^4,x, algorithm="fricas
")
```

```
[Out] integral((B^2*log(e*((b*x + a)/(d*x + c))^n)^2 + 2*A*B*log(e*((b*x + a)/(d*
x + c))^n) + A^2)/(g^4*x^4 + 4*f*g^3*x^3 + 6*f^2*g^2*x^2 + 4*f^3*g*x + f^4)
, x)
```

**giac [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))^2/(g*x+f)^4,x, algorithm="giac")
[Out] Timed out
```

**maple [F]** time = 0.28, size = 0, normalized size = 0.00

$$\int \frac{\left(B \ln \left(e \left(\frac{bx+a}{dx+c}\right)^{n}\right)+A\right)^{2}}{\left(gx+f\right)^{4}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*ln(e*((b*x+a)/(d*x+c))^n)+A)^2/(g*x+f)^4,x)
[Out] int((B*ln(e*((b*x+a)/(d*x+c))^n)+A)^2/(g*x+f)^4,x)
```

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))^2/(g*x+f)^4,x, algorithm="maxima
")
```

```
[Out] 1/3*(2*b^3*log(b*x + a)/(b^3*f^3*g - 3*a*b^2*f^2*g^2 + 3*a^2*b*f*g^3 - a^3*
g^4) - 2*d^3*log(d*x + c)/(d^3*f^3*g - 3*c*d^2*f^2*g^2 + 3*c^2*d*f*g^3 - c^
3*g^4) + 2*(3*(b^3*c*d^2 - a*b^2*d^3)*f^2 - 3*(b^3*c^2*d - a^2*b*d^3)*f*g +
 (b^3*c^3 - a^3*d^3)*g^2)*log(g*x + f)/(b^3*d^3*f^6 + a^3*c^3*g^6 - 3*(b^3*
c*d^2 + a*b^2*d^3)*f^5*g + 3*(b^3*c^2*d + 3*a*b^2*c*d^2 + a^2*b*d^3)*f^4*g^
2 - (b^3*c^3 + 9*a*b^2*c^2*d + 9*a^2*b*c*d^2 + a^3*d^3)*f^3*g^3 + 3*(a*b^2*
c^3 + 3*a^2*b*c^2*d + a^3*c*d^2)*f^2*g^4 - 3*(a^2*b*c^3 + a^3*c^2*d)*f*g^5)
 - (5*(b^2*c*d - a*b*d^2)*f^2 - 3*(b^2*c^2 - a^2*d^2)*f*g + (a*b*c^2 - a^2*
c*d)*g^2 + 2*(2*(b^2*c*d - a*b*d^2)*f*g - (b^2*c^2 - a^2*d^2)*g^2)*x)/(b^2*
d^2*f^6 + a^2*c^2*f^2*g^4 - 2*(b^2*c*d + a*b*d^2)*f^5*g + (b^2*c^2 + 4*a*b*
c*d + a^2*d^2)*f^4*g^2 - 2*(a*b*c^2 + a^2*c*d)*f^3*g^3 + (b^2*d^2*f^4*g^2 +
```

```
 a^2*c^2*g^6 - 2*(b^2*c*d + a*b*d^2)*f^3*g^3 + (b^2*c^2 + 4*a*b*c*d + a^2*d
^2)*f^2*g^4 - 2*(a*b*c^2 + a^2*c*d)*f*g^5)*x^2 + 2*(b^2*d^2*f^5*g + a^2*c^2
*f*g^5 - 2*(b^2*c*d + a*b*d^2)*f^4*g^2 + (b^2*c^2 + 4*a*b*c*d + a^2*d^2)*f^
3*g^3 - 2*(a*b*c^2 + a^2*c*d)*f^2*g^4)*x))*A*B*n - 1/3*B^2*(log((d*x + c)^n
)^2/(g^4*x^3 + 3*f*g^3*x^2 + 3*f^2*g^2*x + f^3*g) + 3*integrate(-1/3*(3*d*g
*x*log(e)^2 + 3*c*g*log(e)^2 + 3*(d*g*x + c*g)*log((b*x + a)^n)^2 + 6*(d*g*
x*log(e) + c*g*log(e))*log((b*x + a)^n) + 2*(d*f*n + (g*n - 3*g*log(e))*d*x
 - 3*c*g*log(e) - 3*(d*g*x + c*g)*log((b*x + a)^n))*log((d*x + c)^n))/(d*g^
5*x^5 + c*f^4*g + (4*d*f*g^4 + c*g^5)*x^4 + 2*(3*d*f^2*g^3 + 2*c*f*g^4)*x^3
 + 2*(2*d*f^3*g^2 + 3*c*f^2*g^3)*x^2 + (d*f^4*g + 4*c*f^3*g^2)*x), x)) - 2/
3*A*B*log(e*(b*x/(d*x + c) + a/(d*x + c))^n)/(g^4*x^3 + 3*f*g^3*x^2 + 3*f^2
*g^2*x + f^3*g) - 1/3*A^2/(g^4*x^3 + 3*f*g^3*x^2 + 3*f^2*g^2*x + f^3*g)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(A+B\,\ln\left(e\,{\left(\frac{a+b\,x}{c+d\,x}\right)}^n\right)\right)^2}{{\left(f+g\,x\right)}^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*log(e*((a + b*x)/(c + d*x))^n))^2/(f + g*x)^4,x)
[Out] int((A + B*log(e*((a + b*x)/(c + d*x))^n))^2/(f + g*x)^4, x)
```

**sympy [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*ln(e*((b*x+a)/(d*x+c))**n))**2/(g*x+f)**4,x)
[Out] Timed out
```

# 3.75 $\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(f+gx)^5}\,dx$

**Optimal**. Leaf size=1208

$$\frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2 b^4}{4g(bf-ag)^4}-\frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{4g(f+gx)^4}+\frac{B(bc-ad)g\left(\left(6d^2f^2-4cdgf+c^2g^2\right)b^2-2adg(4df-cg\ldots\right.}{2(bf-ag)^4(df-cg\ldots}$$

```
[Out] -1/12*B^2*(-a*d+b*c)^2*g^3*n^2*(d*x+c)^2/(-a*g+b*f)^2/(-c*g+d*f)^4/(g*x+f)^
2-1/6*B^2*(-a*d+b*c)^3*g^3*n^2*(d*x+c)/(-a*g+b*f)^3/(-c*g+d*f)^4/(g*x+f)+1/
4*B^2*(-a*d+b*c)^2*g^2*(-3*a*d*g-b*c*g+4*b*d*f)*n^2*(d*x+c)/(-a*g+b*f)^3/(-
c*g+d*f)^4/(g*x+f)+1/6*B*(-a*d+b*c)*g^3*n*(d*x+c)^3*(A+B*ln(e*((b*x+a)/(d*x
+c))^n))/(-a*g+b*f)/(-c*g+d*f)^4/(g*x+f)^3-1/4*B*(-a*d+b*c)*g^2*(-3*a*d*g-b
*c*g+4*b*d*f)*n*(d*x+c)^2*(A+B*ln(e*((b*x+a)/(d*x+c))^n))/(-a*g+b*f)^2/(-c*
g+d*f)^4/(g*x+f)^2+1/2*B*(-a*d+b*c)*g*(3*a^2*d^2*g^2-2*a*b*d*g*(-c*g+4*d*f)
+b^2*(c^2*g^2-4*c*d*f*g+6*d^2*f^2))*n*(b*x+a)*(A+B*ln(e*((b*x+a)/(d*x+c))^n
))/(-a*g+b*f)^4/(-c*g+d*f)^3/(g*x+f)+1/4*b^4*(A+B*ln(e*((b*x+a)/(d*x+c))^n)
)^2/g/(-a*g+b*f)^4-1/4*(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2/g/(g*x+f)^4-1/6*B^
2*(-a*d+b*c)^4*g^3*n^2*ln((b*x+a)/(d*x+c))/(-a*g+b*f)^4/(-c*g+d*f)^4+1/4*B^
2*(-a*d+b*c)^3*g^2*(-3*a*d*g-b*c*g+4*b*d*f)*n^2*ln((b*x+a)/(d*x+c))/(-a*g+b
*f)^4/(-c*g+d*f)^4+1/6*B^2*(-a*d+b*c)^4*g^3*n^2*ln((g*x+f)/(d*x+c))/(-a*g+b
*f)^4/(-c*g+d*f)^4-1/4*B^2*(-a*d+b*c)^3*g^2*(-3*a*d*g-b*c*g+4*b*d*f)*n^2*ln
((g*x+f)/(d*x+c))/(-a*g+b*f)^4/(-c*g+d*f)^4+1/2*B^2*(-a*d+b*c)^2*g*(3*a^2*d
^2*g^2-2*a*b*d*g*(-c*g+4*d*f)+b^2*(c^2*g^2-4*c*d*f*g+6*d^2*f^2))*n^2*ln((g*
x+f)/(d*x+c))/(-a*g+b*f)^4/(-c*g+d*f)^4-1/2*B*(-a*d+b*c)*(-a*d*g-b*c*g+2*b*
d*f)*(2*a*b*d^2*f*g-a^2*d^2*g^2-b^2*(c^2*g^2-2*c*d*f*g+2*d^2*f^2))*n*(A+B*l
n(e*((b*x+a)/(d*x+c))^n))*ln(1-(-c*g+d*f)*(b*x+a)/(-a*g+b*f)/(d*x+c))/(-a*g
+b*f)^4/(-c*g+d*f)^4-1/2*B^2*(-a*d+b*c)*(-a*d*g-b*c*g+2*b*d*f)*(2*a*b*d^2*f
*g-a^2*d^2*g^2-b^2*(c^2*g^2-2*c*d*f*g+2*d^2*f^2))*n^2*polylog(2,(-c*g+d*f)*
(b*x+a)/(-a*g+b*f)/(d*x+c))/(-a*g+b*f)^4/(-c*g+d*f)^4
```

**Rubi** [A] time = 3.55, antiderivative size = 1968, normalized size of antiderivative = 1.63, number of steps used = 41, number of rules used = 11, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.344, Rules used = {2525, 12, 2528, 2524, 2418, 2390, 2301, 2394, 2393, 2391, 72}

result too large to display

Antiderivative was successfully verified.

```
[In] Int[(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2/(f + g*x)^5,x]
```

```
[Out] -(B^2*(b*c - a*d)^2*g*n^2)/(12*(b*f - a*g)^2*(d*f - c*g)^2*(f + g*x)^2) - (
5*B^2*(b*c - a*d)^2*g*(2*b*d*f - b*c*g - a*d*g)*n^2)/(12*(b*f - a*g)^3*(d*f
```

```
 - c*g)^3*(f + g*x)) + (b^3*B^2*(b*c - a*d)*n^2*Log[a + b*x])/(6*(b*f - a*g
)^4*(d*f - c*g)) + (b^2*B^2*(b*c - a*d)*(2*b*d*f - b*c*g - a*d*g)*n^2*Log[a
 + b*x])/(4*(b*f - a*g)^4*(d*f - c*g)^2) + (b*B^2*(b*c - a*d)*(a^2*d^2*g^2
- a*b*d*g*(3*d*f - c*g) + b^2*(3*d^2*f^2 - 3*c*d*f*g + c^2*g^2))*n^2*Log[a
+ b*x])/(2*(b*f - a*g)^4*(d*f - c*g)^3) - (b^4*B^2*n^2*Log[a + b*x]^2)/(4*g
*(b*f - a*g)^4) - (B*(b*c - a*d)*n*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/
(6*(b*f - a*g)*(d*f - c*g)*(f + g*x)^3) - (B*(b*c - a*d)*(2*b*d*f - b*c*g -
 a*d*g)*n*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/(4*(b*f - a*g)^2*(d*f - c
*g)^2*(f + g*x)^2) - (B*(b*c - a*d)*(a^2*d^2*g^2 - a*b*d*g*(3*d*f - c*g) +
b^2*(3*d^2*f^2 - 3*c*d*f*g + c^2*g^2))*n*(A + B*Log[e*((a + b*x)/(c + d*x))
^n]))/(2*(b*f - a*g)^3*(d*f - c*g)^3*(f + g*x)) + (b^4*B*n*Log[a + b*x]*(A
+ B*Log[e*((a + b*x)/(c + d*x))^n]))/(2*g*(b*f - a*g)^4) - (A + B*Log[e*((a
 + b*x)/(c + d*x))^n])^2/(4*g*(f + g*x)^4) - (B^2*d^3*(b*c - a*d)*n^2*Log[c
 + d*x])/(6*(b*f - a*g)*(d*f - c*g)^4) - (B^2*d^2*(b*c - a*d)*(2*b*d*f - b*
c*g - a*d*g)*n^2*Log[c + d*x])/(4*(b*f - a*g)^2*(d*f - c*g)^4) - (B^2*d*(b*
c - a*d)*(a^2*d^2*g^2 - a*b*d*g*(3*d*f - c*g) + b^2*(3*d^2*f^2 - 3*c*d*f*g
+ c^2*g^2))*n^2*Log[c + d*x])/(2*(b*f - a*g)^3*(d*f - c*g)^4) + (B^2*d^4*n^
2*Log[-((d*(a + b*x))/(b*c - a*d))]*Log[c + d*x])/(2*g*(d*f - c*g)^4) - (B*
d^4*n*(A + B*Log[e*((a + b*x)/(c + d*x))^n])*Log[c + d*x])/(2*g*(d*f - c*g)
^4) - (B^2*d^4*n^2*Log[c + d*x]^2)/(4*g*(d*f - c*g)^4) + (b^4*B^2*n^2*Log[a
 + b*x]*Log[(b*(c + d*x))/(b*c - a*d)])/(2*g*(b*f - a*g)^4) + (B^2*(b*c - a
*d)^2*g*(2*b*d*f - b*c*g - a*d*g)^2*n^2*Log[f + g*x])/(4*(b*f - a*g)^4*(d*f
 - c*g)^4) + (2*B^2*(b*c - a*d)^2*g*(a^2*d^2*g^2 - a*b*d*g*(3*d*f - c*g) +
b^2*(3*d^2*f^2 - 3*c*d*f*g + c^2*g^2))*n^2*Log[f + g*x])/(3*(b*f - a*g)^4*(
d*f - c*g)^4) + (B^2*(b*c - a*d)*(2*b*d*f - b*c*g - a*d*g)*(2*a*b*d^2*f*g -
 a^2*d^2*g^2 - b^2*(2*d^2*f^2 - 2*c*d*f*g + c^2*g^2))*n^2*Log[-((g*(a + b*x
))/(b*f - a*g))]*Log[f + g*x])/(2*(b*f - a*g)^4*(d*f - c*g)^4) - (B*(b*c -
a*d)*(2*b*d*f - b*c*g - a*d*g)*(2*a*b*d^2*f*g - a^2*d^2*g^2 - b^2*(2*d^2*f^
2 - 2*c*d*f*g + c^2*g^2))*n*(A + B*Log[e*((a + b*x)/(c + d*x))^n])*Log[f +
g*x])/(2*(b*f - a*g)^4*(d*f - c*g)^4) - (B^2*(b*c - a*d)*(2*b*d*f - b*c*g -
 a*d*g)*(2*a*b*d^2*f*g - a^2*d^2*g^2 - b^2*(2*d^2*f^2 - 2*c*d*f*g + c^2*g^2
))*n^2*Log[-((g*(c + d*x))/(d*f - c*g))]*Log[f + g*x])/(2*(b*f - a*g)^4*(d*
f - c*g)^4) + (b^4*B^2*n^2*PolyLog[2, -((d*(a + b*x))/(b*c - a*d))])/(2*g*(
b*f - a*g)^4) + (B^2*d^4*n^2*PolyLog[2, (b*(c + d*x))/(b*c - a*d)])/(2*g*(d
*f - c*g)^4) + (B^2*(b*c - a*d)*(2*b*d*f - b*c*g - a*d*g)*(2*a*b*d^2*f*g -
a^2*d^2*g^2 - b^2*(2*d^2*f^2 - 2*c*d*f*g + c^2*g^2))*n^2*PolyLog[2, (b*(f +
 g*x))/(b*f - a*g)])/(2*(b*f - a*g)^4*(d*f - c*g)^4) - (B^2*(b*c - a*d)*(2*
b*d*f - b*c*g - a*d*g)*(2*a*b*d^2*f*g - a^2*d^2*g^2 - b^2*(2*d^2*f^2 - 2*c*
d*f*g + c^2*g^2))*n^2*PolyLog[2, (d*(f + g*x))/(d*f - c*g)])/(2*(b*f - a*g)
^4*(d*f - c*g)^4)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 72

```
Int[((e_.) + (f_.)*(x_))^(p_.)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
 x_Symbol] :> Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x]
 /; FreeQ[{a, b, c, d, e, f}, x] && IntegerQ[p]
```

Rule 2301

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

Rule 2390

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_))^(q_.), x_Symbol] :> Dist[1/e, Subst[Int[((f*x)/d)^q*(a + b*Log[c*x^
n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && E
qQ[e*f - d*g, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2393

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + (c*e*x)/g])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2394

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d + e*x
)^n]))/g, x] - Dist[(b*e*n)/g, Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2418

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*(RFx_), x_Sy
mbol] :> With[{u = ExpandIntegrand[(a + b*Log[c*(d + e*x)^n])^p, RFx, x]},
Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d, e, n}, x] && RationalFunctionQ[
RFx, x] && IntegerQ[p]
```

Rule 2524

```
Int[((a_.) + Log[(c_.)*(RFx_)^(p_.)]*(b_.))^(n_.)/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[(Log[d + e*x]*(a + b*Log[c*RFx^p])^n)/e, x] - Dist[(b*n*p)/e
, Int[(Log[d + e*x]*(a + b*Log[c*RFx^p])^(n - 1)*D[RFx, x])/RFx, x], x] /;
FreeQ[{a, b, c, d, e, p}, x] && RationalFunctionQ[RFx, x] && IGtQ[n, 0]
```

Rule 2525

```
Int[((a_.) + Log[(c_.)*(RFx_)^(p_.)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_))^(m_.
), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(a + b*Log[c*RFx^p])^n)/(e*(m + 1))
, x] - Dist[(b*n*p)/(e*(m + 1)), Int[SimplifyIntegrand[((d + e*x)^(m + 1)*(
a + b*Log[c*RFx^p])^(n - 1)*D[RFx, x])/RFx, x], x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && RationalFunctionQ[RFx, x] && IGtQ[n, 0] && (EqQ[n, 1] ||
IntegerQ[m]) && NeQ[m, -1]
```

Rule 2528

```
Int[((a_.) + Log[(c_.)*(RFx_)^(p_.)]*(b_.))^(n_.)*(RGx_), x_Symbol] :> With
[{u = ExpandIntegrand[(a + b*Log[c*RFx^p])^n, RGx, x]}, Int[u, x] /; SumQ[u
]] /; FreeQ[{a, b, c, p}, x] && RationalFunctionQ[RFx, x] && RationalFuncti
onQ[RGx, x] && IGtQ[n, 0]
```

Rubi steps

$$\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(f+gx)^5}\,dx = -\frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{4g(f+gx)^4} + \frac{(Bn)\int \frac{(bc-ad)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(a+bx)(c+dx)(f+gx)^4}\,dx}{2g}$$

$$= -\frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{4g(f+gx)^4} + \frac{(B(bc-ad)n)\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(a+bx)(c+dx)(f+gx)^4}\,dx}{2g}$$

$$= -\frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{4g(f+gx)^4} + \frac{(B(bc-ad)n)\int \left(\frac{b^5\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(bc-ad)(bf-ag)^4(a+bx)} - \frac{d^5\left(A+B\log\left(\cdots\right.\right.}{(bc-ad)(-df+\cdots}\cdots\right.}{\cdots}$$

$$= -\frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{4g(f+gx)^4} + \frac{\left(b^5Bn\right)\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{a+bx}\,dx}{2g(bf-ag)^4} - \frac{\left(Bd^5n\right)\int \frac{A+B\log\left(\cdots\right.}{c+\cdots}}{2g(df-c\cdots}$$

$$= -\frac{B(bc-ad)n\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{6(bf-ag)(df-cg)(f+gx)^3} - \frac{B(bc-ad)(2bdf-bcg-adg)n\left(A+\cdots\right.}{4(bf-ag)^2(df-cg)^2(f+\cdots}$$

$$= -\frac{B(bc-ad)n\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{6(bf-ag)(df-cg)(f+gx)^3} - \frac{B(bc-ad)(2bdf-bcg-adg)n\left(A+\cdots\right.}{4(bf-ag)^2(df-cg)^2(f+\cdots}$$

$$= -\frac{B(bc-ad)n\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{6(bf-ag)(df-cg)(f+gx)^3} - \frac{B(bc-ad)(2bdf-bcg-adg)n\left(A+\cdots\right.}{4(bf-ag)^2(df-cg)^2(f+\cdots}$$

$$= -\frac{B^2(bc-ad)^2gn^2}{12(bf-ag)^2(df-cg)^2(f+gx)^2} - \frac{5B^2(bc-ad)^2g(2bdf-bcg-adg)n^2}{12(bf-ag)^3(df-cg)^3(f+gx)} + \frac{b^3\cdots}{\cdots}$$

$$= -\frac{B^2(bc-ad)^2gn^2}{12(bf-ag)^2(df-cg)^2(f+gx)^2} - \frac{5B^2(bc-ad)^2g(2bdf-bcg-adg)n^2}{12(bf-ag)^3(df-cg)^3(f+gx)} + \frac{b^3\cdots}{\cdots}$$

$$= -\frac{B^2(bc-ad)^2gn^2}{12(bf-ag)^2(df-cg)^2(f+gx)^2} - \frac{5B^2(bc-ad)^2g(2bdf-bcg-adg)n^2}{12(bf-ag)^3(df-cg)^3(f+gx)} + \frac{b^3\cdots}{\cdots}$$

**Mathematica** [A] time = 7.32, size = 1476, normalized size = 1.22

$$B(bc-ad)n\left(\frac{\log(a+bx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)b^4}{(bc-ad)(bf-ag)^4}-\frac{Bn\left(\log^2(a+bx)-2\log\left(\frac{b(c+dx)}{bc-ad}\right)\log(a+bx)-2\text{Li}_2\left(-\frac{d(a+bx)}{bc-ad}\right)\right)b^4}{2(bc-ad)(bf-ag)^4}-\frac{g\left(\left(3d^2f^2-3cdgf+c^2g^2\right)b^2 \cdots}{(b \cdots}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2/(f + g*x)^5,x]
```

```
[Out] -1/4*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2/(g*(f + g*x)^4) + (B*(b*c - a
*d)*n*(-1/3*(g*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/((b*f - a*g)*(d*f -
c*g)*(f + g*x)^3) - (g*(2*b*d*f - b*c*g - a*d*g)*(A + B*Log[e*((a + b*x)/(c
 + d*x))^n]))/(2*(b*f - a*g)^2*(d*f - c*g)^2*(f + g*x)^2) - (g*(a^2*d^2*g^2
 - a*b*d*g*(3*d*f - c*g) + b^2*(3*d^2*f^2 - 3*c*d*f*g + c^2*g^2))*(A + B*Lo
g[e*((a + b*x)/(c + d*x))^n]))/((b*f - a*g)^3*(d*f - c*g)^3*(f + g*x)) + (b
^4*Log[a + b*x]*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/((b*c - a*d)*(b*f -
 a*g)^4) - (d^4*(A + B*Log[e*((a + b*x)/(c + d*x))^n])*Log[c + d*x])/((b*c
- a*d)*(d*f - c*g)^4) + (g*(2*b*d*f - b*c*g - a*d*g)*(2*b^2*d^2*f^2 - 2*b^2
*c*d*f*g - 2*a*b*d^2*f*g + b^2*c^2*g^2 + a^2*d^2*g^2)*(A + B*Log[e*((a + b*
x)/(c + d*x))^n])*Log[f + g*x])/((b*f - a*g)^4*(d*f - c*g)^4) + (B*(b*c - a
*d)*g*(a^2*d^2*g^2 - a*b*d*g*(3*d*f - c*g) + b^2*(3*d^2*f^2 - 3*c*d*f*g + c
^2*g^2))*n*((b*Log[a + b*x])/((b*c - a*d)*(b*f - a*g)) - (d*Log[c + d*x])/(
(b*c - a*d)*(d*f - c*g)) + (g*Log[f + g*x])/((b*f - a*g)*(d*f - c*g))))/((b
*f - a*g)^3*(d*f - c*g)^3) - (B*(b*c - a*d)*g*(2*b*d*f - b*c*g - a*d*g)*n*(
g/((b*f - a*g)*(d*f - c*g)*(f + g*x)) - (b^2*Log[a + b*x])/((b*c - a*d)*(b*
f - a*g)^2) + (d^2*Log[c + d*x])/((b*c - a*d)*(d*f - c*g)^2) - (g*(2*b*d*f
- b*c*g - a*d*g)*Log[f + g*x])/((b*f - a*g)^2*(d*f - c*g)^2)))/(2*(b*f - a*
g)^2*(d*f - c*g)^2) - (B*(b*c - a*d)*g*n*(g/((b*f - a*g)*(d*f - c*g)*(f + g
*x)^2) + (2*g*(2*b*d*f - b*c*g - a*d*g))/((b*f - a*g)^2*(d*f - c*g)^2*(f +
g*x)) - (2*b^3*Log[a + b*x])/((b*c - a*d)*(b*f - a*g)^3) + (2*d^3*Log[c + d
*x])/((b*c - a*d)*(d*f - c*g)^3) - (2*g*(a^2*d^2*g^2 - a*b*d*g*(3*d*f - c*g
) + b^2*(3*d^2*f^2 - 3*c*d*f*g + c^2*g^2))*Log[f + g*x])/((b*f - a*g)^3*(d*
f - c*g)^3)))/(6*(b*f - a*g)*(d*f - c*g)) - (b^4*B*n*(Log[a + b*x]^2 - 2*Lo
g[a + b*x]*Log[(b*(c + d*x))/(b*c - a*d)] - 2*PolyLog[2, -((d*(a + b*x))/(b
*c - a*d))]))/(2*(b*c - a*d)*(b*f - a*g)^4) + (B*d^4*n*(2*Log[-((d*(a + b*x
))/(b*c - a*d))]*Log[c + d*x] - Log[c + d*x]^2 + 2*PolyLog[2, (b*(c + d*x))
/(b*c - a*d)]))/(2*(b*c - a*d)*(d*f - c*g)^4) - (B*g*(2*b*d*f - b*c*g - a*d
*g)*(2*b^2*d^2*f^2 - 2*b^2*c*d*f*g - 2*a*b*d^2*f*g + b^2*c^2*g^2 + a^2*d^2*
g^2)*n*(Log[-((g*(a + b*x))/(b*f - a*g))]*Log[f + g*x] - Log[-((g*(c + d*x)
)/(d*f - c*g))]*Log[f + g*x] + PolyLog[2, (b*(f + g*x))/(b*f - a*g)] - Poly
Log[2, (d*(f + g*x))/(d*f - c*g)]))/((b*f - a*g)^4*(d*f - c*g)^4)))/(2*g)
```

**fricas [F]** time = 1.09, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{B^2\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)^2 + 2\,AB\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right) + A^2}{g^5x^5 + 5\,fg^4x^4 + 10\,f^2g^3x^3 + 10\,f^3g^2x^2 + 5\,f^4gx + f^5}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))^2/(g*x+f)^5,x, algorithm="fricas
")
```

```
[Out] integral((B^2*log(e*((b*x + a)/(d*x + c))^n)^2 + 2*A*B*log(e*((b*x + a)/(d*
x + c))^n) + A^2)/(g^5*x^5 + 5*f*g^4*x^4 + 10*f^2*g^3*x^3 + 10*f^3*g^2*x^2
+ 5*f^4*g*x + f^5), x)
```

**giac [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))^2/(g*x+f)^5,x, algorithm="giac")
```

```
[Out] Timed out
```

**maple [F]** time = 0.29, size = 0, normalized size = 0.00

$$\int \frac{\left(B\ln\left(e\left(\frac{bx+a}{dx+c}\right)^n\right) + A\right)^2}{\left(gx+f\right)^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*ln(e*((b*x+a)/(d*x+c))^n)+A)^2/(g*x+f)^5,x)
```

```
[Out] int((B*ln(e*((b*x+a)/(d*x+c))^n)+A)^2/(g*x+f)^5,x)
```

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))^2/(g*x+f)^5,x, algorithm="maxima
")
```

```
[Out] 1/12*(6*b^4*log(b*x + a)/(b^4*f^4*g - 4*a*b^3*f^3*g^2 + 6*a^2*b^2*f^2*g^3 -
 4*a^3*b*f*g^4 + a^4*g^5) - 6*d^4*log(d*x + c)/(d^4*f^4*g - 4*c*d^3*f^3*g^2
 + 6*c^2*d^2*f^2*g^3 - 4*c^3*d*f*g^4 + c^4*g^5) + 6*(4*(b^4*c*d^3 - a*b^3*d
^4)*f^3 - 6*(b^4*c^2*d^2 - a^2*b^2*d^4)*f^2*g + 4*(b^4*c^3*d - a^3*b*d^4)*f
*g^2 - (b^4*c^4 - a^4*d^4)*g^3)*log(g*x + f)/(b^4*d^4*f^8 + a^4*c^4*g^8 - 4
*(b^4*c*d^3 + a*b^3*d^4)*f^7*g + 2*(3*b^4*c^2*d^2 + 8*a*b^3*c*d^3 + 3*a^2*b
^2*d^4)*f^6*g^2 - 4*(b^4*c^3*d + 6*a*b^3*c^2*d^2 + 6*a^2*b^2*c*d^3 + a^3*b*
d^4)*f^5*g^3 + (b^4*c^4 + 16*a*b^3*c^3*d + 36*a^2*b^2*c^2*d^2 + 16*a^3*b*c*
d^3 + a^4*d^4)*f^4*g^4 - 4*(a*b^3*c^4 + 6*a^2*b^2*c^3*d + 6*a^3*b*c^2*d^2 +
 a^4*c*d^3)*f^3*g^5 + 2*(3*a^2*b^2*c^4 + 8*a^3*b*c^3*d + 3*a^4*c^2*d^2)*f^2
*g^6 - 4*(a^3*b*c^4 + a^4*c^3*d)*f*g^7) - (26*(b^3*c*d^2 - a*b^2*d^3)*f^4 -
 31*(b^3*c^2*d - a^2*b*d^3)*f^3*g + (11*b^3*c^3 + 15*a*b^2*c^2*d - 15*a^2*b
*c*d^2 - 11*a^3*d^3)*f^2*g^2 - 7*(a*b^2*c^3 - a^3*c*d^2)*f*g^3 + 2*(a^2*b*c
^3 - a^3*c^2*d)*g^4 + 6*(3*(b^3*c*d^2 - a*b^2*d^3)*f^2*g^2 - 3*(b^3*c^2*d -
 a^2*b*d^3)*f*g^3 + (b^3*c^3 - a^3*d^3)*g^4)*x^2 + 3*(14*(b^3*c*d^2 - a*b^2
*d^3)*f^3*g - 15*(b^3*c^2*d - a^2*b*d^3)*f^2*g^2 + (5*b^3*c^3 + 3*a*b^2*c^2
*d - 3*a^2*b*c*d^2 - 5*a^3*d^3)*f*g^3 - (a*b^2*c^3 - a^3*c*d^2)*g^4)*x)/(b^
3*d^3*f^9 + a^3*c^3*f^3*g^6 - 3*(b^3*c*d^2 + a*b^2*d^3)*f^8*g + 3*(b^3*c^2*
d + 3*a*b^2*c*d^2 + a^2*b*d^3)*f^7*g^2 - (b^3*c^3 + 9*a*b^2*c^2*d + 9*a^2*b
*c*d^2 + a^3*d^3)*f^6*g^3 + 3*(a*b^2*c^3 + 3*a^2*b*c^2*d + a^3*c*d^2)*f^5*g
^4 - 3*(a^2*b*c^3 + a^3*c^2*d)*f^4*g^5 + (b^3*d^3*f^6*g^3 + a^3*c^3*g^9 - 3
*(b^3*c*d^2 + a*b^2*d^3)*f^5*g^4 + 3*(b^3*c^2*d + 3*a*b^2*c*d^2 + a^2*b*d^3
)*f^4*g^5 - (b^3*c^3 + 9*a*b^2*c^2*d + 9*a^2*b*c*d^2 + a^3*d^3)*f^3*g^6 + 3
*(a*b^2*c^3 + 3*a^2*b*c^2*d + a^3*c*d^2)*f^2*g^7 - 3*(a^2*b*c^3 + a^3*c^2*d
)*f*g^8)*x^3 + 3*(b^3*d^3*f^7*g^2 + a^3*c^3*f*g^8 - 3*(b^3*c*d^2 + a*b^2*d^
3)*f^6*g^3 + 3*(b^3*c^2*d + 3*a*b^2*c*d^2 + a^2*b*d^3)*f^5*g^4 - (b^3*c^3 +
 9*a*b^2*c^2*d + 9*a^2*b*c*d^2 + a^3*d^3)*f^4*g^5 + 3*(a*b^2*c^3 + 3*a^2*b*
c^2*d + a^3*c*d^2)*f^3*g^6 - 3*(a^2*b*c^3 + a^3*c^2*d)*f^2*g^7)*x^2 + 3*(b^
3*d^3*f^8*g + a^3*c^3*f^2*g^7 - 3*(b^3*c*d^2 + a*b^2*d^3)*f^7*g^2 + 3*(b^3*
c^2*d + 3*a*b^2*c*d^2 + a^2*b*d^3)*f^6*g^3 - (b^3*c^3 + 9*a*b^2*c^2*d + 9*a
^2*b*c*d^2 + a^3*d^3)*f^5*g^4 + 3*(a*b^2*c^3 + 3*a^2*b*c^2*d + a^3*c*d^2)*f
^4*g^5 - 3*(a^2*b*c^3 + a^3*c^2*d)*f^3*g^6)*x))*A*B*n - 1/4*B^2*(log((d*x +
 c)^n)^2/(g^5*x^4 + 4*f*g^4*x^3 + 6*f^2*g^3*x^2 + 4*f^3*g^2*x + f^4*g) + 4*
integrate(-1/2*(2*d*g*x*log(e)^2 + 2*c*g*log(e)^2 + 2*(d*g*x + c*g)*log((b*
x + a)^n)^2 + 4*(d*g*x*log(e) + c*g*log(e))*log((b*x + a)^n) + (d*f*n + (g*
n - 4*g*log(e))*d*x - 4*c*g*log(e) - 4*(d*g*x + c*g)*log((b*x + a)^n))*log(
(d*x + c)^n))/(d*g^6*x^6 + c*f^5*g + (5*d*f*g^5 + c*g^6)*x^5 + 5*(2*d*f^2*g
^4 + c*f*g^5)*x^4 + 10*(d*f^3*g^3 + c*f^2*g^4)*x^3 + 5*(d*f^4*g^2 + 2*c*f^3
*g^3)*x^2 + (d*f^5*g + 5*c*f^4*g^2)*x), x)) - 1/2*A*B*log(e*(b*x/(d*x + c)
+ a/(d*x + c))^n)/(g^5*x^4 + 4*f*g^4*x^3 + 6*f^2*g^3*x^2 + 4*f^3*g^2*x + f^
4*g) - 1/4*A^2/(g^5*x^4 + 4*f*g^4*x^3 + 6*f^2*g^3*x^2 + 4*f^3*g^2*x + f^4*g
)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(A+B\,\ln\left(e\left(\frac{a+b\,x}{c+d\,x}\right)^n\right)\right)^2}{\left(f+g\,x\right)^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*log(e*((a + b*x)/(c + d*x))^n))^2/(f + g*x)^5,x)
[Out] int((A + B*log(e*((a + b*x)/(c + d*x))^n))^2/(f + g*x)^5, x)
```

**sympy [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*ln(e*((b*x+a)/(d*x+c))**n))**2/(g*x+f)**5,x)
[Out] Timed out
```

# 3.76 $\int \frac{(f+gx)^2}{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}\,dx$

**Optimal**. Leaf size=35

$$\text{Int}\left(\frac{(f+gx)^2}{B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A},x\right)$$

```
[Out] Unintegrable((g*x+f)^2/(A+B*ln(e*((b*x+a)/(d*x+c))^n)),x)
```

**Rubi [A]** time = 0.19, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{(f+gx)^2}{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(f + g*x)^2/(A + B*Log[e*((a + b*x)/(c + d*x))^n]),x]
```

```
[Out] f^2*Defer[Int][(A + B*Log[e*((a + b*x)/(c + d*x))^n])^(-1), x] + 2*f*g*Defe
r[Int][x/(A + B*Log[e*((a + b*x)/(c + d*x))^n]), x] + g^2*Defer[Int][x^2/(A
 + B*Log[e*((a + b*x)/(c + d*x))^n]), x]
```

Rubi steps

$$\int \frac{(f+gx)^2}{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}\,dx = \int \left(\frac{f^2}{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}+\frac{2fgx}{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}+\frac{g^2x^2}{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}\right) dx$$
$$= f^2\int \frac{1}{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}\,dx+(2fg)\int \frac{x}{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}\,dx+g^2\int \frac{}{A+}$$

**Mathematica [A]** time = 0.43, size = 0, normalized size = 0.00

$$\int \frac{(f+gx)^2}{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(f + g*x)^2/(A + B*Log[e*((a + b*x)/(c + d*x))^n]),x]
```

```
[Out] Integrate[(f + g*x)^2/(A + B*Log[e*((a + b*x)/(c + d*x))^n]), x]
```

**fricas [A]** time = 0.74, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{g^2x^2+2\,fgx+f^2}{B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^2/(A+B*log(e*((b*x+a)/(d*x+c))^n)),x, algorithm="fricas")
```

```
[Out] integral((g^2*x^2 + 2*f*g*x + f^2)/(B*log(e*((b*x + a)/(d*x + c))^n) + A),
x)
```

**giac [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^2/(A+B*log(e*((b*x+a)/(d*x+c))^n)),x, algorithm="giac")
```

```
[Out] Timed out
```

**maple [A]** time = 0.26, size = 0, normalized size = 0.00

$$\int \frac{\left(gx+f\right)^{2}}{B\ln\left(e\left(\frac{bx+a}{dx+c}\right)^{n}\right)+A}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x+f)^2/(B*ln(e*((b*x+a)/(d*x+c))^n)+A),x)
```

```
[Out] int((g*x+f)^2/(B*ln(e*((b*x+a)/(d*x+c))^n)+A),x)
```

**maxima [A]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(gx+f\right)^{2}}{B\log\left(e\left(\frac{bx+a}{dx+c}\right)^{n}\right)+A}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^2/(A+B*log(e*((b*x+a)/(d*x+c))^n)),x, algorithm="maxima")
[Out] integrate((g*x + f)^2/(B*log(e*((b*x + a)/(d*x + c))^n) + A), x)
```

**mupad [A]** time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{\left(f+g\,x\right)^2}{A+B\,\ln\left(e\,{\left(\frac{a+b\,x}{c+d\,x}\right)}^n\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f + g*x)^2/(A + B*log(e*((a + b*x)/(c + d*x))^n)),x)
[Out] int((f + g*x)^2/(A + B*log(e*((a + b*x)/(c + d*x))^n)), x)
```

**sympy [A]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(f+gx\right)^2}{A+B\log\left(e\left(\frac{a}{c+dx}+\frac{bx}{c+dx}\right)^n\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)**2/(A+B*ln(e*((b*x+a)/(d*x+c))**n)),x)
[Out] Integral((f + g*x)**2/(A + B*log(e*(a/(c + d*x) + b*x/(c + d*x))**n)), x)
```

## 3.77 $\int \frac{f+gx}{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}\,dx$

**Optimal**. Leaf size=33

$$\mathrm{Int}\left(\frac{f+gx}{B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A},x\right)$$

```
[Out] Unintegrable((g*x+f)/(A+B*ln(e*((b*x+a)/(d*x+c))^n)),x)
```

**Rubi [A]** time = 0.10, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{f+gx}{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(f + g*x)/(A + B*Log[e*((a + b*x)/(c + d*x))^n]),x]
```

```
[Out] f*Defer[Int][(A + B*Log[e*((a + b*x)/(c + d*x))^n])^(-1), x] + g*Defer[Int]
[x/(A + B*Log[e*((a + b*x)/(c + d*x))^n]), x]
```

Rubi steps

$$\begin{aligned}
\int \frac{f+gx}{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}\,dx &= \int \left(\frac{f}{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}+\frac{gx}{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}\right)dx\\
&= f\int \frac{1}{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}\,dx+g\int \frac{x}{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}\,dx
\end{aligned}$$

**Mathematica [A]** time = 0.29, size = 0, normalized size = 0.00

$$\int \frac{f+gx}{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(f + g*x)/(A + B*Log[e*((a + b*x)/(c + d*x))^n]),x]
[Out] Integrate[(f + g*x)/(A + B*Log[e*((a + b*x)/(c + d*x))^n]), x]
```

**fricas [A]** time = 0.86, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{gx+f}{B\log\left(e\left(\frac{bx+a}{dx+c}\right)^{n}\right)+A},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)/(A+B*log(e*((b*x+a)/(d*x+c))^n)),x, algorithm="fricas")
[Out] integral((g*x + f)/(B*log(e*((b*x + a)/(d*x + c))^n) + A), x)
```

**giac [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)/(A+B*log(e*((b*x+a)/(d*x+c))^n)),x, algorithm="giac")
[Out] Timed out
```

**maple [A]** time = 0.15, size = 0, normalized size = 0.00

$$\int \frac{gx+f}{B\ln\left(e\left(\frac{bx+a}{dx+c}\right)^{n}\right)+A}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x+f)/(B*ln(e*((b*x+a)/(d*x+c))^n)+A),x)
[Out] int((g*x+f)/(B*ln(e*((b*x+a)/(d*x+c))^n)+A),x)
```

**maxima [A]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{gx+f}{B\log\left(e\left(\frac{bx+a}{dx+c}\right)^{n}\right)+A}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)/(A+B*log(e*((b*x+a)/(d*x+c))^n)),x, algorithm="maxima")
[Out] integrate((g*x + f)/(B*log(e*((b*x + a)/(d*x + c))^n) + A), x)
```

**mupad [A]** time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{f+g\,x}{A+B\,\ln\left(e\,{\left(\frac{a+b\,x}{c+d\,x}\right)}^n\right)} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f + g*x)/(A + B*log(e*((a + b*x)/(c + d*x))^n)),x)
[Out] int((f + g*x)/(A + B*log(e*((a + b*x)/(c + d*x))^n)), x)
```

**sympy [A]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{f+gx}{A+B\log\left(e\left(\frac{a}{c+dx}+\frac{bx}{c+dx}\right)^{n}\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)/(A+B*ln(e*((b*x+a)/(d*x+c))**n)),x)
[Out] Integral((f + g*x)/(A + B*log(e*(a/(c + d*x) + b*x/(c + d*x))**n)), x)
```

## 3.78 $\int \frac{1}{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}\,dx$

**Optimal**. Leaf size=27

$$\operatorname{Int}\left(\frac{1}{B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A},x\right)$$

[Out] Unintegrable(1/(A+B*ln(e*((b*x+a)/(d*x+c))^n)),x)

**Rubi [A]** time = 0.02, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{1}{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}\,dx$$

Verification is Not applicable to the result.

[In] Int[(A + B*Log[e*((a + b*x)/(c + d*x))^n])^(-1),x]

[Out] Defer[Int][(A + B*Log[e*((a + b*x)/(c + d*x))^n])^(-1), x]

Rubi steps

$$\int \frac{1}{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}\,dx=\int \frac{1}{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}\,dx$$

**Mathematica [A]** time = 0.02, size = 0, normalized size = 0.00

$$\int \frac{1}{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}\,dx$$

Verification is Not applicable to the result.

[In] Integrate[(A + B*Log[e*((a + b*x)/(c + d*x))^n])^(-1),x]

[Out] Integrate[(A + B*Log[e*((a + b*x)/(c + d*x))^n])^(-1), x]

**fricas [A]** time = 1.07, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{1}{B\log\left(e\left(\frac{bx+a}{dx+c}\right)^{n}\right)+A},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(A+B*log(e*((b*x+a)/(d*x+c))^n)),x, algorithm="fricas")
[Out] integral(1/(B*log(e*((b*x + a)/(d*x + c))^n) + A), x)
```

**giac [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(A+B*log(e*((b*x+a)/(d*x+c))^n)),x, algorithm="giac")
[Out] Timed out
```

**maple [A]** time = 0.12, size = 0, normalized size = 0.00

$$\int \frac{1}{B\ln\left(e\left(\frac{bx+a}{dx+c}\right)^{n}\right)+A}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(B*ln(e*((b*x+a)/(d*x+c))^n)+A),x)
[Out] int(1/(B*ln(e*((b*x+a)/(d*x+c))^n)+A),x)
```

**maxima [A]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{B\log\left(e\left(\frac{bx+a}{dx+c}\right)^{n}\right)+A}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(A+B*log(e*((b*x+a)/(d*x+c))^n)),x, algorithm="maxima")
[Out] integrate(1/(B*log(e*((b*x + a)/(d*x + c))^n) + A), x)
```

**mupad** [A] time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{1}{A+B\,\ln\left(e\,{\left(\frac{a+b\,x}{c+d\,x}\right)}^n\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(A + B*log(e*((a + b*x)/(c + d*x))^n)),x)
[Out] int(1/(A + B*log(e*((a + b*x)/(c + d*x))^n)), x)
```

**sympy** [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(A+B*ln(e*((b*x+a)/(d*x+c))**n)),x)
[Out] Integral(1/(A + B*log(e*((a + b*x)/(c + d*x))**n)), x)
```

## 3.79 $\int \frac{1}{(f+gx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}\,dx$

**Optimal**. Leaf size=35

$$\operatorname{Int}\left(\frac{1}{(f+gx)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)},x\right)$$

```
[Out] Unintegrable(1/(g*x+f)/(A+B*ln(e*((b*x+a)/(d*x+c))^n)),x)
```

**Rubi [A]** time = 0.07, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{1}{(f+gx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[1/((f + g*x)*(A + B*Log[e*((a + b*x)/(c + d*x))^n])),x]
```

```
[Out] Defer[Int][1/((f + g*x)*(A + B*Log[e*((a + b*x)/(c + d*x))^n])), x]
```

Rubi steps

$$\int \frac{1}{(f+gx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}\,dx=\int \frac{1}{(f+gx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}\,dx$$

**Mathematica [A]** time = 0.98, size = 0, normalized size = 0.00

$$\int \frac{1}{(f+gx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[1/((f + g*x)*(A + B*Log[e*((a + b*x)/(c + d*x))^n])),x]
```

```
[Out] Integrate[1/((f + g*x)*(A + B*Log[e*((a + b*x)/(c + d*x))^n])), x]
```

**fricas [A]** time = 0.81, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{1}{Agx + Af + \left(Bgx + Bf\right)\log\left(e\left(\frac{bx+a}{dx+c}\right)^{n}\right)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(g*x+f)/(A+B*log(e*((b*x+a)/(d*x+c))^n)),x, algorithm="fricas")
[Out] integral(1/(A*g*x + A*f + (B*g*x + B*f)*log(e*((b*x + a)/(d*x + c))^n)), x)
```

**giac [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(g*x+f)/(A+B*log(e*((b*x+a)/(d*x+c))^n)),x, algorithm="giac")
[Out] Timed out
```

**maple [A]** time = 0.38, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(gx + f\right)\left(B\ln\left(e\left(\frac{bx+a}{dx+c}\right)^{n}\right) + A\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(g*x+f)/(B*ln(e*((b*x+a)/(d*x+c))^n)+A),x)
[Out] int(1/(g*x+f)/(B*ln(e*((b*x+a)/(d*x+c))^n)+A),x)
```

**maxima [A]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(gx + f\right)\left(B\log\left(e\left(\frac{bx+a}{dx+c}\right)^{n}\right) + A\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(g*x+f)/(A+B*log(e*((b*x+a)/(d*x+c))^n)),x, algorithm="maxima")
[Out] integrate(1/((g*x + f)*(B*log(e*((b*x + a)/(d*x + c))^n) + A)), x)
```

**mupad [A]** time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{1}{\left(f + g\,x\right)\,\left(A + B\,\ln\left(e\,{\left(\frac{a+b\,x}{c+d\,x}\right)}^n\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((f + g*x)*(A + B*log(e*((a + b*x)/(c + d*x))^n))),x)
[Out] int(1/((f + g*x)*(A + B*log(e*((a + b*x)/(c + d*x))^n))), x)
```

**sympy [A]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(A + B\log\left(e\left(\frac{a}{c+dx} + \frac{bx}{c+dx}\right)^n\right)\right)\left(f + gx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(g*x+f)/(A+B*ln(e*((b*x+a)/(d*x+c))**n)),x)
[Out] Integral(1/((A + B*log(e*(a/(c + d*x) + b*x/(c + d*x))**n))*(f + g*x)), x)
```

# 3.80 $\int \frac{1}{(f+gx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}\,dx$

**Optimal**. Leaf size=35

$$\text{Int}\left(\frac{1}{(f+gx)^2\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)},x\right)$$

```
[Out] Unintegrable(1/(g*x+f)^2/(A+B*ln(e*((b*x+a)/(d*x+c))^n)),x)
```

**Rubi [A]** time = 0.07, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{1}{(f+gx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[1/((f + g*x)^2*(A + B*Log[e*((a + b*x)/(c + d*x))^n])),x]
[Out] Defer[Int][1/((f + g*x)^2*(A + B*Log[e*((a + b*x)/(c + d*x))^n])), x]
```

Rubi steps

$$\int \frac{1}{(f+gx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}\,dx=\int \frac{1}{(f+gx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}\,dx$$

**Mathematica [A]** time = 1.10, size = 0, normalized size = 0.00

$$\int \frac{1}{(f+gx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[1/((f + g*x)^2*(A + B*Log[e*((a + b*x)/(c + d*x))^n])),x]
```

```
[Out] Integrate[1/((f + g*x)^2*(A + B*Log[e*((a + b*x)/(c + d*x))^n])), x]
```

**fricas [A]** time = 0.90, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{1}{Ag^2x^2 + 2\,Afgx + Af^2 + \left(Bg^2x^2 + 2\,Bfgx + Bf^2\right)\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(g*x+f)^2/(A+B*log(e*((b*x+a)/(d*x+c))^n)),x, algorithm="fricas
")
```

```
[Out] integral(1/(A*g^2*x^2 + 2*A*f*g*x + A*f^2 + (B*g^2*x^2 + 2*B*f*g*x + B*f^2)
*log(e*((b*x + a)/(d*x + c))^n)), x)
```

**giac [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(g*x+f)^2/(A+B*log(e*((b*x+a)/(d*x+c))^n)),x, algorithm="giac")
```

```
[Out] Timed out
```

**maple [A]** time = 0.29, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(gx+f\right)^2\left(B\ln\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(g*x+f)^2/(B*ln(e*((b*x+a)/(d*x+c))^n)+A),x)
```

```
[Out] int(1/(g*x+f)^2/(B*ln(e*((b*x+a)/(d*x+c))^n)+A),x)
```

**maxima [A]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(gx+f\right)^2\left(B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(g*x+f)^2/(A+B*log(e*((b*x+a)/(d*x+c))^n)),x, algorithm="maxima
")
```

```
[Out] integrate(1/((g*x + f)^2*(B*log(e*((b*x + a)/(d*x + c))^n) + A)), x)
```

**mupad [A]**    time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{1}{\left(f+g\,x\right)^2\,\left(A+B\,\ln\left(e\,{\left(\frac{a+b\,x}{c+d\,x}\right)}^n\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((f + g*x)^2*(A + B*log(e*((a + b*x)/(c + d*x))^n))),x)
```

```
[Out] int(1/((f + g*x)^2*(A + B*log(e*((a + b*x)/(c + d*x))^n))), x)
```

**sympy [F(-1)]**    time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(g*x+f)**2/(A+B*ln(e*((b*x+a)/(d*x+c))**n)),x)
```

```
[Out] Timed out
```

## 3.81 $\int \frac{1}{(f+gx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}\,dx$

**Optimal**. Leaf size=35

$$\text{Int}\left(\frac{1}{(f+gx)^3\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)},x\right)$$

```
[Out] Unintegrable(1/(g*x+f)^3/(A+B*ln(e*((b*x+a)/(d*x+c))^n)),x)
```

**Rubi [A]** time = 0.07, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{1}{(f+gx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[1/((f + g*x)^3*(A + B*Log[e*((a + b*x)/(c + d*x))^n])),x]
[Out] Defer[Int][1/((f + g*x)^3*(A + B*Log[e*((a + b*x)/(c + d*x))^n])), x]
```

Rubi steps

$$\int \frac{1}{(f+gx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}\,dx=\int \frac{1}{(f+gx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}\,dx$$

**Mathematica [A]** time = 11.86, size = 0, normalized size = 0.00

$$\int \frac{1}{(f+gx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[1/((f + g*x)^3*(A + B*Log[e*((a + b*x)/(c + d*x))^n])),x]
```

```
[Out] Integrate[1/((f + g*x)^3*(A + B*Log[e*((a + b*x)/(c + d*x))^n])), x]
```

**fricas [A]** time = 2.88, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{1}{Ag^3x^3 + 3\,Afg^2x^2 + 3\,Af^2gx + Af^3 + \left(Bg^3x^3 + 3\,Bfg^2x^2 + 3\,Bf^2gx + Bf^3\right)\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(g*x+f)^3/(A+B*log(e*((b*x+a)/(d*x+c))^n)),x, algorithm="fricas
")
```

```
[Out] integral(1/(A*g^3*x^3 + 3*A*f*g^2*x^2 + 3*A*f^2*g*x + A*f^3 + (B*g^3*x^3 +
3*B*f*g^2*x^2 + 3*B*f^2*g*x + B*f^3)*log(e*((b*x + a)/(d*x + c))^n)), x)
```

**giac [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(g*x+f)^3/(A+B*log(e*((b*x+a)/(d*x+c))^n)),x, algorithm="giac")
```

```
[Out] Timed out
```

**maple [A]** time = 0.29, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(gx+f\right)^{3}\left(B\ln\left(e\left(\frac{bx+a}{dx+c}\right)^{n}\right)+A\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(g*x+f)^3/(B*ln(e*((b*x+a)/(d*x+c))^n)+A),x)
```

```
[Out] int(1/(g*x+f)^3/(B*ln(e*((b*x+a)/(d*x+c))^n)+A),x)
```

**maxima [A]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(gx+f\right)^{3}\left(B\log\left(e\left(\frac{bx+a}{dx+c}\right)^{n}\right)+A\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(g*x+f)^3/(A+B*log(e*((b*x+a)/(d*x+c))^n)),x, algorithm="maxima
")
```

```
[Out] integrate(1/((g*x + f)^3*(B*log(e*((b*x + a)/(d*x + c))^n) + A)), x)
```

**mupad [A]** time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{1}{\left(f+g\,x\right)^3\,\left(A+B\,\ln\left(e\,{\left(\frac{a+b\,x}{c+d\,x}\right)}^n\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((f + g*x)^3*(A + B*log(e*((a + b*x)/(c + d*x))^n))),x)
```

```
[Out] int(1/((f + g*x)^3*(A + B*log(e*((a + b*x)/(c + d*x))^n))), x)
```

**sympy [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(g*x+f)**3/(A+B*ln(e*((b*x+a)/(d*x+c))**n)),x)
```

```
[Out] Timed out
```

# 3.82 $\int \frac{(f+gx)^2}{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}\,dx$

**Optimal**. Leaf size=35

$$\text{Int}\left(\frac{(f+gx)^2}{\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2},x\right)$$

```
[Out] Unintegrable((g*x+f)^2/(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2,x)
```

**Rubi [A]** time = 0.21, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{(f+gx)^2}{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(f + g*x)^2/(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2,x]
```

```
[Out] f^2*Defer[Int][(A + B*Log[e*((a + b*x)/(c + d*x))^n])^(-2), x] + 2*f*g*Defe
r[Int][x/(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2, x] + g^2*Defer[Int][x^2/
(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2, x]
```

Rubi steps

$$\int \frac{(f+gx)^2}{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}\,dx = \int \left(\frac{f^2}{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}+\frac{2fgx}{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}+\frac{g^2x^2}{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}\right.$$

$$= f^2\int \frac{1}{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}\,dx+(2fg)\int \frac{x}{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}\,dx+$$

**Mathematica** [A] time = 0.94, size = 0, normalized size = 0.00

$$\int \frac{(f+gx)^2}{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(f + g*x)^2/(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2,x]
```

```
[Out] Integrate[(f + g*x)^2/(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2, x]
```

**fricas** [A] time = 0.81, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{g^2x^2+2\,fgx+f^2}{B^2\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)^2+2\,AB\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^2/(A+B*log(e*((b*x+a)/(d*x+c))^n))^2,x, algorithm="fricas
")
```

```
[Out] integral((g^2*x^2 + 2*f*g*x + f^2)/(B^2*log(e*((b*x + a)/(d*x + c))^n)^2 +
2*A*B*log(e*((b*x + a)/(d*x + c))^n) + A^2), x)
```

**giac** [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(gx+f)^2}{\left(B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^2/(A+B*log(e*((b*x+a)/(d*x+c))^n))^2,x, algorithm="giac")
```

```
[Out] integrate((g*x + f)^2/(B*log(e*((b*x + a)/(d*x + c))^n) + A)^2, x)
```

**maple** [A] time = 0.27, size = 0, normalized size = 0.00

$$\int \frac{(gx+f)^2}{\left(B\ln\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x+f)^2/(B*ln(e*((b*x+a)/(d*x+c))^n)+A)^2,x)
```

```
[Out] int((g*x+f)^2/(B*ln(e*((b*x+a)/(d*x+c))^n)+A)^2,x)
```

**maxima [A]** time = 0.00, size = 0, normalized size = 0.00

$$-\frac{bdg^2x^4 + acf^2 + \left(adg^2 + \left(2\,dfg + cg^2\right)b\right)x^3 + \left(\left(2\,dfg + cg^2\right)a + \left(df^2 + 2\,cfg\right)b\right)x^2 + \left(bcf^2 + \left(df^2 + 2\,cfg\right)a\right.}{(bcn - adn)B^2\log\left((bx + a)^n\right) - (bcn - adn)B^2\log\left((dx + c)^n\right) + (bcn - adn)AB + \left(bcn\log(e) - adn\log(e)\right)B}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^2/(A+B*log(e*((b*x+a)/(d*x+c))^n))^2,x, algorithm="maxima
")
```

```
[Out] -(b*d*g^2*x^4 + a*c*f^2 + (a*d*g^2 + (2*d*f*g + c*g^2)*b)*x^3 + ((2*d*f*g +
 c*g^2)*a + (d*f^2 + 2*c*f*g)*b)*x^2 + (b*c*f^2 + (d*f^2 + 2*c*f*g)*a)*x)/(
(b*c*n - a*d*n)*B^2*log((b*x + a)^n) - (b*c*n - a*d*n)*B^2*log((d*x + c)^n)
 + (b*c*n - a*d*n)*A*B + (b*c*n*log(e) - a*d*n*log(e))*B^2) + integrate((4*
b*d*g^2*x^3 + b*c*f^2 + 3*(a*d*g^2 + (2*d*f*g + c*g^2)*b)*x^2 + (d*f^2 + 2*
c*f*g)*a + 2*((2*d*f*g + c*g^2)*a + (d*f^2 + 2*c*f*g)*b)*x)/((b*c*n - a*d*n
)*B^2*log((b*x + a)^n) - (b*c*n - a*d*n)*B^2*log((d*x + c)^n) + (b*c*n - a*
d*n)*A*B + (b*c*n*log(e) - a*d*n*log(e))*B^2), x)
```

**mupad [A]** time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{{\left(f+g\,x\right)}^2}{{\left(A+B\,\ln\left(e\,{\left(\frac{a+b\,x}{c+d\,x}\right)}^n\right)\right)}^2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f + g*x)^2/(A + B*log(e*((a + b*x)/(c + d*x))^n))^2,x)
```

```
[Out] int((f + g*x)^2/(A + B*log(e*((a + b*x)/(c + d*x))^n))^2, x)
```

**sympy [A]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(f + gx\right)^{2}}{\left(A + B\log\left(e\left(\frac{a}{c+dx} + \frac{bx}{c+dx}\right)^{n}\right)\right)^{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)**2/(A+B*ln(e*((b*x+a)/(d*x+c))**n))**2,x)
```

```
[Out] Integral((f + g*x)**2/(A + B*log(e*(a/(c + d*x) + b*x/(c + d*x))**n))**2, x
)
```

# 3.83 $\int \frac{f+gx}{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}\,dx$

**Optimal**. Leaf size=33

$$\text{Int}\left(\frac{f+gx}{\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2},x\right)$$

```
[Out] Unintegrable((g*x+f)/(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2,x)
```

**Rubi [A]** time = 0.11, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{f+gx}{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(f + g*x)/(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2,x]
```

```
[Out] f*Defer[Int][(A + B*Log[e*((a + b*x)/(c + d*x))^n])^(-2), x] + g*Defer[Int]
[x/(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{f+gx}{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}\,dx &= \int \left(\frac{f}{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}+\frac{gx}{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}\right)dx\\
&= f\int \frac{1}{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}\,dx+g\int \frac{x}{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}\,dx
\end{aligned}$$

**Mathematica [A]** time = 0.74, size = 0, normalized size = 0.00

$$\int \frac{f+gx}{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(f + g*x)/(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2,x]
```

```
[Out] Integrate[(f + g*x)/(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2, x]
```

**fricas [A]** time = 0.90, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{gx+f}{B^2\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)^2+2\,AB\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)/(A+B*log(e*((b*x+a)/(d*x+c))^n))^2,x, algorithm="fricas")
```

```
[Out] integral((g*x + f)/(B^2*log(e*((b*x + a)/(d*x + c))^n)^2 + 2*A*B*log(e*((b*
x + a)/(d*x + c))^n) + A^2), x)
```

**giac [A]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{gx+f}{\left(B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)/(A+B*log(e*((b*x+a)/(d*x+c))^n))^2,x, algorithm="giac")
```

```
[Out] integrate((g*x + f)/(B*log(e*((b*x + a)/(d*x + c))^n) + A)^2, x)
```

**maple [A]** time = 0.06, size = 0, normalized size = 0.00

$$\int \frac{gx+f}{\left(B\ln\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x+f)/(B*ln(e*((b*x+a)/(d*x+c))^n)+A)^2,x)
```

```
[Out] int((g*x+f)/(B*ln(e*((b*x+a)/(d*x+c))^n)+A)^2,x)
```

**maxima [A]** time = 0.00, size = 0, normalized size = 0.00

$$-\frac{bdgx^3+acf+\left(adg+\left(df+cg\right)b\right)x^2+\left(bcf+\left(df+cg\right)a\right)x}{(bcn-adn)B^2\log\left((bx+a)^n\right)-(bcn-adn)B^2\log\left((dx+c)^n\right)+(bcn-adn)AB+\left(bcn\log(e)-adn\log(e)\right)B^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)/(A+B*log(e*((b*x+a)/(d*x+c))^n))^2,x, algorithm="maxima")
```

```
[Out] -(b*d*g*x^3 + a*c*f + (a*d*g + (d*f + c*g)*b)*x^2 + (b*c*f + (d*f + c*g)*a)
*x)/((b*c*n - a*d*n)*B^2*log((b*x + a)^n) - (b*c*n - a*d*n)*B^2*log((d*x +
c)^n) + (b*c*n - a*d*n)*A*B + (b*c*n*log(e) - a*d*n*log(e))*B^2) + integrat
e((3*b*d*g*x^2 + b*c*f + (d*f + c*g)*a + 2*(a*d*g + (d*f + c*g)*b)*x)/((b*c
*n - a*d*n)*B^2*log((b*x + a)^n) - (b*c*n - a*d*n)*B^2*log((d*x + c)^n) + (
b*c*n - a*d*n)*A*B + (b*c*n*log(e) - a*d*n*log(e))*B^2), x)
```

**mupad** [A] time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{f+g\,x}{\left(A+B\,\ln\left(e\,{\left(\frac{a+b\,x}{c+d\,x}\right)}^n\right)\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f + g*x)/(A + B*log(e*((a + b*x)/(c + d*x))^n))^2,x)
```

```
[Out] int((f + g*x)/(A + B*log(e*((a + b*x)/(c + d*x))^n))^2, x)
```

**sympy** [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{f+gx}{\left(A+B\log\left(e\left(\frac{a}{c+dx}+\frac{bx}{c+dx}\right)^n\right)\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)/(A+B*ln(e*((b*x+a)/(d*x+c))**n))**2,x)
```

```
[Out] Integral((f + g*x)/(A + B*log(e*(a/(c + d*x) + b*x/(c + d*x))**n))**2, x)
```

## 3.84 $\int \frac{1}{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}\,dx$

**Optimal**. Leaf size=27

$$\text{Int}\left(\frac{1}{\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2},x\right)$$

```
[Out] Unintegrable(1/(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2,x)
```

**Rubi [A]** time = 0.01, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{1}{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(A + B*Log[e*((a + b*x)/(c + d*x))^n])^(-2),x]
[Out] Defer[Int][(A + B*Log[e*((a + b*x)/(c + d*x))^n])^(-2), x]
```

Rubi steps

$$\int \frac{1}{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}\,dx=\int \frac{1}{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}\,dx$$

**Mathematica [A]** time = 0.67, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(A + B*Log[e*((a + b*x)/(c + d*x))^n])^(-2),x]
```

```
[Out] Integrate[(A + B*Log[e*((a + b*x)/(c + d*x))^n])^(-2), x]
```

**fricas [A]** time = 0.88, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{1}{B^2\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)^2+2\,AB\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(A+B*log(e*((b*x+a)/(d*x+c))^n))^2,x, algorithm="fricas")
```

```
[Out] integral(1/(B^2*log(e*((b*x + a)/(d*x + c))^n)^2 + 2*A*B*log(e*((b*x + a)/(
d*x + c))^n) + A^2), x)
```

**giac [A]** time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{1}{\left(B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(A+B*log(e*((b*x+a)/(d*x+c))^n))^2,x, algorithm="giac")
```

```
[Out] integrate((B*log(e*((b*x + a)/(d*x + c))^n) + A)^(-2), x)
```

**maple [A]** time = 0.14, size = 0, normalized size = 0.00

$$\int\frac{1}{\left(B\ln\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(B*ln(e*((b*x+a)/(d*x+c))^n)+A)^2,x)
```

```
[Out] int(1/(B*ln(e*((b*x+a)/(d*x+c))^n)+A)^2,x)
```

**maxima [A]** time = 0.00, size = 0, normalized size = 0.00

$$-\frac{bdx^2+ac+(bc+ad)x}{(bcn-adn)B^2\log\left((bx+a)^n\right)-(bcn-adn)B^2\log\left((dx+c)^n\right)+(bcn-adn)AB+\left(bcn\log(e)-adn\log(e)\right)B^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(A+B*log(e*((b*x+a)/(d*x+c))^n))^2,x, algorithm="maxima")
```

```
[Out] -(b*d*x^2 + a*c + (b*c + a*d)*x)/((b*c*n - a*d*n)*B^2*log((b*x + a)^n) - (b
*c*n - a*d*n)*B^2*log((d*x + c)^n) + (b*c*n - a*d*n)*A*B + (b*c*n*log(e) -
a*d*n*log(e))*B^2) + integrate((2*b*d*x + b*c + a*d)/((b*c*n - a*d*n)*B^2*l
og((b*x + a)^n) - (b*c*n - a*d*n)*B^2*log((d*x + c)^n) + (b*c*n - a*d*n)*A*
B + (b*c*n*log(e) - a*d*n*log(e))*B^2), x)
```

**mupad [A]** time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{1}{\left(A+B\,\ln\left(e\,{\left(\frac{a+b\,x}{c+d\,x}\right)}^n\right)\right)}^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(A + B*log(e*((a + b*x)/(c + d*x))^n))^2,x)
```

```
[Out] int(1/(A + B*log(e*((a + b*x)/(c + d*x))^n))^2, x)
```

**sympy [A]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^{n}\right)\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(A+B*ln(e*((b*x+a)/(d*x+c))**n))**2,x)
```

```
[Out] Integral((A + B*log(e*((a + b*x)/(c + d*x))**n))**(-2), x)
```

# 3.85 $\int \frac{1}{(f+gx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}\,dx$

**Optimal**. Leaf size=35

$$\text{Int}\left(\frac{1}{(f+gx)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2},x\right)$$

```
[Out] Unintegrable(1/(g*x+f)/(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2,x)
```

**Rubi [A]** time = 0.08, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{1}{(f+gx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[1/((f + g*x)*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2),x]
[Out] Defer[Int][1/((f + g*x)*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2), x]
```

Rubi steps

$$\int \frac{1}{(f+gx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}\,dx=\int \frac{1}{(f+gx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}\,dx$$

**Mathematica [A]** time = 1.71, size = 0, normalized size = 0.00

$$\int \frac{1}{(f+gx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[1/((f + g*x)*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2),x]
```

```
[Out] Integrate[1/((f + g*x)*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2), x]
```

**fricas** [A] time = 0.86, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{1}{A^2gx + A^2f + \left(B^2gx + B^2f\right)\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)^2 + 2\left(ABgx + ABf\right)\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(g*x+f)/(A+B*log(e*((b*x+a)/(d*x+c))^n))^2,x, algorithm="fricas
")
```

```
[Out] integral(1/(A^2*g*x + A^2*f + (B^2*g*x + B^2*f)*log(e*((b*x + a)/(d*x + c))
^n)^2 + 2*(A*B*g*x + A*B*f)*log(e*((b*x + a)/(d*x + c))^n)), x)
```

**giac** [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(gx + f\right)\left(B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right) + A\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(g*x+f)/(A+B*log(e*((b*x+a)/(d*x+c))^n))^2,x, algorithm="giac")
```

```
[Out] integrate(1/((g*x + f)*(B*log(e*((b*x + a)/(d*x + c))^n) + A)^2), x)
```

**maple** [A] time = 0.30, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(gx + f\right)\left(B\ln\left(e\left(\frac{bx+a}{dx+c}\right)^n\right) + A\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(g*x+f)/(B*ln(e*((b*x+a)/(d*x+c))^n)+A)^2,x)
```

```
[Out] int(1/(g*x+f)/(B*ln(e*((b*x+a)/(d*x+c))^n)+A)^2,x)
```

**maxima** [A] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{bdx^2 + ac}{\left(bcfn - adfn\right)AB + \left(bcfn\log(e) - adfn\log(e)\right)B^2 + \left(\left(bcgn - adgn\right)AB + \left(bcgn\log(e) - adgn\log(e)\right)B^2\right)x +}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(g*x+f)/(A+B*log(e*((b*x+a)/(d*x+c))^n))^2,x, algorithm="maxima
")
```

```
[Out] -(b*d*x^2 + a*c + (b*c + a*d)*x)/((b*c*f*n - a*d*f*n)*A*B + (b*c*f*n*log(e)
 - a*d*f*n*log(e))*B^2 + ((b*c*g*n - a*d*g*n)*A*B + (b*c*g*n*log(e) - a*d*g
*n*log(e))*B^2)*x + ((b*c*g*n - a*d*g*n)*B^2*x + (b*c*f*n - a*d*f*n)*B^2)*l
og((b*x + a)^n) - ((b*c*g*n - a*d*g*n)*B^2*x + (b*c*f*n - a*d*f*n)*B^2)*log
((d*x + c)^n)) + integrate((b*d*g*x^2 + 2*b*d*f*x + b*c*f + (d*f - c*g)*a)/
((b*c*f^2*n - a*d*f^2*n)*A*B + (b*c*f^2*n*log(e) - a*d*f^2*n*log(e))*B^2 +
((b*c*g^2*n - a*d*g^2*n)*A*B + (b*c*g^2*n*log(e) - a*d*g^2*n*log(e))*B^2)*x
^2 + 2*((b*c*f*g*n - a*d*f*g*n)*A*B + (b*c*f*g*n*log(e) - a*d*f*g*n*log(e))
*B^2)*x + ((b*c*g^2*n - a*d*g^2*n)*B^2*x^2 + 2*(b*c*f*g*n - a*d*f*g*n)*B^2*
x + (b*c*f^2*n - a*d*f^2*n)*B^2)*log((b*x + a)^n) - ((b*c*g^2*n - a*d*g^2*n
)*B^2*x^2 + 2*(b*c*f*g*n - a*d*f*g*n)*B^2*x + (b*c*f^2*n - a*d*f^2*n)*B^2)*
log((d*x + c)^n)), x)
```

**mupad [A]** time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{1}{\left(f+g\,x\right)\,{\left(A+B\,\ln\left(e\,{\left(\frac{a+b\,x}{c+d\,x}\right)}^n\right)\right)}^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((f + g*x)*(A + B*log(e*((a + b*x)/(c + d*x))^n))^2),x)
```

```
[Out] int(1/((f + g*x)*(A + B*log(e*((a + b*x)/(c + d*x))^n))^2), x)
```

**sympy [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(g*x+f)/(A+B*ln(e*((b*x+a)/(d*x+c))**n))**2,x)
```

```
[Out] Timed out
```

# 3.86 $\int \frac{1}{(f+gx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}\,dx$

**Optimal**. Leaf size=35

$$\operatorname{Int}\left(\frac{1}{(f+gx)^2\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2},x\right)$$

[Out] Unintegrable(1/(g*x+f)^2/(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2,x)

**Rubi** [A] time = 0.08, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{1}{(f+gx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}\,dx$$

Verification is Not applicable to the result.

[In] Int[1/((f + g*x)^2*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2),x]

[Out] Defer[Int][1/((f + g*x)^2*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2), x]

Rubi steps

$$\int \frac{1}{(f+gx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}\,dx=\int \frac{1}{(f+gx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}\,dx$$

**Mathematica** [A] time = 3.82, size = 0, normalized size = 0.00

$$\int \frac{1}{(f+gx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[1/((f + g*x)^2*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2),x]
```

```
[Out] Integrate[1/((f + g*x)^2*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2), x]
```

**fricas [A]** time = 1.22, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{1}{A^2g^2x^2 + 2\,A^2fgx + A^2f^2 + \left(B^2g^2x^2 + 2\,B^2fgx + B^2f^2\right)\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)^2 + 2\left(ABg^2x^2 + 2\,ABfgx + \right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(g*x+f)^2/(A+B*log(e*((b*x+a)/(d*x+c))^n))^2,x, algorithm="fric
as")
```

```
[Out] integral(1/(A^2*g^2*x^2 + 2*A^2*f*g*x + A^2*f^2 + (B^2*g^2*x^2 + 2*B^2*f*g*
x + B^2*f^2)*log(e*((b*x + a)/(d*x + c))^n)^2 + 2*(A*B*g^2*x^2 + 2*A*B*f*g*
x + A*B*f^2)*log(e*((b*x + a)/(d*x + c))^n)), x)
```

**giac [A]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(gx+f\right)^2\left(B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(g*x+f)^2/(A+B*log(e*((b*x+a)/(d*x+c))^n))^2,x, algorithm="giac
")
```

```
[Out] integrate(1/((g*x + f)^2*(B*log(e*((b*x + a)/(d*x + c))^n) + A)^2), x)
```

**maple [A]** time = 0.28, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(gx+f\right)^2\left(B\ln\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(g*x+f)^2/(B*ln(e*((b*x+a)/(d*x+c))^n)+A)^2,x)
```

```
[Out] int(1/(g*x+f)^2/(B*ln(e*((b*x+a)/(d*x+c))^n)+A)^2,x)
```

**maxima** [A] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{}{\left(bcf^2n - adf^2n\right)AB + \left(bcf^2n\log(e) - adf^2n\log(e)\right)B^2 + \left(\left(bcg^2n - adg^2n\right)AB + \left(bcg^2n\log(e) - adg^2n\log(e)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(g*x+f)^2/(A+B*log(e*((b*x+a)/(d*x+c))^n))^2,x, algorithm="maxi
ma")
```

```
[Out] -(b*d*x^2 + a*c + (b*c + a*d)*x)/((b*c*f^2*n - a*d*f^2*n)*A*B + (b*c*f^2*n*
log(e) - a*d*f^2*n*log(e))*B^2 + ((b*c*g^2*n - a*d*g^2*n)*A*B + (b*c*g^2*n*
log(e) - a*d*g^2*n*log(e))*B^2)*x^2 + 2*((b*c*f*g*n - a*d*f*g*n)*A*B + (b*c
*f*g*n*log(e) - a*d*f*g*n*log(e))*B^2)*x + ((b*c*g^2*n - a*d*g^2*n)*B^2*x^2
 + 2*(b*c*f*g*n - a*d*f*g*n)*B^2*x + (b*c*f^2*n - a*d*f^2*n)*B^2)*log((b*x
+ a)^n) - ((b*c*g^2*n - a*d*g^2*n)*B^2*x^2 + 2*(b*c*f*g*n - a*d*f*g*n)*B^2*
x + (b*c*f^2*n - a*d*f^2*n)*B^2)*log((d*x + c)^n)) - integrate(-(b*c*f + (d
*f - 2*c*g)*a - (a*d*g - (2*d*f - c*g)*b)*x)/(((b*c*g^3*n - a*d*g^3*n)*A*B
+ (b*c*g^3*n*log(e) - a*d*g^3*n*log(e))*B^2)*x^3 + (b*c*f^3*n - a*d*f^3*n)*
A*B + (b*c*f^3*n*log(e) - a*d*f^3*n*log(e))*B^2 + 3*((b*c*f*g^2*n - a*d*f*g
^2*n)*A*B + (b*c*f*g^2*n*log(e) - a*d*f*g^2*n*log(e))*B^2)*x^2 + 3*((b*c*f^
2*g*n - a*d*f^2*g*n)*A*B + (b*c*f^2*g*n*log(e) - a*d*f^2*g*n*log(e))*B^2)*x
 + ((b*c*g^3*n - a*d*g^3*n)*B^2*x^3 + 3*(b*c*f*g^2*n - a*d*f*g^2*n)*B^2*x^2
 + 3*(b*c*f^2*g*n - a*d*f^2*g*n)*B^2*x + (b*c*f^3*n - a*d*f^3*n)*B^2)*log((
b*x + a)^n) - ((b*c*g^3*n - a*d*g^3*n)*B^2*x^3 + 3*(b*c*f*g^2*n - a*d*f*g^2
*n)*B^2*x^2 + 3*(b*c*f^2*g*n - a*d*f^2*g*n)*B^2*x + (b*c*f^3*n - a*d*f^3*n)
*B^2)*log((d*x + c)^n)), x)
```

**mupad** [A] time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{1}{{\left(f+g\,x\right)}^2\,{\left(A+B\,\ln\left(e\,{\left(\frac{a+b\,x}{c+d\,x}\right)}^n\right)\right)}^2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((f + g*x)^2*(A + B*log(e*((a + b*x)/(c + d*x))^n))^2),x)
```

```
[Out] int(1/((f + g*x)^2*(A + B*log(e*((a + b*x)/(c + d*x))^n))^2), x)
```

**sympy** [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(g*x+f)**2/(A+B*ln(e*((b*x+a)/(d*x+c))**n))**2,x)

[Out] Timed out
```

## 3.87 $\int \frac{1}{(f+gx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2} dx$

**Optimal**. Leaf size=35

$$\text{Int}\left(\frac{1}{(f+gx)^3\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2},x\right)$$

```
[Out] Unintegrable(1/(g*x+f)^3/(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2,x)
```

**Rubi [A]** time = 0.08, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{1}{(f+gx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2} dx$$

Verification is Not applicable to the result.

```
[In] Int[1/((f + g*x)^3*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2),x]
```

```
[Out] Defer[Int][1/((f + g*x)^3*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2), x]
```

Rubi steps

$$\int \frac{1}{(f+gx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2} dx = \int \frac{1}{(f+gx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2} dx$$

**Mathematica [A]** time = 35.46, size = 0, normalized size = 0.00

$$\int \frac{1}{(f+gx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2} dx$$

Verification is Not applicable to the result.

```
[In] Integrate[1/((f + g*x)^3*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2),x]
```

```
[Out] Integrate[1/((f + g*x)^3*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2), x]
```

**fricas [A]** time = 0.83, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{1}{A^2g^3x^3 + 3\,A^2fg^2x^2 + 3\,A^2f^2gx + A^2f^3 + \left(B^2g^3x^3 + 3\,B^2fg^2x^2 + 3\,B^2f^2gx + B^2f^3\right)\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(g*x+f)^3/(A+B*log(e*((b*x+a)/(d*x+c))^n))^2,x, algorithm="fric
as")
```

```
[Out] integral(1/(A^2*g^3*x^3 + 3*A^2*f*g^2*x^2 + 3*A^2*f^2*g*x + A^2*f^3 + (B^2*
g^3*x^3 + 3*B^2*f*g^2*x^2 + 3*B^2*f^2*g*x + B^2*f^3)*log(e*((b*x + a)/(d*x
+ c))^n)^2 + 2*(A*B*g^3*x^3 + 3*A*B*f*g^2*x^2 + 3*A*B*f^2*g*x + A*B*f^3)*lo
g(e*((b*x + a)/(d*x + c))^n)), x)
```

**giac [A]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(gx+f)^3\left(B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(g*x+f)^3/(A+B*log(e*((b*x+a)/(d*x+c))^n))^2,x, algorithm="giac
")
```

```
[Out] integrate(1/((g*x + f)^3*(B*log(e*((b*x + a)/(d*x + c))^n) + A)^2), x)
```

**maple [A]** time = 0.29, size = 0, normalized size = 0.00

$$\int \frac{1}{(gx+f)^3\left(B\ln\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(g*x+f)^3/(B*ln(e*((b*x+a)/(d*x+c))^n)+A)^2,x)
```

```
[Out] int(1/(g*x+f)^3/(B*ln(e*((b*x+a)/(d*x+c))^n)+A)^2,x)
```

**maxima [A]** time = 0.00, size = 0, normalized size = 0.00

$$-\frac{}{\left(\left(bcg^3n-adg^3n\right)AB+\left(bcg^3n\log(e)-adg^3n\log(e)\right)B^2\right)x^3+\left(bcf^3n-adf^3n\right)AB+\left(bcf^3n\log(e)-adf^3n\log\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(g*x+f)^3/(A+B*log(e*((b*x+a)/(d*x+c))^n))^2,x, algorithm="maxi
ma")
```

```
[Out] -(b*d*x^2 + a*c + (b*c + a*d)*x)/(((b*c*g^3*n - a*d*g^3*n)*A*B + (b*c*g^3*n
*log(e) - a*d*g^3*n*log(e))*B^2)*x^3 + (b*c*f^3*n - a*d*f^3*n)*A*B + (b*c*f
^3*n*log(e) - a*d*f^3*n*log(e))*B^2 + 3*((b*c*f*g^2*n - a*d*f*g^2*n)*A*B +
(b*c*f*g^2*n*log(e) - a*d*f*g^2*n*log(e))*B^2)*x^2 + 3*((b*c*f^2*g*n - a*d*
f^2*g*n)*A*B + (b*c*f^2*g*n*log(e) - a*d*f^2*g*n*log(e))*B^2)*x + ((b*c*g^3
*n - a*d*g^3*n)*B^2*x^3 + 3*(b*c*f*g^2*n - a*d*f*g^2*n)*B^2*x^2 + 3*(b*c*f^
2*g*n - a*d*f^2*g*n)*B^2*x + (b*c*f^3*n - a*d*f^3*n)*B^2)*log((b*x + a)^n)
- ((b*c*g^3*n - a*d*g^3*n)*B^2*x^3 + 3*(b*c*f*g^2*n - a*d*f*g^2*n)*B^2*x^2
+ 3*(b*c*f^2*g*n - a*d*f^2*g*n)*B^2*x + (b*c*f^3*n - a*d*f^3*n)*B^2)*log((d
*x + c)^n)) - integrate((b*d*g*x^2 - b*c*f - (d*f - 3*c*g)*a + 2*(a*d*g - (
d*f - c*g)*b)*x)/(((b*c*g^4*n - a*d*g^4*n)*A*B + (b*c*g^4*n*log(e) - a*d*g^
4*n*log(e))*B^2)*x^4 + 4*((b*c*f*g^3*n - a*d*f*g^3*n)*A*B + (b*c*f*g^3*n*lo
g(e) - a*d*f*g^3*n*log(e))*B^2)*x^3 + (b*c*f^4*n - a*d*f^4*n)*A*B + (b*c*f^
4*n*log(e) - a*d*f^4*n*log(e))*B^2 + 6*((b*c*f^2*g^2*n - a*d*f^2*g^2*n)*A*B
 + (b*c*f^2*g^2*n*log(e) - a*d*f^2*g^2*n*log(e))*B^2)*x^2 + 4*((b*c*f^3*g*n
 - a*d*f^3*g*n)*A*B + (b*c*f^3*g*n*log(e) - a*d*f^3*g*n*log(e))*B^2)*x + ((
b*c*g^4*n - a*d*g^4*n)*B^2*x^4 + 4*(b*c*f*g^3*n - a*d*f*g^3*n)*B^2*x^3 + 6*
(b*c*f^2*g^2*n - a*d*f^2*g^2*n)*B^2*x^2 + 4*(b*c*f^3*g*n - a*d*f^3*g*n)*B^2
*x + (b*c*f^4*n - a*d*f^4*n)*B^2)*log((b*x + a)^n) - ((b*c*g^4*n - a*d*g^4*
n)*B^2*x^4 + 4*(b*c*f*g^3*n - a*d*f*g^3*n)*B^2*x^3 + 6*(b*c*f^2*g^2*n - a*d
*f^2*g^2*n)*B^2*x^2 + 4*(b*c*f^3*g*n - a*d*f^3*g*n)*B^2*x + (b*c*f^4*n - a*
d*f^4*n)*B^2)*log((d*x + c)^n)), x)
```

**mupad [A]** time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{1}{\left(f+g\,x\right)^3\,\left(A+B\,\ln\left(e\,\left(\frac{a+b\,x}{c+d\,x}\right)^n\right)\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((f + g*x)^3*(A + B*log(e*((a + b*x)/(c + d*x))^n))^2),x)
```

```
[Out] int(1/((f + g*x)^3*(A + B*log(e*((a + b*x)/(c + d*x))^n))^2), x)
```

<u>**sympy**</u> **[F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(g*x+f)**3/(A+B*ln(e*((b*x+a)/(d*x+c))**n))**2,x)
[Out] Timed out
```

# 3.88 $\int (ag+bgx)^4 \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right) dx$

**Optimal**. Leaf size=180

$$\frac{g^4(a+bx)^5\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{5b}-\frac{Bg^4(bc-ad)^5\log(c+dx)}{5bd^5}+\frac{Bg^4x(bc-ad)^4}{5d^4}-\frac{Bg^4(a+bx)^2(bc-ad)^3}{10bd^3}+\frac{Bg^4(a+$$

```
[Out] 1/5*B*(-a*d+b*c)^4*g^4*x/d^4-1/10*B*(-a*d+b*c)^3*g^4*(b*x+a)^2/b/d^3+1/15*B
*(-a*d+b*c)^2*g^4*(b*x+a)^3/b/d^2-1/20*B*(-a*d+b*c)*g^4*(b*x+a)^4/b/d+1/5*g
^4*(b*x+a)^5*(A+B*ln(e*(b*x+a)/(d*x+c)))/b-1/5*B*(-a*d+b*c)^5*g^4*ln(d*x+c)
/b/d^5
```

**Rubi [A]** time = 0.12, antiderivative size = 180, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {2525, 12, 43}

$$\frac{g^4(a+bx)^5\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{5b}+\frac{Bg^4x(bc-ad)^4}{5d^4}-\frac{Bg^4(a+bx)^2(bc-ad)^3}{10bd^3}+\frac{Bg^4(a+bx)^3(bc-ad)^2}{15bd^2}-\frac{Bg^4(bc-ad}{5}$$

Antiderivative was successfully verified.

```
[In] Int[(a*g + b*g*x)^4*(A + B*Log[(e*(a + b*x))/(c + d*x)]),x]
```

```
[Out] (B*(b*c - a*d)^4*g^4*x)/(5*d^4) - (B*(b*c - a*d)^3*g^4*(a + b*x)^2)/(10*b*d
^3) + (B*(b*c - a*d)^2*g^4*(a + b*x)^3)/(15*b*d^2) - (B*(b*c - a*d)*g^4*(a
+ b*x)^4)/(20*b*d) + (g^4*(a + b*x)^5*(A + B*Log[(e*(a + b*x))/(c + d*x)]))
/(5*b) - (B*(b*c - a*d)^5*g^4*Log[c + d*x])/(5*b*d^5)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2525

```
Int[((a_.) + Log[(c_.)*(RFx_)^(p_.)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_))^(m_.
), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(a + b*Log[c*RFx^p])^n)/(e*(m + 1))
```

```
, x] - Dist[(b*n*p)/(e*(m + 1)), Int[SimplifyIntegrand[((d + e*x)^(m + 1)*(
a + b*Log[c*RFx^p])^(n - 1)*D[RFx, x])/RFx, x], x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && RationalFunctionQ[RFx, x] && IGtQ[n, 0] && (EqQ[n, 1] ||
IntegerQ[m]) && NeQ[m, -1]
```

Rubi steps

$$\int (ag+bgx)^4\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)dx=\frac{g^4(a+bx)^5\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{5b}-\frac{B\int\frac{(bc-ad)g^5(a+bx)^4}{c+dx}dx}{5bg}$$

$$=\frac{g^4(a+bx)^5\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{5b}-\frac{\left(B(bc-ad)g^4\right)\int\frac{(a+bx)^4}{c+dx}dx}{5b}$$

$$=\frac{g^4(a+bx)^5\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{5b}-\frac{\left(B(bc-ad)g^4\right)\int\left(-\frac{b(bc-ad)^3}{d^4}+\right.}{}$$

$$=\frac{B(bc-ad)^4g^4x}{5d^4}-\frac{B(bc-ad)^3g^4(a+bx)^2}{10bd^3}+\frac{B(bc-ad)^2g^4(a+bx)}{15bd^2}$$

**Mathematica [A]** time = 0.10, size = 142, normalized size = 0.79

$$\frac{g^4\left((a+bx)^5\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)-\frac{B(bc-ad)\left(4d^3(a+bx)^3(ad-bc)+6d^2(a+bx)^2(bc-ad)^2-12bdx(bc-ad)^3+12(bc-ad)^4\log(c+dx)+3d^4(a+\right.}{12d^5}}{5b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*g + b*g*x)^4*(A + B*Log[(e*(a + b*x))/(c + d*x)]),x]
```

```
[Out] (g^4*((a + b*x)^5*(A + B*Log[(e*(a + b*x))/(c + d*x)]) - (B*(b*c - a*d)*(-1
2*b*d*(b*c - a*d)^3*x + 6*d^2*(b*c - a*d)^2*(a + b*x)^2 + 4*d^3*(-(b*c) + a
*d)*(a + b*x)^3 + 3*d^4*(a + b*x)^4 + 12*(b*c - a*d)^4*Log[c + d*x]))/(12*d
^5)))/(5*b)
```

**fricas [B]** time = 1.06, size = 431, normalized size = 2.39

$$\frac{12\,Ab^5d^5g^4x^5+12\,Ba^5d^5g^4\log(bx+a)-3\left(Bb^5cd^4-(20\,A+B)ab^4d^5\right)g^4x^4+4\left(Bb^5c^2d^3-5\,Bab^4cd^4+2\,(15\,A\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*g*x+a*g)^4*(A+B*log(e*(b*x+a)/(d*x+c))),x, algorithm="fricas")
```

```
[Out] 1/60*(12*A*b^5*d^5*g^4*x^5 + 12*B*a^5*d^5*g^4*log(b*x + a) - 3*(B*b^5*c*d^4
 - (20*A + B)*a*b^4*d^5)*g^4*x^4 + 4*(B*b^5*c^2*d^3 - 5*B*a*b^4*c*d^4 + 2*(
15*A + 2*B)*a^2*b^3*d^5)*g^4*x^3 - 6*(B*b^5*c^3*d^2 - 5*B*a*b^4*c^2*d^3 + 1
0*B*a^2*b^3*c*d^4 - 2*(10*A + 3*B)*a^3*b^2*d^5)*g^4*x^2 + 12*(B*b^5*c^4*d -
 5*B*a*b^4*c^3*d^2 + 10*B*a^2*b^3*c^2*d^3 - 10*B*a^3*b^2*c*d^4 + (5*A + 4*B
)*a^4*b*d^5)*g^4*x - 12*(B*b^5*c^5 - 5*B*a*b^4*c^4*d + 10*B*a^2*b^3*c^3*d^2
 - 10*B*a^3*b^2*c^2*d^3 + 5*B*a^4*b*c*d^4)*g^4*log(d*x + c) + 12*(B*b^5*d^5
*g^4*x^5 + 5*B*a*b^4*d^5*g^4*x^4 + 10*B*a^2*b^3*d^5*g^4*x^3 + 10*B*a^3*b^2*
d^5*g^4*x^2 + 5*B*a^4*b*d^5*g^4*x)*log((b*e*x + a*e)/(d*x + c)))/(b*d^5)
```

**giac [B]** time = 2.56, size = 5428, normalized size = 30.16

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*g*x+a*g)^4*(A+B*log(e*(b*x+a)/(d*x+c))),x, algorithm="giac")
```

```
[Out] 1/60*(12*B*b^11*c^6*g^4*e^6*log(-b*e + (b*x*e + a*e)*d/(d*x + c)) - 72*B*a*
b^10*c^5*d*g^4*e^6*log(-b*e + (b*x*e + a*e)*d/(d*x + c)) + 180*B*a^2*b^9*c^
4*d^2*g^4*e^6*log(-b*e + (b*x*e + a*e)*d/(d*x + c)) - 240*B*a^3*b^8*c^3*d^3
*g^4*e^6*log(-b*e + (b*x*e + a*e)*d/(d*x + c)) + 180*B*a^4*b^7*c^2*d^4*g^4*
e^6*log(-b*e + (b*x*e + a*e)*d/(d*x + c)) - 72*B*a^5*b^6*c*d^5*g^4*e^6*log(
-b*e + (b*x*e + a*e)*d/(d*x + c)) + 12*B*a^6*b^5*d^6*g^4*e^6*log(-b*e + (b*
x*e + a*e)*d/(d*x + c)) - 60*(b*x*e + a*e)*B*b^10*c^6*d*g^4*e^5*log(-b*e +
(b*x*e + a*e)*d/(d*x + c))/(d*x + c) + 360*(b*x*e + a*e)*B*a*b^9*c^5*d^2*g^
4*e^5*log(-b*e + (b*x*e + a*e)*d/(d*x + c))/(d*x + c) - 900*(b*x*e + a*e)*B
*a^2*b^8*c^4*d^3*g^4*e^5*log(-b*e + (b*x*e + a*e)*d/(d*x + c))/(d*x + c) +
1200*(b*x*e + a*e)*B*a^3*b^7*c^3*d^4*g^4*e^5*log(-b*e + (b*x*e + a*e)*d/(d*
x + c))/(d*x + c) - 900*(b*x*e + a*e)*B*a^4*b^6*c^2*d^5*g^4*e^5*log(-b*e +
(b*x*e + a*e)*d/(d*x + c))/(d*x + c) + 360*(b*x*e + a*e)*B*a^5*b^5*c*d^6*g^
4*e^5*log(-b*e + (b*x*e + a*e)*d/(d*x + c))/(d*x + c) - 60*(b*x*e + a*e)*B*
a^6*b^4*d^7*g^4*e^5*log(-b*e + (b*x*e + a*e)*d/(d*x + c))/(d*x + c) + 120*(
b*x*e + a*e)^2*B*b^9*c^6*d^2*g^4*e^4*log(-b*e + (b*x*e + a*e)*d/(d*x + c))/
(d*x + c)^2 - 720*(b*x*e + a*e)^2*B*a*b^8*c^5*d^3*g^4*e^4*log(-b*e + (b*x*e
 + a*e)*d/(d*x + c))/(d*x + c)^2 + 1800*(b*x*e + a*e)^2*B*a^2*b^7*c^4*d^4*g
^4*e^4*log(-b*e + (b*x*e + a*e)*d/(d*x + c))/(d*x + c)^2 - 2400*(b*x*e + a*
e)^2*B*a^3*b^6*c^3*d^5*g^4*e^4*log(-b*e + (b*x*e + a*e)*d/(d*x + c))/(d*x +
 c)^2 + 1800*(b*x*e + a*e)^2*B*a^4*b^5*c^2*d^6*g^4*e^4*log(-b*e + (b*x*e +
a*e)*d/(d*x + c))/(d*x + c)^2 - 720*(b*x*e + a*e)^2*B*a^5*b^4*c*d^7*g^4*e^4
*log(-b*e + (b*x*e + a*e)*d/(d*x + c))/(d*x + c)^2 + 120*(b*x*e + a*e)^2*B*
a^6*b^3*d^8*g^4*e^4*log(-b*e + (b*x*e + a*e)*d/(d*x + c))/(d*x + c)^2 - 120
*(b*x*e + a*e)^3*B*b^8*c^6*d^3*g^4*e^3*log(-b*e + (b*x*e + a*e)*d/(d*x + c)
)/(d*x + c)^3 + 720*(b*x*e + a*e)^3*B*a*b^7*c^5*d^4*g^4*e^3*log(-b*e + (b*x
*e + a*e)*d/(d*x + c))/(d*x + c)^3 - 1800*(b*x*e + a*e)^3*B*a^2*b^6*c^4*d^5
*g^4*e^3*log(-b*e + (b*x*e + a*e)*d/(d*x + c))/(d*x + c)^3 + 2400*(b*x*e +
```

a*e)^3*B*a^3*b^5*c^3*d^6*g^4*e^3*log(-b*e + (b*x*e + a*e)*d/(d*x + c))/(d*x + c)^3 - 1800*(b*x*e + a*e)^3*B*a^4*b^4*c^2*d^7*g^4*e^3*log(-b*e + (b*x*e + a*e)*d/(d*x + c))/(d*x + c)^3 + 720*(b*x*e + a*e)^3*B*a^5*b^3*c*d^8*g^4*e^3*log(-b*e + (b*x*e + a*e)*d/(d*x + c))/(d*x + c)^3 - 120*(b*x*e + a*e)^3*B*a^6*b^2*d^9*g^4*e^3*log(-b*e + (b*x*e + a*e)*d/(d*x + c))/(d*x + c)^3 + 60*(b*x*e + a*e)^4*B*b^7*c^6*d^4*g^4*e^2*log(-b*e + (b*x*e + a*e)*d/(d*x + c))/(d*x + c)^4 - 360*(b*x*e + a*e)^4*B*a*b^6*c^5*d^5*g^4*e^2*log(-b*e + (b*x*e + a*e)*d/(d*x + c))/(d*x + c)^4 + 900*(b*x*e + a*e)^4*B*a^2*b^5*c^4*d^6*g^4*e^2*log(-b*e + (b*x*e + a*e)*d/(d*x + c))/(d*x + c)^4 - 1200*(b*x*e + a*e)^4*B*a^3*b^4*c^3*d^7*g^4*e^2*log(-b*e + (b*x*e + a*e)*d/(d*x + c))/(d*x + c)^4 + 900*(b*x*e + a*e)^4*B*a^4*b^3*c^2*d^8*g^4*e^2*log(-b*e + (b*x*e + a*e)*d/(d*x + c))/(d*x + c)^4 - 360*(b*x*e + a*e)^4*B*a^5*b^2*c*d^9*g^4*e^2*log(-b*e + (b*x*e + a*e)*d/(d*x + c))/(d*x + c)^4 + 60*(b*x*e + a*e)^4*B*a^6*b*d^10*g^4*e^2*log(-b*e + (b*x*e + a*e)*d/(d*x + c))/(d*x + c)^4 - 12*(b*x*e + a*e)^5*B*b^6*c^6*d^5*g^4*e*log(-b*e + (b*x*e + a*e)*d/(d*x + c))/(d*x + c)^5 + 72*(b*x*e + a*e)^5*B*a*b^5*c^5*d^6*g^4*e*log(-b*e + (b*x*e + a*e)*d/(d*x + c))/(d*x + c)^5 - 180*(b*x*e + a*e)^5*B*a^2*b^4*c^4*d^7*g^4*e*log(-b*e + (b*x*e + a*e)*d/(d*x + c))/(d*x + c)^5 + 240*(b*x*e + a*e)^5*B*a^3*b^3*c^3*d^8*g^4*e*log(-b*e + (b*x*e + a*e)*d/(d*x + c))/(d*x + c)^5 - 180*(b*x*e + a*e)^5*B*a^4*b^2*c^2*d^9*g^4*e*log(-b*e + (b*x*e + a*e)*d/(d*x + c))/(d*x + c)^5 + 72*(b*x*e + a*e)^5*B*a^5*b*c*d^10*g^4*e*log(-b*e + (b*x*e + a*e)*d/(d*x + c))/(d*x + c)^5 - 12*(b*x*e + a*e)^5*B*a^6*d^11*g^4*e*log(-b*e + (b*x*e + a*e)*d/(d*x + c))/(d*x + c)^5 + 12*(b*x*e + a*e)^5*B*b^6*c^6*d^5*g^4*e*log((b*x*e + a*e)/(d*x + c))/(d*x + c)^5 - 72*(b*x*e + a*e)^5*B*a*b^5*c^5*d^6*g^4*e*log((b*x*e + a*e)/(d*x + c))/(d*x + c)^5 + 180*(b*x*e + a*e)^5*B*a^2*b^4*c^4*d^7*g^4*e*log((b*x*e + a*e)/(d*x + c))/(d*x + c)^5 - 240*(b*x*e + a*e)^5*B*a^3*b^3*c^3*d^8*g^4*e*log((b*x*e + a*e)/(d*x + c))/(d*x + c)^5 + 180*(b*x*e + a*e)^5*B*a^4*b^2*c^2*d^9*g^4*e*log((b*x*e + a*e)/(d*x + c))/(d*x + c)^5 - 72*(b*x*e + a*e)^5*B*a^5*b*c*d^10*g^4*e*log((b*x*e + a*e)/(d*x + c))/(d*x + c)^5 + 12*(b*x*e + a*e)^5*B*a^6*d^11*g^4*e*log((b*x*e + a*e)/(d*x + c))/(d*x + c)^5 + 12*A*b^11*c^6*g^4*e^6 + 25*B*b^11*c^6*g^4*e^6 - 72*A*a*b^10*c^5*d*g^4*e^6 - 150*B*a*b^10*c^5*d*g^4*e^6 + 180*A*a^2*b^9*c^4*d^2*g^4*e^6 + 375*B*a^2*b^9*c^4*d^2*g^4*e^6 - 240*A*a^3*b^8*c^3*d^3*g^4*e^6 - 500*B*a^3*b^8*c^3*d^3*g^4*e^6 + 180*A*a^4*b^7*c^2*d^4*g^4*e^6 + 375*B*a^4*b^7*c^2*d^4*g^4*e^6 - 72*A*a^5*b^6*c*d^5*g^4*e^6 - 150*B*a^5*b^6*c*d^5*g^4*e^6 + 12*A*a^6*b^5*d^6*g^4*e^6 + 25*B*a^6*b^5*d^6*g^4*e^6 - 60*(b*x*e + a*e)*A*b^10*c^6*d*g^4*e^5/(d*x + c) - 113*(b*x*e + a*e)*B*b^10*c^6*d*g^4*e^5/(d*x + c) + 360*(b*x*e + a*e)*A*a*b^9*c^5*d^2*g^4*e^5/(d*x + c) + 678*(b*x*e + a*e)*B*a*b^9*c^5*d^2*g^4*e^5/(d*x + c) - 900*(b*x*e + a*e)*A*a^2*b^8*c^4*d^3*g^4*e^5/(d*x + c) - 1695*(b*x*e + a*e)*B*a^2*b^8*c^4*d^3*g^4*e^5/(d*x + c) + 1200*(b*x*e + a*e)*A*a^3*b^7*c^3*d^4*g^4*e^5/(d*x + c) + 2260*(b*x*e + a*e)*B*a^3*b^7*c^3*d^4*g^4*e^5/(d*x + c) - 900*(b*x*e + a*e)*A*a^4*b^6*c^2*d^5*g^4*e^5/(d*x + c) - 1695*(b*x*e + a*e)*B*a^4*b^6*c^2*d^5*g^4*e^5/(d*x + c) + 360*(b*x*e + a*e)*A*a^5*b^5*c*d^6*g^4*e^5/(d*x + c) + 678*(b*x*e + a*e)*B*a^5*b^5*c*d^6*g^4*e^5/(d*x + c) - 60*(b*x*e + a*e)*A*a^

```
6*b^4*d^7*g^4*e^5/(d*x + c) - 113*(b*x*e + a*e)*B*a^6*b^4*d^7*g^4*e^5/(d*x
+ c) + 120*(b*x*e + a*e)^2*A*b^9*c^6*d^2*g^4*e^4/(d*x + c)^2 + 196*(b*x*e +
 a*e)^2*B*b^9*c^6*d^2*g^4*e^4/(d*x + c)^2 - 720*(b*x*e + a*e)^2*A*a*b^8*c^5
*d^3*g^4*e^4/(d*x + c)^2 - 1176*(b*x*e + a*e)^2*B*a*b^8*c^5*d^3*g^4*e^4/(d*
x + c)^2 + 1800*(b*x*e + a*e)^2*A*a^2*b^7*c^4*d^4*g^4*e^4/(d*x + c)^2 + 294
0*(b*x*e + a*e)^2*B*a^2*b^7*c^4*d^4*g^4*e^4/(d*x + c)^2 - 2400*(b*x*e + a*e
)^2*A*a^3*b^6*c^3*d^5*g^4*e^4/(d*x + c)^2 - 3920*(b*x*e + a*e)^2*B*a^3*b^6*
c^3*d^5*g^4*e^4/(d*x + c)^2 + 1800*(b*x*e + a*e)^2*A*a^4*b^5*c^2*d^6*g^4*e^
4/(d*x + c)^2 + 2940*(b*x*e + a*e)^2*B*a^4*b^5*c^2*d^6*g^4*e^4/(d*x + c)^2
- 720*(b*x*e + a*e)^2*A*a^5*b^4*c*d^7*g^4*e^4/(d*x + c)^2 - 1176*(b*x*e + a
*e)^2*B*a^5*b^4*c*d^7*g^4*e^4/(d*x + c)^2 + 120*(b*x*e + a*e)^2*A*a^6*b^3*d
^8*g^4*e^4/(d*x + c)^2 + 196*(b*x*e + a*e)^2*B*a^6*b^3*d^8*g^4*e^4/(d*x + c
)^2 - 120*(b*x*e + a*e)^3*A*b^8*c^6*d^3*g^4*e^3/(d*x + c)^3 - 156*(b*x*e +
a*e)^3*B*b^8*c^6*d^3*g^4*e^3/(d*x + c)^3 + 720*(b*x*e + a*e)^3*A*a*b^7*c^5*
d^4*g^4*e^3/(d*x + c)^3 + 936*(b*x*e + a*e)^3*B*a*b^7*c^5*d^4*g^4*e^3/(d*x
+ c)^3 - 1800*(b*x*e + a*e)^3*A*a^2*b^6*c^4*d^5*g^4*e^3/(d*x + c)^3 - 2340*
(b*x*e + a*e)^3*B*a^2*b^6*c^4*d^5*g^4*e^3/(d*x + c)^3 + 2400*(b*x*e + a*e)^
3*A*a^3*b^5*c^3*d^6*g^4*e^3/(d*x + c)^3 + 3120*(b*x*e + a*e)^3*B*a^3*b^5*c^
3*d^6*g^4*e^3/(d*x + c)^3 - 1800*(b*x*e + a*e)^3*A*a^4*b^4*c^2*d^7*g^4*e^3/
(d*x + c)^3 - 2340*(b*x*e + a*e)^3*B*a^4*b^4*c^2*d^7*g^4*e^3/(d*x + c)^3 +
720*(b*x*e + a*e)^3*A*a^5*b^3*c*d^8*g^4*e^3/(d*x + c)^3 + 936*(b*x*e + a*e)
^3*B*a^5*b^3*c*d^8*g^4*e^3/(d*x + c)^3 - 120*(b*x*e + a*e)^3*A*a^6*b^2*d^9*
g^4*e^3/(d*x + c)^3 - 156*(b*x*e + a*e)^3*B*a^6*b^2*d^9*g^4*e^3/(d*x + c)^3
 + 60*(b*x*e + a*e)^4*A*b^7*c^6*d^4*g^4*e^2/(d*x + c)^4 + 48*(b*x*e + a*e)^
4*B*b^7*c^6*d^4*g^4*e^2/(d*x + c)^4 - 360*(b*x*e + a*e)^4*A*a*b^6*c^5*d^5*g
^4*e^2/(d*x + c)^4 - 288*(b*x*e + a*e)^4*B*a*b^6*c^5*d^5*g^4*e^2/(d*x + c)^
4 + 900*(b*x*e + a*e)^4*A*a^2*b^5*c^4*d^6*g^4*e^2/(d*x + c)^4 + 720*(b*x*e
+ a*e)^4*B*a^2*b^5*c^4*d^6*g^4*e^2/(d*x + c)^4 - 1200*(b*x*e + a*e)^4*A*a^3
*b^4*c^3*d^7*g^4*e^2/(d*x + c)^4 - 960*(b*x*e + a*e)^4*B*a^3*b^4*c^3*d^7*g^
4*e^2/(d*x + c)^4 + 900*(b*x*e + a*e)^4*A*a^4*b^3*c^2*d^8*g^4*e^2/(d*x + c)
^4 + 720*(b*x*e + a*e)^4*B*a^4*b^3*c^2*d^8*g^4*e^2/(d*x + c)^4 - 360*(b*x*e
 + a*e)^4*A*a^5*b^2*c*d^9*g^4*e^2/(d*x + c)^4 - 288*(b*x*e + a*e)^4*B*a^5*b
^2*c*d^9*g^4*e^2/(d*x + c)^4 + 60*(b*x*e + a*e)^4*A*a^6*b*d^10*g^4*e^2/(d*x
 + c)^4 + 48*(b*x*e + a*e)^4*B*a^6*b*d^10*g^4*e^2/(d*x + c)^4)*(b*c/((b*c*e
 - a*d*e)*(b*c - a*d)) - a*d/((b*c*e - a*d*e)*(b*c - a*d)))/(b^6*d^5*e^5 -
5*(b*x*e + a*e)*b^5*d^6*e^4/(d*x + c) + 10*(b*x*e + a*e)^2*b^4*d^7*e^3/(d*x
 + c)^2 - 10*(b*x*e + a*e)^3*b^3*d^8*e^2/(d*x + c)^3 + 5*(b*x*e + a*e)^4*b^
2*d^9*e/(d*x + c)^4 - (b*x*e + a*e)^5*b*d^10/(d*x + c)^5)
```

**maple [B]** time = 0.18, size = 8417, normalized size = 46.76

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*g*x+a*g)^4*(A+B*ln(e*(b*x+a)/(d*x+c))),x)
```

```
[Out] result too large to display
```

**maxima [B]** time = 1.32, size = 623, normalized size = 3.46

$$\frac{1}{5}Ab^4g^4x^5+Aab^3g^4x^4+2Aa^2b^2g^4x^3+2Aa^3bg^4x^2+\left(x\log\left(\frac{bex}{dx+c}+\frac{ae}{dx+c}\right)+\frac{a\log(bx+a)}{b}-\frac{c\log(dx+c)}{d}\right)B$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*g*x+a*g)^4*(A+B*log(e*(b*x+a)/(d*x+c))),x, algorithm="maxima")
```

```
[Out] 1/5*A*b^4*g^4*x^5 + A*a*b^3*g^4*x^4 + 2*A*a^2*b^2*g^4*x^3 + 2*A*a^3*b*g^4*x
^2 + (x*log(b*e*x/(d*x + c) + a*e/(d*x + c)) + a*log(b*x + a)/b - c*log(d*x
 + c)/d)*B*a^4*g^4 + 2*(x^2*log(b*e*x/(d*x + c) + a*e/(d*x + c)) - a^2*log(
b*x + a)/b^2 + c^2*log(d*x + c)/d^2 - (b*c - a*d)*x/(b*d))*B*a^3*b*g^4 + (2
*x^3*log(b*e*x/(d*x + c) + a*e/(d*x + c)) + 2*a^3*log(b*x + a)/b^3 - 2*c^3*
log(d*x + c)/d^3 - ((b^2*c*d - a*b*d^2)*x^2 - 2*(b^2*c^2 - a^2*d^2)*x)/(b^2
*d^2))*B*a^2*b^2*g^4 + 1/6*(6*x^4*log(b*e*x/(d*x + c) + a*e/(d*x + c)) - 6*
a^4*log(b*x + a)/b^4 + 6*c^4*log(d*x + c)/d^4 - (2*(b^3*c*d^2 - a*b^2*d^3)*
x^3 - 3*(b^3*c^2*d - a^2*b*d^3)*x^2 + 6*(b^3*c^3 - a^3*d^3)*x)/(b^3*d^3))*B
*a*b^3*g^4 + 1/60*(12*x^5*log(b*e*x/(d*x + c) + a*e/(d*x + c)) + 12*a^5*log
(b*x + a)/b^5 - 12*c^5*log(d*x + c)/d^5 - (3*(b^4*c*d^3 - a*b^3*d^4)*x^4 -
4*(b^4*c^2*d^2 - a^2*b^2*d^4)*x^3 + 6*(b^4*c^3*d - a^3*b*d^4)*x^2 - 12*(b^4
*c^4 - a^4*d^4)*x)/(b^4*d^4))*B*b^4*g^4 + A*a^4*g^4*x
```

**mupad [B]** time = 4.78, size = 1009, normalized size = 5.61

$$\ln\left(\frac{e\,(a+b\,x)}{c+d\,x}\right)\left(B\,a^4\,g^4\,x+2\,B\,a^3\,b\,g^4\,x^2+2\,B\,a^2\,b^2\,g^4\,x^3+B\,a\,b^3\,g^4\,x^4+\frac{B\,b^4\,g^4\,x^5}{5}\right)-x^3\left(\frac{\left(\frac{b^3\,g^4\,(25\,A\,a\,d+5\,A\,b\,c+B}{5\,d}\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*g + b*g*x)^4*(A + B*log((e*(a + b*x))/(c + d*x))),x)
```

```
[Out] log((e*(a + b*x))/(c + d*x))*((B*b^4*g^4*x^5)/5 + B*a^4*g^4*x + 2*B*a^3*b*g
^4*x^2 + B*a*b^3*g^4*x^4 + 2*B*a^2*b^2*g^4*x^3) - x^3*((((b^3*g^4*(25*A*a*d
```

```
 + 5*A*b*c + B*a*d - B*b*c))/(5*d) - (A*b^3*g^4*(5*a*d + 5*b*c))/(5*d))*(5*
a*d + 5*b*c))/(15*b*d) - (a*b^2*g^4*(10*A*a*d + 5*A*b*c + B*a*d - B*b*c))/(
3*d) + (A*a*b^3*c*g^4)/(3*d)) + x^2*(((5*a*d + 5*b*c)*((((b^3*g^4*(25*A*a*d
 + 5*A*b*c + B*a*d - B*b*c))/(5*d) - (A*b^3*g^4*(5*a*d + 5*b*c))/(5*d))*(5*
a*d + 5*b*c))/(5*b*d) - (a*b^2*g^4*(10*A*a*d + 5*A*b*c + B*a*d - B*b*c))/d
+ (A*a*b^3*c*g^4)/d))/(10*b*d) + (a^2*b*g^4*(5*A*a*d + 5*A*b*c + B*a*d - B*
b*c))/d - (a*c*((b^3*g^4*(25*A*a*d + 5*A*b*c + B*a*d - B*b*c))/(5*d) - (A*b
^3*g^4*(5*a*d + 5*b*c))/(5*d)))/(2*b*d)) + x*((a^3*g^4*(5*A*a*d + 10*A*b*c
+ 2*B*a*d - 2*B*b*c))/d - ((5*a*d + 5*b*c)*(((5*a*d + 5*b*c)*((((b^3*g^4*(2
5*A*a*d + 5*A*b*c + B*a*d - B*b*c))/(5*d) - (A*b^3*g^4*(5*a*d + 5*b*c))/(5*
d))*(5*a*d + 5*b*c))/(5*b*d) - (a*b^2*g^4*(10*A*a*d + 5*A*b*c + B*a*d - B*b
*c))/d + (A*a*b^3*c*g^4)/d))/(5*b*d) + (2*a^2*b*g^4*(5*A*a*d + 5*A*b*c + B*
a*d - B*b*c))/d - (a*c*((b^3*g^4*(25*A*a*d + 5*A*b*c + B*a*d - B*b*c))/(5*d
) - (A*b^3*g^4*(5*a*d + 5*b*c))/(5*d)))/(b*d)))/(5*b*d) + (a*c*((((b^3*g^4*
(25*A*a*d + 5*A*b*c + B*a*d - B*b*c))/(5*d) - (A*b^3*g^4*(5*a*d + 5*b*c))/(
5*d))*(5*a*d + 5*b*c))/(5*b*d) - (a*b^2*g^4*(10*A*a*d + 5*A*b*c + B*a*d - B
*b*c))/d + (A*a*b^3*c*g^4)/d))/(b*d)) + x^4*((b^3*g^4*(25*A*a*d + 5*A*b*c +
 B*a*d - B*b*c))/(20*d) - (A*b^3*g^4*(5*a*d + 5*b*c))/(20*d)) - (log(c + d*
x)*(B*b^4*c^5*g^4 + 5*B*a^4*c*d^4*g^4 - 10*B*a^3*b*c^2*d^3*g^4 + 10*B*a^2*b
^2*c^3*d^2*g^4 - 5*B*a*b^3*c^4*d*g^4))/(5*d^5) + (A*b^4*g^4*x^5)/5 + (B*a^5
*g^4*log(a + b*x))/(5*b)
```

**sympy [B]** time = 6.40, size = 969, normalized size = 5.38

$$\frac{Ab^4g^4x^5}{5}+\frac{Ba^5g^4\log\left(x+\frac{\frac{Ba^6d^5g^4}{b}+5Ba^5cd^4g^4-10Ba^4bc^2d^3g^4+10Ba^3b^2c^3d^2g^4-5Ba^2b^3c^4dg^4+Bab^4c^5g^4}{Ba^5d^5g^4+5Ba^4bcd^4g^4-10Ba^3b^2c^2d^3g^4+10Ba^2b^3c^3d^2g^4-5Bab^4c^4dg^4+Bb^5c^5g^4}\right)}{5b}-\frac{Bcg^4\left(5a^4d^4-10a^3bcd^3\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*g*x+a*g)**4*(A+B*ln(e*(b*x+a)/(d*x+c))),x)
```

```
[Out] A*b**4*g**4*x**5/5 + B*a**5*g**4*log(x + (B*a**6*d**5*g**4/b + 5*B*a**5*c*d
**4*g**4 - 10*B*a**4*b*c**2*d**3*g**4 + 10*B*a**3*b**2*c**3*d**2*g**4 - 5*B
*a**2*b**3*c**4*d*g**4 + B*a*b**4*c**5*g**4)/(B*a**5*d**5*g**4 + 5*B*a**4*b
*c*d**4*g**4 - 10*B*a**3*b**2*c**2*d**3*g**4 + 10*B*a**2*b**3*c**3*d**2*g**
4 - 5*B*a*b**4*c**4*d*g**4 + B*b**5*c**5*g**4))/(5*b) - B*c*g**4*(5*a**4*d*
*4 - 10*a**3*b*c*d**3 + 10*a**2*b**2*c**2*d**2 - 5*a*b**3*c**3*d + b**4*c**
4)*log(x + (6*B*a**5*c*d**4*g**4 - 10*B*a**4*b*c**2*d**3*g**4 + 10*B*a**3*b
**2*c**3*d**2*g**4 - 5*B*a**2*b**3*c**4*d*g**4 + B*a*b**4*c**5*g**4 - B*a*c
*g**4*(5*a**4*d**4 - 10*a**3*b*c*d**3 + 10*a**2*b**2*c**2*d**2 - 5*a*b**3*c
**3*d + b**4*c**4) + B*b*c**2*g**4*(5*a**4*d**4 - 10*a**3*b*c*d**3 + 10*a**
2*b**2*c**2*d**2 - 5*a*b**3*c**3*d + b**4*c**4)/d)/(B*a**5*d**5*g**4 + 5*B*
a**4*b*c*d**4*g**4 - 10*B*a**3*b**2*c**2*d**3*g**4 + 10*B*a**2*b**3*c**3*d*
*2*g**4 - 5*B*a*b**4*c**4*d*g**4 + B*b**5*c**5*g**4))/(5*d**5) + x**4*(A*a*
```

```
b**3*g**4 + B*a*b**3*g**4/20 - B*b**4*c*g**4/(20*d)) + x**3*(2*A*a**2*b**2*
g**4 + 4*B*a**2*b**2*g**4/15 - B*a*b**3*c*g**4/(3*d) + B*b**4*c**2*g**4/(15
*d**2)) + x**2*(2*A*a**3*b*g**4 + 3*B*a**3*b*g**4/5 - B*a**2*b**2*c*g**4/d
+ B*a*b**3*c**2*g**4/(2*d**2) - B*b**4*c**3*g**4/(10*d**3)) + x*(A*a**4*g**
4 + 4*B*a**4*g**4/5 - 2*B*a**3*b*c*g**4/d + 2*B*a**2*b**2*c**2*g**4/d**2 -
B*a*b**3*c**3*g**4/d**3 + B*b**4*c**4*g**4/(5*d**4)) + (B*a**4*g**4*x + 2*B
*a**3*b*g**4*x**2 + 2*B*a**2*b**2*g**4*x**3 + B*a*b**3*g**4*x**4 + B*b**4*g
**4*x**5/5)*log(e*(a + b*x)/(c + d*x))
```

# 3.89 $\int (ag+bgx)^3 \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right) dx$

**Optimal**. Leaf size=149

$$\frac{g^3(a+bx)^4\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{4b}+\frac{Bg^3(bc-ad)^4\log(c+dx)}{4bd^4}-\frac{Bg^3x(bc-ad)^3}{4d^3}+\frac{Bg^3(a+bx)^2(bc-ad)^2}{8bd^2}-\frac{Bg^3(a+$$

```
[Out] -1/4*B*(-a*d+b*c)^3*g^3*x/d^3+1/8*B*(-a*d+b*c)^2*g^3*(b*x+a)^2/b/d^2-1/12*B
*(-a*d+b*c)*g^3*(b*x+a)^3/b/d+1/4*g^3*(b*x+a)^4*(A+B*ln(e*(b*x+a)/(d*x+c)))
/b+1/4*B*(-a*d+b*c)^4*g^3*ln(d*x+c)/b/d^4
```

**Rubi [A]** time = 0.10, antiderivative size = 149, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {2525, 12, 43}

$$\frac{g^3(a+bx)^4\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{4b}-\frac{Bg^3x(bc-ad)^3}{4d^3}+\frac{Bg^3(a+bx)^2(bc-ad)^2}{8bd^2}+\frac{Bg^3(bc-ad)^4\log(c+dx)}{4bd^4}-\frac{Bg^3(a+$$

Antiderivative was successfully verified.

```
[In] Int[(a*g + b*g*x)^3*(A + B*Log[(e*(a + b*x))/(c + d*x)]),x]
```

```
[Out] -(B*(b*c - a*d)^3*g^3*x)/(4*d^3) + (B*(b*c - a*d)^2*g^3*(a + b*x)^2)/(8*b*d
^2) - (B*(b*c - a*d)*g^3*(a + b*x)^3)/(12*b*d) + (g^3*(a + b*x)^4*(A + B*Lo
g[(e*(a + b*x))/(c + d*x)]))/(4*b) + (B*(b*c - a*d)^4*g^3*Log[c + d*x])/(4*
b*d^4)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2525

```
Int[((a_.) + Log[(c_.)*(RFx_)^(p_.)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_))^(m_.
), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(a + b*Log[c*RFx^p])^n)/(e*(m + 1))
, x] - Dist[(b*n*p)/(e*(m + 1)), Int[SimplifyIntegrand[((d + e*x)^(m + 1)*(
```

```
a + b*Log[c*RFx^p])^(n - 1)*D[RFx, x])/RFx, x], x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && RationalFunctionQ[RFx, x] && IGtQ[n, 0] && (EqQ[n, 1] ||
IntegerQ[m]) && NeQ[m, -1]
```

Rubi steps

$$\int (ag+bgx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)dx = \frac{g^3(a+bx)^4\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{4b} - \frac{B\int\frac{(bc-ad)g^4(a+bx)^3}{c+dx}\,dx}{4bg}$$
$$= \frac{g^3(a+bx)^4\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{4b} - \frac{\left(B(bc-ad)g^3\right)\int\frac{(a+bx)^3}{c+dx}\,dx}{4b}$$
$$= \frac{g^3(a+bx)^4\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{4b} - \frac{\left(B(bc-ad)g^3\right)\int\left(\frac{b(bc-ad)^2}{d^3} - \frac{b}{}\right.}{}$$
$$= -\frac{B(bc-ad)^3g^3x}{4d^3} + \frac{B(bc-ad)^2g^3(a+bx)^2}{8bd^2} - \frac{B(bc-ad)g^3(a+bx}{12bd}$$

**Mathematica [A]** time = 0.10, size = 120, normalized size = 0.81

$$\frac{g^3\left((a+bx)^4\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right) - \frac{B(bc-ad)\left(3d^2(a+bx)^2(ad-bc)+6bdx(bc-ad)^2-6(bc-ad)^3\log(c+dx)+2d^3(a+bx)^3\right)}{6d^4}\right)}{4b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*g + b*g*x)^3*(A + B*Log[(e*(a + b*x))/(c + d*x)]),x]
```

```
[Out] (g^3*((a + b*x)^4*(A + B*Log[(e*(a + b*x))/(c + d*x)]) - (B*(b*c - a*d)*(6*
b*d*(b*c - a*d)^2*x + 3*d^2*(-(b*c) + a*d)*(a + b*x)^2 + 2*d^3*(a + b*x)^3
- 6*(b*c - a*d)^3*Log[c + d*x]))/(6*d^4)))/(4*b)
```

**fricas [B]** time = 1.01, size = 318, normalized size = 2.13

$$\frac{6\,Ab^4d^4g^3x^4 + 6\,Ba^4d^4g^3\log(bx+a) - 2\left(Bb^4cd^3 - (12\,A+B)ab^3d^4\right)g^3x^3 + 3\left(Bb^4c^2d^2 - 4\,Bab^3cd^3 + 3\,(4\,A+\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*g*x+a*g)^3*(A+B*log(e*(b*x+a)/(d*x+c))),x, algorithm="fricas")
```

```
[Out] 1/24*(6*A*b^4*d^4*g^3*x^4 + 6*B*a^4*d^4*g^3*log(b*x + a) - 2*(B*b^4*c*d^3 -
 (12*A + B)*a*b^3*d^4)*g^3*x^3 + 3*(B*b^4*c^2*d^2 - 4*B*a*b^3*c*d^3 + 3*(4*
```

```
A + B)*a^2*b^2*d^4)*g^3*x^2 - 6*(B*b^4*c^3*d - 4*B*a*b^3*c^2*d^2 + 6*B*a^2*
b^2*c*d^3 - (4*A + 3*B)*a^3*b*d^4)*g^3*x + 6*(B*b^4*c^4 - 4*B*a*b^3*c^3*d +
 6*B*a^2*b^2*c^2*d^2 - 4*B*a^3*b*c*d^3)*g^3*log(d*x + c) + 6*(B*b^4*d^4*g^3
*x^4 + 4*B*a*b^3*d^4*g^3*x^3 + 6*B*a^2*b^2*d^4*g^3*x^2 + 4*B*a^3*b*d^4*g^3*
x)*log((b*e*x + a*e)/(d*x + c)))/(b*d^4)
```

**giac [B]** time = 1.81, size = 3795, normalized size = 25.47

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*g*x+a*g)^3*(A+B*log(e*(b*x+a)/(d*x+c))),x, algorithm="giac")
```

```
[Out] -1/24*(6*B*b^9*c^5*g^3*e^5*log(-b*e + (b*x*e + a*e)*d/(d*x + c)) - 30*B*a*b
^8*c^4*d*g^3*e^5*log(-b*e + (b*x*e + a*e)*d/(d*x + c)) + 60*B*a^2*b^7*c^3*d
^2*g^3*e^5*log(-b*e + (b*x*e + a*e)*d/(d*x + c)) - 60*B*a^3*b^6*c^2*d^3*g^3
*e^5*log(-b*e + (b*x*e + a*e)*d/(d*x + c)) + 30*B*a^4*b^5*c*d^4*g^3*e^5*log
(-b*e + (b*x*e + a*e)*d/(d*x + c)) - 6*B*a^5*b^4*d^5*g^3*e^5*log(-b*e + (b*
x*e + a*e)*d/(d*x + c)) - 24*(b*x*e + a*e)*B*b^8*c^5*d*g^3*e^4*log(-b*e + (
b*x*e + a*e)*d/(d*x + c))/(d*x + c) + 120*(b*x*e + a*e)*B*a*b^7*c^4*d^2*g^3
*e^4*log(-b*e + (b*x*e + a*e)*d/(d*x + c))/(d*x + c) - 240*(b*x*e + a*e)*B*
a^2*b^6*c^3*d^3*g^3*e^4*log(-b*e + (b*x*e + a*e)*d/(d*x + c))/(d*x + c) + 2
40*(b*x*e + a*e)*B*a^3*b^5*c^2*d^4*g^3*e^4*log(-b*e + (b*x*e + a*e)*d/(d*x
+ c))/(d*x + c) - 120*(b*x*e + a*e)*B*a^4*b^4*c*d^5*g^3*e^4*log(-b*e + (b*x
*e + a*e)*d/(d*x + c))/(d*x + c) + 24*(b*x*e + a*e)*B*a^5*b^3*d^6*g^3*e^4*l
og(-b*e + (b*x*e + a*e)*d/(d*x + c))/(d*x + c) + 36*(b*x*e + a*e)^2*B*b^7*c
^5*d^2*g^3*e^3*log(-b*e + (b*x*e + a*e)*d/(d*x + c))/(d*x + c)^2 - 180*(b*x
*e + a*e)^2*B*a*b^6*c^4*d^3*g^3*e^3*log(-b*e + (b*x*e + a*e)*d/(d*x + c))/(
d*x + c)^2 + 360*(b*x*e + a*e)^2*B*a^2*b^5*c^3*d^4*g^3*e^3*log(-b*e + (b*x*
e + a*e)*d/(d*x + c))/(d*x + c)^2 - 360*(b*x*e + a*e)^2*B*a^3*b^4*c^2*d^5*g
^3*e^3*log(-b*e + (b*x*e + a*e)*d/(d*x + c))/(d*x + c)^2 + 180*(b*x*e + a*e
)^2*B*a^4*b^3*c*d^6*g^3*e^3*log(-b*e + (b*x*e + a*e)*d/(d*x + c))/(d*x + c)
^2 - 36*(b*x*e + a*e)^2*B*a^5*b^2*d^7*g^3*e^3*log(-b*e + (b*x*e + a*e)*d/(d
*x + c))/(d*x + c)^2 - 24*(b*x*e + a*e)^3*B*b^6*c^5*d^3*g^3*e^2*log(-b*e +
(b*x*e + a*e)*d/(d*x + c))/(d*x + c)^3 + 120*(b*x*e + a*e)^3*B*a*b^5*c^4*d^
4*g^3*e^2*log(-b*e + (b*x*e + a*e)*d/(d*x + c))/(d*x + c)^3 - 240*(b*x*e +
a*e)^3*B*a^2*b^4*c^3*d^5*g^3*e^2*log(-b*e + (b*x*e + a*e)*d/(d*x + c))/(d*x
 + c)^3 + 240*(b*x*e + a*e)^3*B*a^3*b^3*c^2*d^6*g^3*e^2*log(-b*e + (b*x*e +
 a*e)*d/(d*x + c))/(d*x + c)^3 - 120*(b*x*e + a*e)^3*B*a^4*b^2*c*d^7*g^3*e^
2*log(-b*e + (b*x*e + a*e)*d/(d*x + c))/(d*x + c)^3 + 24*(b*x*e + a*e)^3*B*
a^5*b*d^8*g^3*e^2*log(-b*e + (b*x*e + a*e)*d/(d*x + c))/(d*x + c)^3 + 6*(b*
x*e + a*e)^4*B*b^5*c^5*d^4*g^3*e*log(-b*e + (b*x*e + a*e)*d/(d*x + c))/(d*x
 + c)^4 - 30*(b*x*e + a*e)^4*B*a*b^4*c^4*d^5*g^3*e*log(-b*e + (b*x*e + a*e)
*d/(d*x + c))/(d*x + c)^4 + 60*(b*x*e + a*e)^4*B*a^2*b^3*c^3*d^6*g^3*e*log(
-b*e + (b*x*e + a*e)*d/(d*x + c))/(d*x + c)^4 - 60*(b*x*e + a*e)^4*B*a^3*b^
```

2*c^2*d^7*g^3*e*log(-b*e + (b*x*e + a*e)*d/(d*x + c))/(d*x + c)^4 + 30*(b*x
*e + a*e)^4*B*a^4*b*c*d^8*g^3*e*log(-b*e + (b*x*e + a*e)*d/(d*x + c))/(d*x
+ c)^4 - 6*(b*x*e + a*e)^4*B*a^5*d^9*g^3*e*log(-b*e + (b*x*e + a*e)*d/(d*x
+ c))/(d*x + c)^4 - 6*(b*x*e + a*e)^4*B*b^5*c^5*d^4*g^3*e*log((b*x*e + a*e)
/(d*x + c))/(d*x + c)^4 + 30*(b*x*e + a*e)^4*B*a*b^4*c^4*d^5*g^3*e*log((b*x
*e + a*e)/(d*x + c))/(d*x + c)^4 - 60*(b*x*e + a*e)^4*B*a^2*b^3*c^3*d^6*g^3
*e*log((b*x*e + a*e)/(d*x + c))/(d*x + c)^4 + 60*(b*x*e + a*e)^4*B*a^3*b^2*
c^2*d^7*g^3*e*log((b*x*e + a*e)/(d*x + c))/(d*x + c)^4 - 30*(b*x*e + a*e)^4
*B*a^4*b*c*d^8*g^3*e*log((b*x*e + a*e)/(d*x + c))/(d*x + c)^4 + 6*(b*x*e +
a*e)^4*B*a^5*d^9*g^3*e*log((b*x*e + a*e)/(d*x + c))/(d*x + c)^4 + 6*A*b^9*c
^5*g^3*e^5 + 11*B*b^9*c^5*g^3*e^5 - 30*A*a*b^8*c^4*d*g^3*e^5 - 55*B*a*b^8*c
^4*d*g^3*e^5 + 60*A*a^2*b^7*c^3*d^2*g^3*e^5 + 110*B*a^2*b^7*c^3*d^2*g^3*e^5
 - 60*A*a^3*b^6*c^2*d^3*g^3*e^5 - 110*B*a^3*b^6*c^2*d^3*g^3*e^5 + 30*A*a^4*
b^5*c*d^4*g^3*e^5 + 55*B*a^4*b^5*c*d^4*g^3*e^5 - 6*A*a^5*b^4*d^5*g^3*e^5 -
11*B*a^5*b^4*d^5*g^3*e^5 - 24*(b*x*e + a*e)*A*b^8*c^5*d*g^3*e^4/(d*x + c) -
 38*(b*x*e + a*e)*B*b^8*c^5*d*g^3*e^4/(d*x + c) + 120*(b*x*e + a*e)*A*a*b^7
*c^4*d^2*g^3*e^4/(d*x + c) + 190*(b*x*e + a*e)*B*a*b^7*c^4*d^2*g^3*e^4/(d*x
 + c) - 240*(b*x*e + a*e)*A*a^2*b^6*c^3*d^3*g^3*e^4/(d*x + c) - 380*(b*x*e
+ a*e)*B*a^2*b^6*c^3*d^3*g^3*e^4/(d*x + c) + 240*(b*x*e + a*e)*A*a^3*b^5*c^
2*d^4*g^3*e^4/(d*x + c) + 380*(b*x*e + a*e)*B*a^3*b^5*c^2*d^4*g^3*e^4/(d*x
+ c) - 120*(b*x*e + a*e)*A*a^4*b^4*c*d^5*g^3*e^4/(d*x + c) - 190*(b*x*e + a
*e)*B*a^4*b^4*c*d^5*g^3*e^4/(d*x + c) + 24*(b*x*e + a*e)*A*a^5*b^3*d^6*g^3*
e^4/(d*x + c) + 38*(b*x*e + a*e)*B*a^5*b^3*d^6*g^3*e^4/(d*x + c) + 36*(b*x*
e + a*e)^2*A*b^7*c^5*d^2*g^3*e^3/(d*x + c)^2 + 45*(b*x*e + a*e)^2*B*b^7*c^5
*d^2*g^3*e^3/(d*x + c)^2 - 180*(b*x*e + a*e)^2*A*a*b^6*c^4*d^3*g^3*e^3/(d*x
 + c)^2 - 225*(b*x*e + a*e)^2*B*a*b^6*c^4*d^3*g^3*e^3/(d*x + c)^2 + 360*(b*
x*e + a*e)^2*A*a^2*b^5*c^3*d^4*g^3*e^3/(d*x + c)^2 + 450*(b*x*e + a*e)^2*B*
a^2*b^5*c^3*d^4*g^3*e^3/(d*x + c)^2 - 360*(b*x*e + a*e)^2*A*a^3*b^4*c^2*d^5
*g^3*e^3/(d*x + c)^2 - 450*(b*x*e + a*e)^2*B*a^3*b^4*c^2*d^5*g^3*e^3/(d*x +
 c)^2 + 180*(b*x*e + a*e)^2*A*a^4*b^3*c*d^6*g^3*e^3/(d*x + c)^2 + 225*(b*x*
e + a*e)^2*B*a^4*b^3*c*d^6*g^3*e^3/(d*x + c)^2 - 36*(b*x*e + a*e)^2*A*a^5*b
^2*d^7*g^3*e^3/(d*x + c)^2 - 45*(b*x*e + a*e)^2*B*a^5*b^2*d^7*g^3*e^3/(d*x
+ c)^2 - 24*(b*x*e + a*e)^3*A*b^6*c^5*d^3*g^3*e^2/(d*x + c)^3 - 18*(b*x*e +
 a*e)^3*B*b^6*c^5*d^3*g^3*e^2/(d*x + c)^3 + 120*(b*x*e + a*e)^3*A*a*b^5*c^4
*d^4*g^3*e^2/(d*x + c)^3 + 90*(b*x*e + a*e)^3*B*a*b^5*c^4*d^4*g^3*e^2/(d*x
+ c)^3 - 240*(b*x*e + a*e)^3*A*a^2*b^4*c^3*d^5*g^3*e^2/(d*x + c)^3 - 180*(b
*x*e + a*e)^3*B*a^2*b^4*c^3*d^5*g^3*e^2/(d*x + c)^3 + 240*(b*x*e + a*e)^3*A
*a^3*b^3*c^2*d^6*g^3*e^2/(d*x + c)^3 + 180*(b*x*e + a*e)^3*B*a^3*b^3*c^2*d^
6*g^3*e^2/(d*x + c)^3 - 120*(b*x*e + a*e)^3*A*a^4*b^2*c*d^7*g^3*e^2/(d*x +
c)^3 - 90*(b*x*e + a*e)^3*B*a^4*b^2*c*d^7*g^3*e^2/(d*x + c)^3 + 24*(b*x*e +
 a*e)^3*A*a^5*b*d^8*g^3*e^2/(d*x + c)^3 + 18*(b*x*e + a*e)^3*B*a^5*b*d^8*g^
3*e^2/(d*x + c)^3)*(b*c/((b*c*e - a*d*e)*(b*c - a*d)) - a*d/((b*c*e - a*d*e
)*(b*c - a*d)))/(b^5*d^4*e^4 - 4*(b*x*e + a*e)*b^4*d^5*e^3/(d*x + c) + 6*(b
*x*e + a*e)^2*b^3*d^6*e^2/(d*x + c)^2 - 4*(b*x*e + a*e)^3*b^2*d^7*e/(d*x +
c)^3 + (b*x*e + a*e)^4*b*d^8/(d*x + c)^4)

**maple [B]** time = 0.16, size = 5556, normalized size = 37.29

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*g*x+a*g)^3*(A+B*ln(e*(b*x+a)/(d*x+c))),x)
```

```
[Out] result too large to display
```

**maxima [B]** time = 1.33, size = 439, normalized size = 2.95

$$\frac{1}{4}Ab^3g^3x^4+Aab^2g^3x^3+\frac{3}{2}Aa^2bg^3x^2+\left(x\log\left(\frac{bex}{dx+c}+\frac{ae}{dx+c}\right)+\frac{a\log(bx+a)}{b}-\frac{c\log(dx+c)}{d}\right)Ba^3g^3+\frac{3}{2}\left(x^2\log\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*g*x+a*g)^3*(A+B*log(e*(b*x+a)/(d*x+c))),x, algorithm="maxima")
```

```
[Out] 1/4*A*b^3*g^3*x^4 + A*a*b^2*g^3*x^3 + 3/2*A*a^2*b*g^3*x^2 + (x*log(b*e*x/(d
*x + c) + a*e/(d*x + c)) + a*log(b*x + a)/b - c*log(d*x + c)/d)*B*a^3*g^3 +
 3/2*(x^2*log(b*e*x/(d*x + c) + a*e/(d*x + c)) - a^2*log(b*x + a)/b^2 + c^2
*log(d*x + c)/d^2 - (b*c - a*d)*x/(b*d))*B*a^2*b*g^3 + 1/2*(2*x^3*log(b*e*x
/(d*x + c) + a*e/(d*x + c)) + 2*a^3*log(b*x + a)/b^3 - 2*c^3*log(d*x + c)/d
^3 - ((b^2*c*d - a*b*d^2)*x^2 - 2*(b^2*c^2 - a^2*d^2)*x)/(b^2*d^2))*B*a*b^2
*g^3 + 1/24*(6*x^4*log(b*e*x/(d*x + c) + a*e/(d*x + c)) - 6*a^4*log(b*x + a
)/b^4 + 6*c^4*log(d*x + c)/d^4 - (2*(b^3*c*d^2 - a*b^2*d^3)*x^3 - 3*(b^3*c^
2*d - a^2*b*d^3)*x^2 + 6*(b^3*c^3 - a^3*d^3)*x)/(b^3*d^3))*B*b^3*g^3 + A*a^
3*g^3*x
```

**mupad [B]** time = 4.64, size = 566, normalized size = 3.80

$$x\left(\frac{(4ad+4bc)\left(\frac{\left(\frac{b^2g^3(16Aad+4Abc+Bad-Bbc)}{4d}-\frac{Ab^2g^3(4ad+4bc)}{4d}\right)(4ad+4bc)}{4bd}-\frac{abg^3(6Aad+4Abc+Bad-Bbc)}{d}+\frac{Aab^2cg^3}{d}\right)}{4bd}+\frac{a^2g\cdots}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*g + b*g*x)^3*(A + B*log((e*(a + b*x))/(c + d*x))),x)
```

```
[Out] x*(((4*a*d + 4*b*c)*((((b^2*g^3*(16*A*a*d + 4*A*b*c + B*a*d - B*b*c))/(4*d)
 - (A*b^2*g^3*(4*a*d + 4*b*c))/(4*d))*(4*a*d + 4*b*c))/(4*b*d) - (a*b*g^3*(
```

```
6*A*a*d + 4*A*b*c + B*a*d - B*b*c))/d + (A*a*b^2*c*g^3)/d))/(4*b*d) + (a^2*
g^3*(8*A*a*d + 12*A*b*c + 3*B*a*d - 3*B*b*c))/(2*d) - (a*c*((b^2*g^3*(16*A*
a*d + 4*A*b*c + B*a*d - B*b*c))/(4*d) - (A*b^2*g^3*(4*a*d + 4*b*c))/(4*d)))
/(b*d)) - x^2*((((b^2*g^3*(16*A*a*d + 4*A*b*c + B*a*d - B*b*c))/(4*d) - (A*
b^2*g^3*(4*a*d + 4*b*c))/(4*d))*(4*a*d + 4*b*c))/(8*b*d) - (a*b*g^3*(6*A*a*
d + 4*A*b*c + B*a*d - B*b*c))/(2*d) + (A*a*b^2*c*g^3)/(2*d)) + log((e*(a +
b*x))/(c + d*x))*((B*b^3*g^3*x^4)/4 + B*a^3*g^3*x + (3*B*a^2*b*g^3*x^2)/2 +
 B*a*b^2*g^3*x^3) + x^3*((b^2*g^3*(16*A*a*d + 4*A*b*c + B*a*d - B*b*c))/(12
*d) - (A*b^2*g^3*(4*a*d + 4*b*c))/(12*d)) + (log(c + d*x)*(B*b^3*c^4*g^3 -
4*B*a^3*c*d^3*g^3 + 6*B*a^2*b*c^2*d^2*g^3 - 4*B*a*b^2*c^3*d*g^3))/(4*d^4) +
 (A*b^3*g^3*x^4)/4 + (B*a^4*g^3*log(a + b*x))/(4*b)
```

**sympy [B]** time = 4.30, size = 706, normalized size = 4.74

$$\frac{Ab^3g^3x^4}{4}+\frac{Ba^4g^3\log\left(x+\frac{\frac{Ba^5d^4g^3}{b}+4Ba^4cd^3g^3-6Ba^3bc^2d^2g^3+4Ba^2b^2c^3dg^3-Bab^3c^4g^3}{Ba^4d^4g^3+4Ba^3bcd^3g^3-6Ba^2b^2c^2d^2g^3+4Bab^3c^3dg^3-Bb^4c^4g^3}\right)}{4b}-\frac{Bcg^3\left(2ad-bc\right)\left(2a^2d^2-2abcd+b^2c\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*g*x+a*g)**3*(A+B*ln(e*(b*x+a)/(d*x+c))),x)
```

```
[Out] A*b**3*g**3*x**4/4 + B*a**4*g**3*log(x + (B*a**5*d**4*g**3/b + 4*B*a**4*c*d
**3*g**3 - 6*B*a**3*b*c**2*d**2*g**3 + 4*B*a**2*b**2*c**3*d*g**3 - B*a*b**3
*c**4*g**3)/(B*a**4*d**4*g**3 + 4*B*a**3*b*c*d**3*g**3 - 6*B*a**2*b**2*c**2
*d**2*g**3 + 4*B*a*b**3*c**3*d*g**3 - B*b**4*c**4*g**3))/(4*b) - B*c*g**3*(
2*a*d - b*c)*(2*a**2*d**2 - 2*a*b*c*d + b**2*c**2)*log(x + (5*B*a**4*c*d**3
*g**3 - 6*B*a**3*b*c**2*d**2*g**3 + 4*B*a**2*b**2*c**3*d*g**3 - B*a*b**3*c*
*4*g**3 - B*a*c*g**3*(2*a*d - b*c)*(2*a**2*d**2 - 2*a*b*c*d + b**2*c**2) +
B*b*c**2*g**3*(2*a*d - b*c)*(2*a**2*d**2 - 2*a*b*c*d + b**2*c**2)/d)/(B*a**
4*d**4*g**3 + 4*B*a**3*b*c*d**3*g**3 - 6*B*a**2*b**2*c**2*d**2*g**3 + 4*B*a
*b**3*c**3*d*g**3 - B*b**4*c**4*g**3))/(4*d**4) + x**3*(A*a*b**2*g**3 + B*a
*b**2*g**3/12 - B*b**3*c*g**3/(12*d)) + x**2*(3*A*a**2*b*g**3/2 + 3*B*a**2*
b*g**3/8 - B*a*b**2*c*g**3/(2*d) + B*b**3*c**2*g**3/(8*d**2)) + x*(A*a**3*g
**3 + 3*B*a**3*g**3/4 - 3*B*a**2*b*c*g**3/(2*d) + B*a*b**2*c**2*g**3/d**2 -
 B*b**3*c**3*g**3/(4*d**3)) + (B*a**3*g**3*x + 3*B*a**2*b*g**3*x**2/2 + B*a
*b**2*g**3*x**3 + B*b**3*g**3*x**4/4)*log(e*(a + b*x)/(c + d*x))
```

## 3.90 $\int (ag+bgx)^2 \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right) dx$

**Optimal**. Leaf size=118

$$\frac{g^2(a+bx)^3\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{3b}-\frac{Bg^2(bc-ad)^3\log(c+dx)}{3bd^3}+\frac{Bg^2x(bc-ad)^2}{3d^2}-\frac{Bg^2(a+bx)^2(bc-ad)}{6bd}$$

```
[Out] 1/3*B*(-a*d+b*c)^2*g^2*x/d^2-1/6*B*(-a*d+b*c)*g^2*(b*x+a)^2/b/d+1/3*g^2*(b*
x+a)^3*(A+B*ln(e*(b*x+a)/(d*x+c)))/b-1/3*B*(-a*d+b*c)^3*g^2*ln(d*x+c)/b/d^3
```

**Rubi [A]** time = 0.08, antiderivative size = 118, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {2525, 12, 43}

$$\frac{g^2(a+bx)^3\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{3b}+\frac{Bg^2x(bc-ad)^2}{3d^2}-\frac{Bg^2(bc-ad)^3\log(c+dx)}{3bd^3}-\frac{Bg^2(a+bx)^2(bc-ad)}{6bd}$$

Antiderivative was successfully verified.

```
[In] Int[(a*g + b*g*x)^2*(A + B*Log[(e*(a + b*x))/(c + d*x)]),x]
```

```
[Out] (B*(b*c - a*d)^2*g^2*x)/(3*d^2) - (B*(b*c - a*d)*g^2*(a + b*x)^2)/(6*b*d) +
 (g^2*(a + b*x)^3*(A + B*Log[(e*(a + b*x))/(c + d*x)]))/(3*b) - (B*(b*c - a
*d)^3*g^2*Log[c + d*x])/(3*b*d^3)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2525

```
Int[((a_.) + Log[(c_.)*(RFx_)^(p_.)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_))^(m_.
), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(a + b*Log[c*RFx^p])^n)/(e*(m + 1))
, x] - Dist[(b*n*p)/(e*(m + 1)), Int[SimplifyIntegrand[((d + e*x)^(m + 1)*(
a + b*Log[c*RFx^p])^(n - 1)*D[RFx, x])/RFx, x], x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && RationalFunctionQ[RFx, x] && IGtQ[n, 0] && (EqQ[n, 1] ||
```

```
IntegerQ[m]) && NeQ[m, -1]
```

Rubi steps

$$\int (ag+bgx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)dx = \frac{g^2(a+bx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{3b} - \frac{B\int\frac{(bc-ad)g^3(a+bx)^2}{c+dx}dx}{3bg}$$

$$= \frac{g^2(a+bx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{3b} - \frac{\left(B(bc-ad)g^2\right)\int\frac{(a+bx)^2}{c+dx}dx}{3b}$$

$$= \frac{g^2(a+bx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{3b} - \frac{\left(B(bc-ad)g^2\right)\int\left(-\frac{b(bc-ad)}{d^2}+\right.}{3b}$$

$$= \frac{B(bc-ad)^2g^2x}{3d^2} - \frac{B(bc-ad)g^2(a+bx)^2}{6bd} + \frac{g^2(a+bx)^3\left(A+B\log\right.}{3b}$$

**Mathematica [A]** time = 0.06, size = 99, normalized size = 0.84

$$\frac{g^2\left(\frac{B(ad-bc)\left(d\left(a^2d+4abdx+b^2x(dx-2c)\right)+2(bc-ad)^2\log(c+dx)\right)}{2d^3}+(a+bx)^3\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)\right)}{3b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*g + b*g*x)^2*(A + B*Log[(e*(a + b*x))/(c + d*x)]),x]
```

```
[Out] (g^2*((a + b*x)^3*(A + B*Log[(e*(a + b*x))/(c + d*x)]) + (B*(-(b*c) + a*d)*
(d*(a^2*d + 4*a*b*d*x + b^2*x*(-2*c + d*x)) + 2*(b*c - a*d)^2*Log[c + d*x])
)/(2*d^3)))/(3*b)
```

**fricas [B]** time = 0.57, size = 222, normalized size = 1.88

$$\frac{2\,Ab^3d^3g^2x^3+2\,Ba^3d^3g^2\log(bx+a)-\left(Bb^3cd^2-(6\,A+B)ab^2d^3\right)g^2x^2+2\left(Bb^3c^2d-3\,Bab^2cd^2+(3\,A+2\,B)a\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*g*x+a*g)^2*(A+B*log(e*(b*x+a)/(d*x+c))),x, algorithm="fricas")
```

```
[Out] 1/6*(2*A*b^3*d^3*g^2*x^3 + 2*B*a^3*d^3*g^2*log(b*x + a) - (B*b^3*c*d^2 - (6
*A + B)*a*b^2*d^3)*g^2*x^2 + 2*(B*b^3*c^2*d - 3*B*a*b^2*c*d^2 + (3*A + 2*B)
*a^2*b*d^3)*g^2*x - 2*(B*b^3*c^3 - 3*B*a*b^2*c^2*d + 3*B*a^2*b*c*d^2)*g^2*l
```

```
og(d*x + c) + 2*(B*b^3*d^3*g^2*x^3 + 3*B*a*b^2*d^3*g^2*x^2 + 3*B*a^2*b*d^3*
g^2*x)*log((b*e*x + a*e)/(d*x + c)))/(b*d^3)
```

**giac [B]** time = 1.29, size = 2450, normalized size = 20.76

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*g*x+a*g)^2*(A+B*log(e*(b*x+a)/(d*x+c))),x, algorithm="giac")
```

```
[Out] 1/6*(2*B*b^7*c^4*g^2*e^4*log(-b*e + (b*x*e + a*e)*d/(d*x + c)) - 8*B*a*b^6*
c^3*d*g^2*e^4*log(-b*e + (b*x*e + a*e)*d/(d*x + c)) + 12*B*a^2*b^5*c^2*d^2*
g^2*e^4*log(-b*e + (b*x*e + a*e)*d/(d*x + c)) - 8*B*a^3*b^4*c*d^3*g^2*e^4*l
og(-b*e + (b*x*e + a*e)*d/(d*x + c)) + 2*B*a^4*b^3*d^4*g^2*e^4*log(-b*e + (
b*x*e + a*e)*d/(d*x + c)) - 6*(b*x*e + a*e)*B*b^6*c^4*d*g^2*e^3*log(-b*e +
(b*x*e + a*e)*d/(d*x + c))/(d*x + c) + 24*(b*x*e + a*e)*B*a*b^5*c^3*d^2*g^2
*e^3*log(-b*e + (b*x*e + a*e)*d/(d*x + c))/(d*x + c) - 36*(b*x*e + a*e)*B*a
^2*b^4*c^2*d^3*g^2*e^3*log(-b*e + (b*x*e + a*e)*d/(d*x + c))/(d*x + c) + 24
*(b*x*e + a*e)*B*a^3*b^3*c*d^4*g^2*e^3*log(-b*e + (b*x*e + a*e)*d/(d*x + c)
)/(d*x + c) - 6*(b*x*e + a*e)*B*a^4*b^2*d^5*g^2*e^3*log(-b*e + (b*x*e + a*e
)*d/(d*x + c))/(d*x + c) + 6*(b*x*e + a*e)^2*B*b^5*c^4*d^2*g^2*e^2*log(-b*e
 + (b*x*e + a*e)*d/(d*x + c))/(d*x + c)^2 - 24*(b*x*e + a*e)^2*B*a*b^4*c^3*
d^3*g^2*e^2*log(-b*e + (b*x*e + a*e)*d/(d*x + c))/(d*x + c)^2 + 36*(b*x*e +
 a*e)^2*B*a^2*b^3*c^2*d^4*g^2*e^2*log(-b*e + (b*x*e + a*e)*d/(d*x + c))/(d*
x + c)^2 - 24*(b*x*e + a*e)^2*B*a^3*b^2*c*d^5*g^2*e^2*log(-b*e + (b*x*e + a
*e)*d/(d*x + c))/(d*x + c)^2 + 6*(b*x*e + a*e)^2*B*a^4*b*d^6*g^2*e^2*log(-b
*e + (b*x*e + a*e)*d/(d*x + c))/(d*x + c)^2 - 2*(b*x*e + a*e)^3*B*b^4*c^4*d
^3*g^2*e*log(-b*e + (b*x*e + a*e)*d/(d*x + c))/(d*x + c)^3 + 8*(b*x*e + a*e
)^3*B*a*b^3*c^3*d^4*g^2*e*log(-b*e + (b*x*e + a*e)*d/(d*x + c))/(d*x + c)^3
 - 12*(b*x*e + a*e)^3*B*a^2*b^2*c^2*d^5*g^2*e*log(-b*e + (b*x*e + a*e)*d/(d
*x + c))/(d*x + c)^3 + 8*(b*x*e + a*e)^3*B*a^3*b*c*d^6*g^2*e*log(-b*e + (b*
x*e + a*e)*d/(d*x + c))/(d*x + c)^3 - 2*(b*x*e + a*e)^3*B*a^4*d^7*g^2*e*log
(-b*e + (b*x*e + a*e)*d/(d*x + c))/(d*x + c)^3 + 2*(b*x*e + a*e)^3*B*b^4*c^
4*d^3*g^2*e*log((b*x*e + a*e)/(d*x + c))/(d*x + c)^3 - 8*(b*x*e + a*e)^3*B*
a*b^3*c^3*d^4*g^2*e*log((b*x*e + a*e)/(d*x + c))/(d*x + c)^3 + 12*(b*x*e +
a*e)^3*B*a^2*b^2*c^2*d^5*g^2*e*log((b*x*e + a*e)/(d*x + c))/(d*x + c)^3 - 8
*(b*x*e + a*e)^3*B*a^3*b*c*d^6*g^2*e*log((b*x*e + a*e)/(d*x + c))/(d*x + c)
^3 + 2*(b*x*e + a*e)^3*B*a^4*d^7*g^2*e*log((b*x*e + a*e)/(d*x + c))/(d*x +
c)^3 + 2*A*b^7*c^4*g^2*e^4 + 3*B*b^7*c^4*g^2*e^4 - 8*A*a*b^6*c^3*d*g^2*e^4
- 12*B*a*b^6*c^3*d*g^2*e^4 + 12*A*a^2*b^5*c^2*d^2*g^2*e^4 + 18*B*a^2*b^5*c^
2*d^2*g^2*e^4 - 8*A*a^3*b^4*c*d^3*g^2*e^4 - 12*B*a^3*b^4*c*d^3*g^2*e^4 + 2*
A*a^4*b^3*d^4*g^2*e^4 + 3*B*a^4*b^3*d^4*g^2*e^4 - 6*(b*x*e + a*e)*A*b^6*c^4
*d*g^2*e^3/(d*x + c) - 7*(b*x*e + a*e)*B*b^6*c^4*d*g^2*e^3/(d*x + c) + 24*(
b*x*e + a*e)*A*a*b^5*c^3*d^2*g^2*e^3/(d*x + c) + 28*(b*x*e + a*e)*B*a*b^5*c
^3*d^2*g^2*e^3/(d*x + c) - 36*(b*x*e + a*e)*A*a^2*b^4*c^2*d^3*g^2*e^3/(d*x
```

```
+ c) - 42*(b*x*e + a*e)*B*a^2*b^4*c^2*d^3*g^2*e^3/(d*x + c) + 24*(b*x*e + a
*e)*A*a^3*b^3*c*d^4*g^2*e^3/(d*x + c) + 28*(b*x*e + a*e)*B*a^3*b^3*c*d^4*g^
2*e^3/(d*x + c) - 6*(b*x*e + a*e)*A*a^4*b^2*d^5*g^2*e^3/(d*x + c) - 7*(b*x*
e + a*e)*B*a^4*b^2*d^5*g^2*e^3/(d*x + c) + 6*(b*x*e + a*e)^2*A*b^5*c^4*d^2*
g^2*e^2/(d*x + c)^2 + 4*(b*x*e + a*e)^2*B*b^5*c^4*d^2*g^2*e^2/(d*x + c)^2 -
 24*(b*x*e + a*e)^2*A*a*b^4*c^3*d^3*g^2*e^2/(d*x + c)^2 - 16*(b*x*e + a*e)^
2*B*a*b^4*c^3*d^3*g^2*e^2/(d*x + c)^2 + 36*(b*x*e + a*e)^2*A*a^2*b^3*c^2*d^
4*g^2*e^2/(d*x + c)^2 + 24*(b*x*e + a*e)^2*B*a^2*b^3*c^2*d^4*g^2*e^2/(d*x +
 c)^2 - 24*(b*x*e + a*e)^2*A*a^3*b^2*c*d^5*g^2*e^2/(d*x + c)^2 - 16*(b*x*e
+ a*e)^2*B*a^3*b^2*c*d^5*g^2*e^2/(d*x + c)^2 + 6*(b*x*e + a*e)^2*A*a^4*b*d^
6*g^2*e^2/(d*x + c)^2 + 4*(b*x*e + a*e)^2*B*a^4*b*d^6*g^2*e^2/(d*x + c)^2)*
(b*c/((b*c*e - a*d*e)*(b*c - a*d)) - a*d/((b*c*e - a*d*e)*(b*c - a*d)))/(b^
4*d^3*e^3 - 3*(b*x*e + a*e)*b^3*d^4*e^2/(d*x + c) + 3*(b*x*e + a*e)^2*b^2*d
^5*e/(d*x + c)^2 - (b*x*e + a*e)^3*b*d^6/(d*x + c)^3)
```

**maple [B]** time = 0.15, size = 3283, normalized size = 27.82

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*g*x+a*g)^2*(A+B*ln(e*(b*x+a)/(d*x+c))),x)
```

```
[Out] e*A*g^2/(d*e/(d*x+c)*a-e/(d*x+c)*b*c)*a^3+2/3*e*B*g^2/(d*e/(d*x+c)*a-e/(d*x
+c)*b*c)*a^3-1/d^3*e*B*g^2*ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))*b^3/(d*e/(d*x+c)
*a-e/(d*x+c)*b*c)*c^3-1/d^3*e^2*B*g^2*ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))/(d*e/
(d*x+c)*a-e/(d*x+c)*b*c)^2*b^4*c^3-1/3/d^3*e^3*B*g^2*ln(b*e/d+(a*d-b*c)*e/d
/(d*x+c))/(d*e/(d*x+c)*a-e/(d*x+c)*b*c)^3*b^5*c^3-4*e*B*g^2*ln(b*e/d+(a*d-b
*c)*e/d/(d*x+c))/(d*e/(d*x+c)*a-e/(d*x+c)*b*c)*a^3/(d*x+c)*c+1/2/d^2*e^2*B*
g^2*b^3/(d*e/(d*x+c)*a-e/(d*x+c)*b*c)^2*a*c^2+1/d^2*e^3*A*g^2*b^4/(d*e/(d*x
+c)*a-e/(d*x+c)*b*c)^3*a*c^2-3/d*e*A*g^2/(d*e/(d*x+c)*a-e/(d*x+c)*b*c)*a^2*
b*c-2/d*e*B*g^2/(d*e/(d*x+c)*a-e/(d*x+c)*b*c)*a^2*b*c+3/d^2*e^2*A*g^2*b^3/(
d*e/(d*x+c)*a-e/(d*x+c)*b*c)^2*a*c^2-3/d*e^2*A*g^2*b^2/(d*e/(d*x+c)*a-e/(d*
x+c)*b*c)^2*a^2*c-1/d*e^3*A*g^2*b^3/(d*e/(d*x+c)*a-e/(d*x+c)*b*c)^3*a^2*c-1
/3/d^3*e^3*A*g^2*b^5/(d*e/(d*x+c)*a-e/(d*x+c)*b*c)^3*c^3-2/3/d^3*e*B*g^2/(d
*e/(d*x+c)*a-e/(d*x+c)*b*c)*b^3*c^3-1/d^3*e*A*g^2/(d*e/(d*x+c)*a-e/(d*x+c)*
b*c)*b^3*c^3-1/6/d^3*e^2*B*g^2*b^4/(d*e/(d*x+c)*a-e/(d*x+c)*b*c)^2*c^3-1/d^
2*B*g^2*ln((b*e/d+(a*d-b*c)*e/d/(d*x+c))*d-b*e)*a*c^2*b+e^2*B*g^2*ln(b*e/d+
(a*d-b*c)*e/d/(d*x+c))/(d*e/(d*x+c)*a-e/(d*x+c)*b*c)^2*a^3*b+1/3*e^3*B*g^2*
ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))/(d*e/(d*x+c)*a-e/(d*x+c)*b*c)^3*a^3*b^2-1/3
*B*g^2/b*ln((b*e/d+(a*d-b*c)*e/d/(d*x+c))*d-b*e)*a^3+1/6*e^2*B*g^2*b/(d*e/(
d*x+c)*a-e/(d*x+c)*b*c)^2*a^3+1/3*e^3*A*g^2*b^2/(d*e/(d*x+c)*a-e/(d*x+c)*b*
c)^3*a^3+e^2*A*g^2*b/(d*e/(d*x+c)*a-e/(d*x+c)*b*c)^2*a^3+1/d*B*g^2*ln((b*e/
d+(a*d-b*c)*e/d/(d*x+c))*d-b*e)*a^2*c+e*B*g^2*ln(b*e/d+(a*d-b*c)*e/d/(d*x+c
))/(d*e/(d*x+c)*a-e/(d*x+c)*b*c)*a^3+1/3/d^3*B*g^2*b^2*ln((b*e/d+(a*d-b*c)*
e/d/(d*x+c))*d-b*e)*c^3+2/d^2*e*B*g^2/(d*e/(d*x+c)*a-e/(d*x+c)*b*c)*a*b^2*c
```

```
^2-1/2/d*e^2*B*g^2*b^2/(d*e/(d*x+c)*a-e/(d*x+c)*b*c)^2*a^2*c+3/d^2*e*A*g^2/
(d*e/(d*x+c)*a-e/(d*x+c)*b*c)*b^2*c^2*a-1/d^3*e^2*A*g^2*b^4/(d*e/(d*x+c)*a-
e/(d*x+c)*b*c)^2*c^3+5*d*e^2*B*g^2*ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))/(d*e/(d*
x+c)*a-e/(d*x+c)*b*c)^2*a^4/(d*x+c)^2*c-2*d^2*e^3*B*g^2*ln(b*e/d+(a*d-b*c)*
e/d/(d*x+c))/(d*e/(d*x+c)*a-e/(d*x+c)*b*c)^3*a^5/(d*x+c)^3*c+6/d*e*B*g^2*ln
(b*e/d+(a*d-b*c)*e/d/(d*x+c))/(d*e/(d*x+c)*a-e/(d*x+c)*b*c)*a^2/(d*x+c)*c^2
*b-5/d^2*e^2*B*g^2*b^3*ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))/(d*e/(d*x+c)*a-e/(d*
x+c)*b*c)^2*c^4/(d*x+c)^2*a-4/d^2*e*B*g^2*ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))*b
^2/(d*e/(d*x+c)*a-e/(d*x+c)*b*c)*c^3/(d*x+c)*a+5/d*e^3*B*g^2*b^3*ln(b*e/d+(
a*d-b*c)*e/d/(d*x+c))/(d*e/(d*x+c)*a-e/(d*x+c)*b*c)^3*c^4/(d*x+c)^3*a^2-2/d
^2*e^3*B*g^2*b^4*ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))/(d*e/(d*x+c)*a-e/(d*x+c)*b
*c)^3*c^5/(d*x+c)^3*a+3/d^2*e^2*B*g^2*ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))/(d*e/
(d*x+c)*a-e/(d*x+c)*b*c)^2*b^3*c^2*a+1/d^2*e^3*B*g^2*ln(b*e/d+(a*d-b*c)*e/d
/(d*x+c))/(d*e/(d*x+c)*a-e/(d*x+c)*b*c)^3*b^4*c^2*a-3/d*e^2*B*g^2*ln(b*e/d+
(a*d-b*c)*e/d/(d*x+c))/(d*e/(d*x+c)*a-e/(d*x+c)*b*c)^2*a^2*b^2*c-1/d*e^3*B*
g^2*ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))/(d*e/(d*x+c)*a-e/(d*x+c)*b*c)^3*a^2*b^3
*c-d^2*e^2*B*g^2/b*ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))/(d*e/(d*x+c)*a-e/(d*x+c)
*b*c)^2*a^5/(d*x+c)^2+1/d^3*e*B*g^2*ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))*b^3/(d*
e/(d*x+c)*a-e/(d*x+c)*b*c)*c^4/(d*x+c)-20/3*e^3*B*g^2*b^2*ln(b*e/d+(a*d-b*c
)*e/d/(d*x+c))/(d*e/(d*x+c)*a-e/(d*x+c)*b*c)^3*c^3/(d*x+c)^3*a^3-10*e^2*B*g
^2*ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))/(d*e/(d*x+c)*a-e/(d*x+c)*b*c)^2*a^3/(d*x
+c)^2*c^2*b-3/d*e*B*g^2*ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))/(d*e/(d*x+c)*a-e/(d
*x+c)*b*c)*a^2*c*b+3/d^2*e*B*g^2*ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))/(d*e/(d*x+
c)*a-e/(d*x+c)*b*c)*a*c^2*b^2+d*e*B*g^2*ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))/b/(
d*e/(d*x+c)*a-e/(d*x+c)*b*c)*a^4/(d*x+c)+1/3*d^3*e^3*B*g^2/b*ln(b*e/d+(a*d-
b*c)*e/d/(d*x+c))/(d*e/(d*x+c)*a-e/(d*x+c)*b*c)^3*a^6/(d*x+c)^3+1/d^3*e^2*B
*g^2*b^4*ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))/(d*e/(d*x+c)*a-e/(d*x+c)*b*c)^2*c^
5/(d*x+c)^2+1/3/d^3*e^3*B*g^2*b^5*ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))/(d*e/(d*x
+c)*a-e/(d*x+c)*b*c)^3*c^6/(d*x+c)^3+10/d*e^2*B*g^2*b^2*ln(b*e/d+(a*d-b*c)*
e/d/(d*x+c))/(d*e/(d*x+c)*a-e/(d*x+c)*b*c)^2*a^2/(d*x+c)^2*c^3+5*d*e^3*B*g^
2*ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))/(d*e/(d*x+c)*a-e/(d*x+c)*b*c)^3*a^4/(d*x+
c)^3*c^2*b
```

**maxima [B]** time = 1.24, size = 280, normalized size = 2.37

$$\frac{1}{3}Ab^2g^2x^3+Aabg^2x^2+\left(x\log\left(\frac{bex}{dx+c}+\frac{ae}{dx+c}\right)+\frac{a\log(bx+a)}{b}-\frac{c\log(dx+c)}{d}\right)Ba^2g^2+\left(x^2\log\left(\frac{bex}{dx+c}+\frac{ae}{dx+c}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*g*x+a*g)^2*(A+B*log(e*(b*x+a)/(d*x+c))),x, algorithm="maxima")
```

```
[Out] 1/3*A*b^2*g^2*x^3 + A*a*b*g^2*x^2 + (x*log(b*e*x/(d*x + c) + a*e/(d*x + c))
 + a*log(b*x + a)/b - c*log(d*x + c)/d)*B*a^2*g^2 + (x^2*log(b*e*x/(d*x + c
) + a*e/(d*x + c)) - a^2*log(b*x + a)/b^2 + c^2*log(d*x + c)/d^2 - (b*c - a
```

```
*d)*x/(b*d))*B*a*b*g^2 + 1/6*(2*x^3*log(b*e*x/(d*x + c) + a*e/(d*x + c)) +
2*a^3*log(b*x + a)/b^3 - 2*c^3*log(d*x + c)/d^3 - ((b^2*c*d - a*b*d^2)*x^2
- 2*(b^2*c^2 - a^2*d^2)*x)/(b^2*d^2))*B*b^2*g^2 + A*a^2*g^2*x
```

**mupad [B]** time = 4.48, size = 290, normalized size = 2.46

$$x^2\left(\frac{b\,g^2\,\left(9\,A\,a\,d+3\,A\,b\,c+B\,a\,d-B\,b\,c\right)}{6\,d}-\frac{A\,b\,g^2\,\left(3\,a\,d+3\,b\,c\right)}{6\,d}\right)-x\left(\frac{\left(3\,a\,d+3\,b\,c\right)\,\left(\frac{b\,g^2\,\left(9\,A\,a\,d+3\,A\,b\,c+B\,a\,d-B\,b\,c\right)}{3\,d}\right.}{3\,b\,d}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*g + b*g*x)^2*(A + B*log((e*(a + b*x))/(c + d*x))),x)
```

```
[Out] x^2*((b*g^2*(9*A*a*d + 3*A*b*c + B*a*d - B*b*c))/(6*d) - (A*b*g^2*(3*a*d +
3*b*c))/(6*d)) - x*(((3*a*d + 3*b*c)*((b*g^2*(9*A*a*d + 3*A*b*c + B*a*d - B
*b*c))/(3*d) - (A*b*g^2*(3*a*d + 3*b*c))/(3*d)))/(3*b*d) - (a*g^2*(3*A*a*d
+ 3*A*b*c + B*a*d - B*b*c))/d + (A*a*b*c*g^2)/d) + log((e*(a + b*x))/(c + d
*x))*((B*b^2*g^2*x^3)/3 + B*a^2*g^2*x + B*a*b*g^2*x^2) - (log(c + d*x)*(B*b
^2*c^3*g^2 + 3*B*a^2*c*d^2*g^2 - 3*B*a*b*c^2*d*g^2))/(3*d^3) + (A*b^2*g^2*x
^3)/3 + (B*a^3*g^2*log(a + b*x))/(3*b)
```

**sympy [B]** time = 2.92, size = 491, normalized size = 4.16

$$\frac{Ab^2g^2x^3}{3}+\frac{Ba^3g^2\log\left(x+\frac{\frac{Ba^4d^3g^2}{b}+3Ba^3cd^2g^2-3Ba^2bc^2dg^2+Bab^2c^3g^2}{Ba^3d^3g^2+3Ba^2bcd^2g^2-3Bab^2c^2dg^2+Bb^3c^3g^2}\right)}{3b}-\frac{Bcg^2\left(3a^2d^2-3abcd+b^2c^2\right)\log\left(x+\frac{4Ba^3cd^2g^2-3\cdots}{\cdots}\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*g*x+a*g)**2*(A+B*ln(e*(b*x+a)/(d*x+c))),x)
```

```
[Out] A*b**2*g**2*x**3/3 + B*a**3*g**2*log(x + (B*a**4*d**3*g**2/b + 3*B*a**3*c*d
**2*g**2 - 3*B*a**2*b*c**2*d*g**2 + B*a*b**2*c**3*g**2)/(B*a**3*d**3*g**2 +
 3*B*a**2*b*c*d**2*g**2 - 3*B*a*b**2*c**2*d*g**2 + B*b**3*c**3*g**2))/(3*b)
 - B*c*g**2*(3*a**2*d**2 - 3*a*b*c*d + b**2*c**2)*log(x + (4*B*a**3*c*d**2*
g**2 - 3*B*a**2*b*c**2*d*g**2 + B*a*b**2*c**3*g**2 - B*a*c*g**2*(3*a**2*d**
2 - 3*a*b*c*d + b**2*c**2) + B*b*c**2*g**2*(3*a**2*d**2 - 3*a*b*c*d + b**2*
c**2)/d)/(B*a**3*d**3*g**2 + 3*B*a**2*b*c*d**2*g**2 - 3*B*a*b**2*c**2*d*g**
2 + B*b**3*c**3*g**2))/(3*d**3) + x**2*(A*a*b*g**2 + B*a*b*g**2/6 - B*b**2*
c*g**2/(6*d)) + x*(A*a**2*g**2 + 2*B*a**2*g**2/3 - B*a*b*c*g**2/d + B*b**2*
c**2*g**2/(3*d**2)) + (B*a**2*g**2*x + B*a*b*g**2*x**2 + B*b**2*g**2*x**3/3
)*log(e*(a + b*x)/(c + d*x))
```

## 3.91 $\int (ag+bgx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)\,dx$

**Optimal**. Leaf size=81

$$\frac{g(a+bx)^2\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{2b}+\frac{Bg(bc-ad)^2\log(c+dx)}{2bd^2}-\frac{Bgx(bc-ad)}{2d}$$

```
[Out] -1/2*B*(-a*d+b*c)*g*x/d+1/2*g*(b*x+a)^2*(A+B*ln(e*(b*x+a)/(d*x+c)))/b+1/2*B
*(-a*d+b*c)^2*g*ln(d*x+c)/b/d^2
```

**Rubi [A]** time = 0.05, antiderivative size = 81, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.107, Rules used = {2525, 12, 43}

$$\frac{g(a+bx)^2\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{2b}+\frac{Bg(bc-ad)^2\log(c+dx)}{2bd^2}-\frac{Bgx(bc-ad)}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[(a*g + b*g*x)*(A + B*Log[(e*(a + b*x))/(c + d*x)]),x]
```

```
[Out] -(B*(b*c - a*d)*g*x)/(2*d) + (g*(a + b*x)^2*(A + B*Log[(e*(a + b*x))/(c + d
*x)]))/(2*b) + (B*(b*c - a*d)^2*g*Log[c + d*x])/(2*b*d^2)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2525

```
Int[((a_.) + Log[(c_.)*(RFx_)^(p_.)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_))^(m_.
), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(a + b*Log[c*RFx^p])^n)/(e*(m + 1))
, x] - Dist[(b*n*p)/(e*(m + 1)), Int[SimplifyIntegrand[((d + e*x)^(m + 1)*(
a + b*Log[c*RFx^p])^(n - 1)*D[RFx, x])/RFx, x], x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && RationalFunctionQ[RFx, x] && IGtQ[n, 0] && (EqQ[n, 1] ||
```

```
IntegerQ[m]) && NeQ[m, -1]
```

Rubi steps

$$\int (ag+bgx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)dx = \frac{g(a+bx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{2b} - \frac{B\int \frac{(bc-ad)g^2(a+bx)}{c+dx}\,dx}{2bg}$$

$$= \frac{g(a+bx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{2b} - \frac{(B(bc-ad)g)\int \frac{a+bx}{c+dx}\,dx}{2b}$$

$$= \frac{g(a+bx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{2b} - \frac{(B(bc-ad)g)\int \left(\frac{b}{d}+\frac{-bc+ad}{d(c+dx)}\right)dx}{2b}$$

$$= -\frac{B(bc-ad)gx}{2d} + \frac{g(a+bx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{2b} + \frac{B(bc-ad)^2 g \,\mathrm{l}}{2bd^2}$$

**Mathematica [A]** time = 0.03, size = 69, normalized size = 0.85

$$\frac{g\left((a+bx)^2\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)+\frac{B(ad-bc)((ad-bc)\log(c+dx)+bdx)}{d^2}\right)}{2b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*g + b*g*x)*(A + B*Log[(e*(a + b*x))/(c + d*x)]),x]
```

```
[Out] (g*((a + b*x)^2*(A + B*Log[(e*(a + b*x))/(c + d*x)]) + (B*(-(b*c) + a*d)*(b
*d*x + (-(b*c) + a*d)*Log[c + d*x]))/d^2))/(2*b)
```

**fricas [A]** time = 0.79, size = 125, normalized size = 1.54

$$\frac{Ab^2d^2gx^2 + Ba^2d^2g\log(bx+a) - \left(Bb^2cd - (2A+B)abd^2\right)gx + \left(Bb^2c^2 - 2Babcd\right)g\log(dx+c) + \left(Bb^2d^2gx^2 +\right.}{2bd^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*g*x+a*g)*(A+B*log(e*(b*x+a)/(d*x+c))),x, algorithm="fricas")
```

```
[Out] 1/2*(A*b^2*d^2*g*x^2 + B*a^2*d^2*g*log(b*x + a) - (B*b^2*c*d - (2*A + B)*a*
b*d^2)*g*x + (B*b^2*c^2 - 2*B*a*b*c*d)*g*log(d*x + c) + (B*b^2*d^2*g*x^2 +
2*B*a*b*d^2*g*x)*log((b*e*x + a*e)/(d*x + c)))/(b*d^2)
```

**giac [B]** time = 0.97, size = 1319, normalized size = 16.28

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*g*x+a*g)*(A+B*log(e*(b*x+a)/(d*x+c))),x, algorithm="giac")
```

```
[Out] -1/2*(B*b^5*c^3*g*e^3*log(-b*e + (b*x*e + a*e)*d/(d*x + c)) - 3*B*a*b^4*c^2
*d*g*e^3*log(-b*e + (b*x*e + a*e)*d/(d*x + c)) + 3*B*a^2*b^3*c*d^2*g*e^3*lo
g(-b*e + (b*x*e + a*e)*d/(d*x + c)) - B*a^3*b^2*d^3*g*e^3*log(-b*e + (b*x*e
 + a*e)*d/(d*x + c)) - 2*(b*x*e + a*e)*B*b^4*c^3*d*g*e^2*log(-b*e + (b*x*e
+ a*e)*d/(d*x + c))/(d*x + c) + 6*(b*x*e + a*e)*B*a*b^3*c^2*d^2*g*e^2*log(-
b*e + (b*x*e + a*e)*d/(d*x + c))/(d*x + c) - 6*(b*x*e + a*e)*B*a^2*b^2*c*d^
3*g*e^2*log(-b*e + (b*x*e + a*e)*d/(d*x + c))/(d*x + c) + 2*(b*x*e + a*e)*B
*a^3*b*d^4*g*e^2*log(-b*e + (b*x*e + a*e)*d/(d*x + c))/(d*x + c) + (b*x*e +
 a*e)^2*B*b^3*c^3*d^2*g*e*log(-b*e + (b*x*e + a*e)*d/(d*x + c))/(d*x + c)^2
 - 3*(b*x*e + a*e)^2*B*a*b^2*c^2*d^3*g*e*log(-b*e + (b*x*e + a*e)*d/(d*x +
c))/(d*x + c)^2 + 3*(b*x*e + a*e)^2*B*a^2*b*c*d^4*g*e*log(-b*e + (b*x*e + a
*e)*d/(d*x + c))/(d*x + c)^2 - (b*x*e + a*e)^2*B*a^3*d^5*g*e*log(-b*e + (b*
x*e + a*e)*d/(d*x + c))/(d*x + c)^2 - (b*x*e + a*e)^2*B*b^3*c^3*d^2*g*e*log
((b*x*e + a*e)/(d*x + c))/(d*x + c)^2 + 3*(b*x*e + a*e)^2*B*a*b^2*c^2*d^3*g
*e*log((b*x*e + a*e)/(d*x + c))/(d*x + c)^2 - 3*(b*x*e + a*e)^2*B*a^2*b*c*d
^4*g*e*log((b*x*e + a*e)/(d*x + c))/(d*x + c)^2 + (b*x*e + a*e)^2*B*a^3*d^5
*g*e*log((b*x*e + a*e)/(d*x + c))/(d*x + c)^2 + A*b^5*c^3*g*e^3 + B*b^5*c^3
*g*e^3 - 3*A*a*b^4*c^2*d*g*e^3 - 3*B*a*b^4*c^2*d*g*e^3 + 3*A*a^2*b^3*c*d^2*
g*e^3 + 3*B*a^2*b^3*c*d^2*g*e^3 - A*a^3*b^2*d^3*g*e^3 - B*a^3*b^2*d^3*g*e^3
 - 2*(b*x*e + a*e)*A*b^4*c^3*d*g*e^2/(d*x + c) - (b*x*e + a*e)*B*b^4*c^3*d*
g*e^2/(d*x + c) + 6*(b*x*e + a*e)*A*a*b^3*c^2*d^2*g*e^2/(d*x + c) + 3*(b*x*
e + a*e)*B*a*b^3*c^2*d^2*g*e^2/(d*x + c) - 6*(b*x*e + a*e)*A*a^2*b^2*c*d^3*
g*e^2/(d*x + c) - 3*(b*x*e + a*e)*B*a^2*b^2*c*d^3*g*e^2/(d*x + c) + 2*(b*x*
e + a*e)*A*a^3*b*d^4*g*e^2/(d*x + c) + (b*x*e + a*e)*B*a^3*b*d^4*g*e^2/(d*x
 + c))*(b*c/((b*c*e - a*d*e)*(b*c - a*d)) - a*d/((b*c*e - a*d*e)*(b*c - a*d
)))/(b^3*d^2*e^2 - 2*(b*x*e + a*e)*b^2*d^3*e/(d*x + c) + (b*x*e + a*e)^2*b*
d^4/(d*x + c)^2)
```

**maple [B]** time = 0.13, size = 1544, normalized size = 19.06

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*g*x+a*g)*(A+B*ln(e*(b*x+a)/(d*x+c))),x)
```

```
[Out] -1/2/d^2*B*g*ln((b*e/d+(a*d-b*c)*e/d/(d*x+c))*d-b*e)*c^2*b+1/d*B*g*ln((b*e/
d+(a*d-b*c)*e/d/(d*x+c))*d-b*e)*c*a+1/2*e^2*A*g*b/(d*e/(d*x+c)*a-e/(d*x+c)*
```

```
b*c)^2*a^2+e*B*g*ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))/(d*e/(d*x+c)*a-e/(d*x+c)*b
*c)*a^2+1/2*e*B*g/(d*e/(d*x+c)*a-e/(d*x+c)*b*c)*a^2-1/2*B*g/b*ln((b*e/d+(a*
d-b*c)*e/d/(d*x+c))*d-b*e)*a^2+e*A*g/(d*e/(d*x+c)*a-e/(d*x+c)*b*c)*a^2-2/d*
e*A*g/(d*e/(d*x+c)*a-e/(d*x+c)*b*c)*b*c*a+1/d^2*e*B*g*ln(b*e/d+(a*d-b*c)*e/
d/(d*x+c))/(d*e/(d*x+c)*a-e/(d*x+c)*b*c)*c^2*b^2+1/2/d^2*e^2*A*g*b^3/(d*e/(
d*x+c)*a-e/(d*x+c)*b*c)^2*c^2+1/d^2*e*A*g/(d*e/(d*x+c)*a-e/(d*x+c)*b*c)*b^2
*c^2+1/2/d^2*e*B*g/(d*e/(d*x+c)*a-e/(d*x+c)*b*c)*b^2*c^2+1/2*e^2*B*g*b*ln(b
*e/d+(a*d-b*c)*e/d/(d*x+c))/(d*e/(d*x+c)*a-e/(d*x+c)*b*c)^2*a^2+1/2/d^2*e^2
*B*g*ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))/(d*e/(d*x+c)*a-e/(d*x+c)*b*c)^2*c^2*b^
3-3*e*B*g*ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))/(d*e/(d*x+c)*a-e/(d*x+c)*b*c)*a^2
/(d*x+c)*c-1/d*e*B*g/(d*e/(d*x+c)*a-e/(d*x+c)*b*c)*a*b*c-1/d*e^2*A*g*b^2/(d
*e/(d*x+c)*a-e/(d*x+c)*b*c)^2*a*c+2/d*e^2*B*g*b^2*ln(b*e/d+(a*d-b*c)*e/d/(d
*x+c))/(d*e/(d*x+c)*a-e/(d*x+c)*b*c)^2*a/(d*x+c)^2*c^3+3/d*e*B*g*ln(b*e/d+(
a*d-b*c)*e/d/(d*x+c))/(d*e/(d*x+c)*a-e/(d*x+c)*b*c)*a/(d*x+c)*c^2*b-3*e^2*B
*g*ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))/(d*e/(d*x+c)*a-e/(d*x+c)*b*c)^2*a^2/(d*x
+c)^2*c^2*b-1/2*d^2*e^2*B*g/b*ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))/(d*e/(d*x+c)*
a-e/(d*x+c)*b*c)^2*a^4/(d*x+c)^2-1/2/d^2*e^2*B*g*ln(b*e/d+(a*d-b*c)*e/d/(d*
x+c))/(d*e/(d*x+c)*a-e/(d*x+c)*b*c)^2*c^4/(d*x+c)^2*b^3-2/d*e*B*g*ln(b*e/d+
(a*d-b*c)*e/d/(d*x+c))/(d*e/(d*x+c)*a-e/(d*x+c)*b*c)*c*b*a-1/d*e^2*B*g*b^2*
ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))/(d*e/(d*x+c)*a-e/(d*x+c)*b*c)^2*a*c-1/d^2*e
*B*g*ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))/(d*e/(d*x+c)*a-e/(d*x+c)*b*c)*c^3/(d*x
+c)*b^2+d*e*B*g*ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))/b/(d*e/(d*x+c)*a-e/(d*x+c)*
b*c)*a^3/(d*x+c)+2*d*e^2*B*g*ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))/(d*e/(d*x+c)*a
-e/(d*x+c)*b*c)^2*a^3/(d*x+c)^2*c
```

**maxima [A]** time = 1.46, size = 144, normalized size = 1.78

$$\frac{1}{2}Abgx^2+\left(x\log\left(\frac{bex}{dx+c}+\frac{ae}{dx+c}\right)+\frac{a\log(bx+a)}{b}-\frac{c\log(dx+c)}{d}\right)Bag+\frac{1}{2}\left(x^2\log\left(\frac{bex}{dx+c}+\frac{ae}{dx+c}\right)-\frac{a^2\log}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*g*x+a*g)*(A+B*log(e*(b*x+a)/(d*x+c))),x, algorithm="maxima")
```

```
[Out] 1/2*A*b*g*x^2 + (x*log(b*e*x/(d*x + c) + a*e/(d*x + c)) + a*log(b*x + a)/b
- c*log(d*x + c)/d)*B*a*g + 1/2*(x^2*log(b*e*x/(d*x + c) + a*e/(d*x + c)) -
 a^2*log(b*x + a)/b^2 + c^2*log(d*x + c)/d^2 - (b*c - a*d)*x/(b*d))*B*b*g +
 A*a*g*x
```

**mupad [B]** time = 4.30, size = 126, normalized size = 1.56

$$x\left(\frac{g\,(4\,A\,a\,d+2\,A\,b\,c+B\,a\,d-B\,b\,c)}{2\,d}-\frac{A\,g\,(2\,a\,d+2\,b\,c)}{2\,d}\right)+\ln\left(\frac{e\,(a+b\,x)}{c+d\,x}\right)\left(\frac{B\,b\,g\,x^2}{2}+B\,a\,g\,x\right)+\frac{\ln(c+d\,x)}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*g + b*g*x)*(A + B*log((e*(a + b*x))/(c + d*x))),x)
```

```
[Out] x*((g*(4*A*a*d + 2*A*b*c + B*a*d - B*b*c))/(2*d) - (A*g*(2*a*d + 2*b*c))/(2
*d)) + log((e*(a + b*x))/(c + d*x))*((B*b*g*x^2)/2 + B*a*g*x) + (log(c + d*
x)*(B*b*c^2*g - 2*B*a*c*d*g))/(2*d^2) + (A*b*g*x^2)/2 + (B*a^2*g*log(a + b*
x))/(2*b)
```

**sympy [B]** time = 1.95, size = 253, normalized size = 3.12

$$\frac{Abgx^2}{2}+\frac{Ba^2g\log\left(x+\frac{\frac{Ba^3d^2g}{b}+2Ba^2cdg-Babc^2g}{Ba^2d^2g+2Babcdg-Bb^2c^2g}\right)}{2b}-\frac{Bcg\left(2ad-bc\right)\log\left(x+\frac{3Ba^2cdg-Babc^2g-Bacg(2ad-bc)+\frac{Bbc^2g(2ad-bc)}{d}}{Ba^2d^2g+2Babcdg-Bb^2c^2g}\right)}{2d^2}+x\left(A\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*g*x+a*g)*(A+B*ln(e*(b*x+a)/(d*x+c))),x)
```

```
[Out] A*b*g*x**2/2 + B*a**2*g*log(x + (B*a**3*d**2*g/b + 2*B*a**2*c*d*g - B*a*b*c
**2*g)/(B*a**2*d**2*g + 2*B*a*b*c*d*g - B*b**2*c**2*g))/(2*b) - B*c*g*(2*a*
d - b*c)*log(x + (3*B*a**2*c*d*g - B*a*b*c**2*g - B*a*c*g*(2*a*d - b*c) + B
*b*c**2*g*(2*a*d - b*c)/d)/(B*a**2*d**2*g + 2*B*a*b*c*d*g - B*b**2*c**2*g))
/(2*d**2) + x*(A*a*g + B*a*g/2 - B*b*c*g/(2*d)) + (B*a*g*x + B*b*g*x**2/2)*
log(e*(a + b*x)/(c + d*x))
```

## 3.92 $\int \frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{ag+bgx}\,dx$

**Optimal**. Leaf size=80

$$\frac{B\text{Li}_2\left(\frac{bc-ad}{d(a+bx)}+1\right)}{bg}-\frac{\log\left(-\frac{bc-ad}{d(a+bx)}\right)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{bg}$$

```
[Out] -ln((a*d-b*c)/d/(b*x+a))*(A+B*ln(e*(b*x+a)/(d*x+c)))/b/g+B*polylog(2,1+(-a*
d+b*c)/d/(b*x+a))/b/g
```

**Rubi [A]** time = 0.22, antiderivative size = 120, normalized size of antiderivative = 1.50, number of steps used = 10, number of rules used = 8, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {2524, 12, 2418, 2390, 2301, 2394, 2393, 2391}

$$\frac{B\text{PolyLog}\left(2,-\frac{d(a+bx)}{bc-ad}\right)}{bg}+\frac{\log(ag+bgx)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{bg}+\frac{B\log(ag+bgx)\log\left(\frac{b(c+dx)}{bc-ad}\right)}{bg}-\frac{B\log^2(g(a+bx))}{2bg}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Log[(e*(a + b*x))/(c + d*x)])/(a*g + b*g*x),x]
```

```
[Out] -(B*Log[g*(a + b*x)]^2)/(2*b*g) + ((A + B*Log[(e*(a + b*x))/(c + d*x)])*Log
[a*g + b*g*x])/(b*g) + (B*Log[(b*(c + d*x))/(b*c - a*d)]*Log[a*g + b*g*x])/
(b*g) + (B*PolyLog[2, -((d*(a + b*x))/(b*c - a*d))])/(b*g)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2301

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

Rule 2390

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_))^(q_.), x_Symbol] :> Dist[1/e, Subst[Int[((f*x)/d)^q*(a + b*Log[c*x^
n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && E
qQ[e*f - d*g, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2393

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + (c*e*x)/g])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2394

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d + e*x
)^n]))/g, x] - Dist[(b*e*n)/g, Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2418

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*(RFx_), x_Sy
mbol] :> With[{u = ExpandIntegrand[(a + b*Log[c*(d + e*x)^n])^p, RFx, x]},
Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d, e, n}, x] && RationalFunctionQ[
RFx, x] && IntegerQ[p]
```

Rule 2524

```
Int[((a_.) + Log[(c_.)*(RFx_)^(p_.)]*(b_.))^(n_.)/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[(Log[d + e*x]*(a + b*Log[c*RFx^p])^n)/e, x] - Dist[(b*n*p)/e
, Int[(Log[d + e*x]*(a + b*Log[c*RFx^p])^(n - 1)*D[RFx, x])/RFx, x], x] /;
FreeQ[{a, b, c, d, e, p}, x] && RationalFunctionQ[RFx, x] && IGtQ[n, 0]
```

Rubi steps

$$\int \frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{ag+bgx}\,dx = \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)\log(ag+bgx)}{bg} - \frac{B\int \frac{(c+dx)\left(-\frac{de(a+bx)}{(c+dx)^2}+\frac{be}{c+dx}\right)\log(ag+bgx)}{e(a+bx)}dx}{bg}$$

$$= \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)\log(ag+bgx)}{bg} - \frac{B\int \frac{(c+dx)\left(-\frac{de(a+bx)}{(c+dx)^2}+\frac{be}{c+dx}\right)\log(ag+bgx)}{a+bx}dx}{beg}$$

$$= \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)\log(ag+bgx)}{bg} - \frac{B\int \left(\frac{be\log(ag+bgx)}{a+bx}-\frac{de\log(ag+bgx)}{c+dx}\right)dx}{beg}$$

$$= \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)\log(ag+bgx)}{bg} - \frac{B\int \frac{\log(ag+bgx)}{a+bx}dx}{g} + \frac{(Bd)\int \frac{\log(ag+bgx)}{c+dx}dx}{bg}$$

$$= \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)\log(ag+bgx)}{bg} + \frac{B\log\left(\frac{b(c+dx)}{bc-ad}\right)\log(ag+bgx)}{bg} - B\int \frac{\log\left(\frac{bg(\cdots}{bcg\cdots}\right.}{ag+\cdots}$$

$$= \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)\log(ag+bgx)}{bg} + \frac{B\log\left(\frac{b(c+dx)}{bc-ad}\right)\log(ag+bgx)}{bg} - \frac{B\,\mathrm{Subst}\left(\int \cdots\right.}{\cdots}$$

$$= -\frac{B\log^2(g(a+bx))}{2bg} + \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)\log(ag+bgx)}{bg} + \frac{B\log\left(\frac{b(c+dx)}{bc-ad}\right)\log(ag\cdots}{bg}$$

**Mathematica [A]** time = 0.04, size = 95, normalized size = 1.19

$$\frac{\log(g(a+bx))\left(2\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+B\log\left(\frac{b(c+dx)}{bc-ad}\right)+A\right)-B\log(g(a+bx))\right)+2B\,\mathrm{Li}_2\left(\frac{d(a+bx)}{ad-bc}\right)}{2bg}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Log[(e*(a + b*x))/(c + d*x)])/(a*g + b*g*x),x]
```

```
[Out] (Log[g*(a + b*x)]*(-(B*Log[g*(a + b*x)]) + 2*(A + B*Log[(e*(a + b*x))/(c +
d*x)] + B*Log[(b*(c + d*x))/(b*c - a*d)])) + 2*B*PolyLog[2, (d*(a + b*x))/(
-(b*c) + a*d)])/(2*b*g)
```

**fricas [F]** time = 0.98, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{B\log\left(\frac{bex+ae}{dx+c}\right)+A}{bgx+ag},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*(b*x+a)/(d*x+c)))/(b*g*x+a*g),x, algorithm="fricas")
[Out] integral((B*log((b*e*x + a*e)/(d*x + c)) + A)/(b*g*x + a*g), x)
```

**giac [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*(b*x+a)/(d*x+c)))/(b*g*x+a*g),x, algorithm="giac")
[Out] Timed out
```

**maple [B]** time = 0.09, size = 602, normalized size = 7.52

$$-\frac{Bad\ln\left(-\frac{-be+\left(\frac{be}{d}+\frac{(ad-bc)e}{(dx+c)d}\right)d}{be}\right)\ln\left(\frac{be}{d}+\frac{(ad-bc)e}{(dx+c)d}\right)}{(ad-bc)bg}+\frac{Bad\ln\left(\frac{be}{d}+\frac{(ad-bc)e}{(dx+c)d}\right)^2}{2(ad-bc)bg}+\frac{Bc\ln\left(-\frac{-be+\left(\frac{be}{d}+\frac{(ad-bc)e}{(dx+c)d}\right)d}{be}\right)\ln\left(\frac{be}{d}+\frac{(ad-bc)e}{(dx+c)d}\right)}{(ad-bc)g}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*ln(e*(b*x+a)/(d*x+c)))/(b*g*x+a*g),x)
[Out] -d/g/(a*d-b*c)*A/b*ln(-b*e+(b/d*e+(a*d-b*c)/(d*x+c)/d*e)*d)*a+1/g/(a*d-b*c)
*A*ln(-b*e+(b/d*e+(a*d-b*c)/(d*x+c)/d*e)*d)*c+d/g/(a*d-b*c)*A/b*ln(b/d*e+(a
*d-b*c)/(d*x+c)/d*e)*a-1/g/(a*d-b*c)*A*ln(b/d*e+(a*d-b*c)/(d*x+c)/d*e)*c-d/
g/(a*d-b*c)*B/b*dilog(-(-b*e+(b/d*e+(a*d-b*c)/(d*x+c)/d*e)*d)/b/e)*a+1/g/(a
*d-b*c)*B*dilog(-(-b*e+(b/d*e+(a*d-b*c)/(d*x+c)/d*e)*d)/b/e)*c-d/g/(a*d-b*c
)*B/b*ln(b/d*e+(a*d-b*c)/(d*x+c)/d*e)*ln(-(-b*e+(b/d*e+(a*d-b*c)/(d*x+c)/d*
e)*d)/b/e)*a+1/g/(a*d-b*c)*B*ln(b/d*e+(a*d-b*c)/(d*x+c)/d*e)*ln(-(-b*e+(b/d
*e+(a*d-b*c)/(d*x+c)/d*e)*d)/b/e)*c+1/2*d/g/(a*d-b*c)*B*ln(b/d*e+(a*d-b*c)/
(d*x+c)/d*e)^2/b*a-1/2/g/(a*d-b*c)*B*ln(b/d*e+(a*d-b*c)/(d*x+c)/d*e)^2*c
```

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$-B\left(\frac{\log(bx+a)\log(dx+c)}{bg}-\int\frac{bdx\log(e)+bc\log(e)+(2bdx+bc+ad)\log(bx+a)}{b^2dgx^2+abcg+\left(b^2cg+abdg\right)x}\,dx\right)+\frac{A\log\left(bgx+ag\right)}{bg}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*(b*x+a)/(d*x+c)))/(b*g*x+a*g),x, algorithm="maxima")
```

```
[Out] -B*(log(b*x + a)*log(d*x + c)/(b*g) - integrate((b*d*x*log(e) + b*c*log(e)
+ (2*b*d*x + b*c + a*d)*log(b*x + a))/(b^2*d*g*x^2 + a*b*c*g + (b^2*c*g + a
*b*d*g)*x), x)) + A*log(b*g*x + a*g)/(b*g)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{A + B\,\ln\left(\frac{e\,(a+b\,x)}{c+d\,x}\right)}{a\,g + b\,g\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*log((e*(a + b*x))/(c + d*x)))/(a*g + b*g*x),x)
```

```
[Out] int((A + B*log((e*(a + b*x))/(c + d*x)))/(a*g + b*g*x), x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{A}{a+bx}\,dx + \int \frac{B\log\left(\frac{ae}{c+dx}+\frac{bex}{c+dx}\right)}{a+bx}\,dx}{g}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*ln(e*(b*x+a)/(d*x+c)))/(b*g*x+a*g),x)
```

```
[Out] (Integral(A/(a + b*x), x) + Integral(B*log(a*e/(c + d*x) + b*e*x/(c + d*x))
/(a + b*x), x))/g
```

# 3.93 $\int \frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{(ag+bgx)^2}\,dx$

**Optimal**. Leaf size=63

$$-\frac{(c+dx)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{g^2(a+bx)(bc-ad)}-\frac{B}{bg^2(a+bx)}$$

```
[Out] -B/b/g^2/(b*x+a)-(d*x+c)*(A+B*ln(e*(b*x+a)/(d*x+c)))/(-a*d+b*c)/g^2/(b*x+a)
```

**Rubi [A]** time = 0.08, antiderivative size = 102, normalized size of antiderivative = 1.62, number of steps used = 4, number of rules used = 3, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {2525, 12, 44}

$$-\frac{B\log\left(\frac{e(a+bx)}{c+dx}\right)+A}{bg^2(a+bx)}-\frac{Bd\log(a+bx)}{bg^2(bc-ad)}+\frac{Bd\log(c+dx)}{bg^2(bc-ad)}-\frac{B}{bg^2(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Log[(e*(a + b*x))/(c + d*x)])/(a*g + b*g*x)^2,x]
```

```
[Out] -(B/(b*g^2*(a + b*x))) - (B*d*Log[a + b*x])/(b*(b*c - a*d)*g^2) - (A + B*Lo
g[(e*(a + b*x))/(c + d*x)])/(b*g^2*(a + b*x)) + (B*d*Log[c + d*x])/(b*(b*c
- a*d)*g^2)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 2525

```
Int[((a_.) + Log[(c_.)*(RFx_)^(p_.)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_))^(m_.
), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(a + b*Log[c*RFx^p])^n)/(e*(m + 1))
, x] - Dist[(b*n*p)/(e*(m + 1)), Int[SimplifyIntegrand[((d + e*x)^(m + 1)*(
a + b*Log[c*RFx^p])^(n - 1)*D[RFx, x])/RFx, x], x], x] /; FreeQ[{a, b, c, d
```

```
, e, m, p}, x] && RationalFunctionQ[RFx, x] && IGtQ[n, 0] && (EqQ[n, 1] ||
IntegerQ[m]) && NeQ[m, -1]
```

Rubi steps

$$\int \frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{(ag+bgx)^2}\,dx = -\frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{bg^2(a+bx)}+\frac{B\int \frac{bc-ad}{g(a+bx)^2(c+dx)}\,dx}{bg}$$
$$= -\frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{bg^2(a+bx)}+\frac{(B(bc-ad))\int \frac{1}{(a+bx)^2(c+dx)}\,dx}{bg^2}$$
$$= -\frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{bg^2(a+bx)}+\frac{(B(bc-ad))\int \left(\frac{b}{(bc-ad)(a+bx)^2}-\frac{bd}{(bc-ad)^2(a+bx)}+\frac{d^2}{(bc-ad)^2(c+dx)}\right)}{bg^2}$$
$$= -\frac{B}{bg^2(a+bx)}-\frac{Bd\log(a+bx)}{b(bc-ad)g^2}-\frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{bg^2(a+bx)}+\frac{Bd\log(c+dx)}{b(bc-ad)g^2}$$

**Mathematica [A]** time = 0.06, size = 105, normalized size = 1.67

$$\frac{aAd+(aBd-bBc)\log\left(\frac{e(a+bx)}{c+dx}\right)-Bd(a+bx)\log(a+bx)+aBd\log(c+dx)+aBd-Abc+bBdx\log(c+dx)-}{bg^2(a+bx)(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Log[(e*(a + b*x))/(c + d*x)])/(a*g + b*g*x)^2,x]
```

```
[Out] (-(A*b*c) - b*B*c + a*A*d + a*B*d - B*d*(a + b*x)*Log[a + b*x] + (-(b*B*c)
+ a*B*d)*Log[(e*(a + b*x))/(c + d*x)] + a*B*d*Log[c + d*x] + b*B*d*x*Log[c
+ d*x])/(b*(b*c - a*d)*g^2*(a + b*x))
```

**fricas [A]** time = 1.37, size = 83, normalized size = 1.32

$$-\frac{(A+B)bc-(A+B)ad+(Bbdx+Bbc)\log\left(\frac{bex+ae}{dx+c}\right)}{\left(b^3c-ab^2d\right)g^2x+\left(ab^2c-a^2bd\right)g^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*(b*x+a)/(d*x+c)))/(b*g*x+a*g)^2,x, algorithm="fricas")
```

```
[Out] -((A + B)*b*c - (A + B)*a*d + (B*b*d*x + B*b*c)*log((b*e*x + a*e)/(d*x + c)
))/((b^3*c - a*b^2*d)*g^2*x + (a*b^2*c - a^2*b*d)*g^2)
```

**giac [A]** time = 1.30, size = 110, normalized size = 1.75

$$-\frac{\left(Be^2\log\left(\frac{bxe+ae}{dx+c}\right)+Ae^2+Be^2\right)(dx+c)\left(\frac{bc}{(bce-ade)(bc-ad)}-\frac{ad}{(bce-ade)(bc-ad)}\right)}{(bxe+ae)g^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*(b*x+a)/(d*x+c)))/(b*g*x+a*g)^2,x, algorithm="giac")
```

```
[Out] -(B*e^2*log((b*x*e + a*e)/(d*x + c)) + A*e^2 + B*e^2)*(d*x + c)*(b*c/((b*c*
e - a*d*e)*(b*c - a*d)) - a*d/((b*c*e - a*d*e)*(b*c - a*d)))/((b*x*e + a*e)
*g^2)
```

**maple [B]** time = 0.05, size = 373, normalized size = 5.92

$$\frac{Bade\ln\left(\frac{be}{d}+\frac{(ad-bc)e}{(dx+c)d}\right)}{(ad-bc)^2\left(\frac{ae}{dx+c}-\frac{bce}{(dx+c)d}+\frac{be}{d}\right)g^2}-\frac{Bbce\ln\left(\frac{be}{d}+\frac{(ad-bc)e}{(dx+c)d}\right)}{(ad-bc)^2\left(\frac{ae}{dx+c}-\frac{bce}{(dx+c)d}+\frac{be}{d}\right)g^2}+\frac{Aade}{(ad-bc)^2\left(\frac{ae}{dx+c}-\frac{bce}{(dx+c)d}+\frac{be}{d}\right)g^2}-\frac{}{(ad-b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*ln(e*(b*x+a)/(d*x+c)))/(b*g*x+a*g)^2,x)
```

```
[Out] d*e/(a*d-b*c)^2/g^2*A/(b/d*e+e/(d*x+c)*a-e/d/(d*x+c)*b*c)*a-e/(a*d-b*c)^2/g
^2*A/(b/d*e+e/(d*x+c)*a-e/d/(d*x+c)*b*c)*b*c+d*e/(a*d-b*c)^2/g^2*B/(b/d*e+e
/(d*x+c)*a-e/d/(d*x+c)*b*c)*ln(b/d*e+(a*d-b*c)/(d*x+c)/d*e)*a-e/(a*d-b*c)^2
/g^2*B/(b/d*e+e/(d*x+c)*a-e/d/(d*x+c)*b*c)*ln(b/d*e+(a*d-b*c)/(d*x+c)/d*e)*
b*c+d*e/(a*d-b*c)^2/g^2*B/(b/d*e+e/(d*x+c)*a-e/d/(d*x+c)*b*c)*a-e/(a*d-b*c)
^2/g^2*B/(b/d*e+e/(d*x+c)*a-e/d/(d*x+c)*b*c)*b*c
```

**maxima [B]** time = 1.09, size = 132, normalized size = 2.10

$$-B\left(\frac{\log\left(\frac{bex}{dx+c}+\frac{ae}{dx+c}\right)}{b^2g^2x+abg^2}+\frac{1}{b^2g^2x+abg^2}+\frac{d\log(bx+a)}{\left(b^2c-abd\right)g^2}-\frac{d\log(dx+c)}{\left(b^2c-abd\right)g^2}\right)-\frac{A}{b^2g^2x+abg^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*(b*x+a)/(d*x+c)))/(b*g*x+a*g)^2,x, algorithm="maxima")
```

```
[Out] -B*(log(b*e*x/(d*x + c) + a*e/(d*x + c))/(b^2*g^2*x + a*b*g^2) + 1/(b^2*g^2
*x + a*b*g^2) + d*log(b*x + a)/((b^2*c - a*b*d)*g^2) - d*log(d*x + c)/((b^2
*c - a*b*d)*g^2)) - A/(b^2*g^2*x + a*b*g^2)
```

**mupad [B]** time = 5.02, size = 104, normalized size = 1.65

$$-\frac{A+B}{x\,b^2\,g^2+a\,b\,g^2}-\frac{B\,\ln\left(\frac{e\,(a+b\,x)}{c+d\,x}\right)}{b^2\,g^2\,\left(x+\frac{a}{b}\right)}-\frac{B\,d\,\mathrm{atan}\left(\frac{b\,c\,2\mathrm{i}+b\,d\,x\,2\mathrm{i}}{a\,d-b\,c}+1\mathrm{i}\right)\,2\mathrm{i}}{b\,g^2\,\left(a\,d-b\,c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*log((e*(a + b*x))/(c + d*x)))/(a*g + b*g*x)^2,x)
```

```
[Out] - (A + B)/(b^2*g^2*x + a*b*g^2) - (B*log((e*(a + b*x))/(c + d*x)))/(b^2*g^2
*(x + a/b)) - (B*d*atan((b*c*2i + b*d*x*2i)/(a*d - b*c) + 1i)*2i)/(b*g^2*(a
*d - b*c))
```

**sympy [B]** time = 1.58, size = 233, normalized size = 3.70

$$-\frac{B\log\left(\frac{e(a+bx)}{c+dx}\right)}{abg^2+b^2g^2x}-\frac{Bd\log\left(x+\frac{-\frac{Ba^2d^3}{ad-bc}+\frac{2Babcd^2}{ad-bc}+Bad^2-\frac{Bb^2c^2d}{ad-bc}+Bbcd}{2Bbd^2}\right)}{bg^2(ad-bc)}+\frac{Bd\log\left(x+\frac{\frac{Ba^2d^3}{ad-bc}-\frac{2Babcd^2}{ad-bc}+Bad^2+\frac{Bb^2c^2d}{ad-bc}+Bbcd}{2Bbd^2}\right)}{bg^2(ad-bc)}+\frac{-A}{abg^2 +}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*ln(e*(b*x+a)/(d*x+c)))/(b*g*x+a*g)**2,x)
```

```
[Out] -B*log(e*(a + b*x)/(c + d*x))/(a*b*g**2 + b**2*g**2*x) - B*d*log(x + (-B*a*
*2*d**3/(a*d - b*c) + 2*B*a*b*c*d**2/(a*d - b*c) + B*a*d**2 - B*b**2*c**2*d
/(a*d - b*c) + B*b*c*d)/(2*B*b*d**2))/(b*g**2*(a*d - b*c)) + B*d*log(x + (B
*a**2*d**3/(a*d - b*c) - 2*B*a*b*c*d**2/(a*d - b*c) + B*a*d**2 + B*b**2*c**
2*d/(a*d - b*c) + B*b*c*d)/(2*B*b*d**2))/(b*g**2*(a*d - b*c)) + (-A - B)/(a
*b*g**2 + b**2*g**2*x)
```

## 3.94 $\int \frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{(ag+bgx)^3}\,dx$

**Optimal**. Leaf size=144

$$-\frac{B\log\left(\frac{e(a+bx)}{c+dx}\right)+A}{2bg^3(a+bx)^2}+\frac{Bd^2\log(a+bx)}{2bg^3(bc-ad)^2}-\frac{Bd^2\log(c+dx)}{2bg^3(bc-ad)^2}+\frac{Bd}{2bg^3(a+bx)(bc-ad)}-\frac{B}{4bg^3(a+bx)^2}$$

```
[Out] -1/4*B/b/g^3/(b*x+a)^2+1/2*B*d/b/(-a*d+b*c)/g^3/(b*x+a)+1/2*B*d^2*ln(b*x+a)
/b/(-a*d+b*c)^2/g^3+1/2*(-A-B*ln(e*(b*x+a)/(d*x+c)))/b/g^3/(b*x+a)^2-1/2*B*
d^2*ln(d*x+c)/b/(-a*d+b*c)^2/g^3
```

**Rubi [A]** time = 0.10, antiderivative size = 144, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {2525, 12, 44}

$$-\frac{B\log\left(\frac{e(a+bx)}{c+dx}\right)+A}{2bg^3(a+bx)^2}+\frac{Bd^2\log(a+bx)}{2bg^3(bc-ad)^2}-\frac{Bd^2\log(c+dx)}{2bg^3(bc-ad)^2}+\frac{Bd}{2bg^3(a+bx)(bc-ad)}-\frac{B}{4bg^3(a+bx)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Log[(e*(a + b*x))/(c + d*x)])/(a*g + b*g*x)^3,x]
```

```
[Out] -B/(4*b*g^3*(a + b*x)^2) + (B*d)/(2*b*(b*c - a*d)*g^3*(a + b*x)) + (B*d^2*L
og[a + b*x])/(2*b*(b*c - a*d)^2*g^3) - (A + B*Log[(e*(a + b*x))/(c + d*x)])
/(2*b*g^3*(a + b*x)^2) - (B*d^2*Log[c + d*x])/(2*b*(b*c - a*d)^2*g^3)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 2525

```
Int[((a_.) + Log[(c_.)*(RFx_)^(p_.)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_))^(m_.
), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(a + b*Log[c*RFx^p])^n)/(e*(m + 1))
```

```
, x] - Dist[(b*n*p)/(e*(m + 1)), Int[SimplifyIntegrand[((d + e*x)^(m + 1)*(
a + b*Log[c*RFx^p])^(n - 1)*D[RFx, x])/RFx, x], x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && RationalFunctionQ[RFx, x] && IGtQ[n, 0] && (EqQ[n, 1] ||
IntegerQ[m]) && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{(ag+bgx)^3}\,dx &= -\frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{2bg^3(a+bx)^2}+\frac{B\int\frac{bc-ad}{g^2(a+bx)^3(c+dx)}\,dx}{2bg}\\ &= -\frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{2bg^3(a+bx)^2}+\frac{(B(bc-ad))\int\frac{1}{(a+bx)^3(c+dx)}\,dx}{2bg^3}\\ &= -\frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{2bg^3(a+bx)^2}+\frac{(B(bc-ad))\int\left(\frac{b}{(bc-ad)(a+bx)^3}-\frac{bd}{(bc-ad)^2(a+bx)^2}+\frac{bd^2}{(bc-ad)^3(a+bx)}\right.}{2bg^3}\\ &= -\frac{B}{4bg^3(a+bx)^2}+\frac{Bd}{2b(bc-ad)g^3(a+bx)}+\frac{Bd^2\log(a+bx)}{2b(bc-ad)^2g^3}-\frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{2bg^3(a+bx)^2}-\end{aligned}$$

**Mathematica [A]** time = 0.13, size = 110, normalized size = 0.76

$$-\frac{2\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)+\frac{B\left(2d^2(a+bx)^2\log(c+dx)+(bc-ad)(b(c-2dx)-3ad)-2d^2(a+bx)^2\log(a+bx)\right)}{(bc-ad)^2}}{4bg^3(a+bx)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Log[(e*(a + b*x))/(c + d*x)])/(a*g + b*g*x)^3,x]
```

```
[Out] -1/4*(2*(A + B*Log[(e*(a + b*x))/(c + d*x)]) + (B*((b*c - a*d)*(-3*a*d + b*
(c - 2*d*x)) - 2*d^2*(a + b*x)^2*Log[a + b*x] + 2*d^2*(a + b*x)^2*Log[c + d
*x]))/(b*c - a*d)^2)/(b*g^3*(a + b*x)^2)
```

**fricas [A]** time = 0.61, size = 217, normalized size = 1.51

$$-\frac{(2\,A+B)b^2c^2-4\,(A+B)abcd+(2\,A+3\,B)a^2d^2-2\left(Bb^2cd-Babd^2\right)x-2\left(Bb^2d^2x^2+2\,Babd^2x-Bb^2c^2+2\right.}{4\left(\left(b^5c^2-2\,ab^4cd+a^2b^3d^2\right)g^3x^2+2\left(ab^4c^2-2\,a^2b^3cd+a^3b^2d^2\right)g^3x+\left(a^2b^3c^2-2\,a^3b^2cd+a^4b\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*(b*x+a)/(d*x+c)))/(b*g*x+a*g)^3,x, algorithm="fricas")
```

```
[Out] -1/4*((2*A + B)*b^2*c^2 - 4*(A + B)*a*b*c*d + (2*A + 3*B)*a^2*d^2 - 2*(B*b^
2*c*d - B*a*b*d^2)*x - 2*(B*b^2*d^2*x^2 + 2*B*a*b*d^2*x - B*b^2*c^2 + 2*B*a
*b*c*d)*log((b*e*x + a*e)/(d*x + c)))/((b^5*c^2 - 2*a*b^4*c*d + a^2*b^3*d^2
)*g^3*x^2 + 2*(a*b^4*c^2 - 2*a^2*b^3*c*d + a^3*b^2*d^2)*g^3*x + (a^2*b^3*c^
2 - 2*a^3*b^2*c*d + a^4*b*d^2)*g^3)
```

**giac [A]** time = 1.69, size = 237, normalized size = 1.65

$$-\frac{\left(2\,Bbe^3\log\left(\frac{bxe+ae}{dx+c}\right)-\frac{4\,(bxe+ae)Bde^2\log\left(\frac{bxe+ae}{dx+c}\right)}{dx+c}+2\,Abe^3+Bbe^3-\frac{4\,(bxe+ae)Ade^2}{dx+c}-\frac{4\,(bxe+ae)Bde^2}{dx+c}\right)\left(\frac{bc}{(bce-ade)(bc-ad)}-\frac{\cdots}{(bce-\cdots}\right.}{4\left(\frac{(bxe+ae)^2bcg^3}{(dx+c)^2}-\frac{(bxe+ae)^2adg^3}{(dx+c)^2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*(b*x+a)/(d*x+c)))/(b*g*x+a*g)^3,x, algorithm="giac")
```

```
[Out] -1/4*(2*B*b*e^3*log((b*x*e + a*e)/(d*x + c)) - 4*(b*x*e + a*e)*B*d*e^2*log(
(b*x*e + a*e)/(d*x + c))/(d*x + c) + 2*A*b*e^3 + B*b*e^3 - 4*(b*x*e + a*e)*
A*d*e^2/(d*x + c) - 4*(b*x*e + a*e)*B*d*e^2/(d*x + c))*(b*c/((b*c*e - a*d*e
)*(b*c - a*d)) - a*d/((b*c*e - a*d*e)*(b*c - a*d)))/((b*x*e + a*e)^2*b*c*g^
3/(d*x + c)^2 - (b*x*e + a*e)^2*a*d*g^3/(d*x + c)^2)
```

**maple [B]** time = 0.05, size = 777, normalized size = 5.40

$$-\frac{Babd\,e^2\ln\left(\frac{be}{d}+\frac{(ad-bc)e}{(dx+c)d}\right)}{2\,(ad-bc)^3\left(\frac{ae}{dx+c}-\frac{bce}{(dx+c)d}+\frac{be}{d}\right)^2g^3}+\frac{B\,b^2c\,e^2\ln\left(\frac{be}{d}+\frac{(ad-bc)e}{(dx+c)d}\right)}{2\,(ad-bc)^3\left(\frac{ae}{dx+c}-\frac{bce}{(dx+c)d}+\frac{be}{d}\right)^2g^3}-\frac{Aabd\,e^2}{2\,(ad-bc)^3\left(\frac{ae}{dx+c}-\frac{bce}{(dx+c)d}+\frac{be}{d}\right)^2g^{\cdots}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*ln(e*(b*x+a)/(d*x+c)))/(b*g*x+a*g)^3,x)
```

```
[Out] d^2*e/(a*d-b*c)^3/g^3*A/(1/(d*x+c)*a*e-1/(d*x+c)*b*c/d*e+b/d*e)*a-d*e/(a*d-
b*c)^3/g^3*A/(1/(d*x+c)*a*e-1/(d*x+c)*b*c/d*e+b/d*e)*b*c-1/2*d*e^2/(a*d-b*c
)^3/g^3*A*b/(1/(d*x+c)*a*e-1/(d*x+c)*b*c/d*e+b/d*e)^2*a+1/2*e^2/(a*d-b*c)^3
/g^3*A*b^2/(1/(d*x+c)*a*e-1/(d*x+c)*b*c/d*e+b/d*e)^2*c+d^2*e/(a*d-b*c)^3/g^
3*B/(1/(d*x+c)*a*e-1/(d*x+c)*b*c/d*e+b/d*e)*ln(b/d*e+(a*d-b*c)/(d*x+c)/d*e)
*a-d*e/(a*d-b*c)^3/g^3*B/(1/(d*x+c)*a*e-1/(d*x+c)*b*c/d*e+b/d*e)*ln(b/d*e+(
a*d-b*c)/(d*x+c)/d*e)*b*c+d^2*e/(a*d-b*c)^3/g^3*B/(1/(d*x+c)*a*e-1/(d*x+c)*
b*c/d*e+b/d*e)*a-d*e/(a*d-b*c)^3/g^3*B/(1/(d*x+c)*a*e-1/(d*x+c)*b*c/d*e+b/d
*e)*b*c-1/2*d*e^2/(a*d-b*c)^3/g^3*B*b/(1/(d*x+c)*a*e-1/(d*x+c)*b*c/d*e+b/d*
e)^2*ln(b/d*e+(a*d-b*c)/(d*x+c)/d*e)*a+1/2*e^2/(a*d-b*c)^3/g^3*B*b^2/(1/(d*
x+c)*a*e-1/(d*x+c)*b*c/d*e+b/d*e)^2*ln(b/d*e+(a*d-b*c)/(d*x+c)/d*e)*c-1/4*d
```

```
*e^2/(a*d-b*c)^3/g^3*B*b/(1/(d*x+c)*a*e-1/(d*x+c)*b*c/d*e+b/d*e)^2*a+1/4*e^
2/(a*d-b*c)^3/g^3*B*b^2/(1/(d*x+c)*a*e-1/(d*x+c)*b*c/d*e+b/d*e)^2*c
```

**maxima [A]** time = 1.26, size = 255, normalized size = 1.77

$$\frac{1}{4}B\left(\frac{2\,bdx-bc+3\,ad}{\left(b^4c-ab^3d\right)g^3x^2+2\left(ab^3c-a^2b^2d\right)g^3x+\left(a^2b^2c-a^3bd\right)g^3}-\frac{2\,\log\left(\frac{bex}{dx+c}+\frac{ae}{dx+c}\right)}{b^3g^3x^2+2\,ab^2g^3x+a^2bg^3}+\frac{2\,d^2\log\left(\right.}{\left(b^3c^2-2\,ab^2c\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*(b*x+a)/(d*x+c)))/(b*g*x+a*g)^3,x, algorithm="maxima")
```

```
[Out] 1/4*B*((2*b*d*x - b*c + 3*a*d)/((b^4*c - a*b^3*d)*g^3*x^2 + 2*(a*b^3*c - a^
2*b^2*d)*g^3*x + (a^2*b^2*c - a^3*b*d)*g^3) - 2*log(b*e*x/(d*x + c) + a*e/(
d*x + c))/(b^3*g^3*x^2 + 2*a*b^2*g^3*x + a^2*b*g^3) + 2*d^2*log(b*x + a)/((
b^3*c^2 - 2*a*b^2*c*d + a^2*b*d^2)*g^3) - 2*d^2*log(d*x + c)/((b^3*c^2 - 2*
a*b^2*c*d + a^2*b*d^2)*g^3)) - 1/2*A/(b^3*g^3*x^2 + 2*a*b^2*g^3*x + a^2*b*g
^3)
```

**mupad [B]** time = 5.04, size = 209, normalized size = 1.45

$$-\frac{\frac{2\,A\,a\,d-2\,A\,b\,c+3\,B\,a\,d-B\,b\,c}{2\,\left(a\,d-b\,c\right)}+\frac{B\,b\,d\,x}{a\,d-b\,c}}{2\,a^2\,b\,g^3+4\,a\,b^2\,g^3\,x+2\,b^3\,g^3\,x^2}-\frac{B\,\ln\left(\frac{e\,\left(a+b\,x\right)}{c+d\,x}\right)}{2\,b^2\,g^3\,\left(2\,a\,x+b\,x^2+\frac{a^2}{b}\right)}-\frac{B\,d^2\,\mathrm{atanh}\left(\frac{2\,b^3\,c^2\,g^3-2\,a^2\,b\,d^2\,g^3}{2\,b\,g^3\,{\left(a\,d-b\,c\right)}^2}-\frac{2\,b\,d\,x}{a\,d-b\,c}\right)}{b\,g^3\,{\left(a\,d-b\,c\right)}^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*log((e*(a + b*x))/(c + d*x)))/(a*g + b*g*x)^3,x)
```

```
[Out] - ((2*A*a*d - 2*A*b*c + 3*B*a*d - B*b*c)/(2*(a*d - b*c)) + (B*b*d*x)/(a*d -
 b*c))/(2*a^2*b*g^3 + 2*b^3*g^3*x^2 + 4*a*b^2*g^3*x) - (B*log((e*(a + b*x))
/(c + d*x)))/(2*b^2*g^3*(2*a*x + b*x^2 + a^2/b)) - (B*d^2*atanh((2*b^3*c^2*
g^3 - 2*a^2*b*d^2*g^3)/(2*b*g^3*(a*d - b*c)^2) - (2*b*d*x)/(a*d - b*c)))/(b
*g^3*(a*d - b*c)^2)
```

**sympy [B]** time = 2.71, size = 422, normalized size = 2.93

$$-\frac{B\log\left(\frac{e(a+bx)}{c+dx}\right)}{2a^2bg^3+4ab^2g^3x+2b^3g^3x^2}-\frac{Bd^2\log\left(x+\frac{-\frac{Ba^3d^5}{(ad-bc)^2}+\frac{3Ba^2bcd^4}{(ad-bc)^2}-\frac{3Bab^2c^2d^3}{(ad-bc)^2}+Bad^3+\frac{Bb^3c^3d^2}{(ad-bc)^2}+Bbcd^2}{2Bbd^3}\right)}{2bg^3\left(ad-bc\right)^2}+\frac{Bd^2\log\left(x+\frac{\frac{Ba^3d^5}{(ad-bc)^2}-}{}\right.}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*ln(e*(b*x+a)/(d*x+c)))/(b*g*x+a*g)**3,x)
```

```
[Out] -B*log(e*(a + b*x)/(c + d*x))/(2*a**2*b*g**3 + 4*a*b**2*g**3*x + 2*b**3*g**
3*x**2) - B*d**2*log(x + (-B*a**3*d**5/(a*d - b*c)**2 + 3*B*a**2*b*c*d**4/(
a*d - b*c)**2 - 3*B*a*b**2*c**2*d**3/(a*d - b*c)**2 + B*a*d**3 + B*b**3*c**
3*d**2/(a*d - b*c)**2 + B*b*c*d**2)/(2*B*b*d**3))/(2*b*g**3*(a*d - b*c)**2)
 + B*d**2*log(x + (B*a**3*d**5/(a*d - b*c)**2 - 3*B*a**2*b*c*d**4/(a*d - b*
c)**2 + 3*B*a*b**2*c**2*d**3/(a*d - b*c)**2 + B*a*d**3 - B*b**3*c**3*d**2/(
a*d - b*c)**2 + B*b*c*d**2)/(2*B*b*d**3))/(2*b*g**3*(a*d - b*c)**2) + (-2*A
*a*d + 2*A*b*c - 3*B*a*d + B*b*c - 2*B*b*d*x)/(4*a**3*b*d*g**3 - 4*a**2*b**
2*c*g**3 + x**2*(4*a*b**3*d*g**3 - 4*b**4*c*g**3) + x*(8*a**2*b**2*d*g**3 -
 8*a*b**3*c*g**3))
```

625

# 3.95 $\int \frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{(ag+bgx)^4}\,dx$

**Optimal**. Leaf size=175

$$-\frac{B\log\left(\frac{e(a+bx)}{c+dx}\right)+A}{3bg^4(a+bx)^3}-\frac{Bd^3\log(a+bx)}{3bg^4(bc-ad)^3}+\frac{Bd^3\log(c+dx)}{3bg^4(bc-ad)^3}-\frac{Bd^2}{3bg^4(a+bx)(bc-ad)^2}+\frac{Bd}{6bg^4(a+bx)^2(bc-ad)}-9bg$$

```
[Out] -1/9*B/b/g^4/(b*x+a)^3+1/6*B*d/b/(-a*d+b*c)/g^4/(b*x+a)^2-1/3*B*d^2/b/(-a*d
+b*c)^2/g^4/(b*x+a)-1/3*B*d^3*ln(b*x+a)/b/(-a*d+b*c)^3/g^4+1/3*(-A-B*ln(e*(
b*x+a)/(d*x+c)))/b/g^4/(b*x+a)^3+1/3*B*d^3*ln(d*x+c)/b/(-a*d+b*c)^3/g^4
```

**Rubi [A]** time = 0.13, antiderivative size = 175, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {2525, 12, 44}

$$-\frac{B\log\left(\frac{e(a+bx)}{c+dx}\right)+A}{3bg^4(a+bx)^3}-\frac{Bd^2}{3bg^4(a+bx)(bc-ad)^2}-\frac{Bd^3\log(a+bx)}{3bg^4(bc-ad)^3}+\frac{Bd^3\log(c+dx)}{3bg^4(bc-ad)^3}+\frac{Bd}{6bg^4(a+bx)^2(bc-ad)}-9bg$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Log[(e*(a + b*x))/(c + d*x)])/(a*g + b*g*x)^4,x]
```

```
[Out] -B/(9*b*g^4*(a + b*x)^3) + (B*d)/(6*b*(b*c - a*d)*g^4*(a + b*x)^2) - (B*d^2
)/(3*b*(b*c - a*d)^2*g^4*(a + b*x)) - (B*d^3*Log[a + b*x])/(3*b*(b*c - a*d)
^3*g^4) - (A + B*Log[(e*(a + b*x))/(c + d*x)])/(3*b*g^4*(a + b*x)^3) + (B*d
^3*Log[c + d*x])/(3*b*(b*c - a*d)^3*g^4)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 2525

```
Int[((a_.) + Log[(c_.)*(RFx_)^(p_.)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_))^(m_.
), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(a + b*Log[c*RFx^p])^n)/(e*(m + 1))
```

```
, x] - Dist[(b*n*p)/(e*(m + 1)), Int[SimplifyIntegrand[((d + e*x)^(m + 1)*(
a + b*Log[c*RFx^p])^(n - 1)*D[RFx, x])/RFx, x], x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && RationalFunctionQ[RFx, x] && IGtQ[n, 0] && (EqQ[n, 1] ||
IntegerQ[m]) && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{(ag+bgx)^4}\,dx &= -\frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{3bg^4(a+bx)^3}+\frac{B\int \frac{bc-ad}{g^3(a+bx)^4(c+dx)}\,dx}{3bg}\\ &= -\frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{3bg^4(a+bx)^3}+\frac{(B(bc-ad))\int \frac{1}{(a+bx)^4(c+dx)}\,dx}{3bg^4}\\ &= -\frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{3bg^4(a+bx)^3}+\frac{(B(bc-ad))\int \left(\frac{b}{(bc-ad)(a+bx)^4}-\frac{bd}{(bc-ad)^2(a+bx)^3}+\frac{bd^2}{(bc-ad)^3(a+bx)^2}-\right.}{3bg^4}\\ &= -\frac{B}{9bg^4(a+bx)^3}+\frac{Bd}{6b(bc-ad)g^4(a+bx)^2}-\frac{Bd^2}{3b(bc-ad)^2g^4(a+bx)}-\frac{Bd^3\log(a+bx}{3b(bc-ad)^3g^4}\end{aligned}$$

**Mathematica [A]** time = 0.16, size = 141, normalized size = 0.81

$$-\frac{\frac{B\left((bc-ad)\left(11a^2d^2+abd(15dx-7c)+b^2\left(2c^2-3cdx+6d^2x^2\right)\right)-6d^3(a+bx)^3\log(c+dx)+6d^3(a+bx)^3\log(a+bx)\right)}{(bc-ad)^3}+6\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{18bg^4(a+bx)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Log[(e*(a + b*x))/(c + d*x)])/(a*g + b*g*x)^4,x]
```

```
[Out] -1/18*(6*(A + B*Log[(e*(a + b*x))/(c + d*x)]) + (B*((b*c - a*d)*(11*a^2*d^2
 + a*b*d*(-7*c + 15*d*x) + b^2*(2*c^2 - 3*c*d*x + 6*d^2*x^2)) + 6*d^3*(a +
b*x)^3*Log[a + b*x] - 6*d^3*(a + b*x)^3*Log[c + d*x]))/(b*c - a*d)^3)/(b*g^
4*(a + b*x)^3)
```

**fricas [B]** time = 0.61, size = 406, normalized size = 2.32

$$-\frac{2\,(3\,A+B)b^3c^3-9\,(2\,A+B)ab^2c^2d+18\,(A+B)a^2bcd^2-(6\,A+11\,B)a^3d^3+6\left(Bb^3cd^2-Bab^2d^3\right)x^2-3\left(Bb^3c^2\right.}{18\left(\left(b^7c^3-3\,ab^6c^2d+3\,a^2b^5cd^2-a^3b^4d^3\right)g^4x^3+3\left(ab^6c^3-3\,a^2b^5c^2d+3\,a^3b^4cd^2-a^4b^3d^3\right)g^4\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*(b*x+a)/(d*x+c)))/(b*g*x+a*g)^4,x, algorithm="fricas")
```

```
[Out] -1/18*(2*(3*A + B)*b^3*c^3 - 9*(2*A + B)*a*b^2*c^2*d + 18*(A + B)*a^2*b*c*d
^2 - (6*A + 11*B)*a^3*d^3 + 6*(B*b^3*c*d^2 - B*a*b^2*d^3)*x^2 - 3*(B*b^3*c^
2*d - 6*B*a*b^2*c*d^2 + 5*B*a^2*b*d^3)*x + 6*(B*b^3*d^3*x^3 + 3*B*a*b^2*d^3
*x^2 + 3*B*a^2*b*d^3*x + B*b^3*c^3 - 3*B*a*b^2*c^2*d + 3*B*a^2*b*c*d^2)*log
((b*e*x + a*e)/(d*x + c)))/((b^7*c^3 - 3*a*b^6*c^2*d + 3*a^2*b^5*c*d^2 - a^
3*b^4*d^3)*g^4*x^3 + 3*(a*b^6*c^3 - 3*a^2*b^5*c^2*d + 3*a^3*b^4*c*d^2 - a^4
*b^3*d^3)*g^4*x^2 + 3*(a^2*b^5*c^3 - 3*a^3*b^4*c^2*d + 3*a^4*b^3*c*d^2 - a^
5*b^2*d^3)*g^4*x + (a^3*b^4*c^3 - 3*a^4*b^3*c^2*d + 3*a^5*b^2*c*d^2 - a^6*b
*d^3)*g^4)
```

**giac [B]** time = 2.13, size = 382, normalized size = 2.18

$$-\frac{\left(6\,Bb^2e^4\log\left(\frac{bxe+ae}{dx+c}\right) - \frac{18\,(bxe+ae)Bbde^3\log\left(\frac{bxe+ae}{dx+c}\right)}{dx+c} + \frac{18\,(bxe+ae)^2Bd^2e^2\log\left(\frac{bxe+ae}{dx+c}\right)}{(dx+c)^2} + 6\,Ab^2e^4 + 2\,Bb^2e^4 - \frac{18\,(bxe+ae)Abde^3}{dx+c}\right.}{18\left(\frac{(bxe+ae)^3b^2c^2g^4}{(dx+c)^3} - \frac{2\,(bxe+ae)^3abcdg^4}{(dx+c)^3} + \right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*(b*x+a)/(d*x+c)))/(b*g*x+a*g)^4,x, algorithm="giac")
```

```
[Out] -1/18*(6*B*b^2*e^4*log((b*x*e + a*e)/(d*x + c)) - 18*(b*x*e + a*e)*B*b*d*e^
3*log((b*x*e + a*e)/(d*x + c))/(d*x + c) + 18*(b*x*e + a*e)^2*B*d^2*e^2*log
((b*x*e + a*e)/(d*x + c))/(d*x + c)^2 + 6*A*b^2*e^4 + 2*B*b^2*e^4 - 18*(b*x
*e + a*e)*A*b*d*e^3/(d*x + c) - 9*(b*x*e + a*e)*B*b*d*e^3/(d*x + c) + 18*(b
*x*e + a*e)^2*A*d^2*e^2/(d*x + c)^2 + 18*(b*x*e + a*e)^2*B*d^2*e^2/(d*x + c
)^2)*(b*c/((b*c*e - a*d*e)*(b*c - a*d)) - a*d/((b*c*e - a*d*e)*(b*c - a*d))
)/((b*x*e + a*e)^3*b^2*c^2*g^4/(d*x + c)^3 - 2*(b*x*e + a*e)^3*a*b*c*d*g^4/
(d*x + c)^3 + (b*x*e + a*e)^3*a^2*d^2*g^4/(d*x + c)^3)
```

**maple [B]** time = 0.05, size = 1191, normalized size = 6.81

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*ln(e*(b*x+a)/(d*x+c)))/(b*g*x+a*g)^4,x)
```

```
[Out] d^3*e/(a*d-b*c)^4/g^4*A/(1/(d*x+c)*a*e-1/(d*x+c)*b*c/d*e+b/d*e)*a-d^2*e/(a*
d-b*c)^4/g^4*A/(1/(d*x+c)*a*e-1/(d*x+c)*b*c/d*e+b/d*e)*b*c-d^2*e^2/(a*d-b*c
)^4/g^4*A*b/(1/(d*x+c)*a*e-1/(d*x+c)*b*c/d*e+b/d*e)^2*a+d*e^2/(a*d-b*c)^4/g
^4*A*b^2/(1/(d*x+c)*a*e-1/(d*x+c)*b*c/d*e+b/d*e)^2*c+1/3*d*e^3/(a*d-b*c)^4/
g^4*A*b^2/(1/(d*x+c)*a*e-1/(d*x+c)*b*c/d*e+b/d*e)^3*a-1/3*e^3/(a*d-b*c)^4/g
^4*A*b^3/(1/(d*x+c)*a*e-1/(d*x+c)*b*c/d*e+b/d*e)^3*c+d^3*e/(a*d-b*c)^4/g^4*
```

```
B/(1/(d*x+c)*a*e-1/(d*x+c)*b*c/d*e+b/d*e)*ln(b/d*e+(a*d-b*c)/(d*x+c)/d*e)*a
-d^2*e/(a*d-b*c)^4/g^4*B/(1/(d*x+c)*a*e-1/(d*x+c)*b*c/d*e+b/d*e)*ln(b/d*e+(
a*d-b*c)/(d*x+c)/d*e)*b*c+d^3*e/(a*d-b*c)^4/g^4*B/(1/(d*x+c)*a*e-1/(d*x+c)*
b*c/d*e+b/d*e)*a-d^2*e/(a*d-b*c)^4/g^4*B/(1/(d*x+c)*a*e-1/(d*x+c)*b*c/d*e+b
/d*e)*b*c-d^2*e^2/(a*d-b*c)^4/g^4*B*b/(1/(d*x+c)*a*e-1/(d*x+c)*b*c/d*e+b/d*
e)^2*ln(b/d*e+(a*d-b*c)/(d*x+c)/d*e)*a+d*e^2/(a*d-b*c)^4/g^4*B*b^2/(1/(d*x+
c)*a*e-1/(d*x+c)*b*c/d*e+b/d*e)^2*ln(b/d*e+(a*d-b*c)/(d*x+c)/d*e)*c-1/2*d^2
*e^2/(a*d-b*c)^4/g^4*B*b/(1/(d*x+c)*a*e-1/(d*x+c)*b*c/d*e+b/d*e)^2*a+1/2*d*
e^2/(a*d-b*c)^4/g^4*B*b^2/(1/(d*x+c)*a*e-1/(d*x+c)*b*c/d*e+b/d*e)^2*c+1/3*d
*e^3/(a*d-b*c)^4/g^4*B*b^2/(1/(d*x+c)*a*e-1/(d*x+c)*b*c/d*e+b/d*e)^3*ln(b/d
*e+(a*d-b*c)/(d*x+c)/d*e)*a-1/3*e^3/(a*d-b*c)^4/g^4*B*b^3/(1/(d*x+c)*a*e-1/
(d*x+c)*b*c/d*e+b/d*e)^3*ln(b/d*e+(a*d-b*c)/(d*x+c)/d*e)*c+1/9*d*e^3/(a*d-b
*c)^4/g^4*B*b^2/(1/(d*x+c)*a*e-1/(d*x+c)*b*c/d*e+b/d*e)^3*a-1/9*e^3/(a*d-b*
c)^4/g^4*B*b^3/(1/(d*x+c)*a*e-1/(d*x+c)*b*c/d*e+b/d*e)^3*c
```

**maxima [B]** time = 1.36, size = 428, normalized size = 2.45

$$-\frac{1}{18}B\left(\frac{6\,b^2d^2x^2+2\,b^2c^2-7\,abcd+11\,a^2d^2-3\,\left(b^2cd-5\,abd^2\right)x}{\left(b^6c^2-2\,ab^5cd+a^2b^4d^2\right)g^4x^3+3\,\left(ab^5c^2-2\,a^2b^4cd+a^3b^3d^2\right)g^4x^2+3\,\left(a^2b^4c^2-2\,a^3b^3cd+a^4b^2d^2\right)g^4x+\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*(b*x+a)/(d*x+c)))/(b*g*x+a*g)^4,x, algorithm="maxima")
```

```
[Out] -1/18*B*((6*b^2*d^2*x^2 + 2*b^2*c^2 - 7*a*b*c*d + 11*a^2*d^2 - 3*(b^2*c*d -
 5*a*b*d^2)*x)/((b^6*c^2 - 2*a*b^5*c*d + a^2*b^4*d^2)*g^4*x^3 + 3*(a*b^5*c^
2 - 2*a^2*b^4*c*d + a^3*b^3*d^2)*g^4*x^2 + 3*(a^2*b^4*c^2 - 2*a^3*b^3*c*d +
 a^4*b^2*d^2)*g^4*x + (a^3*b^3*c^2 - 2*a^4*b^2*c*d + a^5*b*d^2)*g^4) + 6*lo
g(b*e*x/(d*x + c) + a*e/(d*x + c))/(b^4*g^4*x^3 + 3*a*b^3*g^4*x^2 + 3*a^2*b
^2*g^4*x + a^3*b*g^4) + 6*d^3*log(b*x + a)/((b^4*c^3 - 3*a*b^3*c^2*d + 3*a^
2*b^2*c*d^2 - a^3*b*d^3)*g^4) - 6*d^3*log(d*x + c)/((b^4*c^3 - 3*a*b^3*c^2*
d + 3*a^2*b^2*c*d^2 - a^3*b*d^3)*g^4)) - 1/3*A/(b^4*g^4*x^3 + 3*a*b^3*g^4*x
^2 + 3*a^2*b^2*g^4*x + a^3*b*g^4)
```

**mupad [B]** time = 5.58, size = 339, normalized size = 1.94

$$\frac{2\,A\,a\,c\,d}{3\,g^4\,{\left(a\,d-b\,c\right)}^2\,{\left(a+b\,x\right)}^3}-\frac{A\,b\,c^2}{3\,g^4\,{\left(a\,d-b\,c\right)}^2\,{\left(a+b\,x\right)}^3}-\frac{B\,b\,c^2}{9\,g^4\,{\left(a\,d-b\,c\right)}^2\,{\left(a+b\,x\right)}^3}-\frac{A\,a^2\,d^2}{3\,b\,g^4\,{\left(a\,d-b\,c\right)}^2\,{\left(a+b\,x\right)}^3}-\frac{}{18\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*log((e*(a + b*x))/(c + d*x)))/(a*g + b*g*x)^4,x)
```

```
[Out] (2*A*a*c*d)/(3*g^4*(a*d - b*c)^2*(a + b*x)^3) - (B*d^3*atan((a*d*1i + b*c*1
i + b*d*x*2i)/(a*d - b*c))*2i)/(3*b*g^4*(a*d - b*c)^3) - (A*b*c^2)/(3*g^4*(
```

```
a*d - b*c)^2*(a + b*x)^3) - (B*b*c^2)/(9*g^4*(a*d - b*c)^2*(a + b*x)^3) - (
A*a^2*d^2)/(3*b*g^4*(a*d - b*c)^2*(a + b*x)^3) - (11*B*a^2*d^2)/(18*b*g^4*(
a*d - b*c)^2*(a + b*x)^3) - (5*B*a*d^2*x)/(6*g^4*(a*d - b*c)^2*(a + b*x)^3)
 - (B*b*d^2*x^2)/(3*g^4*(a*d - b*c)^2*(a + b*x)^3) - (B*log((e*(a + b*x))/(
c + d*x)))/(3*b*g^4*(a + b*x)^3) + (7*B*a*c*d)/(18*g^4*(a*d - b*c)^2*(a + b
*x)^3) + (B*b*c*d*x)/(6*g^4*(a*d - b*c)^2*(a + b*x)^3)
```

**sympy [B]** time = 4.25, size = 656, normalized size = 3.75

$$-\frac{B\log\left(\frac{e(a+bx)}{c+dx}\right)}{3a^3bg^4+9a^2b^2g^4x+9ab^3g^4x^2+3b^4g^4x^3}-\frac{Bd^3\log\left(x+\frac{-\frac{Ba^4d^7}{(ad-bc)^3}+\frac{4Ba^3bcd^6}{(ad-bc)^3}-\frac{6Ba^2b^2c^2d^5}{(ad-bc)^3}+\frac{4Bab^3c^3d^4}{(ad-bc)^3}+Bad^4-\frac{Bb^4c^4d^3}{(ad-bc)^3}+Bbcd^3}{2Bbd^4}\right)}{3bg^4(ad-bc)^3}+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*ln(e*(b*x+a)/(d*x+c)))/(b*g*x+a*g)**4,x)
```

```
[Out] -B*log(e*(a + b*x)/(c + d*x))/(3*a**3*b*g**4 + 9*a**2*b**2*g**4*x + 9*a*b**
3*g**4*x**2 + 3*b**4*g**4*x**3) - B*d**3*log(x + (-B*a**4*d**7/(a*d - b*c)*
*3 + 4*B*a**3*b*c*d**6/(a*d - b*c)**3 - 6*B*a**2*b**2*c**2*d**5/(a*d - b*c)
**3 + 4*B*a*b**3*c**3*d**4/(a*d - b*c)**3 + B*a*d**4 - B*b**4*c**4*d**3/(a*
d - b*c)**3 + B*b*c*d**3)/(2*B*b*d**4))/(3*b*g**4*(a*d - b*c)**3) + B*d**3*
log(x + (B*a**4*d**7/(a*d - b*c)**3 - 4*B*a**3*b*c*d**6/(a*d - b*c)**3 + 6*
B*a**2*b**2*c**2*d**5/(a*d - b*c)**3 - 4*B*a*b**3*c**3*d**4/(a*d - b*c)**3
+ B*a*d**4 + B*b**4*c**4*d**3/(a*d - b*c)**3 + B*b*c*d**3)/(2*B*b*d**4))/(3
*b*g**4*(a*d - b*c)**3) + (-6*A*a**2*d**2 + 12*A*a*b*c*d - 6*A*b**2*c**2 -
11*B*a**2*d**2 + 7*B*a*b*c*d - 2*B*b**2*c**2 - 6*B*b**2*d**2*x**2 + x*(-15*
B*a*b*d**2 + 3*B*b**2*c*d))/(18*a**5*b*d**2*g**4 - 36*a**4*b**2*c*d*g**4 +
18*a**3*b**3*c**2*g**4 + x**3*(18*a**2*b**4*d**2*g**4 - 36*a*b**5*c*d*g**4
+ 18*b**6*c**2*g**4) + x**2*(54*a**3*b**3*d**2*g**4 - 108*a**2*b**4*c*d*g**
4 + 54*a*b**5*c**2*g**4) + x*(54*a**4*b**2*d**2*g**4 - 108*a**3*b**3*c*d*g*
*4 + 54*a**2*b**4*c**2*g**4))
```

## 3.96 $\int \frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{(ag+bgx)^5}\,dx$

**Optimal**. Leaf size=206

$$-\frac{B\log\left(\frac{e(a+bx)}{c+dx}\right)+A}{4bg^5(a+bx)^4}+\frac{Bd^4\log(a+bx)}{4bg^5(bc-ad)^4}-\frac{Bd^4\log(c+dx)}{4bg^5(bc-ad)^4}+\frac{Bd^3}{4bg^5(a+bx)(bc-ad)^3}-\frac{Bd^2}{8bg^5(a+bx)^2(bc-ad)^2}+\frac{}{12bg}$$

```
[Out] -1/16*B/b/g^5/(b*x+a)^4+1/12*B*d/b/(-a*d+b*c)/g^5/(b*x+a)^3-1/8*B*d^2/b/(-a
*d+b*c)^2/g^5/(b*x+a)^2+1/4*B*d^3/b/(-a*d+b*c)^3/g^5/(b*x+a)+1/4*B*d^4*ln(b
*x+a)/b/(-a*d+b*c)^4/g^5+1/4*(-A-B*ln(e*(b*x+a)/(d*x+c)))/b/g^5/(b*x+a)^4-1
/4*B*d^4*ln(d*x+c)/b/(-a*d+b*c)^4/g^5
```

**Rubi [A]** time = 0.16, antiderivative size = 206, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {2525, 12, 44}

$$-\frac{B\log\left(\frac{e(a+bx)}{c+dx}\right)+A}{4bg^5(a+bx)^4}+\frac{Bd^3}{4bg^5(a+bx)(bc-ad)^3}-\frac{Bd^2}{8bg^5(a+bx)^2(bc-ad)^2}+\frac{Bd^4\log(a+bx)}{4bg^5(bc-ad)^4}-\frac{Bd^4\log(c+dx)}{4bg^5(bc-ad)^4}+\frac{}{12bg}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Log[(e*(a + b*x))/(c + d*x)])/(a*g + b*g*x)^5,x]
```

```
[Out] -B/(16*b*g^5*(a + b*x)^4) + (B*d)/(12*b*(b*c - a*d)*g^5*(a + b*x)^3) - (B*d
^2)/(8*b*(b*c - a*d)^2*g^5*(a + b*x)^2) + (B*d^3)/(4*b*(b*c - a*d)^3*g^5*(a
 + b*x)) + (B*d^4*Log[a + b*x])/(4*b*(b*c - a*d)^4*g^5) - (A + B*Log[(e*(a
+ b*x))/(c + d*x)])/(4*b*g^5*(a + b*x)^4) - (B*d^4*Log[c + d*x])/(4*b*(b*c
- a*d)^4*g^5)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 2525

```
Int[((a_.) + Log[(c_.)*(RFx_)^(p_.)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_))^(m_.
), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(a + b*Log[c*RFx^p])^n)/(e*(m + 1))
, x] - Dist[(b*n*p)/(e*(m + 1)), Int[SimplifyIntegrand[((d + e*x)^(m + 1)*(
a + b*Log[c*RFx^p])^(n - 1)*D[RFx, x])/RFx, x], x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && RationalFunctionQ[RFx, x] && IGtQ[n, 0] && (EqQ[n, 1] ||
IntegerQ[m]) && NeQ[m, -1]
```

Rubi steps

$$\int \frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{(ag+bgx)^5}\,dx = -\frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{4bg^5(a+bx)^4}+\frac{B\int\frac{bc-ad}{g^4(a+bx)^5(c+dx)}\,dx}{4bg}$$
$$= -\frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{4bg^5(a+bx)^4}+\frac{(B(bc-ad))\int\frac{1}{(a+bx)^5(c+dx)}\,dx}{4bg^5}$$
$$= -\frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{4bg^5(a+bx)^4}+\frac{(B(bc-ad))\int\left(\frac{b}{(bc-ad)(a+bx)^5}-\frac{bd}{(bc-ad)^2(a+bx)^4}+\frac{bd^2}{(bc-ad)^3(a+bx)^3}\right.}{4bg^5}$$
$$= -\frac{B}{16bg^5(a+bx)^4}+\frac{Bd}{12b(bc-ad)g^5(a+bx)^3}-\frac{Bd^2}{8b(bc-ad)^2g^5(a+bx)^2}+\frac{}{4b(bc-a}$$

**Mathematica [A]** time = 0.21, size = 158, normalized size = 0.77

$$\frac{\frac{B\left(\frac{12d^3(bc-ad)}{a+bx}-\frac{6d^2(bc-ad)^2}{(a+bx)^2}+\frac{4d(bc-ad)^3}{(a+bx)^3}-\frac{3(bc-ad)^4}{(a+bx)^4}+12d^4\log(a+bx)-12d^4\log(c+dx)\right)}{12(bc-ad)^4}-\frac{B\log\left(\frac{e(a+bx)}{c+dx}\right)+A}{(a+bx)^4}}{4bg^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Log[(e*(a + b*x))/(c + d*x)])/(a*g + b*g*x)^5,x]
```

```
[Out] (-((A + B*Log[(e*(a + b*x))/(c + d*x)])/(a + b*x)^4) + (B*((-3*(b*c - a*d)^
4)/(a + b*x)^4 + (4*d*(b*c - a*d)^3)/(a + b*x)^3 - (6*d^2*(b*c - a*d)^2)/(a
 + b*x)^2 + (12*d^3*(b*c - a*d))/(a + b*x) + 12*d^4*Log[a + b*x] - 12*d^4*L
og[c + d*x]))/(12*(b*c - a*d)^4))/(4*b*g^5)
```

**fricas [B]** time = 3.02, size = 629, normalized size = 3.05

$$-\frac{3(4A+B)b^4c^4-16(3A+B)ab^3c^3d+36(2A+B)a^2b^2c^2d^2-48(A+B)a^3bcd^3+(12A+25B)a^4d^4-12\left(B\right.}{48\left(\left(b^9c^4-4ab^8c^3d+6a^2b^7c^2d^2-4a^3b^6cd^3+a^4b^5d^4\right)g^5x^4+4\left(ab^8c^4-4a^2b^7c^3d+6a^3b^6c^2\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*(b*x+a)/(d*x+c)))/(b*g*x+a*g)^5,x, algorithm="fricas")
```

```
[Out] -1/48*(3*(4*A + B)*b^4*c^4 - 16*(3*A + B)*a*b^3*c^3*d + 36*(2*A + B)*a^2*b^
2*c^2*d^2 - 48*(A + B)*a^3*b*c*d^3 + (12*A + 25*B)*a^4*d^4 - 12*(B*b^4*c*d^
3 - B*a*b^3*d^4)*x^3 + 6*(B*b^4*c^2*d^2 - 8*B*a*b^3*c*d^3 + 7*B*a^2*b^2*d^4
)*x^2 - 4*(B*b^4*c^3*d - 6*B*a*b^3*c^2*d^2 + 18*B*a^2*b^2*c*d^3 - 13*B*a^3*
b*d^4)*x - 12*(B*b^4*d^4*x^4 + 4*B*a*b^3*d^4*x^3 + 6*B*a^2*b^2*d^4*x^2 + 4*
B*a^3*b*d^4*x - B*b^4*c^4 + 4*B*a*b^3*c^3*d - 6*B*a^2*b^2*c^2*d^2 + 4*B*a^3
*b*c*d^3)*log((b*e*x + a*e)/(d*x + c)))/((b^9*c^4 - 4*a*b^8*c^3*d + 6*a^2*b
^7*c^2*d^2 - 4*a^3*b^6*c*d^3 + a^4*b^5*d^4)*g^5*x^4 + 4*(a*b^8*c^4 - 4*a^2*
b^7*c^3*d + 6*a^3*b^6*c^2*d^2 - 4*a^4*b^5*c*d^3 + a^5*b^4*d^4)*g^5*x^3 + 6*
(a^2*b^7*c^4 - 4*a^3*b^6*c^3*d + 6*a^4*b^5*c^2*d^2 - 4*a^5*b^4*c*d^3 + a^6*
b^3*d^4)*g^5*x^2 + 4*(a^3*b^6*c^4 - 4*a^4*b^5*c^3*d + 6*a^5*b^4*c^2*d^2 - 4
*a^6*b^3*c*d^3 + a^7*b^2*d^4)*g^5*x + (a^4*b^5*c^4 - 4*a^5*b^4*c^3*d + 6*a^
6*b^3*c^2*d^2 - 4*a^7*b^2*c*d^3 + a^8*b*d^4)*g^5)
```

**giac [B]** time = 2.04, size = 528, normalized size = 2.56

$$-\frac{\left(12\,Bb^3e^5\log\left(\frac{bxe+ae}{dx+c}\right) - \frac{48\,(bxe+ae)Bb^2de^4\log\left(\frac{bxe+ae}{dx+c}\right)}{dx+c} + \frac{72\,(bxe+ae)^2Bbd^2e^3\log\left(\frac{bxe+ae}{dx+c}\right)}{(dx+c)^2} - \frac{48\,(bxe+ae)^3Bd^3e^2\log\left(\frac{bxe+ae}{dx+c}\right)}{(dx+c)^3} + 12\,Ab^3e\cdots\right.}{48\left(\frac{(bxe+ae)^4b^3c^3g^5}{(dx+c)^4}\cdots\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*(b*x+a)/(d*x+c)))/(b*g*x+a*g)^5,x, algorithm="giac")
```

```
[Out] -1/48*(12*B*b^3*e^5*log((b*x*e + a*e)/(d*x + c)) - 48*(b*x*e + a*e)*B*b^2*d
*e^4*log((b*x*e + a*e)/(d*x + c))/(d*x + c) + 72*(b*x*e + a*e)^2*B*b*d^2*e^
3*log((b*x*e + a*e)/(d*x + c))/(d*x + c)^2 - 48*(b*x*e + a*e)^3*B*d^3*e^2*l
og((b*x*e + a*e)/(d*x + c))/(d*x + c)^3 + 12*A*b^3*e^5 + 3*B*b^3*e^5 - 48*(
b*x*e + a*e)*A*b^2*d*e^4/(d*x + c) - 16*(b*x*e + a*e)*B*b^2*d*e^4/(d*x + c)
 + 72*(b*x*e + a*e)^2*A*b*d^2*e^3/(d*x + c)^2 + 36*(b*x*e + a*e)^2*B*b*d^2*
e^3/(d*x + c)^2 - 48*(b*x*e + a*e)^3*A*d^3*e^2/(d*x + c)^3 - 48*(b*x*e + a*
e)^3*B*d^3*e^2/(d*x + c)^3)*(b*c/((b*c*e - a*d*e)*(b*c - a*d)) - a*d/((b*c*
e - a*d*e)*(b*c - a*d)))/((b*x*e + a*e)^4*b^3*c^3*g^5/(d*x + c)^4 - 3*(b*x*
e + a*e)^4*a*b^2*c^2*d*g^5/(d*x + c)^4 + 3*(b*x*e + a*e)^4*a^2*b*c*d^2*g^5/
(d*x + c)^4 - (b*x*e + a*e)^4*a^3*d^3*g^5/(d*x + c)^4)
```

**maple [B]** time = 0.05, size = 1607, normalized size = 7.80

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*ln(e*(b*x+a)/(d*x+c)))/(b*g*x+a*g)^5,x)
```

```
[Out] d^4*e/(a*d-b*c)^5/g^5*A/(1/(d*x+c)*a*e-1/(d*x+c)*b*c/d*e+b/d*e)*a-d^3*e/(a*
d-b*c)^5/g^5*A/(1/(d*x+c)*a*e-1/(d*x+c)*b*c/d*e+b/d*e)*b*c-3/2*d^3*e^2/(a*d
-b*c)^5/g^5*A*b/(1/(d*x+c)*a*e-1/(d*x+c)*b*c/d*e+b/d*e)^2*a+3/2*d^2*e^2/(a*
d-b*c)^5/g^5*A*b^2/(1/(d*x+c)*a*e-1/(d*x+c)*b*c/d*e+b/d*e)^2*c+d^2*e^3/(a*d
-b*c)^5/g^5*A*b^2/(1/(d*x+c)*a*e-1/(d*x+c)*b*c/d*e+b/d*e)^3*a-d*e^3/(a*d-b*
c)^5/g^5*A*b^3/(1/(d*x+c)*a*e-1/(d*x+c)*b*c/d*e+b/d*e)^3*c-1/4*d*e^4/(a*d-b
*c)^5/g^5*A*b^3/(1/(d*x+c)*a*e-1/(d*x+c)*b*c/d*e+b/d*e)^4*a+1/4*e^4/(a*d-b*
c)^5/g^5*A*b^4/(1/(d*x+c)*a*e-1/(d*x+c)*b*c/d*e+b/d*e)^4*c+d^4*e/(a*d-b*c)^
5/g^5*B/(1/(d*x+c)*a*e-1/(d*x+c)*b*c/d*e+b/d*e)*ln(b/d*e+(a*d-b*c)/(d*x+c)/
d*e)*a-d^3*e/(a*d-b*c)^5/g^5*B/(1/(d*x+c)*a*e-1/(d*x+c)*b*c/d*e+b/d*e)*ln(b
/d*e+(a*d-b*c)/(d*x+c)/d*e)*b*c+d^4*e/(a*d-b*c)^5/g^5*B/(1/(d*x+c)*a*e-1/(d
*x+c)*b*c/d*e+b/d*e)*a-d^3*e/(a*d-b*c)^5/g^5*B/(1/(d*x+c)*a*e-1/(d*x+c)*b*c
/d*e+b/d*e)*b*c-3/2*d^3*e^2/(a*d-b*c)^5/g^5*B*b/(1/(d*x+c)*a*e-1/(d*x+c)*b*
c/d*e+b/d*e)^2*ln(b/d*e+(a*d-b*c)/(d*x+c)/d*e)*a+3/2*d^2*e^2/(a*d-b*c)^5/g^
5*B*b^2/(1/(d*x+c)*a*e-1/(d*x+c)*b*c/d*e+b/d*e)^2*ln(b/d*e+(a*d-b*c)/(d*x+c
)/d*e)*c-3/4*d^3*e^2/(a*d-b*c)^5/g^5*B*b/(1/(d*x+c)*a*e-1/(d*x+c)*b*c/d*e+b
/d*e)^2*a+3/4*d^2*e^2/(a*d-b*c)^5/g^5*B*b^2/(1/(d*x+c)*a*e-1/(d*x+c)*b*c/d*
e+b/d*e)^2*c+d^2*e^3/(a*d-b*c)^5/g^5*B*b^2/(1/(d*x+c)*a*e-1/(d*x+c)*b*c/d*e
+b/d*e)^3*ln(b/d*e+(a*d-b*c)/(d*x+c)/d*e)*a-d*e^3/(a*d-b*c)^5/g^5*B*b^3/(1/
(d*x+c)*a*e-1/(d*x+c)*b*c/d*e+b/d*e)^3*ln(b/d*e+(a*d-b*c)/(d*x+c)/d*e)*c+1/
3*d^2*e^3/(a*d-b*c)^5/g^5*B*b^2/(1/(d*x+c)*a*e-1/(d*x+c)*b*c/d*e+b/d*e)^3*a
-1/3*d*e^3/(a*d-b*c)^5/g^5*B*b^3/(1/(d*x+c)*a*e-1/(d*x+c)*b*c/d*e+b/d*e)^3*
c-1/4*d*e^4/(a*d-b*c)^5/g^5*B*b^3/(1/(d*x+c)*a*e-1/(d*x+c)*b*c/d*e+b/d*e)^4
*ln(b/d*e+(a*d-b*c)/(d*x+c)/d*e)*a+1/4*e^4/(a*d-b*c)^5/g^5*B*b^4/(1/(d*x+c)
*a*e-1/(d*x+c)*b*c/d*e+b/d*e)^4*ln(b/d*e+(a*d-b*c)/(d*x+c)/d*e)*c-1/16*d*e^
4/(a*d-b*c)^5/g^5*B*b^3/(1/(d*x+c)*a*e-1/(d*x+c)*b*c/d*e+b/d*e)^4*a+1/16*e^
4/(a*d-b*c)^5/g^5*B*b^4/(1/(d*x+c)*a*e-1/(d*x+c)*b*c/d*e+b/d*e)^4*c
```

**maxima [B]** time = 1.54, size = 647, normalized size = 3.14

$$\frac{1}{48}B\left(\frac{12\,b^3d^3x^3-3\,b^3c^3+13\,ab^2c^2d-23\,a^2bcd^2}{\left(b^8c^3-3\,ab^7c^2d+3\,a^2b^6cd^2-a^3b^5d^3\right)g^5x^4+4\left(ab^7c^3-3\,a^2b^6c^2d+3\,a^3b^5cd^2-a^4b^4d^3\right)g^5x^3+6\left(a^2b^6c^3\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*(b*x+a)/(d*x+c)))/(b*g*x+a*g)^5,x, algorithm="maxima")
```

```
[Out] 1/48*B*((12*b^3*d^3*x^3 - 3*b^3*c^3 + 13*a*b^2*c^2*d - 23*a^2*b*c*d^2 + 25*
a^3*d^3 - 6*(b^3*c*d^2 - 7*a*b^2*d^3)*x^2 + 4*(b^3*c^2*d - 5*a*b^2*c*d^2 +
13*a^2*b*d^3)*x)/((b^8*c^3 - 3*a*b^7*c^2*d + 3*a^2*b^6*c*d^2 - a^3*b^5*d^3)
*g^5*x^4 + 4*(a*b^7*c^3 - 3*a^2*b^6*c^2*d + 3*a^3*b^5*c*d^2 - a^4*b^4*d^3)*
g^5*x^3 + 6*(a^2*b^6*c^3 - 3*a^3*b^5*c^2*d + 3*a^4*b^4*c*d^2 - a^5*b^3*d^3)
*g^5*x^2 + 4*(a^3*b^5*c^3 - 3*a^4*b^4*c^2*d + 3*a^5*b^3*c*d^2 - a^6*b^2*d^3
```

```
)*g^5*x + (a^4*b^4*c^3 - 3*a^5*b^3*c^2*d + 3*a^6*b^2*c*d^2 - a^7*b*d^3)*g^5
) - 12*log(b*e*x/(d*x + c) + a*e/(d*x + c))/(b^5*g^5*x^4 + 4*a*b^4*g^5*x^3
+ 6*a^2*b^3*g^5*x^2 + 4*a^3*b^2*g^5*x + a^4*b*g^5) + 12*d^4*log(b*x + a)/((
b^5*c^4 - 4*a*b^4*c^3*d + 6*a^2*b^3*c^2*d^2 - 4*a^3*b^2*c*d^3 + a^4*b*d^4)*
g^5) - 12*d^4*log(d*x + c)/((b^5*c^4 - 4*a*b^4*c^3*d + 6*a^2*b^3*c^2*d^2 -
4*a^3*b^2*c*d^3 + a^4*b*d^4)*g^5)) - 1/4*A/(b^5*g^5*x^4 + 4*a*b^4*g^5*x^3 +
 6*a^2*b^3*g^5*x^2 + 4*a^3*b^2*g^5*x + a^4*b*g^5)
```

**mupad [B]** time = 6.17, size = 577, normalized size = 2.80

$$-\frac{\frac{12\,A\,a^3\,d^3-12\,A\,b^3\,c^3+25\,B\,a^3\,d^3-3\,B\,b^3\,c^3+36\,A\,a\,b^2\,c^2\,d-36\,A\,a^2\,b\,c\,d^2+13\,B\,a\,b^2\,c^2\,d-23\,B\,a^2\,b\,c\,d^2}{12\,\left(a^3\,d^3-3\,a^2\,b\,c\,d^2+3\,a\,b^2\,c^2\,d-b^3\,c^3\right)}-\frac{d^2\,x^2\,\left(B\,b^3\,c-7\,B\,a\,b^2\,d\right)}{2\,\left(a^3\,d^3-3\,a^2\,b\,c\,d^2+3\,a\,b^2\,c^2\,d-b^3\,c^3\right)}+\frac{d\,x \cdots}{3\,(\cdots}}{4\,a^4\,b\,g^5+16\,a^3\,b^2\,g^5\,x+24\,a^2\,b^3\,g^5\,x^2+16\,a\,b^4\,g^5\,x^3+4\,b^5\,g \cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*log((e*(a + b*x))/(c + d*x)))/(a*g + b*g*x)^5,x)
```

```
[Out] - ((12*A*a^3*d^3 - 12*A*b^3*c^3 + 25*B*a^3*d^3 - 3*B*b^3*c^3 + 36*A*a*b^2*c
^2*d - 36*A*a^2*b*c*d^2 + 13*B*a*b^2*c^2*d - 23*B*a^2*b*c*d^2)/(12*(a^3*d^3
 - b^3*c^3 + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2)) - (d^2*x^2*(B*b^3*c - 7*B*a*b^
2*d))/(2*(a^3*d^3 - b^3*c^3 + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2)) + (d*x*(B*b^3
*c^2 + 13*B*a^2*b*d^2 - 5*B*a*b^2*c*d))/(3*(a^3*d^3 - b^3*c^3 + 3*a*b^2*c^2
*d - 3*a^2*b*c*d^2)) + (B*b^3*d^3*x^3)/(a^3*d^3 - b^3*c^3 + 3*a*b^2*c^2*d -
 3*a^2*b*c*d^2))/(4*a^4*b*g^5 + 4*b^5*g^5*x^4 + 16*a^3*b^2*g^5*x + 16*a*b^4
*g^5*x^3 + 24*a^2*b^3*g^5*x^2) - (B*log((e*(a + b*x))/(c + d*x)))/(4*b^2*g^
5*(4*a^3*x + a^4/b + b^3*x^4 + 6*a^2*b*x^2 + 4*a*b^2*x^3)) - (B*d^4*atanh((
4*b^5*c^4*g^5 - 4*a^4*b*d^4*g^5 - 8*a*b^4*c^3*d*g^5 + 8*a^3*b^2*c*d^3*g^5)/
(4*b*g^5*(a*d - b*c)^4) - (2*b*d*x*(a^3*d^3 - b^3*c^3 + 3*a*b^2*c^2*d - 3*a
^2*b*c*d^2))/(a*d - b*c)^4))/(2*b*g^5*(a*d - b*c)^4)
```

**sympy [B]** time = 5.93, size = 944, normalized size = 4.58

$$-\frac{B \log{\left(\frac{e(a+bx)}{c+dx} \right)}}{4a^4bg^5+16a^3b^2g^5x+24a^2b^3g^5x^2+16ab^4g^5x^3+4b^5g^5x^4}-\frac{Bd^4 \log{\left(x+\frac{-\frac{Ba^5d^9}{(ad-bc)^4}+\frac{5Ba^4bcd^8}{(ad-bc)^4}-\frac{10Ba^3b^2c^2d^7}{(ad-bc)^4}+\frac{10Ba^2b^3c^3d^6}{(ad-bc)^4}-\frac{5B \cdots}{(\cdots}}{2Bbd^5} \cdots \right)}}{4bg^5(ad-bc)^4} \cdots$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*ln(e*(b*x+a)/(d*x+c)))/(b*g*x+a*g)**5,x)
```

```
[Out] -B*log(e*(a + b*x)/(c + d*x))/(4*a**4*b*g**5 + 16*a**3*b**2*g**5*x + 24*a**
2*b**3*g**5*x**2 + 16*a*b**4*g**5*x**3 + 4*b**5*g**5*x**4) - B*d**4*log(x +
 (-B*a**5*d**9/(a*d - b*c)**4 + 5*B*a**4*b*c*d**8/(a*d - b*c)**4 - 10*B*a**
```

```
3*b**2*c**2*d**7/(a*d - b*c)**4 + 10*B*a**2*b**3*c**3*d**6/(a*d - b*c)**4 -
 5*B*a*b**4*c**4*d**5/(a*d - b*c)**4 + B*a*d**5 + B*b**5*c**5*d**4/(a*d - b
*c)**4 + B*b*c*d**4)/(2*B*b*d**5))/(4*b*g**5*(a*d - b*c)**4) + B*d**4*log(x
 + (B*a**5*d**9/(a*d - b*c)**4 - 5*B*a**4*b*c*d**8/(a*d - b*c)**4 + 10*B*a*
*3*b**2*c**2*d**7/(a*d - b*c)**4 - 10*B*a**2*b**3*c**3*d**6/(a*d - b*c)**4
+ 5*B*a*b**4*c**4*d**5/(a*d - b*c)**4 + B*a*d**5 - B*b**5*c**5*d**4/(a*d -
b*c)**4 + B*b*c*d**4)/(2*B*b*d**5))/(4*b*g**5*(a*d - b*c)**4) + (-12*A*a**3
*d**3 + 36*A*a**2*b*c*d**2 - 36*A*a*b**2*c**2*d + 12*A*b**3*c**3 - 25*B*a**
3*d**3 + 23*B*a**2*b*c*d**2 - 13*B*a*b**2*c**2*d + 3*B*b**3*c**3 - 12*B*b**
3*d**3*x**3 + x**2*(-42*B*a*b**2*d**3 + 6*B*b**3*c*d**2) + x*(-52*B*a**2*b*
d**3 + 20*B*a*b**2*c*d**2 - 4*B*b**3*c**2*d))/(48*a**7*b*d**3*g**5 - 144*a*
*6*b**2*c*d**2*g**5 + 144*a**5*b**3*c**2*d*g**5 - 48*a**4*b**4*c**3*g**5 +
x**4*(48*a**3*b**5*d**3*g**5 - 144*a**2*b**6*c*d**2*g**5 + 144*a*b**7*c**2*
d*g**5 - 48*b**8*c**3*g**5) + x**3*(192*a**4*b**4*d**3*g**5 - 576*a**3*b**5
*c*d**2*g**5 + 576*a**2*b**6*c**2*d*g**5 - 192*a*b**7*c**3*g**5) + x**2*(28
8*a**5*b**3*d**3*g**5 - 864*a**4*b**4*c*d**2*g**5 + 864*a**3*b**5*c**2*d*g*
*5 - 288*a**2*b**6*c**3*g**5) + x*(192*a**6*b**2*d**3*g**5 - 576*a**5*b**3*
c*d**2*g**5 + 576*a**4*b**4*c**2*d*g**5 - 192*a**3*b**5*c**3*g**5))
```

# 3.97 $\int (ag+bgx)^4 \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2 dx$

**Optimal**. Leaf size=365

$$\frac{Bg^4(bc-ad)^5\log\left(\frac{bc-ad}{b(c+dx)}\right)\left(12B\log\left(\frac{e(a+bx)}{c+dx}\right)+12A+25B\right)}{30bd^5}+\frac{Bg^4(a+bx)(bc-ad)^4\left(12B\log\left(\frac{e(a+bx)}{c+dx}\right)+12A+13\right.}{30bd^4}$$

```
[Out] -1/10*B*(-a*d+b*c)*g^4*(b*x+a)^4*(A+B*ln(e*(b*x+a)/(d*x+c)))/b/d+1/5*g^4*(b
*x+a)^5*(A+B*ln(e*(b*x+a)/(d*x+c)))^2/b+1/30*B*(-a*d+b*c)^2*g^4*(b*x+a)^3*(
4*A+B+4*B*ln(e*(b*x+a)/(d*x+c)))/b/d^2-1/60*B*(-a*d+b*c)^3*g^4*(b*x+a)^2*(1
2*A+7*B+12*B*ln(e*(b*x+a)/(d*x+c)))/b/d^3+1/30*B*(-a*d+b*c)^4*g^4*(b*x+a)*(
12*A+13*B+12*B*ln(e*(b*x+a)/(d*x+c)))/b/d^4+1/30*B*(-a*d+b*c)^5*g^4*ln((-a*
d+b*c)/b/(d*x+c))*(12*A+25*B+12*B*ln(e*(b*x+a)/(d*x+c)))/b/d^5+2/5*B^2*(-a*
d+b*c)^5*g^4*polylog(2,d*(b*x+a)/b/(d*x+c))/b/d^5
```

**Rubi [A]** time = 0.85, antiderivative size = 557, normalized size of antiderivative = 1.53, number of steps used = 28, number of rules used = 13, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.406, Rules used = {2525, 12, 2528, 2486, 31, 43, 2524, 2418, 2394, 2393, 2391, 2390, 2301}

$$\frac{2B^2g^4(bc-ad)^5\text{PolyLog}\left(2,\frac{b(c+dx)}{bc-ad}\right)}{5bd^5}-\frac{2Bg^4(bc-ad)^5\log(c+dx)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{5bd^5}-\frac{Bg^4(a+bx)^2(bc-ad)^3\left(\right.}{5bd^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a*g + b*g*x)^4*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2,x]
```

```
[Out] (2*A*B*(b*c - a*d)^4*g^4*x)/(5*d^4) + (13*B^2*(b*c - a*d)^4*g^4*x)/(30*d^4)
 - (7*B^2*(b*c - a*d)^3*g^4*(a + b*x)^2)/(60*b*d^3) + (B^2*(b*c - a*d)^2*g^
4*(a + b*x)^3)/(30*b*d^2) + (2*B^2*(b*c - a*d)^4*g^4*(a + b*x)*Log[(e*(a +
b*x))/(c + d*x)])/(5*b*d^4) - (B*(b*c - a*d)^3*g^4*(a + b*x)^2*(A + B*Log[(
e*(a + b*x))/(c + d*x)]))/(5*b*d^3) + (2*B*(b*c - a*d)^2*g^4*(a + b*x)^3*(A
 + B*Log[(e*(a + b*x))/(c + d*x)]))/(15*b*d^2) - (B*(b*c - a*d)*g^4*(a + b*
x)^4*(A + B*Log[(e*(a + b*x))/(c + d*x)]))/(10*b*d) + (g^4*(a + b*x)^5*(A +
 B*Log[(e*(a + b*x))/(c + d*x)])^2)/(5*b) - (5*B^2*(b*c - a*d)^5*g^4*Log[c
+ d*x])/(6*b*d^5) + (2*B^2*(b*c - a*d)^5*g^4*Log[-((d*(a + b*x))/(b*c - a*d
))]*Log[c + d*x])/(5*b*d^5) - (2*B*(b*c - a*d)^5*g^4*(A + B*Log[(e*(a + b*x
))/(c + d*x)])*Log[c + d*x])/(5*b*d^5) - (B^2*(b*c - a*d)^5*g^4*Log[c + d*x
]^2)/(5*b*d^5) + (2*B^2*(b*c - a*d)^5*g^4*PolyLog[2, (b*(c + d*x))/(b*c - a
*d)])/(5*b*d^5)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2301

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

Rule 2390

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_))^(q_.), x_Symbol] :> Dist[1/e, Subst[Int[((f*x)/d)^q*(a + b*Log[c*x^
n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && E
qQ[e*f - d*g, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2393

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + (c*e*x)/g])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2394

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d + e*x
)^n]))/g, x] - Dist[(b*e*n)/g, Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
```

```
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2418

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*(RFx_), x_Sy
mbol] :> With[{u = ExpandIntegrand[(a + b*Log[c*(d + e*x)^n])^p, RFx, x]},
Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d, e, n}, x] && RationalFunctionQ[
RFx, x] && IntegerQ[p]
```

Rule 2486

```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))
^(r_.)]^(s_.), x_Symbol] :> Simp[((a + b*x)*Log[e*(f*(a + b*x)^p*(c + d*x)^
q)^r]^s)/b, x] + Dist[(q*r*s*(b*c - a*d))/b, Int[Log[e*(f*(a + b*x)^p*(c +
d*x)^q)^r]^(s - 1)/(c + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, p, q, r, s
}, x] && NeQ[b*c - a*d, 0] && EqQ[p + q, 0] && IGtQ[s, 0]
```

Rule 2524

```
Int[((a_.) + Log[(c_.)*(RFx_)^(p_.)]*(b_.))^(n_.)/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[(Log[d + e*x]*(a + b*Log[c*RFx^p])^n)/e, x] - Dist[(b*n*p)/e
, Int[(Log[d + e*x]*(a + b*Log[c*RFx^p])^(n - 1)*D[RFx, x])/RFx, x], x] /;
FreeQ[{a, b, c, d, e, p}, x] && RationalFunctionQ[RFx, x] && IGtQ[n, 0]
```

Rule 2525

```
Int[((a_.) + Log[(c_.)*(RFx_)^(p_.)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_))^(m_.
), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(a + b*Log[c*RFx^p])^n)/(e*(m + 1))
, x] - Dist[(b*n*p)/(e*(m + 1)), Int[SimplifyIntegrand[((d + e*x)^(m + 1)*(
a + b*Log[c*RFx^p])^(n - 1)*D[RFx, x])/RFx, x], x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && RationalFunctionQ[RFx, x] && IGtQ[n, 0] && (EqQ[n, 1] ||
IntegerQ[m]) && NeQ[m, -1]
```

Rule 2528

```
Int[((a_.) + Log[(c_.)*(RFx_)^(p_.)]*(b_.))^(n_.)*(RGx_), x_Symbol] :> With
[{u = ExpandIntegrand[(a + b*Log[c*RFx^p])^n, RGx, x]}, Int[u, x] /; SumQ[u
]] /; FreeQ[{a, b, c, p}, x] && RationalFunctionQ[RFx, x] && RationalFuncti
onQ[RGx, x] && IGtQ[n, 0]
```

Rubi steps

$$\int (ag+bgx)^4\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2\,dx = \frac{g^4(a+bx)^5\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{5b} - \frac{(2B)\int \frac{(bc-ad)g^5(a+bx)^4\left(A+B\log\right.}{c+dx}}{5bg}$$

$$= \frac{g^4(a+bx)^5\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{5b} - \frac{\left(2B(bc-ad)g^4\right)\int \frac{(a+bx)^4\left(A+\right.}{c}}{5b}$$

$$= \frac{g^4(a+bx)^5\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{5b} - \frac{\left(2B(bc-ad)g^4\right)\int\left(-\frac{b(bc-ad)}{}\right.}{}$$

$$= \frac{g^4(a+bx)^5\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{5b} - \frac{\left(2B(bc-ad)g^4\right)\int (a+bx)^3}{5d}$$

$$= \frac{2AB(bc-ad)^4g^4x}{5d^4} - \frac{B(bc-ad)^3g^4(a+bx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{5bd^3}$$

$$= \frac{2AB(bc-ad)^4g^4x}{5d^4} + \frac{2B^2(bc-ad)^4g^4(a+bx)\log\left(\frac{e(a+bx)}{c+dx}\right)}{5bd^4} - \frac{B(b}{}$$

$$= \frac{2AB(bc-ad)^4g^4x}{5d^4} + \frac{2B^2(bc-ad)^4g^4(a+bx)\log\left(\frac{e(a+bx)}{c+dx}\right)}{5bd^4} - \frac{B(b}{}$$

$$= \frac{2AB(bc-ad)^4g^4x}{5d^4} + \frac{13B^2(bc-ad)^4g^4x}{30d^4} - \frac{7B^2(bc-ad)^3g^4(a+b}{60bd^3}$$

$$= \frac{2AB(bc-ad)^4g^4x}{5d^4} + \frac{13B^2(bc-ad)^4g^4x}{30d^4} - \frac{7B^2(bc-ad)^3g^4(a+b}{60bd^3}$$

$$= \frac{2AB(bc-ad)^4g^4x}{5d^4} + \frac{13B^2(bc-ad)^4g^4x}{30d^4} - \frac{7B^2(bc-ad)^3g^4(a+b}{60bd^3}$$

$$= \frac{2AB(bc-ad)^4g^4x}{5d^4} + \frac{13B^2(bc-ad)^4g^4x}{30d^4} - \frac{7B^2(bc-ad)^3g^4(a+b}{60bd^3}$$

**Mathematica [A]** time = 0.49, size = 511, normalized size = 1.40

$$g^4\left(\frac{B(bc-ad)\left(-6d^4(a+bx)^4\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)+8d^3(a+bx)^3(bc-ad)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)-12d^2(a+bx)^2(bc-ad)^2\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)-24(bc-ad)^4\log(\right.}{}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*g + b*g*x)^4*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2,x]
```

```
[Out] (g^4*((a + b*x)^5*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2 + (B*(b*c - a*d)*(
24*A*b*d*(b*c - a*d)^3*x + 24*B*d*(b*c - a*d)^3*(a + b*x)*Log[(e*(a + b*x))
/(c + d*x)] - 12*d^2*(b*c - a*d)^2*(a + b*x)^2*(A + B*Log[(e*(a + b*x))/(c
+ d*x)]) + 8*d^3*(b*c - a*d)*(a + b*x)^3*(A + B*Log[(e*(a + b*x))/(c + d*x)
]) - 6*d^4*(a + b*x)^4*(A + B*Log[(e*(a + b*x))/(c + d*x)]) - 24*B*(b*c - a
*d)^4*Log[c + d*x] - 24*(b*c - a*d)^4*(A + B*Log[(e*(a + b*x))/(c + d*x)])*
Log[c + d*x] + 4*B*(b*c - a*d)^2*(2*b*d*(b*c - a*d)*x - d^2*(a + b*x)^2 - 2
*(b*c - a*d)^2*Log[c + d*x]) + B*(b*c - a*d)*(6*b*d*(b*c - a*d)^2*x + 3*d^2
*(-(b*c) + a*d)*(a + b*x)^2 + 2*d^3*(a + b*x)^3 - 6*(b*c - a*d)^3*Log[c + d
*x]) + 12*B*(b*c - a*d)^3*(b*d*x + (-(b*c) + a*d)*Log[c + d*x]) + 12*B*(b*c
 - a*d)^4*((2*Log[(d*(a + b*x))/(-(b*c) + a*d)] - Log[c + d*x])*Log[c + d*x
] + 2*PolyLog[2, (b*(c + d*x))/(b*c - a*d)])))/(12*d^5)))/(5*b)
```

**fricas [F]** time = 1.85, size = 0, normalized size = 0.00

$$\text{integral}\left(A^2b^4g^4x^4 + 4\,A^2ab^3g^4x^3 + 6\,A^2a^2b^2g^4x^2 + 4\,A^2a^3bg^4x + A^2a^4g^4 + \left(B^2b^4g^4x^4 + 4\,B^2ab^3g^4x^3 + 6\,B^2a^2b\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*g*x+a*g)^4*(A+B*log(e*(b*x+a)/(d*x+c)))^2,x, algorithm="fricas
")
```

```
[Out] integral(A^2*b^4*g^4*x^4 + 4*A^2*a*b^3*g^4*x^3 + 6*A^2*a^2*b^2*g^4*x^2 + 4*
A^2*a^3*b*g^4*x + A^2*a^4*g^4 + (B^2*b^4*g^4*x^4 + 4*B^2*a*b^3*g^4*x^3 + 6*
B^2*a^2*b^2*g^4*x^2 + 4*B^2*a^3*b*g^4*x + B^2*a^4*g^4)*log((b*e*x + a*e)/(d
*x + c))^2 + 2*(A*B*b^4*g^4*x^4 + 4*A*B*a*b^3*g^4*x^3 + 6*A*B*a^2*b^2*g^4*x
^2 + 4*A*B*a^3*b*g^4*x + A*B*a^4*g^4)*log((b*e*x + a*e)/(d*x + c)), x)
```

**giac [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*g*x+a*g)^4*(A+B*log(e*(b*x+a)/(d*x+c)))^2,x, algorithm="giac")
```

```
[Out] Timed out
```

**maple [F]** time = 2.57, size = 0, normalized size = 0.00

$$\int \left(bgx+ag\right)^4 \left(B\ln\left(\frac{(bx+a)\,e}{dx+c}\right)+A\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*g*x+a*g)^4*(A+B*ln(e*(b*x+a)/(d*x+c)))^2,x)
```

```
[Out] int((b*g*x+a*g)^4*(A+B*ln(e*(b*x+a)/(d*x+c)))^2,x)
```

**maxima [B]** time = 2.38, size = 2389, normalized size = 6.55

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*g*x+a*g)^4*(A+B*log(e*(b*x+a)/(d*x+c)))^2,x, algorithm="maxima
")
```

```
[Out] 1/5*A^2*b^4*g^4*x^5 + A^2*a*b^3*g^4*x^4 + 2*A^2*a^2*b^2*g^4*x^3 + 2*A^2*a^3
*b*g^4*x^2 + 2*(x*log(b*e*x/(d*x + c) + a*e/(d*x + c)) + a*log(b*x + a)/b -
 c*log(d*x + c)/d)*A*B*a^4*g^4 + 4*(x^2*log(b*e*x/(d*x + c) + a*e/(d*x + c)
) - a^2*log(b*x + a)/b^2 + c^2*log(d*x + c)/d^2 - (b*c - a*d)*x/(b*d))*A*B*
a^3*b*g^4 + 2*(2*x^3*log(b*e*x/(d*x + c) + a*e/(d*x + c)) + 2*a^3*log(b*x +
 a)/b^3 - 2*c^3*log(d*x + c)/d^3 - ((b^2*c*d - a*b*d^2)*x^2 - 2*(b^2*c^2 -
a^2*d^2)*x)/(b^2*d^2))*A*B*a^2*b^2*g^4 + 1/3*(6*x^4*log(b*e*x/(d*x + c) + a
*e/(d*x + c)) - 6*a^4*log(b*x + a)/b^4 + 6*c^4*log(d*x + c)/d^4 - (2*(b^3*c
*d^2 - a*b^2*d^3)*x^3 - 3*(b^3*c^2*d - a^2*b*d^3)*x^2 + 6*(b^3*c^3 - a^3*d^
3)*x)/(b^3*d^3))*A*B*a*b^3*g^4 + 1/30*(12*x^5*log(b*e*x/(d*x + c) + a*e/(d*
x + c)) + 12*a^5*log(b*x + a)/b^5 - 12*c^5*log(d*x + c)/d^5 - (3*(b^4*c*d^3
 - a*b^3*d^4)*x^4 - 4*(b^4*c^2*d^2 - a^2*b^2*d^4)*x^3 + 6*(b^4*c^3*d - a^3*
b*d^4)*x^2 - 12*(b^4*c^4 - a^4*d^4)*x)/(b^4*d^4))*A*B*b^4*g^4 + A^2*a^4*g^4
*x - 1/30*((12*g^4*log(e) + 25*g^4)*b^4*c^5 - (60*g^4*log(e) + 113*g^4)*a*b
^3*c^4*d + 4*(30*g^4*log(e) + 49*g^4)*a^2*b^2*c^3*d^2 - 12*(10*g^4*log(e) +
 13*g^4)*a^3*b*c^2*d^3 + 12*(5*g^4*log(e) + 4*g^4)*a^4*c*d^4)*B^2*log(d*x +
 c)/d^5 - 2/5*(b^5*c^5*g^4 - 5*a*b^4*c^4*d*g^4 + 10*a^2*b^3*c^3*d^2*g^4 - 1
0*a^3*b^2*c^2*d^3*g^4 + 5*a^4*b*c*d^4*g^4 - a^5*d^5*g^4)*(log(b*x + a)*log(
(b*d*x + a*d)/(b*c - a*d) + 1) + dilog(-(b*d*x + a*d)/(b*c - a*d)))*B^2/(b*
d^5) + 1/60*(12*B^2*b^5*d^5*g^4*x^5*log(e)^2 - 6*(b^5*c*d^4*g^4*log(e) - (1
0*g^4*log(e)^2 + g^4*log(e))*a*b^4*d^5)*B^2*x^4 + 2*((4*g^4*log(e) + g^4)*b
^5*c^2*d^3 - 2*(10*g^4*log(e) + g^4)*a*b^4*c*d^4 + (60*g^4*log(e)^2 + 16*g^
4*log(e) + g^4)*a^2*b^3*d^5)*B^2*x^3 - ((12*g^4*log(e) + 7*g^4)*b^5*c^3*d^2
 - 3*(20*g^4*log(e) + 9*g^4)*a*b^4*c^2*d^3 + 3*(40*g^4*log(e) + 11*g^4)*a^2
*b^3*c*d^4 - (120*g^4*log(e)^2 + 72*g^4*log(e) + 13*g^4)*a^3*b^2*d^5)*B^2*x
^2 + 2*((12*g^4*log(e) + 13*g^4)*b^5*c^4*d - (60*g^4*log(e) + 59*g^4)*a*b^4
*c^3*d^2 + 6*(20*g^4*log(e) + 17*g^4)*a^2*b^3*c^2*d^3 - (120*g^4*log(e) + 7
9*g^4)*a^3*b^2*c*d^4 + (30*g^4*log(e)^2 + 48*g^4*log(e) + 23*g^4)*a^4*b*d^5
)*B^2*x + 12*(B^2*b^5*d^5*g^4*x^5 + 5*B^2*a*b^4*d^5*g^4*x^4 + 10*B^2*a^2*b^
3*d^5*g^4*x^3 + 10*B^2*a^3*b^2*d^5*g^4*x^2 + 5*B^2*a^4*b*d^5*g^4*x + B^2*a^
5*d^5*g^4)*log(b*x + a)^2 + 12*(B^2*b^5*d^5*g^4*x^5 + 5*B^2*a*b^4*d^5*g^4*x
```

```
^4 + 10*B^2*a^2*b^3*d^5*g^4*x^3 + 10*B^2*a^3*b^2*d^5*g^4*x^2 + 5*B^2*a^4*b*
d^5*g^4*x + (b^5*c^5*g^4 - 5*a*b^4*c^4*d*g^4 + 10*a^2*b^3*c^3*d^2*g^4 - 10*
a^3*b^2*c^2*d^3*g^4 + 5*a^4*b*c*d^4*g^4)*B^2)*log(d*x + c)^2 + 2*(12*B^2*b^
5*d^5*g^4*x^5*log(e) - 3*(b^5*c*d^4*g^4 - (20*g^4*log(e) + g^4)*a*b^4*d^5)*
B^2*x^4 + 4*(b^5*c^2*d^3*g^4 - 5*a*b^4*c*d^4*g^4 + 2*(15*g^4*log(e) + 2*g^4
)*a^2*b^3*d^5)*B^2*x^3 - 6*(b^5*c^3*d^2*g^4 - 5*a*b^4*c^2*d^3*g^4 + 10*a^2*
b^3*c*d^4*g^4 - 2*(10*g^4*log(e) + 3*g^4)*a^3*b^2*d^5)*B^2*x^2 + 12*(b^5*c^
4*d*g^4 - 5*a*b^4*c^3*d^2*g^4 + 10*a^2*b^3*c^2*d^3*g^4 - 10*a^3*b^2*c*d^4*g
^4 + (5*g^4*log(e) + 4*g^4)*a^4*b*d^5)*B^2*x + (12*a*b^4*c^4*d*g^4 - 54*a^2
*b^3*c^3*d^2*g^4 + 94*a^3*b^2*c^2*d^3*g^4 - 77*a^4*b*c*d^4*g^4 + (12*g^4*lo
g(e) + 25*g^4)*a^5*d^5)*B^2)*log(b*x + a) - 2*(12*B^2*b^5*d^5*g^4*x^5*log(e
) - 3*(b^5*c*d^4*g^4 - (20*g^4*log(e) + g^4)*a*b^4*d^5)*B^2*x^4 + 4*(b^5*c^
2*d^3*g^4 - 5*a*b^4*c*d^4*g^4 + 2*(15*g^4*log(e) + 2*g^4)*a^2*b^3*d^5)*B^2*
x^3 - 6*(b^5*c^3*d^2*g^4 - 5*a*b^4*c^2*d^3*g^4 + 10*a^2*b^3*c*d^4*g^4 - 2*(
10*g^4*log(e) + 3*g^4)*a^3*b^2*d^5)*B^2*x^2 + 12*(b^5*c^4*d*g^4 - 5*a*b^4*c
^3*d^2*g^4 + 10*a^2*b^3*c^2*d^3*g^4 - 10*a^3*b^2*c*d^4*g^4 + (5*g^4*log(e)
+ 4*g^4)*a^4*b*d^5)*B^2*x + 12*(B^2*b^5*d^5*g^4*x^5 + 5*B^2*a*b^4*d^5*g^4*x
^4 + 10*B^2*a^2*b^3*d^5*g^4*x^3 + 10*B^2*a^3*b^2*d^5*g^4*x^2 + 5*B^2*a^4*b*
d^5*g^4*x + B^2*a^5*d^5*g^4)*log(b*x + a))*log(d*x + c))/(b*d^5)
```

**mupad** **[F]** time = 0.00, size = -1, normalized size = -0.00

$$\int \left(a\,g+b\,g\,x\right)^4\,\left(A+B\,\ln\left(\frac{e\,\left(a+b\,x\right)}{c+d\,x}\right)\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*g + b*g*x)^4*(A + B*log((e*(a + b*x))/(c + d*x)))^2,x)
[Out] int((a*g + b*g*x)^4*(A + B*log((e*(a + b*x))/(c + d*x)))^2, x)
```

**sympy** **[F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*g*x+a*g)**4*(A+B*ln(e*(b*x+a)/(d*x+c)))**2,x)
[Out] Timed out
```

## 3.98 $\int (ag+bgx)^3 \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2 dx$

**Optimal**. Leaf size=309

$$-\frac{Bg^3(bc-ad)^4\log\left(\frac{bc-ad}{b(c+dx)}\right)\left(6B\log\left(\frac{e(a+bx)}{c+dx}\right)+6A+11B\right)}{12bd^4}-\frac{Bg^3(a+bx)(bc-ad)^3\left(6B\log\left(\frac{e(a+bx)}{c+dx}\right)+6A+5B\right)}{12bd^3}$$

```
[Out] -1/6*B*(-a*d+b*c)*g^3*(b*x+a)^3*(A+B*ln(e*(b*x+a)/(d*x+c)))/b/d+1/4*g^3*(b*
x+a)^4*(A+B*ln(e*(b*x+a)/(d*x+c)))^2/b+1/12*B*(-a*d+b*c)^2*g^3*(b*x+a)^2*(3
*A+B+3*B*ln(e*(b*x+a)/(d*x+c)))/b/d^2-1/12*B*(-a*d+b*c)^3*g^3*(b*x+a)*(6*A+
5*B+6*B*ln(e*(b*x+a)/(d*x+c)))/b/d^3-1/12*B*(-a*d+b*c)^4*g^3*ln((-a*d+b*c)/
b/(d*x+c))*(6*A+11*B+6*B*ln(e*(b*x+a)/(d*x+c)))/b/d^4-1/2*B^2*(-a*d+b*c)^4*
g^3*polylog(2,d*(b*x+a)/b/(d*x+c))/b/d^4
```

**Rubi [A]** time = 0.65, antiderivative size = 474, normalized size of antiderivative = 1.53, number of steps used = 24, number of rules used = 13, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.406, Rules used = {2525, 12, 2528, 2486, 31, 43, 2524, 2418, 2394, 2393, 2391, 2390, 2301}

$$-\frac{B^2g^3(bc-ad)^4\text{PolyLog}\left(2,\frac{b(c+dx)}{bc-ad}\right)}{2bd^4}+\frac{Bg^3(bc-ad)^4\log(c+dx)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{2bd^4}+\frac{Bg^3(a+bx)^2(bc-ad)^2}{4bd}$$

Antiderivative was successfully verified.

```
[In] Int[(a*g + b*g*x)^3*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2,x]
```

```
[Out] -(A*B*(b*c - a*d)^3*g^3*x)/(2*d^3) - (5*B^2*(b*c - a*d)^3*g^3*x)/(12*d^3) +
 (B^2*(b*c - a*d)^2*g^3*(a + b*x)^2)/(12*b*d^2) - (B^2*(b*c - a*d)^3*g^3*(a
 + b*x)*Log[(e*(a + b*x))/(c + d*x)])/(2*b*d^3) + (B*(b*c - a*d)^2*g^3*(a +
 b*x)^2*(A + B*Log[(e*(a + b*x))/(c + d*x)]))/(4*b*d^2) - (B*(b*c - a*d)*g^
3*(a + b*x)^3*(A + B*Log[(e*(a + b*x))/(c + d*x)]))/(6*b*d) + (g^3*(a + b*x
)^4*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2)/(4*b) + (11*B^2*(b*c - a*d)^4*g
^3*Log[c + d*x])/(12*b*d^4) - (B^2*(b*c - a*d)^4*g^3*Log[-((d*(a + b*x))/(b
*c - a*d))]*Log[c + d*x])/(2*b*d^4) + (B*(b*c - a*d)^4*g^3*(A + B*Log[(e*(a
 + b*x))/(c + d*x)])*Log[c + d*x])/(2*b*d^4) + (B^2*(b*c - a*d)^4*g^3*Log[c
 + d*x]^2)/(4*b*d^4) - (B^2*(b*c - a*d)^4*g^3*PolyLog[2, (b*(c + d*x))/(b*c
 - a*d)])/(2*b*d^4)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2301

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

Rule 2390

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_))^(q_.), x_Symbol] :> Dist[1/e, Subst[Int[((f*x)/d)^q*(a + b*Log[c*x^
n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && E
qQ[e*f - d*g, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2393

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + (c*e*x)/g])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2394

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d + e*x
)^n]))/g, x] - Dist[(b*e*n)/g, Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2418

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*(RFx_), x_Sy
mbol] :> With[{u = ExpandIntegrand[(a + b*Log[c*(d + e*x)^n])^p, RFx, x]},
Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d, e, n}, x] && RationalFunctionQ[
RFx, x] && IntegerQ[p]
```

Rule 2486

```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))
^(r_.)]^(s_.), x_Symbol] :> Simp[((a + b*x)*Log[e*(f*(a + b*x)^p*(c + d*x)^
q)^r]^s)/b, x] + Dist[(q*r*s*(b*c - a*d))/b, Int[Log[e*(f*(a + b*x)^p*(c +
d*x)^q)^r]^(s - 1)/(c + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, p, q, r, s
}, x] && NeQ[b*c - a*d, 0] && EqQ[p + q, 0] && IGtQ[s, 0]
```

Rule 2524

```
Int[((a_.) + Log[(c_.)*(RFx_)^(p_.)]*(b_.))^(n_.)/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[(Log[d + e*x]*(a + b*Log[c*RFx^p])^n)/e, x] - Dist[(b*n*p)/e
, Int[(Log[d + e*x]*(a + b*Log[c*RFx^p])^(n - 1)*D[RFx, x])/RFx, x], x] /;
FreeQ[{a, b, c, d, e, p}, x] && RationalFunctionQ[RFx, x] && IGtQ[n, 0]
```

Rule 2525

```
Int[((a_.) + Log[(c_.)*(RFx_)^(p_.)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_))^(m_.
), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(a + b*Log[c*RFx^p])^n)/(e*(m + 1))
, x] - Dist[(b*n*p)/(e*(m + 1)), Int[SimplifyIntegrand[((d + e*x)^(m + 1)*(
a + b*Log[c*RFx^p])^(n - 1)*D[RFx, x])/RFx, x], x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && RationalFunctionQ[RFx, x] && IGtQ[n, 0] && (EqQ[n, 1] ||
IntegerQ[m]) && NeQ[m, -1]
```

Rule 2528

```
Int[((a_.) + Log[(c_.)*(RFx_)^(p_.)]*(b_.))^(n_.)*(RGx_), x_Symbol] :> With
[{u = ExpandIntegrand[(a + b*Log[c*RFx^p])^n, RGx, x]}, Int[u, x] /; SumQ[u
]] /; FreeQ[{a, b, c, p}, x] && RationalFunctionQ[RFx, x] && RationalFuncti
onQ[RGx, x] && IGtQ[n, 0]
```

Rubi steps

$$\int (ag+bgx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2 dx = \frac{g^3(a+bx)^4\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{4b} - \frac{B\int \frac{(bc-ad)g^4(a+bx)^3\left(A+B\log\left(\frac{e(a+b}{c+d}\right.}{c+dx}}{2bg}$$

$$= \frac{g^3(a+bx)^4\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{4b} - \frac{\left(B(bc-ad)g^3\right)\int \frac{(a+bx)^3\left(A+B\log\right.}{c+dx}}{2b}$$

$$= \frac{g^3(a+bx)^4\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{4b} - \frac{\left(B(bc-ad)g^3\right)\int\left(\frac{b(bc-ad)^2\left(A+\right.}{d}\right.}{}$$

$$= \frac{g^3(a+bx)^4\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{4b} - \frac{\left(B(bc-ad)g^3\right)\int(a+bx)^2\left(A\right.}{2d}$$

$$= -\frac{AB(bc-ad)^3g^3x}{2d^3} + \frac{B(bc-ad)^2g^3(a+bx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{4bd^2}$$

$$= -\frac{AB(bc-ad)^3g^3x}{2d^3} - \frac{B^2(bc-ad)^3g^3(a+bx)\log\left(\frac{e(a+bx)}{c+dx}\right)}{2bd^3} + \frac{B(bc-}{}$$

$$= -\frac{AB(bc-ad)^3g^3x}{2d^3} - \frac{B^2(bc-ad)^3g^3(a+bx)\log\left(\frac{e(a+bx)}{c+dx}\right)}{2bd^3} + \frac{B(bc-}{}$$

$$= -\frac{AB(bc-ad)^3g^3x}{2d^3} - \frac{5B^2(bc-ad)^3g^3x}{12d^3} + \frac{B^2(bc-ad)^2g^3(a+bx)^2}{12bd^2}$$

$$= -\frac{AB(bc-ad)^3g^3x}{2d^3} - \frac{5B^2(bc-ad)^3g^3x}{12d^3} + \frac{B^2(bc-ad)^2g^3(a+bx)^2}{12bd^2}$$

$$= -\frac{AB(bc-ad)^3g^3x}{2d^3} - \frac{5B^2(bc-ad)^3g^3x}{12d^3} + \frac{B^2(bc-ad)^2g^3(a+bx)^2}{12bd^2}$$

$$= -\frac{AB(bc-ad)^3g^3x}{2d^3} - \frac{5B^2(bc-ad)^3g^3x}{12d^3} + \frac{B^2(bc-ad)^2g^3(a+bx)^2}{12bd^2}$$

**Mathematica [A]** time = 0.37, size = 391, normalized size = 1.27

$$g^3\left((a+bx)^4\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2 - \frac{B(bc-ad)\left(2d^3(a+bx)^3\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)+3d^2(a+bx)^2(ad-bc)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)-6(bc-ad)^3\log(c-\right.}{}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*g + b*g*x)^3*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2,x]
```

```
[Out] (g^3*((a + b*x)^4*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2 - (B*(b*c - a*d)*(
6*A*b*d*(b*c - a*d)^2*x + 6*B*d*(b*c - a*d)^2*(a + b*x)*Log[(e*(a + b*x))/(
c + d*x)] + 3*d^2*(-(b*c) + a*d)*(a + b*x)^2*(A + B*Log[(e*(a + b*x))/(c +
d*x)]) + 2*d^3*(a + b*x)^3*(A + B*Log[(e*(a + b*x))/(c + d*x)]) - 6*B*(b*c
- a*d)^3*Log[c + d*x] - 6*(b*c - a*d)^3*(A + B*Log[(e*(a + b*x))/(c + d*x)]
)*Log[c + d*x] + B*(b*c - a*d)*(2*b*d*(b*c - a*d)*x - d^2*(a + b*x)^2 - 2*(
b*c - a*d)^2*Log[c + d*x]) + 3*B*(b*c - a*d)^2*(b*d*x + (-(b*c) + a*d)*Log[
c + d*x]) + 3*B*(b*c - a*d)^3*((2*Log[(d*(a + b*x))/(-(b*c) + a*d)] - Log[c
 + d*x])*Log[c + d*x] + 2*PolyLog[2, (b*(c + d*x))/(b*c - a*d)])))/(3*d^4))
)/(4*b)
```

**fricas [F]** time = 1.89, size = 0, normalized size = 0.00

$$\text{integral}\left(A^2b^3g^3x^3 + 3\,A^2ab^2g^3x^2 + 3\,A^2a^2bg^3x + A^2a^3g^3 + \left(B^2b^3g^3x^3 + 3\,B^2ab^2g^3x^2 + 3\,B^2a^2bg^3x + B^2a^3g^3\right)\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*g*x+a*g)^3*(A+B*log(e*(b*x+a)/(d*x+c)))^2,x, algorithm="fricas
")
```

```
[Out] integral(A^2*b^3*g^3*x^3 + 3*A^2*a*b^2*g^3*x^2 + 3*A^2*a^2*b*g^3*x + A^2*a^
3*g^3 + (B^2*b^3*g^3*x^3 + 3*B^2*a*b^2*g^3*x^2 + 3*B^2*a^2*b*g^3*x + B^2*a^
3*g^3)*log((b*e*x + a*e)/(d*x + c))^2 + 2*(A*B*b^3*g^3*x^3 + 3*A*B*a*b^2*g^
3*x^2 + 3*A*B*a^2*b*g^3*x + A*B*a^3*g^3)*log((b*e*x + a*e)/(d*x + c)), x)
```

**giac [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*g*x+a*g)^3*(A+B*log(e*(b*x+a)/(d*x+c)))^2,x, algorithm="giac")
```

```
[Out] Timed out
```

**maple [F]** time = 2.18, size = 0, normalized size = 0.00

$$\int \left(bgx + ag\right)^{3} \left(B\ln\left(\frac{(bx+a)\,e}{dx+c}\right) + A\right)^{2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*g*x+a*g)^3*(B*ln((b*x+a)/(d*x+c)*e)+A)^2,x)
```

```
[Out] int((b*g*x+a*g)^3*(B*ln((b*x+a)/(d*x+c)*e)+A)^2,x)
```

**maxima [B]** time = 2.37, size = 1732, normalized size = 5.61

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*g*x+a*g)^3*(A+B*log(e*(b*x+a)/(d*x+c)))^2,x, algorithm="maxima
")
```

```
[Out] 1/4*A^2*b^3*g^3*x^4 + A^2*a*b^2*g^3*x^3 + 3/2*A^2*a^2*b*g^3*x^2 + 2*(x*log(
b*e*x/(d*x + c) + a*e/(d*x + c)) + a*log(b*x + a)/b - c*log(d*x + c)/d)*A*B
*a^3*g^3 + 3*(x^2*log(b*e*x/(d*x + c) + a*e/(d*x + c)) - a^2*log(b*x + a)/b
^2 + c^2*log(d*x + c)/d^2 - (b*c - a*d)*x/(b*d))*A*B*a^2*b*g^3 + (2*x^3*log
(b*e*x/(d*x + c) + a*e/(d*x + c)) + 2*a^3*log(b*x + a)/b^3 - 2*c^3*log(d*x
+ c)/d^3 - ((b^2*c*d - a*b*d^2)*x^2 - 2*(b^2*c^2 - a^2*d^2)*x)/(b^2*d^2))*A
*B*a*b^2*g^3 + 1/12*(6*x^4*log(b*e*x/(d*x + c) + a*e/(d*x + c)) - 6*a^4*log
(b*x + a)/b^4 + 6*c^4*log(d*x + c)/d^4 - (2*(b^3*c*d^2 - a*b^2*d^3)*x^3 - 3
*(b^3*c^2*d - a^2*b*d^3)*x^2 + 6*(b^3*c^3 - a^3*d^3)*x)/(b^3*d^3))*A*B*b^3*
g^3 + A^2*a^3*g^3*x + 1/12*((6*g^3*log(e) + 11*g^3)*b^3*c^4 - 2*(12*g^3*log
(e) + 19*g^3)*a*b^2*c^3*d + 9*(4*g^3*log(e) + 5*g^3)*a^2*b*c^2*d^2 - 6*(4*g
^3*log(e) + 3*g^3)*a^3*c*d^3)*B^2*log(d*x + c)/d^4 + 1/2*(b^4*c^4*g^3 - 4*a
*b^3*c^3*d*g^3 + 6*a^2*b^2*c^2*d^2*g^3 - 4*a^3*b*c*d^3*g^3 + a^4*d^4*g^3)*(
log(b*x + a)*log((b*d*x + a*d)/(b*c - a*d) + 1) + dilog(-(b*d*x + a*d)/(b*c
 - a*d)))*B^2/(b*d^4) + 1/12*(3*B^2*b^4*d^4*g^3*x^4*log(e)^2 - 2*(b^4*c*d^3
*g^3*log(e) - (6*g^3*log(e)^2 + g^3*log(e))*a*b^3*d^4)*B^2*x^3 + ((3*g^3*lo
g(e) + g^3)*b^4*c^2*d^2 - 2*(6*g^3*log(e) + g^3)*a*b^3*c*d^3 + (18*g^3*log(
e)^2 + 9*g^3*log(e) + g^3)*a^2*b^2*d^4)*B^2*x^2 - ((6*g^3*log(e) + 5*g^3)*b
^4*c^3*d - (24*g^3*log(e) + 17*g^3)*a*b^3*c^2*d^2 + (36*g^3*log(e) + 19*g^3
)*a^2*b^2*c*d^3 - (12*g^3*log(e)^2 + 18*g^3*log(e) + 7*g^3)*a^3*b*d^4)*B^2*
x + 3*(B^2*b^4*d^4*g^3*x^4 + 4*B^2*a*b^3*d^4*g^3*x^3 + 6*B^2*a^2*b^2*d^4*g^
3*x^2 + 4*B^2*a^3*b*d^4*g^3*x + B^2*a^4*d^4*g^3)*log(b*x + a)^2 + 3*(B^2*b^
4*d^4*g^3*x^4 + 4*B^2*a*b^3*d^4*g^3*x^3 + 6*B^2*a^2*b^2*d^4*g^3*x^2 + 4*B^2
*a^3*b*d^4*g^3*x - (b^4*c^4*g^3 - 4*a*b^3*c^3*d*g^3 + 6*a^2*b^2*c^2*d^2*g^3
 - 4*a^3*b*c*d^3*g^3)*B^2)*log(d*x + c)^2 + (6*B^2*b^4*d^4*g^3*x^4*log(e) -
 2*(b^4*c*d^3*g^3 - (12*g^3*log(e) + g^3)*a*b^3*d^4)*B^2*x^3 + 3*(b^4*c^2*d
^2*g^3 - 4*a*b^3*c*d^3*g^3 + 3*(4*g^3*log(e) + g^3)*a^2*b^2*d^4)*B^2*x^2 -
6*(b^4*c^3*d*g^3 - 4*a*b^3*c^2*d^2*g^3 + 6*a^2*b^2*c*d^3*g^3 - (4*g^3*log(e
) + 3*g^3)*a^3*b*d^4)*B^2*x - (6*a*b^3*c^3*d*g^3 - 21*a^2*b^2*c^2*d^2*g^3 +
 26*a^3*b*c*d^3*g^3 - (6*g^3*log(e) + 11*g^3)*a^4*d^4)*B^2)*log(b*x + a) -
(6*B^2*b^4*d^4*g^3*x^4*log(e) - 2*(b^4*c*d^3*g^3 - (12*g^3*log(e) + g^3)*a*
b^3*d^4)*B^2*x^3 + 3*(b^4*c^2*d^2*g^3 - 4*a*b^3*c*d^3*g^3 + 3*(4*g^3*log(e)
 + g^3)*a^2*b^2*d^4)*B^2*x^2 - 6*(b^4*c^3*d*g^3 - 4*a*b^3*c^2*d^2*g^3 + 6*a
```

```
^2*b^2*c*d^3*g^3 - (4*g^3*log(e) + 3*g^3)*a^3*b*d^4)*B^2*x + 6*(B^2*b^4*d^4
*g^3*x^4 + 4*B^2*a*b^3*d^4*g^3*x^3 + 6*B^2*a^2*b^2*d^4*g^3*x^2 + 4*B^2*a^3*
b*d^4*g^3*x + B^2*a^4*d^4*g^3)*log(b*x + a))*log(d*x + c))/(b*d^4)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.00

$$\int (a\,g+b\,g\,x)^3\,\left(A+B\,\ln\left(\frac{e\,(a+b\,x)}{c+d\,x}\right)\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*g + b*g*x)^3*(A + B*log((e*(a + b*x))/(c + d*x)))^2,x)
```

```
[Out] int((a*g + b*g*x)^3*(A + B*log((e*(a + b*x))/(c + d*x)))^2, x)
```

**sympy [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*g*x+a*g)**3*(A+B*ln(e*(b*x+a)/(d*x+c)))**2,x)
```

```
[Out] Timed out
```

## 3.99 $\int (ag+bgx)^2 \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2\,dx$

**Optimal**. Leaf size=253

$$\frac{Bg^2(bc-ad)^3\log\left(\frac{bc-ad}{b(c+dx)}\right)\left(2B\log\left(\frac{e(a+bx)}{c+dx}\right)+2A+3B\right)}{3bd^3}+\frac{Bg^2(a+bx)(bc-ad)^2\left(2B\log\left(\frac{e(a+bx)}{c+dx}\right)+2A+B\right)}{3bd^2}-\frac{Bg}{}$$

```
[Out] -1/3*B*(-a*d+b*c)*g^2*(b*x+a)^2*(A+B*ln(e*(b*x+a)/(d*x+c)))/b/d+1/3*g^2*(b*
x+a)^3*(A+B*ln(e*(b*x+a)/(d*x+c)))^2/b+1/3*B*(-a*d+b*c)^2*g^2*(b*x+a)*(2*A+
B+2*B*ln(e*(b*x+a)/(d*x+c)))/b/d^2+1/3*B*(-a*d+b*c)^3*g^2*ln((-a*d+b*c)/b/(
d*x+c))*(2*A+3*B+2*B*ln(e*(b*x+a)/(d*x+c)))/b/d^3+2/3*B^2*(-a*d+b*c)^3*g^2*
polylog(2,d*(b*x+a)/b/(d*x+c))/b/d^3
```

**Rubi [A]** time = 0.55, antiderivative size = 389, normalized size of antiderivative = 1.54, number of steps used = 20, number of rules used = 13, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.406, Rules used = {2525, 12, 2528, 2486, 31, 43, 2524, 2418, 2394, 2393, 2391, 2390, 2301}

$$\frac{2B^2g^2(bc-ad)^3\text{PolyLog}\left(2,\frac{b(c+dx)}{bc-ad}\right)}{3bd^3}-\frac{2Bg^2(bc-ad)^3\log(c+dx)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{3bd^3}+\frac{2ABg^2x(bc-ad)^2}{3d^2}-\frac{Bg^2}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a*g + b*g*x)^2*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2,x]
```

```
[Out] (2*A*B*(b*c - a*d)^2*g^2*x)/(3*d^2) + (B^2*(b*c - a*d)^2*g^2*x)/(3*d^2) + (
2*B^2*(b*c - a*d)^2*g^2*(a + b*x)*Log[(e*(a + b*x))/(c + d*x)])/(3*b*d^2) -
 (B*(b*c - a*d)*g^2*(a + b*x)^2*(A + B*Log[(e*(a + b*x))/(c + d*x)]))/(3*b*
d) + (g^2*(a + b*x)^3*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2)/(3*b) - (B^2*
(b*c - a*d)^3*g^2*Log[c + d*x])/(b*d^3) + (2*B^2*(b*c - a*d)^3*g^2*Log[-((d
*(a + b*x))/(b*c - a*d))]*Log[c + d*x])/(3*b*d^3) - (2*B*(b*c - a*d)^3*g^2*
(A + B*Log[(e*(a + b*x))/(c + d*x)])*Log[c + d*x])/(3*b*d^3) - (B^2*(b*c -
a*d)^3*g^2*Log[c + d*x]^2)/(3*b*d^3) + (2*B^2*(b*c - a*d)^3*g^2*PolyLog[2,
(b*(c + d*x))/(b*c - a*d)])/(3*b*d^3)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2301

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

Rule 2390

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_))^(q_.), x_Symbol] :> Dist[1/e, Subst[Int[((f*x)/d)^q*(a + b*Log[c*x^
n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && E
qQ[e*f - d*g, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2393

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + (c*e*x)/g])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2394

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d + e*x
)^n]))/g, x] - Dist[(b*e*n)/g, Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2418

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*(RFx_), x_Sy
mbol] :> With[{u = ExpandIntegrand[(a + b*Log[c*(d + e*x)^n])^p, RFx, x]},
```

```
Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d, e, n}, x] && RationalFunctionQ[
RFx, x] && IntegerQ[p]
```

Rule 2486

```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))
^(r_.)]^(s_.), x_Symbol] :> Simp[((a + b*x)*Log[e*(f*(a + b*x)^p*(c + d*x)^
q)^r]^s)/b, x] + Dist[(q*r*s*(b*c - a*d))/b, Int[Log[e*(f*(a + b*x)^p*(c +
d*x)^q)^r]^(s - 1)/(c + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, p, q, r, s
}, x] && NeQ[b*c - a*d, 0] && EqQ[p + q, 0] && IGtQ[s, 0]
```

Rule 2524

```
Int[((a_.) + Log[(c_.)*(RFx_)^(p_.)]*(b_.))^(n_.)/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[(Log[d + e*x]*(a + b*Log[c*RFx^p])^n)/e, x] - Dist[(b*n*p)/e
, Int[(Log[d + e*x]*(a + b*Log[c*RFx^p])^(n - 1)*D[RFx, x])/RFx, x], x] /;
FreeQ[{a, b, c, d, e, p}, x] && RationalFunctionQ[RFx, x] && IGtQ[n, 0]
```

Rule 2525

```
Int[((a_.) + Log[(c_.)*(RFx_)^(p_.)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_))^(m_.
), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(a + b*Log[c*RFx^p])^n)/(e*(m + 1))
, x] - Dist[(b*n*p)/(e*(m + 1)), Int[SimplifyIntegrand[((d + e*x)^(m + 1)*(
a + b*Log[c*RFx^p])^(n - 1)*D[RFx, x])/RFx, x], x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && RationalFunctionQ[RFx, x] && IGtQ[n, 0] && (EqQ[n, 1] ||
IntegerQ[m]) && NeQ[m, -1]
```

Rule 2528

```
Int[((a_.) + Log[(c_.)*(RFx_)^(p_.)]*(b_.))^(n_.)*(RGx_), x_Symbol] :> With
[{u = ExpandIntegrand[(a + b*Log[c*RFx^p])^n, RGx, x]}, Int[u, x] /; SumQ[u
]] /; FreeQ[{a, b, c, p}, x] && RationalFunctionQ[RFx, x] && RationalFuncti
onQ[RGx, x] && IGtQ[n, 0]
```

Rubi steps

$$\int (ag+bgx)^2 \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2 dx = \frac{g^2(a+bx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{3b} - \frac{(2B)\int \frac{(bc-ad)g^3(a+bx)^2\left(A+B\log\right.}{c+dx}}{3bg}$$

$$= \frac{g^2(a+bx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{3b} - \frac{\left(2B(bc-ad)g^2\right)\int \frac{(a+bx)^2\left(A+\right.}{c}}{3b}$$

$$= \frac{g^2(a+bx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{3b} - \frac{\left(2B(bc-ad)g^2\right)\int\left(-\frac{b(bc-ad)}{}\right.}{}$$

$$= \frac{g^2(a+bx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{3b} - \frac{\left(2B(bc-ad)g^2\right)\int(a+bx)\left(\right.}{3d}$$

$$= \frac{2AB(bc-ad)^2g^2x}{3d^2} - \frac{B(bc-ad)g^2(a+bx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{3bd}$$

$$= \frac{2AB(bc-ad)^2g^2x}{3d^2} + \frac{2B^2(bc-ad)^2g^2(a+bx)\log\left(\frac{e(a+bx)}{c+dx}\right)}{3bd^2} - \frac{B(b}{}$$

$$= \frac{2AB(bc-ad)^2g^2x}{3d^2} + \frac{2B^2(bc-ad)^2g^2(a+bx)\log\left(\frac{e(a+bx)}{c+dx}\right)}{3bd^2} - \frac{B(b}{}$$

$$= \frac{2AB(bc-ad)^2g^2x}{3d^2} + \frac{B^2(bc-ad)^2g^2x}{3d^2} + \frac{2B^2(bc-ad)^2g^2(a+bx)}{3bd^2}$$

$$= \frac{2AB(bc-ad)^2g^2x}{3d^2} + \frac{B^2(bc-ad)^2g^2x}{3d^2} + \frac{2B^2(bc-ad)^2g^2(a+bx)}{3bd^2}$$

$$= \frac{2AB(bc-ad)^2g^2x}{3d^2} + \frac{B^2(bc-ad)^2g^2x}{3d^2} + \frac{2B^2(bc-ad)^2g^2(a+bx)}{3bd^2}$$

$$= \frac{2AB(bc-ad)^2g^2x}{3d^2} + \frac{B^2(bc-ad)^2g^2x}{3d^2} + \frac{2B^2(bc-ad)^2g^2(a+bx)}{3bd^2}$$

**Mathematica [A]** time = 0.29, size = 287, normalized size = 1.13

$$g^2\left(\frac{B(bc-ad)\left(-d^2(a+bx)^2\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)-2(bc-ad)^2\log(c+dx)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)+2Abdx(bc-ad)+2Bd(a+bx)(bc-ad)\log\left(\frac{e(a+bx)}{c+dx}\right)+B(bc-ad)^2\right.}{d^3}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*g + b*g*x)^2*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2,x]
```

```
[Out] (g^2*((a + b*x)^3*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2 + (B*(b*c - a*d)*(
2*A*b*d*(b*c - a*d)*x + 2*B*d*(b*c - a*d)*(a + b*x)*Log[(e*(a + b*x))/(c +
d*x)] - d^2*(a + b*x)^2*(A + B*Log[(e*(a + b*x))/(c + d*x)]) - 2*B*(b*c - a
*d)^2*Log[c + d*x] - 2*(b*c - a*d)^2*(A + B*Log[(e*(a + b*x))/(c + d*x)])*L
og[c + d*x] + B*(b*c - a*d)*(b*d*x + (-(b*c) + a*d)*Log[c + d*x]) + B*(b*c
- a*d)^2*((2*Log[(d*(a + b*x))/(-(b*c) + a*d)] - Log[c + d*x])*Log[c + d*x]
 + 2*PolyLog[2, (b*(c + d*x))/(b*c - a*d)])))/d^3))/(3*b)
```

**fricas [F]** time = 0.96, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(A^2b^2g^2x^2 + 2\,A^2abg^2x + A^2a^2g^2 + \left(B^2b^2g^2x^2 + 2\,B^2abg^2x + B^2a^2g^2\right)\log\left(\frac{bex+ae}{dx+c}\right)^2 + 2\left(ABb^2g^2x^2 + \right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*g*x+a*g)^2*(A+B*log(e*(b*x+a)/(d*x+c)))^2,x, algorithm="fricas
")
```

```
[Out] integral(A^2*b^2*g^2*x^2 + 2*A^2*a*b*g^2*x + A^2*a^2*g^2 + (B^2*b^2*g^2*x^2
 + 2*B^2*a*b*g^2*x + B^2*a^2*g^2)*log((b*e*x + a*e)/(d*x + c))^2 + 2*(A*B*b
^2*g^2*x^2 + 2*A*B*a*b*g^2*x + A*B*a^2*g^2)*log((b*e*x + a*e)/(d*x + c)), x
)
```

**giac [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*g*x+a*g)^2*(A+B*log(e*(b*x+a)/(d*x+c)))^2,x, algorithm="giac")
```

```
[Out] Timed out
```

**maple [F]** time = 1.90, size = 0, normalized size = 0.00

$$\int \left(bgx+ag\right)^2 \left(B\ln\left(\frac{(bx+a)\,e}{dx+c}\right)+A\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*g*x+a*g)^2*(B*ln((b*x+a)/(d*x+c)*e)+A)^2,x)
```

```
[Out] int((b*g*x+a*g)^2*(B*ln((b*x+a)/(d*x+c)*e)+A)^2,x)
```

**maxima [B]** time = 2.33, size = 1165, normalized size = 4.60

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*g*x+a*g)^2*(A+B*log(e*(b*x+a)/(d*x+c)))^2,x, algorithm="maxima
")
```

```
[Out] 1/3*A^2*b^2*g^2*x^3 + A^2*a*b*g^2*x^2 + 2*(x*log(b*e*x/(d*x + c) + a*e/(d*x
 + c)) + a*log(b*x + a)/b - c*log(d*x + c)/d)*A*B*a^2*g^2 + 2*(x^2*log(b*e*
x/(d*x + c) + a*e/(d*x + c)) - a^2*log(b*x + a)/b^2 + c^2*log(d*x + c)/d^2
- (b*c - a*d)*x/(b*d))*A*B*a*b*g^2 + 1/3*(2*x^3*log(b*e*x/(d*x + c) + a*e/(
d*x + c)) + 2*a^3*log(b*x + a)/b^3 - 2*c^3*log(d*x + c)/d^3 - ((b^2*c*d - a
*b*d^2)*x^2 - 2*(b^2*c^2 - a^2*d^2)*x)/(b^2*d^2))*A*B*b^2*g^2 + A^2*a^2*g^2
*x - 1/3*((2*g^2*log(e) + 3*g^2)*b^2*c^3 - (6*g^2*log(e) + 7*g^2)*a*b*c^2*d
 + 2*(3*g^2*log(e) + 2*g^2)*a^2*c*d^2)*B^2*log(d*x + c)/d^3 - 2/3*(b^3*c^3*
g^2 - 3*a*b^2*c^2*d*g^2 + 3*a^2*b*c*d^2*g^2 - a^3*d^3*g^2)*(log(b*x + a)*lo
g((b*d*x + a*d)/(b*c - a*d) + 1) + dilog(-(b*d*x + a*d)/(b*c - a*d)))*B^2/(
b*d^3) + 1/3*(B^2*b^3*d^3*g^2*x^3*log(e)^2 - (b^3*c*d^2*g^2*log(e) - (3*g^2
*log(e)^2 + g^2*log(e))*a*b^2*d^3)*B^2*x^2 + ((2*g^2*log(e) + g^2)*b^3*c^2*
d - 2*(3*g^2*log(e) + g^2)*a*b^2*c*d^2 + (3*g^2*log(e)^2 + 4*g^2*log(e) + g
^2)*a^2*b*d^3)*B^2*x + (B^2*b^3*d^3*g^2*x^3 + 3*B^2*a*b^2*d^3*g^2*x^2 + 3*B
^2*a^2*b*d^3*g^2*x + B^2*a^3*d^3*g^2)*log(b*x + a)^2 + (B^2*b^3*d^3*g^2*x^3
 + 3*B^2*a*b^2*d^3*g^2*x^2 + 3*B^2*a^2*b*d^3*g^2*x + (b^3*c^3*g^2 - 3*a*b^2
*c^2*d*g^2 + 3*a^2*b*c*d^2*g^2)*B^2)*log(d*x + c)^2 + (2*B^2*b^3*d^3*g^2*x^
3*log(e) - (b^3*c*d^2*g^2 - (6*g^2*log(e) + g^2)*a*b^2*d^3)*B^2*x^2 + 2*(b^
3*c^2*d*g^2 - 3*a*b^2*c*d^2*g^2 + (3*g^2*log(e) + 2*g^2)*a^2*b*d^3)*B^2*x +
 (2*a*b^2*c^2*d*g^2 - 5*a^2*b*c*d^2*g^2 + (2*g^2*log(e) + 3*g^2)*a^3*d^3)*B
^2)*log(b*x + a) - (2*B^2*b^3*d^3*g^2*x^3*log(e) - (b^3*c*d^2*g^2 - (6*g^2*
log(e) + g^2)*a*b^2*d^3)*B^2*x^2 + 2*(b^3*c^2*d*g^2 - 3*a*b^2*c*d^2*g^2 + (
3*g^2*log(e) + 2*g^2)*a^2*b*d^3)*B^2*x + 2*(B^2*b^3*d^3*g^2*x^3 + 3*B^2*a*b
^2*d^3*g^2*x^2 + 3*B^2*a^2*b*d^3*g^2*x + B^2*a^3*d^3*g^2)*log(b*x + a))*log
(d*x + c))/(b*d^3)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.00

$$\int \left(a\,g+b\,g\,x\right)^2\,{\left(A+B\,\ln\left(\frac{e\,\left(a+b\,x\right)}{c+d\,x}\right)\right)}^2 \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*g + b*g*x)^2*(A + B*log((e*(a + b*x))/(c + d*x)))^2,x)
```

```
[Out] int((a*g + b*g*x)^2*(A + B*log((e*(a + b*x))/(c + d*x)))^2, x)
```

**sympy [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*g*x+a*g)**2*(A+B*ln(e*(b*x+a)/(d*x+c)))**2,x)
```

```
[Out] Timed out
```

# 3.100 $\int (ag+bgx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2\,dx$

**Optimal**. Leaf size=180

$$-\frac{Bg(bc-ad)^2\log\left(\frac{bc-ad}{b(c+dx)}\right)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A+B\right)}{bd^2}-\frac{Bg(a+bx)(bc-ad)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{bd}+\frac{g(a+bx)^2\left(B\right.}{}$$

[Out] -B*(-a*d+b*c)*g*(b*x+a)*(A+B*ln(e*(b*x+a)/(d*x+c)))/b/d+1/2*g*(b*x+a)^2*(A+B*ln(e*(b*x+a)/(d*x+c)))^2/b-B*(-a*d+b*c)^2*g*ln((-a*d+b*c)/b/(d*x+c))*(A+B+B*ln(e*(b*x+a)/(d*x+c)))/b/d^2-B^2*(-a*d+b*c)^2*g*polylog(2,d*(b*x+a)/b/(d*x+c))/b/d^2

**Rubi** [A] time = 0.46, antiderivative size = 285, normalized size of antiderivative = 1.58, number of steps used = 16, number of rules used = 12, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {2525, 12, 2528, 2486, 31, 2524, 2418, 2394, 2393, 2391, 2390, 2301}

$$-\frac{B^2g(bc-ad)^2\text{PolyLog}\left(2,\frac{b(c+dx)}{bc-ad}\right)}{bd^2}+\frac{Bg(bc-ad)^2\log(c+dx)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{bd^2}+\frac{g(a+bx)^2\left(B\log\left(\frac{e(a+bx)}{c+dx}\right.\right.}{2b}$$

Antiderivative was successfully verified.

[In] Int[(a*g + b*g*x)*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2,x]

[Out] -((A*B*(b*c - a*d)*g*x)/d) - (B^2*(b*c - a*d)*g*(a + b*x)*Log[(e*(a + b*x))/(c + d*x)])/(b*d) + (g*(a + b*x)^2*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2)/(2*b) + (B^2*(b*c - a*d)^2*g*Log[c + d*x])/(b*d^2) - (B^2*(b*c - a*d)^2*g*Log[-((d*(a + b*x))/(b*c - a*d))]*Log[c + d*x])/(b*d^2) + (B*(b*c - a*d)^2*g*(A + B*Log[(e*(a + b*x))/(c + d*x)])*Log[c + d*x])/(b*d^2) + (B^2*(b*c - a*d)^2*g*Log[c + d*x]^2)/(2*b*d^2) - (B^2*(b*c - a*d)^2*g*PolyLog[2, (b*(c + d*x))/(b*c - a*d)])/(b*d^2)

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 2301

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

Rule 2390

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_))^(q_.), x_Symbol] :> Dist[1/e, Subst[Int[((f*x)/d)^q*(a + b*Log[c*x^
n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && E
qQ[e*f - d*g, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2393

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + (c*e*x)/g])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2394

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d + e*x
)^n]))/g, x] - Dist[(b*e*n)/g, Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2418

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*(RFx_), x_Sy
mbol] :> With[{u = ExpandIntegrand[(a + b*Log[c*(d + e*x)^n])^p, RFx, x]},
Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d, e, n}, x] && RationalFunctionQ[
RFx, x] && IntegerQ[p]
```

Rule 2486

```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))
^(r_.)]^(s_.), x_Symbol] :> Simp[((a + b*x)*Log[e*(f*(a + b*x)^p*(c + d*x)^
q)^r]^s)/b, x] + Dist[(q*r*s*(b*c - a*d))/b, Int[Log[e*(f*(a + b*x)^p*(c +
d*x)^q)^r]^(s - 1)/(c + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, p, q, r, s
}, x] && NeQ[b*c - a*d, 0] && EqQ[p + q, 0] && IGtQ[s, 0]
```

Rule 2524

```
Int[((a_.) + Log[(c_.)*(RFx_)^(p_.)]*(b_.))^(n_.)/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[(Log[d + e*x]*(a + b*Log[c*RFx^p])^n)/e, x] - Dist[(b*n*p)/e
, Int[(Log[d + e*x]*(a + b*Log[c*RFx^p])^(n - 1)*D[RFx, x])/RFx, x], x] /;
FreeQ[{a, b, c, d, e, p}, x] && RationalFunctionQ[RFx, x] && IGtQ[n, 0]
```

Rule 2525

```
Int[((a_.) + Log[(c_.)*(RFx_)^(p_.)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_))^(m_.
), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(a + b*Log[c*RFx^p])^n)/(e*(m + 1))
, x] - Dist[(b*n*p)/(e*(m + 1)), Int[SimplifyIntegrand[((d + e*x)^(m + 1)*(
a + b*Log[c*RFx^p])^(n - 1)*D[RFx, x])/RFx, x], x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && RationalFunctionQ[RFx, x] && IGtQ[n, 0] && (EqQ[n, 1] ||
IntegerQ[m]) && NeQ[m, -1]
```

Rule 2528

```
Int[((a_.) + Log[(c_.)*(RFx_)^(p_.)]*(b_.))^(n_.)*(RGx_), x_Symbol] :> With
[{u = ExpandIntegrand[(a + b*Log[c*RFx^p])^n, RGx, x]}, Int[u, x] /; SumQ[u
]] /; FreeQ[{a, b, c, p}, x] && RationalFunctionQ[RFx, x] && RationalFuncti
onQ[RGx, x] && IGtQ[n, 0]
```

Rubi steps

$$\int (ag+bgx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2 dx = \frac{g(a+bx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{2b} - \frac{B\int \frac{(bc-ad)g^2(a+bx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{c+dx}}{bg}$$

$$= \frac{g(a+bx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{2b} - \frac{(B(bc-ad)g)\int \frac{(a+bx)\left(A+B\log\left(\frac{e(a}{c}\right.\right.}{c+dx}}{b}$$

$$= \frac{g(a+bx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{2b} - \frac{(B(bc-ad)g)\int\left(\frac{b\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right.}{d}\right.}{b}$$

$$= \frac{g(a+bx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{2b} - \frac{(B(bc-ad)g)\int\left(A+B\log\left(\frac{e(a}{c+}\right.\right.}{d}$$

$$= -\frac{AB(bc-ad)gx}{d} + \frac{g(a+bx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{2b} + \frac{B(bc-ad)^2g}{}$$

$$= -\frac{AB(bc-ad)gx}{d} - \frac{B^2(bc-ad)g(a+bx)\log\left(\frac{e(a+bx)}{c+dx}\right)}{bd} + \frac{g(a+bx)^2\left(\right.}{}$$

$$= -\frac{AB(bc-ad)gx}{d} - \frac{B^2(bc-ad)g(a+bx)\log\left(\frac{e(a+bx)}{c+dx}\right)}{bd} + \frac{g(a+bx)^2\left(\right.}{}$$

$$= -\frac{AB(bc-ad)gx}{d} - \frac{B^2(bc-ad)g(a+bx)\log\left(\frac{e(a+bx)}{c+dx}\right)}{bd} + \frac{g(a+bx)^2\left(\right.}{}$$

$$= -\frac{AB(bc-ad)gx}{d} - \frac{B^2(bc-ad)g(a+bx)\log\left(\frac{e(a+bx)}{c+dx}\right)}{bd} + \frac{g(a+bx)^2\left(\right.}{}$$

$$= -\frac{AB(bc-ad)gx}{d} - \frac{B^2(bc-ad)g(a+bx)\log\left(\frac{e(a+bx)}{c+dx}\right)}{bd} + \frac{g(a+bx)^2\left(\right.}{}$$

$$= -\frac{AB(bc-ad)gx}{d} - \frac{B^2(bc-ad)g(a+bx)\log\left(\frac{e(a+bx)}{c+dx}\right)}{bd} + \frac{g(a+bx)^2\left(\right.}{}$$

**Mathematica** [A] time = 0.17, size = 203, normalized size = 1.13

$$\frac{g\left((a+bx)^2\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2 - \frac{B(bc-ad)\left(-2(bc-ad)\log(c+dx)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)+2Bd(a+bx)\log\left(\frac{e(a+bx)}{c+dx}\right)+B(bc-ad)\left(2\text{Li}_2\left(\frac{b(c+dx)}{bc-ad}\right.\right.}{d^2}\right.}{2b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*g + b*g*x)*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2,x]
```

```
[Out] (g*((a + b*x)^2*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2 - (B*(b*c - a*d)*(2*
A*b*d*x + 2*B*d*(a + b*x)*Log[(e*(a + b*x))/(c + d*x)] - 2*B*(b*c - a*d)*Lo
g[c + d*x] - 2*(b*c - a*d)*(A + B*Log[(e*(a + b*x))/(c + d*x)])*Log[c + d*x
] + B*(b*c - a*d)*((2*Log[(d*(a + b*x))/(-(b*c) + a*d)] - Log[c + d*x])*Log
[c + d*x] + 2*PolyLog[2, (b*(c + d*x))/(b*c - a*d)])))/d^2))/(2*b)
```

**fricas [F]** time = 0.88, size = 0, normalized size = 0.00

$$\text{integral}\left(A^2bgx + A^2ag + \left(B^2bgx + B^2ag\right)\log\left(\frac{bex + ae}{dx + c}\right)^2 + 2\left(ABbgx + ABag\right)\log\left(\frac{bex + ae}{dx + c}\right), x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*g*x+a*g)*(A+B*log(e*(b*x+a)/(d*x+c)))^2,x, algorithm="fricas")
```

```
[Out] integral(A^2*b*g*x + A^2*a*g + (B^2*b*g*x + B^2*a*g)*log((b*e*x + a*e)/(d*x
 + c))^2 + 2*(A*B*b*g*x + A*B*a*g)*log((b*e*x + a*e)/(d*x + c)), x)
```

**giac [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*g*x+a*g)*(A+B*log(e*(b*x+a)/(d*x+c)))^2,x, algorithm="giac")
```

```
[Out] Timed out
```

**maple [F]** time = 1.62, size = 0, normalized size = 0.00

$$\int \left(bgx + ag\right)\left(B\ln\left(\frac{(bx + a)e}{dx + c}\right) + A\right)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*g*x+a*g)*(B*ln((b*x+a)/(d*x+c)*e)+A)^2,x)
```

```
[Out] int((b*g*x+a*g)*(B*ln((b*x+a)/(d*x+c)*e)+A)^2,x)
```

**maxima [B]** time = 2.21, size = 611, normalized size = 3.39

$$\frac{1}{2}A^2bgx^2 + 2\left(x\log\left(\frac{bex}{dx + c} + \frac{ae}{dx + c}\right) + \frac{a\log(bx + a)}{b} - \frac{c\log(dx + c)}{d}\right)ABag + \left(x^2\log\left(\frac{bex}{dx + c} + \frac{ae}{dx + c}\right) - \frac{a^2}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*g*x+a*g)*(A+B*log(e*(b*x+a)/(d*x+c)))^2,x, algorithm="maxima")
```

```
[Out] 1/2*A^2*b*g*x^2 + 2*(x*log(b*e*x/(d*x + c) + a*e/(d*x + c)) + a*log(b*x + a
)/b - c*log(d*x + c)/d)*A*B*a*g + (x^2*log(b*e*x/(d*x + c) + a*e/(d*x + c))
 - a^2*log(b*x + a)/b^2 + c^2*log(d*x + c)/d^2 - (b*c - a*d)*x/(b*d))*A*B*b
*g + A^2*a*g*x + ((g*log(e) + g)*b*c^2 - (2*g*log(e) + g)*a*c*d)*B^2*log(d*
x + c)/d^2 + (b^2*c^2*g - 2*a*b*c*d*g + a^2*d^2*g)*(log(b*x + a)*log((b*d*x
 + a*d)/(b*c - a*d) + 1) + dilog(-(b*d*x + a*d)/(b*c - a*d)))*B^2/(b*d^2) +
 1/2*(B^2*b^2*d^2*g*x^2*log(e)^2 - 2*(b^2*c*d*g*log(e) - (g*log(e)^2 + g*lo
g(e))*a*b*d^2)*B^2*x + (B^2*b^2*d^2*g*x^2 + 2*B^2*a*b*d^2*g*x + B^2*a^2*d^2
*g)*log(b*x + a)^2 + (B^2*b^2*d^2*g*x^2 + 2*B^2*a*b*d^2*g*x - (b^2*c^2*g -
2*a*b*c*d*g)*B^2)*log(d*x + c)^2 + 2*(B^2*b^2*d^2*g*x^2*log(e) + ((2*g*log(
e) + g)*a*b*d^2 - b^2*c*d*g)*B^2*x + ((g*log(e) + g)*a^2*d^2 - a*b*c*d*g)*B
^2)*log(b*x + a) - 2*(B^2*b^2*d^2*g*x^2*log(e) + ((2*g*log(e) + g)*a*b*d^2
- b^2*c*d*g)*B^2*x + (B^2*b^2*d^2*g*x^2 + 2*B^2*a*b*d^2*g*x + B^2*a^2*d^2*g
)*log(b*x + a))*log(d*x + c))/(b*d^2)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.01

$$\int \left(a\,g+b\,g\,x\right)\,{\left(A+B\,\ln\left(\frac{e\,\left(a+b\,x\right)}{c+d\,x}\right)\right)}^2 \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*g + b*g*x)*(A + B*log((e*(a + b*x))/(c + d*x)))^2,x)
```

```
[Out] int((a*g + b*g*x)*(A + B*log((e*(a + b*x))/(c + d*x)))^2, x)
```

**sympy [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*g*x+a*g)*(A+B*ln(e*(b*x+a)/(d*x+c)))**2,x)
```

```
[Out] Timed out
```

# 3.101 $\int \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{ag+bgx}\,dx$

**Optimal**. Leaf size=128

$$\frac{2B\text{Li}_2\left(\frac{b(c+dx)}{d(a+bx)}\right)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{bg}-\frac{\log\left(1-\frac{b(c+dx)}{d(a+bx)}\right)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2}{bg}+\frac{2B^2\text{Li}_3\left(\frac{b(c+dx)}{d(a+bx)}\right)}{bg}$$

```
[Out] -(A+B*ln(e*(b*x+a)/(d*x+c)))^2*ln(1-b*(d*x+c)/d/(b*x+a))/b/g+2*B*(A+B*ln(e*
(b*x+a)/(d*x+c)))*polylog(2,b*(d*x+c)/d/(b*x+a))/b/g+2*B^2*polylog(3,b*(d*x
+c)/d/(b*x+a))/b/g
```

**Rubi [B]** time = 3.42, antiderivative size = 728, normalized size of antiderivative = 5.69, number of steps used = 46, number of rules used = 23, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.719, Rules used = {2524, 12, 2528, 2418, 2390, 2301, 2394, 2393, 2391, 6688, 6742, 2500, 2440, 2434, 2433, 2375, 2317, 2374, 6589, 2499, 2302, 30, 2396}

$$\frac{2AB\text{PolyLog}\left(2,-\frac{d(a+bx)}{bc-ad}\right)}{bg}-\frac{2B^2\text{PolyLog}\left(2,-\frac{d(a+bx)}{bc-ad}\right)\left(-\log\left(\frac{e(a+bx)}{c+dx}\right)+\log(a+bx)+\log\left(\frac{1}{c+dx}\right)\right)}{bg}-\frac{2B^2\text{PolyLo}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Log[(e*(a + b*x))/(c + d*x)])^2/(a*g + b*g*x),x]
```

```
[Out] -((A*B*Log[g*(a + b*x)]^2)/(b*g)) + (B^2*Log[g*(a + b*x)]^3)/(3*b*g) - (B^2
*Log[a + b*x]^2*Log[-c - d*x])/(b*g) + (2*B^2*Log[a + b*x]*Log[g*(a + b*x)]
*Log[-c - d*x])/(b*g) - (B^2*Log[g*(a + b*x)]^2*Log[-c - d*x])/(b*g) + (B^2
*Log[-((d*(a + b*x))/(b*c - a*d))]*Log[(c + d*x)^(-1)]^2)/(b*g) - (B^2*Log[
g*(a + b*x)]*Log[(c + d*x)^(-1)]^2)/(b*g) + (B^2*Log[a + b*x]^2*Log[(b*(c +
 d*x))/(b*c - a*d)])/(b*g) + (B^2*Log[g*(a + b*x)]^2*Log[(b*(c + d*x))/(b*c
 - a*d)])/(b*g) + ((A + B*Log[(e*(a + b*x))/(c + d*x)])^2*Log[a*g + b*g*x])
/(b*g) + (2*A*B*Log[(b*(c + d*x))/(b*c - a*d)]*Log[a*g + b*g*x])/(b*g) - (2
*B^2*(Log[a + b*x] + Log[(c + d*x)^(-1)] - Log[(e*(a + b*x))/(c + d*x)])*Lo
g[(b*(c + d*x))/(b*c - a*d)]*Log[a*g + b*g*x])/(b*g) - (B^2*Log[(e*(a + b*x
))/(c + d*x)]*Log[a*g + b*g*x]^2)/(b*g) - (B^2*Log[(b*(c + d*x))/(b*c - a*d
)]*Log[a*g + b*g*x]^2)/(b*g) + (2*A*B*PolyLog[2, -((d*(a + b*x))/(b*c - a*d
))])/(b*g) + (2*B^2*Log[a + b*x]*PolyLog[2, -((d*(a + b*x))/(b*c - a*d))])/
(b*g) - (2*B^2*(Log[a + b*x] + Log[(c + d*x)^(-1)] - Log[(e*(a + b*x))/(c +
 d*x)])*PolyLog[2, -((d*(a + b*x))/(b*c - a*d))])/(b*g) - (2*B^2*Log[(c + d
*x)^(-1)]*PolyLog[2, (b*(c + d*x))/(b*c - a*d)])/(b*g) - (2*B^2*PolyLog[3,
-((d*(a + b*x))/(b*c - a*d))])/(b*g) - (2*B^2*PolyLog[3, (b*(c + d*x))/(b*c
 - a*d)])/(b*g)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2301

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

Rule 2302

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/(x_), x_Symbol] :> Dist[1/(
b*n), Subst[Int[x^p, x], x, a + b*Log[c*x^n]], x] /; FreeQ[{a, b, c, n, p},
 x]
```

Rule 2317

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symb
ol] :> Simp[(Log[1 + (e*x)/d]*(a + b*Log[c*x^n])^p)/e, x] - Dist[(b*n*p)/e,
 Int[(Log[1 + (e*x)/d]*(a + b*Log[c*x^n])^(p - 1))/x, x], x] /; FreeQ[{a, b
, c, d, e, n}, x] && IGtQ[p, 0]
```

Rule 2374

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> -Simp[(PolyLog[2, -(d*f*x^m)]*(a + b*Log[c*x
^n])^p)/m, x] + Dist[(b*n*p)/m, Int[(PolyLog[2, -(d*f*x^m)]*(a + b*Log[c*x^
n])^(p - 1))/x, x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p, 0]
&& EqQ[d*e, 1]
```

Rule 2375

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))^(r_.)]*((a_.) + Log[(c_.)*(x_)^(n_
.)]*(b_.))^(p_.))/(x_), x_Symbol] :> Simp[(Log[d*(e + f*x^m)^r]*(a + b*Log[
c*x^n])^(p + 1))/(b*n*(p + 1)), x] - Dist[(f*m*r)/(b*n*(p + 1)), Int[(x^(m
- 1)*(a + b*Log[c*x^n])^(p + 1))/(e + f*x^m), x], x] /; FreeQ[{a, b, c, d,
e, f, r, m, n}, x] && IGtQ[p, 0] && NeQ[d*e, 1]
```

Rule 2390

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_))^(q_.), x_Symbol] :> Dist[1/e, Subst[Int[((f*x)/d)^q*(a + b*Log[c*x^
n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && E
qQ[e*f - d*g, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2393

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + (c*e*x)/g])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2394

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d + e*x
)^n]))/g, x] - Dist[(b*e*n)/g, Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2396

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)/((f_.) + (g_.
)*(x_)), x_Symbol] :> Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d
+ e*x)^n])^p)/g, x] - Dist[(b*e*n*p)/g, Int[(Log[(e*(f + g*x))/(e*f - d*g)]
*(a + b*Log[c*(d + e*x)^n])^(p - 1))/(d + e*x), x], x] /; FreeQ[{a, b, c, d
, e, f, g, n, p}, x] && NeQ[e*f - d*g, 0] && IGtQ[p, 1]
```

Rule 2418

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*(RFx_), x_Sy
mbol] :> With[{u = ExpandIntegrand[(a + b*Log[c*(d + e*x)^n])^p, RFx, x]},
Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d, e, n}, x] && RationalFunctionQ[
RFx, x] && IntegerQ[p]
```

Rule 2433

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + Log
[(h_.)*((i_.) + (j_.)*(x_))^(m_.)]*(g_.))*((k_.) + (l_.)*(x_))^(r_.), x_Sym
bol] :> Dist[1/e, Subst[Int[((k*x)/d)^r*(a + b*Log[c*x^n])^p*(f + g*Log[h*(
(e*i - d*j)/e + (j*x)/e)^m]), x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e,
```

```
f, g, h, i, j, k, l, n, p, r}, x] && EqQ[e*k - d*l, 0]
```

Rule 2434

```
Int[(((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + Log[(h_.
)*((i_.) + (j_.)*(x_))^(m_.)]*(g_.)))/(x_), x_Symbol] :> Simp[Log[x]*(a + b
*Log[c*(d + e*x)^n])*(f + g*Log[h*(i + j*x)^m]), x] + (-Dist[e*g*m, Int[(Lo
g[x]*(a + b*Log[c*(d + e*x)^n]))/(d + e*x), x], x] - Dist[b*j*n, Int[(Log[x
]*(f + g*Log[h*(i + j*x)^m]))/(i + j*x), x], x]) /; FreeQ[{a, b, c, d, e, f
, g, h, i, j, m, n}, x] && EqQ[e*i - d*j, 0]
```

Rule 2440

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + Log[(h_.)
*((i_.) + (j_.)*(x_))^(m_.)]*(g_.))*((k_) + (l_.)*(x_))^(r_.), x_Symbol] :>
 Dist[1/l, Subst[Int[x^r*(a + b*Log[c*(-((e*k - d*l)/l) + (e*x)/l)^n])*(f +
 g*Log[h*(-((j*k - i*l)/l) + (j*x)/l)^m]), x], x, k + l*x], x] /; FreeQ[{a,
 b, c, d, e, f, g, h, i, j, k, l, m, n}, x] && IntegerQ[r]
```

Rule 2499

```
Int[(Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.)
)^(r_.)]*((s_.) + Log[(i_.)*((g_.) + (h_.)*(x_))^(n_.)]*(t_.))^(m_.))/((j_.
) + (k_.)*(x_)), x_Symbol] :> Simp[((s + t*Log[i*(g + h*x)^n])^(m + 1)*Log[
e*(f*(a + b*x)^p*(c + d*x)^q)^r])/(k*n*t*(m + 1)), x] + (-Dist[(b*p*r)/(k*n
*t*(m + 1)), Int[(s + t*Log[i*(g + h*x)^n])^(m + 1)/(a + b*x), x], x] - Dis
t[(d*q*r)/(k*n*t*(m + 1)), Int[(s + t*Log[i*(g + h*x)^n])^(m + 1)/(c + d*x)
, x], x]) /; FreeQ[{a, b, c, d, e, f, g, h, i, j, k, s, t, m, n, p, q, r},
x] && NeQ[b*c - a*d, 0] && EqQ[h*j - g*k, 0] && IGtQ[m, 0]
```

Rule 2500

```
Int[(Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.)
)^(r_.)]*((s_.) + Log[(i_.)*((g_.) + (h_.)*(x_))^(n_.)]*(t_.)))/((j_.) + (k
_.)*(x_)), x_Symbol] :> Dist[Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r] - Log[(a
+ b*x)^(p*r)] - Log[(c + d*x)^(q*r)], Int[(s + t*Log[i*(g + h*x)^n])/(j + k
*x), x], x] + (Int[(Log[(a + b*x)^(p*r)]*(s + t*Log[i*(g + h*x)^n]))/(j + k
*x), x] + Int[(Log[(c + d*x)^(q*r)]*(s + t*Log[i*(g + h*x)^n]))/(j + k*x),
x]) /; FreeQ[{a, b, c, d, e, f, g, h, i, j, k, s, t, n, p, q, r}, x] && NeQ
[b*c - a*d, 0]
```

Rule 2524

```
Int[((a_.) + Log[(c_.)*(RFx_)^(p_.)]*(b_.))^(n_.)/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[(Log[d + e*x]*(a + b*Log[c*RFx^p])^n)/e, x] - Dist[(b*n*p)/e
```

```
, Int[(Log[d + e*x]*(a + b*Log[c*RFx^p])^(n - 1)*D[RFx, x])/RFx, x], x] /;
FreeQ[{a, b, c, d, e, p}, x] && RationalFunctionQ[RFx, x] && IGtQ[n, 0]
```

Rule 2528

```
Int[((a_.) + Log[(c_.)*(RFx_)^(p_.)]*(b_.))^(n_.)*(RGx_), x_Symbol] :> With
[{u = ExpandIntegrand[(a + b*Log[c*RFx^p])^n, RGx, x]}, Int[u, x] /; SumQ[u
]] /; FreeQ[{a, b, c, p}, x] && RationalFunctionQ[RFx, x] && RationalFuncti
onQ[RGx, x] && IGtQ[n, 0]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6688

```
Int[u_, x_Symbol] :> With[{v = SimplifyIntegrand[u, x]}, Int[v, x] /; Simpl
erIntegrandQ[v, u, x]]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\int \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{ag+bgx}\,dx = \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2\log(ag+bgx)}{bg} - \frac{(2B)\int \frac{(c+dx)\left(-\frac{de(a+bx)}{(c+dx)^2}+\frac{be}{c+dx}\right)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)\mathrm{l}}{e(a+bx)}}{bg}$$

$$= \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2\log(ag+bgx)}{bg} - \frac{(2B)\int \frac{(c+dx)\left(-\frac{de(a+bx)}{(c+dx)^2}+\frac{be}{c+dx}\right)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)\mathrm{l}}{a+bx}}{beg}$$

$$= \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2\log(ag+bgx)}{bg} - \frac{(2B)\int \frac{(bc-ad)e\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)\log(ag+bgx)}{(a+bx)(c+dx)}\,dx}{beg}$$

$$= \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2\log(ag+bgx)}{bg} - \frac{(2B(bc-ad))\int \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)\log(ag+bgx)}{(a+bx)(c+dx)}}{bg}$$

$$= \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2\log(ag+bgx)}{bg} - \frac{(2B(bc-ad))\int \left(\frac{d\left(-A-B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)\log(ag+bg}{(bc-ad)(c+dx)}\right.}{bg}$$

$$= \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2\log(ag+bgx)}{bg} - \frac{(2B)\int \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)\log(ag+bgx)}{a+bx}\,dx}{g} - \frac{(2B}{}$$

$$= \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2\log(ag+bgx)}{bg} - \frac{(2B)\int \left(\frac{A\log(ag+bgx)}{a+bx}+\frac{B\log\left(\frac{e(a+bx)}{c+dx}\right)\log(ag+bg}{a+bx}\right.}{g}$$

$$= \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2\log(ag+bgx)}{bg} - \frac{(2AB)\int \frac{\log(ag+bgx)}{a+bx}\,dx}{g} - \frac{\left(2B^2\right)\int \frac{\log\left(\frac{e(a+bx)}{c+dx}\right.}{a}}{g}$$

$$= \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2\log(ag+bgx)}{bg} + \frac{2AB\log\left(\frac{b(c+dx)}{bc-ad}\right)\log(ag+bgx)}{bg} - \frac{B^2\log}{}$$

$$= \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2\log(ag+bgx)}{bg} + \frac{2AB\log\left(\frac{b(c+dx)}{bc-ad}\right)\log(ag+bgx)}{bg} - \frac{2B^2\left(\mathrm{l}\right.}{}$$

$$= -\frac{AB\log^2(g(a+bx))}{bg} + \frac{2B^2\log(a+bx)\log(g(a+bx))\log(-c-dx)}{bg} - \frac{B^2\log(g(a}{}$$

$$= -\frac{AB\log^2(g(a+bx))}{bg} + \frac{2B^2\log(a+bx)\log(g(a+bx))\log(-c-dx)}{bg} + \frac{B^2\log\left(-\right.}{}$$

**Mathematica [A]** time = 0.61, size = 250, normalized size = 1.95

$$\frac{A^2\log(a+bx)+2AB\log(a+bx)\log\left(\frac{e(a+bx)}{c+dx}\right)-2AB\text{Li}_2\left(\frac{b(c+dx)}{bc-ad}\right)+2AB\log(a+bx)\log\left(\frac{c}{d}+x\right)-2AB\log\left(\frac{c}{d}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Log[(e*(a + b*x))/(c + d*x)])^2/(a*g + b*g*x),x]
```

```
[Out] (A*B*Log[a/b + x]^2 + A^2*Log[a + b*x] - 2*A*B*Log[a/b + x]*Log[a + b*x] +
2*A*B*Log[c/d + x]*Log[a + b*x] - 2*A*B*Log[c/d + x]*Log[(d*(a + b*x))/(-(b
*c) + a*d)] + 2*A*B*Log[a + b*x]*Log[(e*(a + b*x))/(c + d*x)] - B^2*Log[(-(
b*c) + a*d)/(d*(a + b*x))]*Log[(e*(a + b*x))/(c + d*x)]^2 - 2*A*B*PolyLog[2
, (b*(c + d*x))/(b*c - a*d)] + 2*B^2*Log[(e*(a + b*x))/(c + d*x)]*PolyLog[2
, (b*(c + d*x))/(d*(a + b*x))] + 2*B^2*PolyLog[3, (b*(c + d*x))/(d*(a + b*x
))])/(b*g)
```

**fricas [F]** time = 1.64, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{B^2\log\left(\frac{bex+ae}{dx+c}\right)^2+2\,AB\log\left(\frac{bex+ae}{dx+c}\right)+A^2}{bgx+ag},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*(b*x+a)/(d*x+c)))^2/(b*g*x+a*g),x, algorithm="fricas")
```

```
[Out] integral((B^2*log((b*e*x + a*e)/(d*x + c))^2 + 2*A*B*log((b*e*x + a*e)/(d*x
 + c)) + A^2)/(b*g*x + a*g), x)
```

**giac [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*(b*x+a)/(d*x+c)))^2/(b*g*x+a*g),x, algorithm="giac")
```

```
[Out] Timed out
```

**maple [B]** time = 0.06, size = 1186, normalized size = 9.27

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*ln((b*x+a)/(d*x+c)*e)+A)^2/(b*g*x+a*g),x)
```

```
[Out] -d/g/(a*d-b*c)*A^2/b*ln(-b*e+(b/d*e+(a*d-b*c)/(d*x+c)/d*e)*d)*a+1/g/(a*d-b*
c)*A^2*ln(-b*e+(b/d*e+(a*d-b*c)/(d*x+c)/d*e)*d)*c+d/g/(a*d-b*c)*A^2/b*ln(b/
d*e+(a*d-b*c)/(d*x+c)/d*e)*a-1/g/(a*d-b*c)*A^2*ln(b/d*e+(a*d-b*c)/(d*x+c)/d
*e)*c-d/g/(a*d-b*c)*B^2/b*ln(b/d*e+(a*d-b*c)/(d*x+c)/d*e)^2*ln(1-1/b/e*d*(b
/d*e+(a*d-b*c)/(d*x+c)/d*e))*a+1/g/(a*d-b*c)*B^2*ln(b/d*e+(a*d-b*c)/(d*x+c)
/d*e)^2*ln(1-1/b/e*d*(b/d*e+(a*d-b*c)/(d*x+c)/d*e))*c-2*d/g/(a*d-b*c)*B^2/b
*ln(b/d*e+(a*d-b*c)/(d*x+c)/d*e)*polylog(2,1/b/e*d*(b/d*e+(a*d-b*c)/(d*x+c)
/d*e))*a+2/g/(a*d-b*c)*B^2*ln(b/d*e+(a*d-b*c)/(d*x+c)/d*e)*polylog(2,1/b/e*
d*(b/d*e+(a*d-b*c)/(d*x+c)/d*e))*c+2*d/g/(a*d-b*c)*B^2/b*polylog(3,1/b/e*d*
(b/d*e+(a*d-b*c)/(d*x+c)/d*e))*a-2/g/(a*d-b*c)*B^2*polylog(3,1/b/e*d*(b/d*e
+(a*d-b*c)/(d*x+c)/d*e))*c+1/3*d/g/(a*d-b*c)*B^2/b*ln(b/d*e+(a*d-b*c)/(d*x+
c)/d*e)^3*a-1/3/g/(a*d-b*c)*B^2*ln(b/d*e+(a*d-b*c)/(d*x+c)/d*e)^3*c-2*d/g/(
a*d-b*c)*A*B/b*dilog(-(-b*e+(b/d*e+(a*d-b*c)/(d*x+c)/d*e)*d)/b/e)*a+2/g/(a*
d-b*c)*A*B*dilog(-(-b*e+(b/d*e+(a*d-b*c)/(d*x+c)/d*e)*d)/b/e)*c-2*d/g/(a*d-
b*c)*A*B/b*ln(b/d*e+(a*d-b*c)/(d*x+c)/d*e)*ln(-(-b*e+(b/d*e+(a*d-b*c)/(d*x+
c)/d*e)*d)/b/e)*a+2/g/(a*d-b*c)*A*B*ln(b/d*e+(a*d-b*c)/(d*x+c)/d*e)*ln(-(-b
*e+(b/d*e+(a*d-b*c)/(d*x+c)/d*e)*d)/b/e)*c+d/g/(a*d-b*c)*A*B*ln(b/d*e+(a*d-
b*c)/(d*x+c)/d*e)^2/b*a-1/g/(a*d-b*c)*A*B*ln(b/d*e+(a*d-b*c)/(d*x+c)/d*e)^2
*c
```

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$\frac{B^2\log\left(bx+a\right)\log\left(dx+c\right)^2}{bg}+\frac{A^2\log\left(bgx+ag\right)}{bg}-\int-\frac{B^2bc\log(e)^2+2\,ABbc\log(e)+\left(B^2bdx+B^2bc\right)\log\left(bx+a\right)}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*(b*x+a)/(d*x+c)))^2/(b*g*x+a*g),x, algorithm="maxima")
```

```
[Out] B^2*log(b*x + a)*log(d*x + c)^2/(b*g) + A^2*log(b*g*x + a*g)/(b*g) - integr
ate(-(B^2*b*c*log(e)^2 + 2*A*B*b*c*log(e) + (B^2*b*d*x + B^2*b*c)*log(b*x +
 a)^2 + (B^2*b*d*log(e)^2 + 2*A*B*b*d*log(e))*x + 2*(B^2*b*c*log(e) + A*B*b
*c + (B^2*b*d*log(e) + A*B*b*d)*x)*log(b*x + a) - 2*(B^2*b*c*log(e) + A*B*b
*c + (B^2*b*d*log(e) + A*B*b*d)*x + (2*B^2*b*d*x + (b*c + a*d)*B^2)*log(b*x
 + a))*log(d*x + c))/(b^2*d*g*x^2 + a*b*c*g + (b^2*c*g + a*b*d*g)*x), x)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\left(A+B\,\ln\left(\frac{e\,\left(a+b\,x\right)}{c+d\,x}\right)\right)}^2}{a\,g+b\,g\,x} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*log((e*(a + b*x))/(c + d*x)))^2/(a*g + b*g*x),x)
```

```
[Out] int((A + B*log((e*(a + b*x))/(c + d*x)))^2/(a*g + b*g*x), x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{A^2}{a+bx}\,dx + \int \frac{B^2\log\left(\frac{ae}{c+dx}+\frac{bex}{c+dx}\right)^2}{a+bx}\,dx + \int \frac{2AB\log\left(\frac{ae}{c+dx}+\frac{bex}{c+dx}\right)}{a+bx}\,dx}{g}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*ln(e*(b*x+a)/(d*x+c)))**2/(b*g*x+a*g),x)
```

```
[Out] (Integral(A**2/(a + b*x), x) + Integral(B**2*log(a*e/(c + d*x) + b*e*x/(c +
 d*x))**2/(a + b*x), x) + Integral(2*A*B*log(a*e/(c + d*x) + b*e*x/(c + d*x
))/(a + b*x), x))/g
```

# 3.102 $\int \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)^2} dx$

**Optimal**. Leaf size=126

$$-\frac{2B(c+dx)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{g^2(a+bx)(bc-ad)}-\frac{(c+dx)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2}{g^2(a+bx)(bc-ad)}-\frac{2B^2(c+dx)}{g^2(a+bx)(bc-ad)}$$

```
[Out] -2*B^2*(d*x+c)/(-a*d+b*c)/g^2/(b*x+a)-2*B*(d*x+c)*(A+B*ln(e*(b*x+a)/(d*x+c)
))/(-a*d+b*c)/g^2/(b*x+a)-(d*x+c)*(A+B*ln(e*(b*x+a)/(d*x+c)))^2/(-a*d+b*c)/
g^2/(b*x+a)
```

**Rubi [C]** time = 0.77, antiderivative size = 470, normalized size of antiderivative = 3.73, number of steps used = 26, number of rules used = 11, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.344, Rules used = {2525, 12, 2528, 44, 2524, 2418, 2390, 2301, 2394, 2393, 2391}

$$-\frac{2B^2d\text{PolyLog}\left(2,-\frac{d(a+bx)}{bc-ad}\right)}{bg^2(bc-ad)}-\frac{2B^2d\text{PolyLog}\left(2,\frac{b(c+dx)}{bc-ad}\right)}{bg^2(bc-ad)}-\frac{2Bd\log(a+bx)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{bg^2(bc-ad)}-\frac{2B\left(B\log\left(\frac{e(a+b}{c+d}\right.\right.}{bg^2(a+b}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Log[(e*(a + b*x))/(c + d*x)])^2/(a*g + b*g*x)^2,x]
```

```
[Out] (-2*B^2)/(b*g^2*(a + b*x)) - (2*B^2*d*Log[a + b*x])/(b*(b*c - a*d)*g^2) + (
B^2*d*Log[a + b*x]^2)/(b*(b*c - a*d)*g^2) - (2*B*(A + B*Log[(e*(a + b*x))/(
c + d*x)]))/(b*g^2*(a + b*x)) - (2*B*d*Log[a + b*x]*(A + B*Log[(e*(a + b*x)
)/(c + d*x)]))/(b*(b*c - a*d)*g^2) - (A + B*Log[(e*(a + b*x))/(c + d*x)])^2
/(b*g^2*(a + b*x)) + (2*B^2*d*Log[c + d*x])/(b*(b*c - a*d)*g^2) - (2*B^2*d*
Log[-((d*(a + b*x))/(b*c - a*d))]*Log[c + d*x])/(b*(b*c - a*d)*g^2) + (2*B*
d*(A + B*Log[(e*(a + b*x))/(c + d*x)])*Log[c + d*x])/(b*(b*c - a*d)*g^2) +
(B^2*d*Log[c + d*x]^2)/(b*(b*c - a*d)*g^2) - (2*B^2*d*Log[a + b*x]*Log[(b*(
c + d*x))/(b*c - a*d)])/(b*(b*c - a*d)*g^2) - (2*B^2*d*PolyLog[2, -((d*(a +
 b*x))/(b*c - a*d))])/(b*(b*c - a*d)*g^2) - (2*B^2*d*PolyLog[2, (b*(c + d*x
))/(b*c - a*d)])/(b*(b*c - a*d)*g^2)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 2301

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

Rule 2390

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_))^(q_.), x_Symbol] :> Dist[1/e, Subst[Int[((f*x)/d)^q*(a + b*Log[c*x^
n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && E
qQ[e*f - d*g, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2393

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + (c*e*x)/g])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2394

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d + e*x
)^n]))/g, x] - Dist[(b*e*n)/g, Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2418

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*(RFx_), x_Sy
mbol] :> With[{u = ExpandIntegrand[(a + b*Log[c*(d + e*x)^n])^p, RFx, x]},
Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d, e, n}, x] && RationalFunctionQ[
RFx, x] && IntegerQ[p]
```

Rule 2524

```
Int[((a_.) + Log[(c_.)*(RFx_)^(p_.)]*(b_.))^(n_.)/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[(Log[d + e*x]*(a + b*Log[c*RFx^p])^n)/e, x] - Dist[(b*n*p)/e
, Int[(Log[d + e*x]*(a + b*Log[c*RFx^p])^(n - 1)*D[RFx, x])/RFx, x], x] /;
FreeQ[{a, b, c, d, e, p}, x] && RationalFunctionQ[RFx, x] && IGtQ[n, 0]
```

Rule 2525

```
Int[((a_.) + Log[(c_.)*(RFx_)^(p_.)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_))^(m_.
), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(a + b*Log[c*RFx^p])^n)/(e*(m + 1))
, x] - Dist[(b*n*p)/(e*(m + 1)), Int[SimplifyIntegrand[((d + e*x)^(m + 1)*(
a + b*Log[c*RFx^p])^(n - 1)*D[RFx, x])/RFx, x], x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && RationalFunctionQ[RFx, x] && IGtQ[n, 0] && (EqQ[n, 1] ||
IntegerQ[m]) && NeQ[m, -1]
```

Rule 2528

```
Int[((a_.) + Log[(c_.)*(RFx_)^(p_.)]*(b_.))^(n_.)*(RGx_), x_Symbol] :> With
[{u = ExpandIntegrand[(a + b*Log[c*RFx^p])^n, RGx, x]}, Int[u, x] /; SumQ[u
]] /; FreeQ[{a, b, c, p}, x] && RationalFunctionQ[RFx, x] && RationalFuncti
onQ[RGx, x] && IGtQ[n, 0]
```

Rubi steps

$$\int \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)^2}\,dx = -\frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{bg^2(a+bx)} + \frac{(2B)\int \frac{(bc-ad)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{g(a+bx)^2(c+dx)}\,dx}{bg}$$

$$= -\frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{bg^2(a+bx)} + \frac{(2B(bc-ad))\int \frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{(a+bx)^2(c+dx)}\,dx}{bg^2}$$

$$= -\frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{bg^2(a+bx)} + \frac{(2B(bc-ad))\int \left(\frac{b\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(bc-ad)(a+bx)^2} - \frac{bd\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(bc-ad)^2(a+bx)}\right.}{bg^2}$$

$$= -\frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{bg^2(a+bx)} + \frac{(2B)\int \frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{(a+bx)^2}\,dx}{g^2} - \frac{(2Bd)\int \frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{a+bx}\,dx}{(bc-ad)g^2}$$

$$= -\frac{2B\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{bg^2(a+bx)} - \frac{2Bd\log(a+bx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{b(bc-ad)g^2} - \frac{(A+B\log}{bg^2(a}$$

$$= -\frac{2B\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{bg^2(a+bx)} - \frac{2Bd\log(a+bx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{b(bc-ad)g^2} - \frac{(A+B\log}{bg^2(a}$$

$$= -\frac{2B\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{bg^2(a+bx)} - \frac{2Bd\log(a+bx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{b(bc-ad)g^2} - \frac{(A+B\log}{bg^2(a}$$

$$= -\frac{2B^2}{bg^2(a+bx)} - \frac{2B^2d\log(a+bx)}{b(bc-ad)g^2} - \frac{2B\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{bg^2(a+bx)} - \frac{2Bd\log(a+bx)}{b(bc}$$

$$= -\frac{2B^2}{bg^2(a+bx)} - \frac{2B^2d\log(a+bx)}{b(bc-ad)g^2} - \frac{2B\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{bg^2(a+bx)} - \frac{2Bd\log(a+bx)}{b(bc}$$

$$= -\frac{2B^2}{bg^2(a+bx)} - \frac{2B^2d\log(a+bx)}{b(bc-ad)g^2} + \frac{B^2d\log^2(a+bx)}{b(bc-ad)g^2} - \frac{2B\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{bg^2(a+bx)}$$

$$= -\frac{2B^2}{bg^2(a+bx)} - \frac{2B^2d\log(a+bx)}{b(bc-ad)g^2} + \frac{B^2d\log^2(a+bx)}{b(bc-ad)g^2} - \frac{2B\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{bg^2(a+bx)}$$

**Mathematica [C]** time = 0.45, size = 314, normalized size = 2.49

$$-\frac{B\left(2(bc-ad)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)+2d(a+bx)\log(a+bx)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)-2d(a+bx)\log(c+dx)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)-Bd(a+bx)\left(\log(a+bx)\left(\log(a+bx)-\right.\right.\right.}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Log[(e*(a + b*x))/(c + d*x)])^2/(a*g + b*g*x)^2,x]
```

```
[Out] -(((A + B*Log[(e*(a + b*x))/(c + d*x)])^2 + (B*(2*(b*c - a*d)*(A + B*Log[(e
*(a + b*x))/(c + d*x)]) + 2*d*(a + b*x)*Log[a + b*x]*(A + B*Log[(e*(a + b*x
))/(c + d*x)]) - 2*d*(a + b*x)*(A + B*Log[(e*(a + b*x))/(c + d*x)])*Log[c +
 d*x] + 2*B*(b*c - a*d + d*(a + b*x)*Log[a + b*x] - d*(a + b*x)*Log[c + d*x
]) - B*d*(a + b*x)*(Log[a + b*x]*(Log[a + b*x] - 2*Log[(b*(c + d*x))/(b*c -
 a*d)]) - 2*PolyLog[2, (d*(a + b*x))/(-(b*c) + a*d)]) + B*d*(a + b*x)*((2*L
og[(d*(a + b*x))/(-(b*c) + a*d)] - Log[c + d*x])*Log[c + d*x] + 2*PolyLog[2
, (b*(c + d*x))/(b*c - a*d)])))/(b*c - a*d))/(b*g^2*(a + b*x)))
```

**fricas [A]** time = 1.10, size = 150, normalized size = 1.19

$$-\frac{\left(A^2+2\,AB+2\,B^2\right)bc-\left(A^2+2\,AB+2\,B^2\right)ad+\left(B^2bdx+B^2bc\right)\log\left(\frac{bex+ae}{dx+c}\right)^2+2\left(\left(AB+B^2\right)bdx+\left(AB+B^2\right)\right.}{\left(b^3c-ab^2d\right)g^2x+\left(ab^2c-a^2bd\right)g^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*(b*x+a)/(d*x+c)))^2/(b*g*x+a*g)^2,x, algorithm="fricas
")
```

```
[Out] -((A^2 + 2*A*B + 2*B^2)*b*c - (A^2 + 2*A*B + 2*B^2)*a*d + (B^2*b*d*x + B^2*
b*c)*log((b*e*x + a*e)/(d*x + c))^2 + 2*((A*B + B^2)*b*d*x + (A*B + B^2)*b*
c)*log((b*e*x + a*e)/(d*x + c)))/((b^3*c - a*b^2*d)*g^2*x + (a*b^2*c - a^2*
b*d)*g^2)
```

**giac [A]** time = 2.08, size = 176, normalized size = 1.40

$$-\frac{\left(B^2e^2\log\left(\frac{bxe+ae}{dx+c}\right)^2+2\,ABe^2\log\left(\frac{bxe+ae}{dx+c}\right)+2\,B^2e^2\log\left(\frac{bxe+ae}{dx+c}\right)+A^2e^2+2\,ABe^2+2\,B^2e^2\right)(dx+c)\left(\frac{bc}{(bce-ade)(bc-ad)}\right.}{(bxe+ae)g^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*(b*x+a)/(d*x+c)))^2/(b*g*x+a*g)^2,x, algorithm="giac")
```

```
[Out] -(B^2*e^2*log((b*x*e + a*e)/(d*x + c))^2 + 2*A*B*e^2*log((b*x*e + a*e)/(d*x
 + c)) + 2*B^2*e^2*log((b*x*e + a*e)/(d*x + c)) + A^2*e^2 + 2*A*B*e^2 + 2*B
```

```
^2*e^2)*(d*x + c)*(b*c/((b*c*e - a*d*e)*(b*c - a*d)) - a*d/((b*c*e - a*d*e)
*(b*c - a*d)))/((b*x*e + a*e)*g^2)
```

**maple [B]** time = 0.05, size = 828, normalized size = 6.57

$$\frac{B^2ade\ln\left(\frac{be}{d}+\frac{(ad-bc)e}{(dx+c)d}\right)^2}{(ad-bc)^2\left(\frac{ae}{dx+c}-\frac{bce}{(dx+c)d}+\frac{be}{d}\right)g^2}-\frac{B^2bce\ln\left(\frac{be}{d}+\frac{(ad-bc)e}{(dx+c)d}\right)^2}{(ad-bc)^2\left(\frac{ae}{dx+c}-\frac{bce}{(dx+c)d}+\frac{be}{d}\right)g^2}+\frac{2ABade\ln\left(\frac{be}{d}+\frac{(ad-bc)e}{(dx+c)d}\right)}{(ad-bc)^2\left(\frac{ae}{dx+c}-\frac{bce}{(dx+c)d}+\frac{be}{d}\right)g^2}-\frac{2}{(ad-}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*ln((b*x+a)/(d*x+c)*e)+A)^2/(b*g*x+a*g)^2,x)
```

```
[Out] d*e/(a*d-b*c)^2/g^2*A^2/(1/(d*x+c)*a*e-1/(d*x+c)*b*c/d*e+b/d*e)*a-e/(a*d-b*
c)^2/g^2*A^2/(1/(d*x+c)*a*e-1/(d*x+c)*b*c/d*e+b/d*e)*b*c+2*d*e/(a*d-b*c)^2/
g^2*A*B/(1/(d*x+c)*a*e-1/(d*x+c)*b*c/d*e+b/d*e)*ln(b/d*e+(a*d-b*c)/(d*x+c)/
d*e)*a-2*e/(a*d-b*c)^2/g^2*A*B/(1/(d*x+c)*a*e-1/(d*x+c)*b*c/d*e+b/d*e)*ln(b
/d*e+(a*d-b*c)/(d*x+c)/d*e)*b*c+2*d*e/(a*d-b*c)^2/g^2*A*B/(1/(d*x+c)*a*e-1/
(d*x+c)*b*c/d*e+b/d*e)*a-2*e/(a*d-b*c)^2/g^2*A*B/(1/(d*x+c)*a*e-1/(d*x+c)*b
*c/d*e+b/d*e)*b*c+d*e/(a*d-b*c)^2/g^2*B^2/(1/(d*x+c)*a*e-1/(d*x+c)*b*c/d*e+
b/d*e)*ln(b/d*e+(a*d-b*c)/(d*x+c)/d*e)^2*a-e/(a*d-b*c)^2/g^2*B^2/(1/(d*x+c)
*a*e-1/(d*x+c)*b*c/d*e+b/d*e)*ln(b/d*e+(a*d-b*c)/(d*x+c)/d*e)^2*b*c+2*d*e/(
a*d-b*c)^2/g^2*B^2/(1/(d*x+c)*a*e-1/(d*x+c)*b*c/d*e+b/d*e)*ln(b/d*e+(a*d-b*
c)/(d*x+c)/d*e)*a-2*e/(a*d-b*c)^2/g^2*B^2/(1/(d*x+c)*a*e-1/(d*x+c)*b*c/d*e+
b/d*e)*ln(b/d*e+(a*d-b*c)/(d*x+c)/d*e)*b*c+2*d*e/(a*d-b*c)^2/g^2*B^2/(1/(d*
x+c)*a*e-1/(d*x+c)*b*c/d*e+b/d*e)*a-2*e/(a*d-b*c)^2/g^2*B^2/(1/(d*x+c)*a*e-
1/(d*x+c)*b*c/d*e+b/d*e)*b*c
```

**maxima [B]** time = 1.40, size = 416, normalized size = 3.30

$$-\left(2\left(\frac{1}{b^2g^2x+abg^2}+\frac{d\log(bx+a)}{\left(b^2c-abd\right)g^2}-\frac{d\log(dx+c)}{\left(b^2c-abd\right)g^2}\right)\log\left(\frac{bex}{dx+c}+\frac{ae}{dx+c}\right)-\frac{(bdx+ad)\log(bx+a)^2+(bdx}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*(b*x+a)/(d*x+c)))^2/(b*g*x+a*g)^2,x, algorithm="maxima
")
```

```
[Out] -(2*(1/(b^2*g^2*x + a*b*g^2) + d*log(b*x + a)/((b^2*c - a*b*d)*g^2) - d*log
(d*x + c)/((b^2*c - a*b*d)*g^2))*log(b*e*x/(d*x + c) + a*e/(d*x + c)) - ((b
*d*x + a*d)*log(b*x + a)^2 + (b*d*x + a*d)*log(d*x + c)^2 - 2*b*c + 2*a*d -
 2*(b*d*x + a*d)*log(b*x + a) + 2*(b*d*x + a*d - (b*d*x + a*d)*log(b*x + a)
)*log(d*x + c))/(a*b^2*c*g^2 - a^2*b*d*g^2 + (b^3*c*g^2 - a*b^2*d*g^2)*x))*
B^2 - 2*A*B*(log(b*e*x/(d*x + c) + a*e/(d*x + c))/(b^2*g^2*x + a*b*g^2) + 1
```

```
/(b^2*g^2*x + a*b*g^2) + d*log(b*x + a)/((b^2*c - a*b*d)*g^2) - d*log(d*x +
 c)/((b^2*c - a*b*d)*g^2)) - B^2*log(b*e*x/(d*x + c) + a*e/(d*x + c))^2/(b^
2*g^2*x + a*b*g^2) - A^2/(b^2*g^2*x + a*b*g^2)
```

**mupad [B]** time = 5.26, size = 222, normalized size = 1.76

$$-\frac{A^2+2\,A\,B+2\,B^2}{x\,b^2\,g^2+a\,b\,g^2}-\ln\left(\frac{e\,\left(a+b\,x\right)}{c+d\,x}\right)^2\,\left(\frac{B^2}{b^2\,g^2\,\left(x+\frac{a}{b}\right)}-\frac{B^2\,d}{b\,g^2\,\left(a\,d-b\,c\right)}\right)-\frac{\ln\left(\frac{e\,\left(a+b\,x\right)}{c+d\,x}\right)\,\left(\frac{2\,B^2}{b^2\,d\,g^2}+\frac{2\,A\,B}{b^2\,d\,g^2}\right)}{\frac{x}{d}+\frac{a}{b\,d}}-\frac{B\,d\,\mathrm{atan}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*log((e*(a + b*x))/(c + d*x)))^2/(a*g + b*g*x)^2,x)
```

```
[Out] - (A^2 + 2*B^2 + 2*A*B)/(b^2*g^2*x + a*b*g^2) - log((e*(a + b*x))/(c + d*x)
)^2*(B^2/(b^2*g^2*(x + a/b)) - (B^2*d)/(b*g^2*(a*d - b*c))) - (log((e*(a +
b*x))/(c + d*x))*((2*B^2)/(b^2*d*g^2) + (2*A*B)/(b^2*d*g^2)))/(x/d + a/(b*d
)) - (B*d*atan(((2*b*d*x + (b^2*c*g^2 + a*b*d*g^2)/(b*g^2))*1i)/(a*d - b*c)
)*(A + B)*4i)/(b*g^2*(a*d - b*c))
```

**sympy [B]** time = 3.59, size = 434, normalized size = 3.44

$$-\frac{2Bd\left(A+B\right)\log\left(x+\frac{2ABad^2+2ABbcd+2B^2ad^2+2B^2bcd-\frac{2Ba^2d^3(A+B)}{ad-bc}+\frac{4Babcd^2(A+B)}{ad-bc}-\frac{2Bb^2c^2d(A+B)}{ad-bc}}{4ABbd^2+4B^2bd^2}\right)}{bg^2\left(ad-bc\right)}+\frac{2Bd\left(A+B\right)\log\left(x+\frac{2ABad}{}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*ln(e*(b*x+a)/(d*x+c)))**2/(b*g*x+a*g)**2,x)
```

```
[Out] -2*B*d*(A + B)*log(x + (2*A*B*a*d**2 + 2*A*B*b*c*d + 2*B**2*a*d**2 + 2*B**2
*b*c*d - 2*B*a**2*d**3*(A + B)/(a*d - b*c) + 4*B*a*b*c*d**2*(A + B)/(a*d -
b*c) - 2*B*b**2*c**2*d*(A + B)/(a*d - b*c))/(4*A*B*b*d**2 + 4*B**2*b*d**2))
/(b*g**2*(a*d - b*c)) + 2*B*d*(A + B)*log(x + (2*A*B*a*d**2 + 2*A*B*b*c*d +
 2*B**2*a*d**2 + 2*B**2*b*c*d + 2*B*a**2*d**3*(A + B)/(a*d - b*c) - 4*B*a*b
*c*d**2*(A + B)/(a*d - b*c) + 2*B*b**2*c**2*d*(A + B)/(a*d - b*c))/(4*A*B*b
*d**2 + 4*B**2*b*d**2))/(b*g**2*(a*d - b*c)) + (-2*A*B - 2*B**2)*log(e*(a +
 b*x)/(c + d*x))/(a*b*g**2 + b**2*g**2*x) + (B**2*c + B**2*d*x)*log(e*(a +
b*x)/(c + d*x))**2/(a**2*d*g**2 - a*b*c*g**2 + a*b*d*g**2*x - b**2*c*g**2*x
) + (-A**2 - 2*A*B - 2*B**2)/(a*b*g**2 + b**2*g**2*x)
```

# 3.103 $\int \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)^3}\,dx$

**Optimal**. Leaf size=268

$$-\frac{bB(c+dx)^2\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{2g^3(a+bx)^2(bc-ad)^2}+\frac{2Bd(c+dx)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{g^3(a+bx)(bc-ad)^2}-\frac{b(c+dx)^2\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2}{2g^3(a+bx)^2(bc-ad)^2}+\frac{d(c+\cdots}{\cdots}$$

```
[Out] 2*B^2*d*(d*x+c)/(-a*d+b*c)^2/g^3/(b*x+a)-1/4*b*B^2*(d*x+c)^2/(-a*d+b*c)^2/g
^3/(b*x+a)^2+2*B*d*(d*x+c)*(A+B*ln(e*(b*x+a)/(d*x+c)))/(-a*d+b*c)^2/g^3/(b*
x+a)-1/2*b*B*(d*x+c)^2*(A+B*ln(e*(b*x+a)/(d*x+c)))/(-a*d+b*c)^2/g^3/(b*x+a)
^2+d*(d*x+c)*(A+B*ln(e*(b*x+a)/(d*x+c)))^2/(-a*d+b*c)^2/g^3/(b*x+a)-1/2*b*(
d*x+c)^2*(A+B*ln(e*(b*x+a)/(d*x+c)))^2/(-a*d+b*c)^2/g^3/(b*x+a)^2
```

**Rubi [C]** time = 0.91, antiderivative size = 577, normalized size of antiderivative = 2.15, number of steps used = 30, number of rules used = 11, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.344, Rules used = {2525, 12, 2528, 44, 2524, 2418, 2390, 2301, 2394, 2393, 2391}

$$\frac{B^2d^2\text{PolyLog}\left(2,-\frac{d(a+bx)}{bc-ad}\right)}{bg^3(bc-ad)^2}+\frac{B^2d^2\text{PolyLog}\left(2,\frac{b(c+dx)}{bc-ad}\right)}{bg^3(bc-ad)^2}+\frac{Bd^2\log(a+bx)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{bg^3(bc-ad)^2}-\frac{Bd^2\log(c+dx)\cdots}{bg^3(\cdots}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Log[(e*(a + b*x))/(c + d*x)])^2/(a*g + b*g*x)^3,x]
```

```
[Out] -B^2/(4*b*g^3*(a + b*x)^2) + (3*B^2*d)/(2*b*(b*c - a*d)*g^3*(a + b*x)) + (3
*B^2*d^2*Log[a + b*x])/(2*b*(b*c - a*d)^2*g^3) - (B^2*d^2*Log[a + b*x]^2)/(
2*b*(b*c - a*d)^2*g^3) - (B*(A + B*Log[(e*(a + b*x))/(c + d*x)]))/(2*b*g^3*
(a + b*x)^2) + (B*d*(A + B*Log[(e*(a + b*x))/(c + d*x)]))/(b*(b*c - a*d)*g^
3*(a + b*x)) + (B*d^2*Log[a + b*x]*(A + B*Log[(e*(a + b*x))/(c + d*x)]))/(b
*(b*c - a*d)^2*g^3) - (A + B*Log[(e*(a + b*x))/(c + d*x)])^2/(2*b*g^3*(a +
b*x)^2) - (3*B^2*d^2*Log[c + d*x])/(2*b*(b*c - a*d)^2*g^3) + (B^2*d^2*Log[-
((d*(a + b*x))/(b*c - a*d))]*Log[c + d*x])/(b*(b*c - a*d)^2*g^3) - (B*d^2*(
A + B*Log[(e*(a + b*x))/(c + d*x)])*Log[c + d*x])/(b*(b*c - a*d)^2*g^3) - (
B^2*d^2*Log[c + d*x]^2)/(2*b*(b*c - a*d)^2*g^3) + (B^2*d^2*Log[a + b*x]*Log
[(b*(c + d*x))/(b*c - a*d)])/(b*(b*c - a*d)^2*g^3) + (B^2*d^2*PolyLog[2, -(
(d*(a + b*x))/(b*c - a*d))])/(b*(b*c - a*d)^2*g^3) + (B^2*d^2*PolyLog[2, (b
*(c + d*x))/(b*c - a*d)])/(b*(b*c - a*d)^2*g^3)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 2301

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

Rule 2390

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_))^(q_.), x_Symbol] :> Dist[1/e, Subst[Int[((f*x)/d)^q*(a + b*Log[c*x^
n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && E
qQ[e*f - d*g, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2393

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + (c*e*x)/g])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2394

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d + e*x
)^n]))/g, x] - Dist[(b*e*n)/g, Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2418

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*(RFx_), x_Sy
mbol] :> With[{u = ExpandIntegrand[(a + b*Log[c*(d + e*x)^n])^p, RFx, x]},
Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d, e, n}, x] && RationalFunctionQ[
RFx, x] && IntegerQ[p]
```

Rule 2524

```
Int[((a_.) + Log[(c_.)*(RFx_)^(p_.)]*(b_.))^(n_.)/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[(Log[d + e*x]*(a + b*Log[c*RFx^p])^n)/e, x] - Dist[(b*n*p)/e
, Int[(Log[d + e*x]*(a + b*Log[c*RFx^p])^(n - 1)*D[RFx, x])/RFx, x], x] /;
FreeQ[{a, b, c, d, e, p}, x] && RationalFunctionQ[RFx, x] && IGtQ[n, 0]
```

Rule 2525

```
Int[((a_.) + Log[(c_.)*(RFx_)^(p_.)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_))^(m_.
), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(a + b*Log[c*RFx^p])^n)/(e*(m + 1))
, x] - Dist[(b*n*p)/(e*(m + 1)), Int[SimplifyIntegrand[((d + e*x)^(m + 1)*(
a + b*Log[c*RFx^p])^(n - 1)*D[RFx, x])/RFx, x], x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && RationalFunctionQ[RFx, x] && IGtQ[n, 0] && (EqQ[n, 1] ||
IntegerQ[m]) && NeQ[m, -1]
```

Rule 2528

```
Int[((a_.) + Log[(c_.)*(RFx_)^(p_.)]*(b_.))^(n_.)*(RGx_), x_Symbol] :> With
[{u = ExpandIntegrand[(a + b*Log[c*RFx^p])^n, RGx, x]}, Int[u, x] /; SumQ[u
]] /; FreeQ[{a, b, c, p}, x] && RationalFunctionQ[RFx, x] && RationalFuncti
onQ[RGx, x] && IGtQ[n, 0]
```

Rubi steps

$$\int \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)^3}\,dx = -\frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{2bg^3(a+bx)^2} + \frac{B\int \frac{(bc-ad)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{g^2(a+bx)^3(c+dx)}\,dx}{bg}$$

$$= -\frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{2bg^3(a+bx)^2} + \frac{(B(bc-ad))\int \frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{(a+bx)^3(c+dx)}\,dx}{bg^3}$$

$$= -\frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{2bg^3(a+bx)^2} + \frac{(B(bc-ad))\int\left(\frac{b\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(bc-ad)(a+bx)^3} - \frac{bd\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(bc-ad)^2(a+bx)^2} + \cdots}{bg^3}$$

$$= -\frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{2bg^3(a+bx)^2} + \frac{B\int \frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{(a+bx)^3}\,dx}{g^3} + \frac{\left(Bd^2\right)\int \frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{a+bx}\,dx}{(bc-ad)^2g^3} - \cdots$$

$$= -\frac{B\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{2bg^3(a+bx)^2} + \frac{Bd\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{b(bc-ad)g^3(a+bx)} + \frac{Bd^2\log(a+bx)\left(A+B\log\cdots\right.}{b(bc-ad)^2g^3}$$

$$= -\frac{B\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{2bg^3(a+bx)^2} + \frac{Bd\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{b(bc-ad)g^3(a+bx)} + \frac{Bd^2\log(a+bx)\left(A+B\log\cdots\right.}{b(bc-ad)^2g^3}$$

$$= -\frac{B\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{2bg^3(a+bx)^2} + \frac{Bd\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{b(bc-ad)g^3(a+bx)} + \frac{Bd^2\log(a+bx)\left(A+B\log\cdots\right.}{b(bc-ad)^2g^3}$$

$$= -\frac{B^2}{4bg^3(a+bx)^2} + \frac{3B^2d}{2b(bc-ad)g^3(a+bx)} + \frac{3B^2d^2\log(a+bx)}{2b(bc-ad)^2g^3} - \frac{B\left(A+B\log\left(\cdots\right.\right.}{2bg^3(a+bx)\cdots}$$

$$= -\frac{B^2}{4bg^3(a+bx)^2} + \frac{3B^2d}{2b(bc-ad)g^3(a+bx)} + \frac{3B^2d^2\log(a+bx)}{2b(bc-ad)^2g^3} - \frac{B\left(A+B\log\left(\cdots\right.\right.}{2bg^3(a+bx)\cdots}$$

$$= -\frac{B^2}{4bg^3(a+bx)^2} + \frac{3B^2d}{2b(bc-ad)g^3(a+bx)} + \frac{3B^2d^2\log(a+bx)}{2b(bc-ad)^2g^3} - \frac{B^2d^2\log^2(a+bx\cdots}{2b(bc-ad)^2g^3}$$

$$= -\frac{B^2}{4bg^3(a+bx)^2} + \frac{3B^2d}{2b(bc-ad)g^3(a+bx)} + \frac{3B^2d^2\log(a+bx)}{2b(bc-ad)^2g^3} - \frac{B^2d^2\log^2(a+bx\cdots}{2b(bc-ad)^2g^3}$$

**Mathematica [C]** time = 0.46, size = 443, normalized size = 1.65

$$-\frac{B\left(-4d^2(a+bx)^2\log(a+bx)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)+4d^2(a+bx)^2\log(c+dx)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)+2(bc-ad)^2\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)+4d(a+bx)(ad-bc)\left(B\log\left(\right.\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Log[(e*(a + b*x))/(c + d*x)])^2/(a*g + b*g*x)^3,x]
```

```
[Out] -1/4*(2*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2 + (B*(2*(b*c - a*d)^2*(A + B
*Log[(e*(a + b*x))/(c + d*x)]) + 4*d*(-(b*c) + a*d)*(a + b*x)*(A + B*Log[(e
*(a + b*x))/(c + d*x)]) - 4*d^2*(a + b*x)^2*Log[a + b*x]*(A + B*Log[(e*(a +
 b*x))/(c + d*x)]) + 4*d^2*(a + b*x)^2*(A + B*Log[(e*(a + b*x))/(c + d*x)])
*Log[c + d*x] - 4*B*d*(a + b*x)*(b*c - a*d + d*(a + b*x)*Log[a + b*x] - d*(
a + b*x)*Log[c + d*x]) + B*((b*c - a*d)^2 + 2*d*(-(b*c) + a*d)*(a + b*x) -
2*d^2*(a + b*x)^2*Log[a + b*x] + 2*d^2*(a + b*x)^2*Log[c + d*x]) + 2*B*d^2*
(a + b*x)^2*(Log[a + b*x]*(Log[a + b*x] - 2*Log[(b*(c + d*x))/(b*c - a*d)])
 - 2*PolyLog[2, (d*(a + b*x))/(-(b*c) + a*d)]) - 2*B*d^2*(a + b*x)^2*((2*Lo
g[(d*(a + b*x))/(-(b*c) + a*d)] - Log[c + d*x])*Log[c + d*x] + 2*PolyLog[2,
 (b*(c + d*x))/(b*c - a*d)])))/(b*c - a*d)^2)/(b*g^3*(a + b*x)^2)
```

**fricas [A]** time = 0.57, size = 367, normalized size = 1.37

$$-\frac{\left(2A^2+2AB+B^2\right)b^2c^2-4\left(A^2+2AB+2B^2\right)abcd+\left(2A^2+6AB+7B^2\right)a^2d^2-2\left(B^2b^2d^2x^2+2B^2abd^2x\right.}{4\left(\left(b^5c^2\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*(b*x+a)/(d*x+c)))^2/(b*g*x+a*g)^3,x, algorithm="fricas
")
```

```
[Out] -1/4*((2*A^2 + 2*A*B + B^2)*b^2*c^2 - 4*(A^2 + 2*A*B + 2*B^2)*a*b*c*d + (2*
A^2 + 6*A*B + 7*B^2)*a^2*d^2 - 2*(B^2*b^2*d^2*x^2 + 2*B^2*a*b*d^2*x - B^2*b
^2*c^2 + 2*B^2*a*b*c*d)*log((b*e*x + a*e)/(d*x + c))^2 - 2*((2*A*B + 3*B^2)
*b^2*c*d - (2*A*B + 3*B^2)*a*b*d^2)*x - 2*((2*A*B + 3*B^2)*b^2*d^2*x^2 - (2
*A*B + B^2)*b^2*c^2 + 4*(A*B + B^2)*a*b*c*d + 2*(B^2*b^2*c*d + 2*(A*B + B^2
)*a*b*d^2)*x)*log((b*e*x + a*e)/(d*x + c)))/((b^5*c^2 - 2*a*b^4*c*d + a^2*b
^3*d^2)*g^3*x^2 + 2*(a*b^4*c^2 - 2*a^2*b^3*c*d + a^3*b^2*d^2)*g^3*x + (a^2*
b^3*c^2 - 2*a^3*b^2*c*d + a^4*b*d^2)*g^3)
```

**giac [A]** time = 1.75, size = 424, normalized size = 1.58

$$-\frac{\left(2\,B^2be^3\log\left(\frac{bxe+ae}{dx+c}\right)^2-\frac{4\,(bxe+ae)B^2de^2\log\left(\frac{bxe+ae}{dx+c}\right)^2}{dx+c}+4\,ABbe^3\log\left(\frac{bxe+ae}{dx+c}\right)+2\,B^2be^3\log\left(\frac{bxe+ae}{dx+c}\right)-\frac{8\,(bxe+ae)ABde^2\log\left(\cdots\right.}{dx+c}\cdots\right.}{4\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*(b*x+a)/(d*x+c)))^2/(b*g*x+a*g)^3,x, algorithm="giac")
```

```
[Out] -1/4*(2*B^2*b*e^3*log((b*x*e + a*e)/(d*x + c))^2 - 4*(b*x*e + a*e)*B^2*d*e^
2*log((b*x*e + a*e)/(d*x + c))^2/(d*x + c) + 4*A*B*b*e^3*log((b*x*e + a*e)/
(d*x + c)) + 2*B^2*b*e^3*log((b*x*e + a*e)/(d*x + c)) - 8*(b*x*e + a*e)*A*B
*d*e^2*log((b*x*e + a*e)/(d*x + c))/(d*x + c) - 8*(b*x*e + a*e)*B^2*d*e^2*l
og((b*x*e + a*e)/(d*x + c))/(d*x + c) + 2*A^2*b*e^3 + 2*A*B*b*e^3 + B^2*b*e
^3 - 4*(b*x*e + a*e)*A^2*d*e^2/(d*x + c) - 8*(b*x*e + a*e)*A*B*d*e^2/(d*x +
 c) - 8*(b*x*e + a*e)*B^2*d*e^2/(d*x + c))*(b*c/((b*c*e - a*d*e)*(b*c - a*d
)) - a*d/((b*c*e - a*d*e)*(b*c - a*d)))/((b*x*e + a*e)^2*b*c*g^3/(d*x + c)^
2 - (b*x*e + a*e)^2*a*d*g^3/(d*x + c)^2)
```

**maple [B]** time = 0.05, size = 1715, normalized size = 6.40

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*ln((b*x+a)/(d*x+c)*e)+A)^2/(b*g*x+a*g)^3,x)
```

```
[Out] d^2*e/(a*d-b*c)^3/g^3*A^2/(1/(d*x+c)*a*e-1/(d*x+c)*b*c/d*e+b/d*e)*a-d*e/(a*
d-b*c)^3/g^3*A^2/(1/(d*x+c)*a*e-1/(d*x+c)*b*c/d*e+b/d*e)*b*c-1/2*d*e^2/(a*d
-b*c)^3/g^3*A^2*b/(1/(d*x+c)*a*e-1/(d*x+c)*b*c/d*e+b/d*e)^2*a+1/2*e^2/(a*d-
b*c)^3/g^3*A^2*b^2/(1/(d*x+c)*a*e-1/(d*x+c)*b*c/d*e+b/d*e)^2*c+2*d^2*e/(a*d
-b*c)^3/g^3*A*B/(1/(d*x+c)*a*e-1/(d*x+c)*b*c/d*e+b/d*e)*ln(b/d*e+(a*d-b*c)/
(d*x+c)/d*e)*a-2*d*e/(a*d-b*c)^3/g^3*A*B/(1/(d*x+c)*a*e-1/(d*x+c)*b*c/d*e+b
/d*e)*ln(b/d*e+(a*d-b*c)/(d*x+c)/d*e)*b*c+2*d^2*e/(a*d-b*c)^3/g^3*A*B/(1/(d
*x+c)*a*e-1/(d*x+c)*b*c/d*e+b/d*e)*a-2*d*e/(a*d-b*c)^3/g^3*A*B/(1/(d*x+c)*a
*e-1/(d*x+c)*b*c/d*e+b/d*e)*b*c-d*e^2/(a*d-b*c)^3/g^3*A*B*b/(1/(d*x+c)*a*e-
1/(d*x+c)*b*c/d*e+b/d*e)^2*ln(b/d*e+(a*d-b*c)/(d*x+c)/d*e)*a+e^2/(a*d-b*c)^
3/g^3*A*B*b^2/(1/(d*x+c)*a*e-1/(d*x+c)*b*c/d*e+b/d*e)^2*ln(b/d*e+(a*d-b*c)/
(d*x+c)/d*e)*c-1/2*d*e^2/(a*d-b*c)^3/g^3*A*B*b/(1/(d*x+c)*a*e-1/(d*x+c)*b*c
/d*e+b/d*e)^2*a+1/2*e^2/(a*d-b*c)^3/g^3*A*B*b^2/(1/(d*x+c)*a*e-1/(d*x+c)*b*
c/d*e+b/d*e)^2*c+d^2*e/(a*d-b*c)^3/g^3*B^2/(1/(d*x+c)*a*e-1/(d*x+c)*b*c/d*e
+b/d*e)*ln(b/d*e+(a*d-b*c)/(d*x+c)/d*e)^2*a-d*e/(a*d-b*c)^3/g^3*B^2/(1/(d*x
+c)*a*e-1/(d*x+c)*b*c/d*e+b/d*e)*ln(b/d*e+(a*d-b*c)/(d*x+c)/d*e)^2*b*c+2*d^
```

```
2*e/(a*d-b*c)^3/g^3*B^2/(1/(d*x+c)*a*e-1/(d*x+c)*b*c/d*e+b/d*e)*ln(b/d*e+(a
*d-b*c)/(d*x+c)/d*e)*a-2*d*e/(a*d-b*c)^3/g^3*B^2/(1/(d*x+c)*a*e-1/(d*x+c)*b
*c/d*e+b/d*e)*ln(b/d*e+(a*d-b*c)/(d*x+c)/d*e)*b*c+2*d^2*e/(a*d-b*c)^3/g^3*B
^2/(1/(d*x+c)*a*e-1/(d*x+c)*b*c/d*e+b/d*e)*a-2*d*e/(a*d-b*c)^3/g^3*B^2/(1/(
d*x+c)*a*e-1/(d*x+c)*b*c/d*e+b/d*e)*b*c-1/2*d*e^2/(a*d-b*c)^3/g^3*B^2*b/(1/
(d*x+c)*a*e-1/(d*x+c)*b*c/d*e+b/d*e)^2*ln(b/d*e+(a*d-b*c)/(d*x+c)/d*e)^2*a+
1/2*e^2/(a*d-b*c)^3/g^3*B^2*b^2/(1/(d*x+c)*a*e-1/(d*x+c)*b*c/d*e+b/d*e)^2*l
n(b/d*e+(a*d-b*c)/(d*x+c)/d*e)^2*c-1/2*d*e^2/(a*d-b*c)^3/g^3*B^2*b/(1/(d*x+
c)*a*e-1/(d*x+c)*b*c/d*e+b/d*e)^2*ln(b/d*e+(a*d-b*c)/(d*x+c)/d*e)*a+1/2*e^2
/(a*d-b*c)^3/g^3*B^2*b^2/(1/(d*x+c)*a*e-1/(d*x+c)*b*c/d*e+b/d*e)^2*ln(b/d*e
+(a*d-b*c)/(d*x+c)/d*e)*c-1/4*d*e^2/(a*d-b*c)^3/g^3*B^2*b/(1/(d*x+c)*a*e-1/
(d*x+c)*b*c/d*e+b/d*e)^2*a+1/4*e^2/(a*d-b*c)^3/g^3*B^2*b^2/(1/(d*x+c)*a*e-1
/(d*x+c)*b*c/d*e+b/d*e)^2*c
```

**maxima [B]** time = 1.81, size = 848, normalized size = 3.16

$$\frac{1}{4}\left(2\left(\frac{2\,bdx - bc + 3\,ad}{\left(b^4c - ab^3d\right)g^3x^2 + 2\left(ab^3c - a^2b^2d\right)g^3x + \left(a^2b^2c - a^3bd\right)g^3} + \frac{2\,d^2\log\left(bx + a\right)}{\left(b^3c^2 - 2\,ab^2cd + a^2bd^2\right)g^3} - \frac{2\,d^2\log}{\left(b^3c^2 - 2\,ab^2\right.}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*(b*x+a)/(d*x+c)))^2/(b*g*x+a*g)^3,x, algorithm="maxima
")
```

```
[Out] 1/4*(2*((2*b*d*x - b*c + 3*a*d)/((b^4*c - a*b^3*d)*g^3*x^2 + 2*(a*b^3*c - a
^2*b^2*d)*g^3*x + (a^2*b^2*c - a^3*b*d)*g^3) + 2*d^2*log(b*x + a)/((b^3*c^2
 - 2*a*b^2*c*d + a^2*b*d^2)*g^3) - 2*d^2*log(d*x + c)/((b^3*c^2 - 2*a*b^2*c
*d + a^2*b*d^2)*g^3))*log(b*e*x/(d*x + c) + a*e/(d*x + c)) - (b^2*c^2 - 8*a
*b*c*d + 7*a^2*d^2 + 2*(b^2*d^2*x^2 + 2*a*b*d^2*x + a^2*d^2)*log(b*x + a)^2
 + 2*(b^2*d^2*x^2 + 2*a*b*d^2*x + a^2*d^2)*log(d*x + c)^2 - 6*(b^2*c*d - a*
b*d^2)*x - 6*(b^2*d^2*x^2 + 2*a*b*d^2*x + a^2*d^2)*log(b*x + a) + 2*(3*b^2*
d^2*x^2 + 6*a*b*d^2*x + 3*a^2*d^2 - 2*(b^2*d^2*x^2 + 2*a*b*d^2*x + a^2*d^2)
*log(b*x + a))*log(d*x + c))/(a^2*b^3*c^2*g^3 - 2*a^3*b^2*c*d*g^3 + a^4*b*d
^2*g^3 + (b^5*c^2*g^3 - 2*a*b^4*c*d*g^3 + a^2*b^3*d^2*g^3)*x^2 + 2*(a*b^4*c
^2*g^3 - 2*a^2*b^3*c*d*g^3 + a^3*b^2*d^2*g^3)*x))*B^2 + 1/2*A*B*((2*b*d*x -
 b*c + 3*a*d)/((b^4*c - a*b^3*d)*g^3*x^2 + 2*(a*b^3*c - a^2*b^2*d)*g^3*x +
(a^2*b^2*c - a^3*b*d)*g^3) - 2*log(b*e*x/(d*x + c) + a*e/(d*x + c))/(b^3*g^
3*x^2 + 2*a*b^2*g^3*x + a^2*b*g^3) + 2*d^2*log(b*x + a)/((b^3*c^2 - 2*a*b^2
*c*d + a^2*b*d^2)*g^3) - 2*d^2*log(d*x + c)/((b^3*c^2 - 2*a*b^2*c*d + a^2*b
*d^2)*g^3)) - 1/2*B^2*log(b*e*x/(d*x + c) + a*e/(d*x + c))^2/(b^3*g^3*x^2 +
 2*a*b^2*g^3*x + a^2*b*g^3) - 1/2*A^2/(b^3*g^3*x^2 + 2*a*b^2*g^3*x + a^2*b*
g^3)
```

**mupad [B]** time = 5.85, size = 507, normalized size = 1.89

$$-\frac{\frac{2\,A^2\,a\,d-2\,A^2\,b\,c+7\,B^2\,a\,d-B^2\,b\,c+6\,A\,B\,a\,d-2\,A\,B\,b\,c}{2\,(a\,d-b\,c)}+\frac{x\,\left(3\,b\,d\,B^2+2\,A\,b\,d\,B\right)}{a\,d-b\,c}}{2\,a^2\,b\,g^3+4\,a\,b^2\,g^3\,x+2\,b^3\,g^3\,x^2}-\ln\left(\frac{e\,\left(a+b\,x\right)}{c+d\,x}\right)^2\,\left(\frac{B^2}{2\,b^2\,g^3\,\left(2\,a\,x+b\,x^2+\frac{a^2}{b}\right)}-\frac{}{2\,b}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*log((e*(a + b*x))/(c + d*x)))^2/(a*g + b*g*x)^3,x)
```

```
[Out] - ((2*A^2*a*d - 2*A^2*b*c + 7*B^2*a*d - B^2*b*c + 6*A*B*a*d - 2*A*B*b*c)/(2
*(a*d - b*c)) + (x*(3*B^2*b*d + 2*A*B*b*d))/(a*d - b*c))/(2*a^2*b*g^3 + 2*b
^3*g^3*x^2 + 4*a*b^2*g^3*x) - log((e*(a + b*x))/(c + d*x))^2*(B^2/(2*b^2*g^
3*(2*a*x + b*x^2 + a^2/b)) - (B^2*d^2)/(2*b*g^3*(a^2*d^2 + b^2*c^2 - 2*a*b*
c*d))) - (log((e*(a + b*x))/(c + d*x))*((A*B)/(b^2*d*g^3) + (B^2*x*(a*d - b
*c))/(b*g^3*(a^2*d^2 + b^2*c^2 - 2*a*b*c*d)) + (B^2*d^2*((2*a^2*d^2 + b^2*c
^2 - 3*a*b*c*d)/(2*b*d^3) + (a*(a*d - b*c))/(2*b*d^2)))/(b*g^3*(a^2*d^2 + b
^2*c^2 - 2*a*b*c*d))))/((b*x^2)/d + a^2/(b*d) + (2*a*x)/d) - (B*d^2*atan((B
*d^2*(2*b*d*x - (b^3*c^2*g^3 - a^2*b*d^2*g^3)/(b*g^3*(a*d - b*c)))*(2*A + 3
*B)*1i)/((a*d - b*c)*(3*B^2*d^2 + 2*A*B*d^2)))*(2*A + 3*B)*1i)/(b*g^3*(a*d
- b*c)^2)
```

**sympy [B]** time = 6.55, size = 894, normalized size = 3.34

$$-\frac{Bd^2\left(2A+3B\right)\log\left(x+\frac{2ABad^3+2ABbcd^2+3B^2ad^3+3B^2bcd^2-\frac{Ba^3d^5(2A+3B)}{(ad-bc)^2}+\frac{3Ba^2bcd^4(2A+3B)}{(ad-bc)^2}-\frac{3Bab^2c^2d^3(2A+3B)}{(ad-bc)^2}+\frac{Bb^3c^3d^2(2A+3B)}{(ad-bc)^2}}{4ABbd^3+6B^2bd^3}\right)}{2bg^3\left(ad-bc\right)^2}+\frac{Bd^2\,(}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*ln(e*(b*x+a)/(d*x+c)))**2/(b*g*x+a*g)**3,x)
```

```
[Out] -B*d**2*(2*A + 3*B)*log(x + (2*A*B*a*d**3 + 2*A*B*b*c*d**2 + 3*B**2*a*d**3
+ 3*B**2*b*c*d**2 - B*a**3*d**5*(2*A + 3*B)/(a*d - b*c)**2 + 3*B*a**2*b*c*d
**4*(2*A + 3*B)/(a*d - b*c)**2 - 3*B*a*b**2*c**2*d**3*(2*A + 3*B)/(a*d - b*
c)**2 + B*b**3*c**3*d**2*(2*A + 3*B)/(a*d - b*c)**2)/(4*A*B*b*d**3 + 6*B**2
*b*d**3))/(2*b*g**3*(a*d - b*c)**2) + B*d**2*(2*A + 3*B)*log(x + (2*A*B*a*d
**3 + 2*A*B*b*c*d**2 + 3*B**2*a*d**3 + 3*B**2*b*c*d**2 + B*a**3*d**5*(2*A +
 3*B)/(a*d - b*c)**2 - 3*B*a**2*b*c*d**4*(2*A + 3*B)/(a*d - b*c)**2 + 3*B*a
*b**2*c**2*d**3*(2*A + 3*B)/(a*d - b*c)**2 - B*b**3*c**3*d**2*(2*A + 3*B)/(
a*d - b*c)**2)/(4*A*B*b*d**3 + 6*B**2*b*d**3))/(2*b*g**3*(a*d - b*c)**2) +
(2*B**2*a*c*d + 2*B**2*a*d**2*x - B**2*b*c**2 + B**2*b*d**2*x**2)*log(e*(a
```

```
+ b*x)/(c + d*x))**2/(2*a**4*d**2*g**3 - 4*a**3*b*c*d*g**3 + 4*a**3*b*d**2*
g**3*x + 2*a**2*b**2*c**2*g**3 - 8*a**2*b**2*c*d*g**3*x + 2*a**2*b**2*d**2*
g**3*x**2 + 4*a*b**3*c**2*g**3*x - 4*a*b**3*c*d*g**3*x**2 + 2*b**4*c**2*g**
3*x**2) + (-2*A*B*a*d + 2*A*B*b*c - 3*B**2*a*d + B**2*b*c - 2*B**2*b*d*x)*l
og(e*(a + b*x)/(c + d*x))/(2*a**3*b*d*g**3 - 2*a**2*b**2*c*g**3 + 4*a**2*b*
*2*d*g**3*x - 4*a*b**3*c*g**3*x + 2*a*b**3*d*g**3*x**2 - 2*b**4*c*g**3*x**2
) + (-2*A**2*a*d + 2*A**2*b*c - 6*A*B*a*d + 2*A*B*b*c - 7*B**2*a*d + B**2*b
*c + x*(-4*A*B*b*d - 6*B**2*b*d))/(4*a**3*b*d*g**3 - 4*a**2*b**2*c*g**3 + x
**2*(4*a*b**3*d*g**3 - 4*b**4*c*g**3) + x*(8*a**2*b**2*d*g**3 - 8*a*b**3*c*
g**3))
```

# 3.104 $\int \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)^4}\,dx$

**Optimal**. Leaf size=418

$$-\frac{b^2(c+dx)^3\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2}{3g^4(a+bx)^3(bc-ad)^3}-\frac{2b^2B(c+dx)^3\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{9g^4(a+bx)^3(bc-ad)^3}-\frac{d^2(c+dx)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2}{g^4(a+bx)(bc-ad)^3}-\frac{2Ba}{}$$

```
[Out] -2*B^2*d^2*(d*x+c)/(-a*d+b*c)^3/g^4/(b*x+a)+1/2*b*B^2*d*(d*x+c)^2/(-a*d+b*c
)^3/g^4/(b*x+a)^2-2/27*b^2*B^2*(d*x+c)^3/(-a*d+b*c)^3/g^4/(b*x+a)^3-2*B*d^2
*(d*x+c)*(A+B*ln(e*(b*x+a)/(d*x+c)))/(-a*d+b*c)^3/g^4/(b*x+a)+b*B*d*(d*x+c)
^2*(A+B*ln(e*(b*x+a)/(d*x+c)))/(-a*d+b*c)^3/g^4/(b*x+a)^2-2/9*b^2*B*(d*x+c)
^3*(A+B*ln(e*(b*x+a)/(d*x+c)))/(-a*d+b*c)^3/g^4/(b*x+a)^3-d^2*(d*x+c)*(A+B*
ln(e*(b*x+a)/(d*x+c)))^2/(-a*d+b*c)^3/g^4/(b*x+a)+b*d*(d*x+c)^2*(A+B*ln(e*(
b*x+a)/(d*x+c)))^2/(-a*d+b*c)^3/g^4/(b*x+a)^2-1/3*b^2*(d*x+c)^3*(A+B*ln(e*(
b*x+a)/(d*x+c)))^2/(-a*d+b*c)^3/g^4/(b*x+a)^3
```

**Rubi [C]** time = 1.06, antiderivative size = 680, normalized size of antiderivative = 1.63, number of steps used = 34, number of rules used = 11, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.344, Rules used = {2525, 12, 2528, 44, 2524, 2418, 2390, 2301, 2394, 2393, 2391}

$$-\frac{2B^2d^3\text{PolyLog}\left(2,-\frac{d(a+bx)}{bc-ad}\right)}{3bg^4(bc-ad)^3}-\frac{2B^2d^3\text{PolyLog}\left(2,\frac{b(c+dx)}{bc-ad}\right)}{3bg^4(bc-ad)^3}-\frac{2Bd^3\log(a+bx)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{3bg^4(bc-ad)^3}+\frac{2Bd^3\log(c+}{3}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Log[(e*(a + b*x))/(c + d*x)])^2/(a*g + b*g*x)^4,x]
```

```
[Out] (-2*B^2)/(27*b*g^4*(a + b*x)^3) + (5*B^2*d)/(18*b*(b*c - a*d)*g^4*(a + b*x)
^2) - (11*B^2*d^2)/(9*b*(b*c - a*d)^2*g^4*(a + b*x)) - (11*B^2*d^3*Log[a +
b*x])/(9*b*(b*c - a*d)^3*g^4) + (B^2*d^3*Log[a + b*x]^2)/(3*b*(b*c - a*d)^3
*g^4) - (2*B*(A + B*Log[(e*(a + b*x))/(c + d*x)]))/(9*b*g^4*(a + b*x)^3) +
(B*d*(A + B*Log[(e*(a + b*x))/(c + d*x)]))/(3*b*(b*c - a*d)*g^4*(a + b*x)^2
) - (2*B*d^2*(A + B*Log[(e*(a + b*x))/(c + d*x)]))/(3*b*(b*c - a*d)^2*g^4*(
a + b*x)) - (2*B*d^3*Log[a + b*x]*(A + B*Log[(e*(a + b*x))/(c + d*x)]))/(3*
b*(b*c - a*d)^3*g^4) - (A + B*Log[(e*(a + b*x))/(c + d*x)])^2/(3*b*g^4*(a +
 b*x)^3) + (11*B^2*d^3*Log[c + d*x])/(9*b*(b*c - a*d)^3*g^4) - (2*B^2*d^3*L
og[-((d*(a + b*x))/(b*c - a*d))]*Log[c + d*x])/(3*b*(b*c - a*d)^3*g^4) + (2
*B*d^3*(A + B*Log[(e*(a + b*x))/(c + d*x)])*Log[c + d*x])/(3*b*(b*c - a*d)^
3*g^4) + (B^2*d^3*Log[c + d*x]^2)/(3*b*(b*c - a*d)^3*g^4) - (2*B^2*d^3*Log[
a + b*x]*Log[(b*(c + d*x))/(b*c - a*d)])/(3*b*(b*c - a*d)^3*g^4) - (2*B^2*d
```

```
^3*PolyLog[2, -((d*(a + b*x))/(b*c - a*d))])/(3*b*(b*c - a*d)^3*g^4) - (2*B
^2*d^3*PolyLog[2, (b*(c + d*x))/(b*c - a*d)])/(3*b*(b*c - a*d)^3*g^4)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 2301

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

Rule 2390

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_))^(q_.), x_Symbol] :> Dist[1/e, Subst[Int[((f*x)/d)^q*(a + b*Log[c*x^
n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && E
qQ[e*f - d*g, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2393

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + (c*e*x)/g])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2394

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d + e*x
)^n]))/g, x] - Dist[(b*e*n)/g, Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2418

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*(RFx_), x_Sy
mbol] :> With[{u = ExpandIntegrand[(a + b*Log[c*(d + e*x)^n])^p, RFx, x]},
Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d, e, n}, x] && RationalFunctionQ[
RFx, x] && IntegerQ[p]
```

Rule 2524

```
Int[((a_.) + Log[(c_.)*(RFx_)^(p_.)]*(b_.))^(n_.)/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[(Log[d + e*x]*(a + b*Log[c*RFx^p])^n)/e, x] - Dist[(b*n*p)/e
, Int[(Log[d + e*x]*(a + b*Log[c*RFx^p])^(n - 1)*D[RFx, x])/RFx, x], x] /;
FreeQ[{a, b, c, d, e, p}, x] && RationalFunctionQ[RFx, x] && IGtQ[n, 0]
```

Rule 2525

```
Int[((a_.) + Log[(c_.)*(RFx_)^(p_.)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_))^(m_.
), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(a + b*Log[c*RFx^p])^n)/(e*(m + 1))
, x] - Dist[(b*n*p)/(e*(m + 1)), Int[SimplifyIntegrand[((d + e*x)^(m + 1)*(
a + b*Log[c*RFx^p])^(n - 1)*D[RFx, x])/RFx, x], x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && RationalFunctionQ[RFx, x] && IGtQ[n, 0] && (EqQ[n, 1] ||
IntegerQ[m]) && NeQ[m, -1]
```

Rule 2528

```
Int[((a_.) + Log[(c_.)*(RFx_)^(p_.)]*(b_.))^(n_.)*(RGx_), x_Symbol] :> With
[{u = ExpandIntegrand[(a + b*Log[c*RFx^p])^n, RGx, x]}, Int[u, x] /; SumQ[u
]] /; FreeQ[{a, b, c, p}, x] && RationalFunctionQ[RFx, x] && RationalFuncti
onQ[RGx, x] && IGtQ[n, 0]
```

Rubi steps

$$\int \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)^4}\,dx = -\frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{3bg^4(a+bx)^3} + \frac{(2B)\int \frac{(bc-ad)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{g^3(a+bx)^4(c+dx)}\,dx}{3bg}$$

$$= -\frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{3bg^4(a+bx)^3} + \frac{(2B(bc-ad))\int \frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{(a+bx)^4(c+dx)}\,dx}{3bg^4}$$

$$= -\frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{3bg^4(a+bx)^3} + \frac{(2B(bc-ad))\int \left(\frac{b\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(bc-ad)(a+bx)^4} - \frac{bd\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(bc-ad)^2(a+bx)^3}\right.}{}$$

$$= -\frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{3bg^4(a+bx)^3} + \frac{(2B)\int \frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{(a+bx)^4}\,dx}{3g^4} - \frac{\left(2Bd^3\right)\int \frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{a+bx}\,dx}{3(bc-ad)^3g^4}$$

$$= -\frac{2B\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{9bg^4(a+bx)^3} + \frac{Bd\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{3b(bc-ad)g^4(a+bx)^2} - \frac{2Bd^2\left(A+B\log\left(\frac{e(a+bx}{c+dx}\right.\right.}{3b(bc-ad)^2g^4(a+bx}$$

$$= -\frac{2B\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{9bg^4(a+bx)^3} + \frac{Bd\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{3b(bc-ad)g^4(a+bx)^2} - \frac{2Bd^2\left(A+B\log\left(\frac{e(a+bx}{c+dx}\right.\right.}{3b(bc-ad)^2g^4(a+bx}$$

$$= -\frac{2B\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{9bg^4(a+bx)^3} + \frac{Bd\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{3b(bc-ad)g^4(a+bx)^2} - \frac{2Bd^2\left(A+B\log\left(\frac{e(a+bx}{c+dx}\right.\right.}{3b(bc-ad)^2g^4(a+bx}$$

$$= -\frac{2B^2}{27bg^4(a+bx)^3} + \frac{5B^2d}{18b(bc-ad)g^4(a+bx)^2} - \frac{11B^2d^2}{9b(bc-ad)^2g^4(a+bx)} - \frac{11B^2d^3}{9b(bc}$$

$$= -\frac{2B^2}{27bg^4(a+bx)^3} + \frac{5B^2d}{18b(bc-ad)g^4(a+bx)^2} - \frac{11B^2d^2}{9b(bc-ad)^2g^4(a+bx)} - \frac{11B^2d^3}{9b(bc}$$

$$= -\frac{2B^2}{27bg^4(a+bx)^3} + \frac{5B^2d}{18b(bc-ad)g^4(a+bx)^2} - \frac{11B^2d^2}{9b(bc-ad)^2g^4(a+bx)} - \frac{11B^2d^3}{9b(bc}$$

$$= -\frac{2B^2}{27bg^4(a+bx)^3} + \frac{5B^2d}{18b(bc-ad)g^4(a+bx)^2} - \frac{11B^2d^2}{9b(bc-ad)^2g^4(a+bx)} - \frac{11B^2d^3}{9b(bc}$$

**Mathematica [C]** time = 0.69, size = 585, normalized size = 1.40

$$-\frac{B\left(36d^3(a+bx)^3\log(a+bx)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)-36d^3(a+bx)^3\log(c+dx)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)+36d^2(a+bx)^2(bc-ad)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)+12(bc-ad)^3\left(B\log\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Log[(e*(a + b*x))/(c + d*x)])^2/(a*g + b*g*x)^4,x]
```

```
[Out] -1/54*(18*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2 + (B*(12*(b*c - a*d)^3*(A
+ B*Log[(e*(a + b*x))/(c + d*x)]) - 18*d*(b*c - a*d)^2*(a + b*x)*(A + B*Log
[(e*(a + b*x))/(c + d*x)]) + 36*d^2*(b*c - a*d)*(a + b*x)^2*(A + B*Log[(e*(
a + b*x))/(c + d*x)]) + 36*d^3*(a + b*x)^3*Log[a + b*x]*(A + B*Log[(e*(a +
b*x))/(c + d*x)]) - 36*d^3*(a + b*x)^3*(A + B*Log[(e*(a + b*x))/(c + d*x)])
*Log[c + d*x] + 36*B*d^2*(a + b*x)^2*(b*c - a*d + d*(a + b*x)*Log[a + b*x]
- d*(a + b*x)*Log[c + d*x]) - 9*B*d*(a + b*x)*((b*c - a*d)^2 + 2*d*(-(b*c)
+ a*d)*(a + b*x) - 2*d^2*(a + b*x)^2*Log[a + b*x] + 2*d^2*(a + b*x)^2*Log[c
 + d*x]) + 2*B*(2*(b*c - a*d)^3 - 3*d*(b*c - a*d)^2*(a + b*x) + 6*d^2*(b*c
- a*d)*(a + b*x)^2 + 6*d^3*(a + b*x)^3*Log[a + b*x] - 6*d^3*(a + b*x)^3*Log
[c + d*x]) - 18*B*d^3*(a + b*x)^3*(Log[a + b*x]*(Log[a + b*x] - 2*Log[(b*(c
 + d*x))/(b*c - a*d)]) - 2*PolyLog[2, (d*(a + b*x))/(-(b*c) + a*d)]) + 18*B
*d^3*(a + b*x)^3*((2*Log[(d*(a + b*x))/(-(b*c) + a*d)] - Log[c + d*x])*Log[
c + d*x] + 2*PolyLog[2, (b*(c + d*x))/(b*c - a*d)])))/(b*c - a*d)^3)/(b*g^4
*(a + b*x)^3)
```

**fricas [A]** time = 0.69, size = 672, normalized size = 1.61

$$-\frac{2\left(9A^2+6AB+2B^2\right)b^3c^3-27\left(2A^2+2AB+B^2\right)ab^2c^2d+54\left(A^2+2AB+2B^2\right)a^2bcd^2-\left(18A^2+66AB+\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*(b*x+a)/(d*x+c)))^2/(b*g*x+a*g)^4,x, algorithm="fricas
")
```

```
[Out] -1/54*(2*(9*A^2 + 6*A*B + 2*B^2)*b^3*c^3 - 27*(2*A^2 + 2*A*B + B^2)*a*b^2*c
^2*d + 54*(A^2 + 2*A*B + 2*B^2)*a^2*b*c*d^2 - (18*A^2 + 66*A*B + 85*B^2)*a^
3*d^3 + 6*((6*A*B + 11*B^2)*b^3*c*d^2 - (6*A*B + 11*B^2)*a*b^2*d^3)*x^2 + 1
8*(B^2*b^3*d^3*x^3 + 3*B^2*a*b^2*d^3*x^2 + 3*B^2*a^2*b*d^3*x + B^2*b^3*c^3
- 3*B^2*a*b^2*c^2*d + 3*B^2*a^2*b*c*d^2)*log((b*e*x + a*e)/(d*x + c))^2 - 3
*((6*A*B + 5*B^2)*b^3*c^2*d - 18*(2*A*B + 3*B^2)*a*b^2*c*d^2 + (30*A*B + 49
*B^2)*a^2*b*d^3)*x + 6*((6*A*B + 11*B^2)*b^3*d^3*x^3 + 2*(3*A*B + B^2)*b^3*
c^3 - 9*(2*A*B + B^2)*a*b^2*c^2*d + 18*(A*B + B^2)*a^2*b*c*d^2 + 3*(2*B^2*b
^3*c*d^2 + 3*(2*A*B + 3*B^2)*a*b^2*d^3)*x^2 - 3*(B^2*b^3*c^2*d - 6*B^2*a*b^
```

```
2*c*d^2 - 6*(A*B + B^2)*a^2*b*d^3)*x)*log((b*e*x + a*e)/(d*x + c)))/((b^7*c
^3 - 3*a*b^6*c^2*d + 3*a^2*b^5*c*d^2 - a^3*b^4*d^3)*g^4*x^3 + 3*(a*b^6*c^3
- 3*a^2*b^5*c^2*d + 3*a^3*b^4*c*d^2 - a^4*b^3*d^3)*g^4*x^2 + 3*(a^2*b^5*c^3
 - 3*a^3*b^4*c^2*d + 3*a^4*b^3*c*d^2 - a^5*b^2*d^3)*g^4*x + (a^3*b^4*c^3 -
3*a^4*b^3*c^2*d + 3*a^5*b^2*c*d^2 - a^6*b*d^3)*g^4)
```

**giac [A]** time = 1.89, size = 709, normalized size = 1.70

$$-\frac{\left(18\,B^2b^2e^4\log\left(\frac{bxe+ae}{dx+c}\right)^2-\frac{54\,(bxe+ae)B^2bde^3\log\left(\frac{bxe+ae}{dx+c}\right)^2}{dx+c}+\frac{54\,(bxe+ae)^2B^2d^2e^2\log\left(\frac{bxe+ae}{dx+c}\right)^2}{(dx+c)^2}+36\,ABb^2e^4\log\left(\frac{bxe+ae}{dx+c}\right)+12\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*(b*x+a)/(d*x+c)))^2/(b*g*x+a*g)^4,x, algorithm="giac")
```

```
[Out] -1/54*(18*B^2*b^2*e^4*log((b*x*e + a*e)/(d*x + c))^2 - 54*(b*x*e + a*e)*B^2
*b*d*e^3*log((b*x*e + a*e)/(d*x + c))^2/(d*x + c) + 54*(b*x*e + a*e)^2*B^2*
d^2*e^2*log((b*x*e + a*e)/(d*x + c))^2/(d*x + c)^2 + 36*A*B*b^2*e^4*log((b*
x*e + a*e)/(d*x + c)) + 12*B^2*b^2*e^4*log((b*x*e + a*e)/(d*x + c)) - 108*(
b*x*e + a*e)*A*B*b*d*e^3*log((b*x*e + a*e)/(d*x + c))/(d*x + c) - 54*(b*x*e
 + a*e)*B^2*b*d*e^3*log((b*x*e + a*e)/(d*x + c))/(d*x + c) + 108*(b*x*e + a
*e)^2*A*B*d^2*e^2*log((b*x*e + a*e)/(d*x + c))/(d*x + c)^2 + 108*(b*x*e + a
*e)^2*B^2*d^2*e^2*log((b*x*e + a*e)/(d*x + c))/(d*x + c)^2 + 18*A^2*b^2*e^4
 + 12*A*B*b^2*e^4 + 4*B^2*b^2*e^4 - 54*(b*x*e + a*e)*A^2*b*d*e^3/(d*x + c)
- 54*(b*x*e + a*e)*A*B*b*d*e^3/(d*x + c) - 27*(b*x*e + a*e)*B^2*b*d*e^3/(d*
x + c) + 54*(b*x*e + a*e)^2*A^2*d^2*e^2/(d*x + c)^2 + 108*(b*x*e + a*e)^2*A
*B*d^2*e^2/(d*x + c)^2 + 108*(b*x*e + a*e)^2*B^2*d^2*e^2/(d*x + c)^2)*(b*c/
((b*c*e - a*d*e)*(b*c - a*d)) - a*d/((b*c*e - a*d*e)*(b*c - a*d)))/((b*x*e
+ a*e)^3*b^2*c^2*g^4/(d*x + c)^3 - 2*(b*x*e + a*e)^3*a*b*c*d*g^4/(d*x + c)^
3 + (b*x*e + a*e)^3*a^2*d^2*g^4/(d*x + c)^3)
```

**maple [B]** time = 0.05, size = 2624, normalized size = 6.28

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*ln((b*x+a)/(d*x+c)*e)+A)^2/(b*g*x+a*g)^4,x)
```

```
[Out] -1/3*e^3/(a*d-b*c)^4/g^4*A^2*b^3/(1/(d*x+c)*a*e-1/(d*x+c)*b*c/d*e+b/d*e)^3*
c-2/27*e^3/(a*d-b*c)^4/g^4*B^2*b^3/(1/(d*x+c)*a*e-1/(d*x+c)*b*c/d*e+b/d*e)^
3*c+2*d^3*e/(a*d-b*c)^4/g^4*B^2/(1/(d*x+c)*a*e-1/(d*x+c)*b*c/d*e+b/d*e)*a+d
^3*e/(a*d-b*c)^4/g^4*A^2/(1/(d*x+c)*a*e-1/(d*x+c)*b*c/d*e+b/d*e)*a+d*e^2/(a
```

```
*d-b*c)^4/g^4*A^2*b^2/(1/(d*x+c)*a*e-1/(d*x+c)*b*c/d*e+b/d*e)^2*c+1/3*d*e^3
/(a*d-b*c)^4/g^4*A^2*b^2/(1/(d*x+c)*a*e-1/(d*x+c)*b*c/d*e+b/d*e)^3*a-1/2*d^
2*e^2/(a*d-b*c)^4/g^4*B^2*b/(1/(d*x+c)*a*e-1/(d*x+c)*b*c/d*e+b/d*e)^2*a+2*d
^3*e/(a*d-b*c)^4/g^4*B^2/(1/(d*x+c)*a*e-1/(d*x+c)*b*c/d*e+b/d*e)*ln(b/d*e+(
a*d-b*c)/(d*x+c)/d*e)*a-1/3*e^3/(a*d-b*c)^4/g^4*B^2*b^3/(1/(d*x+c)*a*e-1/(d
*x+c)*b*c/d*e+b/d*e)^3*ln(b/d*e+(a*d-b*c)/(d*x+c)/d*e)^2*c+1/2*d*e^2/(a*d-b
*c)^4/g^4*B^2*b^2/(1/(d*x+c)*a*e-1/(d*x+c)*b*c/d*e+b/d*e)^2*c+2/27*d*e^3/(a
*d-b*c)^4/g^4*B^2*b^2/(1/(d*x+c)*a*e-1/(d*x+c)*b*c/d*e+b/d*e)^3*a+d^3*e/(a*
d-b*c)^4/g^4*B^2/(1/(d*x+c)*a*e-1/(d*x+c)*b*c/d*e+b/d*e)*ln(b/d*e+(a*d-b*c)
/(d*x+c)/d*e)^2*a-d^2*e/(a*d-b*c)^4/g^4*A^2/(1/(d*x+c)*a*e-1/(d*x+c)*b*c/d*
e+b/d*e)*b*c+2*d^3*e/(a*d-b*c)^4/g^4*A*B/(1/(d*x+c)*a*e-1/(d*x+c)*b*c/d*e+b
/d*e)*a-2*d^2*e/(a*d-b*c)^4/g^4*B^2/(1/(d*x+c)*a*e-1/(d*x+c)*b*c/d*e+b/d*e)
*b*c-d^2*e^2/(a*d-b*c)^4/g^4*A*B*b/(1/(d*x+c)*a*e-1/(d*x+c)*b*c/d*e+b/d*e)^
2*a-2*d^2*e/(a*d-b*c)^4/g^4*B^2/(1/(d*x+c)*a*e-1/(d*x+c)*b*c/d*e+b/d*e)*ln(
b/d*e+(a*d-b*c)/(d*x+c)/d*e)*b*c-d^2*e^2/(a*d-b*c)^4/g^4*B^2*b/(1/(d*x+c)*a
*e-1/(d*x+c)*b*c/d*e+b/d*e)^2*ln(b/d*e+(a*d-b*c)/(d*x+c)/d*e)*a+d*e^2/(a*d-
b*c)^4/g^4*B^2*b^2/(1/(d*x+c)*a*e-1/(d*x+c)*b*c/d*e+b/d*e)^2*ln(b/d*e+(a*d-
b*c)/(d*x+c)/d*e)*c-2/9*e^3/(a*d-b*c)^4/g^4*B^2*b^3/(1/(d*x+c)*a*e-1/(d*x+c
)*b*c/d*e+b/d*e)^3*ln(b/d*e+(a*d-b*c)/(d*x+c)/d*e)*c-d^2*e^2/(a*d-b*c)^4/g^
4*A^2*b/(1/(d*x+c)*a*e-1/(d*x+c)*b*c/d*e+b/d*e)^2*a-2/9*e^3/(a*d-b*c)^4/g^4
*A*B*b^3/(1/(d*x+c)*a*e-1/(d*x+c)*b*c/d*e+b/d*e)^3*c-2*d^2*e^2/(a*d-b*c)^4/
g^4*A*B*b/(1/(d*x+c)*a*e-1/(d*x+c)*b*c/d*e+b/d*e)^2*ln(b/d*e+(a*d-b*c)/(d*x
+c)/d*e)*a-2*d^2*e/(a*d-b*c)^4/g^4*A*B/(1/(d*x+c)*a*e-1/(d*x+c)*b*c/d*e+b/d
*e)*ln(b/d*e+(a*d-b*c)/(d*x+c)/d*e)*b*c+2/3*d*e^3/(a*d-b*c)^4/g^4*A*B*b^2/(
1/(d*x+c)*a*e-1/(d*x+c)*b*c/d*e+b/d*e)^3*ln(b/d*e+(a*d-b*c)/(d*x+c)/d*e)*a+
2*d*e^2/(a*d-b*c)^4/g^4*A*B*b^2/(1/(d*x+c)*a*e-1/(d*x+c)*b*c/d*e+b/d*e)^2*l
n(b/d*e+(a*d-b*c)/(d*x+c)/d*e)*c-d^2*e/(a*d-b*c)^4/g^4*B^2/(1/(d*x+c)*a*e-1
/(d*x+c)*b*c/d*e+b/d*e)*ln(b/d*e+(a*d-b*c)/(d*x+c)/d*e)^2*b*c-2/3*e^3/(a*d-
b*c)^4/g^4*A*B*b^3/(1/(d*x+c)*a*e-1/(d*x+c)*b*c/d*e+b/d*e)^3*ln(b/d*e+(a*d-
b*c)/(d*x+c)/d*e)*c+1/3*d*e^3/(a*d-b*c)^4/g^4*B^2*b^2/(1/(d*x+c)*a*e-1/(d*x
+c)*b*c/d*e+b/d*e)^3*ln(b/d*e+(a*d-b*c)/(d*x+c)/d*e)^2*a+2/9*d*e^3/(a*d-b*c
)^4/g^4*B^2*b^2/(1/(d*x+c)*a*e-1/(d*x+c)*b*c/d*e+b/d*e)^3*ln(b/d*e+(a*d-b*c
)/(d*x+c)/d*e)*a-d^2*e^2/(a*d-b*c)^4/g^4*B^2*b/(1/(d*x+c)*a*e-1/(d*x+c)*b*c
/d*e+b/d*e)^2*ln(b/d*e+(a*d-b*c)/(d*x+c)/d*e)^2*a+d*e^2/(a*d-b*c)^4/g^4*A*B
*b^2/(1/(d*x+c)*a*e-1/(d*x+c)*b*c/d*e+b/d*e)^2*c+2/9*d*e^3/(a*d-b*c)^4/g^4*
A*B*b^2/(1/(d*x+c)*a*e-1/(d*x+c)*b*c/d*e+b/d*e)^3*a-2*d^2*e/(a*d-b*c)^4/g^4
*A*B/(1/(d*x+c)*a*e-1/(d*x+c)*b*c/d*e+b/d*e)*b*c+2*d^3*e/(a*d-b*c)^4/g^4*A*
B/(1/(d*x+c)*a*e-1/(d*x+c)*b*c/d*e+b/d*e)*ln(b/d*e+(a*d-b*c)/(d*x+c)/d*e)*a
+d*e^2/(a*d-b*c)^4/g^4*B^2*b^2/(1/(d*x+c)*a*e-1/(d*x+c)*b*c/d*e+b/d*e)^2*ln
(b/d*e+(a*d-b*c)/(d*x+c)/d*e)^2*c
```

**maxima [B]** time = 2.45, size = 1419, normalized size = 3.39

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*(b*x+a)/(d*x+c)))^2/(b*g*x+a*g)^4,x, algorithm="maxima
")
```

```
[Out] -1/54*(6*((6*b^2*d^2*x^2 + 2*b^2*c^2 - 7*a*b*c*d + 11*a^2*d^2 - 3*(b^2*c*d
- 5*a*b*d^2)*x)/((b^6*c^2 - 2*a*b^5*c*d + a^2*b^4*d^2)*g^4*x^3 + 3*(a*b^5*c
^2 - 2*a^2*b^4*c*d + a^3*b^3*d^2)*g^4*x^2 + 3*(a^2*b^4*c^2 - 2*a^3*b^3*c*d
+ a^4*b^2*d^2)*g^4*x + (a^3*b^3*c^2 - 2*a^4*b^2*c*d + a^5*b*d^2)*g^4) + 6*d
^3*log(b*x + a)/((b^4*c^3 - 3*a*b^3*c^2*d + 3*a^2*b^2*c*d^2 - a^3*b*d^3)*g^
4) - 6*d^3*log(d*x + c)/((b^4*c^3 - 3*a*b^3*c^2*d + 3*a^2*b^2*c*d^2 - a^3*b
*d^3)*g^4))*log(b*e*x/(d*x + c) + a*e/(d*x + c)) + (4*b^3*c^3 - 27*a*b^2*c^
2*d + 108*a^2*b*c*d^2 - 85*a^3*d^3 + 66*(b^3*c*d^2 - a*b^2*d^3)*x^2 - 18*(b
^3*d^3*x^3 + 3*a*b^2*d^3*x^2 + 3*a^2*b*d^3*x + a^3*d^3)*log(b*x + a)^2 - 18
*(b^3*d^3*x^3 + 3*a*b^2*d^3*x^2 + 3*a^2*b*d^3*x + a^3*d^3)*log(d*x + c)^2 -
 3*(5*b^3*c^2*d - 54*a*b^2*c*d^2 + 49*a^2*b*d^3)*x + 66*(b^3*d^3*x^3 + 3*a*
b^2*d^3*x^2 + 3*a^2*b*d^3*x + a^3*d^3)*log(b*x + a) - 6*(11*b^3*d^3*x^3 + 3
3*a*b^2*d^3*x^2 + 33*a^2*b*d^3*x + 11*a^3*d^3 - 6*(b^3*d^3*x^3 + 3*a*b^2*d^
3*x^2 + 3*a^2*b*d^3*x + a^3*d^3)*log(b*x + a))*log(d*x + c))/(a^3*b^4*c^3*g
^4 - 3*a^4*b^3*c^2*d*g^4 + 3*a^5*b^2*c*d^2*g^4 - a^6*b*d^3*g^4 + (b^7*c^3*g
^4 - 3*a*b^6*c^2*d*g^4 + 3*a^2*b^5*c*d^2*g^4 - a^3*b^4*d^3*g^4)*x^3 + 3*(a*
b^6*c^3*g^4 - 3*a^2*b^5*c^2*d*g^4 + 3*a^3*b^4*c*d^2*g^4 - a^4*b^3*d^3*g^4)*
x^2 + 3*(a^2*b^5*c^3*g^4 - 3*a^3*b^4*c^2*d*g^4 + 3*a^4*b^3*c*d^2*g^4 - a^5*
b^2*d^3*g^4)*x))*B^2 - 1/9*A*B*((6*b^2*d^2*x^2 + 2*b^2*c^2 - 7*a*b*c*d + 11
*a^2*d^2 - 3*(b^2*c*d - 5*a*b*d^2)*x)/((b^6*c^2 - 2*a*b^5*c*d + a^2*b^4*d^2
)*g^4*x^3 + 3*(a*b^5*c^2 - 2*a^2*b^4*c*d + a^3*b^3*d^2)*g^4*x^2 + 3*(a^2*b^
4*c^2 - 2*a^3*b^3*c*d + a^4*b^2*d^2)*g^4*x + (a^3*b^3*c^2 - 2*a^4*b^2*c*d +
 a^5*b*d^2)*g^4) + 6*log(b*e*x/(d*x + c) + a*e/(d*x + c))/(b^4*g^4*x^3 + 3*
a*b^3*g^4*x^2 + 3*a^2*b^2*g^4*x + a^3*b*g^4) + 6*d^3*log(b*x + a)/((b^4*c^3
 - 3*a*b^3*c^2*d + 3*a^2*b^2*c*d^2 - a^3*b*d^3)*g^4) - 6*d^3*log(d*x + c)/(
(b^4*c^3 - 3*a*b^3*c^2*d + 3*a^2*b^2*c*d^2 - a^3*b*d^3)*g^4)) - 1/3*B^2*log
(b*e*x/(d*x + c) + a*e/(d*x + c))^2/(b^4*g^4*x^3 + 3*a*b^3*g^4*x^2 + 3*a^2*
b^2*g^4*x + a^3*b*g^4) - 1/3*A^2/(b^4*g^4*x^3 + 3*a*b^3*g^4*x^2 + 3*a^2*b^2
*g^4*x + a^3*b*g^4)
```

**mupad [B]** time = 7.41, size = 1064, normalized size = 2.55

$$\frac{\frac{18\,A^2\,a^2\,d^2-36\,A^2\,a\,b\,c\,d+18\,A^2\,b^2\,c^2+66\,A\,B\,a^2\,d^2-42\,A\,B\,a\,b\,c\,d+12\,A\,B\,b^2\,c^2+85\,B^2\,a^2\,d^2-23\,B^2\,a\,b\,c\,d+4\,B^2\,b^2\,c^2}{6\,\left(a\,d-b\,c\right)}+\frac{x\,\left(-5\,c\,B^2\,b^2\,d+49\,a\,B^2\,b\,d^2-6\right.}{2\,(a\,d-}}{x\,\left(27\,a^2\,b^3\,c\,g^4-27\,a^3\,b^2\,d\,g^4\right)-x^2\,\left(27\,a^2\,b^3\,d\,g^4-27\,a\,b^4\,c\,g^4\right)+x^3\,\left(9\,b^5\,c\,g^4-9\,a\,b^4\,d\,g^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*log((e*(a + b*x))/(c + d*x)))^2/(a*g + b*g*x)^4,x)
```

```
[Out] ((18*A^2*a^2*d^2 + 18*A^2*b^2*c^2 + 85*B^2*a^2*d^2 + 4*B^2*b^2*c^2 + 66*A*B
```

```
*a^2*d^2 + 12*A*B*b^2*c^2 - 36*A^2*a*b*c*d - 23*B^2*a*b*c*d - 42*A*B*a*b*c*
d)/(6*(a*d - b*c)) + (x*(49*B^2*a*b*d^2 - 5*B^2*b^2*c*d + 30*A*B*a*b*d^2 -
6*A*B*b^2*c*d))/(2*(a*d - b*c)) + (d*x^2*(11*B^2*b^2*d + 6*A*B*b^2*d))/(a*d
 - b*c))/(x*(27*a^2*b^3*c*g^4 - 27*a^3*b^2*d*g^4) - x^2*(27*a^2*b^3*d*g^4 -
 27*a*b^4*c*g^4) + x^3*(9*b^5*c*g^4 - 9*a*b^4*d*g^4) + 9*a^3*b^2*c*g^4 - 9*
a^4*b*d*g^4) - log((e*(a + b*x))/(c + d*x))^2*(B^2/(3*b^2*g^4*(3*a^2*x + a^
3/b + b^2*x^3 + 3*a*b*x^2)) - (B^2*d^3)/(3*b*g^4*(a^3*d^3 - b^3*c^3 + 3*a*b
^2*c^2*d - 3*a^2*b*c*d^2))) - (log((e*(a + b*x))/(c + d*x))*((2*A*B)/(3*b^2
*d*g^4) + (2*B^2*d^3*(a*((3*a^2*d^2 + b^2*c^2 - 4*a*b*c*d)/(6*b*d^3) + (a*(
a*d - b*c))/(3*b*d^2)) + (3*a^3*d^3 - b^3*c^3 + 4*a*b^2*c^2*d - 6*a^2*b*c*d
^2)/(3*b*d^4)))/(3*b*g^4*(a^3*d^3 - b^3*c^3 + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2
)) - (2*B^2*d^3*x^2*((b^2*c - a*b*d)/(3*d^2) - (2*b*(a*d - b*c))/(3*d^2)))/
(3*b*g^4*(a^3*d^3 - b^3*c^3 + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2)) + (2*B^2*d^3*
x*(b*((3*a^2*d^2 + b^2*c^2 - 4*a*b*c*d)/(6*b*d^3) + (a*(a*d - b*c))/(3*b*d^
2)) + (3*a^2*d^2 + b^2*c^2 - 4*a*b*c*d)/(3*d^3) + (2*a*(a*d - b*c))/(3*d^2)
))/(3*b*g^4*(a^3*d^3 - b^3*c^3 + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2))))/((3*a^2*
x)/d + a^3/(b*d) + (b^2*x^3)/d + (3*a*b*x^2)/d) - (B*d^3*atan((B*d^3*((b^4*
c^3*g^4 + a^3*b*d^3*g^4 - a*b^3*c^2*d*g^4 - a^2*b^2*c*d^2*g^4)/(b^3*c^2*g^4
 + a^2*b*d^2*g^4 - 2*a*b^2*c*d*g^4) + 2*b*d*x)*(6*A + 11*B)*(b^3*c^2*g^4 +
a^2*b*d^2*g^4 - 2*a*b^2*c*d*g^4)*1i)/(b*g^4*(a*d - b*c)^3*(11*B^2*d^3 + 6*A
*B*d^3)))*(6*A + 11*B)*2i)/(9*b*g^4*(a*d - b*c)^3)
```

**sympy [B]** time = 34.30, size = 1544, normalized size = 3.69

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*ln(e*(b*x+a)/(d*x+c)))**2/(b*g*x+a*g)**4,x)
```

```
[Out] -B*d**3*(6*A + 11*B)*log(x + (6*A*B*a*d**4 + 6*A*B*b*c*d**3 + 11*B**2*a*d**
4 + 11*B**2*b*c*d**3 - B*a**4*d**7*(6*A + 11*B)/(a*d - b*c)**3 + 4*B*a**3*b
*c*d**6*(6*A + 11*B)/(a*d - b*c)**3 - 6*B*a**2*b**2*c**2*d**5*(6*A + 11*B)/
(a*d - b*c)**3 + 4*B*a*b**3*c**3*d**4*(6*A + 11*B)/(a*d - b*c)**3 - B*b**4*
c**4*d**3*(6*A + 11*B)/(a*d - b*c)**3)/(12*A*B*b*d**4 + 22*B**2*b*d**4))/(9
*b*g**4*(a*d - b*c)**3) + B*d**3*(6*A + 11*B)*log(x + (6*A*B*a*d**4 + 6*A*B
*b*c*d**3 + 11*B**2*a*d**4 + 11*B**2*b*c*d**3 + B*a**4*d**7*(6*A + 11*B)/(a
*d - b*c)**3 - 4*B*a**3*b*c*d**6*(6*A + 11*B)/(a*d - b*c)**3 + 6*B*a**2*b**
2*c**2*d**5*(6*A + 11*B)/(a*d - b*c)**3 - 4*B*a*b**3*c**3*d**4*(6*A + 11*B)
/(a*d - b*c)**3 + B*b**4*c**4*d**3*(6*A + 11*B)/(a*d - b*c)**3)/(12*A*B*b*d
**4 + 22*B**2*b*d**4))/(9*b*g**4*(a*d - b*c)**3) + (3*B**2*a**2*c*d**2 + 3*
B**2*a**2*d**3*x - 3*B**2*a*b*c**2*d + 3*B**2*a*b*d**3*x**2 + B**2*b**2*c**
3 + B**2*b**2*d**3*x**3)*log(e*(a + b*x)/(c + d*x))**2/(3*a**6*d**3*g**4 -
9*a**5*b*c*d**2*g**4 + 9*a**5*b*d**3*g**4*x + 9*a**4*b**2*c**2*d*g**4 - 27*
a**4*b**2*c*d**2*g**4*x + 9*a**4*b**2*d**3*g**4*x**2 - 3*a**3*b**3*c**3*g**
4 + 27*a**3*b**3*c**2*d*g**4*x - 27*a**3*b**3*c*d**2*g**4*x**2 + 3*a**3*b**
```

```
3*d**3*g**4*x**3 - 9*a**2*b**4*c**3*g**4*x + 27*a**2*b**4*c**2*d*g**4*x**2
- 9*a**2*b**4*c*d**2*g**4*x**3 - 9*a*b**5*c**3*g**4*x**2 + 9*a*b**5*c**2*d*
g**4*x**3 - 3*b**6*c**3*g**4*x**3) + (-6*A*B*a**2*d**2 + 12*A*B*a*b*c*d - 6
*A*B*b**2*c**2 - 11*B**2*a**2*d**2 + 7*B**2*a*b*c*d - 15*B**2*a*b*d**2*x -
2*B**2*b**2*c**2 + 3*B**2*b**2*c*d*x - 6*B**2*b**2*d**2*x**2)*log(e*(a + b*
x)/(c + d*x))/(9*a**5*b*d**2*g**4 - 18*a**4*b**2*c*d*g**4 + 27*a**4*b**2*d*
*2*g**4*x + 9*a**3*b**3*c**2*g**4 - 54*a**3*b**3*c*d*g**4*x + 27*a**3*b**3*
d**2*g**4*x**2 + 27*a**2*b**4*c**2*g**4*x - 54*a**2*b**4*c*d*g**4*x**2 + 9*
a**2*b**4*d**2*g**4*x**3 + 27*a*b**5*c**2*g**4*x**2 - 18*a*b**5*c*d*g**4*x*
*3 + 9*b**6*c**2*g**4*x**3) - (18*A**2*a**2*d**2 - 36*A**2*a*b*c*d + 18*A**
2*b**2*c**2 + 66*A*B*a**2*d**2 - 42*A*B*a*b*c*d + 12*A*B*b**2*c**2 + 85*B**
2*a**2*d**2 - 23*B**2*a*b*c*d + 4*B**2*b**2*c**2 + x**2*(36*A*B*b**2*d**2 +
 66*B**2*b**2*d**2) + x*(90*A*B*a*b*d**2 - 18*A*B*b**2*c*d + 147*B**2*a*b*d
**2 - 15*B**2*b**2*c*d))/(54*a**5*b*d**2*g**4 - 108*a**4*b**2*c*d*g**4 + 54
*a**3*b**3*c**2*g**4 + x**3*(54*a**2*b**4*d**2*g**4 - 108*a*b**5*c*d*g**4 +
 54*b**6*c**2*g**4) + x**2*(162*a**3*b**3*d**2*g**4 - 324*a**2*b**4*c*d*g**
4 + 162*a*b**5*c**2*g**4) + x*(162*a**4*b**2*d**2*g**4 - 324*a**3*b**3*c*d*
g**4 + 162*a**2*b**4*c**2*g**4))
```

## 3.105 $\int \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)^5}\,dx$

**Optimal**. Leaf size=575

$$-\frac{b^3(c+dx)^4\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2}{4g^5(a+bx)^4(bc-ad)^4}-\frac{b^3B(c+dx)^4\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{8g^5(a+bx)^4(bc-ad)^4}+\frac{b^2d(c+dx)^3\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2}{g^5(a+bx)^3(bc-ad)^4}+\frac{2b}{}$$

```
[Out] 2*B^2*d^3*(d*x+c)/(-a*d+b*c)^4/g^5/(b*x+a)-3/4*b*B^2*d^2*(d*x+c)^2/(-a*d+b*
c)^4/g^5/(b*x+a)^2+2/9*b^2*B^2*d*(d*x+c)^3/(-a*d+b*c)^4/g^5/(b*x+a)^3-1/32*
b^3*B^2*(d*x+c)^4/(-a*d+b*c)^4/g^5/(b*x+a)^4+2*B*d^3*(d*x+c)*(A+B*ln(e*(b*x
+a)/(d*x+c)))/(-a*d+b*c)^4/g^5/(b*x+a)-3/2*b*B*d^2*(d*x+c)^2*(A+B*ln(e*(b*x
+a)/(d*x+c)))/(-a*d+b*c)^4/g^5/(b*x+a)^2+2/3*b^2*B*d*(d*x+c)^3*(A+B*ln(e*(b
*x+a)/(d*x+c)))/(-a*d+b*c)^4/g^5/(b*x+a)^3-1/8*b^3*B*(d*x+c)^4*(A+B*ln(e*(b
*x+a)/(d*x+c)))/(-a*d+b*c)^4/g^5/(b*x+a)^4+d^3*(d*x+c)*(A+B*ln(e*(b*x+a)/(d
*x+c)))^2/(-a*d+b*c)^4/g^5/(b*x+a)-3/2*b*d^2*(d*x+c)^2*(A+B*ln(e*(b*x+a)/(d
*x+c)))^2/(-a*d+b*c)^4/g^5/(b*x+a)^2+b^2*d*(d*x+c)^3*(A+B*ln(e*(b*x+a)/(d*x
+c)))^2/(-a*d+b*c)^4/g^5/(b*x+a)^3-1/4*b^3*(d*x+c)^4*(A+B*ln(e*(b*x+a)/(d*x
+c)))^2/(-a*d+b*c)^4/g^5/(b*x+a)^4
```

**Rubi [C]** time = 1.23, antiderivative size = 763, normalized size of antiderivative = 1.33, number of steps used = 38, number of rules used = 11, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.344, Rules used = {2525, 12, 2528, 44, 2524, 2418, 2390, 2301, 2394, 2393, 2391}

$$\frac{B^2d^4\text{PolyLog}\left(2,-\frac{d(a+bx)}{bc-ad}\right)}{2bg^5(bc-ad)^4}+\frac{B^2d^4\text{PolyLog}\left(2,\frac{b(c+dx)}{bc-ad}\right)}{2bg^5(bc-ad)^4}+\frac{Bd^4\log(a+bx)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{2bg^5(bc-ad)^4}-\frac{Bd^4\log(c+dx)\left(\right.}{2bg^5(b}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Log[(e*(a + b*x))/(c + d*x)])^2/(a*g + b*g*x)^5,x]
```

```
[Out] -B^2/(32*b*g^5*(a + b*x)^4) + (7*B^2*d)/(72*b*(b*c - a*d)*g^5*(a + b*x)^3)
- (13*B^2*d^2)/(48*b*(b*c - a*d)^2*g^5*(a + b*x)^2) + (25*B^2*d^3)/(24*b*(b
*c - a*d)^3*g^5*(a + b*x)) + (25*B^2*d^4*Log[a + b*x])/(24*b*(b*c - a*d)^4*
g^5) - (B^2*d^4*Log[a + b*x]^2)/(4*b*(b*c - a*d)^4*g^5) - (B*(A + B*Log[(e*
(a + b*x))/(c + d*x)]))/(8*b*g^5*(a + b*x)^4) + (B*d*(A + B*Log[(e*(a + b*x
))/(c + d*x)]))/(6*b*(b*c - a*d)*g^5*(a + b*x)^3) - (B*d^2*(A + B*Log[(e*(a
 + b*x))/(c + d*x)]))/(4*b*(b*c - a*d)^2*g^5*(a + b*x)^2) + (B*d^3*(A + B*L
og[(e*(a + b*x))/(c + d*x)]))/(2*b*(b*c - a*d)^3*g^5*(a + b*x)) + (B*d^4*Lo
g[a + b*x]*(A + B*Log[(e*(a + b*x))/(c + d*x)]))/(2*b*(b*c - a*d)^4*g^5) -
(A + B*Log[(e*(a + b*x))/(c + d*x)])^2/(4*b*g^5*(a + b*x)^4) - (25*B^2*d^4*
Log[c + d*x])/(24*b*(b*c - a*d)^4*g^5) + (B^2*d^4*Log[-((d*(a + b*x))/(b*c
```

```
- a*d))]*Log[c + d*x])/(2*b*(b*c - a*d)^4*g^5) - (B*d^4*(A + B*Log[(e*(a +
b*x))/(c + d*x)])*Log[c + d*x])/(2*b*(b*c - a*d)^4*g^5) - (B^2*d^4*Log[c +
d*x]^2)/(4*b*(b*c - a*d)^4*g^5) + (B^2*d^4*Log[a + b*x]*Log[(b*(c + d*x))/(
b*c - a*d)])/(2*b*(b*c - a*d)^4*g^5) + (B^2*d^4*PolyLog[2, -((d*(a + b*x))/
(b*c - a*d))])/(2*b*(b*c - a*d)^4*g^5) + (B^2*d^4*PolyLog[2, (b*(c + d*x))/
(b*c - a*d)])/(2*b*(b*c - a*d)^4*g^5)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 2301

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

Rule 2390

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_))^(q_.), x_Symbol] :> Dist[1/e, Subst[Int[((f*x)/d)^q*(a + b*Log[c*x^
n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && E
qQ[e*f - d*g, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2393

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + (c*e*x)/g])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2394

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d + e*x
)^n]))/g, x] - Dist[(b*e*n)/g, Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2418

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*(RFx_), x_Sy
mbol] :> With[{u = ExpandIntegrand[(a + b*Log[c*(d + e*x)^n])^p, RFx, x]},
Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d, e, n}, x] && RationalFunctionQ[
RFx, x] && IntegerQ[p]
```

Rule 2524

```
Int[((a_.) + Log[(c_.)*(RFx_)^(p_.)]*(b_.))^(n_.)/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[(Log[d + e*x]*(a + b*Log[c*RFx^p])^n)/e, x] - Dist[(b*n*p)/e
, Int[(Log[d + e*x]*(a + b*Log[c*RFx^p])^(n - 1)*D[RFx, x])/RFx, x], x] /;
FreeQ[{a, b, c, d, e, p}, x] && RationalFunctionQ[RFx, x] && IGtQ[n, 0]
```

Rule 2525

```
Int[((a_.) + Log[(c_.)*(RFx_)^(p_.)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_))^(m_.
), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(a + b*Log[c*RFx^p])^n)/(e*(m + 1))
, x] - Dist[(b*n*p)/(e*(m + 1)), Int[SimplifyIntegrand[((d + e*x)^(m + 1)*(
a + b*Log[c*RFx^p])^(n - 1)*D[RFx, x])/RFx, x], x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && RationalFunctionQ[RFx, x] && IGtQ[n, 0] && (EqQ[n, 1] ||
IntegerQ[m]) && NeQ[m, -1]
```

Rule 2528

```
Int[((a_.) + Log[(c_.)*(RFx_)^(p_.)]*(b_.))^(n_.)*(RGx_), x_Symbol] :> With
[{u = ExpandIntegrand[(a + b*Log[c*RFx^p])^n, RGx, x]}, Int[u, x] /; SumQ[u
]] /; FreeQ[{a, b, c, p}, x] && RationalFunctionQ[RFx, x] && RationalFuncti
onQ[RGx, x] && IGtQ[n, 0]
```

Rubi steps

$$\int \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)^5}\,dx = -\frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{4bg^5(a+bx)^4} + \frac{B\int \frac{(bc-ad)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{g^4(a+bx)^5(c+dx)}\,dx}{2bg}$$

$$= -\frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{4bg^5(a+bx)^4} + \frac{(B(bc-ad))\int \frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{(a+bx)^5(c+dx)}\,dx}{2bg^5}$$

$$= -\frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{4bg^5(a+bx)^4} + \frac{(B(bc-ad))\int \left(\frac{b\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(bc-ad)(a+bx)^5} - \frac{bd\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(bc-ad)^2(a+bx)^4} \cdots\right.}{\cdots}$$

$$= -\frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{4bg^5(a+bx)^4} + \frac{B\int \frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{(a+bx)^5}\,dx}{2g^5} + \frac{\left(Bd^4\right)\int \frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{a+bx}\,dx}{2(bc-ad)^4g^5} - \cdots$$

$$= -\frac{B\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{8bg^5(a+bx)^4} + \frac{Bd\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{6b(bc-ad)g^5(a+bx)^3} - \frac{Bd^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{4b(bc-ad)^2g^5(a+bx)^2} \cdots$$

$$= -\frac{B\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{8bg^5(a+bx)^4} + \frac{Bd\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{6b(bc-ad)g^5(a+bx)^3} - \frac{Bd^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{4b(bc-ad)^2g^5(a+bx)^2} \cdots$$

$$= -\frac{B\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{8bg^5(a+bx)^4} + \frac{Bd\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{6b(bc-ad)g^5(a+bx)^3} - \frac{Bd^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{4b(bc-ad)^2g^5(a+bx)^2} \cdots$$

$$= -\frac{B^2}{32bg^5(a+bx)^4} + \frac{7B^2d}{72b(bc-ad)g^5(a+bx)^3} - \frac{13B^2d^2}{48b(bc-ad)^2g^5(a+bx)^2} + \frac{\cdots}{24b(\cdots}$$

$$= -\frac{B^2}{32bg^5(a+bx)^4} + \frac{7B^2d}{72b(bc-ad)g^5(a+bx)^3} - \frac{13B^2d^2}{48b(bc-ad)^2g^5(a+bx)^2} + \frac{\cdots}{24b(\cdots}$$

$$= -\frac{B^2}{32bg^5(a+bx)^4} + \frac{7B^2d}{72b(bc-ad)g^5(a+bx)^3} - \frac{13B^2d^2}{48b(bc-ad)^2g^5(a+bx)^2} + \frac{\cdots}{24b(\cdots}$$

$$= -\frac{B^2}{32bg^5(a+bx)^4} + \frac{7B^2d}{72b(bc-ad)g^5(a+bx)^3} - \frac{13B^2d^2}{48b(bc-ad)^2g^5(a+bx)^2} + \frac{\cdots}{24b(\cdots}$$

**Mathematica [C]** time = 0.97, size = 748, normalized size = 1.30

$$-\frac{B\left(-144d^4(a+bx)^4\log(a+bx)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)+144d^4(a+bx)^4\log(c+dx)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)+144d^3(a+bx)^3(ad-bc)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)+72d^2(a+b\cdots}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Log[(e*(a + b*x))/(c + d*x)])^2/(a*g + b*g*x)^5,x]
```

```
[Out] -1/288*(72*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2 + (B*(36*(b*c - a*d)^4*(A
 + B*Log[(e*(a + b*x))/(c + d*x)]) + 48*d*(-(b*c) + a*d)^3*(a + b*x)*(A + B
*Log[(e*(a + b*x))/(c + d*x)]) + 72*d^2*(b*c - a*d)^2*(a + b*x)^2*(A + B*Lo
g[(e*(a + b*x))/(c + d*x)]) + 144*d^3*(-(b*c) + a*d)*(a + b*x)^3*(A + B*Log
[(e*(a + b*x))/(c + d*x)]) - 144*d^4*(a + b*x)^4*Log[a + b*x]*(A + B*Log[(e
*(a + b*x))/(c + d*x)]) + 144*d^4*(a + b*x)^4*(A + B*Log[(e*(a + b*x))/(c +
 d*x)])*Log[c + d*x] - 144*B*d^3*(a + b*x)^3*(b*c - a*d + d*(a + b*x)*Log[a
 + b*x] - d*(a + b*x)*Log[c + d*x]) + 36*B*d^2*(a + b*x)^2*((b*c - a*d)^2 +
 2*d*(-(b*c) + a*d)*(a + b*x) - 2*d^2*(a + b*x)^2*Log[a + b*x] + 2*d^2*(a +
 b*x)^2*Log[c + d*x]) - 8*B*d*(a + b*x)*(2*(b*c - a*d)^3 - 3*d*(b*c - a*d)^
2*(a + b*x) + 6*d^2*(b*c - a*d)*(a + b*x)^2 + 6*d^3*(a + b*x)^3*Log[a + b*x
] - 6*d^3*(a + b*x)^3*Log[c + d*x]) + 3*B*(3*(b*c - a*d)^4 + 4*d*(-(b*c) +
a*d)^3*(a + b*x) + 6*d^2*(b*c - a*d)^2*(a + b*x)^2 + 12*d^3*(-(b*c) + a*d)*
(a + b*x)^3 - 12*d^4*(a + b*x)^4*Log[a + b*x] + 12*d^4*(a + b*x)^4*Log[c +
d*x]) + 72*B*d^4*(a + b*x)^4*(Log[a + b*x]*(Log[a + b*x] - 2*Log[(b*(c + d*
x))/(b*c - a*d)]) - 2*PolyLog[2, (d*(a + b*x))/(-(b*c) + a*d)]) - 72*B*d^4*
(a + b*x)^4*((2*Log[(d*(a + b*x))/(-(b*c) + a*d)] - Log[c + d*x])*Log[c + d
*x] + 2*PolyLog[2, (b*(c + d*x))/(b*c - a*d)])))/(b*c - a*d)^4)/(b*g^5*(a +
 b*x)^4)
```

**fricas [A]** time = 1.50, size = 1035, normalized size = 1.80

$$-\frac{9\left(8A^2+4AB+B^2\right)b^4c^4-32\left(9A^2+6AB+2B^2\right)ab^3c^3d+216\left(2A^2+2AB+B^2\right)a^2b^2c^2d^2-288\left(A^2+2A\cdots\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*(b*x+a)/(d*x+c)))^2/(b*g*x+a*g)^5,x, algorithm="fricas
")
```

```
[Out] -1/288*(9*(8*A^2 + 4*A*B + B^2)*b^4*c^4 - 32*(9*A^2 + 6*A*B + 2*B^2)*a*b^3*
c^3*d + 216*(2*A^2 + 2*A*B + B^2)*a^2*b^2*c^2*d^2 - 288*(A^2 + 2*A*B + 2*B^
2)*a^3*b*c*d^3 + (72*A^2 + 300*A*B + 415*B^2)*a^4*d^4 - 12*((12*A*B + 25*B^
2)*b^4*c*d^3 - (12*A*B + 25*B^2)*a*b^3*d^4)*x^3 + 6*((12*A*B + 13*B^2)*b^4*
c^2*d^2 - 16*(6*A*B + 11*B^2)*a*b^3*c*d^3 + (84*A*B + 163*B^2)*a^2*b^2*d^4)
```

```
*x^2 - 72*(B^2*b^4*d^4*x^4 + 4*B^2*a*b^3*d^4*x^3 + 6*B^2*a^2*b^2*d^4*x^2 +
4*B^2*a^3*b*d^4*x - B^2*b^4*c^4 + 4*B^2*a*b^3*c^3*d - 6*B^2*a^2*b^2*c^2*d^2
 + 4*B^2*a^3*b*c*d^3)*log((b*e*x + a*e)/(d*x + c))^2 - 4*((12*A*B + 7*B^2)*
b^4*c^3*d - 12*(6*A*B + 5*B^2)*a*b^3*c^2*d^2 + 108*(2*A*B + 3*B^2)*a^2*b^2*
c*d^3 - (156*A*B + 271*B^2)*a^3*b*d^4)*x - 12*((12*A*B + 25*B^2)*b^4*d^4*x^
4 - 3*(4*A*B + B^2)*b^4*c^4 + 16*(3*A*B + B^2)*a*b^3*c^3*d - 36*(2*A*B + B^
2)*a^2*b^2*c^2*d^2 + 48*(A*B + B^2)*a^3*b*c*d^3 + 4*(3*B^2*b^4*c*d^3 + 2*(6
*A*B + 11*B^2)*a*b^3*d^4)*x^3 - 6*(B^2*b^4*c^2*d^2 - 8*B^2*a*b^3*c*d^3 - 6*
(2*A*B + 3*B^2)*a^2*b^2*d^4)*x^2 + 4*(B^2*b^4*c^3*d - 6*B^2*a*b^3*c^2*d^2 +
 18*B^2*a^2*b^2*c*d^3 + 12*(A*B + B^2)*a^3*b*d^4)*x)*log((b*e*x + a*e)/(d*x
 + c)))/((b^9*c^4 - 4*a*b^8*c^3*d + 6*a^2*b^7*c^2*d^2 - 4*a^3*b^6*c*d^3 + a
^4*b^5*d^4)*g^5*x^4 + 4*(a*b^8*c^4 - 4*a^2*b^7*c^3*d + 6*a^3*b^6*c^2*d^2 -
4*a^4*b^5*c*d^3 + a^5*b^4*d^4)*g^5*x^3 + 6*(a^2*b^7*c^4 - 4*a^3*b^6*c^3*d +
 6*a^4*b^5*c^2*d^2 - 4*a^5*b^4*c*d^3 + a^6*b^3*d^4)*g^5*x^2 + 4*(a^3*b^6*c^
4 - 4*a^4*b^5*c^3*d + 6*a^5*b^4*c^2*d^2 - 4*a^6*b^3*c*d^3 + a^7*b^2*d^4)*g^
5*x + (a^4*b^5*c^4 - 4*a^5*b^4*c^3*d + 6*a^6*b^3*c^2*d^2 - 4*a^7*b^2*c*d^3
+ a^8*b*d^4)*g^5)
```

**giac [A]** time = 2.23, size = 995, normalized size = 1.73

$$-\frac{\left(72\,B^2b^3e^5\log\left(\frac{bxe+ae}{dx+c}\right)^2-\frac{288\,(bxe+ae)B^2b^2de^4\log\left(\frac{bxe+ae}{dx+c}\right)^2}{dx+c}+\frac{432\,(bxe+ae)^2B^2bd^2e^3\log\left(\frac{bxe+ae}{dx+c}\right)^2}{(dx+c)^2}-\frac{288\,(bxe+ae)^3B^2d^3e^2\log\left(\frac{bxe+ae}{dx+c}\right)}{(dx+c)^3}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*(b*x+a)/(d*x+c)))^2/(b*g*x+a*g)^5,x, algorithm="giac")
```

```
[Out] -1/288*(72*B^2*b^3*e^5*log((b*x*e + a*e)/(d*x + c))^2 - 288*(b*x*e + a*e)*B
^2*b^2*d*e^4*log((b*x*e + a*e)/(d*x + c))^2/(d*x + c) + 432*(b*x*e + a*e)^2
*B^2*b*d^2*e^3*log((b*x*e + a*e)/(d*x + c))^2/(d*x + c)^2 - 288*(b*x*e + a*
e)^3*B^2*d^3*e^2*log((b*x*e + a*e)/(d*x + c))^2/(d*x + c)^3 + 144*A*B*b^3*e
^5*log((b*x*e + a*e)/(d*x + c)) + 36*B^2*b^3*e^5*log((b*x*e + a*e)/(d*x + c
)) - 576*(b*x*e + a*e)*A*B*b^2*d*e^4*log((b*x*e + a*e)/(d*x + c))/(d*x + c)
 - 192*(b*x*e + a*e)*B^2*b^2*d*e^4*log((b*x*e + a*e)/(d*x + c))/(d*x + c) +
 864*(b*x*e + a*e)^2*A*B*b*d^2*e^3*log((b*x*e + a*e)/(d*x + c))/(d*x + c)^2
 + 432*(b*x*e + a*e)^2*B^2*b*d^2*e^3*log((b*x*e + a*e)/(d*x + c))/(d*x + c)
^2 - 576*(b*x*e + a*e)^3*A*B*d^3*e^2*log((b*x*e + a*e)/(d*x + c))/(d*x + c)
^3 - 576*(b*x*e + a*e)^3*B^2*d^3*e^2*log((b*x*e + a*e)/(d*x + c))/(d*x + c)
^3 + 72*A^2*b^3*e^5 + 36*A*B*b^3*e^5 + 9*B^2*b^3*e^5 - 288*(b*x*e + a*e)*A^
2*b^2*d*e^4/(d*x + c) - 192*(b*x*e + a*e)*A*B*b^2*d*e^4/(d*x + c) - 64*(b*x
*e + a*e)*B^2*b^2*d*e^4/(d*x + c) + 432*(b*x*e + a*e)^2*A^2*b*d^2*e^3/(d*x
+ c)^2 + 432*(b*x*e + a*e)^2*A*B*b*d^2*e^3/(d*x + c)^2 + 216*(b*x*e + a*e)^
```

```
2*B^2*b*d^2*e^3/(d*x + c)^2 - 288*(b*x*e + a*e)^3*A^2*d^3*e^2/(d*x + c)^3 -
 576*(b*x*e + a*e)^3*A*B*d^3*e^2/(d*x + c)^3 - 576*(b*x*e + a*e)^3*B^2*d^3*
e^2/(d*x + c)^3)*(b*c/((b*c*e - a*d*e)*(b*c - a*d)) - a*d/((b*c*e - a*d*e)*
(b*c - a*d)))/((b*x*e + a*e)^4*b^3*c^3*g^5/(d*x + c)^4 - 3*(b*x*e + a*e)^4*
a*b^2*c^2*d*g^5/(d*x + c)^4 + 3*(b*x*e + a*e)^4*a^2*b*c*d^2*g^5/(d*x + c)^4
 - (b*x*e + a*e)^4*a^3*d^3*g^5/(d*x + c)^4)
```

**maple [B]** time = 0.05, size = 3538, normalized size = 6.15

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*ln((b*x+a)/(d*x+c)*e)+A)^2/(b*g*x+a*g)^5,x)
```

```
[Out] -3/2*d^3*e^2/(a*d-b*c)^5/g^5*A^2*b/(1/(d*x+c)*a*e-1/(d*x+c)*b*c/d*e+b/d*e)^
2*a+3/2*d^2*e^2/(a*d-b*c)^5/g^5*A^2*b^2/(1/(d*x+c)*a*e-1/(d*x+c)*b*c/d*e+b/
d*e)^2*c-2*d^3*e/(a*d-b*c)^5/g^5*B^2/(1/(d*x+c)*a*e-1/(d*x+c)*b*c/d*e+b/d*e
)*b*c-d^3*e/(a*d-b*c)^5/g^5*A^2/(1/(d*x+c)*a*e-1/(d*x+c)*b*c/d*e+b/d*e)*b*c
+2*d^4*e/(a*d-b*c)^5/g^5*A*B/(1/(d*x+c)*a*e-1/(d*x+c)*b*c/d*e+b/d*e)*a+2*d^
4*e/(a*d-b*c)^5/g^5*B^2/(1/(d*x+c)*a*e-1/(d*x+c)*b*c/d*e+b/d*e)*ln(b/d*e+(a
*d-b*c)/(d*x+c)/d*e)*a+d^4*e/(a*d-b*c)^5/g^5*B^2/(1/(d*x+c)*a*e-1/(d*x+c)*b
*c/d*e+b/d*e)*ln(b/d*e+(a*d-b*c)/(d*x+c)/d*e)^2*a+1/4*e^4/(a*d-b*c)^5/g^5*B
^2*b^4/(1/(d*x+c)*a*e-1/(d*x+c)*b*c/d*e+b/d*e)^4*ln(b/d*e+(a*d-b*c)/(d*x+c)
/d*e)^2*c+1/8*e^4/(a*d-b*c)^5/g^5*B^2*b^4/(1/(d*x+c)*a*e-1/(d*x+c)*b*c/d*e+
b/d*e)^4*ln(b/d*e+(a*d-b*c)/(d*x+c)/d*e)*c+d^2*e^3/(a*d-b*c)^5/g^5*A^2*b^2/
(1/(d*x+c)*a*e-1/(d*x+c)*b*c/d*e+b/d*e)^3*a-d*e^3/(a*d-b*c)^5/g^5*A^2*b^3/(
1/(d*x+c)*a*e-1/(d*x+c)*b*c/d*e+b/d*e)^3*c+3/4*d^2*e^2/(a*d-b*c)^5/g^5*B^2*
b^2/(1/(d*x+c)*a*e-1/(d*x+c)*b*c/d*e+b/d*e)^2*c+2/9*d^2*e^3/(a*d-b*c)^5/g^5
*B^2*b^2/(1/(d*x+c)*a*e-1/(d*x+c)*b*c/d*e+b/d*e)^3*a-3/4*d^3*e^2/(a*d-b*c)^
5/g^5*B^2*b/(1/(d*x+c)*a*e-1/(d*x+c)*b*c/d*e+b/d*e)^2*a-1/4*d*e^4/(a*d-b*c)
^5/g^5*A^2*b^3/(1/(d*x+c)*a*e-1/(d*x+c)*b*c/d*e+b/d*e)^4*a-2/9*d*e^3/(a*d-b
*c)^5/g^5*B^2*b^3/(1/(d*x+c)*a*e-1/(d*x+c)*b*c/d*e+b/d*e)^3*c-1/32*d*e^4/(a
*d-b*c)^5/g^5*B^2*b^3/(1/(d*x+c)*a*e-1/(d*x+c)*b*c/d*e+b/d*e)^4*a+1/8*e^4/(
a*d-b*c)^5/g^5*A*B*b^4/(1/(d*x+c)*a*e-1/(d*x+c)*b*c/d*e+b/d*e)^4*c+1/4*e^4/
(a*d-b*c)^5/g^5*A^2*b^4/(1/(d*x+c)*a*e-1/(d*x+c)*b*c/d*e+b/d*e)^4*c+1/32*e^
4/(a*d-b*c)^5/g^5*B^2*b^4/(1/(d*x+c)*a*e-1/(d*x+c)*b*c/d*e+b/d*e)^4*c+d^4*e
/(a*d-b*c)^5/g^5*A^2/(1/(d*x+c)*a*e-1/(d*x+c)*b*c/d*e+b/d*e)*a+2*d^4*e/(a*d
-b*c)^5/g^5*B^2/(1/(d*x+c)*a*e-1/(d*x+c)*b*c/d*e+b/d*e)*a-3*d^3*e^2/(a*d-b*
c)^5/g^5*A*B*b/(1/(d*x+c)*a*e-1/(d*x+c)*b*c/d*e+b/d*e)^2*ln(b/d*e+(a*d-b*c)
/(d*x+c)/d*e)*a-1/2*d*e^4/(a*d-b*c)^5/g^5*A*B*b^3/(1/(d*x+c)*a*e-1/(d*x+c)*
b*c/d*e+b/d*e)^4*ln(b/d*e+(a*d-b*c)/(d*x+c)/d*e)*a+3*d^2*e^2/(a*d-b*c)^5/g^
5*A*B*b^2/(1/(d*x+c)*a*e-1/(d*x+c)*b*c/d*e+b/d*e)^2*ln(b/d*e+(a*d-b*c)/(d*x
+c)/d*e)*c+2*d^2*e^3/(a*d-b*c)^5/g^5*A*B*b^2/(1/(d*x+c)*a*e-1/(d*x+c)*b*c/d
*e+b/d*e)^3*ln(b/d*e+(a*d-b*c)/(d*x+c)/d*e)*a-2*d*e^3/(a*d-b*c)^5/g^5*A*B*b
^3/(1/(d*x+c)*a*e-1/(d*x+c)*b*c/d*e+b/d*e)^3*ln(b/d*e+(a*d-b*c)/(d*x+c)/d*e
```

```
)*c-2*d^3*e/(a*d-b*c)^5/g^5*A*B/(1/(d*x+c)*a*e-1/(d*x+c)*b*c/d*e+b/d*e)*ln(
b/d*e+(a*d-b*c)/(d*x+c)/d*e)*b*c-1/8*d*e^4/(a*d-b*c)^5/g^5*B^2*b^3/(1/(d*x+
c)*a*e-1/(d*x+c)*b*c/d*e+b/d*e)^4*ln(b/d*e+(a*d-b*c)/(d*x+c)/d*e)*a+2*d^4*e
/(a*d-b*c)^5/g^5*A*B/(1/(d*x+c)*a*e-1/(d*x+c)*b*c/d*e+b/d*e)*ln(b/d*e+(a*d-
b*c)/(d*x+c)/d*e)*a-d^3*e/(a*d-b*c)^5/g^5*B^2/(1/(d*x+c)*a*e-1/(d*x+c)*b*c/
d*e+b/d*e)*ln(b/d*e+(a*d-b*c)/(d*x+c)/d*e)^2*b*c-2*d^3*e/(a*d-b*c)^5/g^5*B^
2/(1/(d*x+c)*a*e-1/(d*x+c)*b*c/d*e+b/d*e)*ln(b/d*e+(a*d-b*c)/(d*x+c)/d*e)*b
*c-2*d^3*e/(a*d-b*c)^5/g^5*A*B/(1/(d*x+c)*a*e-1/(d*x+c)*b*c/d*e+b/d*e)*b*c+
3/2*d^2*e^2/(a*d-b*c)^5/g^5*A*B*b^2/(1/(d*x+c)*a*e-1/(d*x+c)*b*c/d*e+b/d*e)
^2*c+2/3*d^2*e^3/(a*d-b*c)^5/g^5*A*B*b^2/(1/(d*x+c)*a*e-1/(d*x+c)*b*c/d*e+b
/d*e)^3*a-2/3*d*e^3/(a*d-b*c)^5/g^5*A*B*b^3/(1/(d*x+c)*a*e-1/(d*x+c)*b*c/d*
e+b/d*e)^3*c-1/8*d*e^4/(a*d-b*c)^5/g^5*A*B*b^3/(1/(d*x+c)*a*e-1/(d*x+c)*b*c
/d*e+b/d*e)^4*a-3/2*d^3*e^2/(a*d-b*c)^5/g^5*A*B*b/(1/(d*x+c)*a*e-1/(d*x+c)*
b*c/d*e+b/d*e)^2*a-3/2*d^3*e^2/(a*d-b*c)^5/g^5*B^2*b/(1/(d*x+c)*a*e-1/(d*x+
c)*b*c/d*e+b/d*e)^2*ln(b/d*e+(a*d-b*c)/(d*x+c)/d*e)*a+3/2*d^2*e^2/(a*d-b*c)
^5/g^5*B^2*b^2/(1/(d*x+c)*a*e-1/(d*x+c)*b*c/d*e+b/d*e)^2*ln(b/d*e+(a*d-b*c)
/(d*x+c)/d*e)*c+3/2*d^2*e^2/(a*d-b*c)^5/g^5*B^2*b^2/(1/(d*x+c)*a*e-1/(d*x+c
)*b*c/d*e+b/d*e)^2*ln(b/d*e+(a*d-b*c)/(d*x+c)/d*e)^2*c-3/2*d^3*e^2/(a*d-b*c
)^5/g^5*B^2*b/(1/(d*x+c)*a*e-1/(d*x+c)*b*c/d*e+b/d*e)^2*ln(b/d*e+(a*d-b*c)/
(d*x+c)/d*e)^2*a+d^2*e^3/(a*d-b*c)^5/g^5*B^2*b^2/(1/(d*x+c)*a*e-1/(d*x+c)*b
*c/d*e+b/d*e)^3*ln(b/d*e+(a*d-b*c)/(d*x+c)/d*e)^2*a+2/3*d^2*e^3/(a*d-b*c)^5
/g^5*B^2*b^2/(1/(d*x+c)*a*e-1/(d*x+c)*b*c/d*e+b/d*e)^3*ln(b/d*e+(a*d-b*c)/(
d*x+c)/d*e)*a-2/3*d*e^3/(a*d-b*c)^5/g^5*B^2*b^3/(1/(d*x+c)*a*e-1/(d*x+c)*b*
c/d*e+b/d*e)^3*ln(b/d*e+(a*d-b*c)/(d*x+c)/d*e)*c-1/4*d*e^4/(a*d-b*c)^5/g^5*
B^2*b^3/(1/(d*x+c)*a*e-1/(d*x+c)*b*c/d*e+b/d*e)^4*ln(b/d*e+(a*d-b*c)/(d*x+c
)/d*e)^2*a+1/2*e^4/(a*d-b*c)^5/g^5*A*B*b^4/(1/(d*x+c)*a*e-1/(d*x+c)*b*c/d*e
+b/d*e)^4*ln(b/d*e+(a*d-b*c)/(d*x+c)/d*e)*c-d*e^3/(a*d-b*c)^5/g^5*B^2*b^3/(
1/(d*x+c)*a*e-1/(d*x+c)*b*c/d*e+b/d*e)^3*ln(b/d*e+(a*d-b*c)/(d*x+c)/d*e)^2*
c
```

**maxima [B]** time = 3.41, size = 2123, normalized size = 3.69

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*(b*x+a)/(d*x+c)))^2/(b*g*x+a*g)^5,x, algorithm="maxima
")
```

```
[Out] 1/288*(12*((12*b^3*d^3*x^3 - 3*b^3*c^3 + 13*a*b^2*c^2*d - 23*a^2*b*c*d^2 +
25*a^3*d^3 - 6*(b^3*c*d^2 - 7*a*b^2*d^3)*x^2 + 4*(b^3*c^2*d - 5*a*b^2*c*d^2
 + 13*a^2*b*d^3)*x)/((b^8*c^3 - 3*a*b^7*c^2*d + 3*a^2*b^6*c*d^2 - a^3*b^5*d
^3)*g^5*x^4 + 4*(a*b^7*c^3 - 3*a^2*b^6*c^2*d + 3*a^3*b^5*c*d^2 - a^4*b^4*d^
3)*g^5*x^3 + 6*(a^2*b^6*c^3 - 3*a^3*b^5*c^2*d + 3*a^4*b^4*c*d^2 - a^5*b^3*d
^3)*g^5*x^2 + 4*(a^3*b^5*c^3 - 3*a^4*b^4*c^2*d + 3*a^5*b^3*c*d^2 - a^6*b^2*
d^3)*g^5*x + (a^4*b^4*c^3 - 3*a^5*b^3*c^2*d + 3*a^6*b^2*c*d^2 - a^7*b*d^3)*
```

```
g^5) + 12*d^4*log(b*x + a)/((b^5*c^4 - 4*a*b^4*c^3*d + 6*a^2*b^3*c^2*d^2 -
4*a^3*b^2*c*d^3 + a^4*b*d^4)*g^5) - 12*d^4*log(d*x + c)/((b^5*c^4 - 4*a*b^4
*c^3*d + 6*a^2*b^3*c^2*d^2 - 4*a^3*b^2*c*d^3 + a^4*b*d^4)*g^5))*log(b*e*x/(
d*x + c) + a*e/(d*x + c)) - (9*b^4*c^4 - 64*a*b^3*c^3*d + 216*a^2*b^2*c^2*d
^2 - 576*a^3*b*c*d^3 + 415*a^4*d^4 - 300*(b^4*c*d^3 - a*b^3*d^4)*x^3 + 6*(1
3*b^4*c^2*d^2 - 176*a*b^3*c*d^3 + 163*a^2*b^2*d^4)*x^2 + 72*(b^4*d^4*x^4 +
4*a*b^3*d^4*x^3 + 6*a^2*b^2*d^4*x^2 + 4*a^3*b*d^4*x + a^4*d^4)*log(b*x + a)
^2 + 72*(b^4*d^4*x^4 + 4*a*b^3*d^4*x^3 + 6*a^2*b^2*d^4*x^2 + 4*a^3*b*d^4*x
+ a^4*d^4)*log(d*x + c)^2 - 4*(7*b^4*c^3*d - 60*a*b^3*c^2*d^2 + 324*a^2*b^2
*c*d^3 - 271*a^3*b*d^4)*x - 300*(b^4*d^4*x^4 + 4*a*b^3*d^4*x^3 + 6*a^2*b^2*
d^4*x^2 + 4*a^3*b*d^4*x + a^4*d^4)*log(b*x + a) + 12*(25*b^4*d^4*x^4 + 100*
a*b^3*d^4*x^3 + 150*a^2*b^2*d^4*x^2 + 100*a^3*b*d^4*x + 25*a^4*d^4 - 12*(b^
4*d^4*x^4 + 4*a*b^3*d^4*x^3 + 6*a^2*b^2*d^4*x^2 + 4*a^3*b*d^4*x + a^4*d^4)*
log(b*x + a))*log(d*x + c))/(a^4*b^5*c^4*g^5 - 4*a^5*b^4*c^3*d*g^5 + 6*a^6*
b^3*c^2*d^2*g^5 - 4*a^7*b^2*c*d^3*g^5 + a^8*b*d^4*g^5 + (b^9*c^4*g^5 - 4*a*
b^8*c^3*d*g^5 + 6*a^2*b^7*c^2*d^2*g^5 - 4*a^3*b^6*c*d^3*g^5 + a^4*b^5*d^4*g
^5)*x^4 + 4*(a*b^8*c^4*g^5 - 4*a^2*b^7*c^3*d*g^5 + 6*a^3*b^6*c^2*d^2*g^5 -
4*a^4*b^5*c*d^3*g^5 + a^5*b^4*d^4*g^5)*x^3 + 6*(a^2*b^7*c^4*g^5 - 4*a^3*b^6
*c^3*d*g^5 + 6*a^4*b^5*c^2*d^2*g^5 - 4*a^5*b^4*c*d^3*g^5 + a^6*b^3*d^4*g^5)
*x^2 + 4*(a^3*b^6*c^4*g^5 - 4*a^4*b^5*c^3*d*g^5 + 6*a^5*b^4*c^2*d^2*g^5 - 4
*a^6*b^3*c*d^3*g^5 + a^7*b^2*d^4*g^5)*x))*B^2 + 1/24*A*B*((12*b^3*d^3*x^3 -
 3*b^3*c^3 + 13*a*b^2*c^2*d - 23*a^2*b*c*d^2 + 25*a^3*d^3 - 6*(b^3*c*d^2 -
7*a*b^2*d^3)*x^2 + 4*(b^3*c^2*d - 5*a*b^2*c*d^2 + 13*a^2*b*d^3)*x)/((b^8*c^
3 - 3*a*b^7*c^2*d + 3*a^2*b^6*c*d^2 - a^3*b^5*d^3)*g^5*x^4 + 4*(a*b^7*c^3 -
 3*a^2*b^6*c^2*d + 3*a^3*b^5*c*d^2 - a^4*b^4*d^3)*g^5*x^3 + 6*(a^2*b^6*c^3
- 3*a^3*b^5*c^2*d + 3*a^4*b^4*c*d^2 - a^5*b^3*d^3)*g^5*x^2 + 4*(a^3*b^5*c^3
 - 3*a^4*b^4*c^2*d + 3*a^5*b^3*c*d^2 - a^6*b^2*d^3)*g^5*x + (a^4*b^4*c^3 -
3*a^5*b^3*c^2*d + 3*a^6*b^2*c*d^2 - a^7*b*d^3)*g^5) - 12*log(b*e*x/(d*x + c
) + a*e/(d*x + c))/(b^5*g^5*x^4 + 4*a*b^4*g^5*x^3 + 6*a^2*b^3*g^5*x^2 + 4*a
^3*b^2*g^5*x + a^4*b*g^5) + 12*d^4*log(b*x + a)/((b^5*c^4 - 4*a*b^4*c^3*d +
 6*a^2*b^3*c^2*d^2 - 4*a^3*b^2*c*d^3 + a^4*b*d^4)*g^5) - 12*d^4*log(d*x + c
)/((b^5*c^4 - 4*a*b^4*c^3*d + 6*a^2*b^3*c^2*d^2 - 4*a^3*b^2*c*d^3 + a^4*b*d
^4)*g^5)) - 1/4*B^2*log(b*e*x/(d*x + c) + a*e/(d*x + c))^2/(b^5*g^5*x^4 + 4
*a*b^4*g^5*x^3 + 6*a^2*b^3*g^5*x^2 + 4*a^3*b^2*g^5*x + a^4*b*g^5) - 1/4*A^2
/(b^5*g^5*x^4 + 4*a*b^4*g^5*x^3 + 6*a^2*b^3*g^5*x^2 + 4*a^3*b^2*g^5*x + a^4
*b*g^5)
```

**mupad [B]** time = 10.30, size = 1881, normalized size = 3.27

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*log((e*(a + b*x))/(c + d*x)))^2/(a*g + b*g*x)^5,x)
```

```
[Out] (B*d^4*atan((B*d^4*(12*A + 25*B)*(24*b^5*c^4*g^5 - 24*a^4*b*d^4*g^5 - 48*a*
```

```
b^4*c^3*d*g^5 + 48*a^3*b^2*c*d^3*g^5)*1i)/(24*b*g^5*(a*d - b*c)^4*(25*B^2*d
^4 + 12*A*B*d^4)) + (B*d^5*x*(12*A + 25*B)*(b^4*c^3*g^5 - a^3*b*d^3*g^5 - 3
*a*b^3*c^2*d*g^5 + 3*a^2*b^2*c*d^2*g^5)*2i)/(g^5*(a*d - b*c)^4*(25*B^2*d^4
+ 12*A*B*d^4)))*(12*A + 25*B)*1i)/(12*b*g^5*(a*d - b*c)^4) - log((e*(a + b*
x))/(c + d*x))^2*(B^2/(4*b^2*g^5*(4*a^3*x + a^4/b + b^3*x^4 + 6*a^2*b*x^2 +
 4*a*b^2*x^3)) - (B^2*d^4)/(4*b*g^5*(a^4*d^4 + b^4*c^4 + 6*a^2*b^2*c^2*d^2
- 4*a*b^3*c^3*d - 4*a^3*b*c*d^3))) - (log((e*(a + b*x))/(c + d*x))*((A*B)/(
2*b^2*d*g^5) + (B^2*d^4*(a*(a*((4*a^2*d^2 + b^2*c^2 - 5*a*b*c*d)/(12*b*d^3)
 + (a*(a*d - b*c))/(4*b*d^2)) + (6*a^3*d^3 - b^3*c^3 + 5*a*b^2*c^2*d - 10*a
^2*b*c*d^2)/(12*b*d^4)) + (4*a^4*d^4 + b^4*c^4 + 10*a^2*b^2*c^2*d^2 - 5*a*b
^3*c^3*d - 10*a^3*b*c*d^3)/(4*b*d^5)))/(2*b*g^5*(a^4*d^4 + b^4*c^4 + 6*a^2*
b^2*c^2*d^2 - 4*a*b^3*c^3*d - 4*a^3*b*c*d^3)) + (B^2*d^4*x^2*(b*(b*((4*a^2*
d^2 + b^2*c^2 - 5*a*b*c*d)/(12*b*d^3) + (a*(a*d - b*c))/(4*b*d^2)) + (4*a^2
*d^2 + b^2*c^2 - 5*a*b*c*d)/(6*d^3) + (a*(a*d - b*c))/(2*d^2)) - a*((b^2*c
- a*b*d)/(4*d^2) - (b*(a*d - b*c))/(2*d^2)) + (b^3*c^2 + 4*a^2*b*d^2 - 5*a*
b^2*c*d)/(4*d^3)))/(2*b*g^5*(a^4*d^4 + b^4*c^4 + 6*a^2*b^2*c^2*d^2 - 4*a*b^
3*c^3*d - 4*a^3*b*c*d^3)) - (B^2*d^4*x^3*(b*((b^2*c - a*b*d)/(4*d^2) - (b*(
a*d - b*c))/(2*d^2)) + (b^3*c - a*b^2*d)/(4*d^2)))/(2*b*g^5*(a^4*d^4 + b^4*
c^4 + 6*a^2*b^2*c^2*d^2 - 4*a*b^3*c^3*d - 4*a^3*b*c*d^3)) + (B^2*d^4*x*(b*(
a*((4*a^2*d^2 + b^2*c^2 - 5*a*b*c*d)/(12*b*d^3) + (a*(a*d - b*c))/(4*b*d^2)
) + (6*a^3*d^3 - b^3*c^3 + 5*a*b^2*c^2*d - 10*a^2*b*c*d^2)/(12*b*d^4)) + a*
(b*((4*a^2*d^2 + b^2*c^2 - 5*a*b*c*d)/(12*b*d^3) + (a*(a*d - b*c))/(4*b*d^2
)) + (4*a^2*d^2 + b^2*c^2 - 5*a*b*c*d)/(6*d^3) + (a*(a*d - b*c))/(2*d^2)) +
 (6*a^3*d^3 - b^3*c^3 + 5*a*b^2*c^2*d - 10*a^2*b*c*d^2)/(4*d^4)))/(2*b*g^5*
(a^4*d^4 + b^4*c^4 + 6*a^2*b^2*c^2*d^2 - 4*a*b^3*c^3*d - 4*a^3*b*c*d^3))))/
((4*a^3*x)/d + a^4/(b*d) + (b^3*x^4)/d + (6*a^2*b*x^2)/d + (4*a*b^2*x^3)/d)
 - ((72*A^2*a^3*d^3 - 72*A^2*b^3*c^3 + 415*B^2*a^3*d^3 - 9*B^2*b^3*c^3 + 30
0*A*B*a^3*d^3 - 36*A*B*b^3*c^3 + 216*A^2*a*b^2*c^2*d - 216*A^2*a^2*b*c*d^2
+ 55*B^2*a*b^2*c^2*d - 161*B^2*a^2*b*c*d^2 + 156*A*B*a*b^2*c^2*d - 276*A*B*
a^2*b*c*d^2)/(12*(a*d - b*c)) + (x^2*(163*B^2*a*b^2*d^3 - 13*B^2*b^3*c*d^2
+ 84*A*B*a*b^2*d^3 - 12*A*B*b^3*c*d^2))/(2*(a*d - b*c)) + (x*(271*B^2*a^2*b
*d^3 + 7*B^2*b^3*c^2*d - 53*B^2*a*b^2*c*d^2 + 156*A*B*a^2*b*d^3 + 12*A*B*b^
3*c^2*d - 60*A*B*a*b^2*c*d^2))/(3*(a*d - b*c)) + (d*x^3*(25*B^2*b^3*d^2 + 1
2*A*B*b^3*d^2))/(a*d - b*c))/(x*(96*a^3*b^4*c^2*g^5 + 96*a^5*b^2*d^2*g^5 -
192*a^4*b^3*c*d*g^5) + x^3*(96*a*b^6*c^2*g^5 + 96*a^3*b^4*d^2*g^5 - 192*a^2
*b^5*c*d*g^5) + x^4*(24*b^7*c^2*g^5 + 24*a^2*b^5*d^2*g^5 - 48*a*b^6*c*d*g^5
) + x^2*(144*a^2*b^5*c^2*g^5 + 144*a^4*b^3*d^2*g^5 - 288*a^3*b^4*c*d*g^5) +
 24*a^6*b*d^2*g^5 + 24*a^4*b^3*c^2*g^5 - 48*a^5*b^2*c*d*g^5)
```

**sympy** [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*ln(e*(b*x+a)/(d*x+c)))**2/(b*g*x+a*g)**5,x)
```

```
[Out] Timed out
```

# 3.106 $\int \frac{\log\left(\frac{d(a+bx)}{b(c+dx)}\right)}{cf+dfx}\,dx$

**Optimal**. Leaf size=28

$$\frac{\text{Li}_2\left(\frac{bc-ad}{b(c+dx)}\right)}{df}$$

[Out] polylog(2,(-a*d+b*c)/b/(d*x+c))/d/f

**Rubi [A]** time = 0.02, antiderivative size = 29, normalized size of antiderivative = 1.04, number of steps used = 1, number of rules used = 1, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.034, Rules used = {2447}

$$\frac{\text{PolyLog}\left(2,1-\frac{d(a+bx)}{b(c+dx)}\right)}{df}$$

Antiderivative was successfully verified.

[In] Int[Log[(d*(a + b*x))/(b*(c + d*x))]/(c*f + d*f*x),x]

[Out] PolyLog[2, 1 - (d*(a + b*x))/(b*(c + d*x))]/(d*f)

Rule 2447

```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[(Pq^m*(1 - u))
/D[u, x]]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

Rubi steps

$$\int \frac{\log\left(\frac{d(a+bx)}{b(c+dx)}\right)}{cf+dfx}\,dx = \frac{\text{Li}_2\left(1-\frac{d(a+bx)}{b(c+dx)}\right)}{df}$$

**Mathematica [B]** time = 0.05, size = 114, normalized size = 4.07

$$\frac{\log\left(\frac{bc-ad}{bc+bdx}\right)\left(2\log\left(\frac{d(a+bx)}{ad-bc}\right)-2\log\left(\frac{d(a+bx)}{b(c+dx)}\right)+\log\left(\frac{bc-ad}{bc+bdx}\right)\right)-2\text{Li}_2\left(\frac{b(c+dx)}{bc-ad}\right)}{2df}$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[(d*(a + b*x))/(b*(c + d*x))]/(c*f + d*f*x),x]
```

```
[Out] (Log[(b*c - a*d)/(b*c + b*d*x)]*(2*Log[(d*(a + b*x))/(-(b*c) + a*d)] - 2*Lo
g[(d*(a + b*x))/(b*(c + d*x))] + Log[(b*c - a*d)/(b*c + b*d*x)]) - 2*PolyLo
g[2, (b*(c + d*x))/(b*c - a*d)])/(2*d*f)
```

**fricas [A]** time = 0.68, size = 30, normalized size = 1.07

$$\frac{\text{Li}_2\left(-\frac{bdx+ad}{bdx+bc}+1\right)}{df}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(d*(b*x+a)/b/(d*x+c))/(d*f*x+c*f),x, algorithm="fricas")
```

```
[Out] dilog(-(b*d*x + a*d)/(b*d*x + b*c) + 1)/(d*f)
```

**giac [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(d*(b*x+a)/b/(d*x+c))/(d*f*x+c*f),x, algorithm="giac")
```

```
[Out] Timed out
```

**maple [A]** time = 0.05, size = 30, normalized size = 1.07

$$\frac{\text{dilog}\left(\frac{ad-bc}{(dx+c)b}+1\right)}{df}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(d*(b*x+a)/b/(d*x+c))/(d*f*x+c*f),x)
```

```
[Out] 1/d*dilog(1+(a*d-b*c)/b/(d*x+c))/f
```

**maxima [B]** time = 1.17, size = 158, normalized size = 5.64

$$-\frac{b\left(\frac{\log(dx+c)^2}{bf}-\frac{2\left(\log(bx+a)\log\left(\frac{bdx+ad}{bc-ad}+1\right)+\text{Li}_2\left(-\frac{bdx+ad}{bc-ad}\right)\right)}{bf}\right)}{2d}-\frac{b\left(\frac{d\log(bx+a)}{b}-\frac{d\log(dx+c)}{b}\right)\log\left(dfx+cf\right)}{d^2f}+\frac{\log\left(dfx+cf\right)\log}{df}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(d*(b*x+a)/b/(d*x+c))/(d*f*x+c*f),x, algorithm="maxima")
```

```
[Out] -1/2*b*(log(d*x + c)^2/(b*f) - 2*(log(b*x + a)*log((b*d*x + a*d)/(b*c - a*d
) + 1) + dilog(-(b*d*x + a*d)/(b*c - a*d)))/(b*f))/d - b*(d*log(b*x + a)/b
- d*log(d*x + c)/b)*log(d*f*x + c*f)/(d^2*f) + log(d*f*x + c*f)*log((b*x +
a)*d/((d*x + c)*b))/(d*f)
```

**mupad [B]** time = 4.25, size = 25, normalized size = 0.89

$$\frac{\mathrm{Li}_2\left(\frac{d\,(a+b\,x)}{b\,(c+d\,x)}\right)}{d\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log((d*(a + b*x))/(b*(c + d*x)))/(c*f + d*f*x),x)
```

```
[Out] dilog((d*(a + b*x))/(b*(c + d*x)))/(d*f)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\log\left(\frac{ad}{bc+bdx}+\frac{bdx}{bc+bdx}\right)}{c+dx}\,dx}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(d*(b*x+a)/b/(d*x+c))/(d*f*x+c*f),x)
```

```
[Out] Integral(log(a*d/(b*c + b*d*x) + b*d*x/(b*c + b*d*x))/(c + d*x), x)/f
```

# 3.107 $\int \frac{\log\left(1+\frac{1}{a+bx}\right)}{a+bx}\,dx$

**Optimal**. Leaf size=15

$$\frac{\text{Li}_2\left(-\frac{1}{a+bx}\right)}{b}$$

```
[Out] polylog(2,-1/(b*x+a))/b
```

**Rubi [A]** time = 0.01, antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.056, Rules used = {2447}

$$\frac{\text{PolyLog}\left(2,-\frac{1}{a+bx}\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[Log[1 + (a + b*x)^(-1)]/(a + b*x),x]
```

```
[Out] PolyLog[2, -(a + b*x)^(-1)]/b
```

Rule 2447

```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[(Pq^m*(1 - u))
/D[u, x]]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

Rubi steps

$$\int \frac{\log\left(1+\frac{1}{a+bx}\right)}{a+bx}\,dx=\frac{\text{Li}_2\left(-\frac{1}{a+bx}\right)}{b}$$

**Mathematica [B]** time = 0.01, size = 140, normalized size = 9.33

$$-\frac{\text{Li}_2(-a-bx)}{b}+\frac{\log^2\left(\frac{ab-(a+1)b}{b(a+bx)}\right)}{2b}+\frac{\log\left(\frac{b(-a-bx-1)}{(-a-1)b+ab}\right)\log\left(\frac{ab-(a+1)b}{b(a+bx)}\right)}{b}-\frac{\log\left(\frac{a+bx+1}{a+bx}\right)\log\left(\frac{ab-(a+1)b}{b(a+bx)}\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[1 + (a + b*x)^(-1)]/(a + b*x),x]
```

```
[Out] (Log[(b*(-1 - a - b*x))/((-1 - a)*b + a*b)]*Log[(a*b - (1 + a)*b)/(b*(a + b
*x))])/b + Log[(a*b - (1 + a)*b)/(b*(a + b*x))]^2/(2*b) - (Log[(a*b - (1 +
a)*b)/(b*(a + b*x))]*Log[(1 + a + b*x)/(a + b*x)])/b - PolyLog[2, -a - b*x]
/b
```

**fricas [A]** time = 0.66, size = 22, normalized size = 1.47

$$\frac{\mathrm{Li}_2\left(-\frac{bx+a+1}{bx+a}+1\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(1+1/(b*x+a))/(b*x+a),x, algorithm="fricas")
```

```
[Out] dilog(-(b*x + a + 1)/(b*x + a) + 1)/b
```

**giac [B]** time = 39.25, size = 320, normalized size = 21.33

$$\frac{1}{2}\left((a+1)b-ab\right)^{2}\left(\frac{\log\left(\frac{|bx+a+1|}{|bx+a|}\right)}{b^{4}}-\frac{\log\left(\left|\frac{bx+a+1}{bx+a}-1\right|\right)}{b^{4}}-\frac{1}{b^{4}\left(\frac{bx+a+1}{bx+a}-1\right)}-\frac{\log\left(\frac{1}{a-\frac{\left(\frac{\left(a-\frac{\left(\frac{(bx+a+1)a}{bx+a}-a-1\right)b}{\frac{(bx+a+1)b}{bx+a}-b}+1\right)a}{a-\frac{\left(\frac{(bx+a+1)a}{bx+a}-a-1\right)b}{\frac{(bx+a+1)b}{bx+a}-b}}-a-1\right)b}{\frac{\left(a-\frac{\left(\frac{(bx+a+1)a}{bx+a}-a-1\right)b}{\frac{(bx+a+1)b}{bx+a}-b}+1\right)b}{a-\frac{\left(\frac{(bx+a+1)a}{bx+a}-a-1\right)b}{\frac{(bx+a+1)b}{bx+a}-b}}-b}}+1\right)}{b^{4}\left(\frac{bx+a+1}{bx+a}-1\right)^{2}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(1+1/(b*x+a))/(b*x+a),x, algorithm="giac")
```

```
[Out] 1/2*((a + 1)*b - a*b)^2*(log(abs(b*x + a + 1)/abs(b*x + a))/b^4 - log(abs((
b*x + a + 1)/(b*x + a) - 1))/b^4 - 1/(b^4*((b*x + a + 1)/(b*x + a) - 1)) -
```

```
log(1/(a - ((a - ((b*x + a + 1)*a/(b*x + a) - a - 1)*b/((b*x + a + 1)*b/(b*
x + a) - b) + 1)*a/(a - ((b*x + a + 1)*a/(b*x + a) - a - 1)*b/((b*x + a + 1
)*b/(b*x + a) - b)) - a - 1)*b/((a - ((b*x + a + 1)*a/(b*x + a) - a - 1)*b/
((b*x + a + 1)*b/(b*x + a) - b) + 1)*b/(a - ((b*x + a + 1)*a/(b*x + a) - a
- 1)*b/((b*x + a + 1)*b/(b*x + a) - b)) - b)) + 1)/(b^4*((b*x + a + 1)/(b*x
 + a) - 1)^2))
```

**maple** [A] time = 0.04, size = 15, normalized size = 1.00

$$\frac{\operatorname{dilog}\left(1+\frac{1}{bx+a}\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(1+1/(b*x+a))/(b*x+a),x)
```

```
[Out] 1/b*dilog(1+1/(b*x+a))
```

**maxima** [B] time = 1.21, size = 61, normalized size = 4.07

$$\frac{2\,\log\left(bx+a+1\right)\log\left(bx+a\right)-\log\left(bx+a\right)^{2}}{2\,b}-\frac{\log\left(bx+a+1\right)\log\left(bx+a\right)+\operatorname{Li}_2\left(-bx-a\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(1+1/(b*x+a))/(b*x+a),x, algorithm="maxima")
```

```
[Out] 1/2*(2*log(b*x + a + 1)*log(b*x + a) - log(b*x + a)^2)/b - (log(b*x + a + 1
)*log(b*x + a) + dilog(-b*x - a))/b
```

**mupad** [B] time = 4.03, size = 15, normalized size = 1.00

$$\frac{\operatorname{polylog}\left(2,-\frac{1}{a+b\,x}\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(1/(a + b*x) + 1)/(a + b*x),x)
```

```
[Out] polylog(2, -1/(a + b*x))/b
```

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\log\left(1+\frac{1}{a+bx}\right)}{a+bx}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(1+1/(b*x+a))/(b*x+a),x)
```

```
[Out] Integral(log(1 + 1/(a + b*x))/(a + b*x), x)
```

## 3.108 $\int \frac{\log\left(1-\frac{1}{a+bx}\right)}{a+bx}\,dx$

**Optimal**. Leaf size=13

$$\frac{\text{Li}_2\left(\frac{1}{a+bx}\right)}{b}$$

```
[Out] polylog(2,1/(b*x+a))/b
```

**Rubi [A]** time = 0.01, antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.050, Rules used = {2447}

$$\frac{\text{PolyLog}\left(2,\frac{1}{a+bx}\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[Log[1 - (a + b*x)^(-1)]/(a + b*x),x]
```

```
[Out] PolyLog[2, (a + b*x)^(-1)]/b
```

Rule 2447

```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[(Pq^m*(1 - u))
/D[u, x]]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

Rubi steps

$$\int \frac{\log\left(1-\frac{1}{a+bx}\right)}{a+bx}\,dx=\frac{\text{Li}_2\left(\frac{1}{a+bx}\right)}{b}$$

**Mathematica [B]** time = 0.02, size = 133, normalized size = 10.23

$$-\frac{\text{Li}_2(a+bx)}{b}+\frac{\log^2\left(\frac{ab-(a-1)b}{b(a+bx)}\right)}{2b}+\frac{\log\left(\frac{b(a+bx-1)}{(a-1)b-ab}\right)\log\left(\frac{ab-(a-1)b}{b(a+bx)}\right)}{b}-\frac{\log\left(\frac{a+bx-1}{a+bx}\right)\log\left(\frac{ab-(a-1)b}{b(a+bx)}\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[1 - (a + b*x)^(-1)]/(a + b*x),x]
```

```
[Out] (Log[(b*(-1 + a + b*x))/((-1 + a)*b - a*b)]*Log[(-((-1 + a)*b) + a*b)/(b*(a
 + b*x))])/b + Log[(-((-1 + a)*b) + a*b)/(b*(a + b*x))]^2/(2*b) - (Log[(-((
-1 + a)*b) + a*b)/(b*(a + b*x))]*Log[(-1 + a + b*x)/(a + b*x)])/b - PolyLog
[2, a + b*x]/b
```

**fricas [A]** time = 0.81, size = 22, normalized size = 1.69

$$\frac{\mathrm{Li}_2\left(-\frac{bx+a-1}{bx+a}+1\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(1-1/(b*x+a))/(b*x+a),x, algorithm="fricas")
```

```
[Out] dilog(-(b*x + a - 1)/(b*x + a) + 1)/b
```

**giac [B]** time = 30.47, size = 322, normalized size = 24.77

$$-\frac{1}{2}\left((a-1)b-ab\right)^{2}\left(\frac{\log\left(\frac{|bx+a-1|}{|bx+a|}\right)}{b^{4}}-\frac{\log\left(\left|\frac{bx+a-1}{bx+a}-1\right|\right)}{b^{4}}-\frac{1}{b^{4}\left(\frac{bx+a-1}{bx+a}-1\right)}-\frac{\log\left(-\frac{1}{a-\frac{\left(\frac{\left(a-\frac{\left(\frac{(bx+a-1)a}{bx+a}-a+1\right)b}{\frac{(bx+a-1)b}{bx+a}-b}-1\right)a}{a-\frac{\left(\frac{(bx+a-1)a}{bx+a}-a+1\right)b}{\frac{(bx+a-1)b}{bx+a}-b}}-a+1\right)b}{\frac{\left(a-\frac{\left(\frac{(bx+a-1)a}{bx+a}-a+1\right)b}{\frac{(bx+a-1)b}{bx+a}-b}-1\right)b}{a-\frac{\left(\frac{(bx+a-1)a}{bx+a}-a+1\right)b}{\frac{(bx+a-1)b}{bx+a}-b}}-b}}+1\right.}{b^{4}\left(\frac{bx+a-1}{bx+a}-1\right)^{2}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(1-1/(b*x+a))/(b*x+a),x, algorithm="giac")
```

```
[Out] -1/2*((a - 1)*b - a*b)^2*(log(abs(b*x + a - 1)/abs(b*x + a))/b^4 - log(abs(
(b*x + a - 1)/(b*x + a) - 1))/b^4 - 1/(b^4*((b*x + a - 1)/(b*x + a) - 1)) -
```

```
 log(-1/(a - ((a - ((b*x + a - 1)*a/(b*x + a) - a + 1)*b/((b*x + a - 1)*b/(
b*x + a) - b) - 1)*a/(a - ((b*x + a - 1)*a/(b*x + a) - a + 1)*b/((b*x + a -
 1)*b/(b*x + a) - b)) - a + 1)*b/((a - ((b*x + a - 1)*a/(b*x + a) - a + 1)*
b/((b*x + a - 1)*b/(b*x + a) - b) - 1)*b/(a - ((b*x + a - 1)*a/(b*x + a) -
a + 1)*b/((b*x + a - 1)*b/(b*x + a) - b)) - b)) + 1)/(b^4*((b*x + a - 1)/(b
*x + a) - 1)^2))
```

**maple [A]** time = 0.04, size = 17, normalized size = 1.31

$$\frac{\text{dilog}\left(1-\frac{1}{bx+a}\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(1-1/(b*x+a))/(b*x+a),x)
```

```
[Out] 1/b*dilog(1-1/(b*x+a))
```

**maxima [B]** time = 1.30, size = 59, normalized size = 4.54

$$-\frac{\log\left(bx+a\right)^{2}-2\,\log\left(bx+a\right)\log\left(bx+a-1\right)}{2\,b}-\frac{\log\left(bx+a\right)\log\left(-bx-a+1\right)+\text{Li}_{2}\left(bx+a\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(1-1/(b*x+a))/(b*x+a),x, algorithm="maxima")
```

```
[Out] -1/2*(log(b*x + a)^2 - 2*log(b*x + a)*log(b*x + a - 1))/b - (log(b*x + a)*l
og(-b*x - a + 1) + dilog(b*x + a))/b
```

**mupad [B]** time = 4.23, size = 13, normalized size = 1.00

$$\frac{\text{polylog}\left(2,\frac{1}{a+b\,x}\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(1 - 1/(a + b*x))/(a + b*x),x)
```

```
[Out] polylog(2, 1/(a + b*x))/b
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\log\left(1-\frac{1}{a+bx}\right)}{a+bx}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(1-1/(b*x+a))/(b*x+a),x)
```

```
[Out] Integral(log(1 - 1/(a + b*x))/(a + b*x), x)
```

# 3.109 $\int \frac{(ag+bgx)^2}{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}\,dx$

**Optimal**. Leaf size=35

$$\text{Int}\left(\frac{(ag+bgx)^2}{B\log\left(\frac{e(a+bx)}{c+dx}\right)+A},x\right)$$

```
[Out] Unintegrable((b*g*x+a*g)^2/(A+B*ln(e*(b*x+a)/(d*x+c))),x)
```

**Rubi [A]** time = 0.20, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{(ag+bgx)^2}{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(a*g + b*g*x)^2/(A + B*Log[(e*(a + b*x))/(c + d*x)]),x]
```

```
[Out] a^2*g^2*Defer[Int][(A + B*Log[(e*(a + b*x))/(c + d*x)])^(-1), x] + 2*a*b*g^
2*Defer[Int][x/(A + B*Log[(e*(a + b*x))/(c + d*x)]), x] + b^2*g^2*Defer[Int
][x^2/(A + B*Log[(e*(a + b*x))/(c + d*x)]), x]
```

Rubi steps

$$\begin{aligned}\int \frac{(ag+bgx)^2}{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}\,dx &= \int \left(\frac{a^2g^2}{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}+\frac{2abg^2x}{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}+\frac{b^2g^2x^2}{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}\right)dx\\ &= \left(a^2g^2\right)\int \frac{1}{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}\,dx+\left(2abg^2\right)\int \frac{x}{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}\,dx+\left(b^2g^2\right)\int \frac{}{A+}\end{aligned}$$

**Mathematica [A]** time = 0.62, size = 0, normalized size = 0.00

$$\int \frac{(ag+bgx)^2}{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(a*g + b*g*x)^2/(A + B*Log[(e*(a + b*x))/(c + d*x)]),x]
```

```
[Out] Integrate[(a*g + b*g*x)^2/(A + B*Log[(e*(a + b*x))/(c + d*x)]), x]
```

**fricas [A]** time = 0.79, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{b^2 g^2 x^2 + 2\,abg^2 x + a^2 g^2}{B\log\left(\frac{bex+ae}{dx+c}\right) + A}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*g*x+a*g)^2/(A+B*log(e*(b*x+a)/(d*x+c))),x, algorithm="fricas")
```

```
[Out] integral((b^2*g^2*x^2 + 2*a*b*g^2*x + a^2*g^2)/(B*log((b*e*x + a*e)/(d*x +
c)) + A), x)
```

**giac [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*g*x+a*g)^2/(A+B*log(e*(b*x+a)/(d*x+c))),x, algorithm="giac")
```

```
[Out] Timed out
```

**maple [A]** time = 1.15, size = 0, normalized size = 0.00

$$\int \frac{\left(bgx + ag\right)^2}{B\ln\left(\frac{(bx+a)e}{dx+c}\right) + A}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*g*x+a*g)^2/(B*ln((b*x+a)/(d*x+c)*e)+A),x)
```

```
[Out] int((b*g*x+a*g)^2/(B*ln((b*x+a)/(d*x+c)*e)+A),x)
```

**maxima [A]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(bgx + ag\right)^2}{B\log\left(\frac{(bx+a)e}{dx+c}\right) + A}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*g*x+a*g)^2/(A+B*log(e*(b*x+a)/(d*x+c))),x, algorithm="maxima")
```

```
[Out] integrate((b*g*x + a*g)^2/(B*log((b*x + a)*e/(d*x + c)) + A), x)
```

**mupad [A]** time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{{\left(a\,g+b\,g\,x\right)}^2}{A+B\,\ln\left(\frac{e\,\left(a+b\,x\right)}{c+d\,x}\right)} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*g + b*g*x)^2/(A + B*log((e*(a + b*x))/(c + d*x))),x)
```

```
[Out] int((a*g + b*g*x)^2/(A + B*log((e*(a + b*x))/(c + d*x))), x)
```

**sympy [A]** time = 0.00, size = 0, normalized size = 0.00

$$g^{2}\left(\int \frac{a^{2}}{A + B\log\left(\frac{ae}{c+dx} + \frac{bex}{c+dx}\right)}\, dx + \int \frac{b^{2}x^{2}}{A + B\log\left(\frac{ae}{c+dx} + \frac{bex}{c+dx}\right)}\, dx + \int \frac{2abx}{A + B\log\left(\frac{ae}{c+dx} + \frac{bex}{c+dx}\right)}\, dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*g*x+a*g)**2/(A+B*ln(e*(b*x+a)/(d*x+c))),x)
```

```
[Out] g**2*(Integral(a**2/(A + B*log(a*e/(c + d*x) + b*e*x/(c + d*x))), x) + Inte
gral(b**2*x**2/(A + B*log(a*e/(c + d*x) + b*e*x/(c + d*x))), x) + Integral(
2*a*b*x/(A + B*log(a*e/(c + d*x) + b*e*x/(c + d*x))), x))
```

# 3.110 $\int \frac{ag+bgx}{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)} \, dx$

**Optimal**. Leaf size=33

$$\text{Int}\left(\frac{ag+bgx}{B\log\left(\frac{e(a+bx)}{c+dx}\right)+A},x\right)$$

```
[Out] Unintegrable((b*g*x+a*g)/(A+B*ln(e*(b*x+a)/(d*x+c))),x)
```

**Rubi [A]** time = 0.10, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{ag+bgx}{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)} \, dx$$

Verification is Not applicable to the result.

```
[In] Int[(a*g + b*g*x)/(A + B*Log[(e*(a + b*x))/(c + d*x)]),x]
```

```
[Out] a*g*Defer[Int][(A + B*Log[(e*(a + b*x))/(c + d*x)])^(-1), x] + b*g*Defer[In
t][x/(A + B*Log[(e*(a + b*x))/(c + d*x)]), x]
```

Rubi steps

$$\begin{aligned}
\int \frac{ag+bgx}{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)} \, dx &= \int \left(\frac{ag}{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}+\frac{bgx}{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}\right) \, dx \\
&= (ag)\int \frac{1}{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)} \, dx+(bg)\int \frac{x}{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)} \, dx
\end{aligned}$$

**Mathematica [A]** time = 0.25, size = 0, normalized size = 0.00

$$\int \frac{ag+bgx}{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)} \, dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(a*g + b*g*x)/(A + B*Log[(e*(a + b*x))/(c + d*x)]),x]
```

```
[Out] Integrate[(a*g + b*g*x)/(A + B*Log[(e*(a + b*x))/(c + d*x)]), x]
```

**fricas [A]** time = 1.44, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{bgx+ag}{B\log\left(\frac{bex+ae}{dx+c}\right)+A}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*g*x+a*g)/(A+B*log(e*(b*x+a)/(d*x+c))),x, algorithm="fricas")
[Out] integral((b*g*x + a*g)/(B*log((b*e*x + a*e)/(d*x + c)) + A), x)
```

**giac [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*g*x+a*g)/(A+B*log(e*(b*x+a)/(d*x+c))),x, algorithm="giac")
[Out] Timed out
```

**maple [A]** time = 1.02, size = 0, normalized size = 0.00

$$\int \frac{bgx+ag}{B\ln\left(\frac{(bx+a)e}{dx+c}\right)+A}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*g*x+a*g)/(B*ln((b*x+a)/(d*x+c)*e)+A),x)
[Out] int((b*g*x+a*g)/(B*ln((b*x+a)/(d*x+c)*e)+A),x)
```

**maxima [A]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{bgx+ag}{B\log\left(\frac{(bx+a)e}{dx+c}\right)+A}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*g*x+a*g)/(A+B*log(e*(b*x+a)/(d*x+c))),x, algorithm="maxima")
[Out] integrate((b*g*x + a*g)/(B*log((b*x + a)*e/(d*x + c)) + A), x)
```

**mupad** [A] time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{a\,g+b\,g\,x}{A+B\,\ln\left(\frac{e\,(a+b\,x)}{c+d\,x}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*g + b*g*x)/(A + B*log((e*(a + b*x))/(c + d*x))),x)
```

```
[Out] int((a*g + b*g*x)/(A + B*log((e*(a + b*x))/(c + d*x))), x)
```

**sympy** [A] time = 0.00, size = 0, normalized size = 0.00

$$g\left(\int \frac{a}{A+B\log\left(\frac{ae}{c+dx}+\frac{bex}{c+dx}\right)}\,dx+\int \frac{bx}{A+B\log\left(\frac{ae}{c+dx}+\frac{bex}{c+dx}\right)}\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*g*x+a*g)/(A+B*ln(e*(b*x+a)/(d*x+c))),x)
```

```
[Out] g*(Integral(a/(A + B*log(a*e/(c + d*x) + b*e*x/(c + d*x))), x) + Integral(b
*x/(A + B*log(a*e/(c + d*x) + b*e*x/(c + d*x))), x))
```

# 3.111 $\int \frac{1}{(ag+bgx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}\,dx$

**Optimal**. Leaf size=35

$$\text{Int}\left(\frac{1}{(ag+bgx)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)},x\right)$$

```
[Out] Unintegrable(1/(b*g*x+a*g)/(A+B*ln(e*(b*x+a)/(d*x+c))),x)
```

**Rubi [A]** time = 0.07, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{1}{(ag+bgx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[1/((a*g + b*g*x)*(A + B*Log[(e*(a + b*x))/(c + d*x)])),x]
[Out] Defer[Int][1/((a*g + b*g*x)*(A + B*Log[(e*(a + b*x))/(c + d*x)])), x]
```

Rubi steps

$$\int \frac{1}{(ag+bgx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}\,dx=\int \frac{1}{(ag+bgx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}\,dx$$

**Mathematica [A]** time = 0.23, size = 0, normalized size = 0.00

$$\int \frac{1}{(ag+bgx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[1/((a*g + b*g*x)*(A + B*Log[(e*(a + b*x))/(c + d*x)])),x]
[Out] Integrate[1/((a*g + b*g*x)*(A + B*Log[(e*(a + b*x))/(c + d*x)])), x]
```

**fricas** [A] time = 1.64, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{1}{Abgx + Aag + \left(Bbgx + Bag\right)\log\left(\frac{bex+ae}{dx+c}\right)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*g*x+a*g)/(A+B*log(e*(b*x+a)/(d*x+c))),x, algorithm="fricas")
```

```
[Out] integral(1/(A*b*g*x + A*a*g + (B*b*g*x + B*a*g)*log((b*e*x + a*e)/(d*x + c)
)), x)
```

**giac** [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*g*x+a*g)/(A+B*log(e*(b*x+a)/(d*x+c))),x, algorithm="giac")
```

```
[Out] Timed out
```

**maple** [A] time = 1.18, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(bgx + ag\right)\left(B\ln\left(\frac{(bx+a)e}{dx+c}\right) + A\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*g*x+a*g)/(B*ln((b*x+a)/(d*x+c)*e)+A),x)
```

```
[Out] int(1/(b*g*x+a*g)/(B*ln((b*x+a)/(d*x+c)*e)+A),x)
```

**maxima** [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(bgx + ag\right)\left(B\log\left(\frac{(bx+a)e}{dx+c}\right) + A\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*g*x+a*g)/(A+B*log(e*(b*x+a)/(d*x+c))),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*g*x + a*g)*(B*log((b*x + a)*e/(d*x + c)) + A)), x)
```

**mupad [A]** time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{1}{\left(a\,g+b\,g\,x\right)\,\left(A+B\,\ln\left(\frac{e\,\left(a+b\,x\right)}{c+d\,x}\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*g + b*g*x)*(A + B*log((e*(a + b*x))/(c + d*x)))),x)
[Out] int(1/((a*g + b*g*x)*(A + B*log((e*(a + b*x))/(c + d*x)))), x)
```

**sympy [A]** time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{1}{Aa+Abx+Ba\log\left(\frac{ae}{c+dx}+\frac{bex}{c+dx}\right)+Bbx\log\left(\frac{ae}{c+dx}+\frac{bex}{c+dx}\right)}\,dx}{g}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*g*x+a*g)/(A+B*ln(e*(b*x+a)/(d*x+c))),x)
[Out] Integral(1/(A*a + A*b*x + B*a*log(a*e/(c + d*x) + b*e*x/(c + d*x)) + B*b*x*
log(a*e/(c + d*x) + b*e*x/(c + d*x))), x)/g
```

# 3.112 $\int \frac{1}{(ag+bgx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}\,dx$

**Optimal**. Leaf size=50

$$\frac{ee^{A/B}\text{Ei}\left(-\frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{B}\right)}{Bg^2(bc-ad)}$$

```
[Out] e*exp(A/B)*Ei((-A-B*ln(e*(b*x+a)/(d*x+c)))/B)/B/(-a*d+b*c)/g^2
```

**Rubi [F]** time = 0.09, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{1}{(ag+bgx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[1/((a*g + b*g*x)^2*(A + B*Log[(e*(a + b*x))/(c + d*x)])),x]
[Out] Defer[Int][1/((a*g + b*g*x)^2*(A + B*Log[(e*(a + b*x))/(c + d*x)])), x]
```

Rubi steps

$$\int \frac{1}{(ag+bgx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}\,dx=\int \frac{1}{(ag+bgx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}\,dx$$

**Mathematica [A]** time = 0.10, size = 52, normalized size = 1.04

$$\frac{ee^{A/B}\text{Ei}\left(-\frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{B}\right)}{bBcg^2-aBdg^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a*g + b*g*x)^2*(A + B*Log[(e*(a + b*x))/(c + d*x)])),x]
[Out] (e*E^(A/B)*ExpIntegralEi[-((A + B*Log[(e*(a + b*x))/(c + d*x)])/B)])/(b*B*c
*g^2 - a*B*d*g^2)
```

**fricas** [A] time = 1.35, size = 47, normalized size = 0.94

$$\frac{ee^{\frac{A}{B}}\,\mathrm{log\_integral}\left(\frac{(dx+c)e^{\left(-\frac{A}{B}\right)}}{bex+ae}\right)}{(Bbc-Bad)g^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*g*x+a*g)^2/(A+B*log(e*(b*x+a)/(d*x+c))),x, algorithm="fricas
")
```

```
[Out] e*e^(A/B)*log_integral((d*x + c)*e^(-A/B)/(b*e*x + a*e))/((B*b*c - B*a*d)*g
^2)
```

**giac** [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*g*x+a*g)^2/(A+B*log(e*(b*x+a)/(d*x+c))),x, algorithm="giac")
```

```
[Out] Timed out
```

**maple** [F] time = 1.26, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(bgx+ag\right)^2\left(B\ln\left(\frac{(bx+a)e}{dx+c}\right)+A\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*g*x+a*g)^2/(B*ln((b*x+a)/(d*x+c)*e)+A),x)
```

```
[Out] int(1/(b*g*x+a*g)^2/(B*ln((b*x+a)/(d*x+c)*e)+A),x)
```

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(bgx+ag\right)^2\left(B\log\left(\frac{(bx+a)e}{dx+c}\right)+A\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*g*x+a*g)^2/(A+B*log(e*(b*x+a)/(d*x+c))),x, algorithm="maxima
")
```

```
[Out] integrate(1/((b*g*x + a*g)^2*(B*log((b*x + a)*e/(d*x + c)) + A)), x)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{\left(a\,g+b\,g\,x\right)^2\,\left(A+B\,\ln\left(\frac{e\,\left(a+b\,x\right)}{c+d\,x}\right)\right)} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*g + b*g*x)^2*(A + B*log((e*(a + b*x))/(c + d*x)))),x)
[Out] int(1/((a*g + b*g*x)^2*(A + B*log((e*(a + b*x))/(c + d*x)))), x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{1}{Aa^{2}+2Aabx+Ab^{2}x^{2}+Ba^{2}\log\left(\frac{ae}{c+dx}+\frac{bex}{c+dx}\right)+2Babx\log\left(\frac{ae}{c+dx}+\frac{bex}{c+dx}\right)+Bb^{2}x^{2}\log\left(\frac{ae}{c+dx}+\frac{bex}{c+dx}\right)}\, dx}{g^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*g*x+a*g)**2/(A+B*ln(e*(b*x+a)/(d*x+c))),x)
[Out] Integral(1/(A*a**2 + 2*A*a*b*x + A*b**2*x**2 + B*a**2*log(a*e/(c + d*x) + b
*e*x/(c + d*x)) + 2*B*a*b*x*log(a*e/(c + d*x) + b*e*x/(c + d*x)) + B*b**2*x
**2*log(a*e/(c + d*x) + b*e*x/(c + d*x))), x)/g**2
```

# 3.113 $\int \frac{1}{(ag+bgx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}\,dx$

**Optimal**. Leaf size=107

$$\frac{be^2e^{\frac{2A}{B}}\text{Ei}\left(-\frac{2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{B}\right)}{Bg^3(bc-ad)^2}-\frac{dee^{A/B}\text{Ei}\left(-\frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{B}\right)}{Bg^3(bc-ad)^2}$$

```
[Out] b*e^2*exp(2*A/B)*Ei(-2*(A+B*ln(e*(b*x+a)/(d*x+c)))/B)/B/(-a*d+b*c)^2/g^3-d*
e*exp(A/B)*Ei((-A-B*ln(e*(b*x+a)/(d*x+c)))/B)/B/(-a*d+b*c)^2/g^3
```

**Rubi** [F] time = 0.07, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{1}{(ag+bgx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[1/((a*g + b*g*x)^3*(A + B*Log[(e*(a + b*x))/(c + d*x)])),x]
[Out] Defer[Int][1/((a*g + b*g*x)^3*(A + B*Log[(e*(a + b*x))/(c + d*x)])), x]
```

Rubi steps

$$\int \frac{1}{(ag+bgx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}\,dx=\int \frac{1}{(ag+bgx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}\,dx$$

**Mathematica** [A] time = 0.17, size = 89, normalized size = 0.83

$$\frac{ee^{A/B}\left(bee^{A/B}\text{Ei}\left(-\frac{2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{B}\right)-d\text{Ei}\left(-\frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{B}\right)\right)}{Bg^3(bc-ad)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a*g + b*g*x)^3*(A + B*Log[(e*(a + b*x))/(c + d*x)])),x]
```

```
[Out] (e*E^(A/B)*(b*e*E^(A/B)*ExpIntegralEi[(-2*(A + B*Log[(e*(a + b*x))/(c + d*x
)]))/B] - d*ExpIntegralEi[-((A + B*Log[(e*(a + b*x))/(c + d*x)])/B)]))/(B*(
b*c - a*d)^2*g^3)
```

**fricas [A]** time = 0.66, size = 130, normalized size = 1.21

$$\frac{be^2e^{\left(\frac{2A}{B}\right)}\operatorname{log\_integral}\left(\frac{(d^2x^2+2\,cdx+c^2)e^{\left(-\frac{2A}{B}\right)}}{b^2e^2x^2+2\,abe^2x+a^2e^2}\right)-dee^{\frac{A}{B}}\operatorname{log\_integral}\left(\frac{(dx+c)e^{\left(-\frac{A}{B}\right)}}{bex+ae}\right)}{(Bb^2c^2-2\,Babcd+Ba^2d^2)g^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*g*x+a*g)^3/(A+B*log(e*(b*x+a)/(d*x+c))),x, algorithm="fricas
")
```

```
[Out] (b*e^2*e^(2*A/B)*log_integral((d^2*x^2 + 2*c*d*x + c^2)*e^(-2*A/B)/(b^2*e^2
*x^2 + 2*a*b*e^2*x + a^2*e^2)) - d*e*e^(A/B)*log_integral((d*x + c)*e^(-A/B
)/(b*e*x + a*e)))/((B*b^2*c^2 - 2*B*a*b*c*d + B*a^2*d^2)*g^3)
```

**giac [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*g*x+a*g)^3/(A+B*log(e*(b*x+a)/(d*x+c))),x, algorithm="giac")
```

```
[Out] Timed out
```

**maple [F]** time = 1.32, size = 0, normalized size = 0.00

$$\int \frac{1}{(bgx+ag)^3\left(B\ln\left(\frac{(bx+a)e}{dx+c}\right)+A\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*g*x+a*g)^3/(B*ln((b*x+a)/(d*x+c)*e)+A),x)
```

```
[Out] int(1/(b*g*x+a*g)^3/(B*ln((b*x+a)/(d*x+c)*e)+A),x)
```

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(bgx+ag)^3\left(B\log\left(\frac{(bx+a)e}{dx+c}\right)+A\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*g*x+a*g)^3/(A+B*log(e*(b*x+a)/(d*x+c))),x, algorithm="maxima
")
```

```
[Out] integrate(1/((b*g*x + a*g)^3*(B*log((b*x + a)*e/(d*x + c)) + A)), x)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\left(a\,g+b\,g\,x\right)^3\,\left(A+B\,\ln\left(\frac{e\,\left(a+b\,x\right)}{c+d\,x}\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*g + b*g*x)^3*(A + B*log((e*(a + b*x))/(c + d*x)))),x)
```

```
[Out] int(1/((a*g + b*g*x)^3*(A + B*log((e*(a + b*x))/(c + d*x)))), x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{1}{Aa^3+3Aa^2bx+3Aab^2x^2+Ab^3x^3+Ba^3\log\left(\frac{ae}{c+dx}+\frac{bex}{c+dx}\right)+3Ba^2bx\log\left(\frac{ae}{c+dx}+\frac{bex}{c+dx}\right)+3Bab^2x^2\log\left(\frac{ae}{c+dx}+\frac{bex}{c+dx}\right)+Bb^3x^3\log\left(\frac{ae}{c+dx}+\frac{bex}{c+dx}\right)}\,dx}{g^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*g*x+a*g)**3/(A+B*ln(e*(b*x+a)/(d*x+c))),x)
```

```
[Out] Integral(1/(A*a**3 + 3*A*a**2*b*x + 3*A*a*b**2*x**2 + A*b**3*x**3 + B*a**3*
log(a*e/(c + d*x) + b*e*x/(c + d*x)) + 3*B*a**2*b*x*log(a*e/(c + d*x) + b*e
*x/(c + d*x)) + 3*B*a*b**2*x**2*log(a*e/(c + d*x) + b*e*x/(c + d*x)) + B*b*
*3*x**3*log(a*e/(c + d*x) + b*e*x/(c + d*x))), x)/g**3
```

## 3.114 $\int \frac{(ag+bgx)^2}{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}\,dx$

**Optimal**. Leaf size=35

$$\text{Int}\left(\frac{(ag+bgx)^2}{\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2},x\right)$$

```
[Out] Unintegrable((b*g*x+a*g)^2/(A+B*ln(e*(b*x+a)/(d*x+c)))^2,x)
```

**Rubi [A]** time = 0.21, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{(ag+bgx)^2}{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(a*g + b*g*x)^2/(A + B*Log[(e*(a + b*x))/(c + d*x)])^2,x]
```

```
[Out] a^2*g^2*Defer[Int][(A + B*Log[(e*(a + b*x))/(c + d*x)])^(-2), x] + 2*a*b*g^
2*Defer[Int][x/(A + B*Log[(e*(a + b*x))/(c + d*x)])^2, x] + b^2*g^2*Defer[I
nt][x^2/(A + B*Log[(e*(a + b*x))/(c + d*x)])^2, x]
```

Rubi steps

$$\int \frac{(ag+bgx)^2}{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}\,dx = \int \left(\frac{a^2g^2}{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}+\frac{2abg^2x}{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}+\frac{b^2g^2x^2}{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}\right) d$$

$$= \left(a^2g^2\right)\int \frac{1}{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}\,dx+\left(2abg^2\right)\int \frac{x}{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}\,dx+\left(b^2\right.$$

**Mathematica [A]** time = 1.35, size = 0, normalized size = 0.00

$$\int \frac{(ag+bgx)^2}{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(a*g + b*g*x)^2/(A + B*Log[(e*(a + b*x))/(c + d*x)])^2,x]
```

```
[Out] Integrate[(a*g + b*g*x)^2/(A + B*Log[(e*(a + b*x))/(c + d*x)])^2, x]
```

**fricas [A]** time = 0.72, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{b^2g^2x^2 + 2\,abg^2x + a^2g^2}{B^2\log\left(\frac{bex+ae}{dx+c}\right)^2 + 2\,AB\log\left(\frac{bex+ae}{dx+c}\right) + A^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*g*x+a*g)^2/(A+B*log(e*(b*x+a)/(d*x+c)))^2,x, algorithm="fricas
")
```

```
[Out] integral((b^2*g^2*x^2 + 2*a*b*g^2*x + a^2*g^2)/(B^2*log((b*e*x + a*e)/(d*x
+ c))^2 + 2*A*B*log((b*e*x + a*e)/(d*x + c)) + A^2), x)
```

**giac [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*g*x+a*g)^2/(A+B*log(e*(b*x+a)/(d*x+c)))^2,x, algorithm="giac")
```

```
[Out] Timed out
```

**maple [A]** time = 1.09, size = 0, normalized size = 0.00

$$\int \frac{\left(bgx + ag\right)^2}{\left(B\ln\left(\frac{(bx+a)e}{dx+c}\right) + A\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*g*x+a*g)^2/(B*ln((b*x+a)/(d*x+c)*e)+A)^2,x)
```

```
[Out] int((b*g*x+a*g)^2/(B*ln((b*x+a)/(d*x+c)*e)+A)^2,x)
```

**maxima [A]** time = 0.00, size = 0, normalized size = 0.00

$$-\frac{b^3dg^2x^4 + a^3cg^2 + \left(b^3cg^2 + 3\,ab^2dg^2\right)x^3 + 3\left(ab^2cg^2 + a^2bdg^2\right)x^2 + \left(3\,a^2bcg^2 + a^3dg^2\right)x}{(bc-ad)B^2\log(bx+a) - (bc-ad)B^2\log(dx+c) + (bc-ad)AB + \left(bc\log(e) - ad\log(e)\right)B^2} + \int \frac{4\,b^3dg}{(bc-ad)B^2\,\mathrm{lo}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*g*x+a*g)^2/(A+B*log(e*(b*x+a)/(d*x+c)))^2,x, algorithm="maxima
")
```

```
[Out] -(b^3*d*g^2*x^4 + a^3*c*g^2 + (b^3*c*g^2 + 3*a*b^2*d*g^2)*x^3 + 3*(a*b^2*c*
g^2 + a^2*b*d*g^2)*x^2 + (3*a^2*b*c*g^2 + a^3*d*g^2)*x)/((b*c - a*d)*B^2*lo
g(b*x + a) - (b*c - a*d)*B^2*log(d*x + c) + (b*c - a*d)*A*B + (b*c*log(e) -
 a*d*log(e))*B^2) + integrate((4*b^3*d*g^2*x^3 + 3*a^2*b*c*g^2 + a^3*d*g^2
+ 3*(b^3*c*g^2 + 3*a*b^2*d*g^2)*x^2 + 6*(a*b^2*c*g^2 + a^2*b*d*g^2)*x)/((b*
c - a*d)*B^2*log(b*x + a) - (b*c - a*d)*B^2*log(d*x + c) + (b*c - a*d)*A*B
+ (b*c*log(e) - a*d*log(e))*B^2), x)
```

**mupad [A]** time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{{\left(a\,g+b\,g\,x\right)}^2}{{\left(A+B\,\ln\left(\frac{e\,\left(a+b\,x\right)}{c+d\,x}\right)\right)}^2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*g + b*g*x)^2/(A + B*log((e*(a + b*x))/(c + d*x)))^2,x)
```

```
[Out] int((a*g + b*g*x)^2/(A + B*log((e*(a + b*x))/(c + d*x)))^2, x)
```

**sympy [A]** time = 0.00, size = 0, normalized size = 0.00

$$\frac{a^3cg^2 + a^3dg^2x + 3a^2bcg^2x + 3a^2bdg^2x^2 + 3ab^2cg^2x^2 + 3ab^2dg^2x^3 + b^3cg^2x^3 + b^3dg^2x^4}{ABad - ABbc + \left(B^2ad - B^2bc\right)\log\left(\frac{e(a+bx)}{c+dx}\right)} - \frac{g^2\left(\int \frac{a^3d}{A+B\log\left(\frac{ae}{c+dx}+\frac{bex}{c+dx}\right)} \cdots\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*g*x+a*g)**2/(A+B*ln(e*(b*x+a)/(d*x+c)))**2,x)
```

```
[Out] (a**3*c*g**2 + a**3*d*g**2*x + 3*a**2*b*c*g**2*x + 3*a**2*b*d*g**2*x**2 + 3
*a*b**2*c*g**2*x**2 + 3*a*b**2*d*g**2*x**3 + b**3*c*g**2*x**3 + b**3*d*g**2
*x**4)/(A*B*a*d - A*B*b*c + (B**2*a*d - B**2*b*c)*log(e*(a + b*x)/(c + d*x)
)) - g**2*(Integral(a**3*d/(A + B*log(a*e/(c + d*x) + b*e*x/(c + d*x))), x)
 + Integral(3*a**2*b*c/(A + B*log(a*e/(c + d*x) + b*e*x/(c + d*x))), x) + I
ntegral(3*b**3*c*x**2/(A + B*log(a*e/(c + d*x) + b*e*x/(c + d*x))), x) + In
tegral(4*b**3*d*x**3/(A + B*log(a*e/(c + d*x) + b*e*x/(c + d*x))), x) + Int
egral(6*a*b**2*c*x/(A + B*log(a*e/(c + d*x) + b*e*x/(c + d*x))), x) + Integ
ral(9*a*b**2*d*x**2/(A + B*log(a*e/(c + d*x) + b*e*x/(c + d*x))), x) + Inte
gral(6*a**2*b*d*x/(A + B*log(a*e/(c + d*x) + b*e*x/(c + d*x))), x))/(B*(a*d
 - b*c))
```

# 3.115 $\int \frac{ag+bgx}{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}\,dx$

**Optimal**. Leaf size=33

$$\text{Int}\left(\frac{ag+bgx}{\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2},x\right)$$

```
[Out] Unintegrable((b*g*x+a*g)/(A+B*ln(e*(b*x+a)/(d*x+c)))^2,x)
```

**Rubi [A]** time = 0.11, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{ag+bgx}{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(a*g + b*g*x)/(A + B*Log[(e*(a + b*x))/(c + d*x)])^2,x]
```

```
[Out] a*g*Defer[Int][(A + B*Log[(e*(a + b*x))/(c + d*x)])^(-2), x] + b*g*Defer[In
t][x/(A + B*Log[(e*(a + b*x))/(c + d*x)])^2, x]
```

Rubi steps

$$\begin{aligned}\int \frac{ag+bgx}{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}\,dx &= \int \left(\frac{ag}{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}+\frac{bgx}{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}\right)dx\\ &= (ag)\int \frac{1}{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}\,dx+(bg)\int \frac{x}{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}\,dx\end{aligned}$$

**Mathematica [A]** time = 0.93, size = 0, normalized size = 0.00

$$\int \frac{ag+bgx}{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(a*g + b*g*x)/(A + B*Log[(e*(a + b*x))/(c + d*x)])^2,x]
```

```
[Out] Integrate[(a*g + b*g*x)/(A + B*Log[(e*(a + b*x))/(c + d*x)])^2, x]
```

**fricas [A]** time = 0.95, size = 0, normalized size = 0.00

$$\operatorname{integral}\left(\frac{bgx + ag}{B^{2}\log\left(\frac{bex+ae}{dx+c}\right)^{2} + 2\,AB\log\left(\frac{bex+ae}{dx+c}\right) + A^{2}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*g*x+a*g)/(A+B*log(e*(b*x+a)/(d*x+c)))^2,x, algorithm="fricas")
```

```
[Out] integral((b*g*x + a*g)/(B^2*log((b*e*x + a*e)/(d*x + c))^2 + 2*A*B*log((b*e
*x + a*e)/(d*x + c)) + A^2), x)
```

**giac [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*g*x+a*g)/(A+B*log(e*(b*x+a)/(d*x+c)))^2,x, algorithm="giac")
```

```
[Out] Timed out
```

**maple [A]** time = 1.08, size = 0, normalized size = 0.00

$$\int \frac{bgx + ag}{\left(B\ln\left(\frac{(bx+a)e}{dx+c}\right) + A\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*g*x+a*g)/(B*ln((b*x+a)/(d*x+c)*e)+A)^2,x)
```

```
[Out] int((b*g*x+a*g)/(B*ln((b*x+a)/(d*x+c)*e)+A)^2,x)
```

**maxima [A]** time = 0.00, size = 0, normalized size = 0.00

$$-\frac{b^{2}dgx^{3} + a^{2}cg + \left(b^{2}cg + 2\,abdg\right)x^{2} + \left(2\,abcg + a^{2}dg\right)x}{(bc-ad)B^{2}\log\left(bx+a\right) - (bc-ad)B^{2}\log\left(dx+c\right) + (bc-ad)AB + \left(bc\log(e) - ad\log(e)\right)B^{2}} + \int \frac{}{(bc-ad)B^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*g*x+a*g)/(A+B*log(e*(b*x+a)/(d*x+c)))^2,x, algorithm="maxima")
```

```
[Out] -(b^2*d*g*x^3 + a^2*c*g + (b^2*c*g + 2*a*b*d*g)*x^2 + (2*a*b*c*g + a^2*d*g)
*x)/((b*c - a*d)*B^2*log(b*x + a) - (b*c - a*d)*B^2*log(d*x + c) + (b*c - a
*d)*A*B + (b*c*log(e) - a*d*log(e))*B^2) + integrate((3*b^2*d*g*x^2 + 2*a*b
*c*g + a^2*d*g + 2*(b^2*c*g + 2*a*b*d*g)*x)/((b*c - a*d)*B^2*log(b*x + a) -
 (b*c - a*d)*B^2*log(d*x + c) + (b*c - a*d)*A*B + (b*c*log(e) - a*d*log(e))
*B^2), x)
```

**mupad [A]** time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{a\,g+b\,g\,x}{{\left(A+B\,\ln\left(\frac{e\,\left(a+b\,x\right)}{c+d\,x}\right)\right)}^2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*g + b*g*x)/(A + B*log((e*(a + b*x))/(c + d*x)))^2,x)
```

```
[Out] int((a*g + b*g*x)/(A + B*log((e*(a + b*x))/(c + d*x)))^2, x)
```

**sympy [A]** time = 0.00, size = 0, normalized size = 0.00

$$\frac{a^2cg + a^2dgx + 2abcgx + 2abdgx^2 + b^2cgx^2 + b^2dgx^3}{ABad - ABbc + \left(B^2ad - B^2bc\right)\log\left(\frac{e(a+bx)}{c+dx}\right)} - \frac{g\left(\int \frac{a^2d}{A+B\log\left(\frac{ae}{c+dx}+\frac{bex}{c+dx}\right)}\,dx + \int \frac{2abc}{A+B\log\left(\frac{ae}{c+dx}+\frac{bex}{c+dx}\right)}\,dx + \int \frac{}{A\cdots}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*g*x+a*g)/(A+B*ln(e*(b*x+a)/(d*x+c)))**2,x)
```

```
[Out] (a**2*c*g + a**2*d*g*x + 2*a*b*c*g*x + 2*a*b*d*g*x**2 + b**2*c*g*x**2 + b**
2*d*g*x**3)/(A*B*a*d - A*B*b*c + (B**2*a*d - B**2*b*c)*log(e*(a + b*x)/(c +
 d*x))) - g*(Integral(a**2*d/(A + B*log(a*e/(c + d*x) + b*e*x/(c + d*x))),
x) + Integral(2*a*b*c/(A + B*log(a*e/(c + d*x) + b*e*x/(c + d*x))), x) + In
tegral(2*b**2*c*x/(A + B*log(a*e/(c + d*x) + b*e*x/(c + d*x))), x) + Integr
al(3*b**2*d*x**2/(A + B*log(a*e/(c + d*x) + b*e*x/(c + d*x))), x) + Integra
l(4*a*b*d*x/(A + B*log(a*e/(c + d*x) + b*e*x/(c + d*x))), x))/(B*(a*d - b*c
))
```

# 3.116 $\int \frac{1}{(ag+bgx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}\,dx$

**Optimal**. Leaf size=35

$$\text{Int}\left(\frac{1}{(ag+bgx)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2},x\right)$$

```
[Out] Unintegrable(1/(b*g*x+a*g)/(A+B*ln(e*(b*x+a)/(d*x+c)))^2,x)
```

**Rubi [A]** time = 0.08, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{1}{(ag+bgx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[1/((a*g + b*g*x)*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2),x]
[Out] Defer[Int][1/((a*g + b*g*x)*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2), x]
```

Rubi steps

$$\int \frac{1}{(ag+bgx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}\,dx=\int \frac{1}{(ag+bgx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}\,dx$$

**Mathematica [A]** time = 0.63, size = 0, normalized size = 0.00

$$\int \frac{1}{(ag+bgx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[1/((a*g + b*g*x)*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2),x]
[Out] Integrate[1/((a*g + b*g*x)*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2), x]
```

**fricas** [A] time = 0.60, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{1}{A^2bgx + A^2ag + \left(B^2bgx + B^2ag\right)\log\left(\frac{bex+ae}{dx+c}\right)^2 + 2\left(ABbgx + ABag\right)\log\left(\frac{bex+ae}{dx+c}\right)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*g*x+a*g)/(A+B*log(e*(b*x+a)/(d*x+c)))^2,x, algorithm="fricas
")
```

```
[Out] integral(1/(A^2*b*g*x + A^2*a*g + (B^2*b*g*x + B^2*a*g)*log((b*e*x + a*e)/(
d*x + c))^2 + 2*(A*B*b*g*x + A*B*a*g)*log((b*e*x + a*e)/(d*x + c))), x)
```

**giac** [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(bgx + ag\right)\left(B\log\left(\frac{(bx+a)e}{dx+c}\right) + A\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*g*x+a*g)/(A+B*log(e*(b*x+a)/(d*x+c)))^2,x, algorithm="giac")
```

```
[Out] integrate(1/((b*g*x + a*g)*(B*log((b*x + a)*e/(d*x + c)) + A)^2), x)
```

**maple** [A] time = 1.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(bgx + ag\right)\left(B\ln\left(\frac{(bx+a)e}{dx+c}\right) + A\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*g*x+a*g)/(B*ln((b*x+a)/(d*x+c)*e)+A)^2,x)
```

```
[Out] int(1/(b*g*x+a*g)/(B*ln((b*x+a)/(d*x+c)*e)+A)^2,x)
```

**maxima** [A] time = 0.00, size = 0, normalized size = 0.00

$$d\int \frac{1}{\left(bcg - adg\right)B^2\log\left(bx + a\right) - \left(bcg - adg\right)B^2\log\left(dx + c\right) + \left(bcg - adg\right)AB + \left(bcg\log(e) - adg\log(e)\right)B^2}\,dx-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*g*x+a*g)/(A+B*log(e*(b*x+a)/(d*x+c)))^2,x, algorithm="maxima
")
```

```
[Out] d*integrate(1/((b*c*g - a*d*g)*B^2*log(b*x + a) - (b*c*g - a*d*g)*B^2*log(d
*x + c) + (b*c*g - a*d*g)*A*B + (b*c*g*log(e) - a*d*g*log(e))*B^2), x) - (d
*x + c)/((b*c*g - a*d*g)*B^2*log(b*x + a) - (b*c*g - a*d*g)*B^2*log(d*x + c
) + (b*c*g - a*d*g)*A*B + (b*c*g*log(e) - a*d*g*log(e))*B^2)
```

**mupad [A]** time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{1}{\left(a\,g+b\,g\,x\right)\,{\left(A+B\,\ln\left(\frac{e\,\left(a+b\,x\right)}{c+d\,x}\right)\right)}^2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*g + b*g*x)*(A + B*log((e*(a + b*x))/(c + d*x)))^2),x)
[Out] int(1/((a*g + b*g*x)*(A + B*log((e*(a + b*x))/(c + d*x)))^2), x)
```

**sympy [A]** time = 0.00, size = 0, normalized size = 0.00

$$\frac{c+dx}{ABadg-ABbcg+\left(B^2adg-B^2bcg\right)\log\left(\frac{e(a+bx)}{c+dx}\right)}-\frac{d\int\frac{1}{A+B\log\left(\frac{ae}{c+dx}+\frac{bex}{c+dx}\right)}\,dx}{Bg\left(ad-bc\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*g*x+a*g)/(A+B*ln(e*(b*x+a)/(d*x+c)))**2,x)
[Out] (c + d*x)/(A*B*a*d*g - A*B*b*c*g + (B**2*a*d*g - B**2*b*c*g)*log(e*(a + b*x
)/(c + d*x))) - d*Integral(1/(A + B*log(a*e/(c + d*x) + b*e*x/(c + d*x))),
x)/(B*g*(a*d - b*c))
```

# 3.117 $\int \frac{1}{(ag+bgx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}\,dx$

**Optimal**. Leaf size=103

$$-\frac{ee^{A/B}\text{Ei}\left(-\frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{B}\right)}{B^2g^2(bc-ad)}-\frac{c+dx}{Bg^2(a+bx)(bc-ad)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}$$

```
[Out] -e*exp(A/B)*Ei((-A-B*ln(e*(b*x+a)/(d*x+c)))/B)/B^2/(-a*d+b*c)/g^2+(-d*x-c)/
B/(-a*d+b*c)/g^2/(b*x+a)/(A+B*ln(e*(b*x+a)/(d*x+c)))
```

**Rubi [F]** time = 0.09, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{1}{(ag+bgx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[1/((a*g + b*g*x)^2*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2),x]
```

```
[Out] Defer[Int][1/((a*g + b*g*x)^2*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2), x]
```

Rubi steps

$$\int \frac{1}{(ag+bgx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}\,dx=\int \frac{1}{(ag+bgx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}\,dx$$

**Mathematica [A]** time = 0.18, size = 87, normalized size = 0.84

$$\frac{ee^{A/B}\text{Ei}\left(-\frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{B}\right)+\frac{B(c+dx)}{(a+bx)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}}{B^2g^2(ad-bc)}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a*g + b*g*x)^2*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2),x]
```

```
[Out] (e*E^(A/B)*ExpIntegralEi[-((A + B*Log[(e*(a + b*x))/(c + d*x)])/B)] + (B*(c
 + d*x))/((a + b*x)*(A + B*Log[(e*(a + b*x))/(c + d*x)])))/(B^2*(-(b*c) + a
*d)*g^2)
```

**fricas [A]** time = 0.74, size = 199, normalized size = 1.93

$$-\frac{Bdx + Bc + \left((Bbex + Bae)e^{\frac{A}{B}}\log\left(\frac{bex+ae}{dx+c}\right) + (Abex + Aae)e^{\frac{A}{B}}\right)\text{log\_integral}\left(\frac{(dx+c)e^{\left(-\frac{A}{B}\right)}}{bex+ae}\right)}{\left(AB^2b^2c - AB^2abd\right)g^2x + \left(AB^2abc - AB^2a^2d\right)g^2 + \left(\left(B^3b^2c - B^3abd\right)g^2x + \left(B^3abc - B^3a^2d\right)g^2\right)\log\left(\frac{bex+ae}{dx+c}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*g*x+a*g)^2/(A+B*log(e*(b*x+a)/(d*x+c)))^2,x, algorithm="fric
as")
```

```
[Out] -(B*d*x + B*c + ((B*b*e*x + B*a*e)*e^(A/B)*log((b*e*x + a*e)/(d*x + c)) + (
A*b*e*x + A*a*e)*e^(A/B))*log_integral((d*x + c)*e^(-A/B)/(b*e*x + a*e)))/(
(A*B^2*b^2*c - A*B^2*a*b*d)*g^2*x + (A*B^2*a*b*c - A*B^2*a^2*d)*g^2 + ((B^3
*b^2*c - B^3*a*b*d)*g^2*x + (B^3*a*b*c - B^3*a^2*d)*g^2)*log((b*e*x + a*e)/
(d*x + c)))
```

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(bgx + ag\right)^2\left(B\log\left(\frac{(bx+a)e}{dx+c}\right) + A\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*g*x+a*g)^2/(A+B*log(e*(b*x+a)/(d*x+c)))^2,x, algorithm="giac
")
```

```
[Out] integrate(1/((b*g*x + a*g)^2*(B*log((b*x + a)*e/(d*x + c)) + A)^2), x)
```

**maple [F]** time = 1.34, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(bgx + ag\right)^2\left(B\ln\left(\frac{(bx+a)e}{dx+c}\right) + A\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*g*x+a*g)^2/(B*ln((b*x+a)/(d*x+c)*e)+A)^2,x)
```

```
[Out] int(1/(b*g*x+a*g)^2/(B*ln((b*x+a)/(d*x+c)*e)+A)^2,x)
```

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$-\frac{}{\left(abcg^2 - a^2dg^2\right)AB + \left(abcg^2\log(e) - a^2dg^2\log(e)\right)B^2 + \left(\left(b^2cg^2 - abdg^2\right)AB + \left(b^2cg^2\log(e) - abdg^2\log(e)\right)B^2\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*g*x+a*g)^2/(A+B*log(e*(b*x+a)/(d*x+c)))^2,x, algorithm="maxi
ma")
```

```
[Out] -(d*x + c)/((a*b*c*g^2 - a^2*d*g^2)*A*B + (a*b*c*g^2*log(e) - a^2*d*g^2*log
(e))*B^2 + ((b^2*c*g^2 - a*b*d*g^2)*A*B + (b^2*c*g^2*log(e) - a*b*d*g^2*log
(e))*B^2)*x + ((b^2*c*g^2 - a*b*d*g^2)*B^2*x + (a*b*c*g^2 - a^2*d*g^2)*B^2)
*log(b*x + a) - ((b^2*c*g^2 - a*b*d*g^2)*B^2*x + (a*b*c*g^2 - a^2*d*g^2)*B^
2)*log(d*x + c)) + integrate(-1/(B^2*a^2*g^2*log(e) + A*B*a^2*g^2 + (B^2*b^
2*g^2*log(e) + A*B*b^2*g^2)*x^2 + 2*(B^2*a*b*g^2*log(e) + A*B*a*b*g^2)*x +
(B^2*b^2*g^2*x^2 + 2*B^2*a*b*g^2*x + B^2*a^2*g^2)*log(b*x + a) - (B^2*b^2*g
^2*x^2 + 2*B^2*a*b*g^2*x + B^2*a^2*g^2)*log(d*x + c)), x)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{{\left(a\,g+b\,g\,x\right)}^2\,{\left(A+B\,\ln\left(\frac{e\,\left(a+b\,x\right)}{c+d\,x}\right)\right)}^2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*g + b*g*x)^2*(A + B*log((e*(a + b*x))/(c + d*x)))^2),x)
```

```
[Out] int(1/((a*g + b*g*x)^2*(A + B*log((e*(a + b*x))/(c + d*x)))^2), x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\frac{c + dx}{ABa^2dg^2 - ABabcg^2 + ABabdg^2x - ABb^2cg^2x + \left(B^2a^2dg^2 - B^2abcg^2 + B^2abdg^2x - B^2b^2cg^2x\right)\log\left(\frac{e(a+bx)}{c+dx}\right)} - \int$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*g*x+a*g)**2/(A+B*ln(e*(b*x+a)/(d*x+c)))**2,x)
```

```
[Out] (c + d*x)/(A*B*a**2*d*g**2 - A*B*a*b*c*g**2 + A*B*a*b*d*g**2*x - A*B*b**2*c
*g**2*x + (B**2*a**2*d*g**2 - B**2*a*b*c*g**2 + B**2*a*b*d*g**2*x - B**2*b*
*2*c*g**2*x)*log(e*(a + b*x)/(c + d*x))) - Integral(1/(A*a**2 + 2*A*a*b*x +
```

```
 A*b**2*x**2 + B*a**2*log(a*e/(c + d*x) + b*e*x/(c + d*x)) + 2*B*a*b*x*log(
a*e/(c + d*x) + b*e*x/(c + d*x)) + B*b**2*x**2*log(a*e/(c + d*x) + b*e*x/(c
 + d*x))), x)/(B*g**2)
```

# 3.118 $\int \frac{1}{(ag+bgx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}\,dx$

**Optimal**. Leaf size=212

$$-\frac{2be^2e^{\frac{2A}{B}}\operatorname{Ei}\left(-\frac{2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{B}\right)}{B^2g^3(bc-ad)^2}+\frac{dee^{A/B}\operatorname{Ei}\left(-\frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{B}\right)}{B^2g^3(bc-ad)^2}-\frac{b(c+dx)^2}{Bg^3(a+bx)^2(bc-ad)^2\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}+\frac{}{Bg^3}$$

```
[Out] -2*b*e^2*exp(2*A/B)*Ei(-2*(A+B*ln(e*(b*x+a)/(d*x+c)))/B)/B^2/(-a*d+b*c)^2/g
^3+d*e*exp(A/B)*Ei((-A-B*ln(e*(b*x+a)/(d*x+c)))/B)/B^2/(-a*d+b*c)^2/g^3+d*(
d*x+c)/B/(-a*d+b*c)^2/g^3/(b*x+a)/(A+B*ln(e*(b*x+a)/(d*x+c)))-b*(d*x+c)^2/B
/(-a*d+b*c)^2/g^3/(b*x+a)^2/(A+B*ln(e*(b*x+a)/(d*x+c)))
```

**Rubi [F]** time = 0.08, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{1}{(ag+bgx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[1/((a*g + b*g*x)^3*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2),x]
```

```
[Out] Defer[Int][1/((a*g + b*g*x)^3*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2), x]
```

Rubi steps

$$\int \frac{1}{(ag+bgx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}\,dx=\int \frac{1}{(ag+bgx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}\,dx$$

**Mathematica [A]** time = 0.65, size = 136, normalized size = 0.64

$$\frac{-2be^2e^{\frac{2A}{B}}\operatorname{Ei}\left(-\frac{2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{B}\right)+dee^{A/B}\operatorname{Ei}\left(-\frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{B}\right)-\frac{B(c+dx)(bc-ad)}{(a+bx)^2\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}}{B^2g^3(bc-ad)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a*g + b*g*x)^3*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2),x]
```

```
[Out] (-2*b*e^2*E^((2*A)/B)*ExpIntegralEi[(-2*(A + B*Log[(e*(a + b*x))/(c + d*x)]
))/B] + d*e*E^(A/B)*ExpIntegralEi[-((A + B*Log[(e*(a + b*x))/(c + d*x)])/B)
] - (B*(b*c - a*d)*(c + d*x))/((a + b*x)^2*(A + B*Log[(e*(a + b*x))/(c + d*
x)])))/(B^2*(b*c - a*d)^2*g^3)
```

**fricas [B]** time = 0.62, size = 570, normalized size = 2.69

$$-\frac{Bbc^2 - Bacd + (Bbcd - Bad^2)x - ((Bb^2dex^2 + 2\,Babdex + Ba^2de)e^{\frac{A}{B}}\log\left(\frac{bex+ae}{dx+c}\right) + (Ab^2dex^2 + 2\,Aabdex + A\ldots}{(AB^2b^4c^2 - 2\,AB^2ab^3cd + AB^2a^2b^2d^2)g^3x^2 + 2\,(AB^2ab^3c^2 - 2\,AB^2a^2b^2cd + AB^2a^3bd^2)g^3x + (AB^2a^2b^2c^2\ldots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*g*x+a*g)^3/(A+B*log(e*(b*x+a)/(d*x+c)))^2,x, algorithm="fric
as")
```

```
[Out] -(B*b*c^2 - B*a*c*d + (B*b*c*d - B*a*d^2)*x - ((B*b^2*d*e*x^2 + 2*B*a*b*d*e
*x + B*a^2*d*e)*e^(A/B)*log((b*e*x + a*e)/(d*x + c)) + (A*b^2*d*e*x^2 + 2*A
*a*b*d*e*x + A*a^2*d*e)*e^(A/B))*log_integral((d*x + c)*e^(-A/B)/(b*e*x + a
*e)) + 2*((B*b^3*e^2*x^2 + 2*B*a*b^2*e^2*x + B*a^2*b*e^2)*e^(2*A/B)*log((b*
e*x + a*e)/(d*x + c)) + (A*b^3*e^2*x^2 + 2*A*a*b^2*e^2*x + A*a^2*b*e^2)*e^(
2*A/B))*log_integral((d^2*x^2 + 2*c*d*x + c^2)*e^(-2*A/B)/(b^2*e^2*x^2 + 2*
a*b*e^2*x + a^2*e^2)))/((A*B^2*b^4*c^2 - 2*A*B^2*a*b^3*c*d + A*B^2*a^2*b^2*
d^2)*g^3*x^2 + 2*(A*B^2*a*b^3*c^2 - 2*A*B^2*a^2*b^2*c*d + A*B^2*a^3*b*d^2)*
g^3*x + (A*B^2*a^2*b^2*c^2 - 2*A*B^2*a^3*b*c*d + A*B^2*a^4*d^2)*g^3 + ((B^3
*b^4*c^2 - 2*B^3*a*b^3*c*d + B^3*a^2*b^2*d^2)*g^3*x^2 + 2*(B^3*a*b^3*c^2 -
2*B^3*a^2*b^2*c*d + B^3*a^3*b*d^2)*g^3*x + (B^3*a^2*b^2*c^2 - 2*B^3*a^3*b*c
*d + B^3*a^4*d^2)*g^3)*log((b*e*x + a*e)/(d*x + c)))
```

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(bgx + ag)^3\left(B\log\left(\frac{(bx+a)e}{dx+c}\right) + A\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*g*x+a*g)^3/(A+B*log(e*(b*x+a)/(d*x+c)))^2,x, algorithm="giac
")
```

```
[Out] integrate(1/((b*g*x + a*g)^3*(B*log((b*x + a)*e/(d*x + c)) + A)^2), x)
```

**maple [F]** time = 1.51, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(bgx+ag\right)^{3}\left(B\ln\left(\frac{(bx+a)e}{dx+c}\right)+A\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*g*x+a*g)^3/(B*ln((b*x+a)/(d*x+c)*e)+A)^2,x)
```

```
[Out] int(1/(b*g*x+a*g)^3/(B*ln((b*x+a)/(d*x+c)*e)+A)^2,x)
```

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$-\frac{}{\left(a^{2}bcg^{3}-a^{3}dg^{3}\right)AB+\left(a^{2}bcg^{3}\log(e)-a^{3}dg^{3}\log(e)\right)B^{2}+\left(\left(b^{3}cg^{3}-ab^{2}dg^{3}\right)AB+\left(b^{3}cg^{3}\log(e)-ab^{2}dg^{3}\log(e\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*g*x+a*g)^3/(A+B*log(e*(b*x+a)/(d*x+c)))^2,x, algorithm="maxi
ma")
```

```
[Out] -(d*x + c)/((a^2*b*c*g^3 - a^3*d*g^3)*A*B + (a^2*b*c*g^3*log(e) - a^3*d*g^3
*log(e))*B^2 + ((b^3*c*g^3 - a*b^2*d*g^3)*A*B + (b^3*c*g^3*log(e) - a*b^2*d
*g^3*log(e))*B^2)*x^2 + 2*((a*b^2*c*g^3 - a^2*b*d*g^3)*A*B + (a*b^2*c*g^3*l
og(e) - a^2*b*d*g^3*log(e))*B^2)*x + ((b^3*c*g^3 - a*b^2*d*g^3)*B^2*x^2 + 2
*(a*b^2*c*g^3 - a^2*b*d*g^3)*B^2*x + (a^2*b*c*g^3 - a^3*d*g^3)*B^2)*log(b*x
 + a) - ((b^3*c*g^3 - a*b^2*d*g^3)*B^2*x^2 + 2*(a*b^2*c*g^3 - a^2*b*d*g^3)*
B^2*x + (a^2*b*c*g^3 - a^3*d*g^3)*B^2)*log(d*x + c)) - integrate((b*d*x + 2
*b*c - a*d)/(((b^4*c*g^3 - a*b^3*d*g^3)*A*B + (b^4*c*g^3*log(e) - a*b^3*d*g
^3*log(e))*B^2)*x^3 + (a^3*b*c*g^3 - a^4*d*g^3)*A*B + (a^3*b*c*g^3*log(e) -
 a^4*d*g^3*log(e))*B^2 + 3*((a*b^3*c*g^3 - a^2*b^2*d*g^3)*A*B + (a*b^3*c*g^
3*log(e) - a^2*b^2*d*g^3*log(e))*B^2)*x^2 + 3*((a^2*b^2*c*g^3 - a^3*b*d*g^3
)*A*B + (a^2*b^2*c*g^3*log(e) - a^3*b*d*g^3*log(e))*B^2)*x + ((b^4*c*g^3 -
a*b^3*d*g^3)*B^2*x^3 + 3*(a*b^3*c*g^3 - a^2*b^2*d*g^3)*B^2*x^2 + 3*(a^2*b^2
*c*g^3 - a^3*b*d*g^3)*B^2*x + (a^3*b*c*g^3 - a^4*d*g^3)*B^2)*log(b*x + a) -
 ((b^4*c*g^3 - a*b^3*d*g^3)*B^2*x^3 + 3*(a*b^3*c*g^3 - a^2*b^2*d*g^3)*B^2*x
^2 + 3*(a^2*b^2*c*g^3 - a^3*b*d*g^3)*B^2*x + (a^3*b*c*g^3 - a^4*d*g^3)*B^2)
*log(d*x + c)), x)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{{\left(a\,g+b\,g\,x\right)}^3\,{\left(A+B\,\ln\left(\frac{e\,\left(a+b\,x\right)}{c+d\,x}\right)\right)}^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*g + b*g*x)^3*(A + B*log((e*(a + b*x))/(c + d*x)))^2),x)
```

```
[Out] int(1/((a*g + b*g*x)^3*(A + B*log((e*(a + b*x))/(c + d*x)))^2), x)
```

**sympy [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*g*x+a*g)**3/(A+B*ln(e*(b*x+a)/(d*x+c)))**2,x)
```

```
[Out] Timed out
```

## 3.119 $\int (ag+bgx)^4 \left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right) dx$

**Optimal**. Leaf size=182

$$\frac{g^4(a+bx)^5\left(B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+A\right)}{5b}-\frac{2Bg^4(bc-ad)^5\log(c+dx)}{5bd^5}+\frac{2Bg^4x(bc-ad)^4}{5d^4}-\frac{Bg^4(a+bx)^2(bc-ad)^3}{5bd^3}+\frac{2Bg^4}{}$$

```
[Out] 2/5*B*(-a*d+b*c)^4*g^4*x/d^4-1/5*B*(-a*d+b*c)^3*g^4*(b*x+a)^2/b/d^3+2/15*B*
(-a*d+b*c)^2*g^4*(b*x+a)^3/b/d^2-1/10*B*(-a*d+b*c)*g^4*(b*x+a)^4/b/d+1/5*g^
4*(b*x+a)^5*(A+B*ln(e*(b*x+a)^2/(d*x+c)^2))/b-2/5*B*(-a*d+b*c)^5*g^4*ln(d*x
+c)/b/d^5
```

**Rubi [A]** time = 0.11, antiderivative size = 182, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.094, Rules used = {2525, 12, 43}

$$\frac{g^4(a+bx)^5\left(B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+A\right)}{5b}+\frac{2Bg^4x(bc-ad)^4}{5d^4}-\frac{Bg^4(a+bx)^2(bc-ad)^3}{5bd^3}+\frac{2Bg^4(a+bx)^3(bc-ad)^2}{15bd^2}-\frac{2Bg^4(bc}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a*g + b*g*x)^4*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2]),x]
```

```
[Out] (2*B*(b*c - a*d)^4*g^4*x)/(5*d^4) - (B*(b*c - a*d)^3*g^4*(a + b*x)^2)/(5*b*
d^3) + (2*B*(b*c - a*d)^2*g^4*(a + b*x)^3)/(15*b*d^2) - (B*(b*c - a*d)*g^4*
(a + b*x)^4)/(10*b*d) + (g^4*(a + b*x)^5*(A + B*Log[(e*(a + b*x)^2)/(c + d*
x)^2]))/(5*b) - (2*B*(b*c - a*d)^5*g^4*Log[c + d*x])/(5*b*d^5)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2525

```
Int[((a_.) + Log[(c_.)*(RFx_)^(p_.)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_))^(m_.
), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(a + b*Log[c*RFx^p])^n)/(e*(m + 1))
```

```
, x] - Dist[(b*n*p)/(e*(m + 1)), Int[SimplifyIntegrand[((d + e*x)^(m + 1)*(
a + b*Log[c*RFx^p])^(n - 1)*D[RFx, x])/RFx, x], x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && RationalFunctionQ[RFx, x] && IGtQ[n, 0] && (EqQ[n, 1] ||
IntegerQ[m]) && NeQ[m, -1]
```

Rubi steps

$$\int (ag+bgx)^4\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)dx=\frac{g^4(a+bx)^5\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}{5b}-\frac{B\int\frac{2(bc-ad)g^5(a+bx)^4}{c+dx}\,dx}{5bg}$$

$$=\frac{g^4(a+bx)^5\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}{5b}-\frac{\left(2B(bc-ad)g^4\right)\int\frac{(a+bx)^4}{c+dx}\,dx}{5b}$$

$$=\frac{g^4(a+bx)^5\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}{5b}-\frac{\left(2B(bc-ad)g^4\right)\int\left(-\frac{b(bc-ad)^3}{d^4}\right.}{}$$

$$=\frac{2B(bc-ad)^4g^4x}{5d^4}-\frac{B(bc-ad)^3g^4(a+bx)^2}{5bd^3}+\frac{2B(bc-ad)^2g^4(a+}{15bd^2}$$

**Mathematica [A]** time = 0.09, size = 144, normalized size = 0.79

$$\frac{g^4\left((a+bx)^5\left(B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+A\right)+\frac{B(bc-ad)\left(4d^3(a+bx)^3(bc-ad)-6d^2(a+bx)^2(bc-ad)^2+12bdx(bc-ad)^3-12(bc-ad)^4\log(c+dx)-3d^4(a\right.}{6d^5}}{5b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*g + b*g*x)^4*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2]),x]
```

```
[Out] (g^4*((a + b*x)^5*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2]) + (B*(b*c - a*d)
*(12*b*d*(b*c - a*d)^3*x - 6*d^2*(b*c - a*d)^2*(a + b*x)^2 + 4*d^3*(b*c - a
*d)*(a + b*x)^3 - 3*d^4*(a + b*x)^4 - 12*(b*c - a*d)^4*Log[c + d*x]))/(6*d^
5)))/(5*b)
```

**fricas [B]** time = 0.78, size = 454, normalized size = 2.49

$$6\,Ab^5d^5g^4x^5+12\,Ba^5d^5g^4\log(bx+a)-3\left(Bb^5cd^4-(10\,A+B)ab^4d^5\right)g^4x^4+4\left(Bb^5c^2d^3-5\,Bab^4cd^4+(15\,A+\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*g*x+a*g)^4*(A+B*log(e*(b*x+a)^2/(d*x+c)^2)),x, algorithm="fric
as")
```

```
[Out] 1/30*(6*A*b^5*d^5*g^4*x^5 + 12*B*a^5*d^5*g^4*log(b*x + a) - 3*(B*b^5*c*d^4
- (10*A + B)*a*b^4*d^5)*g^4*x^4 + 4*(B*b^5*c^2*d^3 - 5*B*a*b^4*c*d^4 + (15*
A + 4*B)*a^2*b^3*d^5)*g^4*x^3 - 6*(B*b^5*c^3*d^2 - 5*B*a*b^4*c^2*d^3 + 10*B
*a^2*b^3*c*d^4 - 2*(5*A + 3*B)*a^3*b^2*d^5)*g^4*x^2 + 6*(2*B*b^5*c^4*d - 10
*B*a*b^4*c^3*d^2 + 20*B*a^2*b^3*c^2*d^3 - 20*B*a^3*b^2*c*d^4 + (5*A + 8*B)*
a^4*b*d^5)*g^4*x - 12*(B*b^5*c^5 - 5*B*a*b^4*c^4*d + 10*B*a^2*b^3*c^3*d^2 -
 10*B*a^3*b^2*c^2*d^3 + 5*B*a^4*b*c*d^4)*g^4*log(d*x + c) + 6*(B*b^5*d^5*g^
4*x^5 + 5*B*a*b^4*d^5*g^4*x^4 + 10*B*a^2*b^3*d^5*g^4*x^3 + 10*B*a^3*b^2*d^5
*g^4*x^2 + 5*B*a^4*b*d^5*g^4*x)*log((b^2*e*x^2 + 2*a*b*e*x + a^2*e)/(d^2*x^
2 + 2*c*d*x + c^2)))/(b*d^5)
```

**giac [B]** time = 72.97, size = 496, normalized size = 2.73

$$\frac{2\,Ba^5g^4\log\left(bx+a\right)}{5\,b}+\frac{1}{5}\left(Ab^4g^4+Bb^4g^4\right)x^5-\frac{\left(Bb^4cg^4-10\,Aab^3dg^4-11\,Bab^3dg^4\right)x^4}{10\,d}+\frac{2\left(Bb^4c^2g^4-5\,Bab^3cdg^4+\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*g*x+a*g)^4*(A+B*log(e*(b*x+a)^2/(d*x+c)^2)),x, algorithm="giac
")
```

```
[Out] 2/5*B*a^5*g^4*log(b*x + a)/b + 1/5*(A*b^4*g^4 + B*b^4*g^4)*x^5 - 1/10*(B*b^
4*c*g^4 - 10*A*a*b^3*d*g^4 - 11*B*a*b^3*d*g^4)*x^4/d + 2/15*(B*b^4*c^2*g^4
- 5*B*a*b^3*c*d*g^4 + 15*A*a^2*b^2*d^2*g^4 + 19*B*a^2*b^2*d^2*g^4)*x^3/d^2
+ 1/5*(B*b^4*g^4*x^5 + 5*B*a*b^3*g^4*x^4 + 10*B*a^2*b^2*g^4*x^3 + 10*B*a^3*
b*g^4*x^2 + 5*B*a^4*g^4*x)*log((b^2*x^2 + 2*a*b*x + a^2)/(d^2*x^2 + 2*c*d*x
 + c^2)) - 1/5*(B*b^4*c^3*g^4 - 5*B*a*b^3*c^2*d*g^4 + 10*B*a^2*b^2*c*d^2*g^
4 - 10*A*a^3*b*d^3*g^4 - 16*B*a^3*b*d^3*g^4)*x^2/d^3 + 1/5*(2*B*b^4*c^4*g^4
 - 10*B*a*b^3*c^3*d*g^4 + 20*B*a^2*b^2*c^2*d^2*g^4 - 20*B*a^3*b*c*d^3*g^4 +
 5*A*a^4*d^4*g^4 + 13*B*a^4*d^4*g^4)*x/d^4 - 2/5*(B*b^4*c^5*g^4 - 5*B*a*b^3
*c^4*d*g^4 + 10*B*a^2*b^2*c^3*d^2*g^4 - 10*B*a^3*b*c^2*d^3*g^4 + 5*B*a^4*c*
d^4*g^4)*log(-d*x - c)/d^5
```

**maple [B]** time = 0.19, size = 1606, normalized size = 8.82

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*g*x+a*g)^4*(A+B*ln(e*(b*x+a)^2/(d*x+c)^2)),x)
```

```
[Out] -40/d^2*B*g^4*b^2/(a*d-b*c)*ln(a/(d*x+c)*d-1/(d*x+c)*b*c+b)*a^3*c^3-12/d^4*
B*g^4/(a*d-b*c)*ln(a/(d*x+c)*d-1/(d*x+c)*b*c+b)*a*c^5*b^4+30/d*B*g^4*b/(a*d
```

```
-b*c)*ln(a/(d*x+c)*d-1/(d*x+c)*b*c+b)*a^4*c^2+30/d^3*B*g^4*b^3/(a*d-b*c)*ln
(a/(d*x+c)*d-1/(d*x+c)*b*c+b)*a^2*c^4+1/d*B*ln(e*(a/(d*x+c)*d-1/(d*x+c)*b*c
+b)^2/d^2)*a^4*c*g^4+2/5/d^5*B*g^4*ln(1/(d*x+c))*c^5*b^4+8/5/d^5*B*g^4*b^4*
c^5*ln(a/(d*x+c)*d-1/(d*x+c)*b*c+b)+1/5/d^5*B*g^4*b^4*ln(e*(a/(d*x+c)*d-1/(
d*x+c)*b*c+b)^2/d^2)*c^5-1/10/d*B*g^4*b^4*c*x^4+2/15/d^2*B*g^4*b^4*c^2*x^3-
1/5/d^3*B*g^4*b^4*c^3*x^2+2/5/d^4*B*g^4*b^4*c^4*x+2/d*B*g^4*ln(1/(d*x+c))*a
^4*c+2*B*g^4*b*ln(e*(a/(d*x+c)*d-1/(d*x+c)*b*c+b)^2/d^2)*a^3*x^2+B*g^4*b^3*
ln(e*(a/(d*x+c)*d-1/(d*x+c)*b*c+b)^2/d^2)*a*x^4-12*B*g^4/(a*d-b*c)*ln(a/(d*
x+c)*d-1/(d*x+c)*b*c+b)*a^5*c+2*B*g^4*b^2*ln(e*(a/(d*x+c)*d-1/(d*x+c)*b*c+b
)^2/d^2)*a^2*x^3+8/d*B*g^4*a^4*ln(a/(d*x+c)*d-1/(d*x+c)*b*c+b)*c-113/30/d^4
*B*g^4*b^3*a*c^4+98/15/d^3*B*g^4*b^2*a^2*c^3-26/5/d^2*B*g^4*b*a^3*c^2+2/d^3
*A*g^4*a^2*b^2*c^3-1/d^4*A*g^4*a*b^3*c^4-2/d^2*A*g^4*a^3*b*c^2+8/5*B*x*a^4*
g^4+1/5*A*g^4*x^5*b^4+A*g^4*x*a^4+5/6/d^5*B*g^4*b^4*c^5-16/d^2*B*g^4*b*a^3*
ln(a/(d*x+c)*d-1/(d*x+c)*b*c+b)*c^2+2/d^3*B*g^4*ln(e*(a/(d*x+c)*d-1/(d*x+c)
*b*c+b)^2/d^2)*a^2*b^2*c^3-2/d^2*B*g^4*ln(e*(a/(d*x+c)*d-1/(d*x+c)*b*c+b)^2
/d^2)*a^3*b*c^2-1/d^4*B*g^4*b^3*ln(e*(a/(d*x+c)*d-1/(d*x+c)*b*c+b)^2/d^2)*a
*c^4+4/d^3*B*g^4*b^2*ln(1/(d*x+c))*a^2*c^3-8/d^4*B*g^4*b^3*c^4*ln(a/(d*x+c)
*d-1/(d*x+c)*b*c+b)*a-2/d^4*B*g^4*b^3*ln(1/(d*x+c))*a*c^4+16/d^3*B*g^4*b^2*
a^2*ln(a/(d*x+c)*d-1/(d*x+c)*b*c+b)*c^3-4/d^2*B*g^4*b*ln(1/(d*x+c))*a^3*c^2
+2/d^5*B*g^4/(a*d-b*c)*ln(a/(d*x+c)*d-1/(d*x+c)*b*c+b)*c^6*b^5+2*d*B*g^4/b/
(a*d-b*c)*ln(a/(d*x+c)*d-1/(d*x+c)*b*c+b)*a^6-2/d*B*g^4*b^2*a^2*x^2*c+4/d^2
*B*g^4*b^2*a^2*x*c^2-2/d^3*B*g^4*b^3*a*x*c^3-4/d*B*g^4*b*a^3*x*c-2/3/d*B*g^
4*b^3*a*x^3*c+1/d^2*B*g^4*b^3*a*x^2*c^2+1/5/d^5*A*g^4*b^4*c^5+1/d*A*g^4*a^4
*c+8/5/d*B*a^4*c*g^4+8/15*B*g^4*b^2*a^2*x^3+1/10*B*g^4*b^3*a*x^4+6/5*B*g^4*
b*a^3*x^2+2*A*g^4*x^3*a^2*b^2+A*g^4*x^4*a*b^3+2*A*g^4*x^2*a^3*b-2/5*B*g^4/b
*ln(1/(d*x+c))*a^5+1/5*B*g^4*b^4*ln(e*(a/(d*x+c)*d-1/(d*x+c)*b*c+b)^2/d^2)*
x^5+B*ln(e*(a/(d*x+c)*d-1/(d*x+c)*b*c+b)^2/d^2)*x*a^4*g^4-8/5*B*g^4/b*a^5*l
n(a/(d*x+c)*d-1/(d*x+c)*b*c+b)
```

**maxima [B]** time = 1.44, size = 885, normalized size = 4.86

$$\frac{1}{5}Ab^4g^4x^5+Aab^3g^4x^4+2Aa^2b^2g^4x^3+2Aa^3bg^4x^2+\left(x\log\left(\frac{b^2ex^2}{d^2x^2+2cdx+c^2}+\frac{2abex}{d^2x^2+2cdx+c^2}+\frac{a^2e}{d^2x^2+2cd}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*g*x+a*g)^4*(A+B*log(e*(b*x+a)^2/(d*x+c)^2)),x, algorithm="maxi
ma")
```

```
[Out] 1/5*A*b^4*g^4*x^5 + A*a*b^3*g^4*x^4 + 2*A*a^2*b^2*g^4*x^3 + 2*A*a^3*b*g^4*x
^2 + (x*log(b^2*e*x^2/(d^2*x^2 + 2*c*d*x + c^2) + 2*a*b*e*x/(d^2*x^2 + 2*c*
d*x + c^2) + a^2*e/(d^2*x^2 + 2*c*d*x + c^2)) + 2*a*log(b*x + a)/b - 2*c*lo
g(d*x + c)/d)*B*a^4*g^4 + 2*(x^2*log(b^2*e*x^2/(d^2*x^2 + 2*c*d*x + c^2) +
2*a*b*e*x/(d^2*x^2 + 2*c*d*x + c^2) + a^2*e/(d^2*x^2 + 2*c*d*x + c^2)) - 2*
a^2*log(b*x + a)/b^2 + 2*c^2*log(d*x + c)/d^2 - 2*(b*c - a*d)*x/(b*d))*B*a^
```

```
3*b*g^4 + 2*(x^3*log(b^2*e*x^2/(d^2*x^2 + 2*c*d*x + c^2) + 2*a*b*e*x/(d^2*x
^2 + 2*c*d*x + c^2) + a^2*e/(d^2*x^2 + 2*c*d*x + c^2)) + 2*a^3*log(b*x + a)
/b^3 - 2*c^3*log(d*x + c)/d^3 - ((b^2*c*d - a*b*d^2)*x^2 - 2*(b^2*c^2 - a^2
*d^2)*x)/(b^2*d^2))*B*a^2*b^2*g^4 + 1/3*(3*x^4*log(b^2*e*x^2/(d^2*x^2 + 2*c
*d*x + c^2) + 2*a*b*e*x/(d^2*x^2 + 2*c*d*x + c^2) + a^2*e/(d^2*x^2 + 2*c*d*
x + c^2)) - 6*a^4*log(b*x + a)/b^4 + 6*c^4*log(d*x + c)/d^4 - (2*(b^3*c*d^2
 - a*b^2*d^3)*x^3 - 3*(b^3*c^2*d - a^2*b*d^3)*x^2 + 6*(b^3*c^3 - a^3*d^3)*x
)/(b^3*d^3))*B*a*b^3*g^4 + 1/30*(6*x^5*log(b^2*e*x^2/(d^2*x^2 + 2*c*d*x + c
^2) + 2*a*b*e*x/(d^2*x^2 + 2*c*d*x + c^2) + a^2*e/(d^2*x^2 + 2*c*d*x + c^2)
) + 12*a^5*log(b*x + a)/b^5 - 12*c^5*log(d*x + c)/d^5 - (3*(b^4*c*d^3 - a*b
^3*d^4)*x^4 - 4*(b^4*c^2*d^2 - a^2*b^2*d^4)*x^3 + 6*(b^4*c^3*d - a^3*b*d^4)
*x^2 - 12*(b^4*c^4 - a^4*d^4)*x)/(b^4*d^4))*B*b^4*g^4 + A*a^4*g^4*x
```

**mupad [B]** time = 4.99, size = 1025, normalized size = 5.63

$$x^2 \left( \frac{(5\,a\,d+5\,b\,c)\left(\frac{\left(\frac{b^3\,g^4\,(25\,A\,a\,d+5\,A\,b\,c+2\,B\,a\,d-2\,B\,b\,c)}{5\,d}-\frac{A\,b^3\,g^4\,(5\,a\,d+5\,b\,c)}{5\,d}\right)(5\,a\,d+5\,b\,c)}{5\,b\,d}-\frac{a\,b^2\,g^4\,(10\,A\,a\,d+5\,A\,b\,c+2\,B\,a\,d-2\,B\,b\,c)}{d}+\frac{A\,a\,b^3\,c\,g^4}{d}\right.}{10\,b\,d}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*g + b*g*x)^4*(A + B*log((e*(a + b*x)^2)/(c + d*x)^2)),x)
```

```
[Out] x^2*(((5*a*d + 5*b*c)*((((b^3*g^4*(25*A*a*d + 5*A*b*c + 2*B*a*d - 2*B*b*c))
/(5*d) - (A*b^3*g^4*(5*a*d + 5*b*c))/(5*d))*(5*a*d + 5*b*c))/(5*b*d) - (a*b
^2*g^4*(10*A*a*d + 5*A*b*c + 2*B*a*d - 2*B*b*c))/d + (A*a*b^3*c*g^4)/d))/(1
0*b*d) + (a^2*b*g^4*(5*A*a*d + 5*A*b*c + 2*B*a*d - 2*B*b*c))/d - (a*c*((b^3
*g^4*(25*A*a*d + 5*A*b*c + 2*B*a*d - 2*B*b*c))/(5*d) - (A*b^3*g^4*(5*a*d +
5*b*c))/(5*d)))/(2*b*d)) - x^3*((((b^3*g^4*(25*A*a*d + 5*A*b*c + 2*B*a*d -
2*B*b*c))/(5*d) - (A*b^3*g^4*(5*a*d + 5*b*c))/(5*d))*(5*a*d + 5*b*c))/(15*b
*d) - (a*b^2*g^4*(10*A*a*d + 5*A*b*c + 2*B*a*d - 2*B*b*c))/(3*d) + (A*a*b^3
*c*g^4)/(3*d)) + x*((a^3*g^4*(5*A*a*d + 10*A*b*c + 4*B*a*d - 4*B*b*c))/d -
((5*a*d + 5*b*c)*(((5*a*d + 5*b*c)*((((b^3*g^4*(25*A*a*d + 5*A*b*c + 2*B*a*
d - 2*B*b*c))/(5*d) - (A*b^3*g^4*(5*a*d + 5*b*c))/(5*d))*(5*a*d + 5*b*c))/(
5*b*d) - (a*b^2*g^4*(10*A*a*d + 5*A*b*c + 2*B*a*d - 2*B*b*c))/d + (A*a*b^3*
c*g^4)/d))/(5*b*d) + (2*a^2*b*g^4*(5*A*a*d + 5*A*b*c + 2*B*a*d - 2*B*b*c))/
d - (a*c*((b^3*g^4*(25*A*a*d + 5*A*b*c + 2*B*a*d - 2*B*b*c))/(5*d) - (A*b^3
```

```
*g^4*(5*a*d + 5*b*c))/(5*d)))/(b*d)))/(5*b*d) + (a*c*((((b^3*g^4*(25*A*a*d
+ 5*A*b*c + 2*B*a*d - 2*B*b*c))/(5*d) - (A*b^3*g^4*(5*a*d + 5*b*c))/(5*d))*
(5*a*d + 5*b*c))/(5*b*d) - (a*b^2*g^4*(10*A*a*d + 5*A*b*c + 2*B*a*d - 2*B*b
*c))/d + (A*a*b^3*c*g^4)/d))/(b*d)) + log((e*(a + b*x)^2)/(c + d*x)^2)*((B*
b^4*g^4*x^5)/5 + B*a^4*g^4*x + 2*B*a^3*b*g^4*x^2 + B*a*b^3*g^4*x^4 + 2*B*a^
2*b^2*g^4*x^3) + x^4*((b^3*g^4*(25*A*a*d + 5*A*b*c + 2*B*a*d - 2*B*b*c))/(2
0*d) - (A*b^3*g^4*(5*a*d + 5*b*c))/(20*d)) - (log(c + d*x)*(2*B*b^4*c^5*g^4
 + 10*B*a^4*c*d^4*g^4 - 20*B*a^3*b*c^2*d^3*g^4 + 20*B*a^2*b^2*c^3*d^2*g^4 -
 10*B*a*b^3*c^4*d*g^4))/(5*d^5) + (A*b^4*g^4*x^5)/5 + (2*B*a^5*g^4*log(a +
b*x))/(5*b)
```

**sympy [B]** time = 6.74, size = 998, normalized size = 5.48

$$\frac{Ab^4g^4x^5}{5}+\frac{2Ba^5g^4\log\left(x+\frac{\frac{2Ba^6d^5g^4}{b}+10Ba^5cd^4g^4-20Ba^4bc^2d^3g^4+20Ba^3b^2c^3d^2g^4-10Ba^2b^3c^4dg^4+2Bab^4c^5g^4}{2Ba^5d^5g^4+10Ba^4bcd^4g^4-20Ba^3b^2c^2d^3g^4+20Ba^2b^3c^3d^2g^4-10Bab^4c^4dg^4+2Bb^5c^5g^4}\right)}{5b}-\frac{2Bcg^4\left(5a^4d^4-10\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*g*x+a*g)**4*(A+B*ln(e*(b*x+a)**2/(d*x+c)**2)),x)
```

```
[Out] A*b**4*g**4*x**5/5 + 2*B*a**5*g**4*log(x + (2*B*a**6*d**5*g**4/b + 10*B*a**
5*c*d**4*g**4 - 20*B*a**4*b*c**2*d**3*g**4 + 20*B*a**3*b**2*c**3*d**2*g**4
- 10*B*a**2*b**3*c**4*d*g**4 + 2*B*a*b**4*c**5*g**4)/(2*B*a**5*d**5*g**4 +
10*B*a**4*b*c*d**4*g**4 - 20*B*a**3*b**2*c**2*d**3*g**4 + 20*B*a**2*b**3*c*
*3*d**2*g**4 - 10*B*a*b**4*c**4*d*g**4 + 2*B*b**5*c**5*g**4))/(5*b) - 2*B*c
*g**4*(5*a**4*d**4 - 10*a**3*b*c*d**3 + 10*a**2*b**2*c**2*d**2 - 5*a*b**3*c
**3*d + b**4*c**4)*log(x + (12*B*a**5*c*d**4*g**4 - 20*B*a**4*b*c**2*d**3*g
**4 + 20*B*a**3*b**2*c**3*d**2*g**4 - 10*B*a**2*b**3*c**4*d*g**4 + 2*B*a*b*
*4*c**5*g**4 - 2*B*a*c*g**4*(5*a**4*d**4 - 10*a**3*b*c*d**3 + 10*a**2*b**2*
c**2*d**2 - 5*a*b**3*c**3*d + b**4*c**4) + 2*B*b*c**2*g**4*(5*a**4*d**4 - 1
0*a**3*b*c*d**3 + 10*a**2*b**2*c**2*d**2 - 5*a*b**3*c**3*d + b**4*c**4)/d)/
(2*B*a**5*d**5*g**4 + 10*B*a**4*b*c*d**4*g**4 - 20*B*a**3*b**2*c**2*d**3*g*
*4 + 20*B*a**2*b**3*c**3*d**2*g**4 - 10*B*a*b**4*c**4*d*g**4 + 2*B*b**5*c**
5*g**4))/(5*d**5) + x**4*(A*a*b**3*g**4 + B*a*b**3*g**4/10 - B*b**4*c*g**4/
(10*d)) + x**3*(2*A*a**2*b**2*g**4 + 8*B*a**2*b**2*g**4/15 - 2*B*a*b**3*c*g
**4/(3*d) + 2*B*b**4*c**2*g**4/(15*d**2)) + x**2*(2*A*a**3*b*g**4 + 6*B*a**
3*b*g**4/5 - 2*B*a**2*b**2*c*g**4/d + B*a*b**3*c**2*g**4/d**2 - B*b**4*c**3
*g**4/(5*d**3)) + x*(A*a**4*g**4 + 8*B*a**4*g**4/5 - 4*B*a**3*b*c*g**4/d +
4*B*a**2*b**2*c**2*g**4/d**2 - 2*B*a*b**3*c**3*g**4/d**3 + 2*B*b**4*c**4*g*
*4/(5*d**4)) + (B*a**4*g**4*x + 2*B*a**3*b*g**4*x**2 + 2*B*a**2*b**2*g**4*x
**3 + B*a*b**3*g**4*x**4 + B*b**4*g**4*x**5/5)*log(e*(a + b*x)**2/(c + d*x)
**2)
```

# 3.120 $\int (ag+bgx)^3 \left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right) dx$

**Optimal**. Leaf size=151

$$\frac{g^3(a+bx)^4\left(B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+A\right)}{4b}+\frac{Bg^3(bc-ad)^4\log(c+dx)}{2bd^4}-\frac{Bg^3x(bc-ad)^3}{2d^3}+\frac{Bg^3(a+bx)^2(bc-ad)^2}{4bd^2}-\frac{Bg^3(a+$$

```
[Out] -1/2*B*(-a*d+b*c)^3*g^3*x/d^3+1/4*B*(-a*d+b*c)^2*g^3*(b*x+a)^2/b/d^2-1/6*B*
(-a*d+b*c)*g^3*(b*x+a)^3/b/d+1/4*g^3*(b*x+a)^4*(A+B*ln(e*(b*x+a)^2/(d*x+c)^
2))/b+1/2*B*(-a*d+b*c)^4*g^3*ln(d*x+c)/b/d^4
```

**Rubi [A]** time = 0.10, antiderivative size = 151, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.094, Rules used = {2525, 12, 43}

$$\frac{g^3(a+bx)^4\left(B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+A\right)}{4b}-\frac{Bg^3x(bc-ad)^3}{2d^3}+\frac{Bg^3(a+bx)^2(bc-ad)^2}{4bd^2}+\frac{Bg^3(bc-ad)^4\log(c+dx)}{2bd^4}-\frac{Bg^3(a+$$

Antiderivative was successfully verified.

```
[In] Int[(a*g + b*g*x)^3*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2]),x]
```

```
[Out] -(B*(b*c - a*d)^3*g^3*x)/(2*d^3) + (B*(b*c - a*d)^2*g^3*(a + b*x)^2)/(4*b*d
^2) - (B*(b*c - a*d)*g^3*(a + b*x)^3)/(6*b*d) + (g^3*(a + b*x)^4*(A + B*Log
[(e*(a + b*x)^2)/(c + d*x)^2]))/(4*b) + (B*(b*c - a*d)^4*g^3*Log[c + d*x])/
(2*b*d^4)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2525

```
Int[((a_.) + Log[(c_.)*(RFx_)^(p_.)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_))^(m_.
), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(a + b*Log[c*RFx^p])^n)/(e*(m + 1))
, x] - Dist[(b*n*p)/(e*(m + 1)), Int[SimplifyIntegrand[((d + e*x)^(m + 1)*(
```

```
a + b*Log[c*RFx^p])^(n - 1)*D[RFx, x])/RFx, x], x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && RationalFunctionQ[RFx, x] && IGtQ[n, 0] && (EqQ[n, 1] ||
IntegerQ[m]) && NeQ[m, -1]
```

Rubi steps

$$\int (ag+bgx)^3\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)dx = \frac{g^3(a+bx)^4\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}{4b} - \frac{B\int \frac{2(bc-ad)g^4(a+bx)^3}{c+dx}\,dx}{4bg}$$
$$= \frac{g^3(a+bx)^4\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}{4b} - \frac{\left(B(bc-ad)g^3\right)\int \frac{(a+bx)^3}{c+dx}\,dx}{2b}$$
$$= \frac{g^3(a+bx)^4\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}{4b} - \frac{\left(B(bc-ad)g^3\right)\int \left(\frac{b(bc-ad)^2}{d^3} - \right.}{}$$
$$= -\frac{B(bc-ad)^3g^3x}{2d^3} + \frac{B(bc-ad)^2g^3(a+bx)^2}{4bd^2} - \frac{B(bc-ad)g^3(a+bx}{6bd}$$

**Mathematica [A]** time = 0.09, size = 122, normalized size = 0.81

$$\frac{g^3\left((a+bx)^4\left(B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+A\right)-\frac{B(bc-ad)\left(3d^2(a+bx)^2(ad-bc)+6bdx(bc-ad)^2-6(bc-ad)^3\log(c+dx)+2d^3(a+bx)^3\right)}{3d^4}\right)}{4b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*g + b*g*x)^3*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2]),x]
```

```
[Out] (g^3*((a + b*x)^4*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2]) - (B*(b*c - a*d)
*(6*b*d*(b*c - a*d)^2*x + 3*d^2*(-(b*c) + a*d)*(a + b*x)^2 + 2*d^3*(a + b*x
)^3 - 6*(b*c - a*d)^3*Log[c + d*x]))/(3*d^4)))/(4*b)
```

**fricas [B]** time = 1.09, size = 341, normalized size = 2.26

$$\frac{3\,Ab^4d^4g^3x^4+6\,Ba^4d^4g^3\log(bx+a)-2\left(Bb^4cd^3-(6\,A+B)ab^3d^4\right)g^3x^3+3\left(Bb^4c^2d^2-4\,Bab^3cd^3+3\,(2\,A+\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*g*x+a*g)^3*(A+B*log(e*(b*x+a)^2/(d*x+c)^2)),x, algorithm="fric
as")
```

```
[Out] 1/12*(3*A*b^4*d^4*g^3*x^4 + 6*B*a^4*d^4*g^3*log(b*x + a) - 2*(B*b^4*c*d^3 -
 (6*A + B)*a*b^3*d^4)*g^3*x^3 + 3*(B*b^4*c^2*d^2 - 4*B*a*b^3*c*d^3 + 3*(2*A
 + B)*a^2*b^2*d^4)*g^3*x^2 - 6*(B*b^4*c^3*d - 4*B*a*b^3*c^2*d^2 + 6*B*a^2*b
^2*c*d^3 - (2*A + 3*B)*a^3*b*d^4)*g^3*x + 6*(B*b^4*c^4 - 4*B*a*b^3*c^3*d +
6*B*a^2*b^2*c^2*d^2 - 4*B*a^3*b*c*d^3)*g^3*log(d*x + c) + 3*(B*b^4*d^4*g^3*
x^4 + 4*B*a*b^3*d^4*g^3*x^3 + 6*B*a^2*b^2*d^4*g^3*x^2 + 4*B*a^3*b*d^4*g^3*x
)*log((b^2*e*x^2 + 2*a*b*e*x + a^2*e)/(d^2*x^2 + 2*c*d*x + c^2)))/(b*d^4)
```

**giac [B]** time = 17.61, size = 361, normalized size = 2.39

$$\frac{Ba^4g^3\log(bx+a)}{2b}+\frac{1}{4}\left(Ab^3g^3+Bb^3g^3\right)x^4-\frac{\left(Bb^3cg^3-6\,Aab^2dg^3-7\,Bab^2dg^3\right)x^3}{6\,d}+\frac{1}{4}\left(Bb^3g^3x^4+4\,Bab^2g^3x^3+6\,B\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*g*x+a*g)^3*(A+B*log(e*(b*x+a)^2/(d*x+c)^2)),x, algorithm="giac
")
```

```
[Out] 1/2*B*a^4*g^3*log(b*x + a)/b + 1/4*(A*b^3*g^3 + B*b^3*g^3)*x^4 - 1/6*(B*b^3
*c*g^3 - 6*A*a*b^2*d*g^3 - 7*B*a*b^2*d*g^3)*x^3/d + 1/4*(B*b^3*g^3*x^4 + 4*
B*a*b^2*g^3*x^3 + 6*B*a^2*b*g^3*x^2 + 4*B*a^3*g^3*x)*log((b^2*x^2 + 2*a*b*x
 + a^2)/(d^2*x^2 + 2*c*d*x + c^2)) + 1/4*(B*b^3*c^2*g^3 - 4*B*a*b^2*c*d*g^3
 + 6*A*a^2*b*d^2*g^3 + 9*B*a^2*b*d^2*g^3)*x^2/d^2 - 1/2*(B*b^3*c^3*g^3 - 4*
B*a*b^2*c^2*d*g^3 + 6*B*a^2*b*c*d^2*g^3 - 2*A*a^3*d^3*g^3 - 5*B*a^3*d^3*g^3
)*x/d^3 + 1/2*(B*b^3*c^4*g^3 - 4*B*a*b^2*c^3*d*g^3 + 6*B*a^2*b*c^2*d^2*g^3
- 4*B*a^3*c*d^3*g^3)*log(d*x + c)/d^4
```

**maple [B]** time = 0.08, size = 1249, normalized size = 8.27

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*g*x+a*g)^3*(A+B*ln(e*(b*x+a)^2/(d*x+c)^2)),x)
```

```
[Out] 1/4*A*g^3*x^4*b^3+3/2*B*x*a^3*g^3+A*g^3*x*a^3+3/2/d*B*a^3*c*g^3+1/d*A*g^3*a
^3*c-11/12/d^4*B*g^3*b^3*c^4-1/4/d^4*A*g^3*b^3*c^4+2/d*B*g^3*ln(1/(d*x+c))*
a^3*c-1/4/d^4*B*g^3*b^3*ln((1/(d*x+c)*a*d-1/(d*x+c)*b*c+b)^2/d^2*e)*c^4-1/6
/d*B*g^3*b^3*c*x^3-1/2/d^4*B*g^3*ln(1/(d*x+c))*c^4*b^3-3/2/d^4*B*g^3*b^3*c^
4*ln(1/(d*x+c)*a*d-1/(d*x+c)*b*c+b)+6/d*B*g^3*a^3*ln(1/(d*x+c)*a*d-1/(d*x+c
)*b*c+b)*c+1/4/d^2*B*g^3*b^3*c^2*x^2-1/2/d^3*B*g^3*b^3*c^3*x-10*B*g^3/(a*d-
b*c)*ln(1/(d*x+c)*a*d-1/(d*x+c)*b*c+b)*a^4*c+B*g^3*b^2*ln((1/(d*x+c)*a*d-1/
(d*x+c)*b*c+b)^2/d^2*e)*a*x^3+3/2*B*g^3*b*ln((1/(d*x+c)*a*d-1/(d*x+c)*b*c+b
)^2/d^2*e)*a^2*x^2+1/d*B*ln((1/(d*x+c)*a*d-1/(d*x+c)*b*c+b)^2/d^2*e)*a^3*c*
g^3+19/6/d^3*B*g^3*b^2*a*c^3-3/2/d^2*A*g^3*a^2*b*c^2-15/4/d^2*B*g^3*b*a^2*c
^2+1/d^3*A*g^3*a*b^2*c^3+10/d^3*B*g^3/(a*d-b*c)*ln(1/(d*x+c)*a*d-1/(d*x+c)*
```

```
b*c+b)*a*c^4*b^3+20/d*B*g^3*b/(a*d-b*c)*ln(1/(d*x+c)*a*d-1/(d*x+c)*b*c+b)*a
^3*c^2-20/d^2*B*g^3*b^2/(a*d-b*c)*ln(1/(d*x+c)*a*d-1/(d*x+c)*b*c+b)*a^2*c^3
+3/4*B*g^3*b*a^2*x^2+1/6*B*g^3*b^2*a*x^3-3/2*B*g^3/b*a^4*ln(1/(d*x+c)*a*d-1
/(d*x+c)*b*c+b)+1/4*B*g^3*b^3*ln((1/(d*x+c)*a*d-1/(d*x+c)*b*c+b)^2/d^2*e)*x
^4-1/2*B*g^3/b*ln(1/(d*x+c))*a^4+B*ln((1/(d*x+c)*a*d-1/(d*x+c)*b*c+b)^2/d^2
*e)*x*a^3*g^3+3/2*A*g^3*x^2*a^2*b+A*g^3*x^3*a*b^2+2*d*B*g^3/b/(a*d-b*c)*ln(
1/(d*x+c)*a*d-1/(d*x+c)*b*c+b)*a^5+6/d^3*B*g^3*b^2*a*ln(1/(d*x+c)*a*d-1/(d*
x+c)*b*c+b)*c^3-2/d^4*B*g^3/(a*d-b*c)*ln(1/(d*x+c)*a*d-1/(d*x+c)*b*c+b)*c^5
*b^4-3/d^2*B*g^3*b*ln(1/(d*x+c))*a^2*c^2+1/d^3*B*g^3*ln((1/(d*x+c)*a*d-1/(d
*x+c)*b*c+b)^2/d^2*e)*a*b^2*c^3-3/2/d^2*B*g^3*ln((1/(d*x+c)*a*d-1/(d*x+c)*b
*c+b)^2/d^2*e)*a^2*b*c^2-3/d*B*g^3*b*a^2*x*c-1/d*B*g^3*b^2*a*x^2*c+2/d^2*B*
g^3*b^2*a*x*c^2-9/d^2*B*g^3*b*a^2*ln(1/(d*x+c)*a*d-1/(d*x+c)*b*c+b)*c^2+2/d
^3*B*g^3*b^2*ln(1/(d*x+c))*a*c^3
```

**maxima [B]** time = 1.60, size = 647, normalized size = 4.28

$$\frac{1}{4}Ab^3g^3x^4+Aab^2g^3x^3+\frac{3}{2}Aa^2bg^3x^2+\left(x\log\left(\frac{b^2ex^2}{d^2x^2+2\,cdx+c^2}+\frac{2\,abex}{d^2x^2+2\,cdx+c^2}+\frac{a^2e}{d^2x^2+2\,cdx+c^2}\right)+\frac{2\,a}{\phantom{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*g*x+a*g)^3*(A+B*log(e*(b*x+a)^2/(d*x+c)^2)),x, algorithm="maxi
ma")
```

```
[Out] 1/4*A*b^3*g^3*x^4 + A*a*b^2*g^3*x^3 + 3/2*A*a^2*b*g^3*x^2 + (x*log(b^2*e*x^
2/(d^2*x^2 + 2*c*d*x + c^2) + 2*a*b*e*x/(d^2*x^2 + 2*c*d*x + c^2) + a^2*e/(
d^2*x^2 + 2*c*d*x + c^2)) + 2*a*log(b*x + a)/b - 2*c*log(d*x + c)/d)*B*a^3*
g^3 + 3/2*(x^2*log(b^2*e*x^2/(d^2*x^2 + 2*c*d*x + c^2) + 2*a*b*e*x/(d^2*x^2
 + 2*c*d*x + c^2) + a^2*e/(d^2*x^2 + 2*c*d*x + c^2)) - 2*a^2*log(b*x + a)/b
^2 + 2*c^2*log(d*x + c)/d^2 - 2*(b*c - a*d)*x/(b*d))*B*a^2*b*g^3 + (x^3*log
(b^2*e*x^2/(d^2*x^2 + 2*c*d*x + c^2) + 2*a*b*e*x/(d^2*x^2 + 2*c*d*x + c^2)
+ a^2*e/(d^2*x^2 + 2*c*d*x + c^2)) + 2*a^3*log(b*x + a)/b^3 - 2*c^3*log(d*x
 + c)/d^3 - ((b^2*c*d - a*b*d^2)*x^2 - 2*(b^2*c^2 - a^2*d^2)*x)/(b^2*d^2))*
B*a*b^2*g^3 + 1/12*(3*x^4*log(b^2*e*x^2/(d^2*x^2 + 2*c*d*x + c^2) + 2*a*b*e
*x/(d^2*x^2 + 2*c*d*x + c^2) + a^2*e/(d^2*x^2 + 2*c*d*x + c^2)) - 6*a^4*log
(b*x + a)/b^4 + 6*c^4*log(d*x + c)/d^4 - (2*(b^3*c*d^2 - a*b^2*d^3)*x^3 - 3
*(b^3*c^2*d - a^2*b*d^3)*x^2 + 6*(b^3*c^3 - a^3*d^3)*x)/(b^3*d^3))*B*b^3*g^
3 + A*a^3*g^3*x
```

**mupad [B]** time = 4.74, size = 567, normalized size = 3.75

$$\ln\left(\frac{e\,(a+b\,x)^2}{(c+d\,x)^2}\right)\left(B\,a^3\,g^3\,x+\frac{3\,B\,a^2\,b\,g^3\,x^2}{2}+B\,a\,b^2\,g^3\,x^3+\frac{B\,b^3\,g^3\,x^4}{4}\right)-x^2\left(\frac{\left(\frac{b^2\,g^3\,(8\,A\,a\,d+2\,A\,b\,c+B\,a\,d-B\,b\,c)}{2\,d}-\frac{A\,b^2\,g^3\,(2}{2}\right.}{4\,b\,d}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*g + b*g*x)^3*(A + B*log((e*(a + b*x)^2)/(c + d*x)^2)),x)
```

```
[Out] log((e*(a + b*x)^2)/(c + d*x)^2)*((B*b^3*g^3*x^4)/4 + B*a^3*g^3*x + (3*B*a^
2*b*g^3*x^2)/2 + B*a*b^2*g^3*x^3) - x^2*((((b^2*g^3*(8*A*a*d + 2*A*b*c + B*
a*d - B*b*c))/(2*d) - (A*b^2*g^3*(2*a*d + 2*b*c))/(2*d))*(2*a*d + 2*b*c))/(
4*b*d) - (a*b*g^3*(3*A*a*d + 2*A*b*c + B*a*d - B*b*c))/d + (A*a*b^2*c*g^3)/
(2*d)) + x*(((2*a*d + 2*b*c)*((((b^2*g^3*(8*A*a*d + 2*A*b*c + B*a*d - B*b*c
))/(2*d) - (A*b^2*g^3*(2*a*d + 2*b*c))/(2*d))*(2*a*d + 2*b*c))/(2*b*d) - (2
*a*b*g^3*(3*A*a*d + 2*A*b*c + B*a*d - B*b*c))/d + (A*a*b^2*c*g^3)/d))/(2*b*
d) + (a^2*g^3*(4*A*a*d + 6*A*b*c + 3*B*a*d - 3*B*b*c))/d - (a*c*((b^2*g^3*(
8*A*a*d + 2*A*b*c + B*a*d - B*b*c))/(2*d) - (A*b^2*g^3*(2*a*d + 2*b*c))/(2*
d)))/(b*d)) + x^3*((b^2*g^3*(8*A*a*d + 2*A*b*c + B*a*d - B*b*c))/(6*d) - (A
*b^2*g^3*(2*a*d + 2*b*c))/(6*d)) + (log(c + d*x)*(B*b^3*c^4*g^3 - 4*B*a^3*c
*d^3*g^3 + 6*B*a^2*b*c^2*d^2*g^3 - 4*B*a*b^2*c^3*d*g^3))/(2*d^4) + (A*b^3*g
^3*x^4)/4 + (B*a^4*g^3*log(a + b*x))/(2*b)
```

**sympy [B]** time = 4.89, size = 707, normalized size = 4.68

$$\frac{Ab^3g^3x^4}{4}+\frac{Ba^4g^3\log\left(x+\frac{\frac{Ba^5d^4g^3}{b}+4Ba^4cd^3g^3-6Ba^3bc^2d^2g^3+4Ba^2b^2c^3dg^3-Bab^3c^4g^3}{Ba^4d^4g^3+4Ba^3bcd^3g^3-6Ba^2b^2c^2d^2g^3+4Bab^3c^3dg^3-Bb^4c^4g^3}\right)}{2b}-\frac{Bcg^3\left(2ad-bc\right)\left(2a^2d^2-2abcd+b^2c^2\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*g*x+a*g)**3*(A+B*ln(e*(b*x+a)**2/(d*x+c)**2)),x)
```

```
[Out] A*b**3*g**3*x**4/4 + B*a**4*g**3*log(x + (B*a**5*d**4*g**3/b + 4*B*a**4*c*d
**3*g**3 - 6*B*a**3*b*c**2*d**2*g**3 + 4*B*a**2*b**2*c**3*d*g**3 - B*a*b**3
*c**4*g**3)/(B*a**4*d**4*g**3 + 4*B*a**3*b*c*d**3*g**3 - 6*B*a**2*b**2*c**2
*d**2*g**3 + 4*B*a*b**3*c**3*d*g**3 - B*b**4*c**4*g**3))/(2*b) - B*c*g**3*(
2*a*d - b*c)*(2*a**2*d**2 - 2*a*b*c*d + b**2*c**2)*log(x + (5*B*a**4*c*d**3
*g**3 - 6*B*a**3*b*c**2*d**2*g**3 + 4*B*a**2*b**2*c**3*d*g**3 - B*a*b**3*c*
*4*g**3 - B*a*c*g**3*(2*a*d - b*c)*(2*a**2*d**2 - 2*a*b*c*d + b**2*c**2) +
```

```
B*b*c**2*g**3*(2*a*d - b*c)*(2*a**2*d**2 - 2*a*b*c*d + b**2*c**2)/d)/(B*a**
4*d**4*g**3 + 4*B*a**3*b*c*d**3*g**3 - 6*B*a**2*b**2*c**2*d**2*g**3 + 4*B*a
*b**3*c**3*d*g**3 - B*b**4*c**4*g**3))/(2*d**4) + x**3*(A*a*b**2*g**3 + B*a
*b**2*g**3/6 - B*b**3*c*g**3/(6*d)) + x**2*(3*A*a**2*b*g**3/2 + 3*B*a**2*b*
g**3/4 - B*a*b**2*c*g**3/d + B*b**3*c**2*g**3/(4*d**2)) + x*(A*a**3*g**3 +
3*B*a**3*g**3/2 - 3*B*a**2*b*c*g**3/d + 2*B*a*b**2*c**2*g**3/d**2 - B*b**3*
c**3*g**3/(2*d**3)) + (B*a**3*g**3*x + 3*B*a**2*b*g**3*x**2/2 + B*a*b**2*g*
*3*x**3 + B*b**3*g**3*x**4/4)*log(e*(a + b*x)**2/(c + d*x)**2)
```

# 3.121 $\int (ag+bgx)^2 \left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)\,dx$

**Optimal**. Leaf size=120

$$\frac{g^2(a+bx)^3\left(B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+A\right)}{3b}-\frac{2Bg^2(bc-ad)^3\log(c+dx)}{3bd^3}+\frac{2Bg^2x(bc-ad)^2}{3d^2}-\frac{Bg^2(a+bx)^2(bc-ad)}{3bd}$$

```
[Out] 2/3*B*(-a*d+b*c)^2*g^2*x/d^2-1/3*B*(-a*d+b*c)*g^2*(b*x+a)^2/b/d+1/3*g^2*(b*
x+a)^3*(A+B*ln(e*(b*x+a)^2/(d*x+c)^2))/b-2/3*B*(-a*d+b*c)^3*g^2*ln(d*x+c)/b
/d^3
```

**Rubi [A]** time = 0.07, antiderivative size = 120, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.094, Rules used = {2525, 12, 43}

$$\frac{g^2(a+bx)^3\left(B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+A\right)}{3b}+\frac{2Bg^2x(bc-ad)^2}{3d^2}-\frac{2Bg^2(bc-ad)^3\log(c+dx)}{3bd^3}-\frac{Bg^2(a+bx)^2(bc-ad)}{3bd}$$

Antiderivative was successfully verified.

```
[In] Int[(a*g + b*g*x)^2*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2]),x]
```

```
[Out] (2*B*(b*c - a*d)^2*g^2*x)/(3*d^2) - (B*(b*c - a*d)*g^2*(a + b*x)^2)/(3*b*d)
 + (g^2*(a + b*x)^3*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2]))/(3*b) - (2*B*
(b*c - a*d)^3*g^2*Log[c + d*x])/(3*b*d^3)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2525

```
Int[((a_.) + Log[(c_.)*(RFx_)^(p_.)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_))^(m_.
), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(a + b*Log[c*RFx^p])^n)/(e*(m + 1))
, x] - Dist[(b*n*p)/(e*(m + 1)), Int[SimplifyIntegrand[((d + e*x)^(m + 1)*(
a + b*Log[c*RFx^p])^(n - 1)*D[RFx, x])/RFx, x], x], x] /; FreeQ[{a, b, c, d
```

```
, e, m, p}, x] && RationalFunctionQ[RFx, x] && IGtQ[n, 0] && (EqQ[n, 1] ||
IntegerQ[m]) && NeQ[m, -1]
```

Rubi steps

$$\int (ag+bgx)^2\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)dx=\frac{g^2(a+bx)^3\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}{3b}-\frac{B\int\frac{2(bc-ad)g^3(a+bx)^2}{c+dx}dx}{3bg}$$
$$=\frac{g^2(a+bx)^3\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}{3b}-\frac{\left(2B(bc-ad)g^2\right)\int\frac{(a+bx)^2}{c+dx}dx}{3b}$$
$$=\frac{g^2(a+bx)^3\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}{3b}-\frac{\left(2B(bc-ad)g^2\right)\int\left(-\frac{b(bc-ad)}{d^2}\right.}{3b}$$
$$=\frac{2B(bc-ad)^2g^2x}{3d^2}-\frac{B(bc-ad)g^2(a+bx)^2}{3bd}+\frac{g^2(a+bx)^3\left(A+B\,\mathrm{l}\right.}{3b}$$

**Mathematica [A]** time = 0.06, size = 98, normalized size = 0.82

$$\frac{g^2\left(\frac{B(ad-bc)\left(d\left(a^2d+4abdx+b^2x(dx-2c)\right)+2(bc-ad)^2\log(c+dx)\right)}{d^3}+(a+bx)^3\left(B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+A\right)\right)}{3b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*g + b*g*x)^2*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2]),x]
```

```
[Out] (g^2*((a + b*x)^3*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2]) + (B*(-(b*c) + a
*d)*(d*(a^2*d + 4*a*b*d*x + b^2*x*(-2*c + d*x)) + 2*(b*c - a*d)^2*Log[c + d
*x]))/d^3))/(3*b)
```

**fricas [B]** time = 0.75, size = 243, normalized size = 2.02

$$\frac{Ab^3d^3g^2x^3+2\,Ba^3d^3g^2\log(bx+a)-\left(Bb^3cd^2-(3\,A+B)ab^2d^3\right)g^2x^2+\left(2\,Bb^3c^2d-6\,Bab^2cd^2+(3\,A+4\,B)a^2b\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*g*x+a*g)^2*(A+B*log(e*(b*x+a)^2/(d*x+c)^2)),x, algorithm="fric
as")
```

```
[Out] 1/3*(A*b^3*d^3*g^2*x^3 + 2*B*a^3*d^3*g^2*log(b*x + a) - (B*b^3*c*d^2 - (3*A
 + B)*a*b^2*d^3)*g^2*x^2 + (2*B*b^3*c^2*d - 6*B*a*b^2*c*d^2 + (3*A + 4*B)*a
^2*b*d^3)*g^2*x - 2*(B*b^3*c^3 - 3*B*a*b^2*c^2*d + 3*B*a^2*b*c*d^2)*g^2*log
(d*x + c) + (B*b^3*d^3*g^2*x^3 + 3*B*a*b^2*d^3*g^2*x^2 + 3*B*a^2*b*d^3*g^2*
x)*log((b^2*e*x^2 + 2*a*b*e*x + a^2*e)/(d^2*x^2 + 2*c*d*x + c^2)))/(b*d^3)
```

**giac [B]** time = 3.43, size = 252, normalized size = 2.10

$$\frac{2\,Ba^3g^2\log\left(bx+a\right)}{3\,b}+\frac{1}{3}\left(Ab^2g^2+Bb^2g^2\right)x^3-\frac{\left(Bb^2cg^2-3\,Aabdg^2-4\,Babdg^2\right)x^2}{3\,d}+\frac{1}{3}\left(Bb^2g^2x^3+3\,Babg^2x^2+3\,Ba\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*g*x+a*g)^2*(A+B*log(e*(b*x+a)^2/(d*x+c)^2)),x, algorithm="giac
")
```

```
[Out] 2/3*B*a^3*g^2*log(b*x + a)/b + 1/3*(A*b^2*g^2 + B*b^2*g^2)*x^3 - 1/3*(B*b^2
*c*g^2 - 3*A*a*b*d*g^2 - 4*B*a*b*d*g^2)*x^2/d + 1/3*(B*b^2*g^2*x^3 + 3*B*a*
b*g^2*x^2 + 3*B*a^2*g^2*x)*log((b^2*x^2 + 2*a*b*x + a^2)/(d^2*x^2 + 2*c*d*x
 + c^2)) + 1/3*(2*B*b^2*c^2*g^2 - 6*B*a*b*c*d*g^2 + 3*A*a^2*d^2*g^2 + 7*B*a
^2*d^2*g^2)*x/d^2 - 2/3*(B*b^2*c^3*g^2 - 3*B*a*b*c^2*d*g^2 + 3*B*a^2*c*d^2*
g^2)*log(-d*x - c)/d^3
```

**maple [B]** time = 0.08, size = 915, normalized size = 7.62

$$\frac{B\,b^2g^2x^3\ln\left(\frac{\left(\frac{ad}{dx+c}-\frac{bc}{dx+c}+b\right)^2e}{d^2}\right)}{3}+\frac{A\,b^2g^2x^3}{3}+\frac{2B\,a^4d\,g^2\ln\left(\frac{ad}{dx+c}-\frac{bc}{dx+c}+b\right)}{\left(ad-bc\right)b}-\frac{8B\,a^3c\,g^2\ln\left(\frac{ad}{dx+c}-\frac{bc}{dx+c}+b\right)}{ad-bc}+\frac{12B\,a^2b\,c}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*g*x+a*g)^2*(A+B*ln(e*(b*x+a)^2/(d*x+c)^2)),x)
```

```
[Out] 1/3*B*g^2*b*a*x^2+B*ln((1/(d*x+c)*a*d-1/(d*x+c)*b*c+b)^2/d^2*e)*x*a^2*g^2+1
/d*A*g^2*a^2*c+1/3/d^3*A*g^2*b^2*c^3+1/d^3*B*g^2*c^3*b^2+4/3/d*B*g^2*a^2*c-
4/3*B*g^2/b*ln(1/(d*x+c)*a*d-1/(d*x+c)*b*c+b)*a^3-2/3*B*g^2/b*ln(1/(d*x+c))
*a^3+1/3*B*g^2*b^2*ln((1/(d*x+c)*a*d-1/(d*x+c)*b*c+b)^2/d^2*e)*x^3+A*g^2*x^
2*a*b+4/3*B*x*a^2*g^2+A*g^2*x*a^2+1/3*A*g^2*x^3*b^2-1/d^2*B*g^2*ln((1/(d*x+
c)*a*d-1/(d*x+c)*b*c+b)^2/d^2*e)*a*b*c^2-2/d*B*g^2*a*b*c*x+2*d*B*g^2/b/(a*d
-b*c)*ln(1/(d*x+c)*a*d-1/(d*x+c)*b*c+b)*a^4-2/d^2*B*g^2*b*ln(1/(d*x+c))*a*c
^2-4/d^2*B*g^2*b*ln(1/(d*x+c)*a*d-1/(d*x+c)*b*c+b)*a*c^2+4/d*B*g^2*ln(1/(d*
x+c)*a*d-1/(d*x+c)*b*c+b)*a^2*c+2/d*B*g^2*ln(1/(d*x+c))*a^2*c+1/d*B*ln((1/(
d*x+c)*a*d-1/(d*x+c)*b*c+b)^2/d^2*e)*a^2*c*g^2+4/3/d^3*B*g^2*ln(1/(d*x+c)*a
*d-1/(d*x+c)*b*c+b)*c^3*b^2-1/d^2*A*g^2*a*b*c^2-8/d^2*B*g^2/(a*d-b*c)*ln(1/
```

```
(d*x+c)*a*d-1/(d*x+c)*b*c+b)*a*c^3*b^2-7/3/d^2*B*g^2*b*a*c^2+12/d*B*g^2*b/(
a*d-b*c)*ln(1/(d*x+c)*a*d-1/(d*x+c)*b*c+b)*a^2*c^2+2/3/d^3*B*g^2*ln(1/(d*x+
c))*c^3*b^2+1/3/d^3*B*g^2*b^2*ln((1/(d*x+c)*a*d-1/(d*x+c)*b*c+b)^2/d^2*e)*c
^3-8*B*g^2/(a*d-b*c)*ln(1/(d*x+c)*a*d-1/(d*x+c)*b*c+b)*a^3*c+B*g^2*b*ln((1/
(d*x+c)*a*d-1/(d*x+c)*b*c+b)^2/d^2*e)*a*x^2+2/3/d^2*B*g^2*c^2*b^2*x-1/3/d*B
*g^2*b^2*c*x^2+2/d^3*B*g^2/(a*d-b*c)*ln(1/(d*x+c)*a*d-1/(d*x+c)*b*c+b)*c^4*
b^3
```

**maxima [B]** time = 1.41, size = 437, normalized size = 3.64

$$\frac{1}{3}Ab^2g^2x^3+Aabg^2x^2+\left(x\log\left(\frac{b^2ex^2}{d^2x^2+2cdx+c^2}+\frac{2abex}{d^2x^2+2cdx+c^2}+\frac{a^2e}{d^2x^2+2cdx+c^2}\right)+\frac{2a\log(bx+a)}{b}-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*g*x+a*g)^2*(A+B*log(e*(b*x+a)^2/(d*x+c)^2)),x, algorithm="maxi
ma")
```

```
[Out] 1/3*A*b^2*g^2*x^3 + A*a*b*g^2*x^2 + (x*log(b^2*e*x^2/(d^2*x^2 + 2*c*d*x + c
^2) + 2*a*b*e*x/(d^2*x^2 + 2*c*d*x + c^2) + a^2*e/(d^2*x^2 + 2*c*d*x + c^2)
) + 2*a*log(b*x + a)/b - 2*c*log(d*x + c)/d)*B*a^2*g^2 + (x^2*log(b^2*e*x^2
/(d^2*x^2 + 2*c*d*x + c^2) + 2*a*b*e*x/(d^2*x^2 + 2*c*d*x + c^2) + a^2*e/(d
^2*x^2 + 2*c*d*x + c^2)) - 2*a^2*log(b*x + a)/b^2 + 2*c^2*log(d*x + c)/d^2
- 2*(b*c - a*d)*x/(b*d))*B*a*b*g^2 + 1/3*(x^3*log(b^2*e*x^2/(d^2*x^2 + 2*c*
d*x + c^2) + 2*a*b*e*x/(d^2*x^2 + 2*c*d*x + c^2) + a^2*e/(d^2*x^2 + 2*c*d*x
 + c^2)) + 2*a^3*log(b*x + a)/b^3 - 2*c^3*log(d*x + c)/d^3 - ((b^2*c*d - a*
b*d^2)*x^2 - 2*(b^2*c^2 - a^2*d^2)*x)/(b^2*d^2))*B*b^2*g^2 + A*a^2*g^2*x
```

**mupad [B]** time = 4.59, size = 296, normalized size = 2.47

$$x^2\left(\frac{b\,g^2\,\left(9\,A\,a\,d+3\,A\,b\,c+2\,B\,a\,d-2\,B\,b\,c\right)}{6\,d}-\frac{A\,b\,g^2\,\left(3\,a\,d+3\,b\,c\right)}{6\,d}\right)-x\left(\frac{\left(3\,a\,d+3\,b\,c\right)\,\left(\frac{b\,g^2\,\left(9\,A\,a\,d+3\,A\,b\,c+2\,B\,a\,d-\right.}{3\,d}\right.}{3\,b\,d}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*g + b*g*x)^2*(A + B*log((e*(a + b*x)^2)/(c + d*x)^2)),x)
```

```
[Out] x^2*((b*g^2*(9*A*a*d + 3*A*b*c + 2*B*a*d - 2*B*b*c))/(6*d) - (A*b*g^2*(3*a*
d + 3*b*c))/(6*d)) - x*(((3*a*d + 3*b*c)*((b*g^2*(9*A*a*d + 3*A*b*c + 2*B*a
*d - 2*B*b*c))/(3*d) - (A*b*g^2*(3*a*d + 3*b*c))/(3*d)))/(3*b*d) - (a*g^2*(
3*A*a*d + 3*A*b*c + 2*B*a*d - 2*B*b*c))/d + (A*a*b*c*g^2)/d) + log((e*(a +
b*x)^2)/(c + d*x)^2)*((B*b^2*g^2*x^3)/3 + B*a^2*g^2*x + B*a*b*g^2*x^2) - (l
og(c + d*x)*(2*B*b^2*c^3*g^2 + 6*B*a^2*c*d^2*g^2 - 6*B*a*b*c^2*d*g^2))/(3*d
^3) + (A*b^2*g^2*x^3)/3 + (2*B*a^3*g^2*log(a + b*x))/(3*b)
```

**sympy [B]** time = 3.51, size = 517, normalized size = 4.31

$$\frac{Ab^2g^2x^3}{3} + \frac{2Ba^3g^2\log\left(x + \frac{\frac{2Ba^4d^3g^2}{b}+6Ba^3cd^2g^2-6Ba^2bc^2dg^2+2Bab^2c^3g^2}{2Ba^3d^3g^2+6Ba^2bcd^2g^2-6Bab^2c^2dg^2+2Bb^3c^3g^2}\right)}{3b} - \frac{2Bcg^2\left(3a^2d^2-3abcd+b^2c^2\right)\log\left(x + \frac{8Ba^3cd^2g^2 \cdots}{\cdots}\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*g*x+a*g)**2*(A+B*ln(e*(b*x+a)**2/(d*x+c)**2)),x)
```

```
[Out] A*b**2*g**2*x**3/3 + 2*B*a**3*g**2*log(x + (2*B*a**4*d**3*g**2/b + 6*B*a**3
*c*d**2*g**2 - 6*B*a**2*b*c**2*d*g**2 + 2*B*a*b**2*c**3*g**2)/(2*B*a**3*d**
3*g**2 + 6*B*a**2*b*c*d**2*g**2 - 6*B*a*b**2*c**2*d*g**2 + 2*B*b**3*c**3*g*
*2))/(3*b) - 2*B*c*g**2*(3*a**2*d**2 - 3*a*b*c*d + b**2*c**2)*log(x + (8*B*
a**3*c*d**2*g**2 - 6*B*a**2*b*c**2*d*g**2 + 2*B*a*b**2*c**3*g**2 - 2*B*a*c*
g**2*(3*a**2*d**2 - 3*a*b*c*d + b**2*c**2) + 2*B*b*c**2*g**2*(3*a**2*d**2 -
 3*a*b*c*d + b**2*c**2)/d)/(2*B*a**3*d**3*g**2 + 6*B*a**2*b*c*d**2*g**2 - 6
*B*a*b**2*c**2*d*g**2 + 2*B*b**3*c**3*g**2))/(3*d**3) + x**2*(A*a*b*g**2 +
B*a*b*g**2/3 - B*b**2*c*g**2/(3*d)) + x*(A*a**2*g**2 + 4*B*a**2*g**2/3 - 2*
B*a*b*c*g**2/d + 2*B*b**2*c**2*g**2/(3*d**2)) + (B*a**2*g**2*x + B*a*b*g**2
*x**2 + B*b**2*g**2*x**3/3)*log(e*(a + b*x)**2/(c + d*x)**2)
```

# 3.122 $\int (ag+bgx)\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)dx$

**Optimal**. Leaf size=78

$$\frac{g(a+bx)^2\left(B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+A\right)}{2b}+\frac{Bg(bc-ad)^2\log(c+dx)}{bd^2}-\frac{Bgx(bc-ad)}{d}$$

```
[Out] -B*(-a*d+b*c)*g*x/d+1/2*g*(b*x+a)^2*(A+B*ln(e*(b*x+a)^2/(d*x+c)^2))/b+B*(-a
*d+b*c)^2*g*ln(d*x+c)/b/d^2
```

**Rubi [A]** time = 0.06, antiderivative size = 78, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {2525, 12, 43}

$$\frac{g(a+bx)^2\left(B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+A\right)}{2b}+\frac{Bg(bc-ad)^2\log(c+dx)}{bd^2}-\frac{Bgx(bc-ad)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[(a*g + b*g*x)*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2]),x]
```

```
[Out] -((B*(b*c - a*d)*g*x)/d) + (g*(a + b*x)^2*(A + B*Log[(e*(a + b*x)^2)/(c + d
*x)^2]))/(2*b) + (B*(b*c - a*d)^2*g*Log[c + d*x])/(b*d^2)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2525

```
Int[((a_.) + Log[(c_.)*(RFx_)^(p_.)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_))^(m_.
), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(a + b*Log[c*RFx^p])^n)/(e*(m + 1))
, x] - Dist[(b*n*p)/(e*(m + 1)), Int[SimplifyIntegrand[((d + e*x)^(m + 1)*(
a + b*Log[c*RFx^p])^(n - 1)*D[RFx, x])/RFx, x], x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && RationalFunctionQ[RFx, x] && IGtQ[n, 0] && (EqQ[n, 1] ||
```

```
IntegerQ[m]) && NeQ[m, -1]
```

Rubi steps

$$\int (ag+bgx)\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)dx = \frac{g(a+bx)^2\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}{2b} - \frac{B\int\frac{2(bc-ad)g^2(a+bx)}{c+dx}dx}{2bg}$$
$$= \frac{g(a+bx)^2\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}{2b} - \frac{(B(bc-ad)g)\int\frac{a+bx}{c+dx}dx}{b}$$
$$= \frac{g(a+bx)^2\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}{2b} - \frac{(B(bc-ad)g)\int\left(\frac{b}{d}+\frac{-bc+ad}{d(c+dx)}\right)dx}{b}$$
$$= -\frac{B(bc-ad)gx}{d} + \frac{g(a+bx)^2\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}{2b} + \frac{B(bc-ad)^2g\,l}{bd^2}$$

**Mathematica [A]** time = 0.05, size = 72, normalized size = 0.92

$$\frac{g\left((a+bx)^2\left(B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+A\right)+\frac{2B(ad-bc)((ad-bc)\log(c+dx)+bdx)}{d^2}\right)}{2b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*g + b*g*x)*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2]),x]
```

```
[Out] (g*((a + b*x)^2*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2]) + (2*B*(-(b*c) + a
*d)*(b*d*x + (-(b*c) + a*d)*Log[c + d*x]))/d^2))/(2*b)
```

**fricas [A]** time = 0.58, size = 148, normalized size = 1.90

$$\frac{Ab^2d^2gx^2 + 2\,Ba^2d^2g\log(bx+a) - 2\left(Bb^2cd-(A+B)abd^2\right)gx + 2\left(Bb^2c^2-2\,Babcd\right)g\log(dx+c) + \left(Bb^2d^2gx^2\right.}{2\,bd^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*g*x+a*g)*(A+B*log(e*(b*x+a)^2/(d*x+c)^2)),x, algorithm="fricas
")
```

```
[Out] 1/2*(A*b^2*d^2*g*x^2 + 2*B*a^2*d^2*g*log(b*x + a) - 2*(B*b^2*c*d - (A + B)*
a*b*d^2)*g*x + 2*(B*b^2*c^2 - 2*B*a*b*c*d)*g*log(d*x + c) + (B*b^2*d^2*g*x^
2 + 2*B*a*b*d^2*g*x)*log((b^2*e*x^2 + 2*a*b*e*x + a^2*e)/(d^2*x^2 + 2*c*d*x
 + c^2)))/(b*d^2)
```

**giac [A]** time = 0.84, size = 131, normalized size = 1.68

$$\frac{Ba^2g\log(bx+a)}{b}+\frac{1}{2}\left(Abg+Bbg\right)x^2+\frac{1}{2}\left(Bbgx^2+2\,Bagx\right)\log\left(\frac{b^2x^2+2\,abx+a^2}{d^2x^2+2\,cdx+c^2}\right)-\frac{\left(Bbcg-Aadg-2\,Badg\right)x}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*g*x+a*g)*(A+B*log(e*(b*x+a)^2/(d*x+c)^2)),x, algorithm="giac")
```

```
[Out] B*a^2*g*log(b*x + a)/b + 1/2*(A*b*g + B*b*g)*x^2 + 1/2*(B*b*g*x^2 + 2*B*a*g
*x)*log((b^2*x^2 + 2*a*b*x + a^2)/(d^2*x^2 + 2*c*d*x + c^2)) - (B*b*c*g - A
*a*d*g - 2*B*a*d*g)*x/d + (B*b*c^2*g - 2*B*a*c*d*g)*log(d*x + c)/d^2
```

**maple [B]** time = 0.07, size = 560, normalized size = 7.18

$$\frac{2B\,a^3dg\ln\left(\frac{ad}{dx+c}-\frac{bc}{dx+c}+b\right)}{(ad-bc)\,b}-\frac{6B\,a^2cg\ln\left(\frac{ad}{dx+c}-\frac{bc}{dx+c}+b\right)}{ad-bc}+\frac{6Bab\,c^2g\ln\left(\frac{ad}{dx+c}-\frac{bc}{dx+c}+b\right)}{(ad-bc)\,d}-\frac{2B\,b^2c^3g\ln\left(\frac{ad}{dx+c}-\right.}{(ad-bc)\,d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*g*x+a*g)*(A+B*ln(e*(b*x+a)^2/(d*x+c)^2)),x)
```

```
[Out] -1/d*g*B*b*c*x+1/d*B*g*a*c+1/d*B*ln((1/(d*x+c)*a*d-1/(d*x+c)*b*c+b)^2/d^2*e
)*a*c*g+1/d*A*g*a*c-1/2/d^2*A*g*b*c^2+1/2*A*g*x^2*b+2/d*B*g*ln(1/(d*x+c))*a
*c+2/d*B*g*ln(1/(d*x+c)*a*d-1/(d*x+c)*b*c+b)*a*c-B*g/b*ln(1/(d*x+c))*a^2+g*
B*a*x+g*A*a*x-1/d^2*B*g*c^2*b-1/d^2*B*g*ln(1/(d*x+c))*c^2*b-B*g/b*ln(1/(d*x
+c)*a*d-1/(d*x+c)*b*c+b)*a^2-1/d^2*B*g*b*ln(1/(d*x+c)*a*d-1/(d*x+c)*b*c+b)*
c^2-2/d^2*B*g/(a*d-b*c)*ln(1/(d*x+c)*a*d-1/(d*x+c)*b*c+b)*c^3*b^2+2*d*B*g/b
/(a*d-b*c)*ln(1/(d*x+c)*a*d-1/(d*x+c)*b*c+b)*a^3+6/d*B*g/(a*d-b*c)*ln(1/(d*
x+c)*a*d-1/(d*x+c)*b*c+b)*a*c^2*b-1/2/d^2*B*g*ln((1/(d*x+c)*a*d-1/(d*x+c)*b
*c+b)^2/d^2*e)*b*c^2+1/2*B*g*b*ln((1/(d*x+c)*a*d-1/(d*x+c)*b*c+b)^2/d^2*e)*
x^2+B*ln((1/(d*x+c)*a*d-1/(d*x+c)*b*c+b)^2/d^2*e)*x*a*g-6*B*g/(a*d-b*c)*ln(
1/(d*x+c)*a*d-1/(d*x+c)*b*c+b)*a^2*c
```

**maxima [B]** time = 1.32, size = 250, normalized size = 3.21

$$\frac{1}{2}\,Abgx^2+\left(x\log\left(\frac{b^2ex^2}{d^2x^2+2\,cdx+c^2}+\frac{2\,abex}{d^2x^2+2\,cdx+c^2}+\frac{a^2e}{d^2x^2+2\,cdx+c^2}\right)+\frac{2\,a\log(bx+a)}{b}-\frac{2\,c\log(dx+}{d}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*g*x+a*g)*(A+B*log(e*(b*x+a)^2/(d*x+c)^2)),x, algorithm="maxima
")
```

```
[Out] 1/2*A*b*g*x^2 + (x*log(b^2*e*x^2/(d^2*x^2 + 2*c*d*x + c^2) + 2*a*b*e*x/(d^2
*x^2 + 2*c*d*x + c^2) + a^2*e/(d^2*x^2 + 2*c*d*x + c^2)) + 2*a*log(b*x + a)
/b - 2*c*log(d*x + c)/d)*B*a*g + 1/2*(x^2*log(b^2*e*x^2/(d^2*x^2 + 2*c*d*x
+ c^2) + 2*a*b*e*x/(d^2*x^2 + 2*c*d*x + c^2) + a^2*e/(d^2*x^2 + 2*c*d*x + c
^2)) - 2*a^2*log(b*x + a)/b^2 + 2*c^2*log(d*x + c)/d^2 - 2*(b*c - a*d)*x/(b
*d))*B*b*g + A*a*g*x
```

**mupad [B]** time = 4.39, size = 120, normalized size = 1.54

$$x\left(\frac{g\,\left(2\,A\,a\,d+A\,b\,c+B\,a\,d-B\,b\,c\right)}{d}-\frac{A\,g\,\left(a\,d+b\,c\right)}{d}\right)+\ln\left(\frac{e\,{\left(a+b\,x\right)}^2}{{\left(c+d\,x\right)}^2}\right)\,\left(\frac{B\,b\,g\,x^2}{2}+B\,a\,g\,x\right)+\frac{A\,b\,g\,x^2}{2}+\frac{B\,a^2\,g\,\ln}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*g + b*g*x)*(A + B*log((e*(a + b*x)^2)/(c + d*x)^2)),x)
```

```
[Out] x*((g*(2*A*a*d + A*b*c + B*a*d - B*b*c))/d - (A*g*(a*d + b*c))/d) + log((e*
(a + b*x)^2)/(c + d*x)^2)*((B*b*g*x^2)/2 + B*a*g*x) + (A*b*g*x^2)/2 + (B*a^
2*g*log(a + b*x))/b - (B*c*g*log(c + d*x)*(2*a*d - b*c))/d^2
```

**sympy [B]** time = 2.02, size = 250, normalized size = 3.21

$$\frac{Abgx^2}{2}+\frac{Ba^2g\log\left(x+\frac{\frac{Ba^3d^2g}{b}+2Ba^2cdg-Babc^2g}{Ba^2d^2g+2Babcdg-Bb^2c^2g}\right)}{b}-\frac{Bcg\left(2ad-bc\right)\log\left(x+\frac{3Ba^2cdg-Babc^2g-Bacg(2ad-bc)+\frac{Bbc^2g(2ad-bc)}{d}}{Ba^2d^2g+2Babcdg-Bb^2c^2g}\right)}{d^2}+x\left(A\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*g*x+a*g)*(A+B*ln(e*(b*x+a)**2/(d*x+c)**2)),x)
```

```
[Out] A*b*g*x**2/2 + B*a**2*g*log(x + (B*a**3*d**2*g/b + 2*B*a**2*c*d*g - B*a*b*c
**2*g)/(B*a**2*d**2*g + 2*B*a*b*c*d*g - B*b**2*c**2*g))/b - B*c*g*(2*a*d -
b*c)*log(x + (3*B*a**2*c*d*g - B*a*b*c**2*g - B*a*c*g*(2*a*d - b*c) + B*b*c
**2*g*(2*a*d - b*c)/d)/(B*a**2*d**2*g + 2*B*a*b*c*d*g - B*b**2*c**2*g))/d**
2 + x*(A*a*g + B*a*g - B*b*c*g/d) + (B*a*g*x + B*b*g*x**2/2)*log(e*(a + b*x
)**2/(c + d*x)**2)
```

# 3.123 $\int \frac{A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}{ag+bgx}\,dx$

**Optimal**. Leaf size=83

$$\frac{2B\text{Li}_2\left(\frac{bc-ad}{d(a+bx)}+1\right)}{bg}-\frac{\log\left(-\frac{bc-ad}{d(a+bx)}\right)\left(B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+A\right)}{bg}$$

```
[Out] -ln((a*d-b*c)/d/(b*x+a))*(A+B*ln(e*(b*x+a)^2/(d*x+c)^2))/b/g+2*B*polylog(2,
1+(-a*d+b*c)/d/(b*x+a))/b/g
```

**Rubi [A]** time = 0.29, antiderivative size = 122, normalized size of antiderivative = 1.47, number of steps used = 10, number of rules used = 8, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {2524, 12, 2418, 2390, 2301, 2394, 2393, 2391}

$$\frac{2B\text{PolyLog}\left(2,-\frac{d(a+bx)}{bc-ad}\right)}{bg}+\frac{\log(ag+bgx)\left(B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+A\right)}{bg}+\frac{2B\log(ag+bgx)\log\left(\frac{b(c+dx)}{bc-ad}\right)}{bg}-\frac{B\log^2(g(a+b$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])/(a*g + b*g*x),x]
```

```
[Out] -((B*Log[g*(a + b*x)]^2)/(b*g)) + ((A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])
*Log[a*g + b*g*x])/(b*g) + (2*B*Log[(b*(c + d*x))/(b*c - a*d)]*Log[a*g + b*
g*x])/(b*g) + (2*B*PolyLog[2, -((d*(a + b*x))/(b*c - a*d))])/(b*g)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2301

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

Rule 2390

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_))^(q_.), x_Symbol] :> Dist[1/e, Subst[Int[((f*x)/d)^q*(a + b*Log[c*x^
n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && E
qQ[e*f - d*g, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2393

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + (c*e*x)/g])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2394

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d + e*x
)^n]))/g, x] - Dist[(b*e*n)/g, Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2418

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*(RFx_), x_Sy
mbol] :> With[{u = ExpandIntegrand[(a + b*Log[c*(d + e*x)^n])^p, RFx, x]},
Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d, e, n}, x] && RationalFunctionQ[
RFx, x] && IntegerQ[p]
```

Rule 2524

```
Int[((a_.) + Log[(c_.)*(RFx_)^(p_.)]*(b_.))^(n_.)/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[(Log[d + e*x]*(a + b*Log[c*RFx^p])^n)/e, x] - Dist[(b*n*p)/e
, Int[(Log[d + e*x]*(a + b*Log[c*RFx^p])^(n - 1)*D[RFx, x])/RFx, x], x] /;
FreeQ[{a, b, c, d, e, p}, x] && RationalFunctionQ[RFx, x] && IGtQ[n, 0]
```

Rubi steps

$$\int \frac{A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}{ag+bgx}\,dx = \frac{\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)\log(ag+bgx)}{bg} - \frac{B\int \frac{(c+dx)^2\left(-\frac{2de(a+bx)^2}{(c+dx)^3}+\frac{2be(a+bx)}{(c+dx)^2}\right)\log(ag+bgx)}{e(a+bx)^2}\,dx}{bg}$$

$$= \frac{\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)\log(ag+bgx)}{bg} - \frac{B\int \frac{(c+dx)^2\left(-\frac{2de(a+bx)^2}{(c+dx)^3}+\frac{2be(a+bx)}{(c+dx)^2}\right)\log(ag+bgx)}{(a+bx)^2}\,dx}{beg}$$

$$= \frac{\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)\log(ag+bgx)}{bg} - \frac{B\int \left(\frac{2be\log(ag+bgx)}{a+bx}-\frac{2de\log(ag+bgx)}{c+dx}\right)dx}{beg}$$

$$= \frac{\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)\log(ag+bgx)}{bg} - \frac{(2B)\int \frac{\log(ag+bgx)}{a+bx}\,dx}{g} + \frac{(2Bd)\int \frac{\log(ag+bgx)}{c+dx}\,d}{bg}$$

$$= \frac{\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)\log(ag+bgx)}{bg} + \frac{2B\log\left(\frac{b(c+dx)}{bc-ad}\right)\log(ag+bgx)}{bg} - (2B)\int \frac{\mathrm{lo}}{}$$

$$= \frac{\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)\log(ag+bgx)}{bg} + \frac{2B\log\left(\frac{b(c+dx)}{bc-ad}\right)\log(ag+bgx)}{bg} - \frac{(2B)\,\mathrm{Subs}}{}$$

$$= -\frac{B\log^2(g(a+bx))}{bg} + \frac{\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)\log(ag+bgx)}{bg} + \frac{2B\log\left(\frac{b(c+dx)}{bc-ad}\right)\log(}{bg}$$

**Mathematica [A]** time = 0.05, size = 88, normalized size = 1.06

$$\frac{\log(a+bx)\left(B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+2B\log\left(\frac{b(c+dx)}{bc-ad}\right)-B\log(a+bx)+A\right)+2B\,\mathrm{Li}_2\left(\frac{d(a+bx)}{ad-bc}\right)}{bg}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])/(a*g + b*g*x),x]
```

```
[Out] (Log[a + b*x]*(A - B*Log[a + b*x] + B*Log[(e*(a + b*x)^2)/(c + d*x)^2] + 2*
B*Log[(b*(c + d*x))/(b*c - a*d)]) + 2*B*PolyLog[2, (d*(a + b*x))/(-(b*c) +
a*d)])/(b*g)
```

**fricas [F]** time = 2.74, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{B\log\left(\frac{b^2ex^2+2\,abex+a^2e}{d^2x^2+2\,cdx+c^2}\right)+A}{bgx+ag}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*(b*x+a)^2/(d*x+c)^2))/(b*g*x+a*g),x, algorithm="fricas
")
```

```
[Out] integral((B*log((b^2*e*x^2 + 2*a*b*e*x + a^2*e)/(d^2*x^2 + 2*c*d*x + c^2))
+ A)/(b*g*x + a*g), x)
```

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{B\log\left(\frac{(bx+a)^2e}{(dx+c)^2}\right)+A}{bgx+ag}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*(b*x+a)^2/(d*x+c)^2))/(b*g*x+a*g),x, algorithm="giac")
```

```
[Out] integrate((B*log((b*x + a)^2*e/(d*x + c)^2) + A)/(b*g*x + a*g), x)
```

**maple [B]** time = 0.13, size = 552, normalized size = 6.65

$$\frac{2Bad\ln\left(\frac{1}{dx+c}\right)\ln\left(\frac{b+\frac{ad-bc}{dx+c}}{b}\right)}{(ad-bc)\,bg}+\frac{Bad\ln\left(\frac{\left(\frac{ad}{dx+c}-\frac{bc}{dx+c}+b\right)^2e}{d^2}\right)\ln\left(b+\frac{ad-bc}{dx+c}\right)}{(ad-bc)\,bg}-\frac{Bad\ln\left(b+\frac{ad-bc}{dx+c}\right)^2}{(ad-bc)\,bg}-\frac{2Bc\ln\left(\frac{1}{dx+c}\right)\ln\left(\frac{b+\frac{ad-bc}{dx+c}}{b}\right.}{(ad-bc)\,g}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*ln(e*(b*x+a)^2/(d*x+c)^2))/(b*g*x+a*g),x)
```

```
[Out] d/g*A/b/(a*d-b*c)*ln(1/(d*x+c)*a*d-1/(d*x+c)*b*c+b)*a-1/g*A/(a*d-b*c)*ln(1/
(d*x+c)*a*d-1/(d*x+c)*b*c+b)*c-1/g*A/b*ln(1/(d*x+c))+d/g*B/b*ln(1/(d*x+c)*(
a*d-b*c)+b)/(a*d-b*c)*ln((1/(d*x+c)*a*d-1/(d*x+c)*b*c+b)^2/d^2*e)*a-1/g*B*l
n(1/(d*x+c)*(a*d-b*c)+b)/(a*d-b*c)*ln((1/(d*x+c)*a*d-1/(d*x+c)*b*c+b)^2/d^2
*e)*c-d/g*B/b/(a*d-b*c)*ln(1/(d*x+c)*(a*d-b*c)+b)^2*a+1/g*B/(a*d-b*c)*ln(1/
(d*x+c)*(a*d-b*c)+b)^2*c-1/g*B/b*ln(1/(d*x+c))*ln((1/(d*x+c)*a*d-1/(d*x+c)*
b*c+b)^2/d^2*e)+2*d/g*B/b*dilog((1/(d*x+c)*(a*d-b*c)+b)/b)/(a*d-b*c)*a-2/g*
B*dilog((1/(d*x+c)*(a*d-b*c)+b)/b)/(a*d-b*c)*c+2*d/g*B/b*ln(1/(d*x+c))*ln((
1/(d*x+c)*(a*d-b*c)+b)/b)/(a*d-b*c)*a-2/g*B*ln(1/(d*x+c))*ln((1/(d*x+c)*(a*
d-b*c)+b)/b)/(a*d-b*c)*c
```

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$-B\left(\frac{2\,\log(bx+a)\log(dx+c)}{bg}-\int\frac{bdx\log(e)+bc\log(e)+2\,(2\,bdx+bc+ad)\log(bx+a)}{b^2dgx^2+abcg+\left(b^2cg+abdg\right)x}\,dx\right)+\frac{A\log\left(bgx+a\right.}{bg}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*(b*x+a)^2/(d*x+c)^2))/(b*g*x+a*g),x, algorithm="maxima
")
```

```
[Out] -B*(2*log(b*x + a)*log(d*x + c)/(b*g) - integrate((b*d*x*log(e) + b*c*log(e
) + 2*(2*b*d*x + b*c + a*d)*log(b*x + a))/(b^2*d*g*x^2 + a*b*c*g + (b^2*c*g
 + a*b*d*g)*x), x)) + A*log(b*g*x + a*g)/(b*g)
```

**mupad** **[F]** time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{A+B\,\ln\left(\frac{e\,{\left(a+b\,x\right)}^2}{{\left(c+d\,x\right)}^2}\right)}{a\,g+b\,g\,x} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*log((e*(a + b*x)^2)/(c + d*x)^2))/(a*g + b*g*x),x)
```

```
[Out] int((A + B*log((e*(a + b*x)^2)/(c + d*x)^2))/(a*g + b*g*x), x)
```

**sympy** **[F]** time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{A}{a+bx}\, dx + \int \frac{B \log\left(\frac{a^2 e}{c^2+2cdx+d^2x^2} + \frac{2abex}{c^2+2cdx+d^2x^2} + \frac{b^2ex^2}{c^2+2cdx+d^2x^2}\right)}{a+bx}\, dx}{g}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*ln(e*(b*x+a)**2/(d*x+c)**2))/(b*g*x+a*g),x)
```

```
[Out] (Integral(A/(a + b*x), x) + Integral(B*log(a**2*e/(c**2 + 2*c*d*x + d**2*x*
*2) + 2*a*b*e*x/(c**2 + 2*c*d*x + d**2*x**2) + b**2*e*x**2/(c**2 + 2*c*d*x
+ d**2*x**2))/(a + b*x), x))/g
```

## 3.124 $\int \frac{A+B \log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}{(ag+bgx)^2} dx$

**Optimal**. Leaf size=65

$$-\frac{(c+dx)\left(B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+A\right)}{g^2(a+bx)(bc-ad)}-\frac{2B}{bg^2(a+bx)}$$

```
[Out] -2*B/b/g^2/(b*x+a)-(d*x+c)*(A+B*ln(e*(b*x+a)^2/(d*x+c)^2))/(-a*d+b*c)/g^2/(
b*x+a)
```

**Rubi [A]** time = 0.08, antiderivative size = 105, normalized size of antiderivative = 1.62, number of steps used = 4, number of rules used = 3, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.094, Rules used = {2525, 12, 44}

$$-\frac{B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+A}{bg^2(a+bx)}-\frac{2Bd\log(a+bx)}{bg^2(bc-ad)}+\frac{2Bd\log(c+dx)}{bg^2(bc-ad)}-\frac{2B}{bg^2(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])/(a*g + b*g*x)^2,x]
```

```
[Out] (-2*B)/(b*g^2*(a + b*x)) - (2*B*d*Log[a + b*x])/(b*(b*c - a*d)*g^2) - (A +
B*Log[(e*(a + b*x)^2)/(c + d*x)^2])/(b*g^2*(a + b*x)) + (2*B*d*Log[c + d*x]
)/(b*(b*c - a*d)*g^2)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 2525

```
Int[((a_.) + Log[(c_.)*(RFx_)^(p_.)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_))^(m_.
), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(a + b*Log[c*RFx^p])^n)/(e*(m + 1))
, x] - Dist[(b*n*p)/(e*(m + 1)), Int[SimplifyIntegrand[((d + e*x)^(m + 1)*(
```

```
a + b*Log[c*RFx^p])^(n - 1)*D[RFx, x])/RFx, x], x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && RationalFunctionQ[RFx, x] && IGtQ[n, 0] && (EqQ[n, 1] ||
IntegerQ[m]) && NeQ[m, -1]
```

Rubi steps

$$\int \frac{A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}{(ag+bgx)^2}\,dx = -\frac{A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}{bg^2(a+bx)}+\frac{B\int\frac{2(bc-ad)}{g(a+bx)^2(c+dx)}\,dx}{bg}$$

$$= -\frac{A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}{bg^2(a+bx)}+\frac{(2B(bc-ad))\int\frac{1}{(a+bx)^2(c+dx)}\,dx}{bg^2}$$

$$= -\frac{A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}{bg^2(a+bx)}+\frac{(2B(bc-ad))\int\left(\frac{b}{(bc-ad)(a+bx)^2}-\frac{bd}{(bc-ad)^2(a+bx)}+\frac{d^2}{(bc-ad)^2(c+dx}\right.}{bg^2}$$

$$= -\frac{2B}{bg^2(a+bx)}-\frac{2Bd\log(a+bx)}{b(bc-ad)g^2}-\frac{A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}{bg^2(a+bx)}+\frac{2Bd\log(c+dx)}{b(bc-ad)g^2}$$

**Mathematica [A]** time = 0.06, size = 111, normalized size = 1.71

$$\frac{2B(bc-ad)\left(-\frac{1}{(a+bx)(bc-ad)}-\frac{d\log(a+bx)}{(bc-ad)^2}+\frac{d\log(c+dx)}{(bc-ad)^2}\right)}{bg^2}-\frac{B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+A}{bg^2(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])/(a*g + b*g*x)^2,x]
```

```
[Out] -((A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])/(b*g^2*(a + b*x))) + (2*B*(b*c -
 a*d)*(-(1/((b*c - a*d)*(a + b*x))) - (d*Log[a + b*x])/(b*c - a*d)^2 + (d*L
og[c + d*x])/(b*c - a*d)^2))/(b*g^2)
```

**fricas [A]** time = 0.68, size = 110, normalized size = 1.69

$$-\frac{(A+2\,B)bc-(A+2\,B)ad+(Bbdx+Bbc)\log\left(\frac{b^2ex^2+2\,abex+a^2e}{d^2x^2+2\,cdx+c^2}\right)}{\left(b^3c-ab^2d\right)g^2x+\left(ab^2c-a^2bd\right)g^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*(b*x+a)^2/(d*x+c)^2))/(b*g*x+a*g)^2,x, algorithm="fric
as")
```

```
[Out] -((A + 2*B)*b*c - (A + 2*B)*a*d + (B*b*d*x + B*b*c)*log((b^2*e*x^2 + 2*a*b*
e*x + a^2*e)/(d^2*x^2 + 2*c*d*x + c^2)))/((b^3*c - a*b^2*d)*g^2*x + (a*b^2*
c - a^2*b*d)*g^2)
```

**giac [B]** time = 0.45, size = 188, normalized size = 2.89

$$\left(2\left(b^2cg^2-abdg^2\right)\left(\frac{d\log\left(\left|\frac{bcg}{bgx+ag}-\frac{adg}{bgx+ag}+d\right|\right)}{b^4c^2g^4-2\,ab^3cdg^4+a^2b^2d^2g^4}-\frac{1}{\left(b^2cg^2-abdg^2\right)\left(bgx+ag\right)bg}\right)-\frac{\log\left(\frac{(bx+a)^2e}{(dx+c)^2}\right)}{\left(bgx+ag\right)bg}\right)B-\frac{A}{\left(bgx+a\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*(b*x+a)^2/(d*x+c)^2))/(b*g*x+a*g)^2,x, algorithm="giac
")
```

```
[Out] (2*(b^2*c*g^2 - a*b*d*g^2)*(d*log(abs(b*c*g/(b*g*x + a*g) - a*d*g/(b*g*x +
a*g) + d))/(b^4*c^2*g^4 - 2*a*b^3*c*d*g^4 + a^2*b^2*d^2*g^4) - 1/((b^2*c*g^
2 - a*b*d*g^2)*(b*g*x + a*g)*b*g)) - log((b*x + a)^2*e/(d*x + c)^2)/((b*g*x
 + a*g)*b*g))*B - A/((b*g*x + a*g)*b*g)
```

**maple [B]** time = 0.08, size = 157, normalized size = 2.42

$$\frac{Bd\ln\left(\frac{\left(\frac{ad}{dx+c}-\frac{bc}{dx+c}+b\right)^2e}{d^2}\right)}{\left(\frac{ad}{dx+c}-\frac{bc}{dx+c}+b\right)(ad-bc)\,g^2}+\frac{Ad}{\left(\frac{ad}{dx+c}-\frac{bc}{dx+c}+b\right)(ad-bc)\,g^2}-\frac{2Bd}{\left(\frac{ad}{dx+c}-\frac{bc}{dx+c}+b\right)(dx+c)\,b\,g^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*ln(e*(b*x+a)^2/(d*x+c)^2))/(b*g*x+a*g)^2,x)
```

```
[Out] d/g^2*A/(1/(d*x+c)*a*d-1/(d*x+c)*b*c+b)/(a*d-b*c)-2*d/g^2/(1/(d*x+c)*a*d-1/
(d*x+c)*b*c+b)*B/b/(d*x+c)+d/g^2/(1/(d*x+c)*a*d-1/(d*x+c)*b*c+b)*B/(a*d-b*c
)*ln((1/(d*x+c)*a*d-1/(d*x+c)*b*c+b)^2/d^2*e)
```

**maxima [B]** time = 1.22, size = 187, normalized size = 2.88

$$-B\left(\frac{\log\left(\frac{b^2ex^2}{d^2x^2+2\,cdx+c^2}+\frac{2\,abex}{d^2x^2+2\,cdx+c^2}+\frac{a^2e}{d^2x^2+2\,cdx+c^2}\right)}{b^2g^2x+abg^2}+\frac{2}{b^2g^2x+abg^2}+\frac{2\,d\log\left(bx+a\right)}{\left(b^2c-abd\right)g^2}-\frac{2\,d\log\left(dx+c\right)}{\left(b^2c-abd\right)g^2}\right)-\frac{}{b^2g^2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*(b*x+a)^2/(d*x+c)^2))/(b*g*x+a*g)^2,x, algorithm="maxi
ma")
```

```
[Out] -B*(log(b^2*e*x^2/(d^2*x^2 + 2*c*d*x + c^2) + 2*a*b*e*x/(d^2*x^2 + 2*c*d*x
+ c^2) + a^2*e/(d^2*x^2 + 2*c*d*x + c^2))/(b^2*g^2*x + a*b*g^2) + 2/(b^2*g^
2*x + a*b*g^2) + 2*d*log(b*x + a)/((b^2*c - a*b*d)*g^2) - 2*d*log(d*x + c)/
((b^2*c - a*b*d)*g^2)) - A/(b^2*g^2*x + a*b*g^2)
```

**mupad [B]** time = 5.25, size = 108, normalized size = 1.66

$$-\frac{A+2\,B}{x\,b^2\,g^2+a\,b\,g^2}-\frac{B\,\ln\left(\frac{e\,{\left(a+b\,x\right)}^2}{{\left(c+d\,x\right)}^2}\right)}{b^2\,g^2\,\left(x+\frac{a}{b}\right)}-\frac{B\,d\,\mathrm{atan}\left(\frac{b\,c\,2{}\mathrm{i}+b\,d\,x\,2{}\mathrm{i}}{a\,d-b\,c}+1{}\mathrm{i}\right)\,4{}\mathrm{i}}{b\,g^2\,\left(a\,d-b\,c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*log((e*(a + b*x)^2)/(c + d*x)^2))/(a*g + b*g*x)^2,x)
```

```
[Out] - (A + 2*B)/(b^2*g^2*x + a*b*g^2) - (B*log((e*(a + b*x)^2)/(c + d*x)^2))/(b
^2*g^2*(x + a/b)) - (B*d*atan((b*c*2i + b*d*x*2i)/(a*d - b*c) + 1i)*4i)/(b*
g^2*(a*d - b*c))
```

**sympy [B]** time = 1.72, size = 255, normalized size = 3.92

$$-\frac{B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}{abg^2+b^2g^2x}-\frac{2Bd\log\left(x+\frac{-\frac{2Ba^2d^3}{ad-bc}+\frac{4Babcd^2}{ad-bc}+2Bad^2-\frac{2Bb^2c^2d}{ad-bc}+2Bbcd}{4Bbd^2}\right)}{bg^2\left(ad-bc\right)}+\frac{2Bd\log\left(x+\frac{\frac{2Ba^2d^3}{ad-bc}-\frac{4Babcd^2}{ad-bc}+2Bad^2+\frac{2Bb^2c^2d}{ad-bc}+2Bbc}{4Bbd^2}\right.}{bg^2\left(ad-bc\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*ln(e*(b*x+a)**2/(d*x+c)**2))/(b*g*x+a*g)**2,x)
```

```
[Out] -B*log(e*(a + b*x)**2/(c + d*x)**2)/(a*b*g**2 + b**2*g**2*x) - 2*B*d*log(x
+ (-2*B*a**2*d**3/(a*d - b*c) + 4*B*a*b*c*d**2/(a*d - b*c) + 2*B*a*d**2 - 2
*B*b**2*c**2*d/(a*d - b*c) + 2*B*b*c*d)/(4*B*b*d**2))/(b*g**2*(a*d - b*c))
+ 2*B*d*log(x + (2*B*a**2*d**3/(a*d - b*c) - 4*B*a*b*c*d**2/(a*d - b*c) + 2
*B*a*d**2 + 2*B*b**2*c**2*d/(a*d - b*c) + 2*B*b*c*d)/(4*B*b*d**2))/(b*g**2*
(a*d - b*c)) + (-A - 2*B)/(a*b*g**2 + b**2*g**2*x)
```

# 3.125 $\int \frac{A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}{(ag+bgx)^3}\,dx$

**Optimal**. Leaf size=138

$$-\frac{B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+A}{2bg^3(a+bx)^2}+\frac{Bd^2\log(a+bx)}{bg^3(bc-ad)^2}-\frac{Bd^2\log(c+dx)}{bg^3(bc-ad)^2}+\frac{Bd}{bg^3(a+bx)(bc-ad)}-\frac{B}{2bg^3(a+bx)^2}$$

```
[Out] -1/2*B/b/g^3/(b*x+a)^2+B*d/b/(-a*d+b*c)/g^3/(b*x+a)+B*d^2*ln(b*x+a)/b/(-a*d
+b*c)^2/g^3+1/2*(-A-B*ln(e*(b*x+a)^2/(d*x+c)^2))/b/g^3/(b*x+a)^2-B*d^2*ln(d
*x+c)/b/(-a*d+b*c)^2/g^3
```

**Rubi [A]** time = 0.09, antiderivative size = 138, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.094, Rules used = {2525, 12, 44}

$$-\frac{B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+A}{2bg^3(a+bx)^2}+\frac{Bd^2\log(a+bx)}{bg^3(bc-ad)^2}-\frac{Bd^2\log(c+dx)}{bg^3(bc-ad)^2}+\frac{Bd}{bg^3(a+bx)(bc-ad)}-\frac{B}{2bg^3(a+bx)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])/(a*g + b*g*x)^3,x]
```

```
[Out] -B/(2*b*g^3*(a + b*x)^2) + (B*d)/(b*(b*c - a*d)*g^3*(a + b*x)) + (B*d^2*Log
[a + b*x])/(b*(b*c - a*d)^2*g^3) - (A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])
/(2*b*g^3*(a + b*x)^2) - (B*d^2*Log[c + d*x])/(b*(b*c - a*d)^2*g^3)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 2525

```
Int[((a_.) + Log[(c_.)*(RFx_)^(p_.)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_))^(m_.
), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(a + b*Log[c*RFx^p])^n)/(e*(m + 1))
```

```
, x] - Dist[(b*n*p)/(e*(m + 1)), Int[SimplifyIntegrand[((d + e*x)^(m + 1)*(
a + b*Log[c*RFx^p])^(n - 1)*D[RFx, x])/RFx, x], x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && RationalFunctionQ[RFx, x] && IGtQ[n, 0] && (EqQ[n, 1] ||
IntegerQ[m]) && NeQ[m, -1]
```

Rubi steps

$$\int \frac{A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}{(ag+bgx)^3}\,dx = -\frac{A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}{2bg^3(a+bx)^2}+\frac{B\int\frac{2(bc-ad)}{g^2(a+bx)^3(c+dx)}\,dx}{2bg}$$
$$= -\frac{A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}{2bg^3(a+bx)^2}+\frac{(B(bc-ad))\int\frac{1}{(a+bx)^3(c+dx)}\,dx}{bg^3}$$
$$= -\frac{A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}{2bg^3(a+bx)^2}+\frac{(B(bc-ad))\int\left(\frac{b}{(bc-ad)(a+bx)^3}-\frac{bd}{(bc-ad)^2(a+bx)^2}+\frac{bd^2}{(bc-ad)^3(a+bx)}\right.}{bg^3}$$
$$= -\frac{B}{2bg^3(a+bx)^2}+\frac{Bd}{b(bc-ad)g^3(a+bx)}+\frac{Bd^2\log(a+bx)}{b(bc-ad)^2g^3}-\frac{A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}{2bg^3(a+bx)^2}-$$

**Mathematica [A]** time = 0.14, size = 109, normalized size = 0.79

$$-\frac{\frac{B\left(2d^2(a+bx)^2\log(c+dx)+(bc-ad)(b(c-2dx)-3ad)-2d^2(a+bx)^2\log(a+bx)\right)}{(bc-ad)^2}+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+A}{2bg^3(a+bx)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])/(a*g + b*g*x)^3,x]
```

```
[Out] -1/2*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2] + (B*((b*c - a*d)*(-3*a*d + b*
(c - 2*d*x)) - 2*d^2*(a + b*x)^2*Log[a + b*x] + 2*d^2*(a + b*x)^2*Log[c + d
*x]))/(b*c - a*d)^2)/(b*g^3*(a + b*x)^2)
```

**fricas [A]** time = 0.61, size = 238, normalized size = 1.72

$$-\frac{(A+B)b^2c^2-2(A+2B)abcd+(A+3B)a^2d^2-2\left(Bb^2cd-Babd^2\right)x-\left(Bb^2d^2x^2+2Babd^2x-Bb^2c^2+2Bab\right.}{2\left(\left(b^5c^2-2ab^4cd+a^2b^3d^2\right)g^3x^2+2\left(ab^4c^2-2a^2b^3cd+a^3b^2d^2\right)g^3x+\left(a^2b^3c^2-2a^3b^2cd+a^4\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*(b*x+a)^2/(d*x+c)^2))/(b*g*x+a*g)^3,x, algorithm="fric
as")
```

```
[Out] -1/2*((A + B)*b^2*c^2 - 2*(A + 2*B)*a*b*c*d + (A + 3*B)*a^2*d^2 - 2*(B*b^2*
c*d - B*a*b*d^2)*x - (B*b^2*d^2*x^2 + 2*B*a*b*d^2*x - B*b^2*c^2 + 2*B*a*b*c
*d)*log((b^2*e*x^2 + 2*a*b*e*x + a^2*e)/(d^2*x^2 + 2*c*d*x + c^2)))/((b^5*c
^2 - 2*a*b^4*c*d + a^2*b^3*d^2)*g^3*x^2 + 2*(a*b^4*c^2 - 2*a^2*b^3*c*d + a^
3*b^2*d^2)*g^3*x + (a^2*b^3*c^2 - 2*a^3*b^2*c*d + a^4*b*d^2)*g^3)
```

**giac [A]** time = 0.30, size = 264, normalized size = 1.91

$$\frac{Bd^2\log(bx+a)}{b^3c^2g^3-2\,ab^2cdg^3+a^2bd^2g^3}-\frac{Bd^2\log(dx+c)}{b^3c^2g^3-2\,ab^2cdg^3+a^2bd^2g^3}-\frac{B\log\left(\frac{b^2x^2+2\,abx+a^2}{d^2x^2+2\,cdx+c^2}\right)}{2\left(b^3g^3x^2+2\,ab^2g^3x+a^2bg^3\right)}+\frac{2\,}{2\left(b^4cg^3x^2-ab^3d\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*(b*x+a)^2/(d*x+c)^2))/(b*g*x+a*g)^3,x, algorithm="giac
")
```

```
[Out] B*d^2*log(b*x + a)/(b^3*c^2*g^3 - 2*a*b^2*c*d*g^3 + a^2*b*d^2*g^3) - B*d^2*
log(d*x + c)/(b^3*c^2*g^3 - 2*a*b^2*c*d*g^3 + a^2*b*d^2*g^3) - 1/2*B*log((b
^2*x^2 + 2*a*b*x + a^2)/(d^2*x^2 + 2*c*d*x + c^2))/(b^3*g^3*x^2 + 2*a*b^2*g
^3*x + a^2*b*g^3) + 1/2*(2*B*b*d*x - A*b*c - 2*B*b*c + A*a*d + 4*B*a*d)/(b^
4*c*g^3*x^2 - a*b^3*d*g^3*x^2 + 2*a*b^3*c*g^3*x - 2*a^2*b^2*d*g^3*x + a^2*b
^2*c*g^3 - a^3*b*d*g^3)
```

**maple [B]** time = 0.12, size = 355, normalized size = 2.57

$$\frac{Bb\,d^2\ln\left(\frac{\left(\frac{ad}{dx+c}-\frac{bc}{dx+c}+b\right)^2e}{d^2}\right)}{2\left(\frac{ad}{dx+c}-\frac{bc}{dx+c}+b\right)^2\left(a^2d^2-2abcd+b^2c^2\right)g^3}-\frac{Ab\,d^2}{2\left(ad-bc\right)^2\left(\frac{ad}{dx+c}-\frac{bc}{dx+c}+b\right)^2g^3}+\frac{B\,d^2\ln\left(\frac{\left(\frac{ad}{dx+c}-\frac{bc}{dx+c}+b\right)^2e}{d^2}\right)}{\left(\frac{ad}{dx+c}-\frac{bc}{dx+c}+b\right)^2\left(ad-bc\right)\left(dx\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*ln(e*(b*x+a)^2/(d*x+c)^2))/(b*g*x+a*g)^3,x)
```

```
[Out] d^2/g^3*A/(a*d-b*c)^2/(1/(d*x+c)*a*d-1/(d*x+c)*b*c+b)-1/2*d^2/g^3*A*b/(a*d-
b*c)^2/(1/(d*x+c)*a*d-1/(d*x+c)*b*c+b)^2-d^2/g^3/(1/(d*x+c)*a*d-1/(d*x+c)*b
*c+b)^2*B/(a*d-b*c)/(d*x+c)-3/2*d^2/g^3/(1/(d*x+c)*a*d-1/(d*x+c)*b*c+b)^2*B
/b/(d*x+c)^2+1/2*d^2/g^3/(1/(d*x+c)*a*d-1/(d*x+c)*b*c+b)^2*b*B/(a^2*d^2-2*a
*b*c*d+b^2*c^2)*ln((1/(d*x+c)*a*d-1/(d*x+c)*b*c+b)^2/d^2*e)+d^2/g^3/(1/(d*x
+c)*a*d-1/(d*x+c)*b*c+b)^2*B/(a*d-b*c)/(d*x+c)*ln((1/(d*x+c)*a*d-1/(d*x+c)*
b*c+b)^2/d^2*e)
```

**maxima [B]** time = 1.24, size = 307, normalized size = 2.22

$$\frac{1}{2}\,B\left(\frac{2\,bdx - bc + 3\,ad}{\left(b^4c - ab^3d\right)g^3x^2 + 2\left(ab^3c - a^2b^2d\right)g^3x + \left(a^2b^2c - a^3bd\right)g^3} - \frac{\log\left(\frac{b^2ex^2}{d^2x^2+2\,cdx+c^2} + \frac{2\,abex}{d^2x^2+2\,cdx+c^2} + \frac{a^2e}{d^2x^2+2\,cdx+\cdots}\right.}{b^3g^3x^2 + 2\,ab^2g^3x + a^2bg^3}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*(b*x+a)^2/(d*x+c)^2))/(b*g*x+a*g)^3,x, algorithm="maxi
ma")
```

```
[Out] 1/2*B*((2*b*d*x - b*c + 3*a*d)/((b^4*c - a*b^3*d)*g^3*x^2 + 2*(a*b^3*c - a^
2*b^2*d)*g^3*x + (a^2*b^2*c - a^3*b*d)*g^3) - log(b^2*e*x^2/(d^2*x^2 + 2*c*
d*x + c^2) + 2*a*b*e*x/(d^2*x^2 + 2*c*d*x + c^2) + a^2*e/(d^2*x^2 + 2*c*d*x
 + c^2))/(b^3*g^3*x^2 + 2*a*b^2*g^3*x + a^2*b*g^3) + 2*d^2*log(b*x + a)/((b
^3*c^2 - 2*a*b^2*c*d + a^2*b*d^2)*g^3) - 2*d^2*log(d*x + c)/((b^3*c^2 - 2*a
*b^2*c*d + a^2*b*d^2)*g^3)) - 1/2*A/(b^3*g^3*x^2 + 2*a*b^2*g^3*x + a^2*b*g^
3)
```

**mupad [B]** time = 5.14, size = 206, normalized size = 1.49

$$-\frac{\frac{A\,a\,d-A\,b\,c+3\,B\,a\,d-B\,b\,c}{2\,(a\,d-b\,c)} + \frac{B\,b\,d\,x}{a\,d-b\,c}}{a^2\,b\,g^3 + 2\,a\,b^2\,g^3\,x + b^3\,g^3\,x^2} - \frac{B\,\ln\left(\frac{e\,(a+b\,x)^2}{(c+d\,x)^2}\right)}{2\,b^2\,g^3\,\left(2\,a\,x + b\,x^2 + \frac{a^2}{b}\right)} - \frac{2\,B\,d^2\,\mathrm{atanh}\left(\frac{b^3\,c^2\,g^3-a^2\,b\,d^2\,g^3}{b\,g^3\,(a\,d-b\,c)^2} - \frac{2\,b\,d\,x}{a\,d-b\,c}\right)}{b\,g^3\,(a\,d-b\,c)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*log((e*(a + b*x)^2)/(c + d*x)^2))/(a*g + b*g*x)^3,x)
```

```
[Out] - ((A*a*d - A*b*c + 3*B*a*d - B*b*c)/(2*(a*d - b*c)) + (B*b*d*x)/(a*d - b*c
))/(a^2*b*g^3 + b^3*g^3*x^2 + 2*a*b^2*g^3*x) - (B*log((e*(a + b*x)^2)/(c +
d*x)^2))/(2*b^2*g^3*(2*a*x + b*x^2 + a^2/b)) - (2*B*d^2*atanh((b^3*c^2*g^3
- a^2*b*d^2*g^3)/(b*g^3*(a*d - b*c)^2) - (2*b*d*x)/(a*d - b*c)))/(b*g^3*(a*
d - b*c)^2)
```

**sympy [B]** time = 2.74, size = 418, normalized size = 3.03

$$-\frac{B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}{2a^2bg^3 + 4ab^2g^3x + 2b^3g^3x^2} - \frac{Bd^2\log\left(x + \frac{-\frac{Ba^3d^5}{(ad-bc)^2} + \frac{3Ba^2bcd^4}{(ad-bc)^2} - \frac{3Bab^2c^2d^3}{(ad-bc)^2} + Bad^3 + \frac{Bb^3c^3d^2}{(ad-bc)^2} + Bbcd^2}{2Bbd^3}\right)}{bg^3\,(ad-bc)^2} + \frac{Bd^2\log\left(x + \frac{\frac{Ba^3d^5}{(ad-bc)^2} - \cdots}{\cdots}\right.}{b\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*ln(e*(b*x+a)**2/(d*x+c)**2))/(b*g*x+a*g)**3,x)
```

```
[Out] -B*log(e*(a + b*x)**2/(c + d*x)**2)/(2*a**2*b*g**3 + 4*a*b**2*g**3*x + 2*b*
*3*g**3*x**2) - B*d**2*log(x + (-B*a**3*d**5/(a*d - b*c)**2 + 3*B*a**2*b*c*
d**4/(a*d - b*c)**2 - 3*B*a*b**2*c**2*d**3/(a*d - b*c)**2 + B*a*d**3 + B*b*
*3*c**3*d**2/(a*d - b*c)**2 + B*b*c*d**2)/(2*B*b*d**3))/(b*g**3*(a*d - b*c)
**2) + B*d**2*log(x + (B*a**3*d**5/(a*d - b*c)**2 - 3*B*a**2*b*c*d**4/(a*d
- b*c)**2 + 3*B*a*b**2*c**2*d**3/(a*d - b*c)**2 + B*a*d**3 - B*b**3*c**3*d*
*2/(a*d - b*c)**2 + B*b*c*d**2)/(2*B*b*d**3))/(b*g**3*(a*d - b*c)**2) + (-A
*a*d + A*b*c - 3*B*a*d + B*b*c - 2*B*b*d*x)/(2*a**3*b*d*g**3 - 2*a**2*b**2*
c*g**3 + x**2*(2*a*b**3*d*g**3 - 2*b**4*c*g**3) + x*(4*a**2*b**2*d*g**3 - 4
*a*b**3*c*g**3))
```

## 3.126 $\int \frac{A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}{(ag+bgx)^4}\,dx$

**Optimal**. Leaf size=177

$$-\frac{B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+A}{3bg^4(a+bx)^3}-\frac{2Bd^3\log(a+bx)}{3bg^4(bc-ad)^3}+\frac{2Bd^3\log(c+dx)}{3bg^4(bc-ad)^3}-\frac{2Bd^2}{3bg^4(a+bx)(bc-ad)^2}+\frac{Bd}{3bg^4(a+bx)^2(bc-ad)}-$$

```
[Out] -2/9*B/b/g^4/(b*x+a)^3+1/3*B*d/b/(-a*d+b*c)/g^4/(b*x+a)^2-2/3*B*d^2/b/(-a*d
+b*c)^2/g^4/(b*x+a)-2/3*B*d^3*ln(b*x+a)/b/(-a*d+b*c)^3/g^4+1/3*(-A-B*ln(e*(
b*x+a)^2/(d*x+c)^2))/b/g^4/(b*x+a)^3+2/3*B*d^3*ln(d*x+c)/b/(-a*d+b*c)^3/g^4
```

**Rubi [A]** time = 0.11, antiderivative size = 177, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.094, Rules used = {2525, 12, 44}

$$-\frac{B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+A}{3bg^4(a+bx)^3}-\frac{2Bd^2}{3bg^4(a+bx)(bc-ad)^2}-\frac{2Bd^3\log(a+bx)}{3bg^4(bc-ad)^3}+\frac{2Bd^3\log(c+dx)}{3bg^4(bc-ad)^3}+\frac{Bd}{3bg^4(a+bx)^2(bc-ad)}-$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])/(a*g + b*g*x)^4,x]
```

```
[Out] (-2*B)/(9*b*g^4*(a + b*x)^3) + (B*d)/(3*b*(b*c - a*d)*g^4*(a + b*x)^2) - (2
*B*d^2)/(3*b*(b*c - a*d)^2*g^4*(a + b*x)) - (2*B*d^3*Log[a + b*x])/(3*b*(b*
c - a*d)^3*g^4) - (A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])/(3*b*g^4*(a + b*
x)^3) + (2*B*d^3*Log[c + d*x])/(3*b*(b*c - a*d)^3*g^4)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 2525

```
Int[((a_.) + Log[(c_.)*(RFx_)^(p_.)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_))^(m_.
), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(a + b*Log[c*RFx^p])^n)/(e*(m + 1))
```

```
, x] - Dist[(b*n*p)/(e*(m + 1)), Int[SimplifyIntegrand[((d + e*x)^(m + 1)*(
a + b*Log[c*RFx^p])^(n - 1)*D[RFx, x])/RFx, x], x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && RationalFunctionQ[RFx, x] && IGtQ[n, 0] && (EqQ[n, 1] ||
IntegerQ[m]) && NeQ[m, -1]
```

Rubi steps

$$\int \frac{A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}{(ag+bgx)^4}\,dx = -\frac{A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}{3bg^4(a+bx)^3}+\frac{B\int \frac{2(bc-ad)}{g^3(a+bx)^4(c+dx)}\,dx}{3bg}$$

$$= -\frac{A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}{3bg^4(a+bx)^3}+\frac{(2B(bc-ad))\int \frac{1}{(a+bx)^4(c+dx)}\,dx}{3bg^4}$$

$$= -\frac{A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}{3bg^4(a+bx)^3}+\frac{(2B(bc-ad))\int \left(\frac{b}{(bc-ad)(a+bx)^4}-\frac{bd}{(bc-ad)^2(a+bx)^3}+\frac{bd^2}{(bc-ad)^3(a+bx)}\right.}{3bg^4}$$

$$= -\frac{2B}{9bg^4(a+bx)^3}+\frac{Bd}{3b(bc-ad)g^4(a+bx)^2}-\frac{2Bd^2}{3b(bc-ad)^2g^4(a+bx)}-\frac{2Bd^3\log(a+}{3b(bc-ad)^3}$$

**Mathematica [A]** time = 0.17, size = 140, normalized size = 0.79

$$-\frac{3\left(B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+A\right)+\frac{B\left(-6d^3(a+bx)^3\log(c+dx)+6d^2(a+bx)^2(bc-ad)-3d(a+bx)(bc-ad)^2+2(bc-ad)^3+6d^3(a+bx)^3\log(a+bx)\right)}{(bc-ad)^3}}{9bg^4(a+bx)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])/(a*g + b*g*x)^4,x]
```

```
[Out] -1/9*(3*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2]) + (B*(2*(b*c - a*d)^3 - 3*
d*(b*c - a*d)^2*(a + b*x) + 6*d^2*(b*c - a*d)*(a + b*x)^2 + 6*d^3*(a + b*x)
^3*Log[a + b*x] - 6*d^3*(a + b*x)^3*Log[c + d*x]))/(b*c - a*d)^3)/(b*g^4*(a
 + b*x)^3)
```

**fricas [B]** time = 0.59, size = 430, normalized size = 2.43

$$-\frac{(3A+2B)b^3c^3-9(A+B)ab^2c^2d+9(A+2B)a^2bcd^2-(3A+11B)a^3d^3+6\left(Bb^3cd^2-Bab^2d^3\right)x^2-3\left(Bb^3c^2\right.}{9\left(\left(b^7c^3-3ab^6c^2d+3a^2b^5cd^2-a^3b^4d^3\right)g^4x^3+3\left(ab^6c^3-3a^2b^5c^2d+3a^3b^4cd^2-a^4b^3d^3\right)\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*(b*x+a)^2/(d*x+c)^2))/(b*g*x+a*g)^4,x, algorithm="fric
as")
```

```
[Out] -1/9*((3*A + 2*B)*b^3*c^3 - 9*(A + B)*a*b^2*c^2*d + 9*(A + 2*B)*a^2*b*c*d^2
 - (3*A + 11*B)*a^3*d^3 + 6*(B*b^3*c*d^2 - B*a*b^2*d^3)*x^2 - 3*(B*b^3*c^2*
d - 6*B*a*b^2*c*d^2 + 5*B*a^2*b*d^3)*x + 3*(B*b^3*d^3*x^3 + 3*B*a*b^2*d^3*x
^2 + 3*B*a^2*b*d^3*x + B*b^3*c^3 - 3*B*a*b^2*c^2*d + 3*B*a^2*b*c*d^2)*log((
b^2*e*x^2 + 2*a*b*e*x + a^2*e)/(d^2*x^2 + 2*c*d*x + c^2)))/((b^7*c^3 - 3*a*
b^6*c^2*d + 3*a^2*b^5*c*d^2 - a^3*b^4*d^3)*g^4*x^3 + 3*(a*b^6*c^3 - 3*a^2*b
^5*c^2*d + 3*a^3*b^4*c*d^2 - a^4*b^3*d^3)*g^4*x^2 + 3*(a^2*b^5*c^3 - 3*a^3*
b^4*c^2*d + 3*a^4*b^3*c*d^2 - a^5*b^2*d^3)*g^4*x + (a^3*b^4*c^3 - 3*a^4*b^3
*c^2*d + 3*a^5*b^2*c*d^2 - a^6*b*d^3)*g^4)
```

**giac [B]** time = 0.33, size = 473, normalized size = 2.67

$$-\frac{2\,Bd^3\log\left(bx+a\right)}{3\left(b^4c^3g^4-3\,ab^3c^2dg^4+3\,a^2b^2cd^2g^4-a^3bd^3g^4\right)}+\frac{2\,Bd^3\log\left(dx+c\right)}{3\left(b^4c^3g^4-3\,ab^3c^2dg^4+3\,a^2b^2cd^2g^4-a^3bd^3g^4\right)}-\frac{\cdots}{3\left(b^4g^4x^3\cdots\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*(b*x+a)^2/(d*x+c)^2))/(b*g*x+a*g)^4,x, algorithm="giac
")
```

```
[Out] -2/3*B*d^3*log(b*x + a)/(b^4*c^3*g^4 - 3*a*b^3*c^2*d*g^4 + 3*a^2*b^2*c*d^2*
g^4 - a^3*b*d^3*g^4) + 2/3*B*d^3*log(d*x + c)/(b^4*c^3*g^4 - 3*a*b^3*c^2*d*
g^4 + 3*a^2*b^2*c*d^2*g^4 - a^3*b*d^3*g^4) - 1/3*B*log((b^2*x^2 + 2*a*b*x +
 a^2)/(d^2*x^2 + 2*c*d*x + c^2))/(b^4*g^4*x^3 + 3*a*b^3*g^4*x^2 + 3*a^2*b^2
*g^4*x + a^3*b*g^4) - 1/9*(6*B*b^2*d^2*x^2 - 3*B*b^2*c*d*x + 15*B*a*b*d^2*x
 + 3*A*b^2*c^2 + 5*B*b^2*c^2 - 6*A*a*b*c*d - 13*B*a*b*c*d + 3*A*a^2*d^2 + 1
4*B*a^2*d^2)/(b^6*c^2*g^4*x^3 - 2*a*b^5*c*d*g^4*x^3 + a^2*b^4*d^2*g^4*x^3 +
 3*a*b^5*c^2*g^4*x^2 - 6*a^2*b^4*c*d*g^4*x^2 + 3*a^3*b^3*d^2*g^4*x^2 + 3*a^
2*b^4*c^2*g^4*x - 6*a^3*b^3*c*d*g^4*x + 3*a^4*b^2*d^2*g^4*x + a^3*b^3*c^2*g
^4 - 2*a^4*b^2*c*d*g^4 + a^5*b*d^2*g^4)
```

**maple [B]** time = 0.16, size = 579, normalized size = 3.27

$$\frac{B\,b^2d^3\ln\left(\frac{\left(\frac{ad}{dx+c}-\frac{bc}{dx+c}+b\right)^2e}{d^2}\right)}{3\left(\frac{ad}{dx+c}-\frac{bc}{dx+c}+b\right)^3\left(a^3d^3-3a^2c\,d^2b+3a\,c^2d\,b^2-b^3c^3\right)g^4}+\frac{A\,b^2d^3}{3\left(ad-bc\right)^3\left(\frac{ad}{dx+c}-\frac{bc}{dx+c}+b\right)^3g^4}+\frac{B\cdots}{\left(\frac{ad}{dx+c}-\frac{bc}{dx+c}+\cdots\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*ln(e*(b*x+a)^2/(d*x+c)^2))/(b*g*x+a*g)^4,x)
```

```
[Out] 1/3*d^3/g^4*A*b^2/(a*d-b*c)^3/(1/(d*x+c)*a*d-1/(d*x+c)*b*c+b)^3+d^3/g^4*A/(
a*d-b*c)^3/(1/(d*x+c)*a*d-1/(d*x+c)*b*c+b)-d^3/g^4*A*b/(a*d-b*c)^3/(1/(d*x+
c)*a*d-1/(d*x+c)*b*c+b)^2-11/9*d^3/g^4/(1/(d*x+c)*a*d-1/(d*x+c)*b*c+b)^3*B/
b/(d*x+c)^3+1/3*d^3/g^4/(1/(d*x+c)*a*d-1/(d*x+c)*b*c+b)^3*b^2*B/(a^3*d^3-3*
a^2*b*c*d^2+3*a*b^2*c^2*d-b^3*c^3)*ln((1/(d*x+c)*a*d-1/(d*x+c)*b*c+b)^2/d^2
*e)-2/3*d^3/g^4/(1/(d*x+c)*a*d-1/(d*x+c)*b*c+b)^3*B*b/(a^2*d^2-2*a*b*c*d+b^
2*c^2)/(d*x+c)-5/3*d^3/g^4/(1/(d*x+c)*a*d-1/(d*x+c)*b*c+b)^3*B/(a*d-b*c)/(d
*x+c)^2+d^3/g^4/(1/(d*x+c)*a*d-1/(d*x+c)*b*c+b)^3*B/(a*d-b*c)/(d*x+c)^2*ln(
(1/(d*x+c)*a*d-1/(d*x+c)*b*c+b)^2/d^2*e)+d^3/g^4/(1/(d*x+c)*a*d-1/(d*x+c)*b
*c+b)^3*B*b/(a^2*d^2-2*a*b*c*d+b^2*c^2)/(d*x+c)*ln((1/(d*x+c)*a*d-1/(d*x+c)
*b*c+b)^2/d^2*e)
```

**maxima [B]** time = 1.39, size = 480, normalized size = 2.71

$$-\frac{1}{9}B\left(\frac{6\,b^2d^2x^2+2\,b^2c^2-7\,abcd+11\,a^2d^2-3\left(b^2cd-5\,abd^2\right)x}{\left(b^6c^2-2\,ab^5cd+a^2b^4d^2\right)g^4x^3+3\left(ab^5c^2-2\,a^2b^4cd+a^3b^3d^2\right)g^4x^2+3\left(a^2b^4c^2-2\,a^3b^3cd+a^4b^2d^2\right)g^4x+\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*(b*x+a)^2/(d*x+c)^2))/(b*g*x+a*g)^4,x, algorithm="maxi
ma")
```

```
[Out] -1/9*B*((6*b^2*d^2*x^2 + 2*b^2*c^2 - 7*a*b*c*d + 11*a^2*d^2 - 3*(b^2*c*d -
5*a*b*d^2)*x)/((b^6*c^2 - 2*a*b^5*c*d + a^2*b^4*d^2)*g^4*x^3 + 3*(a*b^5*c^2
 - 2*a^2*b^4*c*d + a^3*b^3*d^2)*g^4*x^2 + 3*(a^2*b^4*c^2 - 2*a^3*b^3*c*d +
a^4*b^2*d^2)*g^4*x + (a^3*b^3*c^2 - 2*a^4*b^2*c*d + a^5*b*d^2)*g^4) + 3*log
(b^2*e*x^2/(d^2*x^2 + 2*c*d*x + c^2) + 2*a*b*e*x/(d^2*x^2 + 2*c*d*x + c^2)
+ a^2*e/(d^2*x^2 + 2*c*d*x + c^2))/(b^4*g^4*x^3 + 3*a*b^3*g^4*x^2 + 3*a^2*b
^2*g^4*x + a^3*b*g^4) + 6*d^3*log(b*x + a)/((b^4*c^3 - 3*a*b^3*c^2*d + 3*a^
2*b^2*c*d^2 - a^3*b*d^3)*g^4) - 6*d^3*log(d*x + c)/((b^4*c^3 - 3*a*b^3*c^2*
d + 3*a^2*b^2*c*d^2 - a^3*b*d^3)*g^4)) - 1/3*A/(b^4*g^4*x^3 + 3*a*b^3*g^4*x
^2 + 3*a^2*b^2*g^4*x + a^3*b*g^4)
```

**mupad [B]** time = 5.80, size = 341, normalized size = 1.93

$$\frac{2\,A\,a\,c\,d}{3\,g^4\,(a\,d-b\,c)^2\,(a+b\,x)^3}-\frac{A\,b\,c^2}{3\,g^4\,(a\,d-b\,c)^2\,(a+b\,x)^3}-\frac{2\,B\,b\,c^2}{9\,g^4\,(a\,d-b\,c)^2\,(a+b\,x)^3}-\frac{A\,a^2\,d^2}{3\,b\,g^4\,(a\,d-b\,c)^2\,(a+b\,x)^3}-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*log((e*(a + b*x)^2)/(c + d*x)^2))/(a*g + b*g*x)^4,x)
```

```
[Out] (2*A*a*c*d)/(3*g^4*(a*d - b*c)^2*(a + b*x)^3) - (B*d^3*atan((a*d*1i + b*c*1
i + b*d*x*2i)/(a*d - b*c))*4i)/(3*b*g^4*(a*d - b*c)^3) - (A*b*c^2)/(3*g^4*(
```

```
a*d - b*c)^2*(a + b*x)^3) - (2*B*b*c^2)/(9*g^4*(a*d - b*c)^2*(a + b*x)^3) -
 (A*a^2*d^2)/(3*b*g^4*(a*d - b*c)^2*(a + b*x)^3) - (11*B*a^2*d^2)/(9*b*g^4*
(a*d - b*c)^2*(a + b*x)^3) - (5*B*a*d^2*x)/(3*g^4*(a*d - b*c)^2*(a + b*x)^3
) - (2*B*b*d^2*x^2)/(3*g^4*(a*d - b*c)^2*(a + b*x)^3) - (B*log((e*(a + b*x)
^2)/(c + d*x)^2))/(3*b*g^4*(a + b*x)^3) + (7*B*a*c*d)/(9*g^4*(a*d - b*c)^2*
(a + b*x)^3) + (B*b*c*d*x)/(3*g^4*(a*d - b*c)^2*(a + b*x)^3)
```

**sympy [B]** time = 4.24, size = 677, normalized size = 3.82

$$-\frac{B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}{3a^3bg^4+9a^2b^2g^4x+9ab^3g^4x^2+3b^4g^4x^3}-\frac{2Bd^3\log\left(x+\frac{-\frac{2Ba^4d^7}{(ad-bc)^3}+\frac{8Ba^3bcd^6}{(ad-bc)^3}-\frac{12Ba^2b^2c^2d^5}{(ad-bc)^3}+\frac{8Bab^3c^3d^4}{(ad-bc)^3}+2Bad^4-\frac{2Bb^4c^4d^3}{(ad-bc)^3}+2Bbc}{4Bbd^4}\right.}{3bg^4(ad-bc)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*ln(e*(b*x+a)**2/(d*x+c)**2))/(b*g*x+a*g)**4,x)
```

```
[Out] -B*log(e*(a + b*x)**2/(c + d*x)**2)/(3*a**3*b*g**4 + 9*a**2*b**2*g**4*x + 9
*a*b**3*g**4*x**2 + 3*b**4*g**4*x**3) - 2*B*d**3*log(x + (-2*B*a**4*d**7/(a
*d - b*c)**3 + 8*B*a**3*b*c*d**6/(a*d - b*c)**3 - 12*B*a**2*b**2*c**2*d**5/
(a*d - b*c)**3 + 8*B*a*b**3*c**3*d**4/(a*d - b*c)**3 + 2*B*a*d**4 - 2*B*b**
4*c**4*d**3/(a*d - b*c)**3 + 2*B*b*c*d**3)/(4*B*b*d**4))/(3*b*g**4*(a*d - b
*c)**3) + 2*B*d**3*log(x + (2*B*a**4*d**7/(a*d - b*c)**3 - 8*B*a**3*b*c*d**
6/(a*d - b*c)**3 + 12*B*a**2*b**2*c**2*d**5/(a*d - b*c)**3 - 8*B*a*b**3*c**
3*d**4/(a*d - b*c)**3 + 2*B*a*d**4 + 2*B*b**4*c**4*d**3/(a*d - b*c)**3 + 2*
B*b*c*d**3)/(4*B*b*d**4))/(3*b*g**4*(a*d - b*c)**3) + (-3*A*a**2*d**2 + 6*A
*a*b*c*d - 3*A*b**2*c**2 - 11*B*a**2*d**2 + 7*B*a*b*c*d - 2*B*b**2*c**2 - 6
*B*b**2*d**2*x**2 + x*(-15*B*a*b*d**2 + 3*B*b**2*c*d))/(9*a**5*b*d**2*g**4
- 18*a**4*b**2*c*d*g**4 + 9*a**3*b**3*c**2*g**4 + x**3*(9*a**2*b**4*d**2*g*
*4 - 18*a*b**5*c*d*g**4 + 9*b**6*c**2*g**4) + x**2*(27*a**3*b**3*d**2*g**4
- 54*a**2*b**4*c*d*g**4 + 27*a*b**5*c**2*g**4) + x*(27*a**4*b**2*d**2*g**4
- 54*a**3*b**3*c*d*g**4 + 27*a**2*b**4*c**2*g**4))
```

## 3.127 $\int \frac{A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}{(ag+bgx)^5} dx$

**Optimal**. Leaf size=208

$$-\frac{B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+A}{4bg^5(a+bx)^4}+\frac{Bd^4\log(a+bx)}{2bg^5(bc-ad)^4}-\frac{Bd^4\log(c+dx)}{2bg^5(bc-ad)^4}+\frac{Bd^3}{2bg^5(a+bx)(bc-ad)^3}-\frac{Bd^2}{4bg^5(a+bx)^2(bc-ad)^2}+\frac{}{6bg}$$

```
[Out] -1/8*B/b/g^5/(b*x+a)^4+1/6*B*d/b/(-a*d+b*c)/g^5/(b*x+a)^3-1/4*B*d^2/b/(-a*d
+b*c)^2/g^5/(b*x+a)^2+1/2*B*d^3/b/(-a*d+b*c)^3/g^5/(b*x+a)+1/2*B*d^4*ln(b*x
+a)/b/(-a*d+b*c)^4/g^5+1/4*(-A-B*ln(e*(b*x+a)^2/(d*x+c)^2))/b/g^5/(b*x+a)^4
-1/2*B*d^4*ln(d*x+c)/b/(-a*d+b*c)^4/g^5
```

**Rubi [A]** time = 0.14, antiderivative size = 208, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.094, Rules used = {2525, 12, 44}

$$-\frac{B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+A}{4bg^5(a+bx)^4}+\frac{Bd^3}{2bg^5(a+bx)(bc-ad)^3}-\frac{Bd^2}{4bg^5(a+bx)^2(bc-ad)^2}+\frac{Bd^4\log(a+bx)}{2bg^5(bc-ad)^4}-\frac{Bd^4\log(c+dx)}{2bg^5(bc-ad)^4}+\frac{}{6bg}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])/(a*g + b*g*x)^5,x]
```

```
[Out] -B/(8*b*g^5*(a + b*x)^4) + (B*d)/(6*b*(b*c - a*d)*g^5*(a + b*x)^3) - (B*d^2
)/(4*b*(b*c - a*d)^2*g^5*(a + b*x)^2) + (B*d^3)/(2*b*(b*c - a*d)^3*g^5*(a +
 b*x)) + (B*d^4*Log[a + b*x])/(2*b*(b*c - a*d)^4*g^5) - (A + B*Log[(e*(a +
b*x)^2)/(c + d*x)^2])/(4*b*g^5*(a + b*x)^4) - (B*d^4*Log[c + d*x])/(2*b*(b*
c - a*d)^4*g^5)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 2525

```
Int[((a_.) + Log[(c_.)*(RFx_)^(p_.)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_))^(m_.
), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(a + b*Log[c*RFx^p])^n)/(e*(m + 1))
, x] - Dist[(b*n*p)/(e*(m + 1)), Int[SimplifyIntegrand[((d + e*x)^(m + 1)*(
a + b*Log[c*RFx^p])^(n - 1)*D[RFx, x])/RFx, x], x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && RationalFunctionQ[RFx, x] && IGtQ[n, 0] && (EqQ[n, 1] ||
IntegerQ[m]) && NeQ[m, -1]
```

Rubi steps

$$\int \frac{A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}{(ag+bgx)^5}\,dx = -\frac{A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}{4bg^5(a+bx)^4} + \frac{B\int \frac{2(bc-ad)}{g^4(a+bx)^5(c+dx)}\,dx}{4bg}$$

$$= -\frac{A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}{4bg^5(a+bx)^4} + \frac{(B(bc-ad))\int \frac{1}{(a+bx)^5(c+dx)}\,dx}{2bg^5}$$

$$= -\frac{A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}{4bg^5(a+bx)^4} + \frac{(B(bc-ad))\int \left(\frac{b}{(bc-ad)(a+bx)^5} - \frac{bd}{(bc-ad)^2(a+bx)^4} + \frac{bd^2}{(bc-ad)^3(a+bx)}\right.}{2bg^5}$$

$$= -\frac{B}{8bg^5(a+bx)^4} + \frac{Bd}{6b(bc-ad)g^5(a+bx)^3} - \frac{Bd^2}{4b(bc-ad)^2g^5(a+bx)^2} + \frac{B}{2b(bc-ad)}$$

**Mathematica [A]** time = 0.22, size = 162, normalized size = 0.78

$$-\frac{6\left(B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+A\right)+\frac{B\left(12d^4(a+bx)^4\log(c+dx)+12d^3(a+bx)^3(ad-bc)+6d^2(a+bx)^2(bc-ad)^2+4d(a+bx)(ad-bc)^3+3(bc-ad)^4-12d^4(a+b\right.}{(bc-ad)^4}}{24bg^5(a+bx)^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])/(a*g + b*g*x)^5,x]
```

```
[Out] -1/24*(6*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2]) + (B*(3*(b*c - a*d)^4 + 4
*d*(-(b*c) + a*d)^3*(a + b*x) + 6*d^2*(b*c - a*d)^2*(a + b*x)^2 + 12*d^3*(-
(b*c) + a*d)*(a + b*x)^3 - 12*d^4*(a + b*x)^4*Log[a + b*x] + 12*d^4*(a + b*
x)^4*Log[c + d*x]))/(b*c - a*d)^4)/(b*g^5*(a + b*x)^4)
```

**fricas [B]** time = 0.95, size = 654, normalized size = 3.14

$$-\frac{3\,(2\,A+B)b^4c^4-8\,(3\,A+2\,B)ab^3c^3d+36\,(A+B)a^2b^2c^2d^2-24\,(A+2\,B)a^3bcd^3+(6\,A+25\,B)a^4d^4-12\,\left(Bb\right.}{24\left(\left(b^9c^4-4\,ab^8c^3d+6\,a^2b^7c^2d^2-4\,a^3b^6cd^3+a^4b^5d^4\right)g^5x^4+4\left(ab^8c^4-4\,a^2b^7c^3d+6\,a^3b\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*(b*x+a)^2/(d*x+c)^2))/(b*g*x+a*g)^5,x, algorithm="fric
as")
```

```
[Out] -1/24*(3*(2*A + B)*b^4*c^4 - 8*(3*A + 2*B)*a*b^3*c^3*d + 36*(A + B)*a^2*b^2
*c^2*d^2 - 24*(A + 2*B)*a^3*b*c*d^3 + (6*A + 25*B)*a^4*d^4 - 12*(B*b^4*c*d^
3 - B*a*b^3*d^4)*x^3 + 6*(B*b^4*c^2*d^2 - 8*B*a*b^3*c*d^3 + 7*B*a^2*b^2*d^4
)*x^2 - 4*(B*b^4*c^3*d - 6*B*a*b^3*c^2*d^2 + 18*B*a^2*b^2*c*d^3 - 13*B*a^3*
b*d^4)*x - 6*(B*b^4*d^4*x^4 + 4*B*a*b^3*d^4*x^3 + 6*B*a^2*b^2*d^4*x^2 + 4*B
*a^3*b*d^4*x - B*b^4*c^4 + 4*B*a*b^3*c^3*d - 6*B*a^2*b^2*c^2*d^2 + 4*B*a^3*
b*c*d^3)*log((b^2*e*x^2 + 2*a*b*e*x + a^2*e)/(d^2*x^2 + 2*c*d*x + c^2)))/((
b^9*c^4 - 4*a*b^8*c^3*d + 6*a^2*b^7*c^2*d^2 - 4*a^3*b^6*c*d^3 + a^4*b^5*d^4
)*g^5*x^4 + 4*(a*b^8*c^4 - 4*a^2*b^7*c^3*d + 6*a^3*b^6*c^2*d^2 - 4*a^4*b^5*
c*d^3 + a^5*b^4*d^4)*g^5*x^3 + 6*(a^2*b^7*c^4 - 4*a^3*b^6*c^3*d + 6*a^4*b^5
*c^2*d^2 - 4*a^5*b^4*c*d^3 + a^6*b^3*d^4)*g^5*x^2 + 4*(a^3*b^6*c^4 - 4*a^4*
b^5*c^3*d + 6*a^5*b^4*c^2*d^2 - 4*a^6*b^3*c*d^3 + a^7*b^2*d^4)*g^5*x + (a^4
*b^5*c^4 - 4*a^5*b^4*c^3*d + 6*a^6*b^3*c^2*d^2 - 4*a^7*b^2*c*d^3 + a^8*b*d^
4)*g^5)
```

**giac [B]** time = 0.77, size = 419, normalized size = 2.01

$$-\frac{Bd^4\log\left(-\frac{bcg}{bgx+ag}+\frac{adg}{bgx+ag}-d\right)}{2\left(b^5c^4g^5-4\,ab^4c^3dg^5+6\,a^2b^3c^2d^2g^5-4\,a^3b^2cd^3g^5+a^4bd^4g^5\right)}+\frac{Bd^3}{2\left(b^3c^3g^3-3\,ab^2c^2dg^3+3\,a^2bcd^2g^3-a^3d^3g^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*(b*x+a)^2/(d*x+c)^2))/(b*g*x+a*g)^5,x, algorithm="giac
")
```

```
[Out] -1/2*B*d^4*log(-b*c*g/(b*g*x + a*g) + a*d*g/(b*g*x + a*g) - d)/(b^5*c^4*g^5
 - 4*a*b^4*c^3*d*g^5 + 6*a^2*b^3*c^2*d^2*g^5 - 4*a^3*b^2*c*d^3*g^5 + a^4*b*
d^4*g^5) + 1/2*B*d^3/((b^3*c^3*g^3 - 3*a*b^2*c^2*d*g^3 + 3*a^2*b*c*d^2*g^3
- a^3*d^3*g^3)*(b*g*x + a*g)*b*g) - 1/4*B*d^2/((b^2*c^2*g - 2*a*b*c*d*g + a
^2*d^2*g)*(b*g*x + a*g)^2*b*g^2) - 1/4*B*log(b^2/(b^2*c^2*g^2/(b*g*x + a*g)
^2 - 2*a*b*c*d*g^2/(b*g*x + a*g)^2 + a^2*d^2*g^2/(b*g*x + a*g)^2 + 2*b*c*d*
g/(b*g*x + a*g) - 2*a*d^2*g/(b*g*x + a*g) + d^2))/((b*g*x + a*g)^4*b*g) + 1
/6*B*d/((b*g*x + a*g)^3*(b*c - a*d)*b*g^2) - 1/8*(2*A*b^3*g^3 + 3*B*b^3*g^3
)/((b*g*x + a*g)^4*b^4*g^4)
```

**maple [B]** time = 0.22, size = 833, normalized size = 4.00

$$\frac{B\,b^{3}d^{4}\ln\left(\frac{\left(\frac{ad}{dx+c}-\frac{bc}{dx+c}+b\right)^{2}e}{d^{2}}\right)}{4\left(\frac{ad}{dx+c}-\frac{bc}{dx+c}+b\right)^{4}\left(a^{4}d^{4}-4a^{3}c\,d^{3}b+6a^{2}c^{2}d^{2}b^{2}-4a\,c^{3}d\,b^{3}+c^{4}b^{4}\right)g^{5}}-\frac{A\,b^{3}d^{4}}{4\,(ad-bc)^{4}\left(\frac{ad}{dx+c}-\frac{bc}{dx+c}+b\right)^{4}g^{5}}+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*ln(e*(b*x+a)^2/(d*x+c)^2))/(b*g*x+a*g)^5,x)
```

```
[Out] d^4/g^5*A*b^2/(a*d-b*c)^4/(1/(d*x+c)*a*d-1/(d*x+c)*b*c+b)^3-1/4*d^4/g^5*A*b
^3/(a*d-b*c)^4/(1/(d*x+c)*a*d-1/(d*x+c)*b*c+b)^4+d^4/g^5*A/(a*d-b*c)^4/(1/(
d*x+c)*a*d-1/(d*x+c)*b*c+b)-3/2*d^4/g^5*A*b/(a*d-b*c)^4/(1/(d*x+c)*a*d-1/(d
*x+c)*b*c+b)^2-25/24*d^4/g^5/(1/(d*x+c)*a*d-1/(d*x+c)*b*c+b)^4*B/b/(d*x+c)^
4+1/4*d^4/g^5/(1/(d*x+c)*a*d-1/(d*x+c)*b*c+b)^4*b^3*B/(a^4*d^4-4*a^3*b*c*d^
3+6*a^2*b^2*c^2*d^2-4*a*b^3*c^3*d+b^4*c^4)*ln((1/(d*x+c)*a*d-1/(d*x+c)*b*c+
b)^2/d^2*e)-1/2*d^4/g^5/(1/(d*x+c)*a*d-1/(d*x+c)*b*c+b)^4*B*b^2/(a^3*d^3-3*
a^2*b*c*d^2+3*a*b^2*c^2*d-b^3*c^3)/(d*x+c)-7/4*d^4/g^5/(1/(d*x+c)*a*d-1/(d*
x+c)*b*c+b)^4*B*b/(a^2*d^2-2*a*b*c*d+b^2*c^2)/(d*x+c)^2-13/6*d^4/g^5/(1/(d*
x+c)*a*d-1/(d*x+c)*b*c+b)^4*B/(a*d-b*c)/(d*x+c)^3+d^4/g^5/(1/(d*x+c)*a*d-1/
(d*x+c)*b*c+b)^4*B/(a*d-b*c)/(d*x+c)^3*ln((1/(d*x+c)*a*d-1/(d*x+c)*b*c+b)^2
/d^2*e)+3/2*d^4/g^5/(1/(d*x+c)*a*d-1/(d*x+c)*b*c+b)^4*b*B/(a^2*d^2-2*a*b*c*
d+b^2*c^2)/(d*x+c)^2*ln((1/(d*x+c)*a*d-1/(d*x+c)*b*c+b)^2/d^2*e)+d^4/g^5/(1
/(d*x+c)*a*d-1/(d*x+c)*b*c+b)^4*B*b^2/(a^3*d^3-3*a^2*b*c*d^2+3*a*b^2*c^2*d-
b^3*c^3)/(d*x+c)*ln((1/(d*x+c)*a*d-1/(d*x+c)*b*c+b)^2/d^2*e)
```

**maxima [B]** time = 1.62, size = 699, normalized size = 3.36

$$\frac{1}{24}B\left(\frac{12\,b^{3}d^{3}x^{3}-3\,b^{3}c^{3}+13\,ab^{2}c^{2}d-23\,a^{2}bcd^{2}}{\left(b^{8}c^{3}-3\,ab^{7}c^{2}d+3\,a^{2}b^{6}cd^{2}-a^{3}b^{5}d^{3}\right)g^{5}x^{4}+4\left(ab^{7}c^{3}-3\,a^{2}b^{6}c^{2}d+3\,a^{3}b^{5}cd^{2}-a^{4}b^{4}d^{3}\right)g^{5}x^{3}+6\left(a^{2}b^{6}c^{3}\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*(b*x+a)^2/(d*x+c)^2))/(b*g*x+a*g)^5,x, algorithm="maxi
ma")
```

```
[Out] 1/24*B*((12*b^3*d^3*x^3 - 3*b^3*c^3 + 13*a*b^2*c^2*d - 23*a^2*b*c*d^2 + 25*
a^3*d^3 - 6*(b^3*c*d^2 - 7*a*b^2*d^3)*x^2 + 4*(b^3*c^2*d - 5*a*b^2*c*d^2 +
13*a^2*b*d^3)*x)/((b^8*c^3 - 3*a*b^7*c^2*d + 3*a^2*b^6*c*d^2 - a^3*b^5*d^3)
*g^5*x^4 + 4*(a*b^7*c^3 - 3*a^2*b^6*c^2*d + 3*a^3*b^5*c*d^2 - a^4*b^4*d^3)*
g^5*x^3 + 6*(a^2*b^6*c^3 - 3*a^3*b^5*c^2*d + 3*a^4*b^4*c*d^2 - a^5*b^3*d^3)
*g^5*x^2 + 4*(a^3*b^5*c^3 - 3*a^4*b^4*c^2*d + 3*a^5*b^3*c*d^2 - a^6*b^2*d^3
```

```
)*g^5*x + (a^4*b^4*c^3 - 3*a^5*b^3*c^2*d + 3*a^6*b^2*c*d^2 - a^7*b*d^3)*g^5
) - 6*log(b^2*e*x^2/(d^2*x^2 + 2*c*d*x + c^2) + 2*a*b*e*x/(d^2*x^2 + 2*c*d*
x + c^2) + a^2*e/(d^2*x^2 + 2*c*d*x + c^2))/(b^5*g^5*x^4 + 4*a*b^4*g^5*x^3
+ 6*a^2*b^3*g^5*x^2 + 4*a^3*b^2*g^5*x + a^4*b*g^5) + 12*d^4*log(b*x + a)/((
b^5*c^4 - 4*a*b^4*c^3*d + 6*a^2*b^3*c^2*d^2 - 4*a^3*b^2*c*d^3 + a^4*b*d^4)*
g^5) - 12*d^4*log(d*x + c)/((b^5*c^4 - 4*a*b^4*c^3*d + 6*a^2*b^3*c^2*d^2 -
4*a^3*b^2*c*d^3 + a^4*b*d^4)*g^5)) - 1/4*A/(b^5*g^5*x^4 + 4*a*b^4*g^5*x^3 +
 6*a^2*b^3*g^5*x^2 + 4*a^3*b^2*g^5*x + a^4*b*g^5)
```

**mupad [B]** time = 6.51, size = 579, normalized size = 2.78

$$-\frac{\frac{6\,A\,a^3\,d^3-6\,A\,b^3\,c^3+25\,B\,a^3\,d^3-3\,B\,b^3\,c^3+18\,A\,a\,b^2\,c^2\,d-18\,A\,a^2\,b\,c\,d^2+13\,B\,a\,b^2\,c^2\,d-23\,B\,a^2\,b\,c\,d^2}{12\,\left(a^3\,d^3-3\,a^2\,b\,c\,d^2+3\,a\,b^2\,c^2\,d-b^3\,c^3\right)}-\frac{d^2\,x^2\,\left(B\,b^3\,c-7\,B\,a\,b^2\,d\right)}{2\,\left(a^3\,d^3-3\,a^2\,b\,c\,d^2+3\,a\,b^2\,c^2\,d-b^3\,c^3\right)}+\frac{d\,x\,(1}{3\,(a^3}}{2\,a^4\,b\,g^5+8\,a^3\,b^2\,g^5\,x+12\,a^2\,b^3\,g^5\,x^2+8\,a\,b^4\,g^5\,x^3+2\,b^5\,g^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*log((e*(a + b*x)^2)/(c + d*x)^2))/(a*g + b*g*x)^5,x)
```

```
[Out] - ((6*A*a^3*d^3 - 6*A*b^3*c^3 + 25*B*a^3*d^3 - 3*B*b^3*c^3 + 18*A*a*b^2*c^2
*d - 18*A*a^2*b*c*d^2 + 13*B*a*b^2*c^2*d - 23*B*a^2*b*c*d^2)/(12*(a^3*d^3 -
 b^3*c^3 + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2)) - (d^2*x^2*(B*b^3*c - 7*B*a*b^2*
d))/(2*(a^3*d^3 - b^3*c^3 + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2)) + (d*x*(B*b^3*c
^2 + 13*B*a^2*b*d^2 - 5*B*a*b^2*c*d))/(3*(a^3*d^3 - b^3*c^3 + 3*a*b^2*c^2*d
 - 3*a^2*b*c*d^2)) + (B*b^3*d^3*x^3)/(a^3*d^3 - b^3*c^3 + 3*a*b^2*c^2*d - 3
*a^2*b*c*d^2))/(2*a^4*b*g^5 + 2*b^5*g^5*x^4 + 8*a^3*b^2*g^5*x + 8*a*b^4*g^5
*x^3 + 12*a^2*b^3*g^5*x^2) - (B*log((e*(a + b*x)^2)/(c + d*x)^2))/(4*b^2*g^
5*(4*a^3*x + a^4/b + b^3*x^4 + 6*a^2*b*x^2 + 4*a*b^2*x^3)) - (B*d^4*atanh((
2*b^5*c^4*g^5 - 2*a^4*b*d^4*g^5 - 4*a*b^4*c^3*d*g^5 + 4*a^3*b^2*c*d^3*g^5)/
(2*b*g^5*(a*d - b*c)^4) - (2*b*d*x*(a^3*d^3 - b^3*c^3 + 3*a*b^2*c^2*d - 3*a
^2*b*c*d^2))/(a*d - b*c)^4))/(b*g^5*(a*d - b*c)^4)
```

**sympy [B]** time = 5.81, size = 947, normalized size = 4.55

$$-\frac{B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}{4a^4bg^5+16a^3b^2g^5x+24a^2b^3g^5x^2+16ab^4g^5x^3+4b^5g^5x^4}-\frac{Bd^4\log\left(x+\frac{-\frac{Ba^5d^9}{(ad-bc)^4}+\frac{5Ba^4bcd^8}{(ad-bc)^4}-\frac{10Ba^3b^2c^2d^7}{(ad-bc)^4}+\frac{10Ba^2b^3c^3d^6}{(ad-bc)^4}-\frac{5B}{(a}}{2Bbd^5}\right.}{2bg^5(ad-bc)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*ln(e*(b*x+a)**2/(d*x+c)**2))/(b*g*x+a*g)**5,x)
```

```
[Out] -B*log(e*(a + b*x)**2/(c + d*x)**2)/(4*a**4*b*g**5 + 16*a**3*b**2*g**5*x +
24*a**2*b**3*g**5*x**2 + 16*a*b**4*g**5*x**3 + 4*b**5*g**5*x**4) - B*d**4*l
```

```
og(x + (-B*a**5*d**9/(a*d - b*c)**4 + 5*B*a**4*b*c*d**8/(a*d - b*c)**4 - 10
*B*a**3*b**2*c**2*d**7/(a*d - b*c)**4 + 10*B*a**2*b**3*c**3*d**6/(a*d - b*c
)**4 - 5*B*a*b**4*c**4*d**5/(a*d - b*c)**4 + B*a*d**5 + B*b**5*c**5*d**4/(a
*d - b*c)**4 + B*b*c*d**4)/(2*B*b*d**5))/(2*b*g**5*(a*d - b*c)**4) + B*d**4
*log(x + (B*a**5*d**9/(a*d - b*c)**4 - 5*B*a**4*b*c*d**8/(a*d - b*c)**4 + 1
0*B*a**3*b**2*c**2*d**7/(a*d - b*c)**4 - 10*B*a**2*b**3*c**3*d**6/(a*d - b*
c)**4 + 5*B*a*b**4*c**4*d**5/(a*d - b*c)**4 + B*a*d**5 - B*b**5*c**5*d**4/(
a*d - b*c)**4 + B*b*c*d**4)/(2*B*b*d**5))/(2*b*g**5*(a*d - b*c)**4) + (-6*A
*a**3*d**3 + 18*A*a**2*b*c*d**2 - 18*A*a*b**2*c**2*d + 6*A*b**3*c**3 - 25*B
*a**3*d**3 + 23*B*a**2*b*c*d**2 - 13*B*a*b**2*c**2*d + 3*B*b**3*c**3 - 12*B
*b**3*d**3*x**3 + x**2*(-42*B*a*b**2*d**3 + 6*B*b**3*c*d**2) + x*(-52*B*a**
2*b*d**3 + 20*B*a*b**2*c*d**2 - 4*B*b**3*c**2*d))/(24*a**7*b*d**3*g**5 - 72
*a**6*b**2*c*d**2*g**5 + 72*a**5*b**3*c**2*d*g**5 - 24*a**4*b**4*c**3*g**5
+ x**4*(24*a**3*b**5*d**3*g**5 - 72*a**2*b**6*c*d**2*g**5 + 72*a*b**7*c**2*
d*g**5 - 24*b**8*c**3*g**5) + x**3*(96*a**4*b**4*d**3*g**5 - 288*a**3*b**5*
c*d**2*g**5 + 288*a**2*b**6*c**2*d*g**5 - 96*a*b**7*c**3*g**5) + x**2*(144*
a**5*b**3*d**3*g**5 - 432*a**4*b**4*c*d**2*g**5 + 432*a**3*b**5*c**2*d*g**5
 - 144*a**2*b**6*c**3*g**5) + x*(96*a**6*b**2*d**3*g**5 - 288*a**5*b**3*c*d
**2*g**5 + 288*a**4*b**4*c**2*d*g**5 - 96*a**3*b**5*c**3*g**5))
```

# 3.128 $\int (ag+bgx)^4 \left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2 dx$

**Optimal**. Leaf size=377

$$\frac{2Bg^4(bc-ad)^5\log\left(\frac{bc-ad}{b(c+dx)}\right)\left(6B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+6A+25B\right)}{15bd^5}+\frac{2Bg^4(a+bx)(bc-ad)^4\left(6B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+6A+13\right.}{15bd^4}$$

```
[Out] -1/5*B*(-a*d+b*c)*g^4*(b*x+a)^4*(A+B*ln(e*(b*x+a)^2/(d*x+c)^2))/b/d+1/5*g^4
*(b*x+a)^5*(A+B*ln(e*(b*x+a)^2/(d*x+c)^2))^2/b+2/15*B*(-a*d+b*c)^2*g^4*(b*x
+a)^3*(2*A+B+2*B*ln(e*(b*x+a)^2/(d*x+c)^2))/b/d^2-1/15*B*(-a*d+b*c)^3*g^4*(
b*x+a)^2*(6*A+7*B+6*B*ln(e*(b*x+a)^2/(d*x+c)^2))/b/d^3+2/15*B*(-a*d+b*c)^4*
g^4*(b*x+a)*(6*A+13*B+6*B*ln(e*(b*x+a)^2/(d*x+c)^2))/b/d^4+2/15*B*(-a*d+b*c
)^5*g^4*(6*A+25*B+6*B*ln(e*(b*x+a)^2/(d*x+c)^2))*ln((-a*d+b*c)/b/(d*x+c))/b
/d^5+8/5*B^2*(-a*d+b*c)^5*g^4*polylog(2,d*(b*x+a)/b/(d*x+c))/b/d^5
```

**Rubi [A]** time = 0.87, antiderivative size = 569, normalized size of antiderivative = 1.51, number of steps used = 28, number of rules used = 13, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.382, Rules used = {2525, 12, 2528, 2486, 31, 43, 2524, 2418, 2394, 2393, 2391, 2390, 2301}

$$\frac{8B^2g^4(bc-ad)^5\text{PolyLog}\left(2,\frac{b(c+dx)}{bc-ad}\right)}{5bd^5}-\frac{4Bg^4(bc-ad)^5\log(c+dx)\left(B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+A\right)}{5bd^5}-\frac{2Bg^4(a+bx)^2(bc-ad)^3}{5b}$$

Antiderivative was successfully verified.

```
[In] Int[(a*g + b*g*x)^4*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])^2,x]
```

```
[Out] (4*A*B*(b*c - a*d)^4*g^4*x)/(5*d^4) + (26*B^2*(b*c - a*d)^4*g^4*x)/(15*d^4)
 - (7*B^2*(b*c - a*d)^3*g^4*(a + b*x)^2)/(15*b*d^3) + (2*B^2*(b*c - a*d)^2*
g^4*(a + b*x)^3)/(15*b*d^2) + (4*B^2*(b*c - a*d)^4*g^4*(a + b*x)*Log[(e*(a
+ b*x)^2)/(c + d*x)^2])/(5*b*d^4) - (2*B*(b*c - a*d)^3*g^4*(a + b*x)^2*(A +
 B*Log[(e*(a + b*x)^2)/(c + d*x)^2]))/(5*b*d^3) + (4*B*(b*c - a*d)^2*g^4*(a
 + b*x)^3*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2]))/(15*b*d^2) - (B*(b*c -
a*d)*g^4*(a + b*x)^4*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2]))/(5*b*d) + (g
^4*(a + b*x)^5*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])^2)/(5*b) - (10*B^2*
(b*c - a*d)^5*g^4*Log[c + d*x])/(3*b*d^5) + (8*B^2*(b*c - a*d)^5*g^4*Log[-(
(d*(a + b*x))/(b*c - a*d))]*Log[c + d*x])/(5*b*d^5) - (4*B*(b*c - a*d)^5*g^
4*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])*Log[c + d*x])/(5*b*d^5) - (4*B^2
*(b*c - a*d)^5*g^4*Log[c + d*x]^2)/(5*b*d^5) + (8*B^2*(b*c - a*d)^5*g^4*Pol
yLog[2, (b*(c + d*x))/(b*c - a*d)])/(5*b*d^5)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2301

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

Rule 2390

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_))^(q_.), x_Symbol] :> Dist[1/e, Subst[Int[((f*x)/d)^q*(a + b*Log[c*x^
n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && E
qQ[e*f - d*g, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2393

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + (c*e*x)/g])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2394

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d + e*x
)^n]))/g, x] - Dist[(b*e*n)/g, Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
```

```
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2418

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*(RFx_), x_Sy
mbol] :> With[{u = ExpandIntegrand[(a + b*Log[c*(d + e*x)^n])^p, RFx, x]},
Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d, e, n}, x] && RationalFunctionQ[
RFx, x] && IntegerQ[p]
```

Rule 2486

```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))
^(r_.)]^(s_.), x_Symbol] :> Simp[((a + b*x)*Log[e*(f*(a + b*x)^p*(c + d*x)^
q)^r]^s)/b, x] + Dist[(q*r*s*(b*c - a*d))/b, Int[Log[e*(f*(a + b*x)^p*(c +
d*x)^q)^r]^(s - 1)/(c + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, p, q, r, s
}, x] && NeQ[b*c - a*d, 0] && EqQ[p + q, 0] && IGtQ[s, 0]
```

Rule 2524

```
Int[((a_.) + Log[(c_.)*(RFx_)^(p_.)]*(b_.))^(n_.)/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[(Log[d + e*x]*(a + b*Log[c*RFx^p])^n)/e, x] - Dist[(b*n*p)/e
, Int[(Log[d + e*x]*(a + b*Log[c*RFx^p])^(n - 1)*D[RFx, x])/RFx, x], x] /;
FreeQ[{a, b, c, d, e, p}, x] && RationalFunctionQ[RFx, x] && IGtQ[n, 0]
```

Rule 2525

```
Int[((a_.) + Log[(c_.)*(RFx_)^(p_.)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_))^(m_.
), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(a + b*Log[c*RFx^p])^n)/(e*(m + 1))
, x] - Dist[(b*n*p)/(e*(m + 1)), Int[SimplifyIntegrand[((d + e*x)^(m + 1)*(
a + b*Log[c*RFx^p])^(n - 1)*D[RFx, x])/RFx, x], x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && RationalFunctionQ[RFx, x] && IGtQ[n, 0] && (EqQ[n, 1] ||
IntegerQ[m]) && NeQ[m, -1]
```

Rule 2528

```
Int[((a_.) + Log[(c_.)*(RFx_)^(p_.)]*(b_.))^(n_.)*(RGx_), x_Symbol] :> With
[{u = ExpandIntegrand[(a + b*Log[c*RFx^p])^n, RGx, x]}, Int[u, x] /; SumQ[u
]] /; FreeQ[{a, b, c, p}, x] && RationalFunctionQ[RFx, x] && RationalFuncti
onQ[RGx, x] && IGtQ[n, 0]
```

Rubi steps

$$\int (ag+bgx)^4 \left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2 dx = \frac{g^4(a+bx)^5\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}{5b} - \frac{(2B)\int \frac{2(bc-ad)g^5(a+bx)^4\left(A+B\right.}{c+dx}}{5bg}$$

$$= \frac{g^4(a+bx)^5\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}{5b} - \frac{\left(4B(bc-ad)g^4\right)\int \frac{(a+bx)^4\left(A\right.}{}}{5b}$$

$$= \frac{g^4(a+bx)^5\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}{5b} - \frac{\left(4B(bc-ad)g^4\right)\int\left(-\frac{b(bc-a}{}\right.}{}$$

$$= \frac{g^4(a+bx)^5\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}{5b} - \frac{\left(4B(bc-ad)g^4\right)\int(a+bx)}{5}$$

$$= \frac{4AB(bc-ad)^4g^4x}{5d^4} - \frac{2B(bc-ad)^3g^4(a+bx)^2\left(A+B\log\left(\frac{e(a+bx)}{(c+dx)}\right.\right.}{5bd^3}$$

$$= \frac{4AB(bc-ad)^4g^4x}{5d^4} + \frac{4B^2(bc-ad)^4g^4(a+bx)\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}{5bd^4} - \frac{2B}{}$$

$$= \frac{4AB(bc-ad)^4g^4x}{5d^4} + \frac{4B^2(bc-ad)^4g^4(a+bx)\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}{5bd^4} - \frac{2B}{}$$

$$= \frac{4AB(bc-ad)^4g^4x}{5d^4} + \frac{26B^2(bc-ad)^4g^4x}{15d^4} - \frac{7B^2(bc-ad)^3g^4(a+}{15bd^3}$$

$$= \frac{4AB(bc-ad)^4g^4x}{5d^4} + \frac{26B^2(bc-ad)^4g^4x}{15d^4} - \frac{7B^2(bc-ad)^3g^4(a+}{15bd^3}$$

$$= \frac{4AB(bc-ad)^4g^4x}{5d^4} + \frac{26B^2(bc-ad)^4g^4x}{15d^4} - \frac{7B^2(bc-ad)^3g^4(a+}{15bd^3}$$

$$= \frac{4AB(bc-ad)^4g^4x}{5d^4} + \frac{26B^2(bc-ad)^4g^4x}{15d^4} - \frac{7B^2(bc-ad)^3g^4(a+}{15bd^3}$$

**Mathematica [A]** time = 0.46, size = 523, normalized size = 1.39

$$g^4\left(\frac{B(bc-ad)\left(-3d^4(a+bx)^4\left(B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+A\right)+4d^3(a+bx)^3(bc-ad)\left(B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+A\right)-6d^2(a+bx)^2(bc-ad)^2\left(B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+A\right)-12(bc-ad)^4\log\cdots}{\cdots}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*g + b*g*x)^4*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])^2,x]
```

```
[Out] (g^4*((a + b*x)^5*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])^2 + (B*(b*c - a*
d)*(12*A*b*d*(b*c - a*d)^3*x + 12*B*d*(b*c - a*d)^3*(a + b*x)*Log[(e*(a + b
*x)^2)/(c + d*x)^2] - 6*d^2*(b*c - a*d)^2*(a + b*x)^2*(A + B*Log[(e*(a + b*
x)^2)/(c + d*x)^2]) + 4*d^3*(b*c - a*d)*(a + b*x)^3*(A + B*Log[(e*(a + b*x)
^2)/(c + d*x)^2]) - 3*d^4*(a + b*x)^4*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^
2]) - 24*B*(b*c - a*d)^4*Log[c + d*x] - 12*(b*c - a*d)^4*(A + B*Log[(e*(a +
 b*x)^2)/(c + d*x)^2])*Log[c + d*x] + 4*B*(b*c - a*d)^2*(2*b*d*(b*c - a*d)*
x - d^2*(a + b*x)^2 - 2*(b*c - a*d)^2*Log[c + d*x]) + B*(b*c - a*d)*(6*b*d*
(b*c - a*d)^2*x + 3*d^2*(-(b*c) + a*d)*(a + b*x)^2 + 2*d^3*(a + b*x)^3 - 6*
(b*c - a*d)^3*Log[c + d*x]) + 12*B*(b*c - a*d)^3*(b*d*x + (-(b*c) + a*d)*Lo
g[c + d*x]) + 12*B*(b*c - a*d)^4*((2*Log[(d*(a + b*x))/(-(b*c) + a*d)] - Lo
g[c + d*x])*Log[c + d*x] + 2*PolyLog[2, (b*(c + d*x))/(b*c - a*d)])))/(3*d^
5)))/(5*b)
```

**fricas [F]** time = 0.70, size = 0, normalized size = 0.00

$$\text{integral}\left(A^2b^4g^4x^4+4\,A^2ab^3g^4x^3+6\,A^2a^2b^2g^4x^2+4\,A^2a^3bg^4x+A^2a^4g^4+\left(B^2b^4g^4x^4+4\,B^2ab^3g^4x^3+6\,B^2a^2b\cdots\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*g*x+a*g)^4*(A+B*log(e*(b*x+a)^2/(d*x+c)^2))^2,x, algorithm="fr
icas")
```

```
[Out] integral(A^2*b^4*g^4*x^4 + 4*A^2*a*b^3*g^4*x^3 + 6*A^2*a^2*b^2*g^4*x^2 + 4*
A^2*a^3*b*g^4*x + A^2*a^4*g^4 + (B^2*b^4*g^4*x^4 + 4*B^2*a*b^3*g^4*x^3 + 6*
B^2*a^2*b^2*g^4*x^2 + 4*B^2*a^3*b*g^4*x + B^2*a^4*g^4)*log((b^2*e*x^2 + 2*a
*b*e*x + a^2*e)/(d^2*x^2 + 2*c*d*x + c^2))^2 + 2*(A*B*b^4*g^4*x^4 + 4*A*B*a
*b^3*g^4*x^3 + 6*A*B*a^2*b^2*g^4*x^2 + 4*A*B*a^3*b*g^4*x + A*B*a^4*g^4)*log
((b^2*e*x^2 + 2*a*b*e*x + a^2*e)/(d^2*x^2 + 2*c*d*x + c^2)), x)
```

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int\left(bgx+ag\right)^4\left(B\log\left(\frac{(bx+a)^2e}{(dx+c)^2}\right)+A\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*g*x+a*g)^4*(A+B*log(e*(b*x+a)^2/(d*x+c)^2))^2,x, algorithm="gi
ac")
```

```
[Out] integrate((b*g*x + a*g)^4*(B*log((b*x + a)^2*e/(d*x + c)^2) + A)^2, x)
```

**maple [F]** time = 1.37, size = 0, normalized size = 0.00

$$\int \left(bgx+ag\right)^{4} \left(B\ln\left(\frac{\left(bx+a\right)^{2} e}{\left(dx+c\right)^{2}}\right)+A\right)^{2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*g*x+a*g)^4*(A+B*ln(e*(b*x+a)^2/(d*x+c)^2))^2,x)
```

```
[Out] int((b*g*x+a*g)^4*(A+B*ln(e*(b*x+a)^2/(d*x+c)^2))^2,x)
```

**maxima [B]** time = 3.09, size = 2650, normalized size = 7.03

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*g*x+a*g)^4*(A+B*log(e*(b*x+a)^2/(d*x+c)^2))^2,x, algorithm="ma
xima")
```

```
[Out] 1/5*A^2*b^4*g^4*x^5 + A^2*a*b^3*g^4*x^4 + 2*A^2*a^2*b^2*g^4*x^3 + 2*A^2*a^3
*b*g^4*x^2 + 2*(x*log(b^2*e*x^2/(d^2*x^2 + 2*c*d*x + c^2) + 2*a*b*e*x/(d^2*
x^2 + 2*c*d*x + c^2) + a^2*e/(d^2*x^2 + 2*c*d*x + c^2)) + 2*a*log(b*x + a)/
b - 2*c*log(d*x + c)/d)*A*B*a^4*g^4 + 4*(x^2*log(b^2*e*x^2/(d^2*x^2 + 2*c*d
*x + c^2) + 2*a*b*e*x/(d^2*x^2 + 2*c*d*x + c^2) + a^2*e/(d^2*x^2 + 2*c*d*x
+ c^2)) - 2*a^2*log(b*x + a)/b^2 + 2*c^2*log(d*x + c)/d^2 - 2*(b*c - a*d)*x
/(b*d))*A*B*a^3*b*g^4 + 4*(x^3*log(b^2*e*x^2/(d^2*x^2 + 2*c*d*x + c^2) + 2*
a*b*e*x/(d^2*x^2 + 2*c*d*x + c^2) + a^2*e/(d^2*x^2 + 2*c*d*x + c^2)) + 2*a^
3*log(b*x + a)/b^3 - 2*c^3*log(d*x + c)/d^3 - ((b^2*c*d - a*b*d^2)*x^2 - 2*
(b^2*c^2 - a^2*d^2)*x)/(b^2*d^2))*A*B*a^2*b^2*g^4 + 2/3*(3*x^4*log(b^2*e*x^
2/(d^2*x^2 + 2*c*d*x + c^2) + 2*a*b*e*x/(d^2*x^2 + 2*c*d*x + c^2) + a^2*e/(
d^2*x^2 + 2*c*d*x + c^2)) - 6*a^4*log(b*x + a)/b^4 + 6*c^4*log(d*x + c)/d^4
 - (2*(b^3*c*d^2 - a*b^2*d^3)*x^3 - 3*(b^3*c^2*d - a^2*b*d^3)*x^2 + 6*(b^3*
c^3 - a^3*d^3)*x)/(b^3*d^3))*A*B*a*b^3*g^4 + 1/15*(6*x^5*log(b^2*e*x^2/(d^2
*x^2 + 2*c*d*x + c^2) + 2*a*b*e*x/(d^2*x^2 + 2*c*d*x + c^2) + a^2*e/(d^2*x^
2 + 2*c*d*x + c^2)) + 12*a^5*log(b*x + a)/b^5 - 12*c^5*log(d*x + c)/d^5 - (
3*(b^4*c*d^3 - a*b^3*d^4)*x^4 - 4*(b^4*c^2*d^2 - a^2*b^2*d^4)*x^3 + 6*(b^4*
c^3*d - a^3*b*d^4)*x^2 - 12*(b^4*c^4 - a^4*d^4)*x)/(b^4*d^4))*A*B*b^4*g^4 +
 A^2*a^4*g^4*x - 2/15*((6*g^4*log(e) + 25*g^4)*b^4*c^5 - (30*g^4*log(e) + 1
```

```
13*g^4)*a*b^3*c^4*d + 4*(15*g^4*log(e) + 49*g^4)*a^2*b^2*c^3*d^2 - 12*(5*g^
4*log(e) + 13*g^4)*a^3*b*c^2*d^3 + 6*(5*g^4*log(e) + 8*g^4)*a^4*c*d^4)*B^2*
log(d*x + c)/d^5 - 8/5*(b^5*c^5*g^4 - 5*a*b^4*c^4*d*g^4 + 10*a^2*b^3*c^3*d^
2*g^4 - 10*a^3*b^2*c^2*d^3*g^4 + 5*a^4*b*c*d^4*g^4 - a^5*d^5*g^4)*(log(b*x
+ a)*log((b*d*x + a*d)/(b*c - a*d) + 1) + dilog(-(b*d*x + a*d)/(b*c - a*d))
)*B^2/(b*d^5) + 1/15*(3*B^2*b^5*d^5*g^4*x^5*log(e)^2 - 3*(b^5*c*d^4*g^4*log
(e) - (5*g^4*log(e)^2 + g^4*log(e))*a*b^4*d^5)*B^2*x^4 + 2*((2*g^4*log(e) +
 g^4)*b^5*c^2*d^3 - 2*(5*g^4*log(e) + g^4)*a*b^4*c*d^4 + (15*g^4*log(e)^2 +
 8*g^4*log(e) + g^4)*a^2*b^3*d^5)*B^2*x^3 - ((6*g^4*log(e) + 7*g^4)*b^5*c^3
*d^2 - 3*(10*g^4*log(e) + 9*g^4)*a*b^4*c^2*d^3 + 3*(20*g^4*log(e) + 11*g^4)
*a^2*b^3*c*d^4 - (30*g^4*log(e)^2 + 36*g^4*log(e) + 13*g^4)*a^3*b^2*d^5)*B^
2*x^2 + (2*(6*g^4*log(e) + 13*g^4)*b^5*c^4*d - 2*(30*g^4*log(e) + 59*g^4)*a
*b^4*c^3*d^2 + 12*(10*g^4*log(e) + 17*g^4)*a^2*b^3*c^2*d^3 - 2*(60*g^4*log(
e) + 79*g^4)*a^3*b^2*c*d^4 + (15*g^4*log(e)^2 + 48*g^4*log(e) + 46*g^4)*a^4
*b*d^5)*B^2*x + 12*(B^2*b^5*d^5*g^4*x^5 + 5*B^2*a*b^4*d^5*g^4*x^4 + 10*B^2*
a^2*b^3*d^5*g^4*x^3 + 10*B^2*a^3*b^2*d^5*g^4*x^2 + 5*B^2*a^4*b*d^5*g^4*x +
B^2*a^5*d^5*g^4)*log(b*x + a)^2 + 12*(B^2*b^5*d^5*g^4*x^5 + 5*B^2*a*b^4*d^5
*g^4*x^4 + 10*B^2*a^2*b^3*d^5*g^4*x^3 + 10*B^2*a^3*b^2*d^5*g^4*x^2 + 5*B^2*
a^4*b*d^5*g^4*x + (b^5*c^5*g^4 - 5*a*b^4*c^4*d*g^4 + 10*a^2*b^3*c^3*d^2*g^4
 - 10*a^3*b^2*c^2*d^3*g^4 + 5*a^4*b*c*d^4*g^4)*B^2)*log(d*x + c)^2 + 2*(6*B
^2*b^5*d^5*g^4*x^5*log(e) - 3*(b^5*c*d^4*g^4 - (10*g^4*log(e) + g^4)*a*b^4*
d^5)*B^2*x^4 + 4*(b^5*c^2*d^3*g^4 - 5*a*b^4*c*d^4*g^4 + (15*g^4*log(e) + 4*
g^4)*a^2*b^3*d^5)*B^2*x^3 - 6*(b^5*c^3*d^2*g^4 - 5*a*b^4*c^2*d^3*g^4 + 10*a
^2*b^3*c*d^4*g^4 - 2*(5*g^4*log(e) + 3*g^4)*a^3*b^2*d^5)*B^2*x^2 + 6*(2*b^5
*c^4*d*g^4 - 10*a*b^4*c^3*d^2*g^4 + 20*a^2*b^3*c^2*d^3*g^4 - 20*a^3*b^2*c*d
^4*g^4 + (5*g^4*log(e) + 8*g^4)*a^4*b*d^5)*B^2*x + (12*a*b^4*c^4*d*g^4 - 54
*a^2*b^3*c^3*d^2*g^4 + 94*a^3*b^2*c^2*d^3*g^4 - 77*a^4*b*c*d^4*g^4 + (6*g^4
*log(e) + 25*g^4)*a^5*d^5)*B^2)*log(b*x + a) - 2*(6*B^2*b^5*d^5*g^4*x^5*log
(e) - 3*(b^5*c*d^4*g^4 - (10*g^4*log(e) + g^4)*a*b^4*d^5)*B^2*x^4 + 4*(b^5*
c^2*d^3*g^4 - 5*a*b^4*c*d^4*g^4 + (15*g^4*log(e) + 4*g^4)*a^2*b^3*d^5)*B^2*
x^3 - 6*(b^5*c^3*d^2*g^4 - 5*a*b^4*c^2*d^3*g^4 + 10*a^2*b^3*c*d^4*g^4 - 2*(
5*g^4*log(e) + 3*g^4)*a^3*b^2*d^5)*B^2*x^2 + 6*(2*b^5*c^4*d*g^4 - 10*a*b^4*
c^3*d^2*g^4 + 20*a^2*b^3*c^2*d^3*g^4 - 20*a^3*b^2*c*d^4*g^4 + (5*g^4*log(e)
 + 8*g^4)*a^4*b*d^5)*B^2*x + 12*(B^2*b^5*d^5*g^4*x^5 + 5*B^2*a*b^4*d^5*g^4*
x^4 + 10*B^2*a^2*b^3*d^5*g^4*x^3 + 10*B^2*a^3*b^2*d^5*g^4*x^2 + 5*B^2*a^4*b
*d^5*g^4*x + B^2*a^5*d^5*g^4)*log(b*x + a))*log(d*x + c))/(b*d^5)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.00

$$\int \left(a\,g+b\,g\,x\right)^4\,{\left(A+B\,\ln\left(\frac{e\,{\left(a+b\,x\right)}^2}{{\left(c+d\,x\right)}^2}\right)\right)}^2 \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*g + b*g*x)^4*(A + B*log((e*(a + b*x)^2)/(c + d*x)^2))^2,x)
```

```
[Out] int((a*g + b*g*x)^4*(A + B*log((e*(a + b*x)^2)/(c + d*x)^2))^2, x)
```

<u>**sympy**</u> **[F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*g*x+a*g)**4*(A+B*ln(e*(b*x+a)**2/(d*x+c)**2))**2,x)
[Out] Timed out
```

# 3.129 $\int (ag+bgx)^3 \left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2 dx$

**Optimal**. Leaf size=319

$$-\frac{Bg^3(bc-ad)^4\log\left(\frac{bc-ad}{b(c+dx)}\right)\left(3B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+3A+11B\right)}{3bd^4}-\frac{Bg^3(a+bx)(bc-ad)^3\left(3B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+3A+5B\right)}{3bd^3}$$

```
[Out] -1/3*B*(-a*d+b*c)*g^3*(b*x+a)^3*(A+B*ln(e*(b*x+a)^2/(d*x+c)^2))/b/d+1/4*g^3
*(b*x+a)^4*(A+B*ln(e*(b*x+a)^2/(d*x+c)^2))^2/b+1/6*B*(-a*d+b*c)^2*g^3*(b*x+
a)^2*(3*A+2*B+3*B*ln(e*(b*x+a)^2/(d*x+c)^2))/b/d^2-1/3*B*(-a*d+b*c)^3*g^3*(
b*x+a)*(3*A+5*B+3*B*ln(e*(b*x+a)^2/(d*x+c)^2))/b/d^3-1/3*B*(-a*d+b*c)^4*g^3
*(3*A+11*B+3*B*ln(e*(b*x+a)^2/(d*x+c)^2))*ln((-a*d+b*c)/b/(d*x+c))/b/d^4-2*
B^2*(-a*d+b*c)^4*g^3*polylog(2,d*(b*x+a)/b/(d*x+c))/b/d^4
```

**Rubi [A]** time = 0.76, antiderivative size = 470, normalized size of antiderivative = 1.47, number of steps used = 24, number of rules used = 13, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.382, Rules used = {2525, 12, 2528, 2486, 31, 43, 2524, 2418, 2394, 2393, 2391, 2390, 2301}

$$-\frac{2B^2g^3(bc-ad)^4\text{PolyLog}\left(2,\frac{b(c+dx)}{bc-ad}\right)}{bd^4}+\frac{Bg^3(bc-ad)^4\log(c+dx)\left(B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+A\right)}{bd^4}+\frac{Bg^3(a+bx)^2(bc-ad)^2}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[(a*g + b*g*x)^3*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])^2,x]
```

```
[Out] -((A*B*(b*c - a*d)^3*g^3*x)/d^3) - (5*B^2*(b*c - a*d)^3*g^3*x)/(3*d^3) + (B
^2*(b*c - a*d)^2*g^3*(a + b*x)^2)/(3*b*d^2) - (B^2*(b*c - a*d)^3*g^3*(a + b
*x)*Log[(e*(a + b*x)^2)/(c + d*x)^2])/(b*d^3) + (B*(b*c - a*d)^2*g^3*(a + b
*x)^2*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2]))/(2*b*d^2) - (B*(b*c - a*d)*
g^3*(a + b*x)^3*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2]))/(3*b*d) + (g^3*(a
 + b*x)^4*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])^2)/(4*b) + (11*B^2*(b*c
- a*d)^4*g^3*Log[c + d*x])/(3*b*d^4) - (2*B^2*(b*c - a*d)^4*g^3*Log[-((d*(a
 + b*x))/(b*c - a*d))]*Log[c + d*x])/(b*d^4) + (B*(b*c - a*d)^4*g^3*(A + B*
Log[(e*(a + b*x)^2)/(c + d*x)^2])*Log[c + d*x])/(b*d^4) + (B^2*(b*c - a*d)^
4*g^3*Log[c + d*x]^2)/(b*d^4) - (2*B^2*(b*c - a*d)^4*g^3*PolyLog[2, (b*(c +
 d*x))/(b*c - a*d)])/(b*d^4)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2301

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

Rule 2390

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_))^(q_.), x_Symbol] :> Dist[1/e, Subst[Int[((f*x)/d)^q*(a + b*Log[c*x^
n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && E
qQ[e*f - d*g, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2393

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + (c*e*x)/g])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2394

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d + e*x
)^n]))/g, x] - Dist[(b*e*n)/g, Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2418

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*(RFx_), x_Sy
mbol] :> With[{u = ExpandIntegrand[(a + b*Log[c*(d + e*x)^n])^p, RFx, x]},
Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d, e, n}, x] && RationalFunctionQ[
RFx, x] && IntegerQ[p]
```

Rule 2486

```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))
^(r_.)]^(s_.), x_Symbol] :> Simp[((a + b*x)*Log[e*(f*(a + b*x)^p*(c + d*x)^
q)^r]^s)/b, x] + Dist[(q*r*s*(b*c - a*d))/b, Int[Log[e*(f*(a + b*x)^p*(c +
d*x)^q)^r]^(s - 1)/(c + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, p, q, r, s
}, x] && NeQ[b*c - a*d, 0] && EqQ[p + q, 0] && IGtQ[s, 0]
```

Rule 2524

```
Int[((a_.) + Log[(c_.)*(RFx_)^(p_.)]*(b_.))^(n_.)/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[(Log[d + e*x]*(a + b*Log[c*RFx^p])^n)/e, x] - Dist[(b*n*p)/e
, Int[(Log[d + e*x]*(a + b*Log[c*RFx^p])^(n - 1)*D[RFx, x])/RFx, x], x] /;
FreeQ[{a, b, c, d, e, p}, x] && RationalFunctionQ[RFx, x] && IGtQ[n, 0]
```

Rule 2525

```
Int[((a_.) + Log[(c_.)*(RFx_)^(p_.)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_))^(m_.
), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(a + b*Log[c*RFx^p])^n)/(e*(m + 1))
, x] - Dist[(b*n*p)/(e*(m + 1)), Int[SimplifyIntegrand[((d + e*x)^(m + 1)*(
a + b*Log[c*RFx^p])^(n - 1)*D[RFx, x])/RFx, x], x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && RationalFunctionQ[RFx, x] && IGtQ[n, 0] && (EqQ[n, 1] ||
IntegerQ[m]) && NeQ[m, -1]
```

Rule 2528

```
Int[((a_.) + Log[(c_.)*(RFx_)^(p_.)]*(b_.))^(n_.)*(RGx_), x_Symbol] :> With
[{u = ExpandIntegrand[(a + b*Log[c*RFx^p])^n, RGx, x]}, Int[u, x] /; SumQ[u
]] /; FreeQ[{a, b, c, p}, x] && RationalFunctionQ[RFx, x] && RationalFuncti
onQ[RGx, x] && IGtQ[n, 0]
```

Rubi steps

$$\int (ag+bgx)^3 \left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2 dx = \frac{g^3(a+bx)^4\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}{4b} - \frac{B\int \frac{2(bc-ad)g^4(a+bx)^3\left(A+B\log\right.}{c+dx}}{2bg}$$

$$= \frac{g^3(a+bx)^4\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}{4b} - \frac{\left(B(bc-ad)g^3\right)\int \frac{(a+bx)^3\left(A+\right.}{}}{b}$$

$$= \frac{g^3(a+bx)^4\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}{4b} - \frac{\left(B(bc-ad)g^3\right)\int\left(\frac{b(bc-ad)^2(}{}\right.}{}$$

$$= \frac{g^3(a+bx)^4\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}{4b} - \frac{\left(B(bc-ad)g^3\right)\int(a+bx)^2}{d}$$

$$= -\frac{AB(bc-ad)^3g^3x}{d^3} + \frac{B(bc-ad)^2g^3(a+bx)^2\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right.\right.}{2bd^2}$$

$$= -\frac{AB(bc-ad)^3g^3x}{d^3} - \frac{B^2(bc-ad)^3g^3(a+bx)\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}{bd^3} + \frac{B(b}{}$$

$$= -\frac{AB(bc-ad)^3g^3x}{d^3} - \frac{B^2(bc-ad)^3g^3(a+bx)\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}{bd^3} + \frac{B(b}{}$$

$$= -\frac{AB(bc-ad)^3g^3x}{d^3} - \frac{5B^2(bc-ad)^3g^3x}{3d^3} + \frac{B^2(bc-ad)^2g^3(a+bx}{3bd^2}$$

$$= -\frac{AB(bc-ad)^3g^3x}{d^3} - \frac{5B^2(bc-ad)^3g^3x}{3d^3} + \frac{B^2(bc-ad)^2g^3(a+bx}{3bd^2}$$

$$= -\frac{AB(bc-ad)^3g^3x}{d^3} - \frac{5B^2(bc-ad)^3g^3x}{3d^3} + \frac{B^2(bc-ad)^2g^3(a+bx}{3bd^2}$$

$$= -\frac{AB(bc-ad)^3g^3x}{d^3} - \frac{5B^2(bc-ad)^3g^3x}{3d^3} + \frac{B^2(bc-ad)^2g^3(a+bx}{3bd^2}$$

**Mathematica [A]** time = 0.32, size = 402, normalized size = 1.26

$$g^3\left((a+bx)^4\left(B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+A\right)^2-\frac{2B(bc-ad)\left(2d^3(a+bx)^3\left(B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+A\right)+3d^2(a+bx)^2(ad-bc)\left(B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+A\right)-6(bc-ad)^3\log\right.}{}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*g + b*g*x)^3*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])^2,x]
```

```
[Out] (g^3*((a + b*x)^4*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])^2 - (2*B*(b*c -
a*d)*(6*A*b*d*(b*c - a*d)^2*x + 6*B*d*(b*c - a*d)^2*(a + b*x)*Log[(e*(a + b
*x)^2)/(c + d*x)^2] + 3*d^2*(-(b*c) + a*d)*(a + b*x)^2*(A + B*Log[(e*(a + b
*x)^2)/(c + d*x)^2]) + 2*d^3*(a + b*x)^3*(A + B*Log[(e*(a + b*x)^2)/(c + d*
x)^2]) - 12*B*(b*c - a*d)^3*Log[c + d*x] - 6*(b*c - a*d)^3*(A + B*Log[(e*(a
 + b*x)^2)/(c + d*x)^2])*Log[c + d*x] + 2*B*(b*c - a*d)*(2*b*d*(b*c - a*d)*
x - d^2*(a + b*x)^2 - 2*(b*c - a*d)^2*Log[c + d*x]) + 6*B*(b*c - a*d)^2*(b*
d*x + (-(b*c) + a*d)*Log[c + d*x]) + 6*B*(b*c - a*d)^3*((2*Log[(d*(a + b*x)
)/(-(b*c) + a*d)] - Log[c + d*x])*Log[c + d*x] + 2*PolyLog[2, (b*(c + d*x))
/(b*c - a*d)])))/(3*d^4)))/(4*b)
```

**fricas [F]** time = 0.67, size = 0, normalized size = 0.00

$$\text{integral}\left(A^2b^3g^3x^3+3\,A^2ab^2g^3x^2+3\,A^2a^2bg^3x+A^2a^3g^3+\left(B^2b^3g^3x^3+3\,B^2ab^2g^3x^2+3\,B^2a^2bg^3x+B^2a^3g^3\right)\log\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*g*x+a*g)^3*(A+B*log(e*(b*x+a)^2/(d*x+c)^2))^2,x, algorithm="fr
icas")
```

```
[Out] integral(A^2*b^3*g^3*x^3 + 3*A^2*a*b^2*g^3*x^2 + 3*A^2*a^2*b*g^3*x + A^2*a^
3*g^3 + (B^2*b^3*g^3*x^3 + 3*B^2*a*b^2*g^3*x^2 + 3*B^2*a^2*b*g^3*x + B^2*a^
3*g^3)*log((b^2*e*x^2 + 2*a*b*e*x + a^2*e)/(d^2*x^2 + 2*c*d*x + c^2))^2 + 2
*(A*B*b^3*g^3*x^3 + 3*A*B*a*b^2*g^3*x^2 + 3*A*B*a^2*b*g^3*x + A*B*a^3*g^3)*
log((b^2*e*x^2 + 2*a*b*e*x + a^2*e)/(d^2*x^2 + 2*c*d*x + c^2)), x)
```

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int (bgx+ag)^3\left(B\log\left(\frac{(bx+a)^2e}{(dx+c)^2}\right)+A\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*g*x+a*g)^3*(A+B*log(e*(b*x+a)^2/(d*x+c)^2))^2,x, algorithm="gi
ac")
```

```
[Out] integrate((b*g*x + a*g)^3*(B*log((b*x + a)^2*e/(d*x + c)^2) + A)^2, x)
```

**maple [F]** time = 1.20, size = 0, normalized size = 0.00

$$\int \left(bgx + ag\right)^{3} \left(B \ln\left(\frac{(bx+a)^{2} e}{(dx+c)^{2}}\right) + A\right)^{2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*g*x+a*g)^3*(B*ln((b*x+a)^2/(d*x+c)^2*e)+A)^2,x)
```

```
[Out] int((b*g*x+a*g)^3*(B*ln((b*x+a)^2/(d*x+c)^2*e)+A)^2,x)
```

**maxima [B]** time = 2.98, size = 1948, normalized size = 6.11

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*g*x+a*g)^3*(A+B*log(e*(b*x+a)^2/(d*x+c)^2))^2,x, algorithm="ma
xima")
```

```
[Out] 1/4*A^2*b^3*g^3*x^4 + A^2*a*b^2*g^3*x^3 + 3/2*A^2*a^2*b*g^3*x^2 + 2*(x*log(
b^2*e*x^2/(d^2*x^2 + 2*c*d*x + c^2) + 2*a*b*e*x/(d^2*x^2 + 2*c*d*x + c^2) +
 a^2*e/(d^2*x^2 + 2*c*d*x + c^2)) + 2*a*log(b*x + a)/b - 2*c*log(d*x + c)/d
)*A*B*a^3*g^3 + 3*(x^2*log(b^2*e*x^2/(d^2*x^2 + 2*c*d*x + c^2) + 2*a*b*e*x/
(d^2*x^2 + 2*c*d*x + c^2) + a^2*e/(d^2*x^2 + 2*c*d*x + c^2)) - 2*a^2*log(b*
x + a)/b^2 + 2*c^2*log(d*x + c)/d^2 - 2*(b*c - a*d)*x/(b*d))*A*B*a^2*b*g^3
+ 2*(x^3*log(b^2*e*x^2/(d^2*x^2 + 2*c*d*x + c^2) + 2*a*b*e*x/(d^2*x^2 + 2*c
*d*x + c^2) + a^2*e/(d^2*x^2 + 2*c*d*x + c^2)) + 2*a^3*log(b*x + a)/b^3 - 2
*c^3*log(d*x + c)/d^3 - ((b^2*c*d - a*b*d^2)*x^2 - 2*(b^2*c^2 - a^2*d^2)*x)
/(b^2*d^2))*A*B*a*b^2*g^3 + 1/6*(3*x^4*log(b^2*e*x^2/(d^2*x^2 + 2*c*d*x + c
^2) + 2*a*b*e*x/(d^2*x^2 + 2*c*d*x + c^2) + a^2*e/(d^2*x^2 + 2*c*d*x + c^2)
) - 6*a^4*log(b*x + a)/b^4 + 6*c^4*log(d*x + c)/d^4 - (2*(b^3*c*d^2 - a*b^2
*d^3)*x^3 - 3*(b^3*c^2*d - a^2*b*d^3)*x^2 + 6*(b^3*c^3 - a^3*d^3)*x)/(b^3*d
^3))*A*B*b^3*g^3 + A^2*a^3*g^3*x + 1/3*((3*g^3*log(e) + 11*g^3)*b^3*c^4 - 2
*(6*g^3*log(e) + 19*g^3)*a*b^2*c^3*d + 9*(2*g^3*log(e) + 5*g^3)*a^2*b*c^2*d
^2 - 6*(2*g^3*log(e) + 3*g^3)*a^3*c*d^3)*B^2*log(d*x + c)/d^4 + 2*(b^4*c^4*
g^3 - 4*a*b^3*c^3*d*g^3 + 6*a^2*b^2*c^2*d^2*g^3 - 4*a^3*b*c*d^3*g^3 + a^4*d
^4*g^3)*(log(b*x + a)*log((b*d*x + a*d)/(b*c - a*d) + 1) + dilog(-(b*d*x +
a*d)/(b*c - a*d)))*B^2/(b*d^4) + 1/12*(3*B^2*b^4*d^4*g^3*x^4*log(e)^2 - 4*(
b^4*c*d^3*g^3*log(e) - (3*g^3*log(e)^2 + g^3*log(e))*a*b^3*d^4)*B^2*x^3 + 2
*((3*g^3*log(e) + 2*g^3)*b^4*c^2*d^2 - 4*(3*g^3*log(e) + g^3)*a*b^3*c*d^3 +
```

```
(9*g^3*log(e)^2 + 9*g^3*log(e) + 2*g^3)*a^2*b^2*d^4)*B^2*x^2 - 4*((3*g^3*l
og(e) + 5*g^3)*b^4*c^3*d - (12*g^3*log(e) + 17*g^3)*a*b^3*c^2*d^2 + (18*g^3
*log(e) + 19*g^3)*a^2*b^2*c*d^3 - (3*g^3*log(e)^2 + 9*g^3*log(e) + 7*g^3)*a
^3*b*d^4)*B^2*x + 12*(B^2*b^4*d^4*g^3*x^4 + 4*B^2*a*b^3*d^4*g^3*x^3 + 6*B^2
*a^2*b^2*d^4*g^3*x^2 + 4*B^2*a^3*b*d^4*g^3*x + B^2*a^4*d^4*g^3)*log(b*x + a
)^2 + 12*(B^2*b^4*d^4*g^3*x^4 + 4*B^2*a*b^3*d^4*g^3*x^3 + 6*B^2*a^2*b^2*d^4
*g^3*x^2 + 4*B^2*a^3*b*d^4*g^3*x - (b^4*c^4*g^3 - 4*a*b^3*c^3*d*g^3 + 6*a^2
*b^2*c^2*d^2*g^3 - 4*a^3*b*c*d^3*g^3)*B^2)*log(d*x + c)^2 + 4*(3*B^2*b^4*d^
4*g^3*x^4*log(e) - 2*(b^4*c*d^3*g^3 - (6*g^3*log(e) + g^3)*a*b^3*d^4)*B^2*x
^3 + 3*(b^4*c^2*d^2*g^3 - 4*a*b^3*c*d^3*g^3 + 3*(2*g^3*log(e) + g^3)*a^2*b^
2*d^4)*B^2*x^2 - 6*(b^4*c^3*d*g^3 - 4*a*b^3*c^2*d^2*g^3 + 6*a^2*b^2*c*d^3*g
^3 - (2*g^3*log(e) + 3*g^3)*a^3*b*d^4)*B^2*x - (6*a*b^3*c^3*d*g^3 - 21*a^2*
b^2*c^2*d^2*g^3 + 26*a^3*b*c*d^3*g^3 - (3*g^3*log(e) + 11*g^3)*a^4*d^4)*B^2
)*log(b*x + a) - 4*(3*B^2*b^4*d^4*g^3*x^4*log(e) - 2*(b^4*c*d^3*g^3 - (6*g^
3*log(e) + g^3)*a*b^3*d^4)*B^2*x^3 + 3*(b^4*c^2*d^2*g^3 - 4*a*b^3*c*d^3*g^3
 + 3*(2*g^3*log(e) + g^3)*a^2*b^2*d^4)*B^2*x^2 - 6*(b^4*c^3*d*g^3 - 4*a*b^3
*c^2*d^2*g^3 + 6*a^2*b^2*c*d^3*g^3 - (2*g^3*log(e) + 3*g^3)*a^3*b*d^4)*B^2*
x + 6*(B^2*b^4*d^4*g^3*x^4 + 4*B^2*a*b^3*d^4*g^3*x^3 + 6*B^2*a^2*b^2*d^4*g^
3*x^2 + 4*B^2*a^3*b*d^4*g^3*x + B^2*a^4*d^4*g^3)*log(b*x + a))*log(d*x + c)
)/(b*d^4)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.00

$$\int \left(a\,g+b\,g\,x\right)^3\,{\left(A+B\,\ln\left(\frac{e\,{\left(a+b\,x\right)}^2}{{\left(c+d\,x\right)}^2}\right)\right)}^2 \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*g + b*g*x)^3*(A + B*log((e*(a + b*x)^2)/(c + d*x)^2))^2,x)
[Out] int((a*g + b*g*x)^3*(A + B*log((e*(a + b*x)^2)/(c + d*x)^2))^2, x)
```

**sympy [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*g*x+a*g)**3*(A+B*ln(e*(b*x+a)**2/(d*x+c)**2))**2,x)
[Out] Timed out
```

# 3.130 $\int (ag+bgx)^2 \left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2\,dx$

**Optimal**. Leaf size=255

$$\frac{4Bg^2(bc-ad)^3\log\left(\frac{bc-ad}{b(c+dx)}\right)\left(B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+A+3B\right)}{3bd^3}+\frac{4Bg^2(a+bx)(bc-ad)^2\left(B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+A+B\right)}{3bd^2}-\frac{2B}{}$$

```
[Out] -2/3*B*(-a*d+b*c)*g^2*(b*x+a)^2*(A+B*ln(e*(b*x+a)^2/(d*x+c)^2))/b/d+1/3*g^2
*(b*x+a)^3*(A+B*ln(e*(b*x+a)^2/(d*x+c)^2))^2/b+4/3*B*(-a*d+b*c)^2*g^2*(b*x+
a)*(A+B+B*ln(e*(b*x+a)^2/(d*x+c)^2))/b/d^2+4/3*B*(-a*d+b*c)^3*g^2*(A+3*B+B*
ln(e*(b*x+a)^2/(d*x+c)^2))*ln((-a*d+b*c)/b/(d*x+c))/b/d^3+8/3*B^2*(-a*d+b*c
)^3*g^2*polylog(2,d*(b*x+a)/b/(d*x+c))/b/d^3
```

**Rubi [A]** time = 0.62, antiderivative size = 397, normalized size of antiderivative = 1.56, number of steps used = 20, number of rules used = 13, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.382, Rules used = {2525, 12, 2528, 2486, 31, 43, 2524, 2418, 2394, 2393, 2391, 2390, 2301}

$$\frac{8B^2g^2(bc-ad)^3\text{PolyLog}\left(2,\frac{b(c+dx)}{bc-ad}\right)}{3bd^3}-\frac{4Bg^2(bc-ad)^3\log(c+dx)\left(B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+A\right)}{3bd^3}+\frac{4ABg^2x(bc-ad)^2}{3d^2}-\frac{2B}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a*g + b*g*x)^2*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])^2,x]
```

```
[Out] (4*A*B*(b*c - a*d)^2*g^2*x)/(3*d^2) + (4*B^2*(b*c - a*d)^2*g^2*x)/(3*d^2) +
 (4*B^2*(b*c - a*d)^2*g^2*(a + b*x)*Log[(e*(a + b*x)^2)/(c + d*x)^2])/(3*b*
d^2) - (2*B*(b*c - a*d)*g^2*(a + b*x)^2*(A + B*Log[(e*(a + b*x)^2)/(c + d*x
)^2]))/(3*b*d) + (g^2*(a + b*x)^3*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])^
2)/(3*b) - (4*B^2*(b*c - a*d)^3*g^2*Log[c + d*x])/(b*d^3) + (8*B^2*(b*c - a
*d)^3*g^2*Log[-((d*(a + b*x))/(b*c - a*d))]*Log[c + d*x])/(3*b*d^3) - (4*B*
(b*c - a*d)^3*g^2*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])*Log[c + d*x])/(3
*b*d^3) - (4*B^2*(b*c - a*d)^3*g^2*Log[c + d*x]^2)/(3*b*d^3) + (8*B^2*(b*c
- a*d)^3*g^2*PolyLog[2, (b*(c + d*x))/(b*c - a*d)])/(3*b*d^3)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2301

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

Rule 2390

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_))^(q_.), x_Symbol] :> Dist[1/e, Subst[Int[((f*x)/d)^q*(a + b*Log[c*x^
n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && E
qQ[e*f - d*g, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2393

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + (c*e*x)/g])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2394

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d + e*x
)^n]))/g, x] - Dist[(b*e*n)/g, Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2418

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*(RFx_), x_Sy
mbol] :> With[{u = ExpandIntegrand[(a + b*Log[c*(d + e*x)^n])^p, RFx, x]},
```

```
Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d, e, n}, x] && RationalFunctionQ[
RFx, x] && IntegerQ[p]
```

Rule 2486

```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))
^(r_.)]^(s_.), x_Symbol] :> Simp[((a + b*x)*Log[e*(f*(a + b*x)^p*(c + d*x)^
q)^r]^s)/b, x] + Dist[(q*r*s*(b*c - a*d))/b, Int[Log[e*(f*(a + b*x)^p*(c +
d*x)^q)^r]^(s - 1)/(c + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, p, q, r, s
}, x] && NeQ[b*c - a*d, 0] && EqQ[p + q, 0] && IGtQ[s, 0]
```

Rule 2524

```
Int[((a_.) + Log[(c_.)*(RFx_)^(p_.)]*(b_.))^(n_.)/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[(Log[d + e*x]*(a + b*Log[c*RFx^p])^n)/e, x] - Dist[(b*n*p)/e
, Int[(Log[d + e*x]*(a + b*Log[c*RFx^p])^(n - 1)*D[RFx, x])/RFx, x], x] /;
FreeQ[{a, b, c, d, e, p}, x] && RationalFunctionQ[RFx, x] && IGtQ[n, 0]
```

Rule 2525

```
Int[((a_.) + Log[(c_.)*(RFx_)^(p_.)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_))^(m_.
), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(a + b*Log[c*RFx^p])^n)/(e*(m + 1))
, x] - Dist[(b*n*p)/(e*(m + 1)), Int[SimplifyIntegrand[((d + e*x)^(m + 1)*(
a + b*Log[c*RFx^p])^(n - 1)*D[RFx, x])/RFx, x], x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && RationalFunctionQ[RFx, x] && IGtQ[n, 0] && (EqQ[n, 1] ||
IntegerQ[m]) && NeQ[m, -1]
```

Rule 2528

```
Int[((a_.) + Log[(c_.)*(RFx_)^(p_.)]*(b_.))^(n_.)*(RGx_), x_Symbol] :> With
[{u = ExpandIntegrand[(a + b*Log[c*RFx^p])^n, RGx, x]}, Int[u, x] /; SumQ[u
]] /; FreeQ[{a, b, c, p}, x] && RationalFunctionQ[RFx, x] && RationalFuncti
onQ[RGx, x] && IGtQ[n, 0]
```

Rubi steps

$$\int (ag+bgx)^2\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2 dx = \frac{g^2(a+bx)^3\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}{3b} - \frac{(2B)\int \frac{2(bc-ad)g^3(a+bx)^2\left(A+B\log \cdots\right.}{c+dx}}{3bg}\cdots$$

$$= \frac{g^2(a+bx)^3\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}{3b} - \frac{\left(4B(bc-ad)g^2\right)\int \frac{(a+bx)^2\left(A+\cdots\right.}{\cdots}}{3b}\cdots$$

$$= \frac{g^2(a+bx)^3\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}{3b} - \frac{\left(4B(bc-ad)g^2\right)\int\left(-\frac{b(bc-ad)\cdots}{\cdots}\right.}{\cdots}\cdots$$

$$= \frac{g^2(a+bx)^3\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}{3b} - \frac{\left(4B(bc-ad)g^2\right)\int(a+bx)\left(\cdots\right.}{3d}\cdots$$

$$= \frac{4AB(bc-ad)^2g^2x}{3d^2} - \frac{2B(bc-ad)g^2(a+bx)^2\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\cdots\right.}{3bd}\cdots$$

$$= \frac{4AB(bc-ad)^2g^2x}{3d^2} + \frac{4B^2(bc-ad)^2g^2(a+bx)\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}{3bd^2} - \frac{2B(\cdots}{\cdots}\cdots$$

$$= \frac{4AB(bc-ad)^2g^2x}{3d^2} + \frac{4B^2(bc-ad)^2g^2(a+bx)\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}{3bd^2} - \frac{2B(\cdots}{\cdots}\cdots$$

$$= \frac{4AB(bc-ad)^2g^2x}{3d^2} + \frac{4B^2(bc-ad)^2g^2x}{3d^2} + \frac{4B^2(bc-ad)^2g^2(a+bx\cdots}{3bd^2}\cdots$$

$$= \frac{4AB(bc-ad)^2g^2x}{3d^2} + \frac{4B^2(bc-ad)^2g^2x}{3d^2} + \frac{4B^2(bc-ad)^2g^2(a+bx\cdots}{3bd^2}\cdots$$

$$= \frac{4AB(bc-ad)^2g^2x}{3d^2} + \frac{4B^2(bc-ad)^2g^2x}{3d^2} + \frac{4B^2(bc-ad)^2g^2(a+bx\cdots}{3bd^2}\cdots$$

$$= \frac{4AB(bc-ad)^2g^2x}{3d^2} + \frac{4B^2(bc-ad)^2g^2x}{3d^2} + \frac{4B^2(bc-ad)^2g^2(a+bx\cdots}{3bd^2}\cdots$$

**Mathematica [A]** time = 0.22, size = 298, normalized size = 1.17

$$g^2\left(\frac{2B(bc-ad)\left(-d^2(a+bx)^2\left(B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+A\right)-2(bc-ad)^2\log(c+dx)\left(B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+A\right)+2Abdx(bc-ad)+2Bd(a+bx)(bc-ad)\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+2B(b\right.}{d^3}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*g + b*g*x)^2*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])^2,x]
```

```
[Out] (g^2*((a + b*x)^3*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])^2 + (2*B*(b*c -
a*d)*(2*A*b*d*(b*c - a*d)*x + 2*B*d*(b*c - a*d)*(a + b*x)*Log[(e*(a + b*x)^
2)/(c + d*x)^2] - d^2*(a + b*x)^2*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])
- 4*B*(b*c - a*d)^2*Log[c + d*x] - 2*(b*c - a*d)^2*(A + B*Log[(e*(a + b*x)^
2)/(c + d*x)^2])*Log[c + d*x] + 2*B*(b*c - a*d)*(b*d*x + (-(b*c) + a*d)*Log
[c + d*x]) + 2*B*(b*c - a*d)^2*((2*Log[(d*(a + b*x))/(-(b*c) + a*d)] - Log[
c + d*x])*Log[c + d*x] + 2*PolyLog[2, (b*(c + d*x))/(b*c - a*d)])))/d^3))/(
3*b)
```

**fricas [F]** time = 0.51, size = 0, normalized size = 0.00

$$\text{integral}\left(A^2b^2g^2x^2+2\,A^2abg^2x+A^2a^2g^2+\left(B^2b^2g^2x^2+2\,B^2abg^2x+B^2a^2g^2\right)\log\left(\frac{b^2ex^2+2\,abex+a^2e}{d^2x^2+2\,cdx+c^2}\right)^2+2\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*g*x+a*g)^2*(A+B*log(e*(b*x+a)^2/(d*x+c)^2))^2,x, algorithm="fr
icas")
```

```
[Out] integral(A^2*b^2*g^2*x^2 + 2*A^2*a*b*g^2*x + A^2*a^2*g^2 + (B^2*b^2*g^2*x^2
 + 2*B^2*a*b*g^2*x + B^2*a^2*g^2)*log((b^2*e*x^2 + 2*a*b*e*x + a^2*e)/(d^2*
x^2 + 2*c*d*x + c^2))^2 + 2*(A*B*b^2*g^2*x^2 + 2*A*B*a*b*g^2*x + A*B*a^2*g^
2)*log((b^2*e*x^2 + 2*a*b*e*x + a^2*e)/(d^2*x^2 + 2*c*d*x + c^2)), x)
```

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int\left(bgx+ag\right)^2\left(B\log\left(\frac{(bx+a)^2e}{(dx+c)^2}\right)+A\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*g*x+a*g)^2*(A+B*log(e*(b*x+a)^2/(d*x+c)^2))^2,x, algorithm="gi
ac")
```

```
[Out] integrate((b*g*x + a*g)^2*(B*log((b*x + a)^2*e/(d*x + c)^2) + A)^2, x)
```

**maple [F]** time = 1.16, size = 0, normalized size = 0.00

$$\int \left(bgx + ag\right)^2 \left(B \ln\left(\frac{(bx + a)^2 e}{(dx + c)^2}\right) + A\right)^2 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*g*x+a*g)^2*(B*ln((b*x+a)^2/(d*x+c)^2*e)+A)^2,x)
```

```
[Out] int((b*g*x+a*g)^2*(B*ln((b*x+a)^2/(d*x+c)^2*e)+A)^2,x)
```

**maxima [B]** time = 2.63, size = 1326, normalized size = 5.20

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*g*x+a*g)^2*(A+B*log(e*(b*x+a)^2/(d*x+c)^2))^2,x, algorithm="ma
xima")
```

```
[Out] 1/3*A^2*b^2*g^2*x^3 + A^2*a*b*g^2*x^2 + 2*(x*log(b^2*e*x^2/(d^2*x^2 + 2*c*d
*x + c^2) + 2*a*b*e*x/(d^2*x^2 + 2*c*d*x + c^2) + a^2*e/(d^2*x^2 + 2*c*d*x
+ c^2)) + 2*a*log(b*x + a)/b - 2*c*log(d*x + c)/d)*A*B*a^2*g^2 + 2*(x^2*log
(b^2*e*x^2/(d^2*x^2 + 2*c*d*x + c^2) + 2*a*b*e*x/(d^2*x^2 + 2*c*d*x + c^2)
+ a^2*e/(d^2*x^2 + 2*c*d*x + c^2)) - 2*a^2*log(b*x + a)/b^2 + 2*c^2*log(d*x
 + c)/d^2 - 2*(b*c - a*d)*x/(b*d))*A*B*a*b*g^2 + 2/3*(x^3*log(b^2*e*x^2/(d^
2*x^2 + 2*c*d*x + c^2) + 2*a*b*e*x/(d^2*x^2 + 2*c*d*x + c^2) + a^2*e/(d^2*x
^2 + 2*c*d*x + c^2)) + 2*a^3*log(b*x + a)/b^3 - 2*c^3*log(d*x + c)/d^3 - ((
b^2*c*d - a*b*d^2)*x^2 - 2*(b^2*c^2 - a^2*d^2)*x)/(b^2*d^2))*A*B*b^2*g^2 +
A^2*a^2*g^2*x - 4/3*((g^2*log(e) + 3*g^2)*b^2*c^3 - (3*g^2*log(e) + 7*g^2)*
a*b*c^2*d + (3*g^2*log(e) + 4*g^2)*a^2*c*d^2)*B^2*log(d*x + c)/d^3 - 8/3*(b
^3*c^3*g^2 - 3*a*b^2*c^2*d*g^2 + 3*a^2*b*c*d^2*g^2 - a^3*d^3*g^2)*(log(b*x
+ a)*log((b*d*x + a*d)/(b*c - a*d) + 1) + dilog(-(b*d*x + a*d)/(b*c - a*d))
)*B^2/(b*d^3) + 1/3*(B^2*b^3*d^3*g^2*x^3*log(e)^2 - (2*b^3*c*d^2*g^2*log(e)
 - (3*g^2*log(e)^2 + 2*g^2*log(e))*a*b^2*d^3)*B^2*x^2 + (4*(g^2*log(e) + g^
2)*b^3*c^2*d - 4*(3*g^2*log(e) + 2*g^2)*a*b^2*c*d^2 + (3*g^2*log(e)^2 + 8*g
^2*log(e) + 4*g^2)*a^2*b*d^3)*B^2*x + 4*(B^2*b^3*d^3*g^2*x^3 + 3*B^2*a*b^2*
d^3*g^2*x^2 + 3*B^2*a^2*b*d^3*g^2*x + B^2*a^3*d^3*g^2)*log(b*x + a)^2 + 4*(
B^2*b^3*d^3*g^2*x^3 + 3*B^2*a*b^2*d^3*g^2*x^2 + 3*B^2*a^2*b*d^3*g^2*x + (b^
3*c^3*g^2 - 3*a*b^2*c^2*d*g^2 + 3*a^2*b*c*d^2*g^2)*B^2)*log(d*x + c)^2 + 4*
(B^2*b^3*d^3*g^2*x^3*log(e) - (b^3*c*d^2*g^2 - (3*g^2*log(e) + g^2)*a*b^2*d
^3)*B^2*x^2 + (2*b^3*c^2*d*g^2 - 6*a*b^2*c*d^2*g^2 + (3*g^2*log(e) + 4*g^2)
*a^2*b*d^3)*B^2*x + (2*a*b^2*c^2*d*g^2 - 5*a^2*b*c*d^2*g^2 + (g^2*log(e) +
3*g^2)*a^3*d^3)*B^2)*log(b*x + a) - 4*(B^2*b^3*d^3*g^2*x^3*log(e) - (b^3*c*
d^2*g^2 - (3*g^2*log(e) + g^2)*a*b^2*d^3)*B^2*x^2 + (2*b^3*c^2*d*g^2 - 6*a*
b^2*c*d^2*g^2 + (3*g^2*log(e) + 4*g^2)*a^2*b*d^3)*B^2*x + 2*(B^2*b^3*d^3*g^
```

```
2*x^3 + 3*B^2*a*b^2*d^3*g^2*x^2 + 3*B^2*a^2*b*d^3*g^2*x + B^2*a^3*d^3*g^2)*
log(b*x + a))*log(d*x + c))/(b*d^3)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.00

$$\int \left(a\,g+b\,g\,x\right)^2\,{\left(A+B\,\ln\left(\frac{e\,{\left(a+b\,x\right)}^2}{{\left(c+d\,x\right)}^2}\right)\right)}^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*g + b*g*x)^2*(A + B*log((e*(a + b*x)^2)/(c + d*x)^2))^2,x)
[Out] int((a*g + b*g*x)^2*(A + B*log((e*(a + b*x)^2)/(c + d*x)^2))^2, x)
```

**sympy [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*g*x+a*g)**2*(A+B*ln(e*(b*x+a)**2/(d*x+c)**2))**2,x)
[Out] Timed out
```

# 3.131 $\int (ag+bgx)\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2 dx$

**Optimal**. Leaf size=188

$$-\frac{2Bg(bc-ad)^2\log\left(\frac{bc-ad}{b(c+dx)}\right)\left(B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+A+2B\right)}{bd^2}-\frac{2Bg(a+bx)(bc-ad)\left(B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+A\right)}{bd}+\frac{g(a+bx)}{}$$

```
[Out] -2*B*(-a*d+b*c)*g*(b*x+a)*(A+B*ln(e*(b*x+a)^2/(d*x+c)^2))/b/d+1/2*g*(b*x+a)
^2*(A+B*ln(e*(b*x+a)^2/(d*x+c)^2))^2/b-2*B*(-a*d+b*c)^2*g*(A+2*B+B*ln(e*(b*
x+a)^2/(d*x+c)^2))*ln((-a*d+b*c)/b/(d*x+c))/b/d^2-4*B^2*(-a*d+b*c)^2*g*poly
log(2,d*(b*x+a)/b/(d*x+c))/b/d^2
```

**Rubi [A]** time = 0.49, antiderivative size = 291, normalized size of antiderivative = 1.55, number of steps used = 16, number of rules used = 12, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {2525, 12, 2528, 2486, 31, 2524, 2418, 2394, 2393, 2391, 2390, 2301}

$$-\frac{4B^2g(bc-ad)^2\text{PolyLog}\left(2,\frac{b(c+dx)}{bc-ad}\right)}{bd^2}+\frac{2Bg(bc-ad)^2\log(c+dx)\left(B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+A\right)}{bd^2}+\frac{g(a+bx)^2\left(B\log\left(\frac{e(a+bx}{(c+dx}\right.\right.}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[(a*g + b*g*x)*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])^2,x]
```

```
[Out] (-2*A*B*(b*c - a*d)*g*x)/d - (2*B^2*(b*c - a*d)*g*(a + b*x)*Log[(e*(a + b*x
)^2)/(c + d*x)^2])/(b*d) + (g*(a + b*x)^2*(A + B*Log[(e*(a + b*x)^2)/(c + d
*x)^2])^2)/(2*b) + (4*B^2*(b*c - a*d)^2*g*Log[c + d*x])/(b*d^2) - (4*B^2*(b
*c - a*d)^2*g*Log[-((d*(a + b*x))/(b*c - a*d))]*Log[c + d*x])/(b*d^2) + (2*
B*(b*c - a*d)^2*g*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])*Log[c + d*x])/(b
*d^2) + (2*B^2*(b*c - a*d)^2*g*Log[c + d*x]^2)/(b*d^2) - (4*B^2*(b*c - a*d)
^2*g*PolyLog[2, (b*(c + d*x))/(b*c - a*d)])/(b*d^2)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] && !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 2301

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

Rule 2390

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_))^(q_.), x_Symbol] :> Dist[1/e, Subst[Int[((f*x)/d)^q*(a + b*Log[c*x^
n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && E
qQ[e*f - d*g, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2393

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + (c*e*x)/g])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2394

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d + e*x
)^n]))/g, x] - Dist[(b*e*n)/g, Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2418

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*(RFx_), x_Sy
mbol] :> With[{u = ExpandIntegrand[(a + b*Log[c*(d + e*x)^n])^p, RFx, x]},
Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d, e, n}, x] && RationalFunctionQ[
RFx, x] && IntegerQ[p]
```

Rule 2486

```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))
^(r_.)]^(s_.), x_Symbol] :> Simp[((a + b*x)*Log[e*(f*(a + b*x)^p*(c + d*x)^
q)^r]^s)/b, x] + Dist[(q*r*s*(b*c - a*d))/b, Int[Log[e*(f*(a + b*x)^p*(c +
d*x)^q)^r]^(s - 1)/(c + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, p, q, r, s
}, x] && NeQ[b*c - a*d, 0] && EqQ[p + q, 0] && IGtQ[s, 0]
```

Rule 2524

```
Int[((a_.) + Log[(c_.)*(RFx_)^(p_.)]*(b_.))^(n_.)/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[(Log[d + e*x]*(a + b*Log[c*RFx^p])^n)/e, x] - Dist[(b*n*p)/e
, Int[(Log[d + e*x]*(a + b*Log[c*RFx^p])^(n - 1)*D[RFx, x])/RFx, x], x] /;
FreeQ[{a, b, c, d, e, p}, x] && RationalFunctionQ[RFx, x] && IGtQ[n, 0]
```

Rule 2525

```
Int[((a_.) + Log[(c_.)*(RFx_)^(p_.)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_))^(m_.
), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(a + b*Log[c*RFx^p])^n)/(e*(m + 1))
, x] - Dist[(b*n*p)/(e*(m + 1)), Int[SimplifyIntegrand[((d + e*x)^(m + 1)*(
a + b*Log[c*RFx^p])^(n - 1)*D[RFx, x])/RFx, x], x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && RationalFunctionQ[RFx, x] && IGtQ[n, 0] && (EqQ[n, 1] ||
IntegerQ[m]) && NeQ[m, -1]
```

Rule 2528

```
Int[((a_.) + Log[(c_.)*(RFx_)^(p_.)]*(b_.))^(n_.)*(RGx_), x_Symbol] :> With
[{u = ExpandIntegrand[(a + b*Log[c*RFx^p])^n, RGx, x]}, Int[u, x] /; SumQ[u
]] /; FreeQ[{a, b, c, p}, x] && RationalFunctionQ[RFx, x] && RationalFuncti
onQ[RGx, x] && IGtQ[n, 0]
```

Rubi steps

$$\int (ag+bgx)\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2 dx = \frac{g(a+bx)^2\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}{2b} - \frac{B\int \frac{2(bc-ad)g^2(a+bx)\left(A+B\log\left(\frac{e(a\cdots}{(c+\cdots}\right)\right.}{c+dx}}{bg}\cdots$$

$$= \frac{g(a+bx)^2\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}{2b} - \frac{(2B(bc-ad)g)\int \frac{(a+bx)\left(A+B\log\cdots\right.}{c+\cdots}}{b}\cdots$$

$$= \frac{g(a+bx)^2\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}{2b} - \frac{(2B(bc-ad)g)\int\left(\frac{b\left(A+B\log\left(\frac{e\cdots}{\cdots}\right)\right.}{d}\cdots\right.}{\cdots}\cdots$$

$$= \frac{g(a+bx)^2\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}{2b} - \frac{(2B(bc-ad)g)\int\left(A+B\log\cdots\right.}{d}\cdots$$

$$= -\frac{2AB(bc-ad)gx}{d} + \frac{g(a+bx)^2\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}{2b} + \frac{2B(bc-\cdots}{\cdots}\cdots$$

$$= -\frac{2AB(bc-ad)gx}{d} - \frac{2B^2(bc-ad)g(a+bx)\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}{bd} + \frac{g(a+\cdots}{\cdots}\cdots$$

$$= -\frac{2AB(bc-ad)gx}{d} - \frac{2B^2(bc-ad)g(a+bx)\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}{bd} + \frac{g(a+\cdots}{\cdots}\cdots$$

$$= -\frac{2AB(bc-ad)gx}{d} - \frac{2B^2(bc-ad)g(a+bx)\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}{bd} + \frac{g(a+\cdots}{\cdots}\cdots$$

$$= -\frac{2AB(bc-ad)gx}{d} - \frac{2B^2(bc-ad)g(a+bx)\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}{bd} + \frac{g(a+\cdots}{\cdots}\cdots$$

$$= -\frac{2AB(bc-ad)gx}{d} - \frac{2B^2(bc-ad)g(a+bx)\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}{bd} + \frac{g(a+\cdots}{\cdots}\cdots$$

$$= -\frac{2AB(bc-ad)gx}{d} - \frac{2B^2(bc-ad)g(a+bx)\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}{bd} + \frac{g(a+\cdots}{\cdots}\cdots$$

**Mathematica [A]** time = 0.17, size = 207, normalized size = 1.10

$$\frac{g\left((a+bx)^2\left(B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+A\right)^2-\frac{4B(bc-ad)\left(-(bc-ad)\log(c+dx)\left(B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+A\right)+Bd(a+bx)\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+B(bc-ad)\left(2\text{Li}_2\left(\frac{b(c+dx)}{bc-ad}\right)\right.\right.}{d^2}\right.}{2b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*g + b*g*x)*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])^2,x]
```

```
[Out] (g*((a + b*x)^2*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])^2 - (4*B*(b*c - a*
d)*(A*b*d*x + B*d*(a + b*x)*Log[(e*(a + b*x)^2)/(c + d*x)^2] - 2*B*(b*c - a
*d)*Log[c + d*x] - (b*c - a*d)*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])*Log
[c + d*x] + B*(b*c - a*d)*((2*Log[(d*(a + b*x))/(-(b*c) + a*d)] - Log[c + d
*x])*Log[c + d*x] + 2*PolyLog[2, (b*(c + d*x))/(b*c - a*d)])))/d^2))/(2*b)
```

**fricas [F]** time = 0.92, size = 0, normalized size = 0.00

$$\text{integral}\left(A^2bgx+A^2ag+\left(B^2bgx+B^2ag\right)\log\left(\frac{b^2ex^2+2\,abex+a^2e}{d^2x^2+2\,cdx+c^2}\right)^2+2\left(ABbgx+ABag\right)\log\left(\frac{b^2ex^2+2\,abex}{d^2x^2+2\,cdx}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*g*x+a*g)*(A+B*log(e*(b*x+a)^2/(d*x+c)^2))^2,x, algorithm="fric
as")
```

```
[Out] integral(A^2*b*g*x + A^2*a*g + (B^2*b*g*x + B^2*a*g)*log((b^2*e*x^2 + 2*a*b
*e*x + a^2*e)/(d^2*x^2 + 2*c*d*x + c^2))^2 + 2*(A*B*b*g*x + A*B*a*g)*log((b
^2*e*x^2 + 2*a*b*e*x + a^2*e)/(d^2*x^2 + 2*c*d*x + c^2)), x)
```

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int\left(bgx+ag\right)\left(B\log\left(\frac{(bx+a)^2e}{(dx+c)^2}\right)+A\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*g*x+a*g)*(A+B*log(e*(b*x+a)^2/(d*x+c)^2))^2,x, algorithm="giac
")
```

```
[Out] integrate((b*g*x + a*g)*(B*log((b*x + a)^2*e/(d*x + c)^2) + A)^2, x)
```

**maple [F]** time = 0.95, size = 0, normalized size = 0.00

$$\int\left(bgx+ag\right)\left(B\ln\left(\frac{(bx+a)^2\,e}{(dx+c)^2}\right)+A\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*g*x+a*g)*(B*ln((b*x+a)^2/(d*x+c)^2*e)+A)^2,x)
```

```
[Out] int((b*g*x+a*g)*(B*ln((b*x+a)^2/(d*x+c)^2*e)+A)^2,x)
```

**maxima [B]** time = 2.42, size = 727, normalized size = 3.87

$$\frac{1}{2}A^2bgx^2+2\left(x\log\left(\frac{b^2ex^2}{d^2x^2+2\,cdx+c^2}+\frac{2\,abex}{d^2x^2+2\,cdx+c^2}+\frac{a^2e}{d^2x^2+2\,cdx+c^2}\right)+\frac{2\,a\log\left(bx+a\right)}{b}-\frac{2\,c\log\left(d\right.}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*g*x+a*g)*(A+B*log(e*(b*x+a)^2/(d*x+c)^2))^2,x, algorithm="maxi
ma")
```

```
[Out] 1/2*A^2*b*g*x^2 + 2*(x*log(b^2*e*x^2/(d^2*x^2 + 2*c*d*x + c^2) + 2*a*b*e*x/
(d^2*x^2 + 2*c*d*x + c^2) + a^2*e/(d^2*x^2 + 2*c*d*x + c^2)) + 2*a*log(b*x
+ a)/b - 2*c*log(d*x + c)/d)*A*B*a*g + (x^2*log(b^2*e*x^2/(d^2*x^2 + 2*c*d*
x + c^2) + 2*a*b*e*x/(d^2*x^2 + 2*c*d*x + c^2) + a^2*e/(d^2*x^2 + 2*c*d*x +
 c^2)) - 2*a^2*log(b*x + a)/b^2 + 2*c^2*log(d*x + c)/d^2 - 2*(b*c - a*d)*x/
(b*d))*A*B*b*g + A^2*a*g*x + 2*((g*log(e) + 2*g)*b*c^2 - 2*(g*log(e) + g)*a
*c*d)*B^2*log(d*x + c)/d^2 + 4*(b^2*c^2*g - 2*a*b*c*d*g + a^2*d^2*g)*(log(b
*x + a)*log((b*d*x + a*d)/(b*c - a*d) + 1) + dilog(-(b*d*x + a*d)/(b*c - a*
d)))*B^2/(b*d^2) + 1/2*(B^2*b^2*d^2*g*x^2*log(e)^2 - 2*(2*b^2*c*d*g*log(e)
- (g*log(e)^2 + 2*g*log(e))*a*b*d^2)*B^2*x + 4*(B^2*b^2*d^2*g*x^2 + 2*B^2*a
*b*d^2*g*x + B^2*a^2*d^2*g)*log(b*x + a)^2 + 4*(B^2*b^2*d^2*g*x^2 + 2*B^2*a
*b*d^2*g*x - (b^2*c^2*g - 2*a*b*c*d*g)*B^2)*log(d*x + c)^2 + 4*(B^2*b^2*d^2
*g*x^2*log(e) + 2*((g*log(e) + g)*a*b*d^2 - b^2*c*d*g)*B^2*x + ((g*log(e) +
 2*g)*a^2*d^2 - 2*a*b*c*d*g)*B^2)*log(b*x + a) - 4*(B^2*b^2*d^2*g*x^2*log(e
) + 2*((g*log(e) + g)*a*b*d^2 - b^2*c*d*g)*B^2*x + 2*(B^2*b^2*d^2*g*x^2 + 2
*B^2*a*b*d^2*g*x + B^2*a^2*d^2*g)*log(b*x + a))*log(d*x + c))/(b*d^2)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.01

$$\int \left(a\,g+b\,g\,x\right)\,{\left(A+B\,\ln\left(\frac{e\,{\left(a+b\,x\right)}^2}{{\left(c+d\,x\right)}^2}\right)\right)}^2 \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*g + b*g*x)*(A + B*log((e*(a + b*x)^2)/(c + d*x)^2))^2,x)
```

```
[Out] int((a*g + b*g*x)*(A + B*log((e*(a + b*x)^2)/(c + d*x)^2))^2, x)
```

**sympy [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*g*x+a*g)*(A+B*ln(e*(b*x+a)**2/(d*x+c)**2))**2,x)
```

```
[Out] Timed out
```

# 3.132 $\int \frac{\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}{ag+bgx}\,dx$

**Optimal**. Leaf size=132

$$\frac{4B\text{Li}_2\left(\frac{b(c+dx)}{d(a+bx)}\right)\left(B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+A\right)}{bg}-\frac{\log\left(1-\frac{b(c+dx)}{d(a+bx)}\right)\left(B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+A\right)^2}{bg}+\frac{8B^2\text{Li}_3\left(\frac{b(c+dx)}{d(a+bx)}\right)}{bg}$$

```
[Out] -(A+B*ln(e*(b*x+a)^2/(d*x+c)^2))^2*ln(1-b*(d*x+c)/d/(b*x+a))/b/g+4*B*(A+B*l
n(e*(b*x+a)^2/(d*x+c)^2))*polylog(2,b*(d*x+c)/d/(b*x+a))/b/g+8*B^2*polylog(
3,b*(d*x+c)/d/(b*x+a))/b/g
```

**Rubi [B]** time = 4.14, antiderivative size = 749, normalized size of antiderivative = 5.67, number of steps used = 46, number of rules used = 23, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.676, Rules used = {2524, 12, 2528, 2418, 2390, 2301, 2394, 2393, 2391, 6688, 6742, 2500, 2440, 2434, 2433, 2375, 2317, 2374, 6589, 2499, 2302, 30, 2396}

$$\frac{4AB\text{PolyLog}\left(2,-\frac{d(a+bx)}{bc-ad}\right)}{bg}-\frac{4B^2\text{PolyLog}\left(2,-\frac{d(a+bx)}{bc-ad}\right)\left(-\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+\log\left((a+bx)^2\right)+\log\left(\frac{1}{(c+dx)^2}\right)\right)}{bg}-\frac{8B^2\text{I}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])^2/(a*g + b*g*x),x]
```

```
[Out] (-2*A*B*Log[g*(a + b*x)]^2)/(b*g) + (4*B^2*Log[g*(a + b*x)]^3)/(3*b*g) - (4
*B^2*Log[g*(a + b*x)]^2*Log[-c - d*x])/(b*g) + (4*B^2*Log[g*(a + b*x)]*Log[
(a + b*x)^2]*Log[-c - d*x])/(b*g) - (B^2*Log[(a + b*x)^2]^2*Log[-c - d*x])/
(b*g) + (B^2*Log[-((d*(a + b*x))/(b*c - a*d))]*Log[(c + d*x)^(-2)]^2)/(b*g)
 - (B^2*Log[g*(a + b*x)]*Log[(c + d*x)^(-2)]^2)/(b*g) + (4*B^2*Log[g*(a + b
*x)]^2*Log[(b*(c + d*x))/(b*c - a*d)])/(b*g) + (B^2*Log[(a + b*x)^2]^2*Log[
(b*(c + d*x))/(b*c - a*d)])/(b*g) + ((A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2
])^2*Log[a*g + b*g*x])/(b*g) + (4*A*B*Log[(b*(c + d*x))/(b*c - a*d)]*Log[a*
g + b*g*x])/(b*g) - (4*B^2*(Log[(a + b*x)^2] + Log[(c + d*x)^(-2)] - Log[(e
*(a + b*x)^2)/(c + d*x)^2])*Log[(b*(c + d*x))/(b*c - a*d)]*Log[a*g + b*g*x]
)/(b*g) - (2*B^2*Log[(e*(a + b*x)^2)/(c + d*x)^2]*Log[a*g + b*g*x]^2)/(b*g)
 - (4*B^2*Log[(b*(c + d*x))/(b*c - a*d)]*Log[a*g + b*g*x]^2)/(b*g) + (4*A*B
*PolyLog[2, -((d*(a + b*x))/(b*c - a*d))])/(b*g) + (4*B^2*Log[(a + b*x)^2]*
PolyLog[2, -((d*(a + b*x))/(b*c - a*d))])/(b*g) - (4*B^2*(Log[(a + b*x)^2]
+ Log[(c + d*x)^(-2)] - Log[(e*(a + b*x)^2)/(c + d*x)^2])*PolyLog[2, -((d*(
a + b*x))/(b*c - a*d))])/(b*g) - (4*B^2*Log[(c + d*x)^(-2)]*PolyLog[2, (b*(
c + d*x))/(b*c - a*d)])/(b*g) - (8*B^2*PolyLog[3, -((d*(a + b*x))/(b*c - a*
d))])/(b*g) - (8*B^2*PolyLog[3, (b*(c + d*x))/(b*c - a*d)])/(b*g)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2301

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

Rule 2302

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/(x_), x_Symbol] :> Dist[1/(
b*n), Subst[Int[x^p, x], x, a + b*Log[c*x^n]], x] /; FreeQ[{a, b, c, n, p},
 x]
```

Rule 2317

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symb
ol] :> Simp[(Log[1 + (e*x)/d]*(a + b*Log[c*x^n])^p)/e, x] - Dist[(b*n*p)/e,
 Int[(Log[1 + (e*x)/d]*(a + b*Log[c*x^n])^(p - 1))/x, x], x] /; FreeQ[{a, b
, c, d, e, n}, x] && IGtQ[p, 0]
```

Rule 2374

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> -Simp[(PolyLog[2, -(d*f*x^m)]*(a + b*Log[c*x
^n])^p)/m, x] + Dist[(b*n*p)/m, Int[(PolyLog[2, -(d*f*x^m)]*(a + b*Log[c*x^
n])^(p - 1))/x, x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p, 0]
&& EqQ[d*e, 1]
```

Rule 2375

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))^(r_.)]*((a_.) + Log[(c_.)*(x_)^(n_
.)]*(b_.))^(p_.))/(x_), x_Symbol] :> Simp[(Log[d*(e + f*x^m)^r]*(a + b*Log[
c*x^n])^(p + 1))/(b*n*(p + 1)), x] - Dist[(f*m*r)/(b*n*(p + 1)), Int[(x^(m
- 1)*(a + b*Log[c*x^n])^(p + 1))/(e + f*x^m), x], x] /; FreeQ[{a, b, c, d,
e, f, r, m, n}, x] && IGtQ[p, 0] && NeQ[d*e, 1]
```

Rule 2390

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_))^(q_.), x_Symbol] :> Dist[1/e, Subst[Int[((f*x)/d)^q*(a + b*Log[c*x^
n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && E
qQ[e*f - d*g, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2393

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + (c*e*x)/g])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2394

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d + e*x
)^n]))/g, x] - Dist[(b*e*n)/g, Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2396

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)/((f_.) + (g_.
)*(x_)), x_Symbol] :> Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d
+ e*x)^n])^p)/g, x] - Dist[(b*e*n*p)/g, Int[(Log[(e*(f + g*x))/(e*f - d*g)]
*(a + b*Log[c*(d + e*x)^n])^(p - 1))/(d + e*x), x], x] /; FreeQ[{a, b, c, d
, e, f, g, n, p}, x] && NeQ[e*f - d*g, 0] && IGtQ[p, 1]
```

Rule 2418

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*(RFx_), x_Sy
mbol] :> With[{u = ExpandIntegrand[(a + b*Log[c*(d + e*x)^n])^p, RFx, x]},
Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d, e, n}, x] && RationalFunctionQ[
RFx, x] && IntegerQ[p]
```

Rule 2433

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + Log
[(h_.)*((i_.) + (j_.)*(x_))^(m_.)]*(g_.))*((k_.) + (l_.)*(x_))^(r_.), x_Sym
bol] :> Dist[1/e, Subst[Int[((k*x)/d)^r*(a + b*Log[c*x^n])^p*(f + g*Log[h*(
(e*i - d*j)/e + (j*x)/e)^m]), x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e,
```

```
f, g, h, i, j, k, l, n, p, r}, x] && EqQ[e*k - d*l, 0]
```

Rule 2434

```
Int[(((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + Log[(h_.
)*((i_.) + (j_.)*(x_))^(m_.)]*(g_.)))/(x_), x_Symbol] :> Simp[Log[x]*(a + b
*Log[c*(d + e*x)^n])*(f + g*Log[h*(i + j*x)^m]), x] + (-Dist[e*g*m, Int[(Lo
g[x]*(a + b*Log[c*(d + e*x)^n]))/(d + e*x), x], x] - Dist[b*j*n, Int[(Log[x
]*(f + g*Log[h*(i + j*x)^m]))/(i + j*x), x], x]) /; FreeQ[{a, b, c, d, e, f
, g, h, i, j, m, n}, x] && EqQ[e*i - d*j, 0]
```

Rule 2440

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + Log[(h_.)
*((i_.) + (j_.)*(x_))^(m_.)]*(g_.))*((k_) + (l_.)*(x_))^(r_.), x_Symbol] :>
 Dist[1/l, Subst[Int[x^r*(a + b*Log[c*(-((e*k - d*l)/l) + (e*x)/l)^n])*(f +
 g*Log[h*(-((j*k - i*l)/l) + (j*x)/l)^m]), x], x, k + l*x], x] /; FreeQ[{a,
 b, c, d, e, f, g, h, i, j, k, l, m, n}, x] && IntegerQ[r]
```

Rule 2499

```
Int[(Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.)
)^(r_.)]*((s_.) + Log[(i_.)*((g_.) + (h_.)*(x_))^(n_.)]*(t_.))^(m_.))/((j_.
) + (k_.)*(x_)), x_Symbol] :> Simp[((s + t*Log[i*(g + h*x)^n])^(m + 1)*Log[
e*(f*(a + b*x)^p*(c + d*x)^q)^r])/(k*n*t*(m + 1)), x] + (-Dist[(b*p*r)/(k*n
*t*(m + 1)), Int[(s + t*Log[i*(g + h*x)^n])^(m + 1)/(a + b*x), x], x] - Dis
t[(d*q*r)/(k*n*t*(m + 1)), Int[(s + t*Log[i*(g + h*x)^n])^(m + 1)/(c + d*x)
, x], x]) /; FreeQ[{a, b, c, d, e, f, g, h, i, j, k, s, t, m, n, p, q, r},
x] && NeQ[b*c - a*d, 0] && EqQ[h*j - g*k, 0] && IGtQ[m, 0]
```

Rule 2500

```
Int[(Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.)
)^(r_.)]*((s_.) + Log[(i_.)*((g_.) + (h_.)*(x_))^(n_.)]*(t_.)))/((j_.) + (k
_.)*(x_)), x_Symbol] :> Dist[Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r] - Log[(a
+ b*x)^(p*r)] - Log[(c + d*x)^(q*r)], Int[(s + t*Log[i*(g + h*x)^n])/(j + k
*x), x], x] + (Int[(Log[(a + b*x)^(p*r)]*(s + t*Log[i*(g + h*x)^n]))/(j + k
*x), x] + Int[(Log[(c + d*x)^(q*r)]*(s + t*Log[i*(g + h*x)^n]))/(j + k*x),
x]) /; FreeQ[{a, b, c, d, e, f, g, h, i, j, k, s, t, n, p, q, r}, x] && NeQ
[b*c - a*d, 0]
```

Rule 2524

```
Int[((a_.) + Log[(c_.)*(RFx_)^(p_.)]*(b_.))^(n_.)/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[(Log[d + e*x]*(a + b*Log[c*RFx^p])^n)/e, x] - Dist[(b*n*p)/e
```

```
, Int[(Log[d + e*x]*(a + b*Log[c*RFx^p])^(n - 1)*D[RFx, x])/RFx, x], x] /;
FreeQ[{a, b, c, d, e, p}, x] && RationalFunctionQ[RFx, x] && IGtQ[n, 0]
```

Rule 2528

```
Int[((a_.) + Log[(c_.)*(RFx_)^(p_.)]*(b_.))^(n_.)*(RGx_), x_Symbol] :> With
[{u = ExpandIntegrand[(a + b*Log[c*RFx^p])^n, RGx, x]}, Int[u, x] /; SumQ[u
]] /; FreeQ[{a, b, c, p}, x] && RationalFunctionQ[RFx, x] && RationalFuncti
onQ[RGx, x] && IGtQ[n, 0]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6688

```
Int[u_, x_Symbol] :> With[{v = SimplifyIntegrand[u, x]}, Int[v, x] /; Simpl
erIntegrandQ[v, u, x]]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\int \frac{\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}{ag+bgx}\,dx = \frac{\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2\log(ag+bgx)}{bg} - \frac{(2B)\int \frac{(c+dx)^2\left(-\frac{2de(a+bx)^2}{(c+dx)^3}+\frac{2be(a+bx)}{(c+dx)^2}\right)(A+B\log(}{e(a+bx)^2}}{bg}$$

$$= \frac{\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2\log(ag+bgx)}{bg} - \frac{(2B)\int \frac{(c+dx)^2\left(-\frac{2de(a+bx)^2}{(c+dx)^3}+\frac{2be(a+bx)}{(c+dx)^2}\right)(A+B\log(}{(a+bx)^2}}{beg}$$

$$= \frac{\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2\log(ag+bgx)}{bg} - \frac{(2B)\int \frac{2(bc-ad)e\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)\log(ag+bgx)}{(a+bx)(c+dx)}}{beg}$$

$$= \frac{\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2\log(ag+bgx)}{bg} - \frac{(4B(bc-ad))\int \frac{\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)\log(ag+bg}{(a+bx)(c+dx)}}{bg}$$

$$= \frac{\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2\log(ag+bgx)}{bg} - \frac{(4B(bc-ad))\int \left(\frac{d\left(-A-B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)\log(ag}{(bc-ad)(c+dx)}\right.}{bg}$$

$$= \frac{\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2\log(ag+bgx)}{bg} - \frac{(4B)\int \frac{\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)\log(ag+bgx)}{a+bx}\,dx}{g} - \frac{(}{}$$

$$= \frac{\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2\log(ag+bgx)}{bg} - \frac{(4B)\int \left(\frac{A\log(ag+bgx)}{a+bx}+\frac{B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\log(ag-}{a+bx}\right.}{g}$$

$$= \frac{\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2\log(ag+bgx)}{bg} - \frac{(4AB)\int \frac{\log(ag+bgx)}{a+bx}\,dx}{g} - \frac{\left(4B^2\right)\int \frac{\log\left(\frac{e(a+}{(c+d}\right.}{}}{}$$

$$= \frac{\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2\log(ag+bgx)}{bg} + \frac{4AB\log\left(\frac{b(c+dx)}{bc-ad}\right)\log(ag+bgx)}{bg} - \frac{2B^2 l}{}$$

$$= \frac{\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2\log(ag+bgx)}{bg} + \frac{4AB\log\left(\frac{b(c+dx)}{bc-ad}\right)\log(ag+bgx)}{bg} - \frac{4B^2 (}{}$$

$$= -\frac{2AB\log^2(g(a+bx))}{bg} + \frac{4B^2\log(g(a+bx))\log\left((a+bx)^2\right)\log(-c-dx)}{bg} - \frac{B^2 lo}{}$$

**Mathematica [A]** time = 0.32, size = 257, normalized size = 1.95

$$A^2\log(a+bx)+2AB\log(a+bx)\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)-4AB\text{Li}_2\left(\frac{b(c+dx)}{bc-ad}\right)+4AB\log(a+bx)\log\left(\frac{c}{d}+x\right)-4AB\log\left(\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])^2/(a*g + b*g*x),x]
```

```
[Out] (2*A*B*Log[a/b + x]^2 + A^2*Log[a + b*x] - 4*A*B*Log[a/b + x]*Log[a + b*x]
+ 4*A*B*Log[c/d + x]*Log[a + b*x] - 4*A*B*Log[c/d + x]*Log[(d*(a + b*x))/(-
(b*c) + a*d)] + 2*A*B*Log[a + b*x]*Log[(e*(a + b*x)^2)/(c + d*x)^2] - B^2*L
og[(-(b*c) + a*d)/(d*(a + b*x))]*Log[(e*(a + b*x)^2)/(c + d*x)^2]^2 - 4*A*B
*PolyLog[2, (b*(c + d*x))/(b*c - a*d)] + 4*B^2*Log[(e*(a + b*x)^2)/(c + d*x
)^2]*PolyLog[2, (b*(c + d*x))/(d*(a + b*x))] + 8*B^2*PolyLog[3, (b*(c + d*x
))/(d*(a + b*x))])/(b*g)
```

**fricas [F]** time = 2.02, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{B^2\log\left(\frac{b^2ex^2+2\,abex+a^2e}{d^2x^2+2\,cdx+c^2}\right)^2+2\,AB\log\left(\frac{b^2ex^2+2\,abex+a^2e}{d^2x^2+2\,cdx+c^2}\right)+A^2}{bgx+ag},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*(b*x+a)^2/(d*x+c)^2))^2/(b*g*x+a*g),x, algorithm="fric
as")
```

```
[Out] integral((B^2*log((b^2*e*x^2 + 2*a*b*e*x + a^2*e)/(d^2*x^2 + 2*c*d*x + c^2)
)^2 + 2*A*B*log((b^2*e*x^2 + 2*a*b*e*x + a^2*e)/(d^2*x^2 + 2*c*d*x + c^2))
+ A^2)/(b*g*x + a*g), x)
```

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\left(B\log\left(\frac{(bx+a)^2e}{(dx+c)^2}\right)+A\right)^2}{bgx+ag}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*(b*x+a)^2/(d*x+c)^2))^2/(b*g*x+a*g),x, algorithm="giac
")
```

```
[Out] integrate((B*log((b*x + a)^2*e/(d*x + c)^2) + A)^2/(b*g*x + a*g), x)
```

**maple [F]** time = 0.98, size = 0, normalized size = 0.00

$$\int \frac{\left(B \ln \left(\frac{(bx+a)^2 e}{(dx+c)^2}\right) + A\right)^2}{bgx + ag}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*ln((b*x+a)^2/(d*x+c)^2*e)+A)^2/(b*g*x+a*g),x)
```

```
[Out] int((B*ln((b*x+a)^2/(d*x+c)^2*e)+A)^2/(b*g*x+a*g),x)
```

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$\frac{4\,B^2 \log(bx+a)\log(dx+c)^2}{bg} + \frac{A^2 \log\left(bgx+ag\right)}{bg} - \int -\frac{B^2 bc \log(e)^2 + 2\,ABbc\log(e) + 4\left(B^2 bdx + B^2 bc\right)\log(bx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*(b*x+a)^2/(d*x+c)^2))^2/(b*g*x+a*g),x, algorithm="maxi
ma")
```

```
[Out] 4*B^2*log(b*x + a)*log(d*x + c)^2/(b*g) + A^2*log(b*g*x + a*g)/(b*g) - inte
grate(-(B^2*b*c*log(e)^2 + 2*A*B*b*c*log(e) + 4*(B^2*b*d*x + B^2*b*c)*log(b
*x + a)^2 + (B^2*b*d*log(e)^2 + 2*A*B*b*d*log(e))*x + 4*(B^2*b*c*log(e) + A
*B*b*c + (B^2*b*d*log(e) + A*B*b*d)*x)*log(b*x + a) - 4*(B^2*b*c*log(e) + A
*B*b*c + (B^2*b*d*log(e) + A*B*b*d)*x + 2*(2*B^2*b*d*x + (b*c + a*d)*B^2)*l
og(b*x + a))*log(d*x + c))/(b^2*d*g*x^2 + a*b*c*g + (b^2*c*g + a*b*d*g)*x),
 x)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\left(A + B\,\ln\left(\frac{e\,{(a+b\,x)}^2}{{(c+d\,x)}^2}\right)\right)}^2}{a\,g + b\,g\,x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*log((e*(a + b*x)^2)/(c + d*x)^2))^2/(a*g + b*g*x),x)
```

```
[Out] int((A + B*log((e*(a + b*x)^2)/(c + d*x)^2))^2/(a*g + b*g*x), x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{A^2}{a+bx}\, dx + \int \frac{B^2 \log\left(\frac{a^2 e}{c^2+2cdx+d^2x^2} + \frac{2abex}{c^2+2cdx+d^2x^2} + \frac{b^2 e x^2}{c^2+2cdx+d^2x^2}\right)^2}{a+bx}\, dx + \int \frac{2AB \log\left(\frac{a^2 e}{c^2+2cdx+d^2x^2} + \frac{2abex}{c^2+2cdx+d^2x^2} + \frac{b^2 e x^2}{c^2+2cdx+d^2x^2}\right)}{a+bx}\, dx}{g}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*ln(e*(b*x+a)**2/(d*x+c)**2))**2/(b*g*x+a*g),x)
```

```
[Out] (Integral(A**2/(a + b*x), x) + Integral(B**2*log(a**2*e/(c**2 + 2*c*d*x + d
**2*x**2) + 2*a*b*e*x/(c**2 + 2*c*d*x + d**2*x**2) + b**2*e*x**2/(c**2 + 2*
c*d*x + d**2*x**2))**2/(a + b*x), x) + Integral(2*A*B*log(a**2*e/(c**2 + 2*
c*d*x + d**2*x**2) + 2*a*b*e*x/(c**2 + 2*c*d*x + d**2*x**2) + b**2*e*x**2/(
c**2 + 2*c*d*x + d**2*x**2))/(a + b*x), x))/g
```

# 3.133 $\int \frac{\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}{(ag+bgx)^2}\,dx$

**Optimal**. Leaf size=130

$$-\frac{4B(c+dx)\left(B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+A\right)}{g^2(a+bx)(bc-ad)}-\frac{(c+dx)\left(B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+A\right)^2}{g^2(a+bx)(bc-ad)}-\frac{8B^2(c+dx)}{g^2(a+bx)(bc-ad)}$$

```
[Out] -8*B^2*(d*x+c)/(-a*d+b*c)/g^2/(b*x+a)-4*B*(d*x+c)*(A+B*ln(e*(b*x+a)^2/(d*x+
c)^2))/(-a*d+b*c)/g^2/(b*x+a)-(d*x+c)*(A+B*ln(e*(b*x+a)^2/(d*x+c)^2))^2/(-a
*d+b*c)/g^2/(b*x+a)
```

**Rubi [C]** time = 0.89, antiderivative size = 480, normalized size of antiderivative = 3.69, number of steps used = 26, number of rules used = 11, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.324, Rules used = {2525, 12, 2528, 44, 2524, 2418, 2390, 2301, 2394, 2393, 2391}

$$-\frac{8B^2d\text{PolyLog}\left(2,-\frac{d(a+bx)}{bc-ad}\right)}{bg^2(bc-ad)}-\frac{8B^2d\text{PolyLog}\left(2,\frac{b(c+dx)}{bc-ad}\right)}{bg^2(bc-ad)}-\frac{4Bd\log(a+bx)\left(B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+A\right)}{bg^2(bc-ad)}-\frac{4B\left(B\log\left(\frac{e(a+\cdots}{(c+\cdots}\right.\right.}{bg^2(a+\cdots}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])^2/(a*g + b*g*x)^2,x]
```

```
[Out] (-8*B^2)/(b*g^2*(a + b*x)) - (8*B^2*d*Log[a + b*x])/(b*(b*c - a*d)*g^2) + (
4*B^2*d*Log[a + b*x]^2)/(b*(b*c - a*d)*g^2) - (4*B*(A + B*Log[(e*(a + b*x)^
2)/(c + d*x)^2]))/(b*g^2*(a + b*x)) - (4*B*d*Log[a + b*x]*(A + B*Log[(e*(a
+ b*x)^2)/(c + d*x)^2]))/(b*(b*c - a*d)*g^2) - (A + B*Log[(e*(a + b*x)^2)/(
c + d*x)^2])^2/(b*g^2*(a + b*x)) + (8*B^2*d*Log[c + d*x])/(b*(b*c - a*d)*g^
2) - (8*B^2*d*Log[-((d*(a + b*x))/(b*c - a*d))]*Log[c + d*x])/(b*(b*c - a*d
)*g^2) + (4*B*d*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])*Log[c + d*x])/(b*(
b*c - a*d)*g^2) + (4*B^2*d*Log[c + d*x]^2)/(b*(b*c - a*d)*g^2) - (8*B^2*d*L
og[a + b*x]*Log[(b*(c + d*x))/(b*c - a*d)])/(b*(b*c - a*d)*g^2) - (8*B^2*d*
PolyLog[2, -((d*(a + b*x))/(b*c - a*d))])/(b*(b*c - a*d)*g^2) - (8*B^2*d*Po
lyLog[2, (b*(c + d*x))/(b*c - a*d)])/(b*(b*c - a*d)*g^2)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 2301

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

Rule 2390

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_))^(q_.), x_Symbol] :> Dist[1/e, Subst[Int[((f*x)/d)^q*(a + b*Log[c*x^
n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && E
qQ[e*f - d*g, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2393

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + (c*e*x)/g])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2394

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d + e*x
)^n]))/g, x] - Dist[(b*e*n)/g, Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2418

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*(RFx_), x_Sy
mbol] :> With[{u = ExpandIntegrand[(a + b*Log[c*(d + e*x)^n])^p, RFx, x]},
Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d, e, n}, x] && RationalFunctionQ[
RFx, x] && IntegerQ[p]
```

Rule 2524

```
Int[((a_.) + Log[(c_.)*(RFx_)^(p_.)]*(b_.))^(n_.)/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[(Log[d + e*x]*(a + b*Log[c*RFx^p])^n)/e, x] - Dist[(b*n*p)/e
, Int[(Log[d + e*x]*(a + b*Log[c*RFx^p])^(n - 1)*D[RFx, x])/RFx, x], x] /;
FreeQ[{a, b, c, d, e, p}, x] && RationalFunctionQ[RFx, x] && IGtQ[n, 0]
```

Rule 2525

```
Int[((a_.) + Log[(c_.)*(RFx_)^(p_.)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_))^(m_.
), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(a + b*Log[c*RFx^p])^n)/(e*(m + 1))
, x] - Dist[(b*n*p)/(e*(m + 1)), Int[SimplifyIntegrand[((d + e*x)^(m + 1)*(
a + b*Log[c*RFx^p])^(n - 1)*D[RFx, x])/RFx, x], x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && RationalFunctionQ[RFx, x] && IGtQ[n, 0] && (EqQ[n, 1] ||
IntegerQ[m]) && NeQ[m, -1]
```

Rule 2528

```
Int[((a_.) + Log[(c_.)*(RFx_)^(p_.)]*(b_.))^(n_.)*(RGx_), x_Symbol] :> With
[{u = ExpandIntegrand[(a + b*Log[c*RFx^p])^n, RGx, x]}, Int[u, x] /; SumQ[u
]] /; FreeQ[{a, b, c, p}, x] && RationalFunctionQ[RFx, x] && RationalFuncti
onQ[RGx, x] && IGtQ[n, 0]
```

Rubi steps

$$\int \frac{\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}{(ag+bgx)^2}\,dx = -\frac{\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}{bg^2(a+bx)} + \frac{(2B)\int \frac{2(bc-ad)\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}{g(a+bx)^2(c+dx)}\,dx}{bg}$$

$$= -\frac{\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}{bg^2(a+bx)} + \frac{(4B(bc-ad))\int \frac{A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}{(a+bx)^2(c+dx)}\,dx}{bg^2}$$

$$= -\frac{\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}{bg^2(a+bx)} + \frac{(4B(bc-ad))\int \left(\frac{b\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}{(bc-ad)(a+bx)^2} - \frac{bd\left(A+B\log\left(\frac{e(a+bx}{(c+dx}\right.\right.}{(bc-ad)^2(a+bx}\right.}{bg^2}$$

$$= -\frac{\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}{bg^2(a+bx)} + \frac{(4B)\int \frac{A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}{(a+bx)^2}\,dx}{g^2} - \frac{(4Bd)\int \frac{A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}{a+bx}}{(bc-ad)g^2}$$

$$= -\frac{4B\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}{bg^2(a+bx)} - \frac{4Bd\log(a+bx)\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}{b(bc-ad)g^2} - \frac{\left(A+B\right.}{bg}$$

$$= -\frac{4B\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}{bg^2(a+bx)} - \frac{4Bd\log(a+bx)\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}{b(bc-ad)g^2} - \frac{\left(A+B\right.}{bg}$$

$$= -\frac{4B\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}{bg^2(a+bx)} - \frac{4Bd\log(a+bx)\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}{b(bc-ad)g^2} - \frac{\left(A+B\right.}{bg}$$

$$= -\frac{8B^2}{bg^2(a+bx)} - \frac{8B^2d\log(a+bx)}{b(bc-ad)g^2} - \frac{4B\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}{bg^2(a+bx)} - \frac{4Bd\log(a+bx}{b(}$$

$$= -\frac{8B^2}{bg^2(a+bx)} - \frac{8B^2d\log(a+bx)}{b(bc-ad)g^2} - \frac{4B\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}{bg^2(a+bx)} - \frac{4Bd\log(a+bx}{b(}$$

$$= -\frac{8B^2}{bg^2(a+bx)} - \frac{8B^2d\log(a+bx)}{b(bc-ad)g^2} + \frac{4B^2d\log^2(a+bx)}{b(bc-ad)g^2} - \frac{4B\left(A+B\log\left(\frac{e(a+bx}{(c+dx)}\right.\right.}{bg^2(a+bx)}$$

$$= -\frac{8B^2}{bg^2(a+bx)} - \frac{8B^2d\log(a+bx)}{b(bc-ad)g^2} + \frac{4B^2d\log^2(a+bx)}{b(bc-ad)g^2} - \frac{4B\left(A+B\log\left(\frac{e(a+bx}{(c+dx)}\right.\right.}{bg^2(a+bx)}$$

**Mathematica [C]** time = 0.43, size = 321, normalized size = 2.47

$$-\frac{4B\left((bc-ad)\left(B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+A\right)+d(a+bx)\log(a+bx)\left(B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+A\right)-d(a+bx)\log(c+dx)\left(B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+A\right)-Bd(a+bx)\left(\log(a+bx)\left(\log(a+bx)\right.\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])^2/(a*g + b*g*x)^2,x]
```

```
[Out] -(((A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])^2 + (4*B*((b*c - a*d)*(A + B*Lo
g[(e*(a + b*x)^2)/(c + d*x)^2]) + d*(a + b*x)*Log[a + b*x]*(A + B*Log[(e*(a
 + b*x)^2)/(c + d*x)^2]) - d*(a + b*x)*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)
^2])*Log[c + d*x] + 2*B*(b*c - a*d + d*(a + b*x)*Log[a + b*x] - d*(a + b*x)
*Log[c + d*x]) - B*d*(a + b*x)*(Log[a + b*x]*(Log[a + b*x] - 2*Log[(b*(c +
d*x))/(b*c - a*d)]) - 2*PolyLog[2, (d*(a + b*x))/(-(b*c) + a*d)]) + B*d*(a
+ b*x)*((2*Log[(d*(a + b*x))/(-(b*c) + a*d)] - Log[c + d*x])*Log[c + d*x] +
 2*PolyLog[2, (b*(c + d*x))/(b*c - a*d)])))/(b*c - a*d))/(b*g^2*(a + b*x)))
```

**fricas [A]** time = 0.76, size = 200, normalized size = 1.54

$$-\frac{\left(A^2+4\,AB+8\,B^2\right)bc-\left(A^2+4\,AB+8\,B^2\right)ad+\left(B^2bdx+B^2bc\right)\log\left(\frac{b^2ex^2+2\,abex+a^2e}{d^2x^2+2\,cdx+c^2}\right)^2+2\left(\left(AB+2\,B^2\right)bdx+\right.}{\left(b^3c-ab^2d\right)g^2x+\left(ab^2c-a^2bd\right)g^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*(b*x+a)^2/(d*x+c)^2))^2/(b*g*x+a*g)^2,x, algorithm="fr
icas")
```

```
[Out] -((A^2 + 4*A*B + 8*B^2)*b*c - (A^2 + 4*A*B + 8*B^2)*a*d + (B^2*b*d*x + B^2*
b*c)*log((b^2*e*x^2 + 2*a*b*e*x + a^2*e)/(d^2*x^2 + 2*c*d*x + c^2))^2 + 2*(
(A*B + 2*B^2)*b*d*x + (A*B + 2*B^2)*b*c)*log((b^2*e*x^2 + 2*a*b*e*x + a^2*e
)/(d^2*x^2 + 2*c*d*x + c^2)))/((b^3*c - a*b^2*d)*g^2*x + (a*b^2*c - a^2*b*d
)*g^2)
```

**giac [B]** time = 1.74, size = 378, normalized size = 2.91

$$-\left(\frac{B^2d}{b^2cg^2-abdg^2}+\frac{B^2}{\left(bgx+ag\right)bg}\right)\log\left(\frac{b^2}{\frac{b^2c^2g^2}{\left(bgx+ag\right)^2}-\frac{2\,abcdg^2}{\left(bgx+ag\right)^2}+\frac{a^2d^2g^2}{\left(bgx+ag\right)^2}+\frac{2\,bcdg}{bgx+ag}-\frac{2\,ad^2g}{bgx+ag}+d^2}\right)^2+\frac{4\left(ABd+3\,B^2\right.}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*(b*x+a)^2/(d*x+c)^2))^2/(b*g*x+a*g)^2,x, algorithm="gi
ac")
```

```
[Out] -(B^2*d/(b^2*c*g^2 - a*b*d*g^2) + B^2/((b*g*x + a*g)*b*g))*log(b^2/(b^2*c^2
*g^2/(b*g*x + a*g)^2 - 2*a*b*c*d*g^2/(b*g*x + a*g)^2 + a^2*d^2*g^2/(b*g*x +
 a*g)^2 + 2*b*c*d*g/(b*g*x + a*g) - 2*a*d^2*g/(b*g*x + a*g) + d^2))^2 + 4*(
A*B*d + 3*B^2*d)*log(b*c*g/(b*g*x + a*g) - a*d*g/(b*g*x + a*g) + d)/(b^2*c*
g^2 - a*b*d*g^2) - 2*(A*B + 3*B^2)*log(b^2/(b^2*c^2*g^2/(b*g*x + a*g)^2 - 2
*a*b*c*d*g^2/(b*g*x + a*g)^2 + a^2*d^2*g^2/(b*g*x + a*g)^2 + 2*b*c*d*g/(b*g
*x + a*g) - 2*a*d^2*g/(b*g*x + a*g) + d^2))/((b*g*x + a*g)*b*g) - (A^2 + 6*
A*B + 13*B^2)/((b*g*x + a*g)*b*g)
```

**maple [B]** time = 0.10, size = 357, normalized size = 2.75

$$\frac{B^{2} d \ln\left(\frac{\left(\frac{ad}{dx+c}-\frac{bc}{dx+c}+b\right)^{2} e}{d^{2}}\right)^{2}}{\left(\frac{ad}{dx+c}-\frac{bc}{dx+c}+b\right)(ad-bc)\,g^{2}}+\frac{2ABd\ln\left(\frac{\left(\frac{ad}{dx+c}-\frac{bc}{dx+c}+b\right)^{2} e}{d^{2}}\right)}{\left(\frac{ad}{dx+c}-\frac{bc}{dx+c}+b\right)(ad-bc)\,g^{2}}+\frac{4B^{2} d\ln\left(\frac{\left(\frac{ad}{dx+c}-\frac{bc}{dx+c}+b\right)^{2} e}{d^{2}}\right)}{\left(\frac{ad}{dx+c}-\frac{bc}{dx+c}+b\right)(ad-bc)\,g^{2}}+\frac{A^{2} d}{\left(\frac{ad}{dx+c}-\frac{bc}{dx+c}+b\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*ln((b*x+a)^2/(d*x+c)^2*e)+A)^2/(b*g*x+a*g)^2,x)
```

```
[Out] d/g^2*A^2/(1/(d*x+c)*a*d-1/(d*x+c)*b*c+b)/(a*d-b*c)-8*d/g^2/(1/(d*x+c)*a*d-
1/(d*x+c)*b*c+b)*B^2/b/(d*x+c)+4*d/g^2/(1/(d*x+c)*a*d-1/(d*x+c)*b*c+b)*B^2/
(a*d-b*c)*ln((1/(d*x+c)*a*d-1/(d*x+c)*b*c+b)^2/d^2*e)+d/g^2/(1/(d*x+c)*a*d-
1/(d*x+c)*b*c+b)*B^2/(a*d-b*c)*ln((1/(d*x+c)*a*d-1/(d*x+c)*b*c+b)^2/d^2*e)^
2-4*d/g^2/(1/(d*x+c)*a*d-1/(d*x+c)*b*c+b)*A*B/b/(d*x+c)+2*d/g^2/(1/(d*x+c)*
a*d-1/(d*x+c)*b*c+b)*A*B/(a*d-b*c)*ln((1/(d*x+c)*a*d-1/(d*x+c)*b*c+b)^2/d^2
*e)
```

**maxima [B]** time = 1.59, size = 574, normalized size = 4.42

$$-4\left(\left(\frac{1}{b^{2} g^{2} x+abg^{2}}+\frac{d\log\left(bx+a\right)}{\left(b^{2} c-abd\right)g^{2}}-\frac{d\log\left(dx+c\right)}{\left(b^{2} c-abd\right)g^{2}}\right)\log\left(\frac{b^{2} e x^{2}}{d^{2} x^{2}+2\,cdx+c^{2}}+\frac{2\,abex}{d^{2} x^{2}+2\,cdx+c^{2}}+\frac{a^{2} e}{d^{2} x^{2}+2\,cd\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*(b*x+a)^2/(d*x+c)^2))^2/(b*g*x+a*g)^2,x, algorithm="ma
xima")
```

```
[Out] -4*((1/(b^2*g^2*x + a*b*g^2) + d*log(b*x + a)/((b^2*c - a*b*d)*g^2) - d*log
(d*x + c)/((b^2*c - a*b*d)*g^2))*log(b^2*e*x^2/(d^2*x^2 + 2*c*d*x + c^2) +
2*a*b*e*x/(d^2*x^2 + 2*c*d*x + c^2) + a^2*e/(d^2*x^2 + 2*c*d*x + c^2)) - ((
```

```
b*d*x + a*d)*log(b*x + a)^2 + (b*d*x + a*d)*log(d*x + c)^2 - 2*b*c + 2*a*d
- 2*(b*d*x + a*d)*log(b*x + a) + 2*(b*d*x + a*d - (b*d*x + a*d)*log(b*x + a
))*log(d*x + c))/(a*b^2*c*g^2 - a^2*b*d*g^2 + (b^3*c*g^2 - a*b^2*d*g^2)*x))
*B^2 - 2*A*B*(log(b^2*e*x^2/(d^2*x^2 + 2*c*d*x + c^2) + 2*a*b*e*x/(d^2*x^2
+ 2*c*d*x + c^2) + a^2*e/(d^2*x^2 + 2*c*d*x + c^2))/(b^2*g^2*x + a*b*g^2) +
 2/(b^2*g^2*x + a*b*g^2) + 2*d*log(b*x + a)/((b^2*c - a*b*d)*g^2) - 2*d*log
(d*x + c)/((b^2*c - a*b*d)*g^2)) - B^2*log(b^2*e*x^2/(d^2*x^2 + 2*c*d*x + c
^2) + 2*a*b*e*x/(d^2*x^2 + 2*c*d*x + c^2) + a^2*e/(d^2*x^2 + 2*c*d*x + c^2)
)^2/(b^2*g^2*x + a*b*g^2) - A^2/(b^2*g^2*x + a*b*g^2)
```

**mupad [B]** time = 5.97, size = 228, normalized size = 1.75

$$-\ln\left(\frac{e\,(a+b\,x)^2}{(c+d\,x)^2}\right)^2\left(\frac{B^2}{b^2\,g^2\,\left(x+\frac{a}{b}\right)}-\frac{B^2\,d}{b\,g^2\,\left(a\,d-b\,c\right)}\right)-\frac{A^2+4\,A\,B+8\,B^2}{x\,b^2\,g^2+a\,b\,g^2}-\frac{\ln\left(\frac{e\,(a+b\,x)^2}{(c+d\,x)^2}\right)\left(\frac{4\,B^2}{b^2\,d\,g^2}+\frac{2\,A\,B}{b^2\,d\,g^2}\right)}{\frac{x}{d}+\frac{a}{b\,d}}-\frac{B\,d\,\mathrm{atan}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*log((e*(a + b*x)^2)/(c + d*x)^2))^2/(a*g + b*g*x)^2,x)
```

```
[Out] - log((e*(a + b*x)^2)/(c + d*x)^2)^2*(B^2/(b^2*g^2*(x + a/b)) - (B^2*d)/(b*
g^2*(a*d - b*c))) - (A^2 + 8*B^2 + 4*A*B)/(b^2*g^2*x + a*b*g^2) - (log((e*(
a + b*x)^2)/(c + d*x)^2)*((4*B^2)/(b^2*d*g^2) + (2*A*B)/(b^2*d*g^2)))/(x/d
+ a/(b*d)) - (B*d*atan(((2*b*d*x + (b^2*c*g^2 + a*b*d*g^2)/(b*g^2))*1i)/(a*
d - b*c))*(A + 2*B)*8i)/(b*g^2*(a*d - b*c))
```

**sympy [B]** time = 3.66, size = 454, normalized size = 3.49

$$-\frac{4Bd\,(A+2B)\log\left(x+\frac{4ABad^2+4ABbcd+8B^2ad^2+8B^2bcd-\frac{4Ba^2d^3(A+2B)}{ad-bc}+\frac{8Babcd^2(A+2B)}{ad-bc}-\frac{4Bb^2c^2d(A+2B)}{ad-bc}}{8ABbd^2+16B^2bd^2}\right)}{bg^2\,(ad-bc)}+\frac{4Bd\,(A+2B)\log\left(x+\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*ln(e*(b*x+a)**2/(d*x+c)**2))**2/(b*g*x+a*g)**2,x)
```

```
[Out] -4*B*d*(A + 2*B)*log(x + (4*A*B*a*d**2 + 4*A*B*b*c*d + 8*B**2*a*d**2 + 8*B*
*2*b*c*d - 4*B*a**2*d**3*(A + 2*B)/(a*d - b*c) + 8*B*a*b*c*d**2*(A + 2*B)/(
a*d - b*c) - 4*B*b**2*c**2*d*(A + 2*B)/(a*d - b*c))/(8*A*B*b*d**2 + 16*B**2
*b*d**2))/(b*g**2*(a*d - b*c)) + 4*B*d*(A + 2*B)*log(x + (4*A*B*a*d**2 + 4*
A*B*b*c*d + 8*B**2*a*d**2 + 8*B**2*b*c*d + 4*B*a**2*d**3*(A + 2*B)/(a*d - b
*c) - 8*B*a*b*c*d**2*(A + 2*B)/(a*d - b*c) + 4*B*b**2*c**2*d*(A + 2*B)/(a*d
 - b*c))/(8*A*B*b*d**2 + 16*B**2*b*d**2))/(b*g**2*(a*d - b*c)) + (-2*A*B -
4*B**2)*log(e*(a + b*x)**2/(c + d*x)**2)/(a*b*g**2 + b**2*g**2*x) + (B**2*c
```

```
 + B**2*d*x)*log(e*(a + b*x)**2/(c + d*x)**2)**2/(a**2*d*g**2 - a*b*c*g**2
+ a*b*d*g**2*x - b**2*c*g**2*x) + (-A**2 - 4*A*B - 8*B**2)/(a*b*g**2 + b**2
*g**2*x)
```

# 3.134 $\int \frac{\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}{(ag+bgx)^3}\,dx$

**Optimal**. Leaf size=272

$$-\frac{bB(c+dx)^2\left(B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+A\right)}{g^3(a+bx)^2(bc-ad)^2}+\frac{4Bd(c+dx)\left(B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+A\right)}{g^3(a+bx)(bc-ad)^2}-\frac{b(c+dx)^2\left(B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+A\right)^2}{2g^3(a+bx)^2(bc-ad)^2}+\frac{d(c\cdots}{\cdots}$$

```
[Out] 8*B^2*d*(d*x+c)/(-a*d+b*c)^2/g^3/(b*x+a)-b*B^2*(d*x+c)^2/(-a*d+b*c)^2/g^3/(
b*x+a)^2+4*B*d*(d*x+c)*(A+B*ln(e*(b*x+a)^2/(d*x+c)^2))/(-a*d+b*c)^2/g^3/(b*
x+a)-b*B*(d*x+c)^2*(A+B*ln(e*(b*x+a)^2/(d*x+c)^2))/(-a*d+b*c)^2/g^3/(b*x+a)
^2+d*(d*x+c)*(A+B*ln(e*(b*x+a)^2/(d*x+c)^2))^2/(-a*d+b*c)^2/g^3/(b*x+a)-1/2
*b*(d*x+c)^2*(A+B*ln(e*(b*x+a)^2/(d*x+c)^2))^2/(-a*d+b*c)^2/g^3/(b*x+a)^2
```

**Rubi [C]** time = 1.05, antiderivative size = 579, normalized size of antiderivative = 2.13, number of steps used = 30, number of rules used = 11, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.324, Rules used = {2525, 12, 2528, 44, 2524, 2418, 2390, 2301, 2394, 2393, 2391}

$$\frac{4B^2d^2\text{PolyLog}\left(2,-\frac{d(a+bx)}{bc-ad}\right)}{bg^3(bc-ad)^2}+\frac{4B^2d^2\text{PolyLog}\left(2,\frac{b(c+dx)}{bc-ad}\right)}{bg^3(bc-ad)^2}+\frac{2Bd^2\log(a+bx)\left(B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+A\right)}{bg^3(bc-ad)^2}-\frac{2Bd^2\log(c+\cdots}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])^2/(a*g + b*g*x)^3,x]
```

```
[Out] -(B^2/(b*g^3*(a + b*x)^2)) + (6*B^2*d)/(b*(b*c - a*d)*g^3*(a + b*x)) + (6*B
^2*d^2*Log[a + b*x])/(b*(b*c - a*d)^2*g^3) - (2*B^2*d^2*Log[a + b*x]^2)/(b*
(b*c - a*d)^2*g^3) - (B*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2]))/(b*g^3*(a
 + b*x)^2) + (2*B*d*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2]))/(b*(b*c - a*d
)*g^3*(a + b*x)) + (2*B*d^2*Log[a + b*x]*(A + B*Log[(e*(a + b*x)^2)/(c + d*
x)^2]))/(b*(b*c - a*d)^2*g^3) - (A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])^2/
(2*b*g^3*(a + b*x)^2) - (6*B^2*d^2*Log[c + d*x])/(b*(b*c - a*d)^2*g^3) + (4
*B^2*d^2*Log[-((d*(a + b*x))/(b*c - a*d))]*Log[c + d*x])/(b*(b*c - a*d)^2*g
^3) - (2*B*d^2*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])*Log[c + d*x])/(b*(b
*c - a*d)^2*g^3) - (2*B^2*d^2*Log[c + d*x]^2)/(b*(b*c - a*d)^2*g^3) + (4*B^
2*d^2*Log[a + b*x]*Log[(b*(c + d*x))/(b*c - a*d)])/(b*(b*c - a*d)^2*g^3) +
(4*B^2*d^2*PolyLog[2, -((d*(a + b*x))/(b*c - a*d))])/(b*(b*c - a*d)^2*g^3)
+ (4*B^2*d^2*PolyLog[2, (b*(c + d*x))/(b*c - a*d)])/(b*(b*c - a*d)^2*g^3)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 2301

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

Rule 2390

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_))^(q_.), x_Symbol] :> Dist[1/e, Subst[Int[((f*x)/d)^q*(a + b*Log[c*x^
n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && E
qQ[e*f - d*g, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2393

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + (c*e*x)/g])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2394

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d + e*x
)^n]))/g, x] - Dist[(b*e*n)/g, Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2418

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*(RFx_), x_Sy
mbol] :> With[{u = ExpandIntegrand[(a + b*Log[c*(d + e*x)^n])^p, RFx, x]},
Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d, e, n}, x] && RationalFunctionQ[
RFx, x] && IntegerQ[p]
```

Rule 2524

```
Int[((a_.) + Log[(c_.)*(RFx_)^(p_.)]*(b_.))^(n_.)/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[(Log[d + e*x]*(a + b*Log[c*RFx^p])^n)/e, x] - Dist[(b*n*p)/e
, Int[(Log[d + e*x]*(a + b*Log[c*RFx^p])^(n - 1)*D[RFx, x])/RFx, x], x] /;
FreeQ[{a, b, c, d, e, p}, x] && RationalFunctionQ[RFx, x] && IGtQ[n, 0]
```

Rule 2525

```
Int[((a_.) + Log[(c_.)*(RFx_)^(p_.)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_))^(m_.
), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(a + b*Log[c*RFx^p])^n)/(e*(m + 1))
, x] - Dist[(b*n*p)/(e*(m + 1)), Int[SimplifyIntegrand[((d + e*x)^(m + 1)*(
a + b*Log[c*RFx^p])^(n - 1)*D[RFx, x])/RFx, x], x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && RationalFunctionQ[RFx, x] && IGtQ[n, 0] && (EqQ[n, 1] ||
IntegerQ[m]) && NeQ[m, -1]
```

Rule 2528

```
Int[((a_.) + Log[(c_.)*(RFx_)^(p_.)]*(b_.))^(n_.)*(RGx_), x_Symbol] :> With
[{u = ExpandIntegrand[(a + b*Log[c*RFx^p])^n, RGx, x]}, Int[u, x] /; SumQ[u
]] /; FreeQ[{a, b, c, p}, x] && RationalFunctionQ[RFx, x] && RationalFuncti
onQ[RGx, x] && IGtQ[n, 0]
```

Rubi steps

$$\int \frac{\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}{(ag+bgx)^3}\,dx = -\frac{\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}{2bg^3(a+bx)^2} + \frac{B\int \frac{2(bc-ad)\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}{g^2(a+bx)^3(c+dx)}\,dx}{bg}$$

$$= -\frac{\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}{2bg^3(a+bx)^2} + \frac{(2B(bc-ad))\int \frac{A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}{(a+bx)^3(c+dx)}\,dx}{bg^3}$$

$$= -\frac{\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}{2bg^3(a+bx)^2} + \frac{(2B(bc-ad))\int \left(\frac{b\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}{(bc-ad)(a+bx)^3} - \frac{bd\left(A+B\log\left(\frac{e(a+bx}{(c+dx}\right.\right.}{(bc-ad)^2(a+bx)}\cdots\right.}{bg\cdots}$$

$$= -\frac{\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}{2bg^3(a+bx)^2} + \frac{(2B)\int \frac{A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}{(a+bx)^3}\,dx}{g^3} + \frac{\left(2Bd^2\right)\int \frac{A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right.}{a+bx}\cdots}{(bc-ad)^2g^3}$$

$$= -\frac{B\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}{bg^3(a+bx)^2} + \frac{2Bd\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}{b(bc-ad)g^3(a+bx)} + \frac{2Bd^2\log(a+bx)\left(A\cdots\right.}{b(bc-a\cdots}$$

$$= -\frac{B\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}{bg^3(a+bx)^2} + \frac{2Bd\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}{b(bc-ad)g^3(a+bx)} + \frac{2Bd^2\log(a+bx)\left(A\cdots\right.}{b(bc-a\cdots}$$

$$= -\frac{B\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}{bg^3(a+bx)^2} + \frac{2Bd\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}{b(bc-ad)g^3(a+bx)} + \frac{2Bd^2\log(a+bx)\left(A\cdots\right.}{b(bc-a\cdots}$$

$$= -\frac{B^2}{bg^3(a+bx)^2} + \frac{6B^2d}{b(bc-ad)g^3(a+bx)} + \frac{6B^2d^2\log(a+bx)}{b(bc-ad)^2g^3} - \frac{B\left(A+B\log\left(\frac{e(a}{(c}\cdots\right.\right.}{bg^3(a+bx)\cdots}$$

$$= -\frac{B^2}{bg^3(a+bx)^2} + \frac{6B^2d}{b(bc-ad)g^3(a+bx)} + \frac{6B^2d^2\log(a+bx)}{b(bc-ad)^2g^3} - \frac{B\left(A+B\log\left(\frac{e(a}{(c}\cdots\right.\right.}{bg^3(a+bx)\cdots}$$

$$= -\frac{B^2}{bg^3(a+bx)^2} + \frac{6B^2d}{b(bc-ad)g^3(a+bx)} + \frac{6B^2d^2\log(a+bx)}{b(bc-ad)^2g^3} - \frac{2B^2d^2\log^2(a+b\cdots}{b(bc-ad)^2g^3}$$

$$= -\frac{B^2}{bg^3(a+bx)^2} + \frac{6B^2d}{b(bc-ad)g^3(a+bx)} + \frac{6B^2d^2\log(a+bx)}{b(bc-ad)^2g^3} - \frac{2B^2d^2\log^2(a+b\cdots}{b(bc-ad)^2g^3}$$

**Mathematica [C]** time = 0.44, size = 451, normalized size = 1.66

$$-\frac{\frac{2B\left(-2d^2(a+bx)^2\log(a+bx)\left(B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+A\right)+2d^2(a+bx)^2\log(c+dx)\left(B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+A\right)+(bc-ad)^2\left(B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+A\right)+2d(a+bx)(ad-bc)\left(B\log\right.\right.}{}}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])^2/(a*g + b*g*x)^3,x]
```

```
[Out] -1/2*((A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])^2 + (2*B*((b*c - a*d)^2*(A +
 B*Log[(e*(a + b*x)^2)/(c + d*x)^2]) + 2*d*(-(b*c) + a*d)*(a + b*x)*(A + B*
Log[(e*(a + b*x)^2)/(c + d*x)^2]) - 2*d^2*(a + b*x)^2*Log[a + b*x]*(A + B*L
og[(e*(a + b*x)^2)/(c + d*x)^2]) + 2*d^2*(a + b*x)^2*(A + B*Log[(e*(a + b*x
)^2)/(c + d*x)^2])*Log[c + d*x] - 4*B*d*(a + b*x)*(b*c - a*d + d*(a + b*x)*
Log[a + b*x] - d*(a + b*x)*Log[c + d*x]) + B*((b*c - a*d)^2 + 2*d*(-(b*c) +
 a*d)*(a + b*x) - 2*d^2*(a + b*x)^2*Log[a + b*x] + 2*d^2*(a + b*x)^2*Log[c
+ d*x]) + 2*B*d^2*(a + b*x)^2*(Log[a + b*x]*(Log[a + b*x] - 2*Log[(b*(c + d
*x))/(b*c - a*d)]) - 2*PolyLog[2, (d*(a + b*x))/(-(b*c) + a*d)]) - 2*B*d^2*
(a + b*x)^2*((2*Log[(d*(a + b*x))/(-(b*c) + a*d)] - Log[c + d*x])*Log[c + d
*x] + 2*PolyLog[2, (b*(c + d*x))/(b*c - a*d)])))/(b*c - a*d)^2)/(b*g^3*(a +
 b*x)^2)
```

**fricas [A]** time = 0.66, size = 410, normalized size = 1.51

$$-\frac{\left(A^2+2AB+2B^2\right)b^2c^2-2\left(A^2+4AB+8B^2\right)abcd+\left(A^2+6AB+14B^2\right)a^2d^2-\left(B^2b^2d^2x^2+2B^2abd^2x-B^2\right.}{2\left(\left(b^5\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*(b*x+a)^2/(d*x+c)^2))^2/(b*g*x+a*g)^3,x, algorithm="fr
icas")
```

```
[Out] -1/2*((A^2 + 2*A*B + 2*B^2)*b^2*c^2 - 2*(A^2 + 4*A*B + 8*B^2)*a*b*c*d + (A^
2 + 6*A*B + 14*B^2)*a^2*d^2 - (B^2*b^2*d^2*x^2 + 2*B^2*a*b*d^2*x - B^2*b^2*
c^2 + 2*B^2*a*b*c*d)*log((b^2*e*x^2 + 2*a*b*e*x + a^2*e)/(d^2*x^2 + 2*c*d*x
 + c^2))^2 - 4*((A*B + 3*B^2)*b^2*c*d - (A*B + 3*B^2)*a*b*d^2)*x - 2*((A*B
+ 3*B^2)*b^2*d^2*x^2 - (A*B + B^2)*b^2*c^2 + 2*(A*B + 2*B^2)*a*b*c*d + 2*(B
^2*b^2*c*d + (A*B + 2*B^2)*a*b*d^2)*x)*log((b^2*e*x^2 + 2*a*b*e*x + a^2*e)/
(d^2*x^2 + 2*c*d*x + c^2)))/((b^5*c^2 - 2*a*b^4*c*d + a^2*b^3*d^2)*g^3*x^2
+ 2*(a*b^4*c^2 - 2*a^2*b^3*c*d + a^3*b^2*d^2)*g^3*x + (a^2*b^3*c^2 - 2*a^3*
b^2*c*d + a^4*b*d^2)*g^3)
```

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(B\log\left(\frac{(bx+a)^2e}{(dx+c)^2}\right)+A\right)^2}{\left(bgx+ag\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*(b*x+a)^2/(d*x+c)^2))^2/(b*g*x+a*g)^3,x, algorithm="gi
ac")
```

```
[Out] integrate((B*log((b*x + a)^2*e/(d*x + c)^2) + A)^2/(b*g*x + a*g)^3, x)
```

**maple [B]** time = 0.15, size = 815, normalized size = 3.00

$$\frac{B^2b\,d^2\ln\left(\frac{\left(\frac{ad}{dx+c}-\frac{bc}{dx+c}+b\right)^2e}{d^2}\right)^2}{2\left(\frac{ad}{dx+c}-\frac{bc}{dx+c}+b\right)^2\left(a^2d^2-2abcd+b^2c^2\right)g^3}+\frac{ABb\,d^2\ln\left(\frac{\left(\frac{ad}{dx+c}-\frac{bc}{dx+c}+b\right)^2e}{d^2}\right)}{\left(\frac{ad}{dx+c}-\frac{bc}{dx+c}+b\right)^2\left(a^2d^2-2abcd+b^2c^2\right)g^3}+\frac{3B^2b\,d^2\ln}{\left(\frac{ad}{dx+c}-\frac{bc}{dx+c}+b\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*ln((b*x+a)^2/(d*x+c)^2*e)+A)^2/(b*g*x+a*g)^3,x)
```

```
[Out] d^2/g^3*A^2/(a*d-b*c)^2/(1/(d*x+c)*a*d-1/(d*x+c)*b*c+b)-1/2*d^2/g^3*A^2*b/(
a*d-b*c)^2/(1/(d*x+c)*a*d-1/(d*x+c)*b*c+b)^2-7*d^2/g^3/(1/(d*x+c)*a*d-1/(d*
x+c)*b*c+b)^2*B^2/b/(d*x+c)^2+3*d^2/g^3/(1/(d*x+c)*a*d-1/(d*x+c)*b*c+b)^2*b
*B^2/(a^2*d^2-2*a*b*c*d+b^2*c^2)*ln((1/(d*x+c)*a*d-1/(d*x+c)*b*c+b)^2/d^2*e
)-6*d^2/g^3/(1/(d*x+c)*a*d-1/(d*x+c)*b*c+b)^2*B^2/(a*d-b*c)/(d*x+c)+4*d^2/g
^3/(1/(d*x+c)*a*d-1/(d*x+c)*b*c+b)^2*B^2/(a*d-b*c)/(d*x+c)*ln((1/(d*x+c)*a*
d-1/(d*x+c)*b*c+b)^2/d^2*e)+1/2*d^2/g^3/(1/(d*x+c)*a*d-1/(d*x+c)*b*c+b)^2*B
^2*b/(a^2*d^2-2*a*b*c*d+b^2*c^2)*ln((1/(d*x+c)*a*d-1/(d*x+c)*b*c+b)^2/d^2*e
)^2+d^2/g^3/(1/(d*x+c)*a*d-1/(d*x+c)*b*c+b)^2*B^2/(a*d-b*c)/(d*x+c)*ln((1/(
d*x+c)*a*d-1/(d*x+c)*b*c+b)^2/d^2*e)^2-3*d^2/g^3/(1/(d*x+c)*a*d-1/(d*x+c)*b
*c+b)^2*A*B/b/(d*x+c)^2+d^2/g^3/(1/(d*x+c)*a*d-1/(d*x+c)*b*c+b)^2*b*A*B/(a^
2*d^2-2*a*b*c*d+b^2*c^2)*ln((1/(d*x+c)*a*d-1/(d*x+c)*b*c+b)^2/d^2*e)-2*d^2/
g^3/(1/(d*x+c)*a*d-1/(d*x+c)*b*c+b)^2*A*B/(a*d-b*c)/(d*x+c)+2*d^2/g^3/(1/(d
*x+c)*a*d-1/(d*x+c)*b*c+b)^2*A*B/(a*d-b*c)/(d*x+c)*ln((1/(d*x+c)*a*d-1/(d*x
+c)*b*c+b)^2/d^2*e)
```

**maxima [B]** time = 1.94, size = 1001, normalized size = 3.68

$$\left(\left(\left(\frac{2\,bdx-bc+3\,ad}{\left(b^4c-ab^3d\right)g^3x^2+2\left(ab^3c-a^2b^2d\right)g^3x+\left(a^2b^2c-a^3bd\right)g^3}+\frac{2\,d^2\log\left(bx+a\right)}{\left(b^3c^2-2\,ab^2cd+a^2bd^2\right)g^3}-\frac{2\,d^2\log\left(dx\right.}{\left(b^3c^2-2\,ab^2cd+\right.}\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*(b*x+a)^2/(d*x+c)^2))^2/(b*g*x+a*g)^3,x, algorithm="ma
xima")
```

```
[Out] (((2*b*d*x - b*c + 3*a*d)/((b^4*c - a*b^3*d)*g^3*x^2 + 2*(a*b^3*c - a^2*b^2
*d)*g^3*x + (a^2*b^2*c - a^3*b*d)*g^3) + 2*d^2*log(b*x + a)/((b^3*c^2 - 2*a
*b^2*c*d + a^2*b*d^2)*g^3) - 2*d^2*log(d*x + c)/((b^3*c^2 - 2*a*b^2*c*d + a
^2*b*d^2)*g^3))*log(b^2*e*x^2/(d^2*x^2 + 2*c*d*x + c^2) + 2*a*b*e*x/(d^2*x^
2 + 2*c*d*x + c^2) + a^2*e/(d^2*x^2 + 2*c*d*x + c^2)) - (b^2*c^2 - 8*a*b*c*
d + 7*a^2*d^2 + 2*(b^2*d^2*x^2 + 2*a*b*d^2*x + a^2*d^2)*log(b*x + a)^2 + 2*
(b^2*d^2*x^2 + 2*a*b*d^2*x + a^2*d^2)*log(d*x + c)^2 - 6*(b^2*c*d - a*b*d^2
)*x - 6*(b^2*d^2*x^2 + 2*a*b*d^2*x + a^2*d^2)*log(b*x + a) + 2*(3*b^2*d^2*x
^2 + 6*a*b*d^2*x + 3*a^2*d^2 - 2*(b^2*d^2*x^2 + 2*a*b*d^2*x + a^2*d^2)*log(
b*x + a))*log(d*x + c))/(a^2*b^3*c^2*g^3 - 2*a^3*b^2*c*d*g^3 + a^4*b*d^2*g^
3 + (b^5*c^2*g^3 - 2*a*b^4*c*d*g^3 + a^2*b^3*d^2*g^3)*x^2 + 2*(a*b^4*c^2*g^
3 - 2*a^2*b^3*c*d*g^3 + a^3*b^2*d^2*g^3)*x))*B^2 + A*B*((2*b*d*x - b*c + 3*
a*d)/((b^4*c - a*b^3*d)*g^3*x^2 + 2*(a*b^3*c - a^2*b^2*d)*g^3*x + (a^2*b^2*
c - a^3*b*d)*g^3) - log(b^2*e*x^2/(d^2*x^2 + 2*c*d*x + c^2) + 2*a*b*e*x/(d^
2*x^2 + 2*c*d*x + c^2) + a^2*e/(d^2*x^2 + 2*c*d*x + c^2))/(b^3*g^3*x^2 + 2*
a*b^2*g^3*x + a^2*b*g^3) + 2*d^2*log(b*x + a)/((b^3*c^2 - 2*a*b^2*c*d + a^2
*b*d^2)*g^3) - 2*d^2*log(d*x + c)/((b^3*c^2 - 2*a*b^2*c*d + a^2*b*d^2)*g^3)
) - 1/2*B^2*log(b^2*e*x^2/(d^2*x^2 + 2*c*d*x + c^2) + 2*a*b*e*x/(d^2*x^2 +
2*c*d*x + c^2) + a^2*e/(d^2*x^2 + 2*c*d*x + c^2))^2/(b^3*g^3*x^2 + 2*a*b^2*
g^3*x + a^2*b*g^3) - 1/2*A^2/(b^3*g^3*x^2 + 2*a*b^2*g^3*x + a^2*b*g^3)
```

**mupad [B]** time = 5.89, size = 503, normalized size = 1.85

$$-\frac{\frac{A^2\,a\,d-A^2\,b\,c+14\,B^2\,a\,d-2\,B^2\,b\,c+6\,A\,B\,a\,d-2\,A\,B\,b\,c}{2\,\left(a\,d-b\,c\right)}+\frac{2\,x\,\left(3\,b\,d\,B^2+A\,b\,d\,B\right)}{a\,d-b\,c}}{a^2\,b\,g^3+2\,a\,b^2\,g^3\,x+b^3\,g^3\,x^2}-\ln\left(\frac{e\,{\left(a+b\,x\right)}^2}{{\left(c+d\,x\right)}^2}\right)^2\,\left(\frac{B^2}{2\,b^2\,g^3\,\left(2\,a\,x+b\,x^2+\frac{a^2}{b}\right)}-\frac{\phantom{x}}{2\,b\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*log((e*(a + b*x)^2)/(c + d*x)^2))^2/(a*g + b*g*x)^3,x)
```

```
[Out] - ((A^2*a*d - A^2*b*c + 14*B^2*a*d - 2*B^2*b*c + 6*A*B*a*d - 2*A*B*b*c)/(2*
(a*d - b*c)) + (2*x*(3*B^2*b*d + A*B*b*d))/(a*d - b*c))/(a^2*b*g^3 + b^3*g^
3*x^2 + 2*a*b^2*g^3*x) - log((e*(a + b*x)^2)/(c + d*x)^2)^2*(B^2/(2*b^2*g^3
*(2*a*x + b*x^2 + a^2/b)) - (B^2*d^2)/(2*b*g^3*(a^2*d^2 + b^2*c^2 - 2*a*b*c
*d))) - (log((e*(a + b*x)^2)/(c + d*x)^2)*((A*B)/(b^2*d*g^3) + (2*B^2*x*(a*
d - b*c))/(b*g^3*(a^2*d^2 + b^2*c^2 - 2*a*b*c*d)) + (B^2*d^2*((2*a^2*d^2 +
b^2*c^2 - 3*a*b*c*d)/(b*d^3) + (a*(a*d - b*c))/(b*d^2)))/(b*g^3*(a^2*d^2 +
b^2*c^2 - 2*a*b*c*d))))/((b*x^2)/d + a^2/(b*d) + (2*a*x)/d) - (B*d^2*atan((
```

```
B*d^2*(2*b*d*x - (b^3*c^2*g^3 - a^2*b*d^2*g^3)/(b*g^3*(a*d - b*c)))*(A + 3*
B)*2i)/((a*d - b*c)*(6*B^2*d^2 + 2*A*B*d^2)))*(A + 3*B)*4i)/(b*g^3*(a*d - b
*c)^2)
```

**sympy [B]** time = 6.24, size = 879, normalized size = 3.23

$$-\frac{2Bd^2\left(A+3B\right)\log\left(x+\frac{2ABad^3+2ABbcd^2+6B^2ad^3+6B^2bcd^2-\frac{2Ba^3d^5(A+3B)}{(ad-bc)^2}+\frac{6Ba^2bcd^4(A+3B)}{(ad-bc)^2}-\frac{6Bab^2c^2d^3(A+3B)}{(ad-bc)^2}+\frac{2Bb^3c^3d^2(A+3B)}{(ad-bc)^2}}{4ABbd^3+12B^2bd^3}\right)}{bg^3\left(ad-bc\right)^2}+\frac{2Bd^2}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*ln(e*(b*x+a)**2/(d*x+c)**2))**2/(b*g*x+a*g)**3,x)
```

```
[Out] -2*B*d**2*(A + 3*B)*log(x + (2*A*B*a*d**3 + 2*A*B*b*c*d**2 + 6*B**2*a*d**3
+ 6*B**2*b*c*d**2 - 2*B*a**3*d**5*(A + 3*B)/(a*d - b*c)**2 + 6*B*a**2*b*c*d
**4*(A + 3*B)/(a*d - b*c)**2 - 6*B*a*b**2*c**2*d**3*(A + 3*B)/(a*d - b*c)**
2 + 2*B*b**3*c**3*d**2*(A + 3*B)/(a*d - b*c)**2)/(4*A*B*b*d**3 + 12*B**2*b*
d**3))/(b*g**3*(a*d - b*c)**2) + 2*B*d**2*(A + 3*B)*log(x + (2*A*B*a*d**3 +
 2*A*B*b*c*d**2 + 6*B**2*a*d**3 + 6*B**2*b*c*d**2 + 2*B*a**3*d**5*(A + 3*B)
/(a*d - b*c)**2 - 6*B*a**2*b*c*d**4*(A + 3*B)/(a*d - b*c)**2 + 6*B*a*b**2*c
**2*d**3*(A + 3*B)/(a*d - b*c)**2 - 2*B*b**3*c**3*d**2*(A + 3*B)/(a*d - b*c
)**2)/(4*A*B*b*d**3 + 12*B**2*b*d**3))/(b*g**3*(a*d - b*c)**2) + (2*B**2*a*
c*d + 2*B**2*a*d**2*x - B**2*b*c**2 + B**2*b*d**2*x**2)*log(e*(a + b*x)**2/
(c + d*x)**2)**2/(2*a**4*d**2*g**3 - 4*a**3*b*c*d*g**3 + 4*a**3*b*d**2*g**3
*x + 2*a**2*b**2*c**2*g**3 - 8*a**2*b**2*c*d*g**3*x + 2*a**2*b**2*d**2*g**3
*x**2 + 4*a*b**3*c**2*g**3*x - 4*a*b**3*c*d*g**3*x**2 + 2*b**4*c**2*g**3*x*
*2) + (-A*B*a*d + A*B*b*c - 3*B**2*a*d + B**2*b*c - 2*B**2*b*d*x)*log(e*(a
+ b*x)**2/(c + d*x)**2)/(a**3*b*d*g**3 - a**2*b**2*c*g**3 + 2*a**2*b**2*d*g
**3*x - 2*a*b**3*c*g**3*x + a*b**3*d*g**3*x**2 - b**4*c*g**3*x**2) + (-A**2
*a*d + A**2*b*c - 6*A*B*a*d + 2*A*B*b*c - 14*B**2*a*d + 2*B**2*b*c + x*(-4*
A*B*b*d - 12*B**2*b*d))/(2*a**3*b*d*g**3 - 2*a**2*b**2*c*g**3 + x**2*(2*a*b
**3*d*g**3 - 2*b**4*c*g**3) + x*(4*a**2*b**2*d*g**3 - 4*a*b**3*c*g**3))
```

# 3.135 $\int \frac{\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}{(ag+bgx)^4}\,dx$

**Optimal**. Leaf size=429

$$-\frac{b^2(c+dx)^3\left(B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+A\right)^2}{3g^4(a+bx)^3(bc-ad)^3}-\frac{4b^2B(c+dx)^3\left(B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+A\right)}{9g^4(a+bx)^3(bc-ad)^3}-\frac{d^2(c+dx)\left(B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+A\right)^2}{g^4(a+bx)(bc-ad)^3}-\frac{4}{}$$

```
[Out] -8*B^2*d^2*(d*x+c)/(-a*d+b*c)^3/g^4/(b*x+a)+2*b*B^2*d*(d*x+c)^2/(-a*d+b*c)^
3/g^4/(b*x+a)^2-8/27*b^2*B^2*(d*x+c)^3/(-a*d+b*c)^3/g^4/(b*x+a)^3-4*B*d^2*(
d*x+c)*(A+B*ln(e*(b*x+a)^2/(d*x+c)^2))/(-a*d+b*c)^3/g^4/(b*x+a)+2*b*B*d*(d*
x+c)^2*(A+B*ln(e*(b*x+a)^2/(d*x+c)^2))/(-a*d+b*c)^3/g^4/(b*x+a)^2-4/9*b^2*B
*(d*x+c)^3*(A+B*ln(e*(b*x+a)^2/(d*x+c)^2))/(-a*d+b*c)^3/g^4/(b*x+a)^3-d^2*(
d*x+c)*(A+B*ln(e*(b*x+a)^2/(d*x+c)^2))^2/(-a*d+b*c)^3/g^4/(b*x+a)+b*d*(d*x+
c)^2*(A+B*ln(e*(b*x+a)^2/(d*x+c)^2))^2/(-a*d+b*c)^3/g^4/(b*x+a)^2-1/3*b^2*(
d*x+c)^3*(A+B*ln(e*(b*x+a)^2/(d*x+c)^2))^2/(-a*d+b*c)^3/g^4/(b*x+a)^3
```

**Rubi [C]** time = 1.23, antiderivative size = 692, normalized size of antiderivative = 1.61, number of steps used = 34, number of rules used = 11, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.324, Rules used = {2525, 12, 2528, 44, 2524, 2418, 2390, 2301, 2394, 2393, 2391}

$$-\frac{8B^2d^3\text{PolyLog}\left(2,-\frac{d(a+bx)}{bc-ad}\right)}{3bg^4(bc-ad)^3}-\frac{8B^2d^3\text{PolyLog}\left(2,\frac{b(c+dx)}{bc-ad}\right)}{3bg^4(bc-ad)^3}-\frac{4Bd^3\log(a+bx)\left(B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+A\right)}{3bg^4(bc-ad)^3}+\frac{4Bd^3\log(c}{}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])^2/(a*g + b*g*x)^4,x]
```

```
[Out] (-8*B^2)/(27*b*g^4*(a + b*x)^3) + (10*B^2*d)/(9*b*(b*c - a*d)*g^4*(a + b*x)
^2) - (44*B^2*d^2)/(9*b*(b*c - a*d)^2*g^4*(a + b*x)) - (44*B^2*d^3*Log[a +
b*x])/(9*b*(b*c - a*d)^3*g^4) + (4*B^2*d^3*Log[a + b*x]^2)/(3*b*(b*c - a*d)
^3*g^4) - (4*B*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2]))/(9*b*g^4*(a + b*x)
^3) + (2*B*d*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2]))/(3*b*(b*c - a*d)*g^4
*(a + b*x)^2) - (4*B*d^2*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2]))/(3*b*(b*
c - a*d)^2*g^4*(a + b*x)) - (4*B*d^3*Log[a + b*x]*(A + B*Log[(e*(a + b*x)^2
)/(c + d*x)^2]))/(3*b*(b*c - a*d)^3*g^4) - (A + B*Log[(e*(a + b*x)^2)/(c +
d*x)^2])^2/(3*b*g^4*(a + b*x)^3) + (44*B^2*d^3*Log[c + d*x])/(9*b*(b*c - a*
d)^3*g^4) - (8*B^2*d^3*Log[-((d*(a + b*x))/(b*c - a*d))]*Log[c + d*x])/(3*b
*(b*c - a*d)^3*g^4) + (4*B*d^3*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])*Log
[c + d*x])/(3*b*(b*c - a*d)^3*g^4) + (4*B^2*d^3*Log[c + d*x]^2)/(3*b*(b*c -
 a*d)^3*g^4) - (8*B^2*d^3*Log[a + b*x]*Log[(b*(c + d*x))/(b*c - a*d)])/(3*b
```

```
*(b*c - a*d)^3*g^4) - (8*B^2*d^3*PolyLog[2, -((d*(a + b*x))/(b*c - a*d))])/
(3*b*(b*c - a*d)^3*g^4) - (8*B^2*d^3*PolyLog[2, (b*(c + d*x))/(b*c - a*d)])
/(3*b*(b*c - a*d)^3*g^4)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 2301

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

Rule 2390

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_))^(q_.), x_Symbol] :> Dist[1/e, Subst[Int[((f*x)/d)^q*(a + b*Log[c*x^
n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && E
qQ[e*f - d*g, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2393

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + (c*e*x)/g])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2394

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d + e*x
)^n]))/g, x] - Dist[(b*e*n)/g, Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
```

```
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2418

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*(RFx_), x_Sy
mbol] :> With[{u = ExpandIntegrand[(a + b*Log[c*(d + e*x)^n])^p, RFx, x]},
Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d, e, n}, x] && RationalFunctionQ[
RFx, x] && IntegerQ[p]
```

Rule 2524

```
Int[((a_.) + Log[(c_.)*(RFx_)^(p_.)]*(b_.))^(n_.)/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[(Log[d + e*x]*(a + b*Log[c*RFx^p])^n)/e, x] - Dist[(b*n*p)/e
, Int[(Log[d + e*x]*(a + b*Log[c*RFx^p])^(n - 1)*D[RFx, x])/RFx, x], x] /;
FreeQ[{a, b, c, d, e, p}, x] && RationalFunctionQ[RFx, x] && IGtQ[n, 0]
```

Rule 2525

```
Int[((a_.) + Log[(c_.)*(RFx_)^(p_.)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_))^(m_.
), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(a + b*Log[c*RFx^p])^n)/(e*(m + 1))
, x] - Dist[(b*n*p)/(e*(m + 1)), Int[SimplifyIntegrand[((d + e*x)^(m + 1)*(
a + b*Log[c*RFx^p])^(n - 1)*D[RFx, x])/RFx, x], x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && RationalFunctionQ[RFx, x] && IGtQ[n, 0] && (EqQ[n, 1] ||
IntegerQ[m]) && NeQ[m, -1]
```

Rule 2528

```
Int[((a_.) + Log[(c_.)*(RFx_)^(p_.)]*(b_.))^(n_.)*(RGx_), x_Symbol] :> With
[{u = ExpandIntegrand[(a + b*Log[c*RFx^p])^n, RGx, x]}, Int[u, x] /; SumQ[u
]] /; FreeQ[{a, b, c, p}, x] && RationalFunctionQ[RFx, x] && RationalFuncti
onQ[RGx, x] && IGtQ[n, 0]
```

Rubi steps

$$\int \frac{\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}{(ag+bgx)^4}\,dx = -\frac{\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}{3bg^4(a+bx)^3} + \frac{(2B)\int \frac{2(bc-ad)\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}{g^3(a+bx)^4(c+dx)}\,dx}{3bg}$$

$$= -\frac{\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}{3bg^4(a+bx)^3} + \frac{(4B(bc-ad))\int \frac{A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}{(a+bx)^4(c+dx)}\,dx}{3bg^4}$$

$$= -\frac{\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}{3bg^4(a+bx)^3} + \frac{(4B(bc-ad))\int \left(\frac{b\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}{(bc-ad)(a+bx)^4} - \frac{bd\left(A+B\log\left(\frac{e(a+bx}{(c+dx}\right.\right.}{(bc-ad)^2(a+bx)}\right.}{}$$

$$= -\frac{\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}{3bg^4(a+bx)^3} + \frac{(4B)\int \frac{A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}{(a+bx)^4}\,dx}{3g^4} - \frac{\left(4Bd^3\right)\int \frac{A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right.}{a+bx}}{3(bc-ad)^3g^4}$$

$$= -\frac{4B\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}{9bg^4(a+bx)^3} + \frac{2Bd\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}{3b(bc-ad)g^4(a+bx)^2} - \frac{4Bd^2\left(A+B\log\left(\frac{e(}{(}\right.\right.}{3b(bc-ad)^2g^4(a}$$

$$= -\frac{4B\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}{9bg^4(a+bx)^3} + \frac{2Bd\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}{3b(bc-ad)g^4(a+bx)^2} - \frac{4Bd^2\left(A+B\log\left(\frac{e(}{(}\right.\right.}{3b(bc-ad)^2g^4(a}$$

$$= -\frac{4B\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}{9bg^4(a+bx)^3} + \frac{2Bd\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}{3b(bc-ad)g^4(a+bx)^2} - \frac{4Bd^2\left(A+B\log\left(\frac{e(}{(}\right.\right.}{3b(bc-ad)^2g^4(a}$$

$$= -\frac{8B^2}{27bg^4(a+bx)^3} + \frac{10B^2d}{9b(bc-ad)g^4(a+bx)^2} - \frac{44B^2d^2}{9b(bc-ad)^2g^4(a+bx)} - \frac{44B^2d^3}{9b(bc}$$

$$= -\frac{8B^2}{27bg^4(a+bx)^3} + \frac{10B^2d}{9b(bc-ad)g^4(a+bx)^2} - \frac{44B^2d^2}{9b(bc-ad)^2g^4(a+bx)} - \frac{44B^2d^3}{9b(bc}$$

$$= -\frac{8B^2}{27bg^4(a+bx)^3} + \frac{10B^2d}{9b(bc-ad)g^4(a+bx)^2} - \frac{44B^2d^2}{9b(bc-ad)^2g^4(a+bx)} - \frac{44B^2d^3}{9b(bc}$$

$$= -\frac{8B^2}{27bg^4(a+bx)^3} + \frac{10B^2d}{9b(bc-ad)g^4(a+bx)^2} - \frac{44B^2d^2}{9b(bc-ad)^2g^4(a+bx)} - \frac{44B^2d^3}{9b(bc}$$

**Mathematica [C]** time = 0.64, size = 598, normalized size = 1.39

$$-\frac{2B\left(18d^3(a+bx)^3\log(a+bx)\left(B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+A\right)-18d^3(a+bx)^3\log(c+dx)\left(B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+A\right)+18d^2(a+bx)^2(bc-ad)\left(B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+A\right)+6(bc-ad)^3}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])^2/(a*g + b*g*x)^4,x]
```

```
[Out] -1/27*(9*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])^2 + (2*B*(6*(b*c - a*d)^3
*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2]) - 9*d*(b*c - a*d)^2*(a + b*x)*(A
+ B*Log[(e*(a + b*x)^2)/(c + d*x)^2]) + 18*d^2*(b*c - a*d)*(a + b*x)^2*(A +
 B*Log[(e*(a + b*x)^2)/(c + d*x)^2]) + 18*d^3*(a + b*x)^3*Log[a + b*x]*(A +
 B*Log[(e*(a + b*x)^2)/(c + d*x)^2]) - 18*d^3*(a + b*x)^3*(A + B*Log[(e*(a
+ b*x)^2)/(c + d*x)^2])*Log[c + d*x] + 36*B*d^2*(a + b*x)^2*(b*c - a*d + d*
(a + b*x)*Log[a + b*x] - d*(a + b*x)*Log[c + d*x]) - 9*B*d*(a + b*x)*((b*c
- a*d)^2 + 2*d*(-(b*c) + a*d)*(a + b*x) - 2*d^2*(a + b*x)^2*Log[a + b*x] +
2*d^2*(a + b*x)^2*Log[c + d*x]) + 2*B*(2*(b*c - a*d)^3 - 3*d*(b*c - a*d)^2*
(a + b*x) + 6*d^2*(b*c - a*d)*(a + b*x)^2 + 6*d^3*(a + b*x)^3*Log[a + b*x]
- 6*d^3*(a + b*x)^3*Log[c + d*x]) - 18*B*d^3*(a + b*x)^3*(Log[a + b*x]*(Log
[a + b*x] - 2*Log[(b*(c + d*x))/(b*c - a*d)]) - 2*PolyLog[2, (d*(a + b*x))/
(-(b*c) + a*d)]) + 18*B*d^3*(a + b*x)^3*((2*Log[(d*(a + b*x))/(-(b*c) + a*d
)] - Log[c + d*x])*Log[c + d*x] + 2*PolyLog[2, (b*(c + d*x))/(b*c - a*d)]))
)/(b*c - a*d)^3)/(b*g^4*(a + b*x)^3)
```

**fricas [A]** time = 0.92, size = 719, normalized size = 1.68

$$-\frac{\left(9A^2+12AB+8B^2\right)b^3c^3-27\left(A^2+2AB+2B^2\right)ab^2c^2d+27\left(A^2+4AB+8B^2\right)a^2bcd^2-\left(9A^2+66AB+1\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*(b*x+a)^2/(d*x+c)^2))^2/(b*g*x+a*g)^4,x, algorithm="fr
icas")
```

```
[Out] -1/27*((9*A^2 + 12*A*B + 8*B^2)*b^3*c^3 - 27*(A^2 + 2*A*B + 2*B^2)*a*b^2*c^
2*d + 27*(A^2 + 4*A*B + 8*B^2)*a^2*b*c*d^2 - (9*A^2 + 66*A*B + 170*B^2)*a^3
*d^3 + 12*((3*A*B + 11*B^2)*b^3*c*d^2 - (3*A*B + 11*B^2)*a*b^2*d^3)*x^2 + 9
*(B^2*b^3*d^3*x^3 + 3*B^2*a*b^2*d^3*x^2 + 3*B^2*a^2*b*d^3*x + B^2*b^3*c^3 -
 3*B^2*a*b^2*c^2*d + 3*B^2*a^2*b*c*d^2)*log((b^2*e*x^2 + 2*a*b*e*x + a^2*e)
/(d^2*x^2 + 2*c*d*x + c^2))^2 - 6*((3*A*B + 5*B^2)*b^3*c^2*d - 18*(A*B + 3*
B^2)*a*b^2*c*d^2 + (15*A*B + 49*B^2)*a^2*b*d^3)*x + 6*((3*A*B + 11*B^2)*b^3
*d^3*x^3 + (3*A*B + 2*B^2)*b^3*c^3 - 9*(A*B + B^2)*a*b^2*c^2*d + 9*(A*B + 2
```

```
*B^2)*a^2*b*c*d^2 + 3*(2*B^2*b^3*c*d^2 + 3*(A*B + 3*B^2)*a*b^2*d^3)*x^2 - 3
*(B^2*b^3*c^2*d - 6*B^2*a*b^2*c*d^2 - 3*(A*B + 2*B^2)*a^2*b*d^3)*x)*log((b^
2*e*x^2 + 2*a*b*e*x + a^2*e)/(d^2*x^2 + 2*c*d*x + c^2)))/((b^7*c^3 - 3*a*b^
6*c^2*d + 3*a^2*b^5*c*d^2 - a^3*b^4*d^3)*g^4*x^3 + 3*(a*b^6*c^3 - 3*a^2*b^5
*c^2*d + 3*a^3*b^4*c*d^2 - a^4*b^3*d^3)*g^4*x^2 + 3*(a^2*b^5*c^3 - 3*a^3*b^
4*c^2*d + 3*a^4*b^3*c*d^2 - a^5*b^2*d^3)*g^4*x + (a^3*b^4*c^3 - 3*a^4*b^3*c
^2*d + 3*a^5*b^2*c*d^2 - a^6*b*d^3)*g^4)
```

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(B\log\left(\frac{(bx+a)^2e}{(dx+c)^2}\right)+A\right)^2}{\left(bgx+ag\right)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*(b*x+a)^2/(d*x+c)^2))^2/(b*g*x+a*g)^4,x, algorithm="gi
ac")
```

```
[Out] integrate((B*log((b*x + a)^2*e/(d*x + c)^2) + A)^2/(b*g*x + a*g)^4, x)
```

**maple [B]** time = 0.23, size = 1343, normalized size = 3.13

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*ln((b*x+a)^2/(d*x+c)^2*e)+A)^2/(b*g*x+a*g)^4,x)
```

```
[Out] 1/3*d^3/g^4*A^2*b^2/(a*d-b*c)^3/(1/(d*x+c)*a*d-1/(d*x+c)*b*c+b)^3+d^3/g^4*A
^2/(a*d-b*c)^3/(1/(d*x+c)*a*d-1/(d*x+c)*b*c+b)-d^3/g^4*A^2*b/(a*d-b*c)^3/(1
/(d*x+c)*a*d-1/(d*x+c)*b*c+b)^2-170/27*d^3/g^4/(1/(d*x+c)*a*d-1/(d*x+c)*b*c
+b)^3*B^2/b/(d*x+c)^3+22/9*d^3/g^4/(1/(d*x+c)*a*d-1/(d*x+c)*b*c+b)^3*b^2*B^
2/(a^3*d^3-3*a^2*b*c*d^2+3*a*b^2*c^2*d-b^3*c^3)*ln((1/(d*x+c)*a*d-1/(d*x+c)
*b*c+b)^2/d^2*e)-44/9*d^3/g^4/(1/(d*x+c)*a*d-1/(d*x+c)*b*c+b)^3*B^2*b/(a^2*
d^2-2*a*b*c*d+b^2*c^2)/(d*x+c)-98/9*d^3/g^4/(1/(d*x+c)*a*d-1/(d*x+c)*b*c+b)
^3*B^2/(a*d-b*c)/(d*x+c)^2+4*d^3/g^4/(1/(d*x+c)*a*d-1/(d*x+c)*b*c+b)^3*B^2/
(a*d-b*c)/(d*x+c)^2*ln((1/(d*x+c)*a*d-1/(d*x+c)*b*c+b)^2/d^2*e)+1/3*d^3/g^4
/(1/(d*x+c)*a*d-1/(d*x+c)*b*c+b)^3*B^2*b^2/(a^3*d^3-3*a^2*b*c*d^2+3*a*b^2*c
^2*d-b^3*c^3)*ln((1/(d*x+c)*a*d-1/(d*x+c)*b*c+b)^2/d^2*e)^2+d^3/g^4/(1/(d*x
+c)*a*d-1/(d*x+c)*b*c+b)^3*B^2/(a*d-b*c)/(d*x+c)^2*ln((1/(d*x+c)*a*d-1/(d*x
+c)*b*c+b)^2/d^2*e)^2+6*d^3/g^4/(1/(d*x+c)*a*d-1/(d*x+c)*b*c+b)^3*B^2*b/(a^
2*d^2-2*a*b*c*d+b^2*c^2)/(d*x+c)*ln((1/(d*x+c)*a*d-1/(d*x+c)*b*c+b)^2/d^2*e
)+d^3/g^4/(1/(d*x+c)*a*d-1/(d*x+c)*b*c+b)^3*B^2*b/(a^2*d^2-2*a*b*c*d+b^2*c^
2)/(d*x+c)*ln((1/(d*x+c)*a*d-1/(d*x+c)*b*c+b)^2/d^2*e)^2-22/9*d^3/g^4/(1/(d
*x+c)*a*d-1/(d*x+c)*b*c+b)^3*A*B/b/(d*x+c)^3+2/3*d^3/g^4/(1/(d*x+c)*a*d-1/(
```

```
d*x+c)*b*c+b)^3*b^2*A*B/(a^3*d^3-3*a^2*b*c*d^2+3*a*b^2*c^2*d-b^3*c^3)*ln((1
/(d*x+c)*a*d-1/(d*x+c)*b*c+b)^2/d^2*e)-4/3*d^3/g^4/(1/(d*x+c)*a*d-1/(d*x+c)
*b*c+b)^3*A*B*b/(a^2*d^2-2*a*b*c*d+b^2*c^2)/(d*x+c)-10/3*d^3/g^4/(1/(d*x+c)
*a*d-1/(d*x+c)*b*c+b)^3*A*B/(a*d-b*c)/(d*x+c)^2+2*d^3/g^4/(1/(d*x+c)*a*d-1/
(d*x+c)*b*c+b)^3*A*B/(a*d-b*c)/(d*x+c)^2*ln((1/(d*x+c)*a*d-1/(d*x+c)*b*c+b)
^2/d^2*e)+2*d^3/g^4/(1/(d*x+c)*a*d-1/(d*x+c)*b*c+b)^3*A*B*b/(a^2*d^2-2*a*b*
c*d+b^2*c^2)/(d*x+c)*ln((1/(d*x+c)*a*d-1/(d*x+c)*b*c+b)^2/d^2*e)
```

**maxima [B]** time = 2.67, size = 1575, normalized size = 3.67

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*(b*x+a)^2/(d*x+c)^2))^2/(b*g*x+a*g)^4,x, algorithm="ma
xima")
```

```
[Out] -2/27*(3*((6*b^2*d^2*x^2 + 2*b^2*c^2 - 7*a*b*c*d + 11*a^2*d^2 - 3*(b^2*c*d
- 5*a*b*d^2)*x)/((b^6*c^2 - 2*a*b^5*c*d + a^2*b^4*d^2)*g^4*x^3 + 3*(a*b^5*c
^2 - 2*a^2*b^4*c*d + a^3*b^3*d^2)*g^4*x^2 + 3*(a^2*b^4*c^2 - 2*a^3*b^3*c*d
+ a^4*b^2*d^2)*g^4*x + (a^3*b^3*c^2 - 2*a^4*b^2*c*d + a^5*b*d^2)*g^4) + 6*d
^3*log(b*x + a)/((b^4*c^3 - 3*a*b^3*c^2*d + 3*a^2*b^2*c*d^2 - a^3*b*d^3)*g^
4) - 6*d^3*log(d*x + c)/((b^4*c^3 - 3*a*b^3*c^2*d + 3*a^2*b^2*c*d^2 - a^3*b
*d^3)*g^4))*log(b^2*e*x^2/(d^2*x^2 + 2*c*d*x + c^2) + 2*a*b*e*x/(d^2*x^2 +
2*c*d*x + c^2) + a^2*e/(d^2*x^2 + 2*c*d*x + c^2)) + (4*b^3*c^3 - 27*a*b^2*c
^2*d + 108*a^2*b*c*d^2 - 85*a^3*d^3 + 66*(b^3*c*d^2 - a*b^2*d^3)*x^2 - 18*(
b^3*d^3*x^3 + 3*a*b^2*d^3*x^2 + 3*a^2*b*d^3*x + a^3*d^3)*log(b*x + a)^2 - 1
8*(b^3*d^3*x^3 + 3*a*b^2*d^3*x^2 + 3*a^2*b*d^3*x + a^3*d^3)*log(d*x + c)^2
- 3*(5*b^3*c^2*d - 54*a*b^2*c*d^2 + 49*a^2*b*d^3)*x + 66*(b^3*d^3*x^3 + 3*a
*b^2*d^3*x^2 + 3*a^2*b*d^3*x + a^3*d^3)*log(b*x + a) - 6*(11*b^3*d^3*x^3 +
33*a*b^2*d^3*x^2 + 33*a^2*b*d^3*x + 11*a^3*d^3 - 6*(b^3*d^3*x^3 + 3*a*b^2*d
^3*x^2 + 3*a^2*b*d^3*x + a^3*d^3)*log(b*x + a))*log(d*x + c))/(a^3*b^4*c^3*
g^4 - 3*a^4*b^3*c^2*d*g^4 + 3*a^5*b^2*c*d^2*g^4 - a^6*b*d^3*g^4 + (b^7*c^3*
g^4 - 3*a*b^6*c^2*d*g^4 + 3*a^2*b^5*c*d^2*g^4 - a^3*b^4*d^3*g^4)*x^3 + 3*(a
*b^6*c^3*g^4 - 3*a^2*b^5*c^2*d*g^4 + 3*a^3*b^4*c*d^2*g^4 - a^4*b^3*d^3*g^4)
*x^2 + 3*(a^2*b^5*c^3*g^4 - 3*a^3*b^4*c^2*d*g^4 + 3*a^4*b^3*c*d^2*g^4 - a^5
*b^2*d^3*g^4)*x))*B^2 - 2/9*A*B*((6*b^2*d^2*x^2 + 2*b^2*c^2 - 7*a*b*c*d + 1
1*a^2*d^2 - 3*(b^2*c*d - 5*a*b*d^2)*x)/((b^6*c^2 - 2*a*b^5*c*d + a^2*b^4*d^
2)*g^4*x^3 + 3*(a*b^5*c^2 - 2*a^2*b^4*c*d + a^3*b^3*d^2)*g^4*x^2 + 3*(a^2*b
^4*c^2 - 2*a^3*b^3*c*d + a^4*b^2*d^2)*g^4*x + (a^3*b^3*c^2 - 2*a^4*b^2*c*d
+ a^5*b*d^2)*g^4) + 3*log(b^2*e*x^2/(d^2*x^2 + 2*c*d*x + c^2) + 2*a*b*e*x/(
d^2*x^2 + 2*c*d*x + c^2) + a^2*e/(d^2*x^2 + 2*c*d*x + c^2))/(b^4*g^4*x^3 +
3*a*b^3*g^4*x^2 + 3*a^2*b^2*g^4*x + a^3*b*g^4) + 6*d^3*log(b*x + a)/((b^4*c
^3 - 3*a*b^3*c^2*d + 3*a^2*b^2*c*d^2 - a^3*b*d^3)*g^4) - 6*d^3*log(d*x + c)
/((b^4*c^3 - 3*a*b^3*c^2*d + 3*a^2*b^2*c*d^2 - a^3*b*d^3)*g^4)) - 1/3*B^2*l
og(b^2*e*x^2/(d^2*x^2 + 2*c*d*x + c^2) + 2*a*b*e*x/(d^2*x^2 + 2*c*d*x + c^2
```

```
) + a^2*e/(d^2*x^2 + 2*c*d*x + c^2))^2/(b^4*g^4*x^3 + 3*a*b^3*g^4*x^2 + 3*a
^2*b^2*g^4*x + a^3*b*g^4) - 1/3*A^2/(b^4*g^4*x^3 + 3*a*b^3*g^4*x^2 + 3*a^2*
b^2*g^4*x + a^3*b*g^4)
```

**mupad [B]** time = 7.67, size = 1069, normalized size = 2.49

$$\frac{\frac{9\,A^2\,a^2\,d^2-18\,A^2\,a\,b\,c\,d+9\,A^2\,b^2\,c^2+66\,A\,B\,a^2\,d^2-42\,A\,B\,a\,b\,c\,d+12\,A\,B\,b^2\,c^2+170\,B^2\,a^2\,d^2-46\,B^2\,a\,b\,c\,d+8\,B^2\,b^2\,c^2}{3\,(a\,d-b\,c)}+\frac{2\,x\left(-5\,c\,B^2\,b^2\,d+49\,a\,B^2\,b\,d^2-3\cdots\right.}{a\,d-b\cdots}}{x\,\left(27\,a^2\,b^3\,c\,g^4-27\,a^3\,b^2\,d\,g^4\right)-x^2\,\left(27\,a^2\,b^3\,d\,g^4-27\,a\,b^4\,c\,g^4\right)+x^3\,\left(9\,b^5\,c\,g^4-9\,a\,b^4\,d\,g^4\right)\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*log((e*(a + b*x)^2)/(c + d*x)^2))^2/(a*g + b*g*x)^4,x)
```

```
[Out] ((9*A^2*a^2*d^2 + 9*A^2*b^2*c^2 + 170*B^2*a^2*d^2 + 8*B^2*b^2*c^2 + 66*A*B*
a^2*d^2 + 12*A*B*b^2*c^2 - 18*A^2*a*b*c*d - 46*B^2*a*b*c*d - 42*A*B*a*b*c*d
)/(3*(a*d - b*c)) + (2*x*(49*B^2*a*b*d^2 - 5*B^2*b^2*c*d + 15*A*B*a*b*d^2 -
 3*A*B*b^2*c*d))/(a*d - b*c) + (4*d*x^2*(11*B^2*b^2*d + 3*A*B*b^2*d))/(a*d
- b*c))/(x*(27*a^2*b^3*c*g^4 - 27*a^3*b^2*d*g^4) - x^2*(27*a^2*b^3*d*g^4 -
27*a*b^4*c*g^4) + x^3*(9*b^5*c*g^4 - 9*a*b^4*d*g^4) + 9*a^3*b^2*c*g^4 - 9*a
^4*b*d*g^4) - log((e*(a + b*x)^2)/(c + d*x)^2)^2*(B^2/(3*b^2*g^4*(3*a^2*x +
 a^3/b + b^2*x^3 + 3*a*b*x^2)) - (B^2*d^3)/(3*b*g^4*(a^3*d^3 - b^3*c^3 + 3*
a*b^2*c^2*d - 3*a^2*b*c*d^2))) - (log((e*(a + b*x)^2)/(c + d*x)^2)*((2*A*B)
/(3*b^2*d*g^4) + (2*B^2*d^3*(a*((3*a^2*d^2 + b^2*c^2 - 4*a*b*c*d)/(3*b*d^3)
 + (2*a*(a*d - b*c))/(3*b*d^2)) + (2*(3*a^3*d^3 - b^3*c^3 + 4*a*b^2*c^2*d -
 6*a^2*b*c*d^2))/(3*b*d^4)))/(3*b*g^4*(a^3*d^3 - b^3*c^3 + 3*a*b^2*c^2*d -
3*a^2*b*c*d^2)) - (2*B^2*d^3*x^2*((2*(b^2*c - a*b*d))/(3*d^2) - (4*b*(a*d -
 b*c))/(3*d^2)))/(3*b*g^4*(a^3*d^3 - b^3*c^3 + 3*a*b^2*c^2*d - 3*a^2*b*c*d^
2)) + (2*B^2*d^3*x*(b*((3*a^2*d^2 + b^2*c^2 - 4*a*b*c*d)/(3*b*d^3) + (2*a*(
a*d - b*c))/(3*b*d^2)) + (2*(3*a^2*d^2 + b^2*c^2 - 4*a*b*c*d))/(3*d^3) + (4
*a*(a*d - b*c))/(3*d^2)))/(3*b*g^4*(a^3*d^3 - b^3*c^3 + 3*a*b^2*c^2*d - 3*a
^2*b*c*d^2))))/((3*a^2*x)/d + a^3/(b*d) + (b^2*x^3)/d + (3*a*b*x^2)/d) - (B
*d^3*atan((B*d^3*((b^4*c^3*g^4 + a^3*b*d^3*g^4 - a*b^3*c^2*d*g^4 - a^2*b^2*
c*d^2*g^4)/(b^3*c^2*g^4 + a^2*b*d^2*g^4 - 2*a*b^2*c*d*g^4) + 2*b*d*x)*(3*A
+ 11*B)*(b^3*c^2*g^4 + a^2*b*d^2*g^4 - 2*a*b^2*c*d*g^4)*4i)/(b*g^4*(a*d - b
*c)^3*(44*B^2*d^3 + 12*A*B*d^3)))*(3*A + 11*B)*8i)/(9*b*g^4*(a*d - b*c)^3)
```

**sympy [B]** time = 34.03, size = 1561, normalized size = 3.64

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*ln(e*(b*x+a)**2/(d*x+c)**2))**2/(b*g*x+a*g)**4,x)
```

```
[Out] -4*B*d**3*(3*A + 11*B)*log(x + (12*A*B*a*d**4 + 12*A*B*b*c*d**3 + 44*B**2*a
*d**4 + 44*B**2*b*c*d**3 - 4*B*a**4*d**7*(3*A + 11*B)/(a*d - b*c)**3 + 16*B
*a**3*b*c*d**6*(3*A + 11*B)/(a*d - b*c)**3 - 24*B*a**2*b**2*c**2*d**5*(3*A
+ 11*B)/(a*d - b*c)**3 + 16*B*a*b**3*c**3*d**4*(3*A + 11*B)/(a*d - b*c)**3
- 4*B*b**4*c**4*d**3*(3*A + 11*B)/(a*d - b*c)**3)/(24*A*B*b*d**4 + 88*B**2*
b*d**4))/(9*b*g**4*(a*d - b*c)**3) + 4*B*d**3*(3*A + 11*B)*log(x + (12*A*B*
a*d**4 + 12*A*B*b*c*d**3 + 44*B**2*a*d**4 + 44*B**2*b*c*d**3 + 4*B*a**4*d**
7*(3*A + 11*B)/(a*d - b*c)**3 - 16*B*a**3*b*c*d**6*(3*A + 11*B)/(a*d - b*c)
**3 + 24*B*a**2*b**2*c**2*d**5*(3*A + 11*B)/(a*d - b*c)**3 - 16*B*a*b**3*c*
*3*d**4*(3*A + 11*B)/(a*d - b*c)**3 + 4*B*b**4*c**4*d**3*(3*A + 11*B)/(a*d
- b*c)**3)/(24*A*B*b*d**4 + 88*B**2*b*d**4))/(9*b*g**4*(a*d - b*c)**3) + (3
*B**2*a**2*c*d**2 + 3*B**2*a**2*d**3*x - 3*B**2*a*b*c**2*d + 3*B**2*a*b*d**
3*x**2 + B**2*b**2*c**3 + B**2*b**2*d**3*x**3)*log(e*(a + b*x)**2/(c + d*x)
**2)**2/(3*a**6*d**3*g**4 - 9*a**5*b*c*d**2*g**4 + 9*a**5*b*d**3*g**4*x + 9
*a**4*b**2*c**2*d*g**4 - 27*a**4*b**2*c*d**2*g**4*x + 9*a**4*b**2*d**3*g**4
*x**2 - 3*a**3*b**3*c**3*g**4 + 27*a**3*b**3*c**2*d*g**4*x - 27*a**3*b**3*c
*d**2*g**4*x**2 + 3*a**3*b**3*d**3*g**4*x**3 - 9*a**2*b**4*c**3*g**4*x + 27
*a**2*b**4*c**2*d*g**4*x**2 - 9*a**2*b**4*c*d**2*g**4*x**3 - 9*a*b**5*c**3*
g**4*x**2 + 9*a*b**5*c**2*d*g**4*x**3 - 3*b**6*c**3*g**4*x**3) + (-6*A*B*a*
*2*d**2 + 12*A*B*a*b*c*d - 6*A*B*b**2*c**2 - 22*B**2*a**2*d**2 + 14*B**2*a*
b*c*d - 30*B**2*a*b*d**2*x - 4*B**2*b**2*c**2 + 6*B**2*b**2*c*d*x - 12*B**2
*b**2*d**2*x**2)*log(e*(a + b*x)**2/(c + d*x)**2)/(9*a**5*b*d**2*g**4 - 18*
a**4*b**2*c*d*g**4 + 27*a**4*b**2*d**2*g**4*x + 9*a**3*b**3*c**2*g**4 - 54*
a**3*b**3*c*d*g**4*x + 27*a**3*b**3*d**2*g**4*x**2 + 27*a**2*b**4*c**2*g**4
*x - 54*a**2*b**4*c*d*g**4*x**2 + 9*a**2*b**4*d**2*g**4*x**3 + 27*a*b**5*c*
*2*g**4*x**2 - 18*a*b**5*c*d*g**4*x**3 + 9*b**6*c**2*g**4*x**3) - (9*A**2*a
**2*d**2 - 18*A**2*a*b*c*d + 9*A**2*b**2*c**2 + 66*A*B*a**2*d**2 - 42*A*B*a
*b*c*d + 12*A*B*b**2*c**2 + 170*B**2*a**2*d**2 - 46*B**2*a*b*c*d + 8*B**2*b
**2*c**2 + x**2*(36*A*B*b**2*d**2 + 132*B**2*b**2*d**2) + x*(90*A*B*a*b*d**
2 - 18*A*B*b**2*c*d + 294*B**2*a*b*d**2 - 30*B**2*b**2*c*d))/(27*a**5*b*d**
2*g**4 - 54*a**4*b**2*c*d*g**4 + 27*a**3*b**3*c**2*g**4 + x**3*(27*a**2*b**
4*d**2*g**4 - 54*a*b**5*c*d*g**4 + 27*b**6*c**2*g**4) + x**2*(81*a**3*b**3*
d**2*g**4 - 162*a**2*b**4*c*d*g**4 + 81*a*b**5*c**2*g**4) + x*(81*a**4*b**2
*d**2*g**4 - 162*a**3*b**3*c*d*g**4 + 81*a**2*b**4*c**2*g**4))
```

# 3.136 $\int \frac{\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}{(ag+bgx)^5}\,dx$

**Optimal**. Leaf size=587

$$-\frac{b^3(c+dx)^4\left(B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+A\right)^2}{4g^5(a+bx)^4(bc-ad)^4}-\frac{b^3B(c+dx)^4\left(B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+A\right)}{4g^5(a+bx)^4(bc-ad)^4}+\frac{b^2d(c+dx)^3\left(B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+A\right)^2}{g^5(a+bx)^3(bc-ad)^4}$$

```
[Out] 8*B^2*d^3*(d*x+c)/(-a*d+b*c)^4/g^5/(b*x+a)-3*b*B^2*d^2*(d*x+c)^2/(-a*d+b*c)
^4/g^5/(b*x+a)^2+8/9*b^2*B^2*d*(d*x+c)^3/(-a*d+b*c)^4/g^5/(b*x+a)^3-1/8*b^3
*B^2*(d*x+c)^4/(-a*d+b*c)^4/g^5/(b*x+a)^4+4*B*d^3*(d*x+c)*(A+B*ln(e*(b*x+a)
^2/(d*x+c)^2))/(-a*d+b*c)^4/g^5/(b*x+a)-3*b*B*d^2*(d*x+c)^2*(A+B*ln(e*(b*x+
a)^2/(d*x+c)^2))/(-a*d+b*c)^4/g^5/(b*x+a)^2+4/3*b^2*B*d*(d*x+c)^3*(A+B*ln(e
*(b*x+a)^2/(d*x+c)^2))/(-a*d+b*c)^4/g^5/(b*x+a)^3-1/4*b^3*B*(d*x+c)^4*(A+B*
ln(e*(b*x+a)^2/(d*x+c)^2))/(-a*d+b*c)^4/g^5/(b*x+a)^4+d^3*(d*x+c)*(A+B*ln(e
*(b*x+a)^2/(d*x+c)^2))^2/(-a*d+b*c)^4/g^5/(b*x+a)-3/2*b*d^2*(d*x+c)^2*(A+B*
ln(e*(b*x+a)^2/(d*x+c)^2))^2/(-a*d+b*c)^4/g^5/(b*x+a)^2+b^2*d*(d*x+c)^3*(A+
B*ln(e*(b*x+a)^2/(d*x+c)^2))^2/(-a*d+b*c)^4/g^5/(b*x+a)^3-1/4*b^3*(d*x+c)^4
*(A+B*ln(e*(b*x+a)^2/(d*x+c)^2))^2/(-a*d+b*c)^4/g^5/(b*x+a)^4
```

**Rubi [C]** time = 1.39, antiderivative size = 757, normalized size of antiderivative = 1.29, number of steps used = 38, number of rules used = 11, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.324, Rules used = {2525, 12, 2528, 44, 2524, 2418, 2390, 2301, 2394, 2393, 2391}

$$\frac{2B^2d^4\text{PolyLog}\left(2,-\frac{d(a+bx)}{bc-ad}\right)}{bg^5(bc-ad)^4}+\frac{2B^2d^4\text{PolyLog}\left(2,\frac{b(c+dx)}{bc-ad}\right)}{bg^5(bc-ad)^4}+\frac{Bd^4\log(a+bx)\left(B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+A\right)}{bg^5(bc-ad)^4}-\frac{Bd^4\log(c+\dots}{\dots}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])^2/(a*g + b*g*x)^5,x]
```

```
[Out] -B^2/(8*b*g^5*(a + b*x)^4) + (7*B^2*d)/(18*b*(b*c - a*d)*g^5*(a + b*x)^3) -
 (13*B^2*d^2)/(12*b*(b*c - a*d)^2*g^5*(a + b*x)^2) + (25*B^2*d^3)/(6*b*(b*c
 - a*d)^3*g^5*(a + b*x)) + (25*B^2*d^4*Log[a + b*x])/(6*b*(b*c - a*d)^4*g^5
) - (B^2*d^4*Log[a + b*x]^2)/(b*(b*c - a*d)^4*g^5) - (B*(A + B*Log[(e*(a +
b*x)^2)/(c + d*x)^2]))/(4*b*g^5*(a + b*x)^4) + (B*d*(A + B*Log[(e*(a + b*x)
^2)/(c + d*x)^2]))/(3*b*(b*c - a*d)*g^5*(a + b*x)^3) - (B*d^2*(A + B*Log[(e
*(a + b*x)^2)/(c + d*x)^2]))/(2*b*(b*c - a*d)^2*g^5*(a + b*x)^2) + (B*d^3*(
A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2]))/(b*(b*c - a*d)^3*g^5*(a + b*x)) +
(B*d^4*Log[a + b*x]*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2]))/(b*(b*c - a*d
)^4*g^5) - (A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])^2/(4*b*g^5*(a + b*x)^4)
```

```
 - (25*B^2*d^4*Log[c + d*x])/(6*b*(b*c - a*d)^4*g^5) + (2*B^2*d^4*Log[-((d*
(a + b*x))/(b*c - a*d))]*Log[c + d*x])/(b*(b*c - a*d)^4*g^5) - (B*d^4*(A +
B*Log[(e*(a + b*x)^2)/(c + d*x)^2])*Log[c + d*x])/(b*(b*c - a*d)^4*g^5) - (
B^2*d^4*Log[c + d*x]^2)/(b*(b*c - a*d)^4*g^5) + (2*B^2*d^4*Log[a + b*x]*Log
[(b*(c + d*x))/(b*c - a*d)])/(b*(b*c - a*d)^4*g^5) + (2*B^2*d^4*PolyLog[2,
-((d*(a + b*x))/(b*c - a*d))])/(b*(b*c - a*d)^4*g^5) + (2*B^2*d^4*PolyLog[2
, (b*(c + d*x))/(b*c - a*d)])/(b*(b*c - a*d)^4*g^5)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 2301

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

Rule 2390

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_))^(q_.), x_Symbol] :> Dist[1/e, Subst[Int[((f*x)/d)^q*(a + b*Log[c*x^
n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && E
qQ[e*f - d*g, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2393

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + (c*e*x)/g])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2394

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d + e*x
)^n]))/g, x] - Dist[(b*e*n)/g, Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2418

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*(RFx_), x_Sy
mbol] :> With[{u = ExpandIntegrand[(a + b*Log[c*(d + e*x)^n])^p, RFx, x]},
Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d, e, n}, x] && RationalFunctionQ[
RFx, x] && IntegerQ[p]
```

Rule 2524

```
Int[((a_.) + Log[(c_.)*(RFx_)^(p_.)]*(b_.))^(n_.)/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[(Log[d + e*x]*(a + b*Log[c*RFx^p])^n)/e, x] - Dist[(b*n*p)/e
, Int[(Log[d + e*x]*(a + b*Log[c*RFx^p])^(n - 1)*D[RFx, x])/RFx, x], x] /;
FreeQ[{a, b, c, d, e, p}, x] && RationalFunctionQ[RFx, x] && IGtQ[n, 0]
```

Rule 2525

```
Int[((a_.) + Log[(c_.)*(RFx_)^(p_.)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_))^(m_.
), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(a + b*Log[c*RFx^p])^n)/(e*(m + 1))
, x] - Dist[(b*n*p)/(e*(m + 1)), Int[SimplifyIntegrand[((d + e*x)^(m + 1)*(
a + b*Log[c*RFx^p])^(n - 1)*D[RFx, x])/RFx, x], x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && RationalFunctionQ[RFx, x] && IGtQ[n, 0] && (EqQ[n, 1] ||
IntegerQ[m]) && NeQ[m, -1]
```

Rule 2528

```
Int[((a_.) + Log[(c_.)*(RFx_)^(p_.)]*(b_.))^(n_.)*(RGx_), x_Symbol] :> With
[{u = ExpandIntegrand[(a + b*Log[c*RFx^p])^n, RGx, x]}, Int[u, x] /; SumQ[u
]] /; FreeQ[{a, b, c, p}, x] && RationalFunctionQ[RFx, x] && RationalFuncti
onQ[RGx, x] && IGtQ[n, 0]
```

Rubi steps

$$\int \frac{\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}{(ag+bgx)^5}\,dx = -\frac{\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}{4bg^5(a+bx)^4}+\frac{B\int \frac{2(bc-ad)\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}{g^4(a+bx)^5(c+dx)}\,dx}{2bg}$$

$$= -\frac{\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}{4bg^5(a+bx)^4}+\frac{(B(bc-ad))\int \frac{A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}{(a+bx)^5(c+dx)}\,dx}{bg^5}$$

$$= -\frac{\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}{4bg^5(a+bx)^4}+\frac{(B(bc-ad))\int \left(\frac{b\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}{(bc-ad)(a+bx)^5}-\frac{bd\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}{(bc-ad)^2(a+bx)^4}\cdots\right.}{\cdots}$$

$$= -\frac{\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}{4bg^5(a+bx)^4}+\frac{B\int \frac{A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}{(a+bx)^5}\,dx}{g^5}+\frac{\left(Bd^4\right)\int \frac{A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}{a+bx}\,dx}{(bc-ad)^4g^5}-\cdots$$

$$= -\frac{B\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}{4bg^5(a+bx)^4}+\frac{Bd\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}{3b(bc-ad)g^5(a+bx)^3}-\frac{Bd^2\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\cdots\right.\right.}{2b(bc-ad)^2g^5(a+bx)\cdots}$$

$$= -\frac{B\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}{4bg^5(a+bx)^4}+\frac{Bd\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}{3b(bc-ad)g^5(a+bx)^3}-\frac{Bd^2\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\cdots\right.\right.}{2b(bc-ad)^2g^5(a+bx)\cdots}$$

$$= -\frac{B\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}{4bg^5(a+bx)^4}+\frac{Bd\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}{3b(bc-ad)g^5(a+bx)^3}-\frac{Bd^2\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\cdots\right.\right.}{2b(bc-ad)^2g^5(a+bx)\cdots}$$

$$= -\frac{B^2}{8bg^5(a+bx)^4}+\frac{7B^2d}{18b(bc-ad)g^5(a+bx)^3}-\frac{13B^2d^2}{12b(bc-ad)^2g^5(a+bx)^2}+\frac{\cdots}{6b(bc-\cdots}$$

$$= -\frac{B^2}{8bg^5(a+bx)^4}+\frac{7B^2d}{18b(bc-ad)g^5(a+bx)^3}-\frac{13B^2d^2}{12b(bc-ad)^2g^5(a+bx)^2}+\frac{\cdots}{6b(bc-\cdots}$$

$$= -\frac{B^2}{8bg^5(a+bx)^4}+\frac{7B^2d}{18b(bc-ad)g^5(a+bx)^3}-\frac{13B^2d^2}{12b(bc-ad)^2g^5(a+bx)^2}+\frac{\cdots}{6b(bc-\cdots}$$

$$= -\frac{B^2}{8bg^5(a+bx)^4}+\frac{7B^2d}{18b(bc-ad)g^5(a+bx)^3}-\frac{13B^2d^2}{12b(bc-ad)^2g^5(a+bx)^2}+\frac{\cdots}{6b(bc-\cdots}$$

**Mathematica [C]** time = 0.94, size = 762, normalized size = 1.30

$$-\frac{B\left(-72d^4(a+bx)^4\log(a+bx)\left(B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+A\right)+72d^4(a+bx)^4\log(c+dx)\left(B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+A\right)+72d^3(a+bx)^3(ad-bc)\left(B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+A\right)+36d^2(a-\cdots}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])^2/(a*g + b*g*x)^5,x]
```

```
[Out] -1/72*(18*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])^2 + (B*(18*(b*c - a*d)^4
*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2]) + 24*d*(-(b*c) + a*d)^3*(a + b*x)
*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2]) + 36*d^2*(b*c - a*d)^2*(a + b*x)^
2*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2]) + 72*d^3*(-(b*c) + a*d)*(a + b*x
)^3*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2]) - 72*d^4*(a + b*x)^4*Log[a + b
*x]*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2]) + 72*d^4*(a + b*x)^4*(A + B*Lo
g[(e*(a + b*x)^2)/(c + d*x)^2])*Log[c + d*x] - 144*B*d^3*(a + b*x)^3*(b*c -
 a*d + d*(a + b*x)*Log[a + b*x] - d*(a + b*x)*Log[c + d*x]) + 36*B*d^2*(a +
 b*x)^2*((b*c - a*d)^2 + 2*d*(-(b*c) + a*d)*(a + b*x) - 2*d^2*(a + b*x)^2*L
og[a + b*x] + 2*d^2*(a + b*x)^2*Log[c + d*x]) - 8*B*d*(a + b*x)*(2*(b*c - a
*d)^3 - 3*d*(b*c - a*d)^2*(a + b*x) + 6*d^2*(b*c - a*d)*(a + b*x)^2 + 6*d^3
*(a + b*x)^3*Log[a + b*x] - 6*d^3*(a + b*x)^3*Log[c + d*x]) + 3*B*(3*(b*c -
 a*d)^4 + 4*d*(-(b*c) + a*d)^3*(a + b*x) + 6*d^2*(b*c - a*d)^2*(a + b*x)^2
+ 12*d^3*(-(b*c) + a*d)*(a + b*x)^3 - 12*d^4*(a + b*x)^4*Log[a + b*x] + 12*
d^4*(a + b*x)^4*Log[c + d*x]) + 72*B*d^4*(a + b*x)^4*(Log[a + b*x]*(Log[a +
 b*x] - 2*Log[(b*(c + d*x))/(b*c - a*d)]) - 2*PolyLog[2, (d*(a + b*x))/(-(b
*c) + a*d)]) - 72*B*d^4*(a + b*x)^4*((2*Log[(d*(a + b*x))/(-(b*c) + a*d)] -
 Log[c + d*x])*Log[c + d*x] + 2*PolyLog[2, (b*(c + d*x))/(b*c - a*d)])))/(b
*c - a*d)^4)/(b*g^5*(a + b*x)^4)
```

**fricas [A]** time = 1.03, size = 1084, normalized size = 1.85

$$-\frac{9\left(2A^2+2AB+B^2\right)b^4c^4-8\left(9A^2+12AB+8B^2\right)ab^3c^3d+108\left(A^2+2AB+2B^2\right)a^2b^2c^2d^2-72\left(A^2+4A\cdots\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*(b*x+a)^2/(d*x+c)^2))^2/(b*g*x+a*g)^5,x, algorithm="fr
icas")
```

```
[Out] -1/72*(9*(2*A^2 + 2*A*B + B^2)*b^4*c^4 - 8*(9*A^2 + 12*A*B + 8*B^2)*a*b^3*c
^3*d + 108*(A^2 + 2*A*B + 2*B^2)*a^2*b^2*c^2*d^2 - 72*(A^2 + 4*A*B + 8*B^2)
*a^3*b*c*d^3 + (18*A^2 + 150*A*B + 415*B^2)*a^4*d^4 - 12*((6*A*B + 25*B^2)*
b^4*c*d^3 - (6*A*B + 25*B^2)*a*b^3*d^4)*x^3 + 6*((6*A*B + 13*B^2)*b^4*c^2*d
```

```
^2 - 16*(3*A*B + 11*B^2)*a*b^3*c*d^3 + (42*A*B + 163*B^2)*a^2*b^2*d^4)*x^2
- 18*(B^2*b^4*d^4*x^4 + 4*B^2*a*b^3*d^4*x^3 + 6*B^2*a^2*b^2*d^4*x^2 + 4*B^2
*a^3*b*d^4*x - B^2*b^4*c^4 + 4*B^2*a*b^3*c^3*d - 6*B^2*a^2*b^2*c^2*d^2 + 4*
B^2*a^3*b*c*d^3)*log((b^2*e*x^2 + 2*a*b*e*x + a^2*e)/(d^2*x^2 + 2*c*d*x + c
^2))^2 - 4*((6*A*B + 7*B^2)*b^4*c^3*d - 12*(3*A*B + 5*B^2)*a*b^3*c^2*d^2 +
108*(A*B + 3*B^2)*a^2*b^2*c*d^3 - (78*A*B + 271*B^2)*a^3*b*d^4)*x - 6*((6*A
*B + 25*B^2)*b^4*d^4*x^4 - 3*(2*A*B + B^2)*b^4*c^4 + 8*(3*A*B + 2*B^2)*a*b^
3*c^3*d - 36*(A*B + B^2)*a^2*b^2*c^2*d^2 + 24*(A*B + 2*B^2)*a^3*b*c*d^3 + 4
*(3*B^2*b^4*c*d^3 + 2*(3*A*B + 11*B^2)*a*b^3*d^4)*x^3 - 6*(B^2*b^4*c^2*d^2
- 8*B^2*a*b^3*c*d^3 - 6*(A*B + 3*B^2)*a^2*b^2*d^4)*x^2 + 4*(B^2*b^4*c^3*d -
 6*B^2*a*b^3*c^2*d^2 + 18*B^2*a^2*b^2*c*d^3 + 6*(A*B + 2*B^2)*a^3*b*d^4)*x)
*log((b^2*e*x^2 + 2*a*b*e*x + a^2*e)/(d^2*x^2 + 2*c*d*x + c^2)))/((b^9*c^4
- 4*a*b^8*c^3*d + 6*a^2*b^7*c^2*d^2 - 4*a^3*b^6*c*d^3 + a^4*b^5*d^4)*g^5*x^
4 + 4*(a*b^8*c^4 - 4*a^2*b^7*c^3*d + 6*a^3*b^6*c^2*d^2 - 4*a^4*b^5*c*d^3 +
a^5*b^4*d^4)*g^5*x^3 + 6*(a^2*b^7*c^4 - 4*a^3*b^6*c^3*d + 6*a^4*b^5*c^2*d^2
 - 4*a^5*b^4*c*d^3 + a^6*b^3*d^4)*g^5*x^2 + 4*(a^3*b^6*c^4 - 4*a^4*b^5*c^3*
d + 6*a^5*b^4*c^2*d^2 - 4*a^6*b^3*c*d^3 + a^7*b^2*d^4)*g^5*x + (a^4*b^5*c^4
 - 4*a^5*b^4*c^3*d + 6*a^6*b^3*c^2*d^2 - 4*a^7*b^2*c*d^3 + a^8*b*d^4)*g^5)
```

**giac** [A] time = 3.18, size = 874, normalized size = 1.49

$$\frac{1}{4}\left(\frac{B^2d^4}{b^5c^4g^5-4\,ab^4c^3dg^5+6\,a^2b^3c^2d^2g^5-4\,a^3b^2cd^3g^5+a^4bd^4g^5}-\frac{B^2}{\left(bgx+ag\right)^4bg}\right)\log\left(\frac{\quad}{\frac{b^2c^2g^2}{\left(bgx+ag\right)^2}-\frac{2\,abcdg^2}{\left(bgx+ag\right)^2}+\frac{a^2}{(bg}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*(b*x+a)^2/(d*x+c)^2))^2/(b*g*x+a*g)^5,x, algorithm="gi
ac")
```

```
[Out] 1/4*(B^2*d^4/(b^5*c^4*g^5 - 4*a*b^4*c^3*d*g^5 + 6*a^2*b^3*c^2*d^2*g^5 - 4*a
^3*b^2*c*d^3*g^5 + a^4*b*d^4*g^5) - B^2/((b*g*x + a*g)^4*b*g))*log(b^2/(b^2
*c^2*g^2/(b*g*x + a*g)^2 - 2*a*b*c*d*g^2/(b*g*x + a*g)^2 + a^2*d^2*g^2/(b*g
*x + a*g)^2 + 2*b*c*d*g/(b*g*x + a*g) - 2*a*d^2*g/(b*g*x + a*g) + d^2))^2 +
 1/12*(12*B^2*d^3/((b^3*c^3*g^3 - 3*a*b^2*c^2*d*g^3 + 3*a^2*b*c*d^2*g^3 - a
^3*d^3*g^3)*(b*g*x + a*g)*b*g) - 6*B^2*d^2/((b^2*c^2*g - 2*a*b*c*d*g + a^2*
d^2*g)*(b*g*x + a*g)^2*b*g^2) + 4*B^2*d/((b*g*x + a*g)^3*(b*c - a*d)*b*g^2)
 - 3*(2*A*B*b^3*g^3 + 3*B^2*b^3*g^3)/((b*g*x + a*g)^4*b^4*g^4))*log(b^2/(b^
2*c^2*g^2/(b*g*x + a*g)^2 - 2*a*b*c*d*g^2/(b*g*x + a*g)^2 + a^2*d^2*g^2/(b*
g*x + a*g)^2 + 2*b*c*d*g/(b*g*x + a*g) - 2*a*d^2*g/(b*g*x + a*g) + d^2)) -
1/6*(6*A*B*d^4 + 31*B^2*d^4)*log(-b*c*g/(b*g*x + a*g) + a*d*g/(b*g*x + a*g)
 - d)/(b^5*c^4*g^5 - 4*a*b^4*c^3*d*g^5 + 6*a^2*b^3*c^2*d^2*g^5 - 4*a^3*b^2*
c*d^3*g^5 + a^4*b*d^4*g^5) + 1/6*(6*A*B*d^3 + 31*B^2*d^3)/((b^3*c^3*g^3 - 3
*a*b^2*c^2*d*g^3 + 3*a^2*b*c*d^2*g^3 - a^3*d^3*g^3)*(b*g*x + a*g)*b*g) - 1/
```

```
12*(6*A*B*b*d^2 + 19*B^2*b*d^2)/((b^2*c^2*g - 2*a*b*c*d*g + a^2*d^2*g)*(b*g
*x + a*g)^2*b^2*g^2) + 1/18*(6*A*B*b^2*d*g + 13*B^2*b^2*d*g)/((b*g*x + a*g)
^3*(b*c - a*d)*b^3*g^3) - 1/8*(2*A^2*b^3*g^3 + 6*A*B*b^3*g^3 + 5*B^2*b^3*g^
3)/((b*g*x + a*g)^4*b^4*g^4)
```

**maple [B]** time = 0.34, size = 1943, normalized size = 3.31

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*ln((b*x+a)^2/(d*x+c)^2*e)+A)^2/(b*g*x+a*g)^5,x)
```

```
[Out] d^4/g^5*A^2*b^2/(a*d-b*c)^4/(1/(d*x+c)*a*d-1/(d*x+c)*b*c+b)^3-1/4*d^4/g^5*A
^2*b^3/(a*d-b*c)^4/(1/(d*x+c)*a*d-1/(d*x+c)*b*c+b)^4+d^4/g^5*A^2/(a*d-b*c)^
4/(1/(d*x+c)*a*d-1/(d*x+c)*b*c+b)-3/2*d^4/g^5*A^2*b/(a*d-b*c)^4/(1/(d*x+c)*
a*d-1/(d*x+c)*b*c+b)^2-415/72*d^4/g^5/(1/(d*x+c)*a*d-1/(d*x+c)*b*c+b)^4*B^2
/b/(d*x+c)^4+25/12*d^4/g^5/(1/(d*x+c)*a*d-1/(d*x+c)*b*c+b)^4*b^3*B^2/(a^4*d
^4-4*a^3*b*c*d^3+6*a^2*b^2*c^2*d^2-4*a*b^3*c^3*d+b^4*c^4)*ln((1/(d*x+c)*a*d
-1/(d*x+c)*b*c+b)^2/d^2*e)-25/6*d^4/g^5/(1/(d*x+c)*a*d-1/(d*x+c)*b*c+b)^4*B
^2*b^2/(a^3*d^3-3*a^2*b*c*d^2+3*a*b^2*c^2*d-b^3*c^3)/(d*x+c)-163/12*d^4/g^5
/(1/(d*x+c)*a*d-1/(d*x+c)*b*c+b)^4*B^2*b/(a^2*d^2-2*a*b*c*d+b^2*c^2)/(d*x+c
)^2-271/18*d^4/g^5/(1/(d*x+c)*a*d-1/(d*x+c)*b*c+b)^4*B^2/(a*d-b*c)/(d*x+c)^
3+4*d^4/g^5/(1/(d*x+c)*a*d-1/(d*x+c)*b*c+b)^4*B^2/(a*d-b*c)/(d*x+c)^3*ln((1
/(d*x+c)*a*d-1/(d*x+c)*b*c+b)^2/d^2*e)+1/4*d^4/g^5/(1/(d*x+c)*a*d-1/(d*x+c)
*b*c+b)^4*B^2*b^3/(a^4*d^4-4*a^3*b*c*d^3+6*a^2*b^2*c^2*d^2-4*a*b^3*c^3*d+b^
4*c^4)*ln((1/(d*x+c)*a*d-1/(d*x+c)*b*c+b)^2/d^2*e)^2+d^4/g^5/(1/(d*x+c)*a*d
-1/(d*x+c)*b*c+b)^4*B^2/(a*d-b*c)/(d*x+c)^3*ln((1/(d*x+c)*a*d-1/(d*x+c)*b*c
+b)^2/d^2*e)^2+9*d^4/g^5/(1/(d*x+c)*a*d-1/(d*x+c)*b*c+b)^4*B^2*b/(a^2*d^2-2
*a*b*c*d+b^2*c^2)/(d*x+c)^2*ln((1/(d*x+c)*a*d-1/(d*x+c)*b*c+b)^2/d^2*e)+22/
3*d^4/g^5/(1/(d*x+c)*a*d-1/(d*x+c)*b*c+b)^4*B^2*b^2/(a^3*d^3-3*a^2*b*c*d^2+
3*a*b^2*c^2*d-b^3*c^3)/(d*x+c)*ln((1/(d*x+c)*a*d-1/(d*x+c)*b*c+b)^2/d^2*e)+
3/2*d^4/g^5/(1/(d*x+c)*a*d-1/(d*x+c)*b*c+b)^4*B^2*b/(a^2*d^2-2*a*b*c*d+b^2*
c^2)/(d*x+c)^2*ln((1/(d*x+c)*a*d-1/(d*x+c)*b*c+b)^2/d^2*e)^2+d^4/g^5/(1/(d*
x+c)*a*d-1/(d*x+c)*b*c+b)^4*B^2*b^2/(a^3*d^3-3*a^2*b*c*d^2+3*a*b^2*c^2*d-b^
3*c^3)/(d*x+c)*ln((1/(d*x+c)*a*d-1/(d*x+c)*b*c+b)^2/d^2*e)^2-d^4/g^5/(1/(d*
x+c)*a*d-1/(d*x+c)*b*c+b)^4*A*B*b^2/(a^3*d^3-3*a^2*b*c*d^2+3*a*b^2*c^2*d-b^
3*c^3)/(d*x+c)-25/12*d^4/g^5/(1/(d*x+c)*a*d-1/(d*x+c)*b*c+b)^4*A*B/b/(d*x+c
)^4+1/2*d^4/g^5/(1/(d*x+c)*a*d-1/(d*x+c)*b*c+b)^4*b^3*A*B/(a^4*d^4-4*a^3*b*
c*d^3+6*a^2*b^2*c^2*d^2-4*a*b^3*c^3*d+b^4*c^4)*ln((1/(d*x+c)*a*d-1/(d*x+c)*
b*c+b)^2/d^2*e)-7/2*d^4/g^5/(1/(d*x+c)*a*d-1/(d*x+c)*b*c+b)^4*A*B*b/(a^2*d^
2-2*a*b*c*d+b^2*c^2)/(d*x+c)^2-13/3*d^4/g^5/(1/(d*x+c)*a*d-1/(d*x+c)*b*c+b)
^4*A*B/(a*d-b*c)/(d*x+c)^3+2*d^4/g^5/(1/(d*x+c)*a*d-1/(d*x+c)*b*c+b)^4*A*B/
(a*d-b*c)/(d*x+c)^3*ln((1/(d*x+c)*a*d-1/(d*x+c)*b*c+b)^2/d^2*e)+3*d^4/g^5/(
1/(d*x+c)*a*d-1/(d*x+c)*b*c+b)^4*A*B*b/(a^2*d^2-2*a*b*c*d+b^2*c^2)/(d*x+c)^
2*ln((1/(d*x+c)*a*d-1/(d*x+c)*b*c+b)^2/d^2*e)+2*d^4/g^5/(1/(d*x+c)*a*d-1/(d
```

```
*x+c)*b*c+b)^4*A*B*b^2/(a^3*d^3-3*a^2*b*c*d^2+3*a*b^2*c^2*d-b^3*c^3)/(d*x+c
)*ln((1/(d*x+c)*a*d-1/(d*x+c)*b*c+b)^2/d^2*e)
```

**maxima [B]** time = 3.44, size = 2279, normalized size = 3.88

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*(b*x+a)^2/(d*x+c)^2))^2/(b*g*x+a*g)^5,x, algorithm="ma
xima")
```

```
[Out] 1/72*(6*((12*b^3*d^3*x^3 - 3*b^3*c^3 + 13*a*b^2*c^2*d - 23*a^2*b*c*d^2 + 25
*a^3*d^3 - 6*(b^3*c*d^2 - 7*a*b^2*d^3)*x^2 + 4*(b^3*c^2*d - 5*a*b^2*c*d^2 +
 13*a^2*b*d^3)*x)/((b^8*c^3 - 3*a*b^7*c^2*d + 3*a^2*b^6*c*d^2 - a^3*b^5*d^3
)*g^5*x^4 + 4*(a*b^7*c^3 - 3*a^2*b^6*c^2*d + 3*a^3*b^5*c*d^2 - a^4*b^4*d^3)
*g^5*x^3 + 6*(a^2*b^6*c^3 - 3*a^3*b^5*c^2*d + 3*a^4*b^4*c*d^2 - a^5*b^3*d^3
)*g^5*x^2 + 4*(a^3*b^5*c^3 - 3*a^4*b^4*c^2*d + 3*a^5*b^3*c*d^2 - a^6*b^2*d^
3)*g^5*x + (a^4*b^4*c^3 - 3*a^5*b^3*c^2*d + 3*a^6*b^2*c*d^2 - a^7*b*d^3)*g^
5) + 12*d^4*log(b*x + a)/((b^5*c^4 - 4*a*b^4*c^3*d + 6*a^2*b^3*c^2*d^2 - 4*
a^3*b^2*c*d^3 + a^4*b*d^4)*g^5) - 12*d^4*log(d*x + c)/((b^5*c^4 - 4*a*b^4*c
^3*d + 6*a^2*b^3*c^2*d^2 - 4*a^3*b^2*c*d^3 + a^4*b*d^4)*g^5))*log(b^2*e*x^2
/(d^2*x^2 + 2*c*d*x + c^2) + 2*a*b*e*x/(d^2*x^2 + 2*c*d*x + c^2) + a^2*e/(d
^2*x^2 + 2*c*d*x + c^2)) - (9*b^4*c^4 - 64*a*b^3*c^3*d + 216*a^2*b^2*c^2*d^
2 - 576*a^3*b*c*d^3 + 415*a^4*d^4 - 300*(b^4*c*d^3 - a*b^3*d^4)*x^3 + 6*(13
*b^4*c^2*d^2 - 176*a*b^3*c*d^3 + 163*a^2*b^2*d^4)*x^2 + 72*(b^4*d^4*x^4 + 4
*a*b^3*d^4*x^3 + 6*a^2*b^2*d^4*x^2 + 4*a^3*b*d^4*x + a^4*d^4)*log(b*x + a)^
2 + 72*(b^4*d^4*x^4 + 4*a*b^3*d^4*x^3 + 6*a^2*b^2*d^4*x^2 + 4*a^3*b*d^4*x +
 a^4*d^4)*log(d*x + c)^2 - 4*(7*b^4*c^3*d - 60*a*b^3*c^2*d^2 + 324*a^2*b^2*
c*d^3 - 271*a^3*b*d^4)*x - 300*(b^4*d^4*x^4 + 4*a*b^3*d^4*x^3 + 6*a^2*b^2*d
^4*x^2 + 4*a^3*b*d^4*x + a^4*d^4)*log(b*x + a) + 12*(25*b^4*d^4*x^4 + 100*a
*b^3*d^4*x^3 + 150*a^2*b^2*d^4*x^2 + 100*a^3*b*d^4*x + 25*a^4*d^4 - 12*(b^4
*d^4*x^4 + 4*a*b^3*d^4*x^3 + 6*a^2*b^2*d^4*x^2 + 4*a^3*b*d^4*x + a^4*d^4)*l
og(b*x + a))*log(d*x + c))/(a^4*b^5*c^4*g^5 - 4*a^5*b^4*c^3*d*g^5 + 6*a^6*b
^3*c^2*d^2*g^5 - 4*a^7*b^2*c*d^3*g^5 + a^8*b*d^4*g^5 + (b^9*c^4*g^5 - 4*a*b
^8*c^3*d*g^5 + 6*a^2*b^7*c^2*d^2*g^5 - 4*a^3*b^6*c*d^3*g^5 + a^4*b^5*d^4*g^
5)*x^4 + 4*(a*b^8*c^4*g^5 - 4*a^2*b^7*c^3*d*g^5 + 6*a^3*b^6*c^2*d^2*g^5 - 4
*a^4*b^5*c*d^3*g^5 + a^5*b^4*d^4*g^5)*x^3 + 6*(a^2*b^7*c^4*g^5 - 4*a^3*b^6*
c^3*d*g^5 + 6*a^4*b^5*c^2*d^2*g^5 - 4*a^5*b^4*c*d^3*g^5 + a^6*b^3*d^4*g^5)*
x^2 + 4*(a^3*b^6*c^4*g^5 - 4*a^4*b^5*c^3*d*g^5 + 6*a^5*b^4*c^2*d^2*g^5 - 4*
a^6*b^3*c*d^3*g^5 + a^7*b^2*d^4*g^5)*x))*B^2 + 1/12*A*B*((12*b^3*d^3*x^3 -
3*b^3*c^3 + 13*a*b^2*c^2*d - 23*a^2*b*c*d^2 + 25*a^3*d^3 - 6*(b^3*c*d^2 - 7
*a*b^2*d^3)*x^2 + 4*(b^3*c^2*d - 5*a*b^2*c*d^2 + 13*a^2*b*d^3)*x)/((b^8*c^3
 - 3*a*b^7*c^2*d + 3*a^2*b^6*c*d^2 - a^3*b^5*d^3)*g^5*x^4 + 4*(a*b^7*c^3 -
3*a^2*b^6*c^2*d + 3*a^3*b^5*c*d^2 - a^4*b^4*d^3)*g^5*x^3 + 6*(a^2*b^6*c^3 -
 3*a^3*b^5*c^2*d + 3*a^4*b^4*c*d^2 - a^5*b^3*d^3)*g^5*x^2 + 4*(a^3*b^5*c^3
```

```
- 3*a^4*b^4*c^2*d + 3*a^5*b^3*c*d^2 - a^6*b^2*d^3)*g^5*x + (a^4*b^4*c^3 - 3
*a^5*b^3*c^2*d + 3*a^6*b^2*c*d^2 - a^7*b*d^3)*g^5) - 6*log(b^2*e*x^2/(d^2*x
^2 + 2*c*d*x + c^2) + 2*a*b*e*x/(d^2*x^2 + 2*c*d*x + c^2) + a^2*e/(d^2*x^2
+ 2*c*d*x + c^2))/(b^5*g^5*x^4 + 4*a*b^4*g^5*x^3 + 6*a^2*b^3*g^5*x^2 + 4*a^
3*b^2*g^5*x + a^4*b*g^5) + 12*d^4*log(b*x + a)/((b^5*c^4 - 4*a*b^4*c^3*d +
6*a^2*b^3*c^2*d^2 - 4*a^3*b^2*c*d^3 + a^4*b*d^4)*g^5) - 12*d^4*log(d*x + c)
/((b^5*c^4 - 4*a*b^4*c^3*d + 6*a^2*b^3*c^2*d^2 - 4*a^3*b^2*c*d^3 + a^4*b*d^
4)*g^5)) - 1/4*B^2*log(b^2*e*x^2/(d^2*x^2 + 2*c*d*x + c^2) + 2*a*b*e*x/(d^2
*x^2 + 2*c*d*x + c^2) + a^2*e/(d^2*x^2 + 2*c*d*x + c^2))^2/(b^5*g^5*x^4 + 4
*a*b^4*g^5*x^3 + 6*a^2*b^3*g^5*x^2 + 4*a^3*b^2*g^5*x + a^4*b*g^5) - 1/4*A^2
/(b^5*g^5*x^4 + 4*a*b^4*g^5*x^3 + 6*a^2*b^3*g^5*x^2 + 4*a^3*b^2*g^5*x + a^4
*b*g^5)
```

**mupad [B]** time = 10.55, size = 1883, normalized size = 3.21

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*log((e*(a + b*x)^2)/(c + d*x)^2))^2/(a*g + b*g*x)^5,x)
```

```
[Out] (B*d^4*atan((B*d^4*(6*A + 25*B)*(6*b^5*c^4*g^5 - 6*a^4*b*d^4*g^5 - 12*a*b^4
*c^3*d*g^5 + 12*a^3*b^2*c*d^3*g^5)*1i)/(6*b*g^5*(a*d - b*c)^4*(25*B^2*d^4 +
 6*A*B*d^4)) + (B*d^5*x*(6*A + 25*B)*(b^4*c^3*g^5 - a^3*b*d^3*g^5 - 3*a*b^3
*c^2*d*g^5 + 3*a^2*b^2*c*d^2*g^5)*2i)/(g^5*(a*d - b*c)^4*(25*B^2*d^4 + 6*A*
B*d^4)))*(6*A + 25*B)*1i)/(3*b*g^5*(a*d - b*c)^4) - log((e*(a + b*x)^2)/(c
+ d*x)^2)^2*(B^2/(4*b^2*g^5*(4*a^3*x + a^4/b + b^3*x^4 + 6*a^2*b*x^2 + 4*a*
b^2*x^3)) - (B^2*d^4)/(4*b*g^5*(a^4*d^4 + b^4*c^4 + 6*a^2*b^2*c^2*d^2 - 4*a
*b^3*c^3*d - 4*a^3*b*c*d^3))) - (log((e*(a + b*x)^2)/(c + d*x)^2)*((A*B)/(2
*b^2*d*g^5) + (B^2*d^4*(a*(a*((4*a^2*d^2 + b^2*c^2 - 5*a*b*c*d)/(6*b*d^3) +
 (a*(a*d - b*c))/(2*b*d^2)) + (6*a^3*d^3 - b^3*c^3 + 5*a*b^2*c^2*d - 10*a^2
*b*c*d^2)/(6*b*d^4)) + (4*a^4*d^4 + b^4*c^4 + 10*a^2*b^2*c^2*d^2 - 5*a*b^3*
c^3*d - 10*a^3*b*c*d^3)/(2*b*d^5)))/(2*b*g^5*(a^4*d^4 + b^4*c^4 + 6*a^2*b^2
*c^2*d^2 - 4*a*b^3*c^3*d - 4*a^3*b*c*d^3)) + (B^2*d^4*x^2*(b*(b*((4*a^2*d^2
 + b^2*c^2 - 5*a*b*c*d)/(6*b*d^3) + (a*(a*d - b*c))/(2*b*d^2)) + (4*a^2*d^2
 + b^2*c^2 - 5*a*b*c*d)/(3*d^3) + (a*(a*d - b*c))/d^2) - a*((b^2*c - a*b*d)
/(2*d^2) - (b*(a*d - b*c))/d^2) + (b^3*c^2 + 4*a^2*b*d^2 - 5*a*b^2*c*d)/(2*
d^3)))/(2*b*g^5*(a^4*d^4 + b^4*c^4 + 6*a^2*b^2*c^2*d^2 - 4*a*b^3*c^3*d - 4*
a^3*b*c*d^3)) - (B^2*d^4*x^3*(b*((b^2*c - a*b*d)/(2*d^2) - (b*(a*d - b*c))/
d^2) + (b^3*c - a*b^2*d)/(2*d^2)))/(2*b*g^5*(a^4*d^4 + b^4*c^4 + 6*a^2*b^2*
c^2*d^2 - 4*a*b^3*c^3*d - 4*a^3*b*c*d^3)) + (B^2*d^4*x*(b*(a*((4*a^2*d^2 +
b^2*c^2 - 5*a*b*c*d)/(6*b*d^3) + (a*(a*d - b*c))/(2*b*d^2)) + (6*a^3*d^3 -
b^3*c^3 + 5*a*b^2*c^2*d - 10*a^2*b*c*d^2)/(6*b*d^4)) + a*(b*((4*a^2*d^2 + b
^2*c^2 - 5*a*b*c*d)/(6*b*d^3) + (a*(a*d - b*c))/(2*b*d^2)) + (4*a^2*d^2 + b
^2*c^2 - 5*a*b*c*d)/(3*d^3) + (a*(a*d - b*c))/d^2) + (6*a^3*d^3 - b^3*c^3 +
 5*a*b^2*c^2*d - 10*a^2*b*c*d^2)/(2*d^4)))/(2*b*g^5*(a^4*d^4 + b^4*c^4 + 6*
```

```
a^2*b^2*c^2*d^2 - 4*a*b^3*c^3*d - 4*a^3*b*c*d^3))))/((4*a^3*x)/d + a^4/(b*d
) + (b^3*x^4)/d + (6*a^2*b*x^2)/d + (4*a*b^2*x^3)/d) - ((18*A^2*a^3*d^3 - 1
8*A^2*b^3*c^3 + 415*B^2*a^3*d^3 - 9*B^2*b^3*c^3 + 150*A*B*a^3*d^3 - 18*A*B*
b^3*c^3 + 54*A^2*a*b^2*c^2*d - 54*A^2*a^2*b*c*d^2 + 55*B^2*a*b^2*c^2*d - 16
1*B^2*a^2*b*c*d^2 + 78*A*B*a*b^2*c^2*d - 138*A*B*a^2*b*c*d^2)/(12*(a*d - b*
c)) + (x^2*(163*B^2*a*b^2*d^3 - 13*B^2*b^3*c*d^2 + 42*A*B*a*b^2*d^3 - 6*A*B
*b^3*c*d^2))/(2*(a*d - b*c)) + (x*(271*B^2*a^2*b*d^3 + 7*B^2*b^3*c^2*d - 53
*B^2*a*b^2*c*d^2 + 78*A*B*a^2*b*d^3 + 6*A*B*b^3*c^2*d - 30*A*B*a*b^2*c*d^2)
)/(3*(a*d - b*c)) + (d*x^3*(25*B^2*b^3*d^2 + 6*A*B*b^3*d^2))/(a*d - b*c))/(
x*(24*a^3*b^4*c^2*g^5 + 24*a^5*b^2*d^2*g^5 - 48*a^4*b^3*c*d*g^5) + x^3*(24*
a*b^6*c^2*g^5 + 24*a^3*b^4*d^2*g^5 - 48*a^2*b^5*c*d*g^5) + x^4*(6*b^7*c^2*g
^5 + 6*a^2*b^5*d^2*g^5 - 12*a*b^6*c*d*g^5) + x^2*(36*a^2*b^5*c^2*g^5 + 36*a
^4*b^3*d^2*g^5 - 72*a^3*b^4*c*d*g^5) + 6*a^6*b*d^2*g^5 + 6*a^4*b^3*c^2*g^5
- 12*a^5*b^2*c*d*g^5)
```

**sympy [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*ln(e*(b*x+a)**2/(d*x+c)**2))**2/(b*g*x+a*g)**5,x)
```

```
[Out] Timed out
```

# 3.137 $\int \frac{(ag+bgx)^2}{A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}\,dx$

**Optimal**. Leaf size=37

$$\text{Int}\left(\frac{(ag+bgx)^2}{B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+A},x\right)$$

```
[Out] Unintegrable((b*g*x+a*g)^2/(A+B*ln(e*(b*x+a)^2/(d*x+c)^2)),x)
```

**Rubi [A]** time = 0.20, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{(ag+bgx)^2}{A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(a*g + b*g*x)^2/(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2]),x]
```

```
[Out] a^2*g^2*Defer[Int][(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])^(-1), x] + 2*a*
b*g^2*Defer[Int][x/(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2]), x] + b^2*g^2*D
efer[Int][x^2/(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2]), x]
```

Rubi steps

$$\begin{aligned}\int \frac{(ag+bgx)^2}{A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}\,dx &= \int \left(\frac{a^2g^2}{A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}+\frac{2abg^2x}{A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}+\frac{b^2g^2x^2}{A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}\right)dx\\ &= \left(a^2g^2\right)\int \frac{1}{A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}\,dx+\left(2abg^2\right)\int \frac{x}{A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}\,dx+\left(b^2g^2\right)\int\end{aligned}$$

**Mathematica [A]** time = 0.16, size = 0, normalized size = 0.00

$$\int \frac{(ag+bgx)^2}{A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(a*g + b*g*x)^2/(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2]),x]
```

```
[Out] Integrate[(a*g + b*g*x)^2/(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2]), x]
```

**fricas [A]** time = 0.57, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{b^2g^2x^2 + 2\,abg^2x + a^2g^2}{B\log\left(\frac{b^2ex^2+2\,abex+a^2e}{d^2x^2+2\,cdx+c^2}\right) + A}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*g*x+a*g)^2/(A+B*log(e*(b*x+a)^2/(d*x+c)^2)),x, algorithm="fric
as")
```

```
[Out] integral((b^2*g^2*x^2 + 2*a*b*g^2*x + a^2*g^2)/(B*log((b^2*e*x^2 + 2*a*b*e*
x + a^2*e)/(d^2*x^2 + 2*c*d*x + c^2)) + A), x)
```

**giac [A]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(bgx + ag\right)^2}{B\log\left(\frac{(bx+a)^2e}{(dx+c)^2}\right) + A}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*g*x+a*g)^2/(A+B*log(e*(b*x+a)^2/(d*x+c)^2)),x, algorithm="giac
")
```

```
[Out] integrate((b*g*x + a*g)^2/(B*log((b*x + a)^2*e/(d*x + c)^2) + A), x)
```

**maple [A]** time = 0.80, size = 0, normalized size = 0.00

$$\int \frac{\left(bgx + ag\right)^2}{B\ln\left(\frac{(bx+a)^2e}{(dx+c)^2}\right) + A}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*g*x+a*g)^2/(B*ln((b*x+a)^2/(d*x+c)^2*e)+A),x)
```

```
[Out] int((b*g*x+a*g)^2/(B*ln((b*x+a)^2/(d*x+c)^2*e)+A),x)
```

**maxima [A]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(bgx + ag\right)^2}{B\log\left(\frac{(bx+a)^2e}{(dx+c)^2}\right) + A}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*g*x+a*g)^2/(A+B*log(e*(b*x+a)^2/(d*x+c)^2)),x, algorithm="maxi
ma")
```

```
[Out] integrate((b*g*x + a*g)^2/(B*log((b*x + a)^2*e/(d*x + c)^2) + A), x)
```

**mupad [A]** time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{{\left(a\,g+b\,g\,x\right)}^2}{A+B\,\ln\left(\frac{e\,{\left(a+b\,x\right)}^2}{{\left(c+d\,x\right)}^2}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*g + b*g*x)^2/(A + B*log((e*(a + b*x)^2)/(c + d*x)^2)),x)
```

```
[Out] int((a*g + b*g*x)^2/(A + B*log((e*(a + b*x)^2)/(c + d*x)^2)), x)
```

**sympy [A]** time = 0.00, size = 0, normalized size = 0.00

$$g^2\left(\int \frac{a^2}{A+B\log\left(\frac{a^2e}{c^2+2cdx+d^2x^2}+\frac{2abex}{c^2+2cdx+d^2x^2}+\frac{b^2ex^2}{c^2+2cdx+d^2x^2}\right)}\,dx+\int \frac{b^2x^2}{A+B\log\left(\frac{a^2e}{c^2+2cdx+d^2x^2}+\frac{2abex}{c^2+2cdx+d^2x^2}+\frac{}{c^2+}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*g*x+a*g)**2/(A+B*ln(e*(b*x+a)**2/(d*x+c)**2)),x)
```

```
[Out] g**2*(Integral(a**2/(A + B*log(a**2*e/(c**2 + 2*c*d*x + d**2*x**2) + 2*a*b*
e*x/(c**2 + 2*c*d*x + d**2*x**2) + b**2*e*x**2/(c**2 + 2*c*d*x + d**2*x**2)
)), x) + Integral(b**2*x**2/(A + B*log(a**2*e/(c**2 + 2*c*d*x + d**2*x**2)
+ 2*a*b*e*x/(c**2 + 2*c*d*x + d**2*x**2) + b**2*e*x**2/(c**2 + 2*c*d*x + d*
*2*x**2))), x) + Integral(2*a*b*x/(A + B*log(a**2*e/(c**2 + 2*c*d*x + d**2*
x**2) + 2*a*b*e*x/(c**2 + 2*c*d*x + d**2*x**2) + b**2*e*x**2/(c**2 + 2*c*d*
x + d**2*x**2))), x))
```

# 3.138 $\int \frac{ag+bgx}{A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}\,dx$

**Optimal**. Leaf size=35

$$\text{Int}\left(\frac{ag+bgx}{B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+A},x\right)$$

```
[Out] Unintegrable((b*g*x+a*g)/(A+B*ln(e*(b*x+a)^2/(d*x+c)^2)),x)
```

**Rubi [A]** time = 0.10, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{ag+bgx}{A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(a*g + b*g*x)/(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2]),x]
```

```
[Out] a*g*Defer[Int][(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])^(-1), x] + b*g*Defe
r[Int][x/(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2]), x]
```

Rubi steps

$$\begin{aligned}
\int \frac{ag+bgx}{A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}\,dx &= \int \left(\frac{ag}{A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}+\frac{bgx}{A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}\right)dx\\
&= (ag)\int \frac{1}{A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}\,dx+(bg)\int \frac{x}{A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}\,dx
\end{aligned}$$

**Mathematica [A]** time = 0.12, size = 0, normalized size = 0.00

$$\int \frac{ag+bgx}{A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(a*g + b*g*x)/(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2]),x]
```

```
[Out] Integrate[(a*g + b*g*x)/(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2]), x]
```

**fricas [A]** time = 0.92, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{bgx + ag}{B\log\left(\frac{b^2ex^2+2\,abex+a^2e}{d^2x^2+2\,cdx+c^2}\right) + A}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*g*x+a*g)/(A+B*log(e*(b*x+a)^2/(d*x+c)^2)),x, algorithm="fricas
")
```

```
[Out] integral((b*g*x + a*g)/(B*log((b^2*e*x^2 + 2*a*b*e*x + a^2*e)/(d^2*x^2 + 2*
c*d*x + c^2)) + A), x)
```

**giac [A]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{bgx + ag}{B\log\left(\frac{(bx+a)^2e}{(dx+c)^2}\right) + A}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*g*x+a*g)/(A+B*log(e*(b*x+a)^2/(d*x+c)^2)),x, algorithm="giac")
```

```
[Out] integrate((b*g*x + a*g)/(B*log((b*x + a)^2*e/(d*x + c)^2) + A), x)
```

**maple [A]** time = 0.63, size = 0, normalized size = 0.00

$$\int \frac{bgx + ag}{B\ln\left(\frac{(bx+a)^2e}{(dx+c)^2}\right) + A}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*g*x+a*g)/(B*ln((b*x+a)^2/(d*x+c)^2*e)+A),x)
```

```
[Out] int((b*g*x+a*g)/(B*ln((b*x+a)^2/(d*x+c)^2*e)+A),x)
```

**maxima [A]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{bgx + ag}{B\log\left(\frac{(bx+a)^2e}{(dx+c)^2}\right) + A}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*g*x+a*g)/(A+B*log(e*(b*x+a)^2/(d*x+c)^2)),x, algorithm="maxima
")
```

```
[Out] integrate((b*g*x + a*g)/(B*log((b*x + a)^2*e/(d*x + c)^2) + A), x)
```

**mupad [A]** time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{a\,g+b\,g\,x}{A+B\,\ln\left(\frac{e\,{\left(a+b\,x\right)}^2}{{\left(c+d\,x\right)}^2}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*g + b*g*x)/(A + B*log((e*(a + b*x)^2)/(c + d*x)^2)),x)
```

```
[Out] int((a*g + b*g*x)/(A + B*log((e*(a + b*x)^2)/(c + d*x)^2)), x)
```

**sympy [A]** time = 0.00, size = 0, normalized size = 0.00

$$g\left(\int \frac{a}{A+B\log\left(\frac{a^2e}{c^2+2cdx+d^2x^2}+\frac{2abex}{c^2+2cdx+d^2x^2}+\frac{b^2ex^2}{c^2+2cdx+d^2x^2}\right)}\,dx+\int \frac{bx}{A+B\log\left(\frac{a^2e}{c^2+2cdx+d^2x^2}+\frac{2abex}{c^2+2cdx+d^2x^2}+\frac{b^2}{c^2+2cd}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*g*x+a*g)/(A+B*ln(e*(b*x+a)**2/(d*x+c)**2)),x)
```

```
[Out] g*(Integral(a/(A + B*log(a**2*e/(c**2 + 2*c*d*x + d**2*x**2) + 2*a*b*e*x/(c
**2 + 2*c*d*x + d**2*x**2) + b**2*e*x**2/(c**2 + 2*c*d*x + d**2*x**2))), x)
 + Integral(b*x/(A + B*log(a**2*e/(c**2 + 2*c*d*x + d**2*x**2) + 2*a*b*e*x/
(c**2 + 2*c*d*x + d**2*x**2) + b**2*e*x**2/(c**2 + 2*c*d*x + d**2*x**2))),
x))
```

# 3.139 $\int \frac{1}{(ag+bgx)\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}\,dx$

**Optimal**. Leaf size=37

$$\text{Int}\left(\frac{1}{(ag+bgx)\left(B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+A\right)},x\right)$$

```
[Out] Unintegrable(1/(b*g*x+a*g)/(A+B*ln(e*(b*x+a)^2/(d*x+c)^2)),x)
```

**Rubi [A]** time = 0.07, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{1}{(ag+bgx)\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[1/((a*g + b*g*x)*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])),x]
[Out] Defer[Int][1/((a*g + b*g*x)*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])), x]
```

Rubi steps

$$\int \frac{1}{(ag+bgx)\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}\,dx=\int \frac{1}{(ag+bgx)\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}\,dx$$

**Mathematica [A]** time = 0.07, size = 0, normalized size = 0.00

$$\int \frac{1}{(ag+bgx)\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[1/((a*g + b*g*x)*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])),x]
[Out] Integrate[1/((a*g + b*g*x)*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])), x]
```

**fricas** [A] time = 0.74, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{1}{Abgx+Aag+\left(Bbgx+Bag\right)\log\left(\frac{b^2ex^2+2\,abex+a^2e}{d^2x^2+2\,cdx+c^2}\right)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*g*x+a*g)/(A+B*log(e*(b*x+a)^2/(d*x+c)^2)),x, algorithm="fric
as")
```

```
[Out] integral(1/(A*b*g*x + A*a*g + (B*b*g*x + B*a*g)*log((b^2*e*x^2 + 2*a*b*e*x
+ a^2*e)/(d^2*x^2 + 2*c*d*x + c^2))), x)
```

**giac** [A] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{1}{\left(bgx+ag\right)\left(B\log\left(\frac{(bx+a)^2e}{(dx+c)^2}\right)+A\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*g*x+a*g)/(A+B*log(e*(b*x+a)^2/(d*x+c)^2)),x, algorithm="giac
")
```

```
[Out] integrate(1/((b*g*x + a*g)*(B*log((b*x + a)^2*e/(d*x + c)^2) + A)), x)
```

**maple** [A] time = 0.82, size = 0, normalized size = 0.00

$$\int\frac{1}{\left(bgx+ag\right)\left(B\ln\left(\frac{(bx+a)^2e}{(dx+c)^2}\right)+A\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*g*x+a*g)/(B*ln((b*x+a)^2/(d*x+c)^2*e)+A),x)
```

```
[Out] int(1/(b*g*x+a*g)/(B*ln((b*x+a)^2/(d*x+c)^2*e)+A),x)
```

**maxima** [A] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{1}{\left(bgx+ag\right)\left(B\log\left(\frac{(bx+a)^2e}{(dx+c)^2}\right)+A\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*g*x+a*g)/(A+B*log(e*(b*x+a)^2/(d*x+c)^2)),x, algorithm="maxi
ma")
```

```
[Out] integrate(1/((b*g*x + a*g)*(B*log((b*x + a)^2*e/(d*x + c)^2) + A)), x)
```

**mupad [A]** time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{1}{\left(a\,g+b\,g\,x\right)\,\left(A+B\,\ln\left(\frac{e\,{\left(a+b\,x\right)}^2}{{\left(c+d\,x\right)}^2}\right)\right)} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*g + b*g*x)*(A + B*log((e*(a + b*x)^2)/(c + d*x)^2))),x)
```

```
[Out] int(1/((a*g + b*g*x)*(A + B*log((e*(a + b*x)^2)/(c + d*x)^2))), x)
```

**sympy [A]** time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{1}{Aa+Abx+Ba\log\left(\frac{a^2e}{c^2+2cdx+d^2x^2}+\frac{2abex}{c^2+2cdx+d^2x^2}+\frac{b^2ex^2}{c^2+2cdx+d^2x^2}\right)+Bbx\log\left(\frac{a^2e}{c^2+2cdx+d^2x^2}+\frac{2abex}{c^2+2cdx+d^2x^2}+\frac{b^2ex^2}{c^2+2cdx+d^2x^2}\right)}\,dx}{g}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*g*x+a*g)/(A+B*ln(e*(b*x+a)**2/(d*x+c)**2)),x)
```

```
[Out] Integral(1/(A*a + A*b*x + B*a*log(a**2*e/(c**2 + 2*c*d*x + d**2*x**2) + 2*a
*b*e*x/(c**2 + 2*c*d*x + d**2*x**2) + b**2*e*x**2/(c**2 + 2*c*d*x + d**2*x*
*2)) + B*b*x*log(a**2*e/(c**2 + 2*c*d*x + d**2*x**2) + 2*a*b*e*x/(c**2 + 2*
c*d*x + d**2*x**2) + b**2*e*x**2/(c**2 + 2*c*d*x + d**2*x**2))), x)/g
```

# 3.140 $\int \frac{1}{(ag+bgx)^2\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}\,dx$

**Optimal**. Leaf size=94

$$\frac{e^{\frac{A}{2B}}(c+dx)\sqrt{\frac{e(a+bx)^2}{(c+dx)^2}}\,\mathrm{Ei}\left(\frac{-A-B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}{2B}\right)}{2Bg^2(a+bx)(bc-ad)}$$

```
[Out] 1/2*exp(1/2*A/B)*(d*x+c)*Ei(1/2*(-A-B*ln(e*(b*x+a)^2/(d*x+c)^2))/B)*(e*(b*x
+a)^2/(d*x+c)^2)^(1/2)/B/(-a*d+b*c)/g^2/(b*x+a)
```

**Rubi [F]** time = 0.09, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{1}{(ag+bgx)^2\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[1/((a*g + b*g*x)^2*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])),x]
[Out] Defer[Int][1/((a*g + b*g*x)^2*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])), x]
```

Rubi steps

$$\int \frac{1}{(ag+bgx)^2\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}\,dx=\int \frac{1}{(ag+bgx)^2\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}\,dx$$

**Mathematica [F]** time = 0.07, size = 0, normalized size = 0.00

$$\int \frac{1}{(ag+bgx)^2\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[1/((a*g + b*g*x)^2*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])),x]
```

```
[Out] Integrate[1/((a*g + b*g*x)^2*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])), x]
```

**fricas [F]** time = 0.91, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{1}{Ab^2g^2x^2 + 2\,Aabg^2x + Aa^2g^2 + \left(Bb^2g^2x^2 + 2\,Babg^2x + Ba^2g^2\right)\log\left(\frac{b^2ex^2+2\,abex+a^2e}{d^2x^2+2\,cdx+c^2}\right)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*g*x+a*g)^2/(A+B*log(e*(b*x+a)^2/(d*x+c)^2)),x, algorithm="fr
icas")
```

```
[Out] integral(1/(A*b^2*g^2*x^2 + 2*A*a*b*g^2*x + A*a^2*g^2 + (B*b^2*g^2*x^2 + 2*
B*a*b*g^2*x + B*a^2*g^2)*log((b^2*e*x^2 + 2*a*b*e*x + a^2*e)/(d^2*x^2 + 2*c
*d*x + c^2))), x)
```

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(bgx + ag\right)^2\left(B\log\left(\frac{(bx+a)^2e}{(dx+c)^2}\right) + A\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*g*x+a*g)^2/(A+B*log(e*(b*x+a)^2/(d*x+c)^2)),x, algorithm="gi
ac")
```

```
[Out] integrate(1/((b*g*x + a*g)^2*(B*log((b*x + a)^2*e/(d*x + c)^2) + A)), x)
```

**maple [F]** time = 0.88, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(bgx + ag\right)^2\left(B\ln\left(\frac{(bx+a)^2e}{(dx+c)^2}\right) + A\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*g*x+a*g)^2/(B*ln((b*x+a)^2/(d*x+c)^2*e)+A),x)
```

```
[Out] int(1/(b*g*x+a*g)^2/(B*ln((b*x+a)^2/(d*x+c)^2*e)+A),x)
```

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(bgx + ag\right)^2\left(B\log\left(\frac{(bx+a)^2e}{(dx+c)^2}\right) + A\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*g*x+a*g)^2/(A+B*log(e*(b*x+a)^2/(d*x+c)^2)),x, algorithm="ma
xima")
```

```
[Out] integrate(1/((b*g*x + a*g)^2*(B*log((b*x + a)^2*e/(d*x + c)^2) + A)), x)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\left(a\,g+b\,g\,x\right)^2\,\left(A+B\,\ln\left(\frac{e\,{\left(a+b\,x\right)}^2}{{\left(c+d\,x\right)}^2}\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*g + b*g*x)^2*(A + B*log((e*(a + b*x)^2)/(c + d*x)^2))),x)
```

```
[Out] int(1/((a*g + b*g*x)^2*(A + B*log((e*(a + b*x)^2)/(c + d*x)^2))), x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{1}{Aa^2+2Aabx+Ab^2x^2+Ba^2\log\left(\frac{a^2e}{c^2+2cdx+d^2x^2}+\frac{2abex}{c^2+2cdx+d^2x^2}+\frac{b^2ex^2}{c^2+2cdx+d^2x^2}\right)+2Babx\log\left(\frac{a^2e}{c^2+2cdx+d^2x^2}+\frac{2abex}{c^2+2cdx+d^2x^2}+\frac{b^2ex^2}{c^2+2cdx+d^2x^2}\right)+Bb^2x^2\log\left(\frac{\ }{c^2 \ldots}\right.}}{g^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*g*x+a*g)**2/(A+B*ln(e*(b*x+a)**2/(d*x+c)**2)),x)
```

```
[Out] Integral(1/(A*a**2 + 2*A*a*b*x + A*b**2*x**2 + B*a**2*log(a**2*e/(c**2 + 2*
c*d*x + d**2*x**2) + 2*a*b*e*x/(c**2 + 2*c*d*x + d**2*x**2) + b**2*e*x**2/(
c**2 + 2*c*d*x + d**2*x**2)) + 2*B*a*b*x*log(a**2*e/(c**2 + 2*c*d*x + d**2*
x**2) + 2*a*b*e*x/(c**2 + 2*c*d*x + d**2*x**2) + b**2*e*x**2/(c**2 + 2*c*d*
x + d**2*x**2)) + B*b**2*x**2*log(a**2*e/(c**2 + 2*c*d*x + d**2*x**2) + 2*a
*b*e*x/(c**2 + 2*c*d*x + d**2*x**2) + b**2*e*x**2/(c**2 + 2*c*d*x + d**2*x*
*2))), x)/g**2
```

# 3.141 $\int \frac{1}{(ag+bgx)^3\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}\,dx$

**Optimal**. Leaf size=152

$$\frac{bee^{A/B}\text{Ei}\left(-\frac{A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}{B}\right)}{2Bg^3(bc-ad)^2}-\frac{de^{\frac{A}{2B}}(c+dx)\sqrt{\frac{e(a+bx)^2}{(c+dx)^2}}\,\text{Ei}\left(\frac{-A-B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}{2B}\right)}{2Bg^3(a+bx)(bc-ad)^2}$$

```
[Out] 1/2*b*e*exp(A/B)*Ei((-A-B*ln(e*(b*x+a)^2/(d*x+c)^2))/B)/B/(-a*d+b*c)^2/g^3-
1/2*d*exp(1/2*A/B)*(d*x+c)*Ei(1/2*(-A-B*ln(e*(b*x+a)^2/(d*x+c)^2))/B)*(e*(b
*x+a)^2/(d*x+c)^2)^(1/2)/B/(-a*d+b*c)^2/g^3/(b*x+a)
```

**Rubi [F]** time = 0.07, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{1}{(ag+bgx)^3\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[1/((a*g + b*g*x)^3*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])),x]
```

```
[Out] Defer[Int][1/((a*g + b*g*x)^3*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])), x]
```

Rubi steps

$$\int \frac{1}{(ag+bgx)^3\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}\,dx=\int \frac{1}{(ag+bgx)^3\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}\,dx$$

**Mathematica [F]** time = 0.08, size = 0, normalized size = 0.00

$$\int \frac{1}{(ag+bgx)^3\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[1/((a*g + b*g*x)^3*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])),x]
```

```
[Out] Integrate[1/((a*g + b*g*x)^3*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])), x]
```

**fricas [F]** time = 0.82, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{1}{Ab^3g^3x^3 + 3\,Aab^2g^3x^2 + 3\,Aa^2bg^3x + Aa^3g^3 + \left(Bb^3g^3x^3 + 3\,Bab^2g^3x^2 + 3\,Ba^2bg^3x + Ba^3g^3\right)\log\left(\frac{b^2ex^2}{d^2x^2}\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*g*x+a*g)^3/(A+B*log(e*(b*x+a)^2/(d*x+c)^2)),x, algorithm="fr
icas")
```

```
[Out] integral(1/(A*b^3*g^3*x^3 + 3*A*a*b^2*g^3*x^2 + 3*A*a^2*b*g^3*x + A*a^3*g^3
 + (B*b^3*g^3*x^3 + 3*B*a*b^2*g^3*x^2 + 3*B*a^2*b*g^3*x + B*a^3*g^3)*log((b
^2*e*x^2 + 2*a*b*e*x + a^2*e)/(d^2*x^2 + 2*c*d*x + c^2))), x)
```

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(bgx + ag\right)^3\left(B\log\left(\frac{(bx+a)^2e}{(dx+c)^2}\right) + A\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*g*x+a*g)^3/(A+B*log(e*(b*x+a)^2/(d*x+c)^2)),x, algorithm="gi
ac")
```

```
[Out] integrate(1/((b*g*x + a*g)^3*(B*log((b*x + a)^2*e/(d*x + c)^2) + A)), x)
```

**maple [F]** time = 0.96, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(bgx + ag\right)^3\left(B\ln\left(\frac{(bx+a)^2e}{(dx+c)^2}\right) + A\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*g*x+a*g)^3/(B*ln((b*x+a)^2/(d*x+c)^2*e)+A),x)
```

```
[Out] int(1/(b*g*x+a*g)^3/(B*ln((b*x+a)^2/(d*x+c)^2*e)+A),x)
```

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(bgx + ag\right)^3\left(B\log\left(\frac{(bx+a)^2e}{(dx+c)^2}\right) + A\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*g*x+a*g)^3/(A+B*log(e*(b*x+a)^2/(d*x+c)^2)),x, algorithm="ma
xima")
```

```
[Out] integrate(1/((b*g*x + a*g)^3*(B*log((b*x + a)^2*e/(d*x + c)^2) + A)), x)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{{\left(a\,g+b\,g\,x\right)}^3\,\left(A+B\,\ln\left(\frac{e\,{\left(a+b\,x\right)}^2}{{\left(c+d\,x\right)}^2}\right)\right)} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*g + b*g*x)^3*(A + B*log((e*(a + b*x)^2)/(c + d*x)^2))),x)
```

```
[Out] int(1/((a*g + b*g*x)^3*(A + B*log((e*(a + b*x)^2)/(c + d*x)^2))), x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\frac{1}{Aa^3+3Aa^2bx+3Aab^2x^2+Ab^3x^3+Ba^3\log\left(\frac{a^2e}{c^2+2cdx+d^2x^2}+\frac{2abex}{c^2+2cdx+d^2x^2}+\frac{b^2ex^2}{c^2+2cdx+d^2x^2}\right)+3Ba^2bx\log\left(\frac{a^2e}{c^2+2cdx+d^2x^2}+\frac{2abex}{c^2+2cdx+d^2x^2}+\frac{b^2ex^2}{c^2+2cdx+d^2x^2}\right)+}}{g^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*g*x+a*g)**3/(A+B*ln(e*(b*x+a)**2/(d*x+c)**2)),x)
```

```
[Out] Integral(1/(A*a**3 + 3*A*a**2*b*x + 3*A*a*b**2*x**2 + A*b**3*x**3 + B*a**3*
log(a**2*e/(c**2 + 2*c*d*x + d**2*x**2) + 2*a*b*e*x/(c**2 + 2*c*d*x + d**2*
x**2) + b**2*e*x**2/(c**2 + 2*c*d*x + d**2*x**2)) + 3*B*a**2*b*x*log(a**2*e
/(c**2 + 2*c*d*x + d**2*x**2) + 2*a*b*e*x/(c**2 + 2*c*d*x + d**2*x**2) + b*
*2*e*x**2/(c**2 + 2*c*d*x + d**2*x**2)) + 3*B*a*b**2*x**2*log(a**2*e/(c**2
+ 2*c*d*x + d**2*x**2) + 2*a*b*e*x/(c**2 + 2*c*d*x + d**2*x**2) + b**2*e*x*
*2/(c**2 + 2*c*d*x + d**2*x**2)) + B*b**3*x**3*log(a**2*e/(c**2 + 2*c*d*x +
 d**2*x**2) + 2*a*b*e*x/(c**2 + 2*c*d*x + d**2*x**2) + b**2*e*x**2/(c**2 +
2*c*d*x + d**2*x**2))), x)/g**3
```

# 3.142 $\int \frac{(ag+bgx)^2}{\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}\,dx$

**Optimal**. Leaf size=37

$$\text{Int}\left(\frac{(ag+bgx)^2}{\left(B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+A\right)^2},x\right)$$

```
[Out] Unintegrable((b*g*x+a*g)^2/(A+B*ln(e*(b*x+a)^2/(d*x+c)^2))^2,x)
```

**Rubi [A]** time = 0.22, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{(ag+bgx)^2}{\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(a*g + b*g*x)^2/(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])^2,x]
```

```
[Out] a^2*g^2*Defer[Int][(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])^(-2), x] + 2*a*
b*g^2*Defer[Int][x/(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])^2, x] + b^2*g^2
*Defer[Int][x^2/(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])^2, x]
```

Rubi steps

$$\begin{aligned}\int \frac{(ag+bgx)^2}{\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}\,dx &= \int \left(\frac{a^2g^2}{\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}+\frac{2abg^2x}{\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}+\frac{b^2g^2x^2}{\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}\right. \\ &= \left(a^2g^2\right)\int \frac{1}{\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}\,dx+\left(2abg^2\right)\int \frac{x}{\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}\,dx+\left(b\right.\end{aligned}$$

**Mathematica [A]** time = 0.47, size = 0, normalized size = 0.00

$$\int \frac{(ag+bgx)^2}{\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(a*g + b*g*x)^2/(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])^2,x]
```

```
[Out] Integrate[(a*g + b*g*x)^2/(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])^2, x]
```

**fricas [A]** time = 0.73, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{b^2g^2x^2 + 2\,abg^2x + a^2g^2}{B^2\log\left(\frac{b^2ex^2+2\,abex+a^2e}{d^2x^2+2\,cdx+c^2}\right)^2 + 2\,AB\log\left(\frac{b^2ex^2+2\,abex+a^2e}{d^2x^2+2\,cdx+c^2}\right) + A^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*g*x+a*g)^2/(A+B*log(e*(b*x+a)^2/(d*x+c)^2))^2,x, algorithm="fr
icas")
```

```
[Out] integral((b^2*g^2*x^2 + 2*a*b*g^2*x + a^2*g^2)/(B^2*log((b^2*e*x^2 + 2*a*b*
e*x + a^2*e)/(d^2*x^2 + 2*c*d*x + c^2))^2 + 2*A*B*log((b^2*e*x^2 + 2*a*b*e*
x + a^2*e)/(d^2*x^2 + 2*c*d*x + c^2)) + A^2), x)
```

**giac [A]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(bgx + ag)^2}{\left(B\log\left(\frac{(bx+a)^2e}{(dx+c)^2}\right) + A\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*g*x+a*g)^2/(A+B*log(e*(b*x+a)^2/(d*x+c)^2))^2,x, algorithm="gi
ac")
```

```
[Out] integrate((b*g*x + a*g)^2/(B*log((b*x + a)^2*e/(d*x + c)^2) + A)^2, x)
```

**maple [A]** time = 0.76, size = 0, normalized size = 0.00

$$\int \frac{(bgx + ag)^2}{\left(B\ln\left(\frac{(bx+a)^2e}{(dx+c)^2}\right) + A\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*g*x+a*g)^2/(B*ln((b*x+a)^2/(d*x+c)^2*e)+A)^2,x)
```

```
[Out] int((b*g*x+a*g)^2/(B*ln((b*x+a)^2/(d*x+c)^2*e)+A)^2,x)
```

**maxima** [A] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{b^3dg^2x^4 + a^3cg^2 + \left(b^3cg^2 + 3\,ab^2dg^2\right)x^3 + 3\left(ab^2cg^2 + a^2bdg^2\right)x^2 + \left(3\,a^2bcg^2 + a^3dg^2\right)x}{2\left(2\,(bc-ad)B^2\log\left(bx+a\right) - 2\,(bc-ad)B^2\log\left(dx+c\right) + (bc-ad)AB + \left(bc\log(e) - ad\log(e)\right)B^2\right)} + \int \frac{}{2\left(2\,(b\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*g*x+a*g)^2/(A+B*log(e*(b*x+a)^2/(d*x+c)^2))^2,x, algorithm="ma
xima")
```

```
[Out] -1/2*(b^3*d*g^2*x^4 + a^3*c*g^2 + (b^3*c*g^2 + 3*a*b^2*d*g^2)*x^3 + 3*(a*b^
2*c*g^2 + a^2*b*d*g^2)*x^2 + (3*a^2*b*c*g^2 + a^3*d*g^2)*x)/(2*(b*c - a*d)*
B^2*log(b*x + a) - 2*(b*c - a*d)*B^2*log(d*x + c) + (b*c - a*d)*A*B + (b*c*
log(e) - a*d*log(e))*B^2) + integrate(1/2*(4*b^3*d*g^2*x^3 + 3*a^2*b*c*g^2
+ a^3*d*g^2 + 3*(b^3*c*g^2 + 3*a*b^2*d*g^2)*x^2 + 6*(a*b^2*c*g^2 + a^2*b*d*
g^2)*x)/(2*(b*c - a*d)*B^2*log(b*x + a) - 2*(b*c - a*d)*B^2*log(d*x + c) +
(b*c - a*d)*A*B + (b*c*log(e) - a*d*log(e))*B^2), x)
```

**mupad** [A] time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{{\left(a\,g+b\,g\,x\right)}^2}{{\left(A+B\,\ln\left(\frac{e\,{\left(a+b\,x\right)}^2}{{\left(c+d\,x\right)}^2}\right)\right)}^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*g + b*g*x)^2/(A + B*log((e*(a + b*x)^2)/(c + d*x)^2))^2,x)
```

```
[Out] int((a*g + b*g*x)^2/(A + B*log((e*(a + b*x)^2)/(c + d*x)^2))^2, x)
```

**sympy** [A] time = 0.00, size = 0, normalized size = 0.00

$$\frac{a^3cg^2 + a^3dg^2x + 3a^2bcg^2x + 3a^2bdg^2x^2 + 3ab^2cg^2x^2 + 3ab^2dg^2x^3 + b^3cg^2x^3 + b^3dg^2x^4}{2ABad - 2ABbc + \left(2B^2ad - 2B^2bc\right)\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)} - \frac{g^2\left(\int \frac{}{A+B\log\left(\frac{a^2e}{c^2+2cdx+d^2x^2}\right.}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*g*x+a*g)**2/(A+B*ln(e*(b*x+a)**2/(d*x+c)**2))**2,x)
```

```
[Out] (a**3*c*g**2 + a**3*d*g**2*x + 3*a**2*b*c*g**2*x + 3*a**2*b*d*g**2*x**2 + 3
*a*b**2*c*g**2*x**2 + 3*a*b**2*d*g**2*x**3 + b**3*c*g**2*x**3 + b**3*d*g**2
*x**4)/(2*A*B*a*d - 2*A*B*b*c + (2*B**2*a*d - 2*B**2*b*c)*log(e*(a + b*x)**
```

```
2/(c + d*x)**2)) - g**2*(Integral(a**3*d/(A + B*log(a**2*e/(c**2 + 2*c*d*x
+ d**2*x**2) + 2*a*b*e*x/(c**2 + 2*c*d*x + d**2*x**2) + b**2*e*x**2/(c**2 +
 2*c*d*x + d**2*x**2))), x) + Integral(3*a**2*b*c/(A + B*log(a**2*e/(c**2 +
 2*c*d*x + d**2*x**2) + 2*a*b*e*x/(c**2 + 2*c*d*x + d**2*x**2) + b**2*e*x**
2/(c**2 + 2*c*d*x + d**2*x**2))), x) + Integral(3*b**3*c*x**2/(A + B*log(a*
*2*e/(c**2 + 2*c*d*x + d**2*x**2) + 2*a*b*e*x/(c**2 + 2*c*d*x + d**2*x**2)
+ b**2*e*x**2/(c**2 + 2*c*d*x + d**2*x**2))), x) + Integral(4*b**3*d*x**3/(
A + B*log(a**2*e/(c**2 + 2*c*d*x + d**2*x**2) + 2*a*b*e*x/(c**2 + 2*c*d*x +
 d**2*x**2) + b**2*e*x**2/(c**2 + 2*c*d*x + d**2*x**2))), x) + Integral(6*a
*b**2*c*x/(A + B*log(a**2*e/(c**2 + 2*c*d*x + d**2*x**2) + 2*a*b*e*x/(c**2
+ 2*c*d*x + d**2*x**2) + b**2*e*x**2/(c**2 + 2*c*d*x + d**2*x**2))), x) + I
ntegral(9*a*b**2*d*x**2/(A + B*log(a**2*e/(c**2 + 2*c*d*x + d**2*x**2) + 2*
a*b*e*x/(c**2 + 2*c*d*x + d**2*x**2) + b**2*e*x**2/(c**2 + 2*c*d*x + d**2*x
**2))), x) + Integral(6*a**2*b*d*x/(A + B*log(a**2*e/(c**2 + 2*c*d*x + d**2
*x**2) + 2*a*b*e*x/(c**2 + 2*c*d*x + d**2*x**2) + b**2*e*x**2/(c**2 + 2*c*d
*x + d**2*x**2))), x))/(2*B*(a*d - b*c))
```

# 3.143 $\int \frac{ag+bgx}{\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2} dx$

**Optimal**. Leaf size=35

$$\text{Int}\left(\frac{ag+bgx}{\left(B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+A\right)^2},x\right)$$

```
[Out] Unintegrable((b*g*x+a*g)/(A+B*ln(e*(b*x+a)^2/(d*x+c)^2))^2,x)
```

**Rubi [A]** time = 0.11, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{ag+bgx}{\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2} dx$$

Verification is Not applicable to the result.

```
[In] Int[(a*g + b*g*x)/(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])^2,x]
```

```
[Out] a*g*Defer[Int][(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])^(-2), x] + b*g*Defe
r[Int][x/(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])^2, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{ag+bgx}{\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2} dx &= \int \left(\frac{ag}{\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}+\frac{bgx}{\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}\right) dx\\
&= (ag)\int \frac{1}{\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2} dx+(bg)\int \frac{x}{\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2} dx
\end{aligned}$$

**Mathematica [A]** time = 0.34, size = 0, normalized size = 0.00

$$\int \frac{ag+bgx}{\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2} dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(a*g + b*g*x)/(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])^2,x]
```

```
[Out] Integrate[(a*g + b*g*x)/(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])^2, x]
```

**fricas [A]** time = 0.83, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{bgx+ag}{B^2\log\left(\frac{b^2ex^2+2\,abex+a^2e}{d^2x^2+2\,cdx+c^2}\right)^2+2\,AB\log\left(\frac{b^2ex^2+2\,abex+a^2e}{d^2x^2+2\,cdx+c^2}\right)+A^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*g*x+a*g)/(A+B*log(e*(b*x+a)^2/(d*x+c)^2))^2,x, algorithm="fric
as")
```

```
[Out] integral((b*g*x + a*g)/(B^2*log((b^2*e*x^2 + 2*a*b*e*x + a^2*e)/(d^2*x^2 +
2*c*d*x + c^2))^2 + 2*A*B*log((b^2*e*x^2 + 2*a*b*e*x + a^2*e)/(d^2*x^2 + 2*
c*d*x + c^2)) + A^2), x)
```

**giac [A]** time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{bgx+ag}{\left(B\log\left(\frac{(bx+a)^2e}{(dx+c)^2}\right)+A\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*g*x+a*g)/(A+B*log(e*(b*x+a)^2/(d*x+c)^2))^2,x, algorithm="giac
")
```

```
[Out] integrate((b*g*x + a*g)/(B*log((b*x + a)^2*e/(d*x + c)^2) + A)^2, x)
```

**maple [A]** time = 0.75, size = 0, normalized size = 0.00

$$\int\frac{bgx+ag}{\left(B\ln\left(\frac{(bx+a)^2e}{(dx+c)^2}\right)+A\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*g*x+a*g)/(B*ln((b*x+a)^2/(d*x+c)^2*e)+A)^2,x)
```

```
[Out] int((b*g*x+a*g)/(B*ln((b*x+a)^2/(d*x+c)^2*e)+A)^2,x)
```

**maxima [A]** time = 0.00, size = 0, normalized size = 0.00

$$-\frac{b^2dgx^3+a^2cg+\left(b^2cg+2\,abdg\right)x^2+\left(2\,abcg+a^2dg\right)x}{2\left(2\,(bc-ad)B^2\log\left(bx+a\right)-2\,(bc-ad)B^2\log\left(dx+c\right)+(bc-ad)AB+\left(bc\log(e)-ad\log(e)\right)B^2\right)}+\int\frac{}{2\,(2\,(}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*g*x+a*g)/(A+B*log(e*(b*x+a)^2/(d*x+c)^2))^2,x, algorithm="maxi
ma")
```

```
[Out] -1/2*(b^2*d*g*x^3 + a^2*c*g + (b^2*c*g + 2*a*b*d*g)*x^2 + (2*a*b*c*g + a^2*
d*g)*x)/(2*(b*c - a*d)*B^2*log(b*x + a) - 2*(b*c - a*d)*B^2*log(d*x + c) +
(b*c - a*d)*A*B + (b*c*log(e) - a*d*log(e))*B^2) + integrate(1/2*(3*b^2*d*g
*x^2 + 2*a*b*c*g + a^2*d*g + 2*(b^2*c*g + 2*a*b*d*g)*x)/(2*(b*c - a*d)*B^2*
log(b*x + a) - 2*(b*c - a*d)*B^2*log(d*x + c) + (b*c - a*d)*A*B + (b*c*log(
e) - a*d*log(e))*B^2), x)
```

**mupad [A]** time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{a\,g+b\,g\,x}{{\left(A+B\,\ln\left(\frac{e\,{\left(a+b\,x\right)}^2}{{\left(c+d\,x\right)}^2}\right)\right)}^2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*g + b*g*x)/(A + B*log((e*(a + b*x)^2)/(c + d*x)^2))^2,x)
```

```
[Out] int((a*g + b*g*x)/(A + B*log((e*(a + b*x)^2)/(c + d*x)^2))^2, x)
```

**sympy [A]** time = 0.00, size = 0, normalized size = 0.00

$$\frac{a^2cg + a^2dgx + 2abcgx + 2abdgx^2 + b^2cgx^2 + b^2dgx^3}{2ABad - 2ABbc + \left(2B^2ad - 2B^2bc\right)\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)} - \frac{g\left(\int \frac{a^2d}{A+B\log\left(\frac{a^2e}{c^2+2cdx+d^2x^2} + \frac{2abex}{c^2+2cdx+d^2x^2} + \frac{b^2ex^2}{c^2+2cdx+d^2x^2}\right)}\,dx + \int \frac{}{A+B}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*g*x+a*g)/(A+B*ln(e*(b*x+a)**2/(d*x+c)**2))**2,x)
```

```
[Out] (a**2*c*g + a**2*d*g*x + 2*a*b*c*g*x + 2*a*b*d*g*x**2 + b**2*c*g*x**2 + b**
2*d*g*x**3)/(2*A*B*a*d - 2*A*B*b*c + (2*B**2*a*d - 2*B**2*b*c)*log(e*(a + b
*x)**2/(c + d*x)**2)) - g*(Integral(a**2*d/(A + B*log(a**2*e/(c**2 + 2*c*d*
x + d**2*x**2) + 2*a*b*e*x/(c**2 + 2*c*d*x + d**2*x**2) + b**2*e*x**2/(c**2
 + 2*c*d*x + d**2*x**2))), x) + Integral(2*a*b*c/(A + B*log(a**2*e/(c**2 +
2*c*d*x + d**2*x**2) + 2*a*b*e*x/(c**2 + 2*c*d*x + d**2*x**2) + b**2*e*x**2
/(c**2 + 2*c*d*x + d**2*x**2))), x) + Integral(2*b**2*c*x/(A + B*log(a**2*e
/(c**2 + 2*c*d*x + d**2*x**2) + 2*a*b*e*x/(c**2 + 2*c*d*x + d**2*x**2) + b*
*2*e*x**2/(c**2 + 2*c*d*x + d**2*x**2))), x) + Integral(3*b**2*d*x**2/(A +
B*log(a**2*e/(c**2 + 2*c*d*x + d**2*x**2) + 2*a*b*e*x/(c**2 + 2*c*d*x + d**
```

```
2*x**2) + b**2*e*x**2/(c**2 + 2*c*d*x + d**2*x**2))), x) + Integral(4*a*b*d
*x/(A + B*log(a**2*e/(c**2 + 2*c*d*x + d**2*x**2) + 2*a*b*e*x/(c**2 + 2*c*d
*x + d**2*x**2) + b**2*e*x**2/(c**2 + 2*c*d*x + d**2*x**2))), x))/(2*B*(a*d
 - b*c))
```

# 3.144 $\int \frac{1}{(ag+bgx)\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}\,dx$

**Optimal**. Leaf size=37

$$\text{Int}\left(\frac{1}{(ag+bgx)\left(B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+A\right)^2},x\right)$$

```
[Out] Unintegrable(1/(b*g*x+a*g)/(A+B*ln(e*(b*x+a)^2/(d*x+c)^2))^2,x)
```

**Rubi [A]** time = 0.08, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{1}{(ag+bgx)\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[1/((a*g + b*g*x)*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])^2),x]
```

```
[Out] Defer[Int][1/((a*g + b*g*x)*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])^2), x]
```

Rubi steps

$$\int \frac{1}{(ag+bgx)\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}\,dx=\int \frac{1}{(ag+bgx)\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}\,dx$$

**Mathematica [A]** time = 0.14, size = 0, normalized size = 0.00

$$\int \frac{1}{(ag+bgx)\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[1/((a*g + b*g*x)*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])^2),x]
```

```
[Out] Integrate[1/((a*g + b*g*x)*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])^2), x]
```

**fricas [A]** time = 0.82, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{1}{A^2bgx + A^2ag + \left(B^2bgx + B^2ag\right)\log\left(\frac{b^2ex^2+2\,abex+a^2e}{d^2x^2+2\,cdx+c^2}\right)^2 + 2\left(ABbgx + ABag\right)\log\left(\frac{b^2ex^2+2\,abex+a^2e}{d^2x^2+2\,cdx+c^2}\right)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*g*x+a*g)/(A+B*log(e*(b*x+a)^2/(d*x+c)^2))^2,x, algorithm="fr
icas")
```

```
[Out] integral(1/(A^2*b*g*x + A^2*a*g + (B^2*b*g*x + B^2*a*g)*log((b^2*e*x^2 + 2*
a*b*e*x + a^2*e)/(d^2*x^2 + 2*c*d*x + c^2))^2 + 2*(A*B*b*g*x + A*B*a*g)*log
((b^2*e*x^2 + 2*a*b*e*x + a^2*e)/(d^2*x^2 + 2*c*d*x + c^2))), x)
```

**giac [A]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(bgx + ag\right)\left(B\log\left(\frac{(bx+a)^2e}{(dx+c)^2}\right) + A\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*g*x+a*g)/(A+B*log(e*(b*x+a)^2/(d*x+c)^2))^2,x, algorithm="gi
ac")
```

```
[Out] integrate(1/((b*g*x + a*g)*(B*log((b*x + a)^2*e/(d*x + c)^2) + A)^2), x)
```

**maple [A]** time = 0.74, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(bgx + ag\right)\left(B\ln\left(\frac{(bx+a)^2e}{(dx+c)^2}\right) + A\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*g*x+a*g)/(B*ln((b*x+a)^2/(d*x+c)^2*e)+A)^2,x)
```

```
[Out] int(1/(b*g*x+a*g)/(B*ln((b*x+a)^2/(d*x+c)^2*e)+A)^2,x)
```

**maxima [A]** time = 0.00, size = 0, normalized size = 0.00

$$d\int \frac{1}{2\left(2\left(bcg - adg\right)B^2\log\left(bx + a\right) - 2\left(bcg - adg\right)B^2\log\left(dx + c\right) + \left(bcg - adg\right)AB + \left(bcg\log(e) - adg\log(e)\right)\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*g*x+a*g)/(A+B*log(e*(b*x+a)^2/(d*x+c)^2))^2,x, algorithm="ma
xima")
```

```
[Out] d*integrate(1/2/(2*(b*c*g - a*d*g)*B^2*log(b*x + a) - 2*(b*c*g - a*d*g)*B^2
*log(d*x + c) + (b*c*g - a*d*g)*A*B + (b*c*g*log(e) - a*d*g*log(e))*B^2), x
) - 1/2*(d*x + c)/(2*(b*c*g - a*d*g)*B^2*log(b*x + a) - 2*(b*c*g - a*d*g)*B
^2*log(d*x + c) + (b*c*g - a*d*g)*A*B + (b*c*g*log(e) - a*d*g*log(e))*B^2)
```

**mupad [A]** time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{1}{\left(a\,g+b\,g\,x\right)\,{\left(A+B\,\ln\left(\frac{e\,{\left(a+b\,x\right)}^2}{{\left(c+d\,x\right)}^2}\right)\right)}^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*g + b*g*x)*(A + B*log((e*(a + b*x)^2)/(c + d*x)^2))^2),x)
```

```
[Out] int(1/((a*g + b*g*x)*(A + B*log((e*(a + b*x)^2)/(c + d*x)^2))^2), x)
```

**sympy [A]** time = 0.00, size = 0, normalized size = 0.00

$$\frac{c+dx}{2ABadg-2ABbcg+\left(2B^2adg-2B^2bcg\right)\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}-\frac{d\int\frac{1}{A+B\log\left(\frac{a^2e}{c^2+2cdx+d^2x^2}+\frac{2abex}{c^2+2cdx+d^2x^2}+\frac{b^2ex^2}{c^2+2cdx+d^2x^2}\right)}\,dx}{2Bg\left(ad-bc\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*g*x+a*g)/(A+B*ln(e*(b*x+a)**2/(d*x+c)**2))**2,x)
```

```
[Out] (c + d*x)/(2*A*B*a*d*g - 2*A*B*b*c*g + (2*B**2*a*d*g - 2*B**2*b*c*g)*log(e*
(a + b*x)**2/(c + d*x)**2)) - d*Integral(1/(A + B*log(a**2*e/(c**2 + 2*c*d*
x + d**2*x**2) + 2*a*b*e*x/(c**2 + 2*c*d*x + d**2*x**2) + b**2*e*x**2/(c**2
 + 2*c*d*x + d**2*x**2))), x)/(2*B*g*(a*d - b*c))
```

# 3.145 $\int \frac{1}{(ag+bgx)^2\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}\,dx$

**Optimal**. Leaf size=150

$$-\frac{e^{\frac{A}{2B}}(c+dx)\sqrt{\frac{e(a+bx)^2}{(c+dx)^2}}\,\mathrm{Ei}\left(\frac{-A-B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}{2B}\right)}{4B^2g^2(a+bx)(bc-ad)}-\frac{c+dx}{2Bg^2(a+bx)(bc-ad)\left(B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+A\right)}$$

```
[Out] 1/2*(-d*x-c)/B/(-a*d+b*c)/g^2/(b*x+a)/(A+B*ln(e*(b*x+a)^2/(d*x+c)^2))-1/4*e
xp(1/2*A/B)*(d*x+c)*Ei(1/2*(-A-B*ln(e*(b*x+a)^2/(d*x+c)^2))/B)*(e*(b*x+a)^2
/(d*x+c)^2)^(1/2)/B^2/(-a*d+b*c)/g^2/(b*x+a)
```

**Rubi [F]** time = 0.09, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{1}{(ag+bgx)^2\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[1/((a*g + b*g*x)^2*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])^2),x]
```

```
[Out] Defer[Int][1/((a*g + b*g*x)^2*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])^2),
x]
```

Rubi steps

$$\int \frac{1}{(ag+bgx)^2\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}\,dx=\int \frac{1}{(ag+bgx)^2\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}\,dx$$

**Mathematica [F]** time = 0.18, size = 0, normalized size = 0.00

$$\int \frac{1}{(ag+bgx)^2\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[1/((a*g + b*g*x)^2*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])^2),x]
```

```
[Out] Integrate[1/((a*g + b*g*x)^2*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])^2), x
]
```

**fricas [F]** time = 1.44, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{1}{A^2b^2g^2x^2 + 2\,A^2abg^2x + A^2a^2g^2 + \left(B^2b^2g^2x^2 + 2\,B^2abg^2x + B^2a^2g^2\right)\log\left(\frac{b^2ex^2+2\,abex+a^2e}{d^2x^2+2\,cdx+c^2}\right)^2 + 2\left(ABb^2g\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*g*x+a*g)^2/(A+B*log(e*(b*x+a)^2/(d*x+c)^2))^2,x, algorithm="
fricas")
```

```
[Out] integral(1/(A^2*b^2*g^2*x^2 + 2*A^2*a*b*g^2*x + A^2*a^2*g^2 + (B^2*b^2*g^2*
x^2 + 2*B^2*a*b*g^2*x + B^2*a^2*g^2)*log((b^2*e*x^2 + 2*a*b*e*x + a^2*e)/(d
^2*x^2 + 2*c*d*x + c^2))^2 + 2*(A*B*b^2*g^2*x^2 + 2*A*B*a*b*g^2*x + A*B*a^2
*g^2)*log((b^2*e*x^2 + 2*a*b*e*x + a^2*e)/(d^2*x^2 + 2*c*d*x + c^2))), x)
```

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(bgx + ag\right)^2\left(B\log\left(\frac{(bx+a)^2e}{(dx+c)^2}\right) + A\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*g*x+a*g)^2/(A+B*log(e*(b*x+a)^2/(d*x+c)^2))^2,x, algorithm="
giac")
```

```
[Out] integrate(1/((b*g*x + a*g)^2*(B*log((b*x + a)^2*e/(d*x + c)^2) + A)^2), x)
```

**maple [F]** time = 1.05, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(bgx + ag\right)^2\left(B\ln\left(\frac{(bx+a)^2e}{(dx+c)^2}\right) + A\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*g*x+a*g)^2/(B*ln((b*x+a)^2/(d*x+c)^2*e)+A)^2,x)
```

```
[Out] int(1/(b*g*x+a*g)^2/(B*ln((b*x+a)^2/(d*x+c)^2*e)+A)^2,x)
```

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$-\frac{}{2\left(\left(abcg^2 - a^2dg^2\right)AB + \left(abcg^2\log(e) - a^2dg^2\log(e)\right)B^2 + \left(\left(b^2cg^2 - abdg^2\right)AB + \left(b^2cg^2\log(e) - abdg^2\log(e\right.\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*g*x+a*g)^2/(A+B*log(e*(b*x+a)^2/(d*x+c)^2))^2,x, algorithm="
maxima")
```

```
[Out] -1/2*(d*x + c)/((a*b*c*g^2 - a^2*d*g^2)*A*B + (a*b*c*g^2*log(e) - a^2*d*g^2
*log(e))*B^2 + ((b^2*c*g^2 - a*b*d*g^2)*A*B + (b^2*c*g^2*log(e) - a*b*d*g^2
*log(e))*B^2)*x + 2*((b^2*c*g^2 - a*b*d*g^2)*B^2*x + (a*b*c*g^2 - a^2*d*g^2
)*B^2)*log(b*x + a) - 2*((b^2*c*g^2 - a*b*d*g^2)*B^2*x + (a*b*c*g^2 - a^2*d
*g^2)*B^2)*log(d*x + c)) + integrate(-1/2/(B^2*a^2*g^2*log(e) + A*B*a^2*g^2
 + (B^2*b^2*g^2*log(e) + A*B*b^2*g^2)*x^2 + 2*(B^2*a*b*g^2*log(e) + A*B*a*b
*g^2)*x + 2*(B^2*b^2*g^2*x^2 + 2*B^2*a*b*g^2*x + B^2*a^2*g^2)*log(b*x + a)
- 2*(B^2*b^2*g^2*x^2 + 2*B^2*a*b*g^2*x + B^2*a^2*g^2)*log(d*x + c)), x)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\left(a\,g+b\,g\,x\right)^2\,\left(A+B\,\ln\left(\frac{e\,{\left(a+b\,x\right)}^2}{{\left(c+d\,x\right)}^2}\right)\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*g + b*g*x)^2*(A + B*log((e*(a + b*x)^2)/(c + d*x)^2))^2),x)
```

```
[Out] int(1/((a*g + b*g*x)^2*(A + B*log((e*(a + b*x)^2)/(c + d*x)^2))^2), x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\frac{c+dx}{2ABa^2dg^2 - 2ABabcg^2 + 2ABabdg^2x - 2ABb^2cg^2x + \left(2B^2a^2dg^2 - 2B^2abcg^2 + 2B^2abdg^2x - 2B^2b^2cg^2x\right)\log\left(\frac{e}{(}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*g*x+a*g)**2/(A+B*ln(e*(b*x+a)**2/(d*x+c)**2))**2,x)
```

```
[Out] (c + d*x)/(2*A*B*a**2*d*g**2 - 2*A*B*a*b*c*g**2 + 2*A*B*a*b*d*g**2*x - 2*A*
B*b**2*c*g**2*x + (2*B**2*a**2*d*g**2 - 2*B**2*a*b*c*g**2 + 2*B**2*a*b*d*g*
*2*x - 2*B**2*b**2*c*g**2*x)*log(e*(a + b*x)**2/(c + d*x)**2)) - Integral(1
/(A*a**2 + 2*A*a*b*x + A*b**2*x**2 + B*a**2*log(a**2*e/(c**2 + 2*c*d*x + d*
```

```
*2*x**2) + 2*a*b*e*x/(c**2 + 2*c*d*x + d**2*x**2) + b**2*e*x**2/(c**2 + 2*c
*d*x + d**2*x**2)) + 2*B*a*b*x*log(a**2*e/(c**2 + 2*c*d*x + d**2*x**2) + 2*
a*b*e*x/(c**2 + 2*c*d*x + d**2*x**2) + b**2*e*x**2/(c**2 + 2*c*d*x + d**2*x
**2)) + B*b**2*x**2*log(a**2*e/(c**2 + 2*c*d*x + d**2*x**2) + 2*a*b*e*x/(c*
*2 + 2*c*d*x + d**2*x**2) + b**2*e*x**2/(c**2 + 2*c*d*x + d**2*x**2))), x)/
(2*B*g**2)
```

# 3.146 $\int \frac{1}{(ag+bgx)^3\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}\,dx$

**Optimal**. Leaf size=266

$$\frac{de^{\frac{A}{2B}}(c+dx)\sqrt{\frac{e(a+bx)^2}{(c+dx)^2}}\,\mathrm{Ei}\left(\frac{-A-B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}{2B}\right)}{4B^2g^3(a+bx)(bc-ad)^2}-\frac{bee^{A/B}\mathrm{Ei}\left(-\frac{A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}{B}\right)}{2B^2g^3(bc-ad)^2}-\frac{b(c+dx)^2}{2Bg^3(a+bx)^2(bc-ad)^2\left(B\log\left(\frac{e(a+b}{(c+dx}\right.\right.}$$

```
[Out] -1/2*b*e*exp(A/B)*Ei((-A-B*ln(e*(b*x+a)^2/(d*x+c)^2))/B)/B^2/(-a*d+b*c)^2/g
^3+1/2*d*(d*x+c)/B/(-a*d+b*c)^2/g^3/(b*x+a)/(A+B*ln(e*(b*x+a)^2/(d*x+c)^2))
-1/2*b*(d*x+c)^2/B/(-a*d+b*c)^2/g^3/(b*x+a)^2/(A+B*ln(e*(b*x+a)^2/(d*x+c)^2
))+1/4*d*exp(1/2*A/B)*(d*x+c)*Ei(1/2*(-A-B*ln(e*(b*x+a)^2/(d*x+c)^2))/B)*(e
*(b*x+a)^2/(d*x+c)^2)^(1/2)/B^2/(-a*d+b*c)^2/g^3/(b*x+a)
```

**Rubi [F]** time = 0.08, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{1}{(ag+bgx)^3\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[1/((a*g + b*g*x)^3*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])^2),x]
```

```
[Out] Defer[Int][1/((a*g + b*g*x)^3*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])^2),
x]
```

Rubi steps

$$\int \frac{1}{(ag+bgx)^3\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}\,dx=\int \frac{1}{(ag+bgx)^3\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}\,dx$$

**Mathematica [F]** time = 0.35, size = 0, normalized size = 0.00

$$\int \frac{1}{(ag+bgx)^3\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[1/((a*g + b*g*x)^3*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])^2),x]
```

```
[Out] Integrate[1/((a*g + b*g*x)^3*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])^2), x
]
```

**fricas [F]** time = 0.92, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{1}{A^2b^3g^3x^3 + 3\,A^2ab^2g^3x^2 + 3\,A^2a^2bg^3x + A^2a^3g^3 + \left(B^2b^3g^3x^3 + 3\,B^2ab^2g^3x^2 + 3\,B^2a^2bg^3x + B^2a^3g^3\right)\text{l}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*g*x+a*g)^3/(A+B*log(e*(b*x+a)^2/(d*x+c)^2))^2,x, algorithm="
fricas")
```

```
[Out] integral(1/(A^2*b^3*g^3*x^3 + 3*A^2*a*b^2*g^3*x^2 + 3*A^2*a^2*b*g^3*x + A^2
*a^3*g^3 + (B^2*b^3*g^3*x^3 + 3*B^2*a*b^2*g^3*x^2 + 3*B^2*a^2*b*g^3*x + B^2
*a^3*g^3)*log((b^2*e*x^2 + 2*a*b*e*x + a^2*e)/(d^2*x^2 + 2*c*d*x + c^2))^2
+ 2*(A*B*b^3*g^3*x^3 + 3*A*B*a*b^2*g^3*x^2 + 3*A*B*a^2*b*g^3*x + A*B*a^3*g^
3)*log((b^2*e*x^2 + 2*a*b*e*x + a^2*e)/(d^2*x^2 + 2*c*d*x + c^2))), x)
```

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(bgx + ag\right)^3 \left(B\log\left(\frac{(bx+a)^2e}{(dx+c)^2}\right) + A\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*g*x+a*g)^3/(A+B*log(e*(b*x+a)^2/(d*x+c)^2))^2,x, algorithm="
giac")
```

```
[Out] integrate(1/((b*g*x + a*g)^3*(B*log((b*x + a)^2*e/(d*x + c)^2) + A)^2), x)
```

**maple [F]** time = 1.32, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(bgx + ag\right)^3 \left(B\ln\left(\frac{(bx+a)^2e}{(dx+c)^2}\right) + A\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*g*x+a*g)^3/(B*ln((b*x+a)^2/(d*x+c)^2*e)+A)^2,x)
```

```
[Out] int(1/(b*g*x+a*g)^3/(B*ln((b*x+a)^2/(d*x+c)^2*e)+A)^2,x)
```

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$-\frac{}{2\left(\left(a^2bcg^3-a^3dg^3\right)AB+\left(a^2bcg^3\log(e)-a^3dg^3\log(e)\right)B^2+\left(\left(b^3cg^3-ab^2dg^3\right)AB+\left(b^3cg^3\log(e)-ab^2dg^3\log\right.\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*g*x+a*g)^3/(A+B*log(e*(b*x+a)^2/(d*x+c)^2))^2,x, algorithm="
maxima")
```

```
[Out] -1/2*(d*x + c)/((a^2*b*c*g^3 - a^3*d*g^3)*A*B + (a^2*b*c*g^3*log(e) - a^3*d
*g^3*log(e))*B^2 + ((b^3*c*g^3 - a*b^2*d*g^3)*A*B + (b^3*c*g^3*log(e) - a*b
^2*d*g^3*log(e))*B^2)*x^2 + 2*((a*b^2*c*g^3 - a^2*b*d*g^3)*A*B + (a*b^2*c*g
^3*log(e) - a^2*b*d*g^3*log(e))*B^2)*x + 2*((b^3*c*g^3 - a*b^2*d*g^3)*B^2*x
^2 + 2*(a*b^2*c*g^3 - a^2*b*d*g^3)*B^2*x + (a^2*b*c*g^3 - a^3*d*g^3)*B^2)*l
og(b*x + a) - 2*((b^3*c*g^3 - a*b^2*d*g^3)*B^2*x^2 + 2*(a*b^2*c*g^3 - a^2*b
*d*g^3)*B^2*x + (a^2*b*c*g^3 - a^3*d*g^3)*B^2)*log(d*x + c)) - integrate(1/
2*(b*d*x + 2*b*c - a*d)/(((b^4*c*g^3 - a*b^3*d*g^3)*A*B + (b^4*c*g^3*log(e)
 - a*b^3*d*g^3*log(e))*B^2)*x^3 + (a^3*b*c*g^3 - a^4*d*g^3)*A*B + (a^3*b*c*
g^3*log(e) - a^4*d*g^3*log(e))*B^2 + 3*((a*b^3*c*g^3 - a^2*b^2*d*g^3)*A*B +
 (a*b^3*c*g^3*log(e) - a^2*b^2*d*g^3*log(e))*B^2)*x^2 + 3*((a^2*b^2*c*g^3 -
 a^3*b*d*g^3)*A*B + (a^2*b^2*c*g^3*log(e) - a^3*b*d*g^3*log(e))*B^2)*x + 2*
((b^4*c*g^3 - a*b^3*d*g^3)*B^2*x^3 + 3*(a*b^3*c*g^3 - a^2*b^2*d*g^3)*B^2*x^
2 + 3*(a^2*b^2*c*g^3 - a^3*b*d*g^3)*B^2*x + (a^3*b*c*g^3 - a^4*d*g^3)*B^2)*
log(b*x + a) - 2*((b^4*c*g^3 - a*b^3*d*g^3)*B^2*x^3 + 3*(a*b^3*c*g^3 - a^2*
b^2*d*g^3)*B^2*x^2 + 3*(a^2*b^2*c*g^3 - a^3*b*d*g^3)*B^2*x + (a^3*b*c*g^3 -
 a^4*d*g^3)*B^2)*log(d*x + c)), x)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{{\left(a\,g+b\,g\,x\right)}^3\,{\left(A+B\,\ln\left(\frac{e\,{\left(a+b\,x\right)}^2}{{\left(c+d\,x\right)}^2}\right)\right)}^2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*g + b*g*x)^3*(A + B*log((e*(a + b*x)^2)/(c + d*x)^2))^2),x)
```

```
[Out] int(1/((a*g + b*g*x)^3*(A + B*log((e*(a + b*x)^2)/(c + d*x)^2))^2), x)
```

**sympy [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*g*x+a*g)**3/(A+B*ln(e*(b*x+a)**2/(d*x+c)**2))**2,x)
[Out] Timed out
```

# 3.147 $\int (a+bx)^4 \left(A + B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right) dx$

**Optimal**. Leaf size=171

$$\frac{(a+bx)^5 \left(B\log\left(e(a+bx)^n(c+dx)^{-n}\right)+A\right)}{5b}-\frac{Bn(bc-ad)^5\log(c+dx)}{5bd^5}+\frac{Bnx(bc-ad)^4}{5d^4}-\frac{Bn(a+bx)^2(bc-ad)^3}{10bd^3}$$

```
[Out] 1/5*B*(-a*d+b*c)^4*n*x/d^4-1/10*B*(-a*d+b*c)^3*n*(b*x+a)^2/b/d^3+1/15*B*(-a
*d+b*c)^2*n*(b*x+a)^3/b/d^2-1/20*B*(-a*d+b*c)*n*(b*x+a)^4/b/d-1/5*B*(-a*d+b
*c)^5*n*ln(d*x+c)/b/d^5+1/5*(b*x+a)^5*(A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))/b
```

**Rubi [A]** time = 0.18, antiderivative size = 183, normalized size of antiderivative = 1.07, number of steps used = 5, number of rules used = 3, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.097, Rules used = {6742, 2492, 43}

$$\frac{A(a+bx)^5}{5b}+\frac{Bnx(bc-ad)^4}{5d^4}-\frac{Bn(a+bx)^2(bc-ad)^3}{10bd^3}+\frac{Bn(a+bx)^3(bc-ad)^2}{15bd^2}-\frac{Bn(bc-ad)^5\log(c+dx)}{5bd^5}+\frac{B(a+b$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^4*(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n]),x]
```

```
[Out] (B*(b*c - a*d)^4*n*x)/(5*d^4) - (B*(b*c - a*d)^3*n*(a + b*x)^2)/(10*b*d^3)
+ (B*(b*c - a*d)^2*n*(a + b*x)^3)/(15*b*d^2) - (B*(b*c - a*d)*n*(a + b*x)^4
)/(20*b*d) + (A*(a + b*x)^5)/(5*b) - (B*(b*c - a*d)^5*n*Log[c + d*x])/(5*b*
d^5) + (B*(a + b*x)^5*Log[(e*(a + b*x)^n)/(c + d*x)^n])/(5*b)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2492

```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))
^(r_.)]^(s_.)*((g_.) + (h_.)*(x_))^(m_.), x_Symbol] :> Simp[((g + h*x)^(m +
 1)*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^s)/(h*(m + 1)), x] - Dist[(p*r*s*(
b*c - a*d))/(h*(m + 1)), Int[((g + h*x)^(m + 1)*Log[e*(f*(a + b*x)^p*(c + d
*x)^q)^r]^(s - 1))/((a + b*x)*(c + d*x)), x], x] /; FreeQ[{a, b, c, d, e, f
, g, h, m, p, q, r, s}, x] && NeQ[b*c - a*d, 0] && EqQ[p + q, 0] && IGtQ[s,
 0] && NeQ[m, -1]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\int (a+bx)^4 \left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right) dx = \int \left(A(a+bx)^4+B(a+bx)^4\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right) dx$$
$$= \frac{A(a+bx)^5}{5b} + B\int (a+bx)^4\log\left(e(a+bx)^n(c+dx)^{-n}\right) dx$$
$$= \frac{A(a+bx)^5}{5b} + \frac{B(a+bx)^5\log\left(e(a+bx)^n(c+dx)^{-n}\right)}{5b} - \frac{(B(bc}{}$$
$$= \frac{A(a+bx)^5}{5b} + \frac{B(a+bx)^5\log\left(e(a+bx)^n(c+dx)^{-n}\right)}{5b} - \frac{(B(bc}{}$$
$$= \frac{B(bc-ad)^4nx}{5d^4} - \frac{B(bc-ad)^3n(a+bx)^2}{10bd^3} + \frac{B(bc-ad)^2n(a+}{15bd^2}$$

**Mathematica [B]** time = 0.81, size = 364, normalized size = 2.13

$$\frac{-48a^5Bd^5n\log(a+bx)+bdx\left(12a^4d^4(5A+4Bn)+12a^3bd^3(10Adx-10Bcn+3Bdnx)+4a^2b^2d^2\left(30Ad^2x^2+B\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^4*(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n]),x]
```

```
[Out] (b*d*x*(12*a^4*d^4*(5*A + 4*B*n) + 12*a^3*b*d^3*(-10*B*c*n + 10*A*d*x + 3*B
*d*n*x) + 4*a^2*b^2*d^2*(30*A*d^2*x^2 + B*n*(30*c^2 - 15*c*d*x + 4*d^2*x^2)
) + b^4*(12*A*d^4*x^4 + B*c*n*(12*c^3 - 6*c^2*d*x + 4*c*d^2*x^2 - 3*d^3*x^3
)) + a*b^3*d*(60*A*d^3*x^3 + B*n*(-60*c^3 + 30*c^2*d*x - 20*c*d^2*x^2 + 3*d
^3*x^3))) - 48*a^5*B*d^5*n*Log[a + b*x] - 12*B*(b^5*c^5 - 5*a*b^4*c^4*d + 1
0*a^2*b^3*c^3*d^2 - 10*a^3*b^2*c^2*d^3 + 5*a^4*b*c*d^4 - 5*a^5*d^5)*n*Log[c
 + d*x] + 12*B*d^5*(5*a^5 + 5*a^4*b*x + 10*a^3*b^2*x^2 + 10*a^2*b^3*x^3 + 5
*a*b^4*x^4 + b^5*x^5)*Log[(e*(a + b*x)^n)/(c + d*x)^n])/(60*b*d^5)
```

**fricas [B]** time = 0.77, size = 563, normalized size = 3.29

$$\frac{12\,Ab^5d^5x^5+3\left(20\,Aab^4d^5-\left(Bb^5cd^4-Bab^4d^5\right)n\right)x^4+4\left(30\,Aa^2b^3d^5+\left(Bb^5c^2d^3-5\,Bab^4cd^4+4\,Ba^2b^3d^5\right)n\right)x^3}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^4*(A+B*log(e*(b*x+a)^n/((d*x+c)^n))),x, algorithm="fricas
")
```

```
[Out] 1/60*(12*A*b^5*d^5*x^5 + 3*(20*A*a*b^4*d^5 - (B*b^5*c*d^4 - B*a*b^4*d^5)*n)
*x^4 + 4*(30*A*a^2*b^3*d^5 + (B*b^5*c^2*d^3 - 5*B*a*b^4*c*d^4 + 4*B*a^2*b^3
*d^5)*n)*x^3 + 6*(20*A*a^3*b^2*d^5 - (B*b^5*c^3*d^2 - 5*B*a*b^4*c^2*d^3 + 1
0*B*a^2*b^3*c*d^4 - 6*B*a^3*b^2*d^5)*n)*x^2 + 12*(5*A*a^4*b*d^5 + (B*b^5*c^
4*d - 5*B*a*b^4*c^3*d^2 + 10*B*a^2*b^3*c^2*d^3 - 10*B*a^3*b^2*c*d^4 + 4*B*a
^4*b*d^5)*n)*x + 12*(B*b^5*d^5*n*x^5 + 5*B*a*b^4*d^5*n*x^4 + 10*B*a^2*b^3*d
^5*n*x^3 + 10*B*a^3*b^2*d^5*n*x^2 + 5*B*a^4*b*d^5*n*x + B*a^5*d^5*n)*log(b*
x + a) - 12*(B*b^5*d^5*n*x^5 + 5*B*a*b^4*d^5*n*x^4 + 10*B*a^2*b^3*d^5*n*x^3
 + 10*B*a^3*b^2*d^5*n*x^2 + 5*B*a^4*b*d^5*n*x + (B*b^5*c^5 - 5*B*a*b^4*c^4*
d + 10*B*a^2*b^3*c^3*d^2 - 10*B*a^3*b^2*c^2*d^3 + 5*B*a^4*b*c*d^4)*n)*log(d
*x + c) + 12*(B*b^5*d^5*x^5 + 5*B*a*b^4*d^5*x^4 + 10*B*a^2*b^3*d^5*x^3 + 10
*B*a^3*b^2*d^5*x^2 + 5*B*a^4*b*d^5*x)*log(e))/(b*d^5)
```

**giac [B]** time = 11.53, size = 497, normalized size = 2.91

$$\frac{Ba^5 n \log(bx+a)}{5b} + \frac{1}{5}\left(Ab^4 + Bb^4\right)x^5 - \frac{\left(Bb^4cn - Bab^3dn - 20\,Aab^3d - 20\,Bab^3d\right)x^4}{20\,d} + \frac{\left(Bb^4c^2n - 5\,Bab^3cdn + 4\,B\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^4*(A+B*log(e*(b*x+a)^n/((d*x+c)^n))),x, algorithm="giac")
```

```
[Out] 1/5*B*a^5*n*log(b*x + a)/b + 1/5*(A*b^4 + B*b^4)*x^5 - 1/20*(B*b^4*c*n - B*
a*b^3*d*n - 20*A*a*b^3*d - 20*B*a*b^3*d)*x^4/d + 1/15*(B*b^4*c^2*n - 5*B*a*
b^3*c*d*n + 4*B*a^2*b^2*d^2*n + 30*A*a^2*b^2*d^2 + 30*B*a^2*b^2*d^2)*x^3/d^
2 + 1/5*(B*b^4*n*x^5 + 5*B*a*b^3*n*x^4 + 10*B*a^2*b^2*n*x^3 + 10*B*a^3*b*n*
x^2 + 5*B*a^4*n*x)*log(b*x + a) - 1/5*(B*b^4*n*x^5 + 5*B*a*b^3*n*x^4 + 10*B
*a^2*b^2*n*x^3 + 10*B*a^3*b*n*x^2 + 5*B*a^4*n*x)*log(d*x + c) - 1/10*(B*b^4
*c^3*n - 5*B*a*b^3*c^2*d*n + 10*B*a^2*b^2*c*d^2*n - 6*B*a^3*b*d^3*n - 20*A*
a^3*b*d^3 - 20*B*a^3*b*d^3)*x^2/d^3 + 1/5*(B*b^4*c^4*n - 5*B*a*b^3*c^3*d*n
+ 10*B*a^2*b^2*c^2*d^2*n - 10*B*a^3*b*c*d^3*n + 4*B*a^4*d^4*n + 5*A*a^4*d^4
 + 5*B*a^4*d^4)*x/d^4 - 1/5*(B*b^4*c^5*n - 5*B*a*b^3*c^4*d*n + 10*B*a^2*b^2
*c^3*d^2*n - 10*B*a^3*b*c^2*d^3*n + 5*B*a^4*c*d^4*n)*log(-d*x - c)/d^5
```

**maple [C]** time = 0.78, size = 2374, normalized size = 13.88

Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^4*(A+B*ln(e*(b*x+a)^n/((d*x+c)^n))),x)
```

```
[Out] -1/5*(b*x+a)^5*B/b*ln((d*x+c)^n)+ln((b*x+a)^n)*x*B*a^4+1/5*b^4*A*x^5+1/5*b^
4*B*ln(e)*x^5+1/5*b^4*B*x^5*ln((b*x+a)^n)+B*ln(e)*a^4*x+b^3*A*a*x^4+2*b^2*A
```

```
*a^2*x^3+2*b*A*a^3*x^2+A*a^4*x+b^3*B*ln(e)*a*x^4+b^3*B*a*x^4*ln((b*x+a)^n)+
2*b^2*B*ln(e)*a^2*x^3+2*b^2*B*a^2*x^3*ln((b*x+a)^n)+2*b*B*ln(e)*a^3*x^2+2*b
*B*a^3*x^2*ln((b*x+a)^n)+1/5/b*B*ln(d*x+c)*a^5*n-1/2*I*B*Pi*a^4*x*csgn(I*e/
((d*x+c)^n)*(b*x+a)^n)^3-1/10*I*b^4*B*Pi*x^5*csgn(I*(b*x+a)^n/((d*x+c)^n))^
3-1/10*I*b^4*B*Pi*x^5*csgn(I*e/((d*x+c)^n)*(b*x+a)^n)^3+1/20*b^3*B*a*n*x^4-
1/20*b^4/d*B*c*n*x^4+4/15*b^2*B*a^2*n*x^3+1/15*b^4/d^2*B*c^2*n*x^3+3/5*b*B*
a^3*n*x^2-1/10*b^4/d^3*B*c^3*n*x^2+4/5*B*a^4*n*x+1/5*b^4/d^4*B*c^4*n*x-1/5*
b^4/d^5*B*ln(d*x+c)*c^5*n-1/d*B*ln(d*x+c)*a^4*c*n-1/2*I*B*Pi*a^4*x*csgn(I*(
b*x+a)^n/((d*x+c)^n))^3+I*b^2*B*Pi*a^2*x^3*csgn(I*(b*x+a)^n)*csgn(I*(b*x+a)
^n/((d*x+c)^n))^2+I*b^2*B*Pi*a^2*x^3*csgn(I/((d*x+c)^n))*csgn(I*(b*x+a)^n/(
(d*x+c)^n))^2+I*b^2*B*Pi*a^2*x^3*csgn(I*(b*x+a)^n/((d*x+c)^n))*csgn(I*e/((d
*x+c)^n)*(b*x+a)^n)^2+I*b^2*B*Pi*a^2*x^3*csgn(I*e)*csgn(I*e/((d*x+c)^n)*(b*
x+a)^n)^2+I*b*B*Pi*a^3*x^2*csgn(I*(b*x+a)^n)*csgn(I*(b*x+a)^n/((d*x+c)^n))^
2+I*b*B*Pi*a^3*x^2*csgn(I/((d*x+c)^n))*csgn(I*(b*x+a)^n/((d*x+c)^n))^2+I*b*
B*Pi*a^3*x^2*csgn(I*(b*x+a)^n/((d*x+c)^n))*csgn(I*e/((d*x+c)^n)*(b*x+a)^n)^
2+I*b*B*Pi*a^3*x^2*csgn(I*e)*csgn(I*e/((d*x+c)^n)*(b*x+a)^n)^2-1/10*I*b^4*B
*Pi*x^5*csgn(I*e)*csgn(I*(b*x+a)^n/((d*x+c)^n))*csgn(I*e/((d*x+c)^n)*(b*x+a
)^n)-1/10*I*b^4*B*Pi*x^5*csgn(I*(b*x+a)^n)*csgn(I/((d*x+c)^n))*csgn(I*(b*x+
a)^n/((d*x+c)^n))+1/2*I*b^3*B*Pi*a*x^4*csgn(I*(b*x+a)^n)*csgn(I*(b*x+a)^n/(
(d*x+c)^n))^2+1/2*I*b^3*B*Pi*a*x^4*csgn(I/((d*x+c)^n))*csgn(I*(b*x+a)^n/((d
*x+c)^n))^2+1/2*I*b^3*B*Pi*a*x^4*csgn(I*(b*x+a)^n/((d*x+c)^n))*csgn(I*e/((d
*x+c)^n)*(b*x+a)^n)^2+1/2*I*b^3*B*Pi*a*x^4*csgn(I*e)*csgn(I*e/((d*x+c)^n)*(
b*x+a)^n)^2-1/2*I*B*Pi*a^4*x*csgn(I*e)*csgn(I*(b*x+a)^n/((d*x+c)^n))*csgn(I
*e/((d*x+c)^n)*(b*x+a)^n)-1/2*I*B*Pi*a^4*x*csgn(I*(b*x+a)^n)*csgn(I/((d*x+c
)^n))*csgn(I*(b*x+a)^n/((d*x+c)^n))-1/3*b^3/d*B*a*c*n*x^3-b^2/d*B*a^2*c*n*x
^2+1/2*b^3/d^2*B*a*c^2*n*x^2-2*b/d*B*a^3*c*n*x+2*b^2/d^2*B*a^2*c^2*n*x-b^3/
d^3*B*a*c^3*n*x+2*b/d^2*B*ln(d*x+c)*a^3*c^2*n-2*b^2/d^3*B*ln(d*x+c)*a^2*c^3
*n+b^3/d^4*B*ln(d*x+c)*a*c^4*n-I*b*B*Pi*a^3*x^2*csgn(I*e/((d*x+c)^n)*(b*x+a
)^n)^3+1/2*I*B*Pi*a^4*x*csgn(I*(b*x+a)^n)*csgn(I*(b*x+a)^n/((d*x+c)^n))^2+1
/2*I*B*Pi*a^4*x*csgn(I/((d*x+c)^n))*csgn(I*(b*x+a)^n/((d*x+c)^n))^2+1/2*I*B
*Pi*a^4*x*csgn(I*(b*x+a)^n/((d*x+c)^n))*csgn(I*e/((d*x+c)^n)*(b*x+a)^n)^2+1
/2*I*B*Pi*a^4*x*csgn(I*e)*csgn(I*e/((d*x+c)^n)*(b*x+a)^n)^2+1/10*I*b^4*B*Pi
*x^5*csgn(I*(b*x+a)^n)*csgn(I*(b*x+a)^n/((d*x+c)^n))^2+1/10*I*b^4*B*Pi*x^5*
csgn(I/((d*x+c)^n))*csgn(I*(b*x+a)^n/((d*x+c)^n))^2+1/10*I*b^4*B*Pi*x^5*csg
n(I*(b*x+a)^n/((d*x+c)^n))*csgn(I*e/((d*x+c)^n)*(b*x+a)^n)^2+1/10*I*b^4*B*P
i*x^5*csgn(I*e)*csgn(I*e/((d*x+c)^n)*(b*x+a)^n)^2-1/2*I*b^3*B*Pi*a*x^4*csgn
(I*(b*x+a)^n/((d*x+c)^n))^3-1/2*I*b^3*B*Pi*a*x^4*csgn(I*e/((d*x+c)^n)*(b*x+
a)^n)^3-I*b^2*B*Pi*a^2*x^3*csgn(I*(b*x+a)^n/((d*x+c)^n))^3-I*b^2*B*Pi*a^2*x
^3*csgn(I*e/((d*x+c)^n)*(b*x+a)^n)^3-I*b*B*Pi*a^3*x^2*csgn(I*(b*x+a)^n/((d*
x+c)^n))^3+1/5*B*a^5*n/b*ln(-b*x-a)-I*b^2*B*Pi*a^2*x^3*csgn(I*e)*csgn(I*(b*
x+a)^n/((d*x+c)^n))*csgn(I*e/((d*x+c)^n)*(b*x+a)^n)-I*b^2*B*Pi*a^2*x^3*csgn
(I*(b*x+a)^n)*csgn(I/((d*x+c)^n))*csgn(I*(b*x+a)^n/((d*x+c)^n))-I*b*B*Pi*a^
3*x^2*csgn(I*e)*csgn(I*(b*x+a)^n/((d*x+c)^n))*csgn(I*e/((d*x+c)^n)*(b*x+a)^
n)-I*b*B*Pi*a^3*x^2*csgn(I*(b*x+a)^n)*csgn(I/((d*x+c)^n))*csgn(I*(b*x+a)^n/
((d*x+c)^n))-1/2*I*b^3*B*Pi*a*x^4*csgn(I*e)*csgn(I*(b*x+a)^n/((d*x+c)^n))*c
```

```
sgn(I*e/((d*x+c)^n)*(b*x+a)^n)-1/2*I*b^3*B*Pi*a*x^4*csgn(I*(b*x+a)^n)*csgn(
I/((d*x+c)^n))*csgn(I*(b*x+a)^n/((d*x+c)^n))
```

**maxima [B]** time = 1.47, size = 671, normalized size = 3.92

$$\frac{1}{5}Bb^4x^5\log\left(\frac{(bx+a)^ne}{(dx+c)^n}\right)+\frac{1}{5}Ab^4x^5+Bab^3x^4\log\left(\frac{(bx+a)^ne}{(dx+c)^n}\right)+Aab^3x^4+2\,Ba^2b^2x^3\log\left(\frac{(bx+a)^ne}{(dx+c)^n}\right)+2\,Aa^2b^2x^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^4*(A+B*log(e*(b*x+a)^n/((d*x+c)^n))),x, algorithm="maxima
")
```

```
[Out] 1/5*B*b^4*x^5*log((b*x + a)^n*e/(d*x + c)^n) + 1/5*A*b^4*x^5 + B*a*b^3*x^4*
log((b*x + a)^n*e/(d*x + c)^n) + A*a*b^3*x^4 + 2*B*a^2*b^2*x^3*log((b*x + a
)^n*e/(d*x + c)^n) + 2*A*a^2*b^2*x^3 + 2*B*a^3*b*x^2*log((b*x + a)^n*e/(d*x
 + c)^n) + 2*A*a^3*b*x^2 + B*a^4*x*log((b*x + a)^n*e/(d*x + c)^n) + A*a^4*x
 + (a*e*n*log(b*x + a)/b - c*e*n*log(d*x + c)/d)*B*a^4/e - 2*(a^2*e*n*log(b
*x + a)/b^2 - c^2*e*n*log(d*x + c)/d^2 + (b*c*e*n - a*d*e*n)*x/(b*d))*B*a^3
*b/e + (2*a^3*e*n*log(b*x + a)/b^3 - 2*c^3*e*n*log(d*x + c)/d^3 - ((b^2*c*d
*e*n - a*b*d^2*e*n)*x^2 - 2*(b^2*c^2*e*n - a^2*d^2*e*n)*x)/(b^2*d^2))*B*a^2
*b^2/e - 1/6*(6*a^4*e*n*log(b*x + a)/b^4 - 6*c^4*e*n*log(d*x + c)/d^4 + (2*
(b^3*c*d^2*e*n - a*b^2*d^3*e*n)*x^3 - 3*(b^3*c^2*d*e*n - a^2*b*d^3*e*n)*x^2
 + 6*(b^3*c^3*e*n - a^3*d^3*e*n)*x)/(b^3*d^3))*B*a*b^3/e + 1/60*(12*a^5*e*n
*log(b*x + a)/b^5 - 12*c^5*e*n*log(d*x + c)/d^5 - (3*(b^4*c*d^3*e*n - a*b^3
*d^4*e*n)*x^4 - 4*(b^4*c^2*d^2*e*n - a^2*b^2*d^4*e*n)*x^3 + 6*(b^4*c^3*d*e*
n - a^3*b*d^4*e*n)*x^2 - 12*(b^4*c^4*e*n - a^4*d^4*e*n)*x)/(b^4*d^4))*B*b^4
/e
```

**mupad [B]** time = 4.56, size = 936, normalized size = 5.47

$$x^4\left(\frac{b^3\,\left(25\,A\,a\,d+5\,A\,b\,c+B\,a\,d\,n-B\,b\,c\,n\right)}{20\,d}-\frac{A\,b^3\,\left(5\,a\,d+5\,b\,c\right)}{20\,d}\right)-x^3\left(\frac{\left(5\,a\,d+5\,b\,c\right)\,\left(\frac{b^3\,\left(25\,A\,a\,d+5\,A\,b\,c+B\,a\,d\,n-B\right.}{5\,d}\right.}{15\,b\,d}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*log((e*(a + b*x)^n)/(c + d*x)^n))*(a + b*x)^4,x)
```

```
[Out] x^4*((b^3*(25*A*a*d + 5*A*b*c + B*a*d*n - B*b*c*n))/(20*d) - (A*b^3*(5*a*d
+ 5*b*c))/(20*d)) - x^3*(((5*a*d + 5*b*c)*((b^3*(25*A*a*d + 5*A*b*c + B*a*d
*n - B*b*c*n))/(5*d) - (A*b^3*(5*a*d + 5*b*c))/(5*d)))/(15*b*d) - (a*b^2*(1
0*A*a*d + 5*A*b*c + B*a*d*n - B*b*c*n))/(3*d) + (A*a*b^3*c)/(3*d)) + log((e
*(a + b*x)^n)/(c + d*x)^n)*((B*b^4*x^5)/5 + B*a^4*x + 2*B*a^3*b*x^2 + B*a*b
^3*x^4 + 2*B*a^2*b^2*x^3) + x*((a^3*(5*A*a*d + 10*A*b*c + 2*B*a*d*n - 2*B*b
*c*n))/d - ((5*a*d + 5*b*c)*((2*a^2*b*(5*A*a*d + 5*A*b*c + B*a*d*n - B*b*c*
n))/d + ((5*a*d + 5*b*c)*(((5*a*d + 5*b*c)*((b^3*(25*A*a*d + 5*A*b*c + B*a*
d*n - B*b*c*n))/(5*d) - (A*b^3*(5*a*d + 5*b*c))/(5*d)))/(5*b*d) - (a*b^2*(1
0*A*a*d + 5*A*b*c + B*a*d*n - B*b*c*n))/d + (A*a*b^3*c)/d))/(5*b*d) - (a*c*
((b^3*(25*A*a*d + 5*A*b*c + B*a*d*n - B*b*c*n))/(5*d) - (A*b^3*(5*a*d + 5*b
*c))/(5*d)))/(b*d)))/(5*b*d) + (a*c*(((5*a*d + 5*b*c)*((b^3*(25*A*a*d + 5*A
*b*c + B*a*d*n - B*b*c*n))/(5*d) - (A*b^3*(5*a*d + 5*b*c))/(5*d)))/(5*b*d)
- (a*b^2*(10*A*a*d + 5*A*b*c + B*a*d*n - B*b*c*n))/d + (A*a*b^3*c)/d))/(b*d
)) + x^2*((a^2*b*(5*A*a*d + 5*A*b*c + B*a*d*n - B*b*c*n))/d + ((5*a*d + 5*b
*c)*(((5*a*d + 5*b*c)*((b^3*(25*A*a*d + 5*A*b*c + B*a*d*n - B*b*c*n))/(5*d)
 - (A*b^3*(5*a*d + 5*b*c))/(5*d)))/(5*b*d) - (a*b^2*(10*A*a*d + 5*A*b*c + B
*a*d*n - B*b*c*n))/d + (A*a*b^3*c)/d))/(10*b*d) - (a*c*((b^3*(25*A*a*d + 5*
A*b*c + B*a*d*n - B*b*c*n))/(5*d) - (A*b^3*(5*a*d + 5*b*c))/(5*d)))/(2*b*d)
) + (A*b^4*x^5)/5 - (log(c + d*x)*(B*b^4*c^5*n + 5*B*a^4*c*d^4*n + 10*B*a^2
*b^2*c^3*d^2*n - 5*B*a*b^3*c^4*d*n - 10*B*a^3*b*c^2*d^3*n))/(5*d^5) + (B*a^
5*n*log(a + b*x))/(5*b)
```

**sympy [F(-2)]** time = 0.00, size = 0, normalized size = 0.00

Exception raised: HeuristicGCDFailed

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**4*(A+B*ln(e*(b*x+a)**n/((d*x+c)**n))),x)
```

```
[Out] Exception raised: HeuristicGCDFailed
```

# 3.148 $\int (a+bx)^3 \left(A + B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right) dx$

**Optimal**. Leaf size=142

$$\frac{(a+bx)^4\left(B\log\left(e(a+bx)^n(c+dx)^{-n}\right)+A\right)}{4b}+\frac{Bn(bc-ad)^4\log(c+dx)}{4bd^4}-\frac{Bnx(bc-ad)^3}{4d^3}+\frac{Bn(a+bx)^2(bc-ad)^2}{8bd^2}$$

```
[Out] -1/4*B*(-a*d+b*c)^3*n*x/d^3+1/8*B*(-a*d+b*c)^2*n*(b*x+a)^2/b/d^2-1/12*B*(-a
*d+b*c)*n*(b*x+a)^3/b/d+1/4*B*(-a*d+b*c)^4*n*ln(d*x+c)/b/d^4+1/4*(b*x+a)^4*
(A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))/b
```

**Rubi [A]** time = 0.14, antiderivative size = 154, normalized size of antiderivative = 1.08, number of steps used = 5, number of rules used = 3, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.097, Rules used = {6742, 2492, 43}

$$\frac{A(a+bx)^4}{4b}-\frac{Bnx(bc-ad)^3}{4d^3}+\frac{Bn(a+bx)^2(bc-ad)^2}{8bd^2}+\frac{Bn(bc-ad)^4\log(c+dx)}{4bd^4}+\frac{B(a+bx)^4\log\left(e(a+bx)^n(c+\right.}{4b}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^3*(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n]),x]
```

```
[Out] -(B*(b*c - a*d)^3*n*x)/(4*d^3) + (B*(b*c - a*d)^2*n*(a + b*x)^2)/(8*b*d^2)
- (B*(b*c - a*d)*n*(a + b*x)^3)/(12*b*d) + (A*(a + b*x)^4)/(4*b) + (B*(b*c
- a*d)^4*n*Log[c + d*x])/(4*b*d^4) + (B*(a + b*x)^4*Log[(e*(a + b*x)^n)/(c
+ d*x)^n])/(4*b)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2492

```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))
^(r_.)]^(s_.)*((g_.) + (h_.)*(x_))^(m_.), x_Symbol] :> Simp[((g + h*x)^(m +
 1)*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^s)/(h*(m + 1)), x] - Dist[(p*r*s*(
b*c - a*d))/(h*(m + 1)), Int[((g + h*x)^(m + 1)*Log[e*(f*(a + b*x)^p*(c + d
*x)^q)^r]^(s - 1))/((a + b*x)*(c + d*x)), x], x] /; FreeQ[{a, b, c, d, e, f
, g, h, m, p, q, r, s}, x] && NeQ[b*c - a*d, 0] && EqQ[p + q, 0] && IGtQ[s,
 0] && NeQ[m, -1]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\int (a+bx)^3 \left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)\,dx = \int \left(A(a+bx)^3+B(a+bx)^3\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)\,dx$$

$$= \frac{A(a+bx)^4}{4b} + B\int (a+bx)^3\log\left(e(a+bx)^n(c+dx)^{-n}\right)\,dx$$

$$= \frac{A(a+bx)^4}{4b} + \frac{B(a+bx)^4\log\left(e(a+bx)^n(c+dx)^{-n}\right)}{4b} - \frac{(B(bc$$

$$= \frac{A(a+bx)^4}{4b} + \frac{B(a+bx)^4\log\left(e(a+bx)^n(c+dx)^{-n}\right)}{4b} - \frac{(B(bc$$

$$= -\frac{B(bc-ad)^3nx}{4d^3} + \frac{B(bc-ad)^2n(a+bx)^2}{8bd^2} - \frac{B(bc-ad)n(a+}{12bd}$$

**Mathematica [A]** time = 0.49, size = 273, normalized size = 1.92

$$\frac{-18a^4Bd^4n\log(a+bx)+bdx\left(6a^3d^3(4A+3Bn)+9a^2bd^2(4Adx-4Bcn+Bdnx)+2ab^2d\left(12Ad^2x^2+Bn\left(12c^2\right.\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^3*(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n]),x]
```

```
[Out] (b*d*x*(6*a^3*d^3*(4*A + 3*B*n) + 9*a^2*b*d^2*(-4*B*c*n + 4*A*d*x + B*d*n*x
) + b^3*(6*A*d^3*x^3 + B*c*n*(-6*c^2 + 3*c*d*x - 2*d^2*x^2)) + 2*a*b^2*d*(1
2*A*d^2*x^2 + B*n*(12*c^2 - 6*c*d*x + d^2*x^2))) - 18*a^4*B*d^4*n*Log[a + b
*x] + 6*B*(b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 + 4*
a^4*d^4)*n*Log[c + d*x] + 6*B*d^4*(4*a^4 + 4*a^3*b*x + 6*a^2*b^2*x^2 + 4*a*
b^3*x^3 + b^4*x^4)*Log[(e*(a + b*x)^n)/(c + d*x)^n])/(24*b*d^4)
```

**fricas [B]** time = 0.57, size = 417, normalized size = 2.94

$$\frac{6\,Ab^4d^4x^4+2\left(12\,Aab^3d^4-\left(Bb^4cd^3-Bab^3d^4\right)n\right)x^3+3\left(12\,Aa^2b^2d^4+\left(Bb^4c^2d^2-4\,Bab^3cd^3+3\,Ba^2b^2d^4\right)n\right)x^2+}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^3*(A+B*log(e*(b*x+a)^n/((d*x+c)^n))),x, algorithm="fricas
")
```

```
[Out] 1/24*(6*A*b^4*d^4*x^4 + 2*(12*A*a*b^3*d^4 - (B*b^4*c*d^3 - B*a*b^3*d^4)*n)*
x^3 + 3*(12*A*a^2*b^2*d^4 + (B*b^4*c^2*d^2 - 4*B*a*b^3*c*d^3 + 3*B*a^2*b^2*
d^4)*n)*x^2 + 6*(4*A*a^3*b*d^4 - (B*b^4*c^3*d - 4*B*a*b^3*c^2*d^2 + 6*B*a^2
*b^2*c*d^3 - 3*B*a^3*b*d^4)*n)*x + 6*(B*b^4*d^4*n*x^4 + 4*B*a*b^3*d^4*n*x^3
 + 6*B*a^2*b^2*d^4*n*x^2 + 4*B*a^3*b*d^4*n*x + B*a^4*d^4*n)*log(b*x + a) -
6*(B*b^4*d^4*n*x^4 + 4*B*a*b^3*d^4*n*x^3 + 6*B*a^2*b^2*d^4*n*x^2 + 4*B*a^3*
b*d^4*n*x - (B*b^4*c^4 - 4*B*a*b^3*c^3*d + 6*B*a^2*b^2*c^2*d^2 - 4*B*a^3*b*
c*d^3)*n)*log(d*x + c) + 6*(B*b^4*d^4*x^4 + 4*B*a*b^3*d^4*x^3 + 6*B*a^2*b^2
*d^4*x^2 + 4*B*a^3*b*d^4*x)*log(e))/(b*d^4)
```

**giac [B]** time = 3.82, size = 355, normalized size = 2.50

$$\frac{Ba^4n\log(bx+a)}{4b}+\frac{1}{4}\left(Ab^3+Bb^3\right)x^4-\frac{\left(Bb^3cn-Bab^2dn-12\,Aab^2d-12\,Bab^2d\right)x^3}{12\,d}+\frac{1}{4}\left(Bb^3nx^4+4\,Bab^2nx^3+6\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^3*(A+B*log(e*(b*x+a)^n/((d*x+c)^n))),x, algorithm="giac")
```

```
[Out] 1/4*B*a^4*n*log(b*x + a)/b + 1/4*(A*b^3 + B*b^3)*x^4 - 1/12*(B*b^3*c*n - B*
a*b^2*d*n - 12*A*a*b^2*d - 12*B*a*b^2*d)*x^3/d + 1/4*(B*b^3*n*x^4 + 4*B*a*b
^2*n*x^3 + 6*B*a^2*b*n*x^2 + 4*B*a^3*n*x)*log(b*x + a) - 1/4*(B*b^3*n*x^4 +
 4*B*a*b^2*n*x^3 + 6*B*a^2*b*n*x^2 + 4*B*a^3*n*x)*log(d*x + c) + 1/8*(B*b^3
*c^2*n - 4*B*a*b^2*c*d*n + 3*B*a^2*b*d^2*n + 12*A*a^2*b*d^2 + 12*B*a^2*b*d^
2)*x^2/d^2 - 1/4*(B*b^3*c^3*n - 4*B*a*b^2*c^2*d*n + 6*B*a^2*b*c*d^2*n - 3*B
*a^3*d^3*n - 4*A*a^3*d^3 - 4*B*a^3*d^3)*x/d^3 + 1/4*(B*b^3*c^4*n - 4*B*a*b^
2*c^3*d*n + 6*B*a^2*b*c^2*d^2*n - 4*B*a^3*c*d^3*n)*log(d*x + c)/d^4
```

**maple [C]** time = 0.51, size = 1840, normalized size = 12.96

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^3*(A+B*ln(e*(b*x+a)^n/((d*x+c)^n))),x)
```

```
[Out] ln((b*x+a)^n)*x*B*a^3+1/4*b^3*A*x^4+1/4*b^3*B*ln(e)*x^4+1/4*b^3*B*x^4*ln((b
*x+a)^n)+B*ln(e)*a^3*x-1/4*(b*x+a)^4*B/b*ln((d*x+c)^n)+1/12*b^2*B*a*n*x^3-1
/12*b^3/d*B*c*n*x^3-1/2*I*b^2*B*Pi*a*x^3*csgn(I*(b*x+a)^n)*csgn(I/((d*x+c)^
n))*csgn(I*(b*x+a)^n/((d*x+c)^n))-1/2*I*b^2*B*Pi*a*x^3*csgn(I*e)*csgn(I*(b*
x+a)^n/((d*x+c)^n))*csgn(I*e/((d*x+c)^n)*(b*x+a)^n)-3/4*I*b*B*Pi*a^2*x^2*cs
gn(I*(b*x+a)^n)*csgn(I/((d*x+c)^n))*csgn(I*(b*x+a)^n/((d*x+c)^n))-3/4*I*b*B
*Pi*a^2*x^2*csgn(I*e)*csgn(I*(b*x+a)^n/((d*x+c)^n))*csgn(I*e/((d*x+c)^n)*(b
*x+a)^n)-1/2*b^2/d*B*a*c*n*x^2-3/2*b/d*B*a^2*c*n*x+b^2/d^2*B*a*c^2*n*x+1/4*
B*a^4*n/b*ln(-b*x-a)+b^2*A*a*x^3+3/2*b*A*a^2*x^2+A*a^3*x+3/2*b*B*ln(e)*a^2*
x^2+b^2*B*ln(e)*a*x^3+b^2*B*a*x^3*ln((b*x+a)^n)+3/2*b*B*a^2*x^2*ln((b*x+a)^
```

```
n)+1/4/b*B*ln(d*x+c)*a^4*n-1/4*b^3/d^3*B*c^3*n*x-1/d*B*ln(d*x+c)*a^3*c*n+1/
4*b^3/d^4*B*ln(d*x+c)*c^4*n-1/8*I*b^3*B*Pi*x^4*csgn(I*(b*x+a)^n/((d*x+c)^n)
)^3-1/8*I*b^3*B*Pi*x^4*csgn(I*e/((d*x+c)^n)*(b*x+a)^n)^3-1/2*I*B*Pi*a^3*x*c
sgn(I*(b*x+a)^n/((d*x+c)^n))^3-1/2*I*B*Pi*a^3*x*csgn(I*e/((d*x+c)^n)*(b*x+a
)^n)^3+3/4*I*b*B*Pi*a^2*x^2*csgn(I/((d*x+c)^n))*csgn(I*(b*x+a)^n/((d*x+c)^n
))^2+3/4*I*b*B*Pi*a^2*x^2*csgn(I*e)*csgn(I*e/((d*x+c)^n)*(b*x+a)^n)^2+3/4*I
*b*B*Pi*a^2*x^2*csgn(I*(b*x+a)^n/((d*x+c)^n))*csgn(I*e/((d*x+c)^n)*(b*x+a)^
n)^2-1/2*I*B*Pi*a^3*x*csgn(I*(b*x+a)^n)*csgn(I/((d*x+c)^n))*csgn(I*(b*x+a)^
n/((d*x+c)^n))-1/2*I*B*Pi*a^3*x*csgn(I*e)*csgn(I*(b*x+a)^n/((d*x+c)^n))*csg
n(I*e/((d*x+c)^n)*(b*x+a)^n)-1/8*I*b^3*B*Pi*x^4*csgn(I*(b*x+a)^n)*csgn(I/((
d*x+c)^n))*csgn(I*(b*x+a)^n/((d*x+c)^n))-1/8*I*b^3*B*Pi*x^4*csgn(I*e)*csgn(
I*(b*x+a)^n/((d*x+c)^n))*csgn(I*e/((d*x+c)^n)*(b*x+a)^n)+1/2*I*b^2*B*Pi*a*x
^3*csgn(I*(b*x+a)^n)*csgn(I*(b*x+a)^n/((d*x+c)^n))^2+1/2*I*b^2*B*Pi*a*x^3*c
sgn(I/((d*x+c)^n))*csgn(I*(b*x+a)^n/((d*x+c)^n))^2+1/2*I*b^2*B*Pi*a*x^3*csg
n(I*e)*csgn(I*e/((d*x+c)^n)*(b*x+a)^n)^2+1/2*I*b^2*B*Pi*a*x^3*csgn(I*(b*x+a
)^n/((d*x+c)^n))*csgn(I*e/((d*x+c)^n)*(b*x+a)^n)^2+3/4*I*b*B*Pi*a^2*x^2*csg
n(I*(b*x+a)^n)*csgn(I*(b*x+a)^n/((d*x+c)^n))^2+3/8*b*B*a^2*n*x^2+1/8*b^3/d^
2*B*c^2*n*x^2+3/4*B*a^3*n*x+3/2*b/d^2*B*ln(d*x+c)*a^2*c^2*n-b^2/d^3*B*ln(d*
x+c)*a*c^3*n+1/2*I*B*Pi*a^3*x*csgn(I*(b*x+a)^n)*csgn(I*(b*x+a)^n/((d*x+c)^n
))^2+1/2*I*B*Pi*a^3*x*csgn(I/((d*x+c)^n))*csgn(I*(b*x+a)^n/((d*x+c)^n))^2+1
/2*I*B*Pi*a^3*x*csgn(I*e)*csgn(I*e/((d*x+c)^n)*(b*x+a)^n)^2+1/2*I*B*Pi*a^3*
x*csgn(I*(b*x+a)^n/((d*x+c)^n))*csgn(I*e/((d*x+c)^n)*(b*x+a)^n)^2+1/8*I*b^3
*B*Pi*x^4*csgn(I*(b*x+a)^n)*csgn(I*(b*x+a)^n/((d*x+c)^n))^2+1/8*I*b^3*B*Pi*
x^4*csgn(I/((d*x+c)^n))*csgn(I*(b*x+a)^n/((d*x+c)^n))^2+1/8*I*b^3*B*Pi*x^4*
csgn(I*e)*csgn(I*e/((d*x+c)^n)*(b*x+a)^n)^2+1/8*I*b^3*B*Pi*x^4*csgn(I*(b*x+
a)^n/((d*x+c)^n))*csgn(I*e/((d*x+c)^n)*(b*x+a)^n)^2-1/2*I*b^2*B*Pi*a*x^3*cs
gn(I*(b*x+a)^n/((d*x+c)^n))^3-1/2*I*b^2*B*Pi*a*x^3*csgn(I*e/((d*x+c)^n)*(b*
x+a)^n)^3-3/4*I*b*B*Pi*a^2*x^2*csgn(I*(b*x+a)^n/((d*x+c)^n))^3-3/4*I*b*B*Pi
*a^2*x^2*csgn(I*e/((d*x+c)^n)*(b*x+a)^n)^3
```

**maxima [B]** time = 1.40, size = 467, normalized size = 3.29

$$\frac{1}{4}\,Bb^3x^4\log\left(\frac{(bx+a)^n e}{(dx+c)^n}\right)+\frac{1}{4}\,Ab^3x^4+Bab^2x^3\log\left(\frac{(bx+a)^n e}{(dx+c)^n}\right)+Aab^2x^3+\frac{3}{2}\,Ba^2bx^2\log\left(\frac{(bx+a)^n e}{(dx+c)^n}\right)+\frac{3}{2}\,Aa^2bx^2+B$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^3*(A+B*log(e*(b*x+a)^n/((d*x+c)^n))),x, algorithm="maxima
")
```

```
[Out] 1/4*B*b^3*x^4*log((b*x + a)^n*e/(d*x + c)^n) + 1/4*A*b^3*x^4 + B*a*b^2*x^3*
log((b*x + a)^n*e/(d*x + c)^n) + A*a*b^2*x^3 + 3/2*B*a^2*b*x^2*log((b*x + a
)^n*e/(d*x + c)^n) + 3/2*A*a^2*b*x^2 + B*a^3*x*log((b*x + a)^n*e/(d*x + c)^
n) + A*a^3*x + (a*e*n*log(b*x + a)/b - c*e*n*log(d*x + c)/d)*B*a^3/e - 3/2*
(a^2*e*n*log(b*x + a)/b^2 - c^2*e*n*log(d*x + c)/d^2 + (b*c*e*n - a*d*e*n)*
```

```
x/(b*d))*B*a^2*b/e + 1/2*(2*a^3*e*n*log(b*x + a)/b^3 - 2*c^3*e*n*log(d*x +
c)/d^3 - ((b^2*c*d*e*n - a*b*d^2*e*n)*x^2 - 2*(b^2*c^2*e*n - a^2*d^2*e*n)*x
)/(b^2*d^2))*B*a*b^2/e - 1/24*(6*a^4*e*n*log(b*x + a)/b^4 - 6*c^4*e*n*log(d
*x + c)/d^4 + (2*(b^3*c*d^2*e*n - a*b^2*d^3*e*n)*x^3 - 3*(b^3*c^2*d*e*n - a
^2*b*d^3*e*n)*x^2 + 6*(b^3*c^3*e*n - a^3*d^3*e*n)*x)/(b^3*d^3))*B*b^3/e
```

**mupad [B]** time = 4.49, size = 520, normalized size = 3.66

$$x^3\left(\frac{b^2\,\left(16\,A\,a\,d+4\,A\,b\,c+B\,a\,d\,n-B\,b\,c\,n\right)}{12\,d}-\frac{A\,b^2\,\left(4\,a\,d+4\,b\,c\right)}{12\,d}\right)+\ln\left(\frac{e\,{\left(a+b\,x\right)}^n}{{\left(c+d\,x\right)}^n}\right)\,\left(B\,a^3\,x+\frac{3\,B\,a^2\,b\,x^2}{2}+B\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*log((e*(a + b*x)^n)/(c + d*x)^n))*(a + b*x)^3,x)
```

```
[Out] x^3*((b^2*(16*A*a*d + 4*A*b*c + B*a*d*n - B*b*c*n))/(12*d) - (A*b^2*(4*a*d
+ 4*b*c))/(12*d)) + log((e*(a + b*x)^n)/(c + d*x)^n)*((B*b^3*x^4)/4 + B*a^3
*x + (3*B*a^2*b*x^2)/2 + B*a*b^2*x^3) + x*((a^2*(8*A*a*d + 12*A*b*c + 3*B*a
*d*n - 3*B*b*c*n))/(2*d) + ((4*a*d + 4*b*c)*(((4*a*d + 4*b*c)*((b^2*(16*A*a
*d + 4*A*b*c + B*a*d*n - B*b*c*n))/(4*d) - (A*b^2*(4*a*d + 4*b*c))/(4*d)))/
(4*b*d) - (a*b*(6*A*a*d + 4*A*b*c + B*a*d*n - B*b*c*n))/d + (A*a*b^2*c)/d))
/(4*b*d) - (a*c*((b^2*(16*A*a*d + 4*A*b*c + B*a*d*n - B*b*c*n))/(4*d) - (A*
b^2*(4*a*d + 4*b*c))/(4*d)))/(b*d)) - x^2*(((4*a*d + 4*b*c)*((b^2*(16*A*a*d
 + 4*A*b*c + B*a*d*n - B*b*c*n))/(4*d) - (A*b^2*(4*a*d + 4*b*c))/(4*d)))/(8
*b*d) - (a*b*(6*A*a*d + 4*A*b*c + B*a*d*n - B*b*c*n))/(2*d) + (A*a*b^2*c)/(
2*d)) + (A*b^3*x^4)/4 + (log(c + d*x)*(B*b^3*c^4*n - 4*B*a^3*c*d^3*n - 4*B*
a*b^2*c^3*d*n + 6*B*a^2*b*c^2*d^2*n))/(4*d^4) + (B*a^4*n*log(a + b*x))/(4*b
)
```

**sympy [F(-2)]** time = 0.00, size = 0, normalized size = 0.00

Exception raised: HeuristicGCDFailed

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**3*(A+B*ln(e*(b*x+a)**n/((d*x+c)**n))),x)
```

```
[Out] Exception raised: HeuristicGCDFailed
```

# 3.149 $\int (a+bx)^2 \left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)\,dx$

**Optimal**. Leaf size=113

$$\frac{(a+bx)^3\left(B\log\left(e(a+bx)^n(c+dx)^{-n}\right)+A\right)}{3b}-\frac{Bn(bc-ad)^3\log(c+dx)}{3bd^3}+\frac{Bnx(bc-ad)^2}{3d^2}-\frac{Bn(a+bx)^2(bc-ad)}{6bd}$$

```
[Out] 1/3*B*(-a*d+b*c)^2*n*x/d^2-1/6*B*(-a*d+b*c)*n*(b*x+a)^2/b/d-1/3*B*(-a*d+b*c
)^3*n*ln(d*x+c)/b/d^3+1/3*(b*x+a)^3*(A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))/b
```

**Rubi [A]** time = 0.12, antiderivative size = 125, normalized size of antiderivative = 1.11, number of steps used = 5, number of rules used = 3, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.097, Rules used = {6742, 2492, 43}

$$\frac{A(a+bx)^3}{3b}+\frac{Bnx(bc-ad)^2}{3d^2}-\frac{Bn(bc-ad)^3\log(c+dx)}{3bd^3}+\frac{B(a+bx)^3\log\left(e(a+bx)^n(c+dx)^{-n}\right)}{3b}-\frac{Bn(a+bx)^2(bc-}{6bd}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^2*(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n]),x]
```

```
[Out] (B*(b*c - a*d)^2*n*x)/(3*d^2) - (B*(b*c - a*d)*n*(a + b*x)^2)/(6*b*d) + (A*
(a + b*x)^3)/(3*b) - (B*(b*c - a*d)^3*n*Log[c + d*x])/(3*b*d^3) + (B*(a + b
*x)^3*Log[(e*(a + b*x)^n)/(c + d*x)^n])/(3*b)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2492

```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))
^(r_.)]^(s_.)*((g_.) + (h_.)*(x_))^(m_.), x_Symbol] :> Simp[((g + h*x)^(m +
 1)*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^s)/(h*(m + 1)), x] - Dist[(p*r*s*(
b*c - a*d))/(h*(m + 1)), Int[((g + h*x)^(m + 1)*Log[e*(f*(a + b*x)^p*(c + d
*x)^q)^r]^(s - 1))/((a + b*x)*(c + d*x)), x], x] /; FreeQ[{a, b, c, d, e, f
, g, h, m, p, q, r, s}, x] && NeQ[b*c - a*d, 0] && EqQ[p + q, 0] && IGtQ[s,
 0] && NeQ[m, -1]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\int (a+bx)^2 \left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)\,dx = \int \left(A(a+bx)^2+B(a+bx)^2\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)\,dx$$

$$= \frac{A(a+bx)^3}{3b} + B\int (a+bx)^2\log\left(e(a+bx)^n(c+dx)^{-n}\right)\,dx$$

$$= \frac{A(a+bx)^3}{3b} + \frac{B(a+bx)^3\log\left(e(a+bx)^n(c+dx)^{-n}\right)}{3b} - \frac{(B(b$$

$$= \frac{A(a+bx)^3}{3b} + \frac{B(a+bx)^3\log\left(e(a+bx)^n(c+dx)^{-n}\right)}{3b} - \frac{(B(b$$

$$= \frac{B(bc-ad)^2nx}{3d^2} - \frac{B(bc-ad)n(a+bx)^2}{6bd} + \frac{A(a+bx)^3}{3b} - \frac{B(b$$

**Mathematica [A]** time = 0.29, size = 194, normalized size = 1.72

$$\frac{-4a^3Bd^3n\log(a+bx)+bdx\left(2a^2d^2(3A+2Bn)+abd(6Adx-6Bcn+Bdnx)+b^2\left(2Ad^2x^2+Bcn(2c-dx)\right)\right)-$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^2*(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n]),x]
```

```
[Out] (b*d*x*(2*a^2*d^2*(3*A + 2*B*n) + a*b*d*(-6*B*c*n + 6*A*d*x + B*d*n*x) + b^
2*(2*A*d^2*x^2 + B*c*n*(2*c - d*x))) - 4*a^3*B*d^3*n*Log[a + b*x] - 2*B*(b^
3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - 3*a^3*d^3)*n*Log[c + d*x] + 2*B*d^3
*(3*a^3 + 3*a^2*b*x + 3*a*b^2*x^2 + b^3*x^3)*Log[(e*(a + b*x)^n)/(c + d*x)^
n])/(6*b*d^3)
```

**fricas [B]** time = 0.96, size = 282, normalized size = 2.50

$$\frac{2\,Ab^3d^3x^3+\left(6\,Aab^2d^3-\left(Bb^3cd^2-Bab^2d^3\right)n\right)x^2+2\left(3\,Aa^2bd^3+\left(Bb^3c^2d-3\,Bab^2cd^2+2\,Ba^2bd^3\right)n\right)x+2\left(Bb$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2*(A+B*log(e*(b*x+a)^n/((d*x+c)^n))),x, algorithm="fricas
")
```

```
[Out] 1/6*(2*A*b^3*d^3*x^3 + (6*A*a*b^2*d^3 - (B*b^3*c*d^2 - B*a*b^2*d^3)*n)*x^2
+ 2*(3*A*a^2*b*d^3 + (B*b^3*c^2*d - 3*B*a*b^2*c*d^2 + 2*B*a^2*b*d^3)*n)*x +
 2*(B*b^3*d^3*n*x^3 + 3*B*a*b^2*d^3*n*x^2 + 3*B*a^2*b*d^3*n*x + B*a^3*d^3*n
)*log(b*x + a) - 2*(B*b^3*d^3*n*x^3 + 3*B*a*b^2*d^3*n*x^2 + 3*B*a^2*b*d^3*n
```

```
*x + (B*b^3*c^3 - 3*B*a*b^2*c^2*d + 3*B*a^2*b*c*d^2)*n)*log(d*x + c) + 2*(B
*b^3*d^3*x^3 + 3*B*a*b^2*d^3*x^2 + 3*B*a^2*b*d^3*x)*log(e))/(b*d^3)
```

**giac [B]** time = 1.38, size = 235, normalized size = 2.08

$$\frac{Ba^3n\log(bx+a)}{3b}+\frac{1}{3}\left(Ab^2+Bb^2\right)x^3-\frac{\left(Bb^2cn-Babdn-6\,Aabd-6\,Babd\right)x^2}{6d}+\frac{1}{3}\left(Bb^2nx^3+3\,Babnx^2+3\,Ba^2nx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2*(A+B*log(e*(b*x+a)^n/((d*x+c)^n))),x, algorithm="giac")
```

```
[Out] 1/3*B*a^3*n*log(b*x + a)/b + 1/3*(A*b^2 + B*b^2)*x^3 - 1/6*(B*b^2*c*n - B*a
*b*d*n - 6*A*a*b*d - 6*B*a*b*d)*x^2/d + 1/3*(B*b^2*n*x^3 + 3*B*a*b*n*x^2 +
3*B*a^2*n*x)*log(b*x + a) - 1/3*(B*b^2*n*x^3 + 3*B*a*b*n*x^2 + 3*B*a^2*n*x)
*log(d*x + c) + 1/3*(B*b^2*c^2*n - 3*B*a*b*c*d*n + 2*B*a^2*d^2*n + 3*A*a^2*
d^2 + 3*B*a^2*d^2)*x/d^2 - 1/3*(B*b^2*c^3*n - 3*B*a*b*c^2*d*n + 3*B*a^2*c*d
^2*n)*log(-d*x - c)/d^3
```

**maple [C]** time = 0.47, size = 1325, normalized size = 11.73

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^2*(A+B*ln(e*(b*x+a)^n/((d*x+c)^n))),x)
```

```
[Out] -1/3*(b*x+a)^3*B/b*ln((d*x+c)^n)+1/3*b^2*A*x^3+B*ln(e)*a^2*x+1/3*b^2*B*x^3*
ln((b*x+a)^n)+1/3*b^2*B*ln(e)*x^3+ln((b*x+a)^n)*x*B*a^2+1/3*B*a^3*n/b*ln(-b
*x-a)-1/2*I*b*B*Pi*a*x^2*csgn(I*(b*x+a)^n)*csgn(I/((d*x+c)^n))*csgn(I*(b*x+
a)^n/((d*x+c)^n))-1/2*I*b*B*Pi*a*x^2*csgn(I*e)*csgn(I*(b*x+a)^n/((d*x+c)^n)
)*csgn(I*e/((d*x+c)^n)*(b*x+a)^n)-1/2*I*B*Pi*a^2*x*csgn(I*e)*csgn(I*(b*x+a)
^n/((d*x+c)^n))*csgn(I*e/((d*x+c)^n)*(b*x+a)^n)-1/2*I*B*Pi*a^2*x*csgn(I*(b*
x+a)^n)*csgn(I/((d*x+c)^n))*csgn(I*(b*x+a)^n/((d*x+c)^n))-1/6*I*b^2*B*Pi*x^
3*csgn(I*e)*csgn(I*(b*x+a)^n/((d*x+c)^n))*csgn(I*e/((d*x+c)^n)*(b*x+a)^n)-1
/6*I*b^2*B*Pi*x^3*csgn(I*(b*x+a)^n)*csgn(I/((d*x+c)^n))*csgn(I*(b*x+a)^n/((
d*x+c)^n))+1/2*I*b*B*Pi*a*x^2*csgn(I*(b*x+a)^n)*csgn(I*(b*x+a)^n/((d*x+c)^n
))^2+1/2*I*b*B*Pi*a*x^2*csgn(I/((d*x+c)^n))*csgn(I*(b*x+a)^n/((d*x+c)^n))^2
+1/2*I*b*B*Pi*a*x^2*csgn(I*(b*x+a)^n/((d*x+c)^n))*csgn(I*e/((d*x+c)^n)*(b*x
+a)^n)^2+1/2*I*b*B*Pi*a*x^2*csgn(I*e)*csgn(I*e/((d*x+c)^n)*(b*x+a)^n)^2-1/6
*I*b^2*B*Pi*x^3*csgn(I*e/((d*x+c)^n)*(b*x+a)^n)^3-1/2*I*B*Pi*a^2*x*csgn(I*(
b*x+a)^n/((d*x+c)^n))^3-1/2*I*B*Pi*a^2*x*csgn(I*e/((d*x+c)^n)*(b*x+a)^n)^3+
2/3*B*a^2*n*x+1/3*b^2/d^2*B*c^2*n*x-1/3*b^2/d^3*B*ln(d*x+c)*c^3*n-1/d*B*ln(
d*x+c)*a^2*c*n-1/6*I*b^2*B*Pi*x^3*csgn(I*(b*x+a)^n/((d*x+c)^n))^3+1/2*I*B*P
i*a^2*x*csgn(I*(b*x+a)^n)*csgn(I*(b*x+a)^n/((d*x+c)^n))^2+1/2*I*B*Pi*a^2*x*
csgn(I/((d*x+c)^n))*csgn(I*(b*x+a)^n/((d*x+c)^n))^2+1/2*I*B*Pi*a^2*x*csgn(I
```

```
*(b*x+a)^n/((d*x+c)^n))*csgn(I*e/((d*x+c)^n)*(b*x+a)^n)^2+1/2*I*B*Pi*a^2*x*
csgn(I*e)*csgn(I*e/((d*x+c)^n)*(b*x+a)^n)^2+1/6*I*b^2*B*Pi*x^3*csgn(I*(b*x+
a)^n)*csgn(I*(b*x+a)^n/((d*x+c)^n))^2+1/6*I*b^2*B*Pi*x^3*csgn(I/((d*x+c)^n)
)*csgn(I*(b*x+a)^n/((d*x+c)^n))^2+1/6*I*b^2*B*Pi*x^3*csgn(I*(b*x+a)^n/((d*x
+c)^n))*csgn(I*e/((d*x+c)^n)*(b*x+a)^n)^2+1/6*I*b^2*B*Pi*x^3*csgn(I*e)*csgn
(I*e/((d*x+c)^n)*(b*x+a)^n)^2-1/2*I*b*B*Pi*a*x^2*csgn(I*(b*x+a)^n/((d*x+c)^
n))^3-1/2*I*b*B*Pi*a*x^2*csgn(I*e/((d*x+c)^n)*(b*x+a)^n)^3+b*A*a*x^2+A*a^2*
x+1/3/b*B*ln(d*x+c)*a^3*n+b*B*a*x^2*ln((b*x+a)^n)+b*B*ln(e)*a*x^2-b/d*B*a*c
*n*x+b/d^2*B*ln(d*x+c)*a*c^2*n+1/6*b*B*a*n*x^2-1/6*b^2/d*B*c*n*x^2
```

**maxima [B]** time = 1.27, size = 294, normalized size = 2.60

$$\frac{1}{3}Bb^2x^3\log\left(\frac{(bx+a)^n e}{(dx+c)^n}\right)+\frac{1}{3}Ab^2x^3+Babx^2\log\left(\frac{(bx+a)^n e}{(dx+c)^n}\right)+Aabx^2+Ba^2x\log\left(\frac{(bx+a)^n e}{(dx+c)^n}\right)+Aa^2x+\frac{\left(\frac{aen\log(bx+}{b}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2*(A+B*log(e*(b*x+a)^n/((d*x+c)^n))),x, algorithm="maxima
")
```

```
[Out] 1/3*B*b^2*x^3*log((b*x + a)^n*e/(d*x + c)^n) + 1/3*A*b^2*x^3 + B*a*b*x^2*lo
g((b*x + a)^n*e/(d*x + c)^n) + A*a*b*x^2 + B*a^2*x*log((b*x + a)^n*e/(d*x +
 c)^n) + A*a^2*x + (a*e*n*log(b*x + a)/b - c*e*n*log(d*x + c)/d)*B*a^2/e -
(a^2*e*n*log(b*x + a)/b^2 - c^2*e*n*log(d*x + c)/d^2 + (b*c*e*n - a*d*e*n)*
x/(b*d))*B*a*b/e + 1/6*(2*a^3*e*n*log(b*x + a)/b^3 - 2*c^3*e*n*log(d*x + c)
/d^3 - ((b^2*c*d*e*n - a*b*d^2*e*n)*x^2 - 2*(b^2*c^2*e*n - a^2*d^2*e*n)*x)/
(b^2*d^2))*B*b^2/e
```

**mupad [B]** time = 4.24, size = 262, normalized size = 2.32

$$\ln\left(\frac{e\,(a+b\,x)^n}{(c+d\,x)^n}\right)\left(B\,a^2\,x+B\,a\,b\,x^2+\frac{B\,b^2\,x^3}{3}\right)+x^2\left(\frac{b\,(9\,A\,a\,d+3\,A\,b\,c+B\,a\,d\,n-B\,b\,c\,n)}{6\,d}-\frac{A\,b\,(3\,a\,d+3\,b\,c)}{6\,d}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*log((e*(a + b*x)^n)/(c + d*x)^n))*(a + b*x)^2,x)
```

```
[Out] log((e*(a + b*x)^n)/(c + d*x)^n)*((B*b^2*x^3)/3 + B*a^2*x + B*a*b*x^2) + x^
2*((b*(9*A*a*d + 3*A*b*c + B*a*d*n - B*b*c*n))/(6*d) - (A*b*(3*a*d + 3*b*c)
)/(6*d)) - x*((((b*(9*A*a*d + 3*A*b*c + B*a*d*n - B*b*c*n))/(3*d) - (A*b*(3
*a*d + 3*b*c))/(3*d))*(3*a*d + 3*b*c))/(3*b*d) - (a*(3*A*a*d + 3*A*b*c + B*
a*d*n - B*b*c*n))/d + (A*a*b*c)/d) + (A*b^2*x^3)/3 - (log(c + d*x)*(B*b^2*c
^3*n + 3*B*a^2*c*d^2*n - 3*B*a*b*c^2*d*n))/(3*d^3) + (B*a^3*n*log(a + b*x))
/(3*b)
```

<u>**sympy**</u> **[F(-2)]** time = 0.00, size = 0, normalized size = 0.00

Exception raised: HeuristicGCDFailed

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**2*(A+B*ln(e*(b*x+a)**n/((d*x+c)**n))),x)
[Out] Exception raised: HeuristicGCDFailed
```

# 3.150 $\int (a+bx)\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)dx$

**Optimal**. Leaf size=84

$$\frac{(a+bx)^2\left(B\log\left(e(a+bx)^n(c+dx)^{-n}\right)+A\right)}{2b}+\frac{Bn(bc-ad)^2\log(c+dx)}{2bd^2}-\frac{Bnx(bc-ad)}{2d}$$

```
[Out] -1/2*B*(-a*d+b*c)*n*x/d+1/2*B*(-a*d+b*c)^2*n*ln(d*x+c)/b/d^2+1/2*(b*x+a)^2*
(A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))/b
```

**Rubi [A]** time = 0.09, antiderivative size = 96, normalized size of antiderivative = 1.14, number of steps used = 5, number of rules used = 3, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.103, Rules used = {6742, 2492, 43}

$$\frac{A(a+bx)^2}{2b}+\frac{Bn(bc-ad)^2\log(c+dx)}{2bd^2}+\frac{B(a+bx)^2\log\left(e(a+bx)^n(c+dx)^{-n}\right)}{2b}-\frac{Bnx(bc-ad)}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)*(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n]),x]
```

```
[Out] -(B*(b*c - a*d)*n*x)/(2*d) + (A*(a + b*x)^2)/(2*b) + (B*(b*c - a*d)^2*n*Log
[c + d*x])/(2*b*d^2) + (B*(a + b*x)^2*Log[(e*(a + b*x)^n)/(c + d*x)^n])/(2*
b)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2492

```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))
^(r_.)]^(s_.)*((g_.) + (h_.)*(x_))^(m_.), x_Symbol] :> Simp[((g + h*x)^(m +
 1)*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^s)/(h*(m + 1)), x] - Dist[(p*r*s*(
b*c - a*d))/(h*(m + 1)), Int[((g + h*x)^(m + 1)*Log[e*(f*(a + b*x)^p*(c + d
*x)^q)^r]^(s - 1))/((a + b*x)*(c + d*x)), x], x] /; FreeQ[{a, b, c, d, e, f
, g, h, m, p, q, r, s}, x] && NeQ[b*c - a*d, 0] && EqQ[p + q, 0] && IGtQ[s,
 0] && NeQ[m, -1]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\int (a+bx)\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)dx = \int \left(A(a+bx)+B(a+bx)\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)dx$$

$$= \frac{A(a+bx)^2}{2b} + B\int (a+bx)\log\left(e(a+bx)^n(c+dx)^{-n}\right)dx$$

$$= \frac{A(a+bx)^2}{2b} + \frac{B(a+bx)^2\log\left(e(a+bx)^n(c+dx)^{-n}\right)}{2b} - \frac{(B(bc-$$

$$= \frac{A(a+bx)^2}{2b} + \frac{B(a+bx)^2\log\left(e(a+bx)^n(c+dx)^{-n}\right)}{2b} - \frac{(B(bc-$$

$$= -\frac{B(bc-ad)nx}{2d} + \frac{A(a+bx)^2}{2b} + \frac{B(bc-ad)^2n\log(c+dx)}{2bd^2} + \frac{B$$

**Mathematica [A]** time = 0.15, size = 126, normalized size = 1.50

$$\frac{d\left(Bd\left(2a^2+2abx+b^2x^2\right)\log\left(e(a+bx)^n(c+dx)^{-n}\right)+bx(2aAd+aBdn+Abdx-bBcn)\right)+Bn\left(2a^2d^2-2abcd+\right.}{2bd^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)*(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n]),x]
```

```
[Out] (-(a^2*B*d^2*n*Log[a + b*x]) + B*(b^2*c^2 - 2*a*b*c*d + 2*a^2*d^2)*n*Log[c
+ d*x] + d*(b*x*(2*a*A*d - b*B*c*n + a*B*d*n + A*b*d*x) + B*d*(2*a^2 + 2*a*
b*x + b^2*x^2)*Log[(e*(a + b*x)^n)/(c + d*x)^n]))/(2*b*d^2)
```

**fricas [B]** time = 1.11, size = 163, normalized size = 1.94

$$\frac{Ab^2d^2x^2+\left(2\,Aabd^2-\left(Bb^2cd-Babd^2\right)n\right)x+\left(Bb^2d^2nx^2+2\,Babd^2nx+Ba^2d^2n\right)\log(bx+a)-\left(Bb^2d^2nx^2+2\,B\right.}{2\,bd^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*(A+B*log(e*(b*x+a)^n/((d*x+c)^n))),x, algorithm="fricas")
```

```
[Out] 1/2*(A*b^2*d^2*x^2 + (2*A*a*b*d^2 - (B*b^2*c*d - B*a*b*d^2)*n)*x + (B*b^2*d
^2*n*x^2 + 2*B*a*b*d^2*n*x + B*a^2*d^2*n)*log(b*x + a) - (B*b^2*d^2*n*x^2 +
 2*B*a*b*d^2*n*x - (B*b^2*c^2 - 2*B*a*b*c*d)*n)*log(d*x + c) + (B*b^2*d^2*x
^2 + 2*B*a*b*d^2*x)*log(e))/(b*d^2)
```

**giac [A]** time = 0.58, size = 127, normalized size = 1.51

$$\frac{Ba^2n\log(bx+a)}{2\,b}+\frac{1}{2}(Ab+Bb)x^2+\frac{1}{2}\left(Bbnx^2+2\,Banx\right)\log(bx+a)-\frac{1}{2}\left(Bbnx^2+2\,Banx\right)\log(dx+c)-\frac{(Bbcn-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*(A+B*log(e*(b*x+a)^n/((d*x+c)^n))),x, algorithm="giac")
```

```
[Out] 1/2*B*a^2*n*log(b*x + a)/b + 1/2*(A*b + B*b)*x^2 + 1/2*(B*b*n*x^2 + 2*B*a*n
*x)*log(b*x + a) - 1/2*(B*b*n*x^2 + 2*B*a*n*x)*log(d*x + c) - 1/2*(B*b*c*n
- B*a*d*n - 2*A*a*d - 2*B*a*d)*x/d + 1/2*(B*b*c^2*n - 2*B*a*c*d*n)*log(d*x
+ c)/d^2
```

**maple [C]** time = 0.43, size = 817, normalized size = 9.73

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)*(A+B*ln(e*(b*x+a)^n/((d*x+c)^n))),x)
```

```
[Out] ln((b*x+a)^n)*x*B*a-1/2*B*x*(b*x+2*a)*ln((d*x+c)^n)+A*a*x+1/2*A*b*x^2+B*ln(
e)*a*x+1/2*B*ln(e)*b*x^2+1/2*b*B*x^2*ln((b*x+a)^n)+1/2*B*a^2*n/b*ln(-b*x-a)
+1/4*I*b*B*Pi*x^2*csgn(I/((d*x+c)^n))*csgn(I*(b*x+a)^n/((d*x+c)^n))^2+1/4*I
*b*B*Pi*x^2*csgn(I*(b*x+a)^n)*csgn(I*(b*x+a)^n/((d*x+c)^n))^2+1/2*I*B*Pi*a*
x*csgn(I*e)*csgn(I*e/((d*x+c)^n)*(b*x+a)^n)^2+1/2*I*B*Pi*a*x*csgn(I*(b*x+a)
^n/((d*x+c)^n))*csgn(I*e/((d*x+c)^n)*(b*x+a)^n)^2+1/2*I*B*Pi*a*x*csgn(I/((d
*x+c)^n))*csgn(I*(b*x+a)^n/((d*x+c)^n))^2+1/2*I*B*Pi*a*x*csgn(I*(b*x+a)^n)*
csgn(I*(b*x+a)^n/((d*x+c)^n))^2+1/4*I*b*B*Pi*x^2*csgn(I*e)*csgn(I*e/((d*x+c
)^n)*(b*x+a)^n)^2+1/4*I*b*B*Pi*x^2*csgn(I*(b*x+a)^n/((d*x+c)^n))*csgn(I*e/(
(d*x+c)^n)*(b*x+a)^n)^2+1/2*B*n*a*x-1/2*b/d*B*c*n*x-1/d*B*ln(d*x+c)*a*c*n+1
/2*b/d^2*B*ln(d*x+c)*c^2*n-1/4*I*b*B*Pi*x^2*csgn(I*e/((d*x+c)^n)*(b*x+a)^n)
^3-1/4*I*b*B*Pi*x^2*csgn(I*(b*x+a)^n/((d*x+c)^n))^3-1/2*I*B*Pi*a*x*csgn(I*e
/((d*x+c)^n)*(b*x+a)^n)^3-1/2*I*B*Pi*a*x*csgn(I*(b*x+a)^n/((d*x+c)^n))^3-1/
2*I*B*Pi*a*x*csgn(I*e)*csgn(I*(b*x+a)^n/((d*x+c)^n))*csgn(I*e/((d*x+c)^n)*(
b*x+a)^n)-1/2*I*B*Pi*a*x*csgn(I*(b*x+a)^n)*csgn(I/((d*x+c)^n))*csgn(I*(b*x+
a)^n/((d*x+c)^n))-1/4*I*b*B*Pi*x^2*csgn(I*e)*csgn(I*(b*x+a)^n/((d*x+c)^n))*
csgn(I*e/((d*x+c)^n)*(b*x+a)^n)-1/4*I*b*B*Pi*x^2*csgn(I*(b*x+a)^n)*csgn(I/(
(d*x+c)^n))*csgn(I*(b*x+a)^n/((d*x+c)^n))
```

**maxima [A]** time = 1.23, size = 154, normalized size = 1.83

$$\frac{1}{2}\,Bbx^2\log\left(\frac{(bx+a)^n e}{(dx+c)^n}\right)+\frac{1}{2}\,Abx^2+Bax\log\left(\frac{(bx+a)^n e}{(dx+c)^n}\right)+Aax+\frac{\left(\frac{aen\log(bx+a)}{b}-\frac{cen\log(dx+c)}{d}\right)Ba}{e}-\frac{\left(\frac{a^2en\log(bx+a)}{b^2}-\cdots\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*(A+B*log(e*(b*x+a)^n/((d*x+c)^n))),x, algorithm="maxima")
```

```
[Out] 1/2*B*b*x^2*log((b*x + a)^n*e/(d*x + c)^n) + 1/2*A*b*x^2 + B*a*x*log((b*x +
 a)^n*e/(d*x + c)^n) + A*a*x + (a*e*n*log(b*x + a)/b - c*e*n*log(d*x + c)/d
```

```
)*B*a/e - 1/2*(a^2*e*n*log(b*x + a)/b^2 - c^2*e*n*log(d*x + c)/d^2 + (b*c*e
*n - a*d*e*n)*x/(b*d))*B*b/e
```

**mupad [B]** time = 4.28, size = 127, normalized size = 1.51

$$\ln\left(\frac{e\,(a+b\,x)^n}{(c+d\,x)^n}\right)\left(\frac{B\,b\,x^2}{2}+B\,a\,x\right)+x\,\left(\frac{4\,A\,a\,d+2\,A\,b\,c+B\,a\,d\,n-B\,b\,c\,n}{2\,d}-\frac{A\,(2\,a\,d+2\,b\,c)}{2\,d}\right)+\frac{\ln(c+d\,x)\,\left(B\,b\,c^2\right.}{2\,d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*log((e*(a + b*x)^n)/(c + d*x)^n))*(a + b*x),x)
```

```
[Out] log((e*(a + b*x)^n)/(c + d*x)^n)*(B*a*x + (B*b*x^2)/2) + x*((4*A*a*d + 2*A*
b*c + B*a*d*n - B*b*c*n)/(2*d) - (A*(2*a*d + 2*b*c))/(2*d)) + (log(c + d*x)
*(B*b*c^2*n - 2*B*a*c*d*n))/(2*d^2) + (A*b*x^2)/2 + (B*a^2*n*log(a + b*x))/
(2*b)
```

**sympy [F(-2)]** time = 0.00, size = 0, normalized size = 0.00

Exception raised: HeuristicGCDFailed

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*(A+B*ln(e*(b*x+a)**n/((d*x+c)**n))),x)
```

```
[Out] Exception raised: HeuristicGCDFailed
```

# 3.151 $\int \frac{A+B\log(e(a+bx)^n(c+dx)^{-n})}{a+bx}\,dx$

**Optimal**. Leaf size=79

$$\frac{Bn\text{Li}_2\left(\frac{bc-ad}{d(a+bx)}+1\right)}{b}-\frac{\log\left(-\frac{bc-ad}{d(a+bx)}\right)\left(B\log\left(e(a+bx)^n(c+dx)^{-n}\right)+A\right)}{b}$$

```
[Out] -ln((a*d-b*c)/d/(b*x+a))*(A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))/b+B*n*polylog(2,
1+(-a*d+b*c)/d/(b*x+a))/b
```

**Rubi [A]** time = 0.27, antiderivative size = 87, normalized size of antiderivative = 1.10, number of steps used = 7, number of rules used = 6, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.194, Rules used = {6742, 2488, 2411, 2343, 2333, 2315}

$$\frac{Bn\text{PolyLog}\left(2,\frac{bc-ad}{d(a+bx)}+1\right)}{b}+\frac{A\log(a+bx)}{b}-\frac{B\log\left(-\frac{bc-ad}{d(a+bx)}\right)\log\left(e(a+bx)^n(c+dx)^{-n}\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])/(a + b*x),x]
```

```
[Out] (A*Log[a + b*x])/b - (B*Log[-((b*c - a*d)/(d*(a + b*x)))]*Log[(e*(a + b*x)^
n)/(c + d*x)^n])/b + (B*n*PolyLog[2, 1 + (b*c - a*d)/(d*(a + b*x))])/b
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2333

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)/(x_))^(q_.)*(
x_)^(m_.), x_Symbol] :> Int[(e + d*x)^q*(a + b*Log[c*x^n])^p, x] /; FreeQ[{
a, b, c, d, e, m, n, p}, x] && EqQ[m, q] && IntegerQ[q]
```

Rule 2343

```
Int[((a_.) + Log[(c_.)*(x_)^(n_)]*(b_.))/((x_)*((d_) + (e_.)*(x_)^(r_.))),
x_Symbol] :> Dist[1/n, Subst[Int[(a + b*Log[c*x])/(x*(d + e*x^(r/n))), x],
x, x^n], x] /; FreeQ[{a, b, c, d, e, n, r}, x] && IntegerQ[r/n]
```

Rule 2411

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + (g_
.)*(x_))^(q_.)*((h_.) + (i_.)*(x_))^(r_.), x_Symbol] :> Dist[1/e, Subst[Int
```

```
[((g*x)/e)^q*((e*h - d*i)/e + (i*x)/e)^r*(a + b*Log[c*x^n])^p, x], x, d + e
*x], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, n, p, q, r}, x] && EqQ[e*f - d
*g, 0] && (IGtQ[p, 0] || IGtQ[r, 0]) && IntegerQ[2*r]
```

Rule 2488

```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))
^(r_.)]^(s_.)/((g_.) + (h_.)*(x_)), x_Symbol] :> -Simp[(Log[-((b*c - a*d)/(
d*(a + b*x)))]*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^s)/h, x] + Dist[(p*r*s*
(b*c - a*d))/h, Int[(Log[-((b*c - a*d)/(d*(a + b*x)))]*Log[e*(f*(a + b*x)^p
*(c + d*x)^q)^r]^(s - 1))/((a + b*x)*(c + d*x)), x], x] /; FreeQ[{a, b, c,
d, e, f, g, h, p, q, r, s}, x] && NeQ[b*c - a*d, 0] && EqQ[p + q, 0] && EqQ
[b*g - a*h, 0] && IGtQ[s, 0]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\int \frac{A + B\log\left(e(a+bx)^n(c+dx)^{-n}\right)}{a+bx}\,dx = \int \left(\frac{A}{a+bx} + \frac{B\log\left(e(a+bx)^n(c+dx)^{-n}\right)}{a+bx}\right)dx$$
$$= \frac{A\log(a+bx)}{b} + B\int \frac{\log\left(e(a+bx)^n(c+dx)^{-n}\right)}{a+bx}\,dx$$
$$= \frac{A\log(a+bx)}{b} - \frac{B\log\left(-\frac{bc-ad}{d(a+bx)}\right)\log\left(e(a+bx)^n(c+dx)^{-n}\right)}{b} + \frac{(B(bc-$$
$$= \frac{A\log(a+bx)}{b} - \frac{B\log\left(-\frac{bc-ad}{d(a+bx)}\right)\log\left(e(a+bx)^n(c+dx)^{-n}\right)}{b} + \frac{(B(bc-}{}$$
$$= \frac{A\log(a+bx)}{b} - \frac{B\log\left(-\frac{bc-ad}{d(a+bx)}\right)\log\left(e(a+bx)^n(c+dx)^{-n}\right)}{b} - \frac{(B(bc-}{}$$
$$= \frac{A\log(a+bx)}{b} - \frac{B\log\left(-\frac{bc-ad}{d(a+bx)}\right)\log\left(e(a+bx)^n(c+dx)^{-n}\right)}{b} - \frac{(B(bc-}{}$$
$$= \frac{A\log(a+bx)}{b} - \frac{B\log\left(-\frac{bc-ad}{d(a+bx)}\right)\log\left(e(a+bx)^n(c+dx)^{-n}\right)}{b} + \frac{Bn\text{Li}_2(}{}$$

**Mathematica [A]** time = 0.10, size = 129, normalized size = 1.63

$$\frac{2A\log(a+bx) - 2B\log\left(\frac{ad-bc}{d(a+bx)}\right)\left(\log\left(e(a+bx)^n(c+dx)^{-n}\right) + n\log\left(\frac{b(c+dx)}{bc-ad}\right)\right) + 2Bn\text{Li}_2\left(\frac{d(a+bx)}{ad-bc}\right) - Bn\log^2\left(\frac{a}{d}\right.}{2b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])/(a + b*x),x]
```

```
[Out] (-(B*n*Log[(-(b*c) + a*d)/(d*(a + b*x))]^2) + 2*A*Log[a + b*x] - 2*B*Log[(-
(b*c) + a*d)/(d*(a + b*x))]*(n*Log[(b*(c + d*x))/(b*c - a*d)] + Log[(e*(a +
 b*x)^n)/(c + d*x)^n]) + 2*B*n*PolyLog[2, (d*(a + b*x))/(-(b*c) + a*d)])/(2
*b)
```

**fricas [F]** time = 0.99, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{B\log\left(\frac{(bx+a)^n e}{(dx+c)^n}\right) + A}{bx+a}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*(b*x+a)^n/((d*x+c)^n)))/(b*x+a),x, algorithm="fricas")
```

```
[Out] integral((B*log((b*x + a)^n*e/(d*x + c)^n) + A)/(b*x + a), x)
```

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{B \log\left(\frac{(bx+a)^n e}{(dx+c)^n}\right) + A}{bx + a}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*(b*x+a)^n/((d*x+c)^n)))/(b*x+a),x, algorithm="giac")
```

```
[Out] integrate((B*log((b*x + a)^n*e/(d*x + c)^n) + A)/(b*x + a), x)
```

**maple [C]** time = 1.24, size = 523, normalized size = 6.62

$$-\frac{i\pi B \operatorname{csgn}(ie) \operatorname{csgn}\left(i (bx+a)^n (dx+c)^{-n}\right) \operatorname{csgn}\left(ie (bx+a)^n (dx+c)^{-n}\right) \ln(bx+a)}{2b} + \frac{i\pi B \operatorname{csgn}(ie) \operatorname{csgn}\left(ie (bx+\right.}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))/(b*x+a),x)
```

```
[Out] -B/b*ln(b*x+a)*ln((d*x+c)^n)+1/b*B*n*dilog((-a*d+b*c+d*(b*x+a))/(-a*d+b*c))
+1/b*B*n*ln(b*x+a)*ln((-a*d+b*c+d*(b*x+a))/(-a*d+b*c))+1/2*I/b*B*ln(b*x+a)*
Pi*csgn(I*e)*csgn(I*e/((d*x+c)^n)*(b*x+a)^n)^2+1/2*I/b*B*ln(b*x+a)*Pi*csgn(
I*(b*x+a)^n)*csgn(I*(b*x+a)^n/((d*x+c)^n))^2+1/2*I/b*B*ln(b*x+a)*Pi*csgn(I/
((d*x+c)^n))*csgn(I*(b*x+a)^n/((d*x+c)^n))^2+1/2*I/b*B*ln(b*x+a)*Pi*csgn(I*
(b*x+a)^n/((d*x+c)^n))*csgn(I*e/((d*x+c)^n)*(b*x+a)^n)^2+A*ln(b*x+a)/b+1/b*
B*ln(b*x+a)*ln(e)+1/2/b*B/n*ln((b*x+a)^n)^2-1/2*I/b*B*ln(b*x+a)*Pi*csgn(I*(
b*x+a)^n/((d*x+c)^n))^3-1/2*I/b*B*ln(b*x+a)*Pi*csgn(I*e/((d*x+c)^n)*(b*x+a)
^n)^3-1/2*I/b*B*ln(b*x+a)*Pi*csgn(I*e)*csgn(I*(b*x+a)^n/((d*x+c)^n))*csgn(I
*e/((d*x+c)^n)*(b*x+a)^n)-1/2*I/b*B*ln(b*x+a)*Pi*csgn(I*(b*x+a)^n)*csgn(I/(
(d*x+c)^n))*csgn(I*(b*x+a)^n/((d*x+c)^n))
```

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$B\left(\frac{\log(bx+a)\log\left((bx+a)^n\right) - \log(bx+a)\log\left((dx+c)^n\right)}{b} + \int \frac{bdx\log(e) + bc\log(e) - (bcn - adn)\log(bx+}{b^2dx^2 + abc + \left(b^2c + abd\right)x}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*(b*x+a)^n/((d*x+c)^n)))/(b*x+a),x, algorithm="maxima")
```

```
[Out] B*((log(b*x + a)*log((b*x + a)^n) - log(b*x + a)*log((d*x + c)^n))/b + inte
grate((b*d*x*log(e) + b*c*log(e) - (b*c*n - a*d*n)*log(b*x + a))/(b^2*d*x^2
 + a*b*c + (b^2*c + a*b*d)*x), x)) + A*log(b*x + a)/b
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{A + B\ \ln\left(\frac{e\,(a+b\,x)^n}{(c+d\,x)^n}\right)}{a + b\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*log((e*(a + b*x)^n)/(c + d*x)^n))/(a + b*x),x)
```

```
[Out] int((A + B*log((e*(a + b*x)^n)/(c + d*x)^n))/(a + b*x), x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{A + B \log\left(e\,(a + bx)^n\,(c + dx)^{-n}\right)}{a + bx}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*ln(e*(b*x+a)**n/((d*x+c)**n)))/(b*x+a),x)
```

```
[Out] Integral((A + B*log(e*(a + b*x)**n*(c + d*x)**(-n)))/(a + b*x), x)
```

# 3.152 $\int \frac{A+B\log(e(a+bx)^n(c+dx)^{-n})}{(a+bx)^2}\,dx$

**Optimal**. Leaf size=97

$$-\frac{B\log\left(e(a+bx)^n(c+dx)^{-n}\right)+A}{b(a+bx)}-\frac{Bdn\log(a+bx)}{b(bc-ad)}+\frac{Bdn\log(c+dx)}{b(bc-ad)}-\frac{Bn}{b(a+bx)}$$

```
[Out] -B*n/b/(b*x+a)-B*d*n*ln(b*x+a)/b/(-a*d+b*c)+B*d*n*ln(d*x+c)/b/(-a*d+b*c)+(-
A-B*ln(e*(b*x+a)^n/((d*x+c)^n)))/b/(b*x+a)
```

**Rubi** [A] time = 0.08, antiderivative size = 72, normalized size of antiderivative = 0.74, number of steps used = 4, number of rules used = 3, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.097, Rules used = {6742, 2490, 32}

$$-\frac{A}{b(a+bx)}-\frac{B(c+dx)\log\left(e(a+bx)^n(c+dx)^{-n}\right)}{(a+bx)(bc-ad)}-\frac{Bn}{b(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])/(a + b*x)^2,x]
```

```
[Out] -(A/(b*(a + b*x))) - (B*n)/(b*(a + b*x)) - (B*(c + d*x)*Log[(e*(a + b*x)^n)
/(c + d*x)^n])/((b*c - a*d)*(a + b*x))
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 2490

```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))
^(r_.)]^(s_.)/((g_.) + (h_.)*(x_))^2, x_Symbol] :> Simp[((a + b*x)*Log[e*(f
*(a + b*x)^p*(c + d*x)^q)^r]^s)/((b*g - a*h)*(g + h*x)), x] - Dist[(p*r*s*(
b*c - a*d))/(b*g - a*h), Int[Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^(s - 1)/(
(c + d*x)*(g + h*x)), x], x] /; FreeQ[{a, b, c, d, e, f, g, h, p, q, r, s},
 x] && NeQ[b*c - a*d, 0] && EqQ[p + q, 0] && NeQ[b*g - a*h, 0] && IGtQ[s, 0
]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\int \frac{A + B\log\left(e(a+bx)^n(c+dx)^{-n}\right)}{(a+bx)^2}\,dx = \int \left(\frac{A}{(a+bx)^2} + \frac{B\log\left(e(a+bx)^n(c+dx)^{-n}\right)}{(a+bx)^2}\right)dx$$
$$= -\frac{A}{b(a+bx)} + B\int \frac{\log\left(e(a+bx)^n(c+dx)^{-n}\right)}{(a+bx)^2}\,dx$$
$$= -\frac{A}{b(a+bx)} - \frac{B(c+dx)\log\left(e(a+bx)^n(c+dx)^{-n}\right)}{(bc-ad)(a+bx)} + (Bn)\int \frac{1}{(a+bx)^2}$$
$$= -\frac{A}{b(a+bx)} - \frac{Bn}{b(a+bx)} - \frac{B(c+dx)\log\left(e(a+bx)^n(c+dx)^{-n}\right)}{(bc-ad)(a+bx)}$$

**Mathematica [A]** time = 0.09, size = 89, normalized size = 0.92

$$\frac{-(bc-ad)\left(B\log\left(e(a+bx)^n(c+dx)^{-n}\right) + A + Bn\right) + Bdn(a+bx)\log(c+dx) - Bdn(a+bx)\log(a+bx)}{b(a+bx)(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])/(a + b*x)^2,x]
```

```
[Out] (-(B*d*n*(a + b*x)*Log[a + b*x]) + B*d*n*(a + b*x)*Log[c + d*x] - (b*c - a*
d)*(A + B*n + B*Log[(e*(a + b*x)^n)/(c + d*x)^n]))/(b*(b*c - a*d)*(a + b*x)
)
```

**fricas [A]** time = 2.05, size = 107, normalized size = 1.10

$$-\frac{Abc - Aad + (Bbc - Bad)n + (Bbdnx + Bbcn)\log(bx+a) - (Bbdnx + Bbcn)\log(dx+c) + (Bbc - Bad)\log(e}{ab^2c - a^2bd + \left(b^3c - ab^2d\right)x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*(b*x+a)^n/((d*x+c)^n)))/(b*x+a)^2,x, algorithm="fricas
")
```

```
[Out] -(A*b*c - A*a*d + (B*b*c - B*a*d)*n + (B*b*d*n*x + B*b*c*n)*log(b*x + a) -
(B*b*d*n*x + B*b*c*n)*log(d*x + c) + (B*b*c - B*a*d)*log(e))/(a*b^2*c - a^2
*b*d + (b^3*c - a*b^2*d)*x)
```

**giac [A]** time = 0.20, size = 108, normalized size = 1.11

$$-\frac{Bdn\log(bx+a)}{b^2c - abd} + \frac{Bdn\log(dx+c)}{b^2c - abd} - \frac{Bn\log(bx+a)}{b^2x + ab} + \frac{Bn\log(dx+c)}{b^2x + ab} - \frac{Bn + A + B}{b^2x + ab}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*(b*x+a)^n/((d*x+c)^n)))/(b*x+a)^2,x, algorithm="giac")
```

```
[Out] -B*d*n*log(b*x + a)/(b^2*c - a*b*d) + B*d*n*log(d*x + c)/(b^2*c - a*b*d) -
B*n*log(b*x + a)/(b^2*x + a*b) + B*n*log(d*x + c)/(b^2*x + a*b) - (B*n + A
+ B)/(b^2*x + a*b)
```

**maple [C]** time = 0.40, size = 823, normalized size = 8.48

$$\frac{B\ln\left((dx+c)^n\right)}{(bx+a)\,b}-\frac{2Abc+2Bbcn-2Badn-2Bad\ln(e)+2Bbc\ln(e)-2Bad\ln\left((bx+a)^n\right)+2Bbc\ln\left((bx+a)^n\right)-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))/(b*x+a)^2,x)
```

```
[Out] B/b/(b*x+a)*ln((d*x+c)^n)-1/2*(2*A*b*c+2*B*b*c*n-2*B*a*d*n-2*B*ln(e)*a*d+2*
B*ln(e)*b*c-2*B*a*d*ln((b*x+a)^n)+2*B*b*c*ln((b*x+a)^n)-2*A*a*d+I*B*Pi*b*c*
csgn(I*(b*x+a)^n)*csgn(I*(b*x+a)^n/((d*x+c)^n))^2+2*B*ln(-b*x-a)*b*d*n*x-2*
B*ln(d*x+c)*b*d*n*x+I*B*Pi*b*c*csgn(I*e)*csgn(I*e/((d*x+c)^n)*(b*x+a)^n)^2-
I*B*Pi*b*c*csgn(I*(b*x+a)^n)*csgn(I/((d*x+c)^n))*csgn(I*(b*x+a)^n/((d*x+c)^
n))-I*B*Pi*a*d*csgn(I*e)*csgn(I*e/((d*x+c)^n)*(b*x+a)^n)^2-I*B*Pi*a*d*csgn(
I*(b*x+a)^n)*csgn(I*(b*x+a)^n/((d*x+c)^n))^2+2*B*ln(-b*x-a)*a*d*n-2*B*ln(d*
x+c)*a*d*n+I*B*Pi*a*d*csgn(I*e)*csgn(I*(b*x+a)^n/((d*x+c)^n))*csgn(I*e/((d*
x+c)^n)*(b*x+a)^n)+I*B*Pi*a*d*csgn(I*(b*x+a)^n)*csgn(I/((d*x+c)^n))*csgn(I*
(b*x+a)^n/((d*x+c)^n))-I*B*Pi*b*c*csgn(I*e)*csgn(I*(b*x+a)^n/((d*x+c)^n))*c
sgn(I*e/((d*x+c)^n)*(b*x+a)^n)-I*B*Pi*b*c*csgn(I*(b*x+a)^n/((d*x+c)^n))^3-I
*B*Pi*b*c*csgn(I*e/((d*x+c)^n)*(b*x+a)^n)^3+I*B*Pi*b*c*csgn(I/((d*x+c)^n))*
csgn(I*(b*x+a)^n/((d*x+c)^n))^2+I*B*Pi*b*c*csgn(I*(b*x+a)^n/((d*x+c)^n))*cs
gn(I*e/((d*x+c)^n)*(b*x+a)^n)^2+I*B*Pi*a*d*csgn(I*e/((d*x+c)^n)*(b*x+a)^n)^
3+I*B*Pi*a*d*csgn(I*(b*x+a)^n/((d*x+c)^n))^3-I*B*Pi*a*d*csgn(I/((d*x+c)^n))
*csgn(I*(b*x+a)^n/((d*x+c)^n))^2-I*B*Pi*a*d*csgn(I*(b*x+a)^n/((d*x+c)^n))*c
sgn(I*e/((d*x+c)^n)*(b*x+a)^n)^2)/(b*x+a)/b/(-a*d+b*c)
```

**maxima [A]** time = 1.20, size = 116, normalized size = 1.20

$$-\frac{\left(\frac{den\log(bx+a)}{b^2c-abd}-\frac{den\log(dx+c)}{b^2c-abd}+\frac{en}{b^2x+ab}\right)B}{e}-\frac{B\log\left(\frac{(bx+a)^n e}{(dx+c)^n}\right)}{b^2x+ab}-\frac{A}{b^2x+ab}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*(b*x+a)^n/((d*x+c)^n)))/(b*x+a)^2,x, algorithm="maxima
")
```

```
[Out] -(d*e*n*log(b*x + a)/(b^2*c - a*b*d) - d*e*n*log(d*x + c)/(b^2*c - a*b*d) +
 e*n/(b^2*x + a*b))*B/e - B*log((b*x + a)^n*e/(d*x + c)^n)/(b^2*x + a*b) -
A/(b^2*x + a*b)
```

**mupad [B]** time = 4.91, size = 97, normalized size = 1.00

$$-\frac{A+B\,n}{x\,b^2+a\,b}-\frac{B\,\ln\left(\frac{e\,(a+b\,x)^n}{(c+d\,x)^n}\right)}{b\,(a+b\,x)}-\frac{B\,d\,n\,\mathrm{atan}\left(\frac{b\,c\,2\mathrm{i}+b\,d\,x\,2\mathrm{i}}{a\,d-b\,c}+1\mathrm{i}\right)\,2\mathrm{i}}{b\,(a\,d-b\,c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*log((e*(a + b*x)^n)/(c + d*x)^n))/(a + b*x)^2,x)
```

```
[Out] - (A + B*n)/(a*b + b^2*x) - (B*log((e*(a + b*x)^n)/(c + d*x)^n))/(b*(a + b*
x)) - (B*d*n*atan((b*c*2i + b*d*x*2i)/(a*d - b*c) + 1i)*2i)/(b*(a*d - b*c))
```

**sympy [F(-2)]** time = 0.00, size = 0, normalized size = 0.00

Exception raised: HeuristicGCDFailed

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*ln(e*(b*x+a)**n/((d*x+c)**n)))/(b*x+a)**2,x)
```

```
[Out] Exception raised: HeuristicGCDFailed
```

# 3.153 $\int \frac{A+B\log(e(a+bx)^n(c+dx)^{-n})}{(a+bx)^3}\,dx$

**Optimal**. Leaf size=137

$$-\frac{B\log\left(e(a+bx)^n(c+dx)^{-n}\right)+A}{2b(a+bx)^2}+\frac{Bd^2n\log(a+bx)}{2b(bc-ad)^2}-\frac{Bd^2n\log(c+dx)}{2b(bc-ad)^2}+\frac{Bdn}{2b(a+bx)(bc-ad)}-\frac{Bn}{4b(a+bx)^2}$$

```
[Out] -1/4*B*n/b/(b*x+a)^2+1/2*B*d*n/b/(-a*d+b*c)/(b*x+a)+1/2*B*d^2*n*ln(b*x+a)/b
/(-a*d+b*c)^2-1/2*B*d^2*n*ln(d*x+c)/b/(-a*d+b*c)^2+1/2*(-A-B*ln(e*(b*x+a)^n
/((d*x+c)^n)))/b/(b*x+a)^2
```

**Rubi [A]** time = 0.15, antiderivative size = 149, normalized size of antiderivative = 1.09, number of steps used = 5, number of rules used = 3, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.097, Rules used = {6742, 2492, 44}

$$-\frac{A}{2b(a+bx)^2}+\frac{Bd^2n\log(a+bx)}{2b(bc-ad)^2}-\frac{Bd^2n\log(c+dx)}{2b(bc-ad)^2}-\frac{B\log\left(e(a+bx)^n(c+dx)^{-n}\right)}{2b(a+bx)^2}+\frac{Bdn}{2b(a+bx)(bc-ad)}-\frac{Bn}{4b(a+b$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])/(a + b*x)^3,x]
```

```
[Out] -A/(2*b*(a + b*x)^2) - (B*n)/(4*b*(a + b*x)^2) + (B*d*n)/(2*b*(b*c - a*d)*(
a + b*x)) + (B*d^2*n*Log[a + b*x])/(2*b*(b*c - a*d)^2) - (B*d^2*n*Log[c + d
*x])/(2*b*(b*c - a*d)^2) - (B*Log[(e*(a + b*x)^n)/(c + d*x)^n])/(2*b*(a + b
*x)^2)
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 2492

```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))
^(r_.)]^(s_.)*((g_.) + (h_.)*(x_))^(m_.), x_Symbol] :> Simp[((g + h*x)^(m +
 1)*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^s)/(h*(m + 1)), x] - Dist[(p*r*s*(
b*c - a*d))/(h*(m + 1)), Int[((g + h*x)^(m + 1)*Log[e*(f*(a + b*x)^p*(c + d
*x)^q)^r]^(s - 1))/((a + b*x)*(c + d*x)), x], x] /; FreeQ[{a, b, c, d, e, f
, g, h, m, p, q, r, s}, x] && NeQ[b*c - a*d, 0] && EqQ[p + q, 0] && IGtQ[s,
 0] && NeQ[m, -1]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\int \frac{A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)}{(a+bx)^3}\,dx = \int \left(\frac{A}{(a+bx)^3}+\frac{B\log\left(e(a+bx)^n(c+dx)^{-n}\right)}{(a+bx)^3}\right)dx$$
$$= -\frac{A}{2b(a+bx)^2}+B\int \frac{\log\left(e(a+bx)^n(c+dx)^{-n}\right)}{(a+bx)^3}\,dx$$
$$= -\frac{A}{2b(a+bx)^2}-\frac{B\log\left(e(a+bx)^n(c+dx)^{-n}\right)}{2b(a+bx)^2}+\frac{(B(bc-ad)n)\int \frac{1}{(a+bx)^3}}{2b}$$
$$= -\frac{A}{2b(a+bx)^2}-\frac{B\log\left(e(a+bx)^n(c+dx)^{-n}\right)}{2b(a+bx)^2}+\frac{(B(bc-ad)n)\int \Big(\frac{}{(bc-ad}}{}$$
$$= -\frac{A}{2b(a+bx)^2}-\frac{Bn}{4b(a+bx)^2}+\frac{Bdn}{2b(bc-ad)(a+bx)}+\frac{Bd^2n\log(a+bx}{2b(bc-ad)^2}$$

**Mathematica [A]** time = 0.31, size = 121, normalized size = 0.88

$$-\frac{\frac{2A}{(a+bx)^2}+Bn\left(-\frac{2d^2\log(a+bx)}{(bc-ad)^2}+\frac{2d^2\log(c+dx)}{(bc-ad)^2}+\frac{\frac{2d(a+bx)}{ad-bc}+1}{(a+bx)^2}\right)+\frac{2B\log\left(e(a+bx)^n(c+dx)^{-n}\right)}{(a+bx)^2}}{4b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])/(a + b*x)^3,x]
```

```
[Out] -1/4*((2*A)/(a + b*x)^2 + B*n*((1 + (2*d*(a + b*x))/(-(b*c) + a*d))/(a + b*
x)^2 - (2*d^2*Log[a + b*x])/(b*c - a*d)^2 + (2*d^2*Log[c + d*x])/(b*c - a*d
)^2) + (2*B*Log[(e*(a + b*x)^n)/(c + d*x)^n])/(a + b*x)^2)/b
```

**fricas [B]** time = 0.56, size = 296, normalized size = 2.16

$$-\frac{2\,Ab^2c^2-4\,Aabcd+2\,Aa^2d^2-2\left(Bb^2cd-Babd^2\right)nx+\left(Bb^2c^2-4\,Babcd+3\,Ba^2d^2\right)n-2\left(Bb^2d^2nx^2+2\,Baba\right.}{4\left(a^2b^3c^2-2\,a^3b^2cd+a^4bd^2+\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*(b*x+a)^n/((d*x+c)^n)))/(b*x+a)^3,x, algorithm="fricas
")
```

```
[Out] -1/4*(2*A*b^2*c^2 - 4*A*a*b*c*d + 2*A*a^2*d^2 - 2*(B*b^2*c*d - B*a*b*d^2)*n
*x + (B*b^2*c^2 - 4*B*a*b*c*d + 3*B*a^2*d^2)*n - 2*(B*b^2*d^2*n*x^2 + 2*B*a
*b*d^2*n*x - (B*b^2*c^2 - 2*B*a*b*c*d)*n)*log(b*x + a) + 2*(B*b^2*d^2*n*x^2
 + 2*B*a*b*d^2*n*x - (B*b^2*c^2 - 2*B*a*b*c*d)*n)*log(d*x + c) + 2*(B*b^2*c
^2 - 2*B*a*b*c*d + B*a^2*d^2)*log(e))/(a^2*b^3*c^2 - 2*a^3*b^2*c*d + a^4*b*
d^2 + (b^5*c^2 - 2*a*b^4*c*d + a^2*b^3*d^2)*x^2 + 2*(a*b^4*c^2 - 2*a^2*b^3*
c*d + a^3*b^2*d^2)*x)
```

**giac [A]** time = 0.22, size = 239, normalized size = 1.74

$$\frac{Bd^2n\log(bx+a)}{2\left(b^3c^2-2\,ab^2cd+a^2bd^2\right)}-\frac{Bd^2n\log(dx+c)}{2\left(b^3c^2-2\,ab^2cd+a^2bd^2\right)}-\frac{Bn\log(bx+a)}{2\left(b^3x^2+2\,ab^2x+a^2b\right)}+\frac{Bn\log(dx+c)}{2\left(b^3x^2+2\,ab^2x+a^2b\right)}+\frac{2\,Bbd\cdots}{4\left(b\cdots\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*(b*x+a)^n/((d*x+c)^n)))/(b*x+a)^3,x, algorithm="giac")
```

```
[Out] 1/2*B*d^2*n*log(b*x + a)/(b^3*c^2 - 2*a*b^2*c*d + a^2*b*d^2) - 1/2*B*d^2*n*
log(d*x + c)/(b^3*c^2 - 2*a*b^2*c*d + a^2*b*d^2) - 1/2*B*n*log(b*x + a)/(b^
3*x^2 + 2*a*b^2*x + a^2*b) + 1/2*B*n*log(d*x + c)/(b^3*x^2 + 2*a*b^2*x + a^
2*b) + 1/4*(2*B*b*d*n*x - B*b*c*n + 3*B*a*d*n - 2*A*b*c - 2*B*b*c + 2*A*a*d
 + 2*B*a*d)/(b^4*c*x^2 - a*b^3*d*x^2 + 2*a*b^3*c*x - 2*a^2*b^2*d*x + a^2*b^
2*c - a^3*b*d)
```

**maple [C]** time = 0.49, size = 1379, normalized size = 10.07

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))/(b*x+a)^3,x)
```

```
[Out] 1/2*B/b/(b*x+a)^2*ln((d*x+c)^n)-1/4*(2*A*b^2*c^2+2*I*B*Pi*a*b*c*d*csgn(I*e)
*csgn(I*(b*x+a)^n/((d*x+c)^n))*csgn(I*e/((d*x+c)^n)*(b*x+a)^n)+I*B*Pi*a^2*d
^2*csgn(I*(b*x+a)^n)*csgn(I*(b*x+a)^n/((d*x+c)^n))^2+B*c^2*n*b^2+3*B*a^2*d^
2*n+2*A*a^2*d^2+2*B*ln(e)*b^2*c^2+2*B*ln(e)*a^2*d^2+2*B*a^2*d^2*ln((b*x+a)^
n)+2*B*b^2*c^2*ln((b*x+a)^n)+2*I*B*Pi*a*b*c*d*csgn(I*(b*x+a)^n/((d*x+c)^n))
^3+2*B*a*b*d^2*n*x-2*B*b^2*c*d*n*x-4*B*a*c*d*n*b+2*I*B*Pi*a*b*c*d*csgn(I*e/
((d*x+c)^n)*(b*x+a)^n)^3-I*B*Pi*b^2*c^2*csgn(I*e)*csgn(I*(b*x+a)^n/((d*x+c)
^n))*csgn(I*e/((d*x+c)^n)*(b*x+a)^n)-I*B*Pi*a^2*d^2*csgn(I*(b*x+a)^n)*csgn(
I/((d*x+c)^n))*csgn(I*(b*x+a)^n/((d*x+c)^n))-I*B*Pi*a^2*d^2*csgn(I*e/((d*x+
c)^n)*(b*x+a)^n)^3-I*B*Pi*b^2*c^2*csgn(I*(b*x+a)^n/((d*x+c)^n))^3-I*B*Pi*b^
2*c^2*csgn(I*e/((d*x+c)^n)*(b*x+a)^n)^3+I*B*Pi*a^2*d^2*csgn(I*(b*x+a)^n/((d
*x+c)^n))*csgn(I*e/((d*x+c)^n)*(b*x+a)^n)^2+I*B*Pi*b^2*c^2*csgn(I*e)*csgn(I
*e/((d*x+c)^n)*(b*x+a)^n)^2-I*B*Pi*a^2*d^2*csgn(I*(b*x+a)^n/((d*x+c)^n))^3+
2*I*B*Pi*a*b*c*d*csgn(I*(b*x+a)^n)*csgn(I/((d*x+c)^n))*csgn(I*(b*x+a)^n/((d
```

```
*x+c)^n))-2*B*a^2*n*ln(-b*x-a)*d^2+I*B*Pi*a^2*d^2*csgn(I/((d*x+c)^n))*csgn(
I*(b*x+a)^n/((d*x+c)^n))^2-4*A*a*b*c*d-4*B*a*b*c*d*ln((b*x+a)^n)+2*B*ln(d*x
+c)*a^2*d^2*n-2*B*ln(-b*x-a)*b^2*d^2*n*x^2+2*B*ln(d*x+c)*b^2*d^2*n*x^2-2*I*
B*Pi*a*b*c*d*csgn(I*(b*x+a)^n)*csgn(I*(b*x+a)^n/((d*x+c)^n))^2-2*I*B*Pi*a*b
*c*d*csgn(I/((d*x+c)^n))*csgn(I*(b*x+a)^n/((d*x+c)^n))^2+I*B*Pi*b^2*c^2*csg
n(I*(b*x+a)^n)*csgn(I*(b*x+a)^n/((d*x+c)^n))^2+I*B*Pi*b^2*c^2*csgn(I/((d*x+
c)^n))*csgn(I*(b*x+a)^n/((d*x+c)^n))^2+I*B*Pi*b^2*c^2*csgn(I*(b*x+a)^n/((d*
x+c)^n))*csgn(I*e/((d*x+c)^n)*(b*x+a)^n)^2+I*B*Pi*a^2*d^2*csgn(I*e)*csgn(I*
e/((d*x+c)^n)*(b*x+a)^n)^2-2*I*B*Pi*a*b*c*d*csgn(I*(b*x+a)^n/((d*x+c)^n))*c
sgn(I*e/((d*x+c)^n)*(b*x+a)^n)^2-4*B*ln(-b*x-a)*a*b*d^2*n*x+4*B*ln(d*x+c)*a
*b*d^2*n*x-I*B*Pi*a^2*d^2*csgn(I*e)*csgn(I*(b*x+a)^n/((d*x+c)^n))*csgn(I*e/
((d*x+c)^n)*(b*x+a)^n)-2*I*B*Pi*a*b*c*d*csgn(I*e)*csgn(I*e/((d*x+c)^n)*(b*x
+a)^n)^2-4*B*ln(e)*a*b*c*d-I*B*Pi*b^2*c^2*csgn(I*(b*x+a)^n)*csgn(I/((d*x+c)
^n))*csgn(I*(b*x+a)^n/((d*x+c)^n)))/(b*x+a)^2/(-a*d+b*c)^2/b
```

**maxima [A]** time = 1.38, size = 230, normalized size = 1.68

$$\frac{\left(\frac{2\,d^2en\log(bx+a)}{b^3c^2-2\,ab^2cd+a^2bd^2}-\frac{2\,d^2en\log(dx+c)}{b^3c^2-2\,ab^2cd+a^2bd^2}+\frac{2\,bdenx-bcen+3\,aden}{a^2b^2c-a^3bd+\left(b^4c-ab^3d\right)x^2+2\left(ab^3c-a^2b^2d\right)x}\right)B}{4\,e}-\frac{B\log\left(\frac{(bx+a)^ne}{(dx+c)^n}\right)}{2\left(b^3x^2+2\,ab^2x+a^2b\right)}-\frac{}{2\left(b^3x^2+\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*(b*x+a)^n/((d*x+c)^n)))/(b*x+a)^3,x, algorithm="maxima
")
```

```
[Out] 1/4*(2*d^2*e*n*log(b*x + a)/(b^3*c^2 - 2*a*b^2*c*d + a^2*b*d^2) - 2*d^2*e*n
*log(d*x + c)/(b^3*c^2 - 2*a*b^2*c*d + a^2*b*d^2) + (2*b*d*e*n*x - b*c*e*n
+ 3*a*d*e*n)/(a^2*b^2*c - a^3*b*d + (b^4*c - a*b^3*d)*x^2 + 2*(a*b^3*c - a^
2*b^2*d)*x))*B/e - 1/2*B*log((b*x + a)^n*e/(d*x + c)^n)/(b^3*x^2 + 2*a*b^2*
x + a^2*b) - 1/2*A/(b^3*x^2 + 2*a*b^2*x + a^2*b)
```

**mupad [B]** time = 4.66, size = 192, normalized size = 1.40

$$-\frac{\frac{2\,A\,a\,d-2\,A\,b\,c+3\,B\,a\,d\,n-B\,b\,c\,n}{2\,(a\,d-b\,c)}+\frac{B\,b\,d\,n\,x}{a\,d-b\,c}}{2\,a^2\,b+4\,a\,b^2\,x+2\,b^3\,x^2}-\frac{B\,\ln\left(\frac{e\,(a+b\,x)^n}{(c+d\,x)^n}\right)}{2\,b\,\left(a^2+2\,a\,b\,x+b^2\,x^2\right)}-\frac{B\,d^2\,n\,\mathrm{atanh}\left(\frac{2\,b^3\,c^2-2\,a^2\,b\,d^2}{2\,b\,(a\,d-b\,c)^2}-\frac{2\,b\,d\,x}{a\,d-b\,c}\right)}{b\,(a\,d-b\,c)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*log((e*(a + b*x)^n)/(c + d*x)^n))/(a + b*x)^3,x)
```

```
[Out] - ((2*A*a*d - 2*A*b*c + 3*B*a*d*n - B*b*c*n)/(2*(a*d - b*c)) + (B*b*d*n*x)/
(a*d - b*c))/(2*a^2*b + 2*b^3*x^2 + 4*a*b^2*x) - (B*log((e*(a + b*x)^n)/(c
+ d*x)^n))/(2*b*(a^2 + b^2*x^2 + 2*a*b*x)) - (B*d^2*n*atanh((2*b^3*c^2 - 2*
a^2*b*d^2)/(2*b*(a*d - b*c)^2) - (2*b*d*x)/(a*d - b*c)))/(b*(a*d - b*c)^2)
```

**sympy [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*ln(e*(b*x+a)**n/((d*x+c)**n)))/(b*x+a)**3,x)
[Out] Timed out
```

# 3.154 $\int \frac{A+B\log(e(a+bx)^n(c+dx)^{-n})}{(a+bx)^4}\,dx$

**Optimal**. Leaf size=166

$$-\frac{B\log\left(e(a+bx)^n(c+dx)^{-n}\right)+A}{3b(a+bx)^3}-\frac{Bd^3n\log(a+bx)}{3b(bc-ad)^3}+\frac{Bd^3n\log(c+dx)}{3b(bc-ad)^3}-\frac{Bd^2n}{3b(a+bx)(bc-ad)^2}+\frac{Bdn}{6b(a+bx)^2(bc$$

```
[Out] -1/9*B*n/b/(b*x+a)^3+1/6*B*d*n/b/(-a*d+b*c)/(b*x+a)^2-1/3*B*d^2*n/b/(-a*d+b
*c)^2/(b*x+a)-1/3*B*d^3*n*ln(b*x+a)/b/(-a*d+b*c)^3+1/3*B*d^3*n*ln(d*x+c)/b/
(-a*d+b*c)^3+1/3*(-A-B*ln(e*(b*x+a)^n/((d*x+c)^n)))/b/(b*x+a)^3
```

**Rubi [A]** time = 0.17, antiderivative size = 178, normalized size of antiderivative = 1.07, number of steps used = 5, number of rules used = 3, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.097, Rules used = {6742, 2492, 44}

$$-\frac{A}{3b(a+bx)^3}-\frac{Bd^2n}{3b(a+bx)(bc-ad)^2}-\frac{Bd^3n\log(a+bx)}{3b(bc-ad)^3}+\frac{Bd^3n\log(c+dx)}{3b(bc-ad)^3}-\frac{B\log\left(e(a+bx)^n(c+dx)^{-n}\right)}{3b(a+bx)^3}+\frac{}{6b(a}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])/(a + b*x)^4,x]
```

```
[Out] -A/(3*b*(a + b*x)^3) - (B*n)/(9*b*(a + b*x)^3) + (B*d*n)/(6*b*(b*c - a*d)*(
a + b*x)^2) - (B*d^2*n)/(3*b*(b*c - a*d)^2*(a + b*x)) - (B*d^3*n*Log[a + b*
x])/(3*b*(b*c - a*d)^3) + (B*d^3*n*Log[c + d*x])/(3*b*(b*c - a*d)^3) - (B*L
og[(e*(a + b*x)^n)/(c + d*x)^n])/(3*b*(a + b*x)^3)
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 2492

```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))
^(r_.)]^(s_.)*((g_.) + (h_.)*(x_))^(m_.), x_Symbol] :> Simp[((g + h*x)^(m +
 1)*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^s)/(h*(m + 1)), x] - Dist[(p*r*s*(
b*c - a*d))/(h*(m + 1)), Int[((g + h*x)^(m + 1)*Log[e*(f*(a + b*x)^p*(c + d
*x)^q)^r]^(s - 1))/((a + b*x)*(c + d*x)), x], x] /; FreeQ[{a, b, c, d, e, f
, g, h, m, p, q, r, s}, x] && NeQ[b*c - a*d, 0] && EqQ[p + q, 0] && IGtQ[s,
 0] && NeQ[m, -1]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\int \frac{A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)}{(a+bx)^4}\,dx = \int \left(\frac{A}{(a+bx)^4}+\frac{B\log\left(e(a+bx)^n(c+dx)^{-n}\right)}{(a+bx)^4}\right)dx$$
$$= -\frac{A}{3b(a+bx)^3}+B\int \frac{\log\left(e(a+bx)^n(c+dx)^{-n}\right)}{(a+bx)^4}\,dx$$
$$= -\frac{A}{3b(a+bx)^3}-\frac{B\log\left(e(a+bx)^n(c+dx)^{-n}\right)}{3b(a+bx)^3}+\frac{(B(bc-ad)n)\int \frac{1}{(a+bx)^4(c}}{3b}$$
$$= -\frac{A}{3b(a+bx)^3}-\frac{B\log\left(e(a+bx)^n(c+dx)^{-n}\right)}{3b(a+bx)^3}+\frac{(B(bc-ad)n)\int \left(\frac{b}{(bc-ad)(}\right.}{}$$
$$= -\frac{A}{3b(a+bx)^3}-\frac{Bn}{9b(a+bx)^3}+\frac{Bdn}{6b(bc-ad)(a+bx)^2}-\frac{Bd^2n}{3b(bc-ad)^2(a+}$$

**Mathematica [A]** time = 0.38, size = 143, normalized size = 0.86

$$-\frac{\frac{6A}{(a+bx)^3}+Bn\left(\frac{6d^3\log(a+bx)}{(bc-ad)^3}-\frac{6d^3\log(c+dx)}{(bc-ad)^3}+\frac{\frac{6d^2(a+bx)^2}{(bc-ad)^2}+\frac{3d(a+bx)}{ad-bc}+2}{(a+bx)^3}\right)+\frac{6B\log\left(e(a+bx)^n(c+dx)^{-n}\right)}{(a+bx)^3}}{18b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])/(a + b*x)^4,x]
```

```
[Out] -1/18*((6*A)/(a + b*x)^3 + B*n*((2 + (3*d*(a + b*x))/(-(b*c) + a*d) + (6*d^
2*(a + b*x)^2)/(b*c - a*d)^2)/(a + b*x)^3 + (6*d^3*Log[a + b*x])/(b*c - a*d
)^3 - (6*d^3*Log[c + d*x])/(b*c - a*d)^3) + (6*B*Log[(e*(a + b*x)^n)/(c + d
*x)^n])/(a + b*x)^3)/b
```

**fricas [B]** time = 0.61, size = 540, normalized size = 3.25

$$-\frac{6\,Ab^3c^3-18\,Aab^2c^2d+18\,Aa^2bcd^2-6\,Aa^3d^3+6\left(Bb^3cd^2-Bab^2d^3\right)nx^2-3\left(Bb^3c^2d-6\,Bab^2cd^2+5\,Ba^2bd^3\right)n}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*(b*x+a)^n/((d*x+c)^n)))/(b*x+a)^4,x, algorithm="fricas
")
```

```
[Out] -1/18*(6*A*b^3*c^3 - 18*A*a*b^2*c^2*d + 18*A*a^2*b*c*d^2 - 6*A*a^3*d^3 + 6*
(B*b^3*c*d^2 - B*a*b^2*d^3)*n*x^2 - 3*(B*b^3*c^2*d - 6*B*a*b^2*c*d^2 + 5*B*
a^2*b*d^3)*n*x + (2*B*b^3*c^3 - 9*B*a*b^2*c^2*d + 18*B*a^2*b*c*d^2 - 11*B*a
^3*d^3)*n + 6*(B*b^3*d^3*n*x^3 + 3*B*a*b^2*d^3*n*x^2 + 3*B*a^2*b*d^3*n*x +
(B*b^3*c^3 - 3*B*a*b^2*c^2*d + 3*B*a^2*b*c*d^2)*n)*log(b*x + a) - 6*(B*b^3*
d^3*n*x^3 + 3*B*a*b^2*d^3*n*x^2 + 3*B*a^2*b*d^3*n*x + (B*b^3*c^3 - 3*B*a*b^
2*c^2*d + 3*B*a^2*b*c*d^2)*n)*log(d*x + c) + 6*(B*b^3*c^3 - 3*B*a*b^2*c^2*d
 + 3*B*a^2*b*c*d^2 - B*a^3*d^3)*log(e))/(a^3*b^4*c^3 - 3*a^4*b^3*c^2*d + 3*
a^5*b^2*c*d^2 - a^6*b*d^3 + (b^7*c^3 - 3*a*b^6*c^2*d + 3*a^2*b^5*c*d^2 - a^
3*b^4*d^3)*x^3 + 3*(a*b^6*c^3 - 3*a^2*b^5*c^2*d + 3*a^3*b^4*c*d^2 - a^4*b^3
*d^3)*x^2 + 3*(a^2*b^5*c^3 - 3*a^3*b^4*c^2*d + 3*a^4*b^3*c*d^2 - a^5*b^2*d^
3)*x)
```

**giac [B]** time = 0.23, size = 448, normalized size = 2.70

$$-\frac{Bd^3n\log(bx+a)}{3\left(b^4c^3-3\,ab^3c^2d+3\,a^2b^2cd^2-a^3bd^3\right)}+\frac{Bd^3n\log(dx+c)}{3\left(b^4c^3-3\,ab^3c^2d+3\,a^2b^2cd^2-a^3bd^3\right)}-\frac{Bn\log(bx+a)}{3\left(b^4x^3+3\,ab^3x^2+3\,a^2b^2x\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*(b*x+a)^n/((d*x+c)^n)))/(b*x+a)^4,x, algorithm="giac")
```

```
[Out] -1/3*B*d^3*n*log(b*x + a)/(b^4*c^3 - 3*a*b^3*c^2*d + 3*a^2*b^2*c*d^2 - a^3*
b*d^3) + 1/3*B*d^3*n*log(d*x + c)/(b^4*c^3 - 3*a*b^3*c^2*d + 3*a^2*b^2*c*d^
2 - a^3*b*d^3) - 1/3*B*n*log(b*x + a)/(b^4*x^3 + 3*a*b^3*x^2 + 3*a^2*b^2*x
+ a^3*b) + 1/3*B*n*log(d*x + c)/(b^4*x^3 + 3*a*b^3*x^2 + 3*a^2*b^2*x + a^3*
b) - 1/18*(6*B*b^2*d^2*n*x^2 - 3*B*b^2*c*d*n*x + 15*B*a*b*d^2*n*x + 2*B*b^2
*c^2*n - 7*B*a*b*c*d*n + 11*B*a^2*d^2*n + 6*A*b^2*c^2 + 6*B*b^2*c^2 - 12*A*
a*b*c*d - 12*B*a*b*c*d + 6*A*a^2*d^2 + 6*B*a^2*d^2)/(b^6*c^2*x^3 - 2*a*b^5*
c*d*x^3 + a^2*b^4*d^2*x^3 + 3*a*b^5*c^2*x^2 - 6*a^2*b^4*c*d*x^2 + 3*a^3*b^3
*d^2*x^2 + 3*a^2*b^4*c^2*x - 6*a^3*b^3*c*d*x + 3*a^4*b^2*d^2*x + a^3*b^3*c^
2 - 2*a^4*b^2*c*d + a^5*b*d^2)
```

**maple [C]** time = 0.58, size = 1976, normalized size = 11.90

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))/(b*x+a)^4,x)
```

```
[Out] 1/3*B/b/(b*x+a)^3*ln((d*x+c)^n)-1/18*(-6*B*a^3*d^3*ln((b*x+a)^n)+6*B*b^3*c^
3*ln((b*x+a)^n)+6*A*b^3*c^3+9*I*B*Pi*a^2*b*c*d^2*csgn(I*e)*csgn(I*e/((d*x+c
```

```
)^n)*(b*x+a)^n)^2+9*I*B*Pi*a^2*b*c*d^2*csgn(I*(b*x+a)^n)*csgn(I*(b*x+a)^n/(
(d*x+c)^n))^2+2*B*c^3*n*b^3-11*B*a^3*d^3*n-6*A*a^3*d^3+9*I*B*Pi*a*b^2*c^2*d
*csgn(I*(b*x+a)^n)*csgn(I/((d*x+c)^n))*csgn(I*(b*x+a)^n/((d*x+c)^n))+3*I*B*
Pi*b^3*c^3*csgn(I*e)*csgn(I*e/((d*x+c)^n)*(b*x+a)^n)^2+3*I*B*Pi*b^3*c^3*csg
n(I*(b*x+a)^n)*csgn(I*(b*x+a)^n/((d*x+c)^n))^2+3*I*B*Pi*b^3*c^3*csgn(I/((d*
x+c)^n))*csgn(I*(b*x+a)^n/((d*x+c)^n))^2+6*B*ln(-b*x-a)*b^3*d^3*n*x^3-6*B*l
n(d*x+c)*b^3*d^3*n*x^3+3*I*B*Pi*a^3*d^3*csgn(I*e)*csgn(I*(b*x+a)^n/((d*x+c)
^n))*csgn(I*e/((d*x+c)^n)*(b*x+a)^n)-6*B*ln(e)*a^3*d^3+6*B*ln(e)*b^3*c^3-9*
B*a*c^2*d*n*b^2+18*B*a^2*c*d^2*n*b+18*B*ln(-b*x-a)*a*b^2*d^3*n*x^2-18*B*ln(
d*x+c)*a*b^2*d^3*n*x^2+18*B*ln(-b*x-a)*a^2*b*d^3*n*x-18*B*ln(d*x+c)*a^2*b*d
^3*n*x+18*B*ln(e)*a^2*b*c*d^2-18*B*ln(e)*a*b^2*c^2*d+18*B*a*b^2*c*d^2*n*x-3
*I*B*Pi*b^3*c^3*csgn(I*e)*csgn(I*(b*x+a)^n/((d*x+c)^n))*csgn(I*e/((d*x+c)^n
)*(b*x+a)^n)-3*I*B*Pi*b^3*c^3*csgn(I*(b*x+a)^n)*csgn(I/((d*x+c)^n))*csgn(I*
(b*x+a)^n/((d*x+c)^n))-9*I*B*Pi*a^2*b*c*d^2*csgn(I*e)*csgn(I*(b*x+a)^n/((d*
x+c)^n))*csgn(I*e/((d*x+c)^n)*(b*x+a)^n)-9*I*B*Pi*a^2*b*c*d^2*csgn(I*(b*x+a
)^n)*csgn(I/((d*x+c)^n))*csgn(I*(b*x+a)^n/((d*x+c)^n))+9*I*B*Pi*a*b^2*c^2*d
*csgn(I*e)*csgn(I*(b*x+a)^n/((d*x+c)^n))*csgn(I*e/((d*x+c)^n)*(b*x+a)^n)-9*
I*B*Pi*a*b^2*c^2*d*csgn(I*e)*csgn(I*e/((d*x+c)^n)*(b*x+a)^n)^2-9*I*B*Pi*a*b
^2*c^2*d*csgn(I*(b*x+a)^n)*csgn(I*(b*x+a)^n/((d*x+c)^n))^2-9*I*B*Pi*a*b^2*c
^2*d*csgn(I/((d*x+c)^n))*csgn(I*(b*x+a)^n/((d*x+c)^n))^2-9*I*B*Pi*a*b^2*c^2
*d*csgn(I*(b*x+a)^n/((d*x+c)^n))*csgn(I*e/((d*x+c)^n)*(b*x+a)^n)^2-3*I*B*Pi
*a^3*d^3*csgn(I*(b*x+a)^n)*csgn(I*(b*x+a)^n/((d*x+c)^n))^2-3*I*B*Pi*a^3*d^3
*csgn(I/((d*x+c)^n))*csgn(I*(b*x+a)^n/((d*x+c)^n))^2-3*I*B*Pi*a^3*d^3*csgn(
I*(b*x+a)^n/((d*x+c)^n))*csgn(I*e/((d*x+c)^n)*(b*x+a)^n)^2+6*B*a^3*n*ln(-b*
x-a)*d^3-6*B*ln(d*x+c)*a^3*d^3*n+18*A*a^2*b*c*d^2-18*A*a*b^2*c^2*d+3*I*B*Pi
*b^3*c^3*csgn(I*(b*x+a)^n/((d*x+c)^n))*csgn(I*e/((d*x+c)^n)*(b*x+a)^n)^2+3*
I*B*Pi*a^3*d^3*csgn(I*(b*x+a)^n)*csgn(I/((d*x+c)^n))*csgn(I*(b*x+a)^n/((d*x
+c)^n))-6*B*a*b^2*d^3*n*x^2+6*B*b^3*c*d^2*n*x^2-15*B*a^2*b*d^3*n*x+9*I*B*Pi
*a^2*b*c*d^2*csgn(I/((d*x+c)^n))*csgn(I*(b*x+a)^n/((d*x+c)^n))^2+9*I*B*Pi*a
^2*b*c*d^2*csgn(I*(b*x+a)^n/((d*x+c)^n))*csgn(I*e/((d*x+c)^n)*(b*x+a)^n)^2-
3*I*B*Pi*a^3*d^3*csgn(I*e)*csgn(I*e/((d*x+c)^n)*(b*x+a)^n)^2-3*B*b^3*c^2*d*
n*x+18*B*a^2*b*c*d^2*ln((b*x+a)^n)-18*B*a*b^2*c^2*d*ln((b*x+a)^n)+3*I*B*Pi*
a^3*d^3*csgn(I*(b*x+a)^n/((d*x+c)^n))^3+3*I*B*Pi*a^3*d^3*csgn(I*e/((d*x+c)^
n)*(b*x+a)^n)^3-3*I*B*Pi*b^3*c^3*csgn(I*(b*x+a)^n/((d*x+c)^n))^3-3*I*B*Pi*b
^3*c^3*csgn(I*e/((d*x+c)^n)*(b*x+a)^n)^3-9*I*B*Pi*a^2*b*c*d^2*csgn(I*(b*x+a
)^n/((d*x+c)^n))^3-9*I*B*Pi*a^2*b*c*d^2*csgn(I*e/((d*x+c)^n)*(b*x+a)^n)^3+9
*I*B*Pi*a*b^2*c^2*d*csgn(I*(b*x+a)^n/((d*x+c)^n))^3+9*I*B*Pi*a*b^2*c^2*d*cs
gn(I*e/((d*x+c)^n)*(b*x+a)^n)^3)/(b*x+a)^3/(a^2*d^2-2*a*b*c*d+b^2*c^2)/(-a*
d+b*c)/b
```

**maxima [B]** time = 1.29, size = 400, normalized size = 2.41

$$-\frac{\left(\frac{6\,d^3 en\log(bx+a)}{b^4c^3-3\,ab^3c^2d+3\,a^2b^2cd^2-a^3bd^3}-\frac{6\,d^3 en\log(dx+c)}{b^4c^3-3\,ab^3c^2d+3\,a^2b^2cd^2-a^3bd^3}+\frac{6\,b^2d^2enx^2+2\,b^2c^2en-7\,abcden+11\,a^2d^2en-3\,(b^2c}{a^3b^3c^2-2\,a^4b^2cd+a^5bd^2+\left(b^6c^2-2\,ab^5cd+a^2b^4d^2\right)x^3+3\,(ab^5c^2-2\,a^2b^4cd+a}\right.}{18\,e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*(b*x+a)^n/((d*x+c)^n)))/(b*x+a)^4,x, algorithm="maxima
")
```

```
[Out] -1/18*(6*d^3*e*n*log(b*x + a)/(b^4*c^3 - 3*a*b^3*c^2*d + 3*a^2*b^2*c*d^2 -
a^3*b*d^3) - 6*d^3*e*n*log(d*x + c)/(b^4*c^3 - 3*a*b^3*c^2*d + 3*a^2*b^2*c*
d^2 - a^3*b*d^3) + (6*b^2*d^2*e*n*x^2 + 2*b^2*c^2*e*n - 7*a*b*c*d*e*n + 11*
a^2*d^2*e*n - 3*(b^2*c*d*e*n - 5*a*b*d^2*e*n)*x)/(a^3*b^3*c^2 - 2*a^4*b^2*c
*d + a^5*b*d^2 + (b^6*c^2 - 2*a*b^5*c*d + a^2*b^4*d^2)*x^3 + 3*(a*b^5*c^2 -
 2*a^2*b^4*c*d + a^3*b^3*d^2)*x^2 + 3*(a^2*b^4*c^2 - 2*a^3*b^3*c*d + a^4*b^
2*d^2)*x))*B/e - 1/3*B*log((b*x + a)^n*e/(d*x + c)^n)/(b^4*x^3 + 3*a*b^3*x^
2 + 3*a^2*b^2*x + a^3*b) - 1/3*A/(b^4*x^3 + 3*a*b^3*x^2 + 3*a^2*b^2*x + a^3
*b)
```

**mupad [B]** time = 4.91, size = 317, normalized size = 1.91

$$\frac{2\,A\,a\,c\,d}{3\,{\left(a\,d-b\,c\right)}^2\,{\left(a+b\,x\right)}^3}-\frac{A\,b\,c^2}{3\,{\left(a\,d-b\,c\right)}^2\,{\left(a+b\,x\right)}^3}-\frac{B\,\ln\left(\frac{e\,{\left(a+b\,x\right)}^n}{{\left(c+d\,x\right)}^n}\right)}{3\,b\,{\left(a+b\,x\right)}^3}-\frac{A\,a^2\,d^2}{3\,b\,{\left(a\,d-b\,c\right)}^2\,{\left(a+b\,x\right)}^3}-\frac{B\,b\,c^2\,n}{9\,{\left(a\,d-b\,c\right)}^2\,{\left(a+b\,x\right.}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*log((e*(a + b*x)^n)/(c + d*x)^n))/(a + b*x)^4,x)
```

```
[Out] (2*A*a*c*d)/(3*(a*d - b*c)^2*(a + b*x)^3) - (A*b*c^2)/(3*(a*d - b*c)^2*(a +
 b*x)^3) - (B*log((e*(a + b*x)^n)/(c + d*x)^n))/(3*b*(a + b*x)^3) - (A*a^2*
d^2)/(3*b*(a*d - b*c)^2*(a + b*x)^3) - (B*b*c^2*n)/(9*(a*d - b*c)^2*(a + b*
x)^3) - (B*d^3*n*atan((a*d*1i + b*c*1i + b*d*x*2i)/(a*d - b*c))*2i)/(3*b*(a
*d - b*c)^3) - (5*B*a*d^2*n*x)/(6*(a*d - b*c)^2*(a + b*x)^3) - (B*b*d^2*n*x
^2)/(3*(a*d - b*c)^2*(a + b*x)^3) + (7*B*a*c*d*n)/(18*(a*d - b*c)^2*(a + b*
x)^3) - (11*B*a^2*d^2*n)/(18*b*(a*d - b*c)^2*(a + b*x)^3) + (B*b*c*d*n*x)/(
6*(a*d - b*c)^2*(a + b*x)^3)
```

**sympy [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*ln(e*(b*x+a)**n/((d*x+c)**n)))/(b*x+a)**4,x)
```

```
[Out] Timed out
```

# 3.155 $\int \frac{A+B\log(e(a+bx)^n(c+dx)^{-n})}{(a+bx)^5}\,dx$

**Optimal**. Leaf size=195

$$-\frac{B\log\left(e(a+bx)^n(c+dx)^{-n}\right)+A}{4b(a+bx)^4}+\frac{Bd^4n\log(a+bx)}{4b(bc-ad)^4}-\frac{Bd^4n\log(c+dx)}{4b(bc-ad)^4}+\frac{Bd^3n}{4b(a+bx)(bc-ad)^3}-\frac{Bd^2n}{8b(a+bx)^2(bc-$$

```
[Out] -1/16*B*n/b/(b*x+a)^4+1/12*B*d*n/b/(-a*d+b*c)/(b*x+a)^3-1/8*B*d^2*n/b/(-a*d
+b*c)^2/(b*x+a)^2+1/4*B*d^3*n/b/(-a*d+b*c)^3/(b*x+a)+1/4*B*d^4*n*ln(b*x+a)/
b/(-a*d+b*c)^4-1/4*B*d^4*n*ln(d*x+c)/b/(-a*d+b*c)^4+1/4*(-A-B*ln(e*(b*x+a)^
n/((d*x+c)^n)))/b/(b*x+a)^4
```

**Rubi [A]** time = 0.19, antiderivative size = 207, normalized size of antiderivative = 1.06, number of steps used = 5, number of rules used = 3, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.097, Rules used = {6742, 2492, 44}

$$-\frac{A}{4b(a+bx)^4}+\frac{Bd^3n}{4b(a+bx)(bc-ad)^3}-\frac{Bd^2n}{8b(a+bx)^2(bc-ad)^2}+\frac{Bd^4n\log(a+bx)}{4b(bc-ad)^4}-\frac{Bd^4n\log(c+dx)}{4b(bc-ad)^4}-\frac{B\log(e(a+b}{4b(a+}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])/(a + b*x)^5,x]
```

```
[Out] -A/(4*b*(a + b*x)^4) - (B*n)/(16*b*(a + b*x)^4) + (B*d*n)/(12*b*(b*c - a*d)
*(a + b*x)^3) - (B*d^2*n)/(8*b*(b*c - a*d)^2*(a + b*x)^2) + (B*d^3*n)/(4*b*
(b*c - a*d)^3*(a + b*x)) + (B*d^4*n*Log[a + b*x])/(4*b*(b*c - a*d)^4) - (B*
d^4*n*Log[c + d*x])/(4*b*(b*c - a*d)^4) - (B*Log[(e*(a + b*x)^n)/(c + d*x)^
n])/(4*b*(a + b*x)^4)
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 2492

```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))
^(r_.)]^(s_.)*((g_.) + (h_.)*(x_))^(m_.), x_Symbol] :> Simp[((g + h*x)^(m +
 1)*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^s)/(h*(m + 1)), x] - Dist[(p*r*s*(
b*c - a*d))/(h*(m + 1)), Int[((g + h*x)^(m + 1)*Log[e*(f*(a + b*x)^p*(c + d
*x)^q)^r]^(s - 1))/((a + b*x)*(c + d*x)), x], x] /; FreeQ[{a, b, c, d, e, f
, g, h, m, p, q, r, s}, x] && NeQ[b*c - a*d, 0] && EqQ[p + q, 0] && IGtQ[s,
 0] && NeQ[m, -1]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\int \frac{A + B\log\left(e(a+bx)^n(c+dx)^{-n}\right)}{(a+bx)^5}\,dx = \int \left(\frac{A}{(a+bx)^5} + \frac{B\log\left(e(a+bx)^n(c+dx)^{-n}\right)}{(a+bx)^5}\right)dx$$
$$= -\frac{A}{4b(a+bx)^4} + B\int \frac{\log\left(e(a+bx)^n(c+dx)^{-n}\right)}{(a+bx)^5}\,dx$$
$$= -\frac{A}{4b(a+bx)^4} - \frac{B\log\left(e(a+bx)^n(c+dx)^{-n}\right)}{4b(a+bx)^4} + \frac{(B(bc-ad)n)\int \frac{1}{(a+bx)^5 \cdots}}{4b}$$
$$= -\frac{A}{4b(a+bx)^4} - \frac{B\log\left(e(a+bx)^n(c+dx)^{-n}\right)}{4b(a+bx)^4} + \frac{(B(bc-ad)n)\int \Big(\frac{\cdots}{(bc-ad \cdots}}{\cdots}$$
$$= -\frac{A}{4b(a+bx)^4} - \frac{Bn}{16b(a+bx)^4} + \frac{Bdn}{12b(bc-ad)(a+bx)^3} - \frac{Bd^2n \cdots}{8b(bc-ad)^2( \cdots}$$

**Mathematica [A]** time = 0.36, size = 165, normalized size = 0.85

$$-\frac{\frac{12A}{(a+bx)^4} + Bn\left(-\frac{12d^4\log(a+bx)}{(bc-ad)^4} + \frac{12d^4\log(c+dx)}{(bc-ad)^4} + \frac{-\frac{12d^3(a+bx)^3}{(bc-ad)^3} + \frac{6d^2(a+bx)^2}{(bc-ad)^2} + \frac{4d(a+bx)}{ad-bc} + 3}{(a+bx)^4}\right) + \frac{12B\log\left(e(a+bx)^n(c+dx)^{-n}\right)}{(a+bx)^4}}{48b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])/(a + b*x)^5,x]
```

```
[Out] -1/48*((12*A)/(a + b*x)^4 + B*n*((3 + (4*d*(a + b*x))/(-(b*c) + a*d) + (6*d
^2*(a + b*x)^2)/(b*c - a*d)^2 - (12*d^3*(a + b*x)^3)/(b*c - a*d)^3)/(a + b*
x)^4 - (12*d^4*Log[a + b*x])/(b*c - a*d)^4 + (12*d^4*Log[c + d*x])/(b*c - a
*d)^4) + (12*B*Log[(e*(a + b*x)^n)/(c + d*x)^n])/(a + b*x)^4)/b
```

**fricas [B]** time = 1.52, size = 820, normalized size = 4.21

$$-\frac{12\,Ab^4c^4 - 48\,Aab^3c^3d + 72\,Aa^2b^2c^2d^2 - 48\,Aa^3bcd^3 + 12\,Aa^4d^4 - 12\left(Bb^4cd^3 - Bab^3d^4\right)nx^3 + 6\left(Bb^4c^2d^2 - \cdots\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*(b*x+a)^n/((d*x+c)^n)))/(b*x+a)^5,x, algorithm="fricas
")
```

```
[Out] -1/48*(12*A*b^4*c^4 - 48*A*a*b^3*c^3*d + 72*A*a^2*b^2*c^2*d^2 - 48*A*a^3*b*
c*d^3 + 12*A*a^4*d^4 - 12*(B*b^4*c*d^3 - B*a*b^3*d^4)*n*x^3 + 6*(B*b^4*c^2*
d^2 - 8*B*a*b^3*c*d^3 + 7*B*a^2*b^2*d^4)*n*x^2 - 4*(B*b^4*c^3*d - 6*B*a*b^3
*c^2*d^2 + 18*B*a^2*b^2*c*d^3 - 13*B*a^3*b*d^4)*n*x + (3*B*b^4*c^4 - 16*B*a
*b^3*c^3*d + 36*B*a^2*b^2*c^2*d^2 - 48*B*a^3*b*c*d^3 + 25*B*a^4*d^4)*n - 12
*(B*b^4*d^4*n*x^4 + 4*B*a*b^3*d^4*n*x^3 + 6*B*a^2*b^2*d^4*n*x^2 + 4*B*a^3*b
*d^4*n*x - (B*b^4*c^4 - 4*B*a*b^3*c^3*d + 6*B*a^2*b^2*c^2*d^2 - 4*B*a^3*b*c
*d^3)*n)*log(b*x + a) + 12*(B*b^4*d^4*n*x^4 + 4*B*a*b^3*d^4*n*x^3 + 6*B*a^2
*b^2*d^4*n*x^2 + 4*B*a^3*b*d^4*n*x - (B*b^4*c^4 - 4*B*a*b^3*c^3*d + 6*B*a^2
*b^2*c^2*d^2 - 4*B*a^3*b*c*d^3)*n)*log(d*x + c) + 12*(B*b^4*c^4 - 4*B*a*b^3
*c^3*d + 6*B*a^2*b^2*c^2*d^2 - 4*B*a^3*b*c*d^3 + B*a^4*d^4)*log(e))/(a^4*b^
5*c^4 - 4*a^5*b^4*c^3*d + 6*a^6*b^3*c^2*d^2 - 4*a^7*b^2*c*d^3 + a^8*b*d^4 +
 (b^9*c^4 - 4*a*b^8*c^3*d + 6*a^2*b^7*c^2*d^2 - 4*a^3*b^6*c*d^3 + a^4*b^5*d
^4)*x^4 + 4*(a*b^8*c^4 - 4*a^2*b^7*c^3*d + 6*a^3*b^6*c^2*d^2 - 4*a^4*b^5*c*
d^3 + a^5*b^4*d^4)*x^3 + 6*(a^2*b^7*c^4 - 4*a^3*b^6*c^3*d + 6*a^4*b^5*c^2*d
^2 - 4*a^5*b^4*c*d^3 + a^6*b^3*d^4)*x^2 + 4*(a^3*b^6*c^4 - 4*a^4*b^5*c^3*d
+ 6*a^5*b^4*c^2*d^2 - 4*a^6*b^3*c*d^3 + a^7*b^2*d^4)*x)
```

**giac [B]** time = 0.25, size = 710, normalized size = 3.64

$$\frac{Bd^4 n\log(bx+a)}{4\left(b^5c^4 - 4\,ab^4c^3d + 6\,a^2b^3c^2d^2 - 4\,a^3b^2cd^3 + a^4bd^4\right)} - \frac{Bd^4 n\log(dx+c)}{4\left(b^5c^4 - 4\,ab^4c^3d + 6\,a^2b^3c^2d^2 - 4\,a^3b^2cd^3 + a^4bd^4\right)} - \frac{}{4\left(b^5\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*(b*x+a)^n/((d*x+c)^n)))/(b*x+a)^5,x, algorithm="giac")
```

```
[Out] 1/4*B*d^4*n*log(b*x + a)/(b^5*c^4 - 4*a*b^4*c^3*d + 6*a^2*b^3*c^2*d^2 - 4*a
^3*b^2*c*d^3 + a^4*b*d^4) - 1/4*B*d^4*n*log(d*x + c)/(b^5*c^4 - 4*a*b^4*c^3
*d + 6*a^2*b^3*c^2*d^2 - 4*a^3*b^2*c*d^3 + a^4*b*d^4) - 1/4*B*n*log(b*x + a
)/(b^5*x^4 + 4*a*b^4*x^3 + 6*a^2*b^3*x^2 + 4*a^3*b^2*x + a^4*b) + 1/4*B*n*l
og(d*x + c)/(b^5*x^4 + 4*a*b^4*x^3 + 6*a^2*b^3*x^2 + 4*a^3*b^2*x + a^4*b) +
 1/48*(12*B*b^3*d^3*n*x^3 - 6*B*b^3*c*d^2*n*x^2 + 42*B*a*b^2*d^3*n*x^2 + 4*
B*b^3*c^2*d*n*x - 20*B*a*b^2*c*d^2*n*x + 52*B*a^2*b*d^3*n*x - 3*B*b^3*c^3*n
 + 13*B*a*b^2*c^2*d*n - 23*B*a^2*b*c*d^2*n + 25*B*a^3*d^3*n - 12*A*b^3*c^3
- 12*B*b^3*c^3 + 36*A*a*b^2*c^2*d + 36*B*a*b^2*c^2*d - 36*A*a^2*b*c*d^2 - 3
6*B*a^2*b*c*d^2 + 12*A*a^3*d^3 + 12*B*a^3*d^3)/(b^8*c^3*x^4 - 3*a*b^7*c^2*d
*x^4 + 3*a^2*b^6*c*d^2*x^4 - a^3*b^5*d^3*x^4 + 4*a*b^7*c^3*x^3 - 12*a^2*b^6
*c^2*d*x^3 + 12*a^3*b^5*c*d^2*x^3 - 4*a^4*b^4*d^3*x^3 + 6*a^2*b^6*c^3*x^2 -
 18*a^3*b^5*c^2*d*x^2 + 18*a^4*b^4*c*d^2*x^2 - 6*a^5*b^3*d^3*x^2 + 4*a^3*b^
5*c^3*x - 12*a^4*b^4*c^2*d*x + 12*a^5*b^3*c*d^2*x - 4*a^6*b^2*d^3*x + a^4*b
^4*c^3 - 3*a^5*b^3*c^2*d + 3*a^6*b^2*c*d^2 - a^7*b*d^3)
```

**maple [C]** time = 0.67, size = 2583, normalized size = 13.25

Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))/(b*x+a)^5,x)
```

```
[Out] 1/4*B/b/(b*x+a)^4*ln((d*x+c)^n)+1/48*(-12*A*b^4*c^4-3*B*b^4*c^4*n-12*A*a^4*
d^4-24*I*B*Pi*a*b^3*c^3*d*csgn(I*e)*csgn(I*(b*x+a)^n/((d*x+c)^n))*csgn(I*e/
((d*x+c)^n)*(b*x+a)^n)-24*I*B*Pi*a*b^3*c^3*d*csgn(I*(b*x+a)^n)*csgn(I/((d*x
+c)^n))*csgn(I*(b*x+a)^n/((d*x+c)^n))-12*B*a^4*d^4*ln((b*x+a)^n)-12*B*b^4*c
^4*ln((b*x+a)^n)+48*B*a*b^3*c^3*d*ln((b*x+a)^n)-36*I*B*Pi*a^2*b^2*c^2*d^2*c
sgn(I*(b*x+a)^n/((d*x+c)^n))*csgn(I*e/((d*x+c)^n)*(b*x+a)^n)^2-12*B*ln(e)*a
^4*d^4-12*B*ln(e)*b^4*c^4+48*B*a^3*b*c*d^3*n-36*I*B*Pi*a^2*b^2*c^2*d^2*csgn
(I*(b*x+a)^n)*csgn(I*(b*x+a)^n/((d*x+c)^n))^2-6*I*B*Pi*b^4*c^4*csgn(I*e)*cs
gn(I*e/((d*x+c)^n)*(b*x+a)^n)^2-6*I*B*Pi*b^4*c^4*csgn(I*(b*x+a)^n)*csgn(I*(
b*x+a)^n/((d*x+c)^n))^2+48*B*a*b^3*c*d^3*n*x^2+72*B*a^2*b^2*c*d^3*n*x-24*B*
a*b^3*c^2*d^2*n*x+6*I*B*Pi*b^4*c^4*csgn(I*e)*csgn(I*(b*x+a)^n/((d*x+c)^n))*
csgn(I*e/((d*x+c)^n)*(b*x+a)^n)+6*I*B*Pi*b^4*c^4*csgn(I*(b*x+a)^n)*csgn(I/(
(d*x+c)^n))*csgn(I*(b*x+a)^n/((d*x+c)^n))+24*I*B*Pi*a^3*b*c*d^3*csgn(I*(b*x
+a)^n/((d*x+c)^n))*csgn(I*e/((d*x+c)^n)*(b*x+a)^n)^2-6*I*B*Pi*a^4*d^4*csgn(
I*(b*x+a)^n/((d*x+c)^n))*csgn(I*e/((d*x+c)^n)*(b*x+a)^n)^2-36*I*B*Pi*a^2*b^
2*c^2*d^2*csgn(I/((d*x+c)^n))*csgn(I*(b*x+a)^n/((d*x+c)^n))^2-6*I*B*Pi*a^4*
d^4*csgn(I*(b*x+a)^n)*csgn(I*(b*x+a)^n/((d*x+c)^n))^2-6*I*B*Pi*a^4*d^4*csgn
(I/((d*x+c)^n))*csgn(I*(b*x+a)^n/((d*x+c)^n))^2-48*B*ln(d*x+c)*a*b^3*d^4*n*
x^3+48*B*ln(-b*x-a)*a*b^3*d^4*n*x^3-72*B*ln(d*x+c)*a^2*b^2*d^4*n*x^2+72*B*l
n(-b*x-a)*a^2*b^2*d^4*n*x^2-48*B*ln(d*x+c)*a^3*b*d^4*n*x+48*B*ln(-b*x-a)*a^
3*b*d^4*n*x-25*B*a^4*d^4*n+12*B*a^4*n*ln(-b*x-a)*d^4-12*B*ln(d*x+c)*a^4*d^4
*n+6*I*B*Pi*a^4*d^4*csgn(I*(b*x+a)^n)*csgn(I/((d*x+c)^n))*csgn(I*(b*x+a)^n/
((d*x+c)^n))-24*I*B*Pi*a^3*b*c*d^3*csgn(I*(b*x+a)^n/((d*x+c)^n))^3-24*I*B*P
i*a^3*b*c*d^3*csgn(I*e/((d*x+c)^n)*(b*x+a)^n)^3+36*I*B*Pi*a^2*b^2*c^2*d^2*c
sgn(I*(b*x+a)^n/((d*x+c)^n))^3+48*B*ln(e)*a^3*b*c*d^3-72*B*ln(e)*a^2*b^2*c^
2*d^2+48*B*ln(e)*a*b^3*c^3*d+48*A*a^3*b*c*d^3-72*A*a^2*b^2*c^2*d^2+48*A*a*b
^3*c^3*d+6*I*B*Pi*a^4*d^4*csgn(I*e)*csgn(I*(b*x+a)^n/((d*x+c)^n))*csgn(I*e/
((d*x+c)^n)*(b*x+a)^n)+24*I*B*Pi*a*b^3*c^3*d*csgn(I*e)*csgn(I*e/((d*x+c)^n)
*(b*x+a)^n)^2+36*I*B*Pi*a^2*b^2*c^2*d^2*csgn(I*e/((d*x+c)^n)*(b*x+a)^n)^3-2
4*I*B*Pi*a*b^3*c^3*d*csgn(I*(b*x+a)^n/((d*x+c)^n))^3-24*I*B*Pi*a*b^3*c^3*d*
csgn(I*e/((d*x+c)^n)*(b*x+a)^n)^3+24*I*B*Pi*a*b^3*c^3*d*csgn(I*(b*x+a)^n/((
d*x+c)^n))*csgn(I*e/((d*x+c)^n)*(b*x+a)^n)^2+6*I*B*Pi*a^4*d^4*csgn(I*(b*x+a
)^n/((d*x+c)^n))^3+6*I*B*Pi*a^4*d^4*csgn(I*e/((d*x+c)^n)*(b*x+a)^n)^3+6*I*B
*Pi*b^4*c^4*csgn(I*(b*x+a)^n/((d*x+c)^n))^3+6*I*B*Pi*b^4*c^4*csgn(I*e/((d*x
+c)^n)*(b*x+a)^n)^3+48*B*a^3*b*c*d^3*ln((b*x+a)^n)-72*B*a^2*b^2*c^2*d^2*ln(
(b*x+a)^n)-6*I*B*Pi*b^4*c^4*csgn(I/((d*x+c)^n))*csgn(I*(b*x+a)^n/((d*x+c)^n
))^2-6*I*B*Pi*b^4*c^4*csgn(I*(b*x+a)^n/((d*x+c)^n))*csgn(I*e/((d*x+c)^n)*(b
```

```
*x+a)^n)^2-6*I*B*Pi*a^4*d^4*csgn(I*e)*csgn(I*e/((d*x+c)^n)*(b*x+a)^n)^2+24*
I*B*Pi*a^3*b*c*d^3*csgn(I*e)*csgn(I*e/((d*x+c)^n)*(b*x+a)^n)^2-24*I*B*Pi*a^
3*b*c*d^3*csgn(I*e)*csgn(I*(b*x+a)^n/((d*x+c)^n))*csgn(I*e/((d*x+c)^n)*(b*x
+a)^n)-24*I*B*Pi*a^3*b*c*d^3*csgn(I*(b*x+a)^n)*csgn(I/((d*x+c)^n))*csgn(I*(
b*x+a)^n/((d*x+c)^n))+24*I*B*Pi*a^3*b*c*d^3*csgn(I/((d*x+c)^n))*csgn(I*(b*x
+a)^n/((d*x+c)^n))^2-36*B*a^2*b^2*c^2*d^2*n+16*B*a*b^3*c^3*d*n-12*B*a*b^3*d
^4*n*x^3+12*B*b^4*c*d^3*n*x^3-42*B*a^2*b^2*d^4*n*x^2+36*I*B*Pi*a^2*b^2*c^2*
d^2*csgn(I*e)*csgn(I*(b*x+a)^n/((d*x+c)^n))*csgn(I*e/((d*x+c)^n)*(b*x+a)^n)
+24*I*B*Pi*a^3*b*c*d^3*csgn(I*(b*x+a)^n)*csgn(I*(b*x+a)^n/((d*x+c)^n))^2-12
*B*ln(d*x+c)*b^4*d^4*n*x^4+12*B*ln(-b*x-a)*b^4*d^4*n*x^4+36*I*B*Pi*a^2*b^2*
c^2*d^2*csgn(I*(b*x+a)^n)*csgn(I/((d*x+c)^n))*csgn(I*(b*x+a)^n/((d*x+c)^n))
+24*I*B*Pi*a*b^3*c^3*d*csgn(I/((d*x+c)^n))*csgn(I*(b*x+a)^n/((d*x+c)^n))^2+
24*I*B*Pi*a*b^3*c^3*d*csgn(I*(b*x+a)^n)*csgn(I*(b*x+a)^n/((d*x+c)^n))^2-6*B
*b^4*c^2*d^2*n*x^2-52*B*a^3*b*d^4*n*x+4*B*b^4*c^3*d*n*x-36*I*B*Pi*a^2*b^2*c
^2*d^2*csgn(I*e)*csgn(I*e/((d*x+c)^n)*(b*x+a)^n)^2)/(b*x+a)^4/(-a^3*d^3+3*a
^2*b*c*d^2-3*a*b^2*c^2*d+b^3*c^3)/(-a*d+b*c)/b
```

<u>**maxima**</u> **[B]** time = 1.38, size = 618, normalized size = 3.17

$$\left(\frac{12\,d^4en\log(bx+a)}{b^5c^4-4\,ab^4c^3d+6\,a^2b^3c^2d^2-4\,a^3b^2cd^3+a^4bd^4}-\frac{12\,d^4en\log(dx+c)}{b^5c^4-4\,ab^4c^3d+6\,a^2b^3c^2d^2-4\,a^3b^2cd^3+a^4bd^4}+\frac{12\,b^3d^3enx}{a^4b^4c^3-3\,a^5b^3c^2d+3\,a^6b^2cd^2-a^7bd^3+\left(b^8c^3-3\,ab^7c^2\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*(b*x+a)^n/((d*x+c)^n)))/(b*x+a)^5,x, algorithm="maxima
")
```

```
[Out] 1/48*(12*d^4*e*n*log(b*x + a)/(b^5*c^4 - 4*a*b^4*c^3*d + 6*a^2*b^3*c^2*d^2
- 4*a^3*b^2*c*d^3 + a^4*b*d^4) - 12*d^4*e*n*log(d*x + c)/(b^5*c^4 - 4*a*b^4
*c^3*d + 6*a^2*b^3*c^2*d^2 - 4*a^3*b^2*c*d^3 + a^4*b*d^4) + (12*b^3*d^3*e*n
*x^3 - 3*b^3*c^3*e*n + 13*a*b^2*c^2*d*e*n - 23*a^2*b*c*d^2*e*n + 25*a^3*d^3
*e*n - 6*(b^3*c*d^2*e*n - 7*a*b^2*d^3*e*n)*x^2 + 4*(b^3*c^2*d*e*n - 5*a*b^2
*c*d^2*e*n + 13*a^2*b*d^3*e*n)*x)/(a^4*b^4*c^3 - 3*a^5*b^3*c^2*d + 3*a^6*b^
2*c*d^2 - a^7*b*d^3 + (b^8*c^3 - 3*a*b^7*c^2*d + 3*a^2*b^6*c*d^2 - a^3*b^5*
d^3)*x^4 + 4*(a*b^7*c^3 - 3*a^2*b^6*c^2*d + 3*a^3*b^5*c*d^2 - a^4*b^4*d^3)*
x^3 + 6*(a^2*b^6*c^3 - 3*a^3*b^5*c^2*d + 3*a^4*b^4*c*d^2 - a^5*b^3*d^3)*x^2
 + 4*(a^3*b^5*c^3 - 3*a^4*b^4*c^2*d + 3*a^5*b^3*c*d^2 - a^6*b^2*d^3)*x))*B/
e - 1/4*B*log((b*x + a)^n*e/(d*x + c)^n)/(b^5*x^4 + 4*a*b^4*x^3 + 6*a^2*b^3
*x^2 + 4*a^3*b^2*x + a^4*b) - 1/4*A/(b^5*x^4 + 4*a*b^4*x^3 + 6*a^2*b^3*x^2
+ 4*a^3*b^2*x + a^4*b)
```

**mupad [B]** time = 5.33, size = 555, normalized size = 2.85

$$-\frac{\frac{12\,A\,a^3\,d^3-12\,A\,b^3\,c^3+25\,B\,a^3\,d^3\,n-3\,B\,b^3\,c^3\,n+36\,A\,a\,b^2\,c^2\,d-36\,A\,a^2\,b\,c\,d^2+13\,B\,a\,b^2\,c^2\,d\,n-23\,B\,a^2\,b\,c\,d^2\,n}{12\,\left(a^3\,d^3-3\,a^2\,b\,c\,d^2+3\,a\,b^2\,c^2\,d-b^3\,c^3\right)}+\frac{d\,x\,\left(13\,B\,n\,a^2\,b\,d^2-5\,B\,n\,a\,b^2\,c\,d+B\,n}{3\,\left(a^3\,d^3-3\,a^2\,b\,c\,d^2+3\,a\,b^2\,c^2\,d-b\right.}}{4\,a^4\,b+16\,a^3\,b^2\,x+24\,a^2\,b^3\,x^2+16\,a\,b^4\,x^3+4\,b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*log((e*(a + b*x)^n)/(c + d*x)^n))/(a + b*x)^5,x)
```

```
[Out] - ((12*A*a^3*d^3 - 12*A*b^3*c^3 + 25*B*a^3*d^3*n - 3*B*b^3*c^3*n + 36*A*a*b
^2*c^2*d - 36*A*a^2*b*c*d^2 + 13*B*a*b^2*c^2*d*n - 23*B*a^2*b*c*d^2*n)/(12*
(a^3*d^3 - b^3*c^3 + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2)) + (d*x*(B*b^3*c^2*n +
13*B*a^2*b*d^2*n - 5*B*a*b^2*c*d*n))/(3*(a^3*d^3 - b^3*c^3 + 3*a*b^2*c^2*d
- 3*a^2*b*c*d^2)) - (d^2*x^2*(B*b^3*c*n - 7*B*a*b^2*d*n))/(2*(a^3*d^3 - b^3
*c^3 + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2)) + (B*b^3*d^3*n*x^3)/(a^3*d^3 - b^3*c
^3 + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2))/(4*a^4*b + 4*b^5*x^4 + 16*a^3*b^2*x +
16*a*b^4*x^3 + 24*a^2*b^3*x^2) - (B*log((e*(a + b*x)^n)/(c + d*x)^n))/(4*b*
(a^4 + b^4*x^4 + 4*a*b^3*x^3 + 6*a^2*b^2*x^2 + 4*a^3*b*x)) - (B*d^4*n*atanh
((4*b^5*c^4 - 4*a^4*b*d^4 + 8*a^3*b^2*c*d^3 - 8*a*b^4*c^3*d)/(4*b*(a*d - b*
c)^4) - (2*b*d*x*(a^3*d^3 - b^3*c^3 + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2))/(a*d
- b*c)^4))/(2*b*(a*d - b*c)^4)
```

**sympy [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*ln(e*(b*x+a)**n/((d*x+c)**n)))/(b*x+a)**5,x)
```

```
[Out] Timed out
```

# 3.156 $\int (a+bx)^3 \left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^2 dx$

**Optimal**. Leaf size=322

$$-\frac{Bn(bc-ad)^4\log\left(\frac{bc-ad}{b(c+dx)}\right)\left(6B\log\left(e(a+bx)^n(c+dx)^{-n}\right)+6A+11Bn\right)}{12bd^4}-\frac{Bn(a+bx)(bc-ad)^3\left(6B\log\left(e(a+bx\right.\right.}{12bd^3}$$

```
[Out] -1/6*B*(-a*d+b*c)*n*(b*x+a)^3*(A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))/b/d+1/4*(b*
x+a)^4*(A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))^2/b+1/12*B*(-a*d+b*c)^2*n*(b*x+a)^
2*(3*A+B*n+3*B*ln(e*(b*x+a)^n/((d*x+c)^n)))/b/d^2-1/12*B*(-a*d+b*c)^3*n*(b*
x+a)*(6*A+5*B*n+6*B*ln(e*(b*x+a)^n/((d*x+c)^n)))/b/d^3-1/12*B*(-a*d+b*c)^4*
n*ln((-a*d+b*c)/b/(d*x+c))*(6*A+11*B*n+6*B*ln(e*(b*x+a)^n/((d*x+c)^n)))/b/d
^4-1/2*B^2*(-a*d+b*c)^4*n^2*polylog(2,d*(b*x+a)/b/(d*x+c))/b/d^4
```

**Rubi [A]** time = 0.77, antiderivative size = 542, normalized size of antiderivative = 1.68, number of steps used = 21, number of rules used = 11, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {6742, 2492, 43, 2514, 2486, 31, 2488, 2411, 2343, 2333, 2315}

$$-\frac{B^2n^2(bc-ad)^4\text{PolyLog}\left(2,\frac{d(a+bx)}{b(c+dx)}\right)}{2bd^4}+\frac{A^2(a+bx)^4}{4b}-\frac{ABnx(bc-ad)^3}{2d^3}+\frac{ABn(a+bx)^2(bc-ad)^2}{4bd^2}+\frac{ABn(bc-ad)^4 1}{2bd^4}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^3*(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])^2,x]
```

```
[Out] -(A*B*(b*c - a*d)^3*n*x)/(2*d^3) - (5*B^2*(b*c - a*d)^3*n^2*x)/(12*d^3) + (
A*B*(b*c - a*d)^2*n*(a + b*x)^2)/(4*b*d^2) + (B^2*(b*c - a*d)^2*n^2*(a + b*
x)^2)/(12*b*d^2) - (A*B*(b*c - a*d)*n*(a + b*x)^3)/(6*b*d) + (A^2*(a + b*x)
^4)/(4*b) + (A*B*(b*c - a*d)^4*n*Log[c + d*x])/(2*b*d^4) + (11*B^2*(b*c - a
*d)^4*n^2*Log[c + d*x])/(12*b*d^4) - (B^2*(b*c - a*d)^3*n*(a + b*x)*Log[(e*
(a + b*x)^n)/(c + d*x)^n])/(2*b*d^3) + (B^2*(b*c - a*d)^2*n*(a + b*x)^2*Log
[(e*(a + b*x)^n)/(c + d*x)^n])/(4*b*d^2) - (B^2*(b*c - a*d)*n*(a + b*x)^3*L
og[(e*(a + b*x)^n)/(c + d*x)^n])/(6*b*d) + (A*B*(a + b*x)^4*Log[(e*(a + b*x
)^n)/(c + d*x)^n])/(2*b) - (B^2*(b*c - a*d)^4*n*Log[(b*c - a*d)/(b*(c + d*x
))]*Log[(e*(a + b*x)^n)/(c + d*x)^n])/(2*b*d^4) + (B^2*(a + b*x)^4*Log[(e*(
a + b*x)^n)/(c + d*x)^n]^2)/(4*b) - (B^2*(b*c - a*d)^4*n^2*PolyLog[2, (d*(a
 + b*x))/(b*(c + d*x))])/(2*b*d^4)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2333

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)/(x_))^(q_.)*(
x_)^(m_.), x_Symbol] :> Int[(e + d*x)^q*(a + b*Log[c*x^n])^p, x] /; FreeQ[{
a, b, c, d, e, m, n, p}, x] && EqQ[m, q] && IntegerQ[q]
```

Rule 2343

```
Int[((a_.) + Log[(c_.)*(x_)^(n_)]*(b_.))/((x_)*((d_) + (e_.)*(x_)^(r_.))),
x_Symbol] :> Dist[1/n, Subst[Int[(a + b*Log[c*x])/(x*(d + e*x^(r/n))), x],
x, x^n], x] /; FreeQ[{a, b, c, d, e, n, r}, x] && IntegerQ[r/n]
```

Rule 2411

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + (g_
.)*(x_))^(q_.)*((h_.) + (i_.)*(x_))^(r_.), x_Symbol] :> Dist[1/e, Subst[Int
[((g*x)/e)^q*((e*h - d*i)/e + (i*x)/e)^r*(a + b*Log[c*x^n])^p, x], x, d + e
*x], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, n, p, q, r}, x] && EqQ[e*f - d
*g, 0] && (IGtQ[p, 0] || IGtQ[r, 0]) && IntegerQ[2*r]
```

Rule 2486

```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))
^(r_.)]^(s_.), x_Symbol] :> Simp[((a + b*x)*Log[e*(f*(a + b*x)^p*(c + d*x)^
q)^r]^s)/b, x] + Dist[(q*r*s*(b*c - a*d))/b, Int[Log[e*(f*(a + b*x)^p*(c +
d*x)^q)^r]^(s - 1)/(c + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, p, q, r, s
}, x] && NeQ[b*c - a*d, 0] && EqQ[p + q, 0] && IGtQ[s, 0]
```

Rule 2488

```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))
^(r_.)]^(s_.)/((g_.) + (h_.)*(x_)), x_Symbol] :> -Simp[(Log[-((b*c - a*d)/(
d*(a + b*x)))]*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^s)/h, x] + Dist[(p*r*s*
```

```
(b*c - a*d))/h, Int[(Log[-((b*c - a*d)/(d*(a + b*x)))]*Log[e*(f*(a + b*x)^p
*(c + d*x)^q)^r]^(s - 1))/((a + b*x)*(c + d*x)), x], x] /; FreeQ[{a, b, c,
d, e, f, g, h, p, q, r, s}, x] && NeQ[b*c - a*d, 0] && EqQ[p + q, 0] && EqQ
[b*g - a*h, 0] && IGtQ[s, 0]
```

Rule 2492

```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))
^(r_.)]^(s_.)*((g_.) + (h_.)*(x_))^(m_.), x_Symbol] :> Simp[((g + h*x)^(m +
 1)*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^s)/(h*(m + 1)), x] - Dist[(p*r*s*(
b*c - a*d))/(h*(m + 1)), Int[((g + h*x)^(m + 1)*Log[e*(f*(a + b*x)^p*(c + d
*x)^q)^r]^(s - 1))/((a + b*x)*(c + d*x)), x], x] /; FreeQ[{a, b, c, d, e, f
, g, h, m, p, q, r, s}, x] && NeQ[b*c - a*d, 0] && EqQ[p + q, 0] && IGtQ[s,
 0] && NeQ[m, -1]
```

Rule 2514

```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))
^(r_.)]^(s_.)*(RFx_), x_Symbol] :> With[{u = ExpandIntegrand[Log[e*(f*(a +
b*x)^p*(c + d*x)^q)^r]^s, RFx, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c
, d, e, f, p, q, r, s}, x] && RationalFunctionQ[RFx, x] && IGtQ[s, 0]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\int (a+bx)^3 \left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^2 dx = \int \left(A^2(a+bx)^3+2AB(a+bx)^3\log\left(e(a+bx)^n(c+dx)^{-n}\right.\right.$$

$$= \frac{A^2(a+bx)^4}{4b}+(2AB)\int (a+bx)^3\log\left(e(a+bx)^n(c+dx)^{-}\right.$$

$$= \frac{A^2(a+bx)^4}{4b}+\frac{AB(a+bx)^4\log\left(e(a+bx)^n(c+dx)^{-n}\right)}{2b}+$$

$$= \frac{A^2(a+bx)^4}{4b}+\frac{AB(a+bx)^4\log\left(e(a+bx)^n(c+dx)^{-n}\right)}{2b}+$$

$$= -\frac{AB(bc-ad)^3nx}{2d^3}+\frac{AB(bc-ad)^2n(a+bx)^2}{4bd^2}-\frac{AB(bc-a}{}$$

$$= -\frac{AB(bc-ad)^3nx}{2d^3}+\frac{AB(bc-ad)^2n(a+bx)^2}{4bd^2}-\frac{AB(bc-a}{}$$

$$= -\frac{AB(bc-ad)^3nx}{2d^3}+\frac{AB(bc-ad)^2n(a+bx)^2}{4bd^2}-\frac{AB(bc-a}{}$$

$$= -\frac{AB(bc-ad)^3nx}{2d^3}-\frac{5B^2(bc-ad)^3n^2x}{12d^3}+\frac{AB(bc-ad)^2n(}{4bd^2}$$

$$= -\frac{AB(bc-ad)^3nx}{2d^3}-\frac{5B^2(bc-ad)^3n^2x}{12d^3}+\frac{AB(bc-ad)^2n(}{4bd^2}$$

$$= -\frac{AB(bc-ad)^3nx}{2d^3}-\frac{5B^2(bc-ad)^3n^2x}{12d^3}+\frac{AB(bc-ad)^2n(}{4bd^2}$$

**Mathematica [B]** time = 1.80, size = 1709, normalized size = 5.31

$$\frac{3A^2d^4x^4b^4-2ABcd^3nx^3b^4+B^2c^2d^2n^2x^2b^4+3ABc^2d^2nx^2b^4+3B^2c^4n^2\log^2(c+dx)b^4+3B^2d^4x^4\log^2\left(e(a+b\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^3*(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])^2,x]

[Out] (-24*a^4*A*B*d^4*n + 6*a*b^3*B^2*c^3*d*n^2 - 24*a^2*b^2*B^2*c^2*d^2*n^2 + 3
6*a^3*b*B^2*c*d^3*n^2 - 24*a^4*B^2*d^4*n^2 + 12*a^3*A^2*b*d^4*x - 6*A*b^4*B
```

```
*c^3*d*n*x + 24*a*A*b^3*B*c^2*d^2*n*x - 36*a^2*A*b^2*B*c*d^3*n*x + 18*a^3*A
*b*B*d^4*n*x - 5*b^4*B^2*c^3*d*n^2*x + 17*a*b^3*B^2*c^2*d^2*n^2*x - 19*a^2*
b^2*B^2*c*d^3*n^2*x + 7*a^3*b*B^2*d^4*n^2*x + 18*a^2*A^2*b^2*d^4*x^2 + 3*A*
b^4*B*c^2*d^2*n*x^2 - 12*a*A*b^3*B*c*d^3*n*x^2 + 9*a^2*A*b^2*B*d^4*n*x^2 +
b^4*B^2*c^2*d^2*n^2*x^2 - 2*a*b^3*B^2*c*d^3*n^2*x^2 + a^2*b^2*B^2*d^4*n^2*x
^2 + 12*a*A^2*b^3*d^4*x^3 - 2*A*b^4*B*c*d^3*n*x^3 + 2*a*A*b^3*B*d^4*n*x^3 +
 3*A^2*b^4*d^4*x^4 - 3*a^4*B^2*d^4*n^2*Log[a + b*x]^2 + 6*A*b^4*B*c^4*n*Log
[c + d*x] - 24*a*A*b^3*B*c^3*d*n*Log[c + d*x] + 36*a^2*A*b^2*B*c^2*d^2*n*Lo
g[c + d*x] - 24*a^3*A*b*B*c*d^3*n*Log[c + d*x] + 11*b^4*B^2*c^4*n^2*Log[c +
 d*x] - 38*a*b^3*B^2*c^3*d*n^2*Log[c + d*x] + 45*a^2*b^2*B^2*c^2*d^2*n^2*Lo
g[c + d*x] - 18*a^3*b*B^2*c*d^3*n^2*Log[c + d*x] - 24*a^4*B^2*d^4*n^2*Log[c
 + d*x] + 3*b^4*B^2*c^4*n^2*Log[c + d*x]^2 - 12*a*b^3*B^2*c^3*d*n^2*Log[c +
 d*x]^2 + 18*a^2*b^2*B^2*c^2*d^2*n^2*Log[c + d*x]^2 - 12*a^3*b*B^2*c*d^3*n^
2*Log[c + d*x]^2 - 24*a^4*B^2*d^4*n*Log[(e*(a + b*x)^n)/(c + d*x)^n] + 24*a
^3*A*b*B*d^4*x*Log[(e*(a + b*x)^n)/(c + d*x)^n] - 6*b^4*B^2*c^3*d*n*x*Log[(
e*(a + b*x)^n)/(c + d*x)^n] + 24*a*b^3*B^2*c^2*d^2*n*x*Log[(e*(a + b*x)^n)/
(c + d*x)^n] - 36*a^2*b^2*B^2*c*d^3*n*x*Log[(e*(a + b*x)^n)/(c + d*x)^n] +
18*a^3*b*B^2*d^4*n*x*Log[(e*(a + b*x)^n)/(c + d*x)^n] + 36*a^2*A*b^2*B*d^4*
x^2*Log[(e*(a + b*x)^n)/(c + d*x)^n] + 3*b^4*B^2*c^2*d^2*n*x^2*Log[(e*(a +
b*x)^n)/(c + d*x)^n] - 12*a*b^3*B^2*c*d^3*n*x^2*Log[(e*(a + b*x)^n)/(c + d*
x)^n] + 9*a^2*b^2*B^2*d^4*n*x^2*Log[(e*(a + b*x)^n)/(c + d*x)^n] + 24*a*A*b
^3*B*d^4*x^3*Log[(e*(a + b*x)^n)/(c + d*x)^n] - 2*b^4*B^2*c*d^3*n*x^3*Log[(
e*(a + b*x)^n)/(c + d*x)^n] + 2*a*b^3*B^2*d^4*n*x^3*Log[(e*(a + b*x)^n)/(c
+ d*x)^n] + 6*A*b^4*B*d^4*x^4*Log[(e*(a + b*x)^n)/(c + d*x)^n] + 6*b^4*B^2*
c^4*n*Log[c + d*x]*Log[(e*(a + b*x)^n)/(c + d*x)^n] - 24*a*b^3*B^2*c^3*d*n*
Log[c + d*x]*Log[(e*(a + b*x)^n)/(c + d*x)^n] + 36*a^2*b^2*B^2*c^2*d^2*n*Lo
g[c + d*x]*Log[(e*(a + b*x)^n)/(c + d*x)^n] - 24*a^3*b*B^2*c*d^3*n*Log[c +
d*x]*Log[(e*(a + b*x)^n)/(c + d*x)^n] + 12*a^3*b*B^2*d^4*x*Log[(e*(a + b*x)
^n)/(c + d*x)^n]^2 + 18*a^2*b^2*B^2*d^4*x^2*Log[(e*(a + b*x)^n)/(c + d*x)^n
]^2 + 12*a*b^3*B^2*d^4*x^3*Log[(e*(a + b*x)^n)/(c + d*x)^n]^2 + 3*b^4*B^2*d
^4*x^4*Log[(e*(a + b*x)^n)/(c + d*x)^n]^2 + B*n*Log[a + b*x]*(-6*b*B*c*(b^3
*c^3 - 4*a*b^2*c^2*d + 6*a^2*b*c*d^2 - 4*a^3*d^3)*n*Log[c + d*x] + 6*B*(b*c
 - a*d)^4*n*Log[(b*(c + d*x))/(b*c - a*d)] + a*d*(-6*b^3*B*c^3*n + 21*a*b^2
*B*c^2*d*n - 26*a^2*b*B*c*d^2*n + a^3*d^3*(6*A + 35*B*n) + 6*a^3*B*d^3*Log[
(e*(a + b*x)^n)/(c + d*x)^n])) + 6*B^2*(b*c - a*d)^4*n^2*PolyLog[2, (d*(a +
 b*x))/(-(b*c) + a*d)])/(12*b*d^4)
```

**fricas [F]** time = 1.04, size = 0, normalized size = 0.00

$$\text{integral}\left(A^2b^3x^3 + 3\,A^2ab^2x^2 + 3\,A^2a^2bx + A^2a^3 + \left(B^2b^3x^3 + 3\,B^2ab^2x^2 + 3\,B^2a^2bx + B^2a^3\right)\log\left(\frac{(bx+a)^n e}{(dx+c)^n}\right)^2 \cdot\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^3*(A+B*log(e*(b*x+a)^n/((d*x+c)^n)))^2,x, algorithm="fric
```

```
as")
```

```
[Out] integral(A^2*b^3*x^3 + 3*A^2*a*b^2*x^2 + 3*A^2*a^2*b*x + A^2*a^3 + (B^2*b^3
*x^3 + 3*B^2*a*b^2*x^2 + 3*B^2*a^2*b*x + B^2*a^3)*log((b*x + a)^n*e/(d*x +
c)^n)^2 + 2*(A*B*b^3*x^3 + 3*A*B*a*b^2*x^2 + 3*A*B*a^2*b*x + A*B*a^3)*log((
b*x + a)^n*e/(d*x + c)^n), x)
```

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int (bx+a)^3 \left(B\log\left(\frac{(bx+a)^n e}{(dx+c)^n}\right)+A\right)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^3*(A+B*log(e*(b*x+a)^n/((d*x+c)^n)))^2,x, algorithm="giac
")
```

```
[Out] integrate((b*x + a)^3*(B*log((b*x + a)^n*e/(d*x + c)^n) + A)^2, x)
```

**maple [C]** time = 3.55, size = 26948, normalized size = 83.69

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^3*(A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))^2,x)
```

```
[Out] result too large to display
```

**maxima [B]** time = 7.07, size = 1871, normalized size = 5.81

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^3*(A+B*log(e*(b*x+a)^n/((d*x+c)^n)))^2,x, algorithm="maxi
ma")
```

```
[Out] 1/2*A*B*b^3*x^4*log((b*x + a)^n*e/(d*x + c)^n) + 1/4*A^2*b^3*x^4 + 2*A*B*a*
b^2*x^3*log((b*x + a)^n*e/(d*x + c)^n) + A^2*a*b^2*x^3 + 3*A*B*a^2*b*x^2*lo
g((b*x + a)^n*e/(d*x + c)^n) + 3/2*A^2*a^2*b*x^2 + 2*A*B*a^3*x*log((b*x + a
)^n*e/(d*x + c)^n) + A^2*a^3*x + 2*(a*e*n*log(b*x + a)/b - c*e*n*log(d*x +
c)/d)*A*B*a^3/e - 3*(a^2*e*n*log(b*x + a)/b^2 - c^2*e*n*log(d*x + c)/d^2 +
(b*c*e*n - a*d*e*n)*x/(b*d))*A*B*a^2*b/e + (2*a^3*e*n*log(b*x + a)/b^3 - 2*
c^3*e*n*log(d*x + c)/d^3 - ((b^2*c*d*e*n - a*b*d^2*e*n)*x^2 - 2*(b^2*c^2*e*
n - a^2*d^2*e*n)*x)/(b^2*d^2))*A*B*a*b^2/e - 1/12*(6*a^4*e*n*log(b*x + a)/b
^4 - 6*c^4*e*n*log(d*x + c)/d^4 + (2*(b^3*c*d^2*e*n - a*b^2*d^3*e*n)*x^3 -
```

```
3*(b^3*c^2*d*e*n - a^2*b*d^3*e*n)*x^2 + 6*(b^3*c^3*e*n - a^3*d^3*e*n)*x)/(b
^3*d^3))*A*B*b^3/e + 1/12*((11*n^2 + 6*n*log(e))*b^3*c^4 - 2*(19*n^2 + 12*n
*log(e))*a*b^2*c^3*d + 9*(5*n^2 + 4*n*log(e))*a^2*b*c^2*d^2 - 6*(3*n^2 + 4*
n*log(e))*a^3*c*d^3)*B^2*log(d*x + c)/d^4 + 1/2*(b^4*c^4*n^2 - 4*a*b^3*c^3*
d*n^2 + 6*a^2*b^2*c^2*d^2*n^2 - 4*a^3*b*c*d^3*n^2 + a^4*d^4*n^2)*(log(b*x +
 a)*log((b*d*x + a*d)/(b*c - a*d) + 1) + dilog(-(b*d*x + a*d)/(b*c - a*d)))
*B^2/(b*d^4) + 1/12*(3*B^2*b^4*d^4*x^4*log(e)^2 - 3*B^2*a^4*d^4*n^2*log(b*x
 + a)^2 - 2*(b^4*c*d^3*n*log(e) - (n*log(e) + 6*log(e)^2)*a*b^3*d^4)*B^2*x^
3 + ((n^2 + 3*n*log(e))*b^4*c^2*d^2 - 2*(n^2 + 6*n*log(e))*a*b^3*c*d^3 + (n
^2 + 9*n*log(e) + 18*log(e)^2)*a^2*b^2*d^4)*B^2*x^2 - 6*(b^4*c^4*n^2 - 4*a*
b^3*c^3*d*n^2 + 6*a^2*b^2*c^2*d^2*n^2 - 4*a^3*b*c*d^3*n^2)*B^2*log(b*x + a)
*log(d*x + c) + 3*(b^4*c^4*n^2 - 4*a*b^3*c^3*d*n^2 + 6*a^2*b^2*c^2*d^2*n^2
- 4*a^3*b*c*d^3*n^2)*B^2*log(d*x + c)^2 - ((5*n^2 + 6*n*log(e))*b^4*c^3*d -
 (17*n^2 + 24*n*log(e))*a*b^3*c^2*d^2 + (19*n^2 + 36*n*log(e))*a^2*b^2*c*d^
3 - (7*n^2 + 18*n*log(e) + 12*log(e)^2)*a^3*b*d^4)*B^2*x - (6*a*b^3*c^3*d*n
^2 - 21*a^2*b^2*c^2*d^2*n^2 + 26*a^3*b*c*d^3*n^2 - (11*n^2 + 6*n*log(e))*a^
4*d^4)*B^2*log(b*x + a) + 3*(B^2*b^4*d^4*x^4 + 4*B^2*a*b^3*d^4*x^3 + 6*B^2*
a^2*b^2*d^4*x^2 + 4*B^2*a^3*b*d^4*x)*log((b*x + a)^n)^2 + 3*(B^2*b^4*d^4*x^
4 + 4*B^2*a*b^3*d^4*x^3 + 6*B^2*a^2*b^2*d^4*x^2 + 4*B^2*a^3*b*d^4*x)*log((d
*x + c)^n)^2 + (6*B^2*b^4*d^4*x^4*log(e) + 6*B^2*a^4*d^4*n*log(b*x + a) + 2
*(a*b^3*d^4*(n + 12*log(e)) - b^4*c*d^3*n)*B^2*x^3 + 3*(3*a^2*b^2*d^4*(n +
4*log(e)) + b^4*c^2*d^2*n - 4*a*b^3*c*d^3*n)*B^2*x^2 + 6*(a^3*b*d^4*(3*n +
4*log(e)) - b^4*c^3*d*n + 4*a*b^3*c^2*d^2*n - 6*a^2*b^2*c*d^3*n)*B^2*x + 6*
(b^4*c^4*n - 4*a*b^3*c^3*d*n + 6*a^2*b^2*c^2*d^2*n - 4*a^3*b*c*d^3*n)*B^2*l
og(d*x + c))*log((b*x + a)^n) - (6*B^2*b^4*d^4*x^4*log(e) + 6*B^2*a^4*d^4*n
*log(b*x + a) + 2*(a*b^3*d^4*(n + 12*log(e)) - b^4*c*d^3*n)*B^2*x^3 + 3*(3*
a^2*b^2*d^4*(n + 4*log(e)) + b^4*c^2*d^2*n - 4*a*b^3*c*d^3*n)*B^2*x^2 + 6*(
a^3*b*d^4*(3*n + 4*log(e)) - b^4*c^3*d*n + 4*a*b^3*c^2*d^2*n - 6*a^2*b^2*c*
d^3*n)*B^2*x + 6*(b^4*c^4*n - 4*a*b^3*c^3*d*n + 6*a^2*b^2*c^2*d^2*n - 4*a^3
*b*c*d^3*n)*B^2*log(d*x + c) + 6*(B^2*b^4*d^4*x^4 + 4*B^2*a*b^3*d^4*x^3 + 6
*B^2*a^2*b^2*d^4*x^2 + 4*B^2*a^3*b*d^4*x)*log((b*x + a)^n))*log((d*x + c)^n
))/(b*d^4)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.00

$$\int \left(A+B\,\ln\left(\frac{e\,{\left(a+b\,x\right)}^n}{{\left(c+d\,x\right)}^n}\right)\right)^2\,{\left(a+b\,x\right)}^3 \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*log((e*(a + b*x)^n)/(c + d*x)^n))^2*(a + b*x)^3,x)
```

```
[Out] int((A + B*log((e*(a + b*x)^n)/(c + d*x)^n))^2*(a + b*x)^3, x)
```

**sympy [F(-2)]** time = 0.00, size = 0, normalized size = 0.00

Exception raised: HeuristicGCDFailed

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**3*(A+B*ln(e*(b*x+a)**n/((d*x+c)**n)))**2,x)
```

```
[Out] Exception raised: HeuristicGCDFailed
```

# 3.157 $\int (a+bx)^2 \left(A + B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^2\,dx$

**Optimal**. Leaf size=263

$$\frac{Bn(bc-ad)^3\log\left(\frac{bc-ad}{b(c+dx)}\right)\left(2B\log\left(e(a+bx)^n(c+dx)^{-n}\right)+2A+3Bn\right)}{3bd^3}+\frac{Bn(a+bx)(bc-ad)^2\left(2B\log\left(e(a+bx)^n\right.\right.}{3bd^2}$$

```
[Out] -1/3*B*(-a*d+b*c)*n*(b*x+a)^2*(A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))/b/d+1/3*(b*
x+a)^3*(A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))^2/b+1/3*B*(-a*d+b*c)^2*n*(b*x+a)*(
2*A+B*n+2*B*ln(e*(b*x+a)^n/((d*x+c)^n)))/b/d^2+1/3*B*(-a*d+b*c)^3*n*ln((-a*
d+b*c)/b/(d*x+c))*(2*A+3*B*n+2*B*ln(e*(b*x+a)^n/((d*x+c)^n)))/b/d^3+2/3*B^2
*(-a*d+b*c)^3*n^2*polylog(2,d*(b*x+a)/b/(d*x+c))/b/d^3
```

**Rubi [A]** time = 0.63, antiderivative size = 427, normalized size of antiderivative = 1.62, number of steps used = 18, number of rules used = 11, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {6742, 2492, 43, 2514, 2486, 31, 2488, 2411, 2343, 2333, 2315}

$$\frac{2B^2n^2(bc-ad)^3\text{PolyLog}\left(2,\frac{d(a+bx)}{b(c+dx)}\right)}{3bd^3}+\frac{A^2(a+bx)^3}{3b}+\frac{2ABnx(bc-ad)^2}{3d^2}-\frac{2ABn(bc-ad)^3\log(c+dx)}{3bd^3}+\frac{2AB(a+b}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^2*(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])^2,x]
```

```
[Out] (2*A*B*(b*c - a*d)^2*n*x)/(3*d^2) + (B^2*(b*c - a*d)^2*n^2*x)/(3*d^2) - (A*
B*(b*c - a*d)*n*(a + b*x)^2)/(3*b*d) + (A^2*(a + b*x)^3)/(3*b) - (2*A*B*(b*
c - a*d)^3*n*Log[c + d*x])/(3*b*d^3) - (B^2*(b*c - a*d)^3*n^2*Log[c + d*x])
/(b*d^3) + (2*B^2*(b*c - a*d)^2*n*(a + b*x)*Log[(e*(a + b*x)^n)/(c + d*x)^n
])/(3*b*d^2) - (B^2*(b*c - a*d)*n*(a + b*x)^2*Log[(e*(a + b*x)^n)/(c + d*x)
^n])/(3*b*d) + (2*A*B*(a + b*x)^3*Log[(e*(a + b*x)^n)/(c + d*x)^n])/(3*b) +
 (2*B^2*(b*c - a*d)^3*n*Log[(b*c - a*d)/(b*(c + d*x))]*Log[(e*(a + b*x)^n)/
(c + d*x)^n])/(3*b*d^3) + (B^2*(a + b*x)^3*Log[(e*(a + b*x)^n)/(c + d*x)^n]
^2)/(3*b) + (2*B^2*(b*c - a*d)^3*n^2*PolyLog[2, (d*(a + b*x))/(b*(c + d*x))
])/(3*b*d^3)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
```

```
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2333

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)/(x_))^(q_.)*(
x_)^(m_.), x_Symbol] :> Int[(e + d*x)^q*(a + b*Log[c*x^n])^p, x] /; FreeQ[{
a, b, c, d, e, m, n, p}, x] && EqQ[m, q] && IntegerQ[q]
```

Rule 2343

```
Int[((a_.) + Log[(c_.)*(x_)^(n_)]*(b_.))/((x_)*((d_) + (e_.)*(x_)^(r_.))),
x_Symbol] :> Dist[1/n, Subst[Int[(a + b*Log[c*x])/(x*(d + e*x^(r/n))), x],
x, x^n], x] /; FreeQ[{a, b, c, d, e, n, r}, x] && IntegerQ[r/n]
```

Rule 2411

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + (g_
.)*(x_))^(q_.)*((h_.) + (i_.)*(x_))^(r_.), x_Symbol] :> Dist[1/e, Subst[Int
[((g*x)/e)^q*((e*h - d*i)/e + (i*x)/e)^r*(a + b*Log[c*x^n])^p, x], x, d + e
*x], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, n, p, q, r}, x] && EqQ[e*f - d
*g, 0] && (IGtQ[p, 0] || IGtQ[r, 0]) && IntegerQ[2*r]
```

Rule 2486

```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))
^(r_.)]^(s_.), x_Symbol] :> Simp[((a + b*x)*Log[e*(f*(a + b*x)^p*(c + d*x)^
q)^r]^s)/b, x] + Dist[(q*r*s*(b*c - a*d))/b, Int[Log[e*(f*(a + b*x)^p*(c +
d*x)^q)^r]^(s - 1)/(c + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, p, q, r, s
}, x] && NeQ[b*c - a*d, 0] && EqQ[p + q, 0] && IGtQ[s, 0]
```

Rule 2488

```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))
^(r_.)]^(s_.)/((g_.) + (h_.)*(x_)), x_Symbol] :> -Simp[(Log[-((b*c - a*d)/(
d*(a + b*x)))]*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^s)/h, x] + Dist[(p*r*s*
(b*c - a*d))/h, Int[(Log[-((b*c - a*d)/(d*(a + b*x)))]*Log[e*(f*(a + b*x)^p
*(c + d*x)^q)^r]^(s - 1))/((a + b*x)*(c + d*x)), x], x] /; FreeQ[{a, b, c,
d, e, f, g, h, p, q, r, s}, x] && NeQ[b*c - a*d, 0] && EqQ[p + q, 0] && EqQ
```

```
[b*g - a*h, 0] && IGtQ[s, 0]
```

Rule 2492

```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))
^(r_.)]^(s_.)*((g_.) + (h_.)*(x_))^(m_.), x_Symbol] :> Simp[((g + h*x)^(m +
 1)*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^s)/(h*(m + 1)), x] - Dist[(p*r*s*(
b*c - a*d))/(h*(m + 1)), Int[((g + h*x)^(m + 1)*Log[e*(f*(a + b*x)^p*(c + d
*x)^q)^r]^(s - 1))/((a + b*x)*(c + d*x)), x], x] /; FreeQ[{a, b, c, d, e, f
, g, h, m, p, q, r, s}, x] && NeQ[b*c - a*d, 0] && EqQ[p + q, 0] && IGtQ[s,
 0] && NeQ[m, -1]
```

Rule 2514

```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))
^(r_.)]^(s_.)*(RFx_), x_Symbol] :> With[{u = ExpandIntegrand[Log[e*(f*(a +
b*x)^p*(c + d*x)^q)^r]^s, RFx, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c
, d, e, f, p, q, r, s}, x] && RationalFunctionQ[RFx, x] && IGtQ[s, 0]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\int (a+bx)^2 \left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^2 dx = \int \left(A^2(a+bx)^2+2AB(a+bx)^2\log\left(e(a+bx)^n(c+dx)^{-n}\right.\right.$$

$$= \frac{A^2(a+bx)^3}{3b} + (2AB)\int (a+bx)^2\log\left(e(a+bx)^n(c+dx)^{-}\right.$$

$$= \frac{A^2(a+bx)^3}{3b} + \frac{2AB(a+bx)^3\log\left(e(a+bx)^n(c+dx)^{-n}\right)}{3b} +$$

$$= \frac{A^2(a+bx)^3}{3b} + \frac{2AB(a+bx)^3\log\left(e(a+bx)^n(c+dx)^{-n}\right)}{3b} +$$

$$= \frac{2AB(bc-ad)^2nx}{3d^2} - \frac{AB(bc-ad)n(a+bx)^2}{3bd} + \frac{A^2(a+bx)}{3b}$$

$$= \frac{2AB(bc-ad)^2nx}{3d^2} - \frac{AB(bc-ad)n(a+bx)^2}{3bd} + \frac{A^2(a+bx)}{3b}$$

$$= \frac{2AB(bc-ad)^2nx}{3d^2} - \frac{AB(bc-ad)n(a+bx)^2}{3bd} + \frac{A^2(a+bx)}{3b}$$

$$= \frac{2AB(bc-ad)^2nx}{3d^2} + \frac{B^2(bc-ad)^2n^2x}{3d^2} - \frac{AB(bc-ad)n(a+}{3bd}$$

$$= \frac{2AB(bc-ad)^2nx}{3d^2} + \frac{B^2(bc-ad)^2n^2x}{3d^2} - \frac{AB(bc-ad)n(a+}{3bd}$$

$$= \frac{2AB(bc-ad)^2nx}{3d^2} + \frac{B^2(bc-ad)^2n^2x}{3d^2} - \frac{AB(bc-ad)n(a+}{3bd}$$

**Mathematica [B]** time = 1.06, size = 1149, normalized size = 4.37

$$\frac{A^2d^3x^3b^3 - ABcd^2nx^2b^3 - B^2c^3n^2\log^2(c+dx)b^3 + B^2d^3x^3\log^2\left(e(a+bx)^n(c+dx)^{-n}\right)b^3 + B^2c^2dn^2xb^3 + 2ABc}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^2*(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])^2,x]
```

```
[Out] (-6*a^3*A*B*d^3*n - 2*a*b^2*B^2*c^2*d*n^2 + 6*a^2*b*B^2*c*d^2*n^2 - 6*a^3*B
^2*d^3*n^2 + 3*a^2*A^2*b*d^3*x + 2*A*b^3*B*c^2*d*n*x - 6*a*A*b^2*B*c*d^2*n*
```

```
x + 4*a^2*A*b*B*d^3*n*x + b^3*B^2*c^2*d*n^2*x - 2*a*b^2*B^2*c*d^2*n^2*x + a
^2*b*B^2*d^3*n^2*x + 3*a*A^2*b^2*d^3*x^2 - A*b^3*B*c*d^2*n*x^2 + a*A*b^2*B*
d^3*n*x^2 + A^2*b^3*d^3*x^3 - a^3*B^2*d^3*n^2*Log[a + b*x]^2 - 2*A*b^3*B*c^
3*n*Log[c + d*x] + 6*a*A*b^2*B*c^2*d*n*Log[c + d*x] - 6*a^2*A*b*B*c*d^2*n*L
og[c + d*x] - 3*b^3*B^2*c^3*n^2*Log[c + d*x] + 7*a*b^2*B^2*c^2*d*n^2*Log[c
+ d*x] - 4*a^2*b*B^2*c*d^2*n^2*Log[c + d*x] - 6*a^3*B^2*d^3*n^2*Log[c + d*x
] - b^3*B^2*c^3*n^2*Log[c + d*x]^2 + 3*a*b^2*B^2*c^2*d*n^2*Log[c + d*x]^2 -
 3*a^2*b*B^2*c*d^2*n^2*Log[c + d*x]^2 - 6*a^3*B^2*d^3*n*Log[(e*(a + b*x)^n)
/(c + d*x)^n] + 6*a^2*A*b*B*d^3*x*Log[(e*(a + b*x)^n)/(c + d*x)^n] + 2*b^3*
B^2*c^2*d*n*x*Log[(e*(a + b*x)^n)/(c + d*x)^n] - 6*a*b^2*B^2*c*d^2*n*x*Log[
(e*(a + b*x)^n)/(c + d*x)^n] + 4*a^2*b*B^2*d^3*n*x*Log[(e*(a + b*x)^n)/(c +
 d*x)^n] + 6*a*A*b^2*B*d^3*x^2*Log[(e*(a + b*x)^n)/(c + d*x)^n] - b^3*B^2*c
*d^2*n*x^2*Log[(e*(a + b*x)^n)/(c + d*x)^n] + a*b^2*B^2*d^3*n*x^2*Log[(e*(a
 + b*x)^n)/(c + d*x)^n] + 2*A*b^3*B*d^3*x^3*Log[(e*(a + b*x)^n)/(c + d*x)^n
] - 2*b^3*B^2*c^3*n*Log[c + d*x]*Log[(e*(a + b*x)^n)/(c + d*x)^n] + 6*a*b^2
*B^2*c^2*d*n*Log[c + d*x]*Log[(e*(a + b*x)^n)/(c + d*x)^n] - 6*a^2*b*B^2*c*
d^2*n*Log[c + d*x]*Log[(e*(a + b*x)^n)/(c + d*x)^n] + 3*a^2*b*B^2*d^3*x*Log
[(e*(a + b*x)^n)/(c + d*x)^n]^2 + 3*a*b^2*B^2*d^3*x^2*Log[(e*(a + b*x)^n)/(
c + d*x)^n]^2 + b^3*B^2*d^3*x^3*Log[(e*(a + b*x)^n)/(c + d*x)^n]^2 + B*n*Lo
g[a + b*x]*(2*b*B*c*(b^2*c^2 - 3*a*b*c*d + 3*a^2*d^2)*n*Log[c + d*x] - 2*B*
(b*c - a*d)^3*n*Log[(b*(c + d*x))/(b*c - a*d)] + a*d*(2*b^2*B*c^2*n - 5*a*b
*B*c*d*n + a^2*d^2*(2*A + 9*B*n) + 2*a^2*B*d^2*Log[(e*(a + b*x)^n)/(c + d*x
)^n])) - 2*B^2*(b*c - a*d)^3*n^2*PolyLog[2, (d*(a + b*x))/(-(b*c) + a*d)])/
(3*b*d^3)
```

**fricas [F]** time = 0.52, size = 0, normalized size = 0.00

$$\text{integral}\left(A^2b^2x^2 + 2\,A^2abx + A^2a^2 + \left(B^2b^2x^2 + 2\,B^2abx + B^2a^2\right)\log\left(\frac{(bx+a)^n e}{(dx+c)^n}\right)^2 + 2\left(ABb^2x^2 + 2\,ABabx + A\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2*(A+B*log(e*(b*x+a)^n/((d*x+c)^n)))^2,x, algorithm="fric
as")
```

```
[Out] integral(A^2*b^2*x^2 + 2*A^2*a*b*x + A^2*a^2 + (B^2*b^2*x^2 + 2*B^2*a*b*x +
 B^2*a^2)*log((b*x + a)^n*e/(d*x + c)^n)^2 + 2*(A*B*b^2*x^2 + 2*A*B*a*b*x +
 A*B*a^2)*log((b*x + a)^n*e/(d*x + c)^n), x)
```

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int (bx+a)^2\left(B\log\left(\frac{(bx+a)^n e}{(dx+c)^n}\right) + A\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2*(A+B*log(e*(b*x+a)^n/((d*x+c)^n)))^2,x, algorithm="giac
")
```

```
[Out] integrate((b*x + a)^2*(B*log((b*x + a)^n*e/(d*x + c)^n) + A)^2, x)
```

**maple [C]** time = 2.84, size = 19969, normalized size = 75.93

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^2*(A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))^2,x)
```

```
[Out] result too large to display
```

**maxima [B]** time = 7.05, size = 1284, normalized size = 4.88

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2*(A+B*log(e*(b*x+a)^n/((d*x+c)^n)))^2,x, algorithm="maxi
ma")
```

```
[Out] 2/3*A*B*b^2*x^3*log((b*x + a)^n*e/(d*x + c)^n) + 1/3*A^2*b^2*x^3 + 2*A*B*a*
b*x^2*log((b*x + a)^n*e/(d*x + c)^n) + A^2*a*b*x^2 + 2*A*B*a^2*x*log((b*x +
 a)^n*e/(d*x + c)^n) + A^2*a^2*x + 2*(a*e*n*log(b*x + a)/b - c*e*n*log(d*x
+ c)/d)*A*B*a^2/e - 2*(a^2*e*n*log(b*x + a)/b^2 - c^2*e*n*log(d*x + c)/d^2
+ (b*c*e*n - a*d*e*n)*x/(b*d))*A*B*a*b/e + 1/3*(2*a^3*e*n*log(b*x + a)/b^3
- 2*c^3*e*n*log(d*x + c)/d^3 - ((b^2*c*d*e*n - a*b*d^2*e*n)*x^2 - 2*(b^2*c^
2*e*n - a^2*d^2*e*n)*x)/(b^2*d^2))*A*B*b^2/e - 1/3*((3*n^2 + 2*n*log(e))*b^
2*c^3 - (7*n^2 + 6*n*log(e))*a*b*c^2*d + 2*(2*n^2 + 3*n*log(e))*a^2*c*d^2)*
B^2*log(d*x + c)/d^3 - 2/3*(b^3*c^3*n^2 - 3*a*b^2*c^2*d*n^2 + 3*a^2*b*c*d^2
*n^2 - a^3*d^3*n^2)*(log(b*x + a)*log((b*d*x + a*d)/(b*c - a*d) + 1) + dilo
g(-(b*d*x + a*d)/(b*c - a*d)))*B^2/(b*d^3) + 1/3*(B^2*b^3*d^3*x^3*log(e)^2
- B^2*a^3*d^3*n^2*log(b*x + a)^2 - (b^3*c*d^2*n*log(e) - (n*log(e) + 3*log(
e)^2)*a*b^2*d^3)*B^2*x^2 + 2*(b^3*c^3*n^2 - 3*a*b^2*c^2*d*n^2 + 3*a^2*b*c*d
^2*n^2)*B^2*log(b*x + a)*log(d*x + c) - (b^3*c^3*n^2 - 3*a*b^2*c^2*d*n^2 +
3*a^2*b*c*d^2*n^2)*B^2*log(d*x + c)^2 + ((n^2 + 2*n*log(e))*b^3*c^2*d - 2*(
n^2 + 3*n*log(e))*a*b^2*c*d^2 + (n^2 + 4*n*log(e) + 3*log(e)^2)*a^2*b*d^3)*
B^2*x + (2*a*b^2*c^2*d*n^2 - 5*a^2*b*c*d^2*n^2 + (3*n^2 + 2*n*log(e))*a^3*d
^3)*B^2*log(b*x + a) + (B^2*b^3*d^3*x^3 + 3*B^2*a*b^2*d^3*x^2 + 3*B^2*a^2*b
*d^3*x)*log((b*x + a)^n)^2 + (B^2*b^3*d^3*x^3 + 3*B^2*a*b^2*d^3*x^2 + 3*B^2
*a^2*b*d^3*x)*log((d*x + c)^n)^2 + (2*B^2*b^3*d^3*x^3*log(e) + 2*B^2*a^3*d^
3*n*log(b*x + a) + (a*b^2*d^3*(n + 6*log(e)) - b^3*c*d^2*n)*B^2*x^2 + 2*(a^
2*b*d^3*(2*n + 3*log(e)) + b^3*c^2*d*n - 3*a*b^2*c*d^2*n)*B^2*x - 2*(b^3*c^
3*n - 3*a*b^2*c^2*d*n + 3*a^2*b*c*d^2*n)*B^2*log(d*x + c))*log((b*x + a)^n)
```

```
 - (2*B^2*b^3*d^3*x^3*log(e) + 2*B^2*a^3*d^3*n*log(b*x + a) + (a*b^2*d^3*(n
 + 6*log(e)) - b^3*c*d^2*n)*B^2*x^2 + 2*(a^2*b*d^3*(2*n + 3*log(e)) + b^3*c
^2*d*n - 3*a*b^2*c*d^2*n)*B^2*x - 2*(b^3*c^3*n - 3*a*b^2*c^2*d*n + 3*a^2*b*
c*d^2*n)*B^2*log(d*x + c) + 2*(B^2*b^3*d^3*x^3 + 3*B^2*a*b^2*d^3*x^2 + 3*B^
2*a^2*b*d^3*x)*log((b*x + a)^n))*log((d*x + c)^n))/(b*d^3)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.00

$$\int \left(A+B\,\ln\left(\frac{e\,{\left(a+b\,x\right)}^n}{{\left(c+d\,x\right)}^n}\right)\right)^2\,{\left(a+b\,x\right)}^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*log((e*(a + b*x)^n)/(c + d*x)^n))^2*(a + b*x)^2,x)
[Out] int((A + B*log((e*(a + b*x)^n)/(c + d*x)^n))^2*(a + b*x)^2, x)
```

**sympy [F(-2)]** time = 0.00, size = 0, normalized size = 0.00

Exception raised: HeuristicGCDFailed

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**2*(A+B*ln(e*(b*x+a)**n/((d*x+c)**n)))**2,x)
[Out] Exception raised: HeuristicGCDFailed
```

# 3.158 $\int (a+bx)\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^2\,dx$

**Optimal**. Leaf size=195

$$-\frac{Bn(bc-ad)^2\log\left(\frac{bc-ad}{b(c+dx)}\right)\left(B\log\left(e(a+bx)^n(c+dx)^{-n}\right)+A+Bn\right)}{bd^2}-\frac{Bn(a+bx)(bc-ad)\left(B\log\left(e(a+bx)^n(c+\right.\right.}{bd}$$

```
[Out] -B*(-a*d+b*c)*n*(b*x+a)*(A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))/b/d+1/2*(b*x+a)^2
*(A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))^2/b-B*(-a*d+b*c)^2*n*ln((-a*d+b*c)/b/(d*
x+c))*(A+B*n+B*ln(e*(b*x+a)^n/((d*x+c)^n)))/b/d^2-B^2*(-a*d+b*c)^2*n^2*poly
log(2,d*(b*x+a)/b/(d*x+c))/b/d^2
```

**Rubi [A]** time = 0.49, antiderivative size = 308, normalized size of antiderivative = 1.58, number of steps used = 15, number of rules used = 11, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.355, Rules used = {6742, 2492, 43, 2514, 2486, 31, 2488, 2411, 2343, 2333, 2315}

$$-\frac{B^2n^2(bc-ad)^2\text{PolyLog}\left(2,\frac{d(a+bx)}{b(c+dx)}\right)}{bd^2}+\frac{A^2(a+bx)^2}{2b}+\frac{ABn(bc-ad)^2\log(c+dx)}{bd^2}+\frac{AB(a+bx)^2\log\left(e(a+bx)^n(c\right.}{b}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)*(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])^2,x]
```

```
[Out] -((A*B*(b*c - a*d)*n*x)/d) + (A^2*(a + b*x)^2)/(2*b) + (A*B*(b*c - a*d)^2*n
*Log[c + d*x])/(b*d^2) + (B^2*(b*c - a*d)^2*n^2*Log[c + d*x])/(b*d^2) - (B^
2*(b*c - a*d)*n*(a + b*x)*Log[(e*(a + b*x)^n)/(c + d*x)^n])/(b*d) + (A*B*(a
 + b*x)^2*Log[(e*(a + b*x)^n)/(c + d*x)^n])/b - (B^2*(b*c - a*d)^2*n*Log[(b
*c - a*d)/(b*(c + d*x))]*Log[(e*(a + b*x)^n)/(c + d*x)^n])/(b*d^2) + (B^2*(
a + b*x)^2*Log[(e*(a + b*x)^n)/(c + d*x)^n]^2)/(2*b) - (B^2*(b*c - a*d)^2*n
^2*PolyLog[2, (d*(a + b*x))/(b*(c + d*x))])/(b*d^2)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2333

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)/(x_))^(q_.)*(
x_)^(m_.), x_Symbol] :> Int[(e + d*x)^q*(a + b*Log[c*x^n])^p, x] /; FreeQ[{
a, b, c, d, e, m, n, p}, x] && EqQ[m, q] && IntegerQ[q]
```

Rule 2343

```
Int[((a_.) + Log[(c_.)*(x_)^(n_)]*(b_.))/((x_)*((d_) + (e_.)*(x_)^(r_.))),
x_Symbol] :> Dist[1/n, Subst[Int[(a + b*Log[c*x])/(x*(d + e*x^(r/n))), x],
x, x^n], x] /; FreeQ[{a, b, c, d, e, n, r}, x] && IntegerQ[r/n]
```

Rule 2411

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + (g_
.)*(x_))^(q_.)*((h_.) + (i_.)*(x_))^(r_.), x_Symbol] :> Dist[1/e, Subst[Int
[((g*x)/e)^q*((e*h - d*i)/e + (i*x)/e)^r*(a + b*Log[c*x^n])^p, x], x, d + e
*x], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, n, p, q, r}, x] && EqQ[e*f - d
*g, 0] && (IGtQ[p, 0] || IGtQ[r, 0]) && IntegerQ[2*r]
```

Rule 2486

```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))
^(r_.)]^(s_.), x_Symbol] :> Simp[((a + b*x)*Log[e*(f*(a + b*x)^p*(c + d*x)^
q)^r]^s)/b, x] + Dist[(q*r*s*(b*c - a*d))/b, Int[Log[e*(f*(a + b*x)^p*(c +
d*x)^q)^r]^(s - 1)/(c + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, p, q, r, s
}, x] && NeQ[b*c - a*d, 0] && EqQ[p + q, 0] && IGtQ[s, 0]
```

Rule 2488

```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))
^(r_.)]^(s_.)/((g_.) + (h_.)*(x_)), x_Symbol] :> -Simp[(Log[-((b*c - a*d)/(
d*(a + b*x)))]*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^s)/h, x] + Dist[(p*r*s*
(b*c - a*d))/h, Int[(Log[-((b*c - a*d)/(d*(a + b*x)))]*Log[e*(f*(a + b*x)^p
*(c + d*x)^q)^r]^(s - 1))/((a + b*x)*(c + d*x)), x], x] /; FreeQ[{a, b, c,
d, e, f, g, h, p, q, r, s}, x] && NeQ[b*c - a*d, 0] && EqQ[p + q, 0] && EqQ
[b*g - a*h, 0] && IGtQ[s, 0]
```

Rule 2492

```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))
^(r_.)]^(s_.)*((g_.) + (h_.)*(x_))^(m_.), x_Symbol] :> Simp[((g + h*x)^(m +
 1)*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^s)/(h*(m + 1)), x] - Dist[(p*r*s*(
b*c - a*d))/(h*(m + 1)), Int[((g + h*x)^(m + 1)*Log[e*(f*(a + b*x)^p*(c + d
*x)^q)^r]^(s - 1))/((a + b*x)*(c + d*x)), x], x] /; FreeQ[{a, b, c, d, e, f
, g, h, m, p, q, r, s}, x] && NeQ[b*c - a*d, 0] && EqQ[p + q, 0] && IGtQ[s,
 0] && NeQ[m, -1]
```

Rule 2514

```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))
^(r_.)]^(s_.)*(RFx_), x_Symbol] :> With[{u = ExpandIntegrand[Log[e*(f*(a +
b*x)^p*(c + d*x)^q)^r]^s, RFx, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c
, d, e, f, p, q, r, s}, x] && RationalFunctionQ[RFx, x] && IGtQ[s, 0]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\int (a+bx)\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^2\,dx = \int \left(A^2(a+bx)+2AB(a+bx)\log\left(e(a+bx)^n(c+dx)^{-n}\right)+B\right.$$

$$= \frac{A^2(a+bx)^2}{2b}+(2AB)\int (a+bx)\log\left(e(a+bx)^n(c+dx)^{-n}\right)\,d$$

$$= \frac{A^2(a+bx)^2}{2b}+\frac{AB(a+bx)^2\log\left(e(a+bx)^n(c+dx)^{-n}\right)}{b}+\frac{B^2}{}$$

$$= \frac{A^2(a+bx)^2}{2b}+\frac{AB(a+bx)^2\log\left(e(a+bx)^n(c+dx)^{-n}\right)}{b}+\frac{B^2}{}$$

$$= -\frac{AB(bc-ad)nx}{d}+\frac{A^2(a+bx)^2}{2b}+\frac{AB(bc-ad)^2n\log(c+dx}{bd^2}$$

$$= -\frac{AB(bc-ad)nx}{d}+\frac{A^2(a+bx)^2}{2b}+\frac{AB(bc-ad)^2n\log(c+dx}{bd^2}$$

$$= -\frac{AB(bc-ad)nx}{d}+\frac{A^2(a+bx)^2}{2b}+\frac{AB(bc-ad)^2n\log(c+dx}{bd^2}$$

$$= -\frac{AB(bc-ad)nx}{d}+\frac{A^2(a+bx)^2}{2b}+\frac{AB(bc-ad)^2n\log(c+dx}{bd^2}$$

$$= -\frac{AB(bc-ad)nx}{d}+\frac{A^2(a+bx)^2}{2b}+\frac{AB(bc-ad)^2n\log(c+dx}{bd^2}$$

$$= -\frac{AB(bc-ad)nx}{d}+\frac{A^2(a+bx)^2}{2b}+\frac{AB(bc-ad)^2n\log(c+dx}{bd^2}$$

**Mathematica [B]** time = 0.73, size = 656, normalized size = 3.36

$$-\frac{2a^2ABn}{b}-\frac{2a^2B^2n\log\left(e(a+bx)^n(c+dx)^{-n}\right)}{b}-\frac{2a^2B^2n^2\log(c+dx)}{b}-\frac{a^2B^2n^2\log^2(a+bx)}{2b}-\frac{2a^2B^2n^2}{b}+aA^2x+\frac{Bn}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)*(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])^2,x]
```

```
[Out] (-2*a^2*A*B*n)/b - (2*a^2*B^2*n^2)/b + (a*B^2*c*n^2)/d + a*A^2*x + a*A*B*n*
x - (A*b*B*c*n*x)/d + (A^2*b*x^2)/2 - (a^2*B^2*n^2*Log[a + b*x]^2)/(2*b) +
```

```
(A*b*B*c^2*n*Log[c + d*x])/d^2 - (2*a*A*B*c*n*Log[c + d*x])/d - (2*a^2*B^2*
n^2*Log[c + d*x])/b + (b*B^2*c^2*n^2*Log[c + d*x])/d^2 - (a*B^2*c*n^2*Log[c
 + d*x])/d + (b*B^2*c^2*n^2*Log[c + d*x]^2)/(2*d^2) - (a*B^2*c*n^2*Log[c +
d*x]^2)/d - (2*a^2*B^2*n*Log[(e*(a + b*x)^n)/(c + d*x)^n])/b + 2*a*A*B*x*Lo
g[(e*(a + b*x)^n)/(c + d*x)^n] + a*B^2*n*x*Log[(e*(a + b*x)^n)/(c + d*x)^n]
 - (b*B^2*c*n*x*Log[(e*(a + b*x)^n)/(c + d*x)^n])/d + A*b*B*x^2*Log[(e*(a +
 b*x)^n)/(c + d*x)^n] + (b*B^2*c^2*n*Log[c + d*x]*Log[(e*(a + b*x)^n)/(c +
d*x)^n])/d^2 - (2*a*B^2*c*n*Log[c + d*x]*Log[(e*(a + b*x)^n)/(c + d*x)^n])/
d + a*B^2*x*Log[(e*(a + b*x)^n)/(c + d*x)^n]^2 + (b*B^2*x^2*Log[(e*(a + b*x
)^n)/(c + d*x)^n]^2)/2 + (B*n*Log[a + b*x]*(b*B*c*(-(b*c) + 2*a*d)*n*Log[c
+ d*x] + B*(b*c - a*d)^2*n*Log[(b*(c + d*x))/(b*c - a*d)] + a*d*(-(b*B*c*n)
 + a*d*(A + 3*B*n) + a*B*d*Log[(e*(a + b*x)^n)/(c + d*x)^n])))/(b*d^2) + (B
^2*(b*c - a*d)^2*n^2*PolyLog[2, (d*(a + b*x))/(-(b*c) + a*d)])/(b*d^2)
```

**fricas [F]** time = 0.57, size = 0, normalized size = 0.00

$$\text{integral}\left(A^2bx + A^2a + \left(B^2bx + B^2a\right)\log\left(\frac{(bx+a)^n e}{(dx+c)^n}\right)^2 + 2\,(ABbx + ABa)\log\left(\frac{(bx+a)^n e}{(dx+c)^n}\right), x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*(A+B*log(e*(b*x+a)^n/((d*x+c)^n)))^2,x, algorithm="fricas
")
```

```
[Out] integral(A^2*b*x + A^2*a + (B^2*b*x + B^2*a)*log((b*x + a)^n*e/(d*x + c)^n)
^2 + 2*(A*B*b*x + A*B*a)*log((b*x + a)^n*e/(d*x + c)^n), x)
```

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int (bx+a)\left(B\log\left(\frac{(bx+a)^n e}{(dx+c)^n}\right) + A\right)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*(A+B*log(e*(b*x+a)^n/((d*x+c)^n)))^2,x, algorithm="giac")
```

```
[Out] integrate((b*x + a)*(B*log((b*x + a)^n*e/(d*x + c)^n) + A)^2, x)
```

**maple [C]** time = 2.09, size = 10210, normalized size = 52.36

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)*(A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))^2,x)
```

```
[Out] result too large to display
```

**maxima [B]** time = 6.91, size = 779, normalized size = 3.99

$$ABbx^2\log\left(\frac{(bx+a)^n e}{(dx+c)^n}\right)+\frac{1}{2}A^2bx^2+2ABax\log\left(\frac{(bx+a)^n e}{(dx+c)^n}\right)+A^2ax+\frac{2\left(\frac{aen\log(bx+a)}{b}-\frac{cen\log(dx+c)}{d}\right)ABa}{e}-\frac{\left(\frac{a^2en\log(b\right.}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*(A+B*log(e*(b*x+a)^n/((d*x+c)^n)))^2,x, algorithm="maxima
")
```

```
[Out] A*B*b*x^2*log((b*x + a)^n*e/(d*x + c)^n) + 1/2*A^2*b*x^2 + 2*A*B*a*x*log((b
*x + a)^n*e/(d*x + c)^n) + A^2*a*x + 2*(a*e*n*log(b*x + a)/b - c*e*n*log(d*
x + c)/d)*A*B*a/e - (a^2*e*n*log(b*x + a)/b^2 - c^2*e*n*log(d*x + c)/d^2 +
(b*c*e*n - a*d*e*n)*x/(b*d))*A*B*b/e + ((n^2 + n*log(e))*b*c^2 - (n^2 + 2*n
*log(e))*a*c*d)*B^2*log(d*x + c)/d^2 + (b^2*c^2*n^2 - 2*a*b*c*d*n^2 + a^2*d
^2*n^2)*(log(b*x + a)*log((b*d*x + a*d)/(b*c - a*d) + 1) + dilog(-(b*d*x +
a*d)/(b*c - a*d)))*B^2/(b*d^2) - 1/2*(B^2*a^2*d^2*n^2*log(b*x + a)^2 - B^2*
b^2*d^2*x^2*log(e)^2 + 2*(b^2*c^2*n^2 - 2*a*b*c*d*n^2)*B^2*log(b*x + a)*log
(d*x + c) - (b^2*c^2*n^2 - 2*a*b*c*d*n^2)*B^2*log(d*x + c)^2 + 2*(b^2*c*d*n
*log(e) - (n*log(e) + log(e)^2)*a*b*d^2)*B^2*x + 2*(a*b*c*d*n^2 - (n^2 + n*
log(e))*a^2*d^2)*B^2*log(b*x + a) - (B^2*b^2*d^2*x^2 + 2*B^2*a*b*d^2*x)*log
((b*x + a)^n)^2 - (B^2*b^2*d^2*x^2 + 2*B^2*a*b*d^2*x)*log((d*x + c)^n)^2 -
2*(B^2*b^2*d^2*x^2*log(e) + B^2*a^2*d^2*n*log(b*x + a) + (a*b*d^2*(n + 2*lo
g(e)) - b^2*c*d*n)*B^2*x + (b^2*c^2*n - 2*a*b*c*d*n)*B^2*log(d*x + c))*log(
(b*x + a)^n) + 2*(B^2*b^2*d^2*x^2*log(e) + B^2*a^2*d^2*n*log(b*x + a) + (a*
b*d^2*(n + 2*log(e)) - b^2*c*d*n)*B^2*x + (b^2*c^2*n - 2*a*b*c*d*n)*B^2*log
(d*x + c) + (B^2*b^2*d^2*x^2 + 2*B^2*a*b*d^2*x)*log((b*x + a)^n))*log((d*x
+ c)^n))/(b*d^2)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.01

$$\int \left(A+B\,\ln\left(\frac{e\,(a+b\,x)^n}{(c+d\,x)^n}\right)\right)^2\,(a+b\,x)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*log((e*(a + b*x)^n)/(c + d*x)^n))^2*(a + b*x),x)
```

```
[Out] int((A + B*log((e*(a + b*x)^n)/(c + d*x)^n))^2*(a + b*x), x)
```

**sympy [F(-2)]** time = 0.00, size = 0, normalized size = 0.00

Exception raised: HeuristicGCDFailed

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*(A+B*ln(e*(b*x+a)**n/((d*x+c)**n)))**2,x)
```

```
[Out] Exception raised: HeuristicGCDFailed
```

# 3.159 $\int \frac{\left(A+B\log(e(a+bx)^n(c+dx)^{-n})\right)^2}{a+bx}\,dx$

**Optimal**. Leaf size=131

$$\frac{2Bn\text{Li}_2\left(\frac{b(c+dx)}{d(a+bx)}\right)\left(B\log\left(e(a+bx)^n(c+dx)^{-n}\right)+A\right)}{b}-\frac{\log\left(1-\frac{b(c+dx)}{d(a+bx)}\right)\left(B\log\left(e(a+bx)^n(c+dx)^{-n}\right)+A\right)^2}{b}+\frac{2B^2n\cdots}{\cdots}$$

```
[Out] -(A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))^2*ln(1-b*(d*x+c)/d/(b*x+a))/b+2*B*n*(A+B
*ln(e*(b*x+a)^n/((d*x+c)^n)))*polylog(2,b*(d*x+c)/d/(b*x+a))/b+2*B^2*n^2*po
lylog(3,b*(d*x+c)/d/(b*x+a))/b
```

**Rubi [A]** time = 0.50, antiderivative size = 227, normalized size of antiderivative = 1.73, number of steps used = 10, number of rules used = 8, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.242, Rules used = {6742, 2488, 2411, 2343, 2333, 2315, 2506, 6610}

$$\frac{2ABn\text{PolyLog}\left(2,\frac{bc-ad}{d(a+bx)}+1\right)}{b}+\frac{2B^2n\text{PolyLog}\left(2,\frac{bc-ad}{d(a+bx)}+1\right)\log\left(e(a+bx)^n(c+dx)^{-n}\right)}{b}+\frac{2B^2n^2\text{PolyLog}\left(3,\frac{bc-\cdots}{d(a+\cdots}\right.}{b}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])^2/(a + b*x),x]
```

```
[Out] (A^2*Log[a + b*x])/b - (2*A*B*Log[-((b*c - a*d)/(d*(a + b*x)))]*Log[(e*(a +
 b*x)^n)/(c + d*x)^n])/b - (B^2*Log[-((b*c - a*d)/(d*(a + b*x)))]*Log[(e*(a
 + b*x)^n)/(c + d*x)^n]^2)/b + (2*A*B*n*PolyLog[2, 1 + (b*c - a*d)/(d*(a +
b*x))])/b + (2*B^2*n*Log[(e*(a + b*x)^n)/(c + d*x)^n]*PolyLog[2, 1 + (b*c -
 a*d)/(d*(a + b*x))])/b + (2*B^2*n^2*PolyLog[3, 1 + (b*c - a*d)/(d*(a + b*x
))])/b
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2333

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)/(x_))^(q_.)*(
x_)^(m_.), x_Symbol] :> Int[(e + d*x)^q*(a + b*Log[c*x^n])^p, x] /; FreeQ[{
a, b, c, d, e, m, n, p}, x] && EqQ[m, q] && IntegerQ[q]
```

Rule 2343

```
Int[((a_.) + Log[(c_.)*(x_)^(n_)]*(b_.))/((x_)*((d_) + (e_.)*(x_)^(r_.))),
x_Symbol] :> Dist[1/n, Subst[Int[(a + b*Log[c*x])/(x*(d + e*x^(r/n))), x],
```

```
x, x^n], x] /; FreeQ[{a, b, c, d, e, n, r}, x] && IntegerQ[r/n]
```

Rule 2411

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + (g_
.)*(x_))^(q_.)*((h_.) + (i_.)*(x_))^(r_.), x_Symbol] :> Dist[1/e, Subst[Int
[((g*x)/e)^q*((e*h - d*i)/e + (i*x)/e)^r*(a + b*Log[c*x^n])^p, x], x, d + e
*x], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, n, p, q, r}, x] && EqQ[e*f - d
*g, 0] && (IGtQ[p, 0] || IGtQ[r, 0]) && IntegerQ[2*r]
```

Rule 2488

```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))
^(r_.)]^(s_.)/((g_.) + (h_.)*(x_)), x_Symbol] :> -Simp[(Log[-((b*c - a*d)/(
d*(a + b*x)))]*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^s)/h, x] + Dist[(p*r*s*
(b*c - a*d))/h, Int[(Log[-((b*c - a*d)/(d*(a + b*x)))]*Log[e*(f*(a + b*x)^p
*(c + d*x)^q)^r]^(s - 1))/((a + b*x)*(c + d*x)), x], x] /; FreeQ[{a, b, c,
d, e, f, g, h, p, q, r, s}, x] && NeQ[b*c - a*d, 0] && EqQ[p + q, 0] && EqQ
[b*g - a*h, 0] && IGtQ[s, 0]
```

Rule 2506

```
Int[Log[v_]*Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_)
)^(q_.))^(r_.)]^(s_.)*(u_), x_Symbol] :> With[{g = Simplify[((v - 1)*(c + d
*x))/(a + b*x)], h = Simplify[u*(a + b*x)*(c + d*x)]}, -Simp[(h*PolyLog[2,
1 - v]*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^s)/(b*c - a*d), x] + Dist[h*p*r
*s, Int[(PolyLog[2, 1 - v]*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^(s - 1))/((
a + b*x)*(c + d*x)), x], x] /; FreeQ[{g, h}, x]] /; FreeQ[{a, b, c, d, e, f
, p, q, r, s}, x] && NeQ[b*c - a*d, 0] && IGtQ[s, 0] && EqQ[p + q, 0]
```

Rule 6610

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\int \frac{\left(A + B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^2}{a+bx}\,dx = \int \left(\frac{A^2}{a+bx} + \frac{2AB\log\left(e(a+bx)^n(c+dx)^{-n}\right)}{a+bx} + \frac{B^2\log^2\left(e(a+bx)^n(\right.}{a+bx}\right.$$

$$= \frac{A^2\log(a+bx)}{b} + (2AB)\int \frac{\log\left(e(a+bx)^n(c+dx)^{-n}\right)}{a+bx}\,dx + B^2\int \frac{\mathrm{lo}}{}$$

$$= \frac{A^2\log(a+bx)}{b} - \frac{2AB\log\left(-\frac{bc-ad}{d(a+bx)}\right)\log\left(e(a+bx)^n(c+dx)^{-n}\right)}{b} - \frac{B}{}$$

$$= \frac{A^2\log(a+bx)}{b} - \frac{2AB\log\left(-\frac{bc-ad}{d(a+bx)}\right)\log\left(e(a+bx)^n(c+dx)^{-n}\right)}{b} - \frac{B}{}$$

$$= \frac{A^2\log(a+bx)}{b} - \frac{2AB\log\left(-\frac{bc-ad}{d(a+bx)}\right)\log\left(e(a+bx)^n(c+dx)^{-n}\right)}{b} - \frac{B}{}$$

$$= \frac{A^2\log(a+bx)}{b} - \frac{2AB\log\left(-\frac{bc-ad}{d(a+bx)}\right)\log\left(e(a+bx)^n(c+dx)^{-n}\right)}{b} - \frac{B}{}$$

$$= \frac{A^2\log(a+bx)}{b} - \frac{2AB\log\left(-\frac{bc-ad}{d(a+bx)}\right)\log\left(e(a+bx)^n(c+dx)^{-n}\right)}{b} - \frac{B}{}$$

**Mathematica [B]** time = 0.19, size = 269, normalized size = 2.05

$$\frac{A^2\log(a+bx) - 2AB\log\left(\frac{ad-bc}{d(a+bx)}\right)\log\left(e(a+bx)^n(c+dx)^{-n}\right) + 2ABn\,\mathrm{Li}_2\left(\frac{d(a+bx)}{ad-bc}\right) - ABn\log^2\left(\frac{ad-bc}{d(a+bx)}\right) - 2ABn}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])^2/(a + b*x),x]
```

```
[Out] (-(A*B*n*Log[(-(b*c) + a*d)/(d*(a + b*x))]^2) + A^2*Log[a + b*x] - 2*A*B*n*
Log[(-(b*c) + a*d)/(d*(a + b*x))]*Log[(b*(c + d*x))/(b*c - a*d)] - 2*A*B*Lo
g[(-(b*c) + a*d)/(d*(a + b*x))]*Log[(e*(a + b*x)^n)/(c + d*x)^n] - B^2*Log[
(-(b*c) + a*d)/(d*(a + b*x))]*Log[(e*(a + b*x)^n)/(c + d*x)^n]^2 + 2*A*B*n*
PolyLog[2, (d*(a + b*x))/(-(b*c) + a*d)] + 2*B^2*n*Log[(e*(a + b*x)^n)/(c +
 d*x)^n]*PolyLog[2, (b*(c + d*x))/(d*(a + b*x))] + 2*B^2*n^2*PolyLog[3, (b*
(c + d*x))/(d*(a + b*x))])/b
```

**fricas [F]** time = 0.78, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{B^2\log\left(\frac{(bx+a)^n e}{(dx+c)^n}\right)^2 + 2\,AB\log\left(\frac{(bx+a)^n e}{(dx+c)^n}\right) + A^2}{bx+a}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*(b*x+a)^n/((d*x+c)^n)))^2/(b*x+a),x, algorithm="fricas
")
```

```
[Out] integral((B^2*log((b*x + a)^n*e/(d*x + c)^n)^2 + 2*A*B*log((b*x + a)^n*e/(d
*x + c)^n) + A^2)/(b*x + a), x)
```

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(B\log\left(\frac{(bx+a)^n e}{(dx+c)^n}\right) + A\right)^2}{bx+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*(b*x+a)^n/((d*x+c)^n)))^2/(b*x+a),x, algorithm="giac")
```

```
[Out] integrate((B*log((b*x + a)^n*e/(d*x + c)^n) + A)^2/(b*x + a), x)
```

**maple [F]** time = 2.57, size = 0, normalized size = 0.00

$$\int \frac{\left(B\ln\left(e\,(bx+a)^n\,(dx+c)^{-n}\right) + A\right)^2}{bx+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))^2/(b*x+a),x)
```

```
[Out] int((A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))^2/(b*x+a),x)
```

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$\frac{B^2\log(bx+a)\log\left((dx+c)^n\right)^2}{b} + \frac{A^2\log(bx+a)}{b} - \int -\frac{B^2bc\log(e)^2 + 2\,ABbc\log(e) + \left(B^2bdx + B^2bc\right)\log\left((bx\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*(b*x+a)^n/((d*x+c)^n)))^2/(b*x+a),x, algorithm="maxima
")
```

```
[Out] B^2*log(b*x + a)*log((d*x + c)^n)^2/b + A^2*log(b*x + a)/b - integrate(-(B^
2*b*c*log(e)^2 + 2*A*B*b*c*log(e) + (B^2*b*d*x + B^2*b*c)*log((b*x + a)^n)^
2 + (B^2*b*d*log(e)^2 + 2*A*B*b*d*log(e))*x + 2*(B^2*b*c*log(e) + A*B*b*c +
 (B^2*b*d*log(e) + A*B*b*d)*x)*log((b*x + a)^n) - 2*(B^2*b*c*log(e) + A*B*b
*c + (B^2*b*d*log(e) + A*B*b*d)*x + (B^2*b*d*n*x + B^2*a*d*n)*log(b*x + a)
+ (B^2*b*d*x + B^2*b*c)*log((b*x + a)^n))*log((d*x + c)^n))/(b^2*d*x^2 + a*
b*c + (b^2*c + a*b*d)*x), x)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(A+B\,\ln\left(\frac{e\,{\left(a+b\,x\right)}^n}{{\left(c+d\,x\right)}^n}\right)\right)}^2}{a+b\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*log((e*(a + b*x)^n)/(c + d*x)^n))^2/(a + b*x),x)
```

```
[Out] int((A + B*log((e*(a + b*x)^n)/(c + d*x)^n))^2/(a + b*x), x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(A + B \log{\left(e \left(a + bx\right)^{n} \left(c + dx\right)^{- n} \right)}\right)^{2}}{a + bx}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*ln(e*(b*x+a)**n/((d*x+c)**n)))**2/(b*x+a),x)
```

```
[Out] Integral((A + B*log(e*(a + b*x)**n*(c + d*x)**(-n)))**2/(a + b*x), x)
```

# 3.160 $\int \frac{\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^2}{(a+bx)^2}\,dx$

**Optimal**. Leaf size=129

$$-\frac{2Bn(c+dx)\left(B\log\left(e(a+bx)^n(c+dx)^{-n}\right)+A\right)}{(a+bx)(bc-ad)}-\frac{(c+dx)\left(B\log\left(e(a+bx)^n(c+dx)^{-n}\right)+A\right)^2}{(a+bx)(bc-ad)}-\frac{2B^2n^2(c+dx}{(a+bx)(bc-a}$$

```
[Out] -2*B^2*n^2*(d*x+c)/(-a*d+b*c)/(b*x+a)-2*B*n*(d*x+c)*(A+B*ln(e*(b*x+a)^n/((d
*x+c)^n)))/(-a*d+b*c)/(b*x+a)-(d*x+c)*(A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))^2/(
-a*d+b*c)/(b*x+a)
```

**Rubi [A]** time = 0.18, antiderivative size = 189, normalized size of antiderivative = 1.47, number of steps used = 7, number of rules used = 3, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {6742, 2490, 32}

$$-\frac{A^2}{b(a+bx)}-\frac{2AB(c+dx)\log\left(e(a+bx)^n(c+dx)^{-n}\right)}{(a+bx)(bc-ad)}-\frac{2ABn}{b(a+bx)}-\frac{B^2(c+dx)\log^2\left(e(a+bx)^n(c+dx)^{-n}\right)}{(a+bx)(bc-ad)}-\frac{2B^2n(c}{}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])^2/(a + b*x)^2,x]
```

```
[Out] -(A^2/(b*(a + b*x))) - (2*A*B*n)/(b*(a + b*x)) - (2*B^2*n^2)/(b*(a + b*x))
- (2*A*B*(c + d*x)*Log[(e*(a + b*x)^n)/(c + d*x)^n])/((b*c - a*d)*(a + b*x)
) - (2*B^2*n*(c + d*x)*Log[(e*(a + b*x)^n)/(c + d*x)^n])/((b*c - a*d)*(a +
b*x)) - (B^2*(c + d*x)*Log[(e*(a + b*x)^n)/(c + d*x)^n]^2)/((b*c - a*d)*(a
+ b*x))
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 2490

```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))
^(r_.)]^(s_.)/((g_.) + (h_.)*(x_))^2, x_Symbol] :> Simp[((a + b*x)*Log[e*(f
*(a + b*x)^p*(c + d*x)^q)^r]^s)/((b*g - a*h)*(g + h*x)), x] - Dist[(p*r*s*(
b*c - a*d))/(b*g - a*h), Int[Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^(s - 1)/(
(c + d*x)*(g + h*x)), x], x] /; FreeQ[{a, b, c, d, e, f, g, h, p, q, r, s},
 x] && NeQ[b*c - a*d, 0] && EqQ[p + q, 0] && NeQ[b*g - a*h, 0] && IGtQ[s, 0
]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\int \frac{\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^2}{(a+bx)^2}\,dx = \int \left(\frac{A^2}{(a+bx)^2}+\frac{2AB\log\left(e(a+bx)^n(c+dx)^{-n}\right)}{(a+bx)^2}+\frac{B^2\log^2\left(e(a+bx\right.}{(a+bx}\right.$$
$$= -\frac{A^2}{b(a+bx)}+(2AB)\int \frac{\log\left(e(a+bx)^n(c+dx)^{-n}\right)}{(a+bx)^2}\,dx+B^2\int \frac{\log^2\left(e\right.}{}$$
$$= -\frac{A^2}{b(a+bx)}-\frac{2AB(c+dx)\log\left(e(a+bx)^n(c+dx)^{-n}\right)}{(bc-ad)(a+bx)}-\frac{B^2(c+dx)\,\mathrm{l}}{(b}$$
$$= -\frac{A^2}{b(a+bx)}-\frac{2ABn}{b(a+bx)}-\frac{2AB(c+dx)\log\left(e(a+bx)^n(c+dx)^{-n}\right)}{(bc-ad)(a+bx)}-$$
$$= -\frac{A^2}{b(a+bx)}-\frac{2ABn}{b(a+bx)}-\frac{2B^2n^2}{b(a+bx)}-\frac{2AB(c+dx)\log\left(e(a+bx)^n(c\right.}{(bc-ad)(a+bx)}$$

**Mathematica [A]** time = 0.37, size = 236, normalized size = 1.83

$$\frac{-(bc-ad)\left(2B(A+Bn)\log\left(e(a+bx)^n(c+dx)^{-n}\right)+B^2\log^2\left(e(a+bx)^n(c+dx)^{-n}\right)+A^2+2ABn+2B^2n^2\right)-2B}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])^2/(a + b*x)^2,x]
```

```
[Out] (B^2*d*n^2*(a + b*x)*Log[a + b*x]^2 + B^2*d*n^2*(a + b*x)*Log[c + d*x]^2 +
2*B*d*n*(a + b*x)*Log[c + d*x]*(A + B*n + B*Log[(e*(a + b*x)^n)/(c + d*x)^n
]) - 2*B*d*n*(a + b*x)*Log[a + b*x]*(A + B*n + B*n*Log[c + d*x] + B*Log[(e*
(a + b*x)^n)/(c + d*x)^n]) - (b*c - a*d)*(A^2 + 2*A*B*n + 2*B^2*n^2 + 2*B*(
A + B*n)*Log[(e*(a + b*x)^n)/(c + d*x)^n] + B^2*Log[(e*(a + b*x)^n)/(c + d*
x)^n]^2))/(b*(b*c - a*d)*(a + b*x))
```

**fricas [B]** time = 0.84, size = 339, normalized size = 2.63

$$-\frac{A^2bc-A^2ad+2\left(B^2bc-B^2ad\right)n^2+\left(B^2bdn^2x+B^2bcn^2\right)\log(bx+a)^2+\left(B^2bdn^2x+B^2bcn^2\right)\log(dx+c)^2+\left(\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*(b*x+a)^n/((d*x+c)^n)))^2/(b*x+a)^2,x, algorithm="fric
as")
```

```
[Out] -(A^2*b*c - A^2*a*d + 2*(B^2*b*c - B^2*a*d)*n^2 + (B^2*b*d*n^2*x + B^2*b*c*
n^2)*log(b*x + a)^2 + (B^2*b*d*n^2*x + B^2*b*c*n^2)*log(d*x + c)^2 + (B^2*b
*c - B^2*a*d)*log(e)^2 + 2*(A*B*b*c - A*B*a*d)*n + 2*(B^2*b*c*n^2 + A*B*b*c
*n + (B^2*b*d*n^2 + A*B*b*d*n)*x + (B^2*b*d*n*x + B^2*b*c*n)*log(e))*log(b*
x + a) - 2*(B^2*b*c*n^2 + A*B*b*c*n + (B^2*b*d*n^2 + A*B*b*d*n)*x + (B^2*b*
d*n^2*x + B^2*b*c*n^2)*log(b*x + a) + (B^2*b*d*n*x + B^2*b*c*n)*log(e))*log
(d*x + c) + 2*(A*B*b*c - A*B*a*d + (B^2*b*c - B^2*a*d)*n)*log(e))/(a*b^2*c
- a^2*b*d + (b^3*c - a*b^2*d)*x)
```

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(B\log\left(\frac{(bx+a)^n e}{(dx+c)^n}\right)+A\right)^2}{(bx+a)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*(b*x+a)^n/((d*x+c)^n)))^2/(b*x+a)^2,x, algorithm="giac
")
```

```
[Out] integrate((B*log((b*x + a)^n*e/(d*x + c)^n) + A)^2/(b*x + a)^2, x)
```

**maple [C]** time = 2.25, size = 10098, normalized size = 78.28

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))^2/(b*x+a)^2,x)
```

```
[Out] result too large to display
```

**maxima [B]** time = 1.53, size = 449, normalized size = 3.48

$$-B^2\left(\frac{2\left(\frac{den\log(bx+a)}{b^2c-abd}-\frac{den\log(dx+c)}{b^2c-abd}+\frac{en}{b^2x+ab}\right)\log\left(\frac{(bx+a)^n e}{(dx+c)^n}\right)}{e}+\frac{2\,bce^2n^2-2\,ade^2n^2-\left(bde^2n^2x+ade^2n^2\right)\log\left(bx+\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*(b*x+a)^n/((d*x+c)^n)))^2/(b*x+a)^2,x, algorithm="maxi
ma")
```

```
[Out] -B^2*(2*(d*e*n*log(b*x + a)/(b^2*c - a*b*d) - d*e*n*log(d*x + c)/(b^2*c - a
*b*d) + e*n/(b^2*x + a*b))*log((b*x + a)^n*e/(d*x + c)^n)/e + (2*b*c*e^2*n^
```

```
2 - 2*a*d*e^2*n^2 - (b*d*e^2*n^2*x + a*d*e^2*n^2)*log(b*x + a)^2 - (b*d*e^2
*n^2*x + a*d*e^2*n^2)*log(d*x + c)^2 + 2*(b*d*e^2*n^2*x + a*d*e^2*n^2)*log(
b*x + a) - 2*(b*d*e^2*n^2*x + a*d*e^2*n^2 - (b*d*e^2*n^2*x + a*d*e^2*n^2)*l
og(b*x + a))*log(d*x + c))/((a*b^2*c - a^2*b*d + (b^3*c - a*b^2*d)*x)*e^2))
 - B^2*log((b*x + a)^n*e/(d*x + c)^n)^2/(b^2*x + a*b) - 2*(d*e*n*log(b*x +
a)/(b^2*c - a*b*d) - d*e*n*log(d*x + c)/(b^2*c - a*b*d) + e*n/(b^2*x + a*b)
)*A*B/e - 2*A*B*log((b*x + a)^n*e/(d*x + c)^n)/(b^2*x + a*b) - A^2/(b^2*x +
 a*b)
```

**mupad [B]** time = 5.27, size = 200, normalized size = 1.55

$$-\ln\left(\frac{e\,(a+b\,x)^n}{(c+d\,x)^n}\right)\left(\frac{2\,A\,B}{x\,b^2+a\,b}+\frac{2\,B^2\,n}{x\,b^2+a\,b}\right)-\ln\left(\frac{e\,(a+b\,x)^n}{(c+d\,x)^n}\right)^2\left(\frac{B^2}{b\,(a+b\,x)}-\frac{B^2\,d}{b\,(a\,d-b\,c)}\right)-\frac{A^2+2\,A\,B\,n+2\,B^2}{x\,b^2+a\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*log((e*(a + b*x)^n)/(c + d*x)^n))^2/(a + b*x)^2,x)
```

```
[Out] - log((e*(a + b*x)^n)/(c + d*x)^n)*((2*A*B)/(a*b + b^2*x) + (2*B^2*n)/(a*b
+ b^2*x)) - log((e*(a + b*x)^n)/(c + d*x)^n)^2*(B^2/(b*(a + b*x)) - (B^2*d)
/(b*(a*d - b*c))) - (A^2 + 2*B^2*n^2 + 2*A*B*n)/(a*b + b^2*x) - (B*d*n*atan
((((b^2*c + a*b*d)/b + 2*b*d*x)*1i)/(a*d - b*c))*(A + B*n)*4i)/(b*(a*d - b*
c))
```

**sympy [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*ln(e*(b*x+a)**n/((d*x+c)**n)))**2/(b*x+a)**2,x)
```

```
[Out] Timed out
```

# 3.161 $\int \frac{\left(A+B\log(e(a+bx)^n(c+dx)^{-n})\right)^2}{(a+bx)^3}\,dx$

**Optimal**. Leaf size=274

$$-\frac{bBn(c+dx)^2\left(B\log\left(e(a+bx)^n(c+dx)^{-n}\right)+A\right)}{2(a+bx)^2(bc-ad)^2}+\frac{2Bdn(c+dx)\left(B\log\left(e(a+bx)^n(c+dx)^{-n}\right)+A\right)}{(a+bx)(bc-ad)^2}-\frac{b(c+dx)^2}{}$$

```
[Out] 2*B^2*d*n^2*(d*x+c)/(-a*d+b*c)^2/(b*x+a)-1/4*b*B^2*n^2*(d*x+c)^2/(-a*d+b*c)
^2/(b*x+a)^2+2*B*d*n*(d*x+c)*(A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))/(-a*d+b*c)^2
/(b*x+a)-1/2*b*B*n*(d*x+c)^2*(A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))/(-a*d+b*c)^2
/(b*x+a)^2+d*(d*x+c)*(A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))^2/(-a*d+b*c)^2/(b*x+
a)-1/2*b*(d*x+c)^2*(A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))^2/(-a*d+b*c)^2/(b*x+a)
^2
```

**Rubi [A]** time = 0.42, antiderivative size = 411, normalized size of antiderivative = 1.50, number of steps used = 12, number of rules used = 8, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.242, Rules used = {6742, 2492, 44, 2491, 2490, 32, 2509, 37}

$$-\frac{A^2}{2b(a+bx)^2}+\frac{ABd^2n\log(a+bx)}{b(bc-ad)^2}-\frac{ABd^2n\log(c+dx)}{b(bc-ad)^2}-\frac{AB\log\left(e(a+bx)^n(c+dx)^{-n}\right)}{b(a+bx)^2}+\frac{ABdn}{b(a+bx)(bc-ad)}-\frac{}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])^2/(a + b*x)^3,x]
```

```
[Out] -A^2/(2*b*(a + b*x)^2) - (A*B*n)/(2*b*(a + b*x)^2) + (A*B*d*n)/(b*(b*c - a*
d)*(a + b*x)) + (2*B^2*d*n^2)/(b*(b*c - a*d)*(a + b*x)) - (b*B^2*n^2*(c + d
*x)^2)/(4*(b*c - a*d)^2*(a + b*x)^2) + (A*B*d^2*n*Log[a + b*x])/(b*(b*c - a
*d)^2) - (A*B*d^2*n*Log[c + d*x])/(b*(b*c - a*d)^2) - (A*B*Log[(e*(a + b*x)
^n)/(c + d*x)^n])/(b*(a + b*x)^2) + (2*B^2*d*n*(c + d*x)*Log[(e*(a + b*x)^n
)/(c + d*x)^n])/((b*c - a*d)^2*(a + b*x)) - (b*B^2*n*(c + d*x)^2*Log[(e*(a
+ b*x)^n)/(c + d*x)^n])/(2*(b*c - a*d)^2*(a + b*x)^2) + (B^2*d*(c + d*x)*Lo
g[(e*(a + b*x)^n)/(c + d*x)^n]^2)/((b*c - a*d)^2*(a + b*x)) - (b*B^2*(c + d
*x)^2*Log[(e*(a + b*x)^n)/(c + d*x)^n]^2)/(2*(b*c - a*d)^2*(a + b*x)^2)
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
```

```
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] && !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 2490

```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))
^(r_.)]^(s_.)/((g_.) + (h_.)*(x_))^2, x_Symbol] :> Simp[((a + b*x)*Log[e*(f
*(a + b*x)^p*(c + d*x)^q)^r]^s)/((b*g - a*h)*(g + h*x)), x] - Dist[(p*r*s*(
b*c - a*d))/(b*g - a*h), Int[Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^(s - 1)/(
(c + d*x)*(g + h*x)), x], x] /; FreeQ[{a, b, c, d, e, f, g, h, p, q, r, s},
 x] && NeQ[b*c - a*d, 0] && EqQ[p + q, 0] && NeQ[b*g - a*h, 0] && IGtQ[s, 0
]
```

Rule 2491

```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))
^(r_.)]^(s_)/((g_.) + (h_.)*(x_))^3, x_Symbol] :> Dist[d/(d*g - c*h), Int[L
og[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^s/(g + h*x)^2, x], x] - Dist[h/(d*g - c
*h), Int[((c + d*x)*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^s)/(g + h*x)^3, x]
, x] /; FreeQ[{a, b, c, d, e, f, g, h, p, q, r, s}, x] && NeQ[b*c - a*d, 0]
 && EqQ[p + q, 0] && EqQ[b*g - a*h, 0] && NeQ[d*g - c*h, 0] && IGtQ[s, 0]
```

Rule 2492

```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))
^(r_.)]^(s_.)*((g_.) + (h_.)*(x_))^(m_.), x_Symbol] :> Simp[((g + h*x)^(m +
 1)*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^s)/(h*(m + 1)), x] - Dist[(p*r*s*(
b*c - a*d))/(h*(m + 1)), Int[((g + h*x)^(m + 1)*Log[e*(f*(a + b*x)^p*(c + d
*x)^q)^r]^(s - 1))/((a + b*x)*(c + d*x)), x], x] /; FreeQ[{a, b, c, d, e, f
, g, h, m, p, q, r, s}, x] && NeQ[b*c - a*d, 0] && EqQ[p + q, 0] && IGtQ[s,
 0] && NeQ[m, -1]
```

Rule 2509

```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))
^(r_.)]^(s_.)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symb
ol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^(n + 1)*Log[e*(f*(a + b*x)^p*(c +
d*x)^q)^r]^s)/((m + 1)*(b*c - a*d)), x] - Dist[(p*r*s*(b*c - a*d))/((m + 1)
```

```
*(b*c - a*d)), Int[(a + b*x)^m*(c + d*x)^n*Log[e*(f*(a + b*x)^p*(c + d*x)^q
)^r]^(s - 1), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p, q, r, s}, x] && N
eQ[b*c - a*d, 0] && EqQ[p + q, 0] && EqQ[m + n + 2, 0] && NeQ[m, -1] && IGt
Q[s, 0]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\int \frac{\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^2}{(a+bx)^3}\,dx=\int\left(\frac{A^2}{(a+bx)^3}+\frac{2AB\log\left(e(a+bx)^n(c+dx)^{-n}\right)}{(a+bx)^3}+\frac{B^2\log^2\left(e(a+b\right.}{(a+b}\right.$$

$$=-\frac{A^2}{2b(a+bx)^2}+(2AB)\int\frac{\log\left(e(a+bx)^n(c+dx)^{-n}\right)}{(a+bx)^3}\,dx+B^2\int\frac{\log}{}$$

$$=-\frac{A^2}{2b(a+bx)^2}-\frac{AB\log\left(e(a+bx)^n(c+dx)^{-n}\right)}{b(a+bx)^2}+\frac{\left(bB^2\right)\int\frac{(c+dx)\log^2}{}}{bc-}$$

$$=-\frac{A^2}{2b(a+bx)^2}-\frac{AB\log\left(e(a+bx)^n(c+dx)^{-n}\right)}{b(a+bx)^2}+\frac{B^2d(c+dx)\log^2}{(bc-a}$$

$$=-\frac{A^2}{2b(a+bx)^2}-\frac{ABn}{2b(a+bx)^2}+\frac{ABdn}{b(bc-ad)(a+bx)}+\frac{ABd^2n\log(a}{b(bc-ad)}$$

$$=-\frac{A^2}{2b(a+bx)^2}-\frac{ABn}{2b(a+bx)^2}+\frac{ABdn}{b(bc-ad)(a+bx)}+\frac{2B^2dn^2}{b(bc-ad)(a+}$$

**Mathematica [A]** time = 0.52, size = 332, normalized size = 1.21

$$-\frac{(bc-ad)\left(2A^2(bc-ad)+2B(2A(bc-ad)+Bn(-3ad+bc-2bdx))\log\left(e(a+bx)^n(c+dx)^{-n}\right)+2ABn(-3ad\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])^2/(a + b*x)^3,x]
```

```
[Out] -1/4*(2*B^2*d^2*n^2*(a + b*x)^2*Log[a + b*x]^2 + 2*B^2*d^2*n^2*(a + b*x)^2*
Log[c + d*x]^2 + 2*B*d^2*n*(a + b*x)^2*Log[c + d*x]*(2*A + 3*B*n + 2*B*Log[
(e*(a + b*x)^n)/(c + d*x)^n]) - 2*B*d^2*n*(a + b*x)^2*Log[a + b*x]*(2*A + 3
```

```
*B*n + 2*B*n*Log[c + d*x] + 2*B*Log[(e*(a + b*x)^n)/(c + d*x)^n]) + (b*c -
a*d)*(2*A^2*(b*c - a*d) + B^2*n^2*(b*c - 7*a*d - 6*b*d*x) + 2*A*B*n*(b*c -
3*a*d - 2*b*d*x) + 2*B*(2*A*(b*c - a*d) + B*n*(b*c - 3*a*d - 2*b*d*x))*Log[
(e*(a + b*x)^n)/(c + d*x)^n] + 2*B^2*(b*c - a*d)*Log[(e*(a + b*x)^n)/(c + d
*x)^n]^2))/(b*(b*c - a*d)^2*(a + b*x)^2)
```

**fricas [B]** time = 0.77, size = 919, normalized size = 3.35

$$-\frac{2 A^2b^2c^2 - 4 A^2abcd + 2 A^2a^2d^2 + \left(B^2b^2c^2 - 8 B^2abcd + 7 B^2a^2d^2\right)n^2 - 2\left(B^2b^2d^2n^2x^2 + 2 B^2abd^2n^2x - \left(B^2b^2c^2\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*(b*x+a)^n/((d*x+c)^n)))^2/(b*x+a)^3,x, algorithm="fric
as")
```

```
[Out] -1/4*(2*A^2*b^2*c^2 - 4*A^2*a*b*c*d + 2*A^2*a^2*d^2 + (B^2*b^2*c^2 - 8*B^2*
a*b*c*d + 7*B^2*a^2*d^2)*n^2 - 2*(B^2*b^2*d^2*n^2*x^2 + 2*B^2*a*b*d^2*n^2*x
 - (B^2*b^2*c^2 - 2*B^2*a*b*c*d)*n^2)*log(b*x + a)^2 - 2*(B^2*b^2*d^2*n^2*x
^2 + 2*B^2*a*b*d^2*n^2*x - (B^2*b^2*c^2 - 2*B^2*a*b*c*d)*n^2)*log(d*x + c)^
2 + 2*(B^2*b^2*c^2 - 2*B^2*a*b*c*d + B^2*a^2*d^2)*log(e)^2 + 2*(A*B*b^2*c^2
 - 4*A*B*a*b*c*d + 3*A*B*a^2*d^2)*n - 2*(3*(B^2*b^2*c*d - B^2*a*b*d^2)*n^2
+ 2*(A*B*b^2*c*d - A*B*a*b*d^2)*n)*x + 2*((B^2*b^2*c^2 - 4*B^2*a*b*c*d)*n^2
 - (3*B^2*b^2*d^2*n^2 + 2*A*B*b^2*d^2*n)*x^2 + 2*(A*B*b^2*c^2 - 2*A*B*a*b*c
*d)*n - 2*(2*A*B*a*b*d^2*n + (B^2*b^2*c*d + 2*B^2*a*b*d^2)*n^2)*x - 2*(B^2*
b^2*d^2*n*x^2 + 2*B^2*a*b*d^2*n*x - (B^2*b^2*c^2 - 2*B^2*a*b*c*d)*n)*log(e)
)*log(b*x + a) - 2*((B^2*b^2*c^2 - 4*B^2*a*b*c*d)*n^2 - (3*B^2*b^2*d^2*n^2
+ 2*A*B*b^2*d^2*n)*x^2 + 2*(A*B*b^2*c^2 - 2*A*B*a*b*c*d)*n - 2*(2*A*B*a*b*d
^2*n + (B^2*b^2*c*d + 2*B^2*a*b*d^2)*n^2)*x - 2*(B^2*b^2*d^2*n^2*x^2 + 2*B^
2*a*b*d^2*n^2*x - (B^2*b^2*c^2 - 2*B^2*a*b*c*d)*n^2)*log(b*x + a) - 2*(B^2*
b^2*d^2*n*x^2 + 2*B^2*a*b*d^2*n*x - (B^2*b^2*c^2 - 2*B^2*a*b*c*d)*n)*log(e)
)*log(d*x + c) + 2*(2*A*B*b^2*c^2 - 4*A*B*a*b*c*d + 2*A*B*a^2*d^2 - 2*(B^2*
b^2*c*d - B^2*a*b*d^2)*n*x + (B^2*b^2*c^2 - 4*B^2*a*b*c*d + 3*B^2*a^2*d^2)*
n)*log(e))/(a^2*b^3*c^2 - 2*a^3*b^2*c*d + a^4*b*d^2 + (b^5*c^2 - 2*a*b^4*c*
d + a^2*b^3*d^2)*x^2 + 2*(a*b^4*c^2 - 2*a^2*b^3*c*d + a^3*b^2*d^2)*x)
```

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(B\log\left(\frac{(bx+a)^n e}{(dx+c)^n}\right) + A\right)^2}{(bx+a)^3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*(b*x+a)^n/((d*x+c)^n)))^2/(b*x+a)^3,x, algorithm="giac
")
```

```
[Out] integrate((B*log((b*x + a)^n*e/(d*x + c)^n) + A)^2/(b*x + a)^3, x)
```

**maple [C]** time = 3.36, size = 17300, normalized size = 63.14

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))^2/(b*x+a)^3,x)
```

```
[Out] result too large to display
```

**maxima [B]** time = 1.85, size = 899, normalized size = 3.28

$$\frac{1}{4}B^2\left(\frac{2\left(\frac{2d^2en\log(bx+a)}{b^3c^2-2ab^2cd+a^2bd^2}-\frac{2d^2en\log(dx+c)}{b^3c^2-2ab^2cd+a^2bd^2}+\frac{2bdenx-bcen+3aden}{a^2b^2c-a^3bd+\left(b^4c-ab^3d\right)x^2+2\left(ab^3c-a^2b^2d\right)x}\right)\log\left(\frac{(bx+a)^ne}{(dx+c)^n}\right)}{e}-\frac{b^2c^2e^2n^2-8a\cdots}{\cdots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*(b*x+a)^n/((d*x+c)^n)))^2/(b*x+a)^3,x, algorithm="maxi
ma")
```

```
[Out] 1/4*B^2*(2*(2*d^2*e*n*log(b*x + a)/(b^3*c^2 - 2*a*b^2*c*d + a^2*b*d^2) - 2*
d^2*e*n*log(d*x + c)/(b^3*c^2 - 2*a*b^2*c*d + a^2*b*d^2) + (2*b*d*e*n*x - b
*c*e*n + 3*a*d*e*n)/(a^2*b^2*c - a^3*b*d + (b^4*c - a*b^3*d)*x^2 + 2*(a*b^3
*c - a^2*b^2*d)*x))*log((b*x + a)^n*e/(d*x + c)^n)/e - (b^2*c^2*e^2*n^2 - 8
*a*b*c*d*e^2*n^2 + 7*a^2*d^2*e^2*n^2 + 2*(b^2*d^2*e^2*n^2*x^2 + 2*a*b*d^2*e
^2*n^2*x + a^2*d^2*e^2*n^2)*log(b*x + a)^2 + 2*(b^2*d^2*e^2*n^2*x^2 + 2*a*b
*d^2*e^2*n^2*x + a^2*d^2*e^2*n^2)*log(d*x + c)^2 - 6*(b^2*c*d*e^2*n^2 - a*b
*d^2*e^2*n^2)*x - 6*(b^2*d^2*e^2*n^2*x^2 + 2*a*b*d^2*e^2*n^2*x + a^2*d^2*e^
2*n^2)*log(b*x + a) + 2*(3*b^2*d^2*e^2*n^2*x^2 + 6*a*b*d^2*e^2*n^2*x + 3*a^
2*d^2*e^2*n^2 - 2*(b^2*d^2*e^2*n^2*x^2 + 2*a*b*d^2*e^2*n^2*x + a^2*d^2*e^2*
n^2)*log(b*x + a))*log(d*x + c))/((a^2*b^3*c^2 - 2*a^3*b^2*c*d + a^4*b*d^2
+ (b^5*c^2 - 2*a*b^4*c*d + a^2*b^3*d^2)*x^2 + 2*(a*b^4*c^2 - 2*a^2*b^3*c*d
+ a^3*b^2*d^2)*x)*e^2)) - 1/2*B^2*log((b*x + a)^n*e/(d*x + c)^n)^2/(b^3*x^2
 + 2*a*b^2*x + a^2*b) + 1/2*(2*d^2*e*n*log(b*x + a)/(b^3*c^2 - 2*a*b^2*c*d
+ a^2*b*d^2) - 2*d^2*e*n*log(d*x + c)/(b^3*c^2 - 2*a*b^2*c*d + a^2*b*d^2) +
 (2*b*d*e*n*x - b*c*e*n + 3*a*d*e*n)/(a^2*b^2*c - a^3*b*d + (b^4*c - a*b^3*
d)*x^2 + 2*(a*b^3*c - a^2*b^2*d)*x))*A*B/e - A*B*log((b*x + a)^n*e/(d*x + c
)^n)/(b^3*x^2 + 2*a*b^2*x + a^2*b) - 1/2*A^2/(b^3*x^2 + 2*a*b^2*x + a^2*b)
```

**mupad [B]** time = 5.32, size = 444, normalized size = 1.62

$$-\ln\left(\frac{e\,(a+b\,x)^n}{(c+d\,x)^n}\right)^2\left(\frac{B^2}{2\,b\,\left(a^2+2\,a\,b\,x+b^2\,x^2\right)}-\frac{B^2\,d^2}{2\,b\,\left(a^2\,d^2-2\,a\,b\,c\,d+b^2\,c^2\right)}\right)-\frac{\frac{2\,A^2\,a\,d-2\,A^2\,b\,c+7\,B^2\,a\,d\,n^2-B^2\,b\,c\,n^2+6\,A}{2\,(a\,d-b\,c)}}{2\,a^2\,b+4\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*log((e*(a + b*x)^n)/(c + d*x)^n))^2/(a + b*x)^3,x)
```

```
[Out] - log((e*(a + b*x)^n)/(c + d*x)^n)^2*(B^2/(2*b*(a^2 + b^2*x^2 + 2*a*b*x)) -
 (B^2*d^2)/(2*b*(a^2*d^2 + b^2*c^2 - 2*a*b*c*d))) - ((2*A^2*a*d - 2*A^2*b*c
 + 7*B^2*a*d*n^2 - B^2*b*c*n^2 + 6*A*B*a*d*n - 2*A*B*b*c*n)/(2*(a*d - b*c))
 + (d*x*(3*B^2*b*n^2 + 2*A*B*b*n))/(a*d - b*c))/(2*a^2*b + 2*b^3*x^2 + 4*a*
b^2*x) - log((e*(a + b*x)^n)/(c + d*x)^n)*((A*B)/(a^2*b + b^3*x^2 + 2*a*b^2
*x) + (B^2*d^2*((b*n*(a*d - b*c)*(2*a*d - b*c))/(2*d^2) + (b^2*n*x*(a*d - b
*c))/d + (a*b*n*(a*d - b*c))/(2*d)))/(b*(a^2*d^2 + b^2*c^2 - 2*a*b*c*d)*(a^
2*b + b^3*x^2 + 2*a*b^2*x))) - (B*d^2*n*atan(((2*b*d*x - (2*b^3*c^2 - 2*a^2
*b*d^2)/(2*b*(a*d - b*c)))*1i)/(a*d - b*c))*(2*A + 3*B*n)*1i)/(b*(a*d - b*c
)^2)
```

**sympy [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*ln(e*(b*x+a)**n/((d*x+c)**n)))**2/(b*x+a)**3,x)
```

```
[Out] Timed out
```

# 3.162 $\int \frac{\left(A+B\log(e(a+bx)^n(c+dx)^{-n})\right)^2}{(a+bx)^4}\,dx$

**Optimal**. Leaf size=427

$$-\frac{b^2(c+dx)^3\left(B\log\left(e(a+bx)^n(c+dx)^{-n}\right)+A\right)^2}{3(a+bx)^3(bc-ad)^3}-\frac{2b^2Bn(c+dx)^3\left(B\log\left(e(a+bx)^n(c+dx)^{-n}\right)+A\right)}{9(a+bx)^3(bc-ad)^3}-\frac{d^2(c+d\ldots}{}$$

```
[Out] -2*B^2*d^2*n^2*(d*x+c)/(-a*d+b*c)^3/(b*x+a)+1/2*b*B^2*d*n^2*(d*x+c)^2/(-a*d
+b*c)^3/(b*x+a)^2-2/27*b^2*B^2*n^2*(d*x+c)^3/(-a*d+b*c)^3/(b*x+a)^3-2*B*d^2
*n*(d*x+c)*(A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))/(-a*d+b*c)^3/(b*x+a)+b*B*d*n*(
d*x+c)^2*(A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))/(-a*d+b*c)^3/(b*x+a)^2-2/9*b^2*B
*n*(d*x+c)^3*(A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))/(-a*d+b*c)^3/(b*x+a)^3-d^2*(
d*x+c)*(A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))^2/(-a*d+b*c)^3/(b*x+a)+b*d*(d*x+c)
^2*(A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))^2/(-a*d+b*c)^3/(b*x+a)^2-1/3*b^2*(d*x+
c)^3*(A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))^2/(-a*d+b*c)^3/(b*x+a)^3
```

**Rubi [C]** time = 1.21, antiderivative size = 730, normalized size of antiderivative = 1.71, number of steps used = 26, number of rules used = 11, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {6742, 2492, 44, 2514, 2490, 32, 2488, 2411, 2343, 2333, 2315}

$$-\frac{2B^2d^3n^2\text{PolyLog}\left(2,\frac{d(a+bx)}{b(c+dx)}\right)}{3b(bc-ad)^3}-\frac{2B^2d^3n^2\text{PolyLog}\left(2,\frac{bc-ad}{d(a+bx)}+1\right)}{3b(bc-ad)^3}-\frac{A^2}{3b(a+bx)^3}-\frac{2ABd^2n}{3b(a+bx)(bc-ad)^2}-\frac{2ABd^3n\ldots}{3b(b\ldots}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])^2/(a + b*x)^4,x]
```

```
[Out] -A^2/(3*b*(a + b*x)^3) - (2*A*B*n)/(9*b*(a + b*x)^3) - (2*B^2*n^2)/(27*b*(a
 + b*x)^3) + (A*B*d*n)/(3*b*(b*c - a*d)*(a + b*x)^2) + (5*B^2*d*n^2)/(18*b*
(b*c - a*d)*(a + b*x)^2) - (2*A*B*d^2*n)/(3*b*(b*c - a*d)^2*(a + b*x)) - (1
1*B^2*d^2*n^2)/(9*b*(b*c - a*d)^2*(a + b*x)) - (2*A*B*d^3*n*Log[a + b*x])/(
3*b*(b*c - a*d)^3) - (5*B^2*d^3*n^2*Log[a + b*x])/(9*b*(b*c - a*d)^3) + (2*
A*B*d^3*n*Log[c + d*x])/(3*b*(b*c - a*d)^3) + (5*B^2*d^3*n^2*Log[c + d*x])/
(9*b*(b*c - a*d)^3) - (2*A*B*Log[(e*(a + b*x)^n)/(c + d*x)^n])/(3*b*(a + b*
x)^3) - (2*B^2*n*Log[(e*(a + b*x)^n)/(c + d*x)^n])/(9*b*(a + b*x)^3) + (B^2
*d*n*Log[(e*(a + b*x)^n)/(c + d*x)^n])/(3*b*(b*c - a*d)*(a + b*x)^2) - (2*B
^2*d^2*n*(c + d*x)*Log[(e*(a + b*x)^n)/(c + d*x)^n])/(3*(b*c - a*d)^3*(a +
b*x)) + (2*B^2*d^3*n*Log[-((b*c - a*d)/(d*(a + b*x)))]*Log[(e*(a + b*x)^n)/
(c + d*x)^n])/(3*b*(b*c - a*d)^3) - (2*B^2*d^3*n*Log[(b*c - a*d)/(b*(c + d*
x))]*Log[(e*(a + b*x)^n)/(c + d*x)^n])/(3*b*(b*c - a*d)^3) - (B^2*Log[(e*(a
 + b*x)^n)/(c + d*x)^n]^2)/(3*b*(a + b*x)^3) - (2*B^2*d^3*n^2*PolyLog[2, (d
*(a + b*x))/(b*(c + d*x))])/(3*b*(b*c - a*d)^3) - (2*B^2*d^3*n^2*PolyLog[2,
 1 + (b*c - a*d)/(d*(a + b*x))])/(3*b*(b*c - a*d)^3)
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2333

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)/(x_))^(q_.)*(
x_)^(m_.), x_Symbol] :> Int[(e + d*x)^q*(a + b*Log[c*x^n])^p, x] /; FreeQ[{
a, b, c, d, e, m, n, p}, x] && EqQ[m, q] && IntegerQ[q]
```

Rule 2343

```
Int[((a_.) + Log[(c_.)*(x_)^(n_)]*(b_.))/((x_)*((d_) + (e_.)*(x_)^(r_.))),
x_Symbol] :> Dist[1/n, Subst[Int[(a + b*Log[c*x])/(x*(d + e*x^(r/n))), x],
x, x^n], x] /; FreeQ[{a, b, c, d, e, n, r}, x] && IntegerQ[r/n]
```

Rule 2411

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + (g_
.)*(x_))^(q_.)*((h_.) + (i_.)*(x_))^(r_.), x_Symbol] :> Dist[1/e, Subst[Int
[((g*x)/e)^q*((e*h - d*i)/e + (i*x)/e)^r*(a + b*Log[c*x^n])^p, x], x, d + e
*x], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, n, p, q, r}, x] && EqQ[e*f - d
*g, 0] && (IGtQ[p, 0] || IGtQ[r, 0]) && IntegerQ[2*r]
```

Rule 2488

```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))
^(r_.)]^(s_.)/((g_.) + (h_.)*(x_)), x_Symbol] :> -Simp[(Log[-((b*c - a*d)/(
d*(a + b*x)))]*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^s)/h, x] + Dist[(p*r*s*
(b*c - a*d))/h, Int[(Log[-((b*c - a*d)/(d*(a + b*x)))]*Log[e*(f*(a + b*x)^p
*(c + d*x)^q)^r]^(s - 1))/((a + b*x)*(c + d*x)), x], x] /; FreeQ[{a, b, c,
d, e, f, g, h, p, q, r, s}, x] && NeQ[b*c - a*d, 0] && EqQ[p + q, 0] && EqQ
```

```
[b*g - a*h, 0] && IGtQ[s, 0]
```

Rule 2490

```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))
^(r_.)]^(s_.)/((g_.) + (h_.)*(x_))^2, x_Symbol] :> Simp[((a + b*x)*Log[e*(f
*(a + b*x)^p*(c + d*x)^q)^r]^s)/((b*g - a*h)*(g + h*x)), x] - Dist[(p*r*s*(
b*c - a*d))/(b*g - a*h), Int[Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^(s - 1)/(
(c + d*x)*(g + h*x)), x], x] /; FreeQ[{a, b, c, d, e, f, g, h, p, q, r, s},
 x] && NeQ[b*c - a*d, 0] && EqQ[p + q, 0] && NeQ[b*g - a*h, 0] && IGtQ[s, 0
]
```

Rule 2492

```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))
^(r_.)]^(s_.)*((g_.) + (h_.)*(x_))^(m_.), x_Symbol] :> Simp[((g + h*x)^(m +
 1)*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^s)/(h*(m + 1)), x] - Dist[(p*r*s*(
b*c - a*d))/(h*(m + 1)), Int[((g + h*x)^(m + 1)*Log[e*(f*(a + b*x)^p*(c + d
*x)^q)^r]^(s - 1))/((a + b*x)*(c + d*x)), x], x] /; FreeQ[{a, b, c, d, e, f
, g, h, m, p, q, r, s}, x] && NeQ[b*c - a*d, 0] && EqQ[p + q, 0] && IGtQ[s,
 0] && NeQ[m, -1]
```

Rule 2514

```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))
^(r_.)]^(s_.)*(RFx_), x_Symbol] :> With[{u = ExpandIntegrand[Log[e*(f*(a +
b*x)^p*(c + d*x)^q)^r]^s, RFx, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c
, d, e, f, p, q, r, s}, x] && RationalFunctionQ[RFx, x] && IGtQ[s, 0]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\int \frac{\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^2}{(a+bx)^4}\,dx = \int\left(\frac{A^2}{(a+bx)^4}+\frac{2AB\log\left(e(a+bx)^n(c+dx)^{-n}\right)}{(a+bx)^4}+\frac{B^2\log^2\left(e(a+bx)\right.}{(a+bx}\right.$$

$$= -\frac{A^2}{3b(a+bx)^3}+(2AB)\int\frac{\log\left(e(a+bx)^n(c+dx)^{-n}\right)}{(a+bx)^4}\,dx+B^2\int\frac{\log^2}{}$$

$$= -\frac{A^2}{3b(a+bx)^3}-\frac{2AB\log\left(e(a+bx)^n(c+dx)^{-n}\right)}{3b(a+bx)^3}-\frac{B^2\log^2\left(e(a+bx)\right.}{3b(a+b}$$

$$= -\frac{A^2}{3b(a+bx)^3}-\frac{2AB\log\left(e(a+bx)^n(c+dx)^{-n}\right)}{3b(a+bx)^3}-\frac{B^2\log^2\left(e(a+bx)\right.}{3b(a+b}$$

$$= -\frac{A^2}{3b(a+bx)^3}-\frac{2ABn}{9b(a+bx)^3}+\frac{ABdn}{3b(bc-ad)(a+bx)^2}-\frac{2ABd^2}{3b(bc-ad)^2(}$$

$$= -\frac{A^2}{3b(a+bx)^3}-\frac{2ABn}{9b(a+bx)^3}+\frac{ABdn}{3b(bc-ad)(a+bx)^2}-\frac{2ABd^2}{3b(bc-ad)^2(}$$

$$= -\frac{A^2}{3b(a+bx)^3}-\frac{2ABn}{9b(a+bx)^3}+\frac{ABdn}{3b(bc-ad)(a+bx)^2}-\frac{2ABd^2}{3b(bc-ad)^2(}$$

$$= -\frac{A^2}{3b(a+bx)^3}-\frac{2ABn}{9b(a+bx)^3}-\frac{2B^2n^2}{27b(a+bx)^3}+\frac{ABdn}{3b(bc-ad)(a+bx)^2}$$

$$= -\frac{A^2}{3b(a+bx)^3}-\frac{2ABn}{9b(a+bx)^3}-\frac{2B^2n^2}{27b(a+bx)^3}+\frac{ABdn}{3b(bc-ad)(a+bx)^2}$$

$$= -\frac{A^2}{3b(a+bx)^3}-\frac{2ABn}{9b(a+bx)^3}-\frac{2B^2n^2}{27b(a+bx)^3}+\frac{ABdn}{3b(bc-ad)(a+bx)^2}$$

**Mathematica [A]** time = 0.73, size = 432, normalized size = 1.01

$$\frac{-(bc-ad)\left(6B\left(Bn\left(11a^2d^2+abd(15dx-7c)+b^2\left(2c^2-3cdx+6d^2x^2\right)\right)+6A(bc-ad)^2\right)\log\left(e(a+bx)^n(c+dx)\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])^2/(a + b*x)^4,x]
```

```
[Out] (18*B^2*d^3*n^2*(a + b*x)^3*Log[a + b*x]^2 + 18*B^2*d^3*n^2*(a + b*x)^3*Log
[c + d*x]^2 + 6*B*d^3*n*(a + b*x)^3*Log[c + d*x]*(6*A + 11*B*n + 6*B*Log[(e
*(a + b*x)^n)/(c + d*x)^n]) - 6*B*d^3*n*(a + b*x)^3*Log[a + b*x]*(6*A + 11*
B*n + 6*B*n*Log[c + d*x] + 6*B*Log[(e*(a + b*x)^n)/(c + d*x)^n]) - (b*c - a
*d)*(18*A^2*(b*c - a*d)^2 + 6*A*B*n*(11*a^2*d^2 + a*b*d*(-7*c + 15*d*x) + b
^2*(2*c^2 - 3*c*d*x + 6*d^2*x^2)) + B^2*n^2*(85*a^2*d^2 + a*b*d*(-23*c + 14
7*d*x) + b^2*(4*c^2 - 15*c*d*x + 66*d^2*x^2)) + 6*B*(6*A*(b*c - a*d)^2 + B*
n*(11*a^2*d^2 + a*b*d*(-7*c + 15*d*x) + b^2*(2*c^2 - 3*c*d*x + 6*d^2*x^2)))
*Log[(e*(a + b*x)^n)/(c + d*x)^n] + 18*B^2*(b*c - a*d)^2*Log[(e*(a + b*x)^n
)/(c + d*x)^n]^2))/(54*b*(b*c - a*d)^3*(a + b*x)^3)
```

**fricas [B]** time = 1.16, size = 1635, normalized size = 3.83

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*(b*x+a)^n/((d*x+c)^n)))^2/(b*x+a)^4,x, algorithm="fric
as")
```

```
[Out] -1/54*(18*A^2*b^3*c^3 - 54*A^2*a*b^2*c^2*d + 54*A^2*a^2*b*c*d^2 - 18*A^2*a^
3*d^3 + (4*B^2*b^3*c^3 - 27*B^2*a*b^2*c^2*d + 108*B^2*a^2*b*c*d^2 - 85*B^2*
a^3*d^3)*n^2 + 6*(11*(B^2*b^3*c*d^2 - B^2*a*b^2*d^3)*n^2 + 6*(A*B*b^3*c*d^2
 - A*B*a*b^2*d^3)*n)*x^2 + 18*(B^2*b^3*d^3*n^2*x^3 + 3*B^2*a*b^2*d^3*n^2*x^
2 + 3*B^2*a^2*b*d^3*n^2*x + (B^2*b^3*c^3 - 3*B^2*a*b^2*c^2*d + 3*B^2*a^2*b*
c*d^2)*n^2)*log(b*x + a)^2 + 18*(B^2*b^3*d^3*n^2*x^3 + 3*B^2*a*b^2*d^3*n^2*
x^2 + 3*B^2*a^2*b*d^3*n^2*x + (B^2*b^3*c^3 - 3*B^2*a*b^2*c^2*d + 3*B^2*a^2*
b*c*d^2)*n^2)*log(d*x + c)^2 + 18*(B^2*b^3*c^3 - 3*B^2*a*b^2*c^2*d + 3*B^2*
a^2*b*c*d^2 - B^2*a^3*d^3)*log(e)^2 + 6*(2*A*B*b^3*c^3 - 9*A*B*a*b^2*c^2*d
+ 18*A*B*a^2*b*c*d^2 - 11*A*B*a^3*d^3)*n - 3*((5*B^2*b^3*c^2*d - 54*B^2*a*b
^2*c*d^2 + 49*B^2*a^2*b*d^3)*n^2 + 6*(A*B*b^3*c^2*d - 6*A*B*a*b^2*c*d^2 + 5
*A*B*a^2*b*d^3)*n)*x + 6*((11*B^2*b^3*d^3*n^2 + 6*A*B*b^3*d^3*n)*x^3 + (2*B
^2*b^3*c^3 - 9*B^2*a*b^2*c^2*d + 18*B^2*a^2*b*c*d^2)*n^2 + 3*(6*A*B*a*b^2*d
^3*n + (2*B^2*b^3*c*d^2 + 9*B^2*a*b^2*d^3)*n^2)*x^2 + 6*(A*B*b^3*c^3 - 3*A*
B*a*b^2*c^2*d + 3*A*B*a^2*b*c*d^2)*n + 3*(6*A*B*a^2*b*d^3*n - (B^2*b^3*c^2*
d - 6*B^2*a*b^2*c*d^2 - 6*B^2*a^2*b*d^3)*n^2)*x + 6*(B^2*b^3*d^3*n*x^3 + 3*
B^2*a*b^2*d^3*n*x^2 + 3*B^2*a^2*b*d^3*n*x + (B^2*b^3*c^3 - 3*B^2*a*b^2*c^2*
d + 3*B^2*a^2*b*c*d^2)*n)*log(e))*log(b*x + a) - 6*((11*B^2*b^3*d^3*n^2 + 6
*A*B*b^3*d^3*n)*x^3 + (2*B^2*b^3*c^3 - 9*B^2*a*b^2*c^2*d + 18*B^2*a^2*b*c*d
^2)*n^2 + 3*(6*A*B*a*b^2*d^3*n + (2*B^2*b^3*c*d^2 + 9*B^2*a*b^2*d^3)*n^2)*x
^2 + 6*(A*B*b^3*c^3 - 3*A*B*a*b^2*c^2*d + 3*A*B*a^2*b*c*d^2)*n + 3*(6*A*B*a
^2*b*d^3*n - (B^2*b^3*c^2*d - 6*B^2*a*b^2*c*d^2 - 6*B^2*a^2*b*d^3)*n^2)*x +
 6*(B^2*b^3*d^3*n^2*x^3 + 3*B^2*a*b^2*d^3*n^2*x^2 + 3*B^2*a^2*b*d^3*n^2*x +
 (B^2*b^3*c^3 - 3*B^2*a*b^2*c^2*d + 3*B^2*a^2*b*c*d^2)*n^2)*log(b*x + a) +
6*(B^2*b^3*d^3*n*x^3 + 3*B^2*a*b^2*d^3*n*x^2 + 3*B^2*a^2*b*d^3*n*x + (B^2*b
^3*c^3 - 3*B^2*a*b^2*c^2*d + 3*B^2*a^2*b*c*d^2)*n)*log(e))*log(d*x + c) + 6
```

```
*(6*A*B*b^3*c^3 - 18*A*B*a*b^2*c^2*d + 18*A*B*a^2*b*c*d^2 - 6*A*B*a^3*d^3 +
 6*(B^2*b^3*c*d^2 - B^2*a*b^2*d^3)*n*x^2 - 3*(B^2*b^3*c^2*d - 6*B^2*a*b^2*c
*d^2 + 5*B^2*a^2*b*d^3)*n*x + (2*B^2*b^3*c^3 - 9*B^2*a*b^2*c^2*d + 18*B^2*a
^2*b*c*d^2 - 11*B^2*a^3*d^3)*n)*log(e))/(a^3*b^4*c^3 - 3*a^4*b^3*c^2*d + 3*
a^5*b^2*c*d^2 - a^6*b*d^3 + (b^7*c^3 - 3*a*b^6*c^2*d + 3*a^2*b^5*c*d^2 - a^
3*b^4*d^3)*x^3 + 3*(a*b^6*c^3 - 3*a^2*b^5*c^2*d + 3*a^3*b^4*c*d^2 - a^4*b^3
*d^3)*x^2 + 3*(a^2*b^5*c^3 - 3*a^3*b^4*c^2*d + 3*a^4*b^3*c*d^2 - a^5*b^2*d^
3)*x)
```

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(B \log\left(\frac{(bx+a)^n e}{(dx+c)^n}\right) + A\right)^2}{(bx+a)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*(b*x+a)^n/((d*x+c)^n)))^2/(b*x+a)^4,x, algorithm="giac
")
```

```
[Out] integrate((B*log((b*x + a)^n*e/(d*x + c)^n) + A)^2/(b*x + a)^4, x)
```

**maple [C]** time = 4.70, size = 25057, normalized size = 58.68

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))^2/(b*x+a)^4,x)
```

```
[Out] result too large to display
```

**maxima [B]** time = 2.43, size = 1500, normalized size = 3.51

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*(b*x+a)^n/((d*x+c)^n)))^2/(b*x+a)^4,x, algorithm="maxi
ma")
```

```
[Out] -1/54*B^2*(6*(6*d^3*e*n*log(b*x + a)/(b^4*c^3 - 3*a*b^3*c^2*d + 3*a^2*b^2*c
*d^2 - a^3*b*d^3) - 6*d^3*e*n*log(d*x + c)/(b^4*c^3 - 3*a*b^3*c^2*d + 3*a^2
*b^2*c*d^2 - a^3*b*d^3) + (6*b^2*d^2*e*n*x^2 + 2*b^2*c^2*e*n - 7*a*b*c*d*e*
n + 11*a^2*d^2*e*n - 3*(b^2*c*d*e*n - 5*a*b*d^2*e*n)*x)/(a^3*b^3*c^2 - 2*a^
4*b^2*c*d + a^5*b*d^2 + (b^6*c^2 - 2*a*b^5*c*d + a^2*b^4*d^2)*x^3 + 3*(a*b^
5*c^2 - 2*a^2*b^4*c*d + a^3*b^3*d^2)*x^2 + 3*(a^2*b^4*c^2 - 2*a^3*b^3*c*d +
```

```
 a^4*b^2*d^2)*x))*log((b*x + a)^n*e/(d*x + c)^n)/e + (4*b^3*c^3*e^2*n^2 - 2
7*a*b^2*c^2*d*e^2*n^2 + 108*a^2*b*c*d^2*e^2*n^2 - 85*a^3*d^3*e^2*n^2 + 66*(
b^3*c*d^2*e^2*n^2 - a*b^2*d^3*e^2*n^2)*x^2 - 18*(b^3*d^3*e^2*n^2*x^3 + 3*a*
b^2*d^3*e^2*n^2*x^2 + 3*a^2*b*d^3*e^2*n^2*x + a^3*d^3*e^2*n^2)*log(b*x + a)
^2 - 18*(b^3*d^3*e^2*n^2*x^3 + 3*a*b^2*d^3*e^2*n^2*x^2 + 3*a^2*b*d^3*e^2*n^
2*x + a^3*d^3*e^2*n^2)*log(d*x + c)^2 - 3*(5*b^3*c^2*d*e^2*n^2 - 54*a*b^2*c
*d^2*e^2*n^2 + 49*a^2*b*d^3*e^2*n^2)*x + 66*(b^3*d^3*e^2*n^2*x^3 + 3*a*b^2*
d^3*e^2*n^2*x^2 + 3*a^2*b*d^3*e^2*n^2*x + a^3*d^3*e^2*n^2)*log(b*x + a) - 6
*(11*b^3*d^3*e^2*n^2*x^3 + 33*a*b^2*d^3*e^2*n^2*x^2 + 33*a^2*b*d^3*e^2*n^2*
x + 11*a^3*d^3*e^2*n^2 - 6*(b^3*d^3*e^2*n^2*x^3 + 3*a*b^2*d^3*e^2*n^2*x^2 +
 3*a^2*b*d^3*e^2*n^2*x + a^3*d^3*e^2*n^2)*log(b*x + a))*log(d*x + c))/((a^3
*b^4*c^3 - 3*a^4*b^3*c^2*d + 3*a^5*b^2*c*d^2 - a^6*b*d^3 + (b^7*c^3 - 3*a*b
^6*c^2*d + 3*a^2*b^5*c*d^2 - a^3*b^4*d^3)*x^3 + 3*(a*b^6*c^3 - 3*a^2*b^5*c^
2*d + 3*a^3*b^4*c*d^2 - a^4*b^3*d^3)*x^2 + 3*(a^2*b^5*c^3 - 3*a^3*b^4*c^2*d
 + 3*a^4*b^3*c*d^2 - a^5*b^2*d^3)*x)*e^2)) - 1/3*B^2*log((b*x + a)^n*e/(d*x
 + c)^n)^2/(b^4*x^3 + 3*a*b^3*x^2 + 3*a^2*b^2*x + a^3*b) - 1/9*(6*d^3*e*n*l
og(b*x + a)/(b^4*c^3 - 3*a*b^3*c^2*d + 3*a^2*b^2*c*d^2 - a^3*b*d^3) - 6*d^3
*e*n*log(d*x + c)/(b^4*c^3 - 3*a*b^3*c^2*d + 3*a^2*b^2*c*d^2 - a^3*b*d^3) +
 (6*b^2*d^2*e*n*x^2 + 2*b^2*c^2*e*n - 7*a*b*c*d*e*n + 11*a^2*d^2*e*n - 3*(b
^2*c*d*e*n - 5*a*b*d^2*e*n)*x)/(a^3*b^3*c^2 - 2*a^4*b^2*c*d + a^5*b*d^2 + (
b^6*c^2 - 2*a*b^5*c*d + a^2*b^4*d^2)*x^3 + 3*(a*b^5*c^2 - 2*a^2*b^4*c*d + a
^3*b^3*d^2)*x^2 + 3*(a^2*b^4*c^2 - 2*a^3*b^3*c*d + a^4*b^2*d^2)*x))*A*B/e -
 2/3*A*B*log((b*x + a)^n*e/(d*x + c)^n)/(b^4*x^3 + 3*a*b^3*x^2 + 3*a^2*b^2*
x + a^3*b) - 1/3*A^2/(b^4*x^3 + 3*a*b^3*x^2 + 3*a^2*b^2*x + a^3*b)
```

**mupad [B]** time = 6.84, size = 911, normalized size = 2.13

$$\frac{\frac{18\,A^2\,a^2\,d^2-36\,A^2\,a\,b\,c\,d+18\,A^2\,b^2\,c^2+66\,A\,B\,a^2\,d^2\,n-42\,A\,B\,a\,b\,c\,d\,n+12\,A\,B\,b^2\,c^2\,n+85\,B^2\,a^2\,d^2\,n^2-23\,B^2\,a\,b\,c\,d\,n^2+4\,B^2\,b^2\,c^2\,n^2}{6\,(a\,d-b\,c)}+\frac{x\left(-5\,c\,B^2\,b^2\,d\,n\right.}{}}{x^3\,\left(9\,b^5\,c-9\,a\,b^4\,d\right)+x\,\left(27\,a^2\,b^3\,c-27\,a^3\,b^2\,d\right)-x^2\,\left(27\,a^2\,b^3\,d-27\,a\,b\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*log((e*(a + b*x)^n)/(c + d*x)^n))^2/(a + b*x)^4,x)
```

```
[Out] ((18*A^2*a^2*d^2 + 18*A^2*b^2*c^2 + 85*B^2*a^2*d^2*n^2 + 4*B^2*b^2*c^2*n^2
- 36*A^2*a*b*c*d + 66*A*B*a^2*d^2*n + 12*A*B*b^2*c^2*n - 23*B^2*a*b*c*d*n^2
 - 42*A*B*a*b*c*d*n)/(6*(a*d - b*c)) + (x*(49*B^2*a*b*d^2*n^2 - 5*B^2*b^2*c
*d*n^2 + 30*A*B*a*b*d^2*n - 6*A*B*b^2*c*d*n))/(2*(a*d - b*c)) + (d*x^2*(11*
B^2*b^2*d*n^2 + 6*A*B*b^2*d*n))/(a*d - b*c))/(x^3*(9*b^5*c - 9*a*b^4*d) + x
*(27*a^2*b^3*c - 27*a^3*b^2*d) - x^2*(27*a^2*b^3*d - 27*a*b^4*c) + 9*a^3*b^
2*c - 9*a^4*b*d) - log((e*(a + b*x)^n)/(c + d*x)^n)^2*(B^2/(3*b*(a^3 + b^3*
x^3 + 3*a*b^2*x^2 + 3*a^2*b*x)) - (B^2*d^3)/(3*b*(a^3*d^3 - b^3*c^3 + 3*a*b
```

```
^2*c^2*d - 3*a^2*b*c*d^2))) - log((e*(a + b*x)^n)/(c + d*x)^n)*((2*A*B)/(3*
(a^3*b + b^4*x^3 + 3*a^2*b^2*x + 3*a*b^3*x^2)) + (2*B^2*d^3*(a*((b*n*(a*d -
 b*c)*(3*a*d - b*c))/(2*d^2) + (a*b*n*(a*d - b*c))/d) + x*(b*((b*n*(a*d - b
*c)*(3*a*d - b*c))/(2*d^2) + (a*b*n*(a*d - b*c))/d) + (2*a*b^2*n*(a*d - b*c
))/d + (b^2*n*(a*d - b*c)*(3*a*d - b*c))/d^2) + (b*n*(a*d - b*c)*(3*a^2*d^2
 + b^2*c^2 - 3*a*b*c*d))/d^3 + (3*b^3*n*x^2*(a*d - b*c))/d))/(9*b*(a^3*d^3
- b^3*c^3 + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2)*(a^3*b + b^4*x^3 + 3*a^2*b^2*x +
 3*a*b^3*x^2))) - (B*d^3*n*atan((B*d^3*n*(6*A + 11*B*n)*((b^4*c^3 + a^3*b*d
^3 - a^2*b^2*c*d^2 - a*b^3*c^2*d)/(b^3*c^2 + a^2*b*d^2 - 2*a*b^2*c*d) + 2*b
*d*x)*(b^3*c^2 + a^2*b*d^2 - 2*a*b^2*c*d)*1i)/(b*(11*B^2*d^3*n^2 + 6*A*B*d^
3*n)*(a*d - b*c)^3))*(6*A + 11*B*n)*2i)/(9*b*(a*d - b*c)^3)
```

**sympy [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*ln(e*(b*x+a)**n/((d*x+c)**n)))**2/(b*x+a)**4,x)
[Out] Timed out
```

# 3.163 $\int \frac{\left(A+B\log(e(a+bx)^n(c+dx)^{-n})\right)^2}{(a+bx)^5}\,dx$

**Optimal**. Leaf size=587

$$-\frac{b^3(c+dx)^4\left(B\log\left(e(a+bx)^n(c+dx)^{-n}\right)+A\right)^2}{4(a+bx)^4(bc-ad)^4}-\frac{b^3Bn(c+dx)^4\left(B\log\left(e(a+bx)^n(c+dx)^{-n}\right)+A\right)}{8(a+bx)^4(bc-ad)^4}+\frac{b^2d(c+d\cdots}{\cdots}$$

```
[Out] 2*B^2*d^3*n^2*(d*x+c)/(-a*d+b*c)^4/(b*x+a)-3/4*b*B^2*d^2*n^2*(d*x+c)^2/(-a*
d+b*c)^4/(b*x+a)^2+2/9*b^2*B^2*d*n^2*(d*x+c)^3/(-a*d+b*c)^4/(b*x+a)^3-1/32*
b^3*B^2*n^2*(d*x+c)^4/(-a*d+b*c)^4/(b*x+a)^4+2*B*d^3*n*(d*x+c)*(A+B*ln(e*(b
*x+a)^n/((d*x+c)^n)))/(-a*d+b*c)^4/(b*x+a)-3/2*b*B*d^2*n*(d*x+c)^2*(A+B*ln(
e*(b*x+a)^n/((d*x+c)^n)))/(-a*d+b*c)^4/(b*x+a)^2+2/3*b^2*B*d*n*(d*x+c)^3*(A
+B*ln(e*(b*x+a)^n/((d*x+c)^n)))/(-a*d+b*c)^4/(b*x+a)^3-1/8*b^3*B*n*(d*x+c)^
4*(A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))/(-a*d+b*c)^4/(b*x+a)^4+d^3*(d*x+c)*(A+B
*ln(e*(b*x+a)^n/((d*x+c)^n)))^2/(-a*d+b*c)^4/(b*x+a)-3/2*b*d^2*(d*x+c)^2*(A
+B*ln(e*(b*x+a)^n/((d*x+c)^n)))^2/(-a*d+b*c)^4/(b*x+a)^2+b^2*d*(d*x+c)^3*(A
+B*ln(e*(b*x+a)^n/((d*x+c)^n)))^2/(-a*d+b*c)^4/(b*x+a)^3-1/4*b^3*(d*x+c)^4*
(A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))^2/(-a*d+b*c)^4/(b*x+a)^4
```

**Rubi [C]** time = 1.41, antiderivative size = 843, normalized size of antiderivative = 1.44, number of steps used = 29, number of rules used = 11, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {6742, 2492, 44, 2514, 2490, 32, 2488, 2411, 2343, 2333, 2315}

$$\frac{13B^2n^2\log(a+bx)d^4}{24b(bc-ad)^4}+\frac{ABn\log(a+bx)d^4}{2b(bc-ad)^4}-\frac{13B^2n^2\log(c+dx)d^4}{24b(bc-ad)^4}-\frac{ABn\log(c+dx)d^4}{2b(bc-ad)^4}-\frac{B^2n\log\left(-\frac{bc-ad}{d(a+bx)}\right)\log\cdots}{2b(b\cdots}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])^2/(a + b*x)^5,x]
```

```
[Out] -A^2/(4*b*(a + b*x)^4) - (A*B*n)/(8*b*(a + b*x)^4) - (B^2*n^2)/(32*b*(a + b
*x)^4) + (A*B*d*n)/(6*b*(b*c - a*d)*(a + b*x)^3) + (7*B^2*d*n^2)/(72*b*(b*c
 - a*d)*(a + b*x)^3) - (A*B*d^2*n)/(4*b*(b*c - a*d)^2*(a + b*x)^2) - (13*B^
2*d^2*n^2)/(48*b*(b*c - a*d)^2*(a + b*x)^2) + (A*B*d^3*n)/(2*b*(b*c - a*d)^
3*(a + b*x)) + (25*B^2*d^3*n^2)/(24*b*(b*c - a*d)^3*(a + b*x)) + (A*B*d^4*n
*Log[a + b*x])/(2*b*(b*c - a*d)^4) + (13*B^2*d^4*n^2*Log[a + b*x])/(24*b*(b
*c - a*d)^4) - (A*B*d^4*n*Log[c + d*x])/(2*b*(b*c - a*d)^4) - (13*B^2*d^4*n
^2*Log[c + d*x])/(24*b*(b*c - a*d)^4) - (A*B*Log[(e*(a + b*x)^n)/(c + d*x)^
n])/(2*b*(a + b*x)^4) - (B^2*n*Log[(e*(a + b*x)^n)/(c + d*x)^n])/(8*b*(a +
b*x)^4) + (B^2*d*n*Log[(e*(a + b*x)^n)/(c + d*x)^n])/(6*b*(b*c - a*d)*(a +
b*x)^3) - (B^2*d^2*n*Log[(e*(a + b*x)^n)/(c + d*x)^n])/(4*b*(b*c - a*d)^2*(
a + b*x)^2) + (B^2*d^3*n*(c + d*x)*Log[(e*(a + b*x)^n)/(c + d*x)^n])/(2*(b*
```

```
c - a*d)^4*(a + b*x)) - (B^2*d^4*n*Log[-((b*c - a*d)/(d*(a + b*x)))]*Log[(e
*(a + b*x)^n)/(c + d*x)^n])/(2*b*(b*c - a*d)^4) + (B^2*d^4*n*Log[(b*c - a*d
)/(b*(c + d*x))]*Log[(e*(a + b*x)^n)/(c + d*x)^n])/(2*b*(b*c - a*d)^4) - (B
^2*Log[(e*(a + b*x)^n)/(c + d*x)^n]^2)/(4*b*(a + b*x)^4) + (B^2*d^4*n^2*Pol
yLog[2, (d*(a + b*x))/(b*(c + d*x))])/(2*b*(b*c - a*d)^4) + (B^2*d^4*n^2*Po
lyLog[2, 1 + (b*c - a*d)/(d*(a + b*x))])/(2*b*(b*c - a*d)^4)
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2333

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)/(x_))^(q_.)*(
x_)^(m_.), x_Symbol] :> Int[(e + d*x)^q*(a + b*Log[c*x^n])^p, x] /; FreeQ[{
a, b, c, d, e, m, n, p}, x] && EqQ[m, q] && IntegerQ[q]
```

Rule 2343

```
Int[((a_.) + Log[(c_.)*(x_)^(n_)]*(b_.))/((x_)*((d_) + (e_.)*(x_)^(r_.))),
x_Symbol] :> Dist[1/n, Subst[Int[(a + b*Log[c*x])/(x*(d + e*x^(r/n))), x],
x, x^n], x] /; FreeQ[{a, b, c, d, e, n, r}, x] && IntegerQ[r/n]
```

Rule 2411

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + (g_
.)*(x_))^(q_.)*((h_.) + (i_.)*(x_))^(r_.), x_Symbol] :> Dist[1/e, Subst[Int
[((g*x)/e)^q*((e*h - d*i)/e + (i*x)/e)^r*(a + b*Log[c*x^n])^p, x], x, d + e
*x], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, n, p, q, r}, x] && EqQ[e*f - d
*g, 0] && (IGtQ[p, 0] || IGtQ[r, 0]) && IntegerQ[2*r]
```

Rule 2488

```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))
^(r_.)]^(s_.)/((g_.) + (h_.)*(x_)), x_Symbol] :> -Simp[(Log[-((b*c - a*d)/(
d*(a + b*x)))]*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^s)/h, x] + Dist[(p*r*s*
(b*c - a*d))/h, Int[(Log[-((b*c - a*d)/(d*(a + b*x)))]*Log[e*(f*(a + b*x)^p
*(c + d*x)^q)^r]^(s - 1))/((a + b*x)*(c + d*x)), x], x] /; FreeQ[{a, b, c,
d, e, f, g, h, p, q, r, s}, x] && NeQ[b*c - a*d, 0] && EqQ[p + q, 0] && EqQ
[b*g - a*h, 0] && IGtQ[s, 0]
```

Rule 2490

```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))
^(r_.)]^(s_.)/((g_.) + (h_.)*(x_))^2, x_Symbol] :> Simp[((a + b*x)*Log[e*(f
*(a + b*x)^p*(c + d*x)^q)^r]^s)/((b*g - a*h)*(g + h*x)), x] - Dist[(p*r*s*(
b*c - a*d))/(b*g - a*h), Int[Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^(s - 1)/(
(c + d*x)*(g + h*x)), x], x] /; FreeQ[{a, b, c, d, e, f, g, h, p, q, r, s},
 x] && NeQ[b*c - a*d, 0] && EqQ[p + q, 0] && NeQ[b*g - a*h, 0] && IGtQ[s, 0
]
```

Rule 2492

```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))
^(r_.)]^(s_.)*((g_.) + (h_.)*(x_))^(m_.), x_Symbol] :> Simp[((g + h*x)^(m +
 1)*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^s)/(h*(m + 1)), x] - Dist[(p*r*s*(
b*c - a*d))/(h*(m + 1)), Int[((g + h*x)^(m + 1)*Log[e*(f*(a + b*x)^p*(c + d
*x)^q)^r]^(s - 1))/((a + b*x)*(c + d*x)), x], x] /; FreeQ[{a, b, c, d, e, f
, g, h, m, p, q, r, s}, x] && NeQ[b*c - a*d, 0] && EqQ[p + q, 0] && IGtQ[s,
 0] && NeQ[m, -1]
```

Rule 2514

```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))
^(r_.)]^(s_.)*(RFx_), x_Symbol] :> With[{u = ExpandIntegrand[Log[e*(f*(a +
b*x)^p*(c + d*x)^q)^r]^s, RFx, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c
, d, e, f, p, q, r, s}, x] && RationalFunctionQ[RFx, x] && IGtQ[s, 0]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\int \frac{\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^2}{(a+bx)^5}\,dx = \int\Bigg(\frac{A^2}{(a+bx)^5}+\frac{2AB\log\left(e(a+bx)^n(c+dx)^{-n}\right)}{(a+bx)^5}+\frac{B^2\log^2\left(e(a+bx\right.}{(a+bx}$$

$$= -\frac{A^2}{4b(a+bx)^4}+(2AB)\int\frac{\log\left(e(a+bx)^n(c+dx)^{-n}\right)}{(a+bx)^5}\,dx+B^2\int\frac{\log^2}{}$$

$$= -\frac{A^2}{4b(a+bx)^4}-\frac{AB\log\left(e(a+bx)^n(c+dx)^{-n}\right)}{2b(a+bx)^4}-\frac{B^2\log^2\left(e(a+bx)^n\right.}{4b(a+bx}$$

$$= -\frac{A^2}{4b(a+bx)^4}-\frac{AB\log\left(e(a+bx)^n(c+dx)^{-n}\right)}{2b(a+bx)^4}-\frac{B^2\log^2\left(e(a+bx)^n\right.}{4b(a+bx}$$

$$= -\frac{A^2}{4b(a+bx)^4}-\frac{ABn}{8b(a+bx)^4}+\frac{ABdn}{6b(bc-ad)(a+bx)^3}-\frac{ABd^2n}{4b(bc-ad)^2(}$$

$$= -\frac{A^2}{4b(a+bx)^4}-\frac{ABn}{8b(a+bx)^4}+\frac{ABdn}{6b(bc-ad)(a+bx)^3}-\frac{ABd^2n}{4b(bc-ad)^2(}$$

$$= -\frac{A^2}{4b(a+bx)^4}-\frac{ABn}{8b(a+bx)^4}+\frac{ABdn}{6b(bc-ad)(a+bx)^3}-\frac{ABd^2n}{4b(bc-ad)^2(}$$

$$= -\frac{A^2}{4b(a+bx)^4}-\frac{ABn}{8b(a+bx)^4}-\frac{B^2n^2}{32b(a+bx)^4}+\frac{ABdn}{6b(bc-ad)(a+bx)^3}$$

$$= -\frac{A^2}{4b(a+bx)^4}-\frac{ABn}{8b(a+bx)^4}-\frac{B^2n^2}{32b(a+bx)^4}+\frac{ABdn}{6b(bc-ad)(a+bx)^3}$$

$$= -\frac{A^2}{4b(a+bx)^4}-\frac{ABn}{8b(a+bx)^4}-\frac{B^2n^2}{32b(a+bx)^4}+\frac{ABdn}{6b(bc-ad)(a+bx)^3}$$

**Mathematica [A]** time = 0.95, size = 1011, normalized size = 1.72

$$-\frac{9\left(8A^2+4BnA+16B\left(-n\log(a+bx)+n\log(c+dx)+\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)A+B^2n^2+8B^2\left(-n\log(a\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])^2/(a + b*x)^5,x]
```

```
[Out] -1/288*(72*b*B^2*n^2*(-4*a^3*d^3*(c + d*x) + 6*a^2*b*d^2*(c^2 - d^2*x^2) -
4*a*b^2*d*(c^3 + d^3*x^3) + b^3*(c^4 - d^4*x^4))*Log[a + b*x]^2 + 72*b*B^2*
n^2*(-4*a^3*d^3*(c + d*x) + 6*a^2*b*d^2*(c^2 - d^2*x^2) - 4*a*b^2*d*(c^3 +
d^3*x^3) + b^3*(c^4 - d^4*x^4))*Log[c + d*x]^2 - 4*B*d*(b*c - a*d)^3*n*(a +
 b*x)*(12*A + 7*B*n + 12*B*(-(n*Log[a + b*x]) + n*Log[c + d*x] + Log[(e*(a
+ b*x)^n)/(c + d*x)^n])) + 6*B*d^2*(b*c - a*d)^2*n*(a + b*x)^2*(12*A + 13*B
*n + 12*B*(-(n*Log[a + b*x]) + n*Log[c + d*x] + Log[(e*(a + b*x)^n)/(c + d*
x)^n])) - 12*B*d^3*(b*c - a*d)*n*(a + b*x)^3*(12*A + 25*B*n + 12*B*(-(n*Log
[a + b*x]) + n*Log[c + d*x] + Log[(e*(a + b*x)^n)/(c + d*x)^n])) - 12*B*d^4
*n*(a + b*x)^4*Log[a + b*x]*(12*A + 25*B*n + 12*B*(-(n*Log[a + b*x]) + n*Lo
g[c + d*x] + Log[(e*(a + b*x)^n)/(c + d*x)^n])) + 12*B*d^4*n*(a + b*x)^4*Lo
g[c + d*x]*(12*A + 25*B*n + 12*B*(-(n*Log[a + b*x]) + n*Log[c + d*x] + Log[
(e*(a + b*x)^n)/(c + d*x)^n])) + 9*(b*c - a*d)^4*(8*A^2 + 4*A*B*n + B^2*n^2
 + 16*A*B*(-(n*Log[a + b*x]) + n*Log[c + d*x] + Log[(e*(a + b*x)^n)/(c + d*
x)^n]) + 4*B^2*n*(-(n*Log[a + b*x]) + n*Log[c + d*x] + Log[(e*(a + b*x)^n)/
(c + d*x)^n]) + 8*B^2*(-(n*Log[a + b*x]) + n*Log[c + d*x] + Log[(e*(a + b*x
)^n)/(c + d*x)^n])^2) - 12*B*(b*c - a*d)*n*Log[a + b*x]*(4*B*d*(b*c - a*d)^
2*n*(a + b*x) + 6*B*d^2*(-(b*c) + a*d)*n*(a + b*x)^2 + 12*B*d^3*n*(a + b*x)
^3 - 3*(b*c - a*d)^3*(4*A + B*n + 4*B*(-(n*Log[a + b*x]) + n*Log[c + d*x] +
 Log[(e*(a + b*x)^n)/(c + d*x)^n]))) + 12*B*n*Log[c + d*x]*(4*B*d*(b*c - a*
d)^3*n*(a + b*x) - 6*B*d^2*(b*c - a*d)^2*n*(a + b*x)^2 + 12*B*d^3*(b*c - a*
d)*n*(a + b*x)^3 - 12*B*(b*c - a*d)^4*n*Log[a + b*x] + 12*B*d^4*n*(a + b*x)
^4*Log[a + b*x] - 3*(b*c - a*d)^4*(4*A + B*n + 4*B*(-(n*Log[a + b*x]) + n*L
og[c + d*x] + Log[(e*(a + b*x)^n)/(c + d*x)^n]))))/(b*(b*c - a*d)^4*(a + b*
x)^4)
```

**fricas [B]** time = 0.84, size = 2458, normalized size = 4.19

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*(b*x+a)^n/((d*x+c)^n)))^2/(b*x+a)^5,x, algorithm="fric
as")
```

```
[Out] -1/288*(72*A^2*b^4*c^4 - 288*A^2*a*b^3*c^3*d + 432*A^2*a^2*b^2*c^2*d^2 - 28
8*A^2*a^3*b*c*d^3 + 72*A^2*a^4*d^4 - 12*(25*(B^2*b^4*c*d^3 - B^2*a*b^3*d^4)
*n^2 + 12*(A*B*b^4*c*d^3 - A*B*a*b^3*d^4)*n)*x^3 + (9*B^2*b^4*c^4 - 64*B^2*
a*b^3*c^3*d + 216*B^2*a^2*b^2*c^2*d^2 - 576*B^2*a^3*b*c*d^3 + 415*B^2*a^4*d
^4)*n^2 + 6*((13*B^2*b^4*c^2*d^2 - 176*B^2*a*b^3*c*d^3 + 163*B^2*a^2*b^2*d^
4)*n^2 + 12*(A*B*b^4*c^2*d^2 - 8*A*B*a*b^3*c*d^3 + 7*A*B*a^2*b^2*d^4)*n)*x^
2 - 72*(B^2*b^4*d^4*n^2*x^4 + 4*B^2*a*b^3*d^4*n^2*x^3 + 6*B^2*a^2*b^2*d^4*n
^2*x^2 + 4*B^2*a^3*b*d^4*n^2*x - (B^2*b^4*c^4 - 4*B^2*a*b^3*c^3*d + 6*B^2*a
^2*b^2*c^2*d^2 - 4*B^2*a^3*b*c*d^3)*n^2)*log(b*x + a)^2 - 72*(B^2*b^4*d^4*n
^2*x^4 + 4*B^2*a*b^3*d^4*n^2*x^3 + 6*B^2*a^2*b^2*d^4*n^2*x^2 + 4*B^2*a^3*b*
d^4*n^2*x - (B^2*b^4*c^4 - 4*B^2*a*b^3*c^3*d + 6*B^2*a^2*b^2*c^2*d^2 - 4*B^
```

```
2*a^3*b*c*d^3)*n^2)*log(d*x + c)^2 + 72*(B^2*b^4*c^4 - 4*B^2*a*b^3*c^3*d +
6*B^2*a^2*b^2*c^2*d^2 - 4*B^2*a^3*b*c*d^3 + B^2*a^4*d^4)*log(e)^2 + 12*(3*A
*B*b^4*c^4 - 16*A*B*a*b^3*c^3*d + 36*A*B*a^2*b^2*c^2*d^2 - 48*A*B*a^3*b*c*d
^3 + 25*A*B*a^4*d^4)*n - 4*((7*B^2*b^4*c^3*d - 60*B^2*a*b^3*c^2*d^2 + 324*B
^2*a^2*b^2*c*d^3 - 271*B^2*a^3*b*d^4)*n^2 + 12*(A*B*b^4*c^3*d - 6*A*B*a*b^3
*c^2*d^2 + 18*A*B*a^2*b^2*c*d^3 - 13*A*B*a^3*b*d^4)*n)*x - 12*((25*B^2*b^4*
d^4*n^2 + 12*A*B*b^4*d^4*n)*x^4 + 4*(12*A*B*a*b^3*d^4*n + (3*B^2*b^4*c*d^3
+ 22*B^2*a*b^3*d^4)*n^2)*x^3 - (3*B^2*b^4*c^4 - 16*B^2*a*b^3*c^3*d + 36*B^2
*a^2*b^2*c^2*d^2 - 48*B^2*a^3*b*c*d^3)*n^2 + 6*(12*A*B*a^2*b^2*d^4*n - (B^2
*b^4*c^2*d^2 - 8*B^2*a*b^3*c*d^3 - 18*B^2*a^2*b^2*d^4)*n^2)*x^2 - 12*(A*B*b
^4*c^4 - 4*A*B*a*b^3*c^3*d + 6*A*B*a^2*b^2*c^2*d^2 - 4*A*B*a^3*b*c*d^3)*n +
 4*(12*A*B*a^3*b*d^4*n + (B^2*b^4*c^3*d - 6*B^2*a*b^3*c^2*d^2 + 18*B^2*a^2*
b^2*c*d^3 + 12*B^2*a^3*b*d^4)*n^2)*x + 12*(B^2*b^4*d^4*n*x^4 + 4*B^2*a*b^3*
d^4*n*x^3 + 6*B^2*a^2*b^2*d^4*n*x^2 + 4*B^2*a^3*b*d^4*n*x - (B^2*b^4*c^4 -
4*B^2*a*b^3*c^3*d + 6*B^2*a^2*b^2*c^2*d^2 - 4*B^2*a^3*b*c*d^3)*n)*log(e))*l
og(b*x + a) + 12*((25*B^2*b^4*d^4*n^2 + 12*A*B*b^4*d^4*n)*x^4 + 4*(12*A*B*a
*b^3*d^4*n + (3*B^2*b^4*c*d^3 + 22*B^2*a*b^3*d^4)*n^2)*x^3 - (3*B^2*b^4*c^4
 - 16*B^2*a*b^3*c^3*d + 36*B^2*a^2*b^2*c^2*d^2 - 48*B^2*a^3*b*c*d^3)*n^2 +
6*(12*A*B*a^2*b^2*d^4*n - (B^2*b^4*c^2*d^2 - 8*B^2*a*b^3*c*d^3 - 18*B^2*a^2
*b^2*d^4)*n^2)*x^2 - 12*(A*B*b^4*c^4 - 4*A*B*a*b^3*c^3*d + 6*A*B*a^2*b^2*c^
2*d^2 - 4*A*B*a^3*b*c*d^3)*n + 4*(12*A*B*a^3*b*d^4*n + (B^2*b^4*c^3*d - 6*B
^2*a*b^3*c^2*d^2 + 18*B^2*a^2*b^2*c*d^3 + 12*B^2*a^3*b*d^4)*n^2)*x + 12*(B^
2*b^4*d^4*n^2*x^4 + 4*B^2*a*b^3*d^4*n^2*x^3 + 6*B^2*a^2*b^2*d^4*n^2*x^2 + 4
*B^2*a^3*b*d^4*n^2*x - (B^2*b^4*c^4 - 4*B^2*a*b^3*c^3*d + 6*B^2*a^2*b^2*c^2
*d^2 - 4*B^2*a^3*b*c*d^3)*n^2)*log(b*x + a) + 12*(B^2*b^4*d^4*n*x^4 + 4*B^2
*a*b^3*d^4*n*x^3 + 6*B^2*a^2*b^2*d^4*n*x^2 + 4*B^2*a^3*b*d^4*n*x - (B^2*b^4
*c^4 - 4*B^2*a*b^3*c^3*d + 6*B^2*a^2*b^2*c^2*d^2 - 4*B^2*a^3*b*c*d^3)*n)*lo
g(e))*log(d*x + c) + 12*(12*A*B*b^4*c^4 - 48*A*B*a*b^3*c^3*d + 72*A*B*a^2*b
^2*c^2*d^2 - 48*A*B*a^3*b*c*d^3 + 12*A*B*a^4*d^4 - 12*(B^2*b^4*c*d^3 - B^2*
a*b^3*d^4)*n*x^3 + 6*(B^2*b^4*c^2*d^2 - 8*B^2*a*b^3*c*d^3 + 7*B^2*a^2*b^2*d
^4)*n*x^2 - 4*(B^2*b^4*c^3*d - 6*B^2*a*b^3*c^2*d^2 + 18*B^2*a^2*b^2*c*d^3 -
 13*B^2*a^3*b*d^4)*n*x + (3*B^2*b^4*c^4 - 16*B^2*a*b^3*c^3*d + 36*B^2*a^2*b
^2*c^2*d^2 - 48*B^2*a^3*b*c*d^3 + 25*B^2*a^4*d^4)*n)*log(e))/(a^4*b^5*c^4 -
 4*a^5*b^4*c^3*d + 6*a^6*b^3*c^2*d^2 - 4*a^7*b^2*c*d^3 + a^8*b*d^4 + (b^9*c
^4 - 4*a*b^8*c^3*d + 6*a^2*b^7*c^2*d^2 - 4*a^3*b^6*c*d^3 + a^4*b^5*d^4)*x^4
 + 4*(a*b^8*c^4 - 4*a^2*b^7*c^3*d + 6*a^3*b^6*c^2*d^2 - 4*a^4*b^5*c*d^3 + a
^5*b^4*d^4)*x^3 + 6*(a^2*b^7*c^4 - 4*a^3*b^6*c^3*d + 6*a^4*b^5*c^2*d^2 - 4*
a^5*b^4*c*d^3 + a^6*b^3*d^4)*x^2 + 4*(a^3*b^6*c^4 - 4*a^4*b^5*c^3*d + 6*a^5
*b^4*c^2*d^2 - 4*a^6*b^3*c*d^3 + a^7*b^2*d^4)*x)
```

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(B\log\left(\frac{(bx+a)^n e}{(dx+c)^n}\right)+A\right)^2}{(bx+a)^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*(b*x+a)^n/((d*x+c)^n)))^2/(b*x+a)^5,x, algorithm="giac
")
```

```
[Out] integrate((B*log((b*x + a)^n*e/(d*x + c)^n) + A)^2/(b*x + a)^5, x)
```

**maple [C]** time = 5.84, size = 33370, normalized size = 56.85

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))^2/(b*x+a)^5,x)
```

```
[Out] result too large to display
```

**maxima [B]** time = 3.00, size = 2238, normalized size = 3.81

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*(b*x+a)^n/((d*x+c)^n)))^2/(b*x+a)^5,x, algorithm="maxi
ma")
```

```
[Out] 1/288*B^2*(12*(12*d^4*e*n*log(b*x + a)/(b^5*c^4 - 4*a*b^4*c^3*d + 6*a^2*b^3
*c^2*d^2 - 4*a^3*b^2*c*d^3 + a^4*b*d^4) - 12*d^4*e*n*log(d*x + c)/(b^5*c^4
- 4*a*b^4*c^3*d + 6*a^2*b^3*c^2*d^2 - 4*a^3*b^2*c*d^3 + a^4*b*d^4) + (12*b^
3*d^3*e*n*x^3 - 3*b^3*c^3*e*n + 13*a*b^2*c^2*d*e*n - 23*a^2*b*c*d^2*e*n + 2
5*a^3*d^3*e*n - 6*(b^3*c*d^2*e*n - 7*a*b^2*d^3*e*n)*x^2 + 4*(b^3*c^2*d*e*n
- 5*a*b^2*c*d^2*e*n + 13*a^2*b*d^3*e*n)*x)/(a^4*b^4*c^3 - 3*a^5*b^3*c^2*d +
 3*a^6*b^2*c*d^2 - a^7*b*d^3 + (b^8*c^3 - 3*a*b^7*c^2*d + 3*a^2*b^6*c*d^2 -
 a^3*b^5*d^3)*x^4 + 4*(a*b^7*c^3 - 3*a^2*b^6*c^2*d + 3*a^3*b^5*c*d^2 - a^4*
b^4*d^3)*x^3 + 6*(a^2*b^6*c^3 - 3*a^3*b^5*c^2*d + 3*a^4*b^4*c*d^2 - a^5*b^3
*d^3)*x^2 + 4*(a^3*b^5*c^3 - 3*a^4*b^4*c^2*d + 3*a^5*b^3*c*d^2 - a^6*b^2*d^
3)*x))*log((b*x + a)^n*e/(d*x + c)^n)/e - (9*b^4*c^4*e^2*n^2 - 64*a*b^3*c^3
*d*e^2*n^2 + 216*a^2*b^2*c^2*d^2*e^2*n^2 - 576*a^3*b*c*d^3*e^2*n^2 + 415*a^
4*d^4*e^2*n^2 - 300*(b^4*c*d^3*e^2*n^2 - a*b^3*d^4*e^2*n^2)*x^3 + 6*(13*b^4
*c^2*d^2*e^2*n^2 - 176*a*b^3*c*d^3*e^2*n^2 + 163*a^2*b^2*d^4*e^2*n^2)*x^2 +
 72*(b^4*d^4*e^2*n^2*x^4 + 4*a*b^3*d^4*e^2*n^2*x^3 + 6*a^2*b^2*d^4*e^2*n^2*
x^2 + 4*a^3*b*d^4*e^2*n^2*x + a^4*d^4*e^2*n^2)*log(b*x + a)^2 + 72*(b^4*d^4
*e^2*n^2*x^4 + 4*a*b^3*d^4*e^2*n^2*x^3 + 6*a^2*b^2*d^4*e^2*n^2*x^2 + 4*a^3*
b*d^4*e^2*n^2*x + a^4*d^4*e^2*n^2)*log(d*x + c)^2 - 4*(7*b^4*c^3*d*e^2*n^2
- 60*a*b^3*c^2*d^2*e^2*n^2 + 324*a^2*b^2*c*d^3*e^2*n^2 - 271*a^3*b*d^4*e^2*
n^2)*x - 300*(b^4*d^4*e^2*n^2*x^4 + 4*a*b^3*d^4*e^2*n^2*x^3 + 6*a^2*b^2*d^4
*e^2*n^2*x^2 + 4*a^3*b*d^4*e^2*n^2*x + a^4*d^4*e^2*n^2)*log(b*x + a) + 12*(
```

```
25*b^4*d^4*e^2*n^2*x^4 + 100*a*b^3*d^4*e^2*n^2*x^3 + 150*a^2*b^2*d^4*e^2*n^
2*x^2 + 100*a^3*b*d^4*e^2*n^2*x + 25*a^4*d^4*e^2*n^2 - 12*(b^4*d^4*e^2*n^2*
x^4 + 4*a*b^3*d^4*e^2*n^2*x^3 + 6*a^2*b^2*d^4*e^2*n^2*x^2 + 4*a^3*b*d^4*e^2
*n^2*x + a^4*d^4*e^2*n^2)*log(b*x + a))*log(d*x + c))/((a^4*b^5*c^4 - 4*a^5
*b^4*c^3*d + 6*a^6*b^3*c^2*d^2 - 4*a^7*b^2*c*d^3 + a^8*b*d^4 + (b^9*c^4 - 4
*a*b^8*c^3*d + 6*a^2*b^7*c^2*d^2 - 4*a^3*b^6*c*d^3 + a^4*b^5*d^4)*x^4 + 4*(
a*b^8*c^4 - 4*a^2*b^7*c^3*d + 6*a^3*b^6*c^2*d^2 - 4*a^4*b^5*c*d^3 + a^5*b^4
*d^4)*x^3 + 6*(a^2*b^7*c^4 - 4*a^3*b^6*c^3*d + 6*a^4*b^5*c^2*d^2 - 4*a^5*b^
4*c*d^3 + a^6*b^3*d^4)*x^2 + 4*(a^3*b^6*c^4 - 4*a^4*b^5*c^3*d + 6*a^5*b^4*c
^2*d^2 - 4*a^6*b^3*c*d^3 + a^7*b^2*d^4)*x)*e^2)) - 1/4*B^2*log((b*x + a)^n*
e/(d*x + c)^n)^2/(b^5*x^4 + 4*a*b^4*x^3 + 6*a^2*b^3*x^2 + 4*a^3*b^2*x + a^4
*b) + 1/24*(12*d^4*e*n*log(b*x + a)/(b^5*c^4 - 4*a*b^4*c^3*d + 6*a^2*b^3*c^
2*d^2 - 4*a^3*b^2*c*d^3 + a^4*b*d^4) - 12*d^4*e*n*log(d*x + c)/(b^5*c^4 - 4
*a*b^4*c^3*d + 6*a^2*b^3*c^2*d^2 - 4*a^3*b^2*c*d^3 + a^4*b*d^4) + (12*b^3*d
^3*e*n*x^3 - 3*b^3*c^3*e*n + 13*a*b^2*c^2*d*e*n - 23*a^2*b*c*d^2*e*n + 25*a
^3*d^3*e*n - 6*(b^3*c*d^2*e*n - 7*a*b^2*d^3*e*n)*x^2 + 4*(b^3*c^2*d*e*n - 5
*a*b^2*c*d^2*e*n + 13*a^2*b*d^3*e*n)*x)/(a^4*b^4*c^3 - 3*a^5*b^3*c^2*d + 3*
a^6*b^2*c*d^2 - a^7*b*d^3 + (b^8*c^3 - 3*a*b^7*c^2*d + 3*a^2*b^6*c*d^2 - a^
3*b^5*d^3)*x^4 + 4*(a*b^7*c^3 - 3*a^2*b^6*c^2*d + 3*a^3*b^5*c*d^2 - a^4*b^4
*d^3)*x^3 + 6*(a^2*b^6*c^3 - 3*a^3*b^5*c^2*d + 3*a^4*b^4*c*d^2 - a^5*b^3*d^
3)*x^2 + 4*(a^3*b^5*c^3 - 3*a^4*b^4*c^2*d + 3*a^5*b^3*c*d^2 - a^6*b^2*d^3)*
x))*A*B/e - 1/2*A*B*log((b*x + a)^n*e/(d*x + c)^n)/(b^5*x^4 + 4*a*b^4*x^3 +
 6*a^2*b^3*x^2 + 4*a^3*b^2*x + a^4*b) - 1/4*A^2/(b^5*x^4 + 4*a*b^4*x^3 + 6*
a^2*b^3*x^2 + 4*a^3*b^2*x + a^4*b)
```

**mupad [B]** time = 9.61, size = 1579, normalized size = 2.69

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*log((e*(a + b*x)^n)/(c + d*x)^n))^2/(a + b*x)^5,x)
```

```
[Out] (B*d^4*n*atan((B*d^4*n*(12*A + 25*B*n)*((b^5*c^4 - a^4*b*d^4 + 2*a^3*b^2*c*
d^3 - 2*a*b^4*c^3*d)/(b^4*c^3 - a^3*b*d^3 + 3*a^2*b^2*c*d^2 - 3*a*b^3*c^2*d
) + 2*b*d*x)*(b^4*c^3 - a^3*b*d^3 + 3*a^2*b^2*c*d^2 - 3*a*b^3*c^2*d)*1i)/(b
*(25*B^2*d^4*n^2 + 12*A*B*d^4*n)*(a*d - b*c)^4))*(12*A + 25*B*n)*1i)/(12*b*
(a*d - b*c)^4) - log((e*(a + b*x)^n)/(c + d*x)^n)^2*(B^2/(4*b*(a^4 + b^4*x^
4 + 4*a*b^3*x^3 + 6*a^2*b^2*x^2 + 4*a^3*b*x)) - (B^2*d^4)/(4*b*(a^4*d^4 + b
^4*c^4 + 6*a^2*b^2*c^2*d^2 - 4*a*b^3*c^3*d - 4*a^3*b*c*d^3))) - ((72*A^2*a^
3*d^3 - 72*A^2*b^3*c^3 + 415*B^2*a^3*d^3*n^2 - 9*B^2*b^3*c^3*n^2 + 216*A^2*
a*b^2*c^2*d - 216*A^2*a^2*b*c*d^2 + 300*A*B*a^3*d^3*n - 36*A*B*b^3*c^3*n +
55*B^2*a*b^2*c^2*d*n^2 - 161*B^2*a^2*b*c*d^2*n^2 + 156*A*B*a*b^2*c^2*d*n -
276*A*B*a^2*b*c*d^2*n)/(12*(a*d - b*c)) + (x^2*(163*B^2*a*b^2*d^3*n^2 - 13*
B^2*b^3*c*d^2*n^2 + 84*A*B*a*b^2*d^3*n - 12*A*B*b^3*c*d^2*n))/(2*(a*d - b*c
)) + (x*(271*B^2*a^2*b*d^3*n^2 + 7*B^2*b^3*c^2*d*n^2 - 53*B^2*a*b^2*c*d^2*n
```

```
^2 + 156*A*B*a^2*b*d^3*n + 12*A*B*b^3*c^2*d*n - 60*A*B*a*b^2*c*d^2*n))/(3*(
a*d - b*c)) + (d*x^3*(25*B^2*b^3*d^2*n^2 + 12*A*B*b^3*d^2*n))/(a*d - b*c))/
(x*(96*a^3*b^4*c^2 + 96*a^5*b^2*d^2 - 192*a^4*b^3*c*d) + x^3*(96*a*b^6*c^2
+ 96*a^3*b^4*d^2 - 192*a^2*b^5*c*d) + x^4*(24*b^7*c^2 + 24*a^2*b^5*d^2 - 48
*a*b^6*c*d) + x^2*(144*a^2*b^5*c^2 + 144*a^4*b^3*d^2 - 288*a^3*b^4*c*d) + 2
4*a^6*b*d^2 + 24*a^4*b^3*c^2 - 48*a^5*b^2*c*d) - log((e*(a + b*x)^n)/(c + d
*x)^n)*((A*B)/(2*(a^4*b + b^5*x^4 + 4*a^3*b^2*x + 4*a*b^4*x^3 + 6*a^2*b^3*x
^2)) + (B^2*d^4*(x^2*(b*(b*((b*n*(a*d - b*c)*(4*a*d - b*c))/(6*d^2) + (a*b*
n*(a*d - b*c))/(2*d)) + (a*b^2*n*(a*d - b*c))/d + (b^2*n*(a*d - b*c)*(4*a*d
 - b*c))/(3*d^2)) + (3*a*b^3*n*(a*d - b*c))/(2*d) + (b^3*n*(a*d - b*c)*(4*a
*d - b*c))/(2*d^2)) + a*(a*((b*n*(a*d - b*c)*(4*a*d - b*c))/(6*d^2) + (a*b*
n*(a*d - b*c))/(2*d)) + (b*n*(a*d - b*c)*(6*a^2*d^2 + b^2*c^2 - 4*a*b*c*d))
/(6*d^3)) + x*(b*(a*((b*n*(a*d - b*c)*(4*a*d - b*c))/(6*d^2) + (a*b*n*(a*d
- b*c))/(2*d)) + (b*n*(a*d - b*c)*(6*a^2*d^2 + b^2*c^2 - 4*a*b*c*d))/(6*d^3
)) + a*(b*((b*n*(a*d - b*c)*(4*a*d - b*c))/(6*d^2) + (a*b*n*(a*d - b*c))/(2
*d)) + (a*b^2*n*(a*d - b*c))/d + (b^2*n*(a*d - b*c)*(4*a*d - b*c))/(3*d^2))
 + (b^2*n*(a*d - b*c)*(6*a^2*d^2 + b^2*c^2 - 4*a*b*c*d))/(2*d^3)) + (b*n*(a
*d - b*c)*(4*a^3*d^3 - b^3*c^3 + 4*a*b^2*c^2*d - 6*a^2*b*c*d^2))/(2*d^4) +
(2*b^4*n*x^3*(a*d - b*c))/d))/(4*b*(a^4*b + b^5*x^4 + 4*a^3*b^2*x + 4*a*b^4
*x^3 + 6*a^2*b^3*x^2)*(a^4*d^4 + b^4*c^4 + 6*a^2*b^2*c^2*d^2 - 4*a*b^3*c^3*
d - 4*a^3*b*c*d^3)))
```

**sympy [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*ln(e*(b*x+a)**n/((d*x+c)**n)))**2/(b*x+a)**5,x)
[Out] Timed out
```

# 3.164 $\int (a+bx)^3 \left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^3 dx$

**Optimal**. Leaf size=809

$$-\frac{3Bn\log\left(\frac{bc-ad}{b(c+dx)}\right)\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^2(bc-ad)^4}{4bd^4}-\frac{B^3n^3\log\left(\frac{a+bx}{c+dx}\right)(bc-ad)^4}{4bd^4}+\frac{3B^3n^3\log(c+dx)}{2bd^4}$$

```
[Out] -1/4*B^3*(-a*d+b*c)^3*n^3*x/d^3-1/4*B^3*(-a*d+b*c)^4*n^3*ln((b*x+a)/(d*x+c)
)/b/d^4+3/2*B^3*(-a*d+b*c)^4*n^3*ln(d*x+c)/b/d^4-7/4*B^2*(-a*d+b*c)^3*n^2*(
b*x+a)*(A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))/b/d^3+1/4*b*B^2*(-a*d+b*c)^2*n^2*(
d*x+c)^2*(A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))/d^4-9/2*B^2*(-a*d+b*c)^4*n^2*ln(
(-a*d+b*c)/b/(d*x+c))*(A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))/b/d^4-9/4*B*(-a*d+b
*c)^3*n*(b*x+a)*(A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))^2/b/d^3+9/8*b*B*(-a*d+b*c
)^2*n*(d*x+c)^2*(A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))^2/d^4-1/4*b^2*B*(-a*d+b*c
)*n*(d*x+c)^3*(A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))^2/d^4-3/4*B*(-a*d+b*c)^4*n*
ln((-a*d+b*c)/b/(d*x+c))*(A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))^2/b/d^4+1/4*(b*x
+a)^4*(A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))^3/b+7/4*B^2*(-a*d+b*c)^4*n^2*(A+B*l
n(e*(b*x+a)^n/((d*x+c)^n)))*ln(1-b*(d*x+c)/d/(b*x+a))/b/d^4-9/2*B^3*(-a*d+b
*c)^4*n^3*polylog(2,d*(b*x+a)/b/(d*x+c))/b/d^4-3/2*B^2*(-a*d+b*c)^4*n^2*(A+
B*ln(e*(b*x+a)^n/((d*x+c)^n)))*polylog(2,d*(b*x+a)/b/(d*x+c))/b/d^4-7/4*B^3
*(-a*d+b*c)^4*n^3*polylog(2,b*(d*x+c)/d/(b*x+a))/b/d^4+3/2*B^3*(-a*d+b*c)^4
*n^3*polylog(3,d*(b*x+a)/b/(d*x+c))/b/d^4
```

**Rubi [A]** time = 2.40, antiderivative size = 1203, normalized size of antiderivative = 1.49, number of steps used = 56, number of rules used = 13, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.394$, Rules used = {6742, 2492, 43, 2514, 2486, 31, 2488, 2411, 2343, 2333, 2315, 2506, 6610}

$$-\frac{3B^3n\log\left(\frac{bc-ad}{b(c+dx)}\right)\log^2\left(e(a+bx)^n(c+dx)^{-n}\right)(bc-ad)^4}{4bd^4}+\frac{3B^3n^3\log(c+dx)(bc-ad)^4}{2bd^4}+\frac{11AB^2n^2\log(c+dx)(bc}{4bd^4}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^3*(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])^3,x]
```

```
[Out] (-3*A^2*B*(b*c - a*d)^3*n*x)/(4*d^3) - (5*A*B^2*(b*c - a*d)^3*n^2*x)/(4*d^3
) - (B^3*(b*c - a*d)^3*n^3*x)/(4*d^3) + (3*A^2*B*(b*c - a*d)^2*n*(a + b*x)^
2)/(8*b*d^2) + (A*B^2*(b*c - a*d)^2*n^2*(a + b*x)^2)/(4*b*d^2) - (A^2*B*(b*
c - a*d)*n*(a + b*x)^3)/(4*b*d) + (A^3*(a + b*x)^4)/(4*b) + (3*A^2*B*(b*c -
 a*d)^4*n*Log[c + d*x])/(4*b*d^4) + (11*A*B^2*(b*c - a*d)^4*n^2*Log[c + d*x
])/(4*b*d^4) + (3*B^3*(b*c - a*d)^4*n^3*Log[c + d*x])/(2*b*d^4) - (3*A*B^2*
(b*c - a*d)^3*n*(a + b*x)*Log[(e*(a + b*x)^n)/(c + d*x)^n])/(2*b*d^3) - (5*
B^3*(b*c - a*d)^3*n^2*(a + b*x)*Log[(e*(a + b*x)^n)/(c + d*x)^n])/(4*b*d^3)
```

```
 + (3*A*B^2*(b*c - a*d)^2*n*(a + b*x)^2*Log[(e*(a + b*x)^n)/(c + d*x)^n])/(
4*b*d^2) + (B^3*(b*c - a*d)^2*n^2*(a + b*x)^2*Log[(e*(a + b*x)^n)/(c + d*x)
^n])/(4*b*d^2) - (A*B^2*(b*c - a*d)*n*(a + b*x)^3*Log[(e*(a + b*x)^n)/(c +
d*x)^n])/(2*b*d) + (3*A^2*B*(a + b*x)^4*Log[(e*(a + b*x)^n)/(c + d*x)^n])/(
4*b) - (3*A*B^2*(b*c - a*d)^4*n*Log[(b*c - a*d)/(b*(c + d*x))]*Log[(e*(a +
b*x)^n)/(c + d*x)^n])/(2*b*d^4) - (11*B^3*(b*c - a*d)^4*n^2*Log[(b*c - a*d)
/(b*(c + d*x))]*Log[(e*(a + b*x)^n)/(c + d*x)^n])/(4*b*d^4) - (3*B^3*(b*c -
 a*d)^3*n*(a + b*x)*Log[(e*(a + b*x)^n)/(c + d*x)^n]^2)/(4*b*d^3) + (3*B^3*
(b*c - a*d)^2*n*(a + b*x)^2*Log[(e*(a + b*x)^n)/(c + d*x)^n]^2)/(8*b*d^2) -
 (B^3*(b*c - a*d)*n*(a + b*x)^3*Log[(e*(a + b*x)^n)/(c + d*x)^n]^2)/(4*b*d)
 + (3*A*B^2*(a + b*x)^4*Log[(e*(a + b*x)^n)/(c + d*x)^n]^2)/(4*b) - (3*B^3*
(b*c - a*d)^4*n*Log[(b*c - a*d)/(b*(c + d*x))]*Log[(e*(a + b*x)^n)/(c + d*x
)^n]^2)/(4*b*d^4) + (B^3*(a + b*x)^4*Log[(e*(a + b*x)^n)/(c + d*x)^n]^3)/(4
*b) - (3*A*B^2*(b*c - a*d)^4*n^2*PolyLog[2, (d*(a + b*x))/(b*(c + d*x))])/(
2*b*d^4) - (11*B^3*(b*c - a*d)^4*n^3*PolyLog[2, (d*(a + b*x))/(b*(c + d*x))
])/(4*b*d^4) - (3*B^3*(b*c - a*d)^4*n^2*Log[(e*(a + b*x)^n)/(c + d*x)^n]*Po
lyLog[2, 1 - (b*c - a*d)/(b*(c + d*x))])/(2*b*d^4) + (3*B^3*(b*c - a*d)^4*n
^3*PolyLog[3, 1 - (b*c - a*d)/(b*(c + d*x))])/(2*b*d^4)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2333

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)/(x_))^(q_.)*(
x_)^(m_.), x_Symbol] :> Int[(e + d*x)^q*(a + b*Log[c*x^n])^p, x] /; FreeQ[{
a, b, c, d, e, m, n, p}, x] && EqQ[m, q] && IntegerQ[q]
```

Rule 2343

```
Int[((a_.) + Log[(c_.)*(x_)^(n_)]*(b_.))/((x_)*((d_) + (e_.)*(x_)^(r_.))),
x_Symbol] :> Dist[1/n, Subst[Int[(a + b*Log[c*x])/(x*(d + e*x^(r/n))), x],
```

```
x, x^n], x] /; FreeQ[{a, b, c, d, e, n, r}, x] && IntegerQ[r/n]
```

Rule 2411

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + (g_
.)*(x_))^(q_.)*((h_.) + (i_.)*(x_))^(r_.), x_Symbol] :> Dist[1/e, Subst[Int
[((g*x)/e)^q*((e*h - d*i)/e + (i*x)/e)^r*(a + b*Log[c*x^n])^p, x], x, d + e
*x], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, n, p, q, r}, x] && EqQ[e*f - d
*g, 0] && (IGtQ[p, 0] || IGtQ[r, 0]) && IntegerQ[2*r]
```

Rule 2486

```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))
^(r_.)]^(s_.), x_Symbol] :> Simp[((a + b*x)*Log[e*(f*(a + b*x)^p*(c + d*x)^
q)^r]^s)/b, x] + Dist[(q*r*s*(b*c - a*d))/b, Int[Log[e*(f*(a + b*x)^p*(c +
d*x)^q)^r]^(s - 1)/(c + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, p, q, r, s
}, x] && NeQ[b*c - a*d, 0] && EqQ[p + q, 0] && IGtQ[s, 0]
```

Rule 2488

```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))
^(r_.)]^(s_.)/((g_.) + (h_.)*(x_)), x_Symbol] :> -Simp[(Log[-((b*c - a*d)/(
d*(a + b*x)))]*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^s)/h, x] + Dist[(p*r*s*
(b*c - a*d))/h, Int[(Log[-((b*c - a*d)/(d*(a + b*x)))]*Log[e*(f*(a + b*x)^p
*(c + d*x)^q)^r]^(s - 1))/((a + b*x)*(c + d*x)), x], x] /; FreeQ[{a, b, c,
d, e, f, g, h, p, q, r, s}, x] && NeQ[b*c - a*d, 0] && EqQ[p + q, 0] && EqQ
[b*g - a*h, 0] && IGtQ[s, 0]
```

Rule 2492

```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))
^(r_.)]^(s_.)*((g_.) + (h_.)*(x_))^(m_.), x_Symbol] :> Simp[((g + h*x)^(m +
 1)*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^s)/(h*(m + 1)), x] - Dist[(p*r*s*(
b*c - a*d))/(h*(m + 1)), Int[((g + h*x)^(m + 1)*Log[e*(f*(a + b*x)^p*(c + d
*x)^q)^r]^(s - 1))/((a + b*x)*(c + d*x)), x], x] /; FreeQ[{a, b, c, d, e, f
, g, h, m, p, q, r, s}, x] && NeQ[b*c - a*d, 0] && EqQ[p + q, 0] && IGtQ[s,
 0] && NeQ[m, -1]
```

Rule 2506

```
Int[Log[v_]*Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_)
)^(q_.))^(r_.)]^(s_.)*(u_), x_Symbol] :> With[{g = Simplify[((v - 1)*(c + d
*x))/(a + b*x)], h = Simplify[u*(a + b*x)*(c + d*x)]}, -Simp[(h*PolyLog[2,
1 - v]*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^s)/(b*c - a*d), x] + Dist[h*p*r
*s, Int[(PolyLog[2, 1 - v]*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^(s - 1))/((
```

```
a + b*x)*(c + d*x)), x], x] /; FreeQ[{g, h}, x]] /; FreeQ[{a, b, c, d, e, f
, p, q, r, s}, x] && NeQ[b*c - a*d, 0] && IGtQ[s, 0] && EqQ[p + q, 0]
```

Rule 2514

```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))
^(r_.)]^(s_.)*(RFx_), x_Symbol] :> With[{u = ExpandIntegrand[Log[e*(f*(a +
b*x)^p*(c + d*x)^q)^r]^s, RFx, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c
, d, e, f, p, q, r, s}, x] && RationalFunctionQ[RFx, x] && IGtQ[s, 0]
```

Rule 6610

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\int (a+bx)^3 \left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^3\,dx = \int \left(A^3(a+bx)^3+3A^2B(a+bx)^3\log\left(e(a+bx)^n(c+dx)^{-n}\right.\right.$$

$$= \frac{A^3(a+bx)^4}{4b}+\left(3A^2B\right)\int (a+bx)^3\log\left(e(a+bx)^n(c+dx)^{-}\right.$$

$$= \frac{A^3(a+bx)^4}{4b}+\frac{3A^2B(a+bx)^4\log\left(e(a+bx)^n(c+dx)^{-n}\right)}{4b}+$$

$$= \frac{A^3(a+bx)^4}{4b}+\frac{3A^2B(a+bx)^4\log\left(e(a+bx)^n(c+dx)^{-n}\right)}{4b}+$$

$$= -\frac{3A^2B(bc-ad)^3nx}{4d^3}+\frac{3A^2B(bc-ad)^2n(a+bx)^2}{8bd^2}-\frac{A^2B(bc}{}$$

$$= -\frac{3A^2B(bc-ad)^3nx}{4d^3}+\frac{3A^2B(bc-ad)^2n(a+bx)^2}{8bd^2}-\frac{A^2B(bc}{}$$

$$= -\frac{3A^2B(bc-ad)^3nx}{4d^3}+\frac{3A^2B(bc-ad)^2n(a+bx)^2}{8bd^2}-\frac{A^2B(bc}{}$$

$$= -\frac{3A^2B(bc-ad)^3nx}{4d^3}-\frac{5AB^2(bc-ad)^3n^2x}{4d^3}+\frac{3A^2B(bc-ad}{8bd}$$

$$= -\frac{3A^2B(bc-ad)^3nx}{4d^3}-\frac{5AB^2(bc-ad)^3n^2x}{4d^3}+\frac{3A^2B(bc-ad}{8bd}$$

$$= -\frac{3A^2B(bc-ad)^3nx}{4d^3}-\frac{5AB^2(bc-ad)^3n^2x}{4d^3}+\frac{3A^2B(bc-ad}{8bd}$$

$$= -\frac{3A^2B(bc-ad)^3nx}{4d^3}-\frac{5AB^2(bc-ad)^3n^2x}{4d^3}-\frac{B^3(bc-ad)^3n}{4d^3}$$

$$= -\frac{3A^2B(bc-ad)^3nx}{4d^3}-\frac{5AB^2(bc-ad)^3n^2x}{4d^3}-\frac{B^3(bc-ad)^3n}{4d^3}$$

$$= -\frac{3A^2B(bc-ad)^3nx}{4d^3}-\frac{5AB^2(bc-ad)^3n^2x}{4d^3}-\frac{B^3(bc-ad)^3n}{4d^3}$$

**Mathematica [B]** time = 10.01, size = 9054, normalized size = 11.19

Result too large to show

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^3*(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])^3,x]
```

```
[Out] Result too large to show
```

**fricas [F]** time = 0.85, size = 0, normalized size = 0.00

$$\text{integral}\left(A^3b^3x^3 + 3\,A^3ab^2x^2 + 3\,A^3a^2bx + A^3a^3 + \left(B^3b^3x^3 + 3\,B^3ab^2x^2 + 3\,B^3a^2bx + B^3a^3\right)\log\left(\frac{(bx+a)^n e}{(dx+c)^n}\right)^3\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^3*(A+B*log(e*(b*x+a)^n/((d*x+c)^n)))^3,x, algorithm="fric
as")
```

```
[Out] integral(A^3*b^3*x^3 + 3*A^3*a*b^2*x^2 + 3*A^3*a^2*b*x + A^3*a^3 + (B^3*b^3
*x^3 + 3*B^3*a*b^2*x^2 + 3*B^3*a^2*b*x + B^3*a^3)*log((b*x + a)^n*e/(d*x +
c)^n)^3 + 3*(A*B^2*b^3*x^3 + 3*A*B^2*a*b^2*x^2 + 3*A*B^2*a^2*b*x + A*B^2*a^
3)*log((b*x + a)^n*e/(d*x + c)^n)^2 + 3*(A^2*B*b^3*x^3 + 3*A^2*B*a*b^2*x^2
+ 3*A^2*B*a^2*b*x + A^2*B*a^3)*log((b*x + a)^n*e/(d*x + c)^n), x)
```

**giac [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^3*(A+B*log(e*(b*x+a)^n/((d*x+c)^n)))^3,x, algorithm="giac
")
```

```
[Out] Timed out
```

**maple [F]** time = 8.02, size = 0, normalized size = 0.00

$$\int (bx+a)^3 \left(B\ln\left(e\,(bx+a)^n\,(dx+c)^{-n}\right)+A\right)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^3*(A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))^3,x)
```

```
[Out] int((b*x+a)^3*(A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))^3,x)
```

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^3*(A+B*log(e*(b*x+a)^n/((d*x+c)^n)))^3,x, algorithm="maxi
ma")
```

```
[Out] 3/4*A^2*B*b^3*x^4*log((b*x + a)^n*e/(d*x + c)^n) + 1/4*A^3*b^3*x^4 + 3*A^2*
B*a*b^2*x^3*log((b*x + a)^n*e/(d*x + c)^n) + A^3*a*b^2*x^3 + 9/2*A^2*B*a^2*
b*x^2*log((b*x + a)^n*e/(d*x + c)^n) + 3/2*A^3*a^2*b*x^2 + 3*A^2*B*a^3*x*lo
g((b*x + a)^n*e/(d*x + c)^n) + A^3*a^3*x + 3*(a*e*n*log(b*x + a)/b - c*e*n*
log(d*x + c)/d)*A^2*B*a^3/e - 9/2*(a^2*e*n*log(b*x + a)/b^2 - c^2*e*n*log(d
*x + c)/d^2 + (b*c*e*n - a*d*e*n)*x/(b*d))*A^2*B*a^2*b/e + 3/2*(2*a^3*e*n*l
og(b*x + a)/b^3 - 2*c^3*e*n*log(d*x + c)/d^3 - ((b^2*c*d*e*n - a*b*d^2*e*n)
*x^2 - 2*(b^2*c^2*e*n - a^2*d^2*e*n)*x)/(b^2*d^2))*A^2*B*a*b^2/e - 1/8*(6*a
^4*e*n*log(b*x + a)/b^4 - 6*c^4*e*n*log(d*x + c)/d^4 + (2*(b^3*c*d^2*e*n -
a*b^2*d^3*e*n)*x^3 - 3*(b^3*c^2*d*e*n - a^2*b*d^3*e*n)*x^2 + 6*(b^3*c^3*e*n
 - a^3*d^3*e*n)*x)/(b^3*d^3))*A^2*B*b^3/e - 1/8*(2*(B^3*b^4*d^4*x^4 + 4*B^3
*a*b^3*d^4*x^3 + 6*B^3*a^2*b^2*d^4*x^2 + 4*B^3*a^3*b*d^4*x)*log((d*x + c)^n
)^3 - (6*B^3*a^4*d^4*n*log(b*x + a) + 6*(B^3*b^4*d^4*log(e) + A*B^2*b^4*d^4
)*x^4 + 6*(b^4*c^4*n - 4*a*b^3*c^3*d*n + 6*a^2*b^2*c^2*d^2*n - 4*a^3*b*c*d^
3*n)*B^3*log(d*x + c) + 2*(12*A*B^2*a*b^3*d^4 + (a*b^3*d^4*(n + 12*log(e))
- b^4*c*d^3*n)*B^3)*x^3 + 3*(12*A*B^2*a^2*b^2*d^4 + (3*a^2*b^2*d^4*(n + 4*l
og(e)) + b^4*c^2*d^2*n - 4*a*b^3*c*d^3*n)*B^3)*x^2 + 6*(4*A*B^2*a^3*b*d^4 +
 (a^3*b*d^4*(3*n + 4*log(e)) - b^4*c^3*d*n + 4*a*b^3*c^2*d^2*n - 6*a^2*b^2*
c*d^3*n)*B^3)*x + 6*(B^3*b^4*d^4*x^4 + 4*B^3*a*b^3*d^4*x^3 + 6*B^3*a^2*b^2*
d^4*x^2 + 4*B^3*a^3*b*d^4*x)*log((b*x + a)^n))*log((d*x + c)^n)^2)/(b*d^4)
- integrate(-1/4*(4*B^3*a^3*b*c*d^3*log(e)^3 + 12*A*B^2*a^3*b*c*d^3*log(e)^
2 + 4*(B^3*b^4*d^4*log(e)^3 + 3*A*B^2*b^4*d^4*log(e)^2)*x^4 + 4*(3*(b^4*c*d
^3*log(e)^2 + 3*a*b^3*d^4*log(e)^2)*A*B^2 + (b^4*c*d^3*log(e)^3 + 3*a*b^3*d
^4*log(e)^3)*B^3)*x^3 + 4*(B^3*b^4*d^4*x^4 + B^3*a^3*b*c*d^3 + (b^4*c*d^3 +
 3*a*b^3*d^4)*B^3*x^3 + 3*(a*b^3*c*d^3 + a^2*b^2*d^4)*B^3*x^2 + (3*a^2*b^2*
c*d^3 + a^3*b*d^4)*B^3*x)*log((b*x + a)^n)^3 + 12*(3*(a*b^3*c*d^3*log(e)^2
+ a^2*b^2*d^4*log(e)^2)*A*B^2 + (a*b^3*c*d^3*log(e)^3 + a^2*b^2*d^4*log(e)^
3)*B^3)*x^2 + 12*(B^3*a^3*b*c*d^3*log(e) + A*B^2*a^3*b*c*d^3 + (B^3*b^4*d^4
*log(e) + A*B^2*b^4*d^4)*x^4 + ((b^4*c*d^3 + 3*a*b^3*d^4)*A*B^2 + (b^4*c*d^
3*log(e) + 3*a*b^3*d^4*log(e))*B^3)*x^3 + 3*((a*b^3*c*d^3 + a^2*b^2*d^4)*A*
B^2 + (a*b^3*c*d^3*log(e) + a^2*b^2*d^4*log(e))*B^3)*x^2 + ((3*a^2*b^2*c*d^
3 + a^3*b*d^4)*A*B^2 + (3*a^2*b^2*c*d^3*log(e) + a^3*b*d^4*log(e))*B^3)*x)*
log((b*x + a)^n)^2 + 4*(3*(3*a^2*b^2*c*d^3*log(e)^2 + a^3*b*d^4*log(e)^2)*A
*B^2 + (3*a^2*b^2*c*d^3*log(e)^3 + a^3*b*d^4*log(e)^3)*B^3)*x + 12*(B^3*a^3
*b*c*d^3*log(e)^2 + 2*A*B^2*a^3*b*c*d^3*log(e) + (B^3*b^4*d^4*log(e)^2 + 2*
A*B^2*b^4*d^4*log(e))*x^4 + (2*(b^4*c*d^3*log(e) + 3*a*b^3*d^4*log(e))*A*B^
```

```
2 + (b^4*c*d^3*log(e)^2 + 3*a*b^3*d^4*log(e)^2)*B^3)*x^3 + 3*(2*(a*b^3*c*d^
3*log(e) + a^2*b^2*d^4*log(e))*A*B^2 + (a*b^3*c*d^3*log(e)^2 + a^2*b^2*d^4*
log(e)^2)*B^3)*x^2 + (2*(3*a^2*b^2*c*d^3*log(e) + a^3*b*d^4*log(e))*A*B^2 +
 (3*a^2*b^2*c*d^3*log(e)^2 + a^3*b*d^4*log(e)^2)*B^3)*x)*log((b*x + a)^n) -
 (6*B^3*a^4*d^4*n^2*log(b*x + a) + 12*B^3*a^3*b*c*d^3*log(e)^2 + 24*A*B^2*a
^3*b*c*d^3*log(e) + 6*((n*log(e) + 2*log(e)^2)*B^3*b^4*d^4 + A*B^2*b^4*d^4*
(n + 4*log(e)))*x^4 + 6*(b^4*c^4*n^2 - 4*a*b^3*c^3*d*n^2 + 6*a^2*b^2*c^2*d^
2*n^2 - 4*a^3*b*c*d^3*n^2)*B^3*log(d*x + c) + 2*(12*(a*b^3*d^4*(n + 3*log(e
)) + b^4*c*d^3*log(e))*A*B^2 - ((n^2 - 6*log(e)^2)*b^4*c*d^3 - (n^2 + 12*n*
log(e) + 18*log(e)^2)*a*b^3*d^4)*B^3)*x^3 + 3*(12*(a^2*b^2*d^4*(n + 2*log(e
)) + 2*a*b^3*c*d^3*log(e))*A*B^2 + (b^4*c^2*d^2*n^2 - 4*(n^2 - 3*log(e)^2)*
a*b^3*c*d^3 + 3*(n^2 + 4*n*log(e) + 4*log(e)^2)*a^2*b^2*d^4)*B^3)*x^2 + 12*
(B^3*b^4*d^4*x^4 + B^3*a^3*b*c*d^3 + (b^4*c*d^3 + 3*a*b^3*d^4)*B^3*x^3 + 3*
(a*b^3*c*d^3 + a^2*b^2*d^4)*B^3*x^2 + (3*a^2*b^2*c*d^3 + a^3*b*d^4)*B^3*x)*
log((b*x + a)^n)^2 + 6*(4*(a^3*b*d^4*(n + log(e)) + 3*a^2*b^2*c*d^3*log(e))
*A*B^2 - (b^4*c^3*d*n^2 - 4*a*b^3*c^2*d^2*n^2 + 6*(n^2 - log(e)^2)*a^2*b^2*
c*d^3 - (3*n^2 + 4*n*log(e) + 2*log(e)^2)*a^3*b*d^4)*B^3)*x + 6*(4*B^3*a^3*
b*c*d^3*log(e) + 4*A*B^2*a^3*b*c*d^3 + (B^3*b^4*d^4*(n + 4*log(e)) + 4*A*B^
2*b^4*d^4)*x^4 + 4*((b^4*c*d^3 + 3*a*b^3*d^4)*A*B^2 + (a*b^3*d^4*(n + 3*log
(e)) + b^4*c*d^3*log(e))*B^3)*x^3 + 6*(2*(a*b^3*c*d^3 + a^2*b^2*d^4)*A*B^2
+ (a^2*b^2*d^4*(n + 2*log(e)) + 2*a*b^3*c*d^3*log(e))*B^3)*x^2 + 4*((3*a^2*
b^2*c*d^3 + a^3*b*d^4)*A*B^2 + (a^3*b*d^4*(n + log(e)) + 3*a^2*b^2*c*d^3*lo
g(e))*B^3)*x)*log((b*x + a)^n))*log((d*x + c)^n))/(b*d^4*x + b*c*d^3), x)
```

**mupad** **[F]** time = 0.00, size = -1, normalized size = -0.00

$$\int \left(A+B\,\ln\left(\frac{e\,(a+b\,x)^n}{(c+d\,x)^n}\right)\right)^3\,(a+b\,x)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*log((e*(a + b*x)^n)/(c + d*x)^n))^3*(a + b*x)^3,x)
[Out] int((A + B*log((e*(a + b*x)^n)/(c + d*x)^n))^3*(a + b*x)^3, x)
```

**sympy** **[F(-2)]** time = 0.00, size = 0, normalized size = 0.00

Exception raised: HeuristicGCDFailed

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**3*(A+B*ln(e*(b*x+a)**n/((d*x+c)**n)))**3,x)
[Out] Exception raised: HeuristicGCDFailed
```

# 3.165 $\int (a+bx)^2 \left(A + B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^3 dx$

**Optimal**. Leaf size=614

$$\frac{2B^2n^2(bc-ad)^3\text{Li}_2\left(\frac{d(a+bx)}{b(c+dx)}\right)\left(B\log\left(e(a+bx)^n(c+dx)^{-n}\right)+A\right)}{bd^3}+\frac{4B^2n^2(bc-ad)^3\log\left(\frac{bc-ad}{b(c+dx)}\right)\left(B\log\left(e(a+bx)^n\right.\right.}{bd^3}$$

```
[Out] -B^3*(-a*d+b*c)^3*n^3*ln(d*x+c)/b/d^3+B^2*(-a*d+b*c)^2*n^2*(b*x+a)*(A+B*ln(
e*(b*x+a)^n/((d*x+c)^n)))/b/d^2+4*B^2*(-a*d+b*c)^3*n^2*ln((-a*d+b*c)/b/(d*x
+c))*(A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))/b/d^3+2*B*(-a*d+b*c)^2*n*(b*x+a)*(A+
B*ln(e*(b*x+a)^n/((d*x+c)^n)))^2/b/d^2-1/2*b*B*(-a*d+b*c)*n*(d*x+c)^2*(A+B*
ln(e*(b*x+a)^n/((d*x+c)^n)))^2/d^3+B*(-a*d+b*c)^3*n*ln((-a*d+b*c)/b/(d*x+c)
)*(A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))^2/b/d^3+1/3*(b*x+a)^3*(A+B*ln(e*(b*x+a)
^n/((d*x+c)^n)))^3/b-B^2*(-a*d+b*c)^3*n^2*(A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))
*ln(1-b*(d*x+c)/d/(b*x+a))/b/d^3+4*B^3*(-a*d+b*c)^3*n^3*polylog(2,d*(b*x+a)
/b/(d*x+c))/b/d^3+2*B^2*(-a*d+b*c)^3*n^2*(A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))*
polylog(2,d*(b*x+a)/b/(d*x+c))/b/d^3+B^3*(-a*d+b*c)^3*n^3*polylog(2,b*(d*x+
c)/d/(b*x+a))/b/d^3-2*B^3*(-a*d+b*c)^3*n^3*polylog(3,d*(b*x+a)/b/(d*x+c))/b
/d^3
```

**Rubi [A]** time = 1.74, antiderivative size = 915, normalized size of antiderivative = 1.49, number of steps used = 40, number of rules used = 13, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.394$, Rules used = {6742, 2492, 43, 2514, 2486, 31, 2488, 2411, 2343, 2333, 2315, 2506, 6610}

$$\frac{(a+bx)^3\log^3\left(e(a+bx)^n(c+dx)^{-n}\right)B^3}{3b}-\frac{(bc-ad)n(a+bx)^2\log^2\left(e(a+bx)^n(c+dx)^{-n}\right)B^3}{2bd}+\frac{(bc-ad)^2n(a+bx)}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^2*(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])^3,x]
```

```
[Out] (A^2*B*(b*c - a*d)^2*n*x)/d^2 + (A*B^2*(b*c - a*d)^2*n^2*x)/d^2 - (A^2*B*(b
*c - a*d)*n*(a + b*x)^2)/(2*b*d) + (A^3*(a + b*x)^3)/(3*b) - (A^2*B*(b*c -
a*d)^3*n*Log[c + d*x])/(b*d^3) - (3*A*B^2*(b*c - a*d)^3*n^2*Log[c + d*x])/(
b*d^3) - (B^3*(b*c - a*d)^3*n^3*Log[c + d*x])/(b*d^3) + (2*A*B^2*(b*c - a*d
)^2*n*(a + b*x)*Log[(e*(a + b*x)^n)/(c + d*x)^n])/(b*d^2) + (B^3*(b*c - a*d
)^2*n^2*(a + b*x)*Log[(e*(a + b*x)^n)/(c + d*x)^n])/(b*d^2) - (A*B^2*(b*c -
 a*d)*n*(a + b*x)^2*Log[(e*(a + b*x)^n)/(c + d*x)^n])/(b*d) + (A^2*B*(a + b
*x)^3*Log[(e*(a + b*x)^n)/(c + d*x)^n])/b + (2*A*B^2*(b*c - a*d)^3*n*Log[(b
*c - a*d)/(b*(c + d*x))]*Log[(e*(a + b*x)^n)/(c + d*x)^n])/(b*d^3) + (3*B^3
*(b*c - a*d)^3*n^2*Log[(b*c - a*d)/(b*(c + d*x))]*Log[(e*(a + b*x)^n)/(c +
d*x)^n])/(b*d^3) + (B^3*(b*c - a*d)^2*n*(a + b*x)*Log[(e*(a + b*x)^n)/(c +
```

```
d*x)^n]^2)/(b*d^2) - (B^3*(b*c - a*d)*n*(a + b*x)^2*Log[(e*(a + b*x)^n)/(c
+ d*x)^n]^2)/(2*b*d) + (A*B^2*(a + b*x)^3*Log[(e*(a + b*x)^n)/(c + d*x)^n]^
2)/b + (B^3*(b*c - a*d)^3*n*Log[(b*c - a*d)/(b*(c + d*x))]*Log[(e*(a + b*x)
^n)/(c + d*x)^n]^2)/(b*d^3) + (B^3*(a + b*x)^3*Log[(e*(a + b*x)^n)/(c + d*x
)^n]^3)/(3*b) + (2*A*B^2*(b*c - a*d)^3*n^2*PolyLog[2, (d*(a + b*x))/(b*(c +
 d*x))])/(b*d^3) + (3*B^3*(b*c - a*d)^3*n^3*PolyLog[2, (d*(a + b*x))/(b*(c
+ d*x))])/(b*d^3) + (2*B^3*(b*c - a*d)^3*n^2*Log[(e*(a + b*x)^n)/(c + d*x)^
n]*PolyLog[2, 1 - (b*c - a*d)/(b*(c + d*x))])/(b*d^3) - (2*B^3*(b*c - a*d)^
3*n^3*PolyLog[3, 1 - (b*c - a*d)/(b*(c + d*x))])/(b*d^3)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2333

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)/(x_))^(q_.)*(
x_)^(m_.), x_Symbol] :> Int[(e + d*x)^q*(a + b*Log[c*x^n])^p, x] /; FreeQ[{
a, b, c, d, e, m, n, p}, x] && EqQ[m, q] && IntegerQ[q]
```

Rule 2343

```
Int[((a_.) + Log[(c_.)*(x_)^(n_)]*(b_.))/((x_)*((d_) + (e_.)*(x_)^(r_.))),
x_Symbol] :> Dist[1/n, Subst[Int[(a + b*Log[c*x])/(x*(d + e*x^(r/n))), x],
x, x^n], x] /; FreeQ[{a, b, c, d, e, n, r}, x] && IntegerQ[r/n]
```

Rule 2411

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + (g_
.)*(x_))^(q_.)*((h_.) + (i_.)*(x_))^(r_.), x_Symbol] :> Dist[1/e, Subst[Int
[((g*x)/e)^q*((e*h - d*i)/e + (i*x)/e)^r*(a + b*Log[c*x^n])^p, x], x, d + e
*x], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, n, p, q, r}, x] && EqQ[e*f - d
```

```
*g, 0] && (IGtQ[p, 0] || IGtQ[r, 0]) && IntegerQ[2*r]
```

Rule 2486

```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))
^(r_.)]^(s_.), x_Symbol] :> Simp[((a + b*x)*Log[e*(f*(a + b*x)^p*(c + d*x)^
q)^r]^s)/b, x] + Dist[(q*r*s*(b*c - a*d))/b, Int[Log[e*(f*(a + b*x)^p*(c +
d*x)^q)^r]^(s - 1)/(c + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, p, q, r, s
}, x] && NeQ[b*c - a*d, 0] && EqQ[p + q, 0] && IGtQ[s, 0]
```

Rule 2488

```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))
^(r_.)]^(s_.)/((g_.) + (h_.)*(x_)), x_Symbol] :> -Simp[(Log[-((b*c - a*d)/(
d*(a + b*x)))]*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^s)/h, x] + Dist[(p*r*s*
(b*c - a*d))/h, Int[(Log[-((b*c - a*d)/(d*(a + b*x)))]*Log[e*(f*(a + b*x)^p
*(c + d*x)^q)^r]^(s - 1))/((a + b*x)*(c + d*x)), x], x] /; FreeQ[{a, b, c,
d, e, f, g, h, p, q, r, s}, x] && NeQ[b*c - a*d, 0] && EqQ[p + q, 0] && EqQ
[b*g - a*h, 0] && IGtQ[s, 0]
```

Rule 2492

```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))
^(r_.)]^(s_.)*((g_.) + (h_.)*(x_))^(m_.), x_Symbol] :> Simp[((g + h*x)^(m +
 1)*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^s)/(h*(m + 1)), x] - Dist[(p*r*s*(
b*c - a*d))/(h*(m + 1)), Int[((g + h*x)^(m + 1)*Log[e*(f*(a + b*x)^p*(c + d
*x)^q)^r]^(s - 1))/((a + b*x)*(c + d*x)), x], x] /; FreeQ[{a, b, c, d, e, f
, g, h, m, p, q, r, s}, x] && NeQ[b*c - a*d, 0] && EqQ[p + q, 0] && IGtQ[s,
 0] && NeQ[m, -1]
```

Rule 2506

```
Int[Log[v_]*Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_)
)^(q_.))^(r_.)]^(s_.)*(u_), x_Symbol] :> With[{g = Simplify[((v - 1)*(c + d
*x))/(a + b*x)], h = Simplify[u*(a + b*x)*(c + d*x)]}, -Simp[(h*PolyLog[2,
1 - v]*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^s)/(b*c - a*d), x] + Dist[h*p*r
*s, Int[(PolyLog[2, 1 - v]*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^(s - 1))/((
a + b*x)*(c + d*x)), x], x] /; FreeQ[{g, h}, x]] /; FreeQ[{a, b, c, d, e, f
, p, q, r, s}, x] && NeQ[b*c - a*d, 0] && IGtQ[s, 0] && EqQ[p + q, 0]
```

Rule 2514

```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))
^(r_.)]^(s_.)*(RFx_), x_Symbol] :> With[{u = ExpandIntegrand[Log[e*(f*(a +
b*x)^p*(c + d*x)^q)^r]^s, RFx, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c
```

```
, d, e, f, p, q, r, s}, x] && RationalFunctionQ[RFx, x] && IGtQ[s, 0]
```

Rule 6610

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\int (a+bx)^2 \left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^3 dx = \int \left(A^3(a+bx)^2 + 3A^2B(a+bx)^2\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right.$$

$$= \frac{A^3(a+bx)^3}{3b} + \left(3A^2B\right)\int (a+bx)^2\log\left(e(a+bx)^n(c+dx)^{-}\right.$$

$$= \frac{A^3(a+bx)^3}{3b} + \frac{A^2B(a+bx)^3\log\left(e(a+bx)^n(c+dx)^{-n}\right)}{b} + \cdots$$

$$= \frac{A^3(a+bx)^3}{3b} + \frac{A^2B(a+bx)^3\log\left(e(a+bx)^n(c+dx)^{-n}\right)}{b} + \cdots$$

$$= \frac{A^2B(bc-ad)^2nx}{d^2} - \frac{A^2B(bc-ad)n(a+bx)^2}{2bd} + \frac{A^3(a+bx)^3}{3b}$$

$$= \frac{A^2B(bc-ad)^2nx}{d^2} - \frac{A^2B(bc-ad)n(a+bx)^2}{2bd} + \frac{A^3(a+bx)^3}{3b}$$

$$= \frac{A^2B(bc-ad)^2nx}{d^2} - \frac{A^2B(bc-ad)n(a+bx)^2}{2bd} + \frac{A^3(a+bx)^3}{3b}$$

$$= \frac{A^2B(bc-ad)^2nx}{d^2} + \frac{AB^2(bc-ad)^2n^2x}{d^2} - \frac{A^2B(bc-ad)n(a}{2bd}$$

$$= \frac{A^2B(bc-ad)^2nx}{d^2} + \frac{AB^2(bc-ad)^2n^2x}{d^2} - \frac{A^2B(bc-ad)n(a}{2bd}$$

$$= \frac{A^2B(bc-ad)^2nx}{d^2} + \frac{AB^2(bc-ad)^2n^2x}{d^2} - \frac{A^2B(bc-ad)n(a}{2bd}$$

$$= \frac{A^2B(bc-ad)^2nx}{d^2} + \frac{AB^2(bc-ad)^2n^2x}{d^2} - \frac{A^2B(bc-ad)n(a}{2bd}$$

$$= \frac{A^2B(bc-ad)^2nx}{d^2} + \frac{AB^2(bc-ad)^2n^2x}{d^2} - \frac{A^2B(bc-ad)n(a}{2bd}$$

$$= \frac{A^2B(bc-ad)^2nx}{d^2} + \frac{AB^2(bc-ad)^2n^2x}{d^2} - \frac{A^2B(bc-ad)n(a}{2bd}$$

**Mathematica [B]** time = 4.22, size = 5668, normalized size = 9.23

Result too large to show

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^2*(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])^3,x]
[Out] Result too large to show
```

**fricas [F]** time = 0.93, size = 0, normalized size = 0.00

$$\text{integral}\left(A^3b^2x^2 + 2\,A^3abx + A^3a^2 + \left(B^3b^2x^2 + 2\,B^3abx + B^3a^2\right)\log\left(\frac{(bx+a)^n e}{(dx+c)^n}\right)^3 + 3\left(AB^2b^2x^2 + 2\,AB^2abx\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2*(A+B*log(e*(b*x+a)^n/((d*x+c)^n)))^3,x, algorithm="fric
as")
[Out] integral(A^3*b^2*x^2 + 2*A^3*a*b*x + A^3*a^2 + (B^3*b^2*x^2 + 2*B^3*a*b*x +
 B^3*a^2)*log((b*x + a)^n*e/(d*x + c)^n)^3 + 3*(A*B^2*b^2*x^2 + 2*A*B^2*a*b
*x + A*B^2*a^2)*log((b*x + a)^n*e/(d*x + c)^n)^2 + 3*(A^2*B*b^2*x^2 + 2*A^2
*B*a*b*x + A^2*B*a^2)*log((b*x + a)^n*e/(d*x + c)^n), x)
```

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int (bx+a)^2\left(B\log\left(\frac{(bx+a)^n e}{(dx+c)^n}\right)+A\right)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2*(A+B*log(e*(b*x+a)^n/((d*x+c)^n)))^3,x, algorithm="giac
")
[Out] integrate((b*x + a)^2*(B*log((b*x + a)^n*e/(d*x + c)^n) + A)^3, x)
```

**maple [F]** time = 7.01, size = 0, normalized size = 0.00

$$\int (bx+a)^2\left(B\ln\left(e\,(bx+a)^n\,(dx+c)^{-n}\right)+A\right)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^2*(A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))^3,x)
[Out] int((b*x+a)^2*(A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))^3,x)
```

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2*(A+B*log(e*(b*x+a)^n/((d*x+c)^n)))^3,x, algorithm="maxi
ma")
```

```
[Out] A^2*B*b^2*x^3*log((b*x + a)^n*e/(d*x + c)^n) + 1/3*A^3*b^2*x^3 + 3*A^2*B*a*
b*x^2*log((b*x + a)^n*e/(d*x + c)^n) + A^3*a*b*x^2 + 3*A^2*B*a^2*x*log((b*x
 + a)^n*e/(d*x + c)^n) + A^3*a^2*x + 3*(a*e*n*log(b*x + a)/b - c*e*n*log(d*
x + c)/d)*A^2*B*a^2/e - 3*(a^2*e*n*log(b*x + a)/b^2 - c^2*e*n*log(d*x + c)/
d^2 + (b*c*e*n - a*d*e*n)*x/(b*d))*A^2*B*a*b/e + 1/2*(2*a^3*e*n*log(b*x + a
)/b^3 - 2*c^3*e*n*log(d*x + c)/d^3 - ((b^2*c*d*e*n - a*b*d^2*e*n)*x^2 - 2*(
b^2*c^2*e*n - a^2*d^2*e*n)*x)/(b^2*d^2))*A^2*B*b^2/e - 1/6*(2*(B^3*b^3*d^3*
x^3 + 3*B^3*a*b^2*d^3*x^2 + 3*B^3*a^2*b*d^3*x)*log((d*x + c)^n)^3 - 3*(2*B^
3*a^3*d^3*n*log(b*x + a) - 2*(b^3*c^3*n - 3*a*b^2*c^2*d*n + 3*a^2*b*c*d^2*n
)*B^3*log(d*x + c) + 2*(B^3*b^3*d^3*log(e) + A*B^2*b^3*d^3)*x^3 + (6*A*B^2*
a*b^2*d^3 + (a*b^2*d^3*(n + 6*log(e)) - b^3*c*d^2*n)*B^3)*x^2 + 2*(3*A*B^2*
a^2*b*d^3 + (a^2*b*d^3*(2*n + 3*log(e)) + b^3*c^2*d*n - 3*a*b^2*c*d^2*n)*B^
3)*x + 2*(B^3*b^3*d^3*x^3 + 3*B^3*a*b^2*d^3*x^2 + 3*B^3*a^2*b*d^3*x)*log((b
*x + a)^n))*log((d*x + c)^n)^2)/(b*d^3) - integrate(-(B^3*a^2*b*c*d^2*log(e
)^3 + 3*A*B^2*a^2*b*c*d^2*log(e)^2 + (B^3*b^3*d^3*log(e)^3 + 3*A*B^2*b^3*d^
3*log(e)^2)*x^3 + (B^3*b^3*d^3*x^3 + B^3*a^2*b*c*d^2 + (b^3*c*d^2 + 2*a*b^2
*d^3)*B^3*x^2 + (2*a*b^2*c*d^2 + a^2*b*d^3)*B^3*x)*log((b*x + a)^n)^3 + (3*
(b^3*c*d^2*log(e)^2 + 2*a*b^2*d^3*log(e)^2)*A*B^2 + (b^3*c*d^2*log(e)^3 + 2
*a*b^2*d^3*log(e)^3)*B^3)*x^2 + 3*(B^3*a^2*b*c*d^2*log(e) + A*B^2*a^2*b*c*d
^2 + (B^3*b^3*d^3*log(e) + A*B^2*b^3*d^3)*x^3 + ((b^3*c*d^2 + 2*a*b^2*d^3)*
A*B^2 + (b^3*c*d^2*log(e) + 2*a*b^2*d^3*log(e))*B^3)*x^2 + ((2*a*b^2*c*d^2
+ a^2*b*d^3)*A*B^2 + (2*a*b^2*c*d^2*log(e) + a^2*b*d^3*log(e))*B^3)*x)*log(
(b*x + a)^n)^2 + (3*(2*a*b^2*c*d^2*log(e)^2 + a^2*b*d^3*log(e)^2)*A*B^2 + (
2*a*b^2*c*d^2*log(e)^3 + a^2*b*d^3*log(e)^3)*B^3)*x + 3*(B^3*a^2*b*c*d^2*lo
g(e)^2 + 2*A*B^2*a^2*b*c*d^2*log(e) + (B^3*b^3*d^3*log(e)^2 + 2*A*B^2*b^3*d
^3*log(e))*x^3 + (2*(b^3*c*d^2*log(e) + 2*a*b^2*d^3*log(e))*A*B^2 + (b^3*c*
d^2*log(e)^2 + 2*a*b^2*d^3*log(e)^2)*B^3)*x^2 + (2*(2*a*b^2*c*d^2*log(e) +
a^2*b*d^3*log(e))*A*B^2 + (2*a*b^2*c*d^2*log(e)^2 + a^2*b*d^3*log(e)^2)*B^3
)*x)*log((b*x + a)^n) - (2*B^3*a^3*d^3*n^2*log(b*x + a) + 3*B^3*a^2*b*c*d^2
*log(e)^2 + 6*A*B^2*a^2*b*c*d^2*log(e) - 2*(b^3*c^3*n^2 - 3*a*b^2*c^2*d*n^2
 + 3*a^2*b*c*d^2*n^2)*B^3*log(d*x + c) + ((2*n*log(e) + 3*log(e)^2)*B^3*b^3
*d^3 + 2*A*B^2*b^3*d^3*(n + 3*log(e)))*x^3 + (6*(a*b^2*d^3*(n + 2*log(e)) +
 b^3*c*d^2*log(e))*A*B^2 - ((n^2 - 3*log(e)^2)*b^3*c*d^2 - (n^2 + 6*n*log(e
) + 6*log(e)^2)*a*b^2*d^3)*B^3)*x^2 + 3*(B^3*b^3*d^3*x^3 + B^3*a^2*b*c*d^2
+ (b^3*c*d^2 + 2*a*b^2*d^3)*B^3*x^2 + (2*a*b^2*c*d^2 + a^2*b*d^3)*B^3*x)*lo
g((b*x + a)^n)^2 + (6*(a^2*b*d^3*(n + log(e)) + 2*a*b^2*c*d^2*log(e))*A*B^2
```

```
 + (2*b^3*c^2*d*n^2 - 6*(n^2 - log(e)^2)*a*b^2*c*d^2 + (4*n^2 + 6*n*log(e)
+ 3*log(e)^2)*a^2*b*d^3)*B^3)*x + 2*(3*B^3*a^2*b*c*d^2*log(e) + 3*A*B^2*a^2
*b*c*d^2 + (B^3*b^3*d^3*(n + 3*log(e)) + 3*A*B^2*b^3*d^3)*x^3 + 3*((b^3*c*d
^2 + 2*a*b^2*d^3)*A*B^2 + (a*b^2*d^3*(n + 2*log(e)) + b^3*c*d^2*log(e))*B^3
)*x^2 + 3*((2*a*b^2*c*d^2 + a^2*b*d^3)*A*B^2 + (a^2*b*d^3*(n + log(e)) + 2*
a*b^2*c*d^2*log(e))*B^3)*x)*log((b*x + a)^n))*log((d*x + c)^n))/(b*d^3*x +
b*c*d^2), x)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.00

$$\int \left(A + B\,\ln\left(\frac{e\,(a + b\,x)^n}{(c + d\,x)^n}\right)\right)^3 (a + b\,x)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*log((e*(a + b*x)^n)/(c + d*x)^n))^3*(a + b*x)^2,x)
[Out] int((A + B*log((e*(a + b*x)^n)/(c + d*x)^n))^3*(a + b*x)^2, x)
```

**sympy [F(-2)]** time = 0.00, size = 0, normalized size = 0.00

Exception raised: HeuristicGCDFailed

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**2*(A+B*ln(e*(b*x+a)**n/((d*x+c)**n)))**3,x)
[Out] Exception raised: HeuristicGCDFailed
```

# 3.166 $\int(a+bx)\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^3\,dx$

**Optimal**. Leaf size=376

$$-\frac{3B^2n^2(bc-ad)^2\text{Li}_2\left(\frac{d(a+bx)}{b(c+dx)}\right)\left(B\log\left(e(a+bx)^n(c+dx)^{-n}\right)+A\right)}{bd^2}-\frac{3B^2n^2(bc-ad)^2\log\left(\frac{bc-ad}{b(c+dx)}\right)\left(B\log\left(e(a+bx)\right.\right.}{bd^2}$$

```
[Out] -3*B^2*(-a*d+b*c)^2*n^2*ln((-a*d+b*c)/b/(d*x+c))*(A+B*ln(e*(b*x+a)^n/((d*x+
c)^n)))/b/d^2-3/2*B*(-a*d+b*c)*n*(b*x+a)*(A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))^
2/b/d-3/2*B*(-a*d+b*c)^2*n*ln((-a*d+b*c)/b/(d*x+c))*(A+B*ln(e*(b*x+a)^n/((d
*x+c)^n)))^2/b/d^2+1/2*(b*x+a)^2*(A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))^3/b-3*B^
3*(-a*d+b*c)^2*n^3*polylog(2,d*(b*x+a)/b/(d*x+c))/b/d^2-3*B^2*(-a*d+b*c)^2*
n^2*(A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))*polylog(2,d*(b*x+a)/b/(d*x+c))/b/d^2+
3*B^3*(-a*d+b*c)^2*n^3*polylog(3,d*(b*x+a)/b/(d*x+c))/b/d^2
```

**Rubi [A]** time = 1.18, antiderivative size = 700, normalized size of antiderivative = 1.86, number of steps used = 27, number of rules used = 13, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.419, Rules used = {6742, 2492, 43, 2514, 2486, 31, 2488, 2411, 2343, 2333, 2315, 2506, 6610}

$$-\frac{3AB^2n^2(bc-ad)^2\text{PolyLog}\left(2,\frac{d(a+bx)}{b(c+dx)}\right)}{bd^2}-\frac{3B^3n^2(bc-ad)^2\text{PolyLog}\left(2,1-\frac{bc-ad}{b(c+dx)}\right)\log\left(e(a+bx)^n(c+dx)^{-n}\right)}{bd^2}-\frac{3B}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)*(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])^3,x]
```

```
[Out] (-3*A^2*B*(b*c - a*d)*n*x)/(2*d) + (A^3*(a + b*x)^2)/(2*b) + (3*A^2*B*(b*c
- a*d)^2*n*Log[c + d*x])/(2*b*d^2) + (3*A*B^2*(b*c - a*d)^2*n^2*Log[c + d*x
])/(b*d^2) - (3*A*B^2*(b*c - a*d)*n*(a + b*x)*Log[(e*(a + b*x)^n)/(c + d*x)
^n])/(b*d) + (3*A^2*B*(a + b*x)^2*Log[(e*(a + b*x)^n)/(c + d*x)^n])/(2*b) -
 (3*A*B^2*(b*c - a*d)^2*n*Log[(b*c - a*d)/(b*(c + d*x))]*Log[(e*(a + b*x)^n
)/(c + d*x)^n])/(b*d^2) - (3*B^3*(b*c - a*d)^2*n^2*Log[(b*c - a*d)/(b*(c +
d*x))]*Log[(e*(a + b*x)^n)/(c + d*x)^n])/(b*d^2) - (3*B^3*(b*c - a*d)*n*(a
+ b*x)*Log[(e*(a + b*x)^n)/(c + d*x)^n]^2)/(2*b*d) + (3*A*B^2*(a + b*x)^2*L
og[(e*(a + b*x)^n)/(c + d*x)^n]^2)/(2*b) - (3*B^3*(b*c - a*d)^2*n*Log[(b*c
- a*d)/(b*(c + d*x))]*Log[(e*(a + b*x)^n)/(c + d*x)^n]^2)/(2*b*d^2) + (B^3*
(a + b*x)^2*Log[(e*(a + b*x)^n)/(c + d*x)^n]^3)/(2*b) - (3*A*B^2*(b*c - a*d
)^2*n^2*PolyLog[2, (d*(a + b*x))/(b*(c + d*x))])/(b*d^2) - (3*B^3*(b*c - a*
d)^2*n^3*PolyLog[2, (d*(a + b*x))/(b*(c + d*x))])/(b*d^2) - (3*B^3*(b*c - a
*d)^2*n^2*Log[(e*(a + b*x)^n)/(c + d*x)^n]*PolyLog[2, 1 - (b*c - a*d)/(b*(c
 + d*x))])/(b*d^2) + (3*B^3*(b*c - a*d)^2*n^3*PolyLog[3, 1 - (b*c - a*d)/(b
*(c + d*x))])/(b*d^2)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2333

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)/(x_))^(q_.)*(
x_)^(m_.), x_Symbol] :> Int[(e + d*x)^q*(a + b*Log[c*x^n])^p, x] /; FreeQ[{
a, b, c, d, e, m, n, p}, x] && EqQ[m, q] && IntegerQ[q]
```

Rule 2343

```
Int[((a_.) + Log[(c_.)*(x_)^(n_)]*(b_.))/((x_)*((d_) + (e_.)*(x_)^(r_.))),
x_Symbol] :> Dist[1/n, Subst[Int[(a + b*Log[c*x])/(x*(d + e*x^(r/n))), x],
x, x^n], x] /; FreeQ[{a, b, c, d, e, n, r}, x] && IntegerQ[r/n]
```

Rule 2411

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + (g_
.)*(x_))^(q_.)*((h_.) + (i_.)*(x_))^(r_.), x_Symbol] :> Dist[1/e, Subst[Int
[((g*x)/e)^q*((e*h - d*i)/e + (i*x)/e)^r*(a + b*Log[c*x^n])^p, x], x, d + e
*x], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, n, p, q, r}, x] && EqQ[e*f - d
*g, 0] && (IGtQ[p, 0] || IGtQ[r, 0]) && IntegerQ[2*r]
```

Rule 2486

```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))
^(r_.)]^(s_.), x_Symbol] :> Simp[((a + b*x)*Log[e*(f*(a + b*x)^p*(c + d*x)^
q)^r]^s)/b, x] + Dist[(q*r*s*(b*c - a*d))/b, Int[Log[e*(f*(a + b*x)^p*(c +
d*x)^q)^r]^(s - 1)/(c + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, p, q, r, s
}, x] && NeQ[b*c - a*d, 0] && EqQ[p + q, 0] && IGtQ[s, 0]
```

Rule 2488

```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))
^(r_.)]^(s_.)/((g_.) + (h_.)*(x_)), x_Symbol] :> -Simp[(Log[-((b*c - a*d)/(
d*(a + b*x)))]*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^s)/h, x] + Dist[(p*r*s*
(b*c - a*d))/h, Int[(Log[-((b*c - a*d)/(d*(a + b*x)))]*Log[e*(f*(a + b*x)^p
*(c + d*x)^q)^r]^(s - 1))/((a + b*x)*(c + d*x)), x], x] /; FreeQ[{a, b, c,
d, e, f, g, h, p, q, r, s}, x] && NeQ[b*c - a*d, 0] && EqQ[p + q, 0] && EqQ
[b*g - a*h, 0] && IGtQ[s, 0]
```

Rule 2492

```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))
^(r_.)]^(s_.)*((g_.) + (h_.)*(x_))^(m_.), x_Symbol] :> Simp[((g + h*x)^(m +
 1)*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^s)/(h*(m + 1)), x] - Dist[(p*r*s*(
b*c - a*d))/(h*(m + 1)), Int[((g + h*x)^(m + 1)*Log[e*(f*(a + b*x)^p*(c + d
*x)^q)^r]^(s - 1))/((a + b*x)*(c + d*x)), x], x] /; FreeQ[{a, b, c, d, e, f
, g, h, m, p, q, r, s}, x] && NeQ[b*c - a*d, 0] && EqQ[p + q, 0] && IGtQ[s,
 0] && NeQ[m, -1]
```

Rule 2506

```
Int[Log[v_]*Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_)
)^(q_.))^(r_.)]^(s_.)*(u_), x_Symbol] :> With[{g = Simplify[((v - 1)*(c + d
*x))/(a + b*x)], h = Simplify[u*(a + b*x)*(c + d*x)]}, -Simp[(h*PolyLog[2,
1 - v]*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^s)/(b*c - a*d), x] + Dist[h*p*r
*s, Int[(PolyLog[2, 1 - v]*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^(s - 1))/((
a + b*x)*(c + d*x)), x], x] /; FreeQ[{g, h}, x]] /; FreeQ[{a, b, c, d, e, f
, p, q, r, s}, x] && NeQ[b*c - a*d, 0] && IGtQ[s, 0] && EqQ[p + q, 0]
```

Rule 2514

```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))
^(r_.)]^(s_.)*(RFx_), x_Symbol] :> With[{u = ExpandIntegrand[Log[e*(f*(a +
b*x)^p*(c + d*x)^q)^r]^s, RFx, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c
, d, e, f, p, q, r, s}, x] && RationalFunctionQ[RFx, x] && IGtQ[s, 0]
```

Rule 6610

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
```

]

Rubi steps

$$\int (a+bx)\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^3\,dx = \int \left(A^3(a+bx)+3A^2B(a+bx)\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right.$$

$$= \frac{A^3(a+bx)^2}{2b}+\left(3A^2B\right)\int (a+bx)\log\left(e(a+bx)^n(c+dx)^{-n}\right)$$

$$= \frac{A^3(a+bx)^2}{2b}+\frac{3A^2B(a+bx)^2\log\left(e(a+bx)^n(c+dx)^{-n}\right)}{2b}+$$

$$= \frac{A^3(a+bx)^2}{2b}+\frac{3A^2B(a+bx)^2\log\left(e(a+bx)^n(c+dx)^{-n}\right)}{2b}+$$

$$= -\frac{3A^2B(bc-ad)nx}{2d}+\frac{A^3(a+bx)^2}{2b}+\frac{3A^2B(bc-ad)^2n\log(}{2bd^2}$$

$$= -\frac{3A^2B(bc-ad)nx}{2d}+\frac{A^3(a+bx)^2}{2b}+\frac{3A^2B(bc-ad)^2n\log(}{2bd^2}$$

$$= -\frac{3A^2B(bc-ad)nx}{2d}+\frac{A^3(a+bx)^2}{2b}+\frac{3A^2B(bc-ad)^2n\log(}{2bd^2}$$

$$= -\frac{3A^2B(bc-ad)nx}{2d}+\frac{A^3(a+bx)^2}{2b}+\frac{3A^2B(bc-ad)^2n\log(}{2bd^2}$$

$$= -\frac{3A^2B(bc-ad)nx}{2d}+\frac{A^3(a+bx)^2}{2b}+\frac{3A^2B(bc-ad)^2n\log(}{2bd^2}$$

$$= -\frac{3A^2B(bc-ad)nx}{2d}+\frac{A^3(a+bx)^2}{2b}+\frac{3A^2B(bc-ad)^2n\log(}{2bd^2}$$

$$= -\frac{3A^2B(bc-ad)nx}{2d}+\frac{A^3(a+bx)^2}{2b}+\frac{3A^2B(bc-ad)^2n\log(}{2bd^2}$$

**Mathematica [B]** time = 3.09, size = 3813, normalized size = 10.14

Result too large to show

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)*(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])^3,x]
```

```
[Out] (4*a^2*B^3*d^2*n^3*Log[a + b*x]^3 - 6*a^2*B^2*d^2*n^2*Log[a + b*x]^2*(2*A -
 B*n + 2*B*n*Log[c + d*x] + 2*B*Log[(e*(a + b*x)^n)/(c + d*x)^n]) + 6*a^2*B
*d^2*n*Log[a + b*x]*(2*A^2 - 2*A*B*n + B^2*n^2 + 2*B^2*n^2*Log[c + d*x]^2 -
 2*B*(-2*A + B*n)*Log[(e*(a + b*x)^n)/(c + d*x)^n] + 2*B^2*Log[(e*(a + b*x)
^n)/(c + d*x)^n]^2 + 2*B*n*Log[c + d*x]*(2*A - B*n + 2*B*Log[(e*(a + b*x)^n
)/(c + d*x)^n])) + b*(4*B^3*c*(b*c - 2*a*d)*n^3*Log[c + d*x]^3 + 6*B^2*d^2*
n^2*x*(2*a + b*x)*Log[c + d*x]^2*(2*A - B*n + 2*B*Log[(e*(a + b*x)^n)/(c +
d*x)^n]) + 6*B*d^2*n*x*(2*a + b*x)*Log[c + d*x]*(2*A^2 - 2*A*B*n + B^2*n^2
- 2*B*(-2*A + B*n)*Log[(e*(a + b*x)^n)/(c + d*x)^n] + 2*B^2*Log[(e*(a + b*x
)^n)/(c + d*x)^n]^2) + d^2*x*(2*a + b*x)*(4*A^3 - 6*A^2*B*n + 6*A*B^2*n^2 -
 3*B^3*n^3 + 6*B*(2*A^2 - 2*A*B*n + B^2*n^2)*Log[(e*(a + b*x)^n)/(c + d*x)^
n] - 6*B^2*(-2*A + B*n)*Log[(e*(a + b*x)^n)/(c + d*x)^n]^2 + 4*B^3*Log[(e*(
a + b*x)^n)/(c + d*x)^n]^3)))/(8*b*d^2) - (3*B*n*(-8*a*A*b*B*c*d*n + 16*a^2
*A*B*d^2*n - 8*b^2*B^2*c^2*n^2 + 16*a*b*B^2*c*d*n^2 - 8*a^2*B^2*d^2*n^2 + 4
*A^2*b^2*c*d*x - 8*a*A^2*b*d^2*x + 4*a*A*b*B*d^2*n*x - 2*a*b*B^2*d^2*n^2*x
- 2*A^2*b^2*d^2*x^2 + 2*A*b^2*B*d^2*n*x^2 - b^2*B^2*d^2*n^2*x^2 + 8*a*A*b*B
*c*d*n*Log[a + b*x] - 12*a^2*A*B*d^2*n*Log[a + b*x] + 8*a*b*B^2*c*d*n^2*Log
[a + b*x] - 14*a^2*B^2*d^2*n^2*Log[a + b*x] - 4*a*b*B^2*c*d*n^2*Log[a + b*x
]^2 + 6*a^2*B^2*d^2*n^2*Log[a + b*x]^2 - 4*A^2*b^2*c^2*Log[c + d*x] + 8*a*A
^2*b*c*d*Log[c + d*x] - 8*A*b^2*B*c^2*n*Log[c + d*x] + 8*a*A*b*B*c*d*n*Log[
c + d*x] - 8*a*b*B^2*c*d*n^2*Log[c + d*x] + 16*a^2*B^2*d^2*n^2*Log[c + d*x]
 + 8*a*A^2*b*d^2*x*Log[c + d*x] - 8*a*A*b*B*d^2*n*x*Log[c + d*x] + 4*a*b*B^
2*d^2*n^2*x*Log[c + d*x] + 4*A^2*b^2*d^2*x^2*Log[c + d*x] - 4*A*b^2*B*d^2*n
*x^2*Log[c + d*x] + 2*b^2*B^2*d^2*n^2*x^2*Log[c + d*x] + 8*A*b^2*B*c^2*n*Lo
g[a + b*x]*Log[c + d*x] - 16*a*A*b*B*c*d*n*Log[a + b*x]*Log[c + d*x] + 8*a^
2*A*B*d^2*n*Log[a + b*x]*Log[c + d*x] - 4*b^2*B^2*c^2*n^2*Log[a + b*x]*Log[
c + d*x] + 16*a*b*B^2*c*d*n^2*Log[a + b*x]*Log[c + d*x] - 16*a^2*B^2*d^2*n^
2*Log[a + b*x]*Log[c + d*x] - 4*b^2*B^2*c^2*n^2*Log[a + b*x]^2*Log[c + d*x]
 + 8*a*b*B^2*c*d*n^2*Log[a + b*x]^2*Log[c + d*x] - 4*a^2*B^2*d^2*n^2*Log[a
+ b*x]^2*Log[c + d*x] + 12*b^2*B^2*c^2*n^2*Log[(d*(a + b*x))/(-(b*c) + a*d)
]*Log[c + d*x] - 24*a*b*B^2*c*d*n^2*Log[(d*(a + b*x))/(-(b*c) + a*d)]*Log[c
 + d*x] + 12*a^2*B^2*d^2*n^2*Log[(d*(a + b*x))/(-(b*c) + a*d)]*Log[c + d*x]
 - 4*A*b^2*B*c^2*n*Log[c + d*x]^2 + 8*a*A*b*B*c*d*n*Log[c + d*x]^2 - 4*b^2*
B^2*c^2*n^2*Log[c + d*x]^2 + 4*a*b*B^2*c*d*n^2*Log[c + d*x]^2 + 8*a*A*b*B*d
^2*n*x*Log[c + d*x]^2 - 4*a*b*B^2*d^2*n^2*x*Log[c + d*x]^2 + 4*A*b^2*B*d^2*
n*x^2*Log[c + d*x]^2 - 2*b^2*B^2*d^2*n^2*x^2*Log[c + d*x]^2 + 8*b^2*B^2*c^2
*n^2*Log[a + b*x]*Log[c + d*x]^2 - 16*a*b*B^2*c*d*n^2*Log[a + b*x]*Log[c +
d*x]^2 + 8*a^2*B^2*d^2*n^2*Log[a + b*x]*Log[c + d*x]^2 - 4*b^2*B^2*c^2*n^2*
Log[(d*(a + b*x))/(-(b*c) + a*d)]*Log[c + d*x]^2 + 8*a*b*B^2*c*d*n^2*Log[(d
*(a + b*x))/(-(b*c) + a*d)]*Log[c + d*x]^2 - 4*a^2*B^2*d^2*n^2*Log[(d*(a +
b*x))/(-(b*c) + a*d)]*Log[c + d*x]^2 - 8*A*b^2*B*c^2*n*Log[a + b*x]*Log[(b*
```

(c + d*x))/(b*c - a*d)] + 16*a*A*b*B*c*d*n*Log[a + b*x]*Log[(b*(c + d*x))/(b*c - a*d)] - 8*a^2*A*B*d^2*n*Log[a + b*x]*Log[(b*(c + d*x))/(b*c - a*d)] + 4*b^2*B^2*c^2*n^2*Log[a + b*x]*Log[(b*(c + d*x))/(b*c - a*d)] - 8*a*b*B^2*c*d*n^2*Log[a + b*x]*Log[(b*(c + d*x))/(b*c - a*d)] + 4*a^2*B^2*d^2*n^2*Log[a + b*x]*Log[(b*(c + d*x))/(b*c - a*d)] + 4*b^2*B^2*c^2*n^2*Log[a + b*x]^2*Log[(b*(c + d*x))/(b*c - a*d)] - 8*a*b*B^2*c*d*n^2*Log[a + b*x]^2*Log[(b*(c + d*x))/(b*c - a*d)] + 4*a^2*B^2*d^2*n^2*Log[a + b*x]^2*Log[(b*(c + d*x))/(b*c - a*d)] - 8*b^2*B^2*c^2*n^2*Log[a + b*x]*Log[c + d*x]*Log[(b*(c + d*x))/(b*c - a*d)] + 16*a*b*B^2*c*d*n^2*Log[a + b*x]*Log[c + d*x]*Log[(b*(c + d*x))/(b*c - a*d)] - 8*a^2*B^2*d^2*n^2*Log[a + b*x]*Log[c + d*x]*Log[(b*(c + d*x))/(b*c - a*d)] - 8*a*b*B^2*c*d*n*Log[(e*(a + b*x)^n)/(c + d*x)^n] + 16*a^2*B^2*d^2*n*Log[(e*(a + b*x)^n)/(c + d*x)^n] + 8*A*b^2*B*c*d*x*Log[(e*(a + b*x)^n)/(c + d*x)^n] - 16*a*A*b*B*d^2*x*Log[(e*(a + b*x)^n)/(c + d*x)^n] + 4*a*b*B^2*d^2*n*x*Log[(e*(a + b*x)^n)/(c + d*x)^n] - 4*A*b^2*B*d^2*x^2*Log[(e*(a + b*x)^n)/(c + d*x)^n] + 2*b^2*B^2*d^2*n*x^2*Log[(e*(a + b*x)^n)/(c + d*x)^n] + 8*a*b*B^2*c*d*n*Log[a + b*x]*Log[(e*(a + b*x)^n)/(c + d*x)^n] - 12*a^2*B^2*d^2*n*Log[a + b*x]*Log[(e*(a + b*x)^n)/(c + d*x)^n] - 8*A*b^2*B*c^2*Log[c + d*x]*Log[(e*(a + b*x)^n)/(c + d*x)^n] + 16*a*A*b*B*c*d*Log[c + d*x]*Log[(e*(a + b*x)^n)/(c + d*x)^n] - 8*b^2*B^2*c^2*n*Log[c + d*x]*Log[(e*(a + b*x)^n)/(c + d*x)^n] + 8*a*b*B^2*c*d*n*Log[c + d*x]*Log[(e*(a + b*x)^n)/(c + d*x)^n] + 16*a*A*b*B*d^2*x*Log[c + d*x]*Log[(e*(a + b*x)^n)/(c + d*x)^n] - 8*a*b*B^2*d^2*n*x*Log[c + d*x]*Log[(e*(a + b*x)^n)/(c + d*x)^n] + 8*A*b^2*B*d^2*x^2*Log[c + d*x]*Log[(e*(a + b*x)^n)/(c + d*x)^n] - 4*b^2*B^2*d^2*n*x^2*Log[c + d*x]*Log[(e*(a + b*x)^n)/(c + d*x)^n] + 8*b^2*B^2*c^2*n*Log[a + b*x]*Log[c + d*x]*Log[(e*(a + b*x)^n)/(c + d*x)^n] - 16*a*b*B^2*c*d*n*Log[a + b*x]*Log[c + d*x]*Log[(e*(a + b*x)^n)/(c + d*x)^n] + 8*a^2*B^2*d^2*n*Log[a + b*x]*Log[c + d*x]*Log[(e*(a + b*x)^n)/(c + d*x)^n] - 4*b^2*B^2*c^2*n*Log[c + d*x]^2*Log[(e*(a + b*x)^n)/(c + d*x)^n] + 8*a*b*B^2*c*d*n*Log[c + d*x]^2*Log[(e*(a + b*x)^n)/(c + d*x)^n] + 8*a*b*B^2*d^2*n*x*Log[c + d*x]^2*Log[(e*(a + b*x)^n)/(c + d*x)^n] + 4*b^2*B^2*d^2*n*x^2*Log[c + d*x]^2*Log[(e*(a + b*x)^n)/(c + d*x)^n] - 8*b^2*B^2*c^2*n*Log[a + b*x]*Log[(b*(c + d*x))/(b*c - a*d)]*Log[(e*(a + b*x)^n)/(c + d*x)^n] + 16*a*b*B^2*c*d*n*Log[a + b*x]*Log[(b*(c + d*x))/(b*c - a*d)]*Log[(e*(a + b*x)^n)/(c + d*x)^n] - 8*a^2*B^2*d^2*n*Log[a + b*x]*Log[(b*(c + d*x))/(b*c - a*d)]*Log[(e*(a + b*x)^n)/(c + d*x)^n] + 4*b^2*B^2*c*d*x*Log[(e*(a + b*x)^n)/(c + d*x)^n]^2 - 8*a*b*B^2*d^2*x*Log[(e*(a + b*x)^n)/(c + d*x)^n]^2 - 2*b^2*B^2*d^2*x^2*Log[(e*(a + b*x)^n)/(c + d*x)^n]^2 - 4*b^2*B^2*c^2*Log[c + d*x]*Log[(e*(a + b*x)^n)/(c + d*x)^n]^2 + 8*a*b*B^2*c*d*Log[c + d*x]*Log[(e*(a + b*x)^n)/(c + d*x)^n]^2 + 8*a*b*B^2*d^2*x*Log[c + d*x]*Log[(e*(a + b*x)^n)/(c + d*x)^n]^2 + 4*b^2*B^2*d^2*x^2*Log[c + d*x]*Log[(e*(a + b*x)^n)/(c + d*x)^n]^2 - 4*B*(b*c - a*d)^2*n*(2*A - B*n + 2*B*n*Log[c + d*x] + 2*B*Log[(e*(a + b*x)^n)/(c + d*x)^n])*PolyLog[2, (d*(a + b*x))/(-(b*c) + a*d)] - 4*B^2*(b*c - a*d)^2*n^2*(-3 + 2*Log[c + d*x])*PolyLog[2, (b*(c + d*x))/(b*c - a*d)] + 8*b^2*B^2*c^2*n^2*PolyLog[3, (d*(a + b*x))/(-(b*c) + a*d)] - 16*a*b*B^2*c*d*n^2*PolyLog[3, (d*(a + b*x))/(-(b*c) + a*d)] + 8*a^2*B^2*d^2*n^2*PolyLog[3, (d*(

```
a + b*x))/(-(b*c) + a*d)] + 8*b^2*B^2*c^2*n^2*PolyLog[3, (b*(c + d*x))/(b*c
 - a*d)] - 16*a*b*B^2*c*d*n^2*PolyLog[3, (b*(c + d*x))/(b*c - a*d)] + 8*a^2
*B^2*d^2*n^2*PolyLog[3, (b*(c + d*x))/(b*c - a*d)]))/(8*b*d^2)
```

**fricas [F]** time = 0.82, size = 0, normalized size = 0.00

$$\text{integral}\left(A^3bx + A^3a + \left(B^3bx + B^3a\right)\log\left(\frac{(bx+a)^n e}{(dx+c)^n}\right)^3 + 3\left(AB^2bx + AB^2a\right)\log\left(\frac{(bx+a)^n e}{(dx+c)^n}\right)^2 + 3\left(A^2Bbx + A\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*(A+B*log(e*(b*x+a)^n/((d*x+c)^n)))^3,x, algorithm="fricas
")
```

```
[Out] integral(A^3*b*x + A^3*a + (B^3*b*x + B^3*a)*log((b*x + a)^n*e/(d*x + c)^n)
^3 + 3*(A*B^2*b*x + A*B^2*a)*log((b*x + a)^n*e/(d*x + c)^n)^2 + 3*(A^2*B*b*
x + A^2*B*a)*log((b*x + a)^n*e/(d*x + c)^n), x)
```

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int (bx+a)\left(B\log\left(\frac{(bx+a)^n e}{(dx+c)^n}\right) + A\right)^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*(A+B*log(e*(b*x+a)^n/((d*x+c)^n)))^3,x, algorithm="giac")
```

```
[Out] integrate((b*x + a)*(B*log((b*x + a)^n*e/(d*x + c)^n) + A)^3, x)
```

**maple [F]** time = 10.05, size = 0, normalized size = 0.00

$$\int (bx+a)\left(B\ln\left(e\,(bx+a)^n\,(dx+c)^{-n}\right) + A\right)^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)*(A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))^3,x)
```

```
[Out] int((b*x+a)*(A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))^3,x)
```

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*(A+B*log(e*(b*x+a)^n/((d*x+c)^n)))^3,x, algorithm="maxima
")
```

```
[Out] 3/2*A^2*B*b*x^2*log((b*x + a)^n*e/(d*x + c)^n) + 1/2*A^3*b*x^2 + 3*A^2*B*a*
x*log((b*x + a)^n*e/(d*x + c)^n) + A^3*a*x + 3*(a*e*n*log(b*x + a)/b - c*e*
n*log(d*x + c)/d)*A^2*B*a/e - 3/2*(a^2*e*n*log(b*x + a)/b^2 - c^2*e*n*log(d
*x + c)/d^2 + (b*c*e*n - a*d*e*n)*x/(b*d))*A^2*B*b/e - 1/2*((B^3*b^2*d^2*x^
2 + 2*B^3*a*b*d^2*x)*log((d*x + c)^n)^3 - 3*(B^3*a^2*d^2*n*log(b*x + a) + (
b^2*c^2*n - 2*a*b*c*d*n)*B^3*log(d*x + c) + (B^3*b^2*d^2*log(e) + A*B^2*b^2
*d^2)*x^2 + (2*A*B^2*a*b*d^2 + (a*b*d^2*(n + 2*log(e)) - b^2*c*d*n)*B^3)*x
+ (B^3*b^2*d^2*x^2 + 2*B^3*a*b*d^2*x)*log((b*x + a)^n))*log((d*x + c)^n)^2)
/(b*d^2) - integrate(-(B^3*a*b*c*d*log(e)^3 + 3*A*B^2*a*b*c*d*log(e)^2 + (B
^3*b^2*d^2*x^2 + B^3*a*b*c*d + (b^2*c*d + a*b*d^2)*B^3*x)*log((b*x + a)^n)^
3 + (B^3*b^2*d^2*log(e)^3 + 3*A*B^2*b^2*d^2*log(e)^2)*x^2 + 3*(B^3*a*b*c*d*
log(e) + A*B^2*a*b*c*d + (B^3*b^2*d^2*log(e) + A*B^2*b^2*d^2)*x^2 + ((b^2*c
*d + a*b*d^2)*A*B^2 + (b^2*c*d*log(e) + a*b*d^2*log(e))*B^3)*x)*log((b*x +
a)^n)^2 + (3*(b^2*c*d*log(e)^2 + a*b*d^2*log(e)^2)*A*B^2 + (b^2*c*d*log(e)^
3 + a*b*d^2*log(e)^3)*B^3)*x + 3*(B^3*a*b*c*d*log(e)^2 + 2*A*B^2*a*b*c*d*lo
g(e) + (B^3*b^2*d^2*log(e)^2 + 2*A*B^2*b^2*d^2*log(e))*x^2 + (2*(b^2*c*d*lo
g(e) + a*b*d^2*log(e))*A*B^2 + (b^2*c*d*log(e)^2 + a*b*d^2*log(e)^2)*B^3)*x
)*log((b*x + a)^n) - 3*(B^3*a^2*d^2*n^2*log(b*x + a) + B^3*a*b*c*d*log(e)^2
 + 2*A*B^2*a*b*c*d*log(e) + (b^2*c^2*n^2 - 2*a*b*c*d*n^2)*B^3*log(d*x + c)
+ ((n*log(e) + log(e)^2)*B^3*b^2*d^2 + A*B^2*b^2*d^2*(n + 2*log(e)))*x^2 +
(B^3*b^2*d^2*x^2 + B^3*a*b*c*d + (b^2*c*d + a*b*d^2)*B^3*x)*log((b*x + a)^n
)^2 + (2*(a*b*d^2*(n + log(e)) + b^2*c*d*log(e))*A*B^2 - ((n^2 - log(e)^2)*
b^2*c*d - (n^2 + 2*n*log(e) + log(e)^2)*a*b*d^2)*B^3)*x + (2*B^3*a*b*c*d*lo
g(e) + 2*A*B^2*a*b*c*d + (B^3*b^2*d^2*(n + 2*log(e)) + 2*A*B^2*b^2*d^2)*x^2
 + 2*((b^2*c*d + a*b*d^2)*A*B^2 + (a*b*d^2*(n + log(e)) + b^2*c*d*log(e))*B
^3)*x)*log((b*x + a)^n))*log((d*x + c)^n))/(b*d^2*x + b*c*d), x)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.00

$$\int \left(A+B\,\ln\left(\frac{e\,{\left(a+b\,x\right)}^n}{{\left(c+d\,x\right)}^n}\right)\right)^3\,\left(a+b\,x\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*log((e*(a + b*x)^n)/(c + d*x)^n))^3*(a + b*x),x)
```

```
[Out] int((A + B*log((e*(a + b*x)^n)/(c + d*x)^n))^3*(a + b*x), x)
```

**sympy [F(-2)]** time = 0.00, size = 0, normalized size = 0.00

Exception raised: HeuristicGCDFailed

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*(A+B*ln(e*(b*x+a)**n/((d*x+c)**n)))**3,x)
```

```
[Out] Exception raised: HeuristicGCDFailed
```

## 3.167 $\int \frac{\left(A+B\log(e(a+bx)^n(c+dx)^{-n})\right)^3}{a+bx}\,dx$

**Optimal**. Leaf size=186

$$\frac{6B^2n^2\text{Li}_3\left(\frac{b(c+dx)}{d(a+bx)}\right)\left(B\log\left(e(a+bx)^n(c+dx)^{-n}\right)+A\right)}{b}+\frac{3Bn\text{Li}_2\left(\frac{b(c+dx)}{d(a+bx)}\right)\left(B\log\left(e(a+bx)^n(c+dx)^{-n}\right)+A\right)^2}{b}-\frac{\log}{}$$

```
[Out] -(A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))^3*ln(1-b*(d*x+c)/d/(b*x+a))/b+3*B*n*(A+B
*ln(e*(b*x+a)^n/((d*x+c)^n)))^2*polylog(2,b*(d*x+c)/d/(b*x+a))/b+6*B^2*n^2*
(A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))*polylog(3,b*(d*x+c)/d/(b*x+a))/b+6*B^3*n^
3*polylog(4,b*(d*x+c)/d/(b*x+a))/b
```

**Rubi [B]** time = 0.85, antiderivative size = 424, normalized size of antiderivative = 2.28, number of steps used = 14, number of rules used = 9, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {6742, 2488, 2411, 2343, 2333, 2315, 2506, 6610, 2508}

$$\frac{3A^2Bn\text{PolyLog}\left(2,\frac{bc-ad}{d(a+bx)}+1\right)}{b}+\frac{6AB^2n\text{PolyLog}\left(2,\frac{bc-ad}{d(a+bx)}+1\right)\log\left(e(a+bx)^n(c+dx)^{-n}\right)}{b}+\frac{6AB^2n^2\text{PolyLog}\left(3\right.}{b}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])^3/(a + b*x),x]
```

```
[Out] (A^3*Log[a + b*x])/b - (3*A^2*B*Log[-((b*c - a*d)/(d*(a + b*x)))]*Log[(e*(a
 + b*x)^n)/(c + d*x)^n])/b - (3*A*B^2*Log[-((b*c - a*d)/(d*(a + b*x)))]*Log
[(e*(a + b*x)^n)/(c + d*x)^n]^2)/b - (B^3*Log[-((b*c - a*d)/(d*(a + b*x)))]
*Log[(e*(a + b*x)^n)/(c + d*x)^n]^3)/b + (3*A^2*B*n*PolyLog[2, 1 + (b*c - a
*d)/(d*(a + b*x))])/b + (6*A*B^2*n*Log[(e*(a + b*x)^n)/(c + d*x)^n]*PolyLog
[2, 1 + (b*c - a*d)/(d*(a + b*x))])/b + (3*B^3*n*Log[(e*(a + b*x)^n)/(c + d
*x)^n]^2*PolyLog[2, 1 + (b*c - a*d)/(d*(a + b*x))])/b + (6*A*B^2*n^2*PolyLo
g[3, 1 + (b*c - a*d)/(d*(a + b*x))])/b + (6*B^3*n^2*Log[(e*(a + b*x)^n)/(c
+ d*x)^n]*PolyLog[3, 1 + (b*c - a*d)/(d*(a + b*x))])/b + (6*B^3*n^3*PolyLog
[4, 1 + (b*c - a*d)/(d*(a + b*x))])/b
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2333

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)/(x_))^(q_.)*(
x_)^(m_.), x_Symbol] :> Int[(e + d*x)^q*(a + b*Log[c*x^n])^p, x] /; FreeQ[{
```

```
a, b, c, d, e, m, n, p}, x] && EqQ[m, q] && IntegerQ[q]
```

Rule 2343

```
Int[((a_.) + Log[(c_.)*(x_)^(n_)]*(b_.))/((x_)*((d_) + (e_.)*(x_)^(r_.))),
x_Symbol] :> Dist[1/n, Subst[Int[(a + b*Log[c*x])/(x*(d + e*x^(r/n))), x],
x, x^n], x] /; FreeQ[{a, b, c, d, e, n, r}, x] && IntegerQ[r/n]
```

Rule 2411

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + (g_
.)*(x_))^(q_.)*((h_.) + (i_.)*(x_))^(r_.), x_Symbol] :> Dist[1/e, Subst[Int
[((g*x)/e)^q*((e*h - d*i)/e + (i*x)/e)^r*(a + b*Log[c*x^n])^p, x], x, d + e
*x], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, n, p, q, r}, x] && EqQ[e*f - d
*g, 0] && (IGtQ[p, 0] || IGtQ[r, 0]) && IntegerQ[2*r]
```

Rule 2488

```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))
^(r_.)]^(s_.)/((g_.) + (h_.)*(x_)), x_Symbol] :> -Simp[(Log[-((b*c - a*d)/(
d*(a + b*x)))]*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^s)/h, x] + Dist[(p*r*s*
(b*c - a*d))/h, Int[(Log[-((b*c - a*d)/(d*(a + b*x)))]*Log[e*(f*(a + b*x)^p
*(c + d*x)^q)^r]^(s - 1))/((a + b*x)*(c + d*x)), x], x] /; FreeQ[{a, b, c,
d, e, f, g, h, p, q, r, s}, x] && NeQ[b*c - a*d, 0] && EqQ[p + q, 0] && EqQ
[b*g - a*h, 0] && IGtQ[s, 0]
```

Rule 2506

```
Int[Log[v_]*Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_)
)^(q_.))^(r_.)]^(s_.)*(u_), x_Symbol] :> With[{g = Simplify[((v - 1)*(c + d
*x))/(a + b*x)], h = Simplify[u*(a + b*x)*(c + d*x)]}, -Simp[(h*PolyLog[2,
1 - v]*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^s)/(b*c - a*d), x] + Dist[h*p*r
*s, Int[(PolyLog[2, 1 - v]*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^(s - 1))/((
a + b*x)*(c + d*x)), x], x] /; FreeQ[{g, h}, x]] /; FreeQ[{a, b, c, d, e, f
, p, q, r, s}, x] && NeQ[b*c - a*d, 0] && IGtQ[s, 0] && EqQ[p + q, 0]
```

Rule 2508

```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))
^(r_.)]^(s_.)*(u_)*PolyLog[n_, v_], x_Symbol] :> With[{g = Simplify[(v*(c +
 d*x))/(a + b*x)], h = Simplify[u*(a + b*x)*(c + d*x)]}, Simp[(h*PolyLog[n
+ 1, v]*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^s)/(b*c - a*d), x] - Dist[h*p*
r*s, Int[(PolyLog[n + 1, v]*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^(s - 1))/(
(a + b*x)*(c + d*x)), x], x] /; FreeQ[{g, h}, x]] /; FreeQ[{a, b, c, d, e,
```

```
f, n, p, q, r, s}, x] && NeQ[b*c - a*d, 0] && IGtQ[s, 0] && EqQ[p + q, 0]
```

Rule 6610

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\int \frac{\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^3}{a+bx}\,dx = \int\left(\frac{A^3}{a+bx}+\frac{3A^2B\log\left(e(a+bx)^n(c+dx)^{-n}\right)}{a+bx}+\frac{3AB^2\log^2\left(e(a+b\right.}{a+b}\right.$$

$$= \frac{A^3\log(a+bx)}{b}+\left(3A^2B\right)\int\frac{\log\left(e(a+bx)^n(c+dx)^{-n}\right)}{a+bx}\,dx+\left(3AB^2\right.$$

$$= \frac{A^3\log(a+bx)}{b}-\frac{3A^2B\log\left(-\frac{bc-ad}{d(a+bx)}\right)\log\left(e(a+bx)^n(c+dx)^{-n}\right)}{b}-$$

$$= \frac{A^3\log(a+bx)}{b}-\frac{3A^2B\log\left(-\frac{bc-ad}{d(a+bx)}\right)\log\left(e(a+bx)^n(c+dx)^{-n}\right)}{b}-$$

$$= \frac{A^3\log(a+bx)}{b}-\frac{3A^2B\log\left(-\frac{bc-ad}{d(a+bx)}\right)\log\left(e(a+bx)^n(c+dx)^{-n}\right)}{b}-$$

$$= \frac{A^3\log(a+bx)}{b}-\frac{3A^2B\log\left(-\frac{bc-ad}{d(a+bx)}\right)\log\left(e(a+bx)^n(c+dx)^{-n}\right)}{b}-$$

$$= \frac{A^3\log(a+bx)}{b}-\frac{3A^2B\log\left(-\frac{bc-ad}{d(a+bx)}\right)\log\left(e(a+bx)^n(c+dx)^{-n}\right)}{b}-$$

**Mathematica [B]** time = 0.99, size = 2513, normalized size = 13.51

Result too large to show

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])^3/(a + b*x),x]

[Out] (4*A^3*Log[a + b*x] - 6*A^2*B*n*Log[a + b*x]^2 + 4*A*B^2*n^2*Log[a + b*x]^3
 - B^3*n^3*Log[a + b*x]^4 + B^3*n^3*Log[(d*(a + b*x))/(-(b*c) + a*d)]^4 - 4
*B^3*n^3*Log[(d*(a + b*x))/(-(b*c) + a*d)]^3*Log[-((d*(a + b*x))/(b*(c + d*
x)))] + 6*B^3*n^3*Log[(d*(a + b*x))/(-(b*c) + a*d)]^2*Log[-((d*(a + b*x))/(
b*(c + d*x)))]^2 - 4*B^3*n^3*Log[(d*(a + b*x))/(-(b*c) + a*d)]*Log[-((d*(a
+ b*x))/(b*(c + d*x)))]^3 + B^3*n^3*Log[-((d*(a + b*x))/(b*(c + d*x)))]^4 -
 12*A*B^2*n^2*Log[a + b*x]*Log[c + d*x]^2 + 12*B^3*n^3*Log[a + b*x]^2*Log[c
 + d*x]^2 + 12*A*B^2*n^2*Log[(d*(a + b*x))/(-(b*c) + a*d)]*Log[c + d*x]^2 -
 12*B^3*n^3*Log[a + b*x]*Log[(d*(a + b*x))/(-(b*c) + a*d)]*Log[c + d*x]^2 -
 8*B^3*n^3*Log[a + b*x]*Log[c + d*x]^3 + 8*B^3*n^3*Log[(d*(a + b*x))/(-(b*c
) + a*d)]*Log[c + d*x]^3 + 12*A^2*B*n*Log[a + b*x]*Log[(b*(c + d*x))/(b*c -
 a*d)] - 12*A*B^2*n^2*Log[a + b*x]^2*Log[(b*(c + d*x))/(b*c - a*d)] + 4*B^3
*n^3*Log[a + b*x]^3*Log[(b*(c + d*x))/(b*c - a*d)] + 8*B^3*n^3*Log[(d*(a +
b*x))/(-(b*c) + a*d)]^3*Log[(b*(c + d*x))/(b*c - a*d)] - 12*B^3*n^3*Log[(d*
(a + b*x))/(-(b*c) + a*d)]^2*Log[-((d*(a + b*x))/(b*(c + d*x)))]*Log[(b*(c
+ d*x))/(b*c - a*d)] + 24*A*B^2*n^2*Log[a + b*x]*Log[c + d*x]*Log[(b*(c + d
*x))/(b*c - a*d)] - 24*B^3*n^3*Log[a + b*x]^2*Log[c + d*x]*Log[(b*(c + d*x)
)/(b*c - a*d)] + 12*B^3*n^3*Log[a + b*x]*Log[c + d*x]^2*Log[(b*(c + d*x))/(
b*c - a*d)] + 6*B^3*n^3*Log[a + b*x]^2*Log[(b*(c + d*x))/(b*c - a*d)]^2 + 1
2*B^3*n^3*Log[a + b*x]*Log[(d*(a + b*x))/(-(b*c) + a*d)]*Log[(b*(c + d*x))/
(b*c - a*d)]^2 - 18*B^3*n^3*Log[(d*(a + b*x))/(-(b*c) + a*d)]^2*Log[(b*(c +
 d*x))/(b*c - a*d)]^2 + 12*A^2*B*Log[a + b*x]*Log[(e*(a + b*x)^n)/(c + d*x)
^n] - 12*A*B^2*n*Log[a + b*x]^2*Log[(e*(a + b*x)^n)/(c + d*x)^n] + 4*B^3*n^
2*Log[a + b*x]^3*Log[(e*(a + b*x)^n)/(c + d*x)^n] - 12*B^3*n^2*Log[a + b*x]
*Log[c + d*x]^2*Log[(e*(a + b*x)^n)/(c + d*x)^n] + 12*B^3*n^2*Log[(d*(a + b
*x))/(-(b*c) + a*d)]*Log[c + d*x]^2*Log[(e*(a + b*x)^n)/(c + d*x)^n] + 24*A
*B^2*n*Log[a + b*x]*Log[(b*(c + d*x))/(b*c - a*d)]*Log[(e*(a + b*x)^n)/(c +
 d*x)^n] - 12*B^3*n^2*Log[a + b*x]^2*Log[(b*(c + d*x))/(b*c - a*d)]*Log[(e*
(a + b*x)^n)/(c + d*x)^n] + 24*B^3*n^2*Log[a + b*x]*Log[c + d*x]*Log[(b*(c
+ d*x))/(b*c - a*d)]*Log[(e*(a + b*x)^n)/(c + d*x)^n] + 12*A*B^2*Log[a + b*
x]*Log[(e*(a + b*x)^n)/(c + d*x)^n]^2 - 6*B^3*n*Log[a + b*x]^2*Log[(e*(a +
b*x)^n)/(c + d*x)^n]^2 + 12*B^3*n*Log[a + b*x]*Log[(b*(c + d*x))/(b*c - a*d
)]*Log[(e*(a + b*x)^n)/(c + d*x)^n]^2 + 4*B^3*Log[a + b*x]*Log[(e*(a + b*x)
^n)/(c + d*x)^n]^3 - 4*B^3*n^3*Log[-((d*(a + b*x))/(b*(c + d*x)))]^3*Log[(b
*c - a*d)/(b*c + b*d*x)] + 12*B*n*(A^2 + B^2*n^2*Log[(d*(a + b*x))/(-(b*c)
+ a*d)]^2 + B^2*n^2*Log[c + d*x]^2 + 2*B^2*n^2*Log[a + b*x]*Log[(b*(c + d*x
))/(b*c - a*d)] - 2*B^2*n^2*Log[(d*(a + b*x))/(-(b*c) + a*d)]*(Log[-((d*(a
+ b*x))/(b*(c + d*x)))] + Log[(b*(c + d*x))/(b*c - a*d)]) + 2*A*B*Log[(e*(a
 + b*x)^n)/(c + d*x)^n] + B^2*Log[(e*(a + b*x)^n)/(c + d*x)^n]^2 + 2*B*n*Lo
g[c + d*x]*(A - B*n*Log[a + b*x] + B*Log[(e*(a + b*x)^n)/(c + d*x)^n]))*Pol
yLog[2, (d*(a + b*x))/(-(b*c) + a*d)] - 12*B^3*n^3*Log[-((d*(a + b*x))/(b*(
c + d*x)))]^2*PolyLog[2, (d*(a + b*x))/(b*(c + d*x))] + 12*B^3*n^3*Log[(d*(
a + b*x))/(-(b*c) + a*d)]^2*PolyLog[2, (b*(c + d*x))/(b*c - a*d)] - 24*B^3*
```

```
n^3*Log[(d*(a + b*x))/(-(b*c) + a*d)]*Log[-((d*(a + b*x))/(b*(c + d*x)))]*P
olyLog[2, (b*(c + d*x))/(b*c - a*d)] + 12*B^3*n^3*Log[-((d*(a + b*x))/(b*(c
 + d*x)))]^2*PolyLog[2, (b*(c + d*x))/(b*c - a*d)] + 24*A*B^2*n^2*Log[c + d
*x]*PolyLog[2, (b*(c + d*x))/(b*c - a*d)] - 24*B^3*n^3*Log[a + b*x]*Log[c +
 d*x]*PolyLog[2, (b*(c + d*x))/(b*c - a*d)] + 12*B^3*n^3*Log[c + d*x]^2*Pol
yLog[2, (b*(c + d*x))/(b*c - a*d)] + 24*B^3*n^3*Log[a + b*x]*Log[(b*(c + d*
x))/(b*c - a*d)]*PolyLog[2, (b*(c + d*x))/(b*c - a*d)] - 24*B^3*n^3*Log[(d*
(a + b*x))/(-(b*c) + a*d)]*Log[(b*(c + d*x))/(b*c - a*d)]*PolyLog[2, (b*(c
+ d*x))/(b*c - a*d)] + 24*B^3*n^2*Log[c + d*x]*Log[(e*(a + b*x)^n)/(c + d*x
)^n]*PolyLog[2, (b*(c + d*x))/(b*c - a*d)] - 24*A*B^2*n^2*PolyLog[3, (d*(a
+ b*x))/(-(b*c) + a*d)] + 24*B^3*n^3*Log[-((d*(a + b*x))/(b*(c + d*x)))]*Po
lyLog[3, (d*(a + b*x))/(-(b*c) + a*d)] - 24*B^3*n^2*Log[(e*(a + b*x)^n)/(c
+ d*x)^n]*PolyLog[3, (d*(a + b*x))/(-(b*c) + a*d)] + 24*B^3*n^3*Log[-((d*(a
 + b*x))/(b*(c + d*x)))]*PolyLog[3, (d*(a + b*x))/(b*(c + d*x))] - 24*A*B^2
*n^2*PolyLog[3, (b*(c + d*x))/(b*c - a*d)] + 24*B^3*n^3*Log[-((d*(a + b*x))
/(b*(c + d*x)))]*PolyLog[3, (b*(c + d*x))/(b*c - a*d)] - 24*B^3*n^2*Log[(e*
(a + b*x)^n)/(c + d*x)^n]*PolyLog[3, (b*(c + d*x))/(b*c - a*d)] - 24*B^3*n^
3*PolyLog[4, (d*(a + b*x))/(b*(c + d*x))])/(4*b)
```

**fricas [F]** time = 0.73, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{B^3\log\left(\frac{(bx+a)^n e}{(dx+c)^n}\right)^3 + 3\,AB^2\log\left(\frac{(bx+a)^n e}{(dx+c)^n}\right)^2 + 3\,A^2B\log\left(\frac{(bx+a)^n e}{(dx+c)^n}\right) + A^3}{bx+a}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*(b*x+a)^n/((d*x+c)^n)))^3/(b*x+a),x, algorithm="fricas
")
```

```
[Out] integral((B^3*log((b*x + a)^n*e/(d*x + c)^n)^3 + 3*A*B^2*log((b*x + a)^n*e/
(d*x + c)^n)^2 + 3*A^2*B*log((b*x + a)^n*e/(d*x + c)^n) + A^3)/(b*x + a), x
)
```

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(B\log\left(\frac{(bx+a)^n e}{(dx+c)^n}\right) + A\right)^3}{bx+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*(b*x+a)^n/((d*x+c)^n)))^3/(b*x+a),x, algorithm="giac")
```

```
[Out] integrate((B*log((b*x + a)^n*e/(d*x + c)^n) + A)^3/(b*x + a), x)
```

**maple [F]** time = 3.34, size = 0, normalized size = 0.00

$$\int \frac{\left(B\ln\left(e\left(bx+a\right)^{n}\left(dx+c\right)^{-n}\right)+A\right)^{3}}{bx+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))^3/(b*x+a),x)
```

```
[Out] int((A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))^3/(b*x+a),x)
```

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$-\frac{B^{3}\log\left(bx+a\right)\log\left(\left(dx+c\right)^{n}\right)^{3}}{b}+\frac{A^{3}\log\left(bx+a\right)}{b}+\int \frac{B^{3}bc\log(e)^{3}+3\,AB^{2}bc\log(e)^{2}+3\,A^{2}Bbc\log(e)+\left(B^{3}b\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*(b*x+a)^n/((d*x+c)^n)))^3/(b*x+a),x, algorithm="maxima
")
```

```
[Out] -B^3*log(b*x + a)*log((d*x + c)^n)^3/b + A^3*log(b*x + a)/b + integrate((B^
3*b*c*log(e)^3 + 3*A*B^2*b*c*log(e)^2 + 3*A^2*B*b*c*log(e) + (B^3*b*d*x + B
^3*b*c)*log((b*x + a)^n)^3 + 3*(B^3*b*c*log(e) + A*B^2*b*c + (B^3*b*d*log(e
) + A*B^2*b*d)*x)*log((b*x + a)^n)^2 + 3*(B^3*b*c*log(e) + A*B^2*b*c + (B^3
*b*d*log(e) + A*B^2*b*d)*x + (B^3*b*d*n*x + B^3*a*d*n)*log(b*x + a) + (B^3*
b*d*x + B^3*b*c)*log((b*x + a)^n))*log((d*x + c)^n)^2 + (B^3*b*d*log(e)^3 +
 3*A*B^2*b*d*log(e)^2 + 3*A^2*B*b*d*log(e))*x + 3*(B^3*b*c*log(e)^2 + 2*A*B
^2*b*c*log(e) + A^2*B*b*c + (B^3*b*d*log(e)^2 + 2*A*B^2*b*d*log(e) + A^2*B*
b*d)*x)*log((b*x + a)^n) - 3*(B^3*b*c*log(e)^2 + 2*A*B^2*b*c*log(e) + A^2*B
*b*c + (B^3*b*d*x + B^3*b*c)*log((b*x + a)^n)^2 + (B^3*b*d*log(e)^2 + 2*A*B
^2*b*d*log(e) + A^2*B*b*d)*x + 2*(B^3*b*c*log(e) + A*B^2*b*c + (B^3*b*d*log
(e) + A*B^2*b*d)*x)*log((b*x + a)^n))*log((d*x + c)^n))/(b^2*d*x^2 + a*b*c
+ (b^2*c + a*b*d)*x), x)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\left(A+B\,\ln\left(\frac{e\,{\left(a+b\,x\right)}^n}{{\left(c+d\,x\right)}^n}\right)\right)}^3}{a+b\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*log((e*(a + b*x)^n)/(c + d*x)^n))^3/(a + b*x),x)
```

```
[Out] int((A + B*log((e*(a + b*x)^n)/(c + d*x)^n))^3/(a + b*x), x)
```

**sympy [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*ln(e*(b*x+a)**n/((d*x+c)**n)))**3/(b*x+a),x)
```

```
[Out] Timed out
```

# 3.168 $\int \frac{\left(A+B\log(e(a+bx)^n(c+dx)^{-n})\right)^3}{(a+bx)^2}\,dx$

**Optimal**. Leaf size=184

$$-\frac{6B^2n^2(c+dx)\left(B\log\left(e(a+bx)^n(c+dx)^{-n}\right)+A\right)}{(a+bx)(bc-ad)}-\frac{3Bn(c+dx)\left(B\log\left(e(a+bx)^n(c+dx)^{-n}\right)+A\right)^2}{(a+bx)(bc-ad)}-\frac{(c+dx)}{}$$

```
[Out] -6*B^3*n^3*(d*x+c)/(-a*d+b*c)/(b*x+a)-6*B^2*n^2*(d*x+c)*(A+B*ln(e*(b*x+a)^n
/((d*x+c)^n)))/(-a*d+b*c)/(b*x+a)-3*B*n*(d*x+c)*(A+B*ln(e*(b*x+a)^n/((d*x+c
)^n)))^2/(-a*d+b*c)/(b*x+a)-(d*x+c)*(A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))^3/(-a
*d+b*c)/(b*x+a)
```

**Rubi [A]** time = 0.31, antiderivative size = 360, normalized size of antiderivative = 1.96, number of steps used = 11, number of rules used = 3, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {6742, 2490, 32}

$$-\frac{3A^2B(c+dx)\log\left(e(a+bx)^n(c+dx)^{-n}\right)}{(a+bx)(bc-ad)}-\frac{3A^2Bn}{b(a+bx)}-\frac{A^3}{b(a+bx)}-\frac{3AB^2(c+dx)\log^2\left(e(a+bx)^n(c+dx)^{-n}\right)}{(a+bx)(bc-ad)}-\frac{6A}{}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])^3/(a + b*x)^2,x]
```

```
[Out] -(A^3/(b*(a + b*x))) - (3*A^2*B*n)/(b*(a + b*x)) - (6*A*B^2*n^2)/(b*(a + b*
x)) - (6*B^3*n^3)/(b*(a + b*x)) - (3*A^2*B*(c + d*x)*Log[(e*(a + b*x)^n)/(c
 + d*x)^n])/((b*c - a*d)*(a + b*x)) - (6*A*B^2*n*(c + d*x)*Log[(e*(a + b*x)
^n)/(c + d*x)^n])/((b*c - a*d)*(a + b*x)) - (6*B^3*n^2*(c + d*x)*Log[(e*(a
+ b*x)^n)/(c + d*x)^n])/((b*c - a*d)*(a + b*x)) - (3*A*B^2*(c + d*x)*Log[(e
*(a + b*x)^n)/(c + d*x)^n]^2)/((b*c - a*d)*(a + b*x)) - (3*B^3*n*(c + d*x)*
Log[(e*(a + b*x)^n)/(c + d*x)^n]^2)/((b*c - a*d)*(a + b*x)) - (B^3*(c + d*x
)*Log[(e*(a + b*x)^n)/(c + d*x)^n]^3)/((b*c - a*d)*(a + b*x))
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 2490

```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))
^(r_.)]^(s_.)/((g_.) + (h_.)*(x_))^2, x_Symbol] :> Simp[((a + b*x)*Log[e*(f
*(a + b*x)^p*(c + d*x)^q)^r]^s)/((b*g - a*h)*(g + h*x)), x] - Dist[(p*r*s*(
b*c - a*d))/(b*g - a*h), Int[Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^(s - 1)/(
(c + d*x)*(g + h*x)), x], x] /; FreeQ[{a, b, c, d, e, f, g, h, p, q, r, s},
```

```
 x] && NeQ[b*c - a*d, 0] && EqQ[p + q, 0] && NeQ[b*g - a*h, 0] && IGtQ[s, 0
]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\int \frac{\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^3}{(a+bx)^2}\,dx = \int\left(\frac{A^3}{(a+bx)^2}+\frac{3A^2B\log\left(e(a+bx)^n(c+dx)^{-n}\right)}{(a+bx)^2}+\frac{3AB^2\log^2\left(e(a+\right.}{(a+}\right.$$

$$= -\frac{A^3}{b(a+bx)}+\left(3A^2B\right)\int\frac{\log\left(e(a+bx)^n(c+dx)^{-n}\right)}{(a+bx)^2}\,dx+\left(3AB^2\right)\int$$

$$= -\frac{A^3}{b(a+bx)}-\frac{3A^2B(c+dx)\log\left(e(a+bx)^n(c+dx)^{-n}\right)}{(bc-ad)(a+bx)}-\frac{3AB^2(c+d}{}$$

$$= -\frac{A^3}{b(a+bx)}-\frac{3A^2Bn}{b(a+bx)}-\frac{3A^2B(c+dx)\log\left(e(a+bx)^n(c+dx)^{-n}\right)}{(bc-ad)(a+bx)}-$$

$$= -\frac{A^3}{b(a+bx)}-\frac{3A^2Bn}{b(a+bx)}-\frac{6AB^2n^2}{b(a+bx)}-\frac{3A^2B(c+dx)\log\left(e(a+bx)^n\right.}{(bc-ad)(a+bx)}$$

$$= -\frac{A^3}{b(a+bx)}-\frac{3A^2Bn}{b(a+bx)}-\frac{6AB^2n^2}{b(a+bx)}-\frac{6B^3n^3}{b(a+bx)}-\frac{3A^2B(c+dx)\log}{(bc-}$$

**Mathematica [B]** time = 0.79, size = 524, normalized size = 2.85

$$\frac{-3Bdn(a+bx)\log(a+bx)\left(2B(A+Bn)\log\left(e(a+bx)^n(c+dx)^{-n}\right)+2Bn\log(c+dx)\left(B\log\left(e(a+bx)^n(c+dx)^{-}\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])^3/(a + b*x)^2,x]
```

```
[Out] (-(B^3*d*n^3*(a + b*x)*Log[a + b*x]^3) + B^3*d*n^3*(a + b*x)*Log[c + d*x]^3
 + 3*B^2*d*n^2*(a + b*x)*Log[c + d*x]^2*(A + B*n + B*Log[(e*(a + b*x)^n)/(c
 + d*x)^n]) + 3*B^2*d*n^2*(a + b*x)*Log[a + b*x]^2*(A + B*n + B*n*Log[c + d
*x] + B*Log[(e*(a + b*x)^n)/(c + d*x)^n]) + 3*B*d*n*(a + b*x)*Log[c + d*x]*
(A^2 + 2*A*B*n + 2*B^2*n^2 + 2*B*(A + B*n)*Log[(e*(a + b*x)^n)/(c + d*x)^n]
 + B^2*Log[(e*(a + b*x)^n)/(c + d*x)^n]^2) - (b*c - a*d)*(A^3 + 3*A^2*B*n +
 6*A*B^2*n^2 + 6*B^3*n^3 + 3*B*(A^2 + 2*A*B*n + 2*B^2*n^2)*Log[(e*(a + b*x)
```

```
^n)/(c + d*x)^n] + 3*B^2*(A + B*n)*Log[(e*(a + b*x)^n)/(c + d*x)^n]^2 + B^3
*Log[(e*(a + b*x)^n)/(c + d*x)^n]^3) - 3*B*d*n*(a + b*x)*Log[a + b*x]*(A^2
+ 2*A*B*n + 2*B^2*n^2 + B^2*n^2*Log[c + d*x]^2 + 2*B*(A + B*n)*Log[(e*(a +
b*x)^n)/(c + d*x)^n] + B^2*Log[(e*(a + b*x)^n)/(c + d*x)^n]^2 + 2*B*n*Log[c
 + d*x]*(A + B*n + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])))/(b*(b*c - a*d)*(a
+ b*x))
```

**fricas [B]** time = 0.92, size = 825, normalized size = 4.48

$$-\frac{A^3bc - A^3ad + 6\left(B^3bc - B^3ad\right)n^3 + \left(B^3bdn^3x + B^3bcn^3\right)\log\left(bx+a\right)^3 - \left(B^3bdn^3x + B^3bcn^3\right)\log\left(dx+c\right)^3 +}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*(b*x+a)^n/((d*x+c)^n)))^3/(b*x+a)^2,x, algorithm="fric
as")
```

```
[Out] -(A^3*b*c - A^3*a*d + 6*(B^3*b*c - B^3*a*d)*n^3 + (B^3*b*d*n^3*x + B^3*b*c*
n^3)*log(b*x + a)^3 - (B^3*b*d*n^3*x + B^3*b*c*n^3)*log(d*x + c)^3 + (B^3*b
*c - B^3*a*d)*log(e)^3 + 6*(A*B^2*b*c - A*B^2*a*d)*n^2 + 3*(B^3*b*c*n^3 + A
*B^2*b*c*n^2 + (B^3*b*d*n^3 + A*B^2*b*d*n^2)*x + (B^3*b*d*n^2*x + B^3*b*c*n
^2)*log(e))*log(b*x + a)^2 + 3*(B^3*b*c*n^3 + A*B^2*b*c*n^2 + (B^3*b*d*n^3
+ A*B^2*b*d*n^2)*x + (B^3*b*d*n^3*x + B^3*b*c*n^3)*log(b*x + a) + (B^3*b*d*
n^2*x + B^3*b*c*n^2)*log(e))*log(d*x + c)^2 + 3*(A*B^2*b*c - A*B^2*a*d + (B
^3*b*c - B^3*a*d)*n)*log(e)^2 + 3*(A^2*B*b*c - A^2*B*a*d)*n + 3*(2*B^3*b*c*
n^3 + 2*A*B^2*b*c*n^2 + A^2*B*b*c*n + (B^3*b*d*n*x + B^3*b*c*n)*log(e)^2 +
(2*B^3*b*d*n^3 + 2*A*B^2*b*d*n^2 + A^2*B*b*d*n)*x + 2*(B^3*b*c*n^2 + A*B^2*
b*c*n + (B^3*b*d*n^2 + A*B^2*b*d*n)*x)*log(e))*log(b*x + a) - 3*(2*B^3*b*c*
n^3 + 2*A*B^2*b*c*n^2 + A^2*B*b*c*n + (B^3*b*d*n^3*x + B^3*b*c*n^3)*log(b*x
 + a)^2 + (B^3*b*d*n*x + B^3*b*c*n)*log(e)^2 + (2*B^3*b*d*n^3 + 2*A*B^2*b*d
*n^2 + A^2*B*b*d*n)*x + 2*(B^3*b*c*n^3 + A*B^2*b*c*n^2 + (B^3*b*d*n^3 + A*B
^2*b*d*n^2)*x + (B^3*b*d*n^2*x + B^3*b*c*n^2)*log(e))*log(b*x + a) + 2*(B^3
*b*c*n^2 + A*B^2*b*c*n + (B^3*b*d*n^2 + A*B^2*b*d*n)*x)*log(e))*log(d*x + c
) + 3*(A^2*B*b*c - A^2*B*a*d + 2*(B^3*b*c - B^3*a*d)*n^2 + 2*(A*B^2*b*c - A
*B^2*a*d)*n)*log(e))/(a*b^2*c - a^2*b*d + (b^3*c - a*b^2*d)*x)
```

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(B\log\left(\frac{(bx+a)^n e}{(dx+c)^n}\right) + A\right)^3}{(bx+a)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*(b*x+a)^n/((d*x+c)^n)))^3/(b*x+a)^2,x, algorithm="giac
")
```

```
[Out] integrate((B*log((b*x + a)^n*e/(d*x + c)^n) + A)^3/(b*x + a)^2, x)
```

**maple** [C] time = 20.96, size = 69354, normalized size = 376.92

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))^3/(b*x+a)^2,x)
```

```
[Out] result too large to display
```

**maxima** [B] time = 2.16, size = 1129, normalized size = 6.14

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*(b*x+a)^n/((d*x+c)^n)))^3/(b*x+a)^2,x, algorithm="maxi
ma")
```

```
[Out] -B^3*log((b*x + a)^n*e/(d*x + c)^n)^3/(b^2*x + a*b) - (3*(d*e*n*log(b*x + a
)/(b^2*c - a*b*d) - d*e*n*log(d*x + c)/(b^2*c - a*b*d) + e*n/(b^2*x + a*b))
*log((b*x + a)^n*e/(d*x + c)^n)^2/e + (3*(2*b*c*e^2*n^2 - 2*a*d*e^2*n^2 - (
b*d*e^2*n^2*x + a*d*e^2*n^2)*log(b*x + a)^2 - (b*d*e^2*n^2*x + a*d*e^2*n^2)
*log(d*x + c)^2 + 2*(b*d*e^2*n^2*x + a*d*e^2*n^2)*log(b*x + a) - 2*(b*d*e^2
*n^2*x + a*d*e^2*n^2 - (b*d*e^2*n^2*x + a*d*e^2*n^2)*log(b*x + a))*log(d*x
+ c))*log((b*x + a)^n*e/(d*x + c)^n)/((a*b^2*c - a^2*b*d + (b^3*c - a*b^2*d
)*x)*e) + (6*b*c*e^3*n^3 - 6*a*d*e^3*n^3 + (b*d*e^3*n^3*x + a*d*e^3*n^3)*lo
g(b*x + a)^3 - (b*d*e^3*n^3*x + a*d*e^3*n^3)*log(d*x + c)^3 - 3*(b*d*e^3*n^
3*x + a*d*e^3*n^3)*log(b*x + a)^2 - 3*(b*d*e^3*n^3*x + a*d*e^3*n^3 - (b*d*e
^3*n^3*x + a*d*e^3*n^3)*log(b*x + a))*log(d*x + c)^2 + 6*(b*d*e^3*n^3*x + a
*d*e^3*n^3)*log(b*x + a) - 3*(2*b*d*e^3*n^3*x + 2*a*d*e^3*n^3 + (b*d*e^3*n^
3*x + a*d*e^3*n^3)*log(b*x + a)^2 - 2*(b*d*e^3*n^3*x + a*d*e^3*n^3)*log(b*x
 + a))*log(d*x + c))/((a*b^2*c - a^2*b*d + (b^3*c - a*b^2*d)*x)*e^2))/e)*B^
3 - 3*A*B^2*(2*(d*e*n*log(b*x + a)/(b^2*c - a*b*d) - d*e*n*log(d*x + c)/(b^
2*c - a*b*d) + e*n/(b^2*x + a*b))*log((b*x + a)^n*e/(d*x + c)^n)/e + (2*b*c
*e^2*n^2 - 2*a*d*e^2*n^2 - (b*d*e^2*n^2*x + a*d*e^2*n^2)*log(b*x + a)^2 - (
b*d*e^2*n^2*x + a*d*e^2*n^2)*log(d*x + c)^2 + 2*(b*d*e^2*n^2*x + a*d*e^2*n^
2)*log(b*x + a) - 2*(b*d*e^2*n^2*x + a*d*e^2*n^2 - (b*d*e^2*n^2*x + a*d*e^2
*n^2)*log(b*x + a))*log(d*x + c))/((a*b^2*c - a^2*b*d + (b^3*c - a*b^2*d)*x
)*e^2)) - 3*A*B^2*log((b*x + a)^n*e/(d*x + c)^n)^2/(b^2*x + a*b) - 3*(d*e*n
*log(b*x + a)/(b^2*c - a*b*d) - d*e*n*log(d*x + c)/(b^2*c - a*b*d) + e*n/(b
^2*x + a*b))*A^2*B/e - 3*A^2*B*log((b*x + a)^n*e/(d*x + c)^n)/(b^2*x + a*b)
 - A^3/(b^2*x + a*b)
```

**mupad [B]** time = 6.06, size = 474, normalized size = 2.58

$$-\ln\left(\frac{e\,(a+b\,x)^n}{(c+d\,x)^n}\right)\left(\frac{3\,B\,b\,d\,A^2\,x^2+3\,B\,\left(a\,d+b\,c\right)\,A^2\,x+3\,B\,a\,c\,A^2}{b\,(a+b\,x)^2\,\left(c+d\,x\right)}+\frac{6\,d\,\left(n\,B^3+A\,B^2\right)\,\left(b^2\,n\,x^2\,\left(a\,d-b\,c\right)+\frac{a}{}\right.}{b^2\,\left(a\,d-b\,c\right)\,\left(a+b\,x\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*log((e*(a + b*x)^n)/(c + d*x)^n))^3/(a + b*x)^2,x)
```

```
[Out] - log((e*(a + b*x)^n)/(c + d*x)^n)*((3*A^2*B*a*c + 3*A^2*B*x*(a*d + b*c) +
3*A^2*B*b*d*x^2)/(b*(a + b*x)^2*(c + d*x)) + (6*d*(A*B^2 + B^3*n)*(b^2*n*x^
2*(a*d - b*c) + (a*b*c*n*(a*d - b*c))/d + (b*n*x*(a*d + b*c)*(a*d - b*c))/d
))/(b^2*(a*d - b*c)*(a + b*x)^2*(c + d*x))) - (A^3 + 6*B^3*n^3 + 6*A*B^2*n^
2 + 3*A^2*B*n)/(a*b + b^2*x) - log((e*(a + b*x)^n)/(c + d*x)^n)^2*((3*A*B^2
)/(a*b + b^2*x) + (3*B^3*n)/(a*b + b^2*x) - (3*d*(A*B^2 + B^3*n))/(b*(a*d -
 b*c))) - log((e*(a + b*x)^n)/(c + d*x)^n)^3*(B^3/(b*(a + b*x)) - (B^3*d)/(
b*(a*d - b*c))) - (B*d*n*atan((B*d*n*((b^2*c + a*b*d)/b + 2*b*d*x)*(A^2 + 2
*B^2*n^2 + 2*A*B*n)*3i)/((a*d - b*c)*(6*B^3*d*n^3 + 3*A^2*B*d*n + 6*A*B^2*d
*n^2)))*(A^2 + 2*B^2*n^2 + 2*A*B*n)*6i)/(b*(a*d - b*c))
```

**sympy [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*ln(e*(b*x+a)**n/((d*x+c)**n)))**3/(b*x+a)**2,x)
```

```
[Out] Timed out
```

# 3.169 $\int \frac{\left(A+B\log(e(a+bx)^n(c+dx)^{-n})\right)^3}{(a+bx)^3}\,dx$

**Optimal**. Leaf size=390

$$-\frac{3bB^2n^2(c+dx)^2\left(B\log\left(e(a+bx)^n(c+dx)^{-n}\right)+A\right)}{4(a+bx)^2(bc-ad)^2}+\frac{6B^2dn^2(c+dx)\left(B\log\left(e(a+bx)^n(c+dx)^{-n}\right)+A\right)}{(a+bx)(bc-ad)^2}-\frac{3bBn\cdots}{\cdots}$$

```
[Out] 6*B^3*d*n^3*(d*x+c)/(-a*d+b*c)^2/(b*x+a)-3/8*b*B^3*n^3*(d*x+c)^2/(-a*d+b*c)
^2/(b*x+a)^2+6*B^2*d*n^2*(d*x+c)*(A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))/(-a*d+b*
c)^2/(b*x+a)-3/4*b*B^2*n^2*(d*x+c)^2*(A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))/(-a*
d+b*c)^2/(b*x+a)^2+3*B*d*n*(d*x+c)*(A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))^2/(-a*
d+b*c)^2/(b*x+a)-3/4*b*B*n*(d*x+c)^2*(A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))^2/(-
a*d+b*c)^2/(b*x+a)^2+d*(d*x+c)*(A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))^3/(-a*d+b*
c)^2/(b*x+a)-1/2*b*(d*x+c)^2*(A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))^3/(-a*d+b*c)
^2/(b*x+a)^2
```

**Rubi [B]** time = 0.80, antiderivative size = 811, normalized size of antiderivative = 2.08, number of steps used = 21, number of rules used = 8, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.242, Rules used = {6742, 2492, 44, 2491, 2490, 32, 2509, 37}

$$-\frac{A^3}{2b(a+bx)^2}+\frac{3Bd^2n\log(a+bx)A^2}{2b(bc-ad)^2}-\frac{3Bd^2n\log(c+dx)A^2}{2b(bc-ad)^2}-\frac{3B\log\left(e(a+bx)^n(c+dx)^{-n}\right)A^2}{2b(a+bx)^2}+\frac{3BdnA^2}{2b(bc-ad)(a+b\cdots}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])^3/(a + b*x)^3,x]
```

```
[Out] -A^3/(2*b*(a + b*x)^2) - (3*A^2*B*n)/(4*b*(a + b*x)^2) + (3*A^2*B*d*n)/(2*b
*(b*c - a*d)*(a + b*x)) + (6*A*B^2*d*n^2)/(b*(b*c - a*d)*(a + b*x)) + (6*B^
3*d*n^3)/(b*(b*c - a*d)*(a + b*x)) - (3*A*b*B^2*n^2*(c + d*x)^2)/(4*(b*c -
a*d)^2*(a + b*x)^2) - (3*b*B^3*n^3*(c + d*x)^2)/(8*(b*c - a*d)^2*(a + b*x)^
2) + (3*A^2*B*d^2*n*Log[a + b*x])/(2*b*(b*c - a*d)^2) - (3*A^2*B*d^2*n*Log[
c + d*x])/(2*b*(b*c - a*d)^2) - (3*A^2*B*Log[(e*(a + b*x)^n)/(c + d*x)^n])/
(2*b*(a + b*x)^2) + (6*A*B^2*d*n*(c + d*x)*Log[(e*(a + b*x)^n)/(c + d*x)^n]
)/((b*c - a*d)^2*(a + b*x)) + (6*B^3*d*n^2*(c + d*x)*Log[(e*(a + b*x)^n)/(c
 + d*x)^n])/((b*c - a*d)^2*(a + b*x)) - (3*A*b*B^2*n*(c + d*x)^2*Log[(e*(a
+ b*x)^n)/(c + d*x)^n])/(2*(b*c - a*d)^2*(a + b*x)^2) - (3*b*B^3*n^2*(c + d
*x)^2*Log[(e*(a + b*x)^n)/(c + d*x)^n])/(4*(b*c - a*d)^2*(a + b*x)^2) + (3*
A*B^2*d*(c + d*x)*Log[(e*(a + b*x)^n)/(c + d*x)^n]^2)/((b*c - a*d)^2*(a + b
*x)) + (3*B^3*d*n*(c + d*x)*Log[(e*(a + b*x)^n)/(c + d*x)^n]^2)/((b*c - a*d
)^2*(a + b*x)) - (3*A*b*B^2*(c + d*x)^2*Log[(e*(a + b*x)^n)/(c + d*x)^n]^2)
/(2*(b*c - a*d)^2*(a + b*x)^2) - (3*b*B^3*n*(c + d*x)^2*Log[(e*(a + b*x)^n)
/(c + d*x)^n]^2)/(4*(b*c - a*d)^2*(a + b*x)^2) + (B^3*d*(c + d*x)*Log[(e*(a
```

```
 + b*x)^n)/(c + d*x)^n]^3)/((b*c - a*d)^2*(a + b*x)) - (b*B^3*(c + d*x)^2*L
og[(e*(a + b*x)^n)/(c + d*x)^n]^3)/(2*(b*c - a*d)^2*(a + b*x)^2)
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 2490

```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))
^(r_.)]^(s_.)/((g_.) + (h_.)*(x_))^2, x_Symbol] :> Simp[((a + b*x)*Log[e*(f
*(a + b*x)^p*(c + d*x)^q)^r]^s)/((b*g - a*h)*(g + h*x)), x] - Dist[(p*r*s*(
b*c - a*d))/(b*g - a*h), Int[Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^(s - 1)/(
(c + d*x)*(g + h*x)), x], x] /; FreeQ[{a, b, c, d, e, f, g, h, p, q, r, s},
 x] && NeQ[b*c - a*d, 0] && EqQ[p + q, 0] && NeQ[b*g - a*h, 0] && IGtQ[s, 0
]
```

Rule 2491

```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))
^(r_.)]^(s_)/((g_.) + (h_.)*(x_))^3, x_Symbol] :> Dist[d/(d*g - c*h), Int[L
og[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^s/(g + h*x)^2, x], x] - Dist[h/(d*g - c
*h), Int[((c + d*x)*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^s)/(g + h*x)^3, x]
, x] /; FreeQ[{a, b, c, d, e, f, g, h, p, q, r, s}, x] && NeQ[b*c - a*d, 0]
 && EqQ[p + q, 0] && EqQ[b*g - a*h, 0] && NeQ[d*g - c*h, 0] && IGtQ[s, 0]
```

Rule 2492

```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))
^(r_.)]^(s_.)*((g_.) + (h_.)*(x_))^(m_.), x_Symbol] :> Simp[((g + h*x)^(m +
 1)*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^s)/(h*(m + 1)), x] - Dist[(p*r*s*(
```

```
b*c - a*d))/(h*(m + 1)), Int[((g + h*x)^(m + 1)*Log[e*(f*(a + b*x)^p*(c + d
*x)^q)^r]^(s - 1))/((a + b*x)*(c + d*x)), x], x] /; FreeQ[{a, b, c, d, e, f
, g, h, m, p, q, r, s}, x] && NeQ[b*c - a*d, 0] && EqQ[p + q, 0] && IGtQ[s,
 0] && NeQ[m, -1]
```

Rule 2509

```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))
^(r_.)]^(s_.)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symb
ol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^(n + 1)*Log[e*(f*(a + b*x)^p*(c +
d*x)^q)^r]^s)/((m + 1)*(b*c - a*d)), x] - Dist[(p*r*s*(b*c - a*d))/((m + 1)
*(b*c - a*d)), Int[(a + b*x)^m*(c + d*x)^n*Log[e*(f*(a + b*x)^p*(c + d*x)^q
)^r]^(s - 1), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p, q, r, s}, x] && N
eQ[b*c - a*d, 0] && EqQ[p + q, 0] && EqQ[m + n + 2, 0] && NeQ[m, -1] && IGt
Q[s, 0]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\int \frac{\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^3}{(a+bx)^3}\,dx = \int\left(\frac{A^3}{(a+bx)^3}+\frac{3A^2B\log\left(e(a+bx)^n(c+dx)^{-n}\right)}{(a+bx)^3}+\frac{3AB^2\log^2\left(e(a+\right.}{(a+}\right.$$

$$= -\frac{A^3}{2b(a+bx)^2}+\left(3A^2B\right)\int\frac{\log\left(e(a+bx)^n(c+dx)^{-n}\right)}{(a+bx)^3}\,dx+\left(3AB^2\right)$$

$$= -\frac{A^3}{2b(a+bx)^2}-\frac{3A^2B\log\left(e(a+bx)^n(c+dx)^{-n}\right)}{2b(a+bx)^2}+\frac{\left(3AbB^2\right)\int\frac{(c+dx)}{}}{b}$$

$$= -\frac{A^3}{2b(a+bx)^2}-\frac{3A^2B\log\left(e(a+bx)^n(c+dx)^{-n}\right)}{2b(a+bx)^2}+\frac{3AB^2d(c+dx)\log}{(bc-}$$

$$= -\frac{A^3}{2b(a+bx)^2}-\frac{3A^2Bn}{4b(a+bx)^2}+\frac{3A^2Bdn}{2b(bc-ad)(a+bx)}+\frac{3A^2Bd^2n\log(}{2b(bc-ad}$$

$$= -\frac{A^3}{2b(a+bx)^2}-\frac{3A^2Bn}{4b(a+bx)^2}+\frac{3A^2Bdn}{2b(bc-ad)(a+bx)}+\frac{6AB^2dn^2}{b(bc-ad)(a+}$$

$$= -\frac{A^3}{2b(a+bx)^2}-\frac{3A^2Bn}{4b(a+bx)^2}+\frac{3A^2Bdn}{2b(bc-ad)(a+bx)}+\frac{6AB^2dn^2}{b(bc-ad)(a+}$$

**Mathematica [A]** time = 1.18, size = 693, normalized size = 1.78

$$-\frac{-6Bd^2n(a+bx)^2\log(a+bx)\left(2B(2A+3Bn)\log\left(e(a+bx)^n(c+dx)^{-n}\right)+2Bn\log(c+dx)\left(2B\log\left(e(a+bx)^n(\right.\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])^3/(a + b*x)^3,x]
```

```
[Out] -1/8*(-4*B^3*d^2*n^3*(a + b*x)^2*Log[a + b*x]^3 + 4*B^3*d^2*n^3*(a + b*x)^2
*Log[c + d*x]^3 + 6*B^2*d^2*n^2*(a + b*x)^2*Log[c + d*x]^2*(2*A + 3*B*n + 2
*B*Log[(e*(a + b*x)^n)/(c + d*x)^n]) + 6*B^2*d^2*n^2*(a + b*x)^2*Log[a + b*
x]^2*(2*A + 3*B*n + 2*B*n*Log[c + d*x] + 2*B*Log[(e*(a + b*x)^n)/(c + d*x)^
n]) + 6*B*d^2*n*(a + b*x)^2*Log[c + d*x]*(2*A^2 + 6*A*B*n + 7*B^2*n^2 + 2*B
*(2*A + 3*B*n)*Log[(e*(a + b*x)^n)/(c + d*x)^n] + 2*B^2*Log[(e*(a + b*x)^n)
/(c + d*x)^n]^2) + (b*c - a*d)*(4*A^3*(b*c - a*d) + 3*B^3*n^3*(-15*a*d + b*
(c - 14*d*x)) + 6*A*B^2*n^2*(-7*a*d + b*(c - 6*d*x)) + 6*A^2*B*n*(-3*a*d +
b*(c - 2*d*x)) + 6*B*(2*A^2*(b*c - a*d) + B^2*n^2*(-7*a*d + b*(c - 6*d*x))
+ 2*A*B*n*(-3*a*d + b*(c - 2*d*x)))*Log[(e*(a + b*x)^n)/(c + d*x)^n] + 6*B^
2*(2*A*(b*c - a*d) + B*n*(-3*a*d + b*(c - 2*d*x)))*Log[(e*(a + b*x)^n)/(c +
 d*x)^n]^2 + 4*B^3*(b*c - a*d)*Log[(e*(a + b*x)^n)/(c + d*x)^n]^3) - 6*B*d^
2*n*(a + b*x)^2*Log[a + b*x]*(2*A^2 + 6*A*B*n + 7*B^2*n^2 + 2*B^2*n^2*Log[c
 + d*x]^2 + 2*B*(2*A + 3*B*n)*Log[(e*(a + b*x)^n)/(c + d*x)^n] + 2*B^2*Log[
(e*(a + b*x)^n)/(c + d*x)^n]^2 + 2*B*n*Log[c + d*x]*(2*A + 3*B*n + 2*B*Log[
(e*(a + b*x)^n)/(c + d*x)^n])))/(b*(b*c - a*d)^2*(a + b*x)^2)
```

**fricas [B]** time = 0.73, size = 2244, normalized size = 5.75

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*(b*x+a)^n/((d*x+c)^n)))^3/(b*x+a)^3,x, algorithm="fric
as")
```

```
[Out] -1/8*(4*A^3*b^2*c^2 - 8*A^3*a*b*c*d + 4*A^3*a^2*d^2 + 3*(B^3*b^2*c^2 - 16*B
^3*a*b*c*d + 15*B^3*a^2*d^2)*n^3 - 4*(B^3*b^2*d^2*n^3*x^2 + 2*B^3*a*b*d^2*n
^3*x - (B^3*b^2*c^2 - 2*B^3*a*b*c*d)*n^3)*log(b*x + a)^3 + 4*(B^3*b^2*d^2*n
^3*x^2 + 2*B^3*a*b*d^2*n^3*x - (B^3*b^2*c^2 - 2*B^3*a*b*c*d)*n^3)*log(d*x +
 c)^3 + 4*(B^3*b^2*c^2 - 2*B^3*a*b*c*d + B^3*a^2*d^2)*log(e)^3 + 6*(A*B^2*b
^2*c^2 - 8*A*B^2*a*b*c*d + 7*A*B^2*a^2*d^2)*n^2 + 6*((B^3*b^2*c^2 - 4*B^3*a
*b*c*d)*n^3 + 2*(A*B^2*b^2*c^2 - 2*A*B^2*a*b*c*d)*n^2 - (3*B^3*b^2*d^2*n^3
+ 2*A*B^2*b^2*d^2*n^2)*x^2 - 2*(2*A*B^2*a*b*d^2*n^2 + (B^3*b^2*c*d + 2*B^3*
a*b*d^2)*n^3)*x - 2*(B^3*b^2*d^2*n^2*x^2 + 2*B^3*a*b*d^2*n^2*x - (B^3*b^2*c
^2 - 2*B^3*a*b*c*d)*n^2)*log(e))*log(b*x + a)^2 + 6*((B^3*b^2*c^2 - 4*B^3*a
*b*c*d)*n^3 + 2*(A*B^2*b^2*c^2 - 2*A*B^2*a*b*c*d)*n^2 - (3*B^3*b^2*d^2*n^3
```

```
+ 2*A*B^2*b^2*d^2*n^2)*x^2 - 2*(2*A*B^2*a*b*d^2*n^2 + (B^3*b^2*c*d + 2*B^3*
a*b*d^2)*n^3)*x - 2*(B^3*b^2*d^2*n^3*x^2 + 2*B^3*a*b*d^2*n^3*x - (B^3*b^2*c
^2 - 2*B^3*a*b*c*d)*n^3)*log(b*x + a) - 2*(B^3*b^2*d^2*n^2*x^2 + 2*B^3*a*b*
d^2*n^2*x - (B^3*b^2*c^2 - 2*B^3*a*b*c*d)*n^2)*log(e))*log(d*x + c)^2 + 6*(
2*A*B^2*b^2*c^2 - 4*A*B^2*a*b*c*d + 2*A*B^2*a^2*d^2 - 2*(B^3*b^2*c*d - B^3*
a*b*d^2)*n*x + (B^3*b^2*c^2 - 4*B^3*a*b*c*d + 3*B^3*a^2*d^2)*n)*log(e)^2 +
6*(A^2*B*b^2*c^2 - 4*A^2*B*a*b*c*d + 3*A^2*B*a^2*d^2)*n - 6*(7*(B^3*b^2*c*d
 - B^3*a*b*d^2)*n^3 + 6*(A*B^2*b^2*c*d - A*B^2*a*b*d^2)*n^2 + 2*(A^2*B*b^2*
c*d - A^2*B*a*b*d^2)*n)*x + 6*((B^3*b^2*c^2 - 8*B^3*a*b*c*d)*n^3 + 2*(A*B^2
*b^2*c^2 - 4*A*B^2*a*b*c*d)*n^2 - (7*B^3*b^2*d^2*n^3 + 6*A*B^2*b^2*d^2*n^2
+ 2*A^2*B*b^2*d^2*n)*x^2 - 2*(B^3*b^2*d^2*n*x^2 + 2*B^3*a*b*d^2*n*x - (B^3*
b^2*c^2 - 2*B^3*a*b*c*d)*n)*log(e)^2 + 2*(A^2*B*b^2*c^2 - 2*A^2*B*a*b*c*d)*
n - 2*(2*A^2*B*a*b*d^2*n + (3*B^3*b^2*c*d + 4*B^3*a*b*d^2)*n^3 + 2*(A*B^2*b
^2*c*d + 2*A*B^2*a*b*d^2)*n^2)*x + 2*((B^3*b^2*c^2 - 4*B^3*a*b*c*d)*n^2 - (
3*B^3*b^2*d^2*n^2 + 2*A*B^2*b^2*d^2*n)*x^2 + 2*(A*B^2*b^2*c^2 - 2*A*B^2*a*b
*c*d)*n - 2*(2*A*B^2*a*b*d^2*n + (B^3*b^2*c*d + 2*B^3*a*b*d^2)*n^2)*x)*log(
e))*log(b*x + a) - 6*((B^3*b^2*c^2 - 8*B^3*a*b*c*d)*n^3 + 2*(A*B^2*b^2*c^2
- 4*A*B^2*a*b*c*d)*n^2 - (7*B^3*b^2*d^2*n^3 + 6*A*B^2*b^2*d^2*n^2 + 2*A^2*B
*b^2*d^2*n)*x^2 - 2*(B^3*b^2*d^2*n^3*x^2 + 2*B^3*a*b*d^2*n^3*x - (B^3*b^2*c
^2 - 2*B^3*a*b*c*d)*n^3)*log(b*x + a)^2 - 2*(B^3*b^2*d^2*n*x^2 + 2*B^3*a*b*
d^2*n*x - (B^3*b^2*c^2 - 2*B^3*a*b*c*d)*n)*log(e)^2 + 2*(A^2*B*b^2*c^2 - 2*
A^2*B*a*b*c*d)*n - 2*(2*A^2*B*a*b*d^2*n + (3*B^3*b^2*c*d + 4*B^3*a*b*d^2)*n
^3 + 2*(A*B^2*b^2*c*d + 2*A*B^2*a*b*d^2)*n^2)*x + 2*((B^3*b^2*c^2 - 4*B^3*a
*b*c*d)*n^3 + 2*(A*B^2*b^2*c^2 - 2*A*B^2*a*b*c*d)*n^2 - (3*B^3*b^2*d^2*n^3
+ 2*A*B^2*b^2*d^2*n^2)*x^2 - 2*(2*A*B^2*a*b*d^2*n^2 + (B^3*b^2*c*d + 2*B^3*
a*b*d^2)*n^3)*x - 2*(B^3*b^2*d^2*n^2*x^2 + 2*B^3*a*b*d^2*n^2*x - (B^3*b^2*c
^2 - 2*B^3*a*b*c*d)*n^2)*log(e))*log(b*x + a) + 2*((B^3*b^2*c^2 - 4*B^3*a*b
*c*d)*n^2 - (3*B^3*b^2*d^2*n^2 + 2*A*B^2*b^2*d^2*n)*x^2 + 2*(A*B^2*b^2*c^2
- 2*A*B^2*a*b*c*d)*n - 2*(2*A*B^2*a*b*d^2*n + (B^3*b^2*c*d + 2*B^3*a*b*d^2)
*n^2)*x)*log(e))*log(d*x + c) + 6*(2*A^2*B*b^2*c^2 - 4*A^2*B*a*b*c*d + 2*A^
2*B*a^2*d^2 + (B^3*b^2*c^2 - 8*B^3*a*b*c*d + 7*B^3*a^2*d^2)*n^2 + 2*(A*B^2*
b^2*c^2 - 4*A*B^2*a*b*c*d + 3*A*B^2*a^2*d^2)*n - 2*(3*(B^3*b^2*c*d - B^3*a*
b*d^2)*n^2 + 2*(A*B^2*b^2*c*d - A*B^2*a*b*d^2)*n)*x)*log(e))/(a^2*b^3*c^2 -
 2*a^3*b^2*c*d + a^4*b*d^2 + (b^5*c^2 - 2*a*b^4*c*d + a^2*b^3*d^2)*x^2 + 2*
(a*b^4*c^2 - 2*a^2*b^3*c*d + a^3*b^2*d^2)*x)
```

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(B\log\left(\frac{(bx+a)^n e}{(dx+c)^n}\right)+A\right)^3}{(bx+a)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*(b*x+a)^n/((d*x+c)^n)))^3/(b*x+a)^3,x, algorithm="giac
```

```
")
```

```
[Out] integrate((B*log((b*x + a)^n*e/(d*x + c)^n) + A)^3/(b*x + a)^3, x)
```

**maple [C]** time = 32.82, size = 120138, normalized size = 308.05

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))^3/(b*x+a)^3,x)
```

```
[Out] result too large to display
```

**maxima [B]** time = 2.76, size = 2246, normalized size = 5.76

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*(b*x+a)^n/((d*x+c)^n)))^3/(b*x+a)^3,x, algorithm="maxi
ma")
```

```
[Out] -1/2*B^3*log((b*x + a)^n*e/(d*x + c)^n)^3/(b^3*x^2 + 2*a*b^2*x + a^2*b) + 1
/8*(6*(2*d^2*e*n*log(b*x + a)/(b^3*c^2 - 2*a*b^2*c*d + a^2*b*d^2) - 2*d^2*e
*n*log(d*x + c)/(b^3*c^2 - 2*a*b^2*c*d + a^2*b*d^2) + (2*b*d*e*n*x - b*c*e*
n + 3*a*d*e*n)/(a^2*b^2*c - a^3*b*d + (b^4*c - a*b^3*d)*x^2 + 2*(a*b^3*c -
a^2*b^2*d)*x))*log((b*x + a)^n*e/(d*x + c)^n)^2/e - (6*(b^2*c^2*e^2*n^2 - 8
*a*b*c*d*e^2*n^2 + 7*a^2*d^2*e^2*n^2 + 2*(b^2*d^2*e^2*n^2*x^2 + 2*a*b*d^2*e
^2*n^2*x + a^2*d^2*e^2*n^2)*log(b*x + a)^2 + 2*(b^2*d^2*e^2*n^2*x^2 + 2*a*b
*d^2*e^2*n^2*x + a^2*d^2*e^2*n^2)*log(d*x + c)^2 - 6*(b^2*c*d*e^2*n^2 - a*b
*d^2*e^2*n^2)*x - 6*(b^2*d^2*e^2*n^2*x^2 + 2*a*b*d^2*e^2*n^2*x + a^2*d^2*e^
2*n^2)*log(b*x + a) + 2*(3*b^2*d^2*e^2*n^2*x^2 + 6*a*b*d^2*e^2*n^2*x + 3*a^
2*d^2*e^2*n^2 - 2*(b^2*d^2*e^2*n^2*x^2 + 2*a*b*d^2*e^2*n^2*x + a^2*d^2*e^2*
n^2)*log(b*x + a))*log(d*x + c))*log((b*x + a)^n*e/(d*x + c)^n)/((a^2*b^3*c
^2 - 2*a^3*b^2*c*d + a^4*b*d^2 + (b^5*c^2 - 2*a*b^4*c*d + a^2*b^3*d^2)*x^2
+ 2*(a*b^4*c^2 - 2*a^2*b^3*c*d + a^3*b^2*d^2)*x)*e) + (3*b^2*c^2*e^3*n^3 -
48*a*b*c*d*e^3*n^3 + 45*a^2*d^2*e^3*n^3 - 4*(b^2*d^2*e^3*n^3*x^2 + 2*a*b*d^
2*e^3*n^3*x + a^2*d^2*e^3*n^3)*log(b*x + a)^3 + 4*(b^2*d^2*e^3*n^3*x^2 + 2*
a*b*d^2*e^3*n^3*x + a^2*d^2*e^3*n^3)*log(d*x + c)^3 + 18*(b^2*d^2*e^3*n^3*x
^2 + 2*a*b*d^2*e^3*n^3*x + a^2*d^2*e^3*n^3)*log(b*x + a)^2 + 6*(3*b^2*d^2*e
^3*n^3*x^2 + 6*a*b*d^2*e^3*n^3*x + 3*a^2*d^2*e^3*n^3 - 2*(b^2*d^2*e^3*n^3*x
^2 + 2*a*b*d^2*e^3*n^3*x + a^2*d^2*e^3*n^3)*log(b*x + a))*log(d*x + c)^2 -
42*(b^2*c*d*e^3*n^3 - a*b*d^2*e^3*n^3)*x - 42*(b^2*d^2*e^3*n^3*x^2 + 2*a*b*
d^2*e^3*n^3*x + a^2*d^2*e^3*n^3)*log(b*x + a) + 6*(7*b^2*d^2*e^3*n^3*x^2 +
14*a*b*d^2*e^3*n^3*x + 7*a^2*d^2*e^3*n^3 + 2*(b^2*d^2*e^3*n^3*x^2 + 2*a*b*d
^2*e^3*n^3*x + a^2*d^2*e^3*n^3)*log(b*x + a)^2 - 6*(b^2*d^2*e^3*n^3*x^2 + 2
```

```
*a*b*d^2*e^3*n^3*x + a^2*d^2*e^3*n^3)*log(b*x + a))*log(d*x + c))/((a^2*b^3
*c^2 - 2*a^3*b^2*c*d + a^4*b*d^2 + (b^5*c^2 - 2*a*b^4*c*d + a^2*b^3*d^2)*x^
2 + 2*(a*b^4*c^2 - 2*a^2*b^3*c*d + a^3*b^2*d^2)*x)*e^2))/e)*B^3 + 3/4*A*B^2
*(2*(2*d^2*e*n*log(b*x + a)/(b^3*c^2 - 2*a*b^2*c*d + a^2*b*d^2) - 2*d^2*e*n
*log(d*x + c)/(b^3*c^2 - 2*a*b^2*c*d + a^2*b*d^2) + (2*b*d*e*n*x - b*c*e*n
+ 3*a*d*e*n)/(a^2*b^2*c - a^3*b*d + (b^4*c - a*b^3*d)*x^2 + 2*(a*b^3*c - a^
2*b^2*d)*x))*log((b*x + a)^n*e/(d*x + c)^n)/e - (b^2*c^2*e^2*n^2 - 8*a*b*c*
d*e^2*n^2 + 7*a^2*d^2*e^2*n^2 + 2*(b^2*d^2*e^2*n^2*x^2 + 2*a*b*d^2*e^2*n^2*
x + a^2*d^2*e^2*n^2)*log(b*x + a)^2 + 2*(b^2*d^2*e^2*n^2*x^2 + 2*a*b*d^2*e^
2*n^2*x + a^2*d^2*e^2*n^2)*log(d*x + c)^2 - 6*(b^2*c*d*e^2*n^2 - a*b*d^2*e^
2*n^2)*x - 6*(b^2*d^2*e^2*n^2*x^2 + 2*a*b*d^2*e^2*n^2*x + a^2*d^2*e^2*n^2)*
log(b*x + a) + 2*(3*b^2*d^2*e^2*n^2*x^2 + 6*a*b*d^2*e^2*n^2*x + 3*a^2*d^2*e
^2*n^2 - 2*(b^2*d^2*e^2*n^2*x^2 + 2*a*b*d^2*e^2*n^2*x + a^2*d^2*e^2*n^2)*lo
g(b*x + a))*log(d*x + c))/((a^2*b^3*c^2 - 2*a^3*b^2*c*d + a^4*b*d^2 + (b^5*
c^2 - 2*a*b^4*c*d + a^2*b^3*d^2)*x^2 + 2*(a*b^4*c^2 - 2*a^2*b^3*c*d + a^3*b
^2*d^2)*x)*e^2)) - 3/2*A*B^2*log((b*x + a)^n*e/(d*x + c)^n)^2/(b^3*x^2 + 2*
a*b^2*x + a^2*b) + 3/4*(2*d^2*e*n*log(b*x + a)/(b^3*c^2 - 2*a*b^2*c*d + a^2
*b*d^2) - 2*d^2*e*n*log(d*x + c)/(b^3*c^2 - 2*a*b^2*c*d + a^2*b*d^2) + (2*b
*d*e*n*x - b*c*e*n + 3*a*d*e*n)/(a^2*b^2*c - a^3*b*d + (b^4*c - a*b^3*d)*x^
2 + 2*(a*b^3*c - a^2*b^2*d)*x))*A^2*B/e - 3/2*A^2*B*log((b*x + a)^n*e/(d*x
+ c)^n)/(b^3*x^2 + 2*a*b^2*x + a^2*b) - 1/2*A^3/(b^3*x^2 + 2*a*b^2*x + a^2*
b)
```

**mupad [B]** time = 8.99, size = 966, normalized size = 2.48

$$-\ln\left(\frac{e\,(a+b\,x)^n}{(c+d\,x)^n}\right)^3\left(\frac{B^3}{2\,b\,\left(a^2+2\,a\,b\,x+b^2\,x^2\right)}-\frac{B^3\,d^2}{2\,b\,\left(a^2\,d^2-2\,a\,b\,c\,d+b^2\,c^2\right)}\right)-\frac{4\,A^3\,a\,d-4\,A^3\,b\,c+45\,B^3\,a\,d\,n^3-3\,B^3\,b\,c\,n^3+1\ldots}{2\ldots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*log((e*(a + b*x)^n)/(c + d*x)^n))^3/(a + b*x)^3,x)
```

```
[Out] - log((e*(a + b*x)^n)/(c + d*x)^n)^3*(B^3/(2*b*(a^2 + b^2*x^2 + 2*a*b*x)) -
 (B^3*d^2)/(2*b*(a^2*d^2 + b^2*c^2 - 2*a*b*c*d))) - ((4*A^3*a*d - 4*A^3*b*c
 + 45*B^3*a*d*n^3 - 3*B^3*b*c*n^3 + 18*A^2*B*a*d*n - 6*A^2*B*b*c*n + 42*A*B
^2*a*d*n^2 - 6*A*B^2*b*c*n^2)/(2*(a*d - b*c)) + (3*x*(7*B^3*b*d*n^3 + 2*A^2
*B*b*d*n + 6*A*B^2*b*d*n^2))/(a*d - b*c))/(4*a^2*b + 4*b^3*x^2 + 8*a*b^2*x)
 - log((e*(a + b*x)^n)/(c + d*x)^n)^2*((3*A*B^2)/(2*(a^2*b + b^3*x^2 + 2*a*
b^2*x)) - (3*d^2*(2*A*B^2 + 3*B^3*n))/(4*b*(a^2*d^2 + b^2*c^2 - 2*a*b*c*d))
 + (3*B^3*d^2*((b*n*(a*d - b*c)*(2*a*d - b*c))/d^2 + (2*b^2*n*x*(a*d - b*c)
)/d + (a*b*n*(a*d - b*c))/d))/(4*b*(a^2*d^2 + b^2*c^2 - 2*a*b*c*d)*(a^2*b +
 b^3*x^2 + 2*a*b^2*x))) - log((e*(a + b*x)^n)/(c + d*x)^n)*((3*B*a*c*(A^2 -
```

```
 B^2*n^2) + 3*B*x*(a*d + b*c)*(A^2 - B^2*n^2) + 3*B*b*d*x^2*(A^2 - B^2*n^2)
)/(2*b*(a + b*x)^3*(c + d*x)) + (3*d^2*(2*A*B^2 + 3*B^3*n)*(x*(((b*n*(a*d -
 b*c)*(2*a*d - b*c))/d^2 + (a*b*n*(a*d - b*c))/d)*(a*d + b*c) + (2*a*b^2*c*
n*(a*d - b*c))/d) + x^2*(b*d*((b*n*(a*d - b*c)*(2*a*d - b*c))/d^2 + (a*b*n*
(a*d - b*c))/d) + (2*b^2*n*(a*d + b*c)*(a*d - b*c))/d) + a*c*((b*n*(a*d - b
*c)*(2*a*d - b*c))/d^2 + (a*b*n*(a*d - b*c))/d) + 2*b^3*n*x^3*(a*d - b*c)))
/(4*b^2*(a + b*x)^3*(c + d*x)*(a^2*d^2 + b^2*c^2 - 2*a*b*c*d))) - (B*d^2*n*
atan((B*d^2*n*(2*b*d*x - (b^3*c^2 - a^2*b*d^2)/(b*(a*d - b*c)))*(2*A^2 + 7*
B^2*n^2 + 6*A*B*n)*3i)/((a*d - b*c)*(21*B^3*d^2*n^3 + 6*A^2*B*d^2*n + 18*A*
B^2*d^2*n^2)))*(2*A^2 + 7*B^2*n^2 + 6*A*B*n)*3i)/(2*b*(a*d - b*c)^2)
```

**sympy [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*ln(e*(b*x+a)**n/((d*x+c)**n)))**3/(b*x+a)**3,x)
[Out] Timed out
```

## 3.170 $\int \frac{\left(A+B\log(e(a+bx)^n(c+dx)^{-n})\right)^3}{(a+bx)^4}\,dx$

**Optimal**. Leaf size=611

$$-\frac{2b^2B^2n^2(c+dx)^3\left(B\log\left(e(a+bx)^n(c+dx)^{-n}\right)+A\right)}{9(a+bx)^3(bc-ad)^3}-\frac{b^2Bn(c+dx)^3\left(B\log\left(e(a+bx)^n(c+dx)^{-n}\right)+A\right)^2}{3(a+bx)^3(bc-ad)^3}-\frac{b^2(c\cdots}{\cdots}$$

```
[Out] -6*B^3*d^2*n^3*(d*x+c)/(-a*d+b*c)^3/(b*x+a)+3/4*b*B^3*d*n^3*(d*x+c)^2/(-a*d
+b*c)^3/(b*x+a)^2-2/27*b^2*B^3*n^3*(d*x+c)^3/(-a*d+b*c)^3/(b*x+a)^3-6*B^2*d
^2*n^2*(d*x+c)*(A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))/(-a*d+b*c)^3/(b*x+a)+3/2*b
*B^2*d*n^2*(d*x+c)^2*(A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))/(-a*d+b*c)^3/(b*x+a)
^2-2/9*b^2*B^2*n^2*(d*x+c)^3*(A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))/(-a*d+b*c)^3
/(b*x+a)^3-3*B*d^2*n*(d*x+c)*(A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))^2/(-a*d+b*c)
^3/(b*x+a)+3/2*b*B*d*n*(d*x+c)^2*(A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))^2/(-a*d+
b*c)^3/(b*x+a)^2-1/3*b^2*B*n*(d*x+c)^3*(A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))^2/
(-a*d+b*c)^3/(b*x+a)^3-d^2*(d*x+c)*(A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))^3/(-a*
d+b*c)^3/(b*x+a)+b*d*(d*x+c)^2*(A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))^3/(-a*d+b*
c)^3/(b*x+a)^2-1/3*b^2*(d*x+c)^3*(A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))^3/(-a*d+
b*c)^3/(b*x+a)^3
```

**Rubi [C]** time = 3.43, antiderivative size = 1876, normalized size of antiderivative = 3.07, number of steps used = 66, number of rules used = 16, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.485, Rules used = {6742, 2492, 44, 2514, 2490, 32, 2488, 2411, 2343, 2333, 2315, 2491, 2509, 37, 2506, 6610}

result too large to display

Antiderivative was successfully verified.

```
[In] Int[(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])^3/(a + b*x)^4,x]
```

```
[Out] -A^3/(3*b*(a + b*x)^3) - (A^2*B*n)/(3*b*(a + b*x)^3) - (2*A*B^2*n^2)/(9*b*(
a + b*x)^3) - (2*B^3*n^3)/(27*b*(a + b*x)^3) + (A^2*B*d*n)/(2*b*(b*c - a*d)
*(a + b*x)^2) + (5*A*B^2*d*n^2)/(6*b*(b*c - a*d)*(a + b*x)^2) + (5*B^3*d*n^
3)/(18*b*(b*c - a*d)*(a + b*x)^2) - (A^2*B*d^2*n)/(b*(b*c - a*d)^2*(a + b*x
)) - (11*A*B^2*d^2*n^2)/(3*b*(b*c - a*d)^2*(a + b*x)) - (47*B^3*d^2*n^3)/(9
*b*(b*c - a*d)^2*(a + b*x)) + (b*B^3*d*n^3*(c + d*x)^2)/(4*(b*c - a*d)^3*(a
 + b*x)^2) - (A^2*B*d^3*n*Log[a + b*x])/(b*(b*c - a*d)^3) - (5*A*B^2*d^3*n^
2*Log[a + b*x])/(3*b*(b*c - a*d)^3) - (5*B^3*d^3*n^3*Log[a + b*x])/(9*b*(b*
c - a*d)^3) + (A^2*B*d^3*n*Log[c + d*x])/(b*(b*c - a*d)^3) + (5*A*B^2*d^3*n
^2*Log[c + d*x])/(3*b*(b*c - a*d)^3) + (5*B^3*d^3*n^3*Log[c + d*x])/(9*b*(b
*c - a*d)^3) - (A^2*B*Log[(e*(a + b*x)^n)/(c + d*x)^n])/(b*(a + b*x)^3) - (
2*A*B^2*n*Log[(e*(a + b*x)^n)/(c + d*x)^n])/(3*b*(a + b*x)^3) - (2*B^3*n^2*
```

```
Log[(e*(a + b*x)^n)/(c + d*x)^n])/(9*b*(a + b*x)^3) + (A*B^2*d*n*Log[(e*(a
+ b*x)^n)/(c + d*x)^n])/(b*(b*c - a*d)*(a + b*x)^2) + (B^3*d*n^2*Log[(e*(a
+ b*x)^n)/(c + d*x)^n])/(3*b*(b*c - a*d)*(a + b*x)^2) - (2*A*B^2*d^2*n*(c +
 d*x)*Log[(e*(a + b*x)^n)/(c + d*x)^n])/((b*c - a*d)^3*(a + b*x)) - (14*B^3
*d^2*n^2*(c + d*x)*Log[(e*(a + b*x)^n)/(c + d*x)^n])/(3*(b*c - a*d)^3*(a +
b*x)) + (b*B^3*d*n^2*(c + d*x)^2*Log[(e*(a + b*x)^n)/(c + d*x)^n])/(2*(b*c
- a*d)^3*(a + b*x)^2) + (2*A*B^2*d^3*n*Log[-((b*c - a*d)/(d*(a + b*x)))]*Lo
g[(e*(a + b*x)^n)/(c + d*x)^n])/(b*(b*c - a*d)^3) + (2*B^3*d^3*n^2*Log[-((b
*c - a*d)/(d*(a + b*x)))]*Log[(e*(a + b*x)^n)/(c + d*x)^n])/(3*b*(b*c - a*d
)^3) - (2*A*B^2*d^3*n*Log[(b*c - a*d)/(b*(c + d*x))]*Log[(e*(a + b*x)^n)/(c
 + d*x)^n])/(b*(b*c - a*d)^3) - (2*B^3*d^3*n^2*Log[(b*c - a*d)/(b*(c + d*x)
)]*Log[(e*(a + b*x)^n)/(c + d*x)^n])/(3*b*(b*c - a*d)^3) - (A*B^2*Log[(e*(a
 + b*x)^n)/(c + d*x)^n]^2)/(b*(a + b*x)^3) - (B^3*n*Log[(e*(a + b*x)^n)/(c
+ d*x)^n]^2)/(3*b*(a + b*x)^3) - (2*B^3*d^2*n*(c + d*x)*Log[(e*(a + b*x)^n)
/(c + d*x)^n]^2)/((b*c - a*d)^3*(a + b*x)) + (b*B^3*d*n*(c + d*x)^2*Log[(e*
(a + b*x)^n)/(c + d*x)^n]^2)/(2*(b*c - a*d)^3*(a + b*x)^2) + (B^3*d^3*n*Log
[-((b*c - a*d)/(d*(a + b*x)))]*Log[(e*(a + b*x)^n)/(c + d*x)^n]^2)/(b*(b*c
- a*d)^3) - (B^3*d^3*n*Log[(b*c - a*d)/(b*(c + d*x))]*Log[(e*(a + b*x)^n)/(
c + d*x)^n]^2)/(b*(b*c - a*d)^3) - (B^3*Log[(e*(a + b*x)^n)/(c + d*x)^n]^3)
/(3*b*(a + b*x)^3) - (2*A*B^2*d^3*n^2*PolyLog[2, (d*(a + b*x))/(b*(c + d*x)
)])/(b*(b*c - a*d)^3) - (2*B^3*d^3*n^3*PolyLog[2, (d*(a + b*x))/(b*(c + d*x
))])/(3*b*(b*c - a*d)^3) - (2*A*B^2*d^3*n^2*PolyLog[2, 1 + (b*c - a*d)/(d*(
a + b*x))])/(b*(b*c - a*d)^3) - (2*B^3*d^3*n^3*PolyLog[2, 1 + (b*c - a*d)/(
d*(a + b*x))])/(3*b*(b*c - a*d)^3) - (2*B^3*d^3*n^2*Log[(e*(a + b*x)^n)/(c
+ d*x)^n]*PolyLog[2, 1 + (b*c - a*d)/(d*(a + b*x))])/(b*(b*c - a*d)^3) - (2
*B^3*d^3*n^2*Log[(e*(a + b*x)^n)/(c + d*x)^n]*PolyLog[2, 1 - (b*c - a*d)/(b
*(c + d*x))])/(b*(b*c - a*d)^3) - (2*B^3*d^3*n^3*PolyLog[3, 1 + (b*c - a*d)
/(d*(a + b*x))])/(b*(b*c - a*d)^3) + (2*B^3*d^3*n^3*PolyLog[3, 1 - (b*c - a
*d)/(b*(c + d*x))])/(b*(b*c - a*d)^3)
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
```

```
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2333

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)/(x_))^(q_.)*(
x_)^(m_.), x_Symbol] :> Int[(e + d*x)^q*(a + b*Log[c*x^n])^p, x] /; FreeQ[{
a, b, c, d, e, m, n, p}, x] && EqQ[m, q] && IntegerQ[q]
```

Rule 2343

```
Int[((a_.) + Log[(c_.)*(x_)^(n_)]*(b_.))/((x_)*((d_) + (e_.)*(x_)^(r_.))),
x_Symbol] :> Dist[1/n, Subst[Int[(a + b*Log[c*x])/(x*(d + e*x^(r/n))), x],
x, x^n], x] /; FreeQ[{a, b, c, d, e, n, r}, x] && IntegerQ[r/n]
```

Rule 2411

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + (g_
.)*(x_))^(q_.)*((h_.) + (i_.)*(x_))^(r_.), x_Symbol] :> Dist[1/e, Subst[Int
[((g*x)/e)^q*((e*h - d*i)/e + (i*x)/e)^r*(a + b*Log[c*x^n])^p, x], x, d + e
*x], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, n, p, q, r}, x] && EqQ[e*f - d
*g, 0] && (IGtQ[p, 0] || IGtQ[r, 0]) && IntegerQ[2*r]
```

Rule 2488

```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))
^(r_.)]^(s_.)/((g_.) + (h_.)*(x_)), x_Symbol] :> -Simp[(Log[-((b*c - a*d)/(
d*(a + b*x)))]*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^s)/h, x] + Dist[(p*r*s*
(b*c - a*d))/h, Int[(Log[-((b*c - a*d)/(d*(a + b*x)))]*Log[e*(f*(a + b*x)^p
*(c + d*x)^q)^r]^(s - 1))/((a + b*x)*(c + d*x)), x], x] /; FreeQ[{a, b, c,
d, e, f, g, h, p, q, r, s}, x] && NeQ[b*c - a*d, 0] && EqQ[p + q, 0] && EqQ
[b*g - a*h, 0] && IGtQ[s, 0]
```

Rule 2490

```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))
^(r_.)]^(s_.)/((g_.) + (h_.)*(x_))^2, x_Symbol] :> Simp[((a + b*x)*Log[e*(f
*(a + b*x)^p*(c + d*x)^q)^r]^s)/((b*g - a*h)*(g + h*x)), x] - Dist[(p*r*s*(
b*c - a*d))/(b*g - a*h), Int[Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^(s - 1)/(
(c + d*x)*(g + h*x)), x], x] /; FreeQ[{a, b, c, d, e, f, g, h, p, q, r, s},
```

```
x] && NeQ[b*c - a*d, 0] && EqQ[p + q, 0] && NeQ[b*g - a*h, 0] && IGtQ[s, 0
]
```

Rule 2491

```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))
^(r_.)]^(s_)/((g_.) + (h_.)*(x_))^3, x_Symbol] :> Dist[d/(d*g - c*h), Int[L
og[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^s/(g + h*x)^2, x], x] - Dist[h/(d*g - c
*h), Int[((c + d*x)*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^s)/(g + h*x)^3, x]
, x] /; FreeQ[{a, b, c, d, e, f, g, h, p, q, r, s}, x] && NeQ[b*c - a*d, 0]
 && EqQ[p + q, 0] && EqQ[b*g - a*h, 0] && NeQ[d*g - c*h, 0] && IGtQ[s, 0]
```

Rule 2492

```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))
^(r_.)]^(s_.)*((g_.) + (h_.)*(x_))^(m_.), x_Symbol] :> Simp[((g + h*x)^(m +
 1)*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^s)/(h*(m + 1)), x] - Dist[(p*r*s*(
b*c - a*d))/(h*(m + 1)), Int[((g + h*x)^(m + 1)*Log[e*(f*(a + b*x)^p*(c + d
*x)^q)^r]^(s - 1))/((a + b*x)*(c + d*x)), x], x] /; FreeQ[{a, b, c, d, e, f
, g, h, m, p, q, r, s}, x] && NeQ[b*c - a*d, 0] && EqQ[p + q, 0] && IGtQ[s,
 0] && NeQ[m, -1]
```

Rule 2506

```
Int[Log[v_]*Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_)
)^(q_.))^(r_.)]^(s_.)*(u_), x_Symbol] :> With[{g = Simplify[((v - 1)*(c + d
*x))/(a + b*x)], h = Simplify[u*(a + b*x)*(c + d*x)]}, -Simp[(h*PolyLog[2,
1 - v]*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^s)/(b*c - a*d), x] + Dist[h*p*r
*s, Int[(PolyLog[2, 1 - v]*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^(s - 1))/((
a + b*x)*(c + d*x)), x], x] /; FreeQ[{g, h}, x]] /; FreeQ[{a, b, c, d, e, f
, p, q, r, s}, x] && NeQ[b*c - a*d, 0] && IGtQ[s, 0] && EqQ[p + q, 0]
```

Rule 2509

```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))
^(r_.)]^(s_.)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symb
ol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^(n + 1)*Log[e*(f*(a + b*x)^p*(c +
d*x)^q)^r]^s)/((m + 1)*(b*c - a*d)), x] - Dist[(p*r*s*(b*c - a*d))/((m + 1)
*(b*c - a*d)), Int[(a + b*x)^m*(c + d*x)^n*Log[e*(f*(a + b*x)^p*(c + d*x)^q
)^r]^(s - 1), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p, q, r, s}, x] && N
eQ[b*c - a*d, 0] && EqQ[p + q, 0] && EqQ[m + n + 2, 0] && NeQ[m, -1] && IGt
Q[s, 0]
```

Rule 2514

```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))
^(r_.)]^(s_.)*(RFx_), x_Symbol] :> With[{u = ExpandIntegrand[Log[e*(f*(a +
b*x)^p*(c + d*x)^q)^r]^s, RFx, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c
, d, e, f, p, q, r, s}, x] && RationalFunctionQ[RFx, x] && IGtQ[s, 0]
```

Rule 6610

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\int \frac{\left(A + B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^3}{(a+bx)^4}\,dx = \int\Bigg(\frac{A^3}{(a+bx)^4} + \frac{3A^2B\log\left(e(a+bx)^n(c+dx)^{-n}\right)}{(a+bx)^4} + \frac{3AB^2\log^2\left(e(a \ldots}{(a \ldots}$$

$$= -\frac{A^3}{3b(a+bx)^3} + \left(3A^2B\right)\int \frac{\log\left(e(a+bx)^n(c+dx)^{-n}\right)}{(a+bx)^4}\,dx + \left(3AB^2\right) \ldots$$

$$= -\frac{A^3}{3b(a+bx)^3} - \frac{A^2B\log\left(e(a+bx)^n(c+dx)^{-n}\right)}{b(a+bx)^3} - \frac{AB^2\log^2\left(e(a+ \ldots}{b(a+ \ldots}$$

$$= -\frac{A^3}{3b(a+bx)^3} - \frac{A^2B\log\left(e(a+bx)^n(c+dx)^{-n}\right)}{b(a+bx)^3} - \frac{AB^2\log^2\left(e(a+ \ldots}{b(a+ \ldots}$$

$$= -\frac{A^3}{3b(a+bx)^3} - \frac{A^2Bn}{3b(a+bx)^3} + \frac{A^2Bdn}{2b(bc-ad)(a+bx)^2} - \frac{A^2Bd^2 \ldots}{b(bc-ad)^2( \ldots}$$

$$= -\frac{A^3}{3b(a+bx)^3} - \frac{A^2Bn}{3b(a+bx)^3} + \frac{A^2Bdn}{2b(bc-ad)(a+bx)^2} - \frac{A^2Bd^2 \ldots}{b(bc-ad)^2( \ldots}$$

$$= -\frac{A^3}{3b(a+bx)^3} - \frac{A^2Bn}{3b(a+bx)^3} + \frac{A^2Bdn}{2b(bc-ad)(a+bx)^2} - \frac{A^2Bd^2 \ldots}{b(bc-ad)^2( \ldots}$$

$$= -\frac{A^3}{3b(a+bx)^3} - \frac{A^2Bn}{3b(a+bx)^3} - \frac{2AB^2n^2}{9b(a+bx)^3} + \frac{A^2Bdn}{2b(bc-ad)(a+bx)^2} \ldots$$

$$= -\frac{A^3}{3b(a+bx)^3} - \frac{A^2Bn}{3b(a+bx)^3} - \frac{2AB^2n^2}{9b(a+bx)^3} + \frac{A^2Bdn}{2b(bc-ad)(a+bx)^2} \ldots$$

$$= -\frac{A^3}{3b(a+bx)^3} - \frac{A^2Bn}{3b(a+bx)^3} - \frac{2AB^2n^2}{9b(a+bx)^3} + \frac{A^2Bdn}{2b(bc-ad)(a+bx)^2} \ldots$$

$$= -\frac{A^3}{3b(a+bx)^3} - \frac{A^2Bn}{3b(a+bx)^3} - \frac{2AB^2n^2}{9b(a+bx)^3} - \frac{2B^3n^3}{27b(a+bx)^3} + \frac{\ldots}{2b(bc \ldots}$$

$$= -\frac{A^3}{3b(a+bx)^3} - \frac{A^2Bn}{3b(a+bx)^3} - \frac{2AB^2n^2}{9b(a+bx)^3} - \frac{2B^3n^3}{27b(a+bx)^3} + \frac{\ldots}{2b(bc \ldots}$$

$$= -\frac{A^3}{3b(a+bx)^3} - \frac{A^2Bn}{3b(a+bx)^3} - \frac{2AB^2n^2}{9b(a+bx)^3} - \frac{2B^3n^3}{27b(a+bx)^3} + \frac{\ldots}{2b(bc \ldots}$$

**Mathematica [A]** time = 1.50, size = 1003, normalized size = 1.64

$$\frac{-36B^3d^3n^3\log^3(a+bx)(a+bx)^3+36B^3d^3n^3\log^3(c+dx)(a+bx)^3+18B^2d^3n^2\log^2(c+dx)\left(6A+11Bn+6B\log\ldots\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])^3/(a + b*x)^4,x]
```

```
[Out] (-36*B^3*d^3*n^3*(a + b*x)^3*Log[a + b*x]^3 + 36*B^3*d^3*n^3*(a + b*x)^3*Lo
g[c + d*x]^3 + 18*B^2*d^3*n^2*(a + b*x)^3*Log[c + d*x]^2*(6*A + 11*B*n + 6*
B*Log[(e*(a + b*x)^n)/(c + d*x)^n]) + 18*B^2*d^3*n^2*(a + b*x)^3*Log[a + b*
x]^2*(6*A + 11*B*n + 6*B*n*Log[c + d*x] + 6*B*Log[(e*(a + b*x)^n)/(c + d*x)
^n]) + 6*B*d^3*n*(a + b*x)^3*Log[c + d*x]*(18*A^2 + 66*A*B*n + 85*B^2*n^2 +
 6*B*(6*A + 11*B*n)*Log[(e*(a + b*x)^n)/(c + d*x)^n] + 18*B^2*Log[(e*(a + b
*x)^n)/(c + d*x)^n]^2) - (b*c - a*d)*(36*A^3*b^2*c^2 - 72*a*A^3*b*c*d + 36*
a^2*A^3*d^2 + 36*A^2*b^2*B*c^2*n - 126*a*A^2*b*B*c*d*n + 198*a^2*A^2*B*d^2*
n + 24*A*b^2*B^2*c^2*n^2 - 138*a*A*b*B^2*c*d*n^2 + 510*a^2*A*B^2*d^2*n^2 +
8*b^2*B^3*c^2*n^3 - 73*a*b*B^3*c*d*n^3 + 575*a^2*B^3*d^2*n^3 - 54*A^2*b^2*B
*c*d*n*x + 270*a*A^2*b*B*d^2*n*x - 90*A*b^2*B^2*c*d*n^2*x + 882*a*A*b*B^2*d
^2*n^2*x - 57*b^2*B^3*c*d*n^3*x + 1077*a*b*B^3*d^2*n^3*x + 108*A^2*b^2*B*d^
2*n*x^2 + 396*A*b^2*B^2*d^2*n^2*x^2 + 510*b^2*B^3*d^2*n^3*x^2 + 6*B*(18*A^2
*(b*c - a*d)^2 + 6*A*B*n*(11*a^2*d^2 + a*b*d*(-7*c + 15*d*x) + b^2*(2*c^2 -
 3*c*d*x + 6*d^2*x^2)) + B^2*n^2*(85*a^2*d^2 + a*b*d*(-23*c + 147*d*x) + b^
2*(4*c^2 - 15*c*d*x + 66*d^2*x^2)))*Log[(e*(a + b*x)^n)/(c + d*x)^n] + 18*B
^2*(6*A*(b*c - a*d)^2 + B*n*(11*a^2*d^2 + a*b*d*(-7*c + 15*d*x) + b^2*(2*c^
2 - 3*c*d*x + 6*d^2*x^2)))*Log[(e*(a + b*x)^n)/(c + d*x)^n]^2 + 36*B^3*(b*c
 - a*d)^2*Log[(e*(a + b*x)^n)/(c + d*x)^n]^3) - 6*B*d^3*n*(a + b*x)^3*Log[a
 + b*x]*(18*A^2 + 66*A*B*n + 85*B^2*n^2 + 18*B^2*n^2*Log[c + d*x]^2 + 6*B*(
6*A + 11*B*n)*Log[(e*(a + b*x)^n)/(c + d*x)^n] + 18*B^2*Log[(e*(a + b*x)^n)
/(c + d*x)^n]^2 + 6*B*n*Log[c + d*x]*(6*A + 11*B*n + 6*B*Log[(e*(a + b*x)^n
)/(c + d*x)^n])))/(108*b*(b*c - a*d)^3*(a + b*x)^3)
```

**fricas [B]** time = 1.11, size = 4008, normalized size = 6.56

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*(b*x+a)^n/((d*x+c)^n)))^3/(b*x+a)^4,x, algorithm="fric
as")
```

```
[Out] -1/108*(36*A^3*b^3*c^3 - 108*A^3*a*b^2*c^2*d + 108*A^3*a^2*b*c*d^2 - 36*A^3
*a^3*d^3 + (8*B^3*b^3*c^3 - 81*B^3*a*b^2*c^2*d + 648*B^3*a^2*b*c*d^2 - 575*
B^3*a^3*d^3)*n^3 + 36*(B^3*b^3*d^3*n^3*x^3 + 3*B^3*a*b^2*d^3*n^3*x^2 + 3*B^
3*a^2*b*d^3*n^3*x + (B^3*b^3*c^3 - 3*B^3*a*b^2*c^2*d + 3*B^3*a^2*b*c*d^2)*n
```

```
^3)*log(b*x + a)^3 - 36*(B^3*b^3*d^3*n^3*x^3 + 3*B^3*a*b^2*d^3*n^3*x^2 + 3*
B^3*a^2*b*d^3*n^3*x + (B^3*b^3*c^3 - 3*B^3*a*b^2*c^2*d + 3*B^3*a^2*b*c*d^2)
*n^3)*log(d*x + c)^3 + 36*(B^3*b^3*c^3 - 3*B^3*a*b^2*c^2*d + 3*B^3*a^2*b*c*
d^2 - B^3*a^3*d^3)*log(e)^3 + 6*(4*A*B^2*b^3*c^3 - 27*A*B^2*a*b^2*c^2*d + 1
08*A*B^2*a^2*b*c*d^2 - 85*A*B^2*a^3*d^3)*n^2 + 6*(85*(B^3*b^3*c*d^2 - B^3*a
*b^2*d^3)*n^3 + 66*(A*B^2*b^3*c*d^2 - A*B^2*a*b^2*d^3)*n^2 + 18*(A^2*B*b^3*
c*d^2 - A^2*B*a*b^2*d^3)*n)*x^2 + 18*((2*B^3*b^3*c^3 - 9*B^3*a*b^2*c^2*d +
18*B^3*a^2*b*c*d^2)*n^3 + (11*B^3*b^3*d^3*n^3 + 6*A*B^2*b^3*d^3*n^2)*x^3 +
6*(A*B^2*b^3*c^3 - 3*A*B^2*a*b^2*c^2*d + 3*A*B^2*a^2*b*c*d^2)*n^2 + 3*(6*A*
B^2*a*b^2*d^3*n^2 + (2*B^3*b^3*c*d^2 + 9*B^3*a*b^2*d^3)*n^3)*x^2 + 3*(6*A*B
^2*a^2*b*d^3*n^2 - (B^3*b^3*c^2*d - 6*B^3*a*b^2*c*d^2 - 6*B^3*a^2*b*d^3)*n^
3)*x + 6*(B^3*b^3*d^3*n^2*x^3 + 3*B^3*a*b^2*d^3*n^2*x^2 + 3*B^3*a^2*b*d^3*n
^2*x + (B^3*b^3*c^3 - 3*B^3*a*b^2*c^2*d + 3*B^3*a^2*b*c*d^2)*n^2)*log(e))*l
og(b*x + a)^2 + 18*((2*B^3*b^3*c^3 - 9*B^3*a*b^2*c^2*d + 18*B^3*a^2*b*c*d^2
)*n^3 + (11*B^3*b^3*d^3*n^3 + 6*A*B^2*b^3*d^3*n^2)*x^3 + 6*(A*B^2*b^3*c^3 -
 3*A*B^2*a*b^2*c^2*d + 3*A*B^2*a^2*b*c*d^2)*n^2 + 3*(6*A*B^2*a*b^2*d^3*n^2
+ (2*B^3*b^3*c*d^2 + 9*B^3*a*b^2*d^3)*n^3)*x^2 + 3*(6*A*B^2*a^2*b*d^3*n^2 -
 (B^3*b^3*c^2*d - 6*B^3*a*b^2*c*d^2 - 6*B^3*a^2*b*d^3)*n^3)*x + 6*(B^3*b^3*
d^3*n^3*x^3 + 3*B^3*a*b^2*d^3*n^3*x^2 + 3*B^3*a^2*b*d^3*n^3*x + (B^3*b^3*c^
3 - 3*B^3*a*b^2*c^2*d + 3*B^3*a^2*b*c*d^2)*n^3)*log(b*x + a) + 6*(B^3*b^3*d
^3*n^2*x^3 + 3*B^3*a*b^2*d^3*n^2*x^2 + 3*B^3*a^2*b*d^3*n^2*x + (B^3*b^3*c^3
 - 3*B^3*a*b^2*c^2*d + 3*B^3*a^2*b*c*d^2)*n^2)*log(e))*log(d*x + c)^2 + 18*
(6*A*B^2*b^3*c^3 - 18*A*B^2*a*b^2*c^2*d + 18*A*B^2*a^2*b*c*d^2 - 6*A*B^2*a^
3*d^3 + 6*(B^3*b^3*c*d^2 - B^3*a*b^2*d^3)*n*x^2 - 3*(B^3*b^3*c^2*d - 6*B^3*
a*b^2*c*d^2 + 5*B^3*a^2*b*d^3)*n*x + (2*B^3*b^3*c^3 - 9*B^3*a*b^2*c^2*d + 1
8*B^3*a^2*b*c*d^2 - 11*B^3*a^3*d^3)*n)*log(e)^2 + 18*(2*A^2*B*b^3*c^3 - 9*A
^2*B*a*b^2*c^2*d + 18*A^2*B*a^2*b*c*d^2 - 11*A^2*B*a^3*d^3)*n - 3*((19*B^3*
b^3*c^2*d - 378*B^3*a*b^2*c*d^2 + 359*B^3*a^2*b*d^3)*n^3 + 6*(5*A*B^2*b^3*c
^2*d - 54*A*B^2*a*b^2*c*d^2 + 49*A*B^2*a^2*b*d^3)*n^2 + 18*(A^2*B*b^3*c^2*d
 - 6*A^2*B*a*b^2*c*d^2 + 5*A^2*B*a^2*b*d^3)*n)*x + 6*((4*B^3*b^3*c^3 - 27*B
^3*a*b^2*c^2*d + 108*B^3*a^2*b*c*d^2)*n^3 + (85*B^3*b^3*d^3*n^3 + 66*A*B^2*
b^3*d^3*n^2 + 18*A^2*B*b^3*d^3*n)*x^3 + 6*(2*A*B^2*b^3*c^3 - 9*A*B^2*a*b^2*
c^2*d + 18*A*B^2*a^2*b*c*d^2)*n^2 + 3*(18*A^2*B*a*b^2*d^3*n + (22*B^3*b^3*c
*d^2 + 63*B^3*a*b^2*d^3)*n^3 + 6*(2*A*B^2*b^3*c*d^2 + 9*A*B^2*a*b^2*d^3)*n^
2)*x^2 + 18*(B^3*b^3*d^3*n*x^3 + 3*B^3*a*b^2*d^3*n*x^2 + 3*B^3*a^2*b*d^3*n*
x + (B^3*b^3*c^3 - 3*B^3*a*b^2*c^2*d + 3*B^3*a^2*b*c*d^2)*n)*log(e)^2 + 18*
(A^2*B*b^3*c^3 - 3*A^2*B*a*b^2*c^2*d + 3*A^2*B*a^2*b*c*d^2)*n + 3*(18*A^2*B
*a^2*b*d^3*n - (5*B^3*b^3*c^2*d - 54*B^3*a*b^2*c*d^2 - 36*B^3*a^2*b*d^3)*n^
3 - 6*(A*B^2*b^3*c^2*d - 6*A*B^2*a*b^2*c*d^2 - 6*A*B^2*a^2*b*d^3)*n^2)*x +
6*((11*B^3*b^3*d^3*n^2 + 6*A*B^2*b^3*d^3*n)*x^3 + (2*B^3*b^3*c^3 - 9*B^3*a*
b^2*c^2*d + 18*B^3*a^2*b*c*d^2)*n^2 + 3*(6*A*B^2*a*b^2*d^3*n + (2*B^3*b^3*c
*d^2 + 9*B^3*a*b^2*d^3)*n^2)*x^2 + 6*(A*B^2*b^3*c^3 - 3*A*B^2*a*b^2*c^2*d +
 3*A*B^2*a^2*b*c*d^2)*n + 3*(6*A*B^2*a^2*b*d^3*n - (B^3*b^3*c^2*d - 6*B^3*a
*b^2*c*d^2 - 6*B^3*a^2*b*d^3)*n^2)*x)*log(e))*log(b*x + a) - 6*((4*B^3*b^3*
c^3 - 27*B^3*a*b^2*c^2*d + 108*B^3*a^2*b*c*d^2)*n^3 + (85*B^3*b^3*d^3*n^3 +
```

```
66*A*B^2*b^3*d^3*n^2 + 18*A^2*B*b^3*d^3*n)*x^3 + 6*(2*A*B^2*b^3*c^3 - 9*A*
B^2*a*b^2*c^2*d + 18*A*B^2*a^2*b*c*d^2)*n^2 + 3*(18*A^2*B*a*b^2*d^3*n + (22
*B^3*b^3*c*d^2 + 63*B^3*a*b^2*d^3)*n^3 + 6*(2*A*B^2*b^3*c*d^2 + 9*A*B^2*a*b
^2*d^3)*n^2)*x^2 + 18*(B^3*b^3*d^3*n^3*x^3 + 3*B^3*a*b^2*d^3*n^3*x^2 + 3*B^
3*a^2*b*d^3*n^3*x + (B^3*b^3*c^3 - 3*B^3*a*b^2*c^2*d + 3*B^3*a^2*b*c*d^2)*n
^3)*log(b*x + a)^2 + 18*(B^3*b^3*d^3*n*x^3 + 3*B^3*a*b^2*d^3*n*x^2 + 3*B^3*
a^2*b*d^3*n*x + (B^3*b^3*c^3 - 3*B^3*a*b^2*c^2*d + 3*B^3*a^2*b*c*d^2)*n)*lo
g(e)^2 + 18*(A^2*B*b^3*c^3 - 3*A^2*B*a*b^2*c^2*d + 3*A^2*B*a^2*b*c*d^2)*n +
 3*(18*A^2*B*a^2*b*d^3*n - (5*B^3*b^3*c^2*d - 54*B^3*a*b^2*c*d^2 - 36*B^3*a
^2*b*d^3)*n^3 - 6*(A*B^2*b^3*c^2*d - 6*A*B^2*a*b^2*c*d^2 - 6*A*B^2*a^2*b*d^
3)*n^2)*x + 6*((2*B^3*b^3*c^3 - 9*B^3*a*b^2*c^2*d + 18*B^3*a^2*b*c*d^2)*n^3
 + (11*B^3*b^3*d^3*n^3 + 6*A*B^2*b^3*d^3*n^2)*x^3 + 6*(A*B^2*b^3*c^3 - 3*A*
B^2*a*b^2*c^2*d + 3*A*B^2*a^2*b*c*d^2)*n^2 + 3*(6*A*B^2*a*b^2*d^3*n^2 + (2*
B^3*b^3*c*d^2 + 9*B^3*a*b^2*d^3)*n^3)*x^2 + 3*(6*A*B^2*a^2*b*d^3*n^2 - (B^3
*b^3*c^2*d - 6*B^3*a*b^2*c*d^2 - 6*B^3*a^2*b*d^3)*n^3)*x + 6*(B^3*b^3*d^3*n
^2*x^3 + 3*B^3*a*b^2*d^3*n^2*x^2 + 3*B^3*a^2*b*d^3*n^2*x + (B^3*b^3*c^3 - 3
*B^3*a*b^2*c^2*d + 3*B^3*a^2*b*c*d^2)*n^2)*log(e))*log(b*x + a) + 6*((11*B^
3*b^3*d^3*n^2 + 6*A*B^2*b^3*d^3*n)*x^3 + (2*B^3*b^3*c^3 - 9*B^3*a*b^2*c^2*d
 + 18*B^3*a^2*b*c*d^2)*n^2 + 3*(6*A*B^2*a*b^2*d^3*n + (2*B^3*b^3*c*d^2 + 9*
B^3*a*b^2*d^3)*n^2)*x^2 + 6*(A*B^2*b^3*c^3 - 3*A*B^2*a*b^2*c^2*d + 3*A*B^2*
a^2*b*c*d^2)*n + 3*(6*A*B^2*a^2*b*d^3*n - (B^3*b^3*c^2*d - 6*B^3*a*b^2*c*d^
2 - 6*B^3*a^2*b*d^3)*n^2)*x)*log(e))*log(d*x + c) + 6*(18*A^2*B*b^3*c^3 - 5
4*A^2*B*a*b^2*c^2*d + 54*A^2*B*a^2*b*c*d^2 - 18*A^2*B*a^3*d^3 + (4*B^3*b^3*
c^3 - 27*B^3*a*b^2*c^2*d + 108*B^3*a^2*b*c*d^2 - 85*B^3*a^3*d^3)*n^2 + 6*(1
1*(B^3*b^3*c*d^2 - B^3*a*b^2*d^3)*n^2 + 6*(A*B^2*b^3*c*d^2 - A*B^2*a*b^2*d^
3)*n)*x^2 + 6*(2*A*B^2*b^3*c^3 - 9*A*B^2*a*b^2*c^2*d + 18*A*B^2*a^2*b*c*d^2
 - 11*A*B^2*a^3*d^3)*n - 3*((5*B^3*b^3*c^2*d - 54*B^3*a*b^2*c*d^2 + 49*B^3*
a^2*b*d^3)*n^2 + 6*(A*B^2*b^3*c^2*d - 6*A*B^2*a*b^2*c*d^2 + 5*A*B^2*a^2*b*d
^3)*n)*x)*log(e))/(a^3*b^4*c^3 - 3*a^4*b^3*c^2*d + 3*a^5*b^2*c*d^2 - a^6*b*
d^3 + (b^7*c^3 - 3*a*b^6*c^2*d + 3*a^2*b^5*c*d^2 - a^3*b^4*d^3)*x^3 + 3*(a*
b^6*c^3 - 3*a^2*b^5*c^2*d + 3*a^3*b^4*c*d^2 - a^4*b^3*d^3)*x^2 + 3*(a^2*b^5
*c^3 - 3*a^3*b^4*c^2*d + 3*a^4*b^3*c*d^2 - a^5*b^2*d^3)*x)
```

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(B\log\left(\frac{(bx+a)^n e}{(dx+c)^n}\right)+A\right)^3}{(bx+a)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*(b*x+a)^n/((d*x+c)^n)))^3/(b*x+a)^4,x, algorithm="giac
")
```

```
[Out] integrate((B*log((b*x + a)^n*e/(d*x + c)^n) + A)^3/(b*x + a)^4, x)
```

**maple [C]** time = 48.03, size = 175812, normalized size = 287.74

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))^3/(b*x+a)^4,x)
```

```
[Out] result too large to display
```

**maxima [B]** time = 4.17, size = 3630, normalized size = 5.94

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*(b*x+a)^n/((d*x+c)^n)))^3/(b*x+a)^4,x, algorithm="maxi
ma")
```

```
[Out] -1/3*B^3*log((b*x + a)^n*e/(d*x + c)^n)^3/(b^4*x^3 + 3*a*b^3*x^2 + 3*a^2*b^
2*x + a^3*b) - 1/108*(18*(6*d^3*e*n*log(b*x + a)/(b^4*c^3 - 3*a*b^3*c^2*d +
 3*a^2*b^2*c*d^2 - a^3*b*d^3) - 6*d^3*e*n*log(d*x + c)/(b^4*c^3 - 3*a*b^3*c
^2*d + 3*a^2*b^2*c*d^2 - a^3*b*d^3) + (6*b^2*d^2*e*n*x^2 + 2*b^2*c^2*e*n -
7*a*b*c*d*e*n + 11*a^2*d^2*e*n - 3*(b^2*c*d*e*n - 5*a*b*d^2*e*n)*x)/(a^3*b^
3*c^2 - 2*a^4*b^2*c*d + a^5*b*d^2 + (b^6*c^2 - 2*a*b^5*c*d + a^2*b^4*d^2)*x
^3 + 3*(a*b^5*c^2 - 2*a^2*b^4*c*d + a^3*b^3*d^2)*x^2 + 3*(a^2*b^4*c^2 - 2*a
^3*b^3*c*d + a^4*b^2*d^2)*x))*log((b*x + a)^n*e/(d*x + c)^n)^2/e + (6*(4*b^
3*c^3*e^2*n^2 - 27*a*b^2*c^2*d*e^2*n^2 + 108*a^2*b*c*d^2*e^2*n^2 - 85*a^3*d
^3*e^2*n^2 + 66*(b^3*c*d^2*e^2*n^2 - a*b^2*d^3*e^2*n^2)*x^2 - 18*(b^3*d^3*e
^2*n^2*x^3 + 3*a*b^2*d^3*e^2*n^2*x^2 + 3*a^2*b*d^3*e^2*n^2*x + a^3*d^3*e^2*
n^2)*log(b*x + a)^2 - 18*(b^3*d^3*e^2*n^2*x^3 + 3*a*b^2*d^3*e^2*n^2*x^2 + 3
*a^2*b*d^3*e^2*n^2*x + a^3*d^3*e^2*n^2)*log(d*x + c)^2 - 3*(5*b^3*c^2*d*e^2
*n^2 - 54*a*b^2*c*d^2*e^2*n^2 + 49*a^2*b*d^3*e^2*n^2)*x + 66*(b^3*d^3*e^2*n
^2*x^3 + 3*a*b^2*d^3*e^2*n^2*x^2 + 3*a^2*b*d^3*e^2*n^2*x + a^3*d^3*e^2*n^2)
*log(b*x + a) - 6*(11*b^3*d^3*e^2*n^2*x^3 + 33*a*b^2*d^3*e^2*n^2*x^2 + 33*a
^2*b*d^3*e^2*n^2*x + 11*a^3*d^3*e^2*n^2 - 6*(b^3*d^3*e^2*n^2*x^3 + 3*a*b^2*
d^3*e^2*n^2*x^2 + 3*a^2*b*d^3*e^2*n^2*x + a^3*d^3*e^2*n^2)*log(b*x + a))*lo
g(d*x + c))*log((b*x + a)^n*e/(d*x + c)^n)/((a^3*b^4*c^3 - 3*a^4*b^3*c^2*d
+ 3*a^5*b^2*c*d^2 - a^6*b*d^3 + (b^7*c^3 - 3*a*b^6*c^2*d + 3*a^2*b^5*c*d^2
- a^3*b^4*d^3)*x^3 + 3*(a*b^6*c^3 - 3*a^2*b^5*c^2*d + 3*a^3*b^4*c*d^2 - a^4
*b^3*d^3)*x^2 + 3*(a^2*b^5*c^3 - 3*a^3*b^4*c^2*d + 3*a^4*b^3*c*d^2 - a^5*b^
2*d^3)*x)*e) + (8*b^3*c^3*e^3*n^3 - 81*a*b^2*c^2*d*e^3*n^3 + 648*a^2*b*c*d^
2*e^3*n^3 - 575*a^3*d^3*e^3*n^3 + 36*(b^3*d^3*e^3*n^3*x^3 + 3*a*b^2*d^3*e^3
*n^3*x^2 + 3*a^2*b*d^3*e^3*n^3*x + a^3*d^3*e^3*n^3)*log(b*x + a)^3 - 36*(b^
3*d^3*e^3*n^3*x^3 + 3*a*b^2*d^3*e^3*n^3*x^2 + 3*a^2*b*d^3*e^3*n^3*x + a^3*d
^3*e^3*n^3)*log(d*x + c)^3 + 510*(b^3*c*d^2*e^3*n^3 - a*b^2*d^3*e^3*n^3)*x^
```

```
2 - 198*(b^3*d^3*e^3*n^3*x^3 + 3*a*b^2*d^3*e^3*n^3*x^2 + 3*a^2*b*d^3*e^3*n^
3*x + a^3*d^3*e^3*n^3)*log(b*x + a)^2 - 18*(11*b^3*d^3*e^3*n^3*x^3 + 33*a*b
^2*d^3*e^3*n^3*x^2 + 33*a^2*b*d^3*e^3*n^3*x + 11*a^3*d^3*e^3*n^3 - 6*(b^3*d
^3*e^3*n^3*x^3 + 3*a*b^2*d^3*e^3*n^3*x^2 + 3*a^2*b*d^3*e^3*n^3*x + a^3*d^3*
e^3*n^3)*log(b*x + a))*log(d*x + c)^2 - 3*(19*b^3*c^2*d*e^3*n^3 - 378*a*b^2
*c*d^2*e^3*n^3 + 359*a^2*b*d^3*e^3*n^3)*x + 510*(b^3*d^3*e^3*n^3*x^3 + 3*a*
b^2*d^3*e^3*n^3*x^2 + 3*a^2*b*d^3*e^3*n^3*x + a^3*d^3*e^3*n^3)*log(b*x + a)
 - 6*(85*b^3*d^3*e^3*n^3*x^3 + 255*a*b^2*d^3*e^3*n^3*x^2 + 255*a^2*b*d^3*e^
3*n^3*x + 85*a^3*d^3*e^3*n^3 + 18*(b^3*d^3*e^3*n^3*x^3 + 3*a*b^2*d^3*e^3*n^
3*x^2 + 3*a^2*b*d^3*e^3*n^3*x + a^3*d^3*e^3*n^3)*log(b*x + a)^2 - 66*(b^3*d
^3*e^3*n^3*x^3 + 3*a*b^2*d^3*e^3*n^3*x^2 + 3*a^2*b*d^3*e^3*n^3*x + a^3*d^3*
e^3*n^3)*log(b*x + a))*log(d*x + c))/((a^3*b^4*c^3 - 3*a^4*b^3*c^2*d + 3*a^
5*b^2*c*d^2 - a^6*b*d^3 + (b^7*c^3 - 3*a*b^6*c^2*d + 3*a^2*b^5*c*d^2 - a^3*
b^4*d^3)*x^3 + 3*(a*b^6*c^3 - 3*a^2*b^5*c^2*d + 3*a^3*b^4*c*d^2 - a^4*b^3*d
^3)*x^2 + 3*(a^2*b^5*c^3 - 3*a^3*b^4*c^2*d + 3*a^4*b^3*c*d^2 - a^5*b^2*d^3)
*x)*e^2))/e)*B^3 - 1/18*A*B^2*(6*(6*d^3*e*n*log(b*x + a)/(b^4*c^3 - 3*a*b^3
*c^2*d + 3*a^2*b^2*c*d^2 - a^3*b*d^3) - 6*d^3*e*n*log(d*x + c)/(b^4*c^3 - 3
*a*b^3*c^2*d + 3*a^2*b^2*c*d^2 - a^3*b*d^3) + (6*b^2*d^2*e*n*x^2 + 2*b^2*c^
2*e*n - 7*a*b*c*d*e*n + 11*a^2*d^2*e*n - 3*(b^2*c*d*e*n - 5*a*b*d^2*e*n)*x)
/(a^3*b^3*c^2 - 2*a^4*b^2*c*d + a^5*b*d^2 + (b^6*c^2 - 2*a*b^5*c*d + a^2*b^
4*d^2)*x^3 + 3*(a*b^5*c^2 - 2*a^2*b^4*c*d + a^3*b^3*d^2)*x^2 + 3*(a^2*b^4*c
^2 - 2*a^3*b^3*c*d + a^4*b^2*d^2)*x))*log((b*x + a)^n*e/(d*x + c)^n)/e + (4
*b^3*c^3*e^2*n^2 - 27*a*b^2*c^2*d*e^2*n^2 + 108*a^2*b*c*d^2*e^2*n^2 - 85*a^
3*d^3*e^2*n^2 + 66*(b^3*c*d^2*e^2*n^2 - a*b^2*d^3*e^2*n^2)*x^2 - 18*(b^3*d^
3*e^2*n^2*x^3 + 3*a*b^2*d^3*e^2*n^2*x^2 + 3*a^2*b*d^3*e^2*n^2*x + a^3*d^3*e
^2*n^2)*log(b*x + a)^2 - 18*(b^3*d^3*e^2*n^2*x^3 + 3*a*b^2*d^3*e^2*n^2*x^2
+ 3*a^2*b*d^3*e^2*n^2*x + a^3*d^3*e^2*n^2)*log(d*x + c)^2 - 3*(5*b^3*c^2*d*
e^2*n^2 - 54*a*b^2*c*d^2*e^2*n^2 + 49*a^2*b*d^3*e^2*n^2)*x + 66*(b^3*d^3*e^
2*n^2*x^3 + 3*a*b^2*d^3*e^2*n^2*x^2 + 3*a^2*b*d^3*e^2*n^2*x + a^3*d^3*e^2*n
^2)*log(b*x + a) - 6*(11*b^3*d^3*e^2*n^2*x^3 + 33*a*b^2*d^3*e^2*n^2*x^2 + 3
3*a^2*b*d^3*e^2*n^2*x + 11*a^3*d^3*e^2*n^2 - 6*(b^3*d^3*e^2*n^2*x^3 + 3*a*b
^2*d^3*e^2*n^2*x^2 + 3*a^2*b*d^3*e^2*n^2*x + a^3*d^3*e^2*n^2)*log(b*x + a))
*log(d*x + c))/((a^3*b^4*c^3 - 3*a^4*b^3*c^2*d + 3*a^5*b^2*c*d^2 - a^6*b*d^
3 + (b^7*c^3 - 3*a*b^6*c^2*d + 3*a^2*b^5*c*d^2 - a^3*b^4*d^3)*x^3 + 3*(a*b^
6*c^3 - 3*a^2*b^5*c^2*d + 3*a^3*b^4*c*d^2 - a^4*b^3*d^3)*x^2 + 3*(a^2*b^5*c
^3 - 3*a^3*b^4*c^2*d + 3*a^4*b^3*c*d^2 - a^5*b^2*d^3)*x)*e^2)) - A*B^2*log(
(b*x + a)^n*e/(d*x + c)^n)^2/(b^4*x^3 + 3*a*b^3*x^2 + 3*a^2*b^2*x + a^3*b)
- 1/6*(6*d^3*e*n*log(b*x + a)/(b^4*c^3 - 3*a*b^3*c^2*d + 3*a^2*b^2*c*d^2 -
a^3*b*d^3) - 6*d^3*e*n*log(d*x + c)/(b^4*c^3 - 3*a*b^3*c^2*d + 3*a^2*b^2*c*
d^2 - a^3*b*d^3) + (6*b^2*d^2*e*n*x^2 + 2*b^2*c^2*e*n - 7*a*b*c*d*e*n + 11*
a^2*d^2*e*n - 3*(b^2*c*d*e*n - 5*a*b*d^2*e*n)*x)/(a^3*b^3*c^2 - 2*a^4*b^2*c
*d + a^5*b*d^2 + (b^6*c^2 - 2*a*b^5*c*d + a^2*b^4*d^2)*x^3 + 3*(a*b^5*c^2 -
 2*a^2*b^4*c*d + a^3*b^3*d^2)*x^2 + 3*(a^2*b^4*c^2 - 2*a^3*b^3*c*d + a^4*b^
2*d^2)*x))*A^2*B/e - A^2*B*log((b*x + a)^n*e/(d*x + c)^n)/(b^4*x^3 + 3*a*b^
3*x^2 + 3*a^2*b^2*x + a^3*b) - 1/3*A^3/(b^4*x^3 + 3*a*b^3*x^2 + 3*a^2*b^2*x
```

```
 + a^3*b)
```

**mupad [B]** time = 10.73, size = 2069, normalized size = 3.39

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*log((e*(a + b*x)^n)/(c + d*x)^n))^3/(a + b*x)^4,x)
```

```
[Out] ((36*A^3*a^2*d^2 + 36*A^3*b^2*c^2 + 575*B^3*a^2*d^2*n^3 + 8*B^3*b^2*c^2*n^3
 + 198*A^2*B*a^2*d^2*n + 36*A^2*B*b^2*c^2*n - 72*A^3*a*b*c*d + 510*A*B^2*a^
2*d^2*n^2 + 24*A*B^2*b^2*c^2*n^2 - 73*B^3*a*b*c*d*n^3 - 126*A^2*B*a*b*c*d*n
 - 138*A*B^2*a*b*c*d*n^2)/(6*(a*d - b*c)) + (x*(359*B^3*a*b*d^2*n^3 - 19*B^
3*b^2*c*d*n^3 + 90*A^2*B*a*b*d^2*n - 18*A^2*B*b^2*c*d*n + 294*A*B^2*a*b*d^2
*n^2 - 30*A*B^2*b^2*c*d*n^2))/(2*(a*d - b*c)) + (x^2*(85*B^3*b^2*d^2*n^3 +
18*A^2*B*b^2*d^2*n + 66*A*B^2*b^2*d^2*n^2))/(a*d - b*c))/(x^3*(18*b^5*c - 1
8*a*b^4*d) + x*(54*a^2*b^3*c - 54*a^3*b^2*d) - x^2*(54*a^2*b^3*d - 54*a*b^4
*c) + 18*a^3*b^2*c - 18*a^4*b*d) - log((e*(a + b*x)^n)/(c + d*x)^n)^3*(B^3/
(3*b*(a^3 + b^3*x^3 + 3*a*b^2*x^2 + 3*a^2*b*x)) - (B^3*d^3)/(3*b*(a^3*d^3 -
 b^3*c^3 + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2))) - log((e*(a + b*x)^n)/(c + d*x)
^n)^2*((A*B^2)/(a^3*b + b^4*x^3 + 3*a^2*b^2*x + 3*a*b^3*x^2) - (d^3*(6*A*B^
2 + 11*B^3*n))/(6*b*(a^3*d^3 - b^3*c^3 + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2)) +
(B^3*d^3*(a*((b*n*(a*d - b*c)*(3*a*d - b*c))/(6*d^2) + (a*b*n*(a*d - b*c))/
(3*d)) + x*(b*((b*n*(a*d - b*c)*(3*a*d - b*c))/(6*d^2) + (a*b*n*(a*d - b*c)
)/(3*d)) + (2*a*b^2*n*(a*d - b*c))/(3*d) + (b^2*n*(a*d - b*c)*(3*a*d - b*c)
)/(3*d^2)) + (b*n*(a*d - b*c)*(3*a^2*d^2 + b^2*c^2 - 3*a*b*c*d))/(3*d^3) +
(b^3*n*x^2*(a*d - b*c))/d))/(b*(a^3*d^3 - b^3*c^3 + 3*a*b^2*c^2*d - 3*a^2*b
*c*d^2)*(a^3*b + b^4*x^3 + 3*a^2*b^2*x + 3*a*b^3*x^2))) - log((e*(a + b*x)^
n)/(c + d*x)^n)*((x*((a*d + b*c)*(3*A^2*B*a*d - 3*A^2*B*b*c - 6*B^3*a*d*n^2
 + 3*B^3*b*c*n^2) - 3*B^3*a*b*c*d*n^2) + x^2*(b*d*(3*A^2*B*a*d - 3*A^2*B*b*
c - 6*B^3*a*d*n^2 + 3*B^3*b*c*n^2) - 3*B^3*b*d*n^2*(a*d + b*c)) + a*c*(3*A^
2*B*a*d - 3*A^2*B*b*c - 6*B^3*a*d*n^2 + 3*B^3*b*c*n^2) - 3*B^3*b^2*d^2*n^2*
x^3)/(3*b*(a*d - b*c)*(a + b*x)^4*(c + d*x)) + (d^3*(6*A*B^2 + 11*B^3*n)*(x
*((a*((a*b*n*(a*d - b*c)^2)/d + (b*n*(a*d - b*c)^2*(3*a*d - b*c))/(2*d^2))
 + (b*n*(a*d - b*c)^2*(3*a^2*d^2 + b^2*c^2 - 3*a*b*c*d))/d^3)*(a*d + b*c) +
a*c*(b*((a*b*n*(a*d - b*c)^2)/d + (b*n*(a*d - b*c)^2*(3*a*d - b*c))/(2*d^2)
) + (b^2*n*(a*d - b*c)^2*(3*a*d - b*c))/d^2 + (2*a*b^2*n*(a*d - b*c)^2)/d))
 + x^2*((a*d + b*c)*(b*((a*b*n*(a*d - b*c)^2)/d + (b*n*(a*d - b*c)^2*(3*a*d
 - b*c))/(2*d^2)) + (b^2*n*(a*d - b*c)^2*(3*a*d - b*c))/d^2 + (2*a*b^2*n*(a
*d - b*c)^2)/d) + b*d*(a*((a*b*n*(a*d - b*c)^2)/d + (b*n*(a*d - b*c)^2*(3*a
*d - b*c))/(2*d^2)) + (b*n*(a*d - b*c)^2*(3*a^2*d^2 + b^2*c^2 - 3*a*b*c*d))
/d^3) + (3*a*b^3*c*n*(a*d - b*c)^2)/d) + x^3*(b*d*(b*((a*b*n*(a*d - b*c)^2)
/d + (b*n*(a*d - b*c)^2*(3*a*d - b*c))/(2*d^2)) + (b^2*n*(a*d - b*c)^2*(3*a
*d - b*c))/d^2 + (2*a*b^2*n*(a*d - b*c)^2)/d) + (3*b^3*n*(a*d + b*c)*(a*d -
 b*c)^2)/d) + a*c*(a*((a*b*n*(a*d - b*c)^2)/d + (b*n*(a*d - b*c)^2*(3*a*d -
```

```
 b*c))/(2*d^2)) + (b*n*(a*d - b*c)^2*(3*a^2*d^2 + b^2*c^2 - 3*a*b*c*d))/d^3
) + 3*b^4*n*x^4*(a*d - b*c)^2))/(9*b^2*(a*d - b*c)*(a + b*x)^4*(c + d*x)*(a
^3*d^3 - b^3*c^3 + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2))) - (B*d^3*n*atan((B*d^3*
n*((b^4*c^3 + a^3*b*d^3 - a^2*b^2*c*d^2 - a*b^3*c^2*d)/(b^3*c^2 + a^2*b*d^2
 - 2*a*b^2*c*d) + 2*b*d*x)*(18*A^2 + 85*B^2*n^2 + 66*A*B*n)*(b^3*c^2 + a^2*
b*d^2 - 2*a*b^2*c*d)*1i)/(b*(a*d - b*c)^3*(85*B^3*d^3*n^3 + 18*A^2*B*d^3*n
+ 66*A*B^2*d^3*n^2)))*(18*A^2 + 85*B^2*n^2 + 66*A*B*n)*1i)/(9*b*(a*d - b*c)
^3)
```

**sympy [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*ln(e*(b*x+a)**n/((d*x+c)**n)))**3/(b*x+a)**4,x)
```

```
[Out] Timed out
```

## 3.171 $\int \frac{\left(A+B\log(e(a+bx)^n(c+dx)^{-n})\right)^3}{(a+bx)^5}\,dx$

**Optimal**. Leaf size=830

$$-\frac{b^3\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^3(c+dx)^4}{4(bc-ad)^4(a+bx)^4}-\frac{3b^3Bn\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^2(c+dx)^4}{16(bc-ad)^4(a+bx)^4}-\frac{3b^3B^2n}{}$$

```
[Out] 6*B^3*d^3*n^3*(d*x+c)/(-a*d+b*c)^4/(b*x+a)-9/8*b*B^3*d^2*n^3*(d*x+c)^2/(-a*
d+b*c)^4/(b*x+a)^2+2/9*b^2*B^3*d*n^3*(d*x+c)^3/(-a*d+b*c)^4/(b*x+a)^3-3/128
*b^3*B^3*n^3*(d*x+c)^4/(-a*d+b*c)^4/(b*x+a)^4+6*B^2*d^3*n^2*(d*x+c)*(A+B*ln
(e*(b*x+a)^n/((d*x+c)^n)))/(-a*d+b*c)^4/(b*x+a)-9/4*b*B^2*d^2*n^2*(d*x+c)^2
*(A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))/(-a*d+b*c)^4/(b*x+a)^2+2/3*b^2*B^2*d*n^2
*(d*x+c)^3*(A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))/(-a*d+b*c)^4/(b*x+a)^3-3/32*b^
3*B^2*n^2*(d*x+c)^4*(A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))/(-a*d+b*c)^4/(b*x+a)^
4+3*B*d^3*n*(d*x+c)*(A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))^2/(-a*d+b*c)^4/(b*x+a
)-9/4*b*B*d^2*n*(d*x+c)^2*(A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))^2/(-a*d+b*c)^4/
(b*x+a)^2+b^2*B*d*n*(d*x+c)^3*(A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))^2/(-a*d+b*c
)^4/(b*x+a)^3-3/16*b^3*B*n*(d*x+c)^4*(A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))^2/(-
a*d+b*c)^4/(b*x+a)^4+d^3*(d*x+c)*(A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))^3/(-a*d+
b*c)^4/(b*x+a)-3/2*b*d^2*(d*x+c)^2*(A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))^3/(-a*
d+b*c)^4/(b*x+a)^2+b^2*d*(d*x+c)^3*(A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))^3/(-a*
d+b*c)^4/(b*x+a)^3-1/4*b^3*(d*x+c)^4*(A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))^3/(-
a*d+b*c)^4/(b*x+a)^4
```

**Rubi [C]** time = 4.67, antiderivative size = 2173, normalized size of antiderivative = 2.62, number of steps used = 93, number of rules used = 16, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.485, Rules used = {6742, 2492, 44, 2514, 2490, 32, 2488, 2411, 2343, 2333, 2315, 2491, 2509, 37, 2506, 6610}

result too large to display

Antiderivative was successfully verified.

```
[In] Int[(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])^3/(a + b*x)^5,x]
```

```
[Out] -A^3/(4*b*(a + b*x)^4) - (3*A^2*B*n)/(16*b*(a + b*x)^4) - (3*A*B^2*n^2)/(32
*b*(a + b*x)^4) - (3*B^3*n^3)/(128*b*(a + b*x)^4) + (A^2*B*d*n)/(4*b*(b*c -
 a*d)*(a + b*x)^3) + (7*A*B^2*d*n^2)/(24*b*(b*c - a*d)*(a + b*x)^3) + (37*B
^3*d*n^3)/(288*b*(b*c - a*d)*(a + b*x)^3) - (3*A^2*B*d^2*n)/(8*b*(b*c - a*d
)^2*(a + b*x)^2) - (13*A*B^2*d^2*n^2)/(16*b*(b*c - a*d)^2*(a + b*x)^2) - (7
9*B^3*d^2*n^3)/(192*b*(b*c - a*d)^2*(a + b*x)^2) + (3*A^2*B*d^3*n)/(4*b*(b*
c - a*d)^3*(a + b*x)) + (25*A*B^2*d^3*n^2)/(8*b*(b*c - a*d)^3*(a + b*x)) +
(451*B^3*d^3*n^3)/(96*b*(b*c - a*d)^3*(a + b*x)) - (3*b*B^3*d^2*n^3*(c + d*
```

```
x)^2)/(16*(b*c - a*d)^4*(a + b*x)^2) + (3*A^2*B*d^4*n*Log[a + b*x])/(4*b*(b
*c - a*d)^4) + (13*A*B^2*d^4*n^2*Log[a + b*x])/(8*b*(b*c - a*d)^4) + (79*B^
3*d^4*n^3*Log[a + b*x])/(96*b*(b*c - a*d)^4) - (3*A^2*B*d^4*n*Log[c + d*x])
/(4*b*(b*c - a*d)^4) - (13*A*B^2*d^4*n^2*Log[c + d*x])/(8*b*(b*c - a*d)^4)
- (79*B^3*d^4*n^3*Log[c + d*x])/(96*b*(b*c - a*d)^4) - (3*A^2*B*Log[(e*(a +
 b*x)^n)/(c + d*x)^n])/(4*b*(a + b*x)^4) - (3*A*B^2*n*Log[(e*(a + b*x)^n)/(
c + d*x)^n])/(8*b*(a + b*x)^4) - (3*B^3*n^2*Log[(e*(a + b*x)^n)/(c + d*x)^n
])/(32*b*(a + b*x)^4) + (A*B^2*d*n*Log[(e*(a + b*x)^n)/(c + d*x)^n])/(2*b*(
b*c - a*d)*(a + b*x)^3) + (7*B^3*d*n^2*Log[(e*(a + b*x)^n)/(c + d*x)^n])/(2
4*b*(b*c - a*d)*(a + b*x)^3) - (3*A*B^2*d^2*n*Log[(e*(a + b*x)^n)/(c + d*x)
^n])/(4*b*(b*c - a*d)^2*(a + b*x)^2) - (7*B^3*d^2*n^2*Log[(e*(a + b*x)^n)/(
c + d*x)^n])/(16*b*(b*c - a*d)^2*(a + b*x)^2) + (3*A*B^2*d^3*n*(c + d*x)*Lo
g[(e*(a + b*x)^n)/(c + d*x)^n])/(2*(b*c - a*d)^4*(a + b*x)) + (31*B^3*d^3*n
^2*(c + d*x)*Log[(e*(a + b*x)^n)/(c + d*x)^n])/(8*(b*c - a*d)^4*(a + b*x))
- (3*b*B^3*d^2*n^2*(c + d*x)^2*Log[(e*(a + b*x)^n)/(c + d*x)^n])/(8*(b*c -
a*d)^4*(a + b*x)^2) - (3*A*B^2*d^4*n*Log[-((b*c - a*d)/(d*(a + b*x)))]*Log[
(e*(a + b*x)^n)/(c + d*x)^n])/(2*b*(b*c - a*d)^4) - (7*B^3*d^4*n^2*Log[-((b
*c - a*d)/(d*(a + b*x)))]*Log[(e*(a + b*x)^n)/(c + d*x)^n])/(8*b*(b*c - a*d
)^4) + (3*A*B^2*d^4*n*Log[(b*c - a*d)/(b*(c + d*x))]*Log[(e*(a + b*x)^n)/(c
 + d*x)^n])/(2*b*(b*c - a*d)^4) + (7*B^3*d^4*n^2*Log[(b*c - a*d)/(b*(c + d*
x))]*Log[(e*(a + b*x)^n)/(c + d*x)^n])/(8*b*(b*c - a*d)^4) - (3*A*B^2*Log[(
e*(a + b*x)^n)/(c + d*x)^n]^2)/(4*b*(a + b*x)^4) - (3*B^3*n*Log[(e*(a + b*x
)^n)/(c + d*x)^n]^2)/(16*b*(a + b*x)^4) + (B^3*d*n*Log[(e*(a + b*x)^n)/(c +
 d*x)^n]^2)/(4*b*(b*c - a*d)*(a + b*x)^3) + (3*B^3*d^3*n*(c + d*x)*Log[(e*(
a + b*x)^n)/(c + d*x)^n]^2)/(2*(b*c - a*d)^4*(a + b*x)) - (3*b*B^3*d^2*n*(c
 + d*x)^2*Log[(e*(a + b*x)^n)/(c + d*x)^n]^2)/(8*(b*c - a*d)^4*(a + b*x)^2)
 - (3*B^3*d^4*n*Log[-((b*c - a*d)/(d*(a + b*x)))]*Log[(e*(a + b*x)^n)/(c +
d*x)^n]^2)/(4*b*(b*c - a*d)^4) + (3*B^3*d^4*n*Log[(b*c - a*d)/(b*(c + d*x))
]*Log[(e*(a + b*x)^n)/(c + d*x)^n]^2)/(4*b*(b*c - a*d)^4) - (B^3*Log[(e*(a
+ b*x)^n)/(c + d*x)^n]^3)/(4*b*(a + b*x)^4) + (3*A*B^2*d^4*n^2*PolyLog[2, (
d*(a + b*x))/(b*(c + d*x))])/(2*b*(b*c - a*d)^4) + (7*B^3*d^4*n^3*PolyLog[2
, (d*(a + b*x))/(b*(c + d*x))])/(8*b*(b*c - a*d)^4) + (3*A*B^2*d^4*n^2*Poly
Log[2, 1 + (b*c - a*d)/(d*(a + b*x))])/(2*b*(b*c - a*d)^4) + (7*B^3*d^4*n^3
*PolyLog[2, 1 + (b*c - a*d)/(d*(a + b*x))])/(8*b*(b*c - a*d)^4) + (3*B^3*d^
4*n^2*Log[(e*(a + b*x)^n)/(c + d*x)^n]*PolyLog[2, 1 + (b*c - a*d)/(d*(a + b
*x))])/(2*b*(b*c - a*d)^4) + (3*B^3*d^4*n^2*Log[(e*(a + b*x)^n)/(c + d*x)^n
]*PolyLog[2, 1 - (b*c - a*d)/(b*(c + d*x))])/(2*b*(b*c - a*d)^4) + (3*B^3*d
^4*n^3*PolyLog[3, 1 + (b*c - a*d)/(d*(a + b*x))])/(2*b*(b*c - a*d)^4) - (3*
B^3*d^4*n^3*PolyLog[3, 1 - (b*c - a*d)/(b*(c + d*x))])/(2*b*(b*c - a*d)^4)
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2333

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)/(x_))^(q_.)*(
x_)^(m_.), x_Symbol] :> Int[(e + d*x)^q*(a + b*Log[c*x^n])^p, x] /; FreeQ[{
a, b, c, d, e, m, n, p}, x] && EqQ[m, q] && IntegerQ[q]
```

Rule 2343

```
Int[((a_.) + Log[(c_.)*(x_)^(n_)]*(b_.))/((x_)*((d_) + (e_.)*(x_)^(r_.))),
x_Symbol] :> Dist[1/n, Subst[Int[(a + b*Log[c*x])/(x*(d + e*x^(r/n))), x],
x, x^n], x] /; FreeQ[{a, b, c, d, e, n, r}, x] && IntegerQ[r/n]
```

Rule 2411

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + (g_
.)*(x_))^(q_.)*((h_.) + (i_.)*(x_))^(r_.), x_Symbol] :> Dist[1/e, Subst[Int
[((g*x)/e)^q*((e*h - d*i)/e + (i*x)/e)^r*(a + b*Log[c*x^n])^p, x], x, d + e
*x], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, n, p, q, r}, x] && EqQ[e*f - d
*g, 0] && (IGtQ[p, 0] || IGtQ[r, 0]) && IntegerQ[2*r]
```

Rule 2488

```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))
^(r_.)]^(s_.)/((g_.) + (h_.)*(x_)), x_Symbol] :> -Simp[(Log[-((b*c - a*d)/(
d*(a + b*x)))]*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^s)/h, x] + Dist[(p*r*s*
(b*c - a*d))/h, Int[(Log[-((b*c - a*d)/(d*(a + b*x)))]*Log[e*(f*(a + b*x)^p
```

```
*(c + d*x)^q)^r]^(s - 1))/((a + b*x)*(c + d*x)), x], x] /; FreeQ[{a, b, c,
d, e, f, g, h, p, q, r, s}, x] && NeQ[b*c - a*d, 0] && EqQ[p + q, 0] && EqQ
[b*g - a*h, 0] && IGtQ[s, 0]
```

Rule 2490

```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))
^(r_.)]^(s_.)/((g_.) + (h_.)*(x_))^2, x_Symbol] :> Simp[((a + b*x)*Log[e*(f
*(a + b*x)^p*(c + d*x)^q)^r]^s)/((b*g - a*h)*(g + h*x)), x] - Dist[(p*r*s*(
b*c - a*d))/(b*g - a*h), Int[Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^(s - 1)/(
(c + d*x)*(g + h*x)), x], x] /; FreeQ[{a, b, c, d, e, f, g, h, p, q, r, s},
 x] && NeQ[b*c - a*d, 0] && EqQ[p + q, 0] && NeQ[b*g - a*h, 0] && IGtQ[s, 0
]
```

Rule 2491

```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))
^(r_.)]^(s_)/((g_.) + (h_.)*(x_))^3, x_Symbol] :> Dist[d/(d*g - c*h), Int[L
og[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^s/(g + h*x)^2, x], x] - Dist[h/(d*g - c
*h), Int[((c + d*x)*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^s)/(g + h*x)^3, x]
, x] /; FreeQ[{a, b, c, d, e, f, g, h, p, q, r, s}, x] && NeQ[b*c - a*d, 0]
 && EqQ[p + q, 0] && EqQ[b*g - a*h, 0] && NeQ[d*g - c*h, 0] && IGtQ[s, 0]
```

Rule 2492

```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))
^(r_.)]^(s_.)*((g_.) + (h_.)*(x_))^(m_.), x_Symbol] :> Simp[((g + h*x)^(m +
 1)*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^s)/(h*(m + 1)), x] - Dist[(p*r*s*(
b*c - a*d))/(h*(m + 1)), Int[((g + h*x)^(m + 1)*Log[e*(f*(a + b*x)^p*(c + d
*x)^q)^r]^(s - 1))/((a + b*x)*(c + d*x)), x], x] /; FreeQ[{a, b, c, d, e, f
, g, h, m, p, q, r, s}, x] && NeQ[b*c - a*d, 0] && EqQ[p + q, 0] && IGtQ[s,
 0] && NeQ[m, -1]
```

Rule 2506

```
Int[Log[v_]*Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_)
)^(q_.))^(r_.)]^(s_.)*(u_), x_Symbol] :> With[{g = Simplify[((v - 1)*(c + d
*x))/(a + b*x)], h = Simplify[u*(a + b*x)*(c + d*x)]}, -Simp[(h*PolyLog[2,
1 - v]*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^s)/(b*c - a*d), x] + Dist[h*p*r
*s, Int[(PolyLog[2, 1 - v]*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^(s - 1))/((
a + b*x)*(c + d*x)), x], x] /; FreeQ[{g, h}, x]] /; FreeQ[{a, b, c, d, e, f
, p, q, r, s}, x] && NeQ[b*c - a*d, 0] && IGtQ[s, 0] && EqQ[p + q, 0]
```

Rule 2509

```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))
^(r_.)]^(s_.)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symb
ol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^(n + 1)*Log[e*(f*(a + b*x)^p*(c +
d*x)^q)^r]^s)/((m + 1)*(b*c - a*d)), x] - Dist[(p*r*s*(b*c - a*d))/((m + 1)
*(b*c - a*d)), Int[(a + b*x)^m*(c + d*x)^n*Log[e*(f*(a + b*x)^p*(c + d*x)^q
)^r]^(s - 1), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p, q, r, s}, x] && N
eQ[b*c - a*d, 0] && EqQ[p + q, 0] && EqQ[m + n + 2, 0] && NeQ[m, -1] && IGt
Q[s, 0]
```

Rule 2514

```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))
^(r_.)]^(s_.)*(RFx_), x_Symbol] :> With[{u = ExpandIntegrand[Log[e*(f*(a +
b*x)^p*(c + d*x)^q)^r]^s, RFx, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c
, d, e, f, p, q, r, s}, x] && RationalFunctionQ[RFx, x] && IGtQ[s, 0]
```

Rule 6610

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\int \frac{\left(A + B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^3}{(a+bx)^5}\,dx = \int\left(\frac{A^3}{(a+bx)^5} + \frac{3A^2B\log\left(e(a+bx)^n(c+dx)^{-n}\right)}{(a+bx)^5} + \frac{3AB^2\log^2\left(e(a+\right.}{(a+}\right.$$

$$= -\frac{A^3}{4b(a+bx)^4} + \left(3A^2B\right)\int \frac{\log\left(e(a+bx)^n(c+dx)^{-n}\right)}{(a+bx)^5}\,dx + \left(3AB^2\right)$$

$$= -\frac{A^3}{4b(a+bx)^4} - \frac{3A^2B\log\left(e(a+bx)^n(c+dx)^{-n}\right)}{4b(a+bx)^4} - \frac{3AB^2\log^2\left(e(a+\right.}{4b(a+}$$

$$= -\frac{A^3}{4b(a+bx)^4} - \frac{3A^2B\log\left(e(a+bx)^n(c+dx)^{-n}\right)}{4b(a+bx)^4} - \frac{3AB^2\log^2\left(e(a+\right.}{4b(a+}$$

$$= -\frac{A^3}{4b(a+bx)^4} - \frac{3A^2Bn}{16b(a+bx)^4} + \frac{A^2Bdn}{4b(bc-ad)(a+bx)^3} - \frac{3A^2Bd}{8b(bc-ad)^2}$$

$$= -\frac{A^3}{4b(a+bx)^4} - \frac{3A^2Bn}{16b(a+bx)^4} + \frac{A^2Bdn}{4b(bc-ad)(a+bx)^3} - \frac{3A^2Bd}{8b(bc-ad)^2}$$

$$= -\frac{A^3}{4b(a+bx)^4} - \frac{3A^2Bn}{16b(a+bx)^4} + \frac{A^2Bdn}{4b(bc-ad)(a+bx)^3} - \frac{3A^2Bd}{8b(bc-ad)^2}$$

$$= -\frac{A^3}{4b(a+bx)^4} - \frac{3A^2Bn}{16b(a+bx)^4} - \frac{3AB^2n^2}{32b(a+bx)^4} + \frac{A^2Bdn}{4b(bc-ad)(a+bx)^3}$$

$$= -\frac{A^3}{4b(a+bx)^4} - \frac{3A^2Bn}{16b(a+bx)^4} - \frac{3AB^2n^2}{32b(a+bx)^4} + \frac{A^2Bdn}{4b(bc-ad)(a+bx)^3}$$

$$= -\frac{A^3}{4b(a+bx)^4} - \frac{3A^2Bn}{16b(a+bx)^4} - \frac{3AB^2n^2}{32b(a+bx)^4} + \frac{A^2Bdn}{4b(bc-ad)(a+bx)^3}$$

$$= -\frac{A^3}{4b(a+bx)^4} - \frac{3A^2Bn}{16b(a+bx)^4} - \frac{3AB^2n^2}{32b(a+bx)^4} - \frac{3B^3n^3}{128b(a+bx)^4} + \frac{}{4b(}$$

$$= -\frac{A^3}{4b(a+bx)^4} - \frac{3A^2Bn}{16b(a+bx)^4} - \frac{3AB^2n^2}{32b(a+bx)^4} - \frac{3B^3n^3}{128b(a+bx)^4} + \frac{}{4b(}$$

$$= -\frac{A^3}{4b(a+bx)^4} - \frac{3A^2Bn}{16b(a+bx)^4} - \frac{3AB^2n^2}{32b(a+bx)^4} - \frac{3B^3n^3}{128b(a+bx)^4} + \frac{}{4b(}$$

**Mathematica [A]** time = 2.22, size = 1370, normalized size = 1.65

$$-\frac{-288B^3d^4n^3\log^3(a+bx)(a+bx)^4+288B^3d^4n^3\log^3(c+dx)(a+bx)^4+72B^2d^4n^2\log^2(c+dx)\left(12A+25Bn\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])^3/(a + b*x)^5,x]
```

```
[Out] -1/1152*(-288*B^3*d^4*n^3*(a + b*x)^4*Log[a + b*x]^3 + 288*B^3*d^4*n^3*(a +
 b*x)^4*Log[c + d*x]^3 + 72*B^2*d^4*n^2*(a + b*x)^4*Log[c + d*x]^2*(12*A +
25*B*n + 12*B*Log[(e*(a + b*x)^n)/(c + d*x)^n]) + 72*B^2*d^4*n^2*(a + b*x)^
4*Log[a + b*x]^2*(12*A + 25*B*n + 12*B*n*Log[c + d*x] + 12*B*Log[(e*(a + b*
x)^n)/(c + d*x)^n]) + 12*B*d^4*n*(a + b*x)^4*Log[c + d*x]*(72*A^2 + 300*A*B
*n + 415*B^2*n^2 + 12*B*(12*A + 25*B*n)*Log[(e*(a + b*x)^n)/(c + d*x)^n] +
72*B^2*Log[(e*(a + b*x)^n)/(c + d*x)^n]^2) + (b*c - a*d)*(288*A^3*b^3*c^3 -
 864*a*A^3*b^2*c^2*d + 864*a^2*A^3*b*c*d^2 - 288*a^3*A^3*d^3 + 216*A^2*b^3*
B*c^3*n - 936*a*A^2*b^2*B*c^2*d*n + 1656*a^2*A^2*b*B*c*d^2*n - 1800*a^3*A^2
*B*d^3*n + 108*A*b^3*B^2*c^3*n^2 - 660*a*A*b^2*B^2*c^2*d*n^2 + 1932*a^2*A*b
*B^2*c*d^2*n^2 - 4980*a^3*A*B^2*d^3*n^2 + 27*b^3*B^3*c^3*n^3 - 229*a*b^2*B^
3*c^2*d*n^3 + 1067*a^2*b*B^3*c*d^2*n^3 - 5845*a^3*B^3*d^3*n^3 - 288*A^2*b^3
*B*c^2*d*n*x + 1440*a*A^2*b^2*B*c*d^2*n*x - 3744*a^2*A^2*b*B*d^3*n*x - 336*
A*b^3*B^2*c^2*d*n^2*x + 2544*a*A*b^2*B^2*c*d^2*n^2*x - 13008*a^2*A*b*B^2*d^
3*n^2*x - 148*b^3*B^3*c^2*d*n^3*x + 1676*a*b^2*B^3*c*d^2*n^3*x - 16468*a^2*
b*B^3*d^3*n^3*x + 432*A^2*b^3*B*c*d^2*n*x^2 - 3024*a*A^2*b^2*B*d^3*n*x^2 +
936*A*b^3*B^2*c*d^2*n^2*x^2 - 11736*a*A*b^2*B^2*d^3*n^2*x^2 + 690*b^3*B^3*c
*d^2*n^3*x^2 - 15630*a*b^2*B^3*d^3*n^3*x^2 - 864*A^2*b^3*B*d^3*n*x^3 - 3600
*A*b^3*B^2*d^3*n^2*x^3 - 4980*b^3*B^3*d^3*n^3*x^3 + 12*B*(72*A^2*(b*c - a*d
)^3 + B^2*n^2*(-415*a^3*d^3 + a^2*b*d^2*(161*c - 1084*d*x) + a*b^2*d*(-55*c
^2 + 212*c*d*x - 978*d^2*x^2) + b^3*(9*c^3 - 28*c^2*d*x + 78*c*d^2*x^2 - 30
0*d^3*x^3)) + 12*A*B*n*(-25*a^3*d^3 + a^2*b*d^2*(23*c - 52*d*x) + a*b^2*d*(
-13*c^2 + 20*c*d*x - 42*d^2*x^2) + b^3*(3*c^3 - 4*c^2*d*x + 6*c*d^2*x^2 - 1
2*d^3*x^3)))*Log[(e*(a + b*x)^n)/(c + d*x)^n] + 72*B^2*(12*A*(b*c - a*d)^3
+ B*n*(-25*a^3*d^3 + a^2*b*d^2*(23*c - 52*d*x) + a*b^2*d*(-13*c^2 + 20*c*d*
x - 42*d^2*x^2) + b^3*(3*c^3 - 4*c^2*d*x + 6*c*d^2*x^2 - 12*d^3*x^3)))*Log[
(e*(a + b*x)^n)/(c + d*x)^n]^2 + 288*B^3*(b*c - a*d)^3*Log[(e*(a + b*x)^n)/
(c + d*x)^n]^3) - 12*B*d^4*n*(a + b*x)^4*Log[a + b*x]*(72*A^2 + 300*A*B*n +
 415*B^2*n^2 + 72*B^2*n^2*Log[c + d*x]^2 + 12*B*(12*A + 25*B*n)*Log[(e*(a +
 b*x)^n)/(c + d*x)^n] + 72*B^2*Log[(e*(a + b*x)^n)/(c + d*x)^n]^2 + 12*B*n*
Log[c + d*x]*(12*A + 25*B*n + 12*B*Log[(e*(a + b*x)^n)/(c + d*x)^n])))/(b*(
b*c - a*d)^4*(a + b*x)^4)
```

**fricas [B]** time = 1.62, size = 6057, normalized size = 7.30

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*(b*x+a)^n/((d*x+c)^n)))^3/(b*x+a)^5,x, algorithm="fric
as")
```

```
[Out] -1/1152*(288*A^3*b^4*c^4 - 1152*A^3*a*b^3*c^3*d + 1728*A^3*a^2*b^2*c^2*d^2
- 1152*A^3*a^3*b*c*d^3 + 288*A^3*a^4*d^4 + (27*B^3*b^4*c^4 - 256*B^3*a*b^3*
c^3*d + 1296*B^3*a^2*b^2*c^2*d^2 - 6912*B^3*a^3*b*c*d^3 + 5845*B^3*a^4*d^4)
*n^3 - 12*(415*(B^3*b^4*c*d^3 - B^3*a*b^3*d^4)*n^3 + 300*(A*B^2*b^4*c*d^3 -
 A*B^2*a*b^3*d^4)*n^2 + 72*(A^2*B*b^4*c*d^3 - A^2*B*a*b^3*d^4)*n)*x^3 - 288
*(B^3*b^4*d^4*n^3*x^4 + 4*B^3*a*b^3*d^4*n^3*x^3 + 6*B^3*a^2*b^2*d^4*n^3*x^2
 + 4*B^3*a^3*b*d^4*n^3*x - (B^3*b^4*c^4 - 4*B^3*a*b^3*c^3*d + 6*B^3*a^2*b^2
*c^2*d^2 - 4*B^3*a^3*b*c*d^3)*n^3)*log(b*x + a)^3 + 288*(B^3*b^4*d^4*n^3*x^
4 + 4*B^3*a*b^3*d^4*n^3*x^3 + 6*B^3*a^2*b^2*d^4*n^3*x^2 + 4*B^3*a^3*b*d^4*n
^3*x - (B^3*b^4*c^4 - 4*B^3*a*b^3*c^3*d + 6*B^3*a^2*b^2*c^2*d^2 - 4*B^3*a^3
*b*c*d^3)*n^3)*log(d*x + c)^3 + 288*(B^3*b^4*c^4 - 4*B^3*a*b^3*c^3*d + 6*B^
3*a^2*b^2*c^2*d^2 - 4*B^3*a^3*b*c*d^3 + B^3*a^4*d^4)*log(e)^3 + 12*(9*A*B^2
*b^4*c^4 - 64*A*B^2*a*b^3*c^3*d + 216*A*B^2*a^2*b^2*c^2*d^2 - 576*A*B^2*a^3
*b*c*d^3 + 415*A*B^2*a^4*d^4)*n^2 + 6*(5*(23*B^3*b^4*c^2*d^2 - 544*B^3*a*b^
3*c*d^3 + 521*B^3*a^2*b^2*d^4)*n^3 + 12*(13*A*B^2*b^4*c^2*d^2 - 176*A*B^2*a
*b^3*c*d^3 + 163*A*B^2*a^2*b^2*d^4)*n^2 + 72*(A^2*B*b^4*c^2*d^2 - 8*A^2*B*a
*b^3*c*d^3 + 7*A^2*B*a^2*b^2*d^4)*n)*x^2 - 72*((25*B^3*b^4*d^4*n^3 + 12*A*B
^2*b^4*d^4*n^2)*x^4 - (3*B^3*b^4*c^4 - 16*B^3*a*b^3*c^3*d + 36*B^3*a^2*b^2*
c^2*d^2 - 48*B^3*a^3*b*c*d^3)*n^3 + 4*(12*A*B^2*a*b^3*d^4*n^2 + (3*B^3*b^4*
c*d^3 + 22*B^3*a*b^3*d^4)*n^3)*x^3 - 12*(A*B^2*b^4*c^4 - 4*A*B^2*a*b^3*c^3*
d + 6*A*B^2*a^2*b^2*c^2*d^2 - 4*A*B^2*a^3*b*c*d^3)*n^2 + 6*(12*A*B^2*a^2*b^
2*d^4*n^2 - (B^3*b^4*c^2*d^2 - 8*B^3*a*b^3*c*d^3 - 18*B^3*a^2*b^2*d^4)*n^3)
*x^2 + 4*(12*A*B^2*a^3*b*d^4*n^2 + (B^3*b^4*c^3*d - 6*B^3*a*b^3*c^2*d^2 + 1
8*B^3*a^2*b^2*c*d^3 + 12*B^3*a^3*b*d^4)*n^3)*x + 12*(B^3*b^4*d^4*n^2*x^4 +
4*B^3*a*b^3*d^4*n^2*x^3 + 6*B^3*a^2*b^2*d^4*n^2*x^2 + 4*B^3*a^3*b*d^4*n^2*x
 - (B^3*b^4*c^4 - 4*B^3*a*b^3*c^3*d + 6*B^3*a^2*b^2*c^2*d^2 - 4*B^3*a^3*b*c
*d^3)*n^2)*log(e))*log(b*x + a)^2 - 72*((25*B^3*b^4*d^4*n^3 + 12*A*B^2*b^4*
d^4*n^2)*x^4 - (3*B^3*b^4*c^4 - 16*B^3*a*b^3*c^3*d + 36*B^3*a^2*b^2*c^2*d^2
 - 48*B^3*a^3*b*c*d^3)*n^3 + 4*(12*A*B^2*a*b^3*d^4*n^2 + (3*B^3*b^4*c*d^3 +
 22*B^3*a*b^3*d^4)*n^3)*x^3 - 12*(A*B^2*b^4*c^4 - 4*A*B^2*a*b^3*c^3*d + 6*A
*B^2*a^2*b^2*c^2*d^2 - 4*A*B^2*a^3*b*c*d^3)*n^2 + 6*(12*A*B^2*a^2*b^2*d^4*n
^2 - (B^3*b^4*c^2*d^2 - 8*B^3*a*b^3*c*d^3 - 18*B^3*a^2*b^2*d^4)*n^3)*x^2 +
4*(12*A*B^2*a^3*b*d^4*n^2 + (B^3*b^4*c^3*d - 6*B^3*a*b^3*c^2*d^2 + 18*B^3*a
^2*b^2*c*d^3 + 12*B^3*a^3*b*d^4)*n^3)*x + 12*(B^3*b^4*d^4*n^3*x^4 + 4*B^3*a
*b^3*d^4*n^3*x^3 + 6*B^3*a^2*b^2*d^4*n^3*x^2 + 4*B^3*a^3*b*d^4*n^3*x - (B^3
*b^4*c^4 - 4*B^3*a*b^3*c^3*d + 6*B^3*a^2*b^2*c^2*d^2 - 4*B^3*a^3*b*c*d^3)*n
^3)*log(b*x + a) + 12*(B^3*b^4*d^4*n^2*x^4 + 4*B^3*a*b^3*d^4*n^2*x^3 + 6*B^
3*a^2*b^2*d^4*n^2*x^2 + 4*B^3*a^3*b*d^4*n^2*x - (B^3*b^4*c^4 - 4*B^3*a*b^3*
c^3*d + 6*B^3*a^2*b^2*c^2*d^2 - 4*B^3*a^3*b*c*d^3)*n^2)*log(e))*log(d*x + c
)^2 + 72*(12*A*B^2*b^4*c^4 - 48*A*B^2*a*b^3*c^3*d + 72*A*B^2*a^2*b^2*c^2*d^
```

```
2 - 48*A*B^2*a^3*b*c*d^3 + 12*A*B^2*a^4*d^4 - 12*(B^3*b^4*c*d^3 - B^3*a*b^3
*d^4)*n*x^3 + 6*(B^3*b^4*c^2*d^2 - 8*B^3*a*b^3*c*d^3 + 7*B^3*a^2*b^2*d^4)*n
*x^2 - 4*(B^3*b^4*c^3*d - 6*B^3*a*b^3*c^2*d^2 + 18*B^3*a^2*b^2*c*d^3 - 13*B
^3*a^3*b*d^4)*n*x + (3*B^3*b^4*c^4 - 16*B^3*a*b^3*c^3*d + 36*B^3*a^2*b^2*c^
2*d^2 - 48*B^3*a^3*b*c*d^3 + 25*B^3*a^4*d^4)*n)*log(e)^2 + 72*(3*A^2*B*b^4*
c^4 - 16*A^2*B*a*b^3*c^3*d + 36*A^2*B*a^2*b^2*c^2*d^2 - 48*A^2*B*a^3*b*c*d^
3 + 25*A^2*B*a^4*d^4)*n - 4*((37*B^3*b^4*c^3*d - 456*B^3*a*b^3*c^2*d^2 + 45
36*B^3*a^2*b^2*c*d^3 - 4117*B^3*a^3*b*d^4)*n^3 + 12*(7*A*B^2*b^4*c^3*d - 60
*A*B^2*a*b^3*c^2*d^2 + 324*A*B^2*a^2*b^2*c*d^3 - 271*A*B^2*a^3*b*d^4)*n^2 +
 72*(A^2*B*b^4*c^3*d - 6*A^2*B*a*b^3*c^2*d^2 + 18*A^2*B*a^2*b^2*c*d^3 - 13*
A^2*B*a^3*b*d^4)*n)*x - 12*((415*B^3*b^4*d^4*n^3 + 300*A*B^2*b^4*d^4*n^2 +
72*A^2*B*b^4*d^4*n)*x^4 - (9*B^3*b^4*c^4 - 64*B^3*a*b^3*c^3*d + 216*B^3*a^2
*b^2*c^2*d^2 - 576*B^3*a^3*b*c*d^3)*n^3 + 4*(72*A^2*B*a*b^3*d^4*n + 5*(15*B
^3*b^4*c*d^3 + 68*B^3*a*b^3*d^4)*n^3 + 12*(3*A*B^2*b^4*c*d^3 + 22*A*B^2*a*b
^3*d^4)*n^2)*x^3 - 12*(3*A*B^2*b^4*c^4 - 16*A*B^2*a*b^3*c^3*d + 36*A*B^2*a^
2*b^2*c^2*d^2 - 48*A*B^2*a^3*b*c*d^3)*n^2 + 6*(72*A^2*B*a^2*b^2*d^4*n - (13
*B^3*b^4*c^2*d^2 - 176*B^3*a*b^3*c*d^3 - 252*B^3*a^2*b^2*d^4)*n^3 - 12*(A*B
^2*b^4*c^2*d^2 - 8*A*B^2*a*b^3*c*d^3 - 18*A*B^2*a^2*b^2*d^4)*n^2)*x^2 + 72*
(B^3*b^4*d^4*n*x^4 + 4*B^3*a*b^3*d^4*n*x^3 + 6*B^3*a^2*b^2*d^4*n*x^2 + 4*B^
3*a^3*b*d^4*n*x - (B^3*b^4*c^4 - 4*B^3*a*b^3*c^3*d + 6*B^3*a^2*b^2*c^2*d^2
- 4*B^3*a^3*b*c*d^3)*n)*log(e)^2 - 72*(A^2*B*b^4*c^4 - 4*A^2*B*a*b^3*c^3*d
+ 6*A^2*B*a^2*b^2*c^2*d^2 - 4*A^2*B*a^3*b*c*d^3)*n + 4*(72*A^2*B*a^3*b*d^4*
n + (7*B^3*b^4*c^3*d - 60*B^3*a*b^3*c^2*d^2 + 324*B^3*a^2*b^2*c*d^3 + 144*B
^3*a^3*b*d^4)*n^3 + 12*(A*B^2*b^4*c^3*d - 6*A*B^2*a*b^3*c^2*d^2 + 18*A*B^2*
a^2*b^2*c*d^3 + 12*A*B^2*a^3*b*d^4)*n^2)*x + 12*((25*B^3*b^4*d^4*n^2 + 12*A
*B^2*b^4*d^4*n)*x^4 + 4*(12*A*B^2*a*b^3*d^4*n + (3*B^3*b^4*c*d^3 + 22*B^3*a
*b^3*d^4)*n^2)*x^3 - (3*B^3*b^4*c^4 - 16*B^3*a*b^3*c^3*d + 36*B^3*a^2*b^2*c
^2*d^2 - 48*B^3*a^3*b*c*d^3)*n^2 + 6*(12*A*B^2*a^2*b^2*d^4*n - (B^3*b^4*c^2
*d^2 - 8*B^3*a*b^3*c*d^3 - 18*B^3*a^2*b^2*d^4)*n^2)*x^2 - 12*(A*B^2*b^4*c^4
 - 4*A*B^2*a*b^3*c^3*d + 6*A*B^2*a^2*b^2*c^2*d^2 - 4*A*B^2*a^3*b*c*d^3)*n +
 4*(12*A*B^2*a^3*b*d^4*n + (B^3*b^4*c^3*d - 6*B^3*a*b^3*c^2*d^2 + 18*B^3*a^
2*b^2*c*d^3 + 12*B^3*a^3*b*d^4)*n^2)*x)*log(e))*log(b*x + a) + 12*((415*B^3
*b^4*d^4*n^3 + 300*A*B^2*b^4*d^4*n^2 + 72*A^2*B*b^4*d^4*n)*x^4 - (9*B^3*b^4
*c^4 - 64*B^3*a*b^3*c^3*d + 216*B^3*a^2*b^2*c^2*d^2 - 576*B^3*a^3*b*c*d^3)*
n^3 + 4*(72*A^2*B*a*b^3*d^4*n + 5*(15*B^3*b^4*c*d^3 + 68*B^3*a*b^3*d^4)*n^3
 + 12*(3*A*B^2*b^4*c*d^3 + 22*A*B^2*a*b^3*d^4)*n^2)*x^3 - 12*(3*A*B^2*b^4*c
^4 - 16*A*B^2*a*b^3*c^3*d + 36*A*B^2*a^2*b^2*c^2*d^2 - 48*A*B^2*a^3*b*c*d^3
)*n^2 + 6*(72*A^2*B*a^2*b^2*d^4*n - (13*B^3*b^4*c^2*d^2 - 176*B^3*a*b^3*c*d
^3 - 252*B^3*a^2*b^2*d^4)*n^3 - 12*(A*B^2*b^4*c^2*d^2 - 8*A*B^2*a*b^3*c*d^3
 - 18*A*B^2*a^2*b^2*d^4)*n^2)*x^2 + 72*(B^3*b^4*d^4*n^3*x^4 + 4*B^3*a*b^3*d
^4*n^3*x^3 + 6*B^3*a^2*b^2*d^4*n^3*x^2 + 4*B^3*a^3*b*d^4*n^3*x - (B^3*b^4*c
^4 - 4*B^3*a*b^3*c^3*d + 6*B^3*a^2*b^2*c^2*d^2 - 4*B^3*a^3*b*c*d^3)*n^3)*lo
g(b*x + a)^2 + 72*(B^3*b^4*d^4*n*x^4 + 4*B^3*a*b^3*d^4*n*x^3 + 6*B^3*a^2*b^
2*d^4*n*x^2 + 4*B^3*a^3*b*d^4*n*x - (B^3*b^4*c^4 - 4*B^3*a*b^3*c^3*d + 6*B^
3*a^2*b^2*c^2*d^2 - 4*B^3*a^3*b*c*d^3)*n)*log(e)^2 - 72*(A^2*B*b^4*c^4 - 4*
```

```
A^2*B*a*b^3*c^3*d + 6*A^2*B*a^2*b^2*c^2*d^2 - 4*A^2*B*a^3*b*c*d^3)*n + 4*(7
2*A^2*B*a^3*b*d^4*n + (7*B^3*b^4*c^3*d - 60*B^3*a*b^3*c^2*d^2 + 324*B^3*a^2
*b^2*c*d^3 + 144*B^3*a^3*b*d^4)*n^3 + 12*(A*B^2*b^4*c^3*d - 6*A*B^2*a*b^3*c
^2*d^2 + 18*A*B^2*a^2*b^2*c*d^3 + 12*A*B^2*a^3*b*d^4)*n^2)*x + 12*((25*B^3*
b^4*d^4*n^3 + 12*A*B^2*b^4*d^4*n^2)*x^4 - (3*B^3*b^4*c^4 - 16*B^3*a*b^3*c^3
*d + 36*B^3*a^2*b^2*c^2*d^2 - 48*B^3*a^3*b*c*d^3)*n^3 + 4*(12*A*B^2*a*b^3*d
^4*n^2 + (3*B^3*b^4*c*d^3 + 22*B^3*a*b^3*d^4)*n^3)*x^3 - 12*(A*B^2*b^4*c^4
- 4*A*B^2*a*b^3*c^3*d + 6*A*B^2*a^2*b^2*c^2*d^2 - 4*A*B^2*a^3*b*c*d^3)*n^2
+ 6*(12*A*B^2*a^2*b^2*d^4*n^2 - (B^3*b^4*c^2*d^2 - 8*B^3*a*b^3*c*d^3 - 18*B
^3*a^2*b^2*d^4)*n^3)*x^2 + 4*(12*A*B^2*a^3*b*d^4*n^2 + (B^3*b^4*c^3*d - 6*B
^3*a*b^3*c^2*d^2 + 18*B^3*a^2*b^2*c*d^3 + 12*B^3*a^3*b*d^4)*n^3)*x + 12*(B^
3*b^4*d^4*n^2*x^4 + 4*B^3*a*b^3*d^4*n^2*x^3 + 6*B^3*a^2*b^2*d^4*n^2*x^2 + 4
*B^3*a^3*b*d^4*n^2*x - (B^3*b^4*c^4 - 4*B^3*a*b^3*c^3*d + 6*B^3*a^2*b^2*c^2
*d^2 - 4*B^3*a^3*b*c*d^3)*n^2)*log(e))*log(b*x + a) + 12*((25*B^3*b^4*d^4*n
^2 + 12*A*B^2*b^4*d^4*n)*x^4 + 4*(12*A*B^2*a*b^3*d^4*n + (3*B^3*b^4*c*d^3 +
 22*B^3*a*b^3*d^4)*n^2)*x^3 - (3*B^3*b^4*c^4 - 16*B^3*a*b^3*c^3*d + 36*B^3*
a^2*b^2*c^2*d^2 - 48*B^3*a^3*b*c*d^3)*n^2 + 6*(12*A*B^2*a^2*b^2*d^4*n - (B^
3*b^4*c^2*d^2 - 8*B^3*a*b^3*c*d^3 - 18*B^3*a^2*b^2*d^4)*n^2)*x^2 - 12*(A*B^
2*b^4*c^4 - 4*A*B^2*a*b^3*c^3*d + 6*A*B^2*a^2*b^2*c^2*d^2 - 4*A*B^2*a^3*b*c
*d^3)*n + 4*(12*A*B^2*a^3*b*d^4*n + (B^3*b^4*c^3*d - 6*B^3*a*b^3*c^2*d^2 +
18*B^3*a^2*b^2*c*d^3 + 12*B^3*a^3*b*d^4)*n^2)*x)*log(e))*log(d*x + c) + 12*
(72*A^2*B*b^4*c^4 - 288*A^2*B*a*b^3*c^3*d + 432*A^2*B*a^2*b^2*c^2*d^2 - 288
*A^2*B*a^3*b*c*d^3 + 72*A^2*B*a^4*d^4 - 12*(25*(B^3*b^4*c*d^3 - B^3*a*b^3*d
^4)*n^2 + 12*(A*B^2*b^4*c*d^3 - A*B^2*a*b^3*d^4)*n)*x^3 + (9*B^3*b^4*c^4 -
64*B^3*a*b^3*c^3*d + 216*B^3*a^2*b^2*c^2*d^2 - 576*B^3*a^3*b*c*d^3 + 415*B^
3*a^4*d^4)*n^2 + 6*((13*B^3*b^4*c^2*d^2 - 176*B^3*a*b^3*c*d^3 + 163*B^3*a^2
*b^2*d^4)*n^2 + 12*(A*B^2*b^4*c^2*d^2 - 8*A*B^2*a*b^3*c*d^3 + 7*A*B^2*a^2*b
^2*d^4)*n)*x^2 + 12*(3*A*B^2*b^4*c^4 - 16*A*B^2*a*b^3*c^3*d + 36*A*B^2*a^2*
b^2*c^2*d^2 - 48*A*B^2*a^3*b*c*d^3 + 25*A*B^2*a^4*d^4)*n - 4*((7*B^3*b^4*c^
3*d - 60*B^3*a*b^3*c^2*d^2 + 324*B^3*a^2*b^2*c*d^3 - 271*B^3*a^3*b*d^4)*n^2
 + 12*(A*B^2*b^4*c^3*d - 6*A*B^2*a*b^3*c^2*d^2 + 18*A*B^2*a^2*b^2*c*d^3 - 1
3*A*B^2*a^3*b*d^4)*n)*x)*log(e))/(a^4*b^5*c^4 - 4*a^5*b^4*c^3*d + 6*a^6*b^3
*c^2*d^2 - 4*a^7*b^2*c*d^3 + a^8*b*d^4 + (b^9*c^4 - 4*a*b^8*c^3*d + 6*a^2*b
^7*c^2*d^2 - 4*a^3*b^6*c*d^3 + a^4*b^5*d^4)*x^4 + 4*(a*b^8*c^4 - 4*a^2*b^7*
c^3*d + 6*a^3*b^6*c^2*d^2 - 4*a^4*b^5*c*d^3 + a^5*b^4*d^4)*x^3 + 6*(a^2*b^7
*c^4 - 4*a^3*b^6*c^3*d + 6*a^4*b^5*c^2*d^2 - 4*a^5*b^4*c*d^3 + a^6*b^3*d^4)
*x^2 + 4*(a^3*b^6*c^4 - 4*a^4*b^5*c^3*d + 6*a^5*b^4*c^2*d^2 - 4*a^6*b^3*c*d
^3 + a^7*b^2*d^4)*x)
```

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(B\log\left(\frac{(bx+a)^n e}{(dx+c)^n}\right)+A\right)^3}{(bx+a)^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*(b*x+a)^n/((d*x+c)^n)))^3/(b*x+a)^5,x, algorithm="giac
")
```

```
[Out] integrate((B*log((b*x + a)^n*e/(d*x + c)^n) + A)^3/(b*x + a)^5, x)
```

**maple [C]** time = 58.08, size = 236754, normalized size = 285.25

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))^3/(b*x+a)^5,x)
```

```
[Out] result too large to display
```

**maxima [B]** time = 5.79, size = 5280, normalized size = 6.36

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*(b*x+a)^n/((d*x+c)^n)))^3/(b*x+a)^5,x, algorithm="maxi
ma")
```

```
[Out] -1/4*B^3*log((b*x + a)^n*e/(d*x + c)^n)^3/(b^5*x^4 + 4*a*b^4*x^3 + 6*a^2*b^
3*x^2 + 4*a^3*b^2*x + a^4*b) + 1/1152*(72*(12*d^4*e*n*log(b*x + a)/(b^5*c^4
 - 4*a*b^4*c^3*d + 6*a^2*b^3*c^2*d^2 - 4*a^3*b^2*c*d^3 + a^4*b*d^4) - 12*d^
4*e*n*log(d*x + c)/(b^5*c^4 - 4*a*b^4*c^3*d + 6*a^2*b^3*c^2*d^2 - 4*a^3*b^2
*c*d^3 + a^4*b*d^4) + (12*b^3*d^3*e*n*x^3 - 3*b^3*c^3*e*n + 13*a*b^2*c^2*d*
e*n - 23*a^2*b*c*d^2*e*n + 25*a^3*d^3*e*n - 6*(b^3*c*d^2*e*n - 7*a*b^2*d^3*
e*n)*x^2 + 4*(b^3*c^2*d*e*n - 5*a*b^2*c*d^2*e*n + 13*a^2*b*d^3*e*n)*x)/(a^4
*b^4*c^3 - 3*a^5*b^3*c^2*d + 3*a^6*b^2*c*d^2 - a^7*b*d^3 + (b^8*c^3 - 3*a*b
^7*c^2*d + 3*a^2*b^6*c*d^2 - a^3*b^5*d^3)*x^4 + 4*(a*b^7*c^3 - 3*a^2*b^6*c^
2*d + 3*a^3*b^5*c*d^2 - a^4*b^4*d^3)*x^3 + 6*(a^2*b^6*c^3 - 3*a^3*b^5*c^2*d
 + 3*a^4*b^4*c*d^2 - a^5*b^3*d^3)*x^2 + 4*(a^3*b^5*c^3 - 3*a^4*b^4*c^2*d +
3*a^5*b^3*c*d^2 - a^6*b^2*d^3)*x))*log((b*x + a)^n*e/(d*x + c)^n)^2/e - (12
*(9*b^4*c^4*e^2*n^2 - 64*a*b^3*c^3*d*e^2*n^2 + 216*a^2*b^2*c^2*d^2*e^2*n^2
- 576*a^3*b*c*d^3*e^2*n^2 + 415*a^4*d^4*e^2*n^2 - 300*(b^4*c*d^3*e^2*n^2 -
a*b^3*d^4*e^2*n^2)*x^3 + 6*(13*b^4*c^2*d^2*e^2*n^2 - 176*a*b^3*c*d^3*e^2*n^
2 + 163*a^2*b^2*d^4*e^2*n^2)*x^2 + 72*(b^4*d^4*e^2*n^2*x^4 + 4*a*b^3*d^4*e^
2*n^2*x^3 + 6*a^2*b^2*d^4*e^2*n^2*x^2 + 4*a^3*b*d^4*e^2*n^2*x + a^4*d^4*e^2
*n^2)*log(b*x + a)^2 + 72*(b^4*d^4*e^2*n^2*x^4 + 4*a*b^3*d^4*e^2*n^2*x^3 +
6*a^2*b^2*d^4*e^2*n^2*x^2 + 4*a^3*b*d^4*e^2*n^2*x + a^4*d^4*e^2*n^2)*log(d*
x + c)^2 - 4*(7*b^4*c^3*d*e^2*n^2 - 60*a*b^3*c^2*d^2*e^2*n^2 + 324*a^2*b^2*
c*d^3*e^2*n^2 - 271*a^3*b*d^4*e^2*n^2)*x - 300*(b^4*d^4*e^2*n^2*x^4 + 4*a*b
```

```
^3*d^4*e^2*n^2*x^3 + 6*a^2*b^2*d^4*e^2*n^2*x^2 + 4*a^3*b*d^4*e^2*n^2*x + a^
4*d^4*e^2*n^2)*log(b*x + a) + 12*(25*b^4*d^4*e^2*n^2*x^4 + 100*a*b^3*d^4*e^
2*n^2*x^3 + 150*a^2*b^2*d^4*e^2*n^2*x^2 + 100*a^3*b*d^4*e^2*n^2*x + 25*a^4*
d^4*e^2*n^2 - 12*(b^4*d^4*e^2*n^2*x^4 + 4*a*b^3*d^4*e^2*n^2*x^3 + 6*a^2*b^2
*d^4*e^2*n^2*x^2 + 4*a^3*b*d^4*e^2*n^2*x + a^4*d^4*e^2*n^2)*log(b*x + a))*l
og(d*x + c))*log((b*x + a)^n*e/(d*x + c)^n)/((a^4*b^5*c^4 - 4*a^5*b^4*c^3*d
 + 6*a^6*b^3*c^2*d^2 - 4*a^7*b^2*c*d^3 + a^8*b*d^4 + (b^9*c^4 - 4*a*b^8*c^3
*d + 6*a^2*b^7*c^2*d^2 - 4*a^3*b^6*c*d^3 + a^4*b^5*d^4)*x^4 + 4*(a*b^8*c^4
- 4*a^2*b^7*c^3*d + 6*a^3*b^6*c^2*d^2 - 4*a^4*b^5*c*d^3 + a^5*b^4*d^4)*x^3
+ 6*(a^2*b^7*c^4 - 4*a^3*b^6*c^3*d + 6*a^4*b^5*c^2*d^2 - 4*a^5*b^4*c*d^3 +
a^6*b^3*d^4)*x^2 + 4*(a^3*b^6*c^4 - 4*a^4*b^5*c^3*d + 6*a^5*b^4*c^2*d^2 - 4
*a^6*b^3*c*d^3 + a^7*b^2*d^4)*x)*e) + (27*b^4*c^4*e^3*n^3 - 256*a*b^3*c^3*d
*e^3*n^3 + 1296*a^2*b^2*c^2*d^2*e^3*n^3 - 6912*a^3*b*c*d^3*e^3*n^3 + 5845*a
^4*d^4*e^3*n^3 - 4980*(b^4*c*d^3*e^3*n^3 - a*b^3*d^4*e^3*n^3)*x^3 - 288*(b^
4*d^4*e^3*n^3*x^4 + 4*a*b^3*d^4*e^3*n^3*x^3 + 6*a^2*b^2*d^4*e^3*n^3*x^2 + 4
*a^3*b*d^4*e^3*n^3*x + a^4*d^4*e^3*n^3)*log(b*x + a)^3 + 288*(b^4*d^4*e^3*n
^3*x^4 + 4*a*b^3*d^4*e^3*n^3*x^3 + 6*a^2*b^2*d^4*e^3*n^3*x^2 + 4*a^3*b*d^4*
e^3*n^3*x + a^4*d^4*e^3*n^3)*log(d*x + c)^3 + 30*(23*b^4*c^2*d^2*e^3*n^3 -
544*a*b^3*c*d^3*e^3*n^3 + 521*a^2*b^2*d^4*e^3*n^3)*x^2 + 1800*(b^4*d^4*e^3*
n^3*x^4 + 4*a*b^3*d^4*e^3*n^3*x^3 + 6*a^2*b^2*d^4*e^3*n^3*x^2 + 4*a^3*b*d^4
*e^3*n^3*x + a^4*d^4*e^3*n^3)*log(b*x + a)^2 + 72*(25*b^4*d^4*e^3*n^3*x^4 +
 100*a*b^3*d^4*e^3*n^3*x^3 + 150*a^2*b^2*d^4*e^3*n^3*x^2 + 100*a^3*b*d^4*e^
3*n^3*x + 25*a^4*d^4*e^3*n^3 - 12*(b^4*d^4*e^3*n^3*x^4 + 4*a*b^3*d^4*e^3*n^
3*x^3 + 6*a^2*b^2*d^4*e^3*n^3*x^2 + 4*a^3*b*d^4*e^3*n^3*x + a^4*d^4*e^3*n^3
)*log(b*x + a))*log(d*x + c)^2 - 4*(37*b^4*c^3*d*e^3*n^3 - 456*a*b^3*c^2*d^
2*e^3*n^3 + 4536*a^2*b^2*c*d^3*e^3*n^3 - 4117*a^3*b*d^4*e^3*n^3)*x - 4980*(
b^4*d^4*e^3*n^3*x^4 + 4*a*b^3*d^4*e^3*n^3*x^3 + 6*a^2*b^2*d^4*e^3*n^3*x^2 +
 4*a^3*b*d^4*e^3*n^3*x + a^4*d^4*e^3*n^3)*log(b*x + a) + 12*(415*b^4*d^4*e^
3*n^3*x^4 + 1660*a*b^3*d^4*e^3*n^3*x^3 + 2490*a^2*b^2*d^4*e^3*n^3*x^2 + 166
0*a^3*b*d^4*e^3*n^3*x + 415*a^4*d^4*e^3*n^3 + 72*(b^4*d^4*e^3*n^3*x^4 + 4*a
*b^3*d^4*e^3*n^3*x^3 + 6*a^2*b^2*d^4*e^3*n^3*x^2 + 4*a^3*b*d^4*e^3*n^3*x +
a^4*d^4*e^3*n^3)*log(b*x + a)^2 - 300*(b^4*d^4*e^3*n^3*x^4 + 4*a*b^3*d^4*e^
3*n^3*x^3 + 6*a^2*b^2*d^4*e^3*n^3*x^2 + 4*a^3*b*d^4*e^3*n^3*x + a^4*d^4*e^3
*n^3)*log(b*x + a))*log(d*x + c))/((a^4*b^5*c^4 - 4*a^5*b^4*c^3*d + 6*a^6*b
^3*c^2*d^2 - 4*a^7*b^2*c*d^3 + a^8*b*d^4 + (b^9*c^4 - 4*a*b^8*c^3*d + 6*a^2
*b^7*c^2*d^2 - 4*a^3*b^6*c*d^3 + a^4*b^5*d^4)*x^4 + 4*(a*b^8*c^4 - 4*a^2*b^
7*c^3*d + 6*a^3*b^6*c^2*d^2 - 4*a^4*b^5*c*d^3 + a^5*b^4*d^4)*x^3 + 6*(a^2*b
^7*c^4 - 4*a^3*b^6*c^3*d + 6*a^4*b^5*c^2*d^2 - 4*a^5*b^4*c*d^3 + a^6*b^3*d^
4)*x^2 + 4*(a^3*b^6*c^4 - 4*a^4*b^5*c^3*d + 6*a^5*b^4*c^2*d^2 - 4*a^6*b^3*c
*d^3 + a^7*b^2*d^4)*x)*e^2))/e)*B^3 + 1/96*A*B^2*(12*(12*d^4*e*n*log(b*x +
a)/(b^5*c^4 - 4*a*b^4*c^3*d + 6*a^2*b^3*c^2*d^2 - 4*a^3*b^2*c*d^3 + a^4*b*d
^4) - 12*d^4*e*n*log(d*x + c)/(b^5*c^4 - 4*a*b^4*c^3*d + 6*a^2*b^3*c^2*d^2
- 4*a^3*b^2*c*d^3 + a^4*b*d^4) + (12*b^3*d^3*e*n*x^3 - 3*b^3*c^3*e*n + 13*a
*b^2*c^2*d*e*n - 23*a^2*b*c*d^2*e*n + 25*a^3*d^3*e*n - 6*(b^3*c*d^2*e*n - 7
*a*b^2*d^3*e*n)*x^2 + 4*(b^3*c^2*d*e*n - 5*a*b^2*c*d^2*e*n + 13*a^2*b*d^3*e
```

```
*n)*x)/(a^4*b^4*c^3 - 3*a^5*b^3*c^2*d + 3*a^6*b^2*c*d^2 - a^7*b*d^3 + (b^8*
c^3 - 3*a*b^7*c^2*d + 3*a^2*b^6*c*d^2 - a^3*b^5*d^3)*x^4 + 4*(a*b^7*c^3 - 3
*a^2*b^6*c^2*d + 3*a^3*b^5*c*d^2 - a^4*b^4*d^3)*x^3 + 6*(a^2*b^6*c^3 - 3*a^
3*b^5*c^2*d + 3*a^4*b^4*c*d^2 - a^5*b^3*d^3)*x^2 + 4*(a^3*b^5*c^3 - 3*a^4*b
^4*c^2*d + 3*a^5*b^3*c*d^2 - a^6*b^2*d^3)*x))*log((b*x + a)^n*e/(d*x + c)^n
)/e - (9*b^4*c^4*e^2*n^2 - 64*a*b^3*c^3*d*e^2*n^2 + 216*a^2*b^2*c^2*d^2*e^2
*n^2 - 576*a^3*b*c*d^3*e^2*n^2 + 415*a^4*d^4*e^2*n^2 - 300*(b^4*c*d^3*e^2*n
^2 - a*b^3*d^4*e^2*n^2)*x^3 + 6*(13*b^4*c^2*d^2*e^2*n^2 - 176*a*b^3*c*d^3*e
^2*n^2 + 163*a^2*b^2*d^4*e^2*n^2)*x^2 + 72*(b^4*d^4*e^2*n^2*x^4 + 4*a*b^3*d
^4*e^2*n^2*x^3 + 6*a^2*b^2*d^4*e^2*n^2*x^2 + 4*a^3*b*d^4*e^2*n^2*x + a^4*d^
4*e^2*n^2)*log(b*x + a)^2 + 72*(b^4*d^4*e^2*n^2*x^4 + 4*a*b^3*d^4*e^2*n^2*x
^3 + 6*a^2*b^2*d^4*e^2*n^2*x^2 + 4*a^3*b*d^4*e^2*n^2*x + a^4*d^4*e^2*n^2)*l
og(d*x + c)^2 - 4*(7*b^4*c^3*d*e^2*n^2 - 60*a*b^3*c^2*d^2*e^2*n^2 + 324*a^2
*b^2*c*d^3*e^2*n^2 - 271*a^3*b*d^4*e^2*n^2)*x - 300*(b^4*d^4*e^2*n^2*x^4 +
4*a*b^3*d^4*e^2*n^2*x^3 + 6*a^2*b^2*d^4*e^2*n^2*x^2 + 4*a^3*b*d^4*e^2*n^2*x
 + a^4*d^4*e^2*n^2)*log(b*x + a) + 12*(25*b^4*d^4*e^2*n^2*x^4 + 100*a*b^3*d
^4*e^2*n^2*x^3 + 150*a^2*b^2*d^4*e^2*n^2*x^2 + 100*a^3*b*d^4*e^2*n^2*x + 25
*a^4*d^4*e^2*n^2 - 12*(b^4*d^4*e^2*n^2*x^4 + 4*a*b^3*d^4*e^2*n^2*x^3 + 6*a^
2*b^2*d^4*e^2*n^2*x^2 + 4*a^3*b*d^4*e^2*n^2*x + a^4*d^4*e^2*n^2)*log(b*x +
a))*log(d*x + c))/((a^4*b^5*c^4 - 4*a^5*b^4*c^3*d + 6*a^6*b^3*c^2*d^2 - 4*a
^7*b^2*c*d^3 + a^8*b*d^4 + (b^9*c^4 - 4*a*b^8*c^3*d + 6*a^2*b^7*c^2*d^2 - 4
*a^3*b^6*c*d^3 + a^4*b^5*d^4)*x^4 + 4*(a*b^8*c^4 - 4*a^2*b^7*c^3*d + 6*a^3*
b^6*c^2*d^2 - 4*a^4*b^5*c*d^3 + a^5*b^4*d^4)*x^3 + 6*(a^2*b^7*c^4 - 4*a^3*b
^6*c^3*d + 6*a^4*b^5*c^2*d^2 - 4*a^5*b^4*c*d^3 + a^6*b^3*d^4)*x^2 + 4*(a^3*
b^6*c^4 - 4*a^4*b^5*c^3*d + 6*a^5*b^4*c^2*d^2 - 4*a^6*b^3*c*d^3 + a^7*b^2*d
^4)*x)*e^2)) - 3/4*A*B^2*log((b*x + a)^n*e/(d*x + c)^n)^2/(b^5*x^4 + 4*a*b^
4*x^3 + 6*a^2*b^3*x^2 + 4*a^3*b^2*x + a^4*b) + 1/16*(12*d^4*e*n*log(b*x + a
)/(b^5*c^4 - 4*a*b^4*c^3*d + 6*a^2*b^3*c^2*d^2 - 4*a^3*b^2*c*d^3 + a^4*b*d^
4) - 12*d^4*e*n*log(d*x + c)/(b^5*c^4 - 4*a*b^4*c^3*d + 6*a^2*b^3*c^2*d^2 -
 4*a^3*b^2*c*d^3 + a^4*b*d^4) + (12*b^3*d^3*e*n*x^3 - 3*b^3*c^3*e*n + 13*a*
b^2*c^2*d*e*n - 23*a^2*b*c*d^2*e*n + 25*a^3*d^3*e*n - 6*(b^3*c*d^2*e*n - 7*
a*b^2*d^3*e*n)*x^2 + 4*(b^3*c^2*d*e*n - 5*a*b^2*c*d^2*e*n + 13*a^2*b*d^3*e*
n)*x)/(a^4*b^4*c^3 - 3*a^5*b^3*c^2*d + 3*a^6*b^2*c*d^2 - a^7*b*d^3 + (b^8*c
^3 - 3*a*b^7*c^2*d + 3*a^2*b^6*c*d^2 - a^3*b^5*d^3)*x^4 + 4*(a*b^7*c^3 - 3*
a^2*b^6*c^2*d + 3*a^3*b^5*c*d^2 - a^4*b^4*d^3)*x^3 + 6*(a^2*b^6*c^3 - 3*a^3
*b^5*c^2*d + 3*a^4*b^4*c*d^2 - a^5*b^3*d^3)*x^2 + 4*(a^3*b^5*c^3 - 3*a^4*b^
4*c^2*d + 3*a^5*b^3*c*d^2 - a^6*b^2*d^3)*x))*A^2*B/e - 3/4*A^2*B*log((b*x +
 a)^n*e/(d*x + c)^n)/(b^5*x^4 + 4*a*b^4*x^3 + 6*a^2*b^3*x^2 + 4*a^3*b^2*x +
 a^4*b) - 1/4*A^3/(b^5*x^4 + 4*a*b^4*x^3 + 6*a^2*b^3*x^2 + 4*a^3*b^2*x + a^
4*b)
```

**mupad [B]** time = 11.29, size = 4257, normalized size = 5.13

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*log((e*(a + b*x)^n)/(c + d*x)^n))^3/(a + b*x)^5,x)

[Out] log((e*(a + b*x)^n)/(c + d*x)^n)*((x*((a*d + b*c)*(a*((9*B^3*a*d^2*n^2)/2 -
 (3*B^3*b*c*d*n^2)/2) + 13*B^3*a^2*d^2*n^2 + (11*B^3*b^2*c^2*n^2)/2 - 6*A^2
*B*a^2*d^2 - 6*A^2*B*b^2*c^2 - (31*B^3*a*b*c*d*n^2)/2 + 12*A^2*B*a*b*c*d) +
 a*c*(b*((9*B^3*a*d^2*n^2)/2 - (3*B^3*b*c*d*n^2)/2) + (27*B^3*a*b*d^2*n^2)/
2 - (9*B^3*b^2*c*d*n^2)/2)) + x^2*((a*d + b*c)*(b*((9*B^3*a*d^2*n^2)/2 - (3
*B^3*b*c*d*n^2)/2) + (27*B^3*a*b*d^2*n^2)/2 - (9*B^3*b^2*c*d*n^2)/2) + b*d*
(a*((9*B^3*a*d^2*n^2)/2 - (3*B^3*b*c*d*n^2)/2) + 13*B^3*a^2*d^2*n^2 + (11*B
^3*b^2*c^2*n^2)/2 - 6*A^2*B*a^2*d^2 - 6*A^2*B*b^2*c^2 - (31*B^3*a*b*c*d*n^2
)/2 + 12*A^2*B*a*b*c*d) + 6*B^3*a*b^2*c*d^2*n^2) + x^3*(b*d*(b*((9*B^3*a*d^
2*n^2)/2 - (3*B^3*b*c*d*n^2)/2) + (27*B^3*a*b*d^2*n^2)/2 - (9*B^3*b^2*c*d*n
^2)/2) + 6*B^3*b^2*d^2*n^2*(a*d + b*c)) + a*c*(a*((9*B^3*a*d^2*n^2)/2 - (3*
B^3*b*c*d*n^2)/2) + 13*B^3*a^2*d^2*n^2 + (11*B^3*b^2*c^2*n^2)/2 - 6*A^2*B*a
^2*d^2 - 6*A^2*B*b^2*c^2 - (31*B^3*a*b*c*d*n^2)/2 + 12*A^2*B*a*b*c*d) + 6*B
^3*b^3*d^3*n^2*x^4)/(8*b*(a*d - b*c)^2*(a + b*x)^5*(c + d*x)) - (d^4*(12*A*
B^2 + 25*B^3*n)*(x^3*((a*d + b*c)*(b*(b*((2*a*b*n*(a*d - b*c)^3)/d + (2*b*n
*(a*d - b*c)^3*(4*a*d - b*c))/(3*d^2)) + (4*b^2*n*(a*d - b*c)^3*(4*a*d - b*
c))/(3*d^2) + (4*a*b^2*n*(a*d - b*c)^3)/d) + (2*b^3*n*(a*d - b*c)^3*(4*a*d
- b*c))/d^2 + (6*a*b^3*n*(a*d - b*c)^3)/d) + b*d*(b*(a*((2*a*b*n*(a*d - b*c
)^3)/d + (2*b*n*(a*d - b*c)^3*(4*a*d - b*c))/(3*d^2)) + (2*b*n*(a*d - b*c)^
3*(6*a^2*d^2 + b^2*c^2 - 4*a*b*c*d))/(3*d^3)) + a*(b*((2*a*b*n*(a*d - b*c)^
3)/d + (2*b*n*(a*d - b*c)^3*(4*a*d - b*c))/(3*d^2)) + (4*b^2*n*(a*d - b*c)^
3*(4*a*d - b*c))/(3*d^2) + (4*a*b^2*n*(a*d - b*c)^3)/d) + (2*b^2*n*(a*d - b
*c)^3*(6*a^2*d^2 + b^2*c^2 - 4*a*b*c*d))/d^3) + (8*a*b^4*c*n*(a*d - b*c)^3)
/d) + x^2*((a*d + b*c)*(b*(a*((2*a*b*n*(a*d - b*c)^3)/d + (2*b*n*(a*d - b*c
)^3*(4*a*d - b*c))/(3*d^2)) + (2*b*n*(a*d - b*c)^3*(6*a^2*d^2 + b^2*c^2 - 4
*a*b*c*d))/(3*d^3)) + a*(b*((2*a*b*n*(a*d - b*c)^3)/d + (2*b*n*(a*d - b*c)^
3*(4*a*d - b*c))/(3*d^2)) + (4*b^2*n*(a*d - b*c)^3*(4*a*d - b*c))/(3*d^2) +
 (4*a*b^2*n*(a*d - b*c)^3)/d) + (2*b^2*n*(a*d - b*c)^3*(6*a^2*d^2 + b^2*c^2
 - 4*a*b*c*d))/d^3) + a*c*(b*(b*((2*a*b*n*(a*d - b*c)^3)/d + (2*b*n*(a*d -
b*c)^3*(4*a*d - b*c))/(3*d^2)) + (4*b^2*n*(a*d - b*c)^3*(4*a*d - b*c))/(3*d
^2) + (4*a*b^2*n*(a*d - b*c)^3)/d) + (2*b^3*n*(a*d - b*c)^3*(4*a*d - b*c))/
d^2 + (6*a*b^3*n*(a*d - b*c)^3)/d) + b*d*(a*(a*((2*a*b*n*(a*d - b*c)^3)/d +
 (2*b*n*(a*d - b*c)^3*(4*a*d - b*c))/(3*d^2)) + (2*b*n*(a*d - b*c)^3*(6*a^2
*d^2 + b^2*c^2 - 4*a*b*c*d))/(3*d^3)) + (2*b*n*(a*d - b*c)^3*(4*a^3*d^3 - b
^3*c^3 + 4*a*b^2*c^2*d - 6*a^2*b*c*d^2))/d^4)) + x*((a*(a*((2*a*b*n*(a*d -
b*c)^3)/d + (2*b*n*(a*d - b*c)^3*(4*a*d - b*c))/(3*d^2)) + (2*b*n*(a*d - b*
c)^3*(6*a^2*d^2 + b^2*c^2 - 4*a*b*c*d))/(3*d^3)) + (2*b*n*(a*d - b*c)^3*(4*
a^3*d^3 - b^3*c^3 + 4*a*b^2*c^2*d - 6*a^2*b*c*d^2))/d^4)*(a*d + b*c) + a*c*
(b*(a*((2*a*b*n*(a*d - b*c)^3)/d + (2*b*n*(a*d - b*c)^3*(4*a*d - b*c))/(3*d
^2)) + (2*b*n*(a*d - b*c)^3*(6*a^2*d^2 + b^2*c^2 - 4*a*b*c*d))/(3*d^3)) + a
*(b*((2*a*b*n*(a*d - b*c)^3)/d + (2*b*n*(a*d - b*c)^3*(4*a*d - b*c))/(3*d^2
)) + (4*b^2*n*(a*d - b*c)^3*(4*a*d - b*c))/(3*d^2) + (4*a*b^2*n*(a*d - b*c)
^3)/d) + (2*b^2*n*(a*d - b*c)^3*(6*a^2*d^2 + b^2*c^2 - 4*a*b*c*d))/d^3)) +
```

x^4*(b*d*(b*(b*((2*a*b*n*(a*d - b*c)^3)/d + (2*b*n*(a*d - b*c)^3*(4*a*d - b
*c))/(3*d^2)) + (4*b^2*n*(a*d - b*c)^3*(4*a*d - b*c))/(3*d^2) + (4*a*b^2*n*
(a*d - b*c)^3)/d) + (2*b^3*n*(a*d - b*c)^3*(4*a*d - b*c))/d^2 + (6*a*b^3*n*
(a*d - b*c)^3)/d) + (8*b^4*n*(a*d + b*c)*(a*d - b*c)^3)/d) + a*c*(a*(a*((2*
a*b*n*(a*d - b*c)^3)/d + (2*b*n*(a*d - b*c)^3*(4*a*d - b*c))/(3*d^2)) + (2*
b*n*(a*d - b*c)^3*(6*a^2*d^2 + b^2*c^2 - 4*a*b*c*d))/(3*d^3)) + (2*b*n*(a*d
 - b*c)^3*(4*a^3*d^3 - b^3*c^3 + 4*a*b^2*c^2*d - 6*a^2*b*c*d^2))/d^4) + 8*b
^5*n*x^5*(a*d - b*c)^3))/(64*b^2*(a*d - b*c)^2*(a + b*x)^5*(c + d*x)*(a^4*d
^4 + b^4*c^4 + 6*a^2*b^2*c^2*d^2 - 4*a*b^3*c^3*d - 4*a^3*b*c*d^3))) - log((
e*(a + b*x)^n)/(c + d*x)^n)^3*(B^3/(4*b*(a^4 + b^4*x^4 + 4*a*b^3*x^3 + 6*a^
2*b^2*x^2 + 4*a^3*b*x)) - (B^3*d^4)/(4*b*(a^4*d^4 + b^4*c^4 + 6*a^2*b^2*c^2
*d^2 - 4*a*b^3*c^3*d - 4*a^3*b*c*d^3))) - log((e*(a + b*x)^n)/(c + d*x)^n)^
2*((3*A*B^2)/(4*(a^4*b + b^5*x^4 + 4*a^3*b^2*x + 4*a*b^4*x^3 + 6*a^2*b^3*x^
2)) - (d^4*(12*A*B^2 + 25*B^3*n))/(16*b*(a^4*d^4 + b^4*c^4 + 6*a^2*b^2*c^2*
d^2 - 4*a*b^3*c^3*d - 4*a^3*b*c*d^3)) + (3*B^3*d^4*(x^2*(b*(b*((b*n*(a*d -
b*c)*(4*a*d - b*c))/(3*d^2) + (a*b*n*(a*d - b*c))/d) + (2*a*b^2*n*(a*d - b*
c))/d + (2*b^2*n*(a*d - b*c)*(4*a*d - b*c))/(3*d^2)) + (3*a*b^3*n*(a*d - b*
c))/d + (b^3*n*(a*d - b*c)*(4*a*d - b*c))/d^2) + a*(a*((b*n*(a*d - b*c)*(4*
a*d - b*c))/(3*d^2) + (a*b*n*(a*d - b*c))/d) + (b*n*(a*d - b*c)*(6*a^2*d^2
+ b^2*c^2 - 4*a*b*c*d))/(3*d^3)) + x*(b*(a*((b*n*(a*d - b*c)*(4*a*d - b*c))
/(3*d^2) + (a*b*n*(a*d - b*c))/d) + (b*n*(a*d - b*c)*(6*a^2*d^2 + b^2*c^2 -
 4*a*b*c*d))/(3*d^3)) + a*(b*((b*n*(a*d - b*c)*(4*a*d - b*c))/(3*d^2) + (a*
b*n*(a*d - b*c))/d) + (2*a*b^2*n*(a*d - b*c))/d + (2*b^2*n*(a*d - b*c)*(4*a
*d - b*c))/(3*d^2)) + (b^2*n*(a*d - b*c)*(6*a^2*d^2 + b^2*c^2 - 4*a*b*c*d))
/d^3) + (b*n*(a*d - b*c)*(4*a^3*d^3 - b^3*c^3 + 4*a*b^2*c^2*d - 6*a^2*b*c*d
^2))/d^4 + (4*b^4*n*x^3*(a*d - b*c))/d))/(16*b*(a^4*b + b^5*x^4 + 4*a^3*b^2
*x + 4*a*b^4*x^3 + 6*a^2*b^3*x^2)*(a^4*d^4 + b^4*c^4 + 6*a^2*b^2*c^2*d^2 -
4*a*b^3*c^3*d - 4*a^3*b*c*d^3))) - ((288*A^3*a^3*d^3 - 288*A^3*b^3*c^3 + 58
45*B^3*a^3*d^3*n^3 - 27*B^3*b^3*c^3*n^3 + 1800*A^2*B*a^3*d^3*n - 216*A^2*B*
b^3*c^3*n + 864*A^3*a*b^2*c^2*d - 864*A^3*a^2*b*c*d^2 + 4980*A*B^2*a^3*d^3*
n^2 - 108*A*B^2*b^3*c^3*n^2 + 229*B^3*a*b^2*c^2*d*n^3 - 1067*B^3*a^2*b*c*d^
2*n^3 + 660*A*B^2*a*b^2*c^2*d*n^2 - 1932*A*B^2*a^2*b*c*d^2*n^2 + 936*A^2*B*
a*b^2*c^2*d*n - 1656*A^2*B*a^2*b*c*d^2*n)/(12*(a*d - b*c)) + (x^2*(2605*B^3
*a*b^2*d^3*n^3 - 115*B^3*b^3*c*d^2*n^3 + 504*A^2*B*a*b^2*d^3*n - 72*A^2*B*b
^3*c*d^2*n + 1956*A*B^2*a*b^2*d^3*n^2 - 156*A*B^2*b^3*c*d^2*n^2))/(2*(a*d -
 b*c)) + (x*(4117*B^3*a^2*b*d^3*n^3 + 37*B^3*b^3*c^2*d*n^3 - 419*B^3*a*b^2*
c*d^2*n^3 + 936*A^2*B*a^2*b*d^3*n + 72*A^2*B*b^3*c^2*d*n + 3252*A*B^2*a^2*b
*d^3*n^2 + 84*A*B^2*b^3*c^2*d*n^2 - 636*A*B^2*a*b^2*c*d^2*n^2 - 360*A^2*B*a
*b^2*c*d^2*n))/(3*(a*d - b*c)) + (x^3*(415*B^3*b^3*d^3*n^3 + 72*A^2*B*b^3*d
^3*n + 300*A*B^2*b^3*d^3*n^2))/(a*d - b*c))/(x*(384*a^3*b^4*c^2 + 384*a^5*b
^2*d^2 - 768*a^4*b^3*c*d) + x^3*(384*a*b^6*c^2 + 384*a^3*b^4*d^2 - 768*a^2*
b^5*c*d) + x^4*(96*b^7*c^2 + 96*a^2*b^5*d^2 - 192*a*b^6*c*d) + x^2*(576*a^2
*b^5*c^2 + 576*a^4*b^3*d^2 - 1152*a^3*b^4*c*d) + 96*a^6*b*d^2 + 96*a^4*b^3*
c^2 - 192*a^5*b^2*c*d) + (B*d^4*n*atan((B*d^4*n*((b^5*c^4 - a^4*b*d^4 + 2*a
^3*b^2*c*d^3 - 2*a*b^4*c^3*d)/(b^4*c^3 - a^3*b*d^3 + 3*a^2*b^2*c*d^2 - 3*a*

```
b^3*c^2*d) + 2*b*d*x)*(72*A^2 + 415*B^2*n^2 + 300*A*B*n)*(b^4*c^3 - a^3*b*d
^3 + 3*a^2*b^2*c*d^2 - 3*a*b^3*c^2*d)*1i)/(b*(a*d - b*c)^4*(415*B^3*d^4*n^3
 + 72*A^2*B*d^4*n + 300*A*B^2*d^4*n^2)))*(72*A^2 + 415*B^2*n^2 + 300*A*B*n)
*1i)/(48*b*(a*d - b*c)^4)
```

**sympy [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*ln(e*(b*x+a)**n/((d*x+c)**n)))**3/(b*x+a)**5,x)
```

```
[Out] Timed out
```

# 3.172 $\int \frac{1}{(ag+bgx)^2\left(A+B\log(e(a+bx)^n(c+dx)^{-n})\right)}\,dx$

**Optimal**. Leaf size=96

$$\frac{e^{\frac{A}{Bn}}(c+dx)\left(e(a+bx)^n(c+dx)^{-n}\right)^{\frac{1}{n}}\operatorname{Ei}\left(-\frac{A+B\log(e(a+bx)^n(c+dx)^{-n})}{Bn}\right)}{Bg^2n(a+bx)(bc-ad)}$$

[Out] exp(A/B/n)*(d*x+c)*(e*(b*x+a)^n/((d*x+c)^n))^(1/n)*Ei((-A-B*ln(e*(b*x+a)^n/((d*x+c)^n)))/B/n)/B/(-a*d+b*c)/g^2/n/(b*x+a)

**Rubi [F]** time = 0.10, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{1}{(ag+bgx)^2\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)}\,dx$$

Verification is Not applicable to the result.

[In] Int[1/((a*g + b*g*x)^2*(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])),x]

[Out] Defer[Int][1/((a*g + b*g*x)^2*(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])), x]

Rubi steps

$$\int \frac{1}{(ag+bgx)^2\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)}\,dx=\int \frac{1}{(ag+bgx)^2\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)}\,dx$$

**Mathematica [F]** time = 0.09, size = 0, normalized size = 0.00

$$\int \frac{1}{(ag+bgx)^2\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)}\,dx$$

Verification is Not applicable to the result.

[In] Integrate[1/((a*g + b*g*x)^2*(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])),x]

[Out] Integrate[1/((a*g + b*g*x)^2*(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])), x]

**fricas [A]** time = 0.61, size = 62, normalized size = 0.65

$$\frac{e^{\left(\frac{B\log(e)+A}{Bn}\right)}\log\_integral\left(\frac{(dx+c)e^{\left(-\frac{B\log(e)+A}{Bn}\right)}}{bx+a}\right)}{(Bbc-Bad)g^2n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*g*x+a*g)^2/(A+B*log(e*(b*x+a)^n/((d*x+c)^n))),x, algorithm="
fricas")
```

```
[Out] e^((B*log(e) + A)/(B*n))*log_integral((d*x + c)*e^(-(B*log(e) + A)/(B*n))/(
b*x + a))/((B*b*c - B*a*d)*g^2*n)
```

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(bgx+ag)^2\left(B\log\left(\frac{(bx+a)^n e}{(dx+c)^n}\right)+A\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*g*x+a*g)^2/(A+B*log(e*(b*x+a)^n/((d*x+c)^n))),x, algorithm="
giac")
```

```
[Out] integrate(1/((b*g*x + a*g)^2*(B*log((b*x + a)^n*e/(d*x + c)^n) + A)), x)
```

**maple [F]** time = 4.52, size = 0, normalized size = 0.00

$$\int \frac{1}{(bgx+ag)^2\left(B\ln\left(e\,(bx+a)^n\,(dx+c)^{-n}\right)+A\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*g*x+a*g)^2/(A+B*ln(e*(b*x+a)^n/((d*x+c)^n))),x)
```

```
[Out] int(1/(b*g*x+a*g)^2/(A+B*ln(e*(b*x+a)^n/((d*x+c)^n))),x)
```

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(bgx+ag)^2\left(B\log\left(\frac{(bx+a)^n e}{(dx+c)^n}\right)+A\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*g*x+a*g)^2/(A+B*log(e*(b*x+a)^n/((d*x+c)^n))),x, algorithm="
maxima")
```

```
[Out] integrate(1/((b*g*x + a*g)^2*(B*log((b*x + a)^n*e/(d*x + c)^n) + A)), x)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\left(a\,g+b\,g\,x\right)^2\,\left(A+B\,\ln\left(\frac{e\,{\left(a+b\,x\right)}^n}{{\left(c+d\,x\right)}^n}\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*g + b*g*x)^2*(A + B*log((e*(a + b*x)^n)/(c + d*x)^n))),x)
```

```
[Out] int(1/((a*g + b*g*x)^2*(A + B*log((e*(a + b*x)^n)/(c + d*x)^n))), x)
```

**sympy [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*g*x+a*g)**2/(A+B*ln(e*(b*x+a)**n/((d*x+c)**n))),x)
```

```
[Out] Timed out
```

# 3.173 $\int (ag+bgx)^4 \left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)\,dx$

**Optimal**. Leaf size=180

$$\frac{g^4(a+bx)^5\left(B\log\left(\frac{e(c+dx)}{a+bx}\right)+A\right)}{5b}+\frac{Bg^4(bc-ad)^5\log(c+dx)}{5bd^5}-\frac{Bg^4x(bc-ad)^4}{5d^4}+\frac{Bg^4(a+bx)^2(bc-ad)^3}{10bd^3}-\frac{Bg^4(a+$$

```
[Out] -1/5*B*(-a*d+b*c)^4*g^4*x/d^4+1/10*B*(-a*d+b*c)^3*g^4*(b*x+a)^2/b/d^3-1/15*
B*(-a*d+b*c)^2*g^4*(b*x+a)^3/b/d^2+1/20*B*(-a*d+b*c)*g^4*(b*x+a)^4/b/d+1/5*
B*(-a*d+b*c)^5*g^4*ln(d*x+c)/b/d^5+1/5*g^4*(b*x+a)^5*(A+B*ln(e*(d*x+c)/(b*x
+a)))/b
```

**Rubi [A]** time = 0.12, antiderivative size = 180, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {2525, 12, 43}

$$\frac{g^4(a+bx)^5\left(B\log\left(\frac{e(c+dx)}{a+bx}\right)+A\right)}{5b}-\frac{Bg^4x(bc-ad)^4}{5d^4}+\frac{Bg^4(a+bx)^2(bc-ad)^3}{10bd^3}-\frac{Bg^4(a+bx)^3(bc-ad)^2}{15bd^2}+\frac{Bg^4(bc-a$$

Antiderivative was successfully verified.

```
[In] Int[(a*g + b*g*x)^4*(A + B*Log[(e*(c + d*x))/(a + b*x)]),x]
```

```
[Out] -(B*(b*c - a*d)^4*g^4*x)/(5*d^4) + (B*(b*c - a*d)^3*g^4*(a + b*x)^2)/(10*b*
d^3) - (B*(b*c - a*d)^2*g^4*(a + b*x)^3)/(15*b*d^2) + (B*(b*c - a*d)*g^4*(a
 + b*x)^4)/(20*b*d) + (B*(b*c - a*d)^5*g^4*Log[c + d*x])/(5*b*d^5) + (g^4*(
a + b*x)^5*(A + B*Log[(e*(c + d*x))/(a + b*x)]))/(5*b)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2525

```
Int[((a_.) + Log[(c_.)*(RFx_)^(p_.)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_))^(m_.
), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(a + b*Log[c*RFx^p])^n)/(e*(m + 1))
```

```
, x] - Dist[(b*n*p)/(e*(m + 1)), Int[SimplifyIntegrand[((d + e*x)^(m + 1)*(
a + b*Log[c*RFx^p])^(n - 1)*D[RFx, x])/RFx, x], x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && RationalFunctionQ[RFx, x] && IGtQ[n, 0] && (EqQ[n, 1] ||
IntegerQ[m]) && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int (ag+bgx)^4\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)dx &= \frac{g^4(a+bx)^5\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)}{5b}-\frac{B\int\frac{(-bc+ad)g^5(a+bx)^4}{c+dx}\,dx}{5bg}\\ &= \frac{g^4(a+bx)^5\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)}{5b}+\frac{\left(B(bc-ad)g^4\right)\int\frac{(a+bx)^4}{c+dx}\,dx}{5b}\\ &= \frac{g^4(a+bx)^5\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)}{5b}+\frac{\left(B(bc-ad)g^4\right)\int\left(-\frac{b(bc-ad)^3}{d^4}+\right.}{}\\ &= -\frac{B(bc-ad)^4g^4x}{5d^4}+\frac{B(bc-ad)^3g^4(a+bx)^2}{10bd^3}-\frac{B(bc-ad)^2g^4(a+bx}{15bd^2}\end{aligned}$$

**Mathematica [A]** time = 0.10, size = 142, normalized size = 0.79

$$\frac{g^4\left((a+bx)^5\left(B\log\left(\frac{e(c+dx)}{a+bx}\right)+A\right)-\frac{B(ad-bc)\left(4d^3(a+bx)^3(ad-bc)+6d^2(a+bx)^2(bc-ad)^2-12bdx(bc-ad)^3+12(bc-ad)^4\log(c+dx)+3d^4(a+\right.}{12d^5}}{5b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*g + b*g*x)^4*(A + B*Log[(e*(c + d*x))/(a + b*x)]),x]
```

```
[Out] (g^4*(-1/12*(B*(-(b*c) + a*d)*(-12*b*d*(b*c - a*d)^3*x + 6*d^2*(b*c - a*d)^
2*(a + b*x)^2 + 4*d^3*(-(b*c) + a*d)*(a + b*x)^3 + 3*d^4*(a + b*x)^4 + 12*(
b*c - a*d)^4*Log[c + d*x]))/d^5 + (a + b*x)^5*(A + B*Log[(e*(c + d*x))/(a +
 b*x)])))/(5*b)
```

**fricas [B]** time = 0.85, size = 433, normalized size = 2.41

$$\frac{12\,Ab^5d^5g^4x^5-12\,Ba^5d^5g^4\log(bx+a)+3\left(Bb^5cd^4+(20\,A-B)ab^4d^5\right)g^4x^4-4\left(Bb^5c^2d^3-5\,Bab^4cd^4-2\,(15\,A\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*g*x+a*g)^4*(A+B*log(e*(d*x+c)/(b*x+a))),x, algorithm="fricas")
```

```
[Out] 1/60*(12*A*b^5*d^5*g^4*x^5 - 12*B*a^5*d^5*g^4*log(b*x + a) + 3*(B*b^5*c*d^4
 + (20*A - B)*a*b^4*d^5)*g^4*x^4 - 4*(B*b^5*c^2*d^3 - 5*B*a*b^4*c*d^4 - 2*(
15*A - 2*B)*a^2*b^3*d^5)*g^4*x^3 + 6*(B*b^5*c^3*d^2 - 5*B*a*b^4*c^2*d^3 + 1
0*B*a^2*b^3*c*d^4 + 2*(10*A - 3*B)*a^3*b^2*d^5)*g^4*x^2 - 12*(B*b^5*c^4*d -
 5*B*a*b^4*c^3*d^2 + 10*B*a^2*b^3*c^2*d^3 - 10*B*a^3*b^2*c*d^4 - (5*A - 4*B
)*a^4*b*d^5)*g^4*x + 12*(B*b^5*c^5 - 5*B*a*b^4*c^4*d + 10*B*a^2*b^3*c^3*d^2
 - 10*B*a^3*b^2*c^2*d^3 + 5*B*a^4*b*c*d^4)*g^4*log(d*x + c) + 12*(B*b^5*d^5
*g^4*x^5 + 5*B*a*b^4*d^5*g^4*x^4 + 10*B*a^2*b^3*d^5*g^4*x^3 + 10*B*a^3*b^2*
d^5*g^4*x^2 + 5*B*a^4*b*d^5*g^4*x)*log((d*e*x + c*e)/(b*x + a)))/(b*d^5)
```

**giac [B]** time = 1.53, size = 5960, normalized size = 33.11

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*g*x+a*g)^4*(A+B*log(e*(d*x+c)/(b*x+a))),x, algorithm="giac")
```

```
[Out] -1/60*(12*B*b^6*c^6*d^5*g^4*e^6*log(-d*e + (d*x*e + c*e)*b/(b*x + a)) - 72*
B*a*b^5*c^5*d^6*g^4*e^6*log(-d*e + (d*x*e + c*e)*b/(b*x + a)) + 180*B*a^2*b
^4*c^4*d^7*g^4*e^6*log(-d*e + (d*x*e + c*e)*b/(b*x + a)) - 240*B*a^3*b^3*c^
3*d^8*g^4*e^6*log(-d*e + (d*x*e + c*e)*b/(b*x + a)) + 180*B*a^4*b^2*c^2*d^9
*g^4*e^6*log(-d*e + (d*x*e + c*e)*b/(b*x + a)) - 72*B*a^5*b*c*d^10*g^4*e^6*
log(-d*e + (d*x*e + c*e)*b/(b*x + a)) + 12*B*a^6*d^11*g^4*e^6*log(-d*e + (d
*x*e + c*e)*b/(b*x + a)) - 60*(d*x*e + c*e)*B*b^7*c^6*d^4*g^4*e^5*log(-d*e
+ (d*x*e + c*e)*b/(b*x + a))/(b*x + a) + 360*(d*x*e + c*e)*B*a*b^6*c^5*d^5*
g^4*e^5*log(-d*e + (d*x*e + c*e)*b/(b*x + a))/(b*x + a) - 900*(d*x*e + c*e)
*B*a^2*b^5*c^4*d^6*g^4*e^5*log(-d*e + (d*x*e + c*e)*b/(b*x + a))/(b*x + a)
+ 1200*(d*x*e + c*e)*B*a^3*b^4*c^3*d^7*g^4*e^5*log(-d*e + (d*x*e + c*e)*b/(
b*x + a))/(b*x + a) - 900*(d*x*e + c*e)*B*a^4*b^3*c^2*d^8*g^4*e^5*log(-d*e
+ (d*x*e + c*e)*b/(b*x + a))/(b*x + a) + 360*(d*x*e + c*e)*B*a^5*b^2*c*d^9*
g^4*e^5*log(-d*e + (d*x*e + c*e)*b/(b*x + a))/(b*x + a) - 60*(d*x*e + c*e)*
B*a^6*b*d^10*g^4*e^5*log(-d*e + (d*x*e + c*e)*b/(b*x + a))/(b*x + a) + 120*
(d*x*e + c*e)^2*B*b^8*c^6*d^3*g^4*e^4*log(-d*e + (d*x*e + c*e)*b/(b*x + a))
/(b*x + a)^2 - 720*(d*x*e + c*e)^2*B*a*b^7*c^5*d^4*g^4*e^4*log(-d*e + (d*x*
e + c*e)*b/(b*x + a))/(b*x + a)^2 + 1800*(d*x*e + c*e)^2*B*a^2*b^6*c^4*d^5*
g^4*e^4*log(-d*e + (d*x*e + c*e)*b/(b*x + a))/(b*x + a)^2 - 2400*(d*x*e + c
*e)^2*B*a^3*b^5*c^3*d^6*g^4*e^4*log(-d*e + (d*x*e + c*e)*b/(b*x + a))/(b*x
+ a)^2 + 1800*(d*x*e + c*e)^2*B*a^4*b^4*c^2*d^7*g^4*e^4*log(-d*e + (d*x*e +
 c*e)*b/(b*x + a))/(b*x + a)^2 - 720*(d*x*e + c*e)^2*B*a^5*b^3*c*d^8*g^4*e^
4*log(-d*e + (d*x*e + c*e)*b/(b*x + a))/(b*x + a)^2 + 120*(d*x*e + c*e)^2*B
*a^6*b^2*d^9*g^4*e^4*log(-d*e + (d*x*e + c*e)*b/(b*x + a))/(b*x + a)^2 - 12
0*(d*x*e + c*e)^3*B*b^9*c^6*d^2*g^4*e^3*log(-d*e + (d*x*e + c*e)*b/(b*x + a
))/(b*x + a)^3 + 720*(d*x*e + c*e)^3*B*a*b^8*c^5*d^3*g^4*e^3*log(-d*e + (d*
x*e + c*e)*b/(b*x + a))/(b*x + a)^3 - 1800*(d*x*e + c*e)^3*B*a^2*b^7*c^4*d^
4*g^4*e^3*log(-d*e + (d*x*e + c*e)*b/(b*x + a))/(b*x + a)^3 + 2400*(d*x*e +
```

c*e)^3*B*a^3*b^6*c^3*d^5*g^4*e^3*log(-d*e + (d*x*e + c*e)*b/(b*x + a))/(b*x + a)^3 - 1800*(d*x*e + c*e)^3*B*a^4*b^5*c^2*d^6*g^4*e^3*log(-d*e + (d*x*e + c*e)*b/(b*x + a))/(b*x + a)^3 + 720*(d*x*e + c*e)^3*B*a^5*b^4*c*d^7*g^4*e^3*log(-d*e + (d*x*e + c*e)*b/(b*x + a))/(b*x + a)^3 - 120*(d*x*e + c*e)^3*B*a^6*b^3*d^8*g^4*e^3*log(-d*e + (d*x*e + c*e)*b/(b*x + a))/(b*x + a)^3 + 60*(d*x*e + c*e)^4*B*b^10*c^6*d*g^4*e^2*log(-d*e + (d*x*e + c*e)*b/(b*x + a))/(b*x + a)^4 - 360*(d*x*e + c*e)^4*B*a*b^9*c^5*d^2*g^4*e^2*log(-d*e + (d*x*e + c*e)*b/(b*x + a))/(b*x + a)^4 + 900*(d*x*e + c*e)^4*B*a^2*b^8*c^4*d^3*g^4*e^2*log(-d*e + (d*x*e + c*e)*b/(b*x + a))/(b*x + a)^4 - 1200*(d*x*e + c*e)^4*B*a^3*b^7*c^3*d^4*g^4*e^2*log(-d*e + (d*x*e + c*e)*b/(b*x + a))/(b*x + a)^4 + 900*(d*x*e + c*e)^4*B*a^4*b^6*c^2*d^5*g^4*e^2*log(-d*e + (d*x*e + c*e)*b/(b*x + a))/(b*x + a)^4 - 360*(d*x*e + c*e)^4*B*a^5*b^5*c*d^6*g^4*e^2*log(-d*e + (d*x*e + c*e)*b/(b*x + a))/(b*x + a)^4 + 60*(d*x*e + c*e)^4*B*a^6*b^4*d^7*g^4*e^2*log(-d*e + (d*x*e + c*e)*b/(b*x + a))/(b*x + a)^4 - 12*(d*x*e + c*e)^5*B*b^11*c^6*g^4*e*log(-d*e + (d*x*e + c*e)*b/(b*x + a))/(b*x + a)^5 + 72*(d*x*e + c*e)^5*B*a*b^10*c^5*d*g^4*e*log(-d*e + (d*x*e + c*e)*b/(b*x + a))/(b*x + a)^5 - 180*(d*x*e + c*e)^5*B*a^2*b^9*c^4*d^2*g^4*e*log(-d*e + (d*x*e + c*e)*b/(b*x + a))/(b*x + a)^5 + 240*(d*x*e + c*e)^5*B*a^3*b^8*c^3*d^3*g^4*e*log(-d*e + (d*x*e + c*e)*b/(b*x + a))/(b*x + a)^5 - 180*(d*x*e + c*e)^5*B*a^4*b^7*c^2*d^4*g^4*e*log(-d*e + (d*x*e + c*e)*b/(b*x + a))/(b*x + a)^5 + 72*(d*x*e + c*e)^5*B*a^5*b^6*c*d^5*g^4*e*log(-d*e + (d*x*e + c*e)*b/(b*x + a))/(b*x + a)^5 - 12*(d*x*e + c*e)^5*B*a^6*b^5*d^6*g^4*e*log(-d*e + (d*x*e + c*e)*b/(b*x + a))/(b*x + a)^5 + 60*(d*x*e + c*e)*B*b^7*c^6*d^4*g^4*e^5*log((d*x*e + c*e)/(b*x + a))/(b*x + a) - 360*(d*x*e + c*e)*B*a*b^6*c^5*d^5*g^4*e^5*log((d*x*e + c*e)/(b*x + a))/(b*x + a) + 900*(d*x*e + c*e)*B*a^2*b^5*c^4*d^6*g^4*e^5*log((d*x*e + c*e)/(b*x + a))/(b*x + a) - 1200*(d*x*e + c*e)*B*a^3*b^4*c^3*d^7*g^4*e^5*log((d*x*e + c*e)/(b*x + a))/(b*x + a) + 900*(d*x*e + c*e)*B*a^4*b^3*c^2*d^8*g^4*e^5*log((d*x*e + c*e)/(b*x + a))/(b*x + a) - 360*(d*x*e + c*e)*B*a^5*b^2*c*d^9*g^4*e^5*log((d*x*e + c*e)/(b*x + a))/(b*x + a) + 60*(d*x*e + c*e)*B*a^6*b*d^10*g^4*e^5*log((d*x*e + c*e)/(b*x + a))/(b*x + a) - 120*(d*x*e + c*e)^2*B*b^8*c^6*d^3*g^4*e^4*log((d*x*e + c*e)/(b*x + a))/(b*x + a)^2 + 720*(d*x*e + c*e)^2*B*a*b^7*c^5*d^4*g^4*e^4*log((d*x*e + c*e)/(b*x + a))/(b*x + a)^2 - 1800*(d*x*e + c*e)^2*B*a^2*b^6*c^4*d^5*g^4*e^4*log((d*x*e + c*e)/(b*x + a))/(b*x + a)^2 + 2400*(d*x*e + c*e)^2*B*a^3*b^5*c^3*d^6*g^4*e^4*log((d*x*e + c*e)/(b*x + a))/(b*x + a)^2 - 1800*(d*x*e + c*e)^2*B*a^4*b^4*c^2*d^7*g^4*e^4*log((d*x*e + c*e)/(b*x + a))/(b*x + a)^2 + 720*(d*x*e + c*e)^2*B*a^5*b^3*c*d^8*g^4*e^4*log((d*x*e + c*e)/(b*x + a))/(b*x + a)^2 - 120*(d*x*e + c*e)^2*B*a^6*b^2*d^9*g^4*e^4*log((d*x*e + c*e)/(b*x + a))/(b*x + a)^2 + 120*(d*x*e + c*e)^3*B*b^9*c^6*d^2*g^4*e^3*log((d*x*e + c*e)/(b*x + a))/(b*x + a)^3 - 720*(d*x*e + c*e)^3*B*a*b^8*c^5*d^3*g^4*e^3*log((d*x*e + c*e)/(b*x + a))/(b*x + a)^3 + 1800*(d*x*e + c*e)^3*B*a^2*b^7*c^4*d^4*g^4*e^3*log((d*x*e + c*e)/(b*x + a))/(b*x + a)^3 - 2400*(d*x*e + c*e)^3*B*a^3*b^6*c^3*d^5*g^4*e^3*log((d*x*e + c*e)/(b*x + a))/(b*x + a)^3 + 1800*(d*x*e + c*e)^3*B*a^4*b^5*c^2*d^6*g^4*e^3*log((d*x*e + c*e)/(b*x + a))/(b*x + a)^3 - 720*(d*x*e + c*e)^3*B*a^5*b^4*c*d^7*g^4

```
*e^3*log((d*x*e + c*e)/(b*x + a))/(b*x + a)^3 + 120*(d*x*e + c*e)^3*B*a^6*b
^3*d^8*g^4*e^3*log((d*x*e + c*e)/(b*x + a))/(b*x + a)^3 - 60*(d*x*e + c*e)^
4*B*b^10*c^6*d*g^4*e^2*log((d*x*e + c*e)/(b*x + a))/(b*x + a)^4 + 360*(d*x*
e + c*e)^4*B*a*b^9*c^5*d^2*g^4*e^2*log((d*x*e + c*e)/(b*x + a))/(b*x + a)^4
 - 900*(d*x*e + c*e)^4*B*a^2*b^8*c^4*d^3*g^4*e^2*log((d*x*e + c*e)/(b*x + a
))/(b*x + a)^4 + 1200*(d*x*e + c*e)^4*B*a^3*b^7*c^3*d^4*g^4*e^2*log((d*x*e
+ c*e)/(b*x + a))/(b*x + a)^4 - 900*(d*x*e + c*e)^4*B*a^4*b^6*c^2*d^5*g^4*e
^2*log((d*x*e + c*e)/(b*x + a))/(b*x + a)^4 + 360*(d*x*e + c*e)^4*B*a^5*b^5
*c*d^6*g^4*e^2*log((d*x*e + c*e)/(b*x + a))/(b*x + a)^4 - 60*(d*x*e + c*e)^
4*B*a^6*b^4*d^7*g^4*e^2*log((d*x*e + c*e)/(b*x + a))/(b*x + a)^4 + 12*(d*x*
e + c*e)^5*B*b^11*c^6*g^4*e*log((d*x*e + c*e)/(b*x + a))/(b*x + a)^5 - 72*(
d*x*e + c*e)^5*B*a*b^10*c^5*d*g^4*e*log((d*x*e + c*e)/(b*x + a))/(b*x + a)^
5 + 180*(d*x*e + c*e)^5*B*a^2*b^9*c^4*d^2*g^4*e*log((d*x*e + c*e)/(b*x + a)
)/(b*x + a)^5 - 240*(d*x*e + c*e)^5*B*a^3*b^8*c^3*d^3*g^4*e*log((d*x*e + c*
e)/(b*x + a))/(b*x + a)^5 + 180*(d*x*e + c*e)^5*B*a^4*b^7*c^2*d^4*g^4*e*log
((d*x*e + c*e)/(b*x + a))/(b*x + a)^5 - 72*(d*x*e + c*e)^5*B*a^5*b^6*c*d^5*
g^4*e*log((d*x*e + c*e)/(b*x + a))/(b*x + a)^5 + 12*(d*x*e + c*e)^5*B*a^6*b
^5*d^6*g^4*e*log((d*x*e + c*e)/(b*x + a))/(b*x + a)^5 + 12*A*b^6*c^6*d^5*g^
4*e^6 - 25*B*b^6*c^6*d^5*g^4*e^6 - 72*A*a*b^5*c^5*d^6*g^4*e^6 + 150*B*a*b^5
*c^5*d^6*g^4*e^6 + 180*A*a^2*b^4*c^4*d^7*g^4*e^6 - 375*B*a^2*b^4*c^4*d^7*g^
4*e^6 - 240*A*a^3*b^3*c^3*d^8*g^4*e^6 + 500*B*a^3*b^3*c^3*d^8*g^4*e^6 + 180
*A*a^4*b^2*c^2*d^9*g^4*e^6 - 375*B*a^4*b^2*c^2*d^9*g^4*e^6 - 72*A*a^5*b*c*d
^10*g^4*e^6 + 150*B*a^5*b*c*d^10*g^4*e^6 + 12*A*a^6*d^11*g^4*e^6 - 25*B*a^6
*d^11*g^4*e^6 + 77*(d*x*e + c*e)*B*b^7*c^6*d^4*g^4*e^5/(b*x + a) - 462*(d*x
*e + c*e)*B*a*b^6*c^5*d^5*g^4*e^5/(b*x + a) + 1155*(d*x*e + c*e)*B*a^2*b^5*
c^4*d^6*g^4*e^5/(b*x + a) - 1540*(d*x*e + c*e)*B*a^3*b^4*c^3*d^7*g^4*e^5/(b
*x + a) + 1155*(d*x*e + c*e)*B*a^4*b^3*c^2*d^8*g^4*e^5/(b*x + a) - 462*(d*x
*e + c*e)*B*a^5*b^2*c*d^9*g^4*e^5/(b*x + a) + 77*(d*x*e + c*e)*B*a^6*b*d^10
*g^4*e^5/(b*x + a) - 94*(d*x*e + c*e)^2*B*b^8*c^6*d^3*g^4*e^4/(b*x + a)^2 +
 564*(d*x*e + c*e)^2*B*a*b^7*c^5*d^4*g^4*e^4/(b*x + a)^2 - 1410*(d*x*e + c*
e)^2*B*a^2*b^6*c^4*d^5*g^4*e^4/(b*x + a)^2 + 1880*(d*x*e + c*e)^2*B*a^3*b^5
*c^3*d^6*g^4*e^4/(b*x + a)^2 - 1410*(d*x*e + c*e)^2*B*a^4*b^4*c^2*d^7*g^4*e
^4/(b*x + a)^2 + 564*(d*x*e + c*e)^2*B*a^5*b^3*c*d^8*g^4*e^4/(b*x + a)^2 -
94*(d*x*e + c*e)^2*B*a^6*b^2*d^9*g^4*e^4/(b*x + a)^2 + 54*(d*x*e + c*e)^3*B
*b^9*c^6*d^2*g^4*e^3/(b*x + a)^3 - 324*(d*x*e + c*e)^3*B*a*b^8*c^5*d^3*g^4*
e^3/(b*x + a)^3 + 810*(d*x*e + c*e)^3*B*a^2*b^7*c^4*d^4*g^4*e^3/(b*x + a)^3
 - 1080*(d*x*e + c*e)^3*B*a^3*b^6*c^3*d^5*g^4*e^3/(b*x + a)^3 + 810*(d*x*e
+ c*e)^3*B*a^4*b^5*c^2*d^6*g^4*e^3/(b*x + a)^3 - 324*(d*x*e + c*e)^3*B*a^5*
b^4*c*d^7*g^4*e^3/(b*x + a)^3 + 54*(d*x*e + c*e)^3*B*a^6*b^3*d^8*g^4*e^3/(b
*x + a)^3 - 12*(d*x*e + c*e)^4*B*b^10*c^6*d*g^4*e^2/(b*x + a)^4 + 72*(d*x*e
 + c*e)^4*B*a*b^9*c^5*d^2*g^4*e^2/(b*x + a)^4 - 180*(d*x*e + c*e)^4*B*a^2*b
^8*c^4*d^3*g^4*e^2/(b*x + a)^4 + 240*(d*x*e + c*e)^4*B*a^3*b^7*c^3*d^4*g^4*
e^2/(b*x + a)^4 - 180*(d*x*e + c*e)^4*B*a^4*b^6*c^2*d^5*g^4*e^2/(b*x + a)^4
 + 72*(d*x*e + c*e)^4*B*a^5*b^5*c*d^6*g^4*e^2/(b*x + a)^4 - 12*(d*x*e + c*e
)^4*B*a^6*b^4*d^7*g^4*e^2/(b*x + a)^4)*(b*c/((b*c*e - a*d*e)*(b*c - a*d)) -
```

```
 a*d/((b*c*e - a*d*e)*(b*c - a*d)))/(b*d^10*e^5 - 5*(d*x*e + c*e)*b^2*d^9*e
^4/(b*x + a) + 10*(d*x*e + c*e)^2*b^3*d^8*e^3/(b*x + a)^2 - 10*(d*x*e + c*e
)^3*b^4*d^7*e^2/(b*x + a)^3 + 5*(d*x*e + c*e)^4*b^5*d^6*e/(b*x + a)^4 - (d*
x*e + c*e)^5*b^6*d^5/(b*x + a)^5)
```

**maple [B]** time = 0.14, size = 2930, normalized size = 16.28

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*g*x+a*g)^4*(A+B*ln(e*(d*x+c)/(b*x+a))),x)
```

```
[Out] 1/5*b^4*e^5*B*g^4*ln(d*e/b-e*(a*d-b*c)/b/(b*x+a))/(-e/(b*x+a)*a*d+b*e/(b*x+
a)*c)^5*c^5-1/5/b*e^5*A*g^4/(-e/(b*x+a)*a*d+b*e/(b*x+a)*c)^5*a^5*d^5+1/5/b*
B*g^4*ln(b*(d*e/b-e*(a*d-b*c)/b/(b*x+a))-d*e)*a^5+1/4*e^4*B*g^4*d^3/(-e/(b*
x+a)*a*d+b*e/(b*x+a)*c)^4*a^4*c-1/3*e^3*B*g^4*d^2/(-e/(b*x+a)*a*d+b*e/(b*x+
a)*c)^3*a^4*c+1/2*e^2*B*g^4*d/(-e/(b*x+a)*a*d+b*e/(b*x+a)*c)^2*a^4*c+e^5*A*
g^4/(-e/(b*x+a)*a*d+b*e/(b*x+a)*c)^5*a^4*c*d^4-1/20/b*e^4*B*g^4*d^4/(-e/(b*
x+a)*a*d+b*e/(b*x+a)*c)^4*a^5-1/10/b*e^2*B*g^4*d^2/(-e/(b*x+a)*a*d+b*e/(b*x
+a)*c)^2*a^5+1/10*b^4*e^2*B*g^4/d^3/(-e/(b*x+a)*a*d+b*e/(b*x+a)*c)^2*c^5+1/
20*b^4*e^4*B*g^4/d/(-e/(b*x+a)*a*d+b*e/(b*x+a)*c)^4*c^5+b^3*B*g^4/d^4*ln(b*
(d*e/b-e*(a*d-b*c)/b/(b*x+a))-d*e)*a*c^4+2*b*B*g^4/d^2*ln(b*(d*e/b-e*(a*d-b
*c)/b/(b*x+a))-d*e)*a^3*c^2-2*b^2*B*g^4/d^3*ln(b*(d*e/b-e*(a*d-b*c)/b/(b*x+
a))-d*e)*a^2*c^3+1/15/b*e^3*B*g^4*d^3/(-e/(b*x+a)*a*d+b*e/(b*x+a)*c)^3*a^5-
1/15*b^4*e^3*B*g^4/d^2/(-e/(b*x+a)*a*d+b*e/(b*x+a)*c)^3*c^5-1/2*b*e^4*B*g^4
*d^2/(-e/(b*x+a)*a*d+b*e/(b*x+a)*c)^4*a^3*c^2-1/5/b*e^5*B*g^4*ln(d*e/b-e*(a
*d-b*c)/b/(b*x+a))/(-e/(b*x+a)*a*d+b*e/(b*x+a)*c)^5*a^5*d^5+e^5*B*g^4*ln(d*
e/b-e*(a*d-b*c)/b/(b*x+a))/(-e/(b*x+a)*a*d+b*e/(b*x+a)*c)^5*a^4*d^4*c+1/2*b
^2*e^4*B*g^4/(-e/(b*x+a)*a*d+b*e/(b*x+a)*c)^4*a^2*c^3*d+2/3*b*e^3*B*g^4/(-e
/(b*x+a)*a*d+b*e/(b*x+a)*c)^3*a^3*c^2*d+1/3*b^3*e^3*B*g^4/d/(-e/(b*x+a)*a*d
+b*e/(b*x+a)*c)^3*c^4*a-1/2*b^3*e^2*B*g^4/d^2/(-e/(b*x+a)*a*d+b*e/(b*x+a)*c
)^2*c^4*a+b^2*e^2*B*g^4/d/(-e/(b*x+a)*a*d+b*e/(b*x+a)*c)^2*a^2*c^3+b^3*e*B*
g^4/d^3/(-e/(b*x+a)*a*d+b*e/(b*x+a)*c)*c^4*a-2*b*e^5*A*g^4/(-e/(b*x+a)*a*d+
b*e/(b*x+a)*c)^5*a^3*c^2*d^3+2*b^2*e^5*A*g^4/(-e/(b*x+a)*a*d+b*e/(b*x+a)*c)
^5*a^2*c^3*d^2-b^3*e^5*A*g^4/(-e/(b*x+a)*a*d+b*e/(b*x+a)*c)^5*a*c^4*d+2*b*e
*B*g^4/d/(-e/(b*x+a)*a*d+b*e/(b*x+a)*c)*a^3*c^2-2*b^2*e*B*g^4/d^2/(-e/(b*x+
a)*a*d+b*e/(b*x+a)*c)*a^2*c^3-2/3*b^2*e^3*B*g^4/(-e/(b*x+a)*a*d+b*e/(b*x+a)
*c)^3*a^2*c^3-1/5*b^4*e*B*g^4/d^4/(-e/(b*x+a)*a*d+b*e/(b*x+a)*c)*c^5+1/5/b*
e*B*g^4/(-e/(b*x+a)*a*d+b*e/(b*x+a)*c)*a^5*d-1/4*b^3*e^4*B*g^4/(-e/(b*x+a)*
a*d+b*e/(b*x+a)*c)^4*c^4*a-b*e^2*B*g^4/(-e/(b*x+a)*a*d+b*e/(b*x+a)*c)^2*a^3
*c^2+42*b^3*e^5*B*g^4*ln(d*e/b-e*(a*d-b*c)/b/(b*x+a))*d/(-e/(b*x+a)*a*d+b*e
/(b*x+a)*c)^5*a^6*c^4/(b*x+a)^5+9*b*e^5*B*g^4*ln(d*e/b-e*(a*d-b*c)/b/(b*x+a
))*d^3/(-e/(b*x+a)*a*d+b*e/(b*x+a)*c)^5*a^8*c^2/(b*x+a)^5-2*b^8*e^5*B*g^4*l
n(d*e/b-e*(a*d-b*c)/b/(b*x+a))/d^4/(-e/(b*x+a)*a*d+b*e/(b*x+a)*c)^5*c^9/(b*
x+a)^5*a+9*b^7*e^5*B*g^4*ln(d*e/b-e*(a*d-b*c)/b/(b*x+a))/d^3/(-e/(b*x+a)*a*
```

```
d+b*e/(b*x+a)*c)^5*a^2*c^8/(b*x+a)^5+42*b^5*e^5*B*g^4*ln(d*e/b-e*(a*d-b*c)/
b/(b*x+a))/d/(-e/(b*x+a)*a*d+b*e/(b*x+a)*c)^5*a^4*c^6/(b*x+a)^5-24*b^6*e^5*
B*g^4*ln(d*e/b-e*(a*d-b*c)/b/(b*x+a))/d^2/(-e/(b*x+a)*a*d+b*e/(b*x+a)*c)^5*
a^3*c^7/(b*x+a)^5-24*b^2*e^5*B*g^4*ln(d*e/b-e*(a*d-b*c)/b/(b*x+a))*d^2/(-e/
(b*x+a)*a*d+b*e/(b*x+a)*c)^5*a^7*c^3/(b*x+a)^5-2*b*e^5*B*g^4*ln(d*e/b-e*(a*
d-b*c)/b/(b*x+a))*d^3/(-e/(b*x+a)*a*d+b*e/(b*x+a)*c)^5*a^3*c^2-2*e^5*B*g^4*
ln(d*e/b-e*(a*d-b*c)/b/(b*x+a))*d^4/(-e/(b*x+a)*a*d+b*e/(b*x+a)*c)^5*a^9*c/
(b*x+a)^5-b^3*e^5*B*g^4*ln(d*e/b-e*(a*d-b*c)/b/(b*x+a))/(-e/(b*x+a)*a*d+b*e
/(b*x+a)*c)^5*c^4*a*d+1/5/b*e^5*B*g^4*ln(d*e/b-e*(a*d-b*c)/b/(b*x+a))*d^5/(
-e/(b*x+a)*a*d+b*e/(b*x+a)*c)^5*a^10/(b*x+a)^5+1/5*b^9*e^5*B*g^4*ln(d*e/b-e
*(a*d-b*c)/b/(b*x+a))/d^5/(-e/(b*x+a)*a*d+b*e/(b*x+a)*c)^5*c^10/(b*x+a)^5+2
*b^2*e^5*B*g^4*ln(d*e/b-e*(a*d-b*c)/b/(b*x+a))*d^2/(-e/(b*x+a)*a*d+b*e/(b*x
+a)*c)^5*a^2*c^3-252/5*b^4*e^5*B*g^4*ln(d*e/b-e*(a*d-b*c)/b/(b*x+a))/(-e/(b
*x+a)*a*d+b*e/(b*x+a)*c)^5*a^5*c^5/(b*x+a)^5-e*B*g^4/(-e/(b*x+a)*a*d+b*e/(b
*x+a)*c)*a^4*c+1/5*b^4*e^5*A*g^4/(-e/(b*x+a)*a*d+b*e/(b*x+a)*c)^5*c^5-B*g^4
/d*ln(b*(d*e/b-e*(a*d-b*c)/b/(b*x+a))-d*e)*a^4*c-1/5*b^4*B*g^4/d^5*ln(b*(d*
e/b-e*(a*d-b*c)/b/(b*x+a))-d*e)*c^5
```

**maxima [B]** time = 1.26, size = 619, normalized size = 3.44

$$\frac{1}{5}Ab^4g^4x^5+Aab^3g^4x^4+2Aa^2b^2g^4x^3+2Aa^3bg^4x^2+\left(x\log\left(\frac{dex}{bx+a}+\frac{ce}{bx+a}\right)-\frac{a\log(bx+a)}{b}+\frac{c\log(dx+c)}{d}\right)Ba$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*g*x+a*g)^4*(A+B*log(e*(d*x+c)/(b*x+a))),x, algorithm="maxima")
```

```
[Out] 1/5*A*b^4*g^4*x^5 + A*a*b^3*g^4*x^4 + 2*A*a^2*b^2*g^4*x^3 + 2*A*a^3*b*g^4*x
^2 + (x*log(d*e*x/(b*x + a) + c*e/(b*x + a)) - a*log(b*x + a)/b + c*log(d*x
 + c)/d)*B*a^4*g^4 + 2*(x^2*log(d*e*x/(b*x + a) + c*e/(b*x + a)) + a^2*log(
b*x + a)/b^2 - c^2*log(d*x + c)/d^2 + (b*c - a*d)*x/(b*d))*B*a^3*b*g^4 + (2
*x^3*log(d*e*x/(b*x + a) + c*e/(b*x + a)) - 2*a^3*log(b*x + a)/b^3 + 2*c^3*
log(d*x + c)/d^3 + ((b^2*c*d - a*b*d^2)*x^2 - 2*(b^2*c^2 - a^2*d^2)*x)/(b^2
*d^2))*B*a^2*b^2*g^4 + 1/6*(6*x^4*log(d*e*x/(b*x + a) + c*e/(b*x + a)) + 6*
a^4*log(b*x + a)/b^4 - 6*c^4*log(d*x + c)/d^4 + (2*(b^3*c*d^2 - a*b^2*d^3)*
x^3 - 3*(b^3*c^2*d - a^2*b*d^3)*x^2 + 6*(b^3*c^3 - a^3*d^3)*x)/(b^3*d^3))*B
*a*b^3*g^4 + 1/60*(12*x^5*log(d*e*x/(b*x + a) + c*e/(b*x + a)) - 12*a^5*log
(b*x + a)/b^5 + 12*c^5*log(d*x + c)/d^5 + (3*(b^4*c*d^3 - a*b^3*d^4)*x^4 -
4*(b^4*c^2*d^2 - a^2*b^2*d^4)*x^3 + 6*(b^4*c^3*d - a^3*b*d^4)*x^2 - 12*(b^4
*c^4 - a^4*d^4)*x)/(b^4*d^4))*B*b^4*g^4 + A*a^4*g^4*x
```

**mupad [B]** time = 4.86, size = 1008, normalized size = 5.60

$$\ln\left(\frac{e\,(c+d\,x)}{a+b\,x}\right)\left(B\,a^4\,g^4\,x+2\,B\,a^3\,b\,g^4\,x^2+2\,B\,a^2\,b^2\,g^4\,x^3+B\,a\,b^3\,g^4\,x^4+\frac{B\,b^4\,g^4\,x^5}{5}\right)-x^3\left(\left(\frac{b^3\,g^4\,(25\,A\,a\,d+5\,A\,b\,c-B}{5\,d}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*g + b*g*x)^4*(A + B*log((e*(c + d*x))/(a + b*x))),x)
```

```
[Out] log((e*(c + d*x))/(a + b*x))*((B*b^4*g^4*x^5)/5 + B*a^4*g^4*x + 2*B*a^3*b*g
^4*x^2 + B*a*b^3*g^4*x^4 + 2*B*a^2*b^2*g^4*x^3) - x^3*((((b^3*g^4*(25*A*a*d
 + 5*A*b*c - B*a*d + B*b*c))/(5*d) - (A*b^3*g^4*(5*a*d + 5*b*c))/(5*d))*(5*
a*d + 5*b*c))/(15*b*d) - (a*b^2*g^4*(10*A*a*d + 5*A*b*c - B*a*d + B*b*c))/(
3*d) + (A*a*b^3*c*g^4)/(3*d)) + x^2*(((5*a*d + 5*b*c)*((((b^3*g^4*(25*A*a*d
 + 5*A*b*c - B*a*d + B*b*c))/(5*d) - (A*b^3*g^4*(5*a*d + 5*b*c))/(5*d))*(5*
a*d + 5*b*c))/(5*b*d) - (a*b^2*g^4*(10*A*a*d + 5*A*b*c - B*a*d + B*b*c))/d
+ (A*a*b^3*c*g^4)/d))/(10*b*d) + (a^2*b*g^4*(5*A*a*d + 5*A*b*c - B*a*d + B*
b*c))/d - (a*c*((b^3*g^4*(25*A*a*d + 5*A*b*c - B*a*d + B*b*c))/(5*d) - (A*b
^3*g^4*(5*a*d + 5*b*c))/(5*d)))/(2*b*d)) + x*((a^3*g^4*(5*A*a*d + 10*A*b*c
- 2*B*a*d + 2*B*b*c))/d - ((5*a*d + 5*b*c)*(((5*a*d + 5*b*c)*((((b^3*g^4*(2
5*A*a*d + 5*A*b*c - B*a*d + B*b*c))/(5*d) - (A*b^3*g^4*(5*a*d + 5*b*c))/(5*
d))*(5*a*d + 5*b*c))/(5*b*d) - (a*b^2*g^4*(10*A*a*d + 5*A*b*c - B*a*d + B*b
*c))/d + (A*a*b^3*c*g^4)/d))/(5*b*d) + (2*a^2*b*g^4*(5*A*a*d + 5*A*b*c - B*
a*d + B*b*c))/d - (a*c*((b^3*g^4*(25*A*a*d + 5*A*b*c - B*a*d + B*b*c))/(5*d
) - (A*b^3*g^4*(5*a*d + 5*b*c))/(5*d)))/(b*d)))/(5*b*d) + (a*c*((((b^3*g^4*
(25*A*a*d + 5*A*b*c - B*a*d + B*b*c))/(5*d) - (A*b^3*g^4*(5*a*d + 5*b*c))/(
5*d))*(5*a*d + 5*b*c))/(5*b*d) - (a*b^2*g^4*(10*A*a*d + 5*A*b*c - B*a*d + B
*b*c))/d + (A*a*b^3*c*g^4)/d))/(b*d)) + x^4*((b^3*g^4*(25*A*a*d + 5*A*b*c -
 B*a*d + B*b*c))/(20*d) - (A*b^3*g^4*(5*a*d + 5*b*c))/(20*d)) + (log(c + d*
x)*((B*b^4*c^5*g^4)/5 + B*a^4*c*d^4*g^4 - 2*B*a^3*b*c^2*d^3*g^4 + 2*B*a^2*b
^2*c^3*d^2*g^4 - B*a*b^3*c^4*d*g^4))/d^5 + (A*b^4*g^4*x^5)/5 - (B*a^5*g^4*l
og(a + b*x))/(5*b)
```

**sympy [B]** time = 6.52, size = 969, normalized size = 5.38

$$\frac{Ab^4g^4x^5}{5} - \frac{Ba^5g^4\log\left(x + \frac{\frac{Ba^6d^5g^4}{b}+5Ba^5cd^4g^4-10Ba^4bc^2d^3g^4+10Ba^3b^2c^3d^2g^4-5Ba^2b^3c^4dg^4+Bab^4c^5g^4}{Ba^5d^5g^4+5Ba^4bcd^4g^4-10Ba^3b^2c^2d^3g^4+10Ba^2b^3c^3d^2g^4-5Bab^4c^4dg^4+Bb^5c^5g^4}\right)}{5b} + \frac{Bcg^4\left(5a^4d^4 - 10a^3bcd^3\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*g*x+a*g)**4*(A+B*ln(e*(d*x+c)/(b*x+a))),x)
```

```
[Out] A*b**4*g**4*x**5/5 - B*a**5*g**4*log(x + (B*a**6*d**5*g**4/b + 5*B*a**5*c*d
**4*g**4 - 10*B*a**4*b*c**2*d**3*g**4 + 10*B*a**3*b**2*c**3*d**2*g**4 - 5*B
*a**2*b**3*c**4*d*g**4 + B*a*b**4*c**5*g**4)/(B*a**5*d**5*g**4 + 5*B*a**4*b
*c*d**4*g**4 - 10*B*a**3*b**2*c**2*d**3*g**4 + 10*B*a**2*b**3*c**3*d**2*g**
4 - 5*B*a*b**4*c**4*d*g**4 + B*b**5*c**5*g**4))/(5*b) + B*c*g**4*(5*a**4*d*
*4 - 10*a**3*b*c*d**3 + 10*a**2*b**2*c**2*d**2 - 5*a*b**3*c**3*d + b**4*c**
4)*log(x + (6*B*a**5*c*d**4*g**4 - 10*B*a**4*b*c**2*d**3*g**4 + 10*B*a**3*b
**2*c**3*d**2*g**4 - 5*B*a**2*b**3*c**4*d*g**4 + B*a*b**4*c**5*g**4 - B*a*c
*g**4*(5*a**4*d**4 - 10*a**3*b*c*d**3 + 10*a**2*b**2*c**2*d**2 - 5*a*b**3*c
**3*d + b**4*c**4) + B*b*c**2*g**4*(5*a**4*d**4 - 10*a**3*b*c*d**3 + 10*a**
2*b**2*c**2*d**2 - 5*a*b**3*c**3*d + b**4*c**4)/d)/(B*a**5*d**5*g**4 + 5*B*
a**4*b*c*d**4*g**4 - 10*B*a**3*b**2*c**2*d**3*g**4 + 10*B*a**2*b**3*c**3*d*
*2*g**4 - 5*B*a*b**4*c**4*d*g**4 + B*b**5*c**5*g**4))/(5*d**5) + x**4*(A*a*
b**3*g**4 - B*a*b**3*g**4/20 + B*b**4*c*g**4/(20*d)) + x**3*(2*A*a**2*b**2*
g**4 - 4*B*a**2*b**2*g**4/15 + B*a*b**3*c*g**4/(3*d) - B*b**4*c**2*g**4/(15
*d**2)) + x**2*(2*A*a**3*b*g**4 - 3*B*a**3*b*g**4/5 + B*a**2*b**2*c*g**4/d
- B*a*b**3*c**2*g**4/(2*d**2) + B*b**4*c**3*g**4/(10*d**3)) + x*(A*a**4*g**
4 - 4*B*a**4*g**4/5 + 2*B*a**3*b*c*g**4/d - 2*B*a**2*b**2*c**2*g**4/d**2 +
B*a*b**3*c**3*g**4/d**3 - B*b**4*c**4*g**4/(5*d**4)) + (B*a**4*g**4*x + 2*B
*a**3*b*g**4*x**2 + 2*B*a**2*b**2*g**4*x**3 + B*a*b**3*g**4*x**4 + B*b**4*g
**4*x**5/5)*log(e*(c + d*x)/(a + b*x))
```

# 3.174 $\int (ag+bgx)^3 \left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right) dx$

**Optimal**. Leaf size=149

$$\frac{g^3(a+bx)^4\left(B\log\left(\frac{e(c+dx)}{a+bx}\right)+A\right)}{4b}-\frac{Bg^3(bc-ad)^4\log(c+dx)}{4bd^4}+\frac{Bg^3x(bc-ad)^3}{4d^3}-\frac{Bg^3(a+bx)^2(bc-ad)^2}{8bd^2}+\frac{Bg^3(a\cdots}{}$$

```
[Out] 1/4*B*(-a*d+b*c)^3*g^3*x/d^3-1/8*B*(-a*d+b*c)^2*g^3*(b*x+a)^2/b/d^2+1/12*B*
(-a*d+b*c)*g^3*(b*x+a)^3/b/d-1/4*B*(-a*d+b*c)^4*g^3*ln(d*x+c)/b/d^4+1/4*g^3
*(b*x+a)^4*(A+B*ln(e*(d*x+c)/(b*x+a)))/b
```

**Rubi [A]** time = 0.10, antiderivative size = 149, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {2525, 12, 43}

$$\frac{g^3(a+bx)^4\left(B\log\left(\frac{e(c+dx)}{a+bx}\right)+A\right)}{4b}+\frac{Bg^3x(bc-ad)^3}{4d^3}-\frac{Bg^3(a+bx)^2(bc-ad)^2}{8bd^2}-\frac{Bg^3(bc-ad)^4\log(c+dx)}{4bd^4}+\frac{Bg^3(a\cdots}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a*g + b*g*x)^3*(A + B*Log[(e*(c + d*x))/(a + b*x)]),x]
```

```
[Out] (B*(b*c - a*d)^3*g^3*x)/(4*d^3) - (B*(b*c - a*d)^2*g^3*(a + b*x)^2)/(8*b*d^
2) + (B*(b*c - a*d)*g^3*(a + b*x)^3)/(12*b*d) - (B*(b*c - a*d)^4*g^3*Log[c
+ d*x])/(4*b*d^4) + (g^3*(a + b*x)^4*(A + B*Log[(e*(c + d*x))/(a + b*x)]))/
(4*b)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2525

```
Int[((a_.) + Log[(c_.)*(RFx_)^(p_.)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_))^(m_.
), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(a + b*Log[c*RFx^p])^n)/(e*(m + 1))
, x] - Dist[(b*n*p)/(e*(m + 1)), Int[SimplifyIntegrand[((d + e*x)^(m + 1)*(
```

```
a + b*Log[c*RFx^p])^(n - 1)*D[RFx, x])/RFx, x], x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && RationalFunctionQ[RFx, x] && IGtQ[n, 0] && (EqQ[n, 1] ||
IntegerQ[m]) && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int (ag+bgx)^3\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)dx &= \frac{g^3(a+bx)^4\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)}{4b}-\frac{B\int \frac{(-bc+ad)g^4(a+bx)^3}{c+dx}\,dx}{4bg}\\ &= \frac{g^3(a+bx)^4\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)}{4b}+\frac{\left(B(bc-ad)g^3\right)\int \frac{(a+bx)^3}{c+dx}\,dx}{4b}\\ &= \frac{g^3(a+bx)^4\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)}{4b}+\frac{\left(B(bc-ad)g^3\right)\int\left(\frac{b(bc-ad)^2}{d^3}-\frac{b(b}{}\right.}{}\\ &= \frac{B(bc-ad)^3g^3x}{4d^3}-\frac{B(bc-ad)^2g^3(a+bx)^2}{8bd^2}+\frac{B(bc-ad)g^3(a+bx)^3}{12bd}-\end{aligned}$$

**Mathematica [A]** time = 0.08, size = 120, normalized size = 0.81

$$\frac{g^3\left((a+bx)^4\left(B\log\left(\frac{e(c+dx)}{a+bx}\right)+A\right)+\frac{B(bc-ad)\left(3d^2(a+bx)^2(ad-bc)+6bdx(bc-ad)^2-6(bc-ad)^3\log(c+dx)+2d^3(a+bx)^3\right)}{6d^4}\right)}{4b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*g + b*g*x)^3*(A + B*Log[(e*(c + d*x))/(a + b*x)]),x]
```

```
[Out] (g^3*((B*(b*c - a*d)*(6*b*d*(b*c - a*d)^2*x + 3*d^2*(-(b*c) + a*d)*(a + b*x
)^2 + 2*d^3*(a + b*x)^3 - 6*(b*c - a*d)^3*Log[c + d*x]))/(6*d^4) + (a + b*x
)^4*(A + B*Log[(e*(c + d*x))/(a + b*x)])))/(4*b)
```

**fricas [B]** time = 1.12, size = 320, normalized size = 2.15

$$\frac{6\,Ab^4d^4g^3x^4-6\,Ba^4d^4g^3\log(bx+a)+2\left(Bb^4cd^3+(12\,A-B)ab^3d^4\right)g^3x^3-3\left(Bb^4c^2d^2-4\,Bab^3cd^3-3\,(4\,A-B\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*g*x+a*g)^3*(A+B*log(e*(d*x+c)/(b*x+a))),x, algorithm="fricas")
```

```
[Out] 1/24*(6*A*b^4*d^4*g^3*x^4 - 6*B*a^4*d^4*g^3*log(b*x + a) + 2*(B*b^4*c*d^3 +
 (12*A - B)*a*b^3*d^4)*g^3*x^3 - 3*(B*b^4*c^2*d^2 - 4*B*a*b^3*c*d^3 - 3*(4*
```

```
A - B)*a^2*b^2*d^4)*g^3*x^2 + 6*(B*b^4*c^3*d - 4*B*a*b^3*c^2*d^2 + 6*B*a^2*
b^2*c*d^3 + (4*A - 3*B)*a^3*b*d^4)*g^3*x - 6*(B*b^4*c^4 - 4*B*a*b^3*c^3*d +
 6*B*a^2*b^2*c^2*d^2 - 4*B*a^3*b*c*d^3)*g^3*log(d*x + c) + 6*(B*b^4*d^4*g^3
*x^4 + 4*B*a*b^3*d^4*g^3*x^3 + 6*B*a^2*b^2*d^4*g^3*x^2 + 4*B*a^3*b*d^4*g^3*
x)*log((d*e*x + c*e)/(b*x + a)))/(b*d^4)
```

**giac [B]** time = 1.19, size = 4137, normalized size = 27.77

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*g*x+a*g)^3*(A+B*log(e*(d*x+c)/(b*x+a))),x, algorithm="giac")
```

```
[Out] 1/24*(6*B*b^5*c^5*d^4*g^3*e^5*log(-d*e + (d*x*e + c*e)*b/(b*x + a)) - 30*B*
a*b^4*c^4*d^5*g^3*e^5*log(-d*e + (d*x*e + c*e)*b/(b*x + a)) + 60*B*a^2*b^3*
c^3*d^6*g^3*e^5*log(-d*e + (d*x*e + c*e)*b/(b*x + a)) - 60*B*a^3*b^2*c^2*d^
7*g^3*e^5*log(-d*e + (d*x*e + c*e)*b/(b*x + a)) + 30*B*a^4*b*c*d^8*g^3*e^5*
log(-d*e + (d*x*e + c*e)*b/(b*x + a)) - 6*B*a^5*d^9*g^3*e^5*log(-d*e + (d*x
*e + c*e)*b/(b*x + a)) - 24*(d*x*e + c*e)*B*b^6*c^5*d^3*g^3*e^4*log(-d*e +
(d*x*e + c*e)*b/(b*x + a))/(b*x + a) + 120*(d*x*e + c*e)*B*a*b^5*c^4*d^4*g^
3*e^4*log(-d*e + (d*x*e + c*e)*b/(b*x + a))/(b*x + a) - 240*(d*x*e + c*e)*B
*a^2*b^4*c^3*d^5*g^3*e^4*log(-d*e + (d*x*e + c*e)*b/(b*x + a))/(b*x + a) +
240*(d*x*e + c*e)*B*a^3*b^3*c^2*d^6*g^3*e^4*log(-d*e + (d*x*e + c*e)*b/(b*x
 + a))/(b*x + a) - 120*(d*x*e + c*e)*B*a^4*b^2*c*d^7*g^3*e^4*log(-d*e + (d*
x*e + c*e)*b/(b*x + a))/(b*x + a) + 24*(d*x*e + c*e)*B*a^5*b*d^8*g^3*e^4*lo
g(-d*e + (d*x*e + c*e)*b/(b*x + a))/(b*x + a) + 36*(d*x*e + c*e)^2*B*b^7*c^
5*d^2*g^3*e^3*log(-d*e + (d*x*e + c*e)*b/(b*x + a))/(b*x + a)^2 - 180*(d*x*
e + c*e)^2*B*a*b^6*c^4*d^3*g^3*e^3*log(-d*e + (d*x*e + c*e)*b/(b*x + a))/(b
*x + a)^2 + 360*(d*x*e + c*e)^2*B*a^2*b^5*c^3*d^4*g^3*e^3*log(-d*e + (d*x*e
 + c*e)*b/(b*x + a))/(b*x + a)^2 - 360*(d*x*e + c*e)^2*B*a^3*b^4*c^2*d^5*g^
3*e^3*log(-d*e + (d*x*e + c*e)*b/(b*x + a))/(b*x + a)^2 + 180*(d*x*e + c*e)
^2*B*a^4*b^3*c*d^6*g^3*e^3*log(-d*e + (d*x*e + c*e)*b/(b*x + a))/(b*x + a)^
2 - 36*(d*x*e + c*e)^2*B*a^5*b^2*d^7*g^3*e^3*log(-d*e + (d*x*e + c*e)*b/(b*
x + a))/(b*x + a)^2 - 24*(d*x*e + c*e)^3*B*b^8*c^5*d*g^3*e^2*log(-d*e + (d*
x*e + c*e)*b/(b*x + a))/(b*x + a)^3 + 120*(d*x*e + c*e)^3*B*a*b^7*c^4*d^2*g
^3*e^2*log(-d*e + (d*x*e + c*e)*b/(b*x + a))/(b*x + a)^3 - 240*(d*x*e + c*e
)^3*B*a^2*b^6*c^3*d^3*g^3*e^2*log(-d*e + (d*x*e + c*e)*b/(b*x + a))/(b*x +
a)^3 + 240*(d*x*e + c*e)^3*B*a^3*b^5*c^2*d^4*g^3*e^2*log(-d*e + (d*x*e + c*
e)*b/(b*x + a))/(b*x + a)^3 - 120*(d*x*e + c*e)^3*B*a^4*b^4*c*d^5*g^3*e^2*l
og(-d*e + (d*x*e + c*e)*b/(b*x + a))/(b*x + a)^3 + 24*(d*x*e + c*e)^3*B*a^5
*b^3*d^6*g^3*e^2*log(-d*e + (d*x*e + c*e)*b/(b*x + a))/(b*x + a)^3 + 6*(d*x
*e + c*e)^4*B*b^9*c^5*g^3*e*log(-d*e + (d*x*e + c*e)*b/(b*x + a))/(b*x + a)
^4 - 30*(d*x*e + c*e)^4*B*a*b^8*c^4*d*g^3*e*log(-d*e + (d*x*e + c*e)*b/(b*x
 + a))/(b*x + a)^4 + 60*(d*x*e + c*e)^4*B*a^2*b^7*c^3*d^2*g^3*e*log(-d*e +
(d*x*e + c*e)*b/(b*x + a))/(b*x + a)^4 - 60*(d*x*e + c*e)^4*B*a^3*b^6*c^2*d
```

^3*g^3*e*log(-d*e + (d*x*e + c*e)*b/(b*x + a))/(b*x + a)^4 + 30*(d*x*e + c*e)^4*B*a^4*b^5*c*d^4*g^3*e*log(-d*e + (d*x*e + c*e)*b/(b*x + a))/(b*x + a)^4 - 6*(d*x*e + c*e)^4*B*a^5*b^4*d^5*g^3*e*log(-d*e + (d*x*e + c*e)*b/(b*x + a))/(b*x + a)^4 + 24*(d*x*e + c*e)*B*b^6*c^5*d^3*g^3*e^4*log((d*x*e + c*e)/(b*x + a))/(b*x + a) - 120*(d*x*e + c*e)*B*a*b^5*c^4*d^4*g^3*e^4*log((d*x*e + c*e)/(b*x + a))/(b*x + a) + 240*(d*x*e + c*e)*B*a^2*b^4*c^3*d^5*g^3*e^4*log((d*x*e + c*e)/(b*x + a))/(b*x + a) - 240*(d*x*e + c*e)*B*a^3*b^3*c^2*d^6*g^3*e^4*log((d*x*e + c*e)/(b*x + a))/(b*x + a) + 120*(d*x*e + c*e)*B*a^4*b^2*c*d^7*g^3*e^4*log((d*x*e + c*e)/(b*x + a))/(b*x + a) - 24*(d*x*e + c*e)*B*a^5*b*d^8*g^3*e^4*log((d*x*e + c*e)/(b*x + a))/(b*x + a) - 36*(d*x*e + c*e)^2*B*b^7*c^5*d^2*g^3*e^3*log((d*x*e + c*e)/(b*x + a))/(b*x + a)^2 + 180*(d*x*e + c*e)^2*B*a*b^6*c^4*d^3*g^3*e^3*log((d*x*e + c*e)/(b*x + a))/(b*x + a)^2 - 360*(d*x*e + c*e)^2*B*a^2*b^5*c^3*d^4*g^3*e^3*log((d*x*e + c*e)/(b*x + a))/(b*x + a)^2 + 360*(d*x*e + c*e)^2*B*a^3*b^4*c^2*d^5*g^3*e^3*log((d*x*e + c*e)/(b*x + a))/(b*x + a)^2 - 180*(d*x*e + c*e)^2*B*a^4*b^3*c*d^6*g^3*e^3*log((d*x*e + c*e)/(b*x + a))/(b*x + a)^2 + 36*(d*x*e + c*e)^2*B*a^5*b^2*d^7*g^3*e^3*log((d*x*e + c*e)/(b*x + a))/(b*x + a)^2 + 24*(d*x*e + c*e)^3*B*b^8*c^5*d*g^3*e^2*log((d*x*e + c*e)/(b*x + a))/(b*x + a)^3 - 120*(d*x*e + c*e)^3*B*a*b^7*c^4*d^2*g^3*e^2*log((d*x*e + c*e)/(b*x + a))/(b*x + a)^3 + 240*(d*x*e + c*e)^3*B*a^2*b^6*c^3*d^3*g^3*e^2*log((d*x*e + c*e)/(b*x + a))/(b*x + a)^3 - 240*(d*x*e + c*e)^3*B*a^3*b^5*c^2*d^4*g^3*e^2*log((d*x*e + c*e)/(b*x + a))/(b*x + a)^3 + 120*(d*x*e + c*e)^3*B*a^4*b^4*c*d^5*g^3*e^2*log((d*x*e + c*e)/(b*x + a))/(b*x + a)^3 - 24*(d*x*e + c*e)^3*B*a^5*b^3*d^6*g^3*e^2*log((d*x*e + c*e)/(b*x + a))/(b*x + a)^3 - 6*(d*x*e + c*e)^4*B*b^9*c^5*g^3*e*log((d*x*e + c*e)/(b*x + a))/(b*x + a)^4 + 30*(d*x*e + c*e)^4*B*a*b^8*c^4*d*g^3*e*log((d*x*e + c*e)/(b*x + a))/(b*x + a)^4 - 60*(d*x*e + c*e)^4*B*a^2*b^7*c^3*d^2*g^3*e*log((d*x*e + c*e)/(b*x + a))/(b*x + a)^4 + 60*(d*x*e + c*e)^4*B*a^3*b^6*c^2*d^3*g^3*e*log((d*x*e + c*e)/(b*x + a))/(b*x + a)^4 - 30*(d*x*e + c*e)^4*B*a^4*b^5*c*d^4*g^3*e*log((d*x*e + c*e)/(b*x + a))/(b*x + a)^4 + 6*(d*x*e + c*e)^4*B*a^5*b^4*d^5*g^3*e*log((d*x*e + c*e)/(b*x + a))/(b*x + a)^4 + 6*A*b^5*c^5*d^4*g^3*e^5 - 11*B*b^5*c^5*d^4*g^3*e^5 - 30*A*a*b^4*c^4*d^5*g^3*e^5 + 55*B*a*b^4*c^4*d^5*g^3*e^5 + 60*A*a^2*b^3*c^3*d^6*g^3*e^5 - 110*B*a^2*b^3*c^3*d^6*g^3*e^5 - 60*A*a^3*b^2*c^2*d^7*g^3*e^5 + 110*B*a^3*b^2*c^2*d^7*g^3*e^5 + 30*A*a^4*b*c*d^8*g^3*e^5 - 55*B*a^4*b*c*d^8*g^3*e^5 - 6*A*a^5*d^9*g^3*e^5 + 11*B*a^5*d^9*g^3*e^5 + 26*(d*x*e + c*e)*B*b^6*c^5*d^3*g^3*e^4/(b*x + a) - 130*(d*x*e + c*e)*B*a*b^5*c^4*d^4*g^3*e^4/(b*x + a) + 260*(d*x*e + c*e)*B*a^2*b^4*c^3*d^5*g^3*e^4/(b*x + a) - 260*(d*x*e + c*e)*B*a^3*b^3*c^2*d^6*g^3*e^4/(b*x + a) + 130*(d*x*e + c*e)*B*a^4*b^2*c*d^7*g^3*e^4/(b*x + a) - 26*(d*x*e + c*e)*B*a^5*b*d^8*g^3*e^4/(b*x + a) - 21*(d*x*e + c*e)^2*B*b^7*c^5*d^2*g^3*e^3/(b*x + a)^2 + 105*(d*x*e + c*e)^2*B*a*b^6*c^4*d^3*g^3*e^3/(b*x + a)^2 - 210*(d*x*e + c*e)^2*B*a^2*b^5*c^3*d^4*g^3*e^3/(b*x + a)^2 + 210*(d*x*e + c*e)^2*B*a^3*b^4*c^2*d^5*g^3*e^3/(b*x + a)^2 - 105*(d*x*e + c*e)^2*B*a^4*b^3*c*d^6*g^3*e^3/(b*x + a)^2 + 21*(d*x*e + c*e)^2*B*a^5*b^2*d^7*g^3*e^3/(b*x + a)^2 + 6*(d*x*e + c*e)^3*B*b^8*c^5*d*g^3*e^2/(b*x + a)^3 - 30*(d*x*e + c*e)^3*B*a*b^7*c^4*d^2*g^3*e^2/(b*x +

```
 a)^3 + 60*(d*x*e + c*e)^3*B*a^2*b^6*c^3*d^3*g^3*e^2/(b*x + a)^3 - 60*(d*x*
e + c*e)^3*B*a^3*b^5*c^2*d^4*g^3*e^2/(b*x + a)^3 + 30*(d*x*e + c*e)^3*B*a^4
*b^4*c*d^5*g^3*e^2/(b*x + a)^3 - 6*(d*x*e + c*e)^3*B*a^5*b^3*d^6*g^3*e^2/(b
*x + a)^3)*(b*c/((b*c*e - a*d*e)*(b*c - a*d)) - a*d/((b*c*e - a*d*e)*(b*c -
 a*d)))/(b*d^8*e^4 - 4*(d*x*e + c*e)*b^2*d^7*e^3/(b*x + a) + 6*(d*x*e + c*e
)^2*b^3*d^6*e^2/(b*x + a)^2 - 4*(d*x*e + c*e)^3*b^4*d^5*e/(b*x + a)^3 + (d*
x*e + c*e)^4*b^5*d^4/(b*x + a)^4)
```

**maple [B]** time = 0.14, size = 2191, normalized size = 14.70

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*g*x+a*g)^3*(A+B*ln(e*(d*x+c)/(b*x+a))),x)
```

```
[Out] 1/2*b^2*e^2*B*g^3/d/(-e/(b*x+a)*a*d+b*e/(b*x+a)*c)^2*a*c^3-e^4*B*g^3*ln(d*e
/b-e*(a*d-b*c)/b/(b*x+a))/(-e/(b*x+a)*a*d+b*e/(b*x+a)*c)^4*a^3*d^3*c+1/4/b*
e^4*B*g^3*ln(d*e/b-e*(a*d-b*c)/b/(b*x+a))/(-e/(b*x+a)*a*d+b*e/(b*x+a)*c)^4*
a^4*d^4+3/2*b*e^4*B*g^3*ln(d*e/b-e*(a*d-b*c)/b/(b*x+a))/(-e/(b*x+a)*a*d+b*e
/(b*x+a)*c)^4*a^2*c^2*d^2-35/2*b^3*e^4*B*g^3*ln(d*e/b-e*(a*d-b*c)/b/(b*x+a)
)/(-e/(b*x+a)*a*d+b*e/(b*x+a)*c)^4*a^4/(b*x+a)^4*c^4-b^2*e^4*B*g^3*ln(d*e/b
-e*(a*d-b*c)/b/(b*x+a))/(-e/(b*x+a)*a*d+b*e/(b*x+a)*c)^4*c^3*a*d-1/4*b^7*e^
4*B*g^3*ln(d*e/b-e*(a*d-b*c)/b/(b*x+a))/d^4/(-e/(b*x+a)*a*d+b*e/(b*x+a)*c)^
4*c^8/(b*x+a)^4+1/4/b*B*g^3*ln(b*(d*e/b-e*(a*d-b*c)/b/(b*x+a))-d*e)*a^4-e*B
*g^3/(-e/(b*x+a)*a*d+b*e/(b*x+a)*c)*a^3*c-B*g^3/d*ln(b*(d*e/b-e*(a*d-b*c)/b
/(b*x+a))-d*e)*a^3*c+1/4*b^3*B*g^3/d^4*ln(b*(d*e/b-e*(a*d-b*c)/b/(b*x+a))-d
*e)*c^4+1/4*b^3*e^4*A*g^3/(-e/(b*x+a)*a*d+b*e/(b*x+a)*c)^4*c^4-b^2*e^4*A*g^
3/(-e/(b*x+a)*a*d+b*e/(b*x+a)*c)^4*c^3*a*d+1/2*b*e^3*B*g^3*d/(-e/(b*x+a)*a*
d+b*e/(b*x+a)*c)^3*a^2*c^2+3/2*b*e^4*A*g^3/(-e/(b*x+a)*a*d+b*e/(b*x+a)*c)^4
*a^2*c^2*d^2+3/2*b*e*B*g^3/d/(-e/(b*x+a)*a*d+b*e/(b*x+a)*c)*a^2*c^2-b^2*e*B
*g^3/d^2/(-e/(b*x+a)*a*d+b*e/(b*x+a)*c)*c^3*a+1/4*b^3*e*B*g^3/d^3/(-e/(b*x+
a)*a*d+b*e/(b*x+a)*c)*c^4+1/4*b^3*e^4*B*g^3*ln(d*e/b-e*(a*d-b*c)/b/(b*x+a))
/(-e/(b*x+a)*a*d+b*e/(b*x+a)*c)^4*c^4+3/2*b*B*g^3/d^2*ln(b*(d*e/b-e*(a*d-b*
c)/b/(b*x+a))-d*e)*a^2*c^2-1/8*b^3*e^2*B*g^3/d^2/(-e/(b*x+a)*a*d+b*e/(b*x+a
)*c)^2*c^4-3/4*b*e^2*B*g^3/(-e/(b*x+a)*a*d+b*e/(b*x+a)*c)^2*a^2*c^2+2*b^6*e
^4*B*g^3*ln(d*e/b-e*(a*d-b*c)/b/(b*x+a))/d^3/(-e/(b*x+a)*a*d+b*e/(b*x+a)*c)
^4*c^7/(b*x+a)^4*a-7*b^5*e^4*B*g^3*ln(d*e/b-e*(a*d-b*c)/b/(b*x+a))/d^2/(-e/
(b*x+a)*a*d+b*e/(b*x+a)*c)^4*c^6/(b*x+a)^4*a^2+14*b^4*e^4*B*g^3*ln(d*e/b-e*
(a*d-b*c)/b/(b*x+a))/d/(-e/(b*x+a)*a*d+b*e/(b*x+a)*c)^4*a^3/(b*x+a)^4*c^5-7
*b*e^4*B*g^3*ln(d*e/b-e*(a*d-b*c)/b/(b*x+a))*d^2/(-e/(b*x+a)*a*d+b*e/(b*x+a
)*c)^4*a^6/(b*x+a)^4*c^2+14*b^2*e^4*B*g^3*ln(d*e/b-e*(a*d-b*c)/b/(b*x+a))*d
/(-e/(b*x+a)*a*d+b*e/(b*x+a)*c)^4*a^5/(b*x+a)^4*c^3-1/3*e^3*B*g^3*d^2/(-e/(
b*x+a)*a*d+b*e/(b*x+a)*c)^3*a^3*c-e^4*A*g^3/(-e/(b*x+a)*a*d+b*e/(b*x+a)*c)^
4*a^3*d^3*c-1/4/b*e^4*B*g^3*ln(d*e/b-e*(a*d-b*c)/b/(b*x+a))*d^4/(-e/(b*x+a)
*a*d+b*e/(b*x+a)*c)^4*a^8/(b*x+a)^4+2*e^4*B*g^3*ln(d*e/b-e*(a*d-b*c)/b/(b*x
```

```
+a))*d^3/(-e/(b*x+a)*a*d+b*e/(b*x+a)*c)^4*a^7/(b*x+a)^4*c-1/3*b^2*e^3*B*g^3
/(-e/(b*x+a)*a*d+b*e/(b*x+a)*c)^3*a*c^3-1/8/b*e^2*B*g^3*d^2/(-e/(b*x+a)*a*d
+b*e/(b*x+a)*c)^2*a^4+1/12/b*e^3*B*g^3*d^3/(-e/(b*x+a)*a*d+b*e/(b*x+a)*c)^3
*a^4+1/2*e^2*B*g^3/(-e/(b*x+a)*a*d+b*e/(b*x+a)*c)^2*a^3*c*d-b^2*B*g^3/d^3*l
n(b*(d*e/b-e*(a*d-b*c)/b/(b*x+a))-d*e)*a*c^3+1/4/b*e*B*g^3/(-e/(b*x+a)*a*d+
b*e/(b*x+a)*c)*a^4*d+1/12*b^3*e^3*B*g^3/d/(-e/(b*x+a)*a*d+b*e/(b*x+a)*c)^3*
c^4+1/4/b*e^4*A*g^3/(-e/(b*x+a)*a*d+b*e/(b*x+a)*c)^4*a^4*d^4
```

**maxima [B]** time = 1.30, size = 436, normalized size = 2.93

$$\frac{1}{4}Ab^3g^3x^4+Aab^2g^3x^3+\frac{3}{2}Aa^2bg^3x^2+\left(x\log\left(\frac{dex}{bx+a}+\frac{ce}{bx+a}\right)-\frac{a\log(bx+a)}{b}+\frac{c\log(dx+c)}{d}\right)Ba^3g^3+\frac{3}{2}\left(x^2\log\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*g*x+a*g)^3*(A+B*log(e*(d*x+c)/(b*x+a))),x, algorithm="maxima")
```

```
[Out] 1/4*A*b^3*g^3*x^4 + A*a*b^2*g^3*x^3 + 3/2*A*a^2*b*g^3*x^2 + (x*log(d*e*x/(b
*x + a) + c*e/(b*x + a)) - a*log(b*x + a)/b + c*log(d*x + c)/d)*B*a^3*g^3 +
 3/2*(x^2*log(d*e*x/(b*x + a) + c*e/(b*x + a)) + a^2*log(b*x + a)/b^2 - c^2
*log(d*x + c)/d^2 + (b*c - a*d)*x/(b*d))*B*a^2*b*g^3 + 1/2*(2*x^3*log(d*e*x
/(b*x + a) + c*e/(b*x + a)) - 2*a^3*log(b*x + a)/b^3 + 2*c^3*log(d*x + c)/d
^3 + ((b^2*c*d - a*b*d^2)*x^2 - 2*(b^2*c^2 - a^2*d^2)*x)/(b^2*d^2))*B*a*b^2
*g^3 + 1/24*(6*x^4*log(d*e*x/(b*x + a) + c*e/(b*x + a)) + 6*a^4*log(b*x + a
)/b^4 - 6*c^4*log(d*x + c)/d^4 + (2*(b^3*c*d^2 - a*b^2*d^3)*x^3 - 3*(b^3*c^
2*d - a^2*b*d^3)*x^2 + 6*(b^3*c^3 - a^3*d^3)*x)/(b^3*d^3))*B*b^3*g^3 + A*a^
3*g^3*x
```

**mupad [B]** time = 4.69, size = 566, normalized size = 3.80

$$x\left(\frac{(4\,a\,d+4\,b\,c)\left(\frac{\left(\frac{b^2\,g^3\,(16\,A\,a\,d+4\,A\,b\,c-B\,a\,d+B\,b\,c)}{4\,d}-\frac{A\,b^2\,g^3\,(4\,a\,d+4\,b\,c)}{4\,d}\right)(4\,a\,d+4\,b\,c)}{4\,b\,d}-\frac{a\,b\,g^3\,(6\,A\,a\,d+4\,A\,b\,c-B\,a\,d+B\,b\,c)}{d}+\frac{A\,a\,b^2\,c\,g^3}{d}\right)}{4\,b\,d}+\frac{a^2\,g\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*g + b*g*x)^3*(A + B*log((e*(c + d*x))/(a + b*x))),x)
```

```
[Out] x*(((4*a*d + 4*b*c)*((((b^2*g^3*(16*A*a*d + 4*A*b*c - B*a*d + B*b*c))/(4*d)
 - (A*b^2*g^3*(4*a*d + 4*b*c))/(4*d))*(4*a*d + 4*b*c))/(4*b*d) - (a*b*g^3*(
6*A*a*d + 4*A*b*c - B*a*d + B*b*c))/d + (A*a*b^2*c*g^3)/d))/(4*b*d) + (a^2*
g^3*(8*A*a*d + 12*A*b*c - 3*B*a*d + 3*B*b*c))/(2*d) - (a*c*((b^2*g^3*(16*A*
```

```
a*d + 4*A*b*c - B*a*d + B*b*c))/(4*d) - (A*b^2*g^3*(4*a*d + 4*b*c))/(4*d)))
/(b*d)) - x^2*((((b^2*g^3*(16*A*a*d + 4*A*b*c - B*a*d + B*b*c))/(4*d) - (A*
b^2*g^3*(4*a*d + 4*b*c))/(4*d))*(4*a*d + 4*b*c))/(8*b*d) - (a*b*g^3*(6*A*a*
d + 4*A*b*c - B*a*d + B*b*c))/(2*d) + (A*a*b^2*c*g^3)/(2*d)) + log((e*(c +
d*x))/(a + b*x))*((B*b^3*g^3*x^4)/4 + B*a^3*g^3*x + (3*B*a^2*b*g^3*x^2)/2 +
 B*a*b^2*g^3*x^3) + x^3*((b^2*g^3*(16*A*a*d + 4*A*b*c - B*a*d + B*b*c))/(12
*d) - (A*b^2*g^3*(4*a*d + 4*b*c))/(12*d)) - (log(c + d*x)*(B*b^3*c^4*g^3 -
4*B*a^3*c*d^3*g^3 + 6*B*a^2*b*c^2*d^2*g^3 - 4*B*a*b^2*c^3*d*g^3))/(4*d^4) +
 (A*b^3*g^3*x^4)/4 - (B*a^4*g^3*log(a + b*x))/(4*b)
```

**sympy [B]** time = 4.35, size = 706, normalized size = 4.74

$$\frac{Ab^3g^3x^4}{4} - \frac{Ba^4g^3\log\left(x + \frac{\frac{Ba^5d^4g^3}{b} + 4Ba^4cd^3g^3 - 6Ba^3bc^2d^2g^3 + 4Ba^2b^2c^3dg^3 - Bab^3c^4g^3}{Ba^4d^4g^3 + 4Ba^3bcd^3g^3 - 6Ba^2b^2c^2d^2g^3 + 4Bab^3c^3dg^3 - Bb^4c^4g^3}\right)}{4b} + \frac{Bcg^3\left(2ad - bc\right)\left(2a^2d^2 - 2abcd + b^2c\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*g*x+a*g)**3*(A+B*ln(e*(d*x+c)/(b*x+a))),x)
```

```
[Out] A*b**3*g**3*x**4/4 - B*a**4*g**3*log(x + (B*a**5*d**4*g**3/b + 4*B*a**4*c*d
**3*g**3 - 6*B*a**3*b*c**2*d**2*g**3 + 4*B*a**2*b**2*c**3*d*g**3 - B*a*b**3
*c**4*g**3)/(B*a**4*d**4*g**3 + 4*B*a**3*b*c*d**3*g**3 - 6*B*a**2*b**2*c**2
*d**2*g**3 + 4*B*a*b**3*c**3*d*g**3 - B*b**4*c**4*g**3))/(4*b) + B*c*g**3*(
2*a*d - b*c)*(2*a**2*d**2 - 2*a*b*c*d + b**2*c**2)*log(x + (5*B*a**4*c*d**3
*g**3 - 6*B*a**3*b*c**2*d**2*g**3 + 4*B*a**2*b**2*c**3*d*g**3 - B*a*b**3*c*
*4*g**3 - B*a*c*g**3*(2*a*d - b*c)*(2*a**2*d**2 - 2*a*b*c*d + b**2*c**2) +
B*b*c**2*g**3*(2*a*d - b*c)*(2*a**2*d**2 - 2*a*b*c*d + b**2*c**2)/d)/(B*a**
4*d**4*g**3 + 4*B*a**3*b*c*d**3*g**3 - 6*B*a**2*b**2*c**2*d**2*g**3 + 4*B*a
*b**3*c**3*d*g**3 - B*b**4*c**4*g**3))/(4*d**4) + x**3*(A*a*b**2*g**3 - B*a
*b**2*g**3/12 + B*b**3*c*g**3/(12*d)) + x**2*(3*A*a**2*b*g**3/2 - 3*B*a**2*
b*g**3/8 + B*a*b**2*c*g**3/(2*d) - B*b**3*c**2*g**3/(8*d**2)) + x*(A*a**3*g
**3 - 3*B*a**3*g**3/4 + 3*B*a**2*b*c*g**3/(2*d) - B*a*b**2*c**2*g**3/d**2 +
 B*b**3*c**3*g**3/(4*d**3)) + (B*a**3*g**3*x + 3*B*a**2*b*g**3*x**2/2 + B*a
*b**2*g**3*x**3 + B*b**3*g**3*x**4/4)*log(e*(c + d*x)/(a + b*x))
```

# 3.175 $\int (ag+bgx)^2 \left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)\,dx$

**Optimal**. Leaf size=118

$$\frac{g^2(a+bx)^3\left(B\log\left(\frac{e(c+dx)}{a+bx}\right)+A\right)}{3b}+\frac{Bg^2(bc-ad)^3\log(c+dx)}{3bd^3}-\frac{Bg^2x(bc-ad)^2}{3d^2}+\frac{Bg^2(a+bx)^2(bc-ad)}{6bd}$$

```
[Out] -1/3*B*(-a*d+b*c)^2*g^2*x/d^2+1/6*B*(-a*d+b*c)*g^2*(b*x+a)^2/b/d+1/3*B*(-a*
d+b*c)^3*g^2*ln(d*x+c)/b/d^3+1/3*g^2*(b*x+a)^3*(A+B*ln(e*(d*x+c)/(b*x+a)))/
b
```

**Rubi [A]** time = 0.08, antiderivative size = 118, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {2525, 12, 43}

$$\frac{g^2(a+bx)^3\left(B\log\left(\frac{e(c+dx)}{a+bx}\right)+A\right)}{3b}-\frac{Bg^2x(bc-ad)^2}{3d^2}+\frac{Bg^2(bc-ad)^3\log(c+dx)}{3bd^3}+\frac{Bg^2(a+bx)^2(bc-ad)}{6bd}$$

Antiderivative was successfully verified.

```
[In] Int[(a*g + b*g*x)^2*(A + B*Log[(e*(c + d*x))/(a + b*x)]),x]
```

```
[Out] -(B*(b*c - a*d)^2*g^2*x)/(3*d^2) + (B*(b*c - a*d)*g^2*(a + b*x)^2)/(6*b*d)
+ (B*(b*c - a*d)^3*g^2*Log[c + d*x])/(3*b*d^3) + (g^2*(a + b*x)^3*(A + B*Lo
g[(e*(c + d*x))/(a + b*x)]))/(3*b)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2525

```
Int[((a_.) + Log[(c_.)*(RFx_)^(p_.)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_))^(m_.
), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(a + b*Log[c*RFx^p])^n)/(e*(m + 1))
, x] - Dist[(b*n*p)/(e*(m + 1)), Int[SimplifyIntegrand[((d + e*x)^(m + 1)*(
a + b*Log[c*RFx^p])^(n - 1)*D[RFx, x])/RFx, x], x], x] /; FreeQ[{a, b, c, d
```

```
, e, m, p}, x] && RationalFunctionQ[RFx, x] && IGtQ[n, 0] && (EqQ[n, 1] ||
IntegerQ[m]) && NeQ[m, -1]
```

Rubi steps

$$\int (ag+bgx)^2\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)dx=\frac{g^2(a+bx)^3\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)}{3b}-\frac{B\int\frac{(-bc+ad)g^3(a+bx)^2}{c+dx}dx}{3bg}$$
$$=\frac{g^2(a+bx)^3\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)}{3b}+\frac{\left(B(bc-ad)g^2\right)\int\frac{(a+bx)^2}{c+dx}dx}{3b}$$
$$=\frac{g^2(a+bx)^3\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)}{3b}+\frac{\left(B(bc-ad)g^2\right)\int\left(-\frac{b(bc-ad)}{d^2}+\right.}{3b}$$
$$=-\frac{B(bc-ad)^2g^2x}{3d^2}+\frac{B(bc-ad)g^2(a+bx)^2}{6bd}+\frac{B(bc-ad)^3g^2\log(c+}{3bd^3}$$

**Mathematica [A]** time = 0.05, size = 99, normalized size = 0.84

$$\frac{g^2\left(\frac{B(bc-ad)\left(d\left(a^2d+4abdx+b^2x(dx-2c)\right)+2(bc-ad)^2\log(c+dx)\right)}{2d^3}+(a+bx)^3\left(B\log\left(\frac{e(c+dx)}{a+bx}\right)+A\right)\right)}{3b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*g + b*g*x)^2*(A + B*Log[(e*(c + d*x))/(a + b*x)]),x]
```

```
[Out] (g^2*((B*(b*c - a*d)*(d*(a^2*d + 4*a*b*d*x + b^2*x*(-2*c + d*x)) + 2*(b*c -
 a*d)^2*Log[c + d*x]))/(2*d^3) + (a + b*x)^3*(A + B*Log[(e*(c + d*x))/(a +
b*x)])))/(3*b)
```

**fricas [B]** time = 0.83, size = 223, normalized size = 1.89

$$\frac{2\,Ab^3d^3g^2x^3-2\,Ba^3d^3g^2\log(bx+a)+\left(Bb^3cd^2+(6\,A-B)ab^2d^3\right)g^2x^2-2\left(Bb^3c^2d-3\,Bab^2cd^2-(3\,A-2\,B)a\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*g*x+a*g)^2*(A+B*log(e*(d*x+c)/(b*x+a))),x, algorithm="fricas")
```

```
[Out] 1/6*(2*A*b^3*d^3*g^2*x^3 - 2*B*a^3*d^3*g^2*log(b*x + a) + (B*b^3*c*d^2 + (6
*A - B)*a*b^2*d^3)*g^2*x^2 - 2*(B*b^3*c^2*d - 3*B*a*b^2*c*d^2 - (3*A - 2*B)
```

```
*a^2*b*d^3)*g^2*x + 2*(B*b^3*c^3 - 3*B*a*b^2*c^2*d + 3*B*a^2*b*c*d^2)*g^2*l
og(d*x + c) + 2*(B*b^3*d^3*g^2*x^3 + 3*B*a*b^2*d^3*g^2*x^2 + 3*B*a^2*b*d^3*
g^2*x)*log((d*e*x + c*e)/(b*x + a)))/(b*d^3)
```

**giac [B]** time = 0.89, size = 2640, normalized size = 22.37

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*g*x+a*g)^2*(A+B*log(e*(d*x+c)/(b*x+a))),x, algorithm="giac")
```

```
[Out] -1/6*(2*B*b^4*c^4*d^3*g^2*e^4*log(-d*e + (d*x*e + c*e)*b/(b*x + a)) - 8*B*a
*b^3*c^3*d^4*g^2*e^4*log(-d*e + (d*x*e + c*e)*b/(b*x + a)) + 12*B*a^2*b^2*c
^2*d^5*g^2*e^4*log(-d*e + (d*x*e + c*e)*b/(b*x + a)) - 8*B*a^3*b*c*d^6*g^2*
e^4*log(-d*e + (d*x*e + c*e)*b/(b*x + a)) + 2*B*a^4*d^7*g^2*e^4*log(-d*e +
(d*x*e + c*e)*b/(b*x + a)) - 6*(d*x*e + c*e)*B*b^5*c^4*d^2*g^2*e^3*log(-d*e
 + (d*x*e + c*e)*b/(b*x + a))/(b*x + a) + 24*(d*x*e + c*e)*B*a*b^4*c^3*d^3*
g^2*e^3*log(-d*e + (d*x*e + c*e)*b/(b*x + a))/(b*x + a) - 36*(d*x*e + c*e)*
B*a^2*b^3*c^2*d^4*g^2*e^3*log(-d*e + (d*x*e + c*e)*b/(b*x + a))/(b*x + a) +
 24*(d*x*e + c*e)*B*a^3*b^2*c*d^5*g^2*e^3*log(-d*e + (d*x*e + c*e)*b/(b*x +
 a))/(b*x + a) - 6*(d*x*e + c*e)*B*a^4*b*d^6*g^2*e^3*log(-d*e + (d*x*e + c*
e)*b/(b*x + a))/(b*x + a) + 6*(d*x*e + c*e)^2*B*b^6*c^4*d*g^2*e^2*log(-d*e
+ (d*x*e + c*e)*b/(b*x + a))/(b*x + a)^2 - 24*(d*x*e + c*e)^2*B*a*b^5*c^3*d
^2*g^2*e^2*log(-d*e + (d*x*e + c*e)*b/(b*x + a))/(b*x + a)^2 + 36*(d*x*e +
c*e)^2*B*a^2*b^4*c^2*d^3*g^2*e^2*log(-d*e + (d*x*e + c*e)*b/(b*x + a))/(b*x
 + a)^2 - 24*(d*x*e + c*e)^2*B*a^3*b^3*c*d^4*g^2*e^2*log(-d*e + (d*x*e + c*
e)*b/(b*x + a))/(b*x + a)^2 + 6*(d*x*e + c*e)^2*B*a^4*b^2*d^5*g^2*e^2*log(-
d*e + (d*x*e + c*e)*b/(b*x + a))/(b*x + a)^2 - 2*(d*x*e + c*e)^3*B*b^7*c^4*
g^2*e*log(-d*e + (d*x*e + c*e)*b/(b*x + a))/(b*x + a)^3 + 8*(d*x*e + c*e)^3
*B*a*b^6*c^3*d*g^2*e*log(-d*e + (d*x*e + c*e)*b/(b*x + a))/(b*x + a)^3 - 12
*(d*x*e + c*e)^3*B*a^2*b^5*c^2*d^2*g^2*e*log(-d*e + (d*x*e + c*e)*b/(b*x +
a))/(b*x + a)^3 + 8*(d*x*e + c*e)^3*B*a^3*b^4*c*d^3*g^2*e*log(-d*e + (d*x*e
 + c*e)*b/(b*x + a))/(b*x + a)^3 - 2*(d*x*e + c*e)^3*B*a^4*b^3*d^4*g^2*e*lo
g(-d*e + (d*x*e + c*e)*b/(b*x + a))/(b*x + a)^3 + 6*(d*x*e + c*e)*B*b^5*c^4
*d^2*g^2*e^3*log((d*x*e + c*e)/(b*x + a))/(b*x + a) - 24*(d*x*e + c*e)*B*a*
b^4*c^3*d^3*g^2*e^3*log((d*x*e + c*e)/(b*x + a))/(b*x + a) + 36*(d*x*e + c*
e)*B*a^2*b^3*c^2*d^4*g^2*e^3*log((d*x*e + c*e)/(b*x + a))/(b*x + a) - 24*(d
*x*e + c*e)*B*a^3*b^2*c*d^5*g^2*e^3*log((d*x*e + c*e)/(b*x + a))/(b*x + a)
+ 6*(d*x*e + c*e)*B*a^4*b*d^6*g^2*e^3*log((d*x*e + c*e)/(b*x + a))/(b*x + a
) - 6*(d*x*e + c*e)^2*B*b^6*c^4*d*g^2*e^2*log((d*x*e + c*e)/(b*x + a))/(b*x
 + a)^2 + 24*(d*x*e + c*e)^2*B*a*b^5*c^3*d^2*g^2*e^2*log((d*x*e + c*e)/(b*x
 + a))/(b*x + a)^2 - 36*(d*x*e + c*e)^2*B*a^2*b^4*c^2*d^3*g^2*e^2*log((d*x*
e + c*e)/(b*x + a))/(b*x + a)^2 + 24*(d*x*e + c*e)^2*B*a^3*b^3*c*d^4*g^2*e^
2*log((d*x*e + c*e)/(b*x + a))/(b*x + a)^2 - 6*(d*x*e + c*e)^2*B*a^4*b^2*d^
5*g^2*e^2*log((d*x*e + c*e)/(b*x + a))/(b*x + a)^2 + 2*(d*x*e + c*e)^3*B*b^
```

```
7*c^4*g^2*e*log((d*x*e + c*e)/(b*x + a))/(b*x + a)^3 - 8*(d*x*e + c*e)^3*B*
a*b^6*c^3*d*g^2*e*log((d*x*e + c*e)/(b*x + a))/(b*x + a)^3 + 12*(d*x*e + c*
e)^3*B*a^2*b^5*c^2*d^2*g^2*e*log((d*x*e + c*e)/(b*x + a))/(b*x + a)^3 - 8*(
d*x*e + c*e)^3*B*a^3*b^4*c*d^3*g^2*e*log((d*x*e + c*e)/(b*x + a))/(b*x + a)
^3 + 2*(d*x*e + c*e)^3*B*a^4*b^3*d^4*g^2*e*log((d*x*e + c*e)/(b*x + a))/(b*
x + a)^3 + 2*A*b^4*c^4*d^3*g^2*e^4 - 3*B*b^4*c^4*d^3*g^2*e^4 - 8*A*a*b^3*c^
3*d^4*g^2*e^4 + 12*B*a*b^3*c^3*d^4*g^2*e^4 + 12*A*a^2*b^2*c^2*d^5*g^2*e^4 -
 18*B*a^2*b^2*c^2*d^5*g^2*e^4 - 8*A*a^3*b*c*d^6*g^2*e^4 + 12*B*a^3*b*c*d^6*
g^2*e^4 + 2*A*a^4*d^7*g^2*e^4 - 3*B*a^4*d^7*g^2*e^4 + 5*(d*x*e + c*e)*B*b^5
*c^4*d^2*g^2*e^3/(b*x + a) - 20*(d*x*e + c*e)*B*a*b^4*c^3*d^3*g^2*e^3/(b*x
+ a) + 30*(d*x*e + c*e)*B*a^2*b^3*c^2*d^4*g^2*e^3/(b*x + a) - 20*(d*x*e + c
*e)*B*a^3*b^2*c*d^5*g^2*e^3/(b*x + a) + 5*(d*x*e + c*e)*B*a^4*b*d^6*g^2*e^3
/(b*x + a) - 2*(d*x*e + c*e)^2*B*b^6*c^4*d*g^2*e^2/(b*x + a)^2 + 8*(d*x*e +
 c*e)^2*B*a*b^5*c^3*d^2*g^2*e^2/(b*x + a)^2 - 12*(d*x*e + c*e)^2*B*a^2*b^4*
c^2*d^3*g^2*e^2/(b*x + a)^2 + 8*(d*x*e + c*e)^2*B*a^3*b^3*c*d^4*g^2*e^2/(b*
x + a)^2 - 2*(d*x*e + c*e)^2*B*a^4*b^2*d^5*g^2*e^2/(b*x + a)^2)*(b*c/((b*c*
e - a*d*e)*(b*c - a*d)) - a*d/((b*c*e - a*d*e)*(b*c - a*d)))/(b*d^6*e^3 - 3
*(d*x*e + c*e)*b^2*d^5*e^2/(b*x + a) + 3*(d*x*e + c*e)^2*b^3*d^4*e/(b*x + a
)^2 - (d*x*e + c*e)^3*b^4*d^3/(b*x + a)^3)
```

**maple [B]** time = 0.19, size = 1537, normalized size = 13.03

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*g*x+a*g)^2*(A+B*ln(e*(d*x+c)/(b*x+a))),x)
```

```
[Out] -1/6/b*e^2*B*g^2*d^2/(-1/(b*x+a)*a*d*e+1/(b*x+a)*b*c*e)^2*a^3+e^3*A*g^2/(-1
/(b*x+a)*a*d*e+1/(b*x+a)*b*c*e)^3*a^2*c*d^2+1/2*e^2*B*g^2*d/(-1/(b*x+a)*a*d
*e+1/(b*x+a)*b*c*e)^2*a^2*c+1/3*b^2*e^3*B*g^2*ln(1/b*d*e-(a*d-b*c)/(b*x+a)/
b*e)/(-1/(b*x+a)*a*d*e+1/(b*x+a)*b*c*e)^3*c^3+b*B*g^2/d^2*ln(-d*e+(1/b*d*e-
(a*d-b*c)/(b*x+a)/b*e)*b)*a*c^2-1/2*b*e^2*B*g^2/(-1/(b*x+a)*a*d*e+1/(b*x+a)
*b*c*e)^2*c^2*a+1/3/b*e*B*g^2/(-1/(b*x+a)*a*d*e+1/(b*x+a)*b*c*e)*a^3*d-1/3*
b^2*e*B*g^2/d^2/(-1/(b*x+a)*a*d*e+1/(b*x+a)*b*c*e)*c^3+1/6*b^2*e^2*B*g^2/d/
(-1/(b*x+a)*a*d*e+1/(b*x+a)*b*c*e)^2*c^3-1/3/b*e^3*A*g^2/(-1/(b*x+a)*a*d*e+
1/(b*x+a)*b*c*e)^3*a^3*d^3+1/3/b*B*g^2*ln(-d*e+(1/b*d*e-(a*d-b*c)/(b*x+a)/b
*e)*b)*a^3-b*e^3*A*g^2/(-1/(b*x+a)*a*d*e+1/(b*x+a)*b*c*e)^3*a*c^2*d+b*e*B*g
^2/d/(-1/(b*x+a)*a*d*e+1/(b*x+a)*b*c*e)*a*c^2-1/3/b*e^3*B*g^2*ln(1/b*d*e-(a
*d-b*c)/(b*x+a)/b*e)*d^3/(-1/(b*x+a)*a*d*e+1/(b*x+a)*b*c*e)^3*a^3+e^3*B*g^2
*ln(1/b*d*e-(a*d-b*c)/(b*x+a)/b*e)*d^2/(-1/(b*x+a)*a*d*e+1/(b*x+a)*b*c*e)^3
*a^2*c-e*B*g^2/(-1/(b*x+a)*a*d*e+1/(b*x+a)*b*c*e)*a^2*c+1/3*b^2*e^3*A*g^2/(
-1/(b*x+a)*a*d*e+1/(b*x+a)*b*c*e)^3*c^3-1/3*b^2*B*g^2/d^3*ln(-d*e+(1/b*d*e-
(a*d-b*c)/(b*x+a)/b*e)*b)*c^3-B*g^2/d*ln(-d*e+(1/b*d*e-(a*d-b*c)/(b*x+a)/b*
e)*b)*a^2*c+5*b*e^3*B*g^2*ln(1/b*d*e-(a*d-b*c)/(b*x+a)/b*e)*d/(-1/(b*x+a)*a
*d*e+1/(b*x+a)*b*c*e)^3*a^4/(b*x+a)^3*c^2+5*b^3*e^3*B*g^2*ln(1/b*d*e-(a*d-b
```

```
*c)/(b*x+a)/b*e)/d/(-1/(b*x+a)*a*d*e+1/(b*x+a)*b*c*e)^3*a^2/(b*x+a)^3*c^4-2
*b^4*e^3*B*g^2*ln(1/b*d*e-(a*d-b*c)/(b*x+a)/b*e)/d^2/(-1/(b*x+a)*a*d*e+1/(b
*x+a)*b*c*e)^3*c^5/(b*x+a)^3*a+1/3*b^5*e^3*B*g^2*ln(1/b*d*e-(a*d-b*c)/(b*x+
a)/b*e)/d^3/(-1/(b*x+a)*a*d*e+1/(b*x+a)*b*c*e)^3*c^6/(b*x+a)^3-b*e^3*B*g^2*
ln(1/b*d*e-(a*d-b*c)/(b*x+a)/b*e)/(-1/(b*x+a)*a*d*e+1/(b*x+a)*b*c*e)^3*c^2*
a*d-20/3*b^2*e^3*B*g^2*ln(1/b*d*e-(a*d-b*c)/(b*x+a)/b*e)/(-1/(b*x+a)*a*d*e+
1/(b*x+a)*b*c*e)^3*a^3/(b*x+a)^3*c^3-2*e^3*B*g^2*ln(1/b*d*e-(a*d-b*c)/(b*x+
a)/b*e)*d^2/(-1/(b*x+a)*a*d*e+1/(b*x+a)*b*c*e)^3*a^5/(b*x+a)^3*c+1/3/b*e^3*
B*g^2*ln(1/b*d*e-(a*d-b*c)/(b*x+a)/b*e)*d^3/(-1/(b*x+a)*a*d*e+1/(b*x+a)*b*c
*e)^3*a^6/(b*x+a)^3
```

**maxima [B]** time = 1.16, size = 278, normalized size = 2.36

$$\frac{1}{3}Ab^2g^2x^3+Aabg^2x^2+\left(x\log\left(\frac{dex}{bx+a}+\frac{ce}{bx+a}\right)-\frac{a\log(bx+a)}{b}+\frac{c\log(dx+c)}{d}\right)Ba^2g^2+\left(x^2\log\left(\frac{dex}{bx+a}+\frac{ce}{bx+\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*g*x+a*g)^2*(A+B*log(e*(d*x+c)/(b*x+a))),x, algorithm="maxima")
```

```
[Out] 1/3*A*b^2*g^2*x^3 + A*a*b*g^2*x^2 + (x*log(d*e*x/(b*x + a) + c*e/(b*x + a))
 - a*log(b*x + a)/b + c*log(d*x + c)/d)*B*a^2*g^2 + (x^2*log(d*e*x/(b*x + a
) + c*e/(b*x + a)) + a^2*log(b*x + a)/b^2 - c^2*log(d*x + c)/d^2 + (b*c - a
*d)*x/(b*d))*B*a*b*g^2 + 1/6*(2*x^3*log(d*e*x/(b*x + a) + c*e/(b*x + a)) -
2*a^3*log(b*x + a)/b^3 + 2*c^3*log(d*x + c)/d^3 + ((b^2*c*d - a*b*d^2)*x^2
- 2*(b^2*c^2 - a^2*d^2)*x)/(b^2*d^2))*B*b^2*g^2 + A*a^2*g^2*x
```

**mupad [B]** time = 4.57, size = 290, normalized size = 2.46

$$x^2\left(\frac{b\,g^2\,\left(9\,A\,a\,d+3\,A\,b\,c-B\,a\,d+B\,b\,c\right)}{6\,d}-\frac{A\,b\,g^2\,\left(3\,a\,d+3\,b\,c\right)}{6\,d}\right)-x\left(\frac{\left(3\,a\,d+3\,b\,c\right)\,\left(\frac{b\,g^2\,\left(9\,A\,a\,d+3\,A\,b\,c-B\,a\,d+B\,b\,c\right)}{3\,d}-\right.}{3\,b\,d}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*g + b*g*x)^2*(A + B*log((e*(c + d*x))/(a + b*x))),x)
```

```
[Out] x^2*((b*g^2*(9*A*a*d + 3*A*b*c - B*a*d + B*b*c))/(6*d) - (A*b*g^2*(3*a*d +
3*b*c))/(6*d)) - x*(((3*a*d + 3*b*c)*((b*g^2*(9*A*a*d + 3*A*b*c - B*a*d + B
*b*c))/(3*d) - (A*b*g^2*(3*a*d + 3*b*c))/(3*d)))/(3*b*d) - (a*g^2*(3*A*a*d
+ 3*A*b*c - B*a*d + B*b*c))/d + (A*a*b*c*g^2)/d) + log((e*(c + d*x))/(a + b
*x))*((B*b^2*g^2*x^3)/3 + B*a^2*g^2*x + B*a*b*g^2*x^2) + (log(c + d*x)*(B*b
^2*c^3*g^2 + 3*B*a^2*c*d^2*g^2 - 3*B*a*b*c^2*d*g^2))/(3*d^3) + (A*b^2*g^2*x
^3)/3 - (B*a^3*g^2*log(a + b*x))/(3*b)
```

**sympy [B]** time = 2.91, size = 491, normalized size = 4.16

$$\frac{Ab^2g^2x^3}{3} - \frac{Ba^3g^2\log\left(x + \frac{\frac{Ba^4d^3g^2}{b} + 3Ba^3cd^2g^2 - 3Ba^2bc^2dg^2 + Bab^2c^3g^2}{Ba^3d^3g^2 + 3Ba^2bcd^2g^2 - 3Bab^2c^2dg^2 + Bb^3c^3g^2}\right)}{3b} + \frac{Bcg^2\left(3a^2d^2 - 3abcd + b^2c^2\right)\log\left(x + \frac{4Ba^3cd^2g^2 - 3}{}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*g*x+a*g)**2*(A+B*ln(e*(d*x+c)/(b*x+a))),x)
```

```
[Out] A*b**2*g**2*x**3/3 - B*a**3*g**2*log(x + (B*a**4*d**3*g**2/b + 3*B*a**3*c*d
**2*g**2 - 3*B*a**2*b*c**2*d*g**2 + B*a*b**2*c**3*g**2)/(B*a**3*d**3*g**2 +
 3*B*a**2*b*c*d**2*g**2 - 3*B*a*b**2*c**2*d*g**2 + B*b**3*c**3*g**2))/(3*b)
 + B*c*g**2*(3*a**2*d**2 - 3*a*b*c*d + b**2*c**2)*log(x + (4*B*a**3*c*d**2*
g**2 - 3*B*a**2*b*c**2*d*g**2 + B*a*b**2*c**3*g**2 - B*a*c*g**2*(3*a**2*d**
2 - 3*a*b*c*d + b**2*c**2) + B*b*c**2*g**2*(3*a**2*d**2 - 3*a*b*c*d + b**2*
c**2)/d)/(B*a**3*d**3*g**2 + 3*B*a**2*b*c*d**2*g**2 - 3*B*a*b**2*c**2*d*g**
2 + B*b**3*c**3*g**2))/(3*d**3) + x**2*(A*a*b*g**2 - B*a*b*g**2/6 + B*b**2*
c*g**2/(6*d)) + x*(A*a**2*g**2 - 2*B*a**2*g**2/3 + B*a*b*c*g**2/d - B*b**2*
c**2*g**2/(3*d**2)) + (B*a**2*g**2*x + B*a*b*g**2*x**2 + B*b**2*g**2*x**3/3
)*log(e*(c + d*x)/(a + b*x))
```

# 3.176 $\int (ag+bgx)\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)\,dx$

**Optimal**. Leaf size=81

$$\frac{g(a+bx)^2\left(B\log\left(\frac{e(c+dx)}{a+bx}\right)+A\right)}{2b}-\frac{Bg(bc-ad)^2\log(c+dx)}{2bd^2}+\frac{Bgx(bc-ad)}{2d}$$

```
[Out] 1/2*B*(-a*d+b*c)*g*x/d-1/2*B*(-a*d+b*c)^2*g*ln(d*x+c)/b/d^2+1/2*g*(b*x+a)^2
*(A+B*ln(e*(d*x+c)/(b*x+a)))/b
```

**Rubi [A]** time = 0.05, antiderivative size = 81, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.107, Rules used = {2525, 12, 43}

$$\frac{g(a+bx)^2\left(B\log\left(\frac{e(c+dx)}{a+bx}\right)+A\right)}{2b}-\frac{Bg(bc-ad)^2\log(c+dx)}{2bd^2}+\frac{Bgx(bc-ad)}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[(a*g + b*g*x)*(A + B*Log[(e*(c + d*x))/(a + b*x)]),x]
```

```
[Out] (B*(b*c - a*d)*g*x)/(2*d) - (B*(b*c - a*d)^2*g*Log[c + d*x])/(2*b*d^2) + (g
*(a + b*x)^2*(A + B*Log[(e*(c + d*x))/(a + b*x)]))/(2*b)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2525

```
Int[((a_.) + Log[(c_.)*(RFx_)^(p_.)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_))^(m_.
), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(a + b*Log[c*RFx^p])^n)/(e*(m + 1))
, x] - Dist[(b*n*p)/(e*(m + 1)), Int[SimplifyIntegrand[((d + e*x)^(m + 1)*(
a + b*Log[c*RFx^p])^(n - 1)*D[RFx, x])/RFx, x], x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && RationalFunctionQ[RFx, x] && IGtQ[n, 0] && (EqQ[n, 1] ||
```

```
IntegerQ[m]) && NeQ[m, -1]
```

Rubi steps

$$\int (ag+bgx)\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)dx = \frac{g(a+bx)^2\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)}{2b} - \frac{B\int \frac{(bc-ad)g^2(-a-bx)}{c+dx}\,dx}{2bg}$$
$$= \frac{g(a+bx)^2\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)}{2b} - \frac{(B(bc-ad)g)\int \frac{-a-bx}{c+dx}\,dx}{2b}$$
$$= \frac{g(a+bx)^2\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)}{2b} - \frac{(B(bc-ad)g)\int\left(-\frac{b}{d}+\frac{bc-ad}{d(c+dx)}\right) d$$
$$= \frac{B(bc-ad)gx}{2d} - \frac{B(bc-ad)^2g\log(c+dx)}{2bd^2} + \frac{g(a+bx)^2\left(A+B\log\left(\right.\right.}{2b}$$

**Mathematica [A]** time = 0.04, size = 69, normalized size = 0.85

$$\frac{g\left((a+bx)^2\left(B\log\left(\frac{e(c+dx)}{a+bx}\right)+A\right)+\frac{B(bc-ad)((ad-bc)\log(c+dx)+bdx)}{d^2}\right)}{2b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*g + b*g*x)*(A + B*Log[(e*(c + d*x))/(a + b*x)]),x]
```

```
[Out] (g*((B*(b*c - a*d)*(b*d*x + (-(b*c) + a*d)*Log[c + d*x]))/d^2 + (a + b*x)^2
*(A + B*Log[(e*(c + d*x))/(a + b*x)])))/(2*b)
```

**fricas [A]** time = 0.96, size = 127, normalized size = 1.57

$$\frac{Ab^2d^2gx^2 - Ba^2d^2g\log(bx+a) + \left(Bb^2cd + (2A-B)abd^2\right)gx - \left(Bb^2c^2 - 2Babcd\right)g\log(dx+c) + \left(Bb^2d^2gx^2 + \right.}{2bd^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*g*x+a*g)*(A+B*log(e*(d*x+c)/(b*x+a))),x, algorithm="fricas")
```

```
[Out] 1/2*(A*b^2*d^2*g*x^2 - B*a^2*d^2*g*log(b*x + a) + (B*b^2*c*d + (2*A - B)*a*
b*d^2)*g*x - (B*b^2*c^2 - 2*B*a*b*c*d)*g*log(d*x + c) + (B*b^2*d^2*g*x^2 +
2*B*a*b*d^2*g*x)*log((d*e*x + c*e)/(b*x + a)))/(b*d^2)
```

**giac [B]** time = 0.67, size = 1395, normalized size = 17.22

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*g*x+a*g)*(A+B*log(e*(d*x+c)/(b*x+a))),x, algorithm="giac")
```

```
[Out] 1/2*(B*b^3*c^3*d^2*g*e^3*log(-d*e + (d*x*e + c*e)*b/(b*x + a)) - 3*B*a*b^2*
c^2*d^3*g*e^3*log(-d*e + (d*x*e + c*e)*b/(b*x + a)) + 3*B*a^2*b*c*d^4*g*e^3
*log(-d*e + (d*x*e + c*e)*b/(b*x + a)) - B*a^3*d^5*g*e^3*log(-d*e + (d*x*e
+ c*e)*b/(b*x + a)) - 2*(d*x*e + c*e)*B*b^4*c^3*d*g*e^2*log(-d*e + (d*x*e +
 c*e)*b/(b*x + a))/(b*x + a) + 6*(d*x*e + c*e)*B*a*b^3*c^2*d^2*g*e^2*log(-d
*e + (d*x*e + c*e)*b/(b*x + a))/(b*x + a) - 6*(d*x*e + c*e)*B*a^2*b^2*c*d^3
*g*e^2*log(-d*e + (d*x*e + c*e)*b/(b*x + a))/(b*x + a) + 2*(d*x*e + c*e)*B*
a^3*b*d^4*g*e^2*log(-d*e + (d*x*e + c*e)*b/(b*x + a))/(b*x + a) + (d*x*e +
c*e)^2*B*b^5*c^3*g*e*log(-d*e + (d*x*e + c*e)*b/(b*x + a))/(b*x + a)^2 - 3*
(d*x*e + c*e)^2*B*a*b^4*c^2*d*g*e*log(-d*e + (d*x*e + c*e)*b/(b*x + a))/(b*
x + a)^2 + 3*(d*x*e + c*e)^2*B*a^2*b^3*c*d^2*g*e*log(-d*e + (d*x*e + c*e)*b
/(b*x + a))/(b*x + a)^2 - (d*x*e + c*e)^2*B*a^3*b^2*d^3*g*e*log(-d*e + (d*x
*e + c*e)*b/(b*x + a))/(b*x + a)^2 + 2*(d*x*e + c*e)*B*b^4*c^3*d*g*e^2*log(
(d*x*e + c*e)/(b*x + a))/(b*x + a) - 6*(d*x*e + c*e)*B*a*b^3*c^2*d^2*g*e^2*
log((d*x*e + c*e)/(b*x + a))/(b*x + a) + 6*(d*x*e + c*e)*B*a^2*b^2*c*d^3*g*
e^2*log((d*x*e + c*e)/(b*x + a))/(b*x + a) - 2*(d*x*e + c*e)*B*a^3*b*d^4*g*
e^2*log((d*x*e + c*e)/(b*x + a))/(b*x + a) - (d*x*e + c*e)^2*B*b^5*c^3*g*e*
log((d*x*e + c*e)/(b*x + a))/(b*x + a)^2 + 3*(d*x*e + c*e)^2*B*a*b^4*c^2*d*
g*e*log((d*x*e + c*e)/(b*x + a))/(b*x + a)^2 - 3*(d*x*e + c*e)^2*B*a^2*b^3*
c*d^2*g*e*log((d*x*e + c*e)/(b*x + a))/(b*x + a)^2 + (d*x*e + c*e)^2*B*a^3*
b^2*d^3*g*e*log((d*x*e + c*e)/(b*x + a))/(b*x + a)^2 + A*b^3*c^3*d^2*g*e^3
- B*b^3*c^3*d^2*g*e^3 - 3*A*a*b^2*c^2*d^3*g*e^3 + 3*B*a*b^2*c^2*d^3*g*e^3 +
 3*A*a^2*b*c*d^4*g*e^3 - 3*B*a^2*b*c*d^4*g*e^3 - A*a^3*d^5*g*e^3 + B*a^3*d^
5*g*e^3 + (d*x*e + c*e)*B*b^4*c^3*d*g*e^2/(b*x + a) - 3*(d*x*e + c*e)*B*a*b
^3*c^2*d^2*g*e^2/(b*x + a) + 3*(d*x*e + c*e)*B*a^2*b^2*c*d^3*g*e^2/(b*x + a
) - (d*x*e + c*e)*B*a^3*b*d^4*g*e^2/(b*x + a))*(b*c/((b*c*e - a*d*e)*(b*c -
 a*d)) - a*d/((b*c*e - a*d*e)*(b*c - a*d)))/(b*d^4*e^2 - 2*(d*x*e + c*e)*b^
2*d^3*e/(b*x + a) + (d*x*e + c*e)^2*b^3*d^2/(b*x + a)^2)
```

**maple [B]** time = 0.14, size = 951, normalized size = 11.74

$$-\frac{B\,a^{4}d^{2}e^{2}g\ln\left(\frac{de}{b}-\frac{(ad-bc)e}{(bx+a)b}\right)}{2\left(-\frac{ade}{bx+a}+\frac{bce}{bx+a}\right)^{2}(bx+a)^{2}\,b}+\frac{2B\,a^{3}cd\,e^{2}g\ln\left(\frac{de}{b}-\frac{(ad-bc)e}{(bx+a)b}\right)}{\left(-\frac{ade}{bx+a}+\frac{bce}{bx+a}\right)^{2}(bx+a)^{2}}-\frac{3B\,a^{2}b\,c^{2}e^{2}g\ln\left(\frac{de}{b}-\frac{(ad-bc)e}{(bx+a)b}\right)}{\left(-\frac{ade}{bx+a}+\frac{bce}{bx+a}\right)^{2}(bx+a)^{2}}+\frac{2Ba\,b^{2}c^{3}e^{2}g\ln\left(\frac{de}{b}\right.}{\left(-\frac{ade}{bx+a}+\frac{bce}{bx+a}\right)^{2}(b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*g*x+a*g)*(A+B*ln(e*(d*x+c)/(b*x+a))),x)
```

```
[Out] 1/2/b*e^2*A*g/(-1/(b*x+a)*a*d*e+1/(b*x+a)*b*c*e)^2*a^2*d^2-e^2*A*g/(-1/(b*x
+a)*a*d*e+1/(b*x+a)*b*c*e)^2*a*d*c+1/2*b*e^2*A*g/(-1/(b*x+a)*a*d*e+1/(b*x+a
)*b*c*e)^2*c^2+1/2/b*B*g*ln(-d*e+(1/b*d*e-(a*d-b*c)/(b*x+a)/b*e)*b)*a^2-B*g
/d*ln(-d*e+(1/b*d*e-(a*d-b*c)/(b*x+a)/b*e)*b)*a*c+1/2*b*B*g/d^2*ln(-d*e+(1/
b*d*e-(a*d-b*c)/(b*x+a)/b*e)*b)*c^2+1/2/b*e*B*g/(-1/(b*x+a)*a*d*e+1/(b*x+a)
*b*c*e)*a^2*d-e*B*g/(-1/(b*x+a)*a*d*e+1/(b*x+a)*b*c*e)*a*c+1/2*b*e*B*g/d/(-
1/(b*x+a)*a*d*e+1/(b*x+a)*b*c*e)*c^2+1/2/b*e^2*B*g*ln(1/b*d*e-(a*d-b*c)/(b*
x+a)/b*e)*d^2/(-1/(b*x+a)*a*d*e+1/(b*x+a)*b*c*e)^2*a^2-e^2*B*g*ln(1/b*d*e-(
a*d-b*c)/(b*x+a)/b*e)*d/(-1/(b*x+a)*a*d*e+1/(b*x+a)*b*c*e)^2*a*c-1/2/b*e^2*
B*g*ln(1/b*d*e-(a*d-b*c)/(b*x+a)/b*e)*d^2/(-1/(b*x+a)*a*d*e+1/(b*x+a)*b*c*e
)^2*a^4/(b*x+a)^2+2*e^2*B*g*ln(1/b*d*e-(a*d-b*c)/(b*x+a)/b*e)*d/(-1/(b*x+a)
*a*d*e+1/(b*x+a)*b*c*e)^2*a^3/(b*x+a)^2*c-3*b*e^2*B*g*ln(1/b*d*e-(a*d-b*c)/
(b*x+a)/b*e)/(-1/(b*x+a)*a*d*e+1/(b*x+a)*b*c*e)^2*a^2/(b*x+a)^2*c^2+2*b^2*e
^2*B*g*ln(1/b*d*e-(a*d-b*c)/(b*x+a)/b*e)/d/(-1/(b*x+a)*a*d*e+1/(b*x+a)*b*c*
e)^2*a/(b*x+a)^2*c^3+1/2*b*e^2*B*g*ln(1/b*d*e-(a*d-b*c)/(b*x+a)/b*e)/(-1/(b
*x+a)*a*d*e+1/(b*x+a)*b*c*e)^2*c^2-1/2*b^3*e^2*B*g*ln(1/b*d*e-(a*d-b*c)/(b*
x+a)/b*e)/d^2/(-1/(b*x+a)*a*d*e+1/(b*x+a)*b*c*e)^2*c^4/(b*x+a)^2
```

**maxima [A]** time = 1.12, size = 143, normalized size = 1.77

$$\frac{1}{2}Abgx^2+\left(x\log\left(\frac{dex}{bx+a}+\frac{ce}{bx+a}\right)-\frac{a\log(bx+a)}{b}+\frac{c\log(dx+c)}{d}\right)Bag+\frac{1}{2}\left(x^2\log\left(\frac{dex}{bx+a}+\frac{ce}{bx+a}\right)+\frac{a^2\log}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*g*x+a*g)*(A+B*log(e*(d*x+c)/(b*x+a))),x, algorithm="maxima")
```

```
[Out] 1/2*A*b*g*x^2 + (x*log(d*e*x/(b*x + a) + c*e/(b*x + a)) - a*log(b*x + a)/b
+ c*log(d*x + c)/d)*B*a*g + 1/2*(x^2*log(d*e*x/(b*x + a) + c*e/(b*x + a)) +
 a^2*log(b*x + a)/b^2 - c^2*log(d*x + c)/d^2 + (b*c - a*d)*x/(b*d))*B*b*g +
 A*a*g*x
```

**mupad [B]** time = 4.31, size = 126, normalized size = 1.56

$$x\left(\frac{g\left(4AAd+2Abc-Bad+Bbc\right)}{2d}-\frac{Ag\left(2ad+2bc\right)}{2d}\right)+\ln\left(\frac{e\left(c+dx\right)}{a+bx}\right)\left(\frac{Bbgx^2}{2}+Bagx\right)-\frac{\ln(c+dx)\,(}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*g + b*g*x)*(A + B*log((e*(c + d*x))/(a + b*x))),x)
```

```
[Out] x*((g*(4*A*a*d + 2*A*b*c - B*a*d + B*b*c))/(2*d) - (A*g*(2*a*d + 2*b*c))/(2
*d)) + log((e*(c + d*x))/(a + b*x))*((B*b*g*x^2)/2 + B*a*g*x) - (log(c + d*
x)*(B*b*c^2*g - 2*B*a*c*d*g))/(2*d^2) + (A*b*g*x^2)/2 - (B*a^2*g*log(a + b*
x))/(2*b)
```

**sympy [B]** time = 1.92, size = 253, normalized size = 3.12

$$\frac{Abgx^2}{2}-\frac{Ba^2g\log\left(x+\frac{\frac{Ba^3d^2g}{b}+2Ba^2cdg-Babc^2g}{Ba^2d^2g+2Babcdg-Bb^2c^2g}\right)}{2b}+\frac{Bcg\,(2ad-bc)\log\left(x+\frac{3Ba^2cdg-Babc^2g-Bacg(2ad-bc)+\frac{Bbc^2g(2ad-bc)}{d}}{Ba^2d^2g+2Babcdg-Bb^2c^2g}\right)}{2d^2}+x\Big(A$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*g*x+a*g)*(A+B*ln(e*(d*x+c)/(b*x+a))),x)
```

```
[Out] A*b*g*x**2/2 - B*a**2*g*log(x + (B*a**3*d**2*g/b + 2*B*a**2*c*d*g - B*a*b*c
**2*g)/(B*a**2*d**2*g + 2*B*a*b*c*d*g - B*b**2*c**2*g))/(2*b) + B*c*g*(2*a*
d - b*c)*log(x + (3*B*a**2*c*d*g - B*a*b*c**2*g - B*a*c*g*(2*a*d - b*c) + B
*b*c**2*g*(2*a*d - b*c)/d)/(B*a**2*d**2*g + 2*B*a*b*c*d*g - B*b**2*c**2*g))
/(2*d**2) + x*(A*a*g - B*a*g/2 + B*b*c*g/(2*d)) + (B*a*g*x + B*b*g*x**2/2)*
log(e*(c + d*x)/(a + b*x))
```

# 3.177 $\int \frac{A+B\log\left(\frac{e(c+dx)}{a+bx}\right)}{ag+bgx}\,dx$

**Optimal**. Leaf size=81

$$-\frac{\log\left(-\frac{bc-ad}{d(a+bx)}\right)\left(B\log\left(\frac{e(c+dx)}{a+bx}\right)+A\right)}{bg}-\frac{B\text{Li}_2\left(\frac{bc-ad}{d(a+bx)}+1\right)}{bg}$$

```
[Out] -ln((a*d-b*c)/d/(b*x+a))*(A+B*ln(e*(d*x+c)/(b*x+a)))/b/g-B*polylog(2,1+(-a*
d+b*c)/d/(b*x+a))/b/g
```

**Rubi [A]** time = 0.21, antiderivative size = 122, normalized size of antiderivative = 1.51, number of steps used = 10, number of rules used = 8, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {2524, 12, 2418, 2390, 2301, 2394, 2393, 2391}

$$-\frac{B\text{PolyLog}\left(2,-\frac{d(a+bx)}{bc-ad}\right)}{bg}+\frac{\log(ag+bgx)\left(B\log\left(\frac{e(c+dx)}{a+bx}\right)+A\right)}{bg}-\frac{B\log(ag+bgx)\log\left(\frac{b(c+dx)}{bc-ad}\right)}{bg}+\frac{B\log^2(g(a+bx$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Log[(e*(c + d*x))/(a + b*x)])/(a*g + b*g*x),x]
```

```
[Out] (B*Log[g*(a + b*x)]^2)/(2*b*g) - (B*Log[(b*(c + d*x))/(b*c - a*d)]*Log[a*g
+ b*g*x])/(b*g) + ((A + B*Log[(e*(c + d*x))/(a + b*x)])*Log[a*g + b*g*x])/(
b*g) - (B*PolyLog[2, -((d*(a + b*x))/(b*c - a*d))])/(b*g)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2301

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

Rule 2390

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_))^(q_.), x_Symbol] :> Dist[1/e, Subst[Int[((f*x)/d)^q*(a + b*Log[c*x^
n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && E
qQ[e*f - d*g, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2393

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + (c*e*x)/g])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2394

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d + e*x
)^n]))/g, x] - Dist[(b*e*n)/g, Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2418

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*(RFx_), x_Sy
mbol] :> With[{u = ExpandIntegrand[(a + b*Log[c*(d + e*x)^n])^p, RFx, x]},
Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d, e, n}, x] && RationalFunctionQ[
RFx, x] && IntegerQ[p]
```

Rule 2524

```
Int[((a_.) + Log[(c_.)*(RFx_)^(p_.)]*(b_.))^(n_.)/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[(Log[d + e*x]*(a + b*Log[c*RFx^p])^n)/e, x] - Dist[(b*n*p)/e
, Int[(Log[d + e*x]*(a + b*Log[c*RFx^p])^(n - 1)*D[RFx, x])/RFx, x], x] /;
FreeQ[{a, b, c, d, e, p}, x] && RationalFunctionQ[RFx, x] && IGtQ[n, 0]
```

Rubi steps

$$\int \frac{A+B\log\left(\frac{e(c+dx)}{a+bx}\right)}{ag+bgx}\,dx = \frac{\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)\log(ag+bgx)}{bg} - \frac{B\int \frac{(a+bx)\left(\frac{de}{a+bx}-\frac{be(c+dx)}{(a+bx)^2}\right)\log(ag+bgx)}{e(c+dx)}dx}{bg}$$

$$= \frac{\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)\log(ag+bgx)}{bg} - \frac{B\int \frac{(a+bx)\left(\frac{de}{a+bx}-\frac{be(c+dx)}{(a+bx)^2}\right)\log(ag+bgx)}{c+dx}dx}{beg}$$

$$= \frac{\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)\log(ag+bgx)}{bg} - \frac{B\int \left(-\frac{be\log(ag+bgx)}{a+bx}+\frac{de\log(ag+bgx)}{c+dx}\right)dx}{beg}$$

$$= \frac{\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)\log(ag+bgx)}{bg} + \frac{B\int \frac{\log(ag+bgx)}{a+bx}dx}{g} - \frac{(Bd)\int \frac{\log(ag+bgx)}{c+dx}dx}{bg}$$

$$= -\frac{B\log\left(\frac{b(c+dx)}{bc-ad}\right)\log(ag+bgx)}{bg} + \frac{\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)\log(ag+bgx)}{bg} + B\int \frac{\log\left(\frac{b}{b}\right.}{ag}$$

$$= -\frac{B\log\left(\frac{b(c+dx)}{bc-ad}\right)\log(ag+bgx)}{bg} + \frac{\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)\log(ag+bgx)}{bg} + \frac{B\,\mathrm{Subst}\left(\int\right.}{}$$

$$= \frac{B\log^2(g(a+bx))}{2bg} - \frac{B\log\left(\frac{b(c+dx)}{bc-ad}\right)\log(ag+bgx)}{bg} + \frac{\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)\log(ag+}{bg}$$

**Mathematica [A]** time = 0.04, size = 95, normalized size = 1.17

$$\frac{\log(g(a+bx))\left(2\left(B\log\left(\frac{e(c+dx)}{a+bx}\right)-B\log\left(\frac{b(c+dx)}{bc-ad}\right)+A\right)+B\log(g(a+bx))\right)-2B\text{Li}_2\left(\frac{d(a+bx)}{ad-bc}\right)}{2bg}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Log[(e*(c + d*x))/(a + b*x)])/(a*g + b*g*x),x]
```

```
[Out] (Log[g*(a + b*x)]*(B*Log[g*(a + b*x)] + 2*(A - B*Log[(b*(c + d*x))/(b*c - a
*d)] + B*Log[(e*(c + d*x))/(a + b*x)])) - 2*B*PolyLog[2, (d*(a + b*x))/(-(b
*c) + a*d)])/(2*b*g)
```

**fricas [F]** time = 0.77, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{B\log\left(\frac{dex+ce}{bx+a}\right)+A}{bgx+ag}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*(d*x+c)/(b*x+a)))/(b*g*x+a*g),x, algorithm="fricas")
```

```
[Out] integral((B*log((d*e*x + c*e)/(b*x + a)) + A)/(b*g*x + a*g), x)
```

**giac [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*(d*x+c)/(b*x+a)))/(b*g*x+a*g),x, algorithm="giac")
```

```
[Out] Timed out
```

**maple [B]** time = 0.06, size = 419, normalized size = 5.17

$$-\frac{Bad\ln\left(-\frac{-de+\left(\frac{de}{b}-\frac{(ad-bc)e}{(bx+a)b}\right)b}{de}\right)\ln\left(\frac{de}{b}-\frac{(ad-bc)e}{(bx+a)b}\right)}{(ad-bc)\,bg}+\frac{Bc\ln\left(-\frac{-de+\left(\frac{de}{b}-\frac{(ad-bc)e}{(bx+a)b}\right)b}{de}\right)\ln\left(\frac{de}{b}-\frac{(ad-bc)e}{(bx+a)b}\right)}{(ad-bc)\,g}-\frac{Aad\ln\left(-de+\left(\frac{de}{b}-\frac{(a}{(b}\right.\right.}{(ad-bc)\,bg}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*ln(e*(d*x+c)/(b*x+a)))/(b*g*x+a*g),x)
```

```
[Out] -1/b/g/(a*d-b*c)*A*ln(-d*e+(1/b*d*e-(a*d-b*c)/(b*x+a)/b*e)*b)*a*d+1/g/(a*d-
b*c)*A*ln(-d*e+(1/b*d*e-(a*d-b*c)/(b*x+a)/b*e)*b)*c-1/b/g/(a*d-b*c)*B*dilog
(-(-d*e+(1/b*d*e-(a*d-b*c)/(b*x+a)/b*e)*b)/d/e)*a*d+1/g/(a*d-b*c)*B*dilog(-
(-d*e+(1/b*d*e-(a*d-b*c)/(b*x+a)/b*e)*b)/d/e)*c-1/b/g/(a*d-b*c)*B*ln(1/b*d*
e-(a*d-b*c)/(b*x+a)/b*e)*ln(-(-d*e+(1/b*d*e-(a*d-b*c)/(b*x+a)/b*e)*b)/d/e)*
a*d+1/g/(a*d-b*c)*B*ln(1/b*d*e-(a*d-b*c)/(b*x+a)/b*e)*ln(-(-d*e+(1/b*d*e-(a
*d-b*c)/(b*x+a)/b*e)*b)/d/e)*c
```

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$B\left(\frac{\log\left(bx+a\right)\log\left(dx+c\right)}{bg}-\int-\frac{bdx\log(e)+bc\log(e)-\left(2\,bdx+bc+ad\right)\log\left(bx+a\right)}{b^2dgx^2+abcg+\left(b^2cg+abdg\right)x}\,dx\right)+\frac{A\log\left(bgx+ag\right)}{bg}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*(d*x+c)/(b*x+a)))/(b*g*x+a*g),x, algorithm="maxima")
```

```
[Out] B*(log(b*x + a)*log(d*x + c)/(b*g) - integrate(-(b*d*x*log(e) + b*c*log(e)
- (2*b*d*x + b*c + a*d)*log(b*x + a))/(b^2*d*g*x^2 + a*b*c*g + (b^2*c*g + a
*b*d*g)*x), x)) + A*log(b*g*x + a*g)/(b*g)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{A+B\,\ln\left(\frac{e\,(c+d\,x)}{a+b\,x}\right)}{a\,g+b\,g\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*log((e*(c + d*x))/(a + b*x)))/(a*g + b*g*x),x)
```

```
[Out] int((A + B*log((e*(c + d*x))/(a + b*x)))/(a*g + b*g*x), x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{A}{a+bx}\,dx+\int \frac{B\log\left(\frac{ce}{a+bx}+\frac{dex}{a+bx}\right)}{a+bx}\,dx}{g}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*ln(e*(d*x+c)/(b*x+a)))/(b*g*x+a*g),x)
```

```
[Out] (Integral(A/(a + b*x), x) + Integral(B*log(c*e/(a + b*x) + d*e*x/(a + b*x))
/(a + b*x), x))/g
```

# 3.178 $\int \frac{A+B\log\left(\frac{e(c+dx)}{a+bx}\right)}{(ag+bgx)^2}\,dx$

**Optimal**. Leaf size=64

$$-\frac{A-B}{bg^2(a+bx)}-\frac{B(c+dx)\log\left(\frac{e(c+dx)}{a+bx}\right)}{g^2(a+bx)(bc-ad)}$$

```
[Out] (-A+B)/b/g^2/(b*x+a)-B*(d*x+c)*ln(e*(d*x+c)/(b*x+a))/(-a*d+b*c)/g^2/(b*x+a)
```

**Rubi [A]** time = 0.08, antiderivative size = 101, normalized size of antiderivative = 1.58, number of steps used = 4, number of rules used = 3, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {2525, 12, 44}

$$-\frac{B\log\left(\frac{e(c+dx)}{a+bx}\right)+A}{bg^2(a+bx)}+\frac{Bd\log(a+bx)}{bg^2(bc-ad)}-\frac{Bd\log(c+dx)}{bg^2(bc-ad)}+\frac{B}{bg^2(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Log[(e*(c + d*x))/(a + b*x)])/(a*g + b*g*x)^2,x]
```

```
[Out] B/(b*g^2*(a + b*x)) + (B*d*Log[a + b*x])/(b*(b*c - a*d)*g^2) - (B*d*Log[c +
 d*x])/(b*(b*c - a*d)*g^2) - (A + B*Log[(e*(c + d*x))/(a + b*x)])/(b*g^2*(a
 + b*x))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 2525

```
Int[((a_.) + Log[(c_.)*(RFx_)^(p_.)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_))^(m_.
), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(a + b*Log[c*RFx^p])^n)/(e*(m + 1))
, x] - Dist[(b*n*p)/(e*(m + 1)), Int[SimplifyIntegrand[((d + e*x)^(m + 1)*(
a + b*Log[c*RFx^p])^(n - 1)*D[RFx, x])/RFx, x], x], x] /; FreeQ[{a, b, c, d
```

```
, e, m, p}, x] && RationalFunctionQ[RFx, x] && IGtQ[n, 0] && (EqQ[n, 1] ||
IntegerQ[m]) && NeQ[m, -1]
```

Rubi steps

$$
\begin{aligned}
\int \frac{A+B\log\left(\frac{e(c+dx)}{a+bx}\right)}{(ag+bgx)^2}\,dx &= -\frac{A+B\log\left(\frac{e(c+dx)}{a+bx}\right)}{bg^2(a+bx)}+\frac{B\int\frac{-bc+ad}{g(a+bx)^2(c+dx)}\,dx}{bg}\\
&= -\frac{A+B\log\left(\frac{e(c+dx)}{a+bx}\right)}{bg^2(a+bx)}-\frac{(B(bc-ad))\int\frac{1}{(a+bx)^2(c+dx)}\,dx}{bg^2}\\
&= -\frac{A+B\log\left(\frac{e(c+dx)}{a+bx}\right)}{bg^2(a+bx)}-\frac{(B(bc-ad))\int\left(\frac{b}{(bc-ad)(a+bx)^2}-\frac{bd}{(bc-ad)^2(a+bx)}+\frac{d^2}{(bc-ad)^2(c+dx)}\right)d}{bg^2}\\
&= \frac{B}{bg^2(a+bx)}+\frac{Bd\log(a+bx)}{b(bc-ad)g^2}-\frac{Bd\log(c+dx)}{b(bc-ad)g^2}-\frac{A+B\log\left(\frac{e(c+dx)}{a+bx}\right)}{bg^2(a+bx)}
\end{aligned}
$$

**Mathematica [A]** time = 0.05, size = 86, normalized size = 1.34

$$
\frac{-(bc-ad)\left(B\log\left(\frac{e(c+dx)}{a+bx}\right)+A-B\right)-Bd(a+bx)\log(c+dx)+Bd(a+bx)\log(a+bx)}{bg^2(a+bx)(bc-ad)}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Log[(e*(c + d*x))/(a + b*x)])/(a*g + b*g*x)^2,x]
```

```
[Out] (B*d*(a + b*x)*Log[a + b*x] - B*d*(a + b*x)*Log[c + d*x] - (b*c - a*d)*(A -
 B + B*Log[(e*(c + d*x))/(a + b*x)]))/(b*(b*c - a*d)*g^2*(a + b*x))
```

**fricas [A]** time = 1.73, size = 87, normalized size = 1.36

$$
-\frac{(A-B)bc-(A-B)ad+(Bbdx+Bbc)\log\left(\frac{dex+ce}{bx+a}\right)}{\left(b^3c-ab^2d\right)g^2x+\left(ab^2c-a^2bd\right)g^2}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*(d*x+c)/(b*x+a)))/(b*g*x+a*g)^2,x, algorithm="fricas")
```

```
[Out] -((A - B)*b*c - (A - B)*a*d + (B*b*d*x + B*b*c)*log((d*e*x + c*e)/(b*x + a)
))/((b^3*c - a*b^2*d)*g^2*x + (a*b^2*c - a^2*b*d)*g^2)
```

**giac [A]** time = 0.90, size = 126, normalized size = 1.97

$$-\left(\frac{bc}{(bce-ade)(bc-ad)}-\frac{ad}{(bce-ade)(bc-ad)}\right)\left(\frac{(dxe+ce)B\log\left(\frac{dxe+ce}{bx+a}\right)}{(bx+a)g^2}+\frac{(dxe+ce)(A-B)}{(bx+a)g^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*(d*x+c)/(b*x+a)))/(b*g*x+a*g)^2,x, algorithm="giac")
```

```
[Out] -(b*c/((b*c*e - a*d*e)*(b*c - a*d)) - a*d/((b*c*e - a*d*e)*(b*c - a*d)))*((
d*x*e + c*e)*B*log((d*x*e + c*e)/(b*x + a))/((b*x + a)*g^2) + (d*x*e + c*e)
*(A - B)/((b*x + a)*g^2))
```

**maple [B]** time = 0.05, size = 520, normalized size = 8.12

$$-\frac{B\,a^2d^2\ln\left(\frac{de}{b}-\frac{(ad-bc)e}{(bx+a)b}\right)}{(ad-bc)^2\,(bx+a)\,b\,g^2}+\frac{2Bacd\ln\left(\frac{de}{b}-\frac{(ad-bc)e}{(bx+a)b}\right)}{(ad-bc)^2\,(bx+a)\,g^2}-\frac{Bb\,c^2\ln\left(\frac{de}{b}-\frac{(ad-bc)e}{(bx+a)b}\right)}{(ad-bc)^2\,(bx+a)\,g^2}-\frac{A\,a^2d^2}{(ad-bc)^2\,(bx+a)\,b\,g^2}+\frac{2Aac}{(ad-bc)^2\,(b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*ln(e*(d*x+c)/(b*x+a)))/(b*g*x+a*g)^2,x)
```

```
[Out] 1/b/(a*d-b*c)^2/g^2*A*d^2*a-1/(a*d-b*c)^2/g^2*A*d*c-1/b/(a*d-b*c)^2/g^2*A/(
b*x+a)*a^2*d^2+2/(a*d-b*c)^2/g^2*A/(b*x+a)*a*d*c-b/(a*d-b*c)^2/g^2*A/(b*x+a
)*c^2+1/b/(a*d-b*c)^2/g^2*B*ln(1/b*d*e-(a*d-b*c)/(b*x+a)/b*e)*d^2*a-1/(a*d-
b*c)^2/g^2*B*ln(1/b*d*e-(a*d-b*c)/(b*x+a)/b*e)*d*c-1/b/(a*d-b*c)^2/g^2*B*ln
(1/b*d*e-(a*d-b*c)/(b*x+a)/b*e)/(b*x+a)*a^2*d^2+2/(a*d-b*c)^2/g^2*B*ln(1/b*
d*e-(a*d-b*c)/(b*x+a)/b*e)/(b*x+a)*a*d*c-b/(a*d-b*c)^2/g^2*B*ln(1/b*d*e-(a*
d-b*c)/(b*x+a)/b*e)/(b*x+a)*c^2+1/b/(a*d-b*c)^2/g^2*B/(b*x+a)*a^2*d^2-2/(a*
d-b*c)^2/g^2*B/(b*x+a)*a*d*c+b/(a*d-b*c)^2/g^2*B/(b*x+a)*c^2-1/b/(a*d-b*c)^
2/g^2*B*d^2*a+1/(a*d-b*c)^2/g^2*B*d*c
```

**maxima [B]** time = 1.07, size = 134, normalized size = 2.09

$$-B\left(\frac{\log\left(\frac{dex}{bx+a}+\frac{ce}{bx+a}\right)}{b^2g^2x+abg^2}-\frac{1}{b^2g^2x+abg^2}-\frac{d\log(bx+a)}{\left(b^2c-abd\right)g^2}+\frac{d\log(dx+c)}{\left(b^2c-abd\right)g^2}\right)-\frac{A}{b^2g^2x+abg^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*(d*x+c)/(b*x+a)))/(b*g*x+a*g)^2,x, algorithm="maxima")
```

```
[Out] -B*(log(d*e*x/(b*x + a) + c*e/(b*x + a))/(b^2*g^2*x + a*b*g^2) - 1/(b^2*g^2
*x + a*b*g^2) - d*log(b*x + a)/((b^2*c - a*b*d)*g^2) + d*log(d*x + c)/((b^2
*c - a*b*d)*g^2)) - A/(b^2*g^2*x + a*b*g^2)
```

**mupad [B]** time = 5.01, size = 106, normalized size = 1.66

$$-\frac{A-B}{x\,b^2\,g^2+a\,b\,g^2}-\frac{B\,\ln\left(\frac{e\,(c+d\,x)}{a+b\,x}\right)}{b^2\,g^2\,\left(x+\frac{a}{b}\right)}+\frac{B\,d\,\mathrm{atan}\left(\frac{b\,c\,2\mathrm{i}+b\,d\,x\,2\mathrm{i}}{a\,d-b\,c}+1\mathrm{i}\right)\,2\mathrm{i}}{b\,g^2\,\left(a\,d-b\,c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*log((e*(c + d*x))/(a + b*x)))/(a*g + b*g*x)^2,x)
```

```
[Out] (B*d*atan((b*c*2i + b*d*x*2i)/(a*d - b*c) + 1i)*2i)/(b*g^2*(a*d - b*c)) - (
B*log((e*(c + d*x))/(a + b*x)))/(b^2*g^2*(x + a/b)) - (A - B)/(b^2*g^2*x +
a*b*g^2)
```

**sympy [B]** time = 1.54, size = 231, normalized size = 3.61

$$-\frac{B\log\left(\frac{e(c+dx)}{a+bx}\right)}{abg^2+b^2g^2x}+\frac{Bd\log\left(x+\frac{-\frac{Ba^2d^3}{ad-bc}+\frac{2Babcd^2}{ad-bc}+Bad^2-\frac{Bb^2c^2d}{ad-bc}+Bbcd}{2Bbd^2}\right)}{bg^2\left(ad-bc\right)}-\frac{Bd\log\left(x+\frac{\frac{Ba^2d^3}{ad-bc}-\frac{2Babcd^2}{ad-bc}+Bad^2+\frac{Bb^2c^2d}{ad-bc}+Bbcd}{2Bbd^2}\right)}{bg^2\left(ad-bc\right)}+\frac{-A}{abg^2 \cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*ln(e*(d*x+c)/(b*x+a)))/(b*g*x+a*g)**2,x)
```

```
[Out] -B*log(e*(c + d*x)/(a + b*x))/(a*b*g**2 + b**2*g**2*x) + B*d*log(x + (-B*a*
*2*d**3/(a*d - b*c) + 2*B*a*b*c*d**2/(a*d - b*c) + B*a*d**2 - B*b**2*c**2*d
/(a*d - b*c) + B*b*c*d)/(2*B*b*d**2))/(b*g**2*(a*d - b*c)) - B*d*log(x + (B
*a**2*d**3/(a*d - b*c) - 2*B*a*b*c*d**2/(a*d - b*c) + B*a*d**2 + B*b**2*c**
2*d/(a*d - b*c) + B*b*c*d)/(2*B*b*d**2))/(b*g**2*(a*d - b*c)) + (-A + B)/(a
*b*g**2 + b**2*g**2*x)
```

# 3.179 $\int \frac{A+B\log\left(\frac{e(c+dx)}{a+bx}\right)}{(ag+bgx)^3}\,dx$

**Optimal**. Leaf size=144

$$-\frac{B\log\left(\frac{e(c+dx)}{a+bx}\right)+A}{2bg^3(a+bx)^2}-\frac{Bd^2\log(a+bx)}{2bg^3(bc-ad)^2}+\frac{Bd^2\log(c+dx)}{2bg^3(bc-ad)^2}-\frac{Bd}{2bg^3(a+bx)(bc-ad)}+\frac{B}{4bg^3(a+bx)^2}$$

```
[Out] 1/4*B/b/g^3/(b*x+a)^2-1/2*B*d/b/(-a*d+b*c)/g^3/(b*x+a)-1/2*B*d^2*ln(b*x+a)/
b/(-a*d+b*c)^2/g^3+1/2*B*d^2*ln(d*x+c)/b/(-a*d+b*c)^2/g^3+1/2*(-A-B*ln(e*(d
*x+c)/(b*x+a)))/b/g^3/(b*x+a)^2
```

**Rubi [A]** time = 0.10, antiderivative size = 144, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {2525, 12, 44}

$$-\frac{B\log\left(\frac{e(c+dx)}{a+bx}\right)+A}{2bg^3(a+bx)^2}-\frac{Bd^2\log(a+bx)}{2bg^3(bc-ad)^2}+\frac{Bd^2\log(c+dx)}{2bg^3(bc-ad)^2}-\frac{Bd}{2bg^3(a+bx)(bc-ad)}+\frac{B}{4bg^3(a+bx)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Log[(e*(c + d*x))/(a + b*x)])/(a*g + b*g*x)^3,x]
```

```
[Out] B/(4*b*g^3*(a + b*x)^2) - (B*d)/(2*b*(b*c - a*d)*g^3*(a + b*x)) - (B*d^2*Lo
g[a + b*x])/(2*b*(b*c - a*d)^2*g^3) + (B*d^2*Log[c + d*x])/(2*b*(b*c - a*d)
^2*g^3) - (A + B*Log[(e*(c + d*x))/(a + b*x)])/(2*b*g^3*(a + b*x)^2)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 2525

```
Int[((a_.) + Log[(c_.)*(RFx_)^(p_.)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_))^(m_.
), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(a + b*Log[c*RFx^p])^n)/(e*(m + 1))
```

```
, x] - Dist[(b*n*p)/(e*(m + 1)), Int[SimplifyIntegrand[((d + e*x)^(m + 1)*(
a + b*Log[c*RFx^p])^(n - 1)*D[RFx, x])/RFx, x], x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && RationalFunctionQ[RFx, x] && IGtQ[n, 0] && (EqQ[n, 1] ||
IntegerQ[m]) && NeQ[m, -1]
```

Rubi steps

$$\int \frac{A+B\log\left(\frac{e(c+dx)}{a+bx}\right)}{(ag+bgx)^3}\,dx = -\frac{A+B\log\left(\frac{e(c+dx)}{a+bx}\right)}{2bg^3(a+bx)^2}+\frac{B\int \frac{-bc+ad}{g^2(a+bx)^3(c+dx)}\,dx}{2bg}$$

$$= -\frac{A+B\log\left(\frac{e(c+dx)}{a+bx}\right)}{2bg^3(a+bx)^2}-\frac{(B(bc-ad))\int \frac{1}{(a+bx)^3(c+dx)}\,dx}{2bg^3}$$

$$= -\frac{A+B\log\left(\frac{e(c+dx)}{a+bx}\right)}{2bg^3(a+bx)^2}-\frac{(B(bc-ad))\int \left(\frac{b}{(bc-ad)(a+bx)^3}-\frac{bd}{(bc-ad)^2(a+bx)^2}+\frac{bd^2}{(bc-ad)^3(a+bx)}\right.}{2bg^3}$$

$$= \frac{B}{4bg^3(a+bx)^2}-\frac{Bd}{2b(bc-ad)g^3(a+bx)}-\frac{Bd^2\log(a+bx)}{2b(bc-ad)^2g^3}+\frac{Bd^2\log(c+dx)}{2b(bc-ad)^2g^3}-\frac{A+}{2}$$

**Mathematica [A]** time = 0.10, size = 128, normalized size = 0.89

$$-\frac{(bc-ad)\left(-2aAd+2B(bc-ad)\log\left(\frac{e(c+dx)}{a+bx}\right)+3aBd+2Abc-bBc+2bBdx\right)-2Bd^2(a+bx)^2\log(c+dx)+2}{4bg^3(a+bx)^2(bc-ad)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Log[(e*(c + d*x))/(a + b*x)])/(a*g + b*g*x)^3,x]
```

```
[Out] -1/4*(2*B*d^2*(a + b*x)^2*Log[a + b*x] - 2*B*d^2*(a + b*x)^2*Log[c + d*x] +
 (b*c - a*d)*(2*A*b*c - b*B*c - 2*a*A*d + 3*a*B*d + 2*b*B*d*x + 2*B*(b*c -
a*d)*Log[(e*(c + d*x))/(a + b*x)]))/(b*(b*c - a*d)^2*g^3*(a + b*x)^2)
```

**fricas [A]** time = 1.70, size = 221, normalized size = 1.53

$$-\frac{(2\,A-B)b^2c^2-4\,(A-B)abcd+(2\,A-3\,B)a^2d^2+2\left(Bb^2cd-Babd^2\right)x-2\left(Bb^2d^2x^2+2\,Babd^2x-Bb^2c^2+2\,B\right.}{4\left(\left(b^5c^2-2\,ab^4cd+a^2b^3d^2\right)g^3x^2+2\left(ab^4c^2-2\,a^2b^3cd+a^3b^2d^2\right)g^3x+\left(a^2b^3c^2-2\,a^3b^2cd+a^4bd\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*(d*x+c)/(b*x+a)))/(b*g*x+a*g)^3,x, algorithm="fricas")
```

```
[Out] -1/4*((2*A - B)*b^2*c^2 - 4*(A - B)*a*b*c*d + (2*A - 3*B)*a^2*d^2 + 2*(B*b^
2*c*d - B*a*b*d^2)*x - 2*(B*b^2*d^2*x^2 + 2*B*a*b*d^2*x - B*b^2*c^2 + 2*B*a
*b*c*d)*log((d*e*x + c*e)/(b*x + a)))/((b^5*c^2 - 2*a*b^4*c*d + a^2*b^3*d^2
)*g^3*x^2 + 2*(a*b^4*c^2 - 2*a^2*b^3*c*d + a^3*b^2*d^2)*g^3*x + (a^2*b^3*c^
2 - 2*a^3*b^2*c*d + a^4*b*d^2)*g^3)
```

**giac [A]** time = 0.86, size = 254, normalized size = 1.76

$$\frac{\left(\frac{4\,(dxe+ce)Bde\log\left(\frac{dxe+ce}{bx+a}\right)}{bx+a}+\frac{4\,(dxe+ce)Ade}{bx+a}-\frac{4\,(dxe+ce)Bde}{bx+a}-\frac{2\,(dxe+ce)^2Bb\log\left(\frac{dxe+ce}{bx+a}\right)}{(bx+a)^2}-\frac{2\,(dxe+ce)^2Ab}{(bx+a)^2}+\frac{(dxe+ce)^2Bb}{(bx+a)^2}\right)\left(\frac{bc}{(bce-ade)(bc-ad)}\right.}{4\left(bcg^3e-adg^3e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*(d*x+c)/(b*x+a)))/(b*g*x+a*g)^3,x, algorithm="giac")
```

```
[Out] 1/4*(4*(d*x*e + c*e)*B*d*e*log((d*x*e + c*e)/(b*x + a))/(b*x + a) + 4*(d*x*
e + c*e)*A*d*e/(b*x + a) - 4*(d*x*e + c*e)*B*d*e/(b*x + a) - 2*(d*x*e + c*e
)^2*B*b*log((d*x*e + c*e)/(b*x + a))/(b*x + a)^2 - 2*(d*x*e + c*e)^2*A*b/(b
*x + a)^2 + (d*x*e + c*e)^2*B*b/(b*x + a)^2)*(b*c/((b*c*e - a*d*e)*(b*c - a
*d)) - a*d/((b*c*e - a*d*e)*(b*c - a*d)))/(b*c*g^3*e - a*d*g^3*e)
```

**maple [B]** time = 0.05, size = 753, normalized size = 5.23

$$-\frac{B\,a^3d^3\ln\left(\frac{de}{b}-\frac{(ad-bc)e}{(bx+a)b}\right)}{2\,(ad-bc)^3\,(bx+a)^2\,b\,g^3}+\frac{3B\,a^2c\,d^2\ln\left(\frac{de}{b}-\frac{(ad-bc)e}{(bx+a)b}\right)}{2\,(ad-bc)^3\,(bx+a)^2\,g^3}-\frac{3Bab\,c^2d\ln\left(\frac{de}{b}-\frac{(ad-bc)e}{(bx+a)b}\right)}{2\,(ad-bc)^3\,(bx+a)^2\,g^3}+\frac{B\,b^2c^3\ln\left(\frac{de}{b}-\frac{(ad-bc)e}{(bx+a)b}\right)}{2\,(ad-bc)^3\,(bx+a)^2\,g^3}-\cdots$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*ln(e*(d*x+c)/(b*x+a)))/(b*g*x+a*g)^3,x)
```

```
[Out] -1/2/b/(a*d-b*c)^3/g^3*A/(b*x+a)^2*a^3*d^3+1/2/b/(a*d-b*c)^3/g^3*B*ln(1/b*d
*e-(a*d-b*c)/(b*x+a)/b*e)*d^3*a-1/2/(a*d-b*c)^3/g^3*B*ln(1/b*d*e-(a*d-b*c)/
(b*x+a)/b*e)*d^2*c+1/4/b/(a*d-b*c)^3/g^3*B/(b*x+a)^2*a^3*d^3+1/2/b/(a*d-b*c
)^3/g^3*B*d^3/(b*x+a)*a^2+1/2*b^2/(a*d-b*c)^3/g^3*B*ln(1/b*d*e-(a*d-b*c)/(b
*x+a)/b*e)/(b*x+a)^2*c^3+1/2*b/(a*d-b*c)^3/g^3*B*d/(b*x+a)*c^2-3/2*b/(a*d-b
*c)^3/g^3*B*ln(1/b*d*e-(a*d-b*c)/(b*x+a)/b*e)/(b*x+a)^2*a*d*c^2+3/2/(a*d-b*
c)^3/g^3*B*ln(1/b*d*e-(a*d-b*c)/(b*x+a)/b*e)/(b*x+a)^2*a^2*d^2*c+1/2/b/(a*d
-b*c)^3/g^3*A*d^3*a-1/2/(a*d-b*c)^3/g^3*A*d^2*c-3/4/b/(a*d-b*c)^3/g^3*B*d^3
*a+3/4/(a*d-b*c)^3/g^3*B*d^2*c-1/4*b^2/(a*d-b*c)^3/g^3*B/(b*x+a)^2*c^3+1/2*
b^2/(a*d-b*c)^3/g^3*A/(b*x+a)^2*c^3+3/2/(a*d-b*c)^3/g^3*A/(b*x+a)^2*a^2*d^2
*c-3/2*b/(a*d-b*c)^3/g^3*A/(b*x+a)^2*a*d*c^2-1/(a*d-b*c)^3/g^3*B*d^2/(b*x+a
)*c*a-3/4/(a*d-b*c)^3/g^3*B/(b*x+a)^2*a^2*d^2*c+3/4*b/(a*d-b*c)^3/g^3*B/(b*
x+a)^2*c^2*a*d-1/2/b/(a*d-b*c)^3/g^3*B*ln(1/b*d*e-(a*d-b*c)/(b*x+a)/b*e)/(b
*x+a)^2*a^3*d^3
```

**maxima [A]** time = 1.28, size = 255, normalized size = 1.77

$$-\frac{1}{4}\,B\left(\frac{2\,bdx - bc + 3\,ad}{\left(b^4c - ab^3d\right)g^3x^2 + 2\left(ab^3c - a^2b^2d\right)g^3x + \left(a^2b^2c - a^3bd\right)g^3} + \frac{2\,\log\left(\frac{dex}{bx+a} + \frac{ce}{bx+a}\right)}{b^3g^3x^2 + 2\,ab^2g^3x + a^2bg^3} + \frac{2\,d^2\log}{\left(b^3c^2 - 2\,ab^2\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*(d*x+c)/(b*x+a)))/(b*g*x+a*g)^3,x, algorithm="maxima")
```

```
[Out] -1/4*B*((2*b*d*x - b*c + 3*a*d)/((b^4*c - a*b^3*d)*g^3*x^2 + 2*(a*b^3*c - a
^2*b^2*d)*g^3*x + (a^2*b^2*c - a^3*b*d)*g^3) + 2*log(d*e*x/(b*x + a) + c*e/
(b*x + a))/(b^3*g^3*x^2 + 2*a*b^2*g^3*x + a^2*b*g^3) + 2*d^2*log(b*x + a)/(
(b^3*c^2 - 2*a*b^2*c*d + a^2*b*d^2)*g^3) - 2*d^2*log(d*x + c)/((b^3*c^2 - 2
*a*b^2*c*d + a^2*b*d^2)*g^3)) - 1/2*A/(b^3*g^3*x^2 + 2*a*b^2*g^3*x + a^2*b*
g^3)
```

**mupad [B]** time = 5.19, size = 208, normalized size = 1.44

$$\frac{B\,d^2\,\mathrm{atanh}\left(\frac{2\,b^3\,c^2\,g^3-2\,a^2\,b\,d^2\,g^3}{2\,b\,g^3\,(a\,d-b\,c)^2}-\frac{2\,b\,d\,x}{a\,d-b\,c}\right)}{b\,g^3\,(a\,d-b\,c)^2}-\frac{B\,\ln\left(\frac{e\,(c+d\,x)}{a+b\,x}\right)}{2\,b^2\,g^3\,\left(2\,a\,x+b\,x^2+\frac{a^2}{b}\right)}-\frac{\frac{2\,A\,a\,d-2\,A\,b\,c-3\,B\,a\,d+B\,b\,c}{2\,(a\,d-b\,c)}-\frac{B\,b\,d\,x}{a\,d-b\,c}}{2\,a^2\,b\,g^3+4\,a\,b^2\,g^3\,x+2\,b^3\,g^3\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*log((e*(c + d*x))/(a + b*x)))/(a*g + b*g*x)^3,x)
```

```
[Out] (B*d^2*atanh((2*b^3*c^2*g^3 - 2*a^2*b*d^2*g^3)/(2*b*g^3*(a*d - b*c)^2) - (2
*b*d*x)/(a*d - b*c)))/(b*g^3*(a*d - b*c)^2) - (B*log((e*(c + d*x))/(a + b*x
)))/(2*b^2*g^3*(2*a*x + b*x^2 + a^2/b)) - ((2*A*a*d - 2*A*b*c - 3*B*a*d + B
*b*c)/(2*(a*d - b*c)) - (B*b*d*x)/(a*d - b*c))/(2*a^2*b*g^3 + 2*b^3*g^3*x^2
 + 4*a*b^2*g^3*x)
```

**sympy [B]** time = 2.68, size = 422, normalized size = 2.93

$$-\frac{B\log\left(\frac{e(c+dx)}{a+bx}\right)}{2a^2bg^3+4ab^2g^3x+2b^3g^3x^2}+\frac{Bd^2\log\left(x+\frac{-\frac{Ba^3d^5}{(ad-bc)^2}+\frac{3Ba^2bcd^4}{(ad-bc)^2}-\frac{3Bab^2c^2d^3}{(ad-bc)^2}+Bad^3+\frac{Bb^3c^3d^2}{(ad-bc)^2}+Bbcd^2}{2Bbd^3}\right)}{2bg^3(ad-bc)^2}-\frac{Bd^2\log\left(x+\frac{\frac{Ba^3d^5}{(ad-bc)^2}-\cdots}{\cdots}\right)}{2\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*ln(e*(d*x+c)/(b*x+a)))/(b*g*x+a*g)**3,x)
```

```
[Out] -B*log(e*(c + d*x)/(a + b*x))/(2*a**2*b*g**3 + 4*a*b**2*g**3*x + 2*b**3*g**
3*x**2) + B*d**2*log(x + (-B*a**3*d**5/(a*d - b*c)**2 + 3*B*a**2*b*c*d**4/(
a*d - b*c)**2 - 3*B*a*b**2*c**2*d**3/(a*d - b*c)**2 + B*a*d**3 + B*b**3*c**
3*d**2/(a*d - b*c)**2 + B*b*c*d**2)/(2*B*b*d**3))/(2*b*g**3*(a*d - b*c)**2)
 - B*d**2*log(x + (B*a**3*d**5/(a*d - b*c)**2 - 3*B*a**2*b*c*d**4/(a*d - b*
c)**2 + 3*B*a*b**2*c**2*d**3/(a*d - b*c)**2 + B*a*d**3 - B*b**3*c**3*d**2/(
a*d - b*c)**2 + B*b*c*d**2)/(2*B*b*d**3))/(2*b*g**3*(a*d - b*c)**2) + (-2*A
*a*d + 2*A*b*c + 3*B*a*d - B*b*c + 2*B*b*d*x)/(4*a**3*b*d*g**3 - 4*a**2*b**
2*c*g**3 + x**2*(4*a*b**3*d*g**3 - 4*b**4*c*g**3) + x*(8*a**2*b**2*d*g**3 -
 8*a*b**3*c*g**3))
```

# 3.180 $\int \frac{A+B\log\left(\frac{e(c+dx)}{a+bx}\right)}{(ag+bgx)^4}\,dx$

**Optimal**. Leaf size=175

$$-\frac{B\log\left(\frac{e(c+dx)}{a+bx}\right)+A}{3bg^4(a+bx)^3}+\frac{Bd^3\log(a+bx)}{3bg^4(bc-ad)^3}-\frac{Bd^3\log(c+dx)}{3bg^4(bc-ad)^3}+\frac{Bd^2}{3bg^4(a+bx)(bc-ad)^2}-\frac{Bd}{6bg^4(a+bx)^2(bc-ad)}+\frac{}{9bg}$$

```
[Out] 1/9*B/b/g^4/(b*x+a)^3-1/6*B*d/b/(-a*d+b*c)/g^4/(b*x+a)^2+1/3*B*d^2/b/(-a*d+
b*c)^2/g^4/(b*x+a)+1/3*B*d^3*ln(b*x+a)/b/(-a*d+b*c)^3/g^4-1/3*B*d^3*ln(d*x+
c)/b/(-a*d+b*c)^3/g^4+1/3*(-A-B*ln(e*(d*x+c)/(b*x+a)))/b/g^4/(b*x+a)^3
```

**Rubi** [A] time = 0.13, antiderivative size = 175, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {2525, 12, 44}

$$-\frac{B\log\left(\frac{e(c+dx)}{a+bx}\right)+A}{3bg^4(a+bx)^3}+\frac{Bd^2}{3bg^4(a+bx)(bc-ad)^2}+\frac{Bd^3\log(a+bx)}{3bg^4(bc-ad)^3}-\frac{Bd^3\log(c+dx)}{3bg^4(bc-ad)^3}-\frac{Bd}{6bg^4(a+bx)^2(bc-ad)}+\frac{}{9bg}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Log[(e*(c + d*x))/(a + b*x)])/(a*g + b*g*x)^4,x]
```

```
[Out] B/(9*b*g^4*(a + b*x)^3) - (B*d)/(6*b*(b*c - a*d)*g^4*(a + b*x)^2) + (B*d^2)
/(3*b*(b*c - a*d)^2*g^4*(a + b*x)) + (B*d^3*Log[a + b*x])/(3*b*(b*c - a*d)^
3*g^4) - (B*d^3*Log[c + d*x])/(3*b*(b*c - a*d)^3*g^4) - (A + B*Log[(e*(c +
d*x))/(a + b*x)])/(3*b*g^4*(a + b*x)^3)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 2525

```
Int[((a_.) + Log[(c_.)*(RFx_)^(p_.)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_))^(m_.
), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(a + b*Log[c*RFx^p])^n)/(e*(m + 1))
```

```
, x] - Dist[(b*n*p)/(e*(m + 1)), Int[SimplifyIntegrand[((d + e*x)^(m + 1)*(
a + b*Log[c*RFx^p])^(n - 1)*D[RFx, x])/RFx, x], x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && RationalFunctionQ[RFx, x] && IGtQ[n, 0] && (EqQ[n, 1] ||
IntegerQ[m]) && NeQ[m, -1]
```

Rubi steps

$$\int \frac{A+B\log\left(\frac{e(c+dx)}{a+bx}\right)}{(ag+bgx)^4}\,dx = -\frac{A+B\log\left(\frac{e(c+dx)}{a+bx}\right)}{3bg^4(a+bx)^3}+\frac{B\int\frac{-bc+ad}{g^3(a+bx)^4(c+dx)}\,dx}{3bg}$$

$$= -\frac{A+B\log\left(\frac{e(c+dx)}{a+bx}\right)}{3bg^4(a+bx)^3}-\frac{(B(bc-ad))\int\frac{1}{(a+bx)^4(c+dx)}\,dx}{3bg^4}$$

$$= -\frac{A+B\log\left(\frac{e(c+dx)}{a+bx}\right)}{3bg^4(a+bx)^3}-\frac{(B(bc-ad))\int\left(\frac{b}{(bc-ad)(a+bx)^4}-\frac{bd}{(bc-ad)^2(a+bx)^3}+\frac{bd^2}{(bc-ad)^3(a+bx)^2}-\right.}{3bg^4}$$

$$= \frac{B}{9bg^4(a+bx)^3}-\frac{Bd}{6b(bc-ad)g^4(a+bx)^2}+\frac{Bd^2}{3b(bc-ad)^2g^4(a+bx)}+\frac{Bd^3\log(a+bx)}{3b(bc-ad)^3g^4}$$

**Mathematica [A]** time = 0.14, size = 141, normalized size = 0.81

$$\frac{\frac{B\left((bc-ad)\left(11a^2d^2+abd(15dx-7c)+b^2\left(2c^2-3cdx+6d^2x^2\right)\right)-6d^3(a+bx)^3\log(c+dx)+6d^3(a+bx)^3\log(a+bx)\right)}{(bc-ad)^3}-6\left(B\log\left(\frac{e(c+dx)}{a+bx}\right)+A\right)}{18bg^4(a+bx)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Log[(e*(c + d*x))/(a + b*x)])/(a*g + b*g*x)^4,x]
```

```
[Out] ((B*((b*c - a*d)*(11*a^2*d^2 + a*b*d*(-7*c + 15*d*x) + b^2*(2*c^2 - 3*c*d*x
 + 6*d^2*x^2)) + 6*d^3*(a + b*x)^3*Log[a + b*x] - 6*d^3*(a + b*x)^3*Log[c +
 d*x]))/(b*c - a*d)^3 - 6*(A + B*Log[(e*(c + d*x))/(a + b*x)]))/(18*b*g^4*(
a + b*x)^3)
```

**fricas [B]** time = 1.12, size = 412, normalized size = 2.35

$$-\frac{2\,(3\,A-B)b^3c^3-9\,(2\,A-B)ab^2c^2d+18\,(A-B)a^2bcd^2-(6\,A-11\,B)a^3d^3-6\left(Bb^3cd^2-Bab^2d^3\right)x^2+3\left(Bb^3c^2\right.}{18\left(\left(b^7c^3-3\,ab^6c^2d+3\,a^2b^5cd^2-a^3b^4d^3\right)g^4x^3+3\left(ab^6c^3-3\,a^2b^5c^2d+3\,a^3b^4cd^2-a^4b^3d^3\right)g^4x\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*(d*x+c)/(b*x+a)))/(b*g*x+a*g)^4,x, algorithm="fricas")
```

```
[Out] -1/18*(2*(3*A - B)*b^3*c^3 - 9*(2*A - B)*a*b^2*c^2*d + 18*(A - B)*a^2*b*c*d
^2 - (6*A - 11*B)*a^3*d^3 - 6*(B*b^3*c*d^2 - B*a*b^2*d^3)*x^2 + 3*(B*b^3*c^
2*d - 6*B*a*b^2*c*d^2 + 5*B*a^2*b*d^3)*x + 6*(B*b^3*d^3*x^3 + 3*B*a*b^2*d^3
*x^2 + 3*B*a^2*b*d^3*x + B*b^3*c^3 - 3*B*a*b^2*c^2*d + 3*B*a^2*b*c*d^2)*log
((d*e*x + c*e)/(b*x + a)))/((b^7*c^3 - 3*a*b^6*c^2*d + 3*a^2*b^5*c*d^2 - a^
3*b^4*d^3)*g^4*x^3 + 3*(a*b^6*c^3 - 3*a^2*b^5*c^2*d + 3*a^3*b^4*c*d^2 - a^4
*b^3*d^3)*g^4*x^2 + 3*(a^2*b^5*c^3 - 3*a^3*b^4*c^2*d + 3*a^4*b^3*c*d^2 - a^
5*b^2*d^3)*g^4*x + (a^3*b^4*c^3 - 3*a^4*b^3*c^2*d + 3*a^5*b^2*c*d^2 - a^6*b
*d^3)*g^4)
```

**giac [B]** time = 1.06, size = 382, normalized size = 2.18

$$-\frac{\left(\frac{18\,(dxe+ce)Bd^2e^2\log\left(\frac{dxe+ce}{bx+a}\right)}{bx+a}-\frac{18\,(dxe+ce)^2Bbde\log\left(\frac{dxe+ce}{bx+a}\right)}{(bx+a)^2}+\frac{18\,(dxe+ce)Ad^2e^2}{bx+a}-\frac{18\,(dxe+ce)Bd^2e^2}{bx+a}-\frac{18\,(dxe+ce)^2Abde}{(bx+a)^2}+\frac{9\,(dxe+ce)^2 B}{(bx+a)^2}\cdots\right.}{18\left(b^2c^2g^4e^2-2\,abcdg^4e^2+a\cdots\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*(d*x+c)/(b*x+a)))/(b*g*x+a*g)^4,x, algorithm="giac")
```

```
[Out] -1/18*(18*(d*x*e + c*e)*B*d^2*e^2*log((d*x*e + c*e)/(b*x + a))/(b*x + a) -
18*(d*x*e + c*e)^2*B*b*d*e*log((d*x*e + c*e)/(b*x + a))/(b*x + a)^2 + 18*(d
*x*e + c*e)*A*d^2*e^2/(b*x + a) - 18*(d*x*e + c*e)*B*d^2*e^2/(b*x + a) - 18
*(d*x*e + c*e)^2*A*b*d*e/(b*x + a)^2 + 9*(d*x*e + c*e)^2*B*b*d*e/(b*x + a)^
2 + 6*(d*x*e + c*e)^3*B*b^2*log((d*x*e + c*e)/(b*x + a))/(b*x + a)^3 + 6*(d
*x*e + c*e)^3*A*b^2/(b*x + a)^3 - 2*(d*x*e + c*e)^3*B*b^2/(b*x + a)^3)*(b*c
/((b*c*e - a*d*e)*(b*c - a*d)) - a*d/((b*c*e - a*d*e)*(b*c - a*d)))/(b^2*c^
2*g^4*e^2 - 2*a*b*c*d*g^4*e^2 + a^2*d^2*g^4*e^2)
```

**maple [B]** time = 0.05, size = 1012, normalized size = 5.78

$$-\frac{B\,a^4d^4\ln\left(\frac{de}{b}-\frac{(ad-bc)e}{(bx+a)b}\right)}{3\,(ad-bc)^4\,(bx+a)^3\,b\,g^4}+\frac{4B\,a^3c\,d^3\ln\left(\frac{de}{b}-\frac{(ad-bc)e}{(bx+a)b}\right)}{3\,(ad-bc)^4\,(bx+a)^3\,g^4}-\frac{2B\,a^2b\,c^2d^2\ln\left(\frac{de}{b}-\frac{(ad-bc)e}{(bx+a)b}\right)}{(ad-bc)^4\,(bx+a)^3\,g^4}+\frac{4Ba\,b^2c^3d\ln\left(\frac{de}{b}-\frac{(ad-bc)e}{(bx+a)b}\right)}{3\,(ad-bc)^4\,(bx+a)^3\,g\cdots}\cdots$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*ln(e*(d*x+c)/(b*x+a)))/(b*g*x+a*g)^4,x)
```

```
[Out] 1/3/b/(a*d-b*c)^4/g^4*A*d^4*a+4/3/(a*d-b*c)^4/g^4*B*ln(1/b*d*e-(a*d-b*c)/(b
*x+a)/b*e)/(b*x+a)^3*a^3*d^3*c-1/3/b/(a*d-b*c)^4/g^4*B*ln(1/b*d*e-(a*d-b*c)
/(b*x+a)/b*e)/(b*x+a)^3*a^4*d^4+1/2*b/(a*d-b*c)^4/g^4*B*d^2/(b*x+a)^2*c^2*a
-4/9*b^2/(a*d-b*c)^4/g^4*B/(b*x+a)^3*c^3*a*d-1/3/(a*d-b*c)^4/g^4*B*ln(1/b*d
*e-(a*d-b*c)/(b*x+a)/b*e)*d^3*c-1/3*b^3/(a*d-b*c)^4/g^4*A/(b*x+a)^3*c^4+1/9
```

```
*b^3/(a*d-b*c)^4/g^4*B/(b*x+a)^3*c^4+2/3*b/(a*d-b*c)^4/g^4*B/(b*x+a)^3*a^2*
d^2*c^2-2*b/(a*d-b*c)^4/g^4*A/(b*x+a)^3*a^2*d^2*c^2+4/3*b^2/(a*d-b*c)^4/g^4
*A/(b*x+a)^3*a*d*c^3+11/18/(a*d-b*c)^4/g^4*B*d^3*c+1/9/b/(a*d-b*c)^4/g^4*B/
(b*x+a)^3*a^4*d^4-1/3/b/(a*d-b*c)^4/g^4*A/(b*x+a)^3*a^4*d^4+1/3*b/(a*d-b*c)
^4/g^4*B*d^2/(b*x+a)*c^2+1/6/b/(a*d-b*c)^4/g^4*B*d^4/(b*x+a)^2*a^3-1/6*b^2/
(a*d-b*c)^4/g^4*B*d/(b*x+a)^2*c^3+1/3/b/(a*d-b*c)^4/g^4*B*d^4/(b*x+a)*a^2+4
/3/(a*d-b*c)^4/g^4*A/(b*x+a)^3*a^3*d^3*c+1/3/b/(a*d-b*c)^4/g^4*B*ln(1/b*d*e
-(a*d-b*c)/(b*x+a)/b*e)*d^4*a-1/3*b^3/(a*d-b*c)^4/g^4*B*ln(1/b*d*e-(a*d-b*c
)/(b*x+a)/b*e)/(b*x+a)^3*c^4-4/9/(a*d-b*c)^4/g^4*B/(b*x+a)^3*a^3*d^3*c-2/3/
(a*d-b*c)^4/g^4*B*d^3/(b*x+a)*c*a-1/2/(a*d-b*c)^4/g^4*B*d^3/(b*x+a)^2*a^2*c
-1/3/(a*d-b*c)^4/g^4*A*d^3*c-11/18/b/(a*d-b*c)^4/g^4*B*d^4*a-2*b/(a*d-b*c)^
4/g^4*B*ln(1/b*d*e-(a*d-b*c)/(b*x+a)/b*e)/(b*x+a)^3*a^2*d^2*c^2+4/3*b^2/(a*
d-b*c)^4/g^4*B*ln(1/b*d*e-(a*d-b*c)/(b*x+a)/b*e)/(b*x+a)^3*c^3*a*d
```

**maxima [B]** time = 1.37, size = 428, normalized size = 2.45

$$\frac{1}{18}B\left(\frac{6\,b^2d^2x^2+2\,b^2c^2-7\,abcd+11\,a^2d^2-3\left(b^2cd-5\,abd^2\right)x}{\left(b^6c^2-2\,ab^5cd+a^2b^4d^2\right)g^4x^3+3\left(ab^5c^2-2\,a^2b^4cd+a^3b^3d^2\right)g^4x^2+3\left(a^2b^4c^2-2\,a^3b^3cd+a^4b^2d^2\right)g^4x+\left(\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*(d*x+c)/(b*x+a)))/(b*g*x+a*g)^4,x, algorithm="maxima")
```

```
[Out] 1/18*B*((6*b^2*d^2*x^2 + 2*b^2*c^2 - 7*a*b*c*d + 11*a^2*d^2 - 3*(b^2*c*d -
5*a*b*d^2)*x)/((b^6*c^2 - 2*a*b^5*c*d + a^2*b^4*d^2)*g^4*x^3 + 3*(a*b^5*c^2
 - 2*a^2*b^4*c*d + a^3*b^3*d^2)*g^4*x^2 + 3*(a^2*b^4*c^2 - 2*a^3*b^3*c*d +
a^4*b^2*d^2)*g^4*x + (a^3*b^3*c^2 - 2*a^4*b^2*c*d + a^5*b*d^2)*g^4) - 6*log
(d*e*x/(b*x + a) + c*e/(b*x + a))/(b^4*g^4*x^3 + 3*a*b^3*g^4*x^2 + 3*a^2*b^
2*g^4*x + a^3*b*g^4) + 6*d^3*log(b*x + a)/((b^4*c^3 - 3*a*b^3*c^2*d + 3*a^2
*b^2*c*d^2 - a^3*b*d^3)*g^4) - 6*d^3*log(d*x + c)/((b^4*c^3 - 3*a*b^3*c^2*d
 + 3*a^2*b^2*c*d^2 - a^3*b*d^3)*g^4)) - 1/3*A/(b^4*g^4*x^3 + 3*a*b^3*g^4*x^
2 + 3*a^2*b^2*g^4*x + a^3*b*g^4)
```

**mupad [B]** time = 5.88, size = 339, normalized size = 1.94

$$\frac{B\,b\,c^2}{9\,g^4\,{\left(a\,d-b\,c\right)}^2\,{\left(a+b\,x\right)}^3}-\frac{A\,b\,c^2}{3\,g^4\,{\left(a\,d-b\,c\right)}^2\,{\left(a+b\,x\right)}^3}-\frac{B\,\ln\left(\frac{e\,\left(c+d\,x\right)}{a+b\,x}\right)}{3\,b\,g^4\,{\left(a+b\,x\right)}^3}-\frac{A\,a^2\,d^2}{3\,b\,g^4\,{\left(a\,d-b\,c\right)}^2\,{\left(a+b\,x\right)}^3}+\frac{11\,B}{18\,b\,g^4\,\left(a\,d-\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*log((e*(c + d*x))/(a + b*x)))/(a*g + b*g*x)^4,x)
```

```
[Out] (B*d^3*atan((a*d*1i + b*c*1i + b*d*x*2i)/(a*d - b*c))*2i)/(3*b*g^4*(a*d - b
*c)^3) - (B*log((e*(c + d*x))/(a + b*x)))/(3*b*g^4*(a + b*x)^3) - (A*b*c^2)
```

```
/(3*g^4*(a*d - b*c)^2*(a + b*x)^3) + (B*b*c^2)/(9*g^4*(a*d - b*c)^2*(a + b*
x)^3) - (A*a^2*d^2)/(3*b*g^4*(a*d - b*c)^2*(a + b*x)^3) + (11*B*a^2*d^2)/(1
8*b*g^4*(a*d - b*c)^2*(a + b*x)^3) + (5*B*a*d^2*x)/(6*g^4*(a*d - b*c)^2*(a
+ b*x)^3) + (B*b*d^2*x^2)/(3*g^4*(a*d - b*c)^2*(a + b*x)^3) + (2*A*a*c*d)/(
3*g^4*(a*d - b*c)^2*(a + b*x)^3) - (7*B*a*c*d)/(18*g^4*(a*d - b*c)^2*(a + b
*x)^3) - (B*b*c*d*x)/(6*g^4*(a*d - b*c)^2*(a + b*x)^3)
```

**sympy [B]** time = 4.06, size = 656, normalized size = 3.75

$$-\frac{B\log\left(\frac{e(c+dx)}{a+bx}\right)}{3a^3bg^4+9a^2b^2g^4x+9ab^3g^4x^2+3b^4g^4x^3}+\frac{Bd^3\log\left(x+\frac{-\frac{Ba^4d^7}{(ad-bc)^3}+\frac{4Ba^3bcd^6}{(ad-bc)^3}-\frac{6Ba^2b^2c^2d^5}{(ad-bc)^3}+\frac{4Bab^3c^3d^4}{(ad-bc)^3}+Bad^4-\frac{Bb^4c^4d^3}{(ad-bc)^3}+Bbcd^3}{2Bbd^4}\right)}{3bg^4(ad-bc)^3}-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*ln(e*(d*x+c)/(b*x+a)))/(b*g*x+a*g)**4,x)
```

```
[Out] -B*log(e*(c + d*x)/(a + b*x))/(3*a**3*b*g**4 + 9*a**2*b**2*g**4*x + 9*a*b**
3*g**4*x**2 + 3*b**4*g**4*x**3) + B*d**3*log(x + (-B*a**4*d**7/(a*d - b*c)*
*3 + 4*B*a**3*b*c*d**6/(a*d - b*c)**3 - 6*B*a**2*b**2*c**2*d**5/(a*d - b*c)
**3 + 4*B*a*b**3*c**3*d**4/(a*d - b*c)**3 + B*a*d**4 - B*b**4*c**4*d**3/(a*
d - b*c)**3 + B*b*c*d**3)/(2*B*b*d**4))/(3*b*g**4*(a*d - b*c)**3) - B*d**3*
log(x + (B*a**4*d**7/(a*d - b*c)**3 - 4*B*a**3*b*c*d**6/(a*d - b*c)**3 + 6*
B*a**2*b**2*c**2*d**5/(a*d - b*c)**3 - 4*B*a*b**3*c**3*d**4/(a*d - b*c)**3
+ B*a*d**4 + B*b**4*c**4*d**3/(a*d - b*c)**3 + B*b*c*d**3)/(2*B*b*d**4))/(3
*b*g**4*(a*d - b*c)**3) + (-6*A*a**2*d**2 + 12*A*a*b*c*d - 6*A*b**2*c**2 +
11*B*a**2*d**2 - 7*B*a*b*c*d + 2*B*b**2*c**2 + 6*B*b**2*d**2*x**2 + x*(15*B
*a*b*d**2 - 3*B*b**2*c*d))/(18*a**5*b*d**2*g**4 - 36*a**4*b**2*c*d*g**4 + 1
8*a**3*b**3*c**2*g**4 + x**3*(18*a**2*b**4*d**2*g**4 - 36*a*b**5*c*d*g**4 +
 18*b**6*c**2*g**4) + x**2*(54*a**3*b**3*d**2*g**4 - 108*a**2*b**4*c*d*g**4
 + 54*a*b**5*c**2*g**4) + x*(54*a**4*b**2*d**2*g**4 - 108*a**3*b**3*c*d*g**
4 + 54*a**2*b**4*c**2*g**4))
```

# 3.181 $\int \frac{A+B\log\left(\frac{e(c+dx)}{a+bx}\right)}{(ag+bgx)^5}\,dx$

**Optimal**. Leaf size=206

$$-\frac{B\log\left(\frac{e(c+dx)}{a+bx}\right)+A}{4bg^5(a+bx)^4}-\frac{Bd^4\log(a+bx)}{4bg^5(bc-ad)^4}+\frac{Bd^4\log(c+dx)}{4bg^5(bc-ad)^4}-\frac{Bd^3}{4bg^5(a+bx)(bc-ad)^3}+\frac{Bd^2}{8bg^5(a+bx)^2(bc-ad)^2}-\frac{}{12bg}$$

```
[Out] 1/16*B/b/g^5/(b*x+a)^4-1/12*B*d/b/(-a*d+b*c)/g^5/(b*x+a)^3+1/8*B*d^2/b/(-a*
d+b*c)^2/g^5/(b*x+a)^2-1/4*B*d^3/b/(-a*d+b*c)^3/g^5/(b*x+a)-1/4*B*d^4*ln(b*
x+a)/b/(-a*d+b*c)^4/g^5+1/4*B*d^4*ln(d*x+c)/b/(-a*d+b*c)^4/g^5+1/4*(-A-B*ln
(e*(d*x+c)/(b*x+a)))/b/g^5/(b*x+a)^4
```

**Rubi [A]** time = 0.15, antiderivative size = 206, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {2525, 12, 44}

$$-\frac{B\log\left(\frac{e(c+dx)}{a+bx}\right)+A}{4bg^5(a+bx)^4}-\frac{Bd^3}{4bg^5(a+bx)(bc-ad)^3}+\frac{Bd^2}{8bg^5(a+bx)^2(bc-ad)^2}-\frac{Bd^4\log(a+bx)}{4bg^5(bc-ad)^4}+\frac{Bd^4\log(c+dx)}{4bg^5(bc-ad)^4}-\frac{}{12bg}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Log[(e*(c + d*x))/(a + b*x)])/(a*g + b*g*x)^5,x]
```

```
[Out] B/(16*b*g^5*(a + b*x)^4) - (B*d)/(12*b*(b*c - a*d)*g^5*(a + b*x)^3) + (B*d^
2)/(8*b*(b*c - a*d)^2*g^5*(a + b*x)^2) - (B*d^3)/(4*b*(b*c - a*d)^3*g^5*(a
+ b*x)) - (B*d^4*Log[a + b*x])/(4*b*(b*c - a*d)^4*g^5) + (B*d^4*Log[c + d*x
])/(4*b*(b*c - a*d)^4*g^5) - (A + B*Log[(e*(c + d*x))/(a + b*x)])/(4*b*g^5*
(a + b*x)^4)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 2525

```
Int[((a_.) + Log[(c_.)*(RFx_)^(p_.)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_))^(m_.
), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(a + b*Log[c*RFx^p])^n)/(e*(m + 1))
, x] - Dist[(b*n*p)/(e*(m + 1)), Int[SimplifyIntegrand[((d + e*x)^(m + 1)*(
a + b*Log[c*RFx^p])^(n - 1)*D[RFx, x])/RFx, x], x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && RationalFunctionQ[RFx, x] && IGtQ[n, 0] && (EqQ[n, 1] ||
IntegerQ[m]) && NeQ[m, -1]
```

Rubi steps

$$\int \frac{A+B\log\left(\frac{e(c+dx)}{a+bx}\right)}{(ag+bgx)^5}\,dx = -\frac{A+B\log\left(\frac{e(c+dx)}{a+bx}\right)}{4bg^5(a+bx)^4} + \frac{B\int \frac{-bc+ad}{g^4(a+bx)^5(c+dx)}\,dx}{4bg}$$

$$= -\frac{A+B\log\left(\frac{e(c+dx)}{a+bx}\right)}{4bg^5(a+bx)^4} - \frac{(B(bc-ad))\int \frac{1}{(a+bx)^5(c+dx)}\,dx}{4bg^5}$$

$$= -\frac{A+B\log\left(\frac{e(c+dx)}{a+bx}\right)}{4bg^5(a+bx)^4} - \frac{(B(bc-ad))\int \left(\frac{b}{(bc-ad)(a+bx)^5} - \frac{bd}{(bc-ad)^2(a+bx)^4} + \frac{bd^2}{(bc-ad)^3(a+bx)^3}\right.}{4bg^5}$$

$$= \frac{B}{16bg^5(a+bx)^4} - \frac{Bd}{12b(bc-ad)g^5(a+bx)^3} + \frac{Bd^2}{8b(bc-ad)^2g^5(a+bx)^2} - \frac{Bd}{4b(bc-ad)}$$

**Mathematica [A]** time = 0.19, size = 166, normalized size = 0.81

$$\frac{\frac{B(ad-bc)\left(\frac{12d^3(bc-ad)}{a+bx}-\frac{6d^2(bc-ad)^2}{(a+bx)^2}+\frac{4d(bc-ad)^3}{(a+bx)^3}-\frac{3(bc-ad)^4}{(a+bx)^4}+12d^4\log(a+bx)-12d^4\log(c+dx)\right)}{12(bc-ad)^5}-\frac{B\log\left(\frac{e(c+dx)}{a+bx}\right)+A}{(a+bx)^4}}{4bg^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Log[(e*(c + d*x))/(a + b*x)])/(a*g + b*g*x)^5,x]
```

```
[Out] ((B*(-(b*c) + a*d)*((-3*(b*c - a*d)^4)/(a + b*x)^4 + (4*d*(b*c - a*d)^3)/(a
 + b*x)^3 - (6*d^2*(b*c - a*d)^2)/(a + b*x)^2 + (12*d^3*(b*c - a*d))/(a + b
*x) + 12*d^4*Log[a + b*x] - 12*d^4*Log[c + d*x]))/(12*(b*c - a*d)^5) - (A +
 B*Log[(e*(c + d*x))/(a + b*x)])/(a + b*x)^4)/(4*b*g^5)
```

**fricas [B]** time = 0.92, size = 637, normalized size = 3.09

$$-\frac{3\,(4\,A-B)b^4c^4-16\,(3\,A-B)ab^3c^3d+36\,(2\,A-B)a^2b^2c^2d^2-48\,(A-B)a^3bcd^3+(12\,A-25\,B)a^4d^4+12\left(Bb\right.}{48\left(\left(b^9c^4-4\,ab^8c^3d+6\,a^2b^7c^2d^2-4\,a^3b^6cd^3+a^4b^5d^4\right)g^5x^4+4\left(ab^8c^4-4\,a^2b^7c^3d+6\,a^3b^6c^2\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*(d*x+c)/(b*x+a)))/(b*g*x+a*g)^5,x, algorithm="fricas")
```

```
[Out] -1/48*(3*(4*A - B)*b^4*c^4 - 16*(3*A - B)*a*b^3*c^3*d + 36*(2*A - B)*a^2*b^
2*c^2*d^2 - 48*(A - B)*a^3*b*c*d^3 + (12*A - 25*B)*a^4*d^4 + 12*(B*b^4*c*d^
3 - B*a*b^3*d^4)*x^3 - 6*(B*b^4*c^2*d^2 - 8*B*a*b^3*c*d^3 + 7*B*a^2*b^2*d^4
)*x^2 + 4*(B*b^4*c^3*d - 6*B*a*b^3*c^2*d^2 + 18*B*a^2*b^2*c*d^3 - 13*B*a^3*
b*d^4)*x - 12*(B*b^4*d^4*x^4 + 4*B*a*b^3*d^4*x^3 + 6*B*a^2*b^2*d^4*x^2 + 4*
B*a^3*b*d^4*x - B*b^4*c^4 + 4*B*a*b^3*c^3*d - 6*B*a^2*b^2*c^2*d^2 + 4*B*a^3
*b*c*d^3)*log((d*e*x + c*e)/(b*x + a)))/((b^9*c^4 - 4*a*b^8*c^3*d + 6*a^2*b
^7*c^2*d^2 - 4*a^3*b^6*c*d^3 + a^4*b^5*d^4)*g^5*x^4 + 4*(a*b^8*c^4 - 4*a^2*
b^7*c^3*d + 6*a^3*b^6*c^2*d^2 - 4*a^4*b^5*c*d^3 + a^5*b^4*d^4)*g^5*x^3 + 6*
(a^2*b^7*c^4 - 4*a^3*b^6*c^3*d + 6*a^4*b^5*c^2*d^2 - 4*a^5*b^4*c*d^3 + a^6*
b^3*d^4)*g^5*x^2 + 4*(a^3*b^6*c^4 - 4*a^4*b^5*c^3*d + 6*a^5*b^4*c^2*d^2 - 4
*a^6*b^3*c*d^3 + a^7*b^2*d^4)*g^5*x + (a^4*b^5*c^4 - 4*a^5*b^4*c^3*d + 6*a^
6*b^3*c^2*d^2 - 4*a^7*b^2*c*d^3 + a^8*b*d^4)*g^5)
```

**giac [B]** time = 1.39, size = 511, normalized size = 2.48

$$\frac{\left(\frac{48\,(dxe+ce)Bd^3e^3\log\left(\frac{dxe+ce}{bx+a}\right)}{bx+a}-\frac{72\,(dxe+ce)^2Bbd^2e^2\log\left(\frac{dxe+ce}{bx+a}\right)}{(bx+a)^2}+\frac{48\,(dxe+ce)^3Bb^2de\log\left(\frac{dxe+ce}{bx+a}\right)}{(bx+a)^3}+\frac{48\,(dxe+ce)Ad^3e^3}{bx+a}-\frac{48\,(dxe+ce)Bd^3e^3}{bx+a}-7\cdots}{48\left(b^3c\cdots\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*(d*x+c)/(b*x+a)))/(b*g*x+a*g)^5,x, algorithm="giac")
```

```
[Out] 1/48*(48*(d*x*e + c*e)*B*d^3*e^3*log((d*x*e + c*e)/(b*x + a))/(b*x + a) - 7
2*(d*x*e + c*e)^2*B*b*d^2*e^2*log((d*x*e + c*e)/(b*x + a))/(b*x + a)^2 + 48
*(d*x*e + c*e)^3*B*b^2*d*e*log((d*x*e + c*e)/(b*x + a))/(b*x + a)^3 + 48*(d
*x*e + c*e)*A*d^3*e^3/(b*x + a) - 48*(d*x*e + c*e)*B*d^3*e^3/(b*x + a) - 72
*(d*x*e + c*e)^2*A*b*d^2*e^2/(b*x + a)^2 + 36*(d*x*e + c*e)^2*B*b*d^2*e^2/(
b*x + a)^2 + 48*(d*x*e + c*e)^3*A*b^2*d*e/(b*x + a)^3 - 16*(d*x*e + c*e)^3*
B*b^2*d*e/(b*x + a)^3 - 12*(d*x*e + c*e)^4*B*b^3*log((d*x*e + c*e)/(b*x + a
))/(b*x + a)^4 - 12*(d*x*e + c*e)^4*A*b^3/(b*x + a)^4 + 3*(d*x*e + c*e)^4*B
*b^3/(b*x + a)^4)*(b*c/((b*c*e - a*d*e)*(b*c - a*d)) - a*d/((b*c*e - a*d*e)
*(b*c - a*d)))/(b^3*c^3*g^5*e^3 - 3*a*b^2*c^2*d*g^5*e^3 + 3*a^2*b*c*d^2*g^5
*e^3 - a^3*d^3*g^5*e^3)
```

**maple [B]** time = 0.06, size = 1306, normalized size = 6.34

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*ln(e*(d*x+c)/(b*x+a)))/(b*g*x+a*g)^5,x)
```

```
[Out] -5/4*b^3/(a*d-b*c)^5/g^5*B*ln(1/b*d*e-(a*d-b*c)/(b*x+a)/b*e)/(b*x+a)^4*a*d*
c^4+5/4/(a*d-b*c)^5/g^5*B*ln(1/b*d*e-(a*d-b*c)/(b*x+a)/b*e)/(b*x+a)^4*a^4*d
^4*c-1/4/b/(a*d-b*c)^5/g^5*B*ln(1/b*d*e-(a*d-b*c)/(b*x+a)/b*e)/(b*x+a)^4*a^
5*d^5-1/3*b^2/(a*d-b*c)^5/g^5*B*d^2/(b*x+a)^3*c^3*a-5/8*b^2/(a*d-b*c)^5/g^5
*B/(b*x+a)^4*a^2*d^2*c^3+5/16*b^3/(a*d-b*c)^5/g^5*B/(b*x+a)^4*a*d*c^4-1/4/(
a*d-b*c)^5/g^5*B*ln(1/b*d*e-(a*d-b*c)/(b*x+a)/b*e)*d^4*c-1/16*b^4/(a*d-b*c)
^5/g^5*B/(b*x+a)^4*c^5+1/4*b^4/(a*d-b*c)^5/g^5*A/(b*x+a)^4*c^5+5/2*b^2/(a*d
-b*c)^5/g^5*A/(b*x+a)^4*a^2*d^2*c^3+3/8*b/(a*d-b*c)^5/g^5*B*d^3/(b*x+a)^2*c
^2*a+5/8*b/(a*d-b*c)^5/g^5*B/(b*x+a)^4*a^3*d^3*c^2-5/4*b^3/(a*d-b*c)^5/g^5*
A/(b*x+a)^4*c^4*a*d+1/2*b/(a*d-b*c)^5/g^5*B*d^3/(b*x+a)^3*a^2*c^2-5/2*b/(a*
d-b*c)^5/g^5*A/(b*x+a)^4*a^3*d^3*c^2+1/4/b/(a*d-b*c)^5/g^5*A*d^5*a-1/4/(a*d
-b*c)^5/g^5*A*d^4*c-25/48/b/(a*d-b*c)^5/g^5*B*d^5*a+25/48/(a*d-b*c)^5/g^5*B
*d^4*c-1/4/b/(a*d-b*c)^5/g^5*A/(b*x+a)^4*a^5*d^5+1/8/b/(a*d-b*c)^5/g^5*B*d^
5/(b*x+a)^2*a^3+1/12*b^3/(a*d-b*c)^5/g^5*B*d/(b*x+a)^3*c^4+1/4/b/(a*d-b*c)^
5/g^5*B*d^5/(b*x+a)*a^2-5/16/(a*d-b*c)^5/g^5*B/(b*x+a)^4*a^4*d^4*c+5/2*b^2/
(a*d-b*c)^5/g^5*B*ln(1/b*d*e-(a*d-b*c)/(b*x+a)/b*e)/(b*x+a)^4*a^2*d^2*c^3-5
/2*b/(a*d-b*c)^5/g^5*B*ln(1/b*d*e-(a*d-b*c)/(b*x+a)/b*e)/(b*x+a)^4*a^3*d^3*
c^2+1/16/b/(a*d-b*c)^5/g^5*B/(b*x+a)^4*a^5*d^5-1/8*b^2/(a*d-b*c)^5/g^5*B*d^
2/(b*x+a)^2*c^3+1/12/b/(a*d-b*c)^5/g^5*B*d^5/(b*x+a)^3*a^4+5/4/(a*d-b*c)^5/
g^5*A/(b*x+a)^4*a^4*d^4*c+1/4/b/(a*d-b*c)^5/g^5*B*ln(1/b*d*e-(a*d-b*c)/(b*x
+a)/b*e)*d^5*a+1/4*b^4/(a*d-b*c)^5/g^5*B*ln(1/b*d*e-(a*d-b*c)/(b*x+a)/b*e)/
(b*x+a)^4*c^5-3/8/(a*d-b*c)^5/g^5*B*d^4/(b*x+a)^2*a^2*c-1/2/(a*d-b*c)^5/g^5
*B*d^4/(b*x+a)*a*c-1/3/(a*d-b*c)^5/g^5*B*d^4/(b*x+a)^3*a^3*c+1/4*b/(a*d-b*c
)^5/g^5*B*d^3/(b*x+a)*c^2
```

**maxima [B]** time = 1.51, size = 647, normalized size = 3.14

$$-\frac{1}{48}B\left(\frac{12\,b^3d^3x^3-3\,b^3c^3+13\,ab^2c^2d-23\,a^2bc}{\left(b^8c^3-3\,ab^7c^2d+3\,a^2b^6cd^2-a^3b^5d^3\right)g^5x^4+4\left(ab^7c^3-3\,a^2b^6c^2d+3\,a^3b^5cd^2-a^4b^4d^3\right)g^5x^3+6\left(a^2b^6c\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*(d*x+c)/(b*x+a)))/(b*g*x+a*g)^5,x, algorithm="maxima")
```

```
[Out] -1/48*B*((12*b^3*d^3*x^3 - 3*b^3*c^3 + 13*a*b^2*c^2*d - 23*a^2*b*c*d^2 + 25
*a^3*d^3 - 6*(b^3*c*d^2 - 7*a*b^2*d^3)*x^2 + 4*(b^3*c^2*d - 5*a*b^2*c*d^2 +
 13*a^2*b*d^3)*x)/((b^8*c^3 - 3*a*b^7*c^2*d + 3*a^2*b^6*c*d^2 - a^3*b^5*d^3
)*g^5*x^4 + 4*(a*b^7*c^3 - 3*a^2*b^6*c^2*d + 3*a^3*b^5*c*d^2 - a^4*b^4*d^3)
*g^5*x^3 + 6*(a^2*b^6*c^3 - 3*a^3*b^5*c^2*d + 3*a^4*b^4*c*d^2 - a^5*b^3*d^3
)*g^5*x^2 + 4*(a^3*b^5*c^3 - 3*a^4*b^4*c^2*d + 3*a^5*b^3*c*d^2 - a^6*b^2*d^
3)*g^5*x + (a^4*b^4*c^3 - 3*a^5*b^3*c^2*d + 3*a^6*b^2*c*d^2 - a^7*b*d^3)*g^
5) + 12*log(d*e*x/(b*x + a) + c*e/(b*x + a))/(b^5*g^5*x^4 + 4*a*b^4*g^5*x^3
 + 6*a^2*b^3*g^5*x^2 + 4*a^3*b^2*g^5*x + a^4*b*g^5) + 12*d^4*log(b*x + a)/(
```

```
(b^5*c^4 - 4*a*b^4*c^3*d + 6*a^2*b^3*c^2*d^2 - 4*a^3*b^2*c*d^3 + a^4*b*d^4)
*g^5) - 12*d^4*log(d*x + c)/((b^5*c^4 - 4*a*b^4*c^3*d + 6*a^2*b^3*c^2*d^2 -
 4*a^3*b^2*c*d^3 + a^4*b*d^4)*g^5)) - 1/4*A/(b^5*g^5*x^4 + 4*a*b^4*g^5*x^3
+ 6*a^2*b^3*g^5*x^2 + 4*a^3*b^2*g^5*x + a^4*b*g^5)
```

**mupad [B]** time = 6.62, size = 578, normalized size = 2.81

$$\frac{B\,d^4\,\mathrm{atanh}\left(\frac{-4\,a^4\,b\,d^4\,g^5+8\,a^3\,b^2\,c\,d^3\,g^5-8\,a\,b^4\,c^3\,d\,g^5+4\,b^5\,c^4\,g^5}{4\,b\,g^5\,(a\,d-b\,c)^4}-\frac{2\,b\,d\,x\,\left(a^3\,d^3-3\,a^2\,b\,c\,d^2+3\,a\,b^2\,c^2\,d-b^3\,c^3\right)}{(a\,d-b\,c)^4}\right)}{2\,b\,g^5\,(a\,d-b\,c)^4}-\frac{B\,\ln\Big(}{4\,b^2\,g^5\,\Big(4\,a^3\,x+\frac{a^4}{b}+b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*log((e*(c + d*x))/(a + b*x)))/(a*g + b*g*x)^5,x)
```

```
[Out] (B*d^4*atanh((4*b^5*c^4*g^5 - 4*a^4*b*d^4*g^5 - 8*a*b^4*c^3*d*g^5 + 8*a^3*b
^2*c*d^3*g^5)/(4*b*g^5*(a*d - b*c)^4) - (2*b*d*x*(a^3*d^3 - b^3*c^3 + 3*a*b
^2*c^2*d - 3*a^2*b*c*d^2))/(a*d - b*c)^4))/(2*b*g^5*(a*d - b*c)^4) - (B*log
((e*(c + d*x))/(a + b*x)))/(4*b^2*g^5*(4*a^3*x + a^4/b + b^3*x^4 + 6*a^2*b*
x^2 + 4*a*b^2*x^3)) - ((12*A*a^3*d^3 - 12*A*b^3*c^3 - 25*B*a^3*d^3 + 3*B*b^
3*c^3 + 36*A*a*b^2*c^2*d - 36*A*a^2*b*c*d^2 - 13*B*a*b^2*c^2*d + 23*B*a^2*b
*c*d^2)/(12*(a^3*d^3 - b^3*c^3 + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2)) + (d^2*x^2
*(B*b^3*c - 7*B*a*b^2*d))/(2*(a^3*d^3 - b^3*c^3 + 3*a*b^2*c^2*d - 3*a^2*b*c
*d^2)) - (d*x*(B*b^3*c^2 + 13*B*a^2*b*d^2 - 5*B*a*b^2*c*d))/(3*(a^3*d^3 - b
^3*c^3 + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2)) - (B*b^3*d^3*x^3)/(a^3*d^3 - b^3*c
^3 + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2))/(4*a^4*b*g^5 + 4*b^5*g^5*x^4 + 16*a^3*
b^2*g^5*x + 16*a*b^4*g^5*x^3 + 24*a^2*b^3*g^5*x^2)
```

**sympy [B]** time = 5.53, size = 944, normalized size = 4.58

$$-\frac{B\log\left(\frac{e(c+dx)}{a+bx}\right)}{4a^4bg^5+16a^3b^2g^5x+24a^2b^3g^5x^2+16ab^4g^5x^3+4b^5g^5x^4}+\frac{Bd^4\log\left(x+\frac{-\frac{Ba^5d^9}{(ad-bc)^4}+\frac{5Ba^4bcd^8}{(ad-bc)^4}-\frac{10Ba^3b^2c^2d^7}{(ad-bc)^4}+\frac{10Ba^2b^3c^3d^6}{(ad-bc)^4}-\frac{5B}{(}}{2Bbd^5}\right.}{4bg^5\,(ad-bc)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*ln(e*(d*x+c)/(b*x+a)))/(b*g*x+a*g)**5,x)
```

```
[Out] -B*log(e*(c + d*x)/(a + b*x))/(4*a**4*b*g**5 + 16*a**3*b**2*g**5*x + 24*a**
2*b**3*g**5*x**2 + 16*a*b**4*g**5*x**3 + 4*b**5*g**5*x**4) + B*d**4*log(x +
 (-B*a**5*d**9/(a*d - b*c)**4 + 5*B*a**4*b*c*d**8/(a*d - b*c)**4 - 10*B*a**
3*b**2*c**2*d**7/(a*d - b*c)**4 + 10*B*a**2*b**3*c**3*d**6/(a*d - b*c)**4 -
 5*B*a*b**4*c**4*d**5/(a*d - b*c)**4 + B*a*d**5 + B*b**5*c**5*d**4/(a*d - b
*c)**4 + B*b*c*d**4)/(2*B*b*d**5))/(4*b*g**5*(a*d - b*c)**4) - B*d**4*log(x
```

```
 + (B*a**5*d**9/(a*d - b*c)**4 - 5*B*a**4*b*c*d**8/(a*d - b*c)**4 + 10*B*a*
*3*b**2*c**2*d**7/(a*d - b*c)**4 - 10*B*a**2*b**3*c**3*d**6/(a*d - b*c)**4
+ 5*B*a*b**4*c**4*d**5/(a*d - b*c)**4 + B*a*d**5 - B*b**5*c**5*d**4/(a*d -
b*c)**4 + B*b*c*d**4)/(2*B*b*d**5))/(4*b*g**5*(a*d - b*c)**4) + (-12*A*a**3
*d**3 + 36*A*a**2*b*c*d**2 - 36*A*a*b**2*c**2*d + 12*A*b**3*c**3 + 25*B*a**
3*d**3 - 23*B*a**2*b*c*d**2 + 13*B*a*b**2*c**2*d - 3*B*b**3*c**3 + 12*B*b**
3*d**3*x**3 + x**2*(42*B*a*b**2*d**3 - 6*B*b**3*c*d**2) + x*(52*B*a**2*b*d*
*3 - 20*B*a*b**2*c*d**2 + 4*B*b**3*c**2*d))/(48*a**7*b*d**3*g**5 - 144*a**6
*b**2*c*d**2*g**5 + 144*a**5*b**3*c**2*d*g**5 - 48*a**4*b**4*c**3*g**5 + x*
*4*(48*a**3*b**5*d**3*g**5 - 144*a**2*b**6*c*d**2*g**5 + 144*a*b**7*c**2*d*
g**5 - 48*b**8*c**3*g**5) + x**3*(192*a**4*b**4*d**3*g**5 - 576*a**3*b**5*c
*d**2*g**5 + 576*a**2*b**6*c**2*d*g**5 - 192*a*b**7*c**3*g**5) + x**2*(288*
a**5*b**3*d**3*g**5 - 864*a**4*b**4*c*d**2*g**5 + 864*a**3*b**5*c**2*d*g**5
 - 288*a**2*b**6*c**3*g**5) + x*(192*a**6*b**2*d**3*g**5 - 576*a**5*b**3*c*
d**2*g**5 + 576*a**4*b**4*c**2*d*g**5 - 192*a**3*b**5*c**3*g**5))
```

# 3.182 $\int (ag+bgx)^4 \left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2\,dx$

**Optimal**. Leaf size=503

$$-\frac{2Bg^4(bc-ad)^5\log\left(1-\frac{d(a+bx)}{b(c+dx)}\right)\left(B\log\left(\frac{e(c+dx)}{a+bx}\right)+A\right)}{5bd^5}-\frac{2Bg^4(c+dx)(bc-ad)^4\left(B\log\left(\frac{e(c+dx)}{a+bx}\right)+A\right)}{5d^5}+\frac{Bg^4(a+b}{}$$

```
[Out] 13/30*B^2*(-a*d+b*c)^4*g^4*x/d^4-7/60*B^2*(-a*d+b*c)^3*g^4*(b*x+a)^2/b/d^3+
1/30*B^2*(-a*d+b*c)^2*g^4*(b*x+a)^3/b/d^2-5/6*B^2*(-a*d+b*c)^5*g^4*ln(b*x+a
)/b/d^5-13/30*B^2*(-a*d+b*c)^5*g^4*ln((d*x+c)/(b*x+a))/b/d^5+1/5*B*(-a*d+b*
c)^3*g^4*(b*x+a)^2*(A+B*ln(e*(d*x+c)/(b*x+a)))/b/d^3-2/15*B*(-a*d+b*c)^2*g^
4*(b*x+a)^3*(A+B*ln(e*(d*x+c)/(b*x+a)))/b/d^2+1/10*B*(-a*d+b*c)*g^4*(b*x+a)
^4*(A+B*ln(e*(d*x+c)/(b*x+a)))/b/d-2/5*B*(-a*d+b*c)^4*g^4*(d*x+c)*(A+B*ln(e
*(d*x+c)/(b*x+a)))/d^5+1/5*g^4*(b*x+a)^5*(A+B*ln(e*(d*x+c)/(b*x+a)))^2/b-2/
5*B*(-a*d+b*c)^5*g^4*(A+B*ln(e*(d*x+c)/(b*x+a)))*ln(1-d*(b*x+a)/b/(d*x+c))/
b/d^5+2/5*B^2*(-a*d+b*c)^5*g^4*polylog(2,d*(b*x+a)/b/(d*x+c))/b/d^5
```

**Rubi [A]** time = 0.82, antiderivative size = 557, normalized size of antiderivative = 1.11, number of steps used = 28, number of rules used = 13, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}} = 0.406$, Rules used = {2525, 12, 2528, 2486, 31, 43, 2524, 2418, 2394, 2393, 2391, 2390, 2301}

$$\frac{2B^2g^4(bc-ad)^5\text{PolyLog}\left(2,\frac{b(c+dx)}{bc-ad}\right)}{5bd^5}+\frac{2Bg^4(bc-ad)^5\log(c+dx)\left(B\log\left(\frac{e(c+dx)}{a+bx}\right)+A\right)}{5bd^5}+\frac{Bg^4(a+bx)^2(bc-ad)^3\left(\right.}{5bd^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a*g + b*g*x)^4*(A + B*Log[(e*(c + d*x))/(a + b*x)])^2,x]
```

```
[Out] (-2*A*B*(b*c - a*d)^4*g^4*x)/(5*d^4) + (13*B^2*(b*c - a*d)^4*g^4*x)/(30*d^4
) - (7*B^2*(b*c - a*d)^3*g^4*(a + b*x)^2)/(60*b*d^3) + (B^2*(b*c - a*d)^2*g
^4*(a + b*x)^3)/(30*b*d^2) - (5*B^2*(b*c - a*d)^5*g^4*Log[c + d*x])/(6*b*d^
5) + (2*B^2*(b*c - a*d)^5*g^4*Log[-((d*(a + b*x))/(b*c - a*d))]*Log[c + d*x
])/(5*b*d^5) - (B^2*(b*c - a*d)^5*g^4*Log[c + d*x]^2)/(5*b*d^5) - (2*B^2*(b
*c - a*d)^4*g^4*(a + b*x)*Log[(e*(c + d*x))/(a + b*x)])/(5*b*d^4) + (B*(b*c
 - a*d)^3*g^4*(a + b*x)^2*(A + B*Log[(e*(c + d*x))/(a + b*x)]))/(5*b*d^3) -
 (2*B*(b*c - a*d)^2*g^4*(a + b*x)^3*(A + B*Log[(e*(c + d*x))/(a + b*x)]))/(
15*b*d^2) + (B*(b*c - a*d)*g^4*(a + b*x)^4*(A + B*Log[(e*(c + d*x))/(a + b*
x)]))/(10*b*d) + (2*B*(b*c - a*d)^5*g^4*Log[c + d*x]*(A + B*Log[(e*(c + d*x
))/(a + b*x)]))/(5*b*d^5) + (g^4*(a + b*x)^5*(A + B*Log[(e*(c + d*x))/(a +
b*x)])^2)/(5*b) + (2*B^2*(b*c - a*d)^5*g^4*PolyLog[2, (b*(c + d*x))/(b*c -
a*d)])/(5*b*d^5)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2301

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

Rule 2390

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_))^(q_.), x_Symbol] :> Dist[1/e, Subst[Int[((f*x)/d)^q*(a + b*Log[c*x^
n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && E
qQ[e*f - d*g, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2393

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + (c*e*x)/g])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2394

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d + e*x
```

```
)^n]))/g, x] - Dist[(b*e*n)/g, Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2418

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*(RFx_), x_Sy
mbol] :> With[{u = ExpandIntegrand[(a + b*Log[c*(d + e*x)^n])^p, RFx, x]},
Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d, e, n}, x] && RationalFunctionQ[
RFx, x] && IntegerQ[p]
```

Rule 2486

```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))
^(r_.)]^(s_.), x_Symbol] :> Simp[((a + b*x)*Log[e*(f*(a + b*x)^p*(c + d*x)^
q)^r]^s)/b, x] + Dist[(q*r*s*(b*c - a*d))/b, Int[Log[e*(f*(a + b*x)^p*(c +
d*x)^q)^r]^(s - 1)/(c + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, p, q, r, s
}, x] && NeQ[b*c - a*d, 0] && EqQ[p + q, 0] && IGtQ[s, 0]
```

Rule 2524

```
Int[((a_.) + Log[(c_.)*(RFx_)^(p_.)]*(b_.))^(n_.)/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[(Log[d + e*x]*(a + b*Log[c*RFx^p])^n)/e, x] - Dist[(b*n*p)/e
, Int[(Log[d + e*x]*(a + b*Log[c*RFx^p])^(n - 1)*D[RFx, x])/RFx, x], x] /;
FreeQ[{a, b, c, d, e, p}, x] && RationalFunctionQ[RFx, x] && IGtQ[n, 0]
```

Rule 2525

```
Int[((a_.) + Log[(c_.)*(RFx_)^(p_.)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_))^(m_.
), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(a + b*Log[c*RFx^p])^n)/(e*(m + 1))
, x] - Dist[(b*n*p)/(e*(m + 1)), Int[SimplifyIntegrand[((d + e*x)^(m + 1)*(
a + b*Log[c*RFx^p])^(n - 1)*D[RFx, x])/RFx, x], x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && RationalFunctionQ[RFx, x] && IGtQ[n, 0] && (EqQ[n, 1] ||
IntegerQ[m]) && NeQ[m, -1]
```

Rule 2528

```
Int[((a_.) + Log[(c_.)*(RFx_)^(p_.)]*(b_.))^(n_.)*(RGx_), x_Symbol] :> With
[{u = ExpandIntegrand[(a + b*Log[c*RFx^p])^n, RGx, x]}, Int[u, x] /; SumQ[u
]] /; FreeQ[{a, b, c, p}, x] && RationalFunctionQ[RFx, x] && RationalFuncti
onQ[RGx, x] && IGtQ[n, 0]
```

Rubi steps

$$\int (ag+bgx)^4 \left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2 dx = \frac{g^4(a+bx)^5\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2}{5b} - \frac{(2B)\int \frac{(bc-ad)g^5(a+bx)^4\left(-A-B\log\right.}{c+dx}}{5bg}$$

$$= \frac{g^4(a+bx)^5\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2}{5b} - \frac{\left(2B(bc-ad)g^4\right)\int \frac{(a+bx)^4\left(-A\right.}{c}}{5b}$$

$$= \frac{g^4(a+bx)^5\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2}{5b} - \frac{\left(2B(bc-ad)g^4\right)\int\left(-\frac{b(bc-ad)^3}{}\right.}{}$$

$$= \frac{g^4(a+bx)^5\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2}{5b} - \frac{\left(2B(bc-ad)g^4\right)\int(a+bx)^3\left(\right.}{5d}$$

$$= -\frac{2AB(bc-ad)^4g^4x}{5d^4} + \frac{B(bc-ad)^3g^4(a+bx)^2\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right.\right.}{5bd^3}$$

$$= -\frac{2AB(bc-ad)^4g^4x}{5d^4} - \frac{2B^2(bc-ad)^4g^4(a+bx)\log\left(\frac{e(c+dx)}{a+bx}\right)}{5bd^4} + \frac{B(}{}$$

$$= -\frac{2AB(bc-ad)^4g^4x}{5d^4} - \frac{2B^2(bc-ad)^5g^4\log(c+dx)}{5bd^5} - \frac{2B^2(bc-ad}{}$$

$$= -\frac{2AB(bc-ad)^4g^4x}{5d^4} + \frac{13B^2(bc-ad)^4g^4x}{30d^4} - \frac{7B^2(bc-ad)^3g^4(a+}{60bd^3}$$

$$= -\frac{2AB(bc-ad)^4g^4x}{5d^4} + \frac{13B^2(bc-ad)^4g^4x}{30d^4} - \frac{7B^2(bc-ad)^3g^4(a+}{60bd^3}$$

$$= -\frac{2AB(bc-ad)^4g^4x}{5d^4} + \frac{13B^2(bc-ad)^4g^4x}{30d^4} - \frac{7B^2(bc-ad)^3g^4(a+}{60bd^3}$$

$$= -\frac{2AB(bc-ad)^4g^4x}{5d^4} + \frac{13B^2(bc-ad)^4g^4x}{30d^4} - \frac{7B^2(bc-ad)^3g^4(a+}{60bd^3}$$

**Mathematica [A]** time = 0.51, size = 512, normalized size = 1.02

$$g^4\left((a+bx)^5\left(B\log\left(\frac{e(c+dx)}{a+bx}\right)+A\right)^2 - \frac{B(bc-ad)\left(-6d^4(a+bx)^4\left(B\log\left(\frac{e(c+dx)}{a+bx}\right)+A\right)+8d^3(a+bx)^3(bc-ad)\left(B\log\left(\frac{e(c+dx)}{a+bx}\right)+A\right)-12d^2(a+bx)^2\right.}{}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*g + b*g*x)^4*(A + B*Log[(e*(c + d*x))/(a + b*x)])^2,x]
```

```
[Out] (g^4*((a + b*x)^5*(A + B*Log[(e*(c + d*x))/(a + b*x)])^2 - (B*(b*c - a*d)*(
24*A*b*d*(b*c - a*d)^3*x + 24*B*(b*c - a*d)^4*Log[a + b*x] - 4*B*(b*c - a*d
)^2*(2*b*d*(b*c - a*d)*x - d^2*(a + b*x)^2 - 2*(b*c - a*d)^2*Log[c + d*x])
- B*(b*c - a*d)*(6*b*d*(b*c - a*d)^2*x + 3*d^2*(-(b*c) + a*d)*(a + b*x)^2 +
 2*d^3*(a + b*x)^3 - 6*(b*c - a*d)^3*Log[c + d*x]) - 12*B*(b*c - a*d)^3*(b*
d*x + (-(b*c) + a*d)*Log[c + d*x]) + 24*b*B*(b*c - a*d)^3*(c + d*x)*Log[(e*
(c + d*x))/(a + b*x)] - 12*d^2*(b*c - a*d)^2*(a + b*x)^2*(A + B*Log[(e*(c +
 d*x))/(a + b*x)]) + 8*d^3*(b*c - a*d)*(a + b*x)^3*(A + B*Log[(e*(c + d*x))
/(a + b*x)]) - 6*d^4*(a + b*x)^4*(A + B*Log[(e*(c + d*x))/(a + b*x)]) - 24*
(b*c - a*d)^4*Log[c + d*x]*(A + B*Log[(e*(c + d*x))/(a + b*x)]) - 12*B*(b*c
 - a*d)^4*((2*Log[(d*(a + b*x))/(-(b*c) + a*d)] - Log[c + d*x])*Log[c + d*x
] + 2*PolyLog[2, (b*(c + d*x))/(b*c - a*d)])))/(12*d^5)))/(5*b)
```

**fricas [F]** time = 0.63, size = 0, normalized size = 0.00

$$\text{integral}\Bigg(A^2b^4g^4x^4 + 4\,A^2ab^3g^4x^3 + 6\,A^2a^2b^2g^4x^2 + 4\,A^2a^3bg^4x + A^2a^4g^4 + \Big(B^2b^4g^4x^4 + 4\,B^2ab^3g^4x^3 + 6\,B^2a^2b$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*g*x+a*g)^4*(A+B*log(e*(d*x+c)/(b*x+a)))^2,x, algorithm="fricas
")
```

```
[Out] integral(A^2*b^4*g^4*x^4 + 4*A^2*a*b^3*g^4*x^3 + 6*A^2*a^2*b^2*g^4*x^2 + 4*
A^2*a^3*b*g^4*x + A^2*a^4*g^4 + (B^2*b^4*g^4*x^4 + 4*B^2*a*b^3*g^4*x^3 + 6*
B^2*a^2*b^2*g^4*x^2 + 4*B^2*a^3*b*g^4*x + B^2*a^4*g^4)*log((d*e*x + c*e)/(b
*x + a))^2 + 2*(A*B*b^4*g^4*x^4 + 4*A*B*a*b^3*g^4*x^3 + 6*A*B*a^2*b^2*g^4*x
^2 + 4*A*B*a^3*b*g^4*x + A*B*a^4*g^4)*log((d*e*x + c*e)/(b*x + a)), x)
```

**giac [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*g*x+a*g)^4*(A+B*log(e*(d*x+c)/(b*x+a)))^2,x, algorithm="giac")
```

```
[Out] Timed out
```

**maple [F]** time = 2.20, size = 0, normalized size = 0.00

$$\int \left(bgx + ag\right)^4 \left(B\ln\left(\frac{(dx+c)\,e}{bx+a}\right) + A\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*g*x+a*g)^4*(A+B*ln(e*(d*x+c)/(b*x+a)))^2,x)
```

```
[Out] int((b*g*x+a*g)^4*(A+B*ln(e*(d*x+c)/(b*x+a)))^2,x)
```

**maxima [B]** time = 2.53, size = 2395, normalized size = 4.76

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*g*x+a*g)^4*(A+B*log(e*(d*x+c)/(b*x+a)))^2,x, algorithm="maxima
")
```

```
[Out] 1/5*A^2*b^4*g^4*x^5 + A^2*a*b^3*g^4*x^4 + 2*A^2*a^2*b^2*g^4*x^3 + 2*A^2*a^3
*b*g^4*x^2 + 2*(x*log(d*e*x/(b*x + a) + c*e/(b*x + a)) - a*log(b*x + a)/b +
 c*log(d*x + c)/d)*A*B*a^4*g^4 + 4*(x^2*log(d*e*x/(b*x + a) + c*e/(b*x + a)
) + a^2*log(b*x + a)/b^2 - c^2*log(d*x + c)/d^2 + (b*c - a*d)*x/(b*d))*A*B*
a^3*b*g^4 + 2*(2*x^3*log(d*e*x/(b*x + a) + c*e/(b*x + a)) - 2*a^3*log(b*x +
 a)/b^3 + 2*c^3*log(d*x + c)/d^3 + ((b^2*c*d - a*b*d^2)*x^2 - 2*(b^2*c^2 -
a^2*d^2)*x)/(b^2*d^2))*A*B*a^2*b^2*g^4 + 1/3*(6*x^4*log(d*e*x/(b*x + a) + c
*e/(b*x + a)) + 6*a^4*log(b*x + a)/b^4 - 6*c^4*log(d*x + c)/d^4 + (2*(b^3*c
*d^2 - a*b^2*d^3)*x^3 - 3*(b^3*c^2*d - a^2*b*d^3)*x^2 + 6*(b^3*c^3 - a^3*d^
3)*x)/(b^3*d^3))*A*B*a*b^3*g^4 + 1/30*(12*x^5*log(d*e*x/(b*x + a) + c*e/(b*
x + a)) - 12*a^5*log(b*x + a)/b^5 + 12*c^5*log(d*x + c)/d^5 + (3*(b^4*c*d^3
 - a*b^3*d^4)*x^4 - 4*(b^4*c^2*d^2 - a^2*b^2*d^4)*x^3 + 6*(b^4*c^3*d - a^3*
b*d^4)*x^2 - 12*(b^4*c^4 - a^4*d^4)*x)/(b^4*d^4))*A*B*b^4*g^4 + A^2*a^4*g^4
*x + 1/30*((12*g^4*log(e) - 25*g^4)*b^4*c^5 - (60*g^4*log(e) - 113*g^4)*a*b
^3*c^4*d + 4*(30*g^4*log(e) - 49*g^4)*a^2*b^2*c^3*d^2 - 12*(10*g^4*log(e) -
 13*g^4)*a^3*b*c^2*d^3 + 12*(5*g^4*log(e) - 4*g^4)*a^4*c*d^4)*B^2*log(d*x +
 c)/d^5 - 2/5*(b^5*c^5*g^4 - 5*a*b^4*c^4*d*g^4 + 10*a^2*b^3*c^3*d^2*g^4 - 1
0*a^3*b^2*c^2*d^3*g^4 + 5*a^4*b*c*d^4*g^4 - a^5*d^5*g^4)*(log(b*x + a)*log(
(b*d*x + a*d)/(b*c - a*d) + 1) + dilog(-(b*d*x + a*d)/(b*c - a*d)))*B^2/(b*
d^5) + 1/60*(12*B^2*b^5*d^5*g^4*x^5*log(e)^2 + 6*(b^5*c*d^4*g^4*log(e) + (1
0*g^4*log(e)^2 - g^4*log(e))*a*b^4*d^5)*B^2*x^4 - 2*((4*g^4*log(e) - g^4)*b
^5*c^2*d^3 - 2*(10*g^4*log(e) - g^4)*a*b^4*c*d^4 - (60*g^4*log(e)^2 - 16*g^
4*log(e) + g^4)*a^2*b^3*d^5)*B^2*x^3 + ((12*g^4*log(e) - 7*g^4)*b^5*c^3*d^2
 - 3*(20*g^4*log(e) - 9*g^4)*a*b^4*c^2*d^3 + 3*(40*g^4*log(e) - 11*g^4)*a^2
*b^3*c*d^4 + (120*g^4*log(e)^2 - 72*g^4*log(e) + 13*g^4)*a^3*b^2*d^5)*B^2*x
^2 - 2*((12*g^4*log(e) - 13*g^4)*b^5*c^4*d - (60*g^4*log(e) - 59*g^4)*a*b^4
*c^3*d^2 + 6*(20*g^4*log(e) - 17*g^4)*a^2*b^3*c^2*d^3 - (120*g^4*log(e) - 7
9*g^4)*a^3*b^2*c*d^4 - (30*g^4*log(e)^2 - 48*g^4*log(e) + 23*g^4)*a^4*b*d^5
)*B^2*x + 12*(B^2*b^5*d^5*g^4*x^5 + 5*B^2*a*b^4*d^5*g^4*x^4 + 10*B^2*a^2*b^
3*d^5*g^4*x^3 + 10*B^2*a^3*b^2*d^5*g^4*x^2 + 5*B^2*a^4*b*d^5*g^4*x + B^2*a^
5*d^5*g^4)*log(b*x + a)^2 + 12*(B^2*b^5*d^5*g^4*x^5 + 5*B^2*a*b^4*d^5*g^4*x
```

```
^4 + 10*B^2*a^2*b^3*d^5*g^4*x^3 + 10*B^2*a^3*b^2*d^5*g^4*x^2 + 5*B^2*a^4*b*
d^5*g^4*x + (b^5*c^5*g^4 - 5*a*b^4*c^4*d*g^4 + 10*a^2*b^3*c^3*d^2*g^4 - 10*
a^3*b^2*c^2*d^3*g^4 + 5*a^4*b*c*d^4*g^4)*B^2)*log(d*x + c)^2 - 2*(12*B^2*b^
5*d^5*g^4*x^5*log(e) + 3*(b^5*c*d^4*g^4 + (20*g^4*log(e) - g^4)*a*b^4*d^5)*
B^2*x^4 - 4*(b^5*c^2*d^3*g^4 - 5*a*b^4*c*d^4*g^4 - 2*(15*g^4*log(e) - 2*g^4
)*a^2*b^3*d^5)*B^2*x^3 + 6*(b^5*c^3*d^2*g^4 - 5*a*b^4*c^2*d^3*g^4 + 10*a^2*
b^3*c*d^4*g^4 + 2*(10*g^4*log(e) - 3*g^4)*a^3*b^2*d^5)*B^2*x^2 - 12*(b^5*c^
4*d*g^4 - 5*a*b^4*c^3*d^2*g^4 + 10*a^2*b^3*c^2*d^3*g^4 - 10*a^3*b^2*c*d^4*g
^4 - (5*g^4*log(e) - 4*g^4)*a^4*b*d^5)*B^2*x - (12*a*b^4*c^4*d*g^4 - 54*a^2
*b^3*c^3*d^2*g^4 + 94*a^3*b^2*c^2*d^3*g^4 - 77*a^4*b*c*d^4*g^4 - (12*g^4*lo
g(e) - 25*g^4)*a^5*d^5)*B^2)*log(b*x + a) + 2*(12*B^2*b^5*d^5*g^4*x^5*log(e
) + 3*(b^5*c*d^4*g^4 + (20*g^4*log(e) - g^4)*a*b^4*d^5)*B^2*x^4 - 4*(b^5*c^
2*d^3*g^4 - 5*a*b^4*c*d^4*g^4 - 2*(15*g^4*log(e) - 2*g^4)*a^2*b^3*d^5)*B^2*
x^3 + 6*(b^5*c^3*d^2*g^4 - 5*a*b^4*c^2*d^3*g^4 + 10*a^2*b^3*c*d^4*g^4 + 2*(
10*g^4*log(e) - 3*g^4)*a^3*b^2*d^5)*B^2*x^2 - 12*(b^5*c^4*d*g^4 - 5*a*b^4*c
^3*d^2*g^4 + 10*a^2*b^3*c^2*d^3*g^4 - 10*a^3*b^2*c*d^4*g^4 - (5*g^4*log(e)
- 4*g^4)*a^4*b*d^5)*B^2*x - 12*(B^2*b^5*d^5*g^4*x^5 + 5*B^2*a*b^4*d^5*g^4*x
^4 + 10*B^2*a^2*b^3*d^5*g^4*x^3 + 10*B^2*a^3*b^2*d^5*g^4*x^2 + 5*B^2*a^4*b*
d^5*g^4*x + B^2*a^5*d^5*g^4)*log(b*x + a))*log(d*x + c))/(b*d^5)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.00

$$\int \left(a\,g+b\,g\,x\right)^4\,\left(A+B\,\ln\left(\frac{e\,\left(c+d\,x\right)}{a+b\,x}\right)\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*g + b*g*x)^4*(A + B*log((e*(c + d*x))/(a + b*x)))^2,x)
```

```
[Out] int((a*g + b*g*x)^4*(A + B*log((e*(c + d*x))/(a + b*x)))^2, x)
```

**sympy [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*g*x+a*g)**4*(A+B*ln(e*(d*x+c)/(b*x+a)))**2,x)
```

```
[Out] Timed out
```

## 3.183 $\int (ag+bgx)^3 \left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2 dx$

**Optimal**. Leaf size=420

$$\frac{Bg^3(bc-ad)^4\log\left(1-\frac{d(a+bx)}{b(c+dx)}\right)\left(B\log\left(\frac{e(c+dx)}{a+bx}\right)+A\right)}{2bd^4}+\frac{Bg^3(c+dx)(bc-ad)^3\left(B\log\left(\frac{e(c+dx)}{a+bx}\right)+A\right)}{2d^4}-\frac{Bg^3(a+bx)^2}{}$$

```
[Out] -5/12*B^2*(-a*d+b*c)^3*g^3*x/d^3+1/12*B^2*(-a*d+b*c)^2*g^3*(b*x+a)^2/b/d^2+
11/12*B^2*(-a*d+b*c)^4*g^3*ln(b*x+a)/b/d^4+5/12*B^2*(-a*d+b*c)^4*g^3*ln((d*
x+c)/(b*x+a))/b/d^4-1/4*B*(-a*d+b*c)^2*g^3*(b*x+a)^2*(A+B*ln(e*(d*x+c)/(b*x
+a)))/b/d^2+1/6*B*(-a*d+b*c)*g^3*(b*x+a)^3*(A+B*ln(e*(d*x+c)/(b*x+a)))/b/d+
1/2*B*(-a*d+b*c)^3*g^3*(d*x+c)*(A+B*ln(e*(d*x+c)/(b*x+a)))/d^4+1/4*g^3*(b*x
+a)^4*(A+B*ln(e*(d*x+c)/(b*x+a)))^2/b+1/2*B*(-a*d+b*c)^4*g^3*(A+B*ln(e*(d*x
+c)/(b*x+a)))*ln(1-d*(b*x+a)/b/(d*x+c))/b/d^4-1/2*B^2*(-a*d+b*c)^4*g^3*poly
log(2,d*(b*x+a)/b/(d*x+c))/b/d^4
```

**Rubi [A]** time = 0.65, antiderivative size = 474, normalized size of antiderivative = 1.13, number of steps used = 24, number of rules used = 13, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.406, Rules used = {2525, 12, 2528, 2486, 31, 43, 2524, 2418, 2394, 2393, 2391, 2390, 2301}

$$-\frac{B^2g^3(bc-ad)^4\text{PolyLog}\left(2,\frac{b(c+dx)}{bc-ad}\right)}{2bd^4}-\frac{Bg^3(bc-ad)^4\log(c+dx)\left(B\log\left(\frac{e(c+dx)}{a+bx}\right)+A\right)}{2bd^4}-\frac{Bg^3(a+bx)^2(bc-ad)^2}{4bd^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a*g + b*g*x)^3*(A + B*Log[(e*(c + d*x))/(a + b*x)])^2,x]
```

```
[Out] (A*B*(b*c - a*d)^3*g^3*x)/(2*d^3) - (5*B^2*(b*c - a*d)^3*g^3*x)/(12*d^3) +
(B^2*(b*c - a*d)^2*g^3*(a + b*x)^2)/(12*b*d^2) + (11*B^2*(b*c - a*d)^4*g^3*
Log[c + d*x])/(12*b*d^4) - (B^2*(b*c - a*d)^4*g^3*Log[-((d*(a + b*x))/(b*c
- a*d))]*Log[c + d*x])/(2*b*d^4) + (B^2*(b*c - a*d)^4*g^3*Log[c + d*x]^2)/(
4*b*d^4) + (B^2*(b*c - a*d)^3*g^3*(a + b*x)*Log[(e*(c + d*x))/(a + b*x)])/(
2*b*d^3) - (B*(b*c - a*d)^2*g^3*(a + b*x)^2*(A + B*Log[(e*(c + d*x))/(a + b
*x)]))/(4*b*d^2) + (B*(b*c - a*d)*g^3*(a + b*x)^3*(A + B*Log[(e*(c + d*x))/
(a + b*x)]))/(6*b*d) - (B*(b*c - a*d)^4*g^3*Log[c + d*x]*(A + B*Log[(e*(c +
 d*x))/(a + b*x)]))/(2*b*d^4) + (g^3*(a + b*x)^4*(A + B*Log[(e*(c + d*x))/(
a + b*x)])^2)/(4*b) - (B^2*(b*c - a*d)^4*g^3*PolyLog[2, (b*(c + d*x))/(b*c
- a*d)])/(2*b*d^4)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2301

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

Rule 2390

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_))^(q_.), x_Symbol] :> Dist[1/e, Subst[Int[((f*x)/d)^q*(a + b*Log[c*x^
n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && E
qQ[e*f - d*g, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2393

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + (c*e*x)/g])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2394

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d + e*x
)^n]))/g, x] - Dist[(b*e*n)/g, Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
```

```
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2418

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*(RFx_), x_Sy
mbol] :> With[{u = ExpandIntegrand[(a + b*Log[c*(d + e*x)^n])^p, RFx, x]},
Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d, e, n}, x] && RationalFunctionQ[
RFx, x] && IntegerQ[p]
```

Rule 2486

```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))
^(r_.)]^(s_.), x_Symbol] :> Simp[((a + b*x)*Log[e*(f*(a + b*x)^p*(c + d*x)^
q)^r]^s)/b, x] + Dist[(q*r*s*(b*c - a*d))/b, Int[Log[e*(f*(a + b*x)^p*(c +
d*x)^q)^r]^(s - 1)/(c + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, p, q, r, s
}, x] && NeQ[b*c - a*d, 0] && EqQ[p + q, 0] && IGtQ[s, 0]
```

Rule 2524

```
Int[((a_.) + Log[(c_.)*(RFx_)^(p_.)]*(b_.))^(n_.)/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[(Log[d + e*x]*(a + b*Log[c*RFx^p])^n)/e, x] - Dist[(b*n*p)/e
, Int[(Log[d + e*x]*(a + b*Log[c*RFx^p])^(n - 1)*D[RFx, x])/RFx, x], x] /;
FreeQ[{a, b, c, d, e, p}, x] && RationalFunctionQ[RFx, x] && IGtQ[n, 0]
```

Rule 2525

```
Int[((a_.) + Log[(c_.)*(RFx_)^(p_.)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_))^(m_.
), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(a + b*Log[c*RFx^p])^n)/(e*(m + 1))
, x] - Dist[(b*n*p)/(e*(m + 1)), Int[SimplifyIntegrand[((d + e*x)^(m + 1)*(
a + b*Log[c*RFx^p])^(n - 1)*D[RFx, x])/RFx, x], x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && RationalFunctionQ[RFx, x] && IGtQ[n, 0] && (EqQ[n, 1] ||
IntegerQ[m]) && NeQ[m, -1]
```

Rule 2528

```
Int[((a_.) + Log[(c_.)*(RFx_)^(p_.)]*(b_.))^(n_.)*(RGx_), x_Symbol] :> With
[{u = ExpandIntegrand[(a + b*Log[c*RFx^p])^n, RGx, x]}, Int[u, x] /; SumQ[u
]] /; FreeQ[{a, b, c, p}, x] && RationalFunctionQ[RFx, x] && RationalFuncti
onQ[RGx, x] && IGtQ[n, 0]
```

Rubi steps

$$\int (ag+bgx)^3\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2 dx = \frac{g^3(a+bx)^4\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2}{4b} - \frac{B\int \frac{(bc-ad)g^4(a+bx)^3\left(-A-B\log\left(\frac{e(c+}{a+}\right.\right.}{c+dx}}{2bg}$$

$$= \frac{g^3(a+bx)^4\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2}{4b} - \frac{\left(B(bc-ad)g^3\right)\int \frac{(a+bx)^3\left(-A-B\,\mathrm{l}\right.}{c+d}}{2b}$$

$$= \frac{g^3(a+bx)^4\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2}{4b} - \frac{\left(B(bc-ad)g^3\right)\int\left(\frac{b(bc-ad)^2\left(-A-\right.}{}\right.}{}$$

$$= \frac{g^3(a+bx)^4\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2}{4b} - \frac{\left(B(bc-ad)g^3\right)\int(a+bx)^2\left(-\right.}{2d}$$

$$= \frac{AB(bc-ad)^3g^3x}{2d^3} - \frac{B(bc-ad)^2g^3(a+bx)^2\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)}{4bd^2} +$$

$$= \frac{AB(bc-ad)^3g^3x}{2d^3} + \frac{B^2(bc-ad)^3g^3(a+bx)\log\left(\frac{e(c+dx)}{a+bx}\right)}{2bd^3} - \frac{B(bc-a}{}$$

$$= \frac{AB(bc-ad)^3g^3x}{2d^3} + \frac{B^2(bc-ad)^4g^3\log(c+dx)}{2bd^4} + \frac{B^2(bc-ad)^3g^3(a}{2}$$

$$= \frac{AB(bc-ad)^3g^3x}{2d^3} - \frac{5B^2(bc-ad)^3g^3x}{12d^3} + \frac{B^2(bc-ad)^2g^3(a+bx)^2}{12bd^2} +$$

$$= \frac{AB(bc-ad)^3g^3x}{2d^3} - \frac{5B^2(bc-ad)^3g^3x}{12d^3} + \frac{B^2(bc-ad)^2g^3(a+bx)^2}{12bd^2} +$$

$$= \frac{AB(bc-ad)^3g^3x}{2d^3} - \frac{5B^2(bc-ad)^3g^3x}{12d^3} + \frac{B^2(bc-ad)^2g^3(a+bx)^2}{12bd^2} +$$

$$= \frac{AB(bc-ad)^3g^3x}{2d^3} - \frac{5B^2(bc-ad)^3g^3x}{12d^3} + \frac{B^2(bc-ad)^2g^3(a+bx)^2}{12bd^2} +$$

**Mathematica** [A] time = 0.35, size = 392, normalized size = 0.93

$$g^3\left(\frac{B(bc-ad)\left(2d^3(a+bx)^3\left(B\log\left(\frac{e(c+dx)}{a+bx}\right)+A\right)+3d^2(a+bx)^2(ad-bc)\left(B\log\left(\frac{e(c+dx)}{a+bx}\right)+A\right)-6(bc-ad)^3\log(c+dx)\left(B\log\left(\frac{e(c+dx)}{a+bx}\right)+A\right)+6Abdx(bc-ad)^2-B(b\right.}{}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*g + b*g*x)^3*(A + B*Log[(e*(c + d*x))/(a + b*x)])^2,x]
```

```
[Out] (g^3*((a + b*x)^4*(A + B*Log[(e*(c + d*x))/(a + b*x)])^2 + (B*(b*c - a*d)*(
6*A*b*d*(b*c - a*d)^2*x + 6*B*(b*c - a*d)^3*Log[a + b*x] - B*(b*c - a*d)*(2
*b*d*(b*c - a*d)*x - d^2*(a + b*x)^2 - 2*(b*c - a*d)^2*Log[c + d*x]) - 3*B*
(b*c - a*d)^2*(b*d*x + (-(b*c) + a*d)*Log[c + d*x]) + 6*b*B*(b*c - a*d)^2*(
c + d*x)*Log[(e*(c + d*x))/(a + b*x)] + 3*d^2*(-(b*c) + a*d)*(a + b*x)^2*(A
 + B*Log[(e*(c + d*x))/(a + b*x)]) + 2*d^3*(a + b*x)^3*(A + B*Log[(e*(c + d
*x))/(a + b*x)]) - 6*(b*c - a*d)^3*Log[c + d*x]*(A + B*Log[(e*(c + d*x))/(a
 + b*x)]) - 3*B*(b*c - a*d)^3*((2*Log[(d*(a + b*x))/(-(b*c) + a*d)] - Log[c
 + d*x])*Log[c + d*x] + 2*PolyLog[2, (b*(c + d*x))/(b*c - a*d)])))/(3*d^4))
)/(4*b)
```

**fricas [F]** time = 0.71, size = 0, normalized size = 0.00

$$\text{integral}\left(A^2b^3g^3x^3 + 3\,A^2ab^2g^3x^2 + 3\,A^2a^2bg^3x + A^2a^3g^3 + \left(B^2b^3g^3x^3 + 3\,B^2ab^2g^3x^2 + 3\,B^2a^2bg^3x + B^2a^3g^3\right)\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*g*x+a*g)^3*(A+B*log(e*(d*x+c)/(b*x+a)))^2,x, algorithm="fricas
")
```

```
[Out] integral(A^2*b^3*g^3*x^3 + 3*A^2*a*b^2*g^3*x^2 + 3*A^2*a^2*b*g^3*x + A^2*a^
3*g^3 + (B^2*b^3*g^3*x^3 + 3*B^2*a*b^2*g^3*x^2 + 3*B^2*a^2*b*g^3*x + B^2*a^
3*g^3)*log((d*e*x + c*e)/(b*x + a))^2 + 2*(A*B*b^3*g^3*x^3 + 3*A*B*a*b^2*g^
3*x^2 + 3*A*B*a^2*b*g^3*x + A*B*a^3*g^3)*log((d*e*x + c*e)/(b*x + a)), x)
```

**giac [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*g*x+a*g)^3*(A+B*log(e*(d*x+c)/(b*x+a)))^2,x, algorithm="giac")
```

```
[Out] Timed out
```

**maple [F]** time = 1.98, size = 0, normalized size = 0.00

$$\int \left(bgx + ag\right)^3 \left(B\ln\left(\frac{(dx+c)\,e}{bx+a}\right) + A\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*g*x+a*g)^3*(B*ln((d*x+c)/(b*x+a)*e)+A)^2,x)
```

```
[Out] int((b*g*x+a*g)^3*(B*ln((d*x+c)/(b*x+a)*e)+A)^2,x)
```

**maxima [B]** time = 2.16, size = 1735, normalized size = 4.13

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*g*x+a*g)^3*(A+B*log(e*(d*x+c)/(b*x+a)))^2,x, algorithm="maxima
")
```

```
[Out] 1/4*A^2*b^3*g^3*x^4 + A^2*a*b^2*g^3*x^3 + 3/2*A^2*a^2*b*g^3*x^2 + 2*(x*log(
d*e*x/(b*x + a) + c*e/(b*x + a)) - a*log(b*x + a)/b + c*log(d*x + c)/d)*A*B
*a^3*g^3 + 3*(x^2*log(d*e*x/(b*x + a) + c*e/(b*x + a)) + a^2*log(b*x + a)/b
^2 - c^2*log(d*x + c)/d^2 + (b*c - a*d)*x/(b*d))*A*B*a^2*b*g^3 + (2*x^3*log
(d*e*x/(b*x + a) + c*e/(b*x + a)) - 2*a^3*log(b*x + a)/b^3 + 2*c^3*log(d*x
+ c)/d^3 + ((b^2*c*d - a*b*d^2)*x^2 - 2*(b^2*c^2 - a^2*d^2)*x)/(b^2*d^2))*A
*B*a*b^2*g^3 + 1/12*(6*x^4*log(d*e*x/(b*x + a) + c*e/(b*x + a)) + 6*a^4*log
(b*x + a)/b^4 - 6*c^4*log(d*x + c)/d^4 + (2*(b^3*c*d^2 - a*b^2*d^3)*x^3 - 3
*(b^3*c^2*d - a^2*b*d^3)*x^2 + 6*(b^3*c^3 - a^3*d^3)*x)/(b^3*d^3))*A*B*b^3*
g^3 + A^2*a^3*g^3*x - 1/12*((6*g^3*log(e) - 11*g^3)*b^3*c^4 - 2*(12*g^3*log
(e) - 19*g^3)*a*b^2*c^3*d + 9*(4*g^3*log(e) - 5*g^3)*a^2*b*c^2*d^2 - 6*(4*g
^3*log(e) - 3*g^3)*a^3*c*d^3)*B^2*log(d*x + c)/d^4 + 1/2*(b^4*c^4*g^3 - 4*a
*b^3*c^3*d*g^3 + 6*a^2*b^2*c^2*d^2*g^3 - 4*a^3*b*c*d^3*g^3 + a^4*d^4*g^3)*(
log(b*x + a)*log((b*d*x + a*d)/(b*c - a*d) + 1) + dilog(-(b*d*x + a*d)/(b*c
 - a*d)))*B^2/(b*d^4) + 1/12*(3*B^2*b^4*d^4*g^3*x^4*log(e)^2 + 2*(b^4*c*d^3
*g^3*log(e) + (6*g^3*log(e)^2 - g^3*log(e))*a*b^3*d^4)*B^2*x^3 - ((3*g^3*lo
g(e) - g^3)*b^4*c^2*d^2 - 2*(6*g^3*log(e) - g^3)*a*b^3*c*d^3 - (18*g^3*log(
e)^2 - 9*g^3*log(e) + g^3)*a^2*b^2*d^4)*B^2*x^2 + ((6*g^3*log(e) - 5*g^3)*b
^4*c^3*d - (24*g^3*log(e) - 17*g^3)*a*b^3*c^2*d^2 + (36*g^3*log(e) - 19*g^3
)*a^2*b^2*c*d^3 + (12*g^3*log(e)^2 - 18*g^3*log(e) + 7*g^3)*a^3*b*d^4)*B^2*
x + 3*(B^2*b^4*d^4*g^3*x^4 + 4*B^2*a*b^3*d^4*g^3*x^3 + 6*B^2*a^2*b^2*d^4*g^
3*x^2 + 4*B^2*a^3*b*d^4*g^3*x + B^2*a^4*d^4*g^3)*log(b*x + a)^2 + 3*(B^2*b^
4*d^4*g^3*x^4 + 4*B^2*a*b^3*d^4*g^3*x^3 + 6*B^2*a^2*b^2*d^4*g^3*x^2 + 4*B^2
*a^3*b*d^4*g^3*x - (b^4*c^4*g^3 - 4*a*b^3*c^3*d*g^3 + 6*a^2*b^2*c^2*d^2*g^3
 - 4*a^3*b*c*d^3*g^3)*B^2)*log(d*x + c)^2 - (6*B^2*b^4*d^4*g^3*x^4*log(e) +
 2*(b^4*c*d^3*g^3 + (12*g^3*log(e) - g^3)*a*b^3*d^4)*B^2*x^3 - 3*(b^4*c^2*d
^2*g^3 - 4*a*b^3*c*d^3*g^3 - 3*(4*g^3*log(e) - g^3)*a^2*b^2*d^4)*B^2*x^2 +
6*(b^4*c^3*d*g^3 - 4*a*b^3*c^2*d^2*g^3 + 6*a^2*b^2*c*d^3*g^3 + (4*g^3*log(e
) - 3*g^3)*a^3*b*d^4)*B^2*x + (6*a*b^3*c^3*d*g^3 - 21*a^2*b^2*c^2*d^2*g^3 +
 26*a^3*b*c*d^3*g^3 + (6*g^3*log(e) - 11*g^3)*a^4*d^4)*B^2)*log(b*x + a) +
(6*B^2*b^4*d^4*g^3*x^4*log(e) + 2*(b^4*c*d^3*g^3 + (12*g^3*log(e) - g^3)*a*
b^3*d^4)*B^2*x^3 - 3*(b^4*c^2*d^2*g^3 - 4*a*b^3*c*d^3*g^3 - 3*(4*g^3*log(e)
 - g^3)*a^2*b^2*d^4)*B^2*x^2 + 6*(b^4*c^3*d*g^3 - 4*a*b^3*c^2*d^2*g^3 + 6*a
```

```
^2*b^2*c*d^3*g^3 + (4*g^3*log(e) - 3*g^3)*a^3*b*d^4)*B^2*x - 6*(B^2*b^4*d^4
*g^3*x^4 + 4*B^2*a*b^3*d^4*g^3*x^3 + 6*B^2*a^2*b^2*d^4*g^3*x^2 + 4*B^2*a^3*
b*d^4*g^3*x + B^2*a^4*d^4*g^3)*log(b*x + a))*log(d*x + c))/(b*d^4)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.00

$$\int \left(a\,g+b\,g\,x\right)^3\,{\left(A+B\,\ln\left(\frac{e\,\left(c+d\,x\right)}{a+b\,x}\right)\right)}^2 \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*g + b*g*x)^3*(A + B*log((e*(c + d*x))/(a + b*x)))^2,x)
[Out] int((a*g + b*g*x)^3*(A + B*log((e*(c + d*x))/(a + b*x)))^2, x)
```

**sympy [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*g*x+a*g)**3*(A+B*ln(e*(d*x+c)/(b*x+a)))**2,x)
[Out] Timed out
```

# 3.184 $\int (ag+bgx)^2 \left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2 dx$

**Optimal**. Leaf size=335

$$-\frac{2Bg^2(bc-ad)^3\log\left(1-\frac{d(a+bx)}{b(c+dx)}\right)\left(B\log\left(\frac{e(c+dx)}{a+bx}\right)+A\right)}{3bd^3}-\frac{2Bg^2(c+dx)(bc-ad)^2\left(B\log\left(\frac{e(c+dx)}{a+bx}\right)+A\right)}{3d^3}+\frac{Bg^2(a+b}{}$$

```
[Out] 1/3*B^2*(-a*d+b*c)^2*g^2*x/d^2-B^2*(-a*d+b*c)^3*g^2*ln(b*x+a)/b/d^3-1/3*B^2
*(-a*d+b*c)^3*g^2*ln((d*x+c)/(b*x+a))/b/d^3+1/3*B*(-a*d+b*c)*g^2*(b*x+a)^2*
(A+B*ln(e*(d*x+c)/(b*x+a)))/b/d-2/3*B*(-a*d+b*c)^2*g^2*(d*x+c)*(A+B*ln(e*(d
*x+c)/(b*x+a)))/d^3+1/3*g^2*(b*x+a)^3*(A+B*ln(e*(d*x+c)/(b*x+a)))^2/b-2/3*B
*(-a*d+b*c)^3*g^2*(A+B*ln(e*(d*x+c)/(b*x+a)))*ln(1-d*(b*x+a)/b/(d*x+c))/b/d
^3+2/3*B^2*(-a*d+b*c)^3*g^2*polylog(2,d*(b*x+a)/b/(d*x+c))/b/d^3
```

**Rubi [A]** time = 0.56, antiderivative size = 389, normalized size of antiderivative = 1.16, number of steps used = 20, number of rules used = 13, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.406, Rules used = {2525, 12, 2528, 2486, 31, 43, 2524, 2418, 2394, 2393, 2391, 2390, 2301}

$$\frac{2B^2g^2(bc-ad)^3\text{PolyLog}\left(2,\frac{b(c+dx)}{bc-ad}\right)}{3bd^3}+\frac{2Bg^2(bc-ad)^3\log(c+dx)\left(B\log\left(\frac{e(c+dx)}{a+bx}\right)+A\right)}{3bd^3}-\frac{2ABg^2x(bc-ad)^2}{3d^2}+\frac{Bg^2}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a*g + b*g*x)^2*(A + B*Log[(e*(c + d*x))/(a + b*x)])^2,x]
```

```
[Out] (-2*A*B*(b*c - a*d)^2*g^2*x)/(3*d^2) + (B^2*(b*c - a*d)^2*g^2*x)/(3*d^2) -
(B^2*(b*c - a*d)^3*g^2*Log[c + d*x])/(b*d^3) + (2*B^2*(b*c - a*d)^3*g^2*Log
[-((d*(a + b*x))/(b*c - a*d))]*Log[c + d*x])/(3*b*d^3) - (B^2*(b*c - a*d)^3
*g^2*Log[c + d*x]^2)/(3*b*d^3) - (2*B^2*(b*c - a*d)^2*g^2*(a + b*x)*Log[(e*
(c + d*x))/(a + b*x)])/(3*b*d^2) + (B*(b*c - a*d)*g^2*(a + b*x)^2*(A + B*Lo
g[(e*(c + d*x))/(a + b*x)]))/(3*b*d) + (2*B*(b*c - a*d)^3*g^2*Log[c + d*x]*
(A + B*Log[(e*(c + d*x))/(a + b*x)]))/(3*b*d^3) + (g^2*(a + b*x)^3*(A + B*L
og[(e*(c + d*x))/(a + b*x)])^2)/(3*b) + (2*B^2*(b*c - a*d)^3*g^2*PolyLog[2,
 (b*(c + d*x))/(b*c - a*d)])/(3*b*d^3)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2301

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

Rule 2390

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_))^(q_.), x_Symbol] :> Dist[1/e, Subst[Int[((f*x)/d)^q*(a + b*Log[c*x^
n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && E
qQ[e*f - d*g, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2393

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + (c*e*x)/g])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2394

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d + e*x
)^n]))/g, x] - Dist[(b*e*n)/g, Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2418

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*(RFx_), x_Sy
mbol] :> With[{u = ExpandIntegrand[(a + b*Log[c*(d + e*x)^n])^p, RFx, x]},
```

```
Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d, e, n}, x] && RationalFunctionQ[
RFx, x] && IntegerQ[p]
```

Rule 2486

```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))
^(r_.)]^(s_.), x_Symbol] :> Simp[((a + b*x)*Log[e*(f*(a + b*x)^p*(c + d*x)^
q)^r]^s)/b, x] + Dist[(q*r*s*(b*c - a*d))/b, Int[Log[e*(f*(a + b*x)^p*(c +
d*x)^q)^r]^(s - 1)/(c + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, p, q, r, s
}, x] && NeQ[b*c - a*d, 0] && EqQ[p + q, 0] && IGtQ[s, 0]
```

Rule 2524

```
Int[((a_.) + Log[(c_.)*(RFx_)^(p_.)]*(b_.))^(n_.)/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[(Log[d + e*x]*(a + b*Log[c*RFx^p])^n)/e, x] - Dist[(b*n*p)/e
, Int[(Log[d + e*x]*(a + b*Log[c*RFx^p])^(n - 1)*D[RFx, x])/RFx, x], x] /;
FreeQ[{a, b, c, d, e, p}, x] && RationalFunctionQ[RFx, x] && IGtQ[n, 0]
```

Rule 2525

```
Int[((a_.) + Log[(c_.)*(RFx_)^(p_.)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_))^(m_.
), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(a + b*Log[c*RFx^p])^n)/(e*(m + 1))
, x] - Dist[(b*n*p)/(e*(m + 1)), Int[SimplifyIntegrand[((d + e*x)^(m + 1)*(
a + b*Log[c*RFx^p])^(n - 1)*D[RFx, x])/RFx, x], x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && RationalFunctionQ[RFx, x] && IGtQ[n, 0] && (EqQ[n, 1] ||
IntegerQ[m]) && NeQ[m, -1]
```

Rule 2528

```
Int[((a_.) + Log[(c_.)*(RFx_)^(p_.)]*(b_.))^(n_.)*(RGx_), x_Symbol] :> With
[{u = ExpandIntegrand[(a + b*Log[c*RFx^p])^n, RGx, x]}, Int[u, x] /; SumQ[u
]] /; FreeQ[{a, b, c, p}, x] && RationalFunctionQ[RFx, x] && RationalFuncti
onQ[RGx, x] && IGtQ[n, 0]
```

Rubi steps

$$\int (ag+bgx)^2 \left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2 dx = \frac{g^2(a+bx)^3\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2}{3b} - \frac{(2B)\int \frac{(bc-ad)g^3(a+bx)^2\left(-A-B\log\right.}{c+dx}}{3bg}$$

$$= \frac{g^2(a+bx)^3\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2}{3b} - \frac{\left(2B(bc-ad)g^2\right)\int \frac{(a+bx)^2\left(-A\right.}{c}}{3b}$$

$$= \frac{g^2(a+bx)^3\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2}{3b} - \frac{\left(2B(bc-ad)g^2\right)\int\left(-\frac{b(bc-ad)}{}\right.}{}$$

$$= \frac{g^2(a+bx)^3\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2}{3b} - \frac{\left(2B(bc-ad)g^2\right)\int(a+bx)\left(\right.}{3d}$$

$$= -\frac{2AB(bc-ad)^2g^2x}{3d^2} + \frac{B(bc-ad)g^2(a+bx)^2\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right.}{3bd}$$

$$= -\frac{2AB(bc-ad)^2g^2x}{3d^2} - \frac{2B^2(bc-ad)^2g^2(a+bx)\log\left(\frac{e(c+dx)}{a+bx}\right)}{3bd^2} + \frac{B(}{}$$

$$= -\frac{2AB(bc-ad)^2g^2x}{3d^2} - \frac{2B^2(bc-ad)^3g^2\log(c+dx)}{3bd^3} - \frac{2B^2(bc-ad}{}$$

$$= -\frac{2AB(bc-ad)^2g^2x}{3d^2} + \frac{B^2(bc-ad)^2g^2x}{3d^2} - \frac{B^2(bc-ad)^3g^2\log(c+}{bd^3}$$

$$= -\frac{2AB(bc-ad)^2g^2x}{3d^2} + \frac{B^2(bc-ad)^2g^2x}{3d^2} - \frac{B^2(bc-ad)^3g^2\log(c+}{bd^3}$$

$$= -\frac{2AB(bc-ad)^2g^2x}{3d^2} + \frac{B^2(bc-ad)^2g^2x}{3d^2} - \frac{B^2(bc-ad)^3g^2\log(c+}{bd^3}$$

$$= -\frac{2AB(bc-ad)^2g^2x}{3d^2} + \frac{B^2(bc-ad)^2g^2x}{3d^2} - \frac{B^2(bc-ad)^3g^2\log(c+}{bd^3}$$

**Mathematica** [A] time = 0.24, size = 290, normalized size = 0.87

$$g^2\left((a+bx)^3\left(B\log\left(\frac{e(c+dx)}{a+bx}\right)+A\right)^2 - \frac{B(bc-ad)\left(-d^2(a+bx)^2\left(B\log\left(\frac{e(c+dx)}{a+bx}\right)+A\right)-2(bc-ad)^2\log(c+dx)\left(B\log\left(\frac{e(c+dx)}{a+bx}\right)+A\right)+2Abdx(bc-ad\right.}{}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*g + b*g*x)^2*(A + B*Log[(e*(c + d*x))/(a + b*x)])^2,x]
```

```
[Out] (g^2*((a + b*x)^3*(A + B*Log[(e*(c + d*x))/(a + b*x)])^2 - (B*(b*c - a*d)*(
2*A*b*d*(b*c - a*d)*x + 2*B*(b*c - a*d)^2*Log[a + b*x] - B*(b*c - a*d)*(b*d
*x + (-(b*c) + a*d)*Log[c + d*x]) + 2*b*B*(b*c - a*d)*(c + d*x)*Log[(e*(c +
 d*x))/(a + b*x)] - d^2*(a + b*x)^2*(A + B*Log[(e*(c + d*x))/(a + b*x)]) -
2*(b*c - a*d)^2*Log[c + d*x]*(A + B*Log[(e*(c + d*x))/(a + b*x)]) - B*(b*c
- a*d)^2*((2*Log[(d*(a + b*x))/(-(b*c) + a*d)] - Log[c + d*x])*Log[c + d*x]
 + 2*PolyLog[2, (b*(c + d*x))/(b*c - a*d)])))/d^3))/(3*b)
```

**fricas [F]** time = 1.02, size = 0, normalized size = 0.00

$$\text{integral}\left(A^2b^2g^2x^2 + 2\,A^2abg^2x + A^2a^2g^2 + \left(B^2b^2g^2x^2 + 2\,B^2abg^2x + B^2a^2g^2\right)\log\left(\frac{dex + ce}{bx + a}\right)^2 + 2\,\left(ABb^2g^2x^2 +\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*g*x+a*g)^2*(A+B*log(e*(d*x+c)/(b*x+a)))^2,x, algorithm="fricas
")
```

```
[Out] integral(A^2*b^2*g^2*x^2 + 2*A^2*a*b*g^2*x + A^2*a^2*g^2 + (B^2*b^2*g^2*x^2
 + 2*B^2*a*b*g^2*x + B^2*a^2*g^2)*log((d*e*x + c*e)/(b*x + a))^2 + 2*(A*B*b
^2*g^2*x^2 + 2*A*B*a*b*g^2*x + A*B*a^2*g^2)*log((d*e*x + c*e)/(b*x + a)), x
)
```

**giac [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*g*x+a*g)^2*(A+B*log(e*(d*x+c)/(b*x+a)))^2,x, algorithm="giac")
```

```
[Out] Timed out
```

**maple [F]** time = 1.81, size = 0, normalized size = 0.00

$$\int \left(bgx + ag\right)^2 \left(B\ln\left(\frac{(dx + c)\,e}{bx + a}\right) + A\right)^2 \,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*g*x+a*g)^2*(B*ln((d*x+c)/(b*x+a)*e)+A)^2,x)
```

```
[Out] int((b*g*x+a*g)^2*(B*ln((d*x+c)/(b*x+a)*e)+A)^2,x)
```

**maxima [B]** time = 2.09, size = 1172, normalized size = 3.50

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*g*x+a*g)^2*(A+B*log(e*(d*x+c)/(b*x+a)))^2,x, algorithm="maxima
")
```

```
[Out] 1/3*A^2*b^2*g^2*x^3 + A^2*a*b*g^2*x^2 + 2*(x*log(d*e*x/(b*x + a) + c*e/(b*x
 + a)) - a*log(b*x + a)/b + c*log(d*x + c)/d)*A*B*a^2*g^2 + 2*(x^2*log(d*e*
x/(b*x + a) + c*e/(b*x + a)) + a^2*log(b*x + a)/b^2 - c^2*log(d*x + c)/d^2
+ (b*c - a*d)*x/(b*d))*A*B*a*b*g^2 + 1/3*(2*x^3*log(d*e*x/(b*x + a) + c*e/(
b*x + a)) - 2*a^3*log(b*x + a)/b^3 + 2*c^3*log(d*x + c)/d^3 + ((b^2*c*d - a
*b*d^2)*x^2 - 2*(b^2*c^2 - a^2*d^2)*x)/(b^2*d^2))*A*B*b^2*g^2 + A^2*a^2*g^2
*x + 1/3*((2*g^2*log(e) - 3*g^2)*b^2*c^3 - (6*g^2*log(e) - 7*g^2)*a*b*c^2*d
 + 2*(3*g^2*log(e) - 2*g^2)*a^2*c*d^2)*B^2*log(d*x + c)/d^3 - 2/3*(b^3*c^3*
g^2 - 3*a*b^2*c^2*d*g^2 + 3*a^2*b*c*d^2*g^2 - a^3*d^3*g^2)*(log(b*x + a)*lo
g((b*d*x + a*d)/(b*c - a*d) + 1) + dilog(-(b*d*x + a*d)/(b*c - a*d)))*B^2/(
b*d^3) + 1/3*(B^2*b^3*d^3*g^2*x^3*log(e)^2 + (b^3*c*d^2*g^2*log(e) + (3*g^2
*log(e)^2 - g^2*log(e))*a*b^2*d^3)*B^2*x^2 - ((2*g^2*log(e) - g^2)*b^3*c^2*
d - 2*(3*g^2*log(e) - g^2)*a*b^2*c*d^2 - (3*g^2*log(e)^2 - 4*g^2*log(e) + g
^2)*a^2*b*d^3)*B^2*x + (B^2*b^3*d^3*g^2*x^3 + 3*B^2*a*b^2*d^3*g^2*x^2 + 3*B
^2*a^2*b*d^3*g^2*x + B^2*a^3*d^3*g^2)*log(b*x + a)^2 + (B^2*b^3*d^3*g^2*x^3
 + 3*B^2*a*b^2*d^3*g^2*x^2 + 3*B^2*a^2*b*d^3*g^2*x + (b^3*c^3*g^2 - 3*a*b^2
*c^2*d*g^2 + 3*a^2*b*c*d^2*g^2)*B^2)*log(d*x + c)^2 - (2*B^2*b^3*d^3*g^2*x^
3*log(e) + (b^3*c*d^2*g^2 + (6*g^2*log(e) - g^2)*a*b^2*d^3)*B^2*x^2 - 2*(b^
3*c^2*d*g^2 - 3*a*b^2*c*d^2*g^2 - (3*g^2*log(e) - 2*g^2)*a^2*b*d^3)*B^2*x -
 (2*a*b^2*c^2*d*g^2 - 5*a^2*b*c*d^2*g^2 - (2*g^2*log(e) - 3*g^2)*a^3*d^3)*B
^2)*log(b*x + a) + (2*B^2*b^3*d^3*g^2*x^3*log(e) + (b^3*c*d^2*g^2 + (6*g^2*
log(e) - g^2)*a*b^2*d^3)*B^2*x^2 - 2*(b^3*c^2*d*g^2 - 3*a*b^2*c*d^2*g^2 - (
3*g^2*log(e) - 2*g^2)*a^2*b*d^3)*B^2*x - 2*(B^2*b^3*d^3*g^2*x^3 + 3*B^2*a*b
^2*d^3*g^2*x^2 + 3*B^2*a^2*b*d^3*g^2*x + B^2*a^3*d^3*g^2)*log(b*x + a))*log
(d*x + c))/(b*d^3)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.00

$$\int {\left(a\,g+b\,g\,x\right)}^2\,{\left(A+B\,\ln\left(\frac{e\,\left(c+d\,x\right)}{a+b\,x}\right)\right)}^2 \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*g + b*g*x)^2*(A + B*log((e*(c + d*x))/(a + b*x)))^2,x)
```

```
[Out] int((a*g + b*g*x)^2*(A + B*log((e*(c + d*x))/(a + b*x)))^2, x)
```

**sympy [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*g*x+a*g)**2*(A+B*ln(e*(d*x+c)/(b*x+a)))**2,x)
[Out] Timed out
```

# 3.185 $\int (ag+bgx)\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2\,dx$

**Optimal**. Leaf size=202

$$\frac{Bg(bc-ad)^2\log\left(1-\frac{d(a+bx)}{b(c+dx)}\right)\left(B\log\left(\frac{e(c+dx)}{a+bx}\right)+A\right)}{bd^2}+\frac{Bg(c+dx)(bc-ad)\left(B\log\left(\frac{e(c+dx)}{a+bx}\right)+A\right)}{d^2}+\frac{g(a+bx)^2\left(B\log\right.}{}$$

```
[Out] B^2*(-a*d+b*c)^2*g*ln(b*x+a)/b/d^2+B*(-a*d+b*c)*g*(d*x+c)*(A+B*ln(e*(d*x+c)
/(b*x+a)))/d^2+1/2*g*(b*x+a)^2*(A+B*ln(e*(d*x+c)/(b*x+a)))^2/b+B*(-a*d+b*c)
^2*g*(A+B*ln(e*(d*x+c)/(b*x+a)))*ln(1-d*(b*x+a)/b/(d*x+c))/b/d^2-B^2*(-a*d+
b*c)^2*g*polylog(2,d*(b*x+a)/b/(d*x+c))/b/d^2
```

**Rubi [A]** time = 0.42, antiderivative size = 284, normalized size of antiderivative = 1.41, number of steps used = 16, number of rules used = 12, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {2525, 12, 2528, 2486, 31, 2524, 2418, 2394, 2393, 2391, 2390, 2301}

$$-\frac{B^2g(bc-ad)^2\text{PolyLog}\left(2,\frac{b(c+dx)}{bc-ad}\right)}{bd^2}-\frac{Bg(bc-ad)^2\log(c+dx)\left(B\log\left(\frac{e(c+dx)}{a+bx}\right)+A\right)}{bd^2}+\frac{g(a+bx)^2\left(B\log\left(\frac{e(c+dx)}{a+bx}\right.\right.}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[(a*g + b*g*x)*(A + B*Log[(e*(c + d*x))/(a + b*x)])^2,x]
```

```
[Out] (A*B*(b*c - a*d)*g*x)/d + (B^2*(b*c - a*d)^2*g*Log[c + d*x])/(b*d^2) - (B^2
*(b*c - a*d)^2*g*Log[-((d*(a + b*x))/(b*c - a*d))]*Log[c + d*x])/(b*d^2) +
(B^2*(b*c - a*d)^2*g*Log[c + d*x]^2)/(2*b*d^2) + (B^2*(b*c - a*d)*g*(a + b*
x)*Log[(e*(c + d*x))/(a + b*x)])/(b*d) - (B*(b*c - a*d)^2*g*Log[c + d*x]*(A
 + B*Log[(e*(c + d*x))/(a + b*x)]))/(b*d^2) + (g*(a + b*x)^2*(A + B*Log[(e*
(c + d*x))/(a + b*x)])^2)/(2*b) - (B^2*(b*c - a*d)^2*g*PolyLog[2, (b*(c + d
*x))/(b*c - a*d)])/(b*d^2)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 2301

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

Rule 2390

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_))^(q_.), x_Symbol] :> Dist[1/e, Subst[Int[((f*x)/d)^q*(a + b*Log[c*x^
n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && E
qQ[e*f - d*g, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2393

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + (c*e*x)/g])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2394

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d + e*x
)^n]))/g, x] - Dist[(b*e*n)/g, Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2418

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*(RFx_), x_Sy
mbol] :> With[{u = ExpandIntegrand[(a + b*Log[c*(d + e*x)^n])^p, RFx, x]},
Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d, e, n}, x] && RationalFunctionQ[
RFx, x] && IntegerQ[p]
```

Rule 2486

```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))
^(r_.)]^(s_.), x_Symbol] :> Simp[((a + b*x)*Log[e*(f*(a + b*x)^p*(c + d*x)^
q)^r]^s)/b, x] + Dist[(q*r*s*(b*c - a*d))/b, Int[Log[e*(f*(a + b*x)^p*(c +
d*x)^q)^r]^(s - 1)/(c + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, p, q, r, s
}, x] && NeQ[b*c - a*d, 0] && EqQ[p + q, 0] && IGtQ[s, 0]
```

Rule 2524

```
Int[((a_.) + Log[(c_.)*(RFx_)^(p_.)]*(b_.))^(n_.)/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[(Log[d + e*x]*(a + b*Log[c*RFx^p])^n)/e, x] - Dist[(b*n*p)/e
, Int[(Log[d + e*x]*(a + b*Log[c*RFx^p])^(n - 1)*D[RFx, x])/RFx, x], x] /;
FreeQ[{a, b, c, d, e, p}, x] && RationalFunctionQ[RFx, x] && IGtQ[n, 0]
```

Rule 2525

```
Int[((a_.) + Log[(c_.)*(RFx_)^(p_.)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_))^(m_.
), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(a + b*Log[c*RFx^p])^n)/(e*(m + 1))
, x] - Dist[(b*n*p)/(e*(m + 1)), Int[SimplifyIntegrand[((d + e*x)^(m + 1)*(
a + b*Log[c*RFx^p])^(n - 1)*D[RFx, x])/RFx, x], x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && RationalFunctionQ[RFx, x] && IGtQ[n, 0] && (EqQ[n, 1] ||
IntegerQ[m]) && NeQ[m, -1]
```

Rule 2528

```
Int[((a_.) + Log[(c_.)*(RFx_)^(p_.)]*(b_.))^(n_.)*(RGx_), x_Symbol] :> With
[{u = ExpandIntegrand[(a + b*Log[c*RFx^p])^n, RGx, x]}, Int[u, x] /; SumQ[u
]] /; FreeQ[{a, b, c, p}, x] && RationalFunctionQ[RFx, x] && RationalFuncti
onQ[RGx, x] && IGtQ[n, 0]
```

Rubi steps

$$\int (ag+bgx)\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2 dx = \frac{g(a+bx)^2\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2}{2b} - \frac{B\int \frac{(bc-ad)g^2(a+bx)\left(-A-B\log\left(\frac{e(c+dx)}{a+bx}\right)\right.}{c+dx}}{bg}$$

$$= \frac{g(a+bx)^2\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2}{2b} - \frac{(B(bc-ad)g)\int \frac{(a+bx)\left(-A-B\log\left(\frac{e(\right.\right.}{}}{c+dx}}{b}$$

$$= \frac{g(a+bx)^2\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2}{2b} - \frac{(B(bc-ad)g)\int\left(\frac{b\left(-A-B\log\left(\frac{e(c+dx}{a+bx}\right.\right.}{d}\right.}{}$$

$$= \frac{g(a+bx)^2\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2}{2b} - \frac{(B(bc-ad)g)\int\left(-A-B\log\left(\frac{e(}{a}\right.\right.}{d}$$

$$= \frac{AB(bc-ad)gx}{d} - \frac{B(bc-ad)^2 g\log(c+dx)\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)}{bd^2} + \ldots$$

$$= \frac{AB(bc-ad)gx}{d} + \frac{B^2(bc-ad)g(a+bx)\log\left(\frac{e(c+dx)}{a+bx}\right)}{bd} - \frac{B(bc-ad)^2 g}{}$$

$$= \frac{AB(bc-ad)gx}{d} + \frac{B^2(bc-ad)^2 g\log(c+dx)}{bd^2} + \frac{B^2(bc-ad)g(a+bx)}{bd}$$

$$= \frac{AB(bc-ad)gx}{d} + \frac{B^2(bc-ad)^2 g\log(c+dx)}{bd^2} + \frac{B^2(bc-ad)g(a+bx)}{bd}$$

$$= \frac{AB(bc-ad)gx}{d} + \frac{B^2(bc-ad)^2 g\log(c+dx)}{bd^2} - \frac{B^2(bc-ad)^2 g\log\left(-\right.}{bd}$$

$$= \frac{AB(bc-ad)gx}{d} + \frac{B^2(bc-ad)^2 g\log(c+dx)}{bd^2} - \frac{B^2(bc-ad)^2 g\log\left(-\right.}{bd}$$

$$= \frac{AB(bc-ad)gx}{d} + \frac{B^2(bc-ad)^2 g\log(c+dx)}{bd^2} - \frac{B^2(bc-ad)^2 g\log\left(-\right.}{bd}$$

**Mathematica [A]** time = 0.18, size = 203, normalized size = 1.00

$$\frac{g\left(\frac{B(bc-ad)\left(-2(bc-ad)\log(c+dx)\left(B\log\left(\frac{e(c+dx)}{a+bx}\right)+A\right)+2bB(c+dx)\log\left(\frac{e(c+dx)}{a+bx}\right)-B(bc-ad)\left(2\text{Li}_2\left(\frac{b(c+dx)}{bc-ad}\right)+\log(c+dx)\left(2\log\left(\frac{d(a+bx)}{ad-bc}\right)-\log(c+dx)\right)\right)+2B(\right.}{d^2}\right.}{2b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*g + b*g*x)*(A + B*Log[(e*(c + d*x))/(a + b*x)])^2,x]
```

```
[Out] (g*((a + b*x)^2*(A + B*Log[(e*(c + d*x))/(a + b*x)])^2 + (B*(b*c - a*d)*(2*
A*b*d*x + 2*B*(b*c - a*d)*Log[a + b*x] + 2*b*B*(c + d*x)*Log[(e*(c + d*x))/
(a + b*x)] - 2*(b*c - a*d)*Log[c + d*x]*(A + B*Log[(e*(c + d*x))/(a + b*x)]
) - B*(b*c - a*d)*((2*Log[(d*(a + b*x))/(-(b*c) + a*d)] - Log[c + d*x])*Log
[c + d*x] + 2*PolyLog[2, (b*(c + d*x))/(b*c - a*d)])))/d^2))/(2*b)
```

**fricas [F]** time = 0.66, size = 0, normalized size = 0.00

$$\text{integral}\left(A^2bgx + A^2ag + \left(B^2bgx + B^2ag\right)\log\left(\frac{dex + ce}{bx + a}\right)^2 + 2\left(ABbgx + ABag\right)\log\left(\frac{dex + ce}{bx + a}\right), x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*g*x+a*g)*(A+B*log(e*(d*x+c)/(b*x+a)))^2,x, algorithm="fricas")
```

```
[Out] integral(A^2*b*g*x + A^2*a*g + (B^2*b*g*x + B^2*a*g)*log((d*e*x + c*e)/(b*x
 + a))^2 + 2*(A*B*b*g*x + A*B*a*g)*log((d*e*x + c*e)/(b*x + a)), x)
```

**giac [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*g*x+a*g)*(A+B*log(e*(d*x+c)/(b*x+a)))^2,x, algorithm="giac")
```

```
[Out] Timed out
```

**maple [F]** time = 1.53, size = 0, normalized size = 0.00

$$\int \left(bgx + ag\right)\left(B\ln\left(\frac{(dx + c)e}{bx + a}\right) + A\right)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*g*x+a*g)*(B*ln((d*x+c)/(b*x+a)*e)+A)^2,x)
```

```
[Out] int((b*g*x+a*g)*(B*ln((d*x+c)/(b*x+a)*e)+A)^2,x)
```

**maxima [B]** time = 1.97, size = 619, normalized size = 3.06

$$\frac{1}{2}A^2bgx^2+2\left(x\log\left(\frac{dex}{bx + a} + \frac{ce}{bx + a}\right) - \frac{a\log(bx + a)}{b} + \frac{c\log(dx + c)}{d}\right)ABag+\left(x^2\log\left(\frac{dex}{bx + a} + \frac{ce}{bx + a}\right) + \frac{a^2}{\ldots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*g*x+a*g)*(A+B*log(e*(d*x+c)/(b*x+a)))^2,x, algorithm="maxima")
```

```
[Out] 1/2*A^2*b*g*x^2 + 2*(x*log(d*e*x/(b*x + a) + c*e/(b*x + a)) - a*log(b*x + a
)/b + c*log(d*x + c)/d)*A*B*a*g + (x^2*log(d*e*x/(b*x + a) + c*e/(b*x + a))
 + a^2*log(b*x + a)/b^2 - c^2*log(d*x + c)/d^2 + (b*c - a*d)*x/(b*d))*A*B*b
*g + A^2*a*g*x - ((g*log(e) - g)*b*c^2 - (2*g*log(e) - g)*a*c*d)*B^2*log(d*
x + c)/d^2 + (b^2*c^2*g - 2*a*b*c*d*g + a^2*d^2*g)*(log(b*x + a)*log((b*d*x
 + a*d)/(b*c - a*d) + 1) + dilog(-(b*d*x + a*d)/(b*c - a*d)))*B^2/(b*d^2) +
 1/2*(B^2*b^2*d^2*g*x^2*log(e)^2 + 2*(b^2*c*d*g*log(e) + (g*log(e)^2 - g*lo
g(e))*a*b*d^2)*B^2*x + (B^2*b^2*d^2*g*x^2 + 2*B^2*a*b*d^2*g*x + B^2*a^2*d^2
*g)*log(b*x + a)^2 + (B^2*b^2*d^2*g*x^2 + 2*B^2*a*b*d^2*g*x - (b^2*c^2*g -
2*a*b*c*d*g)*B^2)*log(d*x + c)^2 - 2*(B^2*b^2*d^2*g*x^2*log(e) + ((2*g*log(
e) - g)*a*b*d^2 + b^2*c*d*g)*B^2*x + ((g*log(e) - g)*a^2*d^2 + a*b*c*d*g)*B
^2)*log(b*x + a) + 2*(B^2*b^2*d^2*g*x^2*log(e) + ((2*g*log(e) - g)*a*b*d^2
+ b^2*c*d*g)*B^2*x - (B^2*b^2*d^2*g*x^2 + 2*B^2*a*b*d^2*g*x + B^2*a^2*d^2*g
)*log(b*x + a))*log(d*x + c))/(b*d^2)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.00

$$\int (a\,g+b\,g\,x)\,\left(A+B\,\ln\left(\frac{e\,(c+d\,x)}{a+b\,x}\right)\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*g + b*g*x)*(A + B*log((e*(c + d*x))/(a + b*x)))^2,x)
```

```
[Out] int((a*g + b*g*x)*(A + B*log((e*(c + d*x))/(a + b*x)))^2, x)
```

**sympy [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*g*x+a*g)*(A+B*ln(e*(d*x+c)/(b*x+a)))**2,x)
```

```
[Out] Timed out
```

# 3.186 $\int \frac{\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2}{ag+bgx}\,dx$

**Optimal**. Leaf size=128

$$-\frac{2B\text{Li}_2\left(\frac{b(c+dx)}{d(a+bx)}\right)\left(B\log\left(\frac{e(c+dx)}{a+bx}\right)+A\right)}{bg}-\frac{\log\left(-\frac{bc-ad}{d(a+bx)}\right)\left(B\log\left(\frac{e(c+dx)}{a+bx}\right)+A\right)^2}{bg}+\frac{2B^2\text{Li}_3\left(\frac{b(c+dx)}{d(a+bx)}\right)}{bg}$$

```
[Out] -ln((a*d-b*c)/d/(b*x+a))*(A+B*ln(e*(d*x+c)/(b*x+a)))^2/b/g-2*B*(A+B*ln(e*(d
*x+c)/(b*x+a)))*polylog(2,b*(d*x+c)/d/(b*x+a))/b/g+2*B^2*polylog(3,b*(d*x+c
)/d/(b*x+a))/b/g
```

**Rubi [B]** time = 4.42, antiderivative size = 719, normalized size of antiderivative = 5.62, number of steps used = 47, number of rules used = 24, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.750, Rules used = {2524, 12, 2528, 2418, 2390, 2301, 2394, 2393, 2391, 6691, 6741, 6742, 2499, 2302, 30, 2396, 2433, 2374, 6589, 2500, 2440, 2434, 2375, 2317}

$$-\frac{2AB\text{PolyLog}\left(2,-\frac{d(a+bx)}{bc-ad}\right)}{bg}+\frac{2B^2\text{PolyLog}\left(2,-\frac{d(a+bx)}{bc-ad}\right)\left(-\log\left(\frac{e(c+dx)}{a+bx}\right)+\log\left(\frac{1}{a+bx}\right)+\log(c+dx)\right)}{bg}-\frac{2B^2\text{PolyI}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Log[(e*(c + d*x))/(a + b*x)])^2/(a*g + b*g*x),x]
```

```
[Out] (A*B*Log[g*(a + b*x)]^2)/(b*g) + (B^2*Log[g*(a + b*x)]^3)/(3*b*g) - (B^2*Lo
g[(a + b*x)^(-1)]^2*Log[c + d*x])/(b*g) - (2*B^2*Log[(a + b*x)^(-1)]*Log[g*
(a + b*x)]*Log[c + d*x])/(b*g) - (B^2*Log[g*(a + b*x)]^2*Log[c + d*x])/(b*g
) + (B^2*Log[-((d*(a + b*x))/(b*c - a*d))]*Log[c + d*x]^2)/(b*g) - (B^2*Log
[g*(a + b*x)]*Log[c + d*x]^2)/(b*g) + (B^2*Log[(a + b*x)^(-1)]^2*Log[(b*(c
+ d*x))/(b*c - a*d)])/(b*g) + (B^2*Log[g*(a + b*x)]^2*Log[(b*(c + d*x))/(b*
c - a*d)])/(b*g) - (2*A*B*Log[(b*(c + d*x))/(b*c - a*d)]*Log[a*g + b*g*x])/
(b*g) + (2*B^2*Log[(b*(c + d*x))/(b*c - a*d)]*(Log[(a + b*x)^(-1)] + Log[c
+ d*x] - Log[(e*(c + d*x))/(a + b*x)])*Log[a*g + b*g*x])/(b*g) + ((A + B*Lo
g[(e*(c + d*x))/(a + b*x)])^2*Log[a*g + b*g*x])/(b*g) - (B^2*Log[(b*(c + d*
x))/(b*c - a*d)]*Log[a*g + b*g*x]^2)/(b*g) + (B^2*Log[(e*(c + d*x))/(a + b*
x)]*Log[a*g + b*g*x]^2)/(b*g) - (2*A*B*PolyLog[2, -((d*(a + b*x))/(b*c - a*
d))])/(b*g) - (2*B^2*Log[(a + b*x)^(-1)]*PolyLog[2, -((d*(a + b*x))/(b*c -
a*d))])/(b*g) + (2*B^2*(Log[(a + b*x)^(-1)] + Log[c + d*x] - Log[(e*(c + d*
x))/(a + b*x)])*PolyLog[2, -((d*(a + b*x))/(b*c - a*d))])/(b*g) + (2*B^2*Lo
g[c + d*x]*PolyLog[2, (b*(c + d*x))/(b*c - a*d)])/(b*g) - (2*B^2*PolyLog[3,
 -((d*(a + b*x))/(b*c - a*d))])/(b*g) - (2*B^2*PolyLog[3, (b*(c + d*x))/(b*
c - a*d)])/(b*g)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2301

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

Rule 2302

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/(x_), x_Symbol] :> Dist[1/(
b*n), Subst[Int[x^p, x], x, a + b*Log[c*x^n]], x] /; FreeQ[{a, b, c, n, p},
 x]
```

Rule 2317

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symb
ol] :> Simp[(Log[1 + (e*x)/d]*(a + b*Log[c*x^n])^p)/e, x] - Dist[(b*n*p)/e,
 Int[(Log[1 + (e*x)/d]*(a + b*Log[c*x^n])^(p - 1))/x, x], x] /; FreeQ[{a, b
, c, d, e, n}, x] && IGtQ[p, 0]
```

Rule 2374

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> -Simp[(PolyLog[2, -(d*f*x^m)]*(a + b*Log[c*x
^n])^p)/m, x] + Dist[(b*n*p)/m, Int[(PolyLog[2, -(d*f*x^m)]*(a + b*Log[c*x^
n])^(p - 1))/x, x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p, 0]
&& EqQ[d*e, 1]
```

Rule 2375

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))^(r_.)]*((a_.) + Log[(c_.)*(x_)^(n_
.)]*(b_.))^(p_.))/(x_), x_Symbol] :> Simp[(Log[d*(e + f*x^m)^r]*(a + b*Log[
c*x^n])^(p + 1))/(b*n*(p + 1)), x] - Dist[(f*m*r)/(b*n*(p + 1)), Int[(x^(m
- 1)*(a + b*Log[c*x^n])^(p + 1))/(e + f*x^m), x], x] /; FreeQ[{a, b, c, d,
e, f, r, m, n}, x] && IGtQ[p, 0] && NeQ[d*e, 1]
```

Rule 2390

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_))^(q_.), x_Symbol] :> Dist[1/e, Subst[Int[((f*x)/d)^q*(a + b*Log[c*x^
n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && E
qQ[e*f - d*g, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2393

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + (c*e*x)/g])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2394

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d + e*x
)^n]))/g, x] - Dist[(b*e*n)/g, Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2396

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)/((f_.) + (g_.
)*(x_)), x_Symbol] :> Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d
+ e*x)^n])^p)/g, x] - Dist[(b*e*n*p)/g, Int[(Log[(e*(f + g*x))/(e*f - d*g)]
*(a + b*Log[c*(d + e*x)^n])^(p - 1))/(d + e*x), x], x] /; FreeQ[{a, b, c, d
, e, f, g, n, p}, x] && NeQ[e*f - d*g, 0] && IGtQ[p, 1]
```

Rule 2418

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*(RFx_), x_Sy
mbol] :> With[{u = ExpandIntegrand[(a + b*Log[c*(d + e*x)^n])^p, RFx, x]},
Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d, e, n}, x] && RationalFunctionQ[
RFx, x] && IntegerQ[p]
```

Rule 2433

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + Log
[(h_.)*((i_.) + (j_.)*(x_))^(m_.)]*(g_.))*((k_.) + (l_.)*(x_))^(r_.), x_Sym
bol] :> Dist[1/e, Subst[Int[((k*x)/d)^r*(a + b*Log[c*x^n])^p*(f + g*Log[h*(
(e*i - d*j)/e + (j*x)/e)^m]), x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e,
```

```
f, g, h, i, j, k, l, n, p, r}, x] && EqQ[e*k - d*l, 0]
```

Rule 2434

```
Int[(((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + Log[(h_.
)*((i_.) + (j_.)*(x_))^(m_.)]*(g_.)))/(x_), x_Symbol] :> Simp[Log[x]*(a + b
*Log[c*(d + e*x)^n])*(f + g*Log[h*(i + j*x)^m]), x] + (-Dist[e*g*m, Int[(Lo
g[x]*(a + b*Log[c*(d + e*x)^n]))/(d + e*x), x], x] - Dist[b*j*n, Int[(Log[x
]*(f + g*Log[h*(i + j*x)^m]))/(i + j*x), x], x]) /; FreeQ[{a, b, c, d, e, f
, g, h, i, j, m, n}, x] && EqQ[e*i - d*j, 0]
```

Rule 2440

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + Log[(h_.)
*((i_.) + (j_.)*(x_))^(m_.)]*(g_.))*((k_) + (l_.)*(x_))^(r_.), x_Symbol] :>
 Dist[1/l, Subst[Int[x^r*(a + b*Log[c*(-((e*k - d*l)/l) + (e*x)/l)^n])*(f +
 g*Log[h*(-((j*k - i*l)/l) + (j*x)/l)^m]), x], x, k + l*x], x] /; FreeQ[{a,
 b, c, d, e, f, g, h, i, j, k, l, m, n}, x] && IntegerQ[r]
```

Rule 2499

```
Int[(Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.)
)^(r_.)]*((s_.) + Log[(i_.)*((g_.) + (h_.)*(x_))^(n_.)]*(t_.))^(m_.))/((j_.
) + (k_.)*(x_)), x_Symbol] :> Simp[((s + t*Log[i*(g + h*x)^n])^(m + 1)*Log[
e*(f*(a + b*x)^p*(c + d*x)^q)^r])/(k*n*t*(m + 1)), x] + (-Dist[(b*p*r)/(k*n
*t*(m + 1)), Int[(s + t*Log[i*(g + h*x)^n])^(m + 1)/(a + b*x), x], x] - Dis
t[(d*q*r)/(k*n*t*(m + 1)), Int[(s + t*Log[i*(g + h*x)^n])^(m + 1)/(c + d*x)
, x], x]) /; FreeQ[{a, b, c, d, e, f, g, h, i, j, k, s, t, m, n, p, q, r},
x] && NeQ[b*c - a*d, 0] && EqQ[h*j - g*k, 0] && IGtQ[m, 0]
```

Rule 2500

```
Int[(Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.)
)^(r_.)]*((s_.) + Log[(i_.)*((g_.) + (h_.)*(x_))^(n_.)]*(t_.)))/((j_.) + (k
_.)*(x_)), x_Symbol] :> Dist[Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r] - Log[(a
+ b*x)^(p*r)] - Log[(c + d*x)^(q*r)], Int[(s + t*Log[i*(g + h*x)^n])/(j + k
*x), x], x] + (Int[(Log[(a + b*x)^(p*r)]*(s + t*Log[i*(g + h*x)^n]))/(j + k
*x), x] + Int[(Log[(c + d*x)^(q*r)]*(s + t*Log[i*(g + h*x)^n]))/(j + k*x),
x]) /; FreeQ[{a, b, c, d, e, f, g, h, i, j, k, s, t, n, p, q, r}, x] && NeQ
[b*c - a*d, 0]
```

Rule 2524

```
Int[((a_.) + Log[(c_.)*(RFx_)^(p_.)]*(b_.))^(n_.)/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[(Log[d + e*x]*(a + b*Log[c*RFx^p])^n)/e, x] - Dist[(b*n*p)/e
```

```
, Int[(Log[d + e*x]*(a + b*Log[c*RFx^p])^(n - 1)*D[RFx, x])/RFx, x], x] /;
FreeQ[{a, b, c, d, e, p}, x] && RationalFunctionQ[RFx, x] && IGtQ[n, 0]
```

Rule 2528

```
Int[((a_.) + Log[(c_.)*(RFx_)^(p_.)]*(b_.))^(n_.)*(RGx_), x_Symbol] :> With
[{u = ExpandIntegrand[(a + b*Log[c*RFx^p])^n, RGx, x]}, Int[u, x] /; SumQ[u
]] /; FreeQ[{a, b, c, p}, x] && RationalFunctionQ[RFx, x] && RationalFuncti
onQ[RGx, x] && IGtQ[n, 0]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6691

```
Int[(u_)^(m_.)*((a_.)*(u_)^(n_) + (v_))^(p_.)*(w_), x_Symbol] :> Int[u^(m +
 n*p)*(a + v/u^n)^p*w, x] /; FreeQ[{a, m, n}, x] && IntegerQ[p] &&  !GtQ[n,
 0] &&  !FreeQ[v, x]
```

Rule 6741

```
Int[u_, x_Symbol] :> With[{v = NormalizeIntegrand[u, x]}, Int[v, x] /; v =!
= u]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\int \frac{\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2}{ag+bgx}\,dx = \frac{\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2\log(ag+bgx)}{bg} - \frac{(2B)\int \frac{(a+bx)\left(\frac{de}{a+bx}-\frac{be(c+dx)}{(a+bx)^2}\right)\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)\log}{e(c+dx)}}{bg}$$

$$= \frac{\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2\log(ag+bgx)}{bg} - \frac{(2B)\int \frac{(a+bx)\left(\frac{de}{a+bx}-\frac{be(c+dx)}{(a+bx)^2}\right)\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)\log}{c+dx}}{beg}$$

$$= \frac{\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2\log(ag+bgx)}{bg} - \frac{(2B)\int \frac{\left(de-\frac{be(c+dx)}{a+bx}\right)\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)\log(ag+bgx)}{c+dx}}{beg}$$

$$= \frac{\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2\log(ag+bgx)}{bg} - \frac{(2B)\int \frac{(bc-ad)e\left(-A-B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)\log(ag+bgx)}{(a+bx)(c+dx)}\,dx}{beg}$$

$$= \frac{\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2\log(ag+bgx)}{bg} - \frac{(2B(bc-ad))\int \frac{\left(-A-B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)\log(ag+bgx)}{(a+bx)(c+dx)}}{bg}$$

$$= \frac{\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2\log(ag+bgx)}{bg} - \frac{(2B(bc-ad))\int\left(\frac{b\left(-A-B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)\log(ag+bg}{(bc-ad)(a+bx)}\right.}{bg}$$

$$= \frac{\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2\log(ag+bgx)}{bg} - \frac{(2B)\int \frac{\left(-A-B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)\log(ag+bgx)}{a+bx}\,dx}{g} - (2I$$

$$= \frac{\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2\log(ag+bgx)}{bg} - \frac{(2B)\int\left(\frac{A\log(ag+bgx)}{-a-bx}+\frac{B\log\left(\frac{e(c+dx)}{a+bx}\right)\log(ag+bg}{-a-bx}\right.}{g}$$

$$= \frac{\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2\log(ag+bgx)}{bg} - \frac{(2AB)\int \frac{\log(ag+bgx)}{-a-bx}\,dx}{g} - \frac{\left(2B^2\right)\int \frac{\log\left(\frac{e(c+dx)}{a+bx}\right.}{-}}{g}$$

$$= -\frac{2AB\log\left(\frac{b(c+dx)}{bc-ad}\right)\log(ag+bgx)}{bg} + \frac{\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2\log(ag+bgx)}{bg} + \frac{B^2\log}{}$$

$$= -\frac{2AB\log\left(\frac{b(c+dx)}{bc-ad}\right)\log(ag+bgx)}{bg} + \frac{2B^2\log\left(\frac{b(c+dx)}{bc-ad}\right)\left(\log\left(\frac{1}{a+bx}\right)+\log(c+dx)-\right.}{bg}$$

$$= \frac{AB\log^2(g(a+bx))}{bg} - \frac{2B^2\log\left(\frac{1}{a+bx}\right)\log(g(a+bx))\log(c+dx)}{bg} - \frac{B^2\log(g(a+b}{b}$$

**Mathematica** [A] time = 0.29, size = 251, normalized size = 1.96

$$\frac{A^2\log(a+bx)+2AB\log(a+bx)\log\left(\frac{e(c+dx)}{a+bx}\right)+2AB\mathrm{Li}_2\left(\frac{b(c+dx)}{bc-ad}\right)-2AB\log(a+bx)\log\left(\frac{c}{d}+x\right)+2AB\log\left(\frac{c}{d}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Log[(e*(c + d*x))/(a + b*x)])^2/(a*g + b*g*x),x]
```

```
[Out] (-(A*B*Log[a/b + x]^2) + A^2*Log[a + b*x] + 2*A*B*Log[a/b + x]*Log[a + b*x]
 - 2*A*B*Log[c/d + x]*Log[a + b*x] + 2*A*B*Log[c/d + x]*Log[(d*(a + b*x))/(
-(b*c) + a*d)] + 2*A*B*Log[a + b*x]*Log[(e*(c + d*x))/(a + b*x)] - B^2*Log[
(-(b*c) + a*d)/(d*(a + b*x))]*Log[(e*(c + d*x))/(a + b*x)]^2 + 2*A*B*PolyLo
g[2, (b*(c + d*x))/(b*c - a*d)] - 2*B^2*Log[(e*(c + d*x))/(a + b*x)]*PolyLo
g[2, (b*(c + d*x))/(d*(a + b*x))] + 2*B^2*PolyLog[3, (b*(c + d*x))/(d*(a +
b*x))])/(b*g)
```

**fricas** [F] time = 0.69, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{B^2\log\left(\frac{dex+ce}{bx+a}\right)^2+2\,AB\log\left(\frac{dex+ce}{bx+a}\right)+A^2}{bgx+ag},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*(d*x+c)/(b*x+a)))^2/(b*g*x+a*g),x, algorithm="fricas")
```

```
[Out] integral((B^2*log((d*e*x + c*e)/(b*x + a))^2 + 2*A*B*log((d*e*x + c*e)/(b*x
 + a)) + A^2)/(b*g*x + a*g), x)
```

**giac** [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*(d*x+c)/(b*x+a)))^2/(b*g*x+a*g),x, algorithm="giac")
```

```
[Out] Timed out
```

**maple** [B] time = 0.06, size = 906, normalized size = 7.08

$$-\frac{B^2ad\ln\left(-\frac{\left(\frac{de}{b}-\frac{(ad-bc)e}{(bx+a)b}\right)b}{de}+1\right)\ln\left(\frac{de}{b}-\frac{(ad-bc)e}{(bx+a)b}\right)^2}{(ad-bc)\,bg}+\frac{B^2c\ln\left(-\frac{\left(\frac{de}{b}-\frac{(ad-bc)e}{(bx+a)b}\right)b}{de}+1\right)\ln\left(\frac{de}{b}-\frac{(ad-bc)e}{(bx+a)b}\right)^2}{(ad-bc)\,g}-\frac{2ABad\ln\left(-\frac{-de+\cdots}{}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*ln((d*x+c)/(b*x+a)*e)+A)^2/(b*g*x+a*g),x)
```

```
[Out] -1/b/g/(a*d-b*c)*A^2*ln(-d*e+(1/b*d*e-(a*d-b*c)/(b*x+a)/b*e)*b)*a*d+1/g/(a*
d-b*c)*A^2*ln(-d*e+(1/b*d*e-(a*d-b*c)/(b*x+a)/b*e)*b)*c-1/b/g/(a*d-b*c)*B^2
*ln(1/b*d*e-(a*d-b*c)/(b*x+a)/b*e)^2*ln(1-b/d/e*(1/b*d*e-(a*d-b*c)/(b*x+a)/
b*e))*a*d+1/g/(a*d-b*c)*B^2*ln(1/b*d*e-(a*d-b*c)/(b*x+a)/b*e)^2*ln(1-b/d/e*
(1/b*d*e-(a*d-b*c)/(b*x+a)/b*e))*c-2/b/g/(a*d-b*c)*B^2*ln(1/b*d*e-(a*d-b*c)
/(b*x+a)/b*e)*polylog(2,b/d/e*(1/b*d*e-(a*d-b*c)/(b*x+a)/b*e))*a*d+2/g/(a*d
-b*c)*B^2*ln(1/b*d*e-(a*d-b*c)/(b*x+a)/b*e)*polylog(2,b/d/e*(1/b*d*e-(a*d-b
*c)/(b*x+a)/b*e))*c+2/b/g/(a*d-b*c)*B^2*polylog(3,b/d/e*(1/b*d*e-(a*d-b*c)/
(b*x+a)/b*e))*a*d-2/g/(a*d-b*c)*B^2*polylog(3,b/d/e*(1/b*d*e-(a*d-b*c)/(b*x
+a)/b*e))*c-2/b/g/(a*d-b*c)*A*B*dilog(-(-d*e+(1/b*d*e-(a*d-b*c)/(b*x+a)/b*e
)*b)/d/e)*a*d+2/g/(a*d-b*c)*A*B*dilog(-(-d*e+(1/b*d*e-(a*d-b*c)/(b*x+a)/b*e
)*b)/d/e)*c-2/b/g/(a*d-b*c)*A*B*ln(1/b*d*e-(a*d-b*c)/(b*x+a)/b*e)*ln(-(-d*e
+(1/b*d*e-(a*d-b*c)/(b*x+a)/b*e)*b)/d/e)*a*d+2/g/(a*d-b*c)*A*B*ln(1/b*d*e-(
a*d-b*c)/(b*x+a)/b*e)*ln(-(-d*e+(1/b*d*e-(a*d-b*c)/(b*x+a)/b*e)*b)/d/e)*c
```

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$\frac{B^2\log(bx+a)\log(dx+c)^2}{bg}+\frac{A^2\log\left(bgx+ag\right)}{bg}-\int-\frac{B^2bc\log(e)^2+2\,ABbc\log(e)+\left(B^2bdx+B^2bc\right)\log(bx+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*(d*x+c)/(b*x+a)))^2/(b*g*x+a*g),x, algorithm="maxima")
```

```
[Out] B^2*log(b*x + a)*log(d*x + c)^2/(b*g) + A^2*log(b*g*x + a*g)/(b*g) - integr
ate(-(B^2*b*c*log(e)^2 + 2*A*B*b*c*log(e) + (B^2*b*d*x + B^2*b*c)*log(b*x +
 a)^2 + (B^2*b*d*log(e)^2 + 2*A*B*b*d*log(e))*x - 2*(B^2*b*c*log(e) + A*B*b
*c + (B^2*b*d*log(e) + A*B*b*d)*x)*log(b*x + a) + 2*(B^2*b*c*log(e) + A*B*b
*c + (B^2*b*d*log(e) + A*B*b*d)*x - (2*B^2*b*d*x + (b*c + a*d)*B^2)*log(b*x
 + a))*log(d*x + c))/(b^2*d*g*x^2 + a*b*c*g + (b^2*c*g + a*b*d*g)*x), x)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\left(A+B\,\ln\left(\frac{e\,\left(c+d\,x\right)}{a+b\,x}\right)\right)}^2}{a\,g+b\,g\,x} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*log((e*(c + d*x))/(a + b*x)))^2/(a*g + b*g*x),x)
```

```
[Out] int((A + B*log((e*(c + d*x))/(a + b*x)))^2/(a*g + b*g*x), x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{A^2}{a+bx}\,dx + \int \frac{B^2 \log\left(\frac{ce}{a+bx}+\frac{dex}{a+bx}\right)^2}{a+bx}\,dx + \int \frac{2AB\log\left(\frac{ce}{a+bx}+\frac{dex}{a+bx}\right)}{a+bx}\,dx}{g}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*ln(e*(d*x+c)/(b*x+a)))**2/(b*g*x+a*g),x)
```

```
[Out] (Integral(A**2/(a + b*x), x) + Integral(B**2*log(c*e/(a + b*x) + d*e*x/(a +
 b*x))**2/(a + b*x), x) + Integral(2*A*B*log(c*e/(a + b*x) + d*e*x/(a + b*x
))/(a + b*x), x))/g
```

# 3.187 $\int \frac{\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2}{(ag+bgx)^2}\,dx$

**Optimal**. Leaf size=153

$$-\frac{(c+dx)\left(B\log\left(\frac{e(c+dx)}{a+bx}\right)+A\right)^2}{g^2(a+bx)(bc-ad)}+\frac{2AB(c+dx)}{g^2(a+bx)(bc-ad)}+\frac{2B^2(c+dx)\log\left(\frac{e(c+dx)}{a+bx}\right)}{g^2(a+bx)(bc-ad)}-\frac{2B^2(c+dx)}{g^2(a+bx)(bc-ad)}$$

```
[Out] 2*A*B*(d*x+c)/(-a*d+b*c)/g^2/(b*x+a)-2*B^2*(d*x+c)/(-a*d+b*c)/g^2/(b*x+a)+2
*B^2*(d*x+c)*ln(e*(d*x+c)/(b*x+a))/(-a*d+b*c)/g^2/(b*x+a)-(d*x+c)*(A+B*ln(e
*(d*x+c)/(b*x+a)))^2/(-a*d+b*c)/g^2/(b*x+a)
```

**Rubi [C]** time = 0.76, antiderivative size = 470, normalized size of antiderivative = 3.07, number of steps used = 26, number of rules used = 11, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.344, Rules used = {2525, 12, 2528, 44, 2524, 2418, 2390, 2301, 2394, 2393, 2391}

$$-\frac{2B^2d\text{PolyLog}\left(2,-\frac{d(a+bx)}{bc-ad}\right)}{bg^2(bc-ad)}-\frac{2B^2d\text{PolyLog}\left(2,\frac{b(c+dx)}{bc-ad}\right)}{bg^2(bc-ad)}+\frac{2Bd\log(a+bx)\left(B\log\left(\frac{e(c+dx)}{a+bx}\right)+A\right)}{bg^2(bc-ad)}-\frac{2Bd\log(c+dx)}{bg^2}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Log[(e*(c + d*x))/(a + b*x)])^2/(a*g + b*g*x)^2,x]
```

```
[Out] (-2*B^2)/(b*g^2*(a + b*x)) - (2*B^2*d*Log[a + b*x])/(b*(b*c - a*d)*g^2) + (
B^2*d*Log[a + b*x]^2)/(b*(b*c - a*d)*g^2) + (2*B^2*d*Log[c + d*x])/(b*(b*c
 - a*d)*g^2) - (2*B^2*d*Log[-((d*(a + b*x))/(b*c - a*d))]*Log[c + d*x])/(b*(
b*c - a*d)*g^2) + (B^2*d*Log[c + d*x]^2)/(b*(b*c - a*d)*g^2) - (2*B^2*d*Log
[a + b*x]*Log[(b*(c + d*x))/(b*c - a*d)])/(b*(b*c - a*d)*g^2) + (2*B*(A + B
*Log[(e*(c + d*x))/(a + b*x)]))/(b*g^2*(a + b*x)) + (2*B*d*Log[a + b*x]*(A
+ B*Log[(e*(c + d*x))/(a + b*x)]))/(b*(b*c - a*d)*g^2) - (2*B*d*Log[c + d*x
]*(A + B*Log[(e*(c + d*x))/(a + b*x)]))/(b*(b*c - a*d)*g^2) - (A + B*Log[(e
*(c + d*x))/(a + b*x)])^2/(b*g^2*(a + b*x)) - (2*B^2*d*PolyLog[2, -((d*(a +
 b*x))/(b*c - a*d))])/(b*(b*c - a*d)*g^2) - (2*B^2*d*PolyLog[2, (b*(c + d*x
))/(b*c - a*d)])/(b*(b*c - a*d)*g^2)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 2301

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

Rule 2390

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_))^(q_.), x_Symbol] :> Dist[1/e, Subst[Int[((f*x)/d)^q*(a + b*Log[c*x^
n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && E
qQ[e*f - d*g, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2393

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + (c*e*x)/g])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2394

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d + e*x
)^n]))/g, x] - Dist[(b*e*n)/g, Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2418

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*(RFx_), x_Sy
mbol] :> With[{u = ExpandIntegrand[(a + b*Log[c*(d + e*x)^n])^p, RFx, x]},
Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d, e, n}, x] && RationalFunctionQ[
RFx, x] && IntegerQ[p]
```

Rule 2524

```
Int[((a_.) + Log[(c_.)*(RFx_)^(p_.)]*(b_.))^(n_.)/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[(Log[d + e*x]*(a + b*Log[c*RFx^p])^n)/e, x] - Dist[(b*n*p)/e
, Int[(Log[d + e*x]*(a + b*Log[c*RFx^p])^(n - 1)*D[RFx, x])/RFx, x], x] /;
FreeQ[{a, b, c, d, e, p}, x] && RationalFunctionQ[RFx, x] && IGtQ[n, 0]
```

Rule 2525

```
Int[((a_.) + Log[(c_.)*(RFx_)^(p_.)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_))^(m_.
), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(a + b*Log[c*RFx^p])^n)/(e*(m + 1))
, x] - Dist[(b*n*p)/(e*(m + 1)), Int[SimplifyIntegrand[((d + e*x)^(m + 1)*(
a + b*Log[c*RFx^p])^(n - 1)*D[RFx, x])/RFx, x], x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && RationalFunctionQ[RFx, x] && IGtQ[n, 0] && (EqQ[n, 1] ||
IntegerQ[m]) && NeQ[m, -1]
```

Rule 2528

```
Int[((a_.) + Log[(c_.)*(RFx_)^(p_.)]*(b_.))^(n_.)*(RGx_), x_Symbol] :> With
[{u = ExpandIntegrand[(a + b*Log[c*RFx^p])^n, RGx, x]}, Int[u, x] /; SumQ[u
]] /; FreeQ[{a, b, c, p}, x] && RationalFunctionQ[RFx, x] && RationalFuncti
onQ[RGx, x] && IGtQ[n, 0]
```

Rubi steps

$$\int \frac{\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2}{(ag+bgx)^2}\,dx = -\frac{\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2}{bg^2(a+bx)} + \frac{(2B)\int \frac{(bc-ad)\left(-A-B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)}{g(a+bx)^2(c+dx)}\,dx}{bg}$$

$$= -\frac{\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2}{bg^2(a+bx)} + \frac{(2B(bc-ad))\int \frac{-A-B\log\left(\frac{e(c+dx)}{a+bx}\right)}{(a+bx)^2(c+dx)}\,dx}{bg^2}$$

$$= -\frac{\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2}{bg^2(a+bx)} + \frac{(2B(bc-ad))\int \left(\frac{b\left(-A-B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)}{(bc-ad)(a+bx)^2} - \frac{bd\left(-A-B\log\left(\frac{e(c+dx)}{a+bx}\right.\right.}{(bc-ad)^2(a+bx)}\right.}{bg^2}$$

$$= -\frac{\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2}{bg^2(a+bx)} + \frac{(2B)\int \frac{-A-B\log\left(\frac{e(c+dx)}{a+bx}\right)}{(a+bx)^2}\,dx}{g^2} - \frac{(2Bd)\int \frac{-A-B\log\left(\frac{e(c+dx)}{a+bx}\right)}{a+bx}\,dx}{(bc-ad)g^2}$$

$$= \frac{2B\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)}{bg^2(a+bx)} + \frac{2Bd\log(a+bx)\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)}{b(bc-ad)g^2} - \frac{2Bd\log(c+}{}$$

$$= \frac{2B\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)}{bg^2(a+bx)} + \frac{2Bd\log(a+bx)\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)}{b(bc-ad)g^2} - \frac{2Bd\log(c+}{}$$

$$= \frac{2B\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)}{bg^2(a+bx)} + \frac{2Bd\log(a+bx)\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)}{b(bc-ad)g^2} - \frac{2Bd\log(c+}{}$$

$$= -\frac{2B^2}{bg^2(a+bx)} - \frac{2B^2d\log(a+bx)}{b(bc-ad)g^2} + \frac{2B^2d\log(c+dx)}{b(bc-ad)g^2} + \frac{2B\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right.}{bg^2(a+bx)}$$

$$= -\frac{2B^2}{bg^2(a+bx)} - \frac{2B^2d\log(a+bx)}{b(bc-ad)g^2} + \frac{2B^2d\log(c+dx)}{b(bc-ad)g^2} - \frac{2B^2d\log\left(-\frac{d(a+bx)}{bc-ad}\right)\log}{b(bc-ad)g^2}$$

$$= -\frac{2B^2}{bg^2(a+bx)} - \frac{2B^2d\log(a+bx)}{b(bc-ad)g^2} + \frac{B^2d\log^2(a+bx)}{b(bc-ad)g^2} + \frac{2B^2d\log(c+dx)}{b(bc-ad)g^2} - \frac{2B^2}{}$$

$$= -\frac{2B^2}{bg^2(a+bx)} - \frac{2B^2d\log(a+bx)}{b(bc-ad)g^2} + \frac{B^2d\log^2(a+bx)}{b(bc-ad)g^2} + \frac{2B^2d\log(c+dx)}{b(bc-ad)g^2} - \frac{2B^2}{}$$

**Mathematica [C]** time = 0.48, size = 314, normalized size = 2.05

$$-\frac{B\left(-2(bc-ad)\left(B\log\left(\frac{e(c+dx)}{a+bx}\right)+A\right)-2d(a+bx)\log(a+bx)\left(B\log\left(\frac{e(c+dx)}{a+bx}\right)+A\right)+2d(a+bx)\log(c+dx)\left(B\log\left(\frac{e(c+dx)}{a+bx}\right)+A\right)-Bd(a+bx)\left(\log(a+bx)\left(\log(a+bx)\right.\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Log[(e*(c + d*x))/(a + b*x)])^2/(a*g + b*g*x)^2,x]
```

```
[Out] -(((A + B*Log[(e*(c + d*x))/(a + b*x)])^2 + (B*(2*B*(b*c - a*d + d*(a + b*x
)*Log[a + b*x] - d*(a + b*x)*Log[c + d*x]) - 2*(b*c - a*d)*(A + B*Log[(e*(c
 + d*x))/(a + b*x)]) - 2*d*(a + b*x)*Log[a + b*x]*(A + B*Log[(e*(c + d*x))/
(a + b*x)]) + 2*d*(a + b*x)*Log[c + d*x]*(A + B*Log[(e*(c + d*x))/(a + b*x)
]) - B*d*(a + b*x)*(Log[a + b*x]*(Log[a + b*x] - 2*Log[(b*(c + d*x))/(b*c -
 a*d)]) - 2*PolyLog[2, (d*(a + b*x))/(-(b*c) + a*d)]) + B*d*(a + b*x)*((2*L
og[(d*(a + b*x))/(-(b*c) + a*d)] - Log[c + d*x])*Log[c + d*x] + 2*PolyLog[2
, (b*(c + d*x))/(b*c - a*d)])))/(b*c - a*d))/(b*g^2*(a + b*x)))
```

**fricas [A]** time = 0.67, size = 154, normalized size = 1.01

$$-\frac{\left(A^2-2\,AB+2\,B^2\right)bc-\left(A^2-2\,AB+2\,B^2\right)ad+\left(B^2bdx+B^2bc\right)\log\left(\frac{dex+ce}{bx+a}\right)^2+2\left(\left(AB-B^2\right)bdx+\left(AB-B^2\right)\right.}{\left(b^3c-ab^2d\right)g^2x+\left(ab^2c-a^2bd\right)g^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*(d*x+c)/(b*x+a)))^2/(b*g*x+a*g)^2,x, algorithm="fricas
")
```

```
[Out] -((A^2 - 2*A*B + 2*B^2)*b*c - (A^2 - 2*A*B + 2*B^2)*a*d + (B^2*b*d*x + B^2*
b*c)*log((d*e*x + c*e)/(b*x + a))^2 + 2*((A*B - B^2)*b*d*x + (A*B - B^2)*b*
c)*log((d*e*x + c*e)/(b*x + a)))/((b^3*c - a*b^2*d)*g^2*x + (a*b^2*c - a^2*
b*d)*g^2)
```

**giac [A]** time = 1.55, size = 188, normalized size = 1.23

$$-\left(\frac{(dxe+ce)B^2\log\left(\frac{dxe+ce}{bx+a}\right)^2}{(bx+a)g^2}+\frac{2\,(dxe+ce)\left(AB-B^2\right)\log\left(\frac{dxe+ce}{bx+a}\right)}{(bx+a)g^2}+\frac{(dxe+ce)\left(A^2-2\,AB+2\,B^2\right)}{(bx+a)g^2}\right)\left(\frac{bc}{(bce-ade)(\cdots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*(d*x+c)/(b*x+a)))^2/(b*g*x+a*g)^2,x, algorithm="giac")
```

```
[Out] -((d*x*e + c*e)*B^2*log((d*x*e + c*e)/(b*x + a))^2/((b*x + a)*g^2) + 2*(d*x
*e + c*e)*(A*B - B^2)*log((d*x*e + c*e)/(b*x + a))/((b*x + a)*g^2) + (d*x*e
 + c*e)*(A^2 - 2*A*B + 2*B^2)/((b*x + a)*g^2))*(b*c/((b*c*e - a*d*e)*(b*c -
 a*d)) - a*d/((b*c*e - a*d*e)*(b*c - a*d)))
```

**maple [B]** time = 0.05, size = 1251, normalized size = 8.18

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*ln((d*x+c)/(b*x+a)*e)+A)^2/(b*g*x+a*g)^2,x)
```

```
[Out] -2*b/(a*d-b*c)^2/g^2*A*B*ln(1/b*d*e-(a*d-b*c)/(b*x+a)/b*e)/(b*x+a)*c^2-1/b/
(a*d-b*c)^2/g^2*B^2*ln(1/b*d*e-(a*d-b*c)/(b*x+a)/b*e)^2/(b*x+a)*a^2*d^2-2/b
/(a*d-b*c)^2/g^2*A*B*d^2*a+2/(a*d-b*c)^2/g^2*A*B*d*c-1/b/(a*d-b*c)^2/g^2*A^
2/(b*x+a)*a^2*d^2+2*b/(a*d-b*c)^2/g^2*A*B/(b*x+a)*c^2+2/b/(a*d-b*c)^2/g^2*B
^2*d^2*a-2/(a*d-b*c)^2/g^2*B^2*d*c+1/b/(a*d-b*c)^2/g^2*A^2*d^2*a-1/(a*d-b*c
)^2/g^2*A^2*d*c-2*b/(a*d-b*c)^2/g^2*B^2/(b*x+a)*c^2-b/(a*d-b*c)^2/g^2*A^2/(
b*x+a)*c^2+2/(a*d-b*c)^2/g^2*B^2*ln(1/b*d*e-(a*d-b*c)/(b*x+a)/b*e)*d*c-1/(a
*d-b*c)^2/g^2*B^2*ln(1/b*d*e-(a*d-b*c)/(b*x+a)/b*e)^2*d*c-2/b/(a*d-b*c)^2/g
^2*B^2/(b*x+a)*a^2*d^2+4/(a*d-b*c)^2/g^2*B^2/(b*x+a)*a*d*c+2/(a*d-b*c)^2/g^
2*A^2/(b*x+a)*a*d*c+2*b/(a*d-b*c)^2/g^2*B^2*ln(1/b*d*e-(a*d-b*c)/(b*x+a)/b*
e)/(b*x+a)*c^2-2/(a*d-b*c)^2/g^2*A*B*ln(1/b*d*e-(a*d-b*c)/(b*x+a)/b*e)*d*c+
1/b/(a*d-b*c)^2/g^2*B^2*ln(1/b*d*e-(a*d-b*c)/(b*x+a)/b*e)^2*d^2*a-b/(a*d-b*
c)^2/g^2*B^2*ln(1/b*d*e-(a*d-b*c)/(b*x+a)/b*e)^2/(b*x+a)*c^2-2/b/(a*d-b*c)^
2/g^2*B^2*ln(1/b*d*e-(a*d-b*c)/(b*x+a)/b*e)*d^2*a-4/(a*d-b*c)^2/g^2*A*B/(b*
x+a)*a*d*c+2/b/(a*d-b*c)^2/g^2*A*B/(b*x+a)*a^2*d^2-4/(a*d-b*c)^2/g^2*B^2*ln
(1/b*d*e-(a*d-b*c)/(b*x+a)/b*e)/(b*x+a)*a*d*c+2/(a*d-b*c)^2/g^2*B^2*ln(1/b*
d*e-(a*d-b*c)/(b*x+a)/b*e)^2/(b*x+a)*a*d*c+2/b/(a*d-b*c)^2/g^2*B^2*ln(1/b*d
*e-(a*d-b*c)/(b*x+a)/b*e)/(b*x+a)*a^2*d^2+2/b/(a*d-b*c)^2/g^2*A*B*ln(1/b*d*
e-(a*d-b*c)/(b*x+a)/b*e)*d^2*a+4/(a*d-b*c)^2/g^2*A*B*ln(1/b*d*e-(a*d-b*c)/(
b*x+a)/b*e)/(b*x+a)*a*d*c-2/b/(a*d-b*c)^2/g^2*A*B*ln(1/b*d*e-(a*d-b*c)/(b*x
+a)/b*e)/(b*x+a)*a^2*d^2
```

**maxima [B]** time = 1.25, size = 416, normalized size = 2.72

$$\left(2\left(\frac{1}{b^2g^2x+abg^2}+\frac{d\log\left(bx+a\right)}{\left(b^2c-abd\right)g^2}-\frac{d\log\left(dx+c\right)}{\left(b^2c-abd\right)g^2}\right)\log\left(\frac{dex}{bx+a}+\frac{ce}{bx+a}\right)+\frac{\left(bdx+ad\right)\log\left(bx+a\right)^2+\left(bdx+\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*(d*x+c)/(b*x+a)))^2/(b*g*x+a*g)^2,x, algorithm="maxima
")
```

```
[Out] (2*(1/(b^2*g^2*x + a*b*g^2) + d*log(b*x + a)/((b^2*c - a*b*d)*g^2) - d*log(
d*x + c)/((b^2*c - a*b*d)*g^2))*log(d*e*x/(b*x + a) + c*e/(b*x + a)) + ((b*
d*x + a*d)*log(b*x + a)^2 + (b*d*x + a*d)*log(d*x + c)^2 - 2*b*c + 2*a*d -
2*(b*d*x + a*d)*log(b*x + a) + 2*(b*d*x + a*d - (b*d*x + a*d)*log(b*x + a))
*log(d*x + c))/(a*b^2*c*g^2 - a^2*b*d*g^2 + (b^3*c*g^2 - a*b^2*d*g^2)*x))*B
^2 - 2*A*B*(log(d*e*x/(b*x + a) + c*e/(b*x + a))/(b^2*g^2*x + a*b*g^2) - 1/
(b^2*g^2*x + a*b*g^2) - d*log(b*x + a)/((b^2*c - a*b*d)*g^2) + d*log(d*x +
c)/((b^2*c - a*b*d)*g^2)) - B^2*log(d*e*x/(b*x + a) + c*e/(b*x + a))^2/(b^2
*g^2*x + a*b*g^2) - A^2/(b^2*g^2*x + a*b*g^2)
```

**mupad [B]** time = 6.38, size = 223, normalized size = 1.46

$$\frac{\ln\left(\frac{e\,(c+d\,x)}{a+b\,x}\right)\left(\frac{2\,B^2}{b^2\,d\,g^2}-\frac{2\,A\,B}{b^2\,d\,g^2}\right)}{\frac{x}{d}+\frac{a}{b\,d}}-\ln\left(\frac{e\,\left(c+d\,x\right)}{a+b\,x}\right)^2\left(\frac{B^2}{b^2\,g^2\,\left(x+\frac{a}{b}\right)}-\frac{B^2\,d}{b\,g^2\,\left(a\,d-b\,c\right)}\right)-\frac{A^2-2\,A\,B+2\,B^2}{x\,b^2\,g^2+a\,b\,g^2}+\frac{B\,d\,\mathrm{atan}\Bigg(\Big(}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*log((e*(c + d*x))/(a + b*x)))^2/(a*g + b*g*x)^2,x)
```

```
[Out] (log((e*(c + d*x))/(a + b*x))*((2*B^2)/(b^2*d*g^2) - (2*A*B)/(b^2*d*g^2)))/
(x/d + a/(b*d)) - log((e*(c + d*x))/(a + b*x))^2*(B^2/(b^2*g^2*(x + a/b)) -
 (B^2*d)/(b*g^2*(a*d - b*c))) - (A^2 + 2*B^2 - 2*A*B)/(b^2*g^2*x + a*b*g^2)
 + (B*d*atan(((2*b*d*x + (b^2*c*g^2 + a*b*d*g^2)/(b*g^2))*1i)/(a*d - b*c))*
(A - B)*4i)/(b*g^2*(a*d - b*c))
```

**sympy [B]** time = 3.75, size = 430, normalized size = 2.81

$$\frac{2Bd\,(A-B)\log\left(x+\frac{2ABad^2+2ABbcd-2B^2ad^2-2B^2bcd-\frac{2Ba^2d^3(A-B)}{ad-bc}+\frac{4Babcd^2(A-B)}{ad-bc}-\frac{2Bb^2c^2d(A-B)}{ad-bc}}{4ABbd^2-4B^2bd^2}\right)}{bg^2\,(ad-bc)}-\frac{2Bd\,(A-B)\log\Bigg(x+\frac{2ABad^2+2}{}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*ln(e*(d*x+c)/(b*x+a)))**2/(b*g*x+a*g)**2,x)
```

```
[Out] 2*B*d*(A - B)*log(x + (2*A*B*a*d**2 + 2*A*B*b*c*d - 2*B**2*a*d**2 - 2*B**2*
b*c*d - 2*B*a**2*d**3*(A - B)/(a*d - b*c) + 4*B*a*b*c*d**2*(A - B)/(a*d - b
*c) - 2*B*b**2*c**2*d*(A - B)/(a*d - b*c))/(4*A*B*b*d**2 - 4*B**2*b*d**2))/
(b*g**2*(a*d - b*c)) - 2*B*d*(A - B)*log(x + (2*A*B*a*d**2 + 2*A*B*b*c*d -
2*B**2*a*d**2 - 2*B**2*b*c*d + 2*B*a**2*d**3*(A - B)/(a*d - b*c) - 4*B*a*b*
c*d**2*(A - B)/(a*d - b*c) + 2*B*b**2*c**2*d*(A - B)/(a*d - b*c))/(4*A*B*b*
d**2 - 4*B**2*b*d**2))/(b*g**2*(a*d - b*c)) + (-2*A*B + 2*B**2)*log(e*(c +
d*x)/(a + b*x))/(a*b*g**2 + b**2*g**2*x) + (B**2*c + B**2*d*x)*log(e*(c + d
```

```
*x)/(a + b*x))**2/(a**2*d*g**2 - a*b*c*g**2 + a*b*d*g**2*x - b**2*c*g**2*x)
 + (-A**2 + 2*A*B - 2*B**2)/(a*b*g**2 + b**2*g**2*x)
```

# 3.188 $\int \frac{\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2}{(ag+bgx)^3}\,dx$

**Optimal**. Leaf size=296

$$\frac{bB(c+dx)^2\left(B\log\left(\frac{e(c+dx)}{a+bx}\right)+A\right)}{2g^3(a+bx)^2(bc-ad)^2}-\frac{b(c+dx)^2\left(B\log\left(\frac{e(c+dx)}{a+bx}\right)+A\right)^2}{2g^3(a+bx)^2(bc-ad)^2}+\frac{d(c+dx)\left(B\log\left(\frac{e(c+dx)}{a+bx}\right)+A\right)^2}{g^3(a+bx)(bc-ad)^2}-\frac{2ABd(}{g^3(a+bx}$$

```
[Out] -2*A*B*d*(d*x+c)/(-a*d+b*c)^2/g^3/(b*x+a)+2*B^2*d*(d*x+c)/(-a*d+b*c)^2/g^3/
(b*x+a)-1/4*b*B^2*(d*x+c)^2/(-a*d+b*c)^2/g^3/(b*x+a)^2-2*B^2*d*(d*x+c)*ln(e
*(d*x+c)/(b*x+a))/(-a*d+b*c)^2/g^3/(b*x+a)+1/2*b*B*(d*x+c)^2*(A+B*ln(e*(d*x
+c)/(b*x+a)))/(-a*d+b*c)^2/g^3/(b*x+a)^2+d*(d*x+c)*(A+B*ln(e*(d*x+c)/(b*x+a
)))^2/(-a*d+b*c)^2/g^3/(b*x+a)-1/2*b*(d*x+c)^2*(A+B*ln(e*(d*x+c)/(b*x+a)))^
2/(-a*d+b*c)^2/g^3/(b*x+a)^2
```

**Rubi [C]** time = 0.91, antiderivative size = 578, normalized size of antiderivative = 1.95, number of steps used = 30, number of rules used = 11, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.344, Rules used = {2525, 12, 2528, 44, 2524, 2418, 2390, 2301, 2394, 2393, 2391}

$$\frac{B^2d^2\text{PolyLog}\left(2,-\frac{d(a+bx)}{bc-ad}\right)}{bg^3(bc-ad)^2}+\frac{B^2d^2\text{PolyLog}\left(2,\frac{b(c+dx)}{bc-ad}\right)}{bg^3(bc-ad)^2}-\frac{Bd^2\log(a+bx)\left(B\log\left(\frac{e(c+dx)}{a+bx}\right)+A\right)}{bg^3(bc-ad)^2}+\frac{Bd^2\log(c+dx)\left(\right.}{bg^3(b}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Log[(e*(c + d*x))/(a + b*x)])^2/(a*g + b*g*x)^3,x]
```

```
[Out] -B^2/(4*b*g^3*(a + b*x)^2) + (3*B^2*d)/(2*b*(b*c - a*d)*g^3*(a + b*x)) + (3
*B^2*d^2*Log[a + b*x])/(2*b*(b*c - a*d)^2*g^3) - (B^2*d^2*Log[a + b*x]^2)/(
2*b*(b*c - a*d)^2*g^3) - (3*B^2*d^2*Log[c + d*x])/(2*b*(b*c - a*d)^2*g^3) +
 (B^2*d^2*Log[-((d*(a + b*x))/(b*c - a*d))]*Log[c + d*x])/(b*(b*c - a*d)^2*
g^3) - (B^2*d^2*Log[c + d*x]^2)/(2*b*(b*c - a*d)^2*g^3) + (B^2*d^2*Log[a +
b*x]*Log[(b*(c + d*x))/(b*c - a*d)])/(b*(b*c - a*d)^2*g^3) + (B*(A + B*Log[
(e*(c + d*x))/(a + b*x)]))/(2*b*g^3*(a + b*x)^2) - (B*d*(A + B*Log[(e*(c +
d*x))/(a + b*x)]))/(b*(b*c - a*d)*g^3*(a + b*x)) - (B*d^2*Log[a + b*x]*(A +
 B*Log[(e*(c + d*x))/(a + b*x)]))/(b*(b*c - a*d)^2*g^3) + (B*d^2*Log[c + d*
x]*(A + B*Log[(e*(c + d*x))/(a + b*x)]))/(b*(b*c - a*d)^2*g^3) - (A + B*Log
[(e*(c + d*x))/(a + b*x)])^2/(2*b*g^3*(a + b*x)^2) + (B^2*d^2*PolyLog[2, -(
(d*(a + b*x))/(b*c - a*d))])/(b*(b*c - a*d)^2*g^3) + (B^2*d^2*PolyLog[2, (b
*(c + d*x))/(b*c - a*d)])/(b*(b*c - a*d)^2*g^3)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 2301

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

Rule 2390

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_))^(q_.), x_Symbol] :> Dist[1/e, Subst[Int[((f*x)/d)^q*(a + b*Log[c*x^
n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && E
qQ[e*f - d*g, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2393

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + (c*e*x)/g])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2394

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d + e*x
)^n]))/g, x] - Dist[(b*e*n)/g, Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2418

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*(RFx_), x_Sy
mbol] :> With[{u = ExpandIntegrand[(a + b*Log[c*(d + e*x)^n])^p, RFx, x]},
```

```
Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d, e, n}, x] && RationalFunctionQ[
RFx, x] && IntegerQ[p]
```

Rule 2524

```
Int[((a_.) + Log[(c_.)*(RFx_)^(p_.)]*(b_.))^(n_.)/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[(Log[d + e*x]*(a + b*Log[c*RFx^p])^n)/e, x] - Dist[(b*n*p)/e
, Int[(Log[d + e*x]*(a + b*Log[c*RFx^p])^(n - 1)*D[RFx, x])/RFx, x], x] /;
FreeQ[{a, b, c, d, e, p}, x] && RationalFunctionQ[RFx, x] && IGtQ[n, 0]
```

Rule 2525

```
Int[((a_.) + Log[(c_.)*(RFx_)^(p_.)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_))^(m_.
), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(a + b*Log[c*RFx^p])^n)/(e*(m + 1))
, x] - Dist[(b*n*p)/(e*(m + 1)), Int[SimplifyIntegrand[((d + e*x)^(m + 1)*(
a + b*Log[c*RFx^p])^(n - 1)*D[RFx, x])/RFx, x], x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && RationalFunctionQ[RFx, x] && IGtQ[n, 0] && (EqQ[n, 1] ||
IntegerQ[m]) && NeQ[m, -1]
```

Rule 2528

```
Int[((a_.) + Log[(c_.)*(RFx_)^(p_.)]*(b_.))^(n_.)*(RGx_), x_Symbol] :> With
[{u = ExpandIntegrand[(a + b*Log[c*RFx^p])^n, RGx, x]}, Int[u, x] /; SumQ[u
]] /; FreeQ[{a, b, c, p}, x] && RationalFunctionQ[RFx, x] && RationalFuncti
onQ[RGx, x] && IGtQ[n, 0]
```

Rubi steps

$$\int \frac{\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2}{(ag+bgx)^3}\,dx = -\frac{\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2}{2bg^3(a+bx)^2} + \frac{B\int \frac{(bc-ad)\left(-A-B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)}{g^2(a+bx)^3(c+dx)}\,dx}{bg}$$

$$= -\frac{\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2}{2bg^3(a+bx)^2} + \frac{(B(bc-ad))\int \frac{-A-B\log\left(\frac{e(c+dx)}{a+bx}\right)}{(a+bx)^3(c+dx)}\,dx}{bg^3}$$

$$= -\frac{\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2}{2bg^3(a+bx)^2} + \frac{(B(bc-ad))\int \left(\frac{b\left(-A-B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)}{(bc-ad)(a+bx)^3} - \frac{bd\left(-A-B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)}{(bc-ad)^2(a+bx)^2} \cdots}{bg^3}$$

$$= -\frac{\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2}{2bg^3(a+bx)^2} + \frac{B\int \frac{-A-B\log\left(\frac{e(c+dx)}{a+bx}\right)}{(a+bx)^3}\,dx}{g^3} + \frac{\left(Bd^2\right)\int \frac{-A-B\log\left(\frac{e(c+dx)}{a+bx}\right)}{a+bx}\,dx}{(bc-ad)^2g^3} - \cdots$$

$$= \frac{B\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)}{2bg^3(a+bx)^2} - \frac{Bd\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)}{b(bc-ad)g^3(a+bx)} - \frac{Bd^2\log(a+bx)\left(A+B\log \cdots\right.}{b(bc-ad)^2g^3}$$

$$= \frac{B\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)}{2bg^3(a+bx)^2} - \frac{Bd\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)}{b(bc-ad)g^3(a+bx)} - \frac{Bd^2\log(a+bx)\left(A+B\log \cdots\right.}{b(bc-ad)^2g^3}$$

$$= \frac{B\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)}{2bg^3(a+bx)^2} - \frac{Bd\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)}{b(bc-ad)g^3(a+bx)} - \frac{Bd^2\log(a+bx)\left(A+B\log \cdots\right.}{b(bc-ad)^2g^3}$$

$$= -\frac{B^2}{4bg^3(a+bx)^2} + \frac{3B^2d}{2b(bc-ad)g^3(a+bx)} + \frac{3B^2d^2\log(a+bx)}{2b(bc-ad)^2g^3} - \frac{3B^2d^2\log(c+ \cdots}{2b(bc-ad)^2g \cdots}$$

$$= -\frac{B^2}{4bg^3(a+bx)^2} + \frac{3B^2d}{2b(bc-ad)g^3(a+bx)} + \frac{3B^2d^2\log(a+bx)}{2b(bc-ad)^2g^3} - \frac{3B^2d^2\log(c+ \cdots}{2b(bc-ad)^2g \cdots}$$

$$= -\frac{B^2}{4bg^3(a+bx)^2} + \frac{3B^2d}{2b(bc-ad)g^3(a+bx)} + \frac{3B^2d^2\log(a+bx)}{2b(bc-ad)^2g^3} - \frac{B^2d^2\log^2(a+b \cdots}{2b(bc-ad)^2g \cdots}$$

$$= -\frac{B^2}{4bg^3(a+bx)^2} + \frac{3B^2d}{2b(bc-ad)g^3(a+bx)} + \frac{3B^2d^2\log(a+bx)}{2b(bc-ad)^2g^3} - \frac{B^2d^2\log^2(a+b \cdots}{2b(bc-ad)^2g \cdots}$$

**Mathematica [C]** time = 0.44, size = 444, normalized size = 1.50

$$\frac{B\left(-4d^2(a+bx)^2\log(a+bx)\left(B\log\left(\frac{e(c+dx)}{a+bx}\right)+A\right)+4d^2(a+bx)^2\log(c+dx)\left(B\log\left(\frac{e(c+dx)}{a+bx}\right)+A\right)+2(bc-ad)^2\left(B\log\left(\frac{e(c+dx)}{a+bx}\right)+A\right)+4d(a+bx)(ad-bc)\left(B\log\left(\frac{e(c+}{a+}\right.\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Log[(e*(c + d*x))/(a + b*x)])^2/(a*g + b*g*x)^3,x]
```

```
[Out] (-2*(A + B*Log[(e*(c + d*x))/(a + b*x)])^2 + (B*(4*B*d*(a + b*x)*(b*c - a*d
 + d*(a + b*x)*Log[a + b*x] - d*(a + b*x)*Log[c + d*x]) - B*((b*c - a*d)^2
+ 2*d*(-(b*c) + a*d)*(a + b*x) - 2*d^2*(a + b*x)^2*Log[a + b*x] + 2*d^2*(a
+ b*x)^2*Log[c + d*x]) + 2*(b*c - a*d)^2*(A + B*Log[(e*(c + d*x))/(a + b*x)
]) + 4*d*(-(b*c) + a*d)*(a + b*x)*(A + B*Log[(e*(c + d*x))/(a + b*x)]) - 4*
d^2*(a + b*x)^2*Log[a + b*x]*(A + B*Log[(e*(c + d*x))/(a + b*x)]) + 4*d^2*(
a + b*x)^2*Log[c + d*x]*(A + B*Log[(e*(c + d*x))/(a + b*x)]) - 2*B*d^2*(a +
 b*x)^2*(Log[a + b*x]*(Log[a + b*x] - 2*Log[(b*(c + d*x))/(b*c - a*d)]) - 2
*PolyLog[2, (d*(a + b*x))/(-(b*c) + a*d)]) + 2*B*d^2*(a + b*x)^2*((2*Log[(d
*(a + b*x))/(-(b*c) + a*d)] - Log[c + d*x])*Log[c + d*x] + 2*PolyLog[2, (b*
(c + d*x))/(b*c - a*d)])))/(b*c - a*d)^2)/(4*b*g^3*(a + b*x)^2)
```

**fricas [A]** time = 0.82, size = 373, normalized size = 1.26

$$-\frac{\left(2A^2-2AB+B^2\right)b^2c^2-4\left(A^2-2AB+2B^2\right)abcd+\left(2A^2-6AB+7B^2\right)a^2d^2-2\left(B^2b^2d^2x^2+2B^2abd^2x-\right.}{4\left(\left(b^5c^2-2\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*(d*x+c)/(b*x+a)))^2/(b*g*x+a*g)^3,x, algorithm="fricas
")
```

```
[Out] -1/4*((2*A^2 - 2*A*B + B^2)*b^2*c^2 - 4*(A^2 - 2*A*B + 2*B^2)*a*b*c*d + (2*
A^2 - 6*A*B + 7*B^2)*a^2*d^2 - 2*(B^2*b^2*d^2*x^2 + 2*B^2*a*b*d^2*x - B^2*b
^2*c^2 + 2*B^2*a*b*c*d)*log((d*e*x + c*e)/(b*x + a))^2 + 2*((2*A*B - 3*B^2)
*b^2*c*d - (2*A*B - 3*B^2)*a*b*d^2)*x - 2*((2*A*B - 3*B^2)*b^2*d^2*x^2 - (2
*A*B - B^2)*b^2*c^2 + 4*(A*B - B^2)*a*b*c*d - 2*(B^2*b^2*c*d - 2*(A*B - B^2
)*a*b*d^2)*x)*log((d*e*x + c*e)/(b*x + a)))/((b^5*c^2 - 2*a*b^4*c*d + a^2*b
^3*d^2)*g^3*x^2 + 2*(a*b^4*c^2 - 2*a^2*b^3*c*d + a^3*b^2*d^2)*g^3*x + (a^2*
b^3*c^2 - 2*a^3*b^2*c*d + a^4*b*d^2)*g^3)
```

**giac [A]** time = 2.19, size = 493, normalized size = 1.67

$$\left(\frac{4\,(dxe+ce)B^2de\log\left(\frac{dxe+ce}{bx+a}\right)^2}{bx+a}+\frac{8\,(dxe+ce)ABde\log\left(\frac{dxe+ce}{bx+a}\right)}{bx+a}-\frac{8\,(dxe+ce)B^2de\log\left(\frac{dxe+ce}{bx+a}\right)}{bx+a}-\frac{2\,(dxe+ce)^2B^2b\log\left(\frac{dxe+ce}{bx+a}\right)^2}{(bx+a)^2}+\frac{4\,(dxe+ce)A^2de}{bx+a}-\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*(d*x+c)/(b*x+a)))^2/(b*g*x+a*g)^3,x, algorithm="giac")
```

```
[Out] 1/4*(4*(d*x*e + c*e)*B^2*d*e*log((d*x*e + c*e)/(b*x + a))^2/(b*x + a) + 8*(
d*x*e + c*e)*A*B*d*e*log((d*x*e + c*e)/(b*x + a))/(b*x + a) - 8*(d*x*e + c*
e)*B^2*d*e*log((d*x*e + c*e)/(b*x + a))/(b*x + a) - 2*(d*x*e + c*e)^2*B^2*b
*log((d*x*e + c*e)/(b*x + a))^2/(b*x + a)^2 + 4*(d*x*e + c*e)*A^2*d*e/(b*x
+ a) - 8*(d*x*e + c*e)*A*B*d*e/(b*x + a) + 8*(d*x*e + c*e)*B^2*d*e/(b*x + a
) - 4*(d*x*e + c*e)^2*A*B*b*log((d*x*e + c*e)/(b*x + a))/(b*x + a)^2 + 2*(d
*x*e + c*e)^2*B^2*b*log((d*x*e + c*e)/(b*x + a))/(b*x + a)^2 - 2*(d*x*e + c
*e)^2*A^2*b/(b*x + a)^2 + 2*(d*x*e + c*e)^2*A*B*b/(b*x + a)^2 - (d*x*e + c*
e)^2*B^2*b/(b*x + a)^2)*(b*c/((b*c*e - a*d*e)*(b*c - a*d)) - a*d/((b*c*e -
a*d*e)*(b*c - a*d)))/(b*c*g^3*e - a*d*g^3*e)
```

**maple [B]** time = 0.05, size = 1934, normalized size = 6.53

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*ln((d*x+c)/(b*x+a)*e)+A)^2/(b*g*x+a*g)^3,x)
```

```
[Out] b/(a*d-b*c)^3/g^3*B^2*ln(1/b*d*e-(a*d-b*c)/(b*x+a)/b*e)*d/(b*x+a)*c^2-1/2/b
/(a*d-b*c)^3/g^3*B^2*ln(1/b*d*e-(a*d-b*c)/(b*x+a)/b*e)^2/(b*x+a)^2*a^3*d^3+
3/2/(a*d-b*c)^3/g^3*B^2*ln(1/b*d*e-(a*d-b*c)/(b*x+a)/b*e)^2/(b*x+a)^2*a^2*d
^2*c-2/(a*d-b*c)^3/g^3*B^2*ln(1/b*d*e-(a*d-b*c)/(b*x+a)/b*e)*d^2/(b*x+a)*a*
c-3/2/(a*d-b*c)^3/g^3*B^2*ln(1/b*d*e-(a*d-b*c)/(b*x+a)/b*e)/(b*x+a)^2*a^2*d
^2*c-3/4*b/(a*d-b*c)^3/g^3*B^2/(b*x+a)^2*a*d*c^2+b/(a*d-b*c)^3/g^3*A*B*d/(b
*x+a)*c^2+1/2/b/(a*d-b*c)^3/g^3*A*B/(b*x+a)^2*a^3*d^3+1/b/(a*d-b*c)^3/g^3*A
*B*d^3/(b*x+a)*a^2-3/2*b/(a*d-b*c)^3/g^3*A^2/(b*x+a)^2*a*d*c^2-2/(a*d-b*c)^
3/g^3*A*B*d^2/(b*x+a)*c*a+1/2/b/(a*d-b*c)^3/g^3*B^2*ln(1/b*d*e-(a*d-b*c)/(b
*x+a)/b*e)/(b*x+a)^2*a^3*d^3+b^2/(a*d-b*c)^3/g^3*A*B*ln(1/b*d*e-(a*d-b*c)/(
b*x+a)/b*e)/(b*x+a)^2*c^3-1/2/(a*d-b*c)^3/g^3*B^2*ln(1/b*d*e-(a*d-b*c)/(b*x
+a)/b*e)^2*d^2*c+3/2/(a*d-b*c)^3/g^3*B^2*ln(1/b*d*e-(a*d-b*c)/(b*x+a)/b*e)*
d^2*c+1/2*b^2/(a*d-b*c)^3/g^3*A^2/(b*x+a)^2*c^3+1/4*b^2/(a*d-b*c)^3/g^3*B^2
/(b*x+a)^2*c^3-3/2/(a*d-b*c)^3/g^3*A*B/(b*x+a)^2*a^2*d^2*c-1/4/b/(a*d-b*c)^
3/g^3*B^2/(b*x+a)^2*a^3*d^3-3/2/b/(a*d-b*c)^3/g^3*B^2*d^3/(b*x+a)*a^2+3/2/(
```

```
a*d-b*c)^3/g^3*A^2/(b*x+a)^2*a^2*d^2*c+3/4/(a*d-b*c)^3/g^3*B^2/(b*x+a)^2*a^
2*d^2*c-1/2/b/(a*d-b*c)^3/g^3*A^2/(b*x+a)^2*a^3*d^3-3/2*b/(a*d-b*c)^3/g^3*B
^2*d/(b*x+a)*c^2-1/2*b^2/(a*d-b*c)^3/g^3*A*B/(b*x+a)^2*c^3+1/2*b^2/(a*d-b*c
)^3/g^3*B^2*ln(1/b*d*e-(a*d-b*c)/(b*x+a)/b*e)^2/(b*x+a)^2*c^3-3/2/b/(a*d-b*
c)^3/g^3*B^2*ln(1/b*d*e-(a*d-b*c)/(b*x+a)/b*e)*d^3*a+1/2/b/(a*d-b*c)^3/g^3*
B^2*ln(1/b*d*e-(a*d-b*c)/(b*x+a)/b*e)^2*d^3*a-1/2*b^2/(a*d-b*c)^3/g^3*B^2*l
n(1/b*d*e-(a*d-b*c)/(b*x+a)/b*e)/(b*x+a)^2*c^3-1/(a*d-b*c)^3/g^3*A*B*ln(1/b
*d*e-(a*d-b*c)/(b*x+a)/b*e)*d^2*c+1/2/b/(a*d-b*c)^3/g^3*A^2*d^3*a+1/b/(a*d-
b*c)^3/g^3*A*B*ln(1/b*d*e-(a*d-b*c)/(b*x+a)/b*e)*d^3*a+1/b/(a*d-b*c)^3/g^3*
B^2*ln(1/b*d*e-(a*d-b*c)/(b*x+a)/b*e)*d^3/(b*x+a)*a^2+7/4/b/(a*d-b*c)^3/g^3
*B^2*d^3*a-7/4/(a*d-b*c)^3/g^3*B^2*d^2*c+3/(a*d-b*c)^3/g^3*B^2*d^2/(b*x+a)*
a*c-1/2/(a*d-b*c)^3/g^3*A^2*d^2*c-3/2/b/(a*d-b*c)^3/g^3*A*B*d^3*a+3/2/(a*d-
b*c)^3/g^3*A*B*d^2*c+3/(a*d-b*c)^3/g^3*A*B*ln(1/b*d*e-(a*d-b*c)/(b*x+a)/b*e
)/(b*x+a)^2*a^2*d^2*c-3/2*b/(a*d-b*c)^3/g^3*B^2*ln(1/b*d*e-(a*d-b*c)/(b*x+a
)/b*e)^2/(b*x+a)^2*a*d*c^2-1/b/(a*d-b*c)^3/g^3*A*B*ln(1/b*d*e-(a*d-b*c)/(b*
x+a)/b*e)/(b*x+a)^2*a^3*d^3+3/2*b/(a*d-b*c)^3/g^3*A*B/(b*x+a)^2*c^2*a*d+3/2
*b/(a*d-b*c)^3/g^3*B^2*ln(1/b*d*e-(a*d-b*c)/(b*x+a)/b*e)/(b*x+a)^2*a*d*c^2-
3*b/(a*d-b*c)^3/g^3*A*B*ln(1/b*d*e-(a*d-b*c)/(b*x+a)/b*e)/(b*x+a)^2*c^2*a*d
```

**maxima [B]** time = 1.52, size = 847, normalized size = 2.86

$$-\frac{1}{4}\left(2\left(\frac{2\,bdx-bc+3\,ad}{\left(b^4c-ab^3d\right)g^3x^2+2\left(ab^3c-a^2b^2d\right)g^3x+\left(a^2b^2c-a^3bd\right)g^3}+\frac{2\,d^2\log\left(bx+a\right)}{\left(b^3c^2-2\,ab^2cd+a^2bd^2\right)g^3}-\frac{2\,d^2\log}{\left(b^3c^2-2\,ab^2\right.}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*(d*x+c)/(b*x+a)))^2/(b*g*x+a*g)^3,x, algorithm="maxima
")
```

```
[Out] -1/4*(2*((2*b*d*x - b*c + 3*a*d)/((b^4*c - a*b^3*d)*g^3*x^2 + 2*(a*b^3*c -
a^2*b^2*d)*g^3*x + (a^2*b^2*c - a^3*b*d)*g^3) + 2*d^2*log(b*x + a)/((b^3*c^
2 - 2*a*b^2*c*d + a^2*b*d^2)*g^3) - 2*d^2*log(d*x + c)/((b^3*c^2 - 2*a*b^2*
c*d + a^2*b*d^2)*g^3))*log(d*e*x/(b*x + a) + c*e/(b*x + a)) + (b^2*c^2 - 8*
a*b*c*d + 7*a^2*d^2 + 2*(b^2*d^2*x^2 + 2*a*b*d^2*x + a^2*d^2)*log(b*x + a)^
2 + 2*(b^2*d^2*x^2 + 2*a*b*d^2*x + a^2*d^2)*log(d*x + c)^2 - 6*(b^2*c*d - a
*b*d^2)*x - 6*(b^2*d^2*x^2 + 2*a*b*d^2*x + a^2*d^2)*log(b*x + a) + 2*(3*b^2
*d^2*x^2 + 6*a*b*d^2*x + 3*a^2*d^2 - 2*(b^2*d^2*x^2 + 2*a*b*d^2*x + a^2*d^2
)*log(b*x + a))*log(d*x + c))/(a^2*b^3*c^2*g^3 - 2*a^3*b^2*c*d*g^3 + a^4*b*
d^2*g^3 + (b^5*c^2*g^3 - 2*a*b^4*c*d*g^3 + a^2*b^3*d^2*g^3)*x^2 + 2*(a*b^4*
c^2*g^3 - 2*a^2*b^3*c*d*g^3 + a^3*b^2*d^2*g^3)*x))*B^2 - 1/2*A*B*((2*b*d*x
- b*c + 3*a*d)/((b^4*c - a*b^3*d)*g^3*x^2 + 2*(a*b^3*c - a^2*b^2*d)*g^3*x +
 (a^2*b^2*c - a^3*b*d)*g^3) + 2*log(d*e*x/(b*x + a) + c*e/(b*x + a))/(b^3*g
^3*x^2 + 2*a*b^2*g^3*x + a^2*b*g^3) + 2*d^2*log(b*x + a)/((b^3*c^2 - 2*a*b^
2*c*d + a^2*b*d^2)*g^3) - 2*d^2*log(d*x + c)/((b^3*c^2 - 2*a*b^2*c*d + a^2*
```

```
b*d^2)*g^3)) - 1/2*B^2*log(d*e*x/(b*x + a) + c*e/(b*x + a))^2/(b^3*g^3*x^2
+ 2*a*b^2*g^3*x + a^2*b*g^3) - 1/2*A^2/(b^3*g^3*x^2 + 2*a*b^2*g^3*x + a^2*b
*g^3)
```

**mupad [B]** time = 6.00, size = 507, normalized size = 1.71

$$\frac{\ln\left(\frac{e\,(c+d\,x)}{a+b\,x}\right)\left(\frac{B^2\,x\,(a\,d-b\,c)}{b\,g^3\left(a^2\,d^2-2\,a\,b\,c\,d+b^2\,c^2\right)}-\frac{A\,B}{b^2\,d\,g^3}+\frac{B^2\,d^2\left(\frac{2\,a^2\,d^2-3\,a\,b\,c\,d+b^2\,c^2}{2\,b\,d^3}+\frac{a\,(a\,d-b\,c)}{2\,b\,d^2}\right)}{b\,g^3\left(a^2\,d^2-2\,a\,b\,c\,d+b^2\,c^2\right)}\right)}{\frac{b\,x^2}{d}+\frac{a^2}{b\,d}+\frac{2\,a\,x}{d}}-\ln\left(\frac{e\,(c+d\,x)}{a+b\,x}\right)^2\left(\frac{B^2}{2\,b^2\,g^3\left(2\,a\,x+b\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*log((e*(c + d*x))/(a + b*x)))^2/(a*g + b*g*x)^3,x)
```

```
[Out] (log((e*(c + d*x))/(a + b*x))*((B^2*x*(a*d - b*c))/(b*g^3*(a^2*d^2 + b^2*c^
2 - 2*a*b*c*d)) - (A*B)/(b^2*d*g^3) + (B^2*d^2*((2*a^2*d^2 + b^2*c^2 - 3*a*
b*c*d)/(2*b*d^3) + (a*(a*d - b*c))/(2*b*d^2)))/(b*g^3*(a^2*d^2 + b^2*c^2 -
2*a*b*c*d))))/((b*x^2)/d + a^2/(b*d) + (2*a*x)/d) - log((e*(c + d*x))/(a +
b*x))^2*(B^2/(2*b^2*g^3*(2*a*x + b*x^2 + a^2/b)) - (B^2*d^2)/(2*b*g^3*(a^2*
d^2 + b^2*c^2 - 2*a*b*c*d))) - ((2*A^2*a*d - 2*A^2*b*c + 7*B^2*a*d - B^2*b*
c - 6*A*B*a*d + 2*A*B*b*c)/(2*(a*d - b*c)) + (x*(3*B^2*b*d - 2*A*B*b*d))/(a
*d - b*c))/(2*a^2*b*g^3 + 2*b^3*g^3*x^2 + 4*a*b^2*g^3*x) - (B*d^2*atan((B*d
^2*(2*b*d*x - (b^3*c^2*g^3 - a^2*b*d^2*g^3)/(b*g^3*(a*d - b*c)))*(2*A - 3*B
)*1i)/((a*d - b*c)*(3*B^2*d^2 - 2*A*B*d^2)))*(2*A - 3*B)*1i)/(b*g^3*(a*d -
b*c)^2)
```

**sympy [B]** time = 6.55, size = 892, normalized size = 3.01

$$\frac{Bd^2\,(2A-3B)\log\left(x+\frac{2ABad^3+2ABbcd^2-3B^2ad^3-3B^2bcd^2-\frac{Ba^3d^5(2A-3B)}{(ad-bc)^2}+\frac{3Ba^2bcd^4(2A-3B)}{(ad-bc)^2}-\frac{3Bab^2c^2d^3(2A-3B)}{(ad-bc)^2}+\frac{Bb^3c^3d^2(2A-3B)}{(ad-bc)^2}}{4ABbd^3-6B^2bd^3}\right)}{2bg^3\,(ad-bc)^2}-\frac{Bd^2\,(2\ldots}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*ln(e*(d*x+c)/(b*x+a)))**2/(b*g*x+a*g)**3,x)
```

```
[Out] B*d**2*(2*A - 3*B)*log(x + (2*A*B*a*d**3 + 2*A*B*b*c*d**2 - 3*B**2*a*d**3 -
 3*B**2*b*c*d**2 - B*a**3*d**5*(2*A - 3*B)/(a*d - b*c)**2 + 3*B*a**2*b*c*d*
*4*(2*A - 3*B)/(a*d - b*c)**2 - 3*B*a*b**2*c**2*d**3*(2*A - 3*B)/(a*d - b*c
)**2 + B*b**3*c**3*d**2*(2*A - 3*B)/(a*d - b*c)**2)/(4*A*B*b*d**3 - 6*B**2*
b*d**3))/(2*b*g**3*(a*d - b*c)**2) - B*d**2*(2*A - 3*B)*log(x + (2*A*B*a*d*
*3 + 2*A*B*b*c*d**2 - 3*B**2*a*d**3 - 3*B**2*b*c*d**2 + B*a**3*d**5*(2*A -
```

```
3*B)/(a*d - b*c)**2 - 3*B*a**2*b*c*d**4*(2*A - 3*B)/(a*d - b*c)**2 + 3*B*a*
b**2*c**2*d**3*(2*A - 3*B)/(a*d - b*c)**2 - B*b**3*c**3*d**2*(2*A - 3*B)/(a
*d - b*c)**2)/(4*A*B*b*d**3 - 6*B**2*b*d**3))/(2*b*g**3*(a*d - b*c)**2) + (
2*B**2*a*c*d + 2*B**2*a*d**2*x - B**2*b*c**2 + B**2*b*d**2*x**2)*log(e*(c +
 d*x)/(a + b*x))**2/(2*a**4*d**2*g**3 - 4*a**3*b*c*d*g**3 + 4*a**3*b*d**2*g
**3*x + 2*a**2*b**2*c**2*g**3 - 8*a**2*b**2*c*d*g**3*x + 2*a**2*b**2*d**2*g
**3*x**2 + 4*a*b**3*c**2*g**3*x - 4*a*b**3*c*d*g**3*x**2 + 2*b**4*c**2*g**3
*x**2) + (-2*A*B*a*d + 2*A*B*b*c + 3*B**2*a*d - B**2*b*c + 2*B**2*b*d*x)*lo
g(e*(c + d*x)/(a + b*x))/(2*a**3*b*d*g**3 - 2*a**2*b**2*c*g**3 + 4*a**2*b**
2*d*g**3*x - 4*a*b**3*c*g**3*x + 2*a*b**3*d*g**3*x**2 - 2*b**4*c*g**3*x**2)
 + (-2*A**2*a*d + 2*A**2*b*c + 6*A*B*a*d - 2*A*B*b*c - 7*B**2*a*d + B**2*b*
c + x*(4*A*B*b*d - 6*B**2*b*d))/(4*a**3*b*d*g**3 - 4*a**2*b**2*c*g**3 + x**
2*(4*a*b**3*d*g**3 - 4*b**4*c*g**3) + x*(8*a**2*b**2*d*g**3 - 8*a*b**3*c*g*
*3))
```

## 3.189 $\int \frac{\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2}{(ag+bgx)^4}\,dx$

**Optimal**. Leaf size=399

$$\frac{2b^2B(c+dx)^3\left(B\log\left(\frac{e(c+dx)}{a+bx}\right)+A\right)}{9g^4(a+bx)^3(bc-ad)^3}-\frac{2Bd^3\log\left(\frac{c+dx}{a+bx}\right)\left(B\log\left(\frac{e(c+dx)}{a+bx}\right)+A\right)}{3bg^4(bc-ad)^3}+\frac{2Bd^2(c+dx)\left(B\log\left(\frac{e(c+dx)}{a+bx}\right)+A\right)}{g^4(a+bx)(bc-ad)^3}$$

```
[Out] -2*B^2*d^2*(d*x+c)/(-a*d+b*c)^3/g^4/(b*x+a)+1/2*b*B^2*d*(d*x+c)^2/(-a*d+b*c
)^3/g^4/(b*x+a)^2-2/27*b^2*B^2*(d*x+c)^3/(-a*d+b*c)^3/g^4/(b*x+a)^3+1/3*B^2
*d^3*ln((d*x+c)/(b*x+a))^2/b/(-a*d+b*c)^3/g^4+2*B*d^2*(d*x+c)*(A+B*ln(e*(d*
x+c)/(b*x+a)))/(-a*d+b*c)^3/g^4/(b*x+a)-b*B*d*(d*x+c)^2*(A+B*ln(e*(d*x+c)/(
b*x+a)))/(-a*d+b*c)^3/g^4/(b*x+a)^2+2/9*b^2*B*(d*x+c)^3*(A+B*ln(e*(d*x+c)/(
b*x+a)))/(-a*d+b*c)^3/g^4/(b*x+a)^3-2/3*B*d^3*ln((d*x+c)/(b*x+a))*(A+B*ln(e
*(d*x+c)/(b*x+a)))/b/(-a*d+b*c)^3/g^4-1/3*(A+B*ln(e*(d*x+c)/(b*x+a)))^2/b/g
^4/(b*x+a)^3
```

**Rubi [C]** time = 1.07, antiderivative size = 680, normalized size of antiderivative = 1.70, number of steps used = 34, number of rules used = 11, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.344, Rules used = {2525, 12, 2528, 44, 2524, 2418, 2390, 2301, 2394, 2393, 2391}

$$-\frac{2B^2d^3\text{PolyLog}\left(2,-\frac{d(a+bx)}{bc-ad}\right)}{3bg^4(bc-ad)^3}-\frac{2B^2d^3\text{PolyLog}\left(2,\frac{b(c+dx)}{bc-ad}\right)}{3bg^4(bc-ad)^3}+\frac{2Bd^3\log(a+bx)\left(B\log\left(\frac{e(c+dx)}{a+bx}\right)+A\right)}{3bg^4(bc-ad)^3}-\frac{2Bd^3\log(c}{}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Log[(e*(c + d*x))/(a + b*x)])^2/(a*g + b*g*x)^4,x]
```

```
[Out] (-2*B^2)/(27*b*g^4*(a + b*x)^3) + (5*B^2*d)/(18*b*(b*c - a*d)*g^4*(a + b*x)
^2) - (11*B^2*d^2)/(9*b*(b*c - a*d)^2*g^4*(a + b*x)) - (11*B^2*d^3*Log[a +
b*x])/(9*b*(b*c - a*d)^3*g^4) + (B^2*d^3*Log[a + b*x]^2)/(3*b*(b*c - a*d)^3
*g^4) + (11*B^2*d^3*Log[c + d*x])/(9*b*(b*c - a*d)^3*g^4) - (2*B^2*d^3*Log[
-((d*(a + b*x))/(b*c - a*d))]*Log[c + d*x])/(3*b*(b*c - a*d)^3*g^4) + (B^2*
d^3*Log[c + d*x]^2)/(3*b*(b*c - a*d)^3*g^4) - (2*B^2*d^3*Log[a + b*x]*Log[(
b*(c + d*x))/(b*c - a*d)])/(3*b*(b*c - a*d)^3*g^4) + (2*B*(A + B*Log[(e*(c
+ d*x))/(a + b*x)]))/(9*b*g^4*(a + b*x)^3) - (B*d*(A + B*Log[(e*(c + d*x))/
(a + b*x)]))/(3*b*(b*c - a*d)*g^4*(a + b*x)^2) + (2*B*d^2*(A + B*Log[(e*(c
+ d*x))/(a + b*x)]))/(3*b*(b*c - a*d)^2*g^4*(a + b*x)) + (2*B*d^3*Log[a + b
*x]*(A + B*Log[(e*(c + d*x))/(a + b*x)]))/(3*b*(b*c - a*d)^3*g^4) - (2*B*d^
3*Log[c + d*x]*(A + B*Log[(e*(c + d*x))/(a + b*x)]))/(3*b*(b*c - a*d)^3*g^4
) - (A + B*Log[(e*(c + d*x))/(a + b*x)])^2/(3*b*g^4*(a + b*x)^3) - (2*B^2*d
```

```
^3*PolyLog[2, -((d*(a + b*x))/(b*c - a*d))])/(3*b*(b*c - a*d)^3*g^4) - (2*B
^2*d^3*PolyLog[2, (b*(c + d*x))/(b*c - a*d)])/(3*b*(b*c - a*d)^3*g^4)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 2301

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

Rule 2390

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_))^(q_.), x_Symbol] :> Dist[1/e, Subst[Int[((f*x)/d)^q*(a + b*Log[c*x^
n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && E
qQ[e*f - d*g, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2393

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + (c*e*x)/g])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2394

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d + e*x
)^n]))/g, x] - Dist[(b*e*n)/g, Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2418

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*(RFx_), x_Sy
mbol] :> With[{u = ExpandIntegrand[(a + b*Log[c*(d + e*x)^n])^p, RFx, x]},
Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d, e, n}, x] && RationalFunctionQ[
RFx, x] && IntegerQ[p]
```

Rule 2524

```
Int[((a_.) + Log[(c_.)*(RFx_)^(p_.)]*(b_.))^(n_.)/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[(Log[d + e*x]*(a + b*Log[c*RFx^p])^n)/e, x] - Dist[(b*n*p)/e
, Int[(Log[d + e*x]*(a + b*Log[c*RFx^p])^(n - 1)*D[RFx, x])/RFx, x], x] /;
FreeQ[{a, b, c, d, e, p}, x] && RationalFunctionQ[RFx, x] && IGtQ[n, 0]
```

Rule 2525

```
Int[((a_.) + Log[(c_.)*(RFx_)^(p_.)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_))^(m_.
), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(a + b*Log[c*RFx^p])^n)/(e*(m + 1))
, x] - Dist[(b*n*p)/(e*(m + 1)), Int[SimplifyIntegrand[((d + e*x)^(m + 1)*(
a + b*Log[c*RFx^p])^(n - 1)*D[RFx, x])/RFx, x], x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && RationalFunctionQ[RFx, x] && IGtQ[n, 0] && (EqQ[n, 1] ||
IntegerQ[m]) && NeQ[m, -1]
```

Rule 2528

```
Int[((a_.) + Log[(c_.)*(RFx_)^(p_.)]*(b_.))^(n_.)*(RGx_), x_Symbol] :> With
[{u = ExpandIntegrand[(a + b*Log[c*RFx^p])^n, RGx, x]}, Int[u, x] /; SumQ[u
]] /; FreeQ[{a, b, c, p}, x] && RationalFunctionQ[RFx, x] && RationalFuncti
onQ[RGx, x] && IGtQ[n, 0]
```

Rubi steps

$$\int \frac{\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2}{(ag+bgx)^4}\,dx = -\frac{\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2}{3bg^4(a+bx)^3} + \frac{(2B)\int \frac{(bc-ad)\left(-A-B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)}{g^3(a+bx)^4(c+dx)}\,dx}{3bg}$$

$$= -\frac{\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2}{3bg^4(a+bx)^3} + \frac{(2B(bc-ad))\int \frac{-A-B\log\left(\frac{e(c+dx)}{a+bx}\right)}{(a+bx)^4(c+dx)}\,dx}{3bg^4}$$

$$= -\frac{\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2}{3bg^4(a+bx)^3} + \frac{(2B(bc-ad))\int \left(\frac{b\left(-A-B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)}{(bc-ad)(a+bx)^4} - \frac{bd\left(-A-B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)}{(bc-ad)^2(a+bx)^3} \cdots\right.}{\cdots}$$

$$= -\frac{\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2}{3bg^4(a+bx)^3} + \frac{(2B)\int \frac{-A-B\log\left(\frac{e(c+dx)}{a+bx}\right)}{(a+bx)^4}\,dx}{3g^4} - \frac{\left(2Bd^3\right)\int \frac{-A-B\log\left(\frac{e(c+dx)}{a+bx}\right)}{a+bx}\,dx}{3(bc-ad)^3g^4} \cdots$$

$$= \frac{2B\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)}{9bg^4(a+bx)^3} - \frac{Bd\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)}{3b(bc-ad)g^4(a+bx)^2} + \frac{2Bd^2\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)}{3b(bc-ad)^2g^4(a+bx)} \cdots$$

$$= \frac{2B\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)}{9bg^4(a+bx)^3} - \frac{Bd\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)}{3b(bc-ad)g^4(a+bx)^2} + \frac{2Bd^2\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)}{3b(bc-ad)^2g^4(a+bx)} \cdots$$

$$= \frac{2B\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)}{9bg^4(a+bx)^3} - \frac{Bd\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)}{3b(bc-ad)g^4(a+bx)^2} + \frac{2Bd^2\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)}{3b(bc-ad)^2g^4(a+bx)} \cdots$$

$$= -\frac{2B^2}{27bg^4(a+bx)^3} + \frac{5B^2d}{18b(bc-ad)g^4(a+bx)^2} - \frac{11B^2d^2}{9b(bc-ad)^2g^4(a+bx)} - \frac{11B^2d^3\,\mathrm{lo}\cdots}{9b(bc-\cdots}$$

$$= -\frac{2B^2}{27bg^4(a+bx)^3} + \frac{5B^2d}{18b(bc-ad)g^4(a+bx)^2} - \frac{11B^2d^2}{9b(bc-ad)^2g^4(a+bx)} - \frac{11B^2d^3\,\mathrm{lo}\cdots}{9b(bc-\cdots}$$

$$= -\frac{2B^2}{27bg^4(a+bx)^3} + \frac{5B^2d}{18b(bc-ad)g^4(a+bx)^2} - \frac{11B^2d^2}{9b(bc-ad)^2g^4(a+bx)} - \frac{11B^2d^3\,\mathrm{lo}\cdots}{9b(bc-\cdots}$$

$$= -\frac{2B^2}{27bg^4(a+bx)^3} + \frac{5B^2d}{18b(bc-ad)g^4(a+bx)^2} - \frac{11B^2d^2}{9b(bc-ad)^2g^4(a+bx)} - \frac{11B^2d^3\,\mathrm{lo}\cdots}{9b(bc-\cdots}$$

**Mathematica [C]** time = 0.68, size = 585, normalized size = 1.47

$$-\frac{B\left(-36d^3(a+bx)^3\log(a+bx)\left(B\log\left(\frac{e(c+dx)}{a+bx}\right)+A\right)+36d^3(a+bx)^3\log(c+dx)\left(B\log\left(\frac{e(c+dx)}{a+bx}\right)+A\right)+36d^2(a+bx)^2(ad-bc)\left(B\log\left(\frac{e(c+dx)}{a+bx}\right)+A\right)-12(bc-ad)^3(\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Log[(e*(c + d*x))/(a + b*x)])^2/(a*g + b*g*x)^4,x]
```

```
[Out] -1/54*(18*(A + B*Log[(e*(c + d*x))/(a + b*x)])^2 + (B*(36*B*d^2*(a + b*x)^2
*(b*c - a*d + d*(a + b*x)*Log[a + b*x] - d*(a + b*x)*Log[c + d*x]) - 9*B*d*
(a + b*x)*((b*c - a*d)^2 + 2*d*(-(b*c) + a*d)*(a + b*x) - 2*d^2*(a + b*x)^2
*Log[a + b*x] + 2*d^2*(a + b*x)^2*Log[c + d*x]) + 2*B*(2*(b*c - a*d)^3 - 3*
d*(b*c - a*d)^2*(a + b*x) + 6*d^2*(b*c - a*d)*(a + b*x)^2 + 6*d^3*(a + b*x)
^3*Log[a + b*x] - 6*d^3*(a + b*x)^3*Log[c + d*x]) - 12*(b*c - a*d)^3*(A + B
*Log[(e*(c + d*x))/(a + b*x)]) + 18*d*(b*c - a*d)^2*(a + b*x)*(A + B*Log[(e
*(c + d*x))/(a + b*x)]) + 36*d^2*(-(b*c) + a*d)*(a + b*x)^2*(A + B*Log[(e*(
c + d*x))/(a + b*x)]) - 36*d^3*(a + b*x)^3*Log[a + b*x]*(A + B*Log[(e*(c +
d*x))/(a + b*x)]) + 36*d^3*(a + b*x)^3*Log[c + d*x]*(A + B*Log[(e*(c + d*x)
)/(a + b*x)]) - 18*B*d^3*(a + b*x)^3*(Log[a + b*x]*(Log[a + b*x] - 2*Log[(b
*(c + d*x))/(b*c - a*d)]) - 2*PolyLog[2, (d*(a + b*x))/(-(b*c) + a*d)]) + 1
8*B*d^3*(a + b*x)^3*((2*Log[(d*(a + b*x))/(-(b*c) + a*d)] - Log[c + d*x])*L
og[c + d*x] + 2*PolyLog[2, (b*(c + d*x))/(b*c - a*d)])))/(b*c - a*d)^3)/(b*
g^4*(a + b*x)^3)
```

**fricas [A]** time = 2.03, size = 680, normalized size = 1.70

$$-\frac{2\left(9A^2-6AB+2B^2\right)b^3c^3-27\left(2A^2-2AB+B^2\right)ab^2c^2d+54\left(A^2-2AB+2B^2\right)a^2bcd^2-\left(18A^2-66AB\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*(d*x+c)/(b*x+a)))^2/(b*g*x+a*g)^4,x, algorithm="fricas
")
```

```
[Out] -1/54*(2*(9*A^2 - 6*A*B + 2*B^2)*b^3*c^3 - 27*(2*A^2 - 2*A*B + B^2)*a*b^2*c
^2*d + 54*(A^2 - 2*A*B + 2*B^2)*a^2*b*c*d^2 - (18*A^2 - 66*A*B + 85*B^2)*a^
3*d^3 - 6*((6*A*B - 11*B^2)*b^3*c*d^2 - (6*A*B - 11*B^2)*a*b^2*d^3)*x^2 + 1
8*(B^2*b^3*d^3*x^3 + 3*B^2*a*b^2*d^3*x^2 + 3*B^2*a^2*b*d^3*x + B^2*b^3*c^3
- 3*B^2*a*b^2*c^2*d + 3*B^2*a^2*b*c*d^2)*log((d*e*x + c*e)/(b*x + a))^2 + 3
*((6*A*B - 5*B^2)*b^3*c^2*d - 18*(2*A*B - 3*B^2)*a*b^2*c*d^2 + (30*A*B - 49
*B^2)*a^2*b*d^3)*x + 6*((6*A*B - 11*B^2)*b^3*d^3*x^3 + 2*(3*A*B - B^2)*b^3*
c^3 - 9*(2*A*B - B^2)*a*b^2*c^2*d + 18*(A*B - B^2)*a^2*b*c*d^2 - 3*(2*B^2*b
^3*c*d^2 - 3*(2*A*B - 3*B^2)*a*b^2*d^3)*x^2 + 3*(B^2*b^3*c^2*d - 6*B^2*a*b^
```

```
2*c*d^2 + 6*(A*B - B^2)*a^2*b*d^3)*x)*log((d*e*x + c*e)/(b*x + a)))/((b^7*c
^3 - 3*a*b^6*c^2*d + 3*a^2*b^5*c*d^2 - a^3*b^4*d^3)*g^4*x^3 + 3*(a*b^6*c^3
- 3*a^2*b^5*c^2*d + 3*a^3*b^4*c*d^2 - a^4*b^3*d^3)*g^4*x^2 + 3*(a^2*b^5*c^3
 - 3*a^3*b^4*c^2*d + 3*a^4*b^3*c*d^2 - a^5*b^2*d^3)*g^4*x + (a^3*b^4*c^3 -
3*a^4*b^3*c^2*d + 3*a^5*b^2*c*d^2 - a^6*b*d^3)*g^4)
```

**giac [A]** time = 2.74, size = 760, normalized size = 1.90

$$-\frac{\left(\frac{54\,(dxe+ce)B^2d^2e^2\log\left(\frac{dxe+ce}{bx+a}\right)^2}{bx+a}-\frac{54\,(dxe+ce)^2B^2bde\log\left(\frac{dxe+ce}{bx+a}\right)^2}{(bx+a)^2}+\frac{108\,(dxe+ce)ABd^2e^2\log\left(\frac{dxe+ce}{bx+a}\right)}{bx+a}-\frac{108\,(dxe+ce)B^2d^2e^2\log\left(\frac{dxe+ce}{bx+a}\right)}{bx+a}-10}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*(d*x+c)/(b*x+a)))^2/(b*g*x+a*g)^4,x, algorithm="giac")
```

```
[Out] -1/54*(54*(d*x*e + c*e)*B^2*d^2*e^2*log((d*x*e + c*e)/(b*x + a))^2/(b*x + a
) - 54*(d*x*e + c*e)^2*B^2*b*d*e*log((d*x*e + c*e)/(b*x + a))^2/(b*x + a)^2
 + 108*(d*x*e + c*e)*A*B*d^2*e^2*log((d*x*e + c*e)/(b*x + a))/(b*x + a) - 1
08*(d*x*e + c*e)*B^2*d^2*e^2*log((d*x*e + c*e)/(b*x + a))/(b*x + a) - 108*(
d*x*e + c*e)^2*A*B*b*d*e*log((d*x*e + c*e)/(b*x + a))/(b*x + a)^2 + 54*(d*x
*e + c*e)^2*B^2*b*d*e*log((d*x*e + c*e)/(b*x + a))/(b*x + a)^2 + 18*(d*x*e
+ c*e)^3*B^2*b^2*log((d*x*e + c*e)/(b*x + a))^2/(b*x + a)^3 + 54*(d*x*e + c
*e)*A^2*d^2*e^2/(b*x + a) - 108*(d*x*e + c*e)*A*B*d^2*e^2/(b*x + a) + 108*(
d*x*e + c*e)*B^2*d^2*e^2/(b*x + a) - 54*(d*x*e + c*e)^2*A^2*b*d*e/(b*x + a)
^2 + 54*(d*x*e + c*e)^2*A*B*b*d*e/(b*x + a)^2 - 27*(d*x*e + c*e)^2*B^2*b*d*
e/(b*x + a)^2 + 36*(d*x*e + c*e)^3*A*B*b^2*log((d*x*e + c*e)/(b*x + a))/(b*
x + a)^3 - 12*(d*x*e + c*e)^3*B^2*b^2*log((d*x*e + c*e)/(b*x + a))/(b*x + a
)^3 + 18*(d*x*e + c*e)^3*A^2*b^2/(b*x + a)^3 - 12*(d*x*e + c*e)^3*A*B*b^2/(
b*x + a)^3 + 4*(d*x*e + c*e)^3*B^2*b^2/(b*x + a)^3)*(b*c/((b*c*e - a*d*e)*(
b*c - a*d)) - a*d/((b*c*e - a*d*e)*(b*c - a*d)))/(b^2*c^2*g^4*e^2 - 2*a*b*c
*d*g^4*e^2 + a^2*d^2*g^4*e^2)
```

**maple [B]** time = 0.05, size = 2758, normalized size = 6.91

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*ln((d*x+c)/(b*x+a)*e)+A)^2/(b*g*x+a*g)^4,x)
```

```
[Out] -2/3/(a*d-b*c)^4/g^4*A*B*ln(1/b*d*e-(a*d-b*c)/(b*x+a)/b*e)*d^3*c-11/9/b/(a*
d-b*c)^4/g^4*B^2*d^4/(b*x+a)*a^2-11/9*b/(a*d-b*c)^4/g^4*B^2*d^2/(b*x+a)*c^2
-1/3/b/(a*d-b*c)^4/g^4*A^2/(b*x+a)^3*a^4*d^4-11/9/b/(a*d-b*c)^4/g^4*B^2*ln(
1/b*d*e-(a*d-b*c)/(b*x+a)/b*e)*d^4*a+1/3/b/(a*d-b*c)^4/g^4*B^2*ln(1/b*d*e-(
```

a*d-b*c)/(b*x+a)/b*e)^2*d^4*a+2/9*b^3/(a*d-b*c)^4/g^4*B^2*ln(1/b*d*e-(a*d-b
*c)/(b*x+a)/b*e)/(b*x+a)^3*c^4-1/3*b^3/(a*d-b*c)^4/g^4*B^2*ln(1/b*d*e-(a*d-
b*c)/(b*x+a)/b*e)^2/(b*x+a)^3*c^4+85/54/b/(a*d-b*c)^4/g^4*B^2*d^4*a-85/54/(
a*d-b*c)^4/g^4*B^2*d^3*c-11/9/b/(a*d-b*c)^4/g^4*A*B*d^4*a+11/9/(a*d-b*c)^4/
g^4*A*B*d^3*c+2/3*b/(a*d-b*c)^4/g^4*A*B*d^2/(b*x+a)*c^2-2/27*b^3/(a*d-b*c)^
4/g^4*B^2/(b*x+a)^3*c^4-1/3*b^3/(a*d-b*c)^4/g^4*A^2/(b*x+a)^3*c^4-1/3/(a*d-
b*c)^4/g^4*B^2*ln(1/b*d*e-(a*d-b*c)/(b*x+a)/b*e)^2*d^3*c+11/9/(a*d-b*c)^4/g
^4*B^2*ln(1/b*d*e-(a*d-b*c)/(b*x+a)/b*e)*d^3*c+2/9/b/(a*d-b*c)^4/g^4*B^2*ln
(1/b*d*e-(a*d-b*c)/(b*x+a)/b*e)/(b*x+a)^3*a^4*d^4+2/3*b/(a*d-b*c)^4/g^4*B^2
*ln(1/b*d*e-(a*d-b*c)/(b*x+a)/b*e)*d^2/(b*x+a)*c^2-4/9*b/(a*d-b*c)^4/g^4*B^
2/(b*x+a)^3*a^2*d^2*c^2-2/3*b^3/(a*d-b*c)^4/g^4*A*B*ln(1/b*d*e-(a*d-b*c)/(b
*x+a)/b*e)/(b*x+a)^3*c^4+2/3/b/(a*d-b*c)^4/g^4*A*B*ln(1/b*d*e-(a*d-b*c)/(b*
x+a)/b*e)*d^4*a+1/3/b/(a*d-b*c)^4/g^4*A*B*d^4/(b*x+a)^2*a^3+8/27*b^2/(a*d-b
*c)^4/g^4*B^2/(b*x+a)^3*c^3*a*d+1/3/b/(a*d-b*c)^4/g^4*B^2*ln(1/b*d*e-(a*d-b
*c)/(b*x+a)/b*e)*d^4/(b*x+a)^2*a^3-1/3*b^2/(a*d-b*c)^4/g^4*B^2*ln(1/b*d*e-(
a*d-b*c)/(b*x+a)/b*e)*d/(b*x+a)^2*c^3+2/3/b/(a*d-b*c)^4/g^4*B^2*ln(1/b*d*e-
(a*d-b*c)/(b*x+a)/b*e)*d^4/(b*x+a)*a^2+4/3/(a*d-b*c)^4/g^4*B^2*ln(1/b*d*e-(
a*d-b*c)/(b*x+a)/b*e)^2/(b*x+a)^3*a^3*d^3*c-8/9/(a*d-b*c)^4/g^4*B^2*ln(1/b*
d*e-(a*d-b*c)/(b*x+a)/b*e)/(b*x+a)^3*a^3*d^3*c+1/3/b/(a*d-b*c)^4/g^4*A^2*d^
4*a-1/3/(a*d-b*c)^4/g^4*A^2*d^3*c+5/18*b^2/(a*d-b*c)^4/g^4*B^2*d/(b*x+a)^2*
c^3-5/18/b/(a*d-b*c)^4/g^4*B^2*d^4/(b*x+a)^2*a^3+2/9*b^3/(a*d-b*c)^4/g^4*A*
B/(b*x+a)^3*c^4-2/27/b/(a*d-b*c)^4/g^4*B^2/(b*x+a)^3*a^4*d^4+22/9/(a*d-b*c)
^4/g^4*B^2*d^3/(b*x+a)*a*c+4/3/(a*d-b*c)^4/g^4*A^2/(b*x+a)^3*a^3*d^3*c-4/3/
(a*d-b*c)^4/g^4*B^2*ln(1/b*d*e-(a*d-b*c)/(b*x+a)/b*e)*d^3/(b*x+a)*a*c-1/(a*
d-b*c)^4/g^4*B^2*ln(1/b*d*e-(a*d-b*c)/(b*x+a)/b*e)*d^3/(b*x+a)^2*a^2*c-1/3/
b/(a*d-b*c)^4/g^4*B^2*ln(1/b*d*e-(a*d-b*c)/(b*x+a)/b*e)^2/(b*x+a)^3*a^4*d^4
+2/9/b/(a*d-b*c)^4/g^4*A*B/(b*x+a)^3*a^4*d^4+2/3/b/(a*d-b*c)^4/g^4*A*B*d^4/
(b*x+a)*a^2-1/3*b^2/(a*d-b*c)^4/g^4*A*B*d/(b*x+a)^2*c^3-5/6*b/(a*d-b*c)^4/g
^4*B^2*d^2/(b*x+a)^2*c^2*a-1/(a*d-b*c)^4/g^4*A*B*d^3/(b*x+a)^2*a^2*c+4/3*b^
2/(a*d-b*c)^4/g^4*A^2/(b*x+a)^3*c^3*a*d-4/3/(a*d-b*c)^4/g^4*A*B*d^3/(b*x+a)
*c*a-8/9/(a*d-b*c)^4/g^4*A*B/(b*x+a)^3*a^3*d^3*c-2*b/(a*d-b*c)^4/g^4*A^2/(b
*x+a)^3*a^2*d^2*c^2-8/9*b^2/(a*d-b*c)^4/g^4*B^2*ln(1/b*d*e-(a*d-b*c)/(b*x+a
)/b*e)/(b*x+a)^3*c^3*a*d+b/(a*d-b*c)^4/g^4*B^2*ln(1/b*d*e-(a*d-b*c)/(b*x+a)
/b*e)*d^2/(b*x+a)^2*c^2*a+b/(a*d-b*c)^4/g^4*A*B*d^2/(b*x+a)^2*a*c^2+8/27/(a
*d-b*c)^4/g^4*B^2/(b*x+a)^3*a^3*d^3*c+5/6/(a*d-b*c)^4/g^4*B^2*d^3/(b*x+a)^2
*a^2*c+8/3*b^2/(a*d-b*c)^4/g^4*A*B*ln(1/b*d*e-(a*d-b*c)/(b*x+a)/b*e)/(b*x+a
)^3*a*d*c^3-4*b/(a*d-b*c)^4/g^4*A*B*ln(1/b*d*e-(a*d-b*c)/(b*x+a)/b*e)/(b*x+
a)^3*a^2*d^2*c^2-8/9*b^2/(a*d-b*c)^4/g^4*A*B/(b*x+a)^3*a*d*c^3+4/3*b^2/(a*d
-b*c)^4/g^4*B^2*ln(1/b*d*e-(a*d-b*c)/(b*x+a)/b*e)^2/(b*x+a)^3*c^3*a*d-2/3/b
/(a*d-b*c)^4/g^4*A*B*ln(1/b*d*e-(a*d-b*c)/(b*x+a)/b*e)/(b*x+a)^3*a^4*d^4-2*
b/(a*d-b*c)^4/g^4*B^2*ln(1/b*d*e-(a*d-b*c)/(b*x+a)/b*e)^2/(b*x+a)^3*a^2*d^2
*c^2+4/3*b/(a*d-b*c)^4/g^4*A*B/(b*x+a)^3*a^2*d^2*c^2+8/3/(a*d-b*c)^4/g^4*A*
B*ln(1/b*d*e-(a*d-b*c)/(b*x+a)/b*e)/(b*x+a)^3*a^3*d^3*c+4/3*b/(a*d-b*c)^4/g
^4*B^2*ln(1/b*d*e-(a*d-b*c)/(b*x+a)/b*e)/(b*x+a)^3*a^2*d^2*c^2

**maxima [B]** time = 2.17, size = 1420, normalized size = 3.56

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*(d*x+c)/(b*x+a)))^2/(b*g*x+a*g)^4,x, algorithm="maxima
")
```

```
[Out] 1/54*(6*((6*b^2*d^2*x^2 + 2*b^2*c^2 - 7*a*b*c*d + 11*a^2*d^2 - 3*(b^2*c*d -
 5*a*b*d^2)*x)/((b^6*c^2 - 2*a*b^5*c*d + a^2*b^4*d^2)*g^4*x^3 + 3*(a*b^5*c^
2 - 2*a^2*b^4*c*d + a^3*b^3*d^2)*g^4*x^2 + 3*(a^2*b^4*c^2 - 2*a^3*b^3*c*d +
 a^4*b^2*d^2)*g^4*x + (a^3*b^3*c^2 - 2*a^4*b^2*c*d + a^5*b*d^2)*g^4) + 6*d^
3*log(b*x + a)/((b^4*c^3 - 3*a*b^3*c^2*d + 3*a^2*b^2*c*d^2 - a^3*b*d^3)*g^4
) - 6*d^3*log(d*x + c)/((b^4*c^3 - 3*a*b^3*c^2*d + 3*a^2*b^2*c*d^2 - a^3*b*
d^3)*g^4))*log(d*e*x/(b*x + a) + c*e/(b*x + a)) - (4*b^3*c^3 - 27*a*b^2*c^2
*d + 108*a^2*b*c*d^2 - 85*a^3*d^3 + 66*(b^3*c*d^2 - a*b^2*d^3)*x^2 - 18*(b^
3*d^3*x^3 + 3*a*b^2*d^3*x^2 + 3*a^2*b*d^3*x + a^3*d^3)*log(b*x + a)^2 - 18*
(b^3*d^3*x^3 + 3*a*b^2*d^3*x^2 + 3*a^2*b*d^3*x + a^3*d^3)*log(d*x + c)^2 -
3*(5*b^3*c^2*d - 54*a*b^2*c*d^2 + 49*a^2*b*d^3)*x + 66*(b^3*d^3*x^3 + 3*a*b
^2*d^3*x^2 + 3*a^2*b*d^3*x + a^3*d^3)*log(b*x + a) - 6*(11*b^3*d^3*x^3 + 33
*a*b^2*d^3*x^2 + 33*a^2*b*d^3*x + 11*a^3*d^3 - 6*(b^3*d^3*x^3 + 3*a*b^2*d^3
*x^2 + 3*a^2*b*d^3*x + a^3*d^3)*log(b*x + a))*log(d*x + c))/(a^3*b^4*c^3*g^
4 - 3*a^4*b^3*c^2*d*g^4 + 3*a^5*b^2*c*d^2*g^4 - a^6*b*d^3*g^4 + (b^7*c^3*g^
4 - 3*a*b^6*c^2*d*g^4 + 3*a^2*b^5*c*d^2*g^4 - a^3*b^4*d^3*g^4)*x^3 + 3*(a*b
^6*c^3*g^4 - 3*a^2*b^5*c^2*d*g^4 + 3*a^3*b^4*c*d^2*g^4 - a^4*b^3*d^3*g^4)*x
^2 + 3*(a^2*b^5*c^3*g^4 - 3*a^3*b^4*c^2*d*g^4 + 3*a^4*b^3*c*d^2*g^4 - a^5*b
^2*d^3*g^4)*x))*B^2 + 1/9*A*B*((6*b^2*d^2*x^2 + 2*b^2*c^2 - 7*a*b*c*d + 11*
a^2*d^2 - 3*(b^2*c*d - 5*a*b*d^2)*x)/((b^6*c^2 - 2*a*b^5*c*d + a^2*b^4*d^2)
*g^4*x^3 + 3*(a*b^5*c^2 - 2*a^2*b^4*c*d + a^3*b^3*d^2)*g^4*x^2 + 3*(a^2*b^4
*c^2 - 2*a^3*b^3*c*d + a^4*b^2*d^2)*g^4*x + (a^3*b^3*c^2 - 2*a^4*b^2*c*d +
a^5*b*d^2)*g^4) - 6*log(d*e*x/(b*x + a) + c*e/(b*x + a))/(b^4*g^4*x^3 + 3*a
*b^3*g^4*x^2 + 3*a^2*b^2*g^4*x + a^3*b*g^4) + 6*d^3*log(b*x + a)/((b^4*c^3
- 3*a*b^3*c^2*d + 3*a^2*b^2*c*d^2 - a^3*b*d^3)*g^4) - 6*d^3*log(d*x + c)/((
b^4*c^3 - 3*a*b^3*c^2*d + 3*a^2*b^2*c*d^2 - a^3*b*d^3)*g^4)) - 1/3*B^2*log(
d*e*x/(b*x + a) + c*e/(b*x + a))^2/(b^4*g^4*x^3 + 3*a*b^3*g^4*x^2 + 3*a^2*b
^2*g^4*x + a^3*b*g^4) - 1/3*A^2/(b^4*g^4*x^3 + 3*a*b^3*g^4*x^2 + 3*a^2*b^2*
g^4*x + a^3*b*g^4)
```

**mupad [B]** time = 7.70, size = 1064, normalized size = 2.67

$$\frac{\frac{18\,A^2\,a^2\,d^2-36\,A^2\,a\,b\,c\,d+18\,A^2\,b^2\,c^2-66\,A\,B\,a^2\,d^2+42\,A\,B\,a\,b\,c\,d-12\,A\,B\,b^2\,c^2+85\,B^2\,a^2\,d^2-23\,B^2\,a\,b\,c\,d+4\,B^2\,b^2\,c^2}{6\,(a\,d-b\,c)}+\frac{x\,\left(-5\,c\,B^2\,b^2\,d+49\,a\,B^2\,b\,d^2+6\,A\right.}{2\,(a\,d-b\,c}}{x\,\left(27\,a^2\,b^3\,c\,g^4-27\,a^3\,b^2\,d\,g^4\right)-x^2\,\left(27\,a^2\,b^3\,d\,g^4-27\,a\,b^4\,c\,g^4\right)+x^3\,\left(9\,b^5\,c\,g^4-9\,a\,b^4\,d\,g^4\right)+}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*log((e*(c + d*x))/(a + b*x)))^2/(a*g + b*g*x)^4,x)
```

```
[Out] ((18*A^2*a^2*d^2 + 18*A^2*b^2*c^2 + 85*B^2*a^2*d^2 + 4*B^2*b^2*c^2 - 66*A*B
*a^2*d^2 - 12*A*B*b^2*c^2 - 36*A^2*a*b*c*d - 23*B^2*a*b*c*d + 42*A*B*a*b*c*
d)/(6*(a*d - b*c)) + (x*(49*B^2*a*b*d^2 - 5*B^2*b^2*c*d - 30*A*B*a*b*d^2 +
6*A*B*b^2*c*d))/(2*(a*d - b*c)) + (d*x^2*(11*B^2*b^2*d - 6*A*B*b^2*d))/(a*d
 - b*c))/(x*(27*a^2*b^3*c*g^4 - 27*a^3*b^2*d*g^4) - x^2*(27*a^2*b^3*d*g^4 -
 27*a*b^4*c*g^4) + x^3*(9*b^5*c*g^4 - 9*a*b^4*d*g^4) + 9*a^3*b^2*c*g^4 - 9*
a^4*b*d*g^4) - log((e*(c + d*x))/(a + b*x))^2*(B^2/(3*b^2*g^4*(3*a^2*x + a^
3/b + b^2*x^3 + 3*a*b*x^2)) - (B^2*d^3)/(3*b*g^4*(a^3*d^3 - b^3*c^3 + 3*a*b
^2*c^2*d - 3*a^2*b*c*d^2))) - (log((e*(c + d*x))/(a + b*x))*((2*A*B)/(3*b^2
*d*g^4) - (2*B^2*d^3*(a*((3*a^2*d^2 + b^2*c^2 - 4*a*b*c*d)/(6*b*d^3) + (a*(
a*d - b*c))/(3*b*d^2)) + (3*a^3*d^3 - b^3*c^3 + 4*a*b^2*c^2*d - 6*a^2*b*c*d
^2)/(3*b*d^4)))/(3*b*g^4*(a^3*d^3 - b^3*c^3 + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2
)) + (2*B^2*d^3*x^2*((b^2*c - a*b*d)/(3*d^2) - (2*b*(a*d - b*c))/(3*d^2)))/
(3*b*g^4*(a^3*d^3 - b^3*c^3 + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2)) - (2*B^2*d^3*
x*(b*((3*a^2*d^2 + b^2*c^2 - 4*a*b*c*d)/(6*b*d^3) + (a*(a*d - b*c))/(3*b*d^
2)) + (3*a^2*d^2 + b^2*c^2 - 4*a*b*c*d)/(3*d^3) + (2*a*(a*d - b*c))/(3*d^2)
))/(3*b*g^4*(a^3*d^3 - b^3*c^3 + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2))))/((3*a^2*
x)/d + a^3/(b*d) + (b^2*x^3)/d + (3*a*b*x^2)/d) - (B*d^3*atan((B*d^3*((b^4*
c^3*g^4 + a^3*b*d^3*g^4 - a*b^3*c^2*d*g^4 - a^2*b^2*c*d^2*g^4)/(b^3*c^2*g^4
 + a^2*b*d^2*g^4 - 2*a*b^2*c*d*g^4) + 2*b*d*x)*(6*A - 11*B)*(b^3*c^2*g^4 +
a^2*b*d^2*g^4 - 2*a*b^2*c*d*g^4)*1i)/(b*g^4*(a*d - b*c)^3*(11*B^2*d^3 - 6*A
*B*d^3)))*(6*A - 11*B)*2i)/(9*b*g^4*(a*d - b*c)^3)
```

**sympy [B]** time = 34.71, size = 1544, normalized size = 3.87

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*ln(e*(d*x+c)/(b*x+a)))**2/(b*g*x+a*g)**4,x)
```

```
[Out] B*d**3*(6*A - 11*B)*log(x + (6*A*B*a*d**4 + 6*A*B*b*c*d**3 - 11*B**2*a*d**4
 - 11*B**2*b*c*d**3 - B*a**4*d**7*(6*A - 11*B)/(a*d - b*c)**3 + 4*B*a**3*b*
c*d**6*(6*A - 11*B)/(a*d - b*c)**3 - 6*B*a**2*b**2*c**2*d**5*(6*A - 11*B)/(
a*d - b*c)**3 + 4*B*a*b**3*c**3*d**4*(6*A - 11*B)/(a*d - b*c)**3 - B*b**4*c
**4*d**3*(6*A - 11*B)/(a*d - b*c)**3)/(12*A*B*b*d**4 - 22*B**2*b*d**4))/(9*
b*g**4*(a*d - b*c)**3) - B*d**3*(6*A - 11*B)*log(x + (6*A*B*a*d**4 + 6*A*B*
b*c*d**3 - 11*B**2*a*d**4 - 11*B**2*b*c*d**3 + B*a**4*d**7*(6*A - 11*B)/(a*
d - b*c)**3 - 4*B*a**3*b*c*d**6*(6*A - 11*B)/(a*d - b*c)**3 + 6*B*a**2*b**2
*c**2*d**5*(6*A - 11*B)/(a*d - b*c)**3 - 4*B*a*b**3*c**3*d**4*(6*A - 11*B)/
(a*d - b*c)**3 + B*b**4*c**4*d**3*(6*A - 11*B)/(a*d - b*c)**3)/(12*A*B*b*d*
*4 - 22*B**2*b*d**4))/(9*b*g**4*(a*d - b*c)**3) + (3*B**2*a**2*c*d**2 + 3*B
```

```
**2*a**2*d**3*x - 3*B**2*a*b*c**2*d + 3*B**2*a*b*d**3*x**2 + B**2*b**2*c**3
 + B**2*b**2*d**3*x**3)*log(e*(c + d*x)/(a + b*x))**2/(3*a**6*d**3*g**4 - 9
*a**5*b*c*d**2*g**4 + 9*a**5*b*d**3*g**4*x + 9*a**4*b**2*c**2*d*g**4 - 27*a
**4*b**2*c*d**2*g**4*x + 9*a**4*b**2*d**3*g**4*x**2 - 3*a**3*b**3*c**3*g**4
 + 27*a**3*b**3*c**2*d*g**4*x - 27*a**3*b**3*c*d**2*g**4*x**2 + 3*a**3*b**3
*d**3*g**4*x**3 - 9*a**2*b**4*c**3*g**4*x + 27*a**2*b**4*c**2*d*g**4*x**2 -
 9*a**2*b**4*c*d**2*g**4*x**3 - 9*a*b**5*c**3*g**4*x**2 + 9*a*b**5*c**2*d*g
**4*x**3 - 3*b**6*c**3*g**4*x**3) + (-6*A*B*a**2*d**2 + 12*A*B*a*b*c*d - 6*
A*B*b**2*c**2 + 11*B**2*a**2*d**2 - 7*B**2*a*b*c*d + 15*B**2*a*b*d**2*x + 2
*B**2*b**2*c**2 - 3*B**2*b**2*c*d*x + 6*B**2*b**2*d**2*x**2)*log(e*(c + d*x
)/(a + b*x))/(9*a**5*b*d**2*g**4 - 18*a**4*b**2*c*d*g**4 + 27*a**4*b**2*d**
2*g**4*x + 9*a**3*b**3*c**2*g**4 - 54*a**3*b**3*c*d*g**4*x + 27*a**3*b**3*d
**2*g**4*x**2 + 27*a**2*b**4*c**2*g**4*x - 54*a**2*b**4*c*d*g**4*x**2 + 9*a
**2*b**4*d**2*g**4*x**3 + 27*a*b**5*c**2*g**4*x**2 - 18*a*b**5*c*d*g**4*x**
3 + 9*b**6*c**2*g**4*x**3) - (18*A**2*a**2*d**2 - 36*A**2*a*b*c*d + 18*A**2
*b**2*c**2 - 66*A*B*a**2*d**2 + 42*A*B*a*b*c*d - 12*A*B*b**2*c**2 + 85*B**2
*a**2*d**2 - 23*B**2*a*b*c*d + 4*B**2*b**2*c**2 + x**2*(-36*A*B*b**2*d**2 +
 66*B**2*b**2*d**2) + x*(-90*A*B*a*b*d**2 + 18*A*B*b**2*c*d + 147*B**2*a*b*
d**2 - 15*B**2*b**2*c*d))/(54*a**5*b*d**2*g**4 - 108*a**4*b**2*c*d*g**4 + 5
4*a**3*b**3*c**2*g**4 + x**3*(54*a**2*b**4*d**2*g**4 - 108*a*b**5*c*d*g**4
+ 54*b**6*c**2*g**4) + x**2*(162*a**3*b**3*d**2*g**4 - 324*a**2*b**4*c*d*g*
*4 + 162*a*b**5*c**2*g**4) + x*(162*a**4*b**2*d**2*g**4 - 324*a**3*b**3*c*d
*g**4 + 162*a**2*b**4*c**2*g**4))
```

## 3.190 $\int \frac{\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2}{(ag+bgx)^5}\,dx$

**Optimal**. Leaf size=498

$$\frac{b^3B(c+dx)^4\left(B\log\left(\frac{e(c+dx)}{a+bx}\right)+A\right)}{8g^5(a+bx)^4(bc-ad)^4}-\frac{2b^2Bd(c+dx)^3\left(B\log\left(\frac{e(c+dx)}{a+bx}\right)+A\right)}{3g^5(a+bx)^3(bc-ad)^4}+\frac{Bd^4\log\left(\frac{c+dx}{a+bx}\right)\left(B\log\left(\frac{e(c+dx)}{a+bx}\right)+A\right)}{2bg^5(bc-ad)^4}$$

```
[Out] 2*B^2*d^3*(d*x+c)/(-a*d+b*c)^4/g^5/(b*x+a)-3/4*b*B^2*d^2*(d*x+c)^2/(-a*d+b*
c)^4/g^5/(b*x+a)^2+2/9*b^2*B^2*d*(d*x+c)^3/(-a*d+b*c)^4/g^5/(b*x+a)^3-1/32*
b^3*B^2*(d*x+c)^4/(-a*d+b*c)^4/g^5/(b*x+a)^4-1/4*B^2*d^4*ln((d*x+c)/(b*x+a)
)^2/b/(-a*d+b*c)^4/g^5-2*B*d^3*(d*x+c)*(A+B*ln(e*(d*x+c)/(b*x+a)))/(-a*d+b*
c)^4/g^5/(b*x+a)+3/2*b*B*d^2*(d*x+c)^2*(A+B*ln(e*(d*x+c)/(b*x+a)))/(-a*d+b*
c)^4/g^5/(b*x+a)^2-2/3*b^2*B*d*(d*x+c)^3*(A+B*ln(e*(d*x+c)/(b*x+a)))/(-a*d+
b*c)^4/g^5/(b*x+a)^3+1/8*b^3*B*(d*x+c)^4*(A+B*ln(e*(d*x+c)/(b*x+a)))/(-a*d+
b*c)^4/g^5/(b*x+a)^4+1/2*B*d^4*ln((d*x+c)/(b*x+a))*(A+B*ln(e*(d*x+c)/(b*x+a
)))/b/(-a*d+b*c)^4/g^5-1/4*(A+B*ln(e*(d*x+c)/(b*x+a)))^2/b/g^5/(b*x+a)^4
```

**Rubi [C]** time = 1.26, antiderivative size = 763, normalized size of antiderivative = 1.53, number of steps used = 38, number of rules used = 11, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.344, Rules used = {2525, 12, 2528, 44, 2524, 2418, 2390, 2301, 2394, 2393, 2391}

$$\frac{B^2d^4\text{PolyLog}\left(2,-\frac{d(a+bx)}{bc-ad}\right)}{2bg^5(bc-ad)^4}+\frac{B^2d^4\text{PolyLog}\left(2,\frac{b(c+dx)}{bc-ad}\right)}{2bg^5(bc-ad)^4}-\frac{Bd^4\log(a+bx)\left(B\log\left(\frac{e(c+dx)}{a+bx}\right)+A\right)}{2bg^5(bc-ad)^4}+\frac{Bd^4\log(c+dx)}{2bg^5}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Log[(e*(c + d*x))/(a + b*x)])^2/(a*g + b*g*x)^5,x]
```

```
[Out] -B^2/(32*b*g^5*(a + b*x)^4) + (7*B^2*d)/(72*b*(b*c - a*d)*g^5*(a + b*x)^3)
- (13*B^2*d^2)/(48*b*(b*c - a*d)^2*g^5*(a + b*x)^2) + (25*B^2*d^3)/(24*b*(b
*c - a*d)^3*g^5*(a + b*x)) + (25*B^2*d^4*Log[a + b*x])/(24*b*(b*c - a*d)^4*
g^5) - (B^2*d^4*Log[a + b*x]^2)/(4*b*(b*c - a*d)^4*g^5) - (25*B^2*d^4*Log[c
 + d*x])/(24*b*(b*c - a*d)^4*g^5) + (B^2*d^4*Log[-((d*(a + b*x))/(b*c - a*d
))]*Log[c + d*x])/(2*b*(b*c - a*d)^4*g^5) - (B^2*d^4*Log[c + d*x]^2)/(4*b*(
b*c - a*d)^4*g^5) + (B^2*d^4*Log[a + b*x]*Log[(b*(c + d*x))/(b*c - a*d)])/(
2*b*(b*c - a*d)^4*g^5) + (B*(A + B*Log[(e*(c + d*x))/(a + b*x)]))/(8*b*g^5*
(a + b*x)^4) - (B*d*(A + B*Log[(e*(c + d*x))/(a + b*x)]))/(6*b*(b*c - a*d)*
g^5*(a + b*x)^3) + (B*d^2*(A + B*Log[(e*(c + d*x))/(a + b*x)]))/(4*b*(b*c -
 a*d)^2*g^5*(a + b*x)^2) - (B*d^3*(A + B*Log[(e*(c + d*x))/(a + b*x)]))/(2*
b*(b*c - a*d)^3*g^5*(a + b*x)) - (B*d^4*Log[a + b*x]*(A + B*Log[(e*(c + d*x
))/(a + b*x)]))/(2*b*(b*c - a*d)^4*g^5) + (B*d^4*Log[c + d*x]*(A + B*Log[(e
```

```
*(c + d*x))/(a + b*x)]))/(2*b*(b*c - a*d)^4*g^5) - (A + B*Log[(e*(c + d*x))
/(a + b*x)])^2/(4*b*g^5*(a + b*x)^4) + (B^2*d^4*PolyLog[2, -((d*(a + b*x))/
(b*c - a*d))])/(2*b*(b*c - a*d)^4*g^5) + (B^2*d^4*PolyLog[2, (b*(c + d*x))/
(b*c - a*d)])/(2*b*(b*c - a*d)^4*g^5)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 2301

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

Rule 2390

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_))^(q_.), x_Symbol] :> Dist[1/e, Subst[Int[((f*x)/d)^q*(a + b*Log[c*x^
n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && E
qQ[e*f - d*g, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2393

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + (c*e*x)/g])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2394

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d + e*x
)^n]))/g, x] - Dist[(b*e*n)/g, Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
```

```
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2418

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*(RFx_), x_Sy
mbol] :> With[{u = ExpandIntegrand[(a + b*Log[c*(d + e*x)^n])^p, RFx, x]},
Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d, e, n}, x] && RationalFunctionQ[
RFx, x] && IntegerQ[p]
```

Rule 2524

```
Int[((a_.) + Log[(c_.)*(RFx_)^(p_.)]*(b_.))^(n_.)/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[(Log[d + e*x]*(a + b*Log[c*RFx^p])^n)/e, x] - Dist[(b*n*p)/e
, Int[(Log[d + e*x]*(a + b*Log[c*RFx^p])^(n - 1)*D[RFx, x])/RFx, x], x] /;
FreeQ[{a, b, c, d, e, p}, x] && RationalFunctionQ[RFx, x] && IGtQ[n, 0]
```

Rule 2525

```
Int[((a_.) + Log[(c_.)*(RFx_)^(p_.)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_))^(m_.
), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(a + b*Log[c*RFx^p])^n)/(e*(m + 1))
, x] - Dist[(b*n*p)/(e*(m + 1)), Int[SimplifyIntegrand[((d + e*x)^(m + 1)*(
a + b*Log[c*RFx^p])^(n - 1)*D[RFx, x])/RFx, x], x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && RationalFunctionQ[RFx, x] && IGtQ[n, 0] && (EqQ[n, 1] ||
IntegerQ[m]) && NeQ[m, -1]
```

Rule 2528

```
Int[((a_.) + Log[(c_.)*(RFx_)^(p_.)]*(b_.))^(n_.)*(RGx_), x_Symbol] :> With
[{u = ExpandIntegrand[(a + b*Log[c*RFx^p])^n, RGx, x]}, Int[u, x] /; SumQ[u
]] /; FreeQ[{a, b, c, p}, x] && RationalFunctionQ[RFx, x] && RationalFuncti
onQ[RGx, x] && IGtQ[n, 0]
```

Rubi steps

$$\int \frac{\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2}{(ag+bgx)^5}\,dx = -\frac{\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2}{4bg^5(a+bx)^4} + \frac{B\int \frac{(bc-ad)\left(-A-B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)}{g^4(a+bx)^5(c+dx)}\,dx}{2bg}$$

$$= -\frac{\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2}{4bg^5(a+bx)^4} + \frac{(B(bc-ad))\int \frac{-A-B\log\left(\frac{e(c+dx)}{a+bx}\right)}{(a+bx)^5(c+dx)}\,dx}{2bg^5}$$

$$= -\frac{\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2}{4bg^5(a+bx)^4} + \frac{(B(bc-ad))\int \left(\frac{b\left(-A-B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)}{(bc-ad)(a+bx)^5} - \frac{bd\left(-A-B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)}{(bc-ad)^2(a+bx)^4}\right.}{}$$

$$= -\frac{\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2}{4bg^5(a+bx)^4} + \frac{B\int \frac{-A-B\log\left(\frac{e(c+dx)}{a+bx}\right)}{(a+bx)^5}\,dx}{2g^5} + \frac{\left(Bd^4\right)\int \frac{-A-B\log\left(\frac{e(c+dx)}{a+bx}\right)}{a+bx}\,dx}{2(bc-ad)^4g^5} -$$

$$= \frac{B\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)}{8bg^5(a+bx)^4} - \frac{Bd\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)}{6b(bc-ad)g^5(a+bx)^3} + \frac{Bd^2\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)}{4b(bc-ad)^2g^5(a+bx)^2} -$$

$$= \frac{B\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)}{8bg^5(a+bx)^4} - \frac{Bd\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)}{6b(bc-ad)g^5(a+bx)^3} + \frac{Bd^2\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)}{4b(bc-ad)^2g^5(a+bx)^2} -$$

$$= \frac{B\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)}{8bg^5(a+bx)^4} - \frac{Bd\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)}{6b(bc-ad)g^5(a+bx)^3} + \frac{Bd^2\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)}{4b(bc-ad)^2g^5(a+bx)^2} -$$

$$= -\frac{B^2}{32bg^5(a+bx)^4} + \frac{7B^2d}{72b(bc-ad)g^5(a+bx)^3} - \frac{13B^2d^2}{48b(bc-ad)^2g^5(a+bx)^2} + \frac{}{24b(bc}$$

$$= -\frac{B^2}{32bg^5(a+bx)^4} + \frac{7B^2d}{72b(bc-ad)g^5(a+bx)^3} - \frac{13B^2d^2}{48b(bc-ad)^2g^5(a+bx)^2} + \frac{}{24b(bc}$$

$$= -\frac{B^2}{32bg^5(a+bx)^4} + \frac{7B^2d}{72b(bc-ad)g^5(a+bx)^3} - \frac{13B^2d^2}{48b(bc-ad)^2g^5(a+bx)^2} + \frac{}{24b(bc}$$

$$= -\frac{B^2}{32bg^5(a+bx)^4} + \frac{7B^2d}{72b(bc-ad)g^5(a+bx)^3} - \frac{13B^2d^2}{48b(bc-ad)^2g^5(a+bx)^2} + \frac{}{24b(bc}$$

**Mathematica [C]** time = 0.92, size = 748, normalized size = 1.50

$$\frac{B\left(-144d^4(a+bx)^4\log(a+bx)\left(B\log\left(\frac{e(c+dx)}{a+bx}\right)+A\right)+144d^4(a+bx)^4\log(c+dx)\left(B\log\left(\frac{e(c+dx)}{a+bx}\right)+A\right)+144d^3(a+bx)^3(ad-bc)\left(B\log\left(\frac{e(c+dx)}{a+bx}\right)+A\right)+72d^2(a+b\ldots\right.}{\ldots}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Log[(e*(c + d*x))/(a + b*x)])^2/(a*g + b*g*x)^5,x]
```

```
[Out] (-72*(A + B*Log[(e*(c + d*x))/(a + b*x)])^2 + (B*(144*B*d^3*(a + b*x)^3*(b*
c - a*d + d*(a + b*x)*Log[a + b*x] - d*(a + b*x)*Log[c + d*x]) - 36*B*d^2*(
a + b*x)^2*((b*c - a*d)^2 + 2*d*(-(b*c) + a*d)*(a + b*x) - 2*d^2*(a + b*x)^
2*Log[a + b*x] + 2*d^2*(a + b*x)^2*Log[c + d*x]) + 8*B*d*(a + b*x)*(2*(b*c
- a*d)^3 - 3*d*(b*c - a*d)^2*(a + b*x) + 6*d^2*(b*c - a*d)*(a + b*x)^2 + 6*
d^3*(a + b*x)^3*Log[a + b*x] - 6*d^3*(a + b*x)^3*Log[c + d*x]) - 3*B*(3*(b*
c - a*d)^4 + 4*d*(-(b*c) + a*d)^3*(a + b*x) + 6*d^2*(b*c - a*d)^2*(a + b*x)
^2 + 12*d^3*(-(b*c) + a*d)*(a + b*x)^3 - 12*d^4*(a + b*x)^4*Log[a + b*x] +
12*d^4*(a + b*x)^4*Log[c + d*x]) + 36*(b*c - a*d)^4*(A + B*Log[(e*(c + d*x)
)/(a + b*x)]) + 48*d*(-(b*c) + a*d)^3*(a + b*x)*(A + B*Log[(e*(c + d*x))/(a
 + b*x)]) + 72*d^2*(b*c - a*d)^2*(a + b*x)^2*(A + B*Log[(e*(c + d*x))/(a +
b*x)]) + 144*d^3*(-(b*c) + a*d)*(a + b*x)^3*(A + B*Log[(e*(c + d*x))/(a + b
*x)]) - 144*d^4*(a + b*x)^4*Log[a + b*x]*(A + B*Log[(e*(c + d*x))/(a + b*x)
]) + 144*d^4*(a + b*x)^4*Log[c + d*x]*(A + B*Log[(e*(c + d*x))/(a + b*x)])
- 72*B*d^4*(a + b*x)^4*(Log[a + b*x]*(Log[a + b*x] - 2*Log[(b*(c + d*x))/(b
*c - a*d)]) - 2*PolyLog[2, (d*(a + b*x))/(-(b*c) + a*d)]) + 72*B*d^4*(a + b
*x)^4*((2*Log[(d*(a + b*x))/(-(b*c) + a*d)] - Log[c + d*x])*Log[c + d*x] +
2*PolyLog[2, (b*(c + d*x))/(b*c - a*d)])))/(b*c - a*d)^4)/(288*b*g^5*(a + b
*x)^4)
```

**fricas [B]** time = 0.60, size = 1045, normalized size = 2.10

$$-\frac{9\left(8A^2-4AB+B^2\right)b^4c^4-32\left(9A^2-6AB+2B^2\right)ab^3c^3d+216\left(2A^2-2AB+B^2\right)a^2b^2c^2d^2-288\left(A^2-2A\ldots\right.}{\ldots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*(d*x+c)/(b*x+a)))^2/(b*g*x+a*g)^5,x, algorithm="fricas
")
```

```
[Out] -1/288*(9*(8*A^2 - 4*A*B + B^2)*b^4*c^4 - 32*(9*A^2 - 6*A*B + 2*B^2)*a*b^3*
c^3*d + 216*(2*A^2 - 2*A*B + B^2)*a^2*b^2*c^2*d^2 - 288*(A^2 - 2*A*B + 2*B^
2)*a^3*b*c*d^3 + (72*A^2 - 300*A*B + 415*B^2)*a^4*d^4 + 12*((12*A*B - 25*B^
2)*b^4*c*d^3 - (12*A*B - 25*B^2)*a*b^3*d^4)*x^3 - 6*((12*A*B - 13*B^2)*b^4*
c^2*d^2 - 16*(6*A*B - 11*B^2)*a*b^3*c*d^3 + (84*A*B - 163*B^2)*a^2*b^2*d^4)
```

```
*x^2 - 72*(B^2*b^4*d^4*x^4 + 4*B^2*a*b^3*d^4*x^3 + 6*B^2*a^2*b^2*d^4*x^2 +
4*B^2*a^3*b*d^4*x - B^2*b^4*c^4 + 4*B^2*a*b^3*c^3*d - 6*B^2*a^2*b^2*c^2*d^2
 + 4*B^2*a^3*b*c*d^3)*log((d*e*x + c*e)/(b*x + a))^2 + 4*((12*A*B - 7*B^2)*
b^4*c^3*d - 12*(6*A*B - 5*B^2)*a*b^3*c^2*d^2 + 108*(2*A*B - 3*B^2)*a^2*b^2*
c*d^3 - (156*A*B - 271*B^2)*a^3*b*d^4)*x - 12*((12*A*B - 25*B^2)*b^4*d^4*x^
4 - 3*(4*A*B - B^2)*b^4*c^4 + 16*(3*A*B - B^2)*a*b^3*c^3*d - 36*(2*A*B - B^
2)*a^2*b^2*c^2*d^2 + 48*(A*B - B^2)*a^3*b*c*d^3 - 4*(3*B^2*b^4*c*d^3 - 2*(6
*A*B - 11*B^2)*a*b^3*d^4)*x^3 + 6*(B^2*b^4*c^2*d^2 - 8*B^2*a*b^3*c*d^3 + 6*
(2*A*B - 3*B^2)*a^2*b^2*d^4)*x^2 - 4*(B^2*b^4*c^3*d - 6*B^2*a*b^3*c^2*d^2 +
 18*B^2*a^2*b^2*c*d^3 - 12*(A*B - B^2)*a^3*b*d^4)*x)*log((d*e*x + c*e)/(b*x
 + a)))/((b^9*c^4 - 4*a*b^8*c^3*d + 6*a^2*b^7*c^2*d^2 - 4*a^3*b^6*c*d^3 + a
^4*b^5*d^4)*g^5*x^4 + 4*(a*b^8*c^4 - 4*a^2*b^7*c^3*d + 6*a^3*b^6*c^2*d^2 -
4*a^4*b^5*c*d^3 + a^5*b^4*d^4)*g^5*x^3 + 6*(a^2*b^7*c^4 - 4*a^3*b^6*c^3*d +
 6*a^4*b^5*c^2*d^2 - 4*a^5*b^4*c*d^3 + a^6*b^3*d^4)*g^5*x^2 + 4*(a^3*b^6*c^
4 - 4*a^4*b^5*c^3*d + 6*a^5*b^4*c^2*d^2 - 4*a^6*b^3*c*d^3 + a^7*b^2*d^4)*g^
5*x + (a^4*b^5*c^4 - 4*a^5*b^4*c^3*d + 6*a^6*b^3*c^2*d^2 - 4*a^7*b^2*c*d^3
+ a^8*b*d^4)*g^5)
```

**giac [B]** time = 2.34, size = 1029, normalized size = 2.07

$$\left(\frac{288\,(dxe+ce)B^2d^3e^3\log\left(\frac{dxe+ce}{bx+a}\right)^2}{bx+a}-\frac{432\,(dxe+ce)^2B^2bd^2e^2\log\left(\frac{dxe+ce}{bx+a}\right)^2}{(bx+a)^2}+\frac{288\,(dxe+ce)^3B^2b^2de\log\left(\frac{dxe+ce}{bx+a}\right)^2}{(bx+a)^3}+\frac{576\,(dxe+ce)ABd^3e^3\log\left(\frac{dxe+ce}{bx+a}\right)}{bx+a}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*(d*x+c)/(b*x+a)))^2/(b*g*x+a*g)^5,x, algorithm="giac")

[Out] 1/288*(288*(d*x*e + c*e)*B^2*d^3*e^3*log((d*x*e + c*e)/(b*x + a))^2/(b*x +
a) - 432*(d*x*e + c*e)^2*B^2*b*d^2*e^2*log((d*x*e + c*e)/(b*x + a))^2/(b*x
+ a)^2 + 288*(d*x*e + c*e)^3*B^2*b^2*d*e*log((d*x*e + c*e)/(b*x + a))^2/(b*
x + a)^3 + 576*(d*x*e + c*e)*A*B*d^3*e^3*log((d*x*e + c*e)/(b*x + a))/(b*x
+ a) - 576*(d*x*e + c*e)*B^2*d^3*e^3*log((d*x*e + c*e)/(b*x + a))/(b*x + a)
 - 864*(d*x*e + c*e)^2*A*B*b*d^2*e^2*log((d*x*e + c*e)/(b*x + a))/(b*x + a)
^2 + 432*(d*x*e + c*e)^2*B^2*b*d^2*e^2*log((d*x*e + c*e)/(b*x + a))/(b*x +
a)^2 + 576*(d*x*e + c*e)^3*A*B*b^2*d*e*log((d*x*e + c*e)/(b*x + a))/(b*x +
a)^3 - 192*(d*x*e + c*e)^3*B^2*b^2*d*e*log((d*x*e + c*e)/(b*x + a))/(b*x +
a)^3 - 72*(d*x*e + c*e)^4*B^2*b^3*log((d*x*e + c*e)/(b*x + a))^2/(b*x + a)^
4 + 288*(d*x*e + c*e)*A^2*d^3*e^3/(b*x + a) - 576*(d*x*e + c*e)*A*B*d^3*e^3
/(b*x + a) + 576*(d*x*e + c*e)*B^2*d^3*e^3/(b*x + a) - 432*(d*x*e + c*e)^2*
A^2*b*d^2*e^2/(b*x + a)^2 + 432*(d*x*e + c*e)^2*A*B*b*d^2*e^2/(b*x + a)^2 -
 216*(d*x*e + c*e)^2*B^2*b*d^2*e^2/(b*x + a)^2 + 288*(d*x*e + c*e)^3*A^2*b^
2*d*e/(b*x + a)^3 - 192*(d*x*e + c*e)^3*A*B*b^2*d*e/(b*x + a)^3 + 64*(d*x*e
 + c*e)^3*B^2*b^2*d*e/(b*x + a)^3 - 144*(d*x*e + c*e)^4*A*B*b^3*log((d*x*e
```

```
+ c*e)/(b*x + a))/(b*x + a)^4 + 36*(d*x*e + c*e)^4*B^2*b^3*log((d*x*e + c*e
)/(b*x + a))/(b*x + a)^4 - 72*(d*x*e + c*e)^4*A^2*b^3/(b*x + a)^4 + 36*(d*x
*e + c*e)^4*A*B*b^3/(b*x + a)^4 - 9*(d*x*e + c*e)^4*B^2*b^3/(b*x + a)^4)*(b
*c/((b*c*e - a*d*e)*(b*c - a*d)) - a*d/((b*c*e - a*d*e)*(b*c - a*d)))/(b^3*
c^3*g^5*e^3 - 3*a*b^2*c^2*d*g^5*e^3 + 3*a^2*b*c*d^2*g^5*e^3 - a^3*d^3*g^5*e
^3)
```

**maple [B]** time = 0.05, size = 3717, normalized size = 7.46

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*ln((d*x+c)/(b*x+a)*e)+A)^2/(b*g*x+a*g)^5,x)
```

```
[Out] -1/4/(a*d-b*c)^5/g^5*B^2*ln(1/b*d*e-(a*d-b*c)/(b*x+a)/b*e)^2*d^4*c+25/24/(a
*d-b*c)^5/g^5*B^2*ln(1/b*d*e-(a*d-b*c)/(b*x+a)/b*e)*d^4*c+1/32*b^4/(a*d-b*c
)^5/g^5*B^2/(b*x+a)^4*c^5+1/4*b^4/(a*d-b*c)^5/g^5*A^2/(b*x+a)^4*c^5-5/8/(a*
d-b*c)^5/g^5*B^2*ln(1/b*d*e-(a*d-b*c)/(b*x+a)/b*e)/(b*x+a)^4*a^4*d^4*c-7/72
/b/(a*d-b*c)^5/g^5*B^2*d^5/(b*x+a)^3*a^4-1/8*b^4/(a*d-b*c)^5/g^5*A*B/(b*x+a
)^4*c^5-1/4/b/(a*d-b*c)^5/g^5*A^2/(b*x+a)^4*a^5*d^5-1/4/(a*d-b*c)^5/g^5*A^2
*d^4*c+1/4/b/(a*d-b*c)^5/g^5*A^2*d^5*a+415/288/b/(a*d-b*c)^5/g^5*B^2*d^5*a-
2/3/(a*d-b*c)^5/g^5*B^2*ln(1/b*d*e-(a*d-b*c)/(b*x+a)/b*e)*d^4/(b*x+a)^3*a^3
*c-13/16*b/(a*d-b*c)^5/g^5*B^2*d^3/(b*x+a)^2*c^2*a-7/12*b/(a*d-b*c)^5/g^5*B
^2*d^3/(b*x+a)^3*a^2*c^2+5/16*b^2/(a*d-b*c)^5/g^5*B^2/(b*x+a)^4*a^2*d^2*c^3
-5/32*b^3/(a*d-b*c)^5/g^5*B^2/(b*x+a)^4*a*d*c^4+1/4/b/(a*d-b*c)^5/g^5*A*B*d
^5/(b*x+a)^2*a^3+1/2/b/(a*d-b*c)^5/g^5*A*B*d^5/(b*x+a)*a^2+1/8/b/(a*d-b*c)^
5/g^5*A*B/(b*x+a)^4*a^5*d^5-1/4*b^2/(a*d-b*c)^5/g^5*A*B*d^2/(b*x+a)^2*c^3-5
/16*b/(a*d-b*c)^5/g^5*B^2/(b*x+a)^4*a^3*d^3*c^2+1/2*b/(a*d-b*c)^5/g^5*A*B*d
^3/(b*x+a)*c^2-3/4/(a*d-b*c)^5/g^5*A*B*d^4/(b*x+a)^2*a^2*c-2/3/(a*d-b*c)^5/
g^5*A*B*d^4/(b*x+a)^3*a^3*c-415/288/(a*d-b*c)^5/g^5*B^2*d^4*c+25/24/(a*d-b*
c)^5/g^5*A*B*d^4*c-25/24/b/(a*d-b*c)^5/g^5*A*B*d^5*a-3/4/(a*d-b*c)^5/g^5*B^
2*ln(1/b*d*e-(a*d-b*c)/(b*x+a)/b*e)*d^4/(b*x+a)^2*a^2*c-1/(a*d-b*c)^5/g^5*A
*B*d^4/(b*x+a)*a*c+5/2*b^2/(a*d-b*c)^5/g^5*B^2*ln(1/b*d*e-(a*d-b*c)/(b*x+a)
/b*e)^2/(b*x+a)^4*a^2*d^2*c^3+5/2/(a*d-b*c)^5/g^5*A*B*ln(1/b*d*e-(a*d-b*c)/
(b*x+a)/b*e)/(b*x+a)^4*a^4*d^4*c+5/8*b^3/(a*d-b*c)^5/g^5*B^2*ln(1/b*d*e-(a*
d-b*c)/(b*x+a)/b*e)/(b*x+a)^4*a*d*c^4-5/4*b^2/(a*d-b*c)^5/g^5*B^2*ln(1/b*d*
e-(a*d-b*c)/(b*x+a)/b*e)/(b*x+a)^4*a^2*d^2*c^3+3/4*b/(a*d-b*c)^5/g^5*A*B*d^
3/(b*x+a)^2*a*c^2-5/2*b^3/(a*d-b*c)^5/g^5*A*B*ln(1/b*d*e-(a*d-b*c)/(b*x+a)/
b*e)/(b*x+a)^4*a*d*c^4+5/32/(a*d-b*c)^5/g^5*B^2/(b*x+a)^4*a^4*d^4*c+7/18/(a
*d-b*c)^5/g^5*B^2*d^4/(b*x+a)^3*a^3*c+7/18*b^2/(a*d-b*c)^5/g^5*B^2*d^2/(b*x
+a)^3*c^3*a-5/4*b^3/(a*d-b*c)^5/g^5*A^2/(b*x+a)^4*a*d*c^4+1/6*b^3/(a*d-b*c)
^5/g^5*A*B*d/(b*x+a)^3*c^4+1/6/b/(a*d-b*c)^5/g^5*A*B*d^5/(b*x+a)^3*a^4+5/2*
b^2/(a*d-b*c)^5/g^5*A^2/(b*x+a)^4*a^2*d^2*c^3-5/8/(a*d-b*c)^5/g^5*A*B/(b*x+
a)^4*a^4*d^4*c-5/2*b/(a*d-b*c)^5/g^5*A^2/(b*x+a)^4*a^3*d^3*c^2-1/4*b^2/(a*d
-b*c)^5/g^5*B^2*ln(1/b*d*e-(a*d-b*c)/(b*x+a)/b*e)*d^2/(b*x+a)^2*c^3-1/4/b/(
```

```
a*d-b*c)^5/g^5*B^2*ln(1/b*d*e-(a*d-b*c)/(b*x+a)/b*e)^2/(b*x+a)^4*a^5*d^5+1/
8/b/(a*d-b*c)^5/g^5*B^2*ln(1/b*d*e-(a*d-b*c)/(b*x+a)/b*e)/(b*x+a)^4*a^5*d^5
-5*b/(a*d-b*c)^5/g^5*A*B*ln(1/b*d*e-(a*d-b*c)/(b*x+a)/b*e)/(b*x+a)^4*a^3*d^
3*c^2+5*b^2/(a*d-b*c)^5/g^5*A*B*ln(1/b*d*e-(a*d-b*c)/(b*x+a)/b*e)/(b*x+a)^4
*a^2*d^2*c^3+b/(a*d-b*c)^5/g^5*A*B*d^3/(b*x+a)^3*a^2*c^2+3/4*b/(a*d-b*c)^5/
g^5*B^2*ln(1/b*d*e-(a*d-b*c)/(b*x+a)/b*e)*d^3/(b*x+a)^2*a*c^2+b/(a*d-b*c)^5
/g^5*B^2*ln(1/b*d*e-(a*d-b*c)/(b*x+a)/b*e)*d^3/(b*x+a)^3*a^2*c^2-1/2/b/(a*d
-b*c)^5/g^5*A*B*ln(1/b*d*e-(a*d-b*c)/(b*x+a)/b*e)/(b*x+a)^4*a^5*d^5+5/4/(a*
d-b*c)^5/g^5*B^2*ln(1/b*d*e-(a*d-b*c)/(b*x+a)/b*e)^2/(b*x+a)^4*a^4*d^4*c+1/
2/b/(a*d-b*c)^5/g^5*A*B*ln(1/b*d*e-(a*d-b*c)/(b*x+a)/b*e)*d^5*a-1/(a*d-b*c)
^5/g^5*B^2*ln(1/b*d*e-(a*d-b*c)/(b*x+a)/b*e)*d^4/(b*x+a)*c*a+1/2*b^4/(a*d-b
*c)^5/g^5*A*B*ln(1/b*d*e-(a*d-b*c)/(b*x+a)/b*e)/(b*x+a)^4*c^5+1/6/b/(a*d-b*
c)^5/g^5*B^2*ln(1/b*d*e-(a*d-b*c)/(b*x+a)/b*e)*d^5/(b*x+a)^3*a^4+1/2*b/(a*d
-b*c)^5/g^5*B^2*ln(1/b*d*e-(a*d-b*c)/(b*x+a)/b*e)*d^3/(b*x+a)*c^2+1/4/b/(a*
d-b*c)^5/g^5*B^2*ln(1/b*d*e-(a*d-b*c)/(b*x+a)/b*e)*d^5/(b*x+a)^2*a^3+1/6*b^
3/(a*d-b*c)^5/g^5*B^2*ln(1/b*d*e-(a*d-b*c)/(b*x+a)/b*e)*d/(b*x+a)^3*c^4+1/2
/b/(a*d-b*c)^5/g^5*B^2*ln(1/b*d*e-(a*d-b*c)/(b*x+a)/b*e)*d^5/(b*x+a)*a^2-25
/24/b/(a*d-b*c)^5/g^5*B^2*d^5/(b*x+a)*a^2-13/48/b/(a*d-b*c)^5/g^5*B^2*d^5/(
b*x+a)^2*a^3+13/48*b^2/(a*d-b*c)^5/g^5*B^2*d^2/(b*x+a)^2*c^3+5/4/(a*d-b*c)^
5/g^5*A^2/(b*x+a)^4*a^4*d^4*c+25/12/(a*d-b*c)^5/g^5*B^2*d^4/(b*x+a)*a*c+13/
16/(a*d-b*c)^5/g^5*B^2*d^4/(b*x+a)^2*a^2*c-25/24/b/(a*d-b*c)^5/g^5*B^2*ln(1
/b*d*e-(a*d-b*c)/(b*x+a)/b*e)*d^5*a+1/4*b^4/(a*d-b*c)^5/g^5*B^2*ln(1/b*d*e-
(a*d-b*c)/(b*x+a)/b*e)^2/(b*x+a)^4*c^5-1/2/(a*d-b*c)^5/g^5*A*B*ln(1/b*d*e-(
a*d-b*c)/(b*x+a)/b*e)*d^4*c-5/4*b^3/(a*d-b*c)^5/g^5*B^2*ln(1/b*d*e-(a*d-b*c
)/(b*x+a)/b*e)^2/(b*x+a)^4*c^4*a*d-5/2*b/(a*d-b*c)^5/g^5*B^2*ln(1/b*d*e-(a*
d-b*c)/(b*x+a)/b*e)^2/(b*x+a)^4*a^3*d^3*c^2-2/3*b^2/(a*d-b*c)^5/g^5*A*B*d^2
/(b*x+a)^3*a*c^3-25/24*b/(a*d-b*c)^5/g^5*B^2*d^3/(b*x+a)*c^2-1/8*b^4/(a*d-b
*c)^5/g^5*B^2*ln(1/b*d*e-(a*d-b*c)/(b*x+a)/b*e)/(b*x+a)^4*c^5+1/4/b/(a*d-b*
c)^5/g^5*B^2*ln(1/b*d*e-(a*d-b*c)/(b*x+a)/b*e)^2*d^5*a-7/72*b^3/(a*d-b*c)^5
/g^5*B^2*d/(b*x+a)^3*c^4-1/32/b/(a*d-b*c)^5/g^5*B^2/(b*x+a)^4*a^5*d^5-5/4*b
^2/(a*d-b*c)^5/g^5*A*B/(b*x+a)^4*a^2*d^2*c^3-2/3*b^2/(a*d-b*c)^5/g^5*B^2*ln
(1/b*d*e-(a*d-b*c)/(b*x+a)/b*e)*d^2/(b*x+a)^3*a*c^3+5/4*b/(a*d-b*c)^5/g^5*B
^2*ln(1/b*d*e-(a*d-b*c)/(b*x+a)/b*e)/(b*x+a)^4*a^3*d^3*c^2+5/8*b^3/(a*d-b*c
)^5/g^5*A*B/(b*x+a)^4*a*d*c^4+5/4*b/(a*d-b*c)^5/g^5*A*B/(b*x+a)^4*a^3*d^3*c
^2
```

**maxima [B]** time = 2.91, size = 2122, normalized size = 4.26

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*(d*x+c)/(b*x+a)))^2/(b*g*x+a*g)^5,x, algorithm="maxima
")
```

```
[Out] -1/288*(12*((12*b^3*d^3*x^3 - 3*b^3*c^3 + 13*a*b^2*c^2*d - 23*a^2*b*c*d^2 +
```

```
 25*a^3*d^3 - 6*(b^3*c*d^2 - 7*a*b^2*d^3)*x^2 + 4*(b^3*c^2*d - 5*a*b^2*c*d^
2 + 13*a^2*b*d^3)*x)/((b^8*c^3 - 3*a*b^7*c^2*d + 3*a^2*b^6*c*d^2 - a^3*b^5*
d^3)*g^5*x^4 + 4*(a*b^7*c^3 - 3*a^2*b^6*c^2*d + 3*a^3*b^5*c*d^2 - a^4*b^4*d
^3)*g^5*x^3 + 6*(a^2*b^6*c^3 - 3*a^3*b^5*c^2*d + 3*a^4*b^4*c*d^2 - a^5*b^3*
d^3)*g^5*x^2 + 4*(a^3*b^5*c^3 - 3*a^4*b^4*c^2*d + 3*a^5*b^3*c*d^2 - a^6*b^2
*d^3)*g^5*x + (a^4*b^4*c^3 - 3*a^5*b^3*c^2*d + 3*a^6*b^2*c*d^2 - a^7*b*d^3)
*g^5) + 12*d^4*log(b*x + a)/((b^5*c^4 - 4*a*b^4*c^3*d + 6*a^2*b^3*c^2*d^2 -
 4*a^3*b^2*c*d^3 + a^4*b*d^4)*g^5) - 12*d^4*log(d*x + c)/((b^5*c^4 - 4*a*b^
4*c^3*d + 6*a^2*b^3*c^2*d^2 - 4*a^3*b^2*c*d^3 + a^4*b*d^4)*g^5))*log(d*e*x/
(b*x + a) + c*e/(b*x + a)) + (9*b^4*c^4 - 64*a*b^3*c^3*d + 216*a^2*b^2*c^2*
d^2 - 576*a^3*b*c*d^3 + 415*a^4*d^4 - 300*(b^4*c*d^3 - a*b^3*d^4)*x^3 + 6*(
13*b^4*c^2*d^2 - 176*a*b^3*c*d^3 + 163*a^2*b^2*d^4)*x^2 + 72*(b^4*d^4*x^4 +
 4*a*b^3*d^4*x^3 + 6*a^2*b^2*d^4*x^2 + 4*a^3*b*d^4*x + a^4*d^4)*log(b*x + a
)^2 + 72*(b^4*d^4*x^4 + 4*a*b^3*d^4*x^3 + 6*a^2*b^2*d^4*x^2 + 4*a^3*b*d^4*x
 + a^4*d^4)*log(d*x + c)^2 - 4*(7*b^4*c^3*d - 60*a*b^3*c^2*d^2 + 324*a^2*b^
2*c*d^3 - 271*a^3*b*d^4)*x - 300*(b^4*d^4*x^4 + 4*a*b^3*d^4*x^3 + 6*a^2*b^2
*d^4*x^2 + 4*a^3*b*d^4*x + a^4*d^4)*log(b*x + a) + 12*(25*b^4*d^4*x^4 + 100
*a*b^3*d^4*x^3 + 150*a^2*b^2*d^4*x^2 + 100*a^3*b*d^4*x + 25*a^4*d^4 - 12*(b
^4*d^4*x^4 + 4*a*b^3*d^4*x^3 + 6*a^2*b^2*d^4*x^2 + 4*a^3*b*d^4*x + a^4*d^4)
*log(b*x + a))*log(d*x + c))/(a^4*b^5*c^4*g^5 - 4*a^5*b^4*c^3*d*g^5 + 6*a^6
*b^3*c^2*d^2*g^5 - 4*a^7*b^2*c*d^3*g^5 + a^8*b*d^4*g^5 + (b^9*c^4*g^5 - 4*a
*b^8*c^3*d*g^5 + 6*a^2*b^7*c^2*d^2*g^5 - 4*a^3*b^6*c*d^3*g^5 + a^4*b^5*d^4*
g^5)*x^4 + 4*(a*b^8*c^4*g^5 - 4*a^2*b^7*c^3*d*g^5 + 6*a^3*b^6*c^2*d^2*g^5 -
 4*a^4*b^5*c*d^3*g^5 + a^5*b^4*d^4*g^5)*x^3 + 6*(a^2*b^7*c^4*g^5 - 4*a^3*b^
6*c^3*d*g^5 + 6*a^4*b^5*c^2*d^2*g^5 - 4*a^5*b^4*c*d^3*g^5 + a^6*b^3*d^4*g^5
)*x^2 + 4*(a^3*b^6*c^4*g^5 - 4*a^4*b^5*c^3*d*g^5 + 6*a^5*b^4*c^2*d^2*g^5 -
4*a^6*b^3*c*d^3*g^5 + a^7*b^2*d^4*g^5)*x))*B^2 - 1/24*A*B*((12*b^3*d^3*x^3
- 3*b^3*c^3 + 13*a*b^2*c^2*d - 23*a^2*b*c*d^2 + 25*a^3*d^3 - 6*(b^3*c*d^2 -
 7*a*b^2*d^3)*x^2 + 4*(b^3*c^2*d - 5*a*b^2*c*d^2 + 13*a^2*b*d^3)*x)/((b^8*c
^3 - 3*a*b^7*c^2*d + 3*a^2*b^6*c*d^2 - a^3*b^5*d^3)*g^5*x^4 + 4*(a*b^7*c^3
- 3*a^2*b^6*c^2*d + 3*a^3*b^5*c*d^2 - a^4*b^4*d^3)*g^5*x^3 + 6*(a^2*b^6*c^3
 - 3*a^3*b^5*c^2*d + 3*a^4*b^4*c*d^2 - a^5*b^3*d^3)*g^5*x^2 + 4*(a^3*b^5*c^
3 - 3*a^4*b^4*c^2*d + 3*a^5*b^3*c*d^2 - a^6*b^2*d^3)*g^5*x + (a^4*b^4*c^3 -
 3*a^5*b^3*c^2*d + 3*a^6*b^2*c*d^2 - a^7*b*d^3)*g^5) + 12*log(d*e*x/(b*x +
a) + c*e/(b*x + a))/(b^5*g^5*x^4 + 4*a*b^4*g^5*x^3 + 6*a^2*b^3*g^5*x^2 + 4*
a^3*b^2*g^5*x + a^4*b*g^5) + 12*d^4*log(b*x + a)/((b^5*c^4 - 4*a*b^4*c^3*d
+ 6*a^2*b^3*c^2*d^2 - 4*a^3*b^2*c*d^3 + a^4*b*d^4)*g^5) - 12*d^4*log(d*x +
c)/((b^5*c^4 - 4*a*b^4*c^3*d + 6*a^2*b^3*c^2*d^2 - 4*a^3*b^2*c*d^3 + a^4*b*
d^4)*g^5)) - 1/4*B^2*log(d*e*x/(b*x + a) + c*e/(b*x + a))^2/(b^5*g^5*x^4 +
4*a*b^4*g^5*x^3 + 6*a^2*b^3*g^5*x^2 + 4*a^3*b^2*g^5*x + a^4*b*g^5) - 1/4*A^
2/(b^5*g^5*x^4 + 4*a*b^4*g^5*x^3 + 6*a^2*b^3*g^5*x^2 + 4*a^3*b^2*g^5*x + a^
4*b*g^5)
```

**mupad [B]** time = 10.94, size = 1880, normalized size = 3.78

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*log((e*(c + d*x))/(a + b*x)))^2/(a*g + b*g*x)^5,x)
```

```
[Out] (log((e*(c + d*x))/(a + b*x))*((B^2*d^4*(a*(a*((4*a^2*d^2 + b^2*c^2 - 5*a*b
*c*d)/(12*b*d^3) + (a*(a*d - b*c))/(4*b*d^2)) + (6*a^3*d^3 - b^3*c^3 + 5*a*
b^2*c^2*d - 10*a^2*b*c*d^2)/(12*b*d^4)) + (4*a^4*d^4 + b^4*c^4 + 10*a^2*b^2
*c^2*d^2 - 5*a*b^3*c^3*d - 10*a^3*b*c*d^3)/(4*b*d^5)))/(2*b*g^5*(a^4*d^4 +
b^4*c^4 + 6*a^2*b^2*c^2*d^2 - 4*a*b^3*c^3*d - 4*a^3*b*c*d^3)) - (A*B)/(2*b^
2*d*g^5) + (B^2*d^4*x^2*(b*(b*((4*a^2*d^2 + b^2*c^2 - 5*a*b*c*d)/(12*b*d^3)
 + (a*(a*d - b*c))/(4*b*d^2)) + (4*a^2*d^2 + b^2*c^2 - 5*a*b*c*d)/(6*d^3) +
 (a*(a*d - b*c))/(2*d^2)) - a*((b^2*c - a*b*d)/(4*d^2) - (b*(a*d - b*c))/(2
*d^2)) + (b^3*c^2 + 4*a^2*b*d^2 - 5*a*b^2*c*d)/(4*d^3)))/(2*b*g^5*(a^4*d^4
+ b^4*c^4 + 6*a^2*b^2*c^2*d^2 - 4*a*b^3*c^3*d - 4*a^3*b*c*d^3)) - (B^2*d^4*
x^3*(b*((b^2*c - a*b*d)/(4*d^2) - (b*(a*d - b*c))/(2*d^2)) + (b^3*c - a*b^2
*d)/(4*d^2)))/(2*b*g^5*(a^4*d^4 + b^4*c^4 + 6*a^2*b^2*c^2*d^2 - 4*a*b^3*c^3
*d - 4*a^3*b*c*d^3)) + (B^2*d^4*x*(b*(a*((4*a^2*d^2 + b^2*c^2 - 5*a*b*c*d)/
(12*b*d^3) + (a*(a*d - b*c))/(4*b*d^2)) + (6*a^3*d^3 - b^3*c^3 + 5*a*b^2*c^
2*d - 10*a^2*b*c*d^2)/(12*b*d^4)) + a*(b*((4*a^2*d^2 + b^2*c^2 - 5*a*b*c*d)
/(12*b*d^3) + (a*(a*d - b*c))/(4*b*d^2)) + (4*a^2*d^2 + b^2*c^2 - 5*a*b*c*d
)/(6*d^3) + (a*(a*d - b*c))/(2*d^2)) + (6*a^3*d^3 - b^3*c^3 + 5*a*b^2*c^2*d
 - 10*a^2*b*c*d^2)/(4*d^4)))/(2*b*g^5*(a^4*d^4 + b^4*c^4 + 6*a^2*b^2*c^2*d^
2 - 4*a*b^3*c^3*d - 4*a^3*b*c*d^3))))/((4*a^3*x)/d + a^4/(b*d) + (b^3*x^4)/
d + (6*a^2*b*x^2)/d + (4*a*b^2*x^3)/d) - log((e*(c + d*x))/(a + b*x))^2*(B^
2/(4*b^2*g^5*(4*a^3*x + a^4/b + b^3*x^4 + 6*a^2*b*x^2 + 4*a*b^2*x^3)) - (B^
2*d^4)/(4*b*g^5*(a^4*d^4 + b^4*c^4 + 6*a^2*b^2*c^2*d^2 - 4*a*b^3*c^3*d - 4*
a^3*b*c*d^3))) - ((72*A^2*a^3*d^3 - 72*A^2*b^3*c^3 + 415*B^2*a^3*d^3 - 9*B^
2*b^3*c^3 - 300*A*B*a^3*d^3 + 36*A*B*b^3*c^3 + 216*A^2*a*b^2*c^2*d - 216*A^
2*a^2*b*c*d^2 + 55*B^2*a*b^2*c^2*d - 161*B^2*a^2*b*c*d^2 - 156*A*B*a*b^2*c^
2*d + 276*A*B*a^2*b*c*d^2)/(12*(a*d - b*c)) + (x^2*(163*B^2*a*b^2*d^3 - 13*
B^2*b^3*c*d^2 - 84*A*B*a*b^2*d^3 + 12*A*B*b^3*c*d^2))/(2*(a*d - b*c)) + (x*
(271*B^2*a^2*b*d^3 + 7*B^2*b^3*c^2*d - 53*B^2*a*b^2*c*d^2 - 156*A*B*a^2*b*d
^3 - 12*A*B*b^3*c^2*d + 60*A*B*a*b^2*c*d^2))/(3*(a*d - b*c)) + (d*x^3*(25*B
^2*b^3*d^2 - 12*A*B*b^3*d^2))/(a*d - b*c))/(x*(96*a^3*b^4*c^2*g^5 + 96*a^5*
b^2*d^2*g^5 - 192*a^4*b^3*c*d*g^5) + x^3*(96*a*b^6*c^2*g^5 + 96*a^3*b^4*d^2
*g^5 - 192*a^2*b^5*c*d*g^5) + x^4*(24*b^7*c^2*g^5 + 24*a^2*b^5*d^2*g^5 - 48
*a*b^6*c*d*g^5) + x^2*(144*a^2*b^5*c^2*g^5 + 144*a^4*b^3*d^2*g^5 - 288*a^3*
b^4*c*d*g^5) + 24*a^6*b*d^2*g^5 + 24*a^4*b^3*c^2*g^5 - 48*a^5*b^2*c*d*g^5)
+ (B*d^4*atan((B*d^4*(12*A - 25*B)*(24*b^5*c^4*g^5 - 24*a^4*b*d^4*g^5 - 48*
a*b^4*c^3*d*g^5 + 48*a^3*b^2*c*d^3*g^5)*1i)/(24*b*g^5*(a*d - b*c)^4*(25*B^2
*d^4 - 12*A*B*d^4)) + (B*d^5*x*(12*A - 25*B)*(b^4*c^3*g^5 - a^3*b*d^3*g^5 -
 3*a*b^3*c^2*d*g^5 + 3*a^2*b^2*c*d^2*g^5)*2i)/(g^5*(a*d - b*c)^4*(25*B^2*d^
4 - 12*A*B*d^4)))*(12*A - 25*B)*1i)/(12*b*g^5*(a*d - b*c)^4)
```

**sympy** [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*ln(e*(d*x+c)/(b*x+a)))**2/(b*g*x+a*g)**5,x)
```

```
[Out] Timed out
```

## 3.191 $\int \frac{(ag+bgx)^2}{A+B\log\left(\frac{e(c+dx)}{a+bx}\right)}\,dx$

**Optimal**. Leaf size=35

$$\text{Int}\left(\frac{(ag+bgx)^2}{B\log\left(\frac{e(c+dx)}{a+bx}\right)+A},x\right)$$

```
[Out] Unintegrable((b*g*x+a*g)^2/(A+B*ln(e*(d*x+c)/(b*x+a))),x)
```

**Rubi [A]** time = 0.20, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{(ag+bgx)^2}{A+B\log\left(\frac{e(c+dx)}{a+bx}\right)}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(a*g + b*g*x)^2/(A + B*Log[(e*(c + d*x))/(a + b*x)]),x]
```

```
[Out] a^2*g^2*Defer[Int][(A + B*Log[(e*(c + d*x))/(a + b*x)])^(-1), x] + 2*a*b*g^
2*Defer[Int][x/(A + B*Log[(e*(c + d*x))/(a + b*x)]), x] + b^2*g^2*Defer[Int
][x^2/(A + B*Log[(e*(c + d*x))/(a + b*x)]), x]
```

Rubi steps

$$\begin{aligned}\int \frac{(ag+bgx)^2}{A+B\log\left(\frac{e(c+dx)}{a+bx}\right)}\,dx &= \int \left(\frac{a^2g^2}{A+B\log\left(\frac{e(c+dx)}{a+bx}\right)}+\frac{2abg^2x}{A+B\log\left(\frac{e(c+dx)}{a+bx}\right)}+\frac{b^2g^2x^2}{A+B\log\left(\frac{e(c+dx)}{a+bx}\right)}\right)dx\\ &= \left(a^2g^2\right)\int \frac{1}{A+B\log\left(\frac{e(c+dx)}{a+bx}\right)}\,dx+\left(2abg^2\right)\int \frac{x}{A+B\log\left(\frac{e(c+dx)}{a+bx}\right)}\,dx+\left(b^2g^2\right)\int \frac{}{A+}\end{aligned}$$

**Mathematica [A]** time = 0.60, size = 0, normalized size = 0.00

$$\int \frac{(ag+bgx)^2}{A+B\log\left(\frac{e(c+dx)}{a+bx}\right)}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(a*g + b*g*x)^2/(A + B*Log[(e*(c + d*x))/(a + b*x)]),x]
[Out] Integrate[(a*g + b*g*x)^2/(A + B*Log[(e*(c + d*x))/(a + b*x)]), x]
```

**fricas [A]** time = 0.76, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{b^{2}g^{2}x^{2}+2\,abg^{2}x+a^{2}g^{2}}{B\log\left(\frac{dex+ce}{bx+a}\right)+A},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*g*x+a*g)^2/(A+B*log(e*(d*x+c)/(b*x+a))),x, algorithm="fricas")
[Out] integral((b^2*g^2*x^2 + 2*a*b*g^2*x + a^2*g^2)/(B*log((d*e*x + c*e)/(b*x +
a)) + A), x)
```

**giac [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*g*x+a*g)^2/(A+B*log(e*(d*x+c)/(b*x+a))),x, algorithm="giac")
[Out] Timed out
```

**maple [A]** time = 1.17, size = 0, normalized size = 0.00

$$\int \frac{\left(bgx+ag\right)^{2}}{B\ln\left(\frac{(dx+c)e}{bx+a}\right)+A}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*g*x+a*g)^2/(B*ln((d*x+c)/(b*x+a)*e)+A),x)
[Out] int((b*g*x+a*g)^2/(B*ln((d*x+c)/(b*x+a)*e)+A),x)
```

**maxima [A]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(bgx+ag\right)^{2}}{B\log\left(\frac{(dx+c)e}{bx+a}\right)+A}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*g*x+a*g)^2/(A+B*log(e*(d*x+c)/(b*x+a))),x, algorithm="maxima")
```

```
[Out] integrate((b*g*x + a*g)^2/(B*log((d*x + c)*e/(b*x + a)) + A), x)
```

**mupad [A]** time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{\left(a\,g+b\,g\,x\right)^2}{A+B\,\ln\left(\frac{e\,\left(c+d\,x\right)}{a+b\,x}\right)} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*g + b*g*x)^2/(A + B*log((e*(c + d*x))/(a + b*x))),x)
```

```
[Out] int((a*g + b*g*x)^2/(A + B*log((e*(c + d*x))/(a + b*x))), x)
```

**sympy [A]** time = 0.00, size = 0, normalized size = 0.00

$$g^{2} \left(\int \frac{a^{2}}{A + B \log{\left(\frac{c e}{a + b x} + \frac{d e x}{a + b x} \right)}}\, dx + \int \frac{b^{2} x^{2}}{A + B \log{\left(\frac{c e}{a + b x} + \frac{d e x}{a + b x} \right)}}\, dx + \int \frac{2 a b x}{A + B \log{\left(\frac{c e}{a + b x} + \frac{d e x}{a + b x} \right)}}\, dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*g*x+a*g)**2/(A+B*ln(e*(d*x+c)/(b*x+a))),x)
```

```
[Out] g**2*(Integral(a**2/(A + B*log(c*e/(a + b*x) + d*e*x/(a + b*x))), x) + Inte
gral(b**2*x**2/(A + B*log(c*e/(a + b*x) + d*e*x/(a + b*x))), x) + Integral(
2*a*b*x/(A + B*log(c*e/(a + b*x) + d*e*x/(a + b*x))), x))
```

# 3.192 $\int \frac{ag+bgx}{A+B\log\left(\frac{e(c+dx)}{a+bx}\right)} \, dx$

**Optimal**. Leaf size=33

$$\text{Int}\left(\frac{ag+bgx}{B\log\left(\frac{e(c+dx)}{a+bx}\right)+A},x\right)$$

```
[Out] Unintegrable((b*g*x+a*g)/(A+B*ln(e*(d*x+c)/(b*x+a))),x)
```

**Rubi [A]** time = 0.10, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{ag+bgx}{A+B\log\left(\frac{e(c+dx)}{a+bx}\right)} \, dx$$

Verification is Not applicable to the result.

```
[In] Int[(a*g + b*g*x)/(A + B*Log[(e*(c + d*x))/(a + b*x)]),x]
```

```
[Out] a*g*Defer[Int][(A + B*Log[(e*(c + d*x))/(a + b*x)])^(-1), x] + b*g*Defer[In
t][x/(A + B*Log[(e*(c + d*x))/(a + b*x)]), x]
```

Rubi steps

$$\begin{aligned}
\int \frac{ag+bgx}{A+B\log\left(\frac{e(c+dx)}{a+bx}\right)} \, dx &= \int \left(\frac{ag}{A+B\log\left(\frac{e(c+dx)}{a+bx}\right)}+\frac{bgx}{A+B\log\left(\frac{e(c+dx)}{a+bx}\right)}\right) dx \\
&= (ag)\int \frac{1}{A+B\log\left(\frac{e(c+dx)}{a+bx}\right)} \, dx+(bg)\int \frac{x}{A+B\log\left(\frac{e(c+dx)}{a+bx}\right)} \, dx
\end{aligned}$$

**Mathematica [A]** time = 0.25, size = 0, normalized size = 0.00

$$\int \frac{ag+bgx}{A+B\log\left(\frac{e(c+dx)}{a+bx}\right)} \, dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(a*g + b*g*x)/(A + B*Log[(e*(c + d*x))/(a + b*x)]),x]
```

```
[Out] Integrate[(a*g + b*g*x)/(A + B*Log[(e*(c + d*x))/(a + b*x)]), x]
```

**fricas [A]** time = 2.55, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{bgx + ag}{B\log\left(\frac{dex+ce}{bx+a}\right) + A}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*g*x+a*g)/(A+B*log(e*(d*x+c)/(b*x+a))),x, algorithm="fricas")
[Out] integral((b*g*x + a*g)/(B*log((d*e*x + c*e)/(b*x + a)) + A), x)
```

**giac [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*g*x+a*g)/(A+B*log(e*(d*x+c)/(b*x+a))),x, algorithm="giac")
[Out] Timed out
```

**maple [A]** time = 0.93, size = 0, normalized size = 0.00

$$\int \frac{bgx + ag}{B\ln\left(\frac{(dx+c)e}{bx+a}\right) + A}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*g*x+a*g)/(B*ln((d*x+c)/(b*x+a)*e)+A),x)
[Out] int((b*g*x+a*g)/(B*ln((d*x+c)/(b*x+a)*e)+A),x)
```

**maxima [A]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{bgx + ag}{B\log\left(\frac{(dx+c)e}{bx+a}\right) + A}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*g*x+a*g)/(A+B*log(e*(d*x+c)/(b*x+a))),x, algorithm="maxima")
[Out] integrate((b*g*x + a*g)/(B*log((d*x + c)*e/(b*x + a)) + A), x)
```

**mupad** [A] time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{a\,g+b\,g\,x}{A+B\,\ln\left(\frac{e\,\left(c+d\,x\right)}{a+b\,x}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*g + b*g*x)/(A + B*log((e*(c + d*x))/(a + b*x))),x)
[Out] int((a*g + b*g*x)/(A + B*log((e*(c + d*x))/(a + b*x))), x)
```

**sympy** [A] time = 0.00, size = 0, normalized size = 0.00

$$g\left(\int \frac{a}{A+B\log\left(\frac{ce}{a+bx}+\frac{dex}{a+bx}\right)}\,dx+\int \frac{bx}{A+B\log\left(\frac{ce}{a+bx}+\frac{dex}{a+bx}\right)}\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*g*x+a*g)/(A+B*ln(e*(d*x+c)/(b*x+a))),x)
[Out] g*(Integral(a/(A + B*log(c*e/(a + b*x) + d*e*x/(a + b*x))), x) + Integral(b
*x/(A + B*log(c*e/(a + b*x) + d*e*x/(a + b*x))), x))
```

# 3.193 $\int \frac{1}{(ag+bgx)\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)}\,dx$

**Optimal**. Leaf size=35

$$\text{Int}\left(\frac{1}{(ag+bgx)\left(B\log\left(\frac{e(c+dx)}{a+bx}\right)+A\right)},x\right)$$

```
[Out] Unintegrable(1/(b*g*x+a*g)/(A+B*ln(e*(d*x+c)/(b*x+a))),x)
```

**Rubi [A]** time = 0.07, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{1}{(ag+bgx)\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[1/((a*g + b*g*x)*(A + B*Log[(e*(c + d*x))/(a + b*x)])),x]
[Out] Defer[Int][1/((a*g + b*g*x)*(A + B*Log[(e*(c + d*x))/(a + b*x)])), x]
```

Rubi steps

$$\int \frac{1}{(ag+bgx)\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)}\,dx=\int \frac{1}{(ag+bgx)\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)}\,dx$$

**Mathematica [A]** time = 0.25, size = 0, normalized size = 0.00

$$\int \frac{1}{(ag+bgx)\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[1/((a*g + b*g*x)*(A + B*Log[(e*(c + d*x))/(a + b*x)])),x]
[Out] Integrate[1/((a*g + b*g*x)*(A + B*Log[(e*(c + d*x))/(a + b*x)])), x]
```

**fricas** [A] time = 1.00, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{1}{Abgx + Aag + \left(Bbgx + Bag\right)\log\left(\frac{dex+ce}{bx+a}\right)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*g*x+a*g)/(A+B*log(e*(d*x+c)/(b*x+a))),x, algorithm="fricas")
```

```
[Out] integral(1/(A*b*g*x + A*a*g + (B*b*g*x + B*a*g)*log((d*e*x + c*e)/(b*x + a)
)), x)
```

**giac** [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*g*x+a*g)/(A+B*log(e*(d*x+c)/(b*x+a))),x, algorithm="giac")
```

```
[Out] Timed out
```

**maple** [A] time = 1.10, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(bgx + ag\right)\left(B\ln\left(\frac{(dx+c)e}{bx+a}\right) + A\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*g*x+a*g)/(B*ln((d*x+c)/(b*x+a)*e)+A),x)
```

```
[Out] int(1/(b*g*x+a*g)/(B*ln((d*x+c)/(b*x+a)*e)+A),x)
```

**maxima** [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(bgx + ag\right)\left(B\log\left(\frac{(dx+c)e}{bx+a}\right) + A\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*g*x+a*g)/(A+B*log(e*(d*x+c)/(b*x+a))),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*g*x + a*g)*(B*log((d*x + c)*e/(b*x + a)) + A)), x)
```

**mupad [A]** time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{1}{\left(a\,g+b\,g\,x\right)\,\left(A+B\,\ln\left(\frac{e\,\left(c+d\,x\right)}{a+b\,x}\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*g + b*g*x)*(A + B*log((e*(c + d*x))/(a + b*x)))),x)
[Out] int(1/((a*g + b*g*x)*(A + B*log((e*(c + d*x))/(a + b*x)))), x)
```

**sympy [A]** time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{1}{Aa+Abx+Ba\log\left(\frac{ce}{a+bx}+\frac{dex}{a+bx}\right)+Bbx\log\left(\frac{ce}{a+bx}+\frac{dex}{a+bx}\right)}\,dx}{g}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*g*x+a*g)/(A+B*ln(e*(d*x+c)/(b*x+a))),x)
[Out] Integral(1/(A*a + A*b*x + B*a*log(c*e/(a + b*x) + d*e*x/(a + b*x)) + B*b*x*
log(c*e/(a + b*x) + d*e*x/(a + b*x))), x)/g
```

# 3.194 $\int \frac{1}{(ag+bgx)^2\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)}\,dx$

**Optimal**. Leaf size=53

$$-\frac{e^{-\frac{A}{B}}\operatorname{Ei}\left(\frac{A+B\log\left(\frac{e(c+dx)}{a+bx}\right)}{B}\right)}{Beg^2(bc-ad)}$$

```
[Out] -Ei((A+B*ln(e*(d*x+c)/(b*x+a)))/B)/B/(-a*d+b*c)/e/exp(A/B)/g^2
```

**Rubi [F]** time = 0.09, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{1}{(ag+bgx)^2\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[1/((a*g + b*g*x)^2*(A + B*Log[(e*(c + d*x))/(a + b*x)])),x]
[Out] Defer[Int][1/((a*g + b*g*x)^2*(A + B*Log[(e*(c + d*x))/(a + b*x)])), x]
```

Rubi steps

$$\int \frac{1}{(ag+bgx)^2\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)}\,dx=\int \frac{1}{(ag+bgx)^2\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)}\,dx$$

**Mathematica [A]** time = 0.07, size = 50, normalized size = 0.94

$$\frac{e^{-\frac{A}{B}}\operatorname{Ei}\left(\frac{A}{B}+\log\left(\frac{e(c+dx)}{a+bx}\right)\right)}{Beg^2(ad-bc)}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a*g + b*g*x)^2*(A + B*Log[(e*(c + d*x))/(a + b*x)])),x]
[Out] ExpIntegralEi[A/B + Log[(e*(c + d*x))/(a + b*x)]]/(B*(-(b*c) + a*d)*e*E^(A/
B)*g^2)
```

**fricas** [A] time = 0.92, size = 50, normalized size = 0.94

$$-\frac{e^{\left(-\frac{A}{B}\right)}\operatorname{log\_integral}\left(\frac{(dex+ce)e^{\frac{A}{B}}}{bx+a}\right)}{(Bbc-Bad)eg^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*g*x+a*g)^2/(A+B*log(e*(d*x+c)/(b*x+a))),x, algorithm="fricas
")
```

```
[Out] -e^(-A/B)*log_integral((d*e*x + c*e)*e^(A/B)/(b*x + a))/((B*b*c - B*a*d)*e*
g^2)
```

**giac** [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*g*x+a*g)^2/(A+B*log(e*(d*x+c)/(b*x+a))),x, algorithm="giac")
```

```
[Out] Timed out
```

**maple** [A] time = 0.53, size = 69, normalized size = 1.30

$$-\frac{\operatorname{Ei}\left(1,-\ln\left(\frac{de}{b}-\frac{(ad-bc)e}{(bx+a)b}\right)-\frac{A}{B}\right)\mathrm{e}^{-\frac{A}{B}}}{(ad-bc)\,Be\,g^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*g*x+a*g)^2/(B*ln((d*x+c)/(b*x+a)*e)+A),x)
```

```
[Out] -1/e/(a*d-b*c)/g^2/B*exp(-A/B)*Ei(1,-ln(1/b*d*e-(a*d-b*c)/(b*x+a)/b*e)-A/B)
```

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(bgx+ag\right)^2\left(B\log\left(\frac{(dx+c)e}{bx+a}\right)+A\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*g*x+a*g)^2/(A+B*log(e*(d*x+c)/(b*x+a))),x, algorithm="maxima
")
```

```
[Out] integrate(1/((b*g*x + a*g)^2*(B*log((d*x + c)*e/(b*x + a)) + A)), x)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{{\left (a\,g+b\,g\,x\right )}^2\,\left (A+B\,\ln \left (\frac{e\,\left (c+d\,x\right )}{a+b\,x}\right )\right )} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*g + b*g*x)^2*(A + B*log((e*(c + d*x))/(a + b*x)))),x)
```

```
[Out] int(1/((a*g + b*g*x)^2*(A + B*log((e*(c + d*x))/(a + b*x)))), x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{1}{Aa^{2} + 2Aabx + Ab^{2}x^{2} + Ba^{2}\log{\left (\frac{ce}{a + bx} + \frac{dex}{a + bx} \right )} + 2Babx\log{\left (\frac{ce}{a + bx} + \frac{dex}{a + bx} \right )} + Bb^{2}x^{2}\log{\left (\frac{ce}{a + bx} + \frac{dex}{a + bx} \right )}}\, dx}{g^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*g*x+a*g)**2/(A+B*ln(e*(d*x+c)/(b*x+a))),x)
```

```
[Out] Integral(1/(A*a**2 + 2*A*a*b*x + A*b**2*x**2 + B*a**2*log(c*e/(a + b*x) + d
*e*x/(a + b*x)) + 2*B*a*b*x*log(c*e/(a + b*x) + d*e*x/(a + b*x)) + B*b**2*x
**2*log(c*e/(a + b*x) + d*e*x/(a + b*x))), x)/g**2
```

# 3.195 $\int \frac{1}{(ag+bgx)^3\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)}\,dx$

**Optimal**. Leaf size=109

$$\frac{de^{-\frac{A}{B}}\text{Ei}\left(\frac{A+B\log\left(\frac{e(c+dx)}{a+bx}\right)}{B}\right)}{Beg^3(bc-ad)^2}-\frac{be^{-\frac{2A}{B}}\text{Ei}\left(\frac{2\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)}{B}\right)}{Be^2g^3(bc-ad)^2}$$

```
[Out] d*Ei((A+B*ln(e*(d*x+c)/(b*x+a)))/B)/B/(-a*d+b*c)^2/e/exp(A/B)/g^3-b*Ei(2*(A
+B*ln(e*(d*x+c)/(b*x+a)))/B)/B/(-a*d+b*c)^2/e^2/exp(2*A/B)/g^3
```

**Rubi [F]** time = 0.07, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{1}{(ag+bgx)^3\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[1/((a*g + b*g*x)^3*(A + B*Log[(e*(c + d*x))/(a + b*x)])),x]
```

```
[Out] Defer[Int][1/((a*g + b*g*x)^3*(A + B*Log[(e*(c + d*x))/(a + b*x)])), x]
```

Rubi steps

$$\int \frac{1}{(ag+bgx)^3\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)}\,dx=\int \frac{1}{(ag+bgx)^3\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)}\,dx$$

**Mathematica [A]** time = 0.16, size = 89, normalized size = 0.82

$$\frac{e^{-\frac{2A}{B}}\left(dee^{A/B}\text{Ei}\left(\frac{A}{B}+\log\left(\frac{e(c+dx)}{a+bx}\right)\right)-b\text{Ei}\left(\frac{2\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)}{B}\right)\right)}{Be^2g^3(bc-ad)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a*g + b*g*x)^3*(A + B*Log[(e*(c + d*x))/(a + b*x)])),x]
```

```
[Out] (d*e*E^(A/B)*ExpIntegralEi[A/B + Log[(e*(c + d*x))/(a + b*x)]] - b*ExpInteg
ralEi[(2*(A + B*Log[(e*(c + d*x))/(a + b*x)]))/B])/(B*(b*c - a*d)^2*e^2*E^(
(2*A)/B)*g^3)
```

**fricas [A]** time = 0.87, size = 129, normalized size = 1.18

$$\frac{\left(dee^{\frac{A}{B}}\log\_integral\left(\frac{(dex+ce)e^{\frac{A}{B}}}{bx+a}\right)-b\log\_integral\left(\frac{\left(d^2e^2x^2+2\,cde^2x+c^2e^2\right)e^{\left(\frac{2\,A}{B}\right)}}{b^2x^2+2\,abx+a^2}\right)\right)e^{\left(-\frac{2\,A}{B}\right)}}{\left(Bb^2c^2-2\,Babcd+Ba^2d^2\right)e^2g^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*g*x+a*g)^3/(A+B*log(e*(d*x+c)/(b*x+a))),x, algorithm="fricas
")
```

```
[Out] (d*e*e^(A/B)*log_integral((d*e*x + c*e)*e^(A/B)/(b*x + a)) - b*log_integral
((d^2*e^2*x^2 + 2*c*d*e^2*x + c^2*e^2)*e^(2*A/B)/(b^2*x^2 + 2*a*b*x + a^2))
)*e^(-2*A/B)/((B*b^2*c^2 - 2*B*a*b*c*d + B*a^2*d^2)*e^2*g^3)
```

**giac [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*g*x+a*g)^3/(A+B*log(e*(d*x+c)/(b*x+a))),x, algorithm="giac")
```

```
[Out] Timed out
```

**maple [F]** time = 1.15, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(bgx+ag\right)^{3}\left(B\ln\left(\frac{(dx+c)e}{bx+a}\right)+A\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*g*x+a*g)^3/(B*ln((d*x+c)/(b*x+a)*e)+A),x)
```

```
[Out] int(1/(b*g*x+a*g)^3/(B*ln((d*x+c)/(b*x+a)*e)+A),x)
```

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(bgx+ag\right)^{3}\left(B\log\left(\frac{(dx+c)e}{bx+a}\right)+A\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*g*x+a*g)^3/(A+B*log(e*(d*x+c)/(b*x+a))),x, algorithm="maxima
")
```

```
[Out] integrate(1/((b*g*x + a*g)^3*(B*log((d*x + c)*e/(b*x + a)) + A)), x)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\left(a\,g+b\,g\,x\right)^3\,\left(A+B\,\ln\left(\frac{e\,\left(c+d\,x\right)}{a+b\,x}\right)\right)} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*g + b*g*x)^3*(A + B*log((e*(c + d*x))/(a + b*x)))),x)
```

```
[Out] int(1/((a*g + b*g*x)^3*(A + B*log((e*(c + d*x))/(a + b*x)))), x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{1}{Aa^3+3Aa^2bx+3Aab^2x^2+Ab^3x^3+Ba^3\log\left(\frac{ce}{a+bx}+\frac{dex}{a+bx}\right)+3Ba^2bx\log\left(\frac{ce}{a+bx}+\frac{dex}{a+bx}\right)+3Bab^2x^2\log\left(\frac{ce}{a+bx}+\frac{dex}{a+bx}\right)+Bb^3x^3\log\left(\frac{ce}{a+bx}+\frac{dex}{a+bx}\right)}\,dx}{g^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*g*x+a*g)**3/(A+B*ln(e*(d*x+c)/(b*x+a))),x)
```

```
[Out] Integral(1/(A*a**3 + 3*A*a**2*b*x + 3*A*a*b**2*x**2 + A*b**3*x**3 + B*a**3*
log(c*e/(a + b*x) + d*e*x/(a + b*x)) + 3*B*a**2*b*x*log(c*e/(a + b*x) + d*e
*x/(a + b*x)) + 3*B*a*b**2*x**2*log(c*e/(a + b*x) + d*e*x/(a + b*x)) + B*b*
*3*x**3*log(c*e/(a + b*x) + d*e*x/(a + b*x))), x)/g**3
```

# 3.196 $\int \frac{(ag+bgx)^2}{\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2}\,dx$

**Optimal**. Leaf size=35

$$\text{Int}\left(\frac{(ag+bgx)^2}{\left(B\log\left(\frac{e(c+dx)}{a+bx}\right)+A\right)^2},x\right)$$

```
[Out] Unintegrable((b*g*x+a*g)^2/(A+B*ln(e*(d*x+c)/(b*x+a)))^2,x)
```

**Rubi [A]** time = 0.21, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{(ag+bgx)^2}{\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(a*g + b*g*x)^2/(A + B*Log[(e*(c + d*x))/(a + b*x)])^2,x]
```

```
[Out] a^2*g^2*Defer[Int][(A + B*Log[(e*(c + d*x))/(a + b*x)])^(-2), x] + 2*a*b*g^
2*Defer[Int][x/(A + B*Log[(e*(c + d*x))/(a + b*x)])^2, x] + b^2*g^2*Defer[I
nt][x^2/(A + B*Log[(e*(c + d*x))/(a + b*x)])^2, x]
```

Rubi steps

$$\int \frac{(ag+bgx)^2}{\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2}\,dx = \int \left(\frac{a^2g^2}{\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2}+\frac{2abg^2x}{\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2}+\frac{b^2g^2x^2}{\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2}\right) d$$

$$= \left(a^2g^2\right)\int \frac{1}{\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2}\,dx+\left(2abg^2\right)\int \frac{x}{\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2}\,dx+\left(b^2\right.$$

**Mathematica [A]** time = 1.40, size = 0, normalized size = 0.00

$$\int \frac{(ag+bgx)^2}{\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(a*g + b*g*x)^2/(A + B*Log[(e*(c + d*x))/(a + b*x)])^2,x]
```

```
[Out] Integrate[(a*g + b*g*x)^2/(A + B*Log[(e*(c + d*x))/(a + b*x)])^2, x]
```

**fricas [A]** time = 2.14, size = 0, normalized size = 0.00

$$\operatorname{integral}\left(\frac{b^2g^2x^2 + 2\,abg^2x + a^2g^2}{B^2\log\left(\frac{dex+ce}{bx+a}\right)^2 + 2\,AB\log\left(\frac{dex+ce}{bx+a}\right) + A^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*g*x+a*g)^2/(A+B*log(e*(d*x+c)/(b*x+a)))^2,x, algorithm="fricas
")
```

```
[Out] integral((b^2*g^2*x^2 + 2*a*b*g^2*x + a^2*g^2)/(B^2*log((d*e*x + c*e)/(b*x
+ a))^2 + 2*A*B*log((d*e*x + c*e)/(b*x + a)) + A^2), x)
```

**giac [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*g*x+a*g)^2/(A+B*log(e*(d*x+c)/(b*x+a)))^2,x, algorithm="giac")
```

```
[Out] Timed out
```

**maple [A]** time = 1.07, size = 0, normalized size = 0.00

$$\int \frac{\left(bgx + ag\right)^2}{\left(B\ln\left(\frac{(dx+c)e}{bx+a}\right) + A\right)^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*g*x+a*g)^2/(B*ln((d*x+c)/(b*x+a)*e)+A)^2,x)
```

```
[Out] int((b*g*x+a*g)^2/(B*ln((d*x+c)/(b*x+a)*e)+A)^2,x)
```

**maxima [A]** time = 0.00, size = 0, normalized size = 0.00

$$-\frac{b^3dg^2x^4 + a^3cg^2 + \left(b^3cg^2 + 3\,ab^2dg^2\right)x^3 + 3\left(ab^2cg^2 + a^2bdg^2\right)x^2 + \left(3\,a^2bcg^2 + a^3dg^2\right)x}{(bc-ad)B^2\log(bx+a) - (bc-ad)B^2\log(dx+c) - (bc-ad)AB - \left(bc\log(e) - ad\log(e)\right)B^2} + \int \frac{4\,b^3dg}{(bc-ad)B^2\,\mathrm{lc}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*g*x+a*g)^2/(A+B*log(e*(d*x+c)/(b*x+a)))^2,x, algorithm="maxima
")
```

```
[Out] -(b^3*d*g^2*x^4 + a^3*c*g^2 + (b^3*c*g^2 + 3*a*b^2*d*g^2)*x^3 + 3*(a*b^2*c*
g^2 + a^2*b*d*g^2)*x^2 + (3*a^2*b*c*g^2 + a^3*d*g^2)*x)/((b*c - a*d)*B^2*lo
g(b*x + a) - (b*c - a*d)*B^2*log(d*x + c) - (b*c - a*d)*A*B - (b*c*log(e) -
 a*d*log(e))*B^2) + integrate((4*b^3*d*g^2*x^3 + 3*a^2*b*c*g^2 + a^3*d*g^2
+ 3*(b^3*c*g^2 + 3*a*b^2*d*g^2)*x^2 + 6*(a*b^2*c*g^2 + a^2*b*d*g^2)*x)/((b*
c - a*d)*B^2*log(b*x + a) - (b*c - a*d)*B^2*log(d*x + c) - (b*c - a*d)*A*B
- (b*c*log(e) - a*d*log(e))*B^2), x)
```

**mupad** [A] time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{{\left(a\,g+b\,g\,x\right)}^2}{{\left(A+B\,\ln\left(\frac{e\,\left(c+d\,x\right)}{a+b\,x}\right)\right)}^2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*g + b*g*x)^2/(A + B*log((e*(c + d*x))/(a + b*x)))^2,x)
```

```
[Out] int((a*g + b*g*x)^2/(A + B*log((e*(c + d*x))/(a + b*x)))^2, x)
```

**sympy** [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*g*x+a*g)**2/(A+B*ln(e*(d*x+c)/(b*x+a)))**2,x)
```

```
[Out] Timed out
```

# 3.197 $\int \frac{ag+bgx}{\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2}\,dx$

**Optimal**. Leaf size=33

$$\text{Int}\left(\frac{ag+bgx}{\left(B\log\left(\frac{e(c+dx)}{a+bx}\right)+A\right)^2},x\right)$$

```
[Out] Unintegrable((b*g*x+a*g)/(A+B*ln(e*(d*x+c)/(b*x+a)))^2,x)
```

**Rubi [A]** time = 0.11, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{ag+bgx}{\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(a*g + b*g*x)/(A + B*Log[(e*(c + d*x))/(a + b*x)])^2,x]
```

```
[Out] a*g*Defer[Int][(A + B*Log[(e*(c + d*x))/(a + b*x)])^(-2), x] + b*g*Defer[In
t][x/(A + B*Log[(e*(c + d*x))/(a + b*x)])^2, x]
```

Rubi steps

$$\begin{aligned}\int \frac{ag+bgx}{\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2}\,dx &= \int \left(\frac{ag}{\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2}+\frac{bgx}{\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2}\right)dx\\ &= (ag)\int \frac{1}{\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2}\,dx+(bg)\int \frac{x}{\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2}\,dx\end{aligned}$$

**Mathematica [A]** time = 0.97, size = 0, normalized size = 0.00

$$\int \frac{ag+bgx}{\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(a*g + b*g*x)/(A + B*Log[(e*(c + d*x))/(a + b*x)])^2,x]
```

```
[Out] Integrate[(a*g + b*g*x)/(A + B*Log[(e*(c + d*x))/(a + b*x)])^2, x]
```

**fricas [A]** time = 0.81, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{bgx+ag}{B^2\log\left(\frac{dex+ce}{bx+a}\right)^2+2\,AB\log\left(\frac{dex+ce}{bx+a}\right)+A^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*g*x+a*g)/(A+B*log(e*(d*x+c)/(b*x+a)))^2,x, algorithm="fricas")
```

```
[Out] integral((b*g*x + a*g)/(B^2*log((d*e*x + c*e)/(b*x + a))^2 + 2*A*B*log((d*e
*x + c*e)/(b*x + a)) + A^2), x)
```

**giac [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*g*x+a*g)/(A+B*log(e*(d*x+c)/(b*x+a)))^2,x, algorithm="giac")
```

```
[Out] Timed out
```

**maple [A]** time = 1.12, size = 0, normalized size = 0.00

$$\int \frac{bgx+ag}{\left(B\ln\left(\frac{(dx+c)e}{bx+a}\right)+A\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*g*x+a*g)/(B*ln((d*x+c)/(b*x+a)*e)+A)^2,x)
```

```
[Out] int((b*g*x+a*g)/(B*ln((d*x+c)/(b*x+a)*e)+A)^2,x)
```

**maxima [A]** time = 0.00, size = 0, normalized size = 0.00

$$-\frac{b^2dgx^3+a^2cg+\left(b^2cg+2\,abdg\right)x^2+\left(2\,abcg+a^2dg\right)x}{(bc-ad)B^2\log\left(bx+a\right)-(bc-ad)B^2\log\left(dx+c\right)-(bc-ad)AB-\left(bc\log(e)-ad\log(e)\right)B^2}+\int\frac{}{(bc-ad)B^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*g*x+a*g)/(A+B*log(e*(d*x+c)/(b*x+a)))^2,x, algorithm="maxima")
```

```
[Out] -(b^2*d*g*x^3 + a^2*c*g + (b^2*c*g + 2*a*b*d*g)*x^2 + (2*a*b*c*g + a^2*d*g)
*x)/((b*c - a*d)*B^2*log(b*x + a) - (b*c - a*d)*B^2*log(d*x + c) - (b*c - a
*d)*A*B - (b*c*log(e) - a*d*log(e))*B^2) + integrate((3*b^2*d*g*x^2 + 2*a*b
*c*g + a^2*d*g + 2*(b^2*c*g + 2*a*b*d*g)*x)/((b*c - a*d)*B^2*log(b*x + a) -
 (b*c - a*d)*B^2*log(d*x + c) - (b*c - a*d)*A*B - (b*c*log(e) - a*d*log(e))
*B^2), x)
```

**mupad [A]** time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{a\,g+b\,g\,x}{{\left(A+B\,\ln\left(\frac{e\,\left(c+d\,x\right)}{a+b\,x}\right)\right)}^2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*g + b*g*x)/(A + B*log((e*(c + d*x))/(a + b*x)))^2,x)
```

```
[Out] int((a*g + b*g*x)/(A + B*log((e*(c + d*x))/(a + b*x)))^2, x)
```

**sympy [A]** time = 0.00, size = 0, normalized size = 0.00

$$\frac{-a^{2}cg - a^{2}dgx - 2abcgx - 2abdgx^{2} - b^{2}cgx^{2} - b^{2}dgx^{3}}{ABad - ABbc + \left(B^{2}ad - B^{2}bc\right)\log{\left(\frac{e(c+dx)}{a+bx} \right)}} + \frac{g\left(\int \frac{a^{2}d}{A+B\log{\left(\frac{ce}{a+bx}+\frac{dex}{a+bx} \right)}}\, dx + \int \frac{2abc}{A+B\log{\left(\frac{ce}{a+bx}+\frac{dex}{a+bx} \right)}}\, dx + \int \cdots\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*g*x+a*g)/(A+B*ln(e*(d*x+c)/(b*x+a)))**2,x)
```

```
[Out] (-a**2*c*g - a**2*d*g*x - 2*a*b*c*g*x - 2*a*b*d*g*x**2 - b**2*c*g*x**2 - b*
*2*d*g*x**3)/(A*B*a*d - A*B*b*c + (B**2*a*d - B**2*b*c)*log(e*(c + d*x)/(a
+ b*x))) + g*(Integral(a**2*d/(A + B*log(c*e/(a + b*x) + d*e*x/(a + b*x))),
 x) + Integral(2*a*b*c/(A + B*log(c*e/(a + b*x) + d*e*x/(a + b*x))), x) + I
ntegral(2*b**2*c*x/(A + B*log(c*e/(a + b*x) + d*e*x/(a + b*x))), x) + Integ
ral(3*b**2*d*x**2/(A + B*log(c*e/(a + b*x) + d*e*x/(a + b*x))), x) + Integr
al(4*a*b*d*x/(A + B*log(c*e/(a + b*x) + d*e*x/(a + b*x))), x))/(B*(a*d - b*
c))
```

# 3.198 $\int \frac{1}{(ag+bgx)\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2}\,dx$

**Optimal**. Leaf size=35

$$\text{Int}\left(\frac{1}{(ag+bgx)\left(B\log\left(\frac{e(c+dx)}{a+bx}\right)+A\right)^2},x\right)$$

```
[Out] Unintegrable(1/(b*g*x+a*g)/(A+B*ln(e*(d*x+c)/(b*x+a)))^2,x)
```

**Rubi [A]** time = 0.08, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{1}{(ag+bgx)\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[1/((a*g + b*g*x)*(A + B*Log[(e*(c + d*x))/(a + b*x)])^2),x]
[Out] Defer[Int][1/((a*g + b*g*x)*(A + B*Log[(e*(c + d*x))/(a + b*x)])^2), x]
```

Rubi steps

$$\int \frac{1}{(ag+bgx)\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2}\,dx=\int \frac{1}{(ag+bgx)\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2}\,dx$$

**Mathematica [A]** time = 0.52, size = 0, normalized size = 0.00

$$\int \frac{1}{(ag+bgx)\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[1/((a*g + b*g*x)*(A + B*Log[(e*(c + d*x))/(a + b*x)])^2),x]
[Out] Integrate[1/((a*g + b*g*x)*(A + B*Log[(e*(c + d*x))/(a + b*x)])^2), x]
```

**fricas** [A] time = 0.64, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{1}{A^2bgx+A^2ag+\left(B^2bgx+B^2ag\right)\log\left(\frac{dex+ce}{bx+a}\right)^2+2\left(ABbgx+ABag\right)\log\left(\frac{dex+ce}{bx+a}\right)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*g*x+a*g)/(A+B*log(e*(d*x+c)/(b*x+a)))^2,x, algorithm="fricas
")
```

```
[Out] integral(1/(A^2*b*g*x + A^2*a*g + (B^2*b*g*x + B^2*a*g)*log((d*e*x + c*e)/(
b*x + a))^2 + 2*(A*B*b*g*x + A*B*a*g)*log((d*e*x + c*e)/(b*x + a))), x)
```

**giac** [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(bgx+ag\right)\left(B\log\left(\frac{(dx+c)e}{bx+a}\right)+A\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*g*x+a*g)/(A+B*log(e*(d*x+c)/(b*x+a)))^2,x, algorithm="giac")
```

```
[Out] integrate(1/((b*g*x + a*g)*(B*log((d*x + c)*e/(b*x + a)) + A)^2), x)
```

**maple** [A] time = 0.98, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(bgx+ag\right)\left(B\ln\left(\frac{(dx+c)e}{bx+a}\right)+A\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*g*x+a*g)/(B*ln((d*x+c)/(b*x+a)*e)+A)^2,x)
```

```
[Out] int(1/(b*g*x+a*g)/(B*ln((d*x+c)/(b*x+a)*e)+A)^2,x)
```

**maxima** [A] time = 0.00, size = 0, normalized size = 0.00

$$d\int \frac{1}{\left(bcg-adg\right)B^2\log\left(bx+a\right)-\left(bcg-adg\right)B^2\log\left(dx+c\right)-\left(bcg-adg\right)AB-\left(bcg\log(e)-adg\log(e)\right)B^2}\,dx-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*g*x+a*g)/(A+B*log(e*(d*x+c)/(b*x+a)))^2,x, algorithm="maxima
")
```

```
[Out] d*integrate(1/((b*c*g - a*d*g)*B^2*log(b*x + a) - (b*c*g - a*d*g)*B^2*log(d
*x + c) - (b*c*g - a*d*g)*A*B - (b*c*g*log(e) - a*d*g*log(e))*B^2), x) - (d
*x + c)/((b*c*g - a*d*g)*B^2*log(b*x + a) - (b*c*g - a*d*g)*B^2*log(d*x + c
) - (b*c*g - a*d*g)*A*B - (b*c*g*log(e) - a*d*g*log(e))*B^2)
```

**mupad** [A] time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{1}{\left(a\,g+b\,g\,x\right)\,{\left(A+B\,\ln\left(\frac{e\,\left(c+d\,x\right)}{a+b\,x}\right)\right)}^2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*g + b*g*x)*(A + B*log((e*(c + d*x))/(a + b*x)))^2),x)
```

```
[Out] int(1/((a*g + b*g*x)*(A + B*log((e*(c + d*x))/(a + b*x)))^2), x)
```

**sympy** [A] time = 0.00, size = 0, normalized size = 0.00

$$\frac{-c-dx}{ABadg-ABbcg+\left(B^2adg-B^2bcg\right)\log\left(\frac{e(c+dx)}{a+bx}\right)}+\frac{d\int\frac{1}{A+B\log\left(\frac{ce}{a+bx}+\frac{dex}{a+bx}\right)}\,dx}{Bg\left(ad-bc\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*g*x+a*g)/(A+B*ln(e*(d*x+c)/(b*x+a)))**2,x)
```

```
[Out] (-c - d*x)/(A*B*a*d*g - A*B*b*c*g + (B**2*a*d*g - B**2*b*c*g)*log(e*(c + d*
x)/(a + b*x))) + d*Integral(1/(A + B*log(c*e/(a + b*x) + d*e*x/(a + b*x))),
 x)/(B*g*(a*d - b*c))
```

## 3.199 $\int \frac{1}{(ag+bgx)^2\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2}\,dx$

**Optimal**. Leaf size=104

$$\frac{c+dx}{Bg^2(a+bx)(bc-ad)\left(B\log\left(\frac{e(c+dx)}{a+bx}\right)+A\right)}-\frac{e^{-\frac{A}{B}}\text{Ei}\left(\frac{A+B\log\left(\frac{e(c+dx)}{a+bx}\right)}{B}\right)}{B^2eg^2(bc-ad)}$$

```
[Out] -Ei((A+B*ln(e*(d*x+c)/(b*x+a)))/B)/B^2/(-a*d+b*c)/e/exp(A/B)/g^2+(d*x+c)/B/
(-a*d+b*c)/g^2/(b*x+a)/(A+B*ln(e*(d*x+c)/(b*x+a)))
```

**Rubi [F]** time = 0.09, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{1}{(ag+bgx)^2\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[1/((a*g + b*g*x)^2*(A + B*Log[(e*(c + d*x))/(a + b*x)])^2),x]
```

```
[Out] Defer[Int][1/((a*g + b*g*x)^2*(A + B*Log[(e*(c + d*x))/(a + b*x)])^2), x]
```

Rubi steps

$$\int \frac{1}{(ag+bgx)^2\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2}\,dx=\int \frac{1}{(ag+bgx)^2\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2}\,dx$$

**Mathematica [A]** time = 0.12, size = 88, normalized size = 0.85

$$\frac{\frac{e^{-\frac{A}{B}}\text{Ei}\left(\frac{A}{B}+\log\left(\frac{e(c+dx)}{a+bx}\right)\right)}{e}-\frac{B(c+dx)}{(a+bx)\left(B\log\left(\frac{e(c+dx)}{a+bx}\right)+A\right)}}{B^2g^2(ad-bc)}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a*g + b*g*x)^2*(A + B*Log[(e*(c + d*x))/(a + b*x)])^2),x]
```

```
[Out] (ExpIntegralEi[A/B + Log[(e*(c + d*x))/(a + b*x)]]/(e*E^(A/B)) - (B*(c + d*
x))/((a + b*x)*(A + B*Log[(e*(c + d*x))/(a + b*x)])))/(B^2*(-(b*c) + a*d)*g
^2)
```

**fricas [B]** time = 0.57, size = 208, normalized size = 2.00

$$\frac{(Bdex+Bce)e^{\frac{A}{B}}-\left(Abx+Aa+(Bbx+Ba)\log\left(\frac{dex+ce}{bx+a}\right)\right)\text{log\_integral}\left(\frac{(dex+ce)e^{\frac{A}{B}}}{bx+a}\right)}{\left(\left(B^3b^2c-B^3abd\right)eg^2x+\left(B^3abc-B^3a^2d\right)eg^2\right)e^{\frac{A}{B}}\log\left(\frac{dex+ce}{bx+a}\right)+\left(\left(AB^2b^2c-AB^2abd\right)eg^2x+\left(AB^2abc-AB^2a^2d\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*g*x+a*g)^2/(A+B*log(e*(d*x+c)/(b*x+a)))^2,x, algorithm="fric
as")
```

```
[Out] ((B*d*e*x + B*c*e)*e^(A/B) - (A*b*x + A*a + (B*b*x + B*a)*log((d*e*x + c*e)
/(b*x + a)))*log_integral((d*e*x + c*e)*e^(A/B)/(b*x + a)))/(((B^3*b^2*c -
B^3*a*b*d)*e*g^2*x + (B^3*a*b*c - B^3*a^2*d)*e*g^2)*e^(A/B)*log((d*e*x + c*
e)/(b*x + a)) + ((A*B^2*b^2*c - A*B^2*a*b*d)*e*g^2*x + (A*B^2*a*b*c - A*B^2
*a^2*d)*e*g^2)*e^(A/B))
```

**giac [A]** time = 1.34, size = 152, normalized size = 1.46

$$\left(\frac{bc}{(bce-ade)(bc-ad)}-\frac{ad}{(bce-ade)(bc-ad)}\right)\left(\frac{dxe+ce}{\left(B^2g^2\log\left(\frac{dxe+ce}{bx+a}\right)+ABg^2\right)(bx+a)}-\frac{\text{Ei}\left(\frac{A}{B}+\log\left(\frac{dxe+ce}{bx+a}\right)\right)e^{\left(-\frac{A}{B}\right.}}{B^2g^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*g*x+a*g)^2/(A+B*log(e*(d*x+c)/(b*x+a)))^2,x, algorithm="giac
")
```

```
[Out] (b*c/((b*c*e - a*d*e)*(b*c - a*d)) - a*d/((b*c*e - a*d*e)*(b*c - a*d)))*((d
*x*e + c*e)/((B^2*g^2*log((d*x*e + c*e)/(b*x + a)) + A*B*g^2)*(b*x + a)) -
Ei(A/B + log((d*x*e + c*e)/(b*x + a)))*e^(-A/B)/(B^2*g^2))
```

**maple [B]** time = 0.45, size = 258, normalized size = 2.48

$$\frac{ad}{(ad-bc)\left(\ln\left(\frac{de}{b}-\frac{(ad-bc)e}{(bx+a)b}\right)+\frac{A}{B}\right)(bx+a)B^2b\,g^2}-\frac{c}{(ad-bc)\left(\ln\left(\frac{de}{b}-\frac{(ad-bc)e}{(bx+a)b}\right)+\frac{A}{B}\right)(bx+a)B^2g^2}-\frac{\text{Ei}\left(1,-\ln\left(\frac{de}{b}\right.\right.}{(ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*g*x+a*g)^2/(B*ln((d*x+c)/(b*x+a)*e)+A)^2,x)
```

```
[Out] -1/(a*d-b*c)/g^2/B^2/(ln(1/b*d*e-(a*d-b*c)/(b*x+a)/b*e)+A/B)/b*d+1/(a*d-b*c
)/g^2/B^2/(ln(1/b*d*e-(a*d-b*c)/(b*x+a)/b*e)+A/B)/b/(b*x+a)*a*d-1/(a*d-b*c)
/g^2/B^2/(ln(1/b*d*e-(a*d-b*c)/(b*x+a)/b*e)+A/B)/(b*x+a)*c-1/e/(a*d-b*c)/g^
2/B^2*exp(-A/B)*Ei(1,-ln(1/b*d*e-(a*d-b*c)/(b*x+a)/b*e)-A/B)
```

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$\frac{d}{\left(abcg^2 - a^2dg^2\right)AB + \left(abcg^2 \log(e) - a^2dg^2 \log(e)\right)B^2 + \left(\left(b^2cg^2 - abdg^2\right)AB + \left(b^2cg^2 \log(e) - abdg^2 \log(e)\right)B^2\right)x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*g*x+a*g)^2/(A+B*log(e*(d*x+c)/(b*x+a)))^2,x, algorithm="maxi
ma")
```

```
[Out] (d*x + c)/((a*b*c*g^2 - a^2*d*g^2)*A*B + (a*b*c*g^2*log(e) - a^2*d*g^2*log(
e))*B^2 + ((b^2*c*g^2 - a*b*d*g^2)*A*B + (b^2*c*g^2*log(e) - a*b*d*g^2*log(
e))*B^2)*x - ((b^2*c*g^2 - a*b*d*g^2)*B^2*x + (a*b*c*g^2 - a^2*d*g^2)*B^2)*
log(b*x + a) + ((b^2*c*g^2 - a*b*d*g^2)*B^2*x + (a*b*c*g^2 - a^2*d*g^2)*B^2
)*log(d*x + c)) + integrate(1/(B^2*a^2*g^2*log(e) + A*B*a^2*g^2 + (B^2*b^2*
g^2*log(e) + A*B*b^2*g^2)*x^2 + 2*(B^2*a*b*g^2*log(e) + A*B*a*b*g^2)*x - (B
^2*b^2*g^2*x^2 + 2*B^2*a*b*g^2*x + B^2*a^2*g^2)*log(b*x + a) + (B^2*b^2*g^2
*x^2 + 2*B^2*a*b*g^2*x + B^2*a^2*g^2)*log(d*x + c)), x)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{{\left(a\,g+b\,g\,x\right)}^2\,{\left(A+B\,\ln\left(\frac{e\,\left(c+d\,x\right)}{a+b\,x}\right)\right)}^2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*g + b*g*x)^2*(A + B*log((e*(c + d*x))/(a + b*x)))^2),x)
```

```
[Out] int(1/((a*g + b*g*x)^2*(A + B*log((e*(c + d*x))/(a + b*x)))^2), x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\frac{-c - dx}{ABa^2dg^2 - ABabcg^2 + ABabdg^2x - ABb^2cg^2x + \left(B^2a^2dg^2 - B^2abcg^2 + B^2abdg^2x - B^2b^2cg^2x\right)\log\left(\frac{e(c+dx)}{a+bx}\right)} + \int$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*g*x+a*g)**2/(A+B*ln(e*(d*x+c)/(b*x+a)))**2,x)
```

```
[Out] (-c - d*x)/(A*B*a**2*d*g**2 - A*B*a*b*c*g**2 + A*B*a*b*d*g**2*x - A*B*b**2*
c*g**2*x + (B**2*a**2*d*g**2 - B**2*a*b*c*g**2 + B**2*a*b*d*g**2*x - B**2*b
**2*c*g**2*x)*log(e*(c + d*x)/(a + b*x))) + Integral(1/(A*a**2 + 2*A*a*b*x
+ A*b**2*x**2 + B*a**2*log(c*e/(a + b*x) + d*e*x/(a + b*x)) + 2*B*a*b*x*log
(c*e/(a + b*x) + d*e*x/(a + b*x)) + B*b**2*x**2*log(c*e/(a + b*x) + d*e*x/(
a + b*x))), x)/(B*g**2)
```

# 3.200 $\int \frac{1}{(ag+bgx)^3\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2}\,dx$

**Optimal**. Leaf size=159

$$-\frac{2be^{-\frac{2A}{B}}\operatorname{Ei}\left(\frac{2\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)}{B}\right)}{B^2e^2g^3(bc-ad)^2}+\frac{de^{-\frac{A}{B}}\operatorname{Ei}\left(\frac{A+B\log\left(\frac{e(c+dx)}{a+bx}\right)}{B}\right)}{B^2eg^3(bc-ad)^2}+\frac{c+dx}{Bg^3(a+bx)^2(bc-ad)\left(B\log\left(\frac{e(c+dx)}{a+bx}\right)+A\right)}$$

```
[Out] d*Ei((A+B*ln(e*(d*x+c)/(b*x+a)))/B)/B^2/(-a*d+b*c)^2/e/exp(A/B)/g^3-2*b*Ei(
2*(A+B*ln(e*(d*x+c)/(b*x+a)))/B)/B^2/(-a*d+b*c)^2/e^2/exp(2*A/B)/g^3+(d*x+c
)/B/(-a*d+b*c)/g^3/(b*x+a)^2/(A+B*ln(e*(d*x+c)/(b*x+a)))
```

**Rubi [F]** time = 0.08, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{1}{(ag+bgx)^3\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[1/((a*g + b*g*x)^3*(A + B*Log[(e*(c + d*x))/(a + b*x)])^2),x]
[Out] Defer[Int][1/((a*g + b*g*x)^3*(A + B*Log[(e*(c + d*x))/(a + b*x)])^2), x]
```

Rubi steps

$$\int \frac{1}{(ag+bgx)^3\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2}\,dx=\int \frac{1}{(ag+bgx)^3\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2}\,dx$$

**Mathematica [A]** time = 0.41, size = 135, normalized size = 0.85

$$\frac{-\frac{2be^{-\frac{2A}{B}}\operatorname{Ei}\left(\frac{2\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)}{B}\right)}{e^2}+\frac{de^{-\frac{A}{B}}\operatorname{Ei}\left(\frac{A}{B}+\log\left(\frac{e(c+dx)}{a+bx}\right)\right)}{e}+\frac{B(c+dx)(bc-ad)}{(a+bx)^2\left(B\log\left(\frac{e(c+dx)}{a+bx}\right)+A\right)}}{B^2g^3(bc-ad)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a*g + b*g*x)^3*(A + B*Log[(e*(c + d*x))/(a + b*x)])^2),x]
```

```
[Out] ((d*ExpIntegralEi[A/B + Log[(e*(c + d*x))/(a + b*x)]])/(e*E^(A/B)) - (2*b*E
xpIntegralEi[(2*(A + B*Log[(e*(c + d*x))/(a + b*x)]))/B])/(e^2*E^((2*A)/B))
 + (B*(b*c - a*d)*(c + d*x))/((a + b*x)^2*(A + B*Log[(e*(c + d*x))/(a + b*x
)])))/(B^2*(b*c - a*d)^2*g^3)
```

**fricas [B]** time = 0.53, size = 584, normalized size = 3.67

$$\frac{\left(\left(Bbcd - Bad^2\right)e^2x + \left(Bbc^2 - Bacd\right)e^2\right)e^{\left(\frac{2A}{B}\right)} - 2\left(Ab^3x^2 + 2\,Aab^2x + Aa^2b + \left(Bb^3x^2 + 2\,Bab^2x + \right.\right.}{\left(\left(B^3b^4c^2 - 2\,B^3ab^3cd + B^3a^2b^2d^2\right)e^2g^3x^2 + 2\left(B^3ab^3c^2 - 2\,B^3a^2b^2cd + B^3a^3bd^2\right)e^2g^3x + \left(B^3a^2b^2c^2 - 2\,B^3a^3bcd \right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*g*x+a*g)^3/(A+B*log(e*(d*x+c)/(b*x+a)))^2,x, algorithm="fric
as")
```

```
[Out] (((B*b*c*d - B*a*d^2)*e^2*x + (B*b*c^2 - B*a*c*d)*e^2)*e^(2*A/B) - 2*(A*b^3
*x^2 + 2*A*a*b^2*x + A*a^2*b + (B*b^3*x^2 + 2*B*a*b^2*x + B*a^2*b)*log((d*e
*x + c*e)/(b*x + a)))*log_integral((d^2*e^2*x^2 + 2*c*d*e^2*x + c^2*e^2)*e^
(2*A/B)/(b^2*x^2 + 2*a*b*x + a^2)) + ((B*b^2*d*e*x^2 + 2*B*a*b*d*e*x + B*a^
2*d*e)*e^(A/B)*log((d*e*x + c*e)/(b*x + a)) + (A*b^2*d*e*x^2 + 2*A*a*b*d*e*
x + A*a^2*d*e)*e^(A/B))*log_integral((d*e*x + c*e)*e^(A/B)/(b*x + a)))/(((B
^3*b^4*c^2 - 2*B^3*a*b^3*c*d + B^3*a^2*b^2*d^2)*e^2*g^3*x^2 + 2*(B^3*a*b^3*
c^2 - 2*B^3*a^2*b^2*c*d + B^3*a^3*b*d^2)*e^2*g^3*x + (B^3*a^2*b^2*c^2 - 2*B
^3*a^3*b*c*d + B^3*a^4*d^2)*e^2*g^3)*e^(2*A/B)*log((d*e*x + c*e)/(b*x + a))
 + ((A*B^2*b^4*c^2 - 2*A*B^2*a*b^3*c*d + A*B^2*a^2*b^2*d^2)*e^2*g^3*x^2 + 2
*(A*B^2*a*b^3*c^2 - 2*A*B^2*a^2*b^2*c*d + A*B^2*a^3*b*d^2)*e^2*g^3*x + (A*B
^2*a^2*b^2*c^2 - 2*A*B^2*a^3*b*c*d + A*B^2*a^4*d^2)*e^2*g^3)*e^(2*A/B))
```

**giac [B]** time = 1.83, size = 317, normalized size = 1.99

$$\left(\frac{d\mathrm{Ei}\left(\frac{A}{B} + \log\left(\frac{dxe+ce}{bx+a}\right)\right)e^{\left(-\frac{A}{B}+1\right)}}{B^2bcg^3e - B^2adg^3e} - \frac{2\,b\mathrm{Ei}\left(\frac{2\,A}{B} + 2\,\log\left(\frac{dxe+ce}{bx+a}\right)\right)e^{\left(-\frac{2\,A}{B}\right)}}{B^2bcg^3e - B^2adg^3e} - \frac{\frac{(dxe+ce)de}{bx+a} - \frac{(dxe+c}{(bx+}}{B^2bcg^3e\log\left(\frac{dxe+ce}{bx+a}\right) - B^2adg^3e\log\left(\frac{dxe}{bx}\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*g*x+a*g)^3/(A+B*log(e*(d*x+c)/(b*x+a)))^2,x, algorithm="giac
")
```

```
[Out] (d*Ei(A/B + log((d*x*e + c*e)/(b*x + a)))*e^(-A/B + 1)/(B^2*b*c*g^3*e - B^2
*a*d*g^3*e) - 2*b*Ei(2*A/B + 2*log((d*x*e + c*e)/(b*x + a)))*e^(-2*A/B)/(B^
2*b*c*g^3*e - B^2*a*d*g^3*e) - ((d*x*e + c*e)*d*e/(b*x + a) - (d*x*e + c*e)
^2*b/(b*x + a)^2)/(B^2*b*c*g^3*e*log((d*x*e + c*e)/(b*x + a)) - B^2*a*d*g^3
*e*log((d*x*e + c*e)/(b*x + a)) + A*B*b*c*g^3*e - A*B*a*d*g^3*e))*(b*c/((b*
c*e - a*d*e)*(b*c - a*d)) - a*d/((b*c*e - a*d*e)*(b*c - a*d)))
```

**maple [F]** time = 1.49, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(bgx+ag\right)^{3} \left(B\ln\left(\frac{(dx+c)e}{bx+a}\right)+A\right)^{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*g*x+a*g)^3/(B*ln((d*x+c)/(b*x+a)*e)+A)^2,x)
```

```
[Out] int(1/(b*g*x+a*g)^3/(B*ln((d*x+c)/(b*x+a)*e)+A)^2,x)
```

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$\overline{\left(a^2bcg^3-a^3dg^3\right)AB+\left(a^2bcg^3\log(e)-a^3dg^3\log(e)\right)B^2+\left(\left(b^3cg^3-ab^2dg^3\right)AB+\left(b^3cg^3\log(e)-ab^2dg^3\log(e)\right)\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*g*x+a*g)^3/(A+B*log(e*(d*x+c)/(b*x+a)))^2,x, algorithm="maxi
ma")
```

```
[Out] (d*x + c)/((a^2*b*c*g^3 - a^3*d*g^3)*A*B + (a^2*b*c*g^3*log(e) - a^3*d*g^3*
log(e))*B^2 + ((b^3*c*g^3 - a*b^2*d*g^3)*A*B + (b^3*c*g^3*log(e) - a*b^2*d*
g^3*log(e))*B^2)*x^2 + 2*((a*b^2*c*g^3 - a^2*b*d*g^3)*A*B + (a*b^2*c*g^3*lo
g(e) - a^2*b*d*g^3*log(e))*B^2)*x - ((b^3*c*g^3 - a*b^2*d*g^3)*B^2*x^2 + 2*
(a*b^2*c*g^3 - a^2*b*d*g^3)*B^2*x + (a^2*b*c*g^3 - a^3*d*g^3)*B^2)*log(b*x
+ a) + ((b^3*c*g^3 - a*b^2*d*g^3)*B^2*x^2 + 2*(a*b^2*c*g^3 - a^2*b*d*g^3)*B
^2*x + (a^2*b*c*g^3 - a^3*d*g^3)*B^2)*log(d*x + c)) - integrate(-(b*d*x + 2
*b*c - a*d)/(((b^4*c*g^3 - a*b^3*d*g^3)*A*B + (b^4*c*g^3*log(e) - a*b^3*d*g
^3*log(e))*B^2)*x^3 + (a^3*b*c*g^3 - a^4*d*g^3)*A*B + (a^3*b*c*g^3*log(e) -
 a^4*d*g^3*log(e))*B^2 + 3*((a*b^3*c*g^3 - a^2*b^2*d*g^3)*A*B + (a*b^3*c*g^
3*log(e) - a^2*b^2*d*g^3*log(e))*B^2)*x^2 + 3*((a^2*b^2*c*g^3 - a^3*b*d*g^3
)*A*B + (a^2*b^2*c*g^3*log(e) - a^3*b*d*g^3*log(e))*B^2)*x - ((b^4*c*g^3 -
a*b^3*d*g^3)*B^2*x^3 + 3*(a*b^3*c*g^3 - a^2*b^2*d*g^3)*B^2*x^2 + 3*(a^2*b^2
*c*g^3 - a^3*b*d*g^3)*B^2*x + (a^3*b*c*g^3 - a^4*d*g^3)*B^2)*log(b*x + a) +
 ((b^4*c*g^3 - a*b^3*d*g^3)*B^2*x^3 + 3*(a*b^3*c*g^3 - a^2*b^2*d*g^3)*B^2*x
^2 + 3*(a^2*b^2*c*g^3 - a^3*b*d*g^3)*B^2*x + (a^3*b*c*g^3 - a^4*d*g^3)*B^2)
*log(d*x + c)), x)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\left(a\,g+b\,g\,x\right)^3\,\left(A+B\,\ln\left(\frac{e\,\left(c+d\,x\right)}{a+b\,x}\right)\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*g + b*g*x)^3*(A + B*log((e*(c + d*x))/(a + b*x)))^2),x)
[Out] int(1/((a*g + b*g*x)^3*(A + B*log((e*(c + d*x))/(a + b*x)))^2), x)
```

**sympy [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*g*x+a*g)**3/(A+B*ln(e*(d*x+c)/(b*x+a)))**2,x)
[Out] Timed out
```

# 3.201 $\int (ag+bgx)^4 \left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right) dx$

**Optimal**. Leaf size=182

$$\frac{g^4(a+bx)^5\left(B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)+A\right)}{5b}+\frac{2Bg^4(bc-ad)^5\log(c+dx)}{5bd^5}-\frac{2Bg^4x(bc-ad)^4}{5d^4}+\frac{Bg^4(a+bx)^2(bc-ad)^3}{5bd^3}-\frac{2Bg^4}{}$$

```
[Out] -2/5*B*(-a*d+b*c)^4*g^4*x/d^4+1/5*B*(-a*d+b*c)^3*g^4*(b*x+a)^2/b/d^3-2/15*B
*(-a*d+b*c)^2*g^4*(b*x+a)^3/b/d^2+1/10*B*(-a*d+b*c)*g^4*(b*x+a)^4/b/d+2/5*B
*(-a*d+b*c)^5*g^4*ln(d*x+c)/b/d^5+1/5*g^4*(b*x+a)^5*(A+B*ln(e*(d*x+c)^2/(b*
x+a)^2))/b
```

**Rubi [A]** time = 0.12, antiderivative size = 182, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.094, Rules used = {2525, 12, 43}

$$\frac{g^4(a+bx)^5\left(B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)+A\right)}{5b}-\frac{2Bg^4x(bc-ad)^4}{5d^4}+\frac{Bg^4(a+bx)^2(bc-ad)^3}{5bd^3}-\frac{2Bg^4(a+bx)^3(bc-ad)^2}{15bd^2}+\frac{2Bg^4(bc}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a*g + b*g*x)^4*(A + B*Log[(e*(c + d*x)^2)/(a + b*x)^2]),x]
```

```
[Out] (-2*B*(b*c - a*d)^4*g^4*x)/(5*d^4) + (B*(b*c - a*d)^3*g^4*(a + b*x)^2)/(5*b
*d^3) - (2*B*(b*c - a*d)^2*g^4*(a + b*x)^3)/(15*b*d^2) + (B*(b*c - a*d)*g^4
*(a + b*x)^4)/(10*b*d) + (2*B*(b*c - a*d)^5*g^4*Log[c + d*x])/(5*b*d^5) + (
g^4*(a + b*x)^5*(A + B*Log[(e*(c + d*x)^2)/(a + b*x)^2]))/(5*b)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2525

```
Int[((a_.) + Log[(c_.)*(RFx_)^(p_.)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_))^(m_.
), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(a + b*Log[c*RFx^p])^n)/(e*(m + 1))
```

```
, x] - Dist[(b*n*p)/(e*(m + 1)), Int[SimplifyIntegrand[((d + e*x)^(m + 1)*(
a + b*Log[c*RFx^p])^(n - 1)*D[RFx, x])/RFx, x], x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && RationalFunctionQ[RFx, x] && IGtQ[n, 0] && (EqQ[n, 1] ||
IntegerQ[m]) && NeQ[m, -1]
```

Rubi steps

$$\int (ag+bgx)^4\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)dx=\frac{g^4(a+bx)^5\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)}{5b}-\frac{B\int\frac{2(-bc+ad)g^5(a+bx)^4}{c+dx}\,dx}{5bg}$$

$$=\frac{g^4(a+bx)^5\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)}{5b}+\frac{\left(2B(bc-ad)g^4\right)\int\frac{(a+bx)^4}{c+dx}\,dx}{5b}$$

$$=\frac{g^4(a+bx)^5\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)}{5b}+\frac{\left(2B(bc-ad)g^4\right)\int\left(-\frac{b(bc-ad)^3}{d^4}\right.}{}$$

$$=-\frac{2B(bc-ad)^4g^4x}{5d^4}+\frac{B(bc-ad)^3g^4(a+bx)^2}{5bd^3}-\frac{2B(bc-ad)^2g^4(a}{15bd^2}$$

**Mathematica [A]** time = 0.10, size = 144, normalized size = 0.79

$$\frac{g^4\left((a+bx)^5\left(B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)+A\right)-\frac{B(ad-bc)\left(4d^3(a+bx)^3(ad-bc)+6d^2(a+bx)^2(bc-ad)^2-12bdx(bc-ad)^3+12(bc-ad)^4\log(c+dx)+3d^4(a\right.}{6d^5}}{5b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*g + b*g*x)^4*(A + B*Log[(e*(c + d*x)^2)/(a + b*x)^2]),x]
```

```
[Out] (g^4*(-1/6*(B*(-(b*c) + a*d)*(-12*b*d*(b*c - a*d)^3*x + 6*d^2*(b*c - a*d)^2
*(a + b*x)^2 + 4*d^3*(-(b*c) + a*d)*(a + b*x)^3 + 3*d^4*(a + b*x)^4 + 12*(b
*c - a*d)^4*Log[c + d*x]))/d^5 + (a + b*x)^5*(A + B*Log[(e*(c + d*x)^2)/(a
+ b*x)^2])))/(5*b)
```

**fricas [B]** time = 0.70, size = 457, normalized size = 2.51

$$\frac{6\,Ab^5d^5g^4x^5-12\,Ba^5d^5g^4\log(bx+a)+3\left(Bb^5cd^4+(10\,A-B)ab^4d^5\right)g^4x^4-4\left(Bb^5c^2d^3-5\,Bab^4cd^4-(15\,A-\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*g*x+a*g)^4*(A+B*log(e*(d*x+c)^2/(b*x+a)^2)),x, algorithm="fric
as")
```

```
[Out] 1/30*(6*A*b^5*d^5*g^4*x^5 - 12*B*a^5*d^5*g^4*log(b*x + a) + 3*(B*b^5*c*d^4
+ (10*A - B)*a*b^4*d^5)*g^4*x^4 - 4*(B*b^5*c^2*d^3 - 5*B*a*b^4*c*d^4 - (15*
A - 4*B)*a^2*b^3*d^5)*g^4*x^3 + 6*(B*b^5*c^3*d^2 - 5*B*a*b^4*c^2*d^3 + 10*B
*a^2*b^3*c*d^4 + 2*(5*A - 3*B)*a^3*b^2*d^5)*g^4*x^2 - 6*(2*B*b^5*c^4*d - 10
*B*a*b^4*c^3*d^2 + 20*B*a^2*b^3*c^2*d^3 - 20*B*a^3*b^2*c*d^4 - (5*A - 8*B)*
a^4*b*d^5)*g^4*x + 12*(B*b^5*c^5 - 5*B*a*b^4*c^4*d + 10*B*a^2*b^3*c^3*d^2 -
 10*B*a^3*b^2*c^2*d^3 + 5*B*a^4*b*c*d^4)*g^4*log(d*x + c) + 6*(B*b^5*d^5*g^
4*x^5 + 5*B*a*b^4*d^5*g^4*x^4 + 10*B*a^2*b^3*d^5*g^4*x^3 + 10*B*a^3*b^2*d^5
*g^4*x^2 + 5*B*a^4*b*d^5*g^4*x)*log((d^2*e*x^2 + 2*c*d*e*x + c^2*e)/(b^2*x^
2 + 2*a*b*x + a^2)))/(b*d^5)
```

**giac [B]** time = 78.00, size = 493, normalized size = 2.71

$$-\frac{2\,Ba^5g^4\log\left(bx+a\right)}{5\,b}+\frac{1}{5}\left(Ab^4g^4+Bb^4g^4\right)x^5+\frac{\left(Bb^4cg^4+10\,Aab^3dg^4+9\,Bab^3dg^4\right)x^4}{10\,d}-\frac{2\left(Bb^4c^2g^4-5\,Bab^3cdg^4\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*g*x+a*g)^4*(A+B*log(e*(d*x+c)^2/(b*x+a)^2)),x, algorithm="giac
")
```

```
[Out] -2/5*B*a^5*g^4*log(b*x + a)/b + 1/5*(A*b^4*g^4 + B*b^4*g^4)*x^5 + 1/10*(B*b
^4*c*g^4 + 10*A*a*b^3*d*g^4 + 9*B*a*b^3*d*g^4)*x^4/d - 2/15*(B*b^4*c^2*g^4
- 5*B*a*b^3*c*d*g^4 - 15*A*a^2*b^2*d^2*g^4 - 11*B*a^2*b^2*d^2*g^4)*x^3/d^2
+ 1/5*(B*b^4*g^4*x^5 + 5*B*a*b^3*g^4*x^4 + 10*B*a^2*b^2*g^4*x^3 + 10*B*a^3*
b*g^4*x^2 + 5*B*a^4*g^4*x)*log((d^2*x^2 + 2*c*d*x + c^2)/(b^2*x^2 + 2*a*b*x
 + a^2)) + 1/5*(B*b^4*c^3*g^4 - 5*B*a*b^3*c^2*d*g^4 + 10*B*a^2*b^2*c*d^2*g^
4 + 10*A*a^3*b*d^3*g^4 + 4*B*a^3*b*d^3*g^4)*x^2/d^3 - 1/5*(2*B*b^4*c^4*g^4
- 10*B*a*b^3*c^3*d*g^4 + 20*B*a^2*b^2*c^2*d^2*g^4 - 20*B*a^3*b*c*d^3*g^4 -
5*A*a^4*d^4*g^4 + 3*B*a^4*d^4*g^4)*x/d^4 + 2/5*(B*b^4*c^5*g^4 - 5*B*a*b^3*c
^4*d*g^4 + 10*B*a^2*b^2*c^3*d^2*g^4 - 10*B*a^3*b*c^2*d^3*g^4 + 5*B*a^4*c*d^
4*g^4)*log(d*x + c)/d^5
```

**maple [B]** time = 0.13, size = 1030, normalized size = 5.66

$$\frac{B\,b^4g^4x^5\ln\left(\frac{\left(\frac{ad}{bx+a}-\frac{bc}{bx+a}-d\right)^2e}{b^2}\right)}{5}+\frac{A\,b^4g^4x^5}{5}+Ba\,b^3g^4x^4\ln\left(\frac{\left(\frac{ad}{bx+a}-\frac{bc}{bx+a}-d\right)^2e}{b^2}\right)+Aa\,b^3g^4x^4+2B\,a^2b^2g^4x^3\ln\left(\frac{\left(\frac{ad}{bx+a}\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*g*x+a*g)^4*(A+B*ln(e*(d*x+c)^2/(b*x+a)^2)),x)
```

```
[Out] -5/6/b*g^4*B*a^5+1/5/b*A*a^5*g^4+1/5*b^4*A*x^5*g^4-8/5*B*x*a^4*g^4-2/5/b*g^
4*B*a^5*ln(a/(b*x+a)*d-1/(b*x+a)*b*c-d)+1/5*b^4*B*ln(e*(a/(b*x+a)*d-1/(b*x+
a)*b*c-d)^2/b^2)*x^5*g^4+1/5/b*B*ln(e*(a/(b*x+a)*d-1/(b*x+a)*b*c-d)^2/b^2)*
a^5*g^4+B*ln(e*(a/(b*x+a)*d-1/(b*x+a)*b*c-d)^2/b^2)*x*a^4*g^4+2/5/b*g^4*B*a
^5*ln(1/(b*x+a))-8/15*b^2*B*x^3*a^2*g^4-6/5*b*B*x^2*a^3*g^4+b^3*A*x^4*a*g^4
+2*b^2*A*x^3*a^2*g^4+2*b*A*x^2*a^3*g^4-1/10*b^3*B*x^4*a*g^4+2*b^2*B*ln(e*(a
/(b*x+a)*d-1/(b*x+a)*b*c-d)^2/b^2)*x^3*a^2*g^4+2*b*B*ln(e*(a/(b*x+a)*d-1/(b
*x+a)*b*c-d)^2/b^2)*x^2*a^3*g^4+2*g^4*B*a^4/d*ln(a/(b*x+a)*d-1/(b*x+a)*b*c-
d)*c-2*g^4*B*a^4/d*ln(1/(b*x+a))*c+A*a^4*g^4*x+9/5*b^2*g^4*B*c^3/d^3*a^2-47
/15*b*g^4*B*c^2/d^2*a^3-2/5*b^3*g^4*B*c^4/d^4*a+77/30*g^4*B*c/d*a^4+1/5*b^4
*g^4*B*c^3/d^3*x^2-2/15*b^4*g^4*B*c^2/d^2*x^3-2/5*b^4*g^4*B*c^4/d^4*x+1/10*
b^4*g^4*B*c/d*x^4-2/5*b^4*g^4*B*c^5/d^5*ln(1/(b*x+a))+2/5*b^4*g^4*B*c^5/d^5
*ln(a/(b*x+a)*d-1/(b*x+a)*b*c-d)+b^3*B*ln(e*(a/(b*x+a)*d-1/(b*x+a)*b*c-d)^2
/b^2)*x^4*a*g^4-4*b*g^4*B*a^3/d^2*ln(a/(b*x+a)*d-1/(b*x+a)*b*c-d)*c^2+4*b*g
^4*B*a^3/d^2*ln(1/(b*x+a))*c^2+4*b*g^4*B*c/d*x*a^3+2*b^3*g^4*B*a/d^4*ln(1/(
b*x+a))*c^4-4*b^2*g^4*B*a^2/d^3*ln(1/(b*x+a))*c^3+4*b^2*g^4*B*a^2/d^3*ln(a/
(b*x+a)*d-1/(b*x+a)*b*c-d)*c^3-2*b^3*g^4*B*a/d^4*ln(a/(b*x+a)*d-1/(b*x+a)*b
*c-d)*c^4-b^3*g^4*B*c^2/d^2*x^2*a-4*b^2*g^4*B*c^2/d^2*x*a^2+2*b^3*g^4*B*c^3
/d^3*x*a+2/3*b^3*g^4*B*c/d*x^3*a+2*b^2*g^4*B*c/d*x^2*a^2
```

**maxima [B]** time = 1.40, size = 882, normalized size = 4.85

$$\frac{1}{5}Ab^4g^4x^5+Aab^3g^4x^4+2\,Aa^2b^2g^4x^3+2\,Aa^3bg^4x^2+\left(x\log\left(\frac{d^2ex^2}{b^2x^2+2\,abx+a^2}+\frac{2\,cdex}{b^2x^2+2\,abx+a^2}+\frac{c^2e}{b^2x^2+2\,ab\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*g*x+a*g)^4*(A+B*log(e*(d*x+c)^2/(b*x+a)^2)),x, algorithm="maxi
ma")
```

```
[Out] 1/5*A*b^4*g^4*x^5 + A*a*b^3*g^4*x^4 + 2*A*a^2*b^2*g^4*x^3 + 2*A*a^3*b*g^4*x
^2 + (x*log(d^2*e*x^2/(b^2*x^2 + 2*a*b*x + a^2) + 2*c*d*e*x/(b^2*x^2 + 2*a*
b*x + a^2) + c^2*e/(b^2*x^2 + 2*a*b*x + a^2)) - 2*a*log(b*x + a)/b + 2*c*lo
g(d*x + c)/d)*B*a^4*g^4 + 2*(x^2*log(d^2*e*x^2/(b^2*x^2 + 2*a*b*x + a^2) +
2*c*d*e*x/(b^2*x^2 + 2*a*b*x + a^2) + c^2*e/(b^2*x^2 + 2*a*b*x + a^2)) + 2*
a^2*log(b*x + a)/b^2 - 2*c^2*log(d*x + c)/d^2 + 2*(b*c - a*d)*x/(b*d))*B*a^
3*b*g^4 + 2*(x^3*log(d^2*e*x^2/(b^2*x^2 + 2*a*b*x + a^2) + 2*c*d*e*x/(b^2*x
^2 + 2*a*b*x + a^2) + c^2*e/(b^2*x^2 + 2*a*b*x + a^2)) - 2*a^3*log(b*x + a)
/b^3 + 2*c^3*log(d*x + c)/d^3 + ((b^2*c*d - a*b*d^2)*x^2 - 2*(b^2*c^2 - a^2
*d^2)*x)/(b^2*d^2))*B*a^2*b^2*g^4 + 1/3*(3*x^4*log(d^2*e*x^2/(b^2*x^2 + 2*a
*b*x + a^2) + 2*c*d*e*x/(b^2*x^2 + 2*a*b*x + a^2) + c^2*e/(b^2*x^2 + 2*a*b*
x + a^2)) + 6*a^4*log(b*x + a)/b^4 - 6*c^4*log(d*x + c)/d^4 + (2*(b^3*c*d^2
 - a*b^2*d^3)*x^3 - 3*(b^3*c^2*d - a^2*b*d^3)*x^2 + 6*(b^3*c^3 - a^3*d^3)*x
)/(b^3*d^3))*B*a*b^3*g^4 + 1/30*(6*x^5*log(d^2*e*x^2/(b^2*x^2 + 2*a*b*x + a
```

```
^2) + 2*c*d*e*x/(b^2*x^2 + 2*a*b*x + a^2) + c^2*e/(b^2*x^2 + 2*a*b*x + a^2)
) - 12*a^5*log(b*x + a)/b^5 + 12*c^5*log(d*x + c)/d^5 + (3*(b^4*c*d^3 - a*b
^3*d^4)*x^4 - 4*(b^4*c^2*d^2 - a^2*b^2*d^4)*x^3 + 6*(b^4*c^3*d - a^3*b*d^4)
*x^2 - 12*(b^4*c^4 - a^4*d^4)*x)/(b^4*d^4))*B*b^4*g^4 + A*a^4*g^4*x
```

**mupad [B]** time = 4.79, size = 1024, normalized size = 5.63

$$x^2 \left( \frac{(5\,a\,d+5\,b\,c)\left(\frac{\left(\frac{b^3\,g^4\,(25\,A\,a\,d+5\,A\,b\,c-2\,B\,a\,d+2\,B\,b\,c)}{5\,d}-\frac{A\,b^3\,g^4\,(5\,a\,d+5\,b\,c)}{5\,d}\right)(5\,a\,d+5\,b\,c)}{5\,b\,d}-\frac{a\,b^2\,g^4\,(10\,A\,a\,d+5\,A\,b\,c-2\,B\,a\,d+2\,B\,b\,c)}{d}+\frac{A\,a\,b^3\,c\,g^4}{d}\right.}{10\,b\,d}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*g + b*g*x)^4*(A + B*log((e*(c + d*x)^2)/(a + b*x)^2)),x)
```

```
[Out] x^2*(((5*a*d + 5*b*c)*((((b^3*g^4*(25*A*a*d + 5*A*b*c - 2*B*a*d + 2*B*b*c))
/(5*d) - (A*b^3*g^4*(5*a*d + 5*b*c))/(5*d))*(5*a*d + 5*b*c))/(5*b*d) - (a*b
^2*g^4*(10*A*a*d + 5*A*b*c - 2*B*a*d + 2*B*b*c))/d + (A*a*b^3*c*g^4)/d))/(1
0*b*d) + (a^2*b*g^4*(5*A*a*d + 5*A*b*c - 2*B*a*d + 2*B*b*c))/d - (a*c*((b^3
*g^4*(25*A*a*d + 5*A*b*c - 2*B*a*d + 2*B*b*c))/(5*d) - (A*b^3*g^4*(5*a*d +
5*b*c))/(5*d)))/(2*b*d)) - x^3*((((b^3*g^4*(25*A*a*d + 5*A*b*c - 2*B*a*d +
2*B*b*c))/(5*d) - (A*b^3*g^4*(5*a*d + 5*b*c))/(5*d))*(5*a*d + 5*b*c))/(15*b
*d) - (a*b^2*g^4*(10*A*a*d + 5*A*b*c - 2*B*a*d + 2*B*b*c))/(3*d) + (A*a*b^3
*c*g^4)/(3*d)) + x*((a^3*g^4*(5*A*a*d + 10*A*b*c - 4*B*a*d + 4*B*b*c))/d -
((5*a*d + 5*b*c)*(((5*a*d + 5*b*c)*((((b^3*g^4*(25*A*a*d + 5*A*b*c - 2*B*a*
d + 2*B*b*c))/(5*d) - (A*b^3*g^4*(5*a*d + 5*b*c))/(5*d))*(5*a*d + 5*b*c))/(
5*b*d) - (a*b^2*g^4*(10*A*a*d + 5*A*b*c - 2*B*a*d + 2*B*b*c))/d + (A*a*b^3*
c*g^4)/d))/(5*b*d) + (2*a^2*b*g^4*(5*A*a*d + 5*A*b*c - 2*B*a*d + 2*B*b*c))/
d - (a*c*((b^3*g^4*(25*A*a*d + 5*A*b*c - 2*B*a*d + 2*B*b*c))/(5*d) - (A*b^3
*g^4*(5*a*d + 5*b*c))/(5*d)))/(b*d)))/(5*b*d) + (a*c*((((b^3*g^4*(25*A*a*d
+ 5*A*b*c - 2*B*a*d + 2*B*b*c))/(5*d) - (A*b^3*g^4*(5*a*d + 5*b*c))/(5*d))*
(5*a*d + 5*b*c))/(5*b*d) - (a*b^2*g^4*(10*A*a*d + 5*A*b*c - 2*B*a*d + 2*B*b
*c))/d + (A*a*b^3*c*g^4)/d))/(b*d)) + log((e*(c + d*x)^2)/(a + b*x)^2)*((B*
b^4*g^4*x^5)/5 + B*a^4*g^4*x + 2*B*a^3*b*g^4*x^2 + B*a*b^3*g^4*x^4 + 2*B*a^
2*b^2*g^4*x^3) + x^4*((b^3*g^4*(25*A*a*d + 5*A*b*c - 2*B*a*d + 2*B*b*c))/(2
0*d) - (A*b^3*g^4*(5*a*d + 5*b*c))/(20*d)) + (log(c + d*x)*((2*B*b^4*c^5*g^
4)/5 + 2*B*a^4*c*d^4*g^4 - 4*B*a^3*b*c^2*d^3*g^4 + 4*B*a^2*b^2*c^3*d^2*g^4
```

```
- 2*B*a*b^3*c^4*d*g^4))/d^5 + (A*b^4*g^4*x^5)/5 - (2*B*a^5*g^4*log(a + b*x)
)/(5*b)
```

**sympy [B]** time = 6.70, size = 998, normalized size = 5.48

$$\frac{Ab^4g^4x^5}{5}-\frac{2Ba^5g^4\log\left(x+\frac{\frac{2Ba^6d^5g^4}{b}+10Ba^5cd^4g^4-20Ba^4bc^2d^3g^4+20Ba^3b^2c^3d^2g^4-10Ba^2b^3c^4dg^4+2Bab^4c^5g^4}{2Ba^5d^5g^4+10Ba^4bcd^4g^4-20Ba^3b^2c^2d^3g^4+20Ba^2b^3c^3d^2g^4-10Bab^4c^4dg^4+2Bb^5c^5g^4}\right)}{5b}+\frac{2Bcg^4\left(5a^4d^4-10\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*g*x+a*g)**4*(A+B*ln(e*(d*x+c)**2/(b*x+a)**2)),x)
```

```
[Out] A*b**4*g**4*x**5/5 - 2*B*a**5*g**4*log(x + (2*B*a**6*d**5*g**4/b + 10*B*a**
5*c*d**4*g**4 - 20*B*a**4*b*c**2*d**3*g**4 + 20*B*a**3*b**2*c**3*d**2*g**4
- 10*B*a**2*b**3*c**4*d*g**4 + 2*B*a*b**4*c**5*g**4)/(2*B*a**5*d**5*g**4 +
10*B*a**4*b*c*d**4*g**4 - 20*B*a**3*b**2*c**2*d**3*g**4 + 20*B*a**2*b**3*c*
*3*d**2*g**4 - 10*B*a*b**4*c**4*d*g**4 + 2*B*b**5*c**5*g**4))/(5*b) + 2*B*c
*g**4*(5*a**4*d**4 - 10*a**3*b*c*d**3 + 10*a**2*b**2*c**2*d**2 - 5*a*b**3*c
**3*d + b**4*c**4)*log(x + (12*B*a**5*c*d**4*g**4 - 20*B*a**4*b*c**2*d**3*g
**4 + 20*B*a**3*b**2*c**3*d**2*g**4 - 10*B*a**2*b**3*c**4*d*g**4 + 2*B*a*b*
*4*c**5*g**4 - 2*B*a*c*g**4*(5*a**4*d**4 - 10*a**3*b*c*d**3 + 10*a**2*b**2*
c**2*d**2 - 5*a*b**3*c**3*d + b**4*c**4) + 2*B*b*c**2*g**4*(5*a**4*d**4 - 1
0*a**3*b*c*d**3 + 10*a**2*b**2*c**2*d**2 - 5*a*b**3*c**3*d + b**4*c**4)/d)/
(2*B*a**5*d**5*g**4 + 10*B*a**4*b*c*d**4*g**4 - 20*B*a**3*b**2*c**2*d**3*g*
*4 + 20*B*a**2*b**3*c**3*d**2*g**4 - 10*B*a*b**4*c**4*d*g**4 + 2*B*b**5*c**
5*g**4))/(5*d**5) + x**4*(A*a*b**3*g**4 - B*a*b**3*g**4/10 + B*b**4*c*g**4/
(10*d)) + x**3*(2*A*a**2*b**2*g**4 - 8*B*a**2*b**2*g**4/15 + 2*B*a*b**3*c*g
**4/(3*d) - 2*B*b**4*c**2*g**4/(15*d**2)) + x**2*(2*A*a**3*b*g**4 - 6*B*a**
3*b*g**4/5 + 2*B*a**2*b**2*c*g**4/d - B*a*b**3*c**2*g**4/d**2 + B*b**4*c**3
*g**4/(5*d**3)) + x*(A*a**4*g**4 - 8*B*a**4*g**4/5 + 4*B*a**3*b*c*g**4/d -
4*B*a**2*b**2*c**2*g**4/d**2 + 2*B*a*b**3*c**3*g**4/d**3 - 2*B*b**4*c**4*g*
*4/(5*d**4)) + (B*a**4*g**4*x + 2*B*a**3*b*g**4*x**2 + 2*B*a**2*b**2*g**4*x
**3 + B*a*b**3*g**4*x**4 + B*b**4*g**4*x**5/5)*log(e*(c + d*x)**2/(a + b*x)
**2)
```

# 3.202 $\int (ag+bgx)^3 \left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)\,dx$

**Optimal**. Leaf size=151

$$\frac{g^3(a+bx)^4\left(B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)+A\right)}{4b}-\frac{Bg^3(bc-ad)^4\log(c+dx)}{2bd^4}+\frac{Bg^3x(bc-ad)^3}{2d^3}-\frac{Bg^3(a+bx)^2(bc-ad)^2}{4bd^2}+\frac{Bg^3(a+$$

```
[Out] 1/2*B*(-a*d+b*c)^3*g^3*x/d^3-1/4*B*(-a*d+b*c)^2*g^3*(b*x+a)^2/b/d^2+1/6*B*(
-a*d+b*c)*g^3*(b*x+a)^3/b/d-1/2*B*(-a*d+b*c)^4*g^3*ln(d*x+c)/b/d^4+1/4*g^3*
(b*x+a)^4*(A+B*ln(e*(d*x+c)^2/(b*x+a)^2))/b
```

**Rubi [A]** time = 0.10, antiderivative size = 151, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.094, Rules used = {2525, 12, 43}

$$\frac{g^3(a+bx)^4\left(B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)+A\right)}{4b}+\frac{Bg^3x(bc-ad)^3}{2d^3}-\frac{Bg^3(a+bx)^2(bc-ad)^2}{4bd^2}-\frac{Bg^3(bc-ad)^4\log(c+dx)}{2bd^4}+\frac{Bg^3(a+$$

Antiderivative was successfully verified.

```
[In] Int[(a*g + b*g*x)^3*(A + B*Log[(e*(c + d*x)^2)/(a + b*x)^2]),x]
```

```
[Out] (B*(b*c - a*d)^3*g^3*x)/(2*d^3) - (B*(b*c - a*d)^2*g^3*(a + b*x)^2)/(4*b*d^
2) + (B*(b*c - a*d)*g^3*(a + b*x)^3)/(6*b*d) - (B*(b*c - a*d)^4*g^3*Log[c +
 d*x])/(2*b*d^4) + (g^3*(a + b*x)^4*(A + B*Log[(e*(c + d*x)^2)/(a + b*x)^2]
))/(4*b)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2525

```
Int[((a_.) + Log[(c_.)*(RFx_)^(p_.)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_))^(m_.
), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(a + b*Log[c*RFx^p])^n)/(e*(m + 1))
, x] - Dist[(b*n*p)/(e*(m + 1)), Int[SimplifyIntegrand[((d + e*x)^(m + 1)*(
```

```
a + b*Log[c*RFx^p])^(n - 1)*D[RFx, x])/RFx, x], x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && RationalFunctionQ[RFx, x] && IGtQ[n, 0] && (EqQ[n, 1] ||
IntegerQ[m]) && NeQ[m, -1]
```

Rubi steps

$$\int (ag+bgx)^3\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)dx=\frac{g^3(a+bx)^4\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)}{4b}-\frac{B\int\frac{2(-bc+ad)g^4(a+bx)^3}{c+dx}\,dx}{4bg}$$
$$=\frac{g^3(a+bx)^4\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)}{4b}+\frac{\left(B(bc-ad)g^3\right)\int\frac{(a+bx)^3}{c+dx}\,dx}{2b}$$
$$=\frac{g^3(a+bx)^4\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)}{4b}+\frac{\left(B(bc-ad)g^3\right)\int\left(\frac{b(bc-ad)^2}{d^3}-\right.}{}$$
$$=\frac{B(bc-ad)^3g^3x}{2d^3}-\frac{B(bc-ad)^2g^3(a+bx)^2}{4bd^2}+\frac{B(bc-ad)g^3(a+bx)^3}{6bd}$$

**Mathematica [A]** time = 0.08, size = 122, normalized size = 0.81

$$\frac{g^3\left((a+bx)^4\left(B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)+A\right)+\frac{B(bc-ad)\left(3d^2(a+bx)^2(ad-bc)+6bdx(bc-ad)^2-6(bc-ad)^3\log(c+dx)+2d^3(a+bx)^3\right)}{3d^4}\right)}{4b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*g + b*g*x)^3*(A + B*Log[(e*(c + d*x)^2)/(a + b*x)^2]),x]
```

```
[Out] (g^3*((B*(b*c - a*d)*(6*b*d*(b*c - a*d)^2*x + 3*d^2*(-(b*c) + a*d)*(a + b*x
)^2 + 2*d^3*(a + b*x)^3 - 6*(b*c - a*d)^3*Log[c + d*x]))/(3*d^4) + (a + b*x
)^4*(A + B*Log[(e*(c + d*x)^2)/(a + b*x)^2])))/(4*b)
```

**fricas [B]** time = 0.53, size = 343, normalized size = 2.27

$$\frac{3\,Ab^4d^4g^3x^4-6\,Ba^4d^4g^3\log(bx+a)+2\left(Bb^4cd^3+(6\,A-B)ab^3d^4\right)g^3x^3-3\left(Bb^4c^2d^2-4\,Bab^3cd^3-3\,(2\,A-B\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*g*x+a*g)^3*(A+B*log(e*(d*x+c)^2/(b*x+a)^2)),x, algorithm="fric
as")
```

```
[Out] 1/12*(3*A*b^4*d^4*g^3*x^4 - 6*B*a^4*d^4*g^3*log(b*x + a) + 2*(B*b^4*c*d^3 +
 (6*A - B)*a*b^3*d^4)*g^3*x^3 - 3*(B*b^4*c^2*d^2 - 4*B*a*b^3*c*d^3 - 3*(2*A
 - B)*a^2*b^2*d^4)*g^3*x^2 + 6*(B*b^4*c^3*d - 4*B*a*b^3*c^2*d^2 + 6*B*a^2*b
^2*c*d^3 + (2*A - 3*B)*a^3*b*d^4)*g^3*x - 6*(B*b^4*c^4 - 4*B*a*b^3*c^3*d +
6*B*a^2*b^2*c^2*d^2 - 4*B*a^3*b*c*d^3)*g^3*log(d*x + c) + 3*(B*b^4*d^4*g^3*
x^4 + 4*B*a*b^3*d^4*g^3*x^3 + 6*B*a^2*b^2*d^4*g^3*x^2 + 4*B*a^3*b*d^4*g^3*x
)*log((d^2*e*x^2 + 2*c*d*e*x + c^2*e)/(b^2*x^2 + 2*a*b*x + a^2)))/(b*d^4)
```

**giac [B]** time = 18.85, size = 364, normalized size = 2.41

$$-\frac{Ba^4g^3\log\left(bx+a\right)}{2\,b}+\frac{1}{4}\left(Ab^3g^3+Bb^3g^3\right)x^4+\frac{\left(Bb^3cg^3+6\,Aab^2dg^3+5\,Bab^2dg^3\right)x^3}{6\,d}+\frac{1}{4}\left(Bb^3g^3x^4+4\,Bab^2g^3x^3+\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*g*x+a*g)^3*(A+B*log(e*(d*x+c)^2/(b*x+a)^2)),x, algorithm="giac
")
```

```
[Out] -1/2*B*a^4*g^3*log(b*x + a)/b + 1/4*(A*b^3*g^3 + B*b^3*g^3)*x^4 + 1/6*(B*b^
3*c*g^3 + 6*A*a*b^2*d*g^3 + 5*B*a*b^2*d*g^3)*x^3/d + 1/4*(B*b^3*g^3*x^4 + 4
*B*a*b^2*g^3*x^3 + 6*B*a^2*b*g^3*x^2 + 4*B*a^3*g^3*x)*log((d^2*x^2 + 2*c*d*
x + c^2)/(b^2*x^2 + 2*a*b*x + a^2)) - 1/4*(B*b^3*c^2*g^3 - 4*B*a*b^2*c*d*g^
3 - 6*A*a^2*b*d^2*g^3 - 3*B*a^2*b*d^2*g^3)*x^2/d^2 + 1/2*(B*b^3*c^3*g^3 - 4
*B*a*b^2*c^2*d*g^3 + 6*B*a^2*b*c*d^2*g^3 + 2*A*a^3*d^3*g^3 - B*a^3*d^3*g^3)
*x/d^3 - 1/2*(B*b^3*c^4*g^3 - 4*B*a*b^2*c^3*d*g^3 + 6*B*a^2*b*c^2*d^2*g^3 -
 4*B*a^3*c*d^3*g^3)*log(-d*x - c)/d^4
```

**maple [B]** time = 0.07, size = 788, normalized size = 5.22

$$\frac{B\,b^3g^3x^4\ln\left(\frac{\left(\frac{ad}{bx+a}-\frac{bc}{bx+a}-d\right)^2e}{b^2}\right)}{4}+\frac{A\,b^3g^3x^4}{4}+Ba\,b^2g^3x^3\ln\left(\frac{\left(\frac{ad}{bx+a}-\frac{bc}{bx+a}-d\right)^2e}{b^2}\right)+Aa\,b^2g^3x^3+\frac{3B\,a^2b\,g^3x^2\ln\left(\frac{\left(\frac{ad}{bx+a}-\right.}{b^2}\right.}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*g*x+a*g)^3*(A+B*ln(e*(d*x+c)^2/(b*x+a)^2)),x)
```

```
[Out] -3/2*B*x*a^3*g^3+1/4/b*A*a^4*g^3-11/12/b*B*a^4*g^3+1/4*b^3*A*x^4*g^3+A*x*a^
3*g^3+B*ln((1/(b*x+a)*a*d-1/(b*x+a)*b*c-d)^2/b^2*e)*x*a^3*g^3-1/6*b^2*B*x^3
*a*g^3-3/4*b*B*x^2*a^2*g^3+3/2*b*A*x^2*a^2*g^3+1/4*b^3*B*ln((1/(b*x+a)*a*d-
1/(b*x+a)*b*c-d)^2/b^2*e)*x^4*g^3-1/2/b*g^3*B*a^4*ln(1/(b*x+a)*a*d-1/(b*x+a
)*b*c-d)+1/4/b*B*ln((1/(b*x+a)*a*d-1/(b*x+a)*b*c-d)^2/b^2*e)*a^4*g^3+1/2/b*
g^3*B*a^4*ln(1/(b*x+a))+b^2*A*x^3*a*g^3+3/2*b*B*ln((1/(b*x+a)*a*d-1/(b*x+a)
*b*c-d)^2/b^2*e)*x^2*a^2*g^3+13/6*g^3*B*c/d*a^3-7/4*b*g^3*B*c^2/d^2*a^2+1/2
```

```
*b^2*g^3*B*c^3/d^3*a+1/6*b^3*g^3*B*c/d*x^3-1/4*b^3*g^3*B*c^2/d^2*x^2+1/2*b^
3*g^3*B*c^3/d^3*x-1/2*b^3*g^3*B*c^4/d^4*ln(1/(b*x+a)*a*d-1/(b*x+a)*b*c-d)+1
/2*b^3*g^3*B*c^4/d^4*ln(1/(b*x+a))+b^2*B*ln((1/(b*x+a)*a*d-1/(b*x+a)*b*c-d)
^2/b^2*e)*x^3*a*g^3+2*g^3*B*a^3/d*ln(1/(b*x+a)*a*d-1/(b*x+a)*b*c-d)*c-2*g^3
*B*a^3/d*ln(1/(b*x+a))*c-2*b^2*g^3*B*a/d^3*ln(1/(b*x+a))*c^3+3*b*g^3*B*a^2/
d^2*ln(1/(b*x+a))*c^2-3*b*g^3*B*a^2/d^2*ln(1/(b*x+a)*a*d-1/(b*x+a)*b*c-d)*c
^2+2*b^2*g^3*B*a/d^3*ln(1/(b*x+a)*a*d-1/(b*x+a)*b*c-d)*c^3+b^2*g^3*B*c/d*x^
2*a+3*b*g^3*B*c/d*x*a^2-2*b^2*g^3*B*c^2/d^2*x*a
```

**maxima [B]** time = 1.31, size = 645, normalized size = 4.27

$$\frac{1}{4}Ab^3g^3x^4+Aab^2g^3x^3+\frac{3}{2}Aa^2bg^3x^2+\left(x\log\left(\frac{d^2ex^2}{b^2x^2+2abx+a^2}+\frac{2cdex}{b^2x^2+2abx+a^2}+\frac{c^2e}{b^2x^2+2abx+a^2}\right)-\frac{2a}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*g*x+a*g)^3*(A+B*log(e*(d*x+c)^2/(b*x+a)^2)),x, algorithm="maxi
ma")
```

```
[Out] 1/4*A*b^3*g^3*x^4 + A*a*b^2*g^3*x^3 + 3/2*A*a^2*b*g^3*x^2 + (x*log(d^2*e*x^
2/(b^2*x^2 + 2*a*b*x + a^2) + 2*c*d*e*x/(b^2*x^2 + 2*a*b*x + a^2) + c^2*e/(
b^2*x^2 + 2*a*b*x + a^2)) - 2*a*log(b*x + a)/b + 2*c*log(d*x + c)/d)*B*a^3*
g^3 + 3/2*(x^2*log(d^2*e*x^2/(b^2*x^2 + 2*a*b*x + a^2) + 2*c*d*e*x/(b^2*x^2
 + 2*a*b*x + a^2) + c^2*e/(b^2*x^2 + 2*a*b*x + a^2)) + 2*a^2*log(b*x + a)/b
^2 - 2*c^2*log(d*x + c)/d^2 + 2*(b*c - a*d)*x/(b*d))*B*a^2*b*g^3 + (x^3*log
(d^2*e*x^2/(b^2*x^2 + 2*a*b*x + a^2) + 2*c*d*e*x/(b^2*x^2 + 2*a*b*x + a^2)
+ c^2*e/(b^2*x^2 + 2*a*b*x + a^2)) - 2*a^3*log(b*x + a)/b^3 + 2*c^3*log(d*x
 + c)/d^3 + ((b^2*c*d - a*b*d^2)*x^2 - 2*(b^2*c^2 - a^2*d^2)*x)/(b^2*d^2))*
B*a*b^2*g^3 + 1/12*(3*x^4*log(d^2*e*x^2/(b^2*x^2 + 2*a*b*x + a^2) + 2*c*d*e
*x/(b^2*x^2 + 2*a*b*x + a^2) + c^2*e/(b^2*x^2 + 2*a*b*x + a^2)) + 6*a^4*log
(b*x + a)/b^4 - 6*c^4*log(d*x + c)/d^4 + (2*(b^3*c*d^2 - a*b^2*d^3)*x^3 - 3
*(b^3*c^2*d - a^2*b*d^3)*x^2 + 6*(b^3*c^3 - a^3*d^3)*x)/(b^3*d^3))*B*b^3*g^
3 + A*a^3*g^3*x
```

**mupad [B]** time = 4.85, size = 567, normalized size = 3.75

$$\ln\left(\frac{e\,(c+d\,x)^2}{(a+b\,x)^2}\right)\left(B\,a^3\,g^3\,x+\frac{3\,B\,a^2\,b\,g^3\,x^2}{2}+B\,a\,b^2\,g^3\,x^3+\frac{B\,b^3\,g^3\,x^4}{4}\right)-x^2\left(\frac{\left(\frac{b^2\,g^3\,(8\,A\,a\,d+2\,A\,b\,c-B\,a\,d+B\,b\,c)}{2\,d}-\frac{A\,b^2\,g^3}{}\right.}{4\,b\,d}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*g + b*g*x)^3*(A + B*log((e*(c + d*x)^2)/(a + b*x)^2)),x)
```

```
[Out] log((e*(c + d*x)^2)/(a + b*x)^2)*((B*b^3*g^3*x^4)/4 + B*a^3*g^3*x + (3*B*a^
2*b*g^3*x^2)/2 + B*a*b^2*g^3*x^3) - x^2*((((b^2*g^3*(8*A*a*d + 2*A*b*c - B*
a*d + B*b*c))/(2*d) - (A*b^2*g^3*(2*a*d + 2*b*c))/(2*d))*(2*a*d + 2*b*c))/(
4*b*d) - (a*b*g^3*(3*A*a*d + 2*A*b*c - B*a*d + B*b*c))/d + (A*a*b^2*c*g^3)/
(2*d)) + x*(((2*a*d + 2*b*c)*((((b^2*g^3*(8*A*a*d + 2*A*b*c - B*a*d + B*b*c
))/(2*d) - (A*b^2*g^3*(2*a*d + 2*b*c))/(2*d))*(2*a*d + 2*b*c))/(2*b*d) - (2
*a*b*g^3*(3*A*a*d + 2*A*b*c - B*a*d + B*b*c))/d + (A*a*b^2*c*g^3)/d))/(2*b*
d) + (a^2*g^3*(4*A*a*d + 6*A*b*c - 3*B*a*d + 3*B*b*c))/d - (a*c*((b^2*g^3*(
8*A*a*d + 2*A*b*c - B*a*d + B*b*c))/(2*d) - (A*b^2*g^3*(2*a*d + 2*b*c))/(2*
d)))/(b*d)) + x^3*((b^2*g^3*(8*A*a*d + 2*A*b*c - B*a*d + B*b*c))/(6*d) - (A
*b^2*g^3*(2*a*d + 2*b*c))/(6*d)) - (log(c + d*x)*(B*b^3*c^4*g^3 - 4*B*a^3*c
*d^3*g^3 + 6*B*a^2*b*c^2*d^2*g^3 - 4*B*a*b^2*c^3*d*g^3))/(2*d^4) + (A*b^3*g
^3*x^4)/4 - (B*a^4*g^3*log(a + b*x))/(2*b)
```

**sympy [B]** time = 4.35, size = 707, normalized size = 4.68

$$\frac{Ab^3g^3x^4}{4} - \frac{Ba^4g^3\log\left(x+\frac{\frac{Ba^5d^4g^3}{b}+4Ba^4cd^3g^3-6Ba^3bc^2d^2g^3+4Ba^2b^2c^3dg^3-Bab^3c^4g^3}{Ba^4d^4g^3+4Ba^3bcd^3g^3-6Ba^2b^2c^2d^2g^3+4Bab^3c^3dg^3-Bb^4c^4g^3}\right)}{2b} + \frac{Bcg^3\left(2ad-bc\right)\left(2a^2d^2-2abcd+b^2c^2\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*g*x+a*g)**3*(A+B*ln(e*(d*x+c)**2/(b*x+a)**2)),x)
```

```
[Out] A*b**3*g**3*x**4/4 - B*a**4*g**3*log(x + (B*a**5*d**4*g**3/b + 4*B*a**4*c*d
**3*g**3 - 6*B*a**3*b*c**2*d**2*g**3 + 4*B*a**2*b**2*c**3*d*g**3 - B*a*b**3
*c**4*g**3)/(B*a**4*d**4*g**3 + 4*B*a**3*b*c*d**3*g**3 - 6*B*a**2*b**2*c**2
*d**2*g**3 + 4*B*a*b**3*c**3*d*g**3 - B*b**4*c**4*g**3))/(2*b) + B*c*g**3*(
2*a*d - b*c)*(2*a**2*d**2 - 2*a*b*c*d + b**2*c**2)*log(x + (5*B*a**4*c*d**3
*g**3 - 6*B*a**3*b*c**2*d**2*g**3 + 4*B*a**2*b**2*c**3*d*g**3 - B*a*b**3*c*
*4*g**3 - B*a*c*g**3*(2*a*d - b*c)*(2*a**2*d**2 - 2*a*b*c*d + b**2*c**2) +
B*b*c**2*g**3*(2*a*d - b*c)*(2*a**2*d**2 - 2*a*b*c*d + b**2*c**2)/d)/(B*a**
4*d**4*g**3 + 4*B*a**3*b*c*d**3*g**3 - 6*B*a**2*b**2*c**2*d**2*g**3 + 4*B*a
*b**3*c**3*d*g**3 - B*b**4*c**4*g**3))/(2*d**4) + x**3*(A*a*b**2*g**3 - B*a
*b**2*g**3/6 + B*b**3*c*g**3/(6*d)) + x**2*(3*A*a**2*b*g**3/2 - 3*B*a**2*b*
g**3/4 + B*a*b**2*c*g**3/d - B*b**3*c**2*g**3/(4*d**2)) + x*(A*a**3*g**3 -
3*B*a**3*g**3/2 + 3*B*a**2*b*c*g**3/d - 2*B*a*b**2*c**2*g**3/d**2 + B*b**3*
c**3*g**3/(2*d**3)) + (B*a**3*g**3*x + 3*B*a**2*b*g**3*x**2/2 + B*a*b**2*g*
*3*x**3 + B*b**3*g**3*x**4/4)*log(e*(c + d*x)**2/(a + b*x)**2)
```

# 3.203 $\int (ag+bgx)^2 \left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right) dx$

**Optimal**. Leaf size=120

$$\frac{g^2(a+bx)^3\left(B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)+A\right)}{3b}+\frac{2Bg^2(bc-ad)^3\log(c+dx)}{3bd^3}-\frac{2Bg^2x(bc-ad)^2}{3d^2}+\frac{Bg^2(a+bx)^2(bc-ad)}{3bd}$$

```
[Out] -2/3*B*(-a*d+b*c)^2*g^2*x/d^2+1/3*B*(-a*d+b*c)*g^2*(b*x+a)^2/b/d+2/3*B*(-a*
d+b*c)^3*g^2*ln(d*x+c)/b/d^3+1/3*g^2*(b*x+a)^3*(A+B*ln(e*(d*x+c)^2/(b*x+a)^
2))/b
```

**Rubi [A]** time = 0.08, antiderivative size = 120, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.094, Rules used = {2525, 12, 43}

$$\frac{g^2(a+bx)^3\left(B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)+A\right)}{3b}-\frac{2Bg^2x(bc-ad)^2}{3d^2}+\frac{2Bg^2(bc-ad)^3\log(c+dx)}{3bd^3}+\frac{Bg^2(a+bx)^2(bc-ad)}{3bd}$$

Antiderivative was successfully verified.

```
[In] Int[(a*g + b*g*x)^2*(A + B*Log[(e*(c + d*x)^2)/(a + b*x)^2]),x]
```

```
[Out] (-2*B*(b*c - a*d)^2*g^2*x)/(3*d^2) + (B*(b*c - a*d)*g^2*(a + b*x)^2)/(3*b*d
) + (2*B*(b*c - a*d)^3*g^2*Log[c + d*x])/(3*b*d^3) + (g^2*(a + b*x)^3*(A +
B*Log[(e*(c + d*x)^2)/(a + b*x)^2]))/(3*b)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2525

```
Int[((a_.) + Log[(c_.)*(RFx_)^(p_.)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_))^(m_.
), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(a + b*Log[c*RFx^p])^n)/(e*(m + 1))
, x] - Dist[(b*n*p)/(e*(m + 1)), Int[SimplifyIntegrand[((d + e*x)^(m + 1)*(
a + b*Log[c*RFx^p])^(n - 1)*D[RFx, x])/RFx, x], x], x] /; FreeQ[{a, b, c, d
```

```
, e, m, p}, x] && RationalFunctionQ[RFx, x] && IGtQ[n, 0] && (EqQ[n, 1] ||
IntegerQ[m]) && NeQ[m, -1]
```

Rubi steps

$$\int (ag+bgx)^2\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)dx=\frac{g^2(a+bx)^3\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)}{3b}-\frac{B\int\frac{2(-bc+ad)g^3(a+bx)^2}{c+dx}dx}{3bg}$$
$$=\frac{g^2(a+bx)^3\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)}{3b}+\frac{\left(2B(bc-ad)g^2\right)\int\frac{(a+bx)^2}{c+dx}dx}{3b}$$
$$=\frac{g^2(a+bx)^3\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)}{3b}+\frac{\left(2B(bc-ad)g^2\right)\int\left(-\frac{b(bc-ad)}{d^2}+\right.}{3b}$$
$$=-\frac{2B(bc-ad)^2g^2x}{3d^2}+\frac{B(bc-ad)g^2(a+bx)^2}{3bd}+\frac{2B(bc-ad)^3g^2\log(c}{3bd^3}$$

**Mathematica [A]** time = 0.05, size = 98, normalized size = 0.82

$$\frac{g^2\left(\frac{B(bc-ad)\left(d\left(a^2d+4abdx+b^2x(dx-2c)\right)+2(bc-ad)^2\log(c+dx)\right)}{d^3}+(a+bx)^3\left(B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)+A\right)\right)}{3b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*g + b*g*x)^2*(A + B*Log[(e*(c + d*x)^2)/(a + b*x)^2]),x]
```

```
[Out] (g^2*((B*(b*c - a*d)*(d*(a^2*d + 4*a*b*d*x + b^2*x*(-2*c + d*x)) + 2*(b*c -
 a*d)^2*Log[c + d*x]))/d^3 + (a + b*x)^3*(A + B*Log[(e*(c + d*x)^2)/(a + b*
x)^2])))/(3*b)
```

**fricas [B]** time = 0.60, size = 245, normalized size = 2.04

$$\frac{Ab^3d^3g^2x^3-2\,Ba^3d^3g^2\log(bx+a)+\left(Bb^3cd^2+(3\,A-B)ab^2d^3\right)g^2x^2-\left(2\,Bb^3c^2d-6\,Bab^2cd^2-(3\,A-4\,B)a^2bd\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*g*x+a*g)^2*(A+B*log(e*(d*x+c)^2/(b*x+a)^2)),x, algorithm="fric
as")
```

```
[Out] 1/3*(A*b^3*d^3*g^2*x^3 - 2*B*a^3*d^3*g^2*log(b*x + a) + (B*b^3*c*d^2 + (3*A
 - B)*a*b^2*d^3)*g^2*x^2 - (2*B*b^3*c^2*d - 6*B*a*b^2*c*d^2 - (3*A - 4*B)*a
^2*b*d^3)*g^2*x + 2*(B*b^3*c^3 - 3*B*a*b^2*c^2*d + 3*B*a^2*b*c*d^2)*g^2*log
(d*x + c) + (B*b^3*d^3*g^2*x^3 + 3*B*a*b^2*d^3*g^2*x^2 + 3*B*a^2*b*d^3*g^2*
x)*log((d^2*e*x^2 + 2*c*d*e*x + c^2*e)/(b^2*x^2 + 2*a*b*x + a^2)))/(b*d^3)
```

**giac [B]** time = 3.51, size = 248, normalized size = 2.07

$$-\frac{2\,Ba^3g^2\log\left(bx+a\right)}{3\,b}+\frac{1}{3}\left(Ab^2g^2+Bb^2g^2\right)x^3+\frac{\left(Bb^2cg^2+3\,Aabdg^2+2\,Babdg^2\right)x^2}{3\,d}+\frac{1}{3}\left(Bb^2g^2x^3+3\,Babg^2x^2+\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*g*x+a*g)^2*(A+B*log(e*(d*x+c)^2/(b*x+a)^2)),x, algorithm="giac
")
```

```
[Out] -2/3*B*a^3*g^2*log(b*x + a)/b + 1/3*(A*b^2*g^2 + B*b^2*g^2)*x^3 + 1/3*(B*b^
2*c*g^2 + 3*A*a*b*d*g^2 + 2*B*a*b*d*g^2)*x^2/d + 1/3*(B*b^2*g^2*x^3 + 3*B*a
*b*g^2*x^2 + 3*B*a^2*g^2*x)*log((d^2*x^2 + 2*c*d*x + c^2)/(b^2*x^2 + 2*a*b*
x + a^2)) - 1/3*(2*B*b^2*c^2*g^2 - 6*B*a*b*c*d*g^2 - 3*A*a^2*d^2*g^2 + B*a^
2*d^2*g^2)*x/d^2 + 2/3*(B*b^2*c^3*g^2 - 3*B*a*b*c^2*d*g^2 + 3*B*a^2*c*d^2*g
^2)*log(d*x + c)/d^3
```

**maple [B]** time = 0.07, size = 569, normalized size = 4.74

$$\frac{B\,b^2g^2x^3\ln\left(\frac{\left(\frac{ad}{bx+a}-\frac{bc}{bx+a}-d\right)^2e}{b^2}\right)}{3}+\frac{A\,b^2g^2x^3}{3}+Bab\,g^2x^2\ln\left(\frac{\left(\frac{ad}{bx+a}-\frac{bc}{bx+a}-d\right)^2e}{b^2}\right)+Aab\,g^2x^2+B\,a^2g^2x\ln\left(\frac{\left(\frac{ad}{bx+a}-\frac{bc}{bx+a}\right.}{b^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*g*x+a*g)^2*(A+B*ln(e*(d*x+c)^2/(b*x+a)^2)),x)
```

```
[Out] 5/3*B*a^2*c/d*g^2+2/3/b*g^2*B*a^3*ln(1/(b*x+a))+1/3*A*b^2*g^2*x^3-2/3*B*a*b
*c^2/d^2*g^2-2/3*b^2*g^2*B*c^3/d^3*ln(1/(b*x+a))+2*B*a*b*c/d*g^2*x+2*b*g^2*
B*a/d^2*ln(1/(b*x+a))*c^2-2*g^2*B*a^2/d*ln(1/(b*x+a))*c+1/3*b^2*B*ln((1/(b*
x+a)*a*d-1/(b*x+a)*b*c-d)^2/b^2*e)*x^3*g^2+b*B*ln((1/(b*x+a)*a*d-1/(b*x+a)*
b*c-d)^2/b^2*e)*x^2*a*g^2+B*ln((1/(b*x+a)*a*d-1/(b*x+a)*b*c-d)^2/b^2*e)*x*a
^2*g^2+1/3/b*B*ln((1/(b*x+a)*a*d-1/(b*x+a)*b*c-d)^2/b^2*e)*a^3*g^2-2/3/b*g^
2*B*a^3*ln(1/(b*x+a)*a*d-1/(b*x+a)*b*c-d)+2/3*b^2*g^2*B*c^3/d^3*ln(1/(b*x+a
)*a*d-1/(b*x+a)*b*c-d)-1/3*B*a*b*g^2*x^2+A*a*b*g^2*x^2+A*a^2*g^2*x-4/3*B*a^
2*g^2*x+1/3/b*A*a^3*g^2-1/b*g^2*B*a^3-2/3*B*b^2*c^2/d^2*g^2*x+1/3*B*b^2*c/d
*g^2*x^2+2*g^2*B*a^2/d*ln(1/(b*x+a)*a*d-1/(b*x+a)*b*c-d)*c-2*b*g^2*B*a/d^2*
ln(1/(b*x+a)*a*d-1/(b*x+a)*b*c-d)*c^2
```

**maxima [B]** time = 1.51, size = 436, normalized size = 3.63

$$\frac{1}{3}\,Ab^2g^2x^3 + Aabg^2x^2 + \left(x\log\left(\frac{d^2ex^2}{b^2x^2+2\,abx+a^2} + \frac{2\,cdex}{b^2x^2+2\,abx+a^2} + \frac{c^2e}{b^2x^2+2\,abx+a^2}\right) - \frac{2\,a\log\left(bx+a\right)}{b} + $$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*g*x+a*g)^2*(A+B*log(e*(d*x+c)^2/(b*x+a)^2)),x, algorithm="maxi
ma")
```

```
[Out] 1/3*A*b^2*g^2*x^3 + A*a*b*g^2*x^2 + (x*log(d^2*e*x^2/(b^2*x^2 + 2*a*b*x + a
^2) + 2*c*d*e*x/(b^2*x^2 + 2*a*b*x + a^2) + c^2*e/(b^2*x^2 + 2*a*b*x + a^2)
) - 2*a*log(b*x + a)/b + 2*c*log(d*x + c)/d)*B*a^2*g^2 + (x^2*log(d^2*e*x^2
/(b^2*x^2 + 2*a*b*x + a^2) + 2*c*d*e*x/(b^2*x^2 + 2*a*b*x + a^2) + c^2*e/(b
^2*x^2 + 2*a*b*x + a^2)) + 2*a^2*log(b*x + a)/b^2 - 2*c^2*log(d*x + c)/d^2
+ 2*(b*c - a*d)*x/(b*d))*B*a*b*g^2 + 1/3*(x^3*log(d^2*e*x^2/(b^2*x^2 + 2*a*
b*x + a^2) + 2*c*d*e*x/(b^2*x^2 + 2*a*b*x + a^2) + c^2*e/(b^2*x^2 + 2*a*b*x
 + a^2)) - 2*a^3*log(b*x + a)/b^3 + 2*c^3*log(d*x + c)/d^3 + ((b^2*c*d - a*
b*d^2)*x^2 - 2*(b^2*c^2 - a^2*d^2)*x)/(b^2*d^2))*B*b^2*g^2 + A*a^2*g^2*x
```

**mupad [B]** time = 4.65, size = 296, normalized size = 2.47

$$x^2\left(\frac{b\,g^2\,\left(9\,A\,a\,d+3\,A\,b\,c-2\,B\,a\,d+2\,B\,b\,c\right)}{6\,d} - \frac{A\,b\,g^2\,\left(3\,a\,d+3\,b\,c\right)}{6\,d}\right) - x\left(\frac{\left(3\,a\,d+3\,b\,c\right)\left(\frac{b\,g^2\,\left(9\,A\,a\,d+3\,A\,b\,c-2\,B\,a\,d+2\right.}{3\,d}\right.}{3\,b\,d}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*g + b*g*x)^2*(A + B*log((e*(c + d*x)^2)/(a + b*x)^2)),x)
```

```
[Out] x^2*((b*g^2*(9*A*a*d + 3*A*b*c - 2*B*a*d + 2*B*b*c))/(6*d) - (A*b*g^2*(3*a*
d + 3*b*c))/(6*d)) - x*(((3*a*d + 3*b*c)*((b*g^2*(9*A*a*d + 3*A*b*c - 2*B*a
*d + 2*B*b*c))/(3*d) - (A*b*g^2*(3*a*d + 3*b*c))/(3*d)))/(3*b*d) - (a*g^2*(
3*A*a*d + 3*A*b*c - 2*B*a*d + 2*B*b*c))/d + (A*a*b*c*g^2)/d) + log((e*(c +
d*x)^2)/(a + b*x)^2)*((B*b^2*g^2*x^3)/3 + B*a^2*g^2*x + B*a*b*g^2*x^2) + (l
og(c + d*x)*(2*B*b^2*c^3*g^2 + 6*B*a^2*c*d^2*g^2 - 6*B*a*b*c^2*d*g^2))/(3*d
^3) + (A*b^2*g^2*x^3)/3 - (2*B*a^3*g^2*log(a + b*x))/(3*b)
```

**sympy [B]** time = 3.13, size = 517, normalized size = 4.31

$$\frac{Ab^2g^2x^3}{3} - \frac{2Ba^3g^2\log\left(x + \frac{\frac{2Ba^4d^3g^2}{b}+6Ba^3cd^2g^2-6Ba^2bc^2dg^2+2Bab^2c^3g^2}{2Ba^3d^3g^2+6Ba^2bcd^2g^2-6Bab^2c^2dg^2+2Bb^3c^3g^2}\right)}{3b} + \frac{2Bcg^2\left(3a^2d^2-3abcd+b^2c^2\right)\log\left(x + \frac{8Ba^3cd^2g^2}{}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*g*x+a*g)**2*(A+B*ln(e*(d*x+c)**2/(b*x+a)**2)),x)
```

```
[Out] A*b**2*g**2*x**3/3 - 2*B*a**3*g**2*log(x + (2*B*a**4*d**3*g**2/b + 6*B*a**3
*c*d**2*g**2 - 6*B*a**2*b*c**2*d*g**2 + 2*B*a*b**2*c**3*g**2)/(2*B*a**3*d**
3*g**2 + 6*B*a**2*b*c*d**2*g**2 - 6*B*a*b**2*c**2*d*g**2 + 2*B*b**3*c**3*g*
*2))/(3*b) + 2*B*c*g**2*(3*a**2*d**2 - 3*a*b*c*d + b**2*c**2)*log(x + (8*B*
a**3*c*d**2*g**2 - 6*B*a**2*b*c**2*d*g**2 + 2*B*a*b**2*c**3*g**2 - 2*B*a*c*
g**2*(3*a**2*d**2 - 3*a*b*c*d + b**2*c**2) + 2*B*b*c**2*g**2*(3*a**2*d**2 -
 3*a*b*c*d + b**2*c**2)/d)/(2*B*a**3*d**3*g**2 + 6*B*a**2*b*c*d**2*g**2 - 6
*B*a*b**2*c**2*d*g**2 + 2*B*b**3*c**3*g**2))/(3*d**3) + x**2*(A*a*b*g**2 -
B*a*b*g**2/3 + B*b**2*c*g**2/(3*d)) + x*(A*a**2*g**2 - 4*B*a**2*g**2/3 + 2*
B*a*b*c*g**2/d - 2*B*b**2*c**2*g**2/(3*d**2)) + (B*a**2*g**2*x + B*a*b*g**2
*x**2 + B*b**2*g**2*x**3/3)*log(e*(c + d*x)**2/(a + b*x)**2)
```

# 3.204 $\int (ag+bgx)\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)\,dx$

**Optimal**. Leaf size=78

$$\frac{g(a+bx)^2\left(B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)+A\right)}{2b}-\frac{Bg(bc-ad)^2\log(c+dx)}{bd^2}+\frac{Bgx(bc-ad)}{d}$$

```
[Out] B*(-a*d+b*c)*g*x/d-B*(-a*d+b*c)^2*g*ln(d*x+c)/b/d^2+1/2*g*(b*x+a)^2*(A+B*ln
(e*(d*x+c)^2/(b*x+a)^2))/b
```

**Rubi [A]** time = 0.05, antiderivative size = 78, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {2525, 12, 43}

$$\frac{g(a+bx)^2\left(B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)+A\right)}{2b}-\frac{Bg(bc-ad)^2\log(c+dx)}{bd^2}+\frac{Bgx(bc-ad)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[(a*g + b*g*x)*(A + B*Log[(e*(c + d*x)^2)/(a + b*x)^2]),x]
```

```
[Out] (B*(b*c - a*d)*g*x)/d - (B*(b*c - a*d)^2*g*Log[c + d*x])/(b*d^2) + (g*(a +
b*x)^2*(A + B*Log[(e*(c + d*x)^2)/(a + b*x)^2]))/(2*b)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2525

```
Int[((a_.) + Log[(c_.)*(RFx_)^(p_.)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_))^(m_.
), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(a + b*Log[c*RFx^p])^n)/(e*(m + 1))
, x] - Dist[(b*n*p)/(e*(m + 1)), Int[SimplifyIntegrand[((d + e*x)^(m + 1)*(
a + b*Log[c*RFx^p])^(n - 1)*D[RFx, x])/RFx, x], x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && RationalFunctionQ[RFx, x] && IGtQ[n, 0] && (EqQ[n, 1] ||
```

```
IntegerQ[m]) && NeQ[m, -1]
```

Rubi steps

$$\int (ag+bgx)\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)dx = \frac{g(a+bx)^2\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)}{2b} - \frac{B\int \frac{2(bc-ad)g^2(-a-bx)}{c+dx}\,dx}{2bg}$$
$$= \frac{g(a+bx)^2\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)}{2b} - \frac{(B(bc-ad)g)\int \frac{-a-bx}{c+dx}\,dx}{b}$$
$$= \frac{g(a+bx)^2\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)}{2b} - \frac{(B(bc-ad)g)\int \left(-\frac{b}{d}+\frac{bc-ad}{d(c+dx)}\right)$$
$$= \frac{B(bc-ad)gx}{d} - \frac{B(bc-ad)^2g\log(c+dx)}{bd^2} + \frac{g(a+bx)^2\left(A+B\log\right.}{2b}$$

**Mathematica [A]** time = 0.04, size = 72, normalized size = 0.92

$$\frac{g\left((a+bx)^2\left(B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)+A\right)-\frac{2B(ad-bc)((ad-bc)\log(c+dx)+bdx)}{d^2}\right)}{2b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*g + b*g*x)*(A + B*Log[(e*(c + d*x)^2)/(a + b*x)^2]),x]
```

```
[Out] (g*((-2*B*(-(b*c) + a*d)*(b*d*x + (-(b*c) + a*d)*Log[c + d*x]))/d^2 + (a +
b*x)^2*(A + B*Log[(e*(c + d*x)^2)/(a + b*x)^2])))/(2*b)
```

**fricas [A]** time = 0.64, size = 149, normalized size = 1.91

$$\frac{Ab^2d^2gx^2 - 2\,Ba^2d^2g\log(bx+a) + 2\left(Bb^2cd+(A-B)abd^2\right)gx - 2\left(Bb^2c^2-2\,Babcd\right)g\log(dx+c) + \left(Bb^2d^2g\right.}{2\,bd^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*g*x+a*g)*(A+B*log(e*(d*x+c)^2/(b*x+a)^2)),x, algorithm="fricas
")
```

```
[Out] 1/2*(A*b^2*d^2*g*x^2 - 2*B*a^2*d^2*g*log(b*x + a) + 2*(B*b^2*c*d + (A - B)*
a*b*d^2)*g*x - 2*(B*b^2*c^2 - 2*B*a*b*c*d)*g*log(d*x + c) + (B*b^2*d^2*g*x^
2 + 2*B*a*b*d^2*g*x)*log((d^2*e*x^2 + 2*c*d*e*x + c^2*e)/(b^2*x^2 + 2*a*b*x
 + a^2)))/(b*d^2)
```

**giac [A]** time = 0.90, size = 128, normalized size = 1.64

$$-\frac{Ba^2g\log(bx+a)}{b}+\frac{1}{2}\left(Abg+Bbg\right)x^2+\frac{1}{2}\left(Bbgx^2+2\,Bagx\right)\log\left(\frac{d^2x^2+2\,cdx+c^2}{b^2x^2+2\,abx+a^2}\right)+\frac{\left(Bbcg+Aadg\right)x}{d}-\frac{\left(Bbc^2g\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*g*x+a*g)*(A+B*log(e*(d*x+c)^2/(b*x+a)^2)),x, algorithm="giac")
```

```
[Out] -B*a^2*g*log(b*x + a)/b + 1/2*(A*b*g + B*b*g)*x^2 + 1/2*(B*b*g*x^2 + 2*B*a*
g*x)*log((d^2*x^2 + 2*c*d*x + c^2)/(b^2*x^2 + 2*a*b*x + a^2)) + (B*b*c*g +
A*a*d*g)*x/d - (B*b*c^2*g - 2*B*a*c*d*g)*log(-d*x - c)/d^2
```

**maple [B]** time = 0.06, size = 340, normalized size = 4.36

$$\frac{Bbg\,x^2\ln\left(\frac{\left(\frac{ad}{bx+a}-\frac{bc}{bx+a}-d\right)^2e}{b^2}\right)}{2}+\frac{Abg\,x^2}{2}+Bagx\ln\left(\frac{\left(\frac{ad}{bx+a}-\frac{bc}{bx+a}-d\right)^2e}{b^2}\right)+Aagx+\frac{B\,a^2g\ln\left(\frac{1}{bx+a}\right)}{b}+\frac{B\,a^2g\ln\left(\frac{\left(\frac{ad}{bx+a}-\frac{bc}{bx+a}\right.}{b^2}\right.}{2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*g*x+a*g)*(A+B*ln(e*(d*x+c)^2/(b*x+a)^2)),x)
```

```
[Out] 1/2*b*A*x^2*g+A*x*a*g+1/2/b*A*a^2*g+1/2*b*B*ln((1/(b*x+a)*a*d-1/(b*x+a)*b*c
-d)^2/b^2*e)*x^2*g+B*ln((1/(b*x+a)*a*d-1/(b*x+a)*b*c-d)^2/b^2*e)*x*a*g+1/2/
b*B*ln((1/(b*x+a)*a*d-1/(b*x+a)*b*c-d)^2/b^2*e)*a^2*g-1/b*g*B*ln(1/(b*x+a)*
a*d-1/(b*x+a)*b*c-d)*a^2+2*g*B/d*ln(1/(b*x+a)*a*d-1/(b*x+a)*b*c-d)*a*c-b*g*
B/d^2*ln(1/(b*x+a)*a*d-1/(b*x+a)*b*c-d)*c^2+1/b*g*B*ln(1/(b*x+a))*a^2-2*g*B
/d*ln(1/(b*x+a))*a*c+b*g*B/d^2*ln(1/(b*x+a))*c^2-B*x*a*g-1/b*g*B*a^2+b*g*B/
d*c*x+g*B/d*a*c
```

**maxima [B]** time = 1.44, size = 250, normalized size = 3.21

$$\frac{1}{2}\,Abgx^2+\left(x\log\left(\frac{d^2ex^2}{b^2x^2+2\,abx+a^2}+\frac{2\,cdex}{b^2x^2+2\,abx+a^2}+\frac{c^2e}{b^2x^2+2\,abx+a^2}\right)-\frac{2\,a\log(bx+a)}{b}+\frac{2\,c\log(dx+}{d}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*g*x+a*g)*(A+B*log(e*(d*x+c)^2/(b*x+a)^2)),x, algorithm="maxima
")
```

```
[Out] 1/2*A*b*g*x^2 + (x*log(d^2*e*x^2/(b^2*x^2 + 2*a*b*x + a^2) + 2*c*d*e*x/(b^2
*x^2 + 2*a*b*x + a^2) + c^2*e/(b^2*x^2 + 2*a*b*x + a^2)) - 2*a*log(b*x + a)
/b + 2*c*log(d*x + c)/d)*B*a*g + 1/2*(x^2*log(d^2*e*x^2/(b^2*x^2 + 2*a*b*x
```

```
+ a^2) + 2*c*d*e*x/(b^2*x^2 + 2*a*b*x + a^2) + c^2*e/(b^2*x^2 + 2*a*b*x + a
^2)) + 2*a^2*log(b*x + a)/b^2 - 2*c^2*log(d*x + c)/d^2 + 2*(b*c - a*d)*x/(b
*d))*B*b*g + A*a*g*x
```

**mupad [B]** time = 4.38, size = 120, normalized size = 1.54

$$x\left(\frac{g\,\left(2\,A\,a\,d+A\,b\,c-B\,a\,d+B\,b\,c\right)}{d}-\frac{A\,g\,\left(a\,d+b\,c\right)}{d}\right)+\ln\left(\frac{e\,{\left(c+d\,x\right)}^2}{{\left(a+b\,x\right)}^2}\right)\,\left(\frac{B\,b\,g\,x^2}{2}+B\,a\,g\,x\right)+\frac{A\,b\,g\,x^2}{2}-\frac{B\,a^2\,g}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*g + b*g*x)*(A + B*log((e*(c + d*x)^2)/(a + b*x)^2)),x)
```

```
[Out] x*((g*(2*A*a*d + A*b*c - B*a*d + B*b*c))/d - (A*g*(a*d + b*c))/d) + log((e*
(c + d*x)^2)/(a + b*x)^2)*((B*b*g*x^2)/2 + B*a*g*x) + (A*b*g*x^2)/2 - (B*a^
2*g*log(a + b*x))/b + (B*c*g*log(c + d*x)*(2*a*d - b*c))/d^2
```

**sympy [B]** time = 1.92, size = 250, normalized size = 3.21

$$\frac{Abgx^2}{2}-\frac{Ba^2g\log\left(x+\frac{\frac{Ba^3d^2g}{b}+2Ba^2cdg-Babc^2g}{Ba^2d^2g+2Babcdg-Bb^2c^2g}\right)}{b}+\frac{Bcg\left(2ad-bc\right)\log\left(x+\frac{3Ba^2cdg-Babc^2g-Bacg(2ad-bc)+\frac{Bbc^2g(2ad-bc)}{d}}{Ba^2d^2g+2Babcdg-Bb^2c^2g}\right)}{d^2}+x\left(\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*g*x+a*g)*(A+B*ln(e*(d*x+c)**2/(b*x+a)**2)),x)
```

```
[Out] A*b*g*x**2/2 - B*a**2*g*log(x + (B*a**3*d**2*g/b + 2*B*a**2*c*d*g - B*a*b*c
**2*g)/(B*a**2*d**2*g + 2*B*a*b*c*d*g - B*b**2*c**2*g))/b + B*c*g*(2*a*d -
b*c)*log(x + (3*B*a**2*c*d*g - B*a*b*c**2*g - B*a*c*g*(2*a*d - b*c) + B*b*c
**2*g*(2*a*d - b*c)/d)/(B*a**2*d**2*g + 2*B*a*b*c*d*g - B*b**2*c**2*g))/d**
2 + x*(A*a*g - B*a*g + B*b*c*g/d) + (B*a*g*x + B*b*g*x**2/2)*log(e*(c + d*x
)**2/(a + b*x)**2)
```

# 3.205 $\int \frac{A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)}{ag+bgx}\,dx$

**Optimal**. Leaf size=83

$$-\frac{\log\left(-\frac{bc-ad}{d(a+bx)}\right)\left(B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)+A\right)}{bg}-\frac{2B\text{Li}_2\left(\frac{bc-ad}{d(a+bx)}+1\right)}{bg}$$

```
[Out] -ln((a*d-b*c)/d/(b*x+a))*(A+B*ln(e*(d*x+c)^2/(b*x+a)^2))/b/g-2*B*polylog(2,
1+(-a*d+b*c)/d/(b*x+a))/b/g
```

**Rubi [A]** time = 0.29, antiderivative size = 121, normalized size of antiderivative = 1.46, number of steps used = 10, number of rules used = 8, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {2524, 12, 2418, 2390, 2301, 2394, 2393, 2391}

$$-\frac{2B\text{PolyLog}\left(2,-\frac{d(a+bx)}{bc-ad}\right)}{bg}+\frac{\log(ag+bgx)\left(B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)+A\right)}{bg}-\frac{2B\log(ag+bgx)\log\left(\frac{b(c+dx)}{bc-ad}\right)}{bg}+\frac{B\log^2(g(a+b}{bg}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Log[(e*(c + d*x)^2)/(a + b*x)^2])/(a*g + b*g*x),x]
```

```
[Out] (B*Log[g*(a + b*x)]^2)/(b*g) - (2*B*Log[(b*(c + d*x))/(b*c - a*d)]*Log[a*g
+ b*g*x])/(b*g) + ((A + B*Log[(e*(c + d*x)^2)/(a + b*x)^2])*Log[a*g + b*g*x
])/(b*g) - (2*B*PolyLog[2, -((d*(a + b*x))/(b*c - a*d))])/(b*g)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2301

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

Rule 2390

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_))^(q_.), x_Symbol] :> Dist[1/e, Subst[Int[((f*x)/d)^q*(a + b*Log[c*x^
n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && E
qQ[e*f - d*g, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2393

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + (c*e*x)/g])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2394

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d + e*x
)^n]))/g, x] - Dist[(b*e*n)/g, Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2418

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*(RFx_), x_Sy
mbol] :> With[{u = ExpandIntegrand[(a + b*Log[c*(d + e*x)^n])^p, RFx, x]},
Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d, e, n}, x] && RationalFunctionQ[
RFx, x] && IntegerQ[p]
```

Rule 2524

```
Int[((a_.) + Log[(c_.)*(RFx_)^(p_.)]*(b_.))^(n_.)/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[(Log[d + e*x]*(a + b*Log[c*RFx^p])^n)/e, x] - Dist[(b*n*p)/e
, Int[(Log[d + e*x]*(a + b*Log[c*RFx^p])^(n - 1)*D[RFx, x])/RFx, x], x] /;
FreeQ[{a, b, c, d, e, p}, x] && RationalFunctionQ[RFx, x] && IGtQ[n, 0]
```

Rubi steps

$$\int \frac{A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)}{ag+bgx}\,dx = \frac{\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)\log(ag+bgx)}{bg} - \frac{B\int \frac{(a+bx)^2\left(\frac{2de(c+dx)}{(a+bx)^2}-\frac{2be(c+dx)^2}{(a+bx)^3}\right)\log(ag+bgx)}{e(c+dx)^2}dx}{bg}$$

$$= \frac{\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)\log(ag+bgx)}{bg} - \frac{B\int \frac{(a+bx)^2\left(\frac{2de(c+dx)}{(a+bx)^2}-\frac{2be(c+dx)^2}{(a+bx)^3}\right)\log(ag+bgx)}{(c+dx)^2}dx}{beg}$$

$$= \frac{\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)\log(ag+bgx)}{bg} - \frac{B\int\left(-\frac{2be\log(ag+bgx)}{a+bx}+\frac{2de\log(ag+bgx)}{c+dx}\right)dx}{beg}$$

$$= \frac{\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)\log(ag+bgx)}{bg} + \frac{(2B)\int\frac{\log(ag+bgx)}{a+bx}dx}{g} - \frac{(2Bd)\int\frac{\log(ag+bgx)}{c+dx}dx}{bg}$$

$$= -\frac{2B\log\left(\frac{b(c+dx)}{bc-ad}\right)\log(ag+bgx)}{bg} + \frac{\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)\log(ag+bgx)}{bg} + (2B)\int \text{lo}\ldots$$

$$= -\frac{2B\log\left(\frac{b(c+dx)}{bc-ad}\right)\log(ag+bgx)}{bg} + \frac{\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)\log(ag+bgx)}{bg} + \frac{(2B)\,\text{Subs}\ldots}{}$$

$$= \frac{B\log^2(g(a+bx))}{bg} - \frac{2B\log\left(\frac{b(c+dx)}{bc-ad}\right)\log(ag+bgx)}{bg} + \frac{\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)\log(ag\ldots}{bg}$$

**Mathematica [A]** time = 0.04, size = 87, normalized size = 1.05

$$\frac{\log(a+bx)\left(B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)-2B\log\left(\frac{b(c+dx)}{bc-ad}\right)+B\log(a+bx)+A\right)-2B\text{Li}_2\left(\frac{d(a+bx)}{ad-bc}\right)}{bg}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Log[(e*(c + d*x)^2)/(a + b*x)^2])/(a*g + b*g*x),x]
```

```
[Out] (Log[a + b*x]*(A + B*Log[a + b*x] - 2*B*Log[(b*(c + d*x))/(b*c - a*d)] + B*
Log[(e*(c + d*x)^2)/(a + b*x)^2]) - 2*B*PolyLog[2, (d*(a + b*x))/(-(b*c) +
a*d)])/(b*g)
```

**fricas [F]** time = 2.56, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{B\log\left(\frac{d^2ex^2+2\,cdex+c^2e}{b^2x^2+2\,abx+a^2}\right)+A}{bgx+ag},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*(d*x+c)^2/(b*x+a)^2))/(b*g*x+a*g),x, algorithm="fricas
")
```

```
[Out] integral((B*log((d^2*e*x^2 + 2*c*d*e*x + c^2*e)/(b^2*x^2 + 2*a*b*x + a^2))
+ A)/(b*g*x + a*g), x)
```

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{B\log\left(\frac{(dx+c)^2 e}{(bx+a)^2}\right)+A}{bgx+ag}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*(d*x+c)^2/(b*x+a)^2))/(b*g*x+a*g),x, algorithm="giac")
```

```
[Out] integrate((B*log((d*x + c)^2*e/(b*x + a)^2) + A)/(b*g*x + a*g), x)
```

**maple [B]** time = 0.06, size = 265, normalized size = 3.19

$$\frac{2Bad\ln\left(\frac{1}{bx+a}\right)\ln\left(-\frac{-d+\frac{ad-bc}{bx+a}}{d}\right)}{(ad-bc)\,bg}-\frac{2Bc\ln\left(\frac{1}{bx+a}\right)\ln\left(-\frac{-d+\frac{ad-bc}{bx+a}}{d}\right)}{(ad-bc)\,g}+\frac{2Bad\,\mathrm{dilog}\left(-\frac{-d+\frac{ad-bc}{bx+a}}{d}\right)}{(ad-bc)\,bg}-\frac{2Bc\,\mathrm{dilog}\left(-\frac{-d+\frac{ad-bc}{bx+a}}{d}\right)}{(ad-bc)\,g}-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*ln(e*(d*x+c)^2/(b*x+a)^2))/(b*g*x+a*g),x)
```

```
[Out] -1/b/g*A*ln(1/(b*x+a))-1/b/g*B*ln(1/(b*x+a))*ln((1/(b*x+a)*a*d-1/(b*x+a)*b*
c-d)^2/b^2*e)+2/b/g*B*dilog(-(1/(b*x+a)*(a*d-b*c)-d)/d)/(a*d-b*c)*a*d-2/g*B
*dilog(-(1/(b*x+a)*(a*d-b*c)-d)/d)/(a*d-b*c)*c+2/b/g*B*ln(1/(b*x+a))*ln(-(1
/(b*x+a)*(a*d-b*c)-d)/d)/(a*d-b*c)*a*d-2/g*B*ln(1/(b*x+a))*ln(-(1/(b*x+a)*(
a*d-b*c)-d)/d)/(a*d-b*c)*c
```

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$B\left(\frac{2\log(bx+a)\log(dx+c)}{bg}-\int -\frac{bdx\log(e)+bc\log(e)-2\,(2\,bdx+bc+ad)\log(bx+a)}{b^2dgx^2+abcg+\left(b^2cg+abdg\right)x}\,dx\right)+\frac{A\log\left(bgx+\right.}{bg}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*(d*x+c)^2/(b*x+a)^2))/(b*g*x+a*g),x, algorithm="maxima
")
```

```
[Out] B*(2*log(b*x + a)*log(d*x + c)/(b*g) - integrate(-(b*d*x*log(e) + b*c*log(e
) - 2*(2*b*d*x + b*c + a*d)*log(b*x + a))/(b^2*d*g*x^2 + a*b*c*g + (b^2*c*g
 + a*b*d*g)*x), x)) + A*log(b*g*x + a*g)/(b*g)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{A + B\,\ln\left(\frac{e\,(c+d\,x)^2}{(a+b\,x)^2}\right)}{a\,g + b\,g\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*log((e*(c + d*x)^2)/(a + b*x)^2))/(a*g + b*g*x),x)
```

```
[Out] int((A + B*log((e*(c + d*x)^2)/(a + b*x)^2))/(a*g + b*g*x), x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{A}{a+bx}\,dx + \int \frac{B\log\left(\frac{c^2e}{a^2+2abx+b^2x^2} + \frac{2cdex}{a^2+2abx+b^2x^2} + \frac{d^2ex^2}{a^2+2abx+b^2x^2}\right)}{a+bx}\,dx}{g}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*ln(e*(d*x+c)**2/(b*x+a)**2))/(b*g*x+a*g),x)
```

```
[Out] (Integral(A/(a + b*x), x) + Integral(B*log(c**2*e/(a**2 + 2*a*b*x + b**2*x*
*2) + 2*c*d*e*x/(a**2 + 2*a*b*x + b**2*x**2) + d**2*e*x**2/(a**2 + 2*a*b*x
+ b**2*x**2))/(a + b*x), x))/g
```

# 3.206 $\int \frac{A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)}{(ag+bgx)^2}\,dx$

**Optimal**. Leaf size=102

$$-\frac{A(c+dx)}{g^2(a+bx)(bc-ad)}-\frac{B(c+dx)\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)}{g^2(a+bx)(bc-ad)}+\frac{2B(c+dx)}{g^2(a+bx)(bc-ad)}$$

```
[Out] -A*(d*x+c)/(-a*d+b*c)/g^2/(b*x+a)+2*B*(d*x+c)/(-a*d+b*c)/g^2/(b*x+a)-B*(d*x
+c)*ln(e*(d*x+c)^2/(b*x+a)^2)/(-a*d+b*c)/g^2/(b*x+a)
```

**Rubi [A]** time = 0.08, antiderivative size = 105, normalized size of antiderivative = 1.03, number of steps used = 4, number of rules used = 3, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.094, Rules used = {2525, 12, 44}

$$-\frac{B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)+A}{bg^2(a+bx)}+\frac{2Bd\log(a+bx)}{bg^2(bc-ad)}-\frac{2Bd\log(c+dx)}{bg^2(bc-ad)}+\frac{2B}{bg^2(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Log[(e*(c + d*x)^2)/(a + b*x)^2])/(a*g + b*g*x)^2,x]
```

```
[Out] (2*B)/(b*g^2*(a + b*x)) + (2*B*d*Log[a + b*x])/(b*(b*c - a*d)*g^2) - (2*B*d
*Log[c + d*x])/(b*(b*c - a*d)*g^2) - (A + B*Log[(e*(c + d*x)^2)/(a + b*x)^2
])/(b*g^2*(a + b*x))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 2525

```
Int[((a_.) + Log[(c_.)*(RFx_)^(p_.)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_))^(m_.
), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(a + b*Log[c*RFx^p])^n)/(e*(m + 1))
, x] - Dist[(b*n*p)/(e*(m + 1)), Int[SimplifyIntegrand[((d + e*x)^(m + 1)*(
```

```
a + b*Log[c*RFx^p])^(n - 1)*D[RFx, x])/RFx, x], x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && RationalFunctionQ[RFx, x] && IGtQ[n, 0] && (EqQ[n, 1] ||
IntegerQ[m]) && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)}{(ag+bgx)^2}\,dx &= -\frac{A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)}{bg^2(a+bx)}+\frac{B\int\frac{2(-bc+ad)}{g(a+bx)^2(c+dx)}\,dx}{bg}\\ &= -\frac{A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)}{bg^2(a+bx)}-\frac{(2B(bc-ad))\int\frac{1}{(a+bx)^2(c+dx)}\,dx}{bg^2}\\ &= -\frac{A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)}{bg^2(a+bx)}-\frac{(2B(bc-ad))\int\left(\frac{b}{(bc-ad)(a+bx)^2}-\frac{bd}{(bc-ad)^2(a+bx)}+\frac{d^2}{(bc-ad)^2(c+dx)}\right)}{bg^2}\\ &= \frac{2B}{bg^2(a+bx)}+\frac{2Bd\log(a+bx)}{b(bc-ad)g^2}-\frac{2Bd\log(c+dx)}{b(bc-ad)g^2}-\frac{A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)}{bg^2(a+bx)}\end{aligned}$$

**Mathematica [A]** time = 0.05, size = 89, normalized size = 0.87

$$\frac{-(bc-ad)\left(B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)+A-2B\right)-2Bd(a+bx)\log(c+dx)+2Bd(a+bx)\log(a+bx)}{bg^2(a+bx)(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Log[(e*(c + d*x)^2)/(a + b*x)^2])/(a*g + b*g*x)^2,x]
```

```
[Out] (2*B*d*(a + b*x)*Log[a + b*x] - 2*B*d*(a + b*x)*Log[c + d*x] - (b*c - a*d)*
(A - 2*B + B*Log[(e*(c + d*x)^2)/(a + b*x)^2]))/(b*(b*c - a*d)*g^2*(a + b*x
))
```

**fricas [A]** time = 0.49, size = 110, normalized size = 1.08

$$-\frac{(A-2\,B)bc-(A-2\,B)ad+(Bbdx+Bbc)\log\left(\frac{d^2ex^2+2\,cdex+c^2e}{b^2x^2+2\,abx+a^2}\right)}{\left(b^3c-ab^2d\right)g^2x+\left(ab^2c-a^2bd\right)g^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*(d*x+c)^2/(b*x+a)^2))/(b*g*x+a*g)^2,x, algorithm="fric
as")
```

```
[Out] -((A - 2*B)*b*c - (A - 2*B)*a*d + (B*b*d*x + B*b*c)*log((d^2*e*x^2 + 2*c*d*
e*x + c^2*e)/(b^2*x^2 + 2*a*b*x + a^2)))/((b^3*c - a*b^2*d)*g^2*x + (a*b^2*
c - a^2*b*d)*g^2)
```

**giac [A]** time = 0.40, size = 188, normalized size = 1.84

$$-\left(2\left(b^2cg^2-abdg^2\right)\left(\frac{d\log\left(\left|\frac{bcg}{bgx+ag}-\frac{adg}{bgx+ag}+d\right|\right)}{b^4c^2g^4-2\,ab^3cdg^4+a^2b^2d^2g^4}-\frac{1}{\left(b^2cg^2-abdg^2\right)\left(bgx+ag\right)bg}\right)+\frac{\log\left(\frac{(dx+c)^2e}{(bx+a)^2}\right)}{\left(bgx+ag\right)bg}\right)B-\frac{}{\left(bgx\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*(d*x+c)^2/(b*x+a)^2))/(b*g*x+a*g)^2,x, algorithm="giac
")
```

```
[Out] -(2*(b^2*c*g^2 - a*b*d*g^2)*(d*log(abs(b*c*g/(b*g*x + a*g) - a*d*g/(b*g*x +
 a*g) + d))/(b^4*c^2*g^4 - 2*a*b^3*c*d*g^4 + a^2*b^2*d^2*g^4) - 1/((b^2*c*g
^2 - a*b*d*g^2)*(b*g*x + a*g)*b*g)) + log((d*x + c)^2*e/(b*x + a)^2)/((b*g*
x + a*g)*b*g))*B - A/((b*g*x + a*g)*b*g)
```

**maple [B]** time = 0.05, size = 212, normalized size = 2.08

$$\frac{2Ba\,d^2\ln\left(\frac{ad}{bx+a}-\frac{bc}{bx+a}-d\right)}{(ad-bc)^2\,b\,g^2}-\frac{2Bcd\ln\left(\frac{ad}{bx+a}-\frac{bc}{bx+a}-d\right)}{(ad-bc)^2\,g^2}+\frac{2Bad}{(ad-bc)\,(bx+a)\,b\,g^2}-\frac{2Bc}{(ad-bc)\,(bx+a)\,g^2}-\frac{B\ln\left(\left(\frac{ad}{bx+}\right.\right.}{(bx+}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*ln(e*(d*x+c)^2/(b*x+a)^2))/(b*g*x+a*g)^2,x)
```

```
[Out] -1/b/g^2*A/(b*x+a)-1/b/g^2*B/(b*x+a)*ln((1/(b*x+a)*a*d-1/(b*x+a)*b*c-d)^2/b
^2*e)+2/b/g^2*B/(a*d-b*c)/(b*x+a)*a*d-2/g^2*B/(a*d-b*c)/(b*x+a)*c+2/b/g^2*B
*d^2/(a*d-b*c)^2*ln(1/(b*x+a)*a*d-1/(b*x+a)*b*c-d)*a-2/g^2*B*d/(a*d-b*c)^2*
ln(1/(b*x+a)*a*d-1/(b*x+a)*b*c-d)*c
```

**maxima [A]** time = 1.08, size = 187, normalized size = 1.83

$$-B\left(\frac{\log\left(\frac{d^2ex^2}{b^2x^2+2\,abx+a^2}+\frac{2\,cdex}{b^2x^2+2\,abx+a^2}+\frac{c^2e}{b^2x^2+2\,abx+a^2}\right)}{b^2g^2x+abg^2}-\frac{2}{b^2g^2x+abg^2}-\frac{2\,d\log\left(bx+a\right)}{\left(b^2c-abd\right)g^2}+\frac{2\,d\log\left(dx+c\right)}{\left(b^2c-abd\right)g^2}\right)-\frac{}{b^2g^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*(d*x+c)^2/(b*x+a)^2))/(b*g*x+a*g)^2,x, algorithm="maxi
ma")
```

```
[Out] -B*(log(d^2*e*x^2/(b^2*x^2 + 2*a*b*x + a^2) + 2*c*d*e*x/(b^2*x^2 + 2*a*b*x
+ a^2) + c^2*e/(b^2*x^2 + 2*a*b*x + a^2))/(b^2*g^2*x + a*b*g^2) - 2/(b^2*g^
2*x + a*b*g^2) - 2*d*log(b*x + a)/((b^2*c - a*b*d)*g^2) + 2*d*log(d*x + c)/
((b^2*c - a*b*d)*g^2)) - A/(b^2*g^2*x + a*b*g^2)
```

**mupad [B]** time = 5.94, size = 108, normalized size = 1.06

$$-\frac{A-2\,B}{x\,b^2\,g^2+a\,b\,g^2}-\frac{B\,\ln\left(\frac{e\,{\left(c+d\,x\right)}^2}{{\left(a+b\,x\right)}^2}\right)}{b^2\,g^2\,\left(x+\frac{a}{b}\right)}+\frac{B\,d\,\mathrm{atan}\left(\frac{b\,c\,2\mathrm{i}+b\,d\,x\,2\mathrm{i}}{a\,d-b\,c}+1\mathrm{i}\right)\,4\mathrm{i}}{b\,g^2\,\left(a\,d-b\,c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*log((e*(c + d*x)^2)/(a + b*x)^2))/(a*g + b*g*x)^2,x)
```

```
[Out] (B*d*atan((b*c*2i + b*d*x*2i)/(a*d - b*c) + 1i)*4i)/(b*g^2*(a*d - b*c)) - (
B*log((e*(c + d*x)^2)/(a + b*x)^2))/(b^2*g^2*(x + a/b)) - (A - 2*B)/(b^2*g^
2*x + a*b*g^2)
```

**sympy [B]** time = 1.64, size = 253, normalized size = 2.48

$$-\frac{B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)}{abg^2+b^2g^2x}+\frac{2Bd\log\left(x+\frac{-\frac{2Ba^2d^3}{ad-bc}+\frac{4Babcd^2}{ad-bc}+2Bad^2-\frac{2Bb^2c^2d}{ad-bc}+2Bbcd}{4Bbd^2}\right)}{bg^2\,(ad-bc)}-\frac{2Bd\log\left(x+\frac{\frac{2Ba^2d^3}{ad-bc}-\frac{4Babcd^2}{ad-bc}+2Bad^2+\frac{2Bb^2c^2d}{ad-bc}+2Bbcd}{4Bbd^2}\right)}{bg^2\,(ad-bc)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*ln(e*(d*x+c)**2/(b*x+a)**2))/(b*g*x+a*g)**2,x)
```

```
[Out] -B*log(e*(c + d*x)**2/(a + b*x)**2)/(a*b*g**2 + b**2*g**2*x) + 2*B*d*log(x
+ (-2*B*a**2*d**3/(a*d - b*c) + 4*B*a*b*c*d**2/(a*d - b*c) + 2*B*a*d**2 - 2
*B*b**2*c**2*d/(a*d - b*c) + 2*B*b*c*d)/(4*B*b*d**2))/(b*g**2*(a*d - b*c))
- 2*B*d*log(x + (2*B*a**2*d**3/(a*d - b*c) - 4*B*a*b*c*d**2/(a*d - b*c) + 2
*B*a*d**2 + 2*B*b**2*c**2*d/(a*d - b*c) + 2*B*b*c*d)/(4*B*b*d**2))/(b*g**2*
(a*d - b*c)) + (-A + 2*B)/(a*b*g**2 + b**2*g**2*x)
```

# 3.207 $\int \frac{A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)}{(ag+bgx)^3}\,dx$

**Optimal**. Leaf size=139

$$-\frac{B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)+A}{2bg^3(a+bx)^2}-\frac{Bd^2\log(a+bx)}{bg^3(bc-ad)^2}+\frac{Bd^2\log(c+dx)}{bg^3(bc-ad)^2}-\frac{Bd}{bg^3(a+bx)(bc-ad)}+\frac{B}{2bg^3(a+bx)^2}$$

```
[Out] 1/2*B/b/g^3/(b*x+a)^2-B*d/b/(-a*d+b*c)/g^3/(b*x+a)-B*d^2*ln(b*x+a)/b/(-a*d+
b*c)^2/g^3+B*d^2*ln(d*x+c)/b/(-a*d+b*c)^2/g^3+1/2*(-A-B*ln(e*(d*x+c)^2/(b*x
+a)^2))/b/g^3/(b*x+a)^2
```

**Rubi [A]** time = 0.10, antiderivative size = 139, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.094, Rules used = {2525, 12, 44}

$$-\frac{B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)+A}{2bg^3(a+bx)^2}-\frac{Bd^2\log(a+bx)}{bg^3(bc-ad)^2}+\frac{Bd^2\log(c+dx)}{bg^3(bc-ad)^2}-\frac{Bd}{bg^3(a+bx)(bc-ad)}+\frac{B}{2bg^3(a+bx)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Log[(e*(c + d*x)^2)/(a + b*x)^2])/(a*g + b*g*x)^3,x]
```

```
[Out] B/(2*b*g^3*(a + b*x)^2) - (B*d)/(b*(b*c - a*d)*g^3*(a + b*x)) - (B*d^2*Log[
a + b*x])/(b*(b*c - a*d)^2*g^3) + (B*d^2*Log[c + d*x])/(b*(b*c - a*d)^2*g^3
) - (A + B*Log[(e*(c + d*x)^2)/(a + b*x)^2])/(2*b*g^3*(a + b*x)^2)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 2525

```
Int[((a_.) + Log[(c_.)*(RFx_)^(p_.)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_))^(m_.
), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(a + b*Log[c*RFx^p])^n)/(e*(m + 1))
```

```
, x] - Dist[(b*n*p)/(e*(m + 1)), Int[SimplifyIntegrand[((d + e*x)^(m + 1)*(
a + b*Log[c*RFx^p])^(n - 1)*D[RFx, x])/RFx, x], x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && RationalFunctionQ[RFx, x] && IGtQ[n, 0] && (EqQ[n, 1] ||
IntegerQ[m]) && NeQ[m, -1]
```

Rubi steps

$$\int \frac{A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)}{(ag+bgx)^3}\,dx = -\frac{A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)}{2bg^3(a+bx)^2} + \frac{B\int \frac{2(-bc+ad)}{g^2(a+bx)^3(c+dx)}\,dx}{2bg}$$

$$= -\frac{A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)}{2bg^3(a+bx)^2} - \frac{(B(bc-ad))\int \frac{1}{(a+bx)^3(c+dx)}\,dx}{bg^3}$$

$$= -\frac{A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)}{2bg^3(a+bx)^2} - \frac{(B(bc-ad))\int \left(\frac{b}{(bc-ad)(a+bx)^3} - \frac{bd}{(bc-ad)^2(a+bx)^2} + \frac{bd^2}{(bc-ad)^3(a+bx)} -\right.}{bg^3}$$

$$= \frac{B}{2bg^3(a+bx)^2} - \frac{Bd}{b(bc-ad)g^3(a+bx)} - \frac{Bd^2\log(a+bx)}{b(bc-ad)^2g^3} + \frac{Bd^2\log(c+dx)}{b(bc-ad)^2g^3} - \frac{A+B}{2b}$$

**Mathematica [A]** time = 0.09, size = 128, normalized size = 0.92

$$-\frac{(bc-ad)\left(-aAd+B(bc-ad)\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)+3aBd+Abc-bBc+2bBdx\right)-2Bd^2(a+bx)^2\log(c+dx)+2Bd^2(}{2bg^3(a+bx)^2(bc-ad)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Log[(e*(c + d*x)^2)/(a + b*x)^2])/(a*g + b*g*x)^3,x]
```

```
[Out] -1/2*(2*B*d^2*(a + b*x)^2*Log[a + b*x] - 2*B*d^2*(a + b*x)^2*Log[c + d*x] +
 (b*c - a*d)*(A*b*c - b*B*c - a*A*d + 3*a*B*d + 2*b*B*d*x + B*(b*c - a*d)*L
og[(e*(c + d*x)^2)/(a + b*x)^2]))/(b*(b*c - a*d)^2*g^3*(a + b*x)^2)
```

**fricas [A]** time = 0.65, size = 240, normalized size = 1.73

$$-\frac{(A-B)b^2c^2-2\,(A-2\,B)abcd+(A-3\,B)a^2d^2+2\left(Bb^2cd-Babd^2\right)x-\left(Bb^2d^2x^2+2\,Babd^2x-Bb^2c^2+2\,Babc\right.}{2\left(\left(b^5c^2-2\,ab^4cd+a^2b^3d^2\right)g^3x^2+2\left(ab^4c^2-2\,a^2b^3cd+a^3b^2d^2\right)g^3x+\left(a^2b^3c^2-2\,a^3b^2cd+a^4b\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*(d*x+c)^2/(b*x+a)^2))/(b*g*x+a*g)^3,x, algorithm="fric
as")
```

```
[Out] -1/2*((A - B)*b^2*c^2 - 2*(A - 2*B)*a*b*c*d + (A - 3*B)*a^2*d^2 + 2*(B*b^2*
c*d - B*a*b*d^2)*x - (B*b^2*d^2*x^2 + 2*B*a*b*d^2*x - B*b^2*c^2 + 2*B*a*b*c
*d)*log((d^2*e*x^2 + 2*c*d*e*x + c^2*e)/(b^2*x^2 + 2*a*b*x + a^2)))/((b^5*c
^2 - 2*a*b^4*c*d + a^2*b^3*d^2)*g^3*x^2 + 2*(a*b^4*c^2 - 2*a^2*b^3*c*d + a^
3*b^2*d^2)*g^3*x + (a^2*b^3*c^2 - 2*a^3*b^2*c*d + a^4*b*d^2)*g^3)
```

**giac [A]** time = 0.34, size = 259, normalized size = 1.86

$$-\frac{Bd^2\log(bx+a)}{b^3c^2g^3-2\,ab^2cdg^3+a^2bd^2g^3}+\frac{Bd^2\log(dx+c)}{b^3c^2g^3-2\,ab^2cdg^3+a^2bd^2g^3}-\frac{B\log\left(\frac{d^2x^2+2\,cdx+c^2}{b^2x^2+2\,abx+a^2}\right)}{2\left(b^3g^3x^2+2\,ab^2g^3x+a^2bg^3\right)}-\frac{}{2\left(b^4cg^3x^2-ab\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*(d*x+c)^2/(b*x+a)^2))/(b*g*x+a*g)^3,x, algorithm="giac
")
```

```
[Out] -B*d^2*log(b*x + a)/(b^3*c^2*g^3 - 2*a*b^2*c*d*g^3 + a^2*b*d^2*g^3) + B*d^2
*log(d*x + c)/(b^3*c^2*g^3 - 2*a*b^2*c*d*g^3 + a^2*b*d^2*g^3) - 1/2*B*log((
d^2*x^2 + 2*c*d*x + c^2)/(b^2*x^2 + 2*a*b*x + a^2))/(b^3*g^3*x^2 + 2*a*b^2*
g^3*x + a^2*b*g^3) - 1/2*(2*B*b*d*x + A*b*c - A*a*d + 2*B*a*d)/(b^4*c*g^3*x
^2 - a*b^3*d*g^3*x^2 + 2*a*b^3*c*g^3*x - 2*a^2*b^2*d*g^3*x + a^2*b^2*c*g^3
- a^3*b*d*g^3)
```

**maple [B]** time = 0.06, size = 300, normalized size = 2.16

$$\frac{Ba\,d^3\ln\left(\frac{ad}{bx+a}-\frac{bc}{bx+a}-d\right)}{(ad-bc)^3\,b\,g^3}-\frac{Bc\,d^2\ln\left(\frac{ad}{bx+a}-\frac{bc}{bx+a}-d\right)}{(ad-bc)^3\,g^3}+\frac{B\,a^2d^2}{2\,(ad-bc)^2\,(bx+a)^2\,b\,g^3}-\frac{Bacd}{(ad-bc)^2\,(bx+a)^2\,g^3}+\frac{}{2\,(ad\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*ln(e*(d*x+c)^2/(b*x+a)^2))/(b*g*x+a*g)^3,x)
```

```
[Out] -1/2/b/(b*x+a)^2/g^3*A-1/2/b/g^3*B/(b*x+a)^2*ln((1/(b*x+a)*a*d-1/(b*x+a)*b*
c-d)^2/b^2*e)+1/2/b/g^3*B/(a*d-b*c)^2/(b*x+a)^2*a^2*d^2-1/g^3*B/(a*d-b*c)^2
/(b*x+a)^2*a*d*c+1/2*b/g^3*B/(a*d-b*c)^2/(b*x+a)^2*c^2+1/b/g^3*B/(a*d-b*c)^
2/(b*x+a)*d^2*a-1/g^3*B/(a*d-b*c)^2/(b*x+a)*d*c+1/b/g^3*B*d^3/(a*d-b*c)^3*l
n(1/(b*x+a)*a*d-1/(b*x+a)*b*c-d)*a-1/g^3*B*d^2/(a*d-b*c)^3*ln(1/(b*x+a)*a*d
-1/(b*x+a)*b*c-d)*c
```

**maxima [B]** time = 1.16, size = 306, normalized size = 2.20

$$-\frac{1}{2}B\left(\frac{2\,bdx-bc+3\,ad}{\left(b^4c-ab^3d\right)g^3x^2+2\left(ab^3c-a^2b^2d\right)g^3x+\left(a^2b^2c-a^3bd\right)g^3}+\frac{\log\left(\frac{d^2ex^2}{b^2x^2+2\,abx+a^2}+\frac{2\,cdex}{b^2x^2+2\,abx+a^2}+\frac{c^2e}{b^2x^2+2\,abx+}\right.}{b^3g^3x^2+2\,ab^2g^3x+a^2bg^3}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*(d*x+c)^2/(b*x+a)^2))/(b*g*x+a*g)^3,x, algorithm="maxi
ma")
```

```
[Out] -1/2*B*((2*b*d*x - b*c + 3*a*d)/((b^4*c - a*b^3*d)*g^3*x^2 + 2*(a*b^3*c - a
^2*b^2*d)*g^3*x + (a^2*b^2*c - a^3*b*d)*g^3) + log(d^2*e*x^2/(b^2*x^2 + 2*a
*b*x + a^2) + 2*c*d*e*x/(b^2*x^2 + 2*a*b*x + a^2) + c^2*e/(b^2*x^2 + 2*a*b*
x + a^2))/(b^3*g^3*x^2 + 2*a*b^2*g^3*x + a^2*b*g^3) + 2*d^2*log(b*x + a)/((
b^3*c^2 - 2*a*b^2*c*d + a^2*b*d^2)*g^3) - 2*d^2*log(d*x + c)/((b^3*c^2 - 2*
a*b^2*c*d + a^2*b*d^2)*g^3)) - 1/2*A/(b^3*g^3*x^2 + 2*a*b^2*g^3*x + a^2*b*g
^3)
```

**mupad [B]** time = 5.93, size = 206, normalized size = 1.48

$$\frac{2\,B\,d^2\,\mathrm{atanh}\left(\frac{b^3\,c^2\,g^3-a^2\,b\,d^2\,g^3}{b\,g^3\,{\left(a\,d-b\,c\right)}^2}-\frac{2\,b\,d\,x}{a\,d-b\,c}\right)}{b\,g^3\,{\left(a\,d-b\,c\right)}^2}-\frac{B\,\ln\left(\frac{e\,{\left(c+d\,x\right)}^2}{{\left(a+b\,x\right)}^2}\right)}{2\,b^2\,g^3\,\left(2\,a\,x+b\,x^2+\frac{a^2}{b}\right)}-\frac{\frac{A\,a\,d-A\,b\,c-3\,B\,a\,d+B\,b\,c}{2\,\left(a\,d-b\,c\right)}-\frac{B\,b\,d\,x}{a\,d-b\,c}}{a^2\,b\,g^3+2\,a\,b^2\,g^3\,x+b^3\,g^3\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*log((e*(c + d*x)^2)/(a + b*x)^2))/(a*g + b*g*x)^3,x)
```

```
[Out] (2*B*d^2*atanh((b^3*c^2*g^3 - a^2*b*d^2*g^3)/(b*g^3*(a*d - b*c)^2) - (2*b*d
*x)/(a*d - b*c)))/(b*g^3*(a*d - b*c)^2) - (B*log((e*(c + d*x)^2)/(a + b*x)^
2))/(2*b^2*g^3*(2*a*x + b*x^2 + a^2/b)) - ((A*a*d - A*b*c - 3*B*a*d + B*b*c
)/(2*(a*d - b*c)) - (B*b*d*x)/(a*d - b*c))/(a^2*b*g^3 + b^3*g^3*x^2 + 2*a*b
^2*g^3*x)
```

**sympy [B]** time = 2.60, size = 418, normalized size = 3.01

$$-\frac{B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)}{2a^2bg^3+4ab^2g^3x+2b^3g^3x^2}+\frac{Bd^2\log\left(x+\frac{-\frac{Ba^3d^5}{(ad-bc)^2}+\frac{3Ba^2bcd^4}{(ad-bc)^2}-\frac{3Bab^2c^2d^3}{(ad-bc)^2}+Bad^3+\frac{Bb^3c^3d^2}{(ad-bc)^2}+Bbcd^2}{2Bbd^3}\right)}{bg^3\left(ad-bc\right)^2}-\frac{Bd^2\log\left(x+\frac{\frac{Ba^3d^5}{(ad-bc)^2}-\frac{3B}{(a}}{}\right.}{bg}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*ln(e*(d*x+c)**2/(b*x+a)**2))/(b*g*x+a*g)**3,x)
```

```
[Out] -B*log(e*(c + d*x)**2/(a + b*x)**2)/(2*a**2*b*g**3 + 4*a*b**2*g**3*x + 2*b*
*3*g**3*x**2) + B*d**2*log(x + (-B*a**3*d**5/(a*d - b*c)**2 + 3*B*a**2*b*c*
d**4/(a*d - b*c)**2 - 3*B*a*b**2*c**2*d**3/(a*d - b*c)**2 + B*a*d**3 + B*b*
*3*c**3*d**2/(a*d - b*c)**2 + B*b*c*d**2)/(2*B*b*d**3))/(b*g**3*(a*d - b*c)
**2) - B*d**2*log(x + (B*a**3*d**5/(a*d - b*c)**2 - 3*B*a**2*b*c*d**4/(a*d
- b*c)**2 + 3*B*a*b**2*c**2*d**3/(a*d - b*c)**2 + B*a*d**3 - B*b**3*c**3*d*
*2/(a*d - b*c)**2 + B*b*c*d**2)/(2*B*b*d**3))/(b*g**3*(a*d - b*c)**2) + (-A
*a*d + A*b*c + 3*B*a*d - B*b*c + 2*B*b*d*x)/(2*a**3*b*d*g**3 - 2*a**2*b**2*
c*g**3 + x**2*(2*a*b**3*d*g**3 - 2*b**4*c*g**3) + x*(4*a**2*b**2*d*g**3 - 4
*a*b**3*c*g**3))
```

## 3.208 $\int \frac{A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)}{(ag+bgx)^4}\,dx$

**Optimal**. Leaf size=177

$$-\frac{B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)+A}{3bg^4(a+bx)^3}+\frac{2Bd^3\log(a+bx)}{3bg^4(bc-ad)^3}-\frac{2Bd^3\log(c+dx)}{3bg^4(bc-ad)^3}+\frac{2Bd^2}{3bg^4(a+bx)(bc-ad)^2}-\frac{Bd}{3bg^4(a+bx)^2(bc-ad)}+\frac{}{9b}$$

```
[Out] 2/9*B/b/g^4/(b*x+a)^3-1/3*B*d/b/(-a*d+b*c)/g^4/(b*x+a)^2+2/3*B*d^2/b/(-a*d+
b*c)^2/g^4/(b*x+a)+2/3*B*d^3*ln(b*x+a)/b/(-a*d+b*c)^3/g^4-2/3*B*d^3*ln(d*x+
c)/b/(-a*d+b*c)^3/g^4+1/3*(-A-B*ln(e*(d*x+c)^2/(b*x+a)^2))/b/g^4/(b*x+a)^3
```

**Rubi [A]** time = 0.12, antiderivative size = 177, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.094, Rules used = {2525, 12, 44}

$$-\frac{B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)+A}{3bg^4(a+bx)^3}+\frac{2Bd^2}{3bg^4(a+bx)(bc-ad)^2}+\frac{2Bd^3\log(a+bx)}{3bg^4(bc-ad)^3}-\frac{2Bd^3\log(c+dx)}{3bg^4(bc-ad)^3}-\frac{Bd}{3bg^4(a+bx)^2(bc-ad)}+\frac{}{9b}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Log[(e*(c + d*x)^2)/(a + b*x)^2])/(a*g + b*g*x)^4,x]
```

```
[Out] (2*B)/(9*b*g^4*(a + b*x)^3) - (B*d)/(3*b*(b*c - a*d)*g^4*(a + b*x)^2) + (2*
B*d^2)/(3*b*(b*c - a*d)^2*g^4*(a + b*x)) + (2*B*d^3*Log[a + b*x])/(3*b*(b*c
 - a*d)^3*g^4) - (2*B*d^3*Log[c + d*x])/(3*b*(b*c - a*d)^3*g^4) - (A + B*Lo
g[(e*(c + d*x)^2)/(a + b*x)^2])/(3*b*g^4*(a + b*x)^3)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 2525

```
Int[((a_.) + Log[(c_.)*(RFx_)^(p_.)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_))^(m_.
), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(a + b*Log[c*RFx^p])^n)/(e*(m + 1))
```

```
, x] - Dist[(b*n*p)/(e*(m + 1)), Int[SimplifyIntegrand[((d + e*x)^(m + 1)*(
a + b*Log[c*RFx^p])^(n - 1)*D[RFx, x])/RFx, x], x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && RationalFunctionQ[RFx, x] && IGtQ[n, 0] && (EqQ[n, 1] ||
IntegerQ[m]) && NeQ[m, -1]
```

Rubi steps

$$\int \frac{A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)}{(ag+bgx)^4}\,dx = -\frac{A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)}{3bg^4(a+bx)^3} + \frac{B\int \frac{2(-bc+ad)}{g^3(a+bx)^4(c+dx)}\,dx}{3bg}$$

$$= -\frac{A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)}{3bg^4(a+bx)^3} - \frac{(2B(bc-ad))\int \frac{1}{(a+bx)^4(c+dx)}\,dx}{3bg^4}$$

$$= -\frac{A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)}{3bg^4(a+bx)^3} - \frac{(2B(bc-ad))\int \left(\frac{b}{(bc-ad)(a+bx)^4} - \frac{bd}{(bc-ad)^2(a+bx)^3} + \frac{bd^2}{(bc-ad)^3(a+b}\right.}{3bg^4}$$

$$= \frac{2B}{9bg^4(a+bx)^3} - \frac{Bd}{3b(bc-ad)g^4(a+bx)^2} + \frac{2Bd^2}{3b(bc-ad)^2g^4(a+bx)} + \frac{2Bd^3\log(a+}{3b(bc-ad)^3}$$

**Mathematica [A]** time = 0.10, size = 140, normalized size = 0.79

$$\frac{\frac{B\left(-6d^3(a+bx)^3\log(c+dx)+6d^2(a+bx)^2(bc-ad)-3d(a+bx)(bc-ad)^2+2(bc-ad)^3+6d^3(a+bx)^3\log(a+bx)\right)}{(bc-ad)^3} - 3\left(B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)+A\right)}{9bg^4(a+bx)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Log[(e*(c + d*x)^2)/(a + b*x)^2])/(a*g + b*g*x)^4,x]
```

```
[Out] ((B*(2*(b*c - a*d)^3 - 3*d*(b*c - a*d)^2*(a + b*x) + 6*d^2*(b*c - a*d)*(a +
 b*x)^2 + 6*d^3*(a + b*x)^3*Log[a + b*x] - 6*d^3*(a + b*x)^3*Log[c + d*x]))
/(b*c - a*d)^3 - 3*(A + B*Log[(e*(c + d*x)^2)/(a + b*x)^2]))/(9*b*g^4*(a +
b*x)^3)
```

**fricas [B]** time = 0.80, size = 432, normalized size = 2.44

$$-\frac{(3A-2B)b^3c^3 - 9(A-B)ab^2c^2d + 9(A-2B)a^2bcd^2 - (3A-11B)a^3d^3 - 6\left(Bb^3cd^2 - Bab^2d^3\right)x^2 + 3\left(Bb^3c^2\right.}{9\left(\left(b^7c^3 - 3ab^6c^2d + 3a^2b^5cd^2 - a^3b^4d^3\right)g^4x^3 + 3\left(ab^6c^3 - 3a^2b^5c^2d + 3a^3b^4cd^2 - a^4b^3d^3\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*(d*x+c)^2/(b*x+a)^2))/(b*g*x+a*g)^4,x, algorithm="fric
as")
```

```
[Out] -1/9*((3*A - 2*B)*b^3*c^3 - 9*(A - B)*a*b^2*c^2*d + 9*(A - 2*B)*a^2*b*c*d^2
 - (3*A - 11*B)*a^3*d^3 - 6*(B*b^3*c*d^2 - B*a*b^2*d^3)*x^2 + 3*(B*b^3*c^2*
d - 6*B*a*b^2*c*d^2 + 5*B*a^2*b*d^3)*x + 3*(B*b^3*d^3*x^3 + 3*B*a*b^2*d^3*x
^2 + 3*B*a^2*b*d^3*x + B*b^3*c^3 - 3*B*a*b^2*c^2*d + 3*B*a^2*b*c*d^2)*log((
d^2*e*x^2 + 2*c*d*e*x + c^2*e)/(b^2*x^2 + 2*a*b*x + a^2)))/((b^7*c^3 - 3*a*
b^6*c^2*d + 3*a^2*b^5*c*d^2 - a^3*b^4*d^3)*g^4*x^3 + 3*(a*b^6*c^3 - 3*a^2*b
^5*c^2*d + 3*a^3*b^4*c*d^2 - a^4*b^3*d^3)*g^4*x^2 + 3*(a^2*b^5*c^3 - 3*a^3*
b^4*c^2*d + 3*a^4*b^3*c*d^2 - a^5*b^2*d^3)*g^4*x + (a^3*b^4*c^3 - 3*a^4*b^3
*c^2*d + 3*a^5*b^2*c*d^2 - a^6*b*d^3)*g^4)
```

**giac [B]** time = 0.33, size = 473, normalized size = 2.67

$$\frac{2\,Bd^3\log\left(bx+a\right)}{3\left(b^4c^3g^4-3\,ab^3c^2dg^4+3\,a^2b^2cd^2g^4-a^3bd^3g^4\right)}-\frac{2\,Bd^3\log\left(dx+c\right)}{3\left(b^4c^3g^4-3\,ab^3c^2dg^4+3\,a^2b^2cd^2g^4-a^3bd^3g^4\right)}-\frac{B}{3\left(b^4g^4x^3+\cdots\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*(d*x+c)^2/(b*x+a)^2))/(b*g*x+a*g)^4,x, algorithm="giac
")
```

```
[Out] 2/3*B*d^3*log(b*x + a)/(b^4*c^3*g^4 - 3*a*b^3*c^2*d*g^4 + 3*a^2*b^2*c*d^2*g
^4 - a^3*b*d^3*g^4) - 2/3*B*d^3*log(d*x + c)/(b^4*c^3*g^4 - 3*a*b^3*c^2*d*g
^4 + 3*a^2*b^2*c*d^2*g^4 - a^3*b*d^3*g^4) - 1/3*B*log((d^2*x^2 + 2*c*d*x +
c^2)/(b^2*x^2 + 2*a*b*x + a^2))/(b^4*g^4*x^3 + 3*a*b^3*g^4*x^2 + 3*a^2*b^2*
g^4*x + a^3*b*g^4) + 1/9*(6*B*b^2*d^2*x^2 - 3*B*b^2*c*d*x + 15*B*a*b*d^2*x
- 3*A*b^2*c^2 - B*b^2*c^2 + 6*A*a*b*c*d - B*a*b*c*d - 3*A*a^2*d^2 + 8*B*a^2
*d^2)/(b^6*c^2*g^4*x^3 - 2*a*b^5*c*d*g^4*x^3 + a^2*b^4*d^2*g^4*x^3 + 3*a*b^
5*c^2*g^4*x^2 - 6*a^2*b^4*c*d*g^4*x^2 + 3*a^3*b^3*d^2*g^4*x^2 + 3*a^2*b^4*c
^2*g^4*x - 6*a^3*b^3*c*d*g^4*x + 3*a^4*b^2*d^2*g^4*x + a^3*b^3*c^2*g^4 - 2*
a^4*b^2*c*d*g^4 + a^5*b*d^2*g^4)
```

**maple [B]** time = 0.06, size = 427, normalized size = 2.41

$$\frac{2B\,a^3d^3}{9\left(ad-bc\right)^3\left(bx+a\right)^3b\,g^4}-\frac{2B\,a^2c\,d^2}{3\left(ad-bc\right)^3\left(bx+a\right)^3g^4}+\frac{2Bab\,c^2d}{3\left(ad-bc\right)^3\left(bx+a\right)^3g^4}+\frac{2Ba\,d^4\ln\left(\frac{ad}{bx+a}-\frac{bc}{bx+a}-d\right)}{3\left(ad-bc\right)^4b\,g^4}-\frac{\cdots}{9\left(ad\cdots\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*ln(e*(d*x+c)^2/(b*x+a)^2))/(b*g*x+a*g)^4,x)
```

```
[Out] -1/3/b/(b*x+a)^3/g^4*A-1/3/b/g^4*B/(b*x+a)^3*ln((1/(b*x+a)*a*d-1/(b*x+a)*b*
c-d)^2/b^2*e)+2/9/b/g^4*B*a^3*d^3/(a*d-b*c)^3/(b*x+a)^3-2/3/g^4*B*a^2*d^2/(
a*d-b*c)^3/(b*x+a)^3*c+2/3*b/g^4*B*a*d/(a*d-b*c)^3/(b*x+a)^3*c^2+1/3/b/g^4*
B*a^2*d^3/(a*d-b*c)^3/(b*x+a)^2-2/3/g^4*B*a*d^2/(a*d-b*c)^3/(b*x+a)^2*c+2/3
/b/g^4*B*a*d^3/(a*d-b*c)^3/(b*x+a)+2/3/b/g^4*B*a*d^4/(a*d-b*c)^4*ln(1/(b*x+
a)*a*d-1/(b*x+a)*b*c-d)-2/9*b^2/g^4*B*c^3/(a*d-b*c)^3/(b*x+a)^3+1/3*b/g^4*B
*c^2/(a*d-b*c)^3/(b*x+a)^2*d-2/3/g^4*B*c/(a*d-b*c)^3/(b*x+a)*d^2-2/3/g^4*B*
c*d^3/(a*d-b*c)^4*ln(1/(b*x+a)*a*d-1/(b*x+a)*b*c-d)
```

**maxima [B]** time = 1.22, size = 480, normalized size = 2.71

$$\frac{1}{9}B\left(\frac{6\,b^2d^2x^2+2\,b^2c^2-7\,abcd+11\,a^2d^2-3\left(b^2cd-5\,abd^2\right)x}{\left(b^6c^2-2\,ab^5cd+a^2b^4d^2\right)g^4x^3+3\left(ab^5c^2-2\,a^2b^4cd+a^3b^3d^2\right)g^4x^2+3\left(a^2b^4c^2-2\,a^3b^3cd+a^4b^2d^2\right)g^4x+\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*(d*x+c)^2/(b*x+a)^2))/(b*g*x+a*g)^4,x, algorithm="maxi
ma")
```

```
[Out] 1/9*B*((6*b^2*d^2*x^2 + 2*b^2*c^2 - 7*a*b*c*d + 11*a^2*d^2 - 3*(b^2*c*d - 5
*a*b*d^2)*x)/((b^6*c^2 - 2*a*b^5*c*d + a^2*b^4*d^2)*g^4*x^3 + 3*(a*b^5*c^2
- 2*a^2*b^4*c*d + a^3*b^3*d^2)*g^4*x^2 + 3*(a^2*b^4*c^2 - 2*a^3*b^3*c*d + a
^4*b^2*d^2)*g^4*x + (a^3*b^3*c^2 - 2*a^4*b^2*c*d + a^5*b*d^2)*g^4) - 3*log(
d^2*e*x^2/(b^2*x^2 + 2*a*b*x + a^2) + 2*c*d*e*x/(b^2*x^2 + 2*a*b*x + a^2) +
 c^2*e/(b^2*x^2 + 2*a*b*x + a^2))/(b^4*g^4*x^3 + 3*a*b^3*g^4*x^2 + 3*a^2*b^
2*g^4*x + a^3*b*g^4) + 6*d^3*log(b*x + a)/((b^4*c^3 - 3*a*b^3*c^2*d + 3*a^2
*b^2*c*d^2 - a^3*b*d^3)*g^4) - 6*d^3*log(d*x + c)/((b^4*c^3 - 3*a*b^3*c^2*d
 + 3*a^2*b^2*c*d^2 - a^3*b*d^3)*g^4)) - 1/3*A/(b^4*g^4*x^3 + 3*a*b^3*g^4*x^
2 + 3*a^2*b^2*g^4*x + a^3*b*g^4)
```

**mupad [B]** time = 6.73, size = 341, normalized size = 1.93

$$\frac{2\,B\,b\,c^2}{9\,g^4\,{\left(a\,d-b\,c\right)}^2\,{\left(a+b\,x\right)}^3}-\frac{A\,b\,c^2}{3\,g^4\,{\left(a\,d-b\,c\right)}^2\,{\left(a+b\,x\right)}^3}-\frac{B\,\ln\left(\frac{e\,{\left(c+d\,x\right)}^2}{{\left(a+b\,x\right)}^2}\right)}{3\,b\,g^4\,{\left(a+b\,x\right)}^3}-\frac{A\,a^2\,d^2}{3\,b\,g^4\,{\left(a\,d-b\,c\right)}^2\,{\left(a+b\,x\right)}^3}+\frac{11}{9\,b\,g^4\,(a\,d-}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*log((e*(c + d*x)^2)/(a + b*x)^2))/(a*g + b*g*x)^4,x)
```

```
[Out] (B*d^3*atan((a*d*1i + b*c*1i + b*d*x*2i)/(a*d - b*c))*4i)/(3*b*g^4*(a*d - b
*c)^3) - (B*log((e*(c + d*x)^2)/(a + b*x)^2))/(3*b*g^4*(a + b*x)^3) - (A*b*
c^2)/(3*g^4*(a*d - b*c)^2*(a + b*x)^3) + (2*B*b*c^2)/(9*g^4*(a*d - b*c)^2*(
a + b*x)^3) - (A*a^2*d^2)/(3*b*g^4*(a*d - b*c)^2*(a + b*x)^3) + (11*B*a^2*d
^2)/(9*b*g^4*(a*d - b*c)^2*(a + b*x)^3) + (5*B*a*d^2*x)/(3*g^4*(a*d - b*c)^
```

2*(a + b*x)^3) + (2*B*b*d^2*x^2)/(3*g^4*(a*d - b*c)^2*(a + b*x)^3) + (2*A*a*c*d)/(3*g^4*(a*d - b*c)^2*(a + b*x)^3) - (7*B*a*c*d)/(9*g^4*(a*d - b*c)^2*(a + b*x)^3) - (B*b*c*d*x)/(3*g^4*(a*d - b*c)^2*(a + b*x)^3)

**sympy [B]** time = 4.06, size = 677, normalized size = 3.82

$$-\frac{B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)}{3a^3bg^4+9a^2b^2g^4x+9ab^3g^4x^2+3b^4g^4x^3}+\frac{2Bd^3\log\left(x+\frac{-\frac{2Ba^4d^7}{(ad-bc)^3}+\frac{8Ba^3bcd^6}{(ad-bc)^3}-\frac{12Ba^2b^2c^2d^5}{(ad-bc)^3}+\frac{8Bab^3c^3d^4}{(ad-bc)^3}+2Bad^4-\frac{2Bb^4c^4d^3}{(ad-bc)^3}+2Bbcd}{4Bbd^4}\right)}{3bg^4(ad-bc)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*ln(e*(d*x+c)**2/(b*x+a)**2))/(b*g*x+a*g)**4,x)
```

```
[Out] -B*log(e*(c + d*x)**2/(a + b*x)**2)/(3*a**3*b*g**4 + 9*a**2*b**2*g**4*x + 9
*a*b**3*g**4*x**2 + 3*b**4*g**4*x**3) + 2*B*d**3*log(x + (-2*B*a**4*d**7/(a
*d - b*c)**3 + 8*B*a**3*b*c*d**6/(a*d - b*c)**3 - 12*B*a**2*b**2*c**2*d**5/
(a*d - b*c)**3 + 8*B*a*b**3*c**3*d**4/(a*d - b*c)**3 + 2*B*a*d**4 - 2*B*b**
4*c**4*d**3/(a*d - b*c)**3 + 2*B*b*c*d**3)/(4*B*b*d**4))/(3*b*g**4*(a*d - b
*c)**3) - 2*B*d**3*log(x + (2*B*a**4*d**7/(a*d - b*c)**3 - 8*B*a**3*b*c*d**
6/(a*d - b*c)**3 + 12*B*a**2*b**2*c**2*d**5/(a*d - b*c)**3 - 8*B*a*b**3*c**
3*d**4/(a*d - b*c)**3 + 2*B*a*d**4 + 2*B*b**4*c**4*d**3/(a*d - b*c)**3 + 2*
B*b*c*d**3)/(4*B*b*d**4))/(3*b*g**4*(a*d - b*c)**3) + (-3*A*a**2*d**2 + 6*A
*a*b*c*d - 3*A*b**2*c**2 + 11*B*a**2*d**2 - 7*B*a*b*c*d + 2*B*b**2*c**2 + 6
*B*b**2*d**2*x**2 + x*(15*B*a*b*d**2 - 3*B*b**2*c*d))/(9*a**5*b*d**2*g**4 -
 18*a**4*b**2*c*d*g**4 + 9*a**3*b**3*c**2*g**4 + x**3*(9*a**2*b**4*d**2*g**
4 - 18*a*b**5*c*d*g**4 + 9*b**6*c**2*g**4) + x**2*(27*a**3*b**3*d**2*g**4 -
 54*a**2*b**4*c*d*g**4 + 27*a*b**5*c**2*g**4) + x*(27*a**4*b**2*d**2*g**4 -
 54*a**3*b**3*c*d*g**4 + 27*a**2*b**4*c**2*g**4))
```

# 3.209 $\int \frac{A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)}{(ag+bgx)^5}\,dx$

**Optimal**. Leaf size=208

$$-\frac{B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)+A}{4bg^5(a+bx)^4}-\frac{Bd^4\log(a+bx)}{2bg^5(bc-ad)^4}+\frac{Bd^4\log(c+dx)}{2bg^5(bc-ad)^4}-\frac{Bd^3}{2bg^5(a+bx)(bc-ad)^3}+\frac{Bd^2}{4bg^5(a+bx)^2(bc-ad)^2}-6b$$

```
[Out] 1/8*B/b/g^5/(b*x+a)^4-1/6*B*d/b/(-a*d+b*c)/g^5/(b*x+a)^3+1/4*B*d^2/b/(-a*d+
b*c)^2/g^5/(b*x+a)^2-1/2*B*d^3/b/(-a*d+b*c)^3/g^5/(b*x+a)-1/2*B*d^4*ln(b*x+
a)/b/(-a*d+b*c)^4/g^5+1/2*B*d^4*ln(d*x+c)/b/(-a*d+b*c)^4/g^5+1/4*(-A-B*ln(e
*(d*x+c)^2/(b*x+a)^2))/b/g^5/(b*x+a)^4
```

**Rubi [A]** time = 0.14, antiderivative size = 208, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.094, Rules used = {2525, 12, 44}

$$-\frac{B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)+A}{4bg^5(a+bx)^4}-\frac{Bd^3}{2bg^5(a+bx)(bc-ad)^3}+\frac{Bd^2}{4bg^5(a+bx)^2(bc-ad)^2}-\frac{Bd^4\log(a+bx)}{2bg^5(bc-ad)^4}+\frac{Bd^4\log(c+dx)}{2bg^5(bc-ad)^4}-6b$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Log[(e*(c + d*x)^2)/(a + b*x)^2])/(a*g + b*g*x)^5,x]
```

```
[Out] B/(8*b*g^5*(a + b*x)^4) - (B*d)/(6*b*(b*c - a*d)*g^5*(a + b*x)^3) + (B*d^2)
/(4*b*(b*c - a*d)^2*g^5*(a + b*x)^2) - (B*d^3)/(2*b*(b*c - a*d)^3*g^5*(a +
b*x)) - (B*d^4*Log[a + b*x])/(2*b*(b*c - a*d)^4*g^5) + (B*d^4*Log[c + d*x])
/(2*b*(b*c - a*d)^4*g^5) - (A + B*Log[(e*(c + d*x)^2)/(a + b*x)^2])/(4*b*g^
5*(a + b*x)^4)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
 + n + 2, 0])
```

Rule 2525

```
Int[((a_.) + Log[(c_.)*(RFx_)^(p_.)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_))^(m_.
), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(a + b*Log[c*RFx^p])^n)/(e*(m + 1))
, x] - Dist[(b*n*p)/(e*(m + 1)), Int[SimplifyIntegrand[((d + e*x)^(m + 1)*(
a + b*Log[c*RFx^p])^(n - 1)*D[RFx, x])/RFx, x], x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && RationalFunctionQ[RFx, x] && IGtQ[n, 0] && (EqQ[n, 1] ||
IntegerQ[m]) && NeQ[m, -1]
```

Rubi steps

$$\int \frac{A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)}{(ag+bgx)^5}\,dx = -\frac{A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)}{4bg^5(a+bx)^4}+\frac{B\int \frac{2(-bc+ad)}{g^4(a+bx)^5(c+dx)}\,dx}{4bg}$$

$$= -\frac{A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)}{4bg^5(a+bx)^4}-\frac{(B(bc-ad))\int \frac{1}{(a+bx)^5(c+dx)}\,dx}{2bg^5}$$

$$= -\frac{A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)}{4bg^5(a+bx)^4}-\frac{(B(bc-ad))\int \left(\frac{b}{(bc-ad)(a+bx)^5}-\frac{bd}{(bc-ad)^2(a+bx)^4}+\frac{bd^2}{(bc-ad)^3(a+bx)^3}\right.}{2bg^5}$$

$$= \frac{B}{8bg^5(a+bx)^4}-\frac{Bd}{6b(bc-ad)g^5(a+bx)^3}+\frac{Bd^2}{4b(bc-ad)^2g^5(a+bx)^2}-\frac{Bd^3}{2b(bc-ad)^3g^5}$$

**Mathematica [A]** time = 0.17, size = 162, normalized size = 0.78

$$\frac{\frac{B\left(12d^4(a+bx)^4\log(c+dx)+12d^3(a+bx)^3(ad-bc)+6d^2(a+bx)^2(bc-ad)^2+4d(a+bx)(ad-bc)^3+3(bc-ad)^4-12d^4(a+bx)^4\log(a+bx)\right)}{(bc-ad)^4}-6\left(B\log\left(\frac{e(c+dx)}{(a+bx)}\right.\right.}{24bg^5(a+bx)^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Log[(e*(c + d*x)^2)/(a + b*x)^2])/(a*g + b*g*x)^5,x]
```

```
[Out] ((B*(3*(b*c - a*d)^4 + 4*d*(-(b*c) + a*d)^3*(a + b*x) + 6*d^2*(b*c - a*d)^2
*(a + b*x)^2 + 12*d^3*(-(b*c) + a*d)*(a + b*x)^3 - 12*d^4*(a + b*x)^4*Log[a
 + b*x] + 12*d^4*(a + b*x)^4*Log[c + d*x]))/(b*c - a*d)^4 - 6*(A + B*Log[(e
*(c + d*x)^2)/(a + b*x)^2]))/(24*b*g^5*(a + b*x)^4)
```

**fricas [B]** time = 1.51, size = 658, normalized size = 3.16

$$-\frac{3\,(2\,A-B)b^4c^4-8\,(3\,A-2\,B)ab^3c^3d+36\,(A-B)a^2b^2c^2d^2-24\,(A-2\,B)a^3bcd^3+(6\,A-25\,B)a^4d^4+12\left(Bb^4c\right.}{24\left(\left(b^9c^4-4\,ab^8c^3d+6\,a^2b^7c^2d^2-4\,a^3b^6cd^3+a^4b^5d^4\right)g^5x^4+4\left(ab^8c^4-4\,a^2b^7c^3d+6\,a^3b^6\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*(d*x+c)^2/(b*x+a)^2))/(b*g*x+a*g)^5,x, algorithm="fric
as")
```

```
[Out] -1/24*(3*(2*A - B)*b^4*c^4 - 8*(3*A - 2*B)*a*b^3*c^3*d + 36*(A - B)*a^2*b^2
*c^2*d^2 - 24*(A - 2*B)*a^3*b*c*d^3 + (6*A - 25*B)*a^4*d^4 + 12*(B*b^4*c*d^
3 - B*a*b^3*d^4)*x^3 - 6*(B*b^4*c^2*d^2 - 8*B*a*b^3*c*d^3 + 7*B*a^2*b^2*d^4
)*x^2 + 4*(B*b^4*c^3*d - 6*B*a*b^3*c^2*d^2 + 18*B*a^2*b^2*c*d^3 - 13*B*a^3*
b*d^4)*x - 6*(B*b^4*d^4*x^4 + 4*B*a*b^3*d^4*x^3 + 6*B*a^2*b^2*d^4*x^2 + 4*B
*a^3*b*d^4*x - B*b^4*c^4 + 4*B*a*b^3*c^3*d - 6*B*a^2*b^2*c^2*d^2 + 4*B*a^3*
b*c*d^3)*log((d^2*e*x^2 + 2*c*d*e*x + c^2*e)/(b^2*x^2 + 2*a*b*x + a^2)))/((
b^9*c^4 - 4*a*b^8*c^3*d + 6*a^2*b^7*c^2*d^2 - 4*a^3*b^6*c*d^3 + a^4*b^5*d^4
)*g^5*x^4 + 4*(a*b^8*c^4 - 4*a^2*b^7*c^3*d + 6*a^3*b^6*c^2*d^2 - 4*a^4*b^5*
c*d^3 + a^5*b^4*d^4)*g^5*x^3 + 6*(a^2*b^7*c^4 - 4*a^3*b^6*c^3*d + 6*a^4*b^5
*c^2*d^2 - 4*a^5*b^4*c*d^3 + a^6*b^3*d^4)*g^5*x^2 + 4*(a^3*b^6*c^4 - 4*a^4*
b^5*c^3*d + 6*a^5*b^4*c^2*d^2 - 4*a^6*b^3*c*d^3 + a^7*b^2*d^4)*g^5*x + (a^4
*b^5*c^4 - 4*a^5*b^4*c^3*d + 6*a^6*b^3*c^2*d^2 - 4*a^7*b^2*c*d^3 + a^8*b*d^
4)*g^5)
```

**giac [B]** time = 0.69, size = 416, normalized size = 2.00

$$\frac{Bd^4\log\left(-\frac{bcg}{bgx+ag}+\frac{adg}{bgx+ag}-d\right)}{2\left(b^5c^4g^5-4\,ab^4c^3dg^5+6\,a^2b^3c^2d^2g^5-4\,a^3b^2cd^3g^5+a^4bd^4g^5\right)}-\frac{Bd^3}{2\left(b^3c^3g^3-3\,ab^2c^2dg^3+3\,a^2bcd^2g^3-a^3d^3g^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*(d*x+c)^2/(b*x+a)^2))/(b*g*x+a*g)^5,x, algorithm="giac
")
```

```
[Out] 1/2*B*d^4*log(-b*c*g/(b*g*x + a*g) + a*d*g/(b*g*x + a*g) - d)/(b^5*c^4*g^5
- 4*a*b^4*c^3*d*g^5 + 6*a^2*b^3*c^2*d^2*g^5 - 4*a^3*b^2*c*d^3*g^5 + a^4*b*d
^4*g^5) - 1/2*B*d^3/((b^3*c^3*g^3 - 3*a*b^2*c^2*d*g^3 + 3*a^2*b*c*d^2*g^3 -
 a^3*d^3*g^3)*(b*g*x + a*g)*b*g) + 1/4*B*d^2/((b^2*c^2*g - 2*a*b*c*d*g + a^
2*d^2*g)*(b*g*x + a*g)^2*b*g^2) - 1/4*B*log((b^2*c^2*g^2/(b*g*x + a*g)^2 -
2*a*b*c*d*g^2/(b*g*x + a*g)^2 + a^2*d^2*g^2/(b*g*x + a*g)^2 + 2*b*c*d*g/(b*
g*x + a*g) - 2*a*d^2*g/(b*g*x + a*g) + d^2)/b^2)/((b*g*x + a*g)^4*b*g) - 1/
6*B*d/((b*g*x + a*g)^3*(b*c - a*d)*b*g^2) - 1/8*(2*A*b^3*g^3 + B*b^3*g^3)/(
(b*g*x + a*g)^4*b^4*g^4)
```

**maple [B]** time = 0.06, size = 587, normalized size = 2.82

$$\frac{B\,a^4d^4}{8\,(ad-bc)^4\,(bx+a)^4\,b\,g^5}-\frac{B\,a^3c\,d^3}{2\,(ad-bc)^4\,(bx+a)^4\,g^5}+\frac{3B\,a^2b\,c^2d^2}{4\,(ad-bc)^4\,(bx+a)^4\,g^5}-\frac{Ba\,b^2c^3d}{2\,(ad-bc)^4\,(bx+a)^4\,g^5}+\frac{}{8\,(ad-}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*ln(e*(d*x+c)^2/(b*x+a)^2))/(b*g*x+a*g)^5,x)
```

```
[Out] -1/4/b/(b*x+a)^4/g^5*A-1/4/b/g^5*B/(b*x+a)^4*ln((1/(b*x+a)*a*d-1/(b*x+a)*b*
c-d)^2/b^2*e)+1/8/b/g^5*B*a^4*d^4/(a*d-b*c)^4/(b*x+a)^4-1/2/g^5*B*a^3*d^3/(
a*d-b*c)^4/(b*x+a)^4*c+3/4*b/g^5*B*a^2*d^2/(a*d-b*c)^4/(b*x+a)^4*c^2-1/2*b^
2/g^5*B*a*d/(a*d-b*c)^4/(b*x+a)^4*c^3+1/6/b/g^5*B*a^3*d^4/(a*d-b*c)^4/(b*x+
a)^3-1/2/g^5*B*a^2*d^3/(a*d-b*c)^4/(b*x+a)^3*c+1/2*b/g^5*B*a*d^2/(a*d-b*c)^
4/(b*x+a)^3*c^2+1/4/b/g^5*B*a^2*d^4/(a*d-b*c)^4/(b*x+a)^2-1/2/g^5*B*a*d^3/(
a*d-b*c)^4/(b*x+a)^2*c+1/2/b/g^5*B*a*d^4/(a*d-b*c)^4/(b*x+a)+1/2/b/g^5*B*a*
d^5/(a*d-b*c)^5*ln(1/(b*x+a)*a*d-1/(b*x+a)*b*c-d)+1/8*b^3/g^5*B*c^4/(a*d-b*
c)^4/(b*x+a)^4-1/6*b^2/g^5*B*c^3/(a*d-b*c)^4/(b*x+a)^3*d+1/4*b/g^5*B*c^2/(a
*d-b*c)^4/(b*x+a)^2*d^2-1/2/g^5*B*c/(a*d-b*c)^4/(b*x+a)*d^3-1/2/g^5*B*c*d^4
/(a*d-b*c)^5*ln(1/(b*x+a)*a*d-1/(b*x+a)*b*c-d)
```

**maxima [B]** time = 1.38, size = 699, normalized size = 3.36

$$-\frac{1}{24}\,B\left(\frac{12\,b^3d^3x^3-3\,b^3c^3+13\,ab^2c^2d-23\,a^2bcd^2}{\left(b^8c^3-3\,ab^7c^2d+3\,a^2b^6cd^2-a^3b^5d^3\right)g^5x^4+4\left(ab^7c^3-3\,a^2b^6c^2d+3\,a^3b^5cd^2-a^4b^4d^3\right)g^5x^3+6\left(a^2b^6c^3\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*(d*x+c)^2/(b*x+a)^2))/(b*g*x+a*g)^5,x, algorithm="maxi
ma")
```

```
[Out] -1/24*B*((12*b^3*d^3*x^3 - 3*b^3*c^3 + 13*a*b^2*c^2*d - 23*a^2*b*c*d^2 + 25
*a^3*d^3 - 6*(b^3*c*d^2 - 7*a*b^2*d^3)*x^2 + 4*(b^3*c^2*d - 5*a*b^2*c*d^2 +
 13*a^2*b*d^3)*x)/((b^8*c^3 - 3*a*b^7*c^2*d + 3*a^2*b^6*c*d^2 - a^3*b^5*d^3
)*g^5*x^4 + 4*(a*b^7*c^3 - 3*a^2*b^6*c^2*d + 3*a^3*b^5*c*d^2 - a^4*b^4*d^3)
*g^5*x^3 + 6*(a^2*b^6*c^3 - 3*a^3*b^5*c^2*d + 3*a^4*b^4*c*d^2 - a^5*b^3*d^3
)*g^5*x^2 + 4*(a^3*b^5*c^3 - 3*a^4*b^4*c^2*d + 3*a^5*b^3*c*d^2 - a^6*b^2*d^
3)*g^5*x + (a^4*b^4*c^3 - 3*a^5*b^3*c^2*d + 3*a^6*b^2*c*d^2 - a^7*b*d^3)*g^
5) + 6*log(d^2*e*x^2/(b^2*x^2 + 2*a*b*x + a^2) + 2*c*d*e*x/(b^2*x^2 + 2*a*b
*x + a^2) + c^2*e/(b^2*x^2 + 2*a*b*x + a^2))/(b^5*g^5*x^4 + 4*a*b^4*g^5*x^3
 + 6*a^2*b^3*g^5*x^2 + 4*a^3*b^2*g^5*x + a^4*b*g^5) + 12*d^4*log(b*x + a)/(
(b^5*c^4 - 4*a*b^4*c^3*d + 6*a^2*b^3*c^2*d^2 - 4*a^3*b^2*c*d^3 + a^4*b*d^4)
```

```
*g^5) - 12*d^4*log(d*x + c)/((b^5*c^4 - 4*a*b^4*c^3*d + 6*a^2*b^3*c^2*d^2 -
 4*a^3*b^2*c*d^3 + a^4*b*d^4)*g^5)) - 1/4*A/(b^5*g^5*x^4 + 4*a*b^4*g^5*x^3
+ 6*a^2*b^3*g^5*x^2 + 4*a^3*b^2*g^5*x + a^4*b*g^5)
```

**mupad [B]** time = 7.90, size = 579, normalized size = 2.78

$$\frac{B\,d^4\,\mathrm{atanh}\left(\frac{-2\,a^4\,b\,d^4\,g^5+4\,a^3\,b^2\,c\,d^3\,g^5-4\,a\,b^4\,c^3\,d\,g^5+2\,b^5\,c^4\,g^5}{2\,b\,g^5\,(a\,d-b\,c)^4}-\frac{2\,b\,d\,x\,\left(a^3\,d^3-3\,a^2\,b\,c\,d^2+3\,a\,b^2\,c^2\,d-b^3\,c^3\right)}{(a\,d-b\,c)^4}\right)}{b\,g^5\,(a\,d-b\,c)^4}-\frac{B\,\ln}{4\,b^2\,g^5\,\left(4\,a^3\,x+\frac{a^4}{b}+b\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*log((e*(c + d*x)^2)/(a + b*x)^2))/(a*g + b*g*x)^5,x)
```

```
[Out] (B*d^4*atanh((2*b^5*c^4*g^5 - 2*a^4*b*d^4*g^5 - 4*a*b^4*c^3*d*g^5 + 4*a^3*b
^2*c*d^3*g^5)/(2*b*g^5*(a*d - b*c)^4) - (2*b*d*x*(a^3*d^3 - b^3*c^3 + 3*a*b
^2*c^2*d - 3*a^2*b*c*d^2))/(a*d - b*c)^4))/(b*g^5*(a*d - b*c)^4) - (B*log((
e*(c + d*x)^2)/(a + b*x)^2))/(4*b^2*g^5*(4*a^3*x + a^4/b + b^3*x^4 + 6*a^2*
b*x^2 + 4*a*b^2*x^3)) - ((6*A*a^3*d^3 - 6*A*b^3*c^3 - 25*B*a^3*d^3 + 3*B*b^
3*c^3 + 18*A*a*b^2*c^2*d - 18*A*a^2*b*c*d^2 - 13*B*a*b^2*c^2*d + 23*B*a^2*b
*c*d^2)/(12*(a^3*d^3 - b^3*c^3 + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2)) + (d^2*x^2
*(B*b^3*c - 7*B*a*b^2*d))/(2*(a^3*d^3 - b^3*c^3 + 3*a*b^2*c^2*d - 3*a^2*b*c
*d^2)) - (d*x*(B*b^3*c^2 + 13*B*a^2*b*d^2 - 5*B*a*b^2*c*d))/(3*(a^3*d^3 - b
^3*c^3 + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2)) - (B*b^3*d^3*x^3)/(a^3*d^3 - b^3*c
^3 + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2))/(2*a^4*b*g^5 + 2*b^5*g^5*x^4 + 8*a^3*b
^2*g^5*x + 8*a*b^4*g^5*x^3 + 12*a^2*b^3*g^5*x^2)
```

**sympy [B]** time = 5.68, size = 947, normalized size = 4.55

$$-\frac{B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)}{4a^4bg^5+16a^3b^2g^5x+24a^2b^3g^5x^2+16ab^4g^5x^3+4b^5g^5x^4}+\frac{Bd^4\log\left(x+\frac{-\frac{Ba^5d^9}{(ad-bc)^4}+\frac{5Ba^4bcd^8}{(ad-bc)^4}-\frac{10Ba^3b^2c^2d^7}{(ad-bc)^4}+\frac{10Ba^2b^3c^3d^6}{(ad-bc)^4}-}{2Bbd^5}\right.}{2bg^5\,(ad-bc)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*ln(e*(d*x+c)**2/(b*x+a)**2))/(b*g*x+a*g)**5,x)
```

```
[Out] -B*log(e*(c + d*x)**2/(a + b*x)**2)/(4*a**4*b*g**5 + 16*a**3*b**2*g**5*x +
24*a**2*b**3*g**5*x**2 + 16*a*b**4*g**5*x**3 + 4*b**5*g**5*x**4) + B*d**4*l
og(x + (-B*a**5*d**9/(a*d - b*c)**4 + 5*B*a**4*b*c*d**8/(a*d - b*c)**4 - 10
*B*a**3*b**2*c**2*d**7/(a*d - b*c)**4 + 10*B*a**2*b**3*c**3*d**6/(a*d - b*c
)**4 - 5*B*a*b**4*c**4*d**5/(a*d - b*c)**4 + B*a*d**5 + B*b**5*c**5*d**4/(a
*d - b*c)**4 + B*b*c*d**4)/(2*B*b*d**5))/(2*b*g**5*(a*d - b*c)**4) - B*d**4
*log(x + (B*a**5*d**9/(a*d - b*c)**4 - 5*B*a**4*b*c*d**8/(a*d - b*c)**4 + 1
```

```
0*B*a**3*b**2*c**2*d**7/(a*d - b*c)**4 - 10*B*a**2*b**3*c**3*d**6/(a*d - b*
c)**4 + 5*B*a*b**4*c**4*d**5/(a*d - b*c)**4 + B*a*d**5 - B*b**5*c**5*d**4/(
a*d - b*c)**4 + B*b*c*d**4)/(2*B*b*d**5))/(2*b*g**5*(a*d - b*c)**4) + (-6*A
*a**3*d**3 + 18*A*a**2*b*c*d**2 - 18*A*a*b**2*c**2*d + 6*A*b**3*c**3 + 25*B
*a**3*d**3 - 23*B*a**2*b*c*d**2 + 13*B*a*b**2*c**2*d - 3*B*b**3*c**3 + 12*B
*b**3*d**3*x**3 + x**2*(42*B*a*b**2*d**3 - 6*B*b**3*c*d**2) + x*(52*B*a**2*
b*d**3 - 20*B*a*b**2*c*d**2 + 4*B*b**3*c**2*d))/(24*a**7*b*d**3*g**5 - 72*a
**6*b**2*c*d**2*g**5 + 72*a**5*b**3*c**2*d*g**5 - 24*a**4*b**4*c**3*g**5 +
x**4*(24*a**3*b**5*d**3*g**5 - 72*a**2*b**6*c*d**2*g**5 + 72*a*b**7*c**2*d*
g**5 - 24*b**8*c**3*g**5) + x**3*(96*a**4*b**4*d**3*g**5 - 288*a**3*b**5*c*
d**2*g**5 + 288*a**2*b**6*c**2*d*g**5 - 96*a*b**7*c**3*g**5) + x**2*(144*a*
*5*b**3*d**3*g**5 - 432*a**4*b**4*c*d**2*g**5 + 432*a**3*b**5*c**2*d*g**5 -
 144*a**2*b**6*c**3*g**5) + x*(96*a**6*b**2*d**3*g**5 - 288*a**5*b**3*c*d**
2*g**5 + 288*a**4*b**4*c**2*d*g**5 - 96*a**3*b**5*c**3*g**5))
```

# 3.210 $\int (ag+bgx)^4 \left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2 dx$

**Optimal**. Leaf size=515

$$-\frac{4Bg^4(bc-ad)^5\log\left(1-\frac{d(a+bx)}{b(c+dx)}\right)\left(B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)+A\right)}{5bd^5}-\frac{4Bg^4(c+dx)(bc-ad)^4\left(B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)+A\right)}{5d^5}+\frac{2Bg^4(a}{}$$

```
[Out] 26/15*B^2*(-a*d+b*c)^4*g^4*x/d^4-7/15*B^2*(-a*d+b*c)^3*g^4*(b*x+a)^2/b/d^3+
2/15*B^2*(-a*d+b*c)^2*g^4*(b*x+a)^3/b/d^2-10/3*B^2*(-a*d+b*c)^5*g^4*ln(b*x+
a)/b/d^5-26/15*B^2*(-a*d+b*c)^5*g^4*ln((d*x+c)/(b*x+a))/b/d^5+2/5*B*(-a*d+b
*c)^3*g^4*(b*x+a)^2*(A+B*ln(e*(d*x+c)^2/(b*x+a)^2))/b/d^3-4/15*B*(-a*d+b*c)
^2*g^4*(b*x+a)^3*(A+B*ln(e*(d*x+c)^2/(b*x+a)^2))/b/d^2+1/5*B*(-a*d+b*c)*g^4
*(b*x+a)^4*(A+B*ln(e*(d*x+c)^2/(b*x+a)^2))/b/d-4/5*B*(-a*d+b*c)^4*g^4*(d*x+
c)*(A+B*ln(e*(d*x+c)^2/(b*x+a)^2))/d^5+1/5*g^4*(b*x+a)^5*(A+B*ln(e*(d*x+c)^
2/(b*x+a)^2))^2/b-4/5*B*(-a*d+b*c)^5*g^4*(A+B*ln(e*(d*x+c)^2/(b*x+a)^2))*ln
(1-d*(b*x+a)/b/(d*x+c))/b/d^5+8/5*B^2*(-a*d+b*c)^5*g^4*polylog(2,d*(b*x+a)/
b/(d*x+c))/b/d^5
```

**Rubi [A]** time = 0.86, antiderivative size = 569, normalized size of antiderivative = 1.10, number of steps used = 28, number of rules used = 13, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.382, Rules used = {2525, 12, 2528, 2486, 31, 43, 2524, 2418, 2394, 2393, 2391, 2390, 2301}

$$\frac{8B^2g^4(bc-ad)^5\text{PolyLog}\left(2,\frac{b(c+dx)}{bc-ad}\right)}{5bd^5}+\frac{4Bg^4(bc-ad)^5\log(c+dx)\left(B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)+A\right)}{5bd^5}+\frac{2Bg^4(a+bx)^2(bc-ad}{5}$$

Antiderivative was successfully verified.

```
[In] Int[(a*g + b*g*x)^4*(A + B*Log[(e*(c + d*x)^2)/(a + b*x)^2])^2,x]
```

```
[Out] (-4*A*B*(b*c - a*d)^4*g^4*x)/(5*d^4) + (26*B^2*(b*c - a*d)^4*g^4*x)/(15*d^4
) - (7*B^2*(b*c - a*d)^3*g^4*(a + b*x)^2)/(15*b*d^3) + (2*B^2*(b*c - a*d)^2
*g^4*(a + b*x)^3)/(15*b*d^2) - (10*B^2*(b*c - a*d)^5*g^4*Log[c + d*x])/(3*b
*d^5) + (8*B^2*(b*c - a*d)^5*g^4*Log[-((d*(a + b*x))/(b*c - a*d))]*Log[c +
d*x])/(5*b*d^5) - (4*B^2*(b*c - a*d)^5*g^4*Log[c + d*x]^2)/(5*b*d^5) - (4*B
^2*(b*c - a*d)^4*g^4*(a + b*x)*Log[(e*(c + d*x)^2)/(a + b*x)^2])/(5*b*d^4)
+ (2*B*(b*c - a*d)^3*g^4*(a + b*x)^2*(A + B*Log[(e*(c + d*x)^2)/(a + b*x)^2
]))/(5*b*d^3) - (4*B*(b*c - a*d)^2*g^4*(a + b*x)^3*(A + B*Log[(e*(c + d*x)^
2)/(a + b*x)^2]))/(15*b*d^2) + (B*(b*c - a*d)*g^4*(a + b*x)^4*(A + B*Log[(e
*(c + d*x)^2)/(a + b*x)^2]))/(5*b*d) + (4*B*(b*c - a*d)^5*g^4*Log[c + d*x]*
(A + B*Log[(e*(c + d*x)^2)/(a + b*x)^2]))/(5*b*d^5) + (g^4*(a + b*x)^5*(A +
```

```
 B*Log[(e*(c + d*x)^2)/(a + b*x)^2])^2)/(5*b) + (8*B^2*(b*c - a*d)^5*g^4*Po
lyLog[2, (b*(c + d*x))/(b*c - a*d)])/(5*b*d^5)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2301

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

Rule 2390

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_))^(q_.), x_Symbol] :> Dist[1/e, Subst[Int[((f*x)/d)^q*(a + b*Log[c*x^
n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && E
qQ[e*f - d*g, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2393

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + (c*e*x)/g])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2394

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d + e*x
)^n]))/g, x] - Dist[(b*e*n)/g, Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2418

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*(RFx_), x_Sy
mbol] :> With[{u = ExpandIntegrand[(a + b*Log[c*(d + e*x)^n])^p, RFx, x]},
Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d, e, n}, x] && RationalFunctionQ[
RFx, x] && IntegerQ[p]
```

Rule 2486

```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))
^(r_.)]^(s_.), x_Symbol] :> Simp[((a + b*x)*Log[e*(f*(a + b*x)^p*(c + d*x)^
q)^r]^s)/b, x] + Dist[(q*r*s*(b*c - a*d))/b, Int[Log[e*(f*(a + b*x)^p*(c +
d*x)^q)^r]^(s - 1)/(c + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, p, q, r, s
}, x] && NeQ[b*c - a*d, 0] && EqQ[p + q, 0] && IGtQ[s, 0]
```

Rule 2524

```
Int[((a_.) + Log[(c_.)*(RFx_)^(p_.)]*(b_.))^(n_.)/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[(Log[d + e*x]*(a + b*Log[c*RFx^p])^n)/e, x] - Dist[(b*n*p)/e
, Int[(Log[d + e*x]*(a + b*Log[c*RFx^p])^(n - 1)*D[RFx, x])/RFx, x], x] /;
FreeQ[{a, b, c, d, e, p}, x] && RationalFunctionQ[RFx, x] && IGtQ[n, 0]
```

Rule 2525

```
Int[((a_.) + Log[(c_.)*(RFx_)^(p_.)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_))^(m_.
), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(a + b*Log[c*RFx^p])^n)/(e*(m + 1))
, x] - Dist[(b*n*p)/(e*(m + 1)), Int[SimplifyIntegrand[((d + e*x)^(m + 1)*(
a + b*Log[c*RFx^p])^(n - 1)*D[RFx, x])/RFx, x], x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && RationalFunctionQ[RFx, x] && IGtQ[n, 0] && (EqQ[n, 1] ||
IntegerQ[m]) && NeQ[m, -1]
```

Rule 2528

```
Int[((a_.) + Log[(c_.)*(RFx_)^(p_.)]*(b_.))^(n_.)*(RGx_), x_Symbol] :> With
[{u = ExpandIntegrand[(a + b*Log[c*RFx^p])^n, RGx, x]}, Int[u, x] /; SumQ[u
]] /; FreeQ[{a, b, c, p}, x] && RationalFunctionQ[RFx, x] && RationalFuncti
onQ[RGx, x] && IGtQ[n, 0]
```

Rubi steps

$$\int (ag+bgx)^4\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2dx = \frac{g^4(a+bx)^5\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2}{5b} - \frac{(2B)\int \frac{2(bc-ad)g^5(a+bx)^4\left(-A-B\,1\ldots\right.}{c+dx}}{5bg}\ldots$$

$$= \frac{g^4(a+bx)^5\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2}{5b} - \frac{\left(4B(bc-ad)g^4\right)\int \frac{(a+bx)^4\left(-A\ldots\right.}{\ldots}}{5b}\ldots$$

$$= \frac{g^4(a+bx)^5\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2}{5b} - \frac{\left(4B(bc-ad)g^4\right)\int\left(-\frac{b(bc-ad)\ldots}{\ldots}\right.}{\ldots}\ldots$$

$$= \frac{g^4(a+bx)^5\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2}{5b} - \frac{\left(4B(bc-ad)g^4\right)\int (a+bx)^3\ldots}{5d\ldots}\ldots$$

$$= -\frac{4AB(bc-ad)^4g^4x}{5d^4} + \frac{2B(bc-ad)^3g^4(a+bx)^2\left(A+B\log\left(\frac{e(c+dx)\ldots}{(a+bx)\ldots}\right.\right.}{5bd^3}\ldots$$

$$= -\frac{4AB(bc-ad)^4g^4x}{5d^4} - \frac{4B^2(bc-ad)^4g^4(a+bx)\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)}{5bd^4} + \frac{2\ldots}{\ldots}\ldots$$

$$= -\frac{4AB(bc-ad)^4g^4x}{5d^4} - \frac{8B^2(bc-ad)^5g^4\log(c+dx)}{5bd^5} - \frac{4B^2(bc-ad\ldots}{\ldots}\ldots$$

$$= -\frac{4AB(bc-ad)^4g^4x}{5d^4} + \frac{26B^2(bc-ad)^4g^4x}{15d^4} - \frac{7B^2(bc-ad)^3g^4(a+\ldots}{15bd^3}\ldots$$

$$= -\frac{4AB(bc-ad)^4g^4x}{5d^4} + \frac{26B^2(bc-ad)^4g^4x}{15d^4} - \frac{7B^2(bc-ad)^3g^4(a+\ldots}{15bd^3}\ldots$$

$$= -\frac{4AB(bc-ad)^4g^4x}{5d^4} + \frac{26B^2(bc-ad)^4g^4x}{15d^4} - \frac{7B^2(bc-ad)^3g^4(a+\ldots}{15bd^3}\ldots$$

$$= -\frac{4AB(bc-ad)^4g^4x}{5d^4} + \frac{26B^2(bc-ad)^4g^4x}{15d^4} - \frac{7B^2(bc-ad)^3g^4(a+\ldots}{15bd^3}\ldots$$

**Mathematica [A]** time = 0.47, size = 524, normalized size = 1.02

$$g^4\left((a+bx)^5\left(B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)+A\right)^2-\frac{B(bc-ad)\left(-3d^4(a+bx)^4\left(B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)+A\right)+4d^3(a+bx)^3(bc-ad)\left(B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)+A\right)-6d^2(a+bx\right.}{}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*g + b*g*x)^4*(A + B*Log[(e*(c + d*x)^2)/(a + b*x)^2])^2,x]
```

```
[Out] (g^4*((a + b*x)^5*(A + B*Log[(e*(c + d*x)^2)/(a + b*x)^2])^2 - (B*(b*c - a*
d)*(12*A*b*d*(b*c - a*d)^3*x + 24*B*(b*c - a*d)^4*Log[c + d*x] - 4*B*(b*c -
 a*d)^2*(2*b*d*(b*c - a*d)*x - d^2*(a + b*x)^2 - 2*(b*c - a*d)^2*Log[c + d*
x]) - B*(b*c - a*d)*(6*b*d*(b*c - a*d)^2*x + 3*d^2*(-(b*c) + a*d)*(a + b*x)
^2 + 2*d^3*(a + b*x)^3 - 6*(b*c - a*d)^3*Log[c + d*x]) - 12*B*(b*c - a*d)^3
*(b*d*x + (-(b*c) + a*d)*Log[c + d*x]) + 12*B*d*(b*c - a*d)^3*(a + b*x)*Log
[(e*(c + d*x)^2)/(a + b*x)^2] - 6*d^2*(b*c - a*d)^2*(a + b*x)^2*(A + B*Log[
(e*(c + d*x)^2)/(a + b*x)^2]) + 4*d^3*(b*c - a*d)*(a + b*x)^3*(A + B*Log[(e
*(c + d*x)^2)/(a + b*x)^2]) - 3*d^4*(a + b*x)^4*(A + B*Log[(e*(c + d*x)^2)/
(a + b*x)^2]) - 12*(b*c - a*d)^4*Log[c + d*x]*(A + B*Log[(e*(c + d*x)^2)/(a
 + b*x)^2]) - 12*B*(b*c - a*d)^4*((2*Log[(d*(a + b*x))/(-(b*c) + a*d)] - Lo
g[c + d*x])*Log[c + d*x] + 2*PolyLog[2, (b*(c + d*x))/(b*c - a*d)])))/(3*d^
5)))/(5*b)
```

**fricas [F]** time = 0.71, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(A^2b^4g^4x^4+4\,A^2ab^3g^4x^3+6\,A^2a^2b^2g^4x^2+4\,A^2a^3bg^4x+A^2a^4g^4+\left(B^2b^4g^4x^4+4\,B^2ab^3g^4x^3+6\,B^2a\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*g*x+a*g)^4*(A+B*log(e*(d*x+c)^2/(b*x+a)^2))^2,x, algorithm="fr
icas")
```

```
[Out] integral(A^2*b^4*g^4*x^4 + 4*A^2*a*b^3*g^4*x^3 + 6*A^2*a^2*b^2*g^4*x^2 + 4*
A^2*a^3*b*g^4*x + A^2*a^4*g^4 + (B^2*b^4*g^4*x^4 + 4*B^2*a*b^3*g^4*x^3 + 6*
B^2*a^2*b^2*g^4*x^2 + 4*B^2*a^3*b*g^4*x + B^2*a^4*g^4)*log((d^2*e*x^2 + 2*c
*d*e*x + c^2*e)/(b^2*x^2 + 2*a*b*x + a^2))^2 + 2*(A*B*b^4*g^4*x^4 + 4*A*B*a
*b^3*g^4*x^3 + 6*A*B*a^2*b^2*g^4*x^2 + 4*A*B*a^3*b*g^4*x + A*B*a^4*g^4)*log
((d^2*e*x^2 + 2*c*d*e*x + c^2*e)/(b^2*x^2 + 2*a*b*x + a^2)), x)
```

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int\left(bgx+ag\right)^4\left(B\log\left(\frac{(dx+c)^2e}{(bx+a)^2}\right)+A\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*g*x+a*g)^4*(A+B*log(e*(d*x+c)^2/(b*x+a)^2))^2,x, algorithm="gi
ac")
```

```
[Out] integrate((b*g*x + a*g)^4*(B*log((d*x + c)^2*e/(b*x + a)^2) + A)^2, x)
```

**maple** [F] time = 1.36, size = 0, normalized size = 0.00

$$\int \left(bgx+ag\right)^{4}\left(B\ln\left(\frac{\left(dx+c\right)^{2}e}{\left(bx+a\right)^{2}}\right)+A\right)^{2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*g*x+a*g)^4*(A+B*ln(e*(d*x+c)^2/(b*x+a)^2))^2,x)
```

```
[Out] int((b*g*x+a*g)^4*(A+B*ln(e*(d*x+c)^2/(b*x+a)^2))^2,x)
```

**maxima** [B] time = 2.60, size = 2660, normalized size = 5.17

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*g*x+a*g)^4*(A+B*log(e*(d*x+c)^2/(b*x+a)^2))^2,x, algorithm="ma
xima")
```

```
[Out] 1/5*A^2*b^4*g^4*x^5 + A^2*a*b^3*g^4*x^4 + 2*A^2*a^2*b^2*g^4*x^3 + 2*A^2*a^3
*b*g^4*x^2 + 2*(x*log(d^2*e*x^2/(b^2*x^2 + 2*a*b*x + a^2) + 2*c*d*e*x/(b^2*
x^2 + 2*a*b*x + a^2) + c^2*e/(b^2*x^2 + 2*a*b*x + a^2)) - 2*a*log(b*x + a)/
b + 2*c*log(d*x + c)/d)*A*B*a^4*g^4 + 4*(x^2*log(d^2*e*x^2/(b^2*x^2 + 2*a*b
*x + a^2) + 2*c*d*e*x/(b^2*x^2 + 2*a*b*x + a^2) + c^2*e/(b^2*x^2 + 2*a*b*x
+ a^2)) + 2*a^2*log(b*x + a)/b^2 - 2*c^2*log(d*x + c)/d^2 + 2*(b*c - a*d)*x
/(b*d))*A*B*a^3*b*g^4 + 4*(x^3*log(d^2*e*x^2/(b^2*x^2 + 2*a*b*x + a^2) + 2*
c*d*e*x/(b^2*x^2 + 2*a*b*x + a^2) + c^2*e/(b^2*x^2 + 2*a*b*x + a^2)) - 2*a^
3*log(b*x + a)/b^3 + 2*c^3*log(d*x + c)/d^3 + ((b^2*c*d - a*b*d^2)*x^2 - 2*
(b^2*c^2 - a^2*d^2)*x)/(b^2*d^2))*A*B*a^2*b^2*g^4 + 2/3*(3*x^4*log(d^2*e*x^
2/(b^2*x^2 + 2*a*b*x + a^2) + 2*c*d*e*x/(b^2*x^2 + 2*a*b*x + a^2) + c^2*e/(
b^2*x^2 + 2*a*b*x + a^2)) + 6*a^4*log(b*x + a)/b^4 - 6*c^4*log(d*x + c)/d^4
 + (2*(b^3*c*d^2 - a*b^2*d^3)*x^3 - 3*(b^3*c^2*d - a^2*b*d^3)*x^2 + 6*(b^3*
c^3 - a^3*d^3)*x)/(b^3*d^3))*A*B*a*b^3*g^4 + 1/15*(6*x^5*log(d^2*e*x^2/(b^2
*x^2 + 2*a*b*x + a^2) + 2*c*d*e*x/(b^2*x^2 + 2*a*b*x + a^2) + c^2*e/(b^2*x^
2 + 2*a*b*x + a^2)) - 12*a^5*log(b*x + a)/b^5 + 12*c^5*log(d*x + c)/d^5 + (
3*(b^4*c*d^3 - a*b^3*d^4)*x^4 - 4*(b^4*c^2*d^2 - a^2*b^2*d^4)*x^3 + 6*(b^4*
c^3*d - a^3*b*d^4)*x^2 - 12*(b^4*c^4 - a^4*d^4)*x)/(b^4*d^4))*A*B*b^4*g^4 +
 A^2*a^4*g^4*x + 2/15*((6*g^4*log(e) - 25*g^4)*b^4*c^5 - (30*g^4*log(e) - 1
```

```
13*g^4)*a*b^3*c^4*d + 4*(15*g^4*log(e) - 49*g^4)*a^2*b^2*c^3*d^2 - 12*(5*g^
4*log(e) - 13*g^4)*a^3*b*c^2*d^3 + 6*(5*g^4*log(e) - 8*g^4)*a^4*c*d^4)*B^2*
log(d*x + c)/d^5 - 8/5*(b^5*c^5*g^4 - 5*a*b^4*c^4*d*g^4 + 10*a^2*b^3*c^3*d^
2*g^4 - 10*a^3*b^2*c^2*d^3*g^4 + 5*a^4*b*c*d^4*g^4 - a^5*d^5*g^4)*(log(b*x
+ a)*log((b*d*x + a*d)/(b*c - a*d) + 1) + dilog(-(b*d*x + a*d)/(b*c - a*d))
)*B^2/(b*d^5) + 1/15*(3*B^2*b^5*d^5*g^4*x^5*log(e)^2 + 3*(b^5*c*d^4*g^4*log
(e) + (5*g^4*log(e)^2 - g^4*log(e))*a*b^4*d^5)*B^2*x^4 - 2*((2*g^4*log(e) -
 g^4)*b^5*c^2*d^3 - 2*(5*g^4*log(e) - g^4)*a*b^4*c*d^4 - (15*g^4*log(e)^2 -
 8*g^4*log(e) + g^4)*a^2*b^3*d^5)*B^2*x^3 + ((6*g^4*log(e) - 7*g^4)*b^5*c^3
*d^2 - 3*(10*g^4*log(e) - 9*g^4)*a*b^4*c^2*d^3 + 3*(20*g^4*log(e) - 11*g^4)
*a^2*b^3*c*d^4 + (30*g^4*log(e)^2 - 36*g^4*log(e) + 13*g^4)*a^3*b^2*d^5)*B^
2*x^2 - (2*(6*g^4*log(e) - 13*g^4)*b^5*c^4*d - 2*(30*g^4*log(e) - 59*g^4)*a
*b^4*c^3*d^2 + 12*(10*g^4*log(e) - 17*g^4)*a^2*b^3*c^2*d^3 - 2*(60*g^4*log(
e) - 79*g^4)*a^3*b^2*c*d^4 - (15*g^4*log(e)^2 - 48*g^4*log(e) + 46*g^4)*a^4
*b*d^5)*B^2*x + 12*(B^2*b^5*d^5*g^4*x^5 + 5*B^2*a*b^4*d^5*g^4*x^4 + 10*B^2*
a^2*b^3*d^5*g^4*x^3 + 10*B^2*a^3*b^2*d^5*g^4*x^2 + 5*B^2*a^4*b*d^5*g^4*x +
B^2*a^5*d^5*g^4)*log(b*x + a)^2 + 12*(B^2*b^5*d^5*g^4*x^5 + 5*B^2*a*b^4*d^5
*g^4*x^4 + 10*B^2*a^2*b^3*d^5*g^4*x^3 + 10*B^2*a^3*b^2*d^5*g^4*x^2 + 5*B^2*
a^4*b*d^5*g^4*x + (b^5*c^5*g^4 - 5*a*b^4*c^4*d*g^4 + 10*a^2*b^3*c^3*d^2*g^4
 - 10*a^3*b^2*c^2*d^3*g^4 + 5*a^4*b*c*d^4*g^4)*B^2)*log(d*x + c)^2 - 2*(6*B
^2*b^5*d^5*g^4*x^5*log(e) + 3*(b^5*c*d^4*g^4 + (10*g^4*log(e) - g^4)*a*b^4*
d^5)*B^2*x^4 - 4*(b^5*c^2*d^3*g^4 - 5*a*b^4*c*d^4*g^4 - (15*g^4*log(e) - 4*
g^4)*a^2*b^3*d^5)*B^2*x^3 + 6*(b^5*c^3*d^2*g^4 - 5*a*b^4*c^2*d^3*g^4 + 10*a
^2*b^3*c*d^4*g^4 + 2*(5*g^4*log(e) - 3*g^4)*a^3*b^2*d^5)*B^2*x^2 - 6*(2*b^5
*c^4*d*g^4 - 10*a*b^4*c^3*d^2*g^4 + 20*a^2*b^3*c^2*d^3*g^4 - 20*a^3*b^2*c*d
^4*g^4 - (5*g^4*log(e) - 8*g^4)*a^4*b*d^5)*B^2*x - (12*a*b^4*c^4*d*g^4 - 54
*a^2*b^3*c^3*d^2*g^4 + 94*a^3*b^2*c^2*d^3*g^4 - 77*a^4*b*c*d^4*g^4 - (6*g^4
*log(e) - 25*g^4)*a^5*d^5)*B^2)*log(b*x + a) + 2*(6*B^2*b^5*d^5*g^4*x^5*log
(e) + 3*(b^5*c*d^4*g^4 + (10*g^4*log(e) - g^4)*a*b^4*d^5)*B^2*x^4 - 4*(b^5*
c^2*d^3*g^4 - 5*a*b^4*c*d^4*g^4 - (15*g^4*log(e) - 4*g^4)*a^2*b^3*d^5)*B^2*
x^3 + 6*(b^5*c^3*d^2*g^4 - 5*a*b^4*c^2*d^3*g^4 + 10*a^2*b^3*c*d^4*g^4 + 2*(
5*g^4*log(e) - 3*g^4)*a^3*b^2*d^5)*B^2*x^2 - 6*(2*b^5*c^4*d*g^4 - 10*a*b^4*
c^3*d^2*g^4 + 20*a^2*b^3*c^2*d^3*g^4 - 20*a^3*b^2*c*d^4*g^4 - (5*g^4*log(e)
 - 8*g^4)*a^4*b*d^5)*B^2*x - 12*(B^2*b^5*d^5*g^4*x^5 + 5*B^2*a*b^4*d^5*g^4*
x^4 + 10*B^2*a^2*b^3*d^5*g^4*x^3 + 10*B^2*a^3*b^2*d^5*g^4*x^2 + 5*B^2*a^4*b
*d^5*g^4*x + B^2*a^5*d^5*g^4)*log(b*x + a))*log(d*x + c))/(b*d^5)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.00

$$\int \left(a\,g+b\,g\,x\right)^4\,{\left(A+B\,\ln\left(\frac{e\,{\left(c+d\,x\right)}^2}{{\left(a+b\,x\right)}^2}\right)\right)}^2 \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*g + b*g*x)^4*(A + B*log((e*(c + d*x)^2)/(a + b*x)^2))^2,x)
```

```
[Out] int((a*g + b*g*x)^4*(A + B*log((e*(c + d*x)^2)/(a + b*x)^2))^2, x)
```

**sympy [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*g*x+a*g)**4*(A+B*ln(e*(d*x+c)**2/(b*x+a)**2))**2,x)
[Out] Timed out
```

# 3.211 $\int (ag+bgx)^3 \left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2 dx$

**Optimal**. Leaf size=422

$$\frac{Bg^3(bc-ad)^4\log\left(1-\frac{d(a+bx)}{b(c+dx)}\right)\left(B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)+A\right)}{bd^4}+\frac{Bg^3(c+dx)(bc-ad)^3\left(B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)+A\right)}{d^4}-\frac{Bg^3(a+bx}{}$$

```
[Out] -5/3*B^2*(-a*d+b*c)^3*g^3*x/d^3+1/3*B^2*(-a*d+b*c)^2*g^3*(b*x+a)^2/b/d^2+11
/3*B^2*(-a*d+b*c)^4*g^3*ln(b*x+a)/b/d^4+5/3*B^2*(-a*d+b*c)^4*g^3*ln((d*x+c)
/(b*x+a))/b/d^4-1/2*B*(-a*d+b*c)^2*g^3*(b*x+a)^2*(A+B*ln(e*(d*x+c)^2/(b*x+a
)^2))/b/d^2+1/3*B*(-a*d+b*c)*g^3*(b*x+a)^3*(A+B*ln(e*(d*x+c)^2/(b*x+a)^2))/
b/d+B*(-a*d+b*c)^3*g^3*(d*x+c)*(A+B*ln(e*(d*x+c)^2/(b*x+a)^2))/d^4+1/4*g^3*
(b*x+a)^4*(A+B*ln(e*(d*x+c)^2/(b*x+a)^2))^2/b+B*(-a*d+b*c)^4*g^3*(A+B*ln(e*
(d*x+c)^2/(b*x+a)^2))*ln(1-d*(b*x+a)/b/(d*x+c))/b/d^4-2*B^2*(-a*d+b*c)^4*g^
3*polylog(2,d*(b*x+a)/b/(d*x+c))/b/d^4
```

**Rubi [A]** time = 0.74, antiderivative size = 469, normalized size of antiderivative = 1.11, number of steps used = 24, number of rules used = 13, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.382, Rules used = {2525, 12, 2528, 2486, 31, 43, 2524, 2418, 2394, 2393, 2391, 2390, 2301}

$$-\frac{2B^2g^3(bc-ad)^4\text{PolyLog}\left(2,\frac{b(c+dx)}{bc-ad}\right)}{bd^4}-\frac{Bg^3(bc-ad)^4\log(c+dx)\left(B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)+A\right)}{bd^4}-\frac{Bg^3(a+bx)^2(bc-ad)}{2\ldots}$$

Antiderivative was successfully verified.

```
[In] Int[(a*g + b*g*x)^3*(A + B*Log[(e*(c + d*x)^2)/(a + b*x)^2])^2,x]
```

```
[Out] (A*B*(b*c - a*d)^3*g^3*x)/d^3 - (5*B^2*(b*c - a*d)^3*g^3*x)/(3*d^3) + (B^2*
(b*c - a*d)^2*g^3*(a + b*x)^2)/(3*b*d^2) + (11*B^2*(b*c - a*d)^4*g^3*Log[c
+ d*x])/(3*b*d^4) - (2*B^2*(b*c - a*d)^4*g^3*Log[-((d*(a + b*x))/(b*c - a*d
))]*Log[c + d*x])/(b*d^4) + (B^2*(b*c - a*d)^4*g^3*Log[c + d*x]^2)/(b*d^4)
+ (B^2*(b*c - a*d)^3*g^3*(a + b*x)*Log[(e*(c + d*x)^2)/(a + b*x)^2])/(b*d^3
) - (B*(b*c - a*d)^2*g^3*(a + b*x)^2*(A + B*Log[(e*(c + d*x)^2)/(a + b*x)^2
]))/(2*b*d^2) + (B*(b*c - a*d)*g^3*(a + b*x)^3*(A + B*Log[(e*(c + d*x)^2)/(
a + b*x)^2]))/(3*b*d) - (B*(b*c - a*d)^4*g^3*Log[c + d*x]*(A + B*Log[(e*(c
+ d*x)^2)/(a + b*x)^2]))/(b*d^4) + (g^3*(a + b*x)^4*(A + B*Log[(e*(c + d*x)
^2)/(a + b*x)^2])^2)/(4*b) - (2*B^2*(b*c - a*d)^4*g^3*PolyLog[2, (b*(c + d*
x))/(b*c - a*d)])/(b*d^4)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2301

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

Rule 2390

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_))^(q_.), x_Symbol] :> Dist[1/e, Subst[Int[((f*x)/d)^q*(a + b*Log[c*x^
n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && E
qQ[e*f - d*g, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2393

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + (c*e*x)/g])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2394

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d + e*x
)^n]))/g, x] - Dist[(b*e*n)/g, Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
```

```
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2418

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*(RFx_), x_Sy
mbol] :> With[{u = ExpandIntegrand[(a + b*Log[c*(d + e*x)^n])^p, RFx, x]},
Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d, e, n}, x] && RationalFunctionQ[
RFx, x] && IntegerQ[p]
```

Rule 2486

```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))
^(r_.)]^(s_.), x_Symbol] :> Simp[((a + b*x)*Log[e*(f*(a + b*x)^p*(c + d*x)^
q)^r]^s)/b, x] + Dist[(q*r*s*(b*c - a*d))/b, Int[Log[e*(f*(a + b*x)^p*(c +
d*x)^q)^r]^(s - 1)/(c + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, p, q, r, s
}, x] && NeQ[b*c - a*d, 0] && EqQ[p + q, 0] && IGtQ[s, 0]
```

Rule 2524

```
Int[((a_.) + Log[(c_.)*(RFx_)^(p_.)]*(b_.))^(n_.)/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[(Log[d + e*x]*(a + b*Log[c*RFx^p])^n)/e, x] - Dist[(b*n*p)/e
, Int[(Log[d + e*x]*(a + b*Log[c*RFx^p])^(n - 1)*D[RFx, x])/RFx, x], x] /;
FreeQ[{a, b, c, d, e, p}, x] && RationalFunctionQ[RFx, x] && IGtQ[n, 0]
```

Rule 2525

```
Int[((a_.) + Log[(c_.)*(RFx_)^(p_.)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_))^(m_.
), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(a + b*Log[c*RFx^p])^n)/(e*(m + 1))
, x] - Dist[(b*n*p)/(e*(m + 1)), Int[SimplifyIntegrand[((d + e*x)^(m + 1)*(
a + b*Log[c*RFx^p])^(n - 1)*D[RFx, x])/RFx, x], x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && RationalFunctionQ[RFx, x] && IGtQ[n, 0] && (EqQ[n, 1] ||
IntegerQ[m]) && NeQ[m, -1]
```

Rule 2528

```
Int[((a_.) + Log[(c_.)*(RFx_)^(p_.)]*(b_.))^(n_.)*(RGx_), x_Symbol] :> With
[{u = ExpandIntegrand[(a + b*Log[c*RFx^p])^n, RGx, x]}, Int[u, x] /; SumQ[u
]] /; FreeQ[{a, b, c, p}, x] && RationalFunctionQ[RFx, x] && RationalFuncti
onQ[RGx, x] && IGtQ[n, 0]
```

Rubi steps

$$\int (ag+bgx)^3\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2 dx = \frac{g^3(a+bx)^4\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2}{4b} - \frac{B\int \frac{2(bc-ad)g^4(a+bx)^3\left(-A-B\log\left(\right.\right.}{c+dx}}{2bg}$$

$$= \frac{g^3(a+bx)^4\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2}{4b} - \frac{\left(B(bc-ad)g^3\right)\int \frac{(a+bx)^3\left(-A-\right.}{c}}{b}$$

$$= \frac{g^3(a+bx)^4\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2}{4b} - \frac{\left(B(bc-ad)g^3\right)\int\left(\frac{b(bc-ad)^2\left(-\right.}{}\right.}{}$$

$$= \frac{g^3(a+bx)^4\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2}{4b} - \frac{\left(B(bc-ad)g^3\right)\int(a+bx)^2\left(\right.}{d}$$

$$= \frac{AB(bc-ad)^3g^3x}{d^3} - \frac{B(bc-ad)^2g^3(a+bx)^2\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)}{2bd^2}$$

$$= \frac{AB(bc-ad)^3g^3x}{d^3} + \frac{B^2(bc-ad)^3g^3(a+bx)\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)}{bd^3} - \frac{B(bc-}{}$$

$$= \frac{AB(bc-ad)^3g^3x}{d^3} + \frac{2B^2(bc-ad)^4g^3\log(c+dx)}{bd^4} + \frac{B^2(bc-ad)^3g^3}{}$$

$$= \frac{AB(bc-ad)^3g^3x}{d^3} - \frac{5B^2(bc-ad)^3g^3x}{3d^3} + \frac{B^2(bc-ad)^2g^3(a+bx)^2}{3bd^2}$$

$$= \frac{AB(bc-ad)^3g^3x}{d^3} - \frac{5B^2(bc-ad)^3g^3x}{3d^3} + \frac{B^2(bc-ad)^2g^3(a+bx)^2}{3bd^2}$$

$$= \frac{AB(bc-ad)^3g^3x}{d^3} - \frac{5B^2(bc-ad)^3g^3x}{3d^3} + \frac{B^2(bc-ad)^2g^3(a+bx)^2}{3bd^2}$$

$$= \frac{AB(bc-ad)^3g^3x}{d^3} - \frac{5B^2(bc-ad)^3g^3x}{3d^3} + \frac{B^2(bc-ad)^2g^3(a+bx)^2}{3bd^2}$$

**Mathematica [A]** time = 0.32, size = 402, normalized size = 0.95

$$g^3\left(\frac{2B(bc-ad)\left(2d^3(a+bx)^3\left(B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)+A\right)+3d^2(a+bx)^2(ad-bc)\left(B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)+A\right)-6(bc-ad)^3\log(c+dx)\left(B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)+A\right)+6Abdx(bc-ad}{}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*g + b*g*x)^3*(A + B*Log[(e*(c + d*x)^2)/(a + b*x)^2])^2,x]
```

```
[Out] (g^3*((a + b*x)^4*(A + B*Log[(e*(c + d*x)^2)/(a + b*x)^2])^2 + (2*B*(b*c -
a*d)*(6*A*b*d*(b*c - a*d)^2*x + 12*B*(b*c - a*d)^3*Log[c + d*x] - 2*B*(b*c
- a*d)*(2*b*d*(b*c - a*d)*x - d^2*(a + b*x)^2 - 2*(b*c - a*d)^2*Log[c + d*x
]) - 6*B*(b*c - a*d)^2*(b*d*x + (-(b*c) + a*d)*Log[c + d*x]) + 6*B*d*(b*c -
 a*d)^2*(a + b*x)*Log[(e*(c + d*x)^2)/(a + b*x)^2] + 3*d^2*(-(b*c) + a*d)*(
a + b*x)^2*(A + B*Log[(e*(c + d*x)^2)/(a + b*x)^2]) + 2*d^3*(a + b*x)^3*(A
+ B*Log[(e*(c + d*x)^2)/(a + b*x)^2]) - 6*(b*c - a*d)^3*Log[c + d*x]*(A + B
*Log[(e*(c + d*x)^2)/(a + b*x)^2]) - 6*B*(b*c - a*d)^3*((2*Log[(d*(a + b*x)
)/(-(b*c) + a*d)] - Log[c + d*x])*Log[c + d*x] + 2*PolyLog[2, (b*(c + d*x))
/(b*c - a*d)])))/(3*d^4)))/(4*b)
```

**fricas [F]** time = 0.65, size = 0, normalized size = 0.00

$$\text{integral}\left(A^2b^3g^3x^3 + 3\,A^2ab^2g^3x^2 + 3\,A^2a^2bg^3x + A^2a^3g^3 + \left(B^2b^3g^3x^3 + 3\,B^2ab^2g^3x^2 + 3\,B^2a^2bg^3x + B^2a^3g^3\right)\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*g*x+a*g)^3*(A+B*log(e*(d*x+c)^2/(b*x+a)^2))^2,x, algorithm="fr
icas")
```

```
[Out] integral(A^2*b^3*g^3*x^3 + 3*A^2*a*b^2*g^3*x^2 + 3*A^2*a^2*b*g^3*x + A^2*a^
3*g^3 + (B^2*b^3*g^3*x^3 + 3*B^2*a*b^2*g^3*x^2 + 3*B^2*a^2*b*g^3*x + B^2*a^
3*g^3)*log((d^2*e*x^2 + 2*c*d*e*x + c^2*e)/(b^2*x^2 + 2*a*b*x + a^2))^2 + 2
*(A*B*b^3*g^3*x^3 + 3*A*B*a*b^2*g^3*x^2 + 3*A*B*a^2*b*g^3*x + A*B*a^3*g^3)*
log((d^2*e*x^2 + 2*c*d*e*x + c^2*e)/(b^2*x^2 + 2*a*b*x + a^2)), x)
```

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \left(bgx + ag\right)^3 \left(B\log\left(\frac{(dx + c)^2 e}{(bx + a)^2}\right) + A\right)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*g*x+a*g)^3*(A+B*log(e*(d*x+c)^2/(b*x+a)^2))^2,x, algorithm="gi
ac")
```

```
[Out] integrate((b*g*x + a*g)^3*(B*log((d*x + c)^2*e/(b*x + a)^2) + A)^2, x)
```

**maple [F]** time = 1.24, size = 0, normalized size = 0.00

$$\int \left(bgx + ag\right)^3 \left(B \ln \left(\frac{(dx + c)^2 e}{(bx + a)^2}\right) + A\right)^2 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*g*x+a*g)^3*(B*ln((d*x+c)^2/(b*x+a)^2*e)+A)^2,x)
```

```
[Out] int((b*g*x+a*g)^3*(B*ln((d*x+c)^2/(b*x+a)^2*e)+A)^2,x)
```

**maxima [B]** time = 2.60, size = 1950, normalized size = 4.62

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*g*x+a*g)^3*(A+B*log(e*(d*x+c)^2/(b*x+a)^2))^2,x, algorithm="ma
xima")
```

```
[Out] 1/4*A^2*b^3*g^3*x^4 + A^2*a*b^2*g^3*x^3 + 3/2*A^2*a^2*b*g^3*x^2 + 2*(x*log(
d^2*e*x^2/(b^2*x^2 + 2*a*b*x + a^2) + 2*c*d*e*x/(b^2*x^2 + 2*a*b*x + a^2) +
 c^2*e/(b^2*x^2 + 2*a*b*x + a^2)) - 2*a*log(b*x + a)/b + 2*c*log(d*x + c)/d
)*A*B*a^3*g^3 + 3*(x^2*log(d^2*e*x^2/(b^2*x^2 + 2*a*b*x + a^2) + 2*c*d*e*x/
(b^2*x^2 + 2*a*b*x + a^2) + c^2*e/(b^2*x^2 + 2*a*b*x + a^2)) + 2*a^2*log(b*
x + a)/b^2 - 2*c^2*log(d*x + c)/d^2 + 2*(b*c - a*d)*x/(b*d))*A*B*a^2*b*g^3
+ 2*(x^3*log(d^2*e*x^2/(b^2*x^2 + 2*a*b*x + a^2) + 2*c*d*e*x/(b^2*x^2 + 2*a
*b*x + a^2) + c^2*e/(b^2*x^2 + 2*a*b*x + a^2)) - 2*a^3*log(b*x + a)/b^3 + 2
*c^3*log(d*x + c)/d^3 + ((b^2*c*d - a*b*d^2)*x^2 - 2*(b^2*c^2 - a^2*d^2)*x)
/(b^2*d^2))*A*B*a*b^2*g^3 + 1/6*(3*x^4*log(d^2*e*x^2/(b^2*x^2 + 2*a*b*x + a
^2) + 2*c*d*e*x/(b^2*x^2 + 2*a*b*x + a^2) + c^2*e/(b^2*x^2 + 2*a*b*x + a^2)
) + 6*a^4*log(b*x + a)/b^4 - 6*c^4*log(d*x + c)/d^4 + (2*(b^3*c*d^2 - a*b^2
*d^3)*x^3 - 3*(b^3*c^2*d - a^2*b*d^3)*x^2 + 6*(b^3*c^3 - a^3*d^3)*x)/(b^3*d
^3))*A*B*b^3*g^3 + A^2*a^3*g^3*x - 1/3*((3*g^3*log(e) - 11*g^3)*b^3*c^4 - 2
*(6*g^3*log(e) - 19*g^3)*a*b^2*c^3*d + 9*(2*g^3*log(e) - 5*g^3)*a^2*b*c^2*d
^2 - 6*(2*g^3*log(e) - 3*g^3)*a^3*c*d^3)*B^2*log(d*x + c)/d^4 + 2*(b^4*c^4*
g^3 - 4*a*b^3*c^3*d*g^3 + 6*a^2*b^2*c^2*d^2*g^3 - 4*a^3*b*c*d^3*g^3 + a^4*d
^4*g^3)*(log(b*x + a)*log((b*d*x + a*d)/(b*c - a*d) + 1) + dilog(-(b*d*x +
a*d)/(b*c - a*d)))*B^2/(b*d^4) + 1/12*(3*B^2*b^4*d^4*g^3*x^4*log(e)^2 + 4*(
b^4*c*d^3*g^3*log(e) + (3*g^3*log(e)^2 - g^3*log(e))*a*b^3*d^4)*B^2*x^3 - 2
*((3*g^3*log(e) - 2*g^3)*b^4*c^2*d^2 - 4*(3*g^3*log(e) - g^3)*a*b^3*c*d^3 -
```

```
(9*g^3*log(e)^2 - 9*g^3*log(e) + 2*g^3)*a^2*b^2*d^4)*B^2*x^2 + 4*((3*g^3*l
og(e) - 5*g^3)*b^4*c^3*d - (12*g^3*log(e) - 17*g^3)*a*b^3*c^2*d^2 + (18*g^3
*log(e) - 19*g^3)*a^2*b^2*c*d^3 + (3*g^3*log(e)^2 - 9*g^3*log(e) + 7*g^3)*a
^3*b*d^4)*B^2*x + 12*(B^2*b^4*d^4*g^3*x^4 + 4*B^2*a*b^3*d^4*g^3*x^3 + 6*B^2
*a^2*b^2*d^4*g^3*x^2 + 4*B^2*a^3*b*d^4*g^3*x + B^2*a^4*d^4*g^3)*log(b*x + a
)^2 + 12*(B^2*b^4*d^4*g^3*x^4 + 4*B^2*a*b^3*d^4*g^3*x^3 + 6*B^2*a^2*b^2*d^4
*g^3*x^2 + 4*B^2*a^3*b*d^4*g^3*x - (b^4*c^4*g^3 - 4*a*b^3*c^3*d*g^3 + 6*a^2
*b^2*c^2*d^2*g^3 - 4*a^3*b*c*d^3*g^3)*B^2)*log(d*x + c)^2 - 4*(3*B^2*b^4*d^
4*g^3*x^4*log(e) + 2*(b^4*c*d^3*g^3 + (6*g^3*log(e) - g^3)*a*b^3*d^4)*B^2*x
^3 - 3*(b^4*c^2*d^2*g^3 - 4*a*b^3*c*d^3*g^3 - 3*(2*g^3*log(e) - g^3)*a^2*b^
2*d^4)*B^2*x^2 + 6*(b^4*c^3*d*g^3 - 4*a*b^3*c^2*d^2*g^3 + 6*a^2*b^2*c*d^3*g
^3 + (2*g^3*log(e) - 3*g^3)*a^3*b*d^4)*B^2*x + (6*a*b^3*c^3*d*g^3 - 21*a^2*
b^2*c^2*d^2*g^3 + 26*a^3*b*c*d^3*g^3 + (3*g^3*log(e) - 11*g^3)*a^4*d^4)*B^2
)*log(b*x + a) + 4*(3*B^2*b^4*d^4*g^3*x^4*log(e) + 2*(b^4*c*d^3*g^3 + (6*g^
3*log(e) - g^3)*a*b^3*d^4)*B^2*x^3 - 3*(b^4*c^2*d^2*g^3 - 4*a*b^3*c*d^3*g^3
 - 3*(2*g^3*log(e) - g^3)*a^2*b^2*d^4)*B^2*x^2 + 6*(b^4*c^3*d*g^3 - 4*a*b^3
*c^2*d^2*g^3 + 6*a^2*b^2*c*d^3*g^3 + (2*g^3*log(e) - 3*g^3)*a^3*b*d^4)*B^2*
x - 6*(B^2*b^4*d^4*g^3*x^4 + 4*B^2*a*b^3*d^4*g^3*x^3 + 6*B^2*a^2*b^2*d^4*g^
3*x^2 + 4*B^2*a^3*b*d^4*g^3*x + B^2*a^4*d^4*g^3)*log(b*x + a))*log(d*x + c)
)/(b*d^4)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.00

$$\int \left(a g + b g x\right)^3 \left(A + B \ln\left(\frac{e (c + d x)^2}{(a + b x)^2}\right)\right)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*g + b*g*x)^3*(A + B*log((e*(c + d*x)^2)/(a + b*x)^2))^2,x)
[Out] int((a*g + b*g*x)^3*(A + B*log((e*(c + d*x)^2)/(a + b*x)^2))^2, x)
```

**sympy [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*g*x+a*g)**3*(A+B*ln(e*(d*x+c)**2/(b*x+a)**2))**2,x)
[Out] Timed out
```

# 3.212 $\int (ag+bgx)^2 \left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2 dx$

**Optimal**. Leaf size=343

$$-\frac{4Bg^2(bc-ad)^3\log\left(1-\frac{d(a+bx)}{b(c+dx)}\right)\left(B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)+A\right)}{3bd^3}-\frac{4Bg^2(c+dx)(bc-ad)^2\left(B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)+A\right)}{3d^3}+\frac{2Bg^2(a\cdots}{}$$

```
[Out] 4/3*B^2*(-a*d+b*c)^2*g^2*x/d^2-4*B^2*(-a*d+b*c)^3*g^2*ln(b*x+a)/b/d^3-4/3*B
^2*(-a*d+b*c)^3*g^2*ln((d*x+c)/(b*x+a))/b/d^3+2/3*B*(-a*d+b*c)*g^2*(b*x+a)^
2*(A+B*ln(e*(d*x+c)^2/(b*x+a)^2))/b/d-4/3*B*(-a*d+b*c)^2*g^2*(d*x+c)*(A+B*l
n(e*(d*x+c)^2/(b*x+a)^2))/d^3+1/3*g^2*(b*x+a)^3*(A+B*ln(e*(d*x+c)^2/(b*x+a)
^2))^2/b-4/3*B*(-a*d+b*c)^3*g^2*(A+B*ln(e*(d*x+c)^2/(b*x+a)^2))*ln(1-d*(b*x
+a)/b/(d*x+c))/b/d^3+8/3*B^2*(-a*d+b*c)^3*g^2*polylog(2,d*(b*x+a)/b/(d*x+c)
)/b/d^3
```

**Rubi [A]** time = 0.63, antiderivative size = 397, normalized size of antiderivative = 1.16, number of steps used = 20, number of rules used = 13, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.382, Rules used = {2525, 12, 2528, 2486, 31, 43, 2524, 2418, 2394, 2393, 2391, 2390, 2301}

$$\frac{8B^2g^2(bc-ad)^3\text{PolyLog}\left(2,\frac{b(c+dx)}{bc-ad}\right)}{3bd^3}+\frac{4Bg^2(bc-ad)^3\log(c+dx)\left(B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)+A\right)}{3bd^3}-\frac{4ABg^2x(bc-ad)^2}{3d^2}+\frac{2Bg^2(a\cdots}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a*g + b*g*x)^2*(A + B*Log[(e*(c + d*x)^2)/(a + b*x)^2])^2,x]
```

```
[Out] (-4*A*B*(b*c - a*d)^2*g^2*x)/(3*d^2) + (4*B^2*(b*c - a*d)^2*g^2*x)/(3*d^2)
- (4*B^2*(b*c - a*d)^3*g^2*Log[c + d*x])/(b*d^3) + (8*B^2*(b*c - a*d)^3*g^2
*Log[-((d*(a + b*x))/(b*c - a*d))]*Log[c + d*x])/(3*b*d^3) - (4*B^2*(b*c -
a*d)^3*g^2*Log[c + d*x]^2)/(3*b*d^3) - (4*B^2*(b*c - a*d)^2*g^2*(a + b*x)*L
og[(e*(c + d*x)^2)/(a + b*x)^2])/(3*b*d^2) + (2*B*(b*c - a*d)*g^2*(a + b*x)
^2*(A + B*Log[(e*(c + d*x)^2)/(a + b*x)^2]))/(3*b*d) + (4*B*(b*c - a*d)^3*g
^2*Log[c + d*x]*(A + B*Log[(e*(c + d*x)^2)/(a + b*x)^2]))/(3*b*d^3) + (g^2*
(a + b*x)^3*(A + B*Log[(e*(c + d*x)^2)/(a + b*x)^2])^2)/(3*b) + (8*B^2*(b*c
 - a*d)^3*g^2*PolyLog[2, (b*(c + d*x))/(b*c - a*d)])/(3*b*d^3)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2301

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

Rule 2390

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_))^(q_.), x_Symbol] :> Dist[1/e, Subst[Int[((f*x)/d)^q*(a + b*Log[c*x^
n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && E
qQ[e*f - d*g, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2393

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + (c*e*x)/g])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2394

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d + e*x
)^n]))/g, x] - Dist[(b*e*n)/g, Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2418

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*(RFx_), x_Sy
mbol] :> With[{u = ExpandIntegrand[(a + b*Log[c*(d + e*x)^n])^p, RFx, x]},
```

```
Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d, e, n}, x] && RationalFunctionQ[
RFx, x] && IntegerQ[p]
```

Rule 2486

```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))
^(r_.)]^(s_.), x_Symbol] :> Simp[((a + b*x)*Log[e*(f*(a + b*x)^p*(c + d*x)^
q)^r]^s)/b, x] + Dist[(q*r*s*(b*c - a*d))/b, Int[Log[e*(f*(a + b*x)^p*(c +
d*x)^q)^r]^(s - 1)/(c + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, p, q, r, s
}, x] && NeQ[b*c - a*d, 0] && EqQ[p + q, 0] && IGtQ[s, 0]
```

Rule 2524

```
Int[((a_.) + Log[(c_.)*(RFx_)^(p_.)]*(b_.))^(n_.)/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[(Log[d + e*x]*(a + b*Log[c*RFx^p])^n)/e, x] - Dist[(b*n*p)/e
, Int[(Log[d + e*x]*(a + b*Log[c*RFx^p])^(n - 1)*D[RFx, x])/RFx, x], x] /;
FreeQ[{a, b, c, d, e, p}, x] && RationalFunctionQ[RFx, x] && IGtQ[n, 0]
```

Rule 2525

```
Int[((a_.) + Log[(c_.)*(RFx_)^(p_.)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_))^(m_.
), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(a + b*Log[c*RFx^p])^n)/(e*(m + 1))
, x] - Dist[(b*n*p)/(e*(m + 1)), Int[SimplifyIntegrand[((d + e*x)^(m + 1)*(
a + b*Log[c*RFx^p])^(n - 1)*D[RFx, x])/RFx, x], x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && RationalFunctionQ[RFx, x] && IGtQ[n, 0] && (EqQ[n, 1] ||
IntegerQ[m]) && NeQ[m, -1]
```

Rule 2528

```
Int[((a_.) + Log[(c_.)*(RFx_)^(p_.)]*(b_.))^(n_.)*(RGx_), x_Symbol] :> With
[{u = ExpandIntegrand[(a + b*Log[c*RFx^p])^n, RGx, x]}, Int[u, x] /; SumQ[u
]] /; FreeQ[{a, b, c, p}, x] && RationalFunctionQ[RFx, x] && RationalFuncti
onQ[RGx, x] && IGtQ[n, 0]
```

Rubi steps

$$\int (ag+bgx)^2 \left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2 dx = \frac{g^2(a+bx)^3\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2}{3b} - \frac{(2B)\int \frac{2(bc-ad)g^3(a+bx)^2\left(-A-\cdots\right.}{c+dx}}{3bg}$$

$$= \frac{g^2(a+bx)^3\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2}{3b} - \frac{\left(4B(bc-ad)g^2\right)\int \frac{(a+bx)^2\left(-\cdots\right.}{\cdots}}{3b}$$

$$= \frac{g^2(a+bx)^3\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2}{3b} - \frac{\left(4B(bc-ad)g^2\right)\int\left(-\frac{b(bc-a\cdots}{\cdots}\right.}{\cdots}$$

$$= \frac{g^2(a+bx)^3\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2}{3b} - \frac{\left(4B(bc-ad)g^2\right)\int(a+bx)\cdots}{3\cdots}$$

$$= -\frac{4AB(bc-ad)^2g^2x}{3d^2} + \frac{2B(bc-ad)g^2(a+bx)^2\left(A+B\log\left(\frac{e(c+dx\cdots}{(a+bx\cdots}\right.\right.}{3bd}$$

$$= -\frac{4AB(bc-ad)^2g^2x}{3d^2} - \frac{4B^2(bc-ad)^2g^2(a+bx)\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)}{3bd^2} + \cdots$$

$$= -\frac{4AB(bc-ad)^2g^2x}{3d^2} - \frac{8B^2(bc-ad)^3g^2\log(c+dx)}{3bd^3} - \frac{4B^2(bc-a\cdots}{\cdots}$$

$$= -\frac{4AB(bc-ad)^2g^2x}{3d^2} + \frac{4B^2(bc-ad)^2g^2x}{3d^2} - \frac{4B^2(bc-ad)^3g^2\log(\cdots}{bd^3}$$

$$= -\frac{4AB(bc-ad)^2g^2x}{3d^2} + \frac{4B^2(bc-ad)^2g^2x}{3d^2} - \frac{4B^2(bc-ad)^3g^2\log(\cdots}{bd^3}$$

$$= -\frac{4AB(bc-ad)^2g^2x}{3d^2} + \frac{4B^2(bc-ad)^2g^2x}{3d^2} - \frac{4B^2(bc-ad)^3g^2\log(\cdots}{bd^3}$$

$$= -\frac{4AB(bc-ad)^2g^2x}{3d^2} + \frac{4B^2(bc-ad)^2g^2x}{3d^2} - \frac{4B^2(bc-ad)^3g^2\log(\cdots}{bd^3}$$

**Mathematica [A]** time = 0.23, size = 298, normalized size = 0.87

$$g^2\left((a+bx)^3\left(B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)+A\right)^2-\frac{2B(bc-ad)\left(-d^2(a+bx)^2\left(B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)+A\right)-2(bc-ad)^2\log(c+dx)\left(B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)+A\right)+2Abdx(bc-\cdots\right.}{\cdots}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*g + b*g*x)^2*(A + B*Log[(e*(c + d*x)^2)/(a + b*x)^2])^2,x]
```

```
[Out] (g^2*((a + b*x)^3*(A + B*Log[(e*(c + d*x)^2)/(a + b*x)^2])^2 - (2*B*(b*c -
a*d)*(2*A*b*d*(b*c - a*d)*x + 4*B*(b*c - a*d)^2*Log[c + d*x] - 2*B*(b*c - a
*d)*(b*d*x + (-(b*c) + a*d)*Log[c + d*x]) + 2*B*d*(b*c - a*d)*(a + b*x)*Log
[(e*(c + d*x)^2)/(a + b*x)^2] - d^2*(a + b*x)^2*(A + B*Log[(e*(c + d*x)^2)/
(a + b*x)^2]) - 2*(b*c - a*d)^2*Log[c + d*x]*(A + B*Log[(e*(c + d*x)^2)/(a
+ b*x)^2]) - 2*B*(b*c - a*d)^2*((2*Log[(d*(a + b*x))/(-(b*c) + a*d)] - Log[
c + d*x])*Log[c + d*x] + 2*PolyLog[2, (b*(c + d*x))/(b*c - a*d)])))/d^3))/(
3*b)
```

**fricas [F]** time = 2.20, size = 0, normalized size = 0.00

$$\text{integral}\left(A^2b^2g^2x^2+2\,A^2abg^2x+A^2a^2g^2+\left(B^2b^2g^2x^2+2\,B^2abg^2x+B^2a^2g^2\right)\log\left(\frac{d^2ex^2+2\,cdex+c^2e}{b^2x^2+2\,abx+a^2}\right)^2+2\left(\cdots\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*g*x+a*g)^2*(A+B*log(e*(d*x+c)^2/(b*x+a)^2))^2,x, algorithm="fr
icas")
```

```
[Out] integral(A^2*b^2*g^2*x^2 + 2*A^2*a*b*g^2*x + A^2*a^2*g^2 + (B^2*b^2*g^2*x^2
 + 2*B^2*a*b*g^2*x + B^2*a^2*g^2)*log((d^2*e*x^2 + 2*c*d*e*x + c^2*e)/(b^2*
x^2 + 2*a*b*x + a^2))^2 + 2*(A*B*b^2*g^2*x^2 + 2*A*B*a*b*g^2*x + A*B*a^2*g^
2)*log((d^2*e*x^2 + 2*c*d*e*x + c^2*e)/(b^2*x^2 + 2*a*b*x + a^2)), x)
```

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int\left(bgx+ag\right)^2\left(B\log\left(\frac{\left(dx+c\right)^2e}{\left(bx+a\right)^2}\right)+A\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*g*x+a*g)^2*(A+B*log(e*(d*x+c)^2/(b*x+a)^2))^2,x, algorithm="gi
ac")
```

```
[Out] integrate((b*g*x + a*g)^2*(B*log((d*x + c)^2*e/(b*x + a)^2) + A)^2, x)
```

**maple [F]** time = 1.15, size = 0, normalized size = 0.00

$$\int \left(bgx+ag\right)^{2} \left(B\ln\left(\frac{\left(dx+c\right)^{2} e}{\left(bx+a\right)^{2}}\right)+A\right)^{2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*g*x+a*g)^2*(B*ln((d*x+c)^2/(b*x+a)^2*e)+A)^2,x)
```

```
[Out] int((b*g*x+a*g)^2*(B*ln((d*x+c)^2/(b*x+a)^2*e)+A)^2,x)
```

**maxima [B]** time = 1.70, size = 1333, normalized size = 3.89

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*g*x+a*g)^2*(A+B*log(e*(d*x+c)^2/(b*x+a)^2))^2,x, algorithm="ma
xima")
```

```
[Out] 1/3*A^2*b^2*g^2*x^3 + A^2*a*b*g^2*x^2 + 2*(x*log(d^2*e*x^2/(b^2*x^2 + 2*a*b
*x + a^2) + 2*c*d*e*x/(b^2*x^2 + 2*a*b*x + a^2) + c^2*e/(b^2*x^2 + 2*a*b*x
+ a^2)) - 2*a*log(b*x + a)/b + 2*c*log(d*x + c)/d)*A*B*a^2*g^2 + 2*(x^2*log
(d^2*e*x^2/(b^2*x^2 + 2*a*b*x + a^2) + 2*c*d*e*x/(b^2*x^2 + 2*a*b*x + a^2)
+ c^2*e/(b^2*x^2 + 2*a*b*x + a^2)) + 2*a^2*log(b*x + a)/b^2 - 2*c^2*log(d*x
 + c)/d^2 + 2*(b*c - a*d)*x/(b*d))*A*B*a*b*g^2 + 2/3*(x^3*log(d^2*e*x^2/(b^
2*x^2 + 2*a*b*x + a^2) + 2*c*d*e*x/(b^2*x^2 + 2*a*b*x + a^2) + c^2*e/(b^2*x
^2 + 2*a*b*x + a^2)) - 2*a^3*log(b*x + a)/b^3 + 2*c^3*log(d*x + c)/d^3 + ((
b^2*c*d - a*b*d^2)*x^2 - 2*(b^2*c^2 - a^2*d^2)*x)/(b^2*d^2))*A*B*b^2*g^2 +
A^2*a^2*g^2*x + 4/3*((g^2*log(e) - 3*g^2)*b^2*c^3 - (3*g^2*log(e) - 7*g^2)*
a*b*c^2*d + (3*g^2*log(e) - 4*g^2)*a^2*c*d^2)*B^2*log(d*x + c)/d^3 - 8/3*(b
^3*c^3*g^2 - 3*a*b^2*c^2*d*g^2 + 3*a^2*b*c*d^2*g^2 - a^3*d^3*g^2)*(log(b*x
+ a)*log((b*d*x + a*d)/(b*c - a*d) + 1) + dilog(-(b*d*x + a*d)/(b*c - a*d))
)*B^2/(b*d^3) + 1/3*(B^2*b^3*d^3*g^2*x^3*log(e)^2 + (2*b^3*c*d^2*g^2*log(e)
 + (3*g^2*log(e)^2 - 2*g^2*log(e))*a*b^2*d^3)*B^2*x^2 - (4*(g^2*log(e) - g^
2)*b^3*c^2*d - 4*(3*g^2*log(e) - 2*g^2)*a*b^2*c*d^2 - (3*g^2*log(e)^2 - 8*g
^2*log(e) + 4*g^2)*a^2*b*d^3)*B^2*x + 4*(B^2*b^3*d^3*g^2*x^3 + 3*B^2*a*b^2*
d^3*g^2*x^2 + 3*B^2*a^2*b*d^3*g^2*x + B^2*a^3*d^3*g^2)*log(b*x + a)^2 + 4*(
B^2*b^3*d^3*g^2*x^3 + 3*B^2*a*b^2*d^3*g^2*x^2 + 3*B^2*a^2*b*d^3*g^2*x + (b^
3*c^3*g^2 - 3*a*b^2*c^2*d*g^2 + 3*a^2*b*c*d^2*g^2)*B^2)*log(d*x + c)^2 - 4*
(B^2*b^3*d^3*g^2*x^3*log(e) + (b^3*c*d^2*g^2 + (3*g^2*log(e) - g^2)*a*b^2*d
^3)*B^2*x^2 - (2*b^3*c^2*d*g^2 - 6*a*b^2*c*d^2*g^2 - (3*g^2*log(e) - 4*g^2)
*a^2*b*d^3)*B^2*x - (2*a*b^2*c^2*d*g^2 - 5*a^2*b*c*d^2*g^2 - (g^2*log(e) -
3*g^2)*a^3*d^3)*B^2)*log(b*x + a) + 4*(B^2*b^3*d^3*g^2*x^3*log(e) + (b^3*c*
d^2*g^2 + (3*g^2*log(e) - g^2)*a*b^2*d^3)*B^2*x^2 - (2*b^3*c^2*d*g^2 - 6*a*
b^2*c*d^2*g^2 - (3*g^2*log(e) - 4*g^2)*a^2*b*d^3)*B^2*x - 2*(B^2*b^3*d^3*g^
```

```
2*x^3 + 3*B^2*a*b^2*d^3*g^2*x^2 + 3*B^2*a^2*b*d^3*g^2*x + B^2*a^3*d^3*g^2)*
log(b*x + a))*log(d*x + c))/(b*d^3)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.00

$$\int \left(a\,g+b\,g\,x\right)^2\,{\left(A+B\,\ln\left(\frac{e\,{\left(c+d\,x\right)}^2}{{\left(a+b\,x\right)}^2}\right)\right)}^2 \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*g + b*g*x)^2*(A + B*log((e*(c + d*x)^2)/(a + b*x)^2))^2,x)
[Out] int((a*g + b*g*x)^2*(A + B*log((e*(c + d*x)^2)/(a + b*x)^2))^2, x)
```

**sympy [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*g*x+a*g)**2*(A+B*ln(e*(d*x+c)**2/(b*x+a)**2))**2,x)
[Out] Timed out
```

# 3.213 $\int (ag+bgx)\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2\,dx$

**Optimal**. Leaf size=211

$$\frac{2Bg(bc-ad)^2\log\left(1-\frac{d(a+bx)}{b(c+dx)}\right)\left(B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)+A\right)}{bd^2}+\frac{2Bg(c+dx)(bc-ad)\left(B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)+A\right)}{d^2}+\frac{g(a+bx)^2}{}$$

[Out] 4*B^2*(-a*d+b*c)^2*g*ln(b*x+a)/b/d^2+2*B*(-a*d+b*c)*g*(d*x+c)*(A+B*ln(e*(d*x+c)^2/(b*x+a)^2))/d^2+1/2*g*(b*x+a)^2*(A+B*ln(e*(d*x+c)^2/(b*x+a)^2))^2/b+2*B*(-a*d+b*c)^2*g*(A+B*ln(e*(d*x+c)^2/(b*x+a)^2))*ln(1-d*(b*x+a)/b/(d*x+c))/b/d^2-4*B^2*(-a*d+b*c)^2*g*polylog(2,d*(b*x+a)/b/(d*x+c))/b/d^2

**Rubi [A]** time = 0.50, antiderivative size = 291, normalized size of antiderivative = 1.38, number of steps used = 16, number of rules used = 12, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {2525, 12, 2528, 2486, 31, 2524, 2418, 2394, 2393, 2391, 2390, 2301}

$$-\frac{4B^2g(bc-ad)^2\text{PolyLog}\left(2,\frac{b(c+dx)}{bc-ad}\right)}{bd^2}-\frac{2Bg(bc-ad)^2\log(c+dx)\left(B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)+A\right)}{bd^2}+\frac{g(a+bx)^2\left(B\log\left(\frac{e(c+}{(a+}\right.\right.}{2b}$$

Antiderivative was successfully verified.

[In] Int[(a*g + b*g*x)*(A + B*Log[(e*(c + d*x)^2)/(a + b*x)^2])^2,x]

[Out] (2*A*B*(b*c - a*d)*g*x)/d + (4*B^2*(b*c - a*d)^2*g*Log[c + d*x])/(b*d^2) - (4*B^2*(b*c - a*d)^2*g*Log[-((d*(a + b*x))/(b*c - a*d))]*Log[c + d*x])/(b*d^2) + (2*B^2*(b*c - a*d)^2*g*Log[c + d*x]^2)/(b*d^2) + (2*B^2*(b*c - a*d)*g*(a + b*x)*Log[(e*(c + d*x)^2)/(a + b*x)^2])/(b*d) - (2*B*(b*c - a*d)^2*g*Log[c + d*x]*(A + B*Log[(e*(c + d*x)^2)/(a + b*x)^2]))/(b*d^2) + (g*(a + b*x)^2*(A + B*Log[(e*(c + d*x)^2)/(a + b*x)^2])^2)/(2*b) - (4*B^2*(b*c - a*d)^2*g*PolyLog[2, (b*(c + d*x))/(b*c - a*d)])/(b*d^2)

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !MatchQ[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 2301

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

Rule 2390

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_))^(q_.), x_Symbol] :> Dist[1/e, Subst[Int[((f*x)/d)^q*(a + b*Log[c*x^
n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && E
qQ[e*f - d*g, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2393

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + (c*e*x)/g])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2394

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d + e*x
)^n]))/g, x] - Dist[(b*e*n)/g, Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2418

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*(RFx_), x_Sy
mbol] :> With[{u = ExpandIntegrand[(a + b*Log[c*(d + e*x)^n])^p, RFx, x]},
Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d, e, n}, x] && RationalFunctionQ[
RFx, x] && IntegerQ[p]
```

Rule 2486

```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))
^(r_.)]^(s_.), x_Symbol] :> Simp[((a + b*x)*Log[e*(f*(a + b*x)^p*(c + d*x)^
q)^r]^s)/b, x] + Dist[(q*r*s*(b*c - a*d))/b, Int[Log[e*(f*(a + b*x)^p*(c +
d*x)^q)^r]^(s - 1)/(c + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, p, q, r, s
}, x] && NeQ[b*c - a*d, 0] && EqQ[p + q, 0] && IGtQ[s, 0]
```

Rule 2524

```
Int[((a_.) + Log[(c_.)*(RFx_)^(p_.)]*(b_.))^(n_.)/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[(Log[d + e*x]*(a + b*Log[c*RFx^p])^n)/e, x] - Dist[(b*n*p)/e
, Int[(Log[d + e*x]*(a + b*Log[c*RFx^p])^(n - 1)*D[RFx, x])/RFx, x], x] /;
FreeQ[{a, b, c, d, e, p}, x] && RationalFunctionQ[RFx, x] && IGtQ[n, 0]
```

Rule 2525

```
Int[((a_.) + Log[(c_.)*(RFx_)^(p_.)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_))^(m_.
), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(a + b*Log[c*RFx^p])^n)/(e*(m + 1))
, x] - Dist[(b*n*p)/(e*(m + 1)), Int[SimplifyIntegrand[((d + e*x)^(m + 1)*(
a + b*Log[c*RFx^p])^(n - 1)*D[RFx, x])/RFx, x], x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && RationalFunctionQ[RFx, x] && IGtQ[n, 0] && (EqQ[n, 1] ||
IntegerQ[m]) && NeQ[m, -1]
```

Rule 2528

```
Int[((a_.) + Log[(c_.)*(RFx_)^(p_.)]*(b_.))^(n_.)*(RGx_), x_Symbol] :> With
[{u = ExpandIntegrand[(a + b*Log[c*RFx^p])^n, RGx, x]}, Int[u, x] /; SumQ[u
]] /; FreeQ[{a, b, c, p}, x] && RationalFunctionQ[RFx, x] && RationalFuncti
onQ[RGx, x] && IGtQ[n, 0]
```

Rubi steps

$$\int (ag+bgx)\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2 dx = \frac{g(a+bx)^2\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2}{2b} - \frac{B\int \frac{2(bc-ad)g^2(a+bx)\left(-A-B\log\left(\frac{e(c+\ldots}{(a+\ldots}\right.\right.}{c+dx}}{bg} \ldots$$

$$= \frac{g(a+bx)^2\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2}{2b} - \frac{(2B(bc-ad)g)\int \frac{(a+bx)\left(-A-B\log\ldots\right.}{c+dx}}{b} \ldots$$

$$= \frac{g(a+bx)^2\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2}{2b} - \frac{(2B(bc-ad)g)\int\left(\frac{b\left(-A-B\log\left(\frac{e(\ldots}{(\ldots}\right.\right.}{d}\ldots\right.}{\ldots} \ldots$$

$$= \frac{g(a+bx)^2\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2}{2b} - \frac{(2B(bc-ad)g)\int\left(-A-B\log\ldots\right.}{d} \ldots$$

$$= \frac{2AB(bc-ad)gx}{d} - \frac{2B(bc-ad)^2 g\log(c+dx)\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)}{bd^2} \ldots$$

$$= \frac{2AB(bc-ad)gx}{d} + \frac{2B^2(bc-ad)g(a+bx)\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)}{bd} - \frac{2B(bc-\ldots}{\ldots} \ldots$$

$$= \frac{2AB(bc-ad)gx}{d} + \frac{4B^2(bc-ad)^2 g\log(c+dx)}{bd^2} + \frac{2B^2(bc-ad)g(a\ldots}{b\ldots} \ldots$$

$$= \frac{2AB(bc-ad)gx}{d} + \frac{4B^2(bc-ad)^2 g\log(c+dx)}{bd^2} + \frac{2B^2(bc-ad)g(a\ldots}{b\ldots} \ldots$$

$$= \frac{2AB(bc-ad)gx}{d} + \frac{4B^2(bc-ad)^2 g\log(c+dx)}{bd^2} - \frac{4B^2(bc-ad)^2 g\log\ldots}{\ldots} \ldots$$

$$= \frac{2AB(bc-ad)gx}{d} + \frac{4B^2(bc-ad)^2 g\log(c+dx)}{bd^2} - \frac{4B^2(bc-ad)^2 g\log\ldots}{\ldots} \ldots$$

$$= \frac{2AB(bc-ad)gx}{d} + \frac{4B^2(bc-ad)^2 g\log(c+dx)}{bd^2} - \frac{4B^2(bc-ad)^2 g\log\ldots}{\ldots} \ldots$$

**Mathematica** [A] time = 0.17, size = 195, normalized size = 0.92

$$\frac{g\left(\frac{4B(bc-ad)\left(-(bc-ad)\log(c+dx)\left(B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)+2B\log\left(\frac{d(a+bx)}{ad-bc}\right)+A-2B\right)+Bd(a+bx)\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)+(2aBd-2bBc)\mathrm{Li}_2\left(\frac{b(c+dx)}{bc-ad}\right)+B(bc-ad)\log^2(c+}{d^2}\right.}{2b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*g + b*g*x)*(A + B*Log[(e*(c + d*x)^2)/(a + b*x)^2])^2,x]
```

```
[Out] (g*((a + b*x)^2*(A + B*Log[(e*(c + d*x)^2)/(a + b*x)^2])^2 + (4*B*(b*c - a*
d)*(A*b*d*x + B*(b*c - a*d)*Log[c + d*x]^2 + B*d*(a + b*x)*Log[(e*(c + d*x)
^2)/(a + b*x)^2] - (b*c - a*d)*Log[c + d*x]*(A - 2*B + 2*B*Log[(d*(a + b*x)
)/(-(b*c) + a*d)] + B*Log[(e*(c + d*x)^2)/(a + b*x)^2]) + (-2*b*B*c + 2*a*B
*d)*PolyLog[2, (b*(c + d*x))/(b*c - a*d)]))/d^2))/(2*b)
```

**fricas** [F] time = 0.54, size = 0, normalized size = 0.00

$$\text{integral}\left(A^2bgx + A^2ag + \left(B^2bgx + B^2ag\right)\log\left(\frac{d^2ex^2 + 2\,cdex + c^2e}{b^2x^2 + 2\,abx + a^2}\right)^2 + 2\left(ABbgx + ABag\right)\log\left(\frac{d^2ex^2 + 2\,cd}{b^2x^2 + 2\,ab}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*g*x+a*g)*(A+B*log(e*(d*x+c)^2/(b*x+a)^2))^2,x, algorithm="fric
as")
```

```
[Out] integral(A^2*b*g*x + A^2*a*g + (B^2*b*g*x + B^2*a*g)*log((d^2*e*x^2 + 2*c*d
*e*x + c^2*e)/(b^2*x^2 + 2*a*b*x + a^2))^2 + 2*(A*B*b*g*x + A*B*a*g)*log((d
^2*e*x^2 + 2*c*d*e*x + c^2*e)/(b^2*x^2 + 2*a*b*x + a^2)), x)
```

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(bgx + ag\right)\left(B\log\left(\frac{(dx + c)^2 e}{(bx + a)^2}\right) + A\right)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*g*x+a*g)*(A+B*log(e*(d*x+c)^2/(b*x+a)^2))^2,x, algorithm="giac
")
```

```
[Out] integrate((b*g*x + a*g)*(B*log((d*x + c)^2*e/(b*x + a)^2) + A)^2, x)
```

**maple** [F] time = 0.92, size = 0, normalized size = 0.00

$$\int \left(bgx + ag\right)\left(B\ln\left(\frac{(dx + c)^2 e}{(bx + a)^2}\right) + A\right)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*g*x+a*g)*(B*ln((d*x+c)^2/(b*x+a)^2*e)+A)^2,x)
```

```
[Out] int((b*g*x+a*g)*(B*ln((d*x+c)^2/(b*x+a)^2*e)+A)^2,x)
```

**maxima [B]** time = 2.09, size = 730, normalized size = 3.46

$$\frac{1}{2}A^2bgx^2+2\left(x\log\left(\frac{d^2ex^2}{b^2x^2+2\,abx+a^2}+\frac{2\,cdex}{b^2x^2+2\,abx+a^2}+\frac{c^2e}{b^2x^2+2\,abx+a^2}\right)-\frac{2\,a\log\left(bx+a\right)}{b}+\frac{2\,c\log\left(dx\right.}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*g*x+a*g)*(A+B*log(e*(d*x+c)^2/(b*x+a)^2))^2,x, algorithm="maxi
ma")
```

```
[Out] 1/2*A^2*b*g*x^2 + 2*(x*log(d^2*e*x^2/(b^2*x^2 + 2*a*b*x + a^2) + 2*c*d*e*x/
(b^2*x^2 + 2*a*b*x + a^2) + c^2*e/(b^2*x^2 + 2*a*b*x + a^2)) - 2*a*log(b*x
+ a)/b + 2*c*log(d*x + c)/d)*A*B*a*g + (x^2*log(d^2*e*x^2/(b^2*x^2 + 2*a*b*
x + a^2) + 2*c*d*e*x/(b^2*x^2 + 2*a*b*x + a^2) + c^2*e/(b^2*x^2 + 2*a*b*x +
 a^2)) + 2*a^2*log(b*x + a)/b^2 - 2*c^2*log(d*x + c)/d^2 + 2*(b*c - a*d)*x/
(b*d))*A*B*b*g + A^2*a*g*x - 2*((g*log(e) - 2*g)*b*c^2 - 2*(g*log(e) - g)*a
*c*d)*B^2*log(d*x + c)/d^2 + 4*(b^2*c^2*g - 2*a*b*c*d*g + a^2*d^2*g)*(log(b
*x + a)*log((b*d*x + a*d)/(b*c - a*d) + 1) + dilog(-(b*d*x + a*d)/(b*c - a*
d)))*B^2/(b*d^2) + 1/2*(B^2*b^2*d^2*g*x^2*log(e)^2 + 2*(2*b^2*c*d*g*log(e)
+ (g*log(e)^2 - 2*g*log(e))*a*b*d^2)*B^2*x + 4*(B^2*b^2*d^2*g*x^2 + 2*B^2*a
*b*d^2*g*x + B^2*a^2*d^2*g)*log(b*x + a)^2 + 4*(B^2*b^2*d^2*g*x^2 + 2*B^2*a
*b*d^2*g*x - (b^2*c^2*g - 2*a*b*c*d*g)*B^2)*log(d*x + c)^2 - 4*(B^2*b^2*d^2
*g*x^2*log(e) + 2*((g*log(e) - g)*a*b*d^2 + b^2*c*d*g)*B^2*x + ((g*log(e) -
 2*g)*a^2*d^2 + 2*a*b*c*d*g)*B^2)*log(b*x + a) + 4*(B^2*b^2*d^2*g*x^2*log(e
) + 2*((g*log(e) - g)*a*b*d^2 + b^2*c*d*g)*B^2*x - 2*(B^2*b^2*d^2*g*x^2 + 2
*B^2*a*b*d^2*g*x + B^2*a^2*d^2*g)*log(b*x + a))*log(d*x + c))/(b*d^2)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.00

$$\int \left(a\,g+b\,g\,x\right)\,{\left(A+B\,\ln\left(\frac{e\,{\left(c+d\,x\right)}^2}{{\left(a+b\,x\right)}^2}\right)\right)}^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*g + b*g*x)*(A + B*log((e*(c + d*x)^2)/(a + b*x)^2))^2,x)
```

```
[Out] int((a*g + b*g*x)*(A + B*log((e*(c + d*x)^2)/(a + b*x)^2))^2, x)
```

**sympy [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*g*x+a*g)*(A+B*ln(e*(d*x+c)**2/(b*x+a)**2))**2,x)
```

```
[Out] Timed out
```

# 3.214 $\int \frac{\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2}{ag+bgx}\,dx$

**Optimal**. Leaf size=132

$$-\frac{4B\text{Li}_2\left(\frac{b(c+dx)}{d(a+bx)}\right)\left(B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)+A\right)}{bg}-\frac{\log\left(-\frac{bc-ad}{d(a+bx)}\right)\left(B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)+A\right)^2}{bg}+\frac{8B^2\text{Li}_3\left(\frac{b(c+dx)}{d(a+bx)}\right)}{bg}$$

```
[Out] -ln((a*d-b*c)/d/(b*x+a))*(A+B*ln(e*(d*x+c)^2/(b*x+a)^2))^2/b/g-4*B*(A+B*ln(
e*(d*x+c)^2/(b*x+a)^2))*polylog(2,b*(d*x+c)/d/(b*x+a))/b/g+8*B^2*polylog(3,
b*(d*x+c)/d/(b*x+a))/b/g
```

**Rubi [B]** time = 4.08, antiderivative size = 740, normalized size of antiderivative = 5.61, number of steps used = 46, number of rules used = 23, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.676, Rules used = {2524, 12, 2528, 2418, 2390, 2301, 2394, 2393, 2391, 6688, 6742, 2499, 2302, 30, 2396, 2433, 2374, 6589, 2500, 2440, 2434, 2375, 2317}

$$-\frac{4AB\text{PolyLog}\left(2,-\frac{d(a+bx)}{bc-ad}\right)}{bg}+\frac{4B^2\text{PolyLog}\left(2,-\frac{d(a+bx)}{bc-ad}\right)\left(-\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)+\log\left(\frac{1}{(a+bx)^2}\right)+\log\left((c+dx)^2\right)\right)}{bg}-\frac{8B^2\text{I}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Log[(e*(c + d*x)^2)/(a + b*x)^2])^2/(a*g + b*g*x),x]
```

```
[Out] (2*A*B*Log[g*(a + b*x)]^2)/(b*g) + (4*B^2*Log[g*(a + b*x)]^3)/(3*b*g) - (B^
2*Log[(a + b*x)^(-2)]^2*Log[c + d*x])/(b*g) - (4*B^2*Log[(a + b*x)^(-2)]*Lo
g[g*(a + b*x)]*Log[c + d*x])/(b*g) - (4*B^2*Log[g*(a + b*x)]^2*Log[c + d*x]
)/(b*g) + (B^2*Log[(a + b*x)^(-2)]^2*Log[(b*(c + d*x))/(b*c - a*d)])/(b*g)
+ (4*B^2*Log[g*(a + b*x)]^2*Log[(b*(c + d*x))/(b*c - a*d)])/(b*g) + (B^2*Lo
g[-((d*(a + b*x))/(b*c - a*d))]*Log[(c + d*x)^2]^2)/(b*g) - (B^2*Log[g*(a +
 b*x)]*Log[(c + d*x)^2]^2)/(b*g) - (4*A*B*Log[(b*(c + d*x))/(b*c - a*d)]*Lo
g[a*g + b*g*x])/(b*g) + (4*B^2*Log[(b*(c + d*x))/(b*c - a*d)]*(Log[(a + b*x
)^(-2)] + Log[(c + d*x)^2] - Log[(e*(c + d*x)^2)/(a + b*x)^2])*Log[a*g + b*
g*x])/(b*g) + ((A + B*Log[(e*(c + d*x)^2)/(a + b*x)^2])^2*Log[a*g + b*g*x])
/(b*g) - (4*B^2*Log[(b*(c + d*x))/(b*c - a*d)]*Log[a*g + b*g*x]^2)/(b*g) +
(2*B^2*Log[(e*(c + d*x)^2)/(a + b*x)^2]*Log[a*g + b*g*x]^2)/(b*g) - (4*A*B*
PolyLog[2, -((d*(a + b*x))/(b*c - a*d))])/(b*g) - (4*B^2*Log[(a + b*x)^(-2)
]*PolyLog[2, -((d*(a + b*x))/(b*c - a*d))])/(b*g) + (4*B^2*(Log[(a + b*x)^(
-2)] + Log[(c + d*x)^2] - Log[(e*(c + d*x)^2)/(a + b*x)^2])*PolyLog[2, -((d
*(a + b*x))/(b*c - a*d))])/(b*g) + (4*B^2*Log[(c + d*x)^2]*PolyLog[2, (b*(c
 + d*x))/(b*c - a*d)])/(b*g) - (8*B^2*PolyLog[3, -((d*(a + b*x))/(b*c - a*d
))])/(b*g) - (8*B^2*PolyLog[3, (b*(c + d*x))/(b*c - a*d)])/(b*g)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2301

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

Rule 2302

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/(x_), x_Symbol] :> Dist[1/(
b*n), Subst[Int[x^p, x], x, a + b*Log[c*x^n]], x] /; FreeQ[{a, b, c, n, p},
 x]
```

Rule 2317

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symb
ol] :> Simp[(Log[1 + (e*x)/d]*(a + b*Log[c*x^n])^p)/e, x] - Dist[(b*n*p)/e,
 Int[(Log[1 + (e*x)/d]*(a + b*Log[c*x^n])^(p - 1))/x, x], x] /; FreeQ[{a, b
, c, d, e, n}, x] && IGtQ[p, 0]
```

Rule 2374

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> -Simp[(PolyLog[2, -(d*f*x^m)]*(a + b*Log[c*x
^n])^p)/m, x] + Dist[(b*n*p)/m, Int[(PolyLog[2, -(d*f*x^m)]*(a + b*Log[c*x^
n])^(p - 1))/x, x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p, 0]
&& EqQ[d*e, 1]
```

Rule 2375

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))^(r_.)]*((a_.) + Log[(c_.)*(x_)^(n_
.)]*(b_.))^(p_.))/(x_), x_Symbol] :> Simp[(Log[d*(e + f*x^m)^r]*(a + b*Log[
c*x^n])^(p + 1))/(b*n*(p + 1)), x] - Dist[(f*m*r)/(b*n*(p + 1)), Int[(x^(m
- 1)*(a + b*Log[c*x^n])^(p + 1))/(e + f*x^m), x], x] /; FreeQ[{a, b, c, d,
e, f, r, m, n}, x] && IGtQ[p, 0] && NeQ[d*e, 1]
```

Rule 2390

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_))^(q_.), x_Symbol] :> Dist[1/e, Subst[Int[((f*x)/d)^q*(a + b*Log[c*x^
n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && E
qQ[e*f - d*g, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2393

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + (c*e*x)/g])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2394

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d + e*x
)^n]))/g, x] - Dist[(b*e*n)/g, Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2396

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)/((f_.) + (g_.
)*(x_)), x_Symbol] :> Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d
+ e*x)^n])^p)/g, x] - Dist[(b*e*n*p)/g, Int[(Log[(e*(f + g*x))/(e*f - d*g)]
*(a + b*Log[c*(d + e*x)^n])^(p - 1))/(d + e*x), x], x] /; FreeQ[{a, b, c, d
, e, f, g, n, p}, x] && NeQ[e*f - d*g, 0] && IGtQ[p, 1]
```

Rule 2418

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*(RFx_), x_Sy
mbol] :> With[{u = ExpandIntegrand[(a + b*Log[c*(d + e*x)^n])^p, RFx, x]},
Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d, e, n}, x] && RationalFunctionQ[
RFx, x] && IntegerQ[p]
```

Rule 2433

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + Log
[(h_.)*((i_.) + (j_.)*(x_))^(m_.)]*(g_.))*((k_.) + (l_.)*(x_))^(r_.), x_Sym
bol] :> Dist[1/e, Subst[Int[((k*x)/d)^r*(a + b*Log[c*x^n])^p*(f + g*Log[h*(
(e*i - d*j)/e + (j*x)/e)^m]), x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e,
```

```
f, g, h, i, j, k, l, n, p, r}, x] && EqQ[e*k - d*l, 0]
```

Rule 2434

```
Int[(((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + Log[(h_.
)*((i_.) + (j_.)*(x_))^(m_.)]*(g_.)))/(x_), x_Symbol] :> Simp[Log[x]*(a + b
*Log[c*(d + e*x)^n])*(f + g*Log[h*(i + j*x)^m]), x] + (-Dist[e*g*m, Int[(Lo
g[x]*(a + b*Log[c*(d + e*x)^n]))/(d + e*x), x], x] - Dist[b*j*n, Int[(Log[x
]*(f + g*Log[h*(i + j*x)^m]))/(i + j*x), x], x]) /; FreeQ[{a, b, c, d, e, f
, g, h, i, j, m, n}, x] && EqQ[e*i - d*j, 0]
```

Rule 2440

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + Log[(h_.)
*((i_.) + (j_.)*(x_))^(m_.)]*(g_.))*((k_) + (l_.)*(x_))^(r_.), x_Symbol] :>
 Dist[1/l, Subst[Int[x^r*(a + b*Log[c*(-((e*k - d*l)/l) + (e*x)/l)^n])*(f +
 g*Log[h*(-((j*k - i*l)/l) + (j*x)/l)^m]), x], x, k + l*x], x] /; FreeQ[{a,
 b, c, d, e, f, g, h, i, j, k, l, m, n}, x] && IntegerQ[r]
```

Rule 2499

```
Int[(Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.)
)^(r_.)]*((s_.) + Log[(i_.)*((g_.) + (h_.)*(x_))^(n_.)]*(t_.))^(m_.))/((j_.
) + (k_.)*(x_)), x_Symbol] :> Simp[((s + t*Log[i*(g + h*x)^n])^(m + 1)*Log[
e*(f*(a + b*x)^p*(c + d*x)^q)^r])/(k*n*t*(m + 1)), x] + (-Dist[(b*p*r)/(k*n
*t*(m + 1)), Int[(s + t*Log[i*(g + h*x)^n])^(m + 1)/(a + b*x), x], x] - Dis
t[(d*q*r)/(k*n*t*(m + 1)), Int[(s + t*Log[i*(g + h*x)^n])^(m + 1)/(c + d*x)
, x], x]) /; FreeQ[{a, b, c, d, e, f, g, h, i, j, k, s, t, m, n, p, q, r},
x] && NeQ[b*c - a*d, 0] && EqQ[h*j - g*k, 0] && IGtQ[m, 0]
```

Rule 2500

```
Int[(Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.)
)^(r_.)]*((s_.) + Log[(i_.)*((g_.) + (h_.)*(x_))^(n_.)]*(t_.)))/((j_.) + (k
_.)*(x_)), x_Symbol] :> Dist[Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r] - Log[(a
+ b*x)^(p*r)] - Log[(c + d*x)^(q*r)], Int[(s + t*Log[i*(g + h*x)^n])/(j + k
*x), x], x] + (Int[(Log[(a + b*x)^(p*r)]*(s + t*Log[i*(g + h*x)^n]))/(j + k
*x), x] + Int[(Log[(c + d*x)^(q*r)]*(s + t*Log[i*(g + h*x)^n]))/(j + k*x),
x]) /; FreeQ[{a, b, c, d, e, f, g, h, i, j, k, s, t, n, p, q, r}, x] && NeQ
[b*c - a*d, 0]
```

Rule 2524

```
Int[((a_.) + Log[(c_.)*(RFx_)^(p_.)]*(b_.))^(n_.)/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[(Log[d + e*x]*(a + b*Log[c*RFx^p])^n)/e, x] - Dist[(b*n*p)/e
```

```
, Int[(Log[d + e*x]*(a + b*Log[c*RFx^p])^(n - 1)*D[RFx, x])/RFx, x], x] /;
FreeQ[{a, b, c, d, e, p}, x] && RationalFunctionQ[RFx, x] && IGtQ[n, 0]
```

Rule 2528

```
Int[((a_.) + Log[(c_.)*(RFx_)^(p_.)]*(b_.))^(n_.)*(RGx_), x_Symbol] :> With
[{u = ExpandIntegrand[(a + b*Log[c*RFx^p])^n, RGx, x]}, Int[u, x] /; SumQ[u
]] /; FreeQ[{a, b, c, p}, x] && RationalFunctionQ[RFx, x] && RationalFuncti
onQ[RGx, x] && IGtQ[n, 0]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6688

```
Int[u_, x_Symbol] :> With[{v = SimplifyIntegrand[u, x]}, Int[v, x] /; Simpl
erIntegrandQ[v, u, x]]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\int \frac{\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2}{ag+bgx}\,dx = \frac{\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2\log(ag+bgx)}{bg} - \frac{(2B)\int \frac{(a+bx)^2\left(\frac{2de(c+dx)}{(a+bx)^2}-\frac{2be(c+dx)^2}{(a+bx)^3}\right)\left(A+B\log\left(\right.\right.}{e(c+dx)^2}}{bg}$$

$$= \frac{\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2\log(ag+bgx)}{bg} - \frac{(2B)\int \frac{(a+bx)^2\left(\frac{2de(c+dx)}{(a+bx)^2}-\frac{2be(c+dx)^2}{(a+bx)^3}\right)\left(A+B\log\left(\right.\right.}{(c+dx)^2}}{beg}$$

$$= \frac{\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2\log(ag+bgx)}{bg} - \frac{(2B)\int \frac{2(bc-ad)e\left(-A-B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)\log(ag+b}{(a+bx)(c+dx)}}{beg}$$

$$= \frac{\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2\log(ag+bgx)}{bg} - \frac{(4B(bc-ad))\int \frac{\left(-A-B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)\log(ag}{(a+bx)(c+dx)}}{bg}$$

$$= \frac{\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2\log(ag+bgx)}{bg} - \frac{(4B(bc-ad))\int\left(\frac{b\left(-A-B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)\log(}{(bc-ad)(a+bx)}\right.}{b}$$

$$= \frac{\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2\log(ag+bgx)}{bg} - \frac{(4B)\int \frac{\left(-A-B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)\log(ag+bgx)}{a+bx}\,dx}{g} -$$

$$= \frac{\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2\log(ag+bgx)}{bg} - \frac{(4B)\int\left(\frac{A\log(ag+bgx)}{-a-bx}+\frac{B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\log(a}{-a-bx}\right.}{g}$$

$$= \frac{\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2\log(ag+bgx)}{bg} - \frac{(4AB)\int \frac{\log(ag+bgx)}{-a-bx}\,dx}{g} - \frac{\left(4B^2\right)\int \frac{\log\left(\frac{e(c}{(a}\right.}{}}{}$$

$$= -\frac{4AB\log\left(\frac{b(c+dx)}{bc-ad}\right)\log(ag+bgx)}{bg} + \frac{\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2\log(ag+bgx)}{bg} + \frac{2I}{}$$

$$= -\frac{4AB\log\left(\frac{b(c+dx)}{bc-ad}\right)\log(ag+bgx)}{bg} + \frac{4B^2\log\left(\frac{b(c+dx)}{bc-ad}\right)\left(\log\left(\frac{1}{(a+bx)^2}\right)+\log\left((c+\right.\right.}{bg}$$

$$= \frac{2AB\log^2(g(a+bx))}{} - \frac{4B^2\log\left(\frac{1}{(a+bx)^2}\right)\log(g(a+bx))\log(c+dx)}{} - \frac{B^2\log(g(}{}$$

**Mathematica [A]** time = 0.33, size = 257, normalized size = 1.95

$$\frac{A^2\log(a+bx)+2AB\log(a+bx)\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)+4AB\text{Li}_2\left(\frac{b(c+dx)}{bc-ad}\right)-4AB\log(a+bx)\log\left(\frac{c}{d}+x\right)+4AB\log\left(\frac{c}{d}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Log[(e*(c + d*x)^2)/(a + b*x)^2])^2/(a*g + b*g*x),x]
```

```
[Out] (-2*A*B*Log[a/b + x]^2 + A^2*Log[a + b*x] + 4*A*B*Log[a/b + x]*Log[a + b*x]
 - 4*A*B*Log[c/d + x]*Log[a + b*x] + 4*A*B*Log[c/d + x]*Log[(d*(a + b*x))/(
-(b*c) + a*d)] + 2*A*B*Log[a + b*x]*Log[(e*(c + d*x)^2)/(a + b*x)^2] - B^2*
Log[(-(b*c) + a*d)/(d*(a + b*x))]*Log[(e*(c + d*x)^2)/(a + b*x)^2]^2 + 4*A*
B*PolyLog[2, (b*(c + d*x))/(b*c - a*d)] - 4*B^2*Log[(e*(c + d*x)^2)/(a + b*
x)^2]*PolyLog[2, (b*(c + d*x))/(d*(a + b*x))] + 8*B^2*PolyLog[3, (b*(c + d*
x))/(d*(a + b*x))])/(b*g)
```

**fricas [F]** time = 1.43, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{B^2\log\left(\frac{d^2ex^2+2\,cdex+c^2e}{b^2x^2+2\,abx+a^2}\right)^2+2\,AB\log\left(\frac{d^2ex^2+2\,cdex+c^2e}{b^2x^2+2\,abx+a^2}\right)+A^2}{bgx+ag},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*(d*x+c)^2/(b*x+a)^2))^2/(b*g*x+a*g),x, algorithm="fric
as")
```

```
[Out] integral((B^2*log((d^2*e*x^2 + 2*c*d*e*x + c^2*e)/(b^2*x^2 + 2*a*b*x + a^2)
)^2 + 2*A*B*log((d^2*e*x^2 + 2*c*d*e*x + c^2*e)/(b^2*x^2 + 2*a*b*x + a^2))
+ A^2)/(b*g*x + a*g), x)
```

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\left(B\log\left(\frac{(dx+c)^2e}{(bx+a)^2}\right)+A\right)^2}{bgx+ag}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*(d*x+c)^2/(b*x+a)^2))^2/(b*g*x+a*g),x, algorithm="giac
")
```

```
[Out] integrate((B*log((d*x + c)^2*e/(b*x + a)^2) + A)^2/(b*g*x + a*g), x)
```

**maple [F]** time = 1.00, size = 0, normalized size = 0.00

$$\int \frac{\left(B\ln\left(\frac{(dx+c)^2 e}{(bx+a)^2}\right)+A\right)^2}{bgx+ag}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*ln((d*x+c)^2/(b*x+a)^2*e)+A)^2/(b*g*x+a*g),x)
```

```
[Out] int((B*ln((d*x+c)^2/(b*x+a)^2*e)+A)^2/(b*g*x+a*g),x)
```

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$\frac{4\,B^2\log(bx+a)\log(dx+c)^2}{bg}+\frac{A^2\log\left(bgx+ag\right)}{bg}-\int -\frac{B^2bc\log(e)^2+2\,ABbc\log(e)+4\left(B^2bdx+B^2bc\right)\log(b}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*(d*x+c)^2/(b*x+a)^2))^2/(b*g*x+a*g),x, algorithm="maxi
ma")
```

```
[Out] 4*B^2*log(b*x + a)*log(d*x + c)^2/(b*g) + A^2*log(b*g*x + a*g)/(b*g) - inte
grate(-(B^2*b*c*log(e)^2 + 2*A*B*b*c*log(e) + 4*(B^2*b*d*x + B^2*b*c)*log(b
*x + a)^2 + (B^2*b*d*log(e)^2 + 2*A*B*b*d*log(e))*x - 4*(B^2*b*c*log(e) + A
*B*b*c + (B^2*b*d*log(e) + A*B*b*d)*x)*log(b*x + a) + 4*(B^2*b*c*log(e) + A
*B*b*c + (B^2*b*d*log(e) + A*B*b*d)*x - 2*(2*B^2*b*d*x + (b*c + a*d)*B^2)*l
og(b*x + a))*log(d*x + c))/(b^2*d*g*x^2 + a*b*c*g + (b^2*c*g + a*b*d*g)*x),
 x)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\left(A+B\,\ln\left(\frac{e\,{\left(c+d\,x\right)}^2}{{\left(a+b\,x\right)}^2}\right)\right)}^2}{a\,g+b\,g\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*log((e*(c + d*x)^2)/(a + b*x)^2))^2/(a*g + b*g*x),x)
```

```
[Out] int((A + B*log((e*(c + d*x)^2)/(a + b*x)^2))^2/(a*g + b*g*x), x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{A^2}{a+bx}\,dx+\int \frac{B^2\log\left(\frac{c^2e}{a^2+2abx+b^2x^2}+\frac{2cdex}{a^2+2abx+b^2x^2}+\frac{d^2ex^2}{a^2+2abx+b^2x^2}\right)^2}{a+bx}\,dx+\int \frac{2AB\log\left(\frac{c^2e}{a^2+2abx+b^2x^2}+\frac{2cdex}{a^2+2abx+b^2x^2}+\frac{d^2ex^2}{a^2+2abx+b^2x^2}\right)}{a+bx}\,dx}{g}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*ln(e*(d*x+c)**2/(b*x+a)**2))**2/(b*g*x+a*g),x)
```

```
[Out] (Integral(A**2/(a + b*x), x) + Integral(B**2*log(c**2*e/(a**2 + 2*a*b*x + b
**2*x**2) + 2*c*d*e*x/(a**2 + 2*a*b*x + b**2*x**2) + d**2*e*x**2/(a**2 + 2*
a*b*x + b**2*x**2))**2/(a + b*x), x) + Integral(2*A*B*log(c**2*e/(a**2 + 2*
a*b*x + b**2*x**2) + 2*c*d*e*x/(a**2 + 2*a*b*x + b**2*x**2) + d**2*e*x**2/(
a**2 + 2*a*b*x + b**2*x**2))/(a + b*x), x))/g
```

## 3.215 $\int \frac{\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2}{(ag+bgx)^2}\,dx$

**Optimal**. Leaf size=157

$$-\frac{(c+dx)\left(B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)+A\right)^2}{g^2(a+bx)(bc-ad)}+\frac{4AB(c+dx)}{g^2(a+bx)(bc-ad)}+\frac{4B^2(c+dx)\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)}{g^2(a+bx)(bc-ad)}-\frac{8B^2(c+dx)}{g^2(a+bx)(bc-ad)}$$

```
[Out] 4*A*B*(d*x+c)/(-a*d+b*c)/g^2/(b*x+a)-8*B^2*(d*x+c)/(-a*d+b*c)/g^2/(b*x+a)+4
*B^2*(d*x+c)*ln(e*(d*x+c)^2/(b*x+a)^2)/(-a*d+b*c)/g^2/(b*x+a)-(d*x+c)*(A+B*
ln(e*(d*x+c)^2/(b*x+a)^2))^2/(-a*d+b*c)/g^2/(b*x+a)
```

**Rubi [C]** time = 0.92, antiderivative size = 480, normalized size of antiderivative = 3.06, number of steps used = 26, number of rules used = 11, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.324, Rules used = {2525, 12, 2528, 44, 2524, 2418, 2390, 2301, 2394, 2393, 2391}

$$-\frac{8B^2d\text{PolyLog}\left(2,-\frac{d(a+bx)}{bc-ad}\right)}{bg^2(bc-ad)}-\frac{8B^2d\text{PolyLog}\left(2,\frac{b(c+dx)}{bc-ad}\right)}{bg^2(bc-ad)}+\frac{4Bd\log(a+bx)\left(B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)+A\right)}{bg^2(bc-ad)}-\frac{4Bd\log(c+d\ldots}{\ldots}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Log[(e*(c + d*x)^2)/(a + b*x)^2])^2/(a*g + b*g*x)^2,x]
```

```
[Out] (-8*B^2)/(b*g^2*(a + b*x)) - (8*B^2*d*Log[a + b*x])/(b*(b*c - a*d)*g^2) + (
4*B^2*d*Log[a + b*x]^2)/(b*(b*c - a*d)*g^2) + (8*B^2*d*Log[c + d*x])/(b*(b*
c - a*d)*g^2) - (8*B^2*d*Log[-((d*(a + b*x))/(b*c - a*d))]*Log[c + d*x])/(b
*(b*c - a*d)*g^2) + (4*B^2*d*Log[c + d*x]^2)/(b*(b*c - a*d)*g^2) - (8*B^2*d
*Log[a + b*x]*Log[(b*(c + d*x))/(b*c - a*d)])/(b*(b*c - a*d)*g^2) + (4*B*(A
 + B*Log[(e*(c + d*x)^2)/(a + b*x)^2]))/(b*g^2*(a + b*x)) + (4*B*d*Log[a +
b*x]*(A + B*Log[(e*(c + d*x)^2)/(a + b*x)^2]))/(b*(b*c - a*d)*g^2) - (4*B*d
*Log[c + d*x]*(A + B*Log[(e*(c + d*x)^2)/(a + b*x)^2]))/(b*(b*c - a*d)*g^2)
 - (A + B*Log[(e*(c + d*x)^2)/(a + b*x)^2])^2/(b*g^2*(a + b*x)) - (8*B^2*d*
PolyLog[2, -((d*(a + b*x))/(b*c - a*d))])/(b*(b*c - a*d)*g^2) - (8*B^2*d*Po
lyLog[2, (b*(c + d*x))/(b*c - a*d)])/(b*(b*c - a*d)*g^2)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 2301

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

Rule 2390

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_))^(q_.), x_Symbol] :> Dist[1/e, Subst[Int[((f*x)/d)^q*(a + b*Log[c*x^
n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && E
qQ[e*f - d*g, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2393

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + (c*e*x)/g])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2394

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d + e*x
)^n]))/g, x] - Dist[(b*e*n)/g, Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2418

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*(RFx_), x_Sy
mbol] :> With[{u = ExpandIntegrand[(a + b*Log[c*(d + e*x)^n])^p, RFx, x]},
Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d, e, n}, x] && RationalFunctionQ[
RFx, x] && IntegerQ[p]
```

Rule 2524

```
Int[((a_.) + Log[(c_.)*(RFx_)^(p_.)]*(b_.))^(n_.)/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[(Log[d + e*x]*(a + b*Log[c*RFx^p])^n)/e, x] - Dist[(b*n*p)/e
, Int[(Log[d + e*x]*(a + b*Log[c*RFx^p])^(n - 1)*D[RFx, x])/RFx, x], x] /;
FreeQ[{a, b, c, d, e, p}, x] && RationalFunctionQ[RFx, x] && IGtQ[n, 0]
```

Rule 2525

```
Int[((a_.) + Log[(c_.)*(RFx_)^(p_.)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_))^(m_.
), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(a + b*Log[c*RFx^p])^n)/(e*(m + 1))
, x] - Dist[(b*n*p)/(e*(m + 1)), Int[SimplifyIntegrand[((d + e*x)^(m + 1)*(
a + b*Log[c*RFx^p])^(n - 1)*D[RFx, x])/RFx, x], x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && RationalFunctionQ[RFx, x] && IGtQ[n, 0] && (EqQ[n, 1] ||
IntegerQ[m]) && NeQ[m, -1]
```

Rule 2528

```
Int[((a_.) + Log[(c_.)*(RFx_)^(p_.)]*(b_.))^(n_.)*(RGx_), x_Symbol] :> With
[{u = ExpandIntegrand[(a + b*Log[c*RFx^p])^n, RGx, x]}, Int[u, x] /; SumQ[u
]] /; FreeQ[{a, b, c, p}, x] && RationalFunctionQ[RFx, x] && RationalFuncti
onQ[RGx, x] && IGtQ[n, 0]
```

Rubi steps

$$\int \frac{\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2}{(ag+bgx)^2}\,dx = -\frac{\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2}{bg^2(a+bx)} + \frac{(2B)\int \frac{2(bc-ad)\left(-A-B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)}{g(a+bx)^2(c+dx)}\,dx}{bg}$$

$$= -\frac{\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2}{bg^2(a+bx)} + \frac{(4B(bc-ad))\int \frac{-A-B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)}{(a+bx)^2(c+dx)}\,dx}{bg^2}$$

$$= -\frac{\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2}{bg^2(a+bx)} + \frac{(4B(bc-ad))\int \left(\frac{b\left(-A-B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)}{(bc-ad)(a+bx)^2} - \frac{bd\left(-A-B\log\left(\frac{e(c+dx}{(a+bx}\right.\right.}{(bc-ad)^2(a+bx)}\right.}{bg^2}$$

$$= -\frac{\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2}{bg^2(a+bx)} + \frac{(4B)\int \frac{-A-B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)}{(a+bx)^2}\,dx}{g^2} - \frac{(4Bd)\int \frac{-A-B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)}{a+bx}}{(bc-ad)g^2}$$

$$= \frac{4B\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)}{bg^2(a+bx)} + \frac{4Bd\log(a+bx)\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)}{b(bc-ad)g^2} - \frac{4Bd\log(c}{}$$

$$= \frac{4B\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)}{bg^2(a+bx)} + \frac{4Bd\log(a+bx)\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)}{b(bc-ad)g^2} - \frac{4Bd\log(c}{}$$

$$= \frac{4B\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)}{bg^2(a+bx)} + \frac{4Bd\log(a+bx)\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)}{b(bc-ad)g^2} - \frac{4Bd\log(c}{}$$

$$= -\frac{8B^2}{bg^2(a+bx)} - \frac{8B^2d\log(a+bx)}{b(bc-ad)g^2} + \frac{8B^2d\log(c+dx)}{b(bc-ad)g^2} + \frac{4B\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right.}{bg^2(a+bx)}$$

$$= -\frac{8B^2}{bg^2(a+bx)} - \frac{8B^2d\log(a+bx)}{b(bc-ad)g^2} + \frac{8B^2d\log(c+dx)}{b(bc-ad)g^2} - \frac{8B^2d\log\left(-\frac{d(a+bx)}{bc-ad}\right)\log(}{b(bc-ad)g^2}$$

$$= -\frac{8B^2}{bg^2(a+bx)} - \frac{8B^2d\log(a+bx)}{b(bc-ad)g^2} + \frac{4B^2d\log^2(a+bx)}{b(bc-ad)g^2} + \frac{8B^2d\log(c+dx)}{b(bc-ad)g^2} - \frac{8B}{}$$

$$= -\frac{8B^2}{bg^2(a+bx)} - \frac{8B^2d\log(a+bx)}{b(bc-ad)g^2} + \frac{4B^2d\log^2(a+bx)}{b(bc-ad)g^2} + \frac{8B^2d\log(c+dx)}{b(bc-ad)g^2} - \frac{8B}{}$$

**Mathematica [C]** time = 0.46, size = 322, normalized size = 2.05

$$-\frac{4B\left(-(bc-ad)\left(B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)+A\right)-d(a+bx)\log(a+bx)\left(B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)+A\right)+d(a+bx)\log(c+dx)\left(B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)+A\right)-Bd(a+bx)\left(\log(a+bx)\left(\log(a+\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Log[(e*(c + d*x)^2)/(a + b*x)^2])^2/(a*g + b*g*x)^2,x]
```

```
[Out] -(((A + B*Log[(e*(c + d*x)^2)/(a + b*x)^2])^2 + (4*B*(2*B*(b*c - a*d + d*(a
 + b*x)*Log[a + b*x] - d*(a + b*x)*Log[c + d*x]) - (b*c - a*d)*(A + B*Log[(
e*(c + d*x)^2)/(a + b*x)^2]) - d*(a + b*x)*Log[a + b*x]*(A + B*Log[(e*(c +
d*x)^2)/(a + b*x)^2]) + d*(a + b*x)*Log[c + d*x]*(A + B*Log[(e*(c + d*x)^2)
/(a + b*x)^2]) - B*d*(a + b*x)*(Log[a + b*x]*(Log[a + b*x] - 2*Log[(b*(c +
d*x))/(b*c - a*d)]) - 2*PolyLog[2, (d*(a + b*x))/(-(b*c) + a*d)]) + B*d*(a
+ b*x)*((2*Log[(d*(a + b*x))/(-(b*c) + a*d)] - Log[c + d*x])*Log[c + d*x] +
 2*PolyLog[2, (b*(c + d*x))/(b*c - a*d)])))/(b*c - a*d))/(b*g^2*(a + b*x)))
```

**fricas [A]** time = 0.74, size = 200, normalized size = 1.27

$$-\frac{\left(A^2-4\,AB+8\,B^2\right)bc-\left(A^2-4\,AB+8\,B^2\right)ad+\left(B^2bdx+B^2bc\right)\log\left(\frac{d^2ex^2+2\,cdex+c^2e}{b^2x^2+2\,abx+a^2}\right)^2+2\left(\left(AB-2\,B^2\right)bdx+\right.}{\left(b^3c-ab^2d\right)g^2x+\left(ab^2c-a^2bd\right)g^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*(d*x+c)^2/(b*x+a)^2))^2/(b*g*x+a*g)^2,x, algorithm="fr
icas")
```

```
[Out] -((A^2 - 4*A*B + 8*B^2)*b*c - (A^2 - 4*A*B + 8*B^2)*a*d + (B^2*b*d*x + B^2*
b*c)*log((d^2*e*x^2 + 2*c*d*e*x + c^2*e)/(b^2*x^2 + 2*a*b*x + a^2))^2 + 2*(
(A*B - 2*B^2)*b*d*x + (A*B - 2*B^2)*b*c)*log((d^2*e*x^2 + 2*c*d*e*x + c^2*e
)/(b^2*x^2 + 2*a*b*x + a^2)))/((b^3*c - a*b^2*d)*g^2*x + (a*b^2*c - a^2*b*d
)*g^2)
```

**giac [B]** time = 1.28, size = 374, normalized size = 2.38

$$-\left(\frac{B^2d}{b^2cg^2-abdg^2}+\frac{B^2}{\left(bgx+ag\right)bg}\right)\log\left(\frac{\frac{b^2c^2g^2}{\left(bgx+ag\right)^2}-\frac{2\,abcdg^2}{\left(bgx+ag\right)^2}+\frac{a^2d^2g^2}{\left(bgx+ag\right)^2}+\frac{2\,bcdg}{bgx+ag}-\frac{2\,ad^2g}{bgx+ag}+d^2}{b^2}\right)^2-\frac{4\left(ABd-B^2d\right.}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*(d*x+c)^2/(b*x+a)^2))^2/(b*g*x+a*g)^2,x, algorithm="gi
ac")
```

```
[Out] -(B^2*d/(b^2*c*g^2 - a*b*d*g^2) + B^2/((b*g*x + a*g)*b*g))*log((b^2*c^2*g^2
/(b*g*x + a*g)^2 - 2*a*b*c*d*g^2/(b*g*x + a*g)^2 + a^2*d^2*g^2/(b*g*x + a*g
)^2 + 2*b*c*d*g/(b*g*x + a*g) - 2*a*d^2*g/(b*g*x + a*g) + d^2)/b^2)^2 - 4*(
A*B*d - B^2*d)*log(b*c*g/(b*g*x + a*g) - a*d*g/(b*g*x + a*g) + d)/(b^2*c*g^
2 - a*b*d*g^2) - 2*(A*B - B^2)*log((b^2*c^2*g^2/(b*g*x + a*g)^2 - 2*a*b*c*d
*g^2/(b*g*x + a*g)^2 + a^2*d^2*g^2/(b*g*x + a*g)^2 + 2*b*c*d*g/(b*g*x + a*g
) - 2*a*d^2*g/(b*g*x + a*g) + d^2)/b^2)/((b*g*x + a*g)*b*g) - (A^2 - 2*A*B
+ 5*B^2)/((b*g*x + a*g)*b*g)
```

**maple [B]** time = 0.06, size = 452, normalized size = 2.88

$$\frac{4ABa\,d^{2}\ln\left(\frac{ad}{bx+a}-\frac{bc}{bx+a}-d\right)}{(ad-bc)^{2}\,b\,g^{2}}-\frac{4ABcd\ln\left(\frac{ad}{bx+a}-\frac{bc}{bx+a}-d\right)}{(ad-bc)^{2}\,g^{2}}+\frac{B^{2}d\ln\left(\frac{\left(\frac{ad}{bx+a}-\frac{bc}{bx+a}-d\right)^{2}e}{b^{2}}\right)^{2}}{(ad-bc)\,b\,g^{2}}+\frac{4ABad}{(ad-bc)\,(bx+a)\,b\,g^{2}}-\cdots$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*ln((d*x+c)^2/(b*x+a)^2*e)+A)^2/(b*g*x+a*g)^2,x)
```

```
[Out] -1/b/g^2*A^2/(b*x+a)-1/b/g^2/(b*x+a)*B^2*ln((1/(b*x+a)*a*d-1/(b*x+a)*b*c-d)
^2/b^2*e)^2-8/b/g^2*B^2/(b*x+a)+4/b/g^2*B^2/(b*x+a)*ln((1/(b*x+a)*a*d-1/(b*
x+a)*b*c-d)^2/b^2*e)-4/b/g^2*B^2*d/(a*d-b*c)*ln((1/(b*x+a)*a*d-1/(b*x+a)*b*
c-d)^2/b^2*e)+1/b/g^2*B^2*d/(a*d-b*c)*ln((1/(b*x+a)*a*d-1/(b*x+a)*b*c-d)^2/
b^2*e)^2-2/b/g^2*A*B/(b*x+a)*ln((1/(b*x+a)*a*d-1/(b*x+a)*b*c-d)^2/b^2*e)+4/
b/g^2*A*B/(a*d-b*c)/(b*x+a)*a*d-4/g^2*A*B/(a*d-b*c)/(b*x+a)*c+4/b/g^2*A*B*d
^2/(a*d-b*c)^2*ln(1/(b*x+a)*a*d-1/(b*x+a)*b*c-d)*a-4/g^2*A*B*d/(a*d-b*c)^2*
ln(1/(b*x+a)*a*d-1/(b*x+a)*b*c-d)*c
```

**maxima [B]** time = 1.16, size = 573, normalized size = 3.65

$$4\left(\left(\left(\frac{1}{b^{2}g^{2}x+abg^{2}}+\frac{d\log(bx+a)}{\left(b^{2}c-abd\right)g^{2}}-\frac{d\log(dx+c)}{\left(b^{2}c-abd\right)g^{2}}\right)\log\left(\frac{d^{2}ex^{2}}{b^{2}x^{2}+2\,abx+a^{2}}+\frac{2\,cdex}{b^{2}x^{2}+2\,abx+a^{2}}+\frac{c^{2}e}{b^{2}x^{2}+2\,abx+\cdots}\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*(d*x+c)^2/(b*x+a)^2))^2/(b*g*x+a*g)^2,x, algorithm="ma
xima")
```

```
[Out] 4*((1/(b^2*g^2*x + a*b*g^2) + d*log(b*x + a)/((b^2*c - a*b*d)*g^2) - d*log(
d*x + c)/((b^2*c - a*b*d)*g^2))*log(d^2*e*x^2/(b^2*x^2 + 2*a*b*x + a^2) + 2
*c*d*e*x/(b^2*x^2 + 2*a*b*x + a^2) + c^2*e/(b^2*x^2 + 2*a*b*x + a^2)) + ((b
```

```
*d*x + a*d)*log(b*x + a)^2 + (b*d*x + a*d)*log(d*x + c)^2 - 2*b*c + 2*a*d -
 2*(b*d*x + a*d)*log(b*x + a) + 2*(b*d*x + a*d - (b*d*x + a*d)*log(b*x + a)
)*log(d*x + c))/(a*b^2*c*g^2 - a^2*b*d*g^2 + (b^3*c*g^2 - a*b^2*d*g^2)*x))*
B^2 - 2*A*B*(log(d^2*e*x^2/(b^2*x^2 + 2*a*b*x + a^2) + 2*c*d*e*x/(b^2*x^2 +
 2*a*b*x + a^2) + c^2*e/(b^2*x^2 + 2*a*b*x + a^2))/(b^2*g^2*x + a*b*g^2) -
2/(b^2*g^2*x + a*b*g^2) - 2*d*log(b*x + a)/((b^2*c - a*b*d)*g^2) + 2*d*log(
d*x + c)/((b^2*c - a*b*d)*g^2)) - B^2*log(d^2*e*x^2/(b^2*x^2 + 2*a*b*x + a^
2) + 2*c*d*e*x/(b^2*x^2 + 2*a*b*x + a^2) + c^2*e/(b^2*x^2 + 2*a*b*x + a^2))
^2/(b^2*g^2*x + a*b*g^2) - A^2/(b^2*g^2*x + a*b*g^2)
```

**mupad [B]** time = 6.49, size = 227, normalized size = 1.45

$$\frac{\ln\left(\frac{e\,(c+d\,x)^2}{(a+b\,x)^2}\right)\left(\frac{4\,B^2}{b^2\,d\,g^2}-\frac{2\,A\,B}{b^2\,d\,g^2}\right)}{\frac{x}{d}+\frac{a}{b\,d}}-\frac{A^2-4\,A\,B+8\,B^2}{x\,b^2\,g^2+a\,b\,g^2}-\ln\left(\frac{e\,(c+d\,x)^2}{(a+b\,x)^2}\right)^2\left(\frac{B^2}{b^2\,g^2\,\left(x+\frac{a}{b}\right)}-\frac{B^2\,d}{b\,g^2\,(a\,d-b\,c)}\right)+\frac{B\,d\,\mathrm{atan}\ldots}{\ldots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*log((e*(c + d*x)^2)/(a + b*x)^2))^2/(a*g + b*g*x)^2,x)
```

```
[Out] (log((e*(c + d*x)^2)/(a + b*x)^2)*((4*B^2)/(b^2*d*g^2) - (2*A*B)/(b^2*d*g^2
)))/(x/d + a/(b*d)) - (A^2 + 8*B^2 - 4*A*B)/(b^2*g^2*x + a*b*g^2) - log((e*
(c + d*x)^2)/(a + b*x)^2)^2*(B^2/(b^2*g^2*(x + a/b)) - (B^2*d)/(b*g^2*(a*d
- b*c))) + (B*d*atan(((2*b*d*x + (b^2*c*g^2 + a*b*d*g^2)/(b*g^2))*1i)/(a*d
- b*c))*(A - 2*B)*8i)/(b*g^2*(a*d - b*c))
```

**sympy [B]** time = 3.80, size = 450, normalized size = 2.87

$$\frac{4Bd\,(A-2B)\log\left(x+\frac{4ABad^2+4ABbcd-8B^2ad^2-8B^2bcd-\frac{4Ba^2d^3(A-2B)}{ad-bc}+\frac{8Babcd^2(A-2B)}{ad-bc}-\frac{4Bb^2c^2d(A-2B)}{ad-bc}}{8ABbd^2-16B^2bd^2}\right)}{bg^2\,(ad-bc)}-\frac{4Bd\,(A-2B)\log\left(x+\ldots\right)}{\ldots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*ln(e*(d*x+c)**2/(b*x+a)**2))**2/(b*g*x+a*g)**2,x)
```

```
[Out] 4*B*d*(A - 2*B)*log(x + (4*A*B*a*d**2 + 4*A*B*b*c*d - 8*B**2*a*d**2 - 8*B**
2*b*c*d - 4*B*a**2*d**3*(A - 2*B)/(a*d - b*c) + 8*B*a*b*c*d**2*(A - 2*B)/(a
*d - b*c) - 4*B*b**2*c**2*d*(A - 2*B)/(a*d - b*c))/(8*A*B*b*d**2 - 16*B**2*
b*d**2))/(b*g**2*(a*d - b*c)) - 4*B*d*(A - 2*B)*log(x + (4*A*B*a*d**2 + 4*A
*B*b*c*d - 8*B**2*a*d**2 - 8*B**2*b*c*d + 4*B*a**2*d**3*(A - 2*B)/(a*d - b*
c) - 8*B*a*b*c*d**2*(A - 2*B)/(a*d - b*c) + 4*B*b**2*c**2*d*(A - 2*B)/(a*d
- b*c))/(8*A*B*b*d**2 - 16*B**2*b*d**2))/(b*g**2*(a*d - b*c)) + (-2*A*B + 4
*B**2)*log(e*(c + d*x)**2/(a + b*x)**2)/(a*b*g**2 + b**2*g**2*x) + (B**2*c
```

```
+ B**2*d*x)*log(e*(c + d*x)**2/(a + b*x)**2)**2/(a**2*d*g**2 - a*b*c*g**2 +
 a*b*d*g**2*x - b**2*c*g**2*x) + (-A**2 + 4*A*B - 8*B**2)/(a*b*g**2 + b**2*
g**2*x)
```

## 3.216 $\int \frac{\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2}{(ag+bgx)^3}\,dx$

**Optimal**. Leaf size=299

$$\frac{bB(c+dx)^2\left(B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)+A\right)}{g^3(a+bx)^2(bc-ad)^2}-\frac{b(c+dx)^2\left(B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)+A\right)^2}{2g^3(a+bx)^2(bc-ad)^2}+\frac{d(c+dx)\left(B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)+A\right)^2}{g^3(a+bx)(bc-ad)^2}-\frac{4A}{g^3(a}$$

```
[Out] -4*A*B*d*(d*x+c)/(-a*d+b*c)^2/g^3/(b*x+a)+8*B^2*d*(d*x+c)/(-a*d+b*c)^2/g^3/
(b*x+a)-b*B^2*(d*x+c)^2/(-a*d+b*c)^2/g^3/(b*x+a)^2-4*B^2*d*(d*x+c)*ln(e*(d*
x+c)^2/(b*x+a)^2)/(-a*d+b*c)^2/g^3/(b*x+a)+b*B*(d*x+c)^2*(A+B*ln(e*(d*x+c)^
2/(b*x+a)^2))/(-a*d+b*c)^2/g^3/(b*x+a)^2+d*(d*x+c)*(A+B*ln(e*(d*x+c)^2/(b*x
+a)^2))^2/(-a*d+b*c)^2/g^3/(b*x+a)-1/2*b*(d*x+c)^2*(A+B*ln(e*(d*x+c)^2/(b*x
+a)^2))^2/(-a*d+b*c)^2/g^3/(b*x+a)^2
```

**Rubi [C]** time = 1.08, antiderivative size = 578, normalized size of antiderivative = 1.93, number of steps used = 30, number of rules used = 11, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.324, Rules used = {2525, 12, 2528, 44, 2524, 2418, 2390, 2301, 2394, 2393, 2391}

$$\frac{4B^2d^2\text{PolyLog}\left(2,-\frac{d(a+bx)}{bc-ad}\right)}{bg^3(bc-ad)^2}+\frac{4B^2d^2\text{PolyLog}\left(2,\frac{b(c+dx)}{bc-ad}\right)}{bg^3(bc-ad)^2}-\frac{2Bd^2\log(a+bx)\left(B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)+A\right)}{bg^3(bc-ad)^2}+\frac{2Bd^2\log(c}{}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Log[(e*(c + d*x)^2)/(a + b*x)^2])^2/(a*g + b*g*x)^3,x]
```

```
[Out] -(B^2/(b*g^3*(a + b*x)^2)) + (6*B^2*d)/(b*(b*c - a*d)*g^3*(a + b*x)) + (6*B
^2*d^2*Log[a + b*x])/(b*(b*c - a*d)^2*g^3) - (2*B^2*d^2*Log[a + b*x]^2)/(b*
(b*c - a*d)^2*g^3) - (6*B^2*d^2*Log[c + d*x])/(b*(b*c - a*d)^2*g^3) + (4*B^
2*d^2*Log[-((d*(a + b*x))/(b*c - a*d))]*Log[c + d*x])/(b*(b*c - a*d)^2*g^3)
 - (2*B^2*d^2*Log[c + d*x]^2)/(b*(b*c - a*d)^2*g^3) + (4*B^2*d^2*Log[a + b*
x]*Log[(b*(c + d*x))/(b*c - a*d)])/(b*(b*c - a*d)^2*g^3) + (B*(A + B*Log[(e
*(c + d*x)^2)/(a + b*x)^2]))/(b*g^3*(a + b*x)^2) - (2*B*d*(A + B*Log[(e*(c
+ d*x)^2)/(a + b*x)^2]))/(b*(b*c - a*d)*g^3*(a + b*x)) - (2*B*d^2*Log[a + b
*x]*(A + B*Log[(e*(c + d*x)^2)/(a + b*x)^2]))/(b*(b*c - a*d)^2*g^3) + (2*B*
d^2*Log[c + d*x]*(A + B*Log[(e*(c + d*x)^2)/(a + b*x)^2]))/(b*(b*c - a*d)^2
*g^3) - (A + B*Log[(e*(c + d*x)^2)/(a + b*x)^2])^2/(2*b*g^3*(a + b*x)^2) +
(4*B^2*d^2*PolyLog[2, -((d*(a + b*x))/(b*c - a*d))])/(b*(b*c - a*d)^2*g^3)
+ (4*B^2*d^2*PolyLog[2, (b*(c + d*x))/(b*c - a*d)])/(b*(b*c - a*d)^2*g^3)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 2301

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

Rule 2390

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_))^(q_.), x_Symbol] :> Dist[1/e, Subst[Int[((f*x)/d)^q*(a + b*Log[c*x^
n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && E
qQ[e*f - d*g, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2393

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + (c*e*x)/g])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2394

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d + e*x
)^n]))/g, x] - Dist[(b*e*n)/g, Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2418

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*(RFx_), x_Sy
mbol] :> With[{u = ExpandIntegrand[(a + b*Log[c*(d + e*x)^n])^p, RFx, x]},
```

```
Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d, e, n}, x] && RationalFunctionQ[
RFx, x] && IntegerQ[p]
```

Rule 2524

```
Int[((a_.) + Log[(c_.)*(RFx_)^(p_.)]*(b_.))^(n_.)/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[(Log[d + e*x]*(a + b*Log[c*RFx^p])^n)/e, x] - Dist[(b*n*p)/e
, Int[(Log[d + e*x]*(a + b*Log[c*RFx^p])^(n - 1)*D[RFx, x])/RFx, x], x] /;
FreeQ[{a, b, c, d, e, p}, x] && RationalFunctionQ[RFx, x] && IGtQ[n, 0]
```

Rule 2525

```
Int[((a_.) + Log[(c_.)*(RFx_)^(p_.)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_))^(m_.
), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(a + b*Log[c*RFx^p])^n)/(e*(m + 1))
, x] - Dist[(b*n*p)/(e*(m + 1)), Int[SimplifyIntegrand[((d + e*x)^(m + 1)*(
a + b*Log[c*RFx^p])^(n - 1)*D[RFx, x])/RFx, x], x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && RationalFunctionQ[RFx, x] && IGtQ[n, 0] && (EqQ[n, 1] ||
IntegerQ[m]) && NeQ[m, -1]
```

Rule 2528

```
Int[((a_.) + Log[(c_.)*(RFx_)^(p_.)]*(b_.))^(n_.)*(RGx_), x_Symbol] :> With
[{u = ExpandIntegrand[(a + b*Log[c*RFx^p])^n, RGx, x]}, Int[u, x] /; SumQ[u
]] /; FreeQ[{a, b, c, p}, x] && RationalFunctionQ[RFx, x] && RationalFuncti
onQ[RGx, x] && IGtQ[n, 0]
```

Rubi steps

$$\int \frac{\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2}{(ag+bgx)^3}\,dx = -\frac{\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2}{2bg^3(a+bx)^2} + \frac{B\int \frac{2(bc-ad)\left(-A-B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)}{g^2(a+bx)^3(c+dx)}\,dx}{bg}$$

$$= -\frac{\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2}{2bg^3(a+bx)^2} + \frac{(2B(bc-ad))\int \frac{-A-B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)}{(a+bx)^3(c+dx)}\,dx}{bg^3}$$

$$= -\frac{\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2}{2bg^3(a+bx)^2} + \frac{(2B(bc-ad))\int \left(\frac{b\left(-A-B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)}{(bc-ad)(a+bx)^3} - \frac{bd\left(-A-B\log\left(\frac{e(c+dx}{(a+bx}\right.\right.}{(bc-ad)^2(a+bx)^2}\right.}{bg}$$

$$= -\frac{\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2}{2bg^3(a+bx)^2} + \frac{(2B)\int \frac{-A-B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)}{(a+bx)^3}\,dx}{g^3} + \frac{\left(2Bd^2\right)\int \frac{-A-B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right.}{a+bx}}{(bc-ad)^2g^3}$$

$$= \frac{B\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)}{bg^3(a+bx)^2} - \frac{2Bd\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)}{b(bc-ad)g^3(a+bx)} - \frac{2Bd^2\log(a+bx)\left(A+B\right.}{b(bc-ad)^2g}$$

$$= \frac{B\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)}{bg^3(a+bx)^2} - \frac{2Bd\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)}{b(bc-ad)g^3(a+bx)} - \frac{2Bd^2\log(a+bx)\left(A+B\right.}{b(bc-ad)^2g}$$

$$= \frac{B\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)}{bg^3(a+bx)^2} - \frac{2Bd\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)}{b(bc-ad)g^3(a+bx)} - \frac{2Bd^2\log(a+bx)\left(A+B\right.}{b(bc-ad)^2g}$$

$$= -\frac{B^2}{bg^3(a+bx)^2} + \frac{6B^2d}{b(bc-ad)g^3(a+bx)} + \frac{6B^2d^2\log(a+bx)}{b(bc-ad)^2g^3} - \frac{6B^2d^2\log(c+dx)}{b(bc-ad)^2g^3}$$

$$= -\frac{B^2}{bg^3(a+bx)^2} + \frac{6B^2d}{b(bc-ad)g^3(a+bx)} + \frac{6B^2d^2\log(a+bx)}{b(bc-ad)^2g^3} - \frac{6B^2d^2\log(c+dx)}{b(bc-ad)^2g^3}$$

$$= -\frac{B^2}{bg^3(a+bx)^2} + \frac{6B^2d}{b(bc-ad)g^3(a+bx)} + \frac{6B^2d^2\log(a+bx)}{b(bc-ad)^2g^3} - \frac{2B^2d^2\log^2(a+bx)}{b(bc-ad)^2g^3}$$

$$= -\frac{B^2}{bg^3(a+bx)^2} + \frac{6B^2d}{b(bc-ad)g^3(a+bx)} + \frac{6B^2d^2\log(a+bx)}{b(bc-ad)^2g^3} - \frac{2B^2d^2\log^2(a+bx)}{b(bc-ad)^2g^3}$$

**Mathematica [C]** time = 0.46, size = 452, normalized size = 1.51

$$-\frac{\left(B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)+A\right)^2-\frac{2B\left(-2d^2(a+bx)^2\log(a+bx)\left(B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)+A\right)+2d^2(a+bx)^2\log(c+dx)\left(B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)+A\right)+(bc-ad)^2\left(B\log\left(\frac{e(}{(}\right.\right.\right.}{}}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Log[(e*(c + d*x)^2)/(a + b*x)^2])^2/(a*g + b*g*x)^3,x]
```

```
[Out] -1/2*((A + B*Log[(e*(c + d*x)^2)/(a + b*x)^2])^2 - (2*B*(4*B*d*(a + b*x)*(b
*c - a*d + d*(a + b*x)*Log[a + b*x] - d*(a + b*x)*Log[c + d*x]) - B*((b*c -
 a*d)^2 + 2*d*(-(b*c) + a*d)*(a + b*x) - 2*d^2*(a + b*x)^2*Log[a + b*x] + 2
*d^2*(a + b*x)^2*Log[c + d*x]) + (b*c - a*d)^2*(A + B*Log[(e*(c + d*x)^2)/(
a + b*x)^2]) + 2*d*(-(b*c) + a*d)*(a + b*x)*(A + B*Log[(e*(c + d*x)^2)/(a +
 b*x)^2]) - 2*d^2*(a + b*x)^2*Log[a + b*x]*(A + B*Log[(e*(c + d*x)^2)/(a +
b*x)^2]) + 2*d^2*(a + b*x)^2*Log[c + d*x]*(A + B*Log[(e*(c + d*x)^2)/(a + b
*x)^2]) - 2*B*d^2*(a + b*x)^2*(Log[a + b*x]*(Log[a + b*x] - 2*Log[(b*(c + d
*x))/(b*c - a*d)]) - 2*PolyLog[2, (d*(a + b*x))/(-(b*c) + a*d)]) + 2*B*d^2*
(a + b*x)^2*((2*Log[(d*(a + b*x))/(-(b*c) + a*d)] - Log[c + d*x])*Log[c + d
*x] + 2*PolyLog[2, (b*(c + d*x))/(b*c - a*d)])))/(b*c - a*d)^2)/(b*g^3*(a +
 b*x)^2)
```

**fricas [A]** time = 1.04, size = 413, normalized size = 1.38

$$-\frac{\left(A^2-2AB+2B^2\right)b^2c^2-2\left(A^2-4AB+8B^2\right)abcd+\left(A^2-6AB+14B^2\right)a^2d^2-\left(B^2b^2d^2x^2+2B^2abd^2x-B\right.}{2\left(\left(b^5\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*(d*x+c)^2/(b*x+a)^2))^2/(b*g*x+a*g)^3,x, algorithm="fr
icas")
```

```
[Out] -1/2*((A^2 - 2*A*B + 2*B^2)*b^2*c^2 - 2*(A^2 - 4*A*B + 8*B^2)*a*b*c*d + (A^
2 - 6*A*B + 14*B^2)*a^2*d^2 - (B^2*b^2*d^2*x^2 + 2*B^2*a*b*d^2*x - B^2*b^2*
c^2 + 2*B^2*a*b*c*d)*log((d^2*e*x^2 + 2*c*d*e*x + c^2*e)/(b^2*x^2 + 2*a*b*x
 + a^2))^2 + 4*((A*B - 3*B^2)*b^2*c*d - (A*B - 3*B^2)*a*b*d^2)*x - 2*((A*B
- 3*B^2)*b^2*d^2*x^2 - (A*B - B^2)*b^2*c^2 + 2*(A*B - 2*B^2)*a*b*c*d - 2*(B
^2*b^2*c*d - (A*B - 2*B^2)*a*b*d^2)*x)*log((d^2*e*x^2 + 2*c*d*e*x + c^2*e)/
(b^2*x^2 + 2*a*b*x + a^2)))/((b^5*c^2 - 2*a*b^4*c*d + a^2*b^3*d^2)*g^3*x^2
+ 2*(a*b^4*c^2 - 2*a^2*b^3*c*d + a^3*b^2*d^2)*g^3*x + (a^2*b^3*c^2 - 2*a^3*
b^2*c*d + a^4*b*d^2)*g^3)
```

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(B\log\left(\frac{(dx+c)^2e}{(bx+a)^2}\right)+A\right)^2}{\left(bgx+ag\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*(d*x+c)^2/(b*x+a)^2))^2/(b*g*x+a*g)^3,x, algorithm="gi
ac")
```

```
[Out] integrate((B*log((d*x + c)^2*e/(b*x + a)^2) + A)^2/(b*g*x + a*g)^3, x)
```

**maple [B]** time = 0.06, size = 664, normalized size = 2.22

$$\frac{2ABa\,d^3\ln\left(\frac{ad}{bx+a}-\frac{bc}{bx+a}-d\right)}{(ad-bc)^3\,b\,g^3}-\frac{2ABc\,d^2\ln\left(\frac{ad}{bx+a}-\frac{bc}{bx+a}-d\right)}{(ad-bc)^3\,g^3}+\frac{AB\,a^2d^2}{(ad-bc)^2\,(bx+a)^2\,b\,g^3}-\frac{2ABacd}{(ad-bc)^2\,(bx+a)^2\,g^3}+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*ln((d*x+c)^2/(b*x+a)^2*e)+A)^2/(b*g*x+a*g)^3,x)
```

```
[Out] -1/2/b/(b*x+a)^2/g^3*A^2-1/b/g^3*B^2/(b*x+a)^2+1/b/g^3*B^2/(b*x+a)^2*ln((1/
(b*x+a)*a*d-1/(b*x+a)*b*c-d)^2/b^2*e)-1/2/b/g^3*B^2/(b*x+a)^2*ln((1/(b*x+a)
*a*d-1/(b*x+a)*b*c-d)^2/b^2*e)^2-6/b/g^3*B^2*d/(a*d-b*c)/(b*x+a)-3/b/g^3*B^
2*d^2/(a^2*d^2-2*a*b*c*d+b^2*c^2)*ln((1/(b*x+a)*a*d-1/(b*x+a)*b*c-d)^2/b^2*
e)+1/2/b/g^3*B^2*d^2/(a^2*d^2-2*a*b*c*d+b^2*c^2)*ln((1/(b*x+a)*a*d-1/(b*x+a
)*b*c-d)^2/b^2*e)^2+2/b/g^3*B^2*d/(a*d-b*c)/(b*x+a)*ln((1/(b*x+a)*a*d-1/(b*
x+a)*b*c-d)^2/b^2*e)-1/b/g^3*A*B/(b*x+a)^2*ln((1/(b*x+a)*a*d-1/(b*x+a)*b*c-
d)^2/b^2*e)+1/b/g^3*A*B/(a*d-b*c)^2/(b*x+a)^2*a^2*d^2-2/g^3*A*B/(a*d-b*c)^2
/(b*x+a)^2*a*d*c+b/g^3*A*B/(a*d-b*c)^2/(b*x+a)^2*c^2+2/b/g^3*A*B/(a*d-b*c)^
2/(b*x+a)*d^2*a-2/g^3*A*B/(a*d-b*c)^2/(b*x+a)*d*c+2/b/g^3*A*B*d^3/(a*d-b*c)
^3*ln(1/(b*x+a)*a*d-1/(b*x+a)*b*c-d)*a-2/g^3*A*B*d^2/(a*d-b*c)^3*ln(1/(b*x+
a)*a*d-1/(b*x+a)*b*c-d)*c
```

**maxima [B]** time = 1.48, size = 1001, normalized size = 3.35

$$-\left(\left(\left(\frac{2\,bdx-bc+3\,ad}{\left(b^4c-ab^3d\right)g^3x^2+2\left(ab^3c-a^2b^2d\right)g^3x+\left(a^2b^2c-a^3bd\right)g^3}+\frac{2\,d^2\log\left(bx+a\right)}{\left(b^3c^2-2\,ab^2cd+a^2bd^2\right)g^3}-\frac{2\,d^2\log\left(dx\right.}{\left(b^3c^2-2\,ab^2cd+\right.}\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*(d*x+c)^2/(b*x+a)^2))^2/(b*g*x+a*g)^3,x, algorithm="ma
xima")
```

```
[Out] -((((2*b*d*x - b*c + 3*a*d)/((b^4*c - a*b^3*d)*g^3*x^2 + 2*(a*b^3*c - a^2*b^
2*d)*g^3*x + (a^2*b^2*c - a^3*b*d)*g^3) + 2*d^2*log(b*x + a)/((b^3*c^2 - 2*
a*b^2*c*d + a^2*b*d^2)*g^3) - 2*d^2*log(d*x + c)/((b^3*c^2 - 2*a*b^2*c*d +
a^2*b*d^2)*g^3))*log(d^2*e*x^2/(b^2*x^2 + 2*a*b*x + a^2) + 2*c*d*e*x/(b^2*x
^2 + 2*a*b*x + a^2) + c^2*e/(b^2*x^2 + 2*a*b*x + a^2)) + (b^2*c^2 - 8*a*b*c
*d + 7*a^2*d^2 + 2*(b^2*d^2*x^2 + 2*a*b*d^2*x + a^2*d^2)*log(b*x + a)^2 + 2
*(b^2*d^2*x^2 + 2*a*b*d^2*x + a^2*d^2)*log(d*x + c)^2 - 6*(b^2*c*d - a*b*d^
2)*x - 6*(b^2*d^2*x^2 + 2*a*b*d^2*x + a^2*d^2)*log(b*x + a) + 2*(3*b^2*d^2*
x^2 + 6*a*b*d^2*x + 3*a^2*d^2 - 2*(b^2*d^2*x^2 + 2*a*b*d^2*x + a^2*d^2)*log
(b*x + a))*log(d*x + c))/(a^2*b^3*c^2*g^3 - 2*a^3*b^2*c*d*g^3 + a^4*b*d^2*g
^3 + (b^5*c^2*g^3 - 2*a*b^4*c*d*g^3 + a^2*b^3*d^2*g^3)*x^2 + 2*(a*b^4*c^2*g
^3 - 2*a^2*b^3*c*d*g^3 + a^3*b^2*d^2*g^3)*x))*B^2 - A*B*((2*b*d*x - b*c + 3
*a*d)/((b^4*c - a*b^3*d)*g^3*x^2 + 2*(a*b^3*c - a^2*b^2*d)*g^3*x + (a^2*b^2
*c - a^3*b*d)*g^3) + log(d^2*e*x^2/(b^2*x^2 + 2*a*b*x + a^2) + 2*c*d*e*x/(b
^2*x^2 + 2*a*b*x + a^2) + c^2*e/(b^2*x^2 + 2*a*b*x + a^2))/(b^3*g^3*x^2 + 2
*a*b^2*g^3*x + a^2*b*g^3) + 2*d^2*log(b*x + a)/((b^3*c^2 - 2*a*b^2*c*d + a^
2*b*d^2)*g^3) - 2*d^2*log(d*x + c)/((b^3*c^2 - 2*a*b^2*c*d + a^2*b*d^2)*g^3
)) - 1/2*B^2*log(d^2*e*x^2/(b^2*x^2 + 2*a*b*x + a^2) + 2*c*d*e*x/(b^2*x^2 +
 2*a*b*x + a^2) + c^2*e/(b^2*x^2 + 2*a*b*x + a^2))^2/(b^3*g^3*x^2 + 2*a*b^2
*g^3*x + a^2*b*g^3) - 1/2*A^2/(b^3*g^3*x^2 + 2*a*b^2*g^3*x + a^2*b*g^3)
```

**mupad [B]** time = 6.71, size = 504, normalized size = 1.69

$$\frac{\ln\left(\frac{e\,(c+d\,x)^2}{(a+b\,x)^2}\right)\left(\frac{2\,B^2\,x\,(a\,d-b\,c)}{b\,g^3\,\left(a^2\,d^2-2\,a\,b\,c\,d+b^2\,c^2\right)}-\frac{A\,B}{b^2\,d\,g^3}+\frac{B^2\,d^2\left(\frac{2\,a^2\,d^2-3\,a\,b\,c\,d+b^2\,c^2}{b\,d^3}+\frac{a\,(a\,d-b\,c)}{b\,d^2}\right)}{b\,g^3\,\left(a^2\,d^2-2\,a\,b\,c\,d+b^2\,c^2\right)}\right)}{\frac{b\,x^2}{d}+\frac{a^2}{b\,d}+\frac{2\,a\,x}{d}}-\ln\left(\frac{e\,(c+d\,x)^2}{(a+b\,x)^2}\right)^2\left(\frac{B^2}{2\,b^2\,g^3\,\left(2\,a\,x+\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*log((e*(c + d*x)^2)/(a + b*x)^2))^2/(a*g + b*g*x)^3,x)
```

```
[Out] (log((e*(c + d*x)^2)/(a + b*x)^2)*((2*B^2*x*(a*d - b*c))/(b*g^3*(a^2*d^2 +
b^2*c^2 - 2*a*b*c*d)) - (A*B)/(b^2*d*g^3) + (B^2*d^2*((2*a^2*d^2 + b^2*c^2
- 3*a*b*c*d)/(b*d^3) + (a*(a*d - b*c))/(b*d^2)))/(b*g^3*(a^2*d^2 + b^2*c^2
- 2*a*b*c*d))))/((b*x^2)/d + a^2/(b*d) + (2*a*x)/d) - log((e*(c + d*x)^2)/(
a + b*x)^2)^2*(B^2/(2*b^2*g^3*(2*a*x + b*x^2 + a^2/b)) - (B^2*d^2)/(2*b*g^3
*(a^2*d^2 + b^2*c^2 - 2*a*b*c*d))) - ((A^2*a*d - A^2*b*c + 14*B^2*a*d - 2*B
^2*b*c - 6*A*B*a*d + 2*A*B*b*c)/(2*(a*d - b*c)) + (2*x*(3*B^2*b*d - A*B*b*d
))/(a*d - b*c))/(a^2*b*g^3 + b^3*g^3*x^2 + 2*a*b^2*g^3*x) - (B*d^2*atan((B*
d^2*(2*b*d*x - (b^3*c^2*g^3 - a^2*b*d^2*g^3)/(b*g^3*(a*d - b*c)))*(A - 3*B)
```

```
*2i)/((a*d - b*c)*(6*B^2*d^2 - 2*A*B*d^2)))*(A - 3*B)*4i)/(b*g^3*(a*d - b*c
)^2)
```

**sympy [B]**   time = 6.65, size = 877, normalized size = 2.93

$$\frac{2Bd^2\left(A-3B\right)\log\left(x+\frac{2ABad^3+2ABbcd^2-6B^2ad^3-6B^2bcd^2-\frac{2Ba^3d^5(A-3B)}{(ad-bc)^2}+\frac{6Ba^2bcd^4(A-3B)}{(ad-bc)^2}-\frac{6Bab^2c^2d^3(A-3B)}{(ad-bc)^2}+\frac{2Bb^3c^3d^2(A-3B)}{(ad-bc)^2}}{4ABbd^3-12B^2bd^3}\right)}{bg^3\left(ad-bc\right)^2}-\frac{2Bd^2\left(A\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*ln(e*(d*x+c)**2/(b*x+a)**2))**2/(b*g*x+a*g)**3,x)
```

```
[Out] 2*B*d**2*(A - 3*B)*log(x + (2*A*B*a*d**3 + 2*A*B*b*c*d**2 - 6*B**2*a*d**3 -
 6*B**2*b*c*d**2 - 2*B*a**3*d**5*(A - 3*B)/(a*d - b*c)**2 + 6*B*a**2*b*c*d*
*4*(A - 3*B)/(a*d - b*c)**2 - 6*B*a*b**2*c**2*d**3*(A - 3*B)/(a*d - b*c)**2
 + 2*B*b**3*c**3*d**2*(A - 3*B)/(a*d - b*c)**2)/(4*A*B*b*d**3 - 12*B**2*b*d
**3))/(b*g**3*(a*d - b*c)**2) - 2*B*d**2*(A - 3*B)*log(x + (2*A*B*a*d**3 +
2*A*B*b*c*d**2 - 6*B**2*a*d**3 - 6*B**2*b*c*d**2 + 2*B*a**3*d**5*(A - 3*B)/
(a*d - b*c)**2 - 6*B*a**2*b*c*d**4*(A - 3*B)/(a*d - b*c)**2 + 6*B*a*b**2*c*
*2*d**3*(A - 3*B)/(a*d - b*c)**2 - 2*B*b**3*c**3*d**2*(A - 3*B)/(a*d - b*c)
**2)/(4*A*B*b*d**3 - 12*B**2*b*d**3))/(b*g**3*(a*d - b*c)**2) + (2*B**2*a*c
*d + 2*B**2*a*d**2*x - B**2*b*c**2 + B**2*b*d**2*x**2)*log(e*(c + d*x)**2/(
a + b*x)**2)**2/(2*a**4*d**2*g**3 - 4*a**3*b*c*d*g**3 + 4*a**3*b*d**2*g**3*
x + 2*a**2*b**2*c**2*g**3 - 8*a**2*b**2*c*d*g**3*x + 2*a**2*b**2*d**2*g**3*
x**2 + 4*a*b**3*c**2*g**3*x - 4*a*b**3*c*d*g**3*x**2 + 2*b**4*c**2*g**3*x**
2) + (-A*B*a*d + A*B*b*c + 3*B**2*a*d - B**2*b*c + 2*B**2*b*d*x)*log(e*(c +
 d*x)**2/(a + b*x)**2)/(a**3*b*d*g**3 - a**2*b**2*c*g**3 + 2*a**2*b**2*d*g*
*3*x - 2*a*b**3*c*g**3*x + a*b**3*d*g**3*x**2 - b**4*c*g**3*x**2) + (-A**2*
a*d + A**2*b*c + 6*A*B*a*d - 2*A*B*b*c - 14*B**2*a*d + 2*B**2*b*c + x*(4*A*
B*b*d - 12*B**2*b*d))/(2*a**3*b*d*g**3 - 2*a**2*b**2*c*g**3 + x**2*(2*a*b**
3*d*g**3 - 2*b**4*c*g**3) + x*(4*a**2*b**2*d*g**3 - 4*a*b**3*c*g**3))
```

# 3.217 $\int \frac{\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2}{(ag+bgx)^4}\,dx$

**Optimal**. Leaf size=407

$$\frac{4b^2B(c+dx)^3\left(B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)+A\right)}{9g^4(a+bx)^3(bc-ad)^3}-\frac{4Bd^3\log\left(\frac{c+dx}{a+bx}\right)\left(B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)+A\right)}{3bg^4(bc-ad)^3}+\frac{4Bd^2(c+dx)\left(B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)+A\right)}{g^4(a+bx)(bc-ad)^3}$$

```
[Out] -8*B^2*d^2*(d*x+c)/(-a*d+b*c)^3/g^4/(b*x+a)+2*b*B^2*d*(d*x+c)^2/(-a*d+b*c)^
3/g^4/(b*x+a)^2-8/27*b^2*B^2*(d*x+c)^3/(-a*d+b*c)^3/g^4/(b*x+a)^3+4/3*B^2*d
^3*ln((d*x+c)/(b*x+a))^2/b/(-a*d+b*c)^3/g^4+4*B*d^2*(d*x+c)*(A+B*ln(e*(d*x+
c)^2/(b*x+a)^2))/(-a*d+b*c)^3/g^4/(b*x+a)-2*b*B*d*(d*x+c)^2*(A+B*ln(e*(d*x+
c)^2/(b*x+a)^2))/(-a*d+b*c)^3/g^4/(b*x+a)^2+4/9*b^2*B*(d*x+c)^3*(A+B*ln(e*(
d*x+c)^2/(b*x+a)^2))/(-a*d+b*c)^3/g^4/(b*x+a)^3-4/3*B*d^3*ln((d*x+c)/(b*x+a
))*(A+B*ln(e*(d*x+c)^2/(b*x+a)^2))/b/(-a*d+b*c)^3/g^4-1/3*(A+B*ln(e*(d*x+c)
^2/(b*x+a)^2))^2/b/g^4/(b*x+a)^3
```

**Rubi [C]** time = 1.23, antiderivative size = 692, normalized size of antiderivative = 1.70, number of steps used = 34, number of rules used = 11, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.324, Rules used = {2525, 12, 2528, 44, 2524, 2418, 2390, 2301, 2394, 2393, 2391}

$$-\frac{8B^2d^3\text{PolyLog}\left(2,-\frac{d(a+bx)}{bc-ad}\right)}{3bg^4(bc-ad)^3}-\frac{8B^2d^3\text{PolyLog}\left(2,\frac{b(c+dx)}{bc-ad}\right)}{3bg^4(bc-ad)^3}+\frac{4Bd^3\log(a+bx)\left(B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)+A\right)}{3bg^4(bc-ad)^3}-\frac{4Bd^3\log(}{}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Log[(e*(c + d*x)^2)/(a + b*x)^2])^2/(a*g + b*g*x)^4,x]
```

```
[Out] (-8*B^2)/(27*b*g^4*(a + b*x)^3) + (10*B^2*d)/(9*b*(b*c - a*d)*g^4*(a + b*x)
^2) - (44*B^2*d^2)/(9*b*(b*c - a*d)^2*g^4*(a + b*x)) - (44*B^2*d^3*Log[a +
b*x])/(9*b*(b*c - a*d)^3*g^4) + (4*B^2*d^3*Log[a + b*x]^2)/(3*b*(b*c - a*d)
^3*g^4) + (44*B^2*d^3*Log[c + d*x])/(9*b*(b*c - a*d)^3*g^4) - (8*B^2*d^3*Lo
g[-((d*(a + b*x))/(b*c - a*d))]*Log[c + d*x])/(3*b*(b*c - a*d)^3*g^4) + (4*
B^2*d^3*Log[c + d*x]^2)/(3*b*(b*c - a*d)^3*g^4) - (8*B^2*d^3*Log[a + b*x]*L
og[(b*(c + d*x))/(b*c - a*d)])/(3*b*(b*c - a*d)^3*g^4) + (4*B*(A + B*Log[(e
*(c + d*x)^2)/(a + b*x)^2]))/(9*b*g^4*(a + b*x)^3) - (2*B*d*(A + B*Log[(e*(
c + d*x)^2)/(a + b*x)^2]))/(3*b*(b*c - a*d)*g^4*(a + b*x)^2) + (4*B*d^2*(A
+ B*Log[(e*(c + d*x)^2)/(a + b*x)^2]))/(3*b*(b*c - a*d)^2*g^4*(a + b*x)) +
(4*B*d^3*Log[a + b*x]*(A + B*Log[(e*(c + d*x)^2)/(a + b*x)^2]))/(3*b*(b*c -
 a*d)^3*g^4) - (4*B*d^3*Log[c + d*x]*(A + B*Log[(e*(c + d*x)^2)/(a + b*x)^2
]))/(3*b*(b*c - a*d)^3*g^4) - (A + B*Log[(e*(c + d*x)^2)/(a + b*x)^2])^2/(3
```

```
*b*g^4*(a + b*x)^3) - (8*B^2*d^3*PolyLog[2, -((d*(a + b*x))/(b*c - a*d))])/
(3*b*(b*c - a*d)^3*g^4) - (8*B^2*d^3*PolyLog[2, (b*(c + d*x))/(b*c - a*d)])
/(3*b*(b*c - a*d)^3*g^4)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 2301

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

Rule 2390

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_))^(q_.), x_Symbol] :> Dist[1/e, Subst[Int[((f*x)/d)^q*(a + b*Log[c*x^
n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && E
qQ[e*f - d*g, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2393

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + (c*e*x)/g])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2394

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d + e*x
)^n]))/g, x] - Dist[(b*e*n)/g, Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
```

```
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2418

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*(RFx_), x_Sy
mbol] :> With[{u = ExpandIntegrand[(a + b*Log[c*(d + e*x)^n])^p, RFx, x]},
Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d, e, n}, x] && RationalFunctionQ[
RFx, x] && IntegerQ[p]
```

Rule 2524

```
Int[((a_.) + Log[(c_.)*(RFx_)^(p_.)]*(b_.))^(n_.)/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[(Log[d + e*x]*(a + b*Log[c*RFx^p])^n)/e, x] - Dist[(b*n*p)/e
, Int[(Log[d + e*x]*(a + b*Log[c*RFx^p])^(n - 1)*D[RFx, x])/RFx, x], x] /;
FreeQ[{a, b, c, d, e, p}, x] && RationalFunctionQ[RFx, x] && IGtQ[n, 0]
```

Rule 2525

```
Int[((a_.) + Log[(c_.)*(RFx_)^(p_.)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_))^(m_.
), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(a + b*Log[c*RFx^p])^n)/(e*(m + 1))
, x] - Dist[(b*n*p)/(e*(m + 1)), Int[SimplifyIntegrand[((d + e*x)^(m + 1)*(
a + b*Log[c*RFx^p])^(n - 1)*D[RFx, x])/RFx, x], x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && RationalFunctionQ[RFx, x] && IGtQ[n, 0] && (EqQ[n, 1] ||
IntegerQ[m]) && NeQ[m, -1]
```

Rule 2528

```
Int[((a_.) + Log[(c_.)*(RFx_)^(p_.)]*(b_.))^(n_.)*(RGx_), x_Symbol] :> With
[{u = ExpandIntegrand[(a + b*Log[c*RFx^p])^n, RGx, x]}, Int[u, x] /; SumQ[u
]] /; FreeQ[{a, b, c, p}, x] && RationalFunctionQ[RFx, x] && RationalFuncti
onQ[RGx, x] && IGtQ[n, 0]
```

Rubi steps

$$\int \frac{\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2}{(ag+bgx)^4}dx = -\frac{\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2}{3bg^4(a+bx)^3} + \frac{(2B)\int \frac{2(bc-ad)\left(-A-B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)}{g^3(a+bx)^4(c+dx)}dx}{3bg}$$

$$= -\frac{\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2}{3bg^4(a+bx)^3} + \frac{(4B(bc-ad))\int \frac{-A-B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)}{(a+bx)^4(c+dx)}dx}{3bg^4}$$

$$= -\frac{\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2}{3bg^4(a+bx)^3} + \frac{(4B(bc-ad))\int \left(\frac{b\left(-A-B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)}{(bc-ad)(a+bx)^4} - \frac{bd\left(-A-B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)}{(bc-ad)^2(a+bx)^3} \cdots\right.}{\cdots}$$

$$= -\frac{\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2}{3bg^4(a+bx)^3} + \frac{(4B)\int \frac{-A-B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)}{(a+bx)^4}dx}{3g^4} - \frac{\left(4Bd^3\right)\int \frac{-A-B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)}{a+bx} \cdots}{3(bc-ad)^3g^4}$$

$$= \frac{4B\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)}{9bg^4(a+bx)^3} - \frac{2Bd\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)}{3b(bc-ad)g^4(a+bx)^2} + \frac{4Bd^2\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)}{3b(bc-ad)^2g^4(a+bx)} \cdots$$

$$= \frac{4B\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)}{9bg^4(a+bx)^3} - \frac{2Bd\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)}{3b(bc-ad)g^4(a+bx)^2} + \frac{4Bd^2\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)}{3b(bc-ad)^2g^4(a+bx)} \cdots$$

$$= \frac{4B\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)}{9bg^4(a+bx)^3} - \frac{2Bd\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)}{3b(bc-ad)g^4(a+bx)^2} + \frac{4Bd^2\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)}{3b(bc-ad)^2g^4(a+bx)} \cdots$$

$$= -\frac{8B^2}{27bg^4(a+bx)^3} + \frac{10B^2d}{9b(bc-ad)g^4(a+bx)^2} - \frac{44B^2d^2}{9b(bc-ad)^2g^4(a+bx)} - \frac{44B^2d^3\log\cdots}{9b(bc-\cdots}$$

$$= -\frac{8B^2}{27bg^4(a+bx)^3} + \frac{10B^2d}{9b(bc-ad)g^4(a+bx)^2} - \frac{44B^2d^2}{9b(bc-ad)^2g^4(a+bx)} - \frac{44B^2d^3\log\cdots}{9b(bc-\cdots}$$

$$= -\frac{8B^2}{27bg^4(a+bx)^3} + \frac{10B^2d}{9b(bc-ad)g^4(a+bx)^2} - \frac{44B^2d^2}{9b(bc-ad)^2g^4(a+bx)} - \frac{44B^2d^3\log\cdots}{9b(bc-\cdots}$$

$$= -\frac{8B^2}{27bg^4(a+bx)^3} + \frac{10B^2d}{9b(bc-ad)g^4(a+bx)^2} - \frac{44B^2d^2}{9b(bc-ad)^2g^4(a+bx)} - \frac{44B^2d^3\log\cdots}{9b(bc-\cdots}$$

**Mathematica [C]** time = 0.76, size = 598, normalized size = 1.47

$$-\frac{2B\Big(-18d^3(a+bx)^3\log(a+bx)\Big(B\log\Big(\frac{e(c+dx)^2}{(a+bx)^2}\Big)+A\Big)+18d^3(a+bx)^3\log(c+dx)\Big(B\log\Big(\frac{e(c+dx)^2}{(a+bx)^2}\Big)+A\Big)+18d^2(a+bx)^2(ad-bc)\Big(B\log\Big(\frac{e(c+dx)^2}{(a+bx)^2}\Big)+A\Big)-6(bc-a}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Log[(e*(c + d*x)^2)/(a + b*x)^2])^2/(a*g + b*g*x)^4,x]
```

```
[Out] -1/27*(9*(A + B*Log[(e*(c + d*x)^2)/(a + b*x)^2])^2 + (2*B*(36*B*d^2*(a + b
*x)^2*(b*c - a*d + d*(a + b*x)*Log[a + b*x] - d*(a + b*x)*Log[c + d*x]) - 9
*B*d*(a + b*x)*((b*c - a*d)^2 + 2*d*(-(b*c) + a*d)*(a + b*x) - 2*d^2*(a + b
*x)^2*Log[a + b*x] + 2*d^2*(a + b*x)^2*Log[c + d*x]) + 2*B*(2*(b*c - a*d)^3
 - 3*d*(b*c - a*d)^2*(a + b*x) + 6*d^2*(b*c - a*d)*(a + b*x)^2 + 6*d^3*(a +
 b*x)^3*Log[a + b*x] - 6*d^3*(a + b*x)^3*Log[c + d*x]) - 6*(b*c - a*d)^3*(A
 + B*Log[(e*(c + d*x)^2)/(a + b*x)^2]) + 9*d*(b*c - a*d)^2*(a + b*x)*(A + B
*Log[(e*(c + d*x)^2)/(a + b*x)^2]) + 18*d^2*(-(b*c) + a*d)*(a + b*x)^2*(A +
 B*Log[(e*(c + d*x)^2)/(a + b*x)^2]) - 18*d^3*(a + b*x)^3*Log[a + b*x]*(A +
 B*Log[(e*(c + d*x)^2)/(a + b*x)^2]) + 18*d^3*(a + b*x)^3*Log[c + d*x]*(A +
 B*Log[(e*(c + d*x)^2)/(a + b*x)^2]) - 18*B*d^3*(a + b*x)^3*(Log[a + b*x]*(
Log[a + b*x] - 2*Log[(b*(c + d*x))/(b*c - a*d)]) - 2*PolyLog[2, (d*(a + b*x
))/(-(b*c) + a*d)]) + 18*B*d^3*(a + b*x)^3*((2*Log[(d*(a + b*x))/(-(b*c) +
a*d)] - Log[c + d*x])*Log[c + d*x] + 2*PolyLog[2, (b*(c + d*x))/(b*c - a*d)
])))/(b*c - a*d)^3)/(b*g^4*(a + b*x)^3)
```

**fricas [A]** time = 0.67, size = 721, normalized size = 1.77

$$-\frac{\big(9A^2-12AB+8B^2\big)b^3c^3-27\big(A^2-2AB+2B^2\big)ab^2c^2d+27\big(A^2-4AB+8B^2\big)a^2bcd^2-\big(9A^2-66AB+}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*(d*x+c)^2/(b*x+a)^2))^2/(b*g*x+a*g)^4,x, algorithm="fr
icas")
```

```
[Out] -1/27*((9*A^2 - 12*A*B + 8*B^2)*b^3*c^3 - 27*(A^2 - 2*A*B + 2*B^2)*a*b^2*c^
2*d + 27*(A^2 - 4*A*B + 8*B^2)*a^2*b*c*d^2 - (9*A^2 - 66*A*B + 170*B^2)*a^3
*d^3 - 12*((3*A*B - 11*B^2)*b^3*c*d^2 - (3*A*B - 11*B^2)*a*b^2*d^3)*x^2 + 9
*(B^2*b^3*d^3*x^3 + 3*B^2*a*b^2*d^3*x^2 + 3*B^2*a^2*b*d^3*x + B^2*b^3*c^3 -
 3*B^2*a*b^2*c^2*d + 3*B^2*a^2*b*c*d^2)*log((d^2*e*x^2 + 2*c*d*e*x + c^2*e)
/(b^2*x^2 + 2*a*b*x + a^2))^2 + 6*((3*A*B - 5*B^2)*b^3*c^2*d - 18*(A*B - 3*
B^2)*a*b^2*c*d^2 + (15*A*B - 49*B^2)*a^2*b*d^3)*x + 6*((3*A*B - 11*B^2)*b^3
*d^3*x^3 + (3*A*B - 2*B^2)*b^3*c^3 - 9*(A*B - B^2)*a*b^2*c^2*d + 9*(A*B - 2
```

```
*B^2)*a^2*b*c*d^2 - 3*(2*B^2*b^3*c*d^2 - 3*(A*B - 3*B^2)*a*b^2*d^3)*x^2 + 3
*(B^2*b^3*c^2*d - 6*B^2*a*b^2*c*d^2 + 3*(A*B - 2*B^2)*a^2*b*d^3)*x)*log((d^
2*e*x^2 + 2*c*d*e*x + c^2*e)/(b^2*x^2 + 2*a*b*x + a^2)))/((b^7*c^3 - 3*a*b^
6*c^2*d + 3*a^2*b^5*c*d^2 - a^3*b^4*d^3)*g^4*x^3 + 3*(a*b^6*c^3 - 3*a^2*b^5
*c^2*d + 3*a^3*b^4*c*d^2 - a^4*b^3*d^3)*g^4*x^2 + 3*(a^2*b^5*c^3 - 3*a^3*b^
4*c^2*d + 3*a^4*b^3*c*d^2 - a^5*b^2*d^3)*g^4*x + (a^3*b^4*c^3 - 3*a^4*b^3*c
^2*d + 3*a^5*b^2*c*d^2 - a^6*b*d^3)*g^4)
```

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(B\log\left(\frac{(dx+c)^2e}{(bx+a)^2}\right)+A\right)^2}{\left(bgx+ag\right)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*(d*x+c)^2/(b*x+a)^2))^2/(b*g*x+a*g)^4,x, algorithm="gi
ac")
```

```
[Out] integrate((B*log((d*x + c)^2*e/(b*x + a)^2) + A)^2/(b*g*x + a*g)^4, x)
```

**maple [B]** time = 0.07, size = 947, normalized size = 2.33

$$\frac{4AB\,a^3d^3}{9\left(ad-bc\right)^3\left(bx+a\right)^3b\,g^4}-\frac{4AB\,a^2c\,d^2}{3\left(ad-bc\right)^3\left(bx+a\right)^3g^4}+\frac{4ABab\,c^2d}{3\left(ad-bc\right)^3\left(bx+a\right)^3g^4}+\frac{4ABa\,d^4\ln\left(\frac{ad}{bx+a}-\frac{bc}{bx+a}-d\right)}{3\left(ad-bc\right)^4b\,g^4}-\frac{}{9(}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*ln((d*x+c)^2/(b*x+a)^2*e)+A)^2/(b*g*x+a*g)^4,x)
```

```
[Out] -1/3/b/(b*x+a)^3/g^4*A^2-8/27/b/g^4*B^2/(b*x+a)^3+4/9/b/g^4*B^2/(b*x+a)^3*l
n((1/(b*x+a)*a*d-1/(b*x+a)*b*c-d)^2/b^2*e)-1/3/b/g^4*B^2/(b*x+a)^3*ln((1/(b
*x+a)*a*d-1/(b*x+a)*b*c-d)^2/b^2*e)^2-10/9/b/g^4*B^2*d/(a*d-b*c)/(b*x+a)^2-
44/9/b/g^4*B^2*d^2/(a^2*d^2-2*a*b*c*d+b^2*c^2)/(b*x+a)-22/9/b/g^4*d^3*B^2/(
a^3*d^3-3*a^2*b*c*d^2+3*a*b^2*c^2*d-b^3*c^3)*ln((1/(b*x+a)*a*d-1/(b*x+a)*b*
c-d)^2/b^2*e)+1/3/b/g^4*d^3*B^2/(a^3*d^3-3*a^2*b*c*d^2+3*a*b^2*c^2*d-b^3*c^
3)*ln((1/(b*x+a)*a*d-1/(b*x+a)*b*c-d)^2/b^2*e)^2+2/3/b/g^4*B^2*d/(a*d-b*c)/
(b*x+a)^2*ln((1/(b*x+a)*a*d-1/(b*x+a)*b*c-d)^2/b^2*e)+4/3/b/g^4*B^2*d^2/(a^
2*d^2-2*a*b*c*d+b^2*c^2)/(b*x+a)*ln((1/(b*x+a)*a*d-1/(b*x+a)*b*c-d)^2/b^2*e
)-2/3/b/g^4*A*B/(b*x+a)^3*ln((1/(b*x+a)*a*d-1/(b*x+a)*b*c-d)^2/b^2*e)+4/9/b
/g^4*A*B*a^3*d^3/(a*d-b*c)^3/(b*x+a)^3-4/3/g^4*A*B*a^2*d^2/(a*d-b*c)^3/(b*x
+a)^3*c+4/3*b/g^4*A*B*a*d/(a*d-b*c)^3/(b*x+a)^3*c^2+2/3/b/g^4*A*B*a^2*d^3/(
a*d-b*c)^3/(b*x+a)^2-4/3/g^4*A*B*a*d^2/(a*d-b*c)^3/(b*x+a)^2*c+4/3/b/g^4*A*
```

```
B*a*d^3/(a*d-b*c)^3/(b*x+a)+4/3/b/g^4*A*B*a*d^4/(a*d-b*c)^4*ln(1/(b*x+a)*a*
d-1/(b*x+a)*b*c-d)-4/9*b^2/g^4*A*B*c^3/(a*d-b*c)^3/(b*x+a)^3+2/3*b/g^4*A*B*
c^2/(a*d-b*c)^3/(b*x+a)^2*d-4/3/g^4*A*B*c/(a*d-b*c)^3/(b*x+a)*d^2-4/3/g^4*A
*B*c*d^3/(a*d-b*c)^4*ln(1/(b*x+a)*a*d-1/(b*x+a)*b*c-d)
```

**maxima [B]** time = 1.90, size = 1576, normalized size = 3.87

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*(d*x+c)^2/(b*x+a)^2))^2/(b*g*x+a*g)^4,x, algorithm="ma
xima")
```

```
[Out] 2/27*(3*((6*b^2*d^2*x^2 + 2*b^2*c^2 - 7*a*b*c*d + 11*a^2*d^2 - 3*(b^2*c*d -
 5*a*b*d^2)*x)/((b^6*c^2 - 2*a*b^5*c*d + a^2*b^4*d^2)*g^4*x^3 + 3*(a*b^5*c^
2 - 2*a^2*b^4*c*d + a^3*b^3*d^2)*g^4*x^2 + 3*(a^2*b^4*c^2 - 2*a^3*b^3*c*d +
 a^4*b^2*d^2)*g^4*x + (a^3*b^3*c^2 - 2*a^4*b^2*c*d + a^5*b*d^2)*g^4) + 6*d^
3*log(b*x + a)/((b^4*c^3 - 3*a*b^3*c^2*d + 3*a^2*b^2*c*d^2 - a^3*b*d^3)*g^4
) - 6*d^3*log(d*x + c)/((b^4*c^3 - 3*a*b^3*c^2*d + 3*a^2*b^2*c*d^2 - a^3*b*
d^3)*g^4))*log(d^2*e*x^2/(b^2*x^2 + 2*a*b*x + a^2) + 2*c*d*e*x/(b^2*x^2 + 2
*a*b*x + a^2) + c^2*e/(b^2*x^2 + 2*a*b*x + a^2)) - (4*b^3*c^3 - 27*a*b^2*c^
2*d + 108*a^2*b*c*d^2 - 85*a^3*d^3 + 66*(b^3*c*d^2 - a*b^2*d^3)*x^2 - 18*(b
^3*d^3*x^3 + 3*a*b^2*d^3*x^2 + 3*a^2*b*d^3*x + a^3*d^3)*log(b*x + a)^2 - 18
*(b^3*d^3*x^3 + 3*a*b^2*d^3*x^2 + 3*a^2*b*d^3*x + a^3*d^3)*log(d*x + c)^2 -
 3*(5*b^3*c^2*d - 54*a*b^2*c*d^2 + 49*a^2*b*d^3)*x + 66*(b^3*d^3*x^3 + 3*a*
b^2*d^3*x^2 + 3*a^2*b*d^3*x + a^3*d^3)*log(b*x + a) - 6*(11*b^3*d^3*x^3 + 3
3*a*b^2*d^3*x^2 + 33*a^2*b*d^3*x + 11*a^3*d^3 - 6*(b^3*d^3*x^3 + 3*a*b^2*d^
3*x^2 + 3*a^2*b*d^3*x + a^3*d^3)*log(b*x + a))*log(d*x + c))/(a^3*b^4*c^3*g
^4 - 3*a^4*b^3*c^2*d*g^4 + 3*a^5*b^2*c*d^2*g^4 - a^6*b*d^3*g^4 + (b^7*c^3*g
^4 - 3*a*b^6*c^2*d*g^4 + 3*a^2*b^5*c*d^2*g^4 - a^3*b^4*d^3*g^4)*x^3 + 3*(a*
b^6*c^3*g^4 - 3*a^2*b^5*c^2*d*g^4 + 3*a^3*b^4*c*d^2*g^4 - a^4*b^3*d^3*g^4)*
x^2 + 3*(a^2*b^5*c^3*g^4 - 3*a^3*b^4*c^2*d*g^4 + 3*a^4*b^3*c*d^2*g^4 - a^5*
b^2*d^3*g^4)*x))*B^2 + 2/9*A*B*((6*b^2*d^2*x^2 + 2*b^2*c^2 - 7*a*b*c*d + 11
*a^2*d^2 - 3*(b^2*c*d - 5*a*b*d^2)*x)/((b^6*c^2 - 2*a*b^5*c*d + a^2*b^4*d^2
)*g^4*x^3 + 3*(a*b^5*c^2 - 2*a^2*b^4*c*d + a^3*b^3*d^2)*g^4*x^2 + 3*(a^2*b^
4*c^2 - 2*a^3*b^3*c*d + a^4*b^2*d^2)*g^4*x + (a^3*b^3*c^2 - 2*a^4*b^2*c*d +
 a^5*b*d^2)*g^4) - 3*log(d^2*e*x^2/(b^2*x^2 + 2*a*b*x + a^2) + 2*c*d*e*x/(b
^2*x^2 + 2*a*b*x + a^2) + c^2*e/(b^2*x^2 + 2*a*b*x + a^2))/(b^4*g^4*x^3 + 3
*a*b^3*g^4*x^2 + 3*a^2*b^2*g^4*x + a^3*b*g^4) + 6*d^3*log(b*x + a)/((b^4*c^
3 - 3*a*b^3*c^2*d + 3*a^2*b^2*c*d^2 - a^3*b*d^3)*g^4) - 6*d^3*log(d*x + c)/
((b^4*c^3 - 3*a*b^3*c^2*d + 3*a^2*b^2*c*d^2 - a^3*b*d^3)*g^4)) - 1/3*B^2*lo
g(d^2*e*x^2/(b^2*x^2 + 2*a*b*x + a^2) + 2*c*d*e*x/(b^2*x^2 + 2*a*b*x + a^2)
 + c^2*e/(b^2*x^2 + 2*a*b*x + a^2))^2/(b^4*g^4*x^3 + 3*a*b^3*g^4*x^2 + 3*a^
2*b^2*g^4*x + a^3*b*g^4) - 1/3*A^2/(b^4*g^4*x^3 + 3*a*b^3*g^4*x^2 + 3*a^2*b
^2*g^4*x + a^3*b*g^4)
```

**mupad [B]** time = 9.08, size = 1069, normalized size = 2.63

$$\frac{\frac{9\,A^2\,a^2\,d^2-18\,A^2\,a\,b\,c\,d+9\,A^2\,b^2\,c^2-66\,A\,B\,a^2\,d^2+42\,A\,B\,a\,b\,c\,d-12\,A\,B\,b^2\,c^2+170\,B^2\,a^2\,d^2-46\,B^2\,a\,b\,c\,d+8\,B^2\,b^2\,c^2}{3\,(a\,d-b\,c)}+\frac{2\,x\,\left(-5\,c\,B^2\,b^2\,d+49\,a\,B^2\,b\,d^2+3\,A\right.}{a\,d-b\,c}}{x\,\left(27\,a^2\,b^3\,c\,g^4-27\,a^3\,b^2\,d\,g^4\right)-x^2\,\left(27\,a^2\,b^3\,d\,g^4-27\,a\,b^4\,c\,g^4\right)+x^3\,\left(9\,b^5\,c\,g^4-9\,a\,b^4\,d\,g^4\right)+}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*log((e*(c + d*x)^2)/(a + b*x)^2))^2/(a*g + b*g*x)^4,x)
```

```
[Out] ((9*A^2*a^2*d^2 + 9*A^2*b^2*c^2 + 170*B^2*a^2*d^2 + 8*B^2*b^2*c^2 - 66*A*B*
a^2*d^2 - 12*A*B*b^2*c^2 - 18*A^2*a*b*c*d - 46*B^2*a*b*c*d + 42*A*B*a*b*c*d
)/(3*(a*d - b*c)) + (2*x*(49*B^2*a*b*d^2 - 5*B^2*b^2*c*d - 15*A*B*a*b*d^2 +
 3*A*B*b^2*c*d))/(a*d - b*c) + (4*d*x^2*(11*B^2*b^2*d - 3*A*B*b^2*d))/(a*d
- b*c))/(x*(27*a^2*b^3*c*g^4 - 27*a^3*b^2*d*g^4) - x^2*(27*a^2*b^3*d*g^4 -
27*a*b^4*c*g^4) + x^3*(9*b^5*c*g^4 - 9*a*b^4*d*g^4) + 9*a^3*b^2*c*g^4 - 9*a
^4*b*d*g^4) - log((e*(c + d*x)^2)/(a + b*x)^2)^2*(B^2/(3*b^2*g^4*(3*a^2*x +
 a^3/b + b^2*x^3 + 3*a*b*x^2)) - (B^2*d^3)/(3*b*g^4*(a^3*d^3 - b^3*c^3 + 3*
a*b^2*c^2*d - 3*a^2*b*c*d^2))) - (log((e*(c + d*x)^2)/(a + b*x)^2)*((2*A*B)
/(3*b^2*d*g^4) - (2*B^2*d^3*(a*((3*a^2*d^2 + b^2*c^2 - 4*a*b*c*d)/(3*b*d^3)
 + (2*a*(a*d - b*c))/(3*b*d^2)) + (2*(3*a^3*d^3 - b^3*c^3 + 4*a*b^2*c^2*d -
 6*a^2*b*c*d^2))/(3*b*d^4)))/(3*b*g^4*(a^3*d^3 - b^3*c^3 + 3*a*b^2*c^2*d -
3*a^2*b*c*d^2)) + (2*B^2*d^3*x^2*((2*(b^2*c - a*b*d))/(3*d^2) - (4*b*(a*d -
 b*c))/(3*d^2)))/(3*b*g^4*(a^3*d^3 - b^3*c^3 + 3*a*b^2*c^2*d - 3*a^2*b*c*d^
2)) - (2*B^2*d^3*x*(b*((3*a^2*d^2 + b^2*c^2 - 4*a*b*c*d)/(3*b*d^3) + (2*a*(
a*d - b*c))/(3*b*d^2)) + (2*(3*a^2*d^2 + b^2*c^2 - 4*a*b*c*d))/(3*d^3) + (4
*a*(a*d - b*c))/(3*d^2)))/(3*b*g^4*(a^3*d^3 - b^3*c^3 + 3*a*b^2*c^2*d - 3*a
^2*b*c*d^2))))/((3*a^2*x)/d + a^3/(b*d) + (b^2*x^3)/d + (3*a*b*x^2)/d) - (B
*d^3*atan((B*d^3*((b^4*c^3*g^4 + a^3*b*d^3*g^4 - a*b^3*c^2*d*g^4 - a^2*b^2*
c*d^2*g^4)/(b^3*c^2*g^4 + a^2*b*d^2*g^4 - 2*a*b^2*c*d*g^4) + 2*b*d*x)*(3*A
- 11*B)*(b^3*c^2*g^4 + a^2*b*d^2*g^4 - 2*a*b^2*c*d*g^4)*4i)/(b*g^4*(a*d - b
*c)^3*(44*B^2*d^3 - 12*A*B*d^3)))*(3*A - 11*B)*8i)/(9*b*g^4*(a*d - b*c)^3)
```

**sympy [B]** time = 35.55, size = 1561, normalized size = 3.84

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*ln(e*(d*x+c)**2/(b*x+a)**2))**2/(b*g*x+a*g)**4,x)
```

```
[Out] 4*B*d**3*(3*A - 11*B)*log(x + (12*A*B*a*d**4 + 12*A*B*b*c*d**3 - 44*B**2*a*
d**4 - 44*B**2*b*c*d**3 - 4*B*a**4*d**7*(3*A - 11*B)/(a*d - b*c)**3 + 16*B*
a**3*b*c*d**6*(3*A - 11*B)/(a*d - b*c)**3 - 24*B*a**2*b**2*c**2*d**5*(3*A -
```

```
 11*B)/(a*d - b*c)**3 + 16*B*a*b**3*c**3*d**4*(3*A - 11*B)/(a*d - b*c)**3 -
 4*B*b**4*c**4*d**3*(3*A - 11*B)/(a*d - b*c)**3)/(24*A*B*b*d**4 - 88*B**2*b
*d**4))/(9*b*g**4*(a*d - b*c)**3) - 4*B*d**3*(3*A - 11*B)*log(x + (12*A*B*a
*d**4 + 12*A*B*b*c*d**3 - 44*B**2*a*d**4 - 44*B**2*b*c*d**3 + 4*B*a**4*d**7
*(3*A - 11*B)/(a*d - b*c)**3 - 16*B*a**3*b*c*d**6*(3*A - 11*B)/(a*d - b*c)*
*3 + 24*B*a**2*b**2*c**2*d**5*(3*A - 11*B)/(a*d - b*c)**3 - 16*B*a*b**3*c**
3*d**4*(3*A - 11*B)/(a*d - b*c)**3 + 4*B*b**4*c**4*d**3*(3*A - 11*B)/(a*d -
 b*c)**3)/(24*A*B*b*d**4 - 88*B**2*b*d**4))/(9*b*g**4*(a*d - b*c)**3) + (3*
B**2*a**2*c*d**2 + 3*B**2*a**2*d**3*x - 3*B**2*a*b*c**2*d + 3*B**2*a*b*d**3
*x**2 + B**2*b**2*c**3 + B**2*b**2*d**3*x**3)*log(e*(c + d*x)**2/(a + b*x)*
*2)**2/(3*a**6*d**3*g**4 - 9*a**5*b*c*d**2*g**4 + 9*a**5*b*d**3*g**4*x + 9*
a**4*b**2*c**2*d*g**4 - 27*a**4*b**2*c*d**2*g**4*x + 9*a**4*b**2*d**3*g**4*
x**2 - 3*a**3*b**3*c**3*g**4 + 27*a**3*b**3*c**2*d*g**4*x - 27*a**3*b**3*c*
d**2*g**4*x**2 + 3*a**3*b**3*d**3*g**4*x**3 - 9*a**2*b**4*c**3*g**4*x + 27*
a**2*b**4*c**2*d*g**4*x**2 - 9*a**2*b**4*c*d**2*g**4*x**3 - 9*a*b**5*c**3*g
**4*x**2 + 9*a*b**5*c**2*d*g**4*x**3 - 3*b**6*c**3*g**4*x**3) + (-6*A*B*a**
2*d**2 + 12*A*B*a*b*c*d - 6*A*B*b**2*c**2 + 22*B**2*a**2*d**2 - 14*B**2*a*b
*c*d + 30*B**2*a*b*d**2*x + 4*B**2*b**2*c**2 - 6*B**2*b**2*c*d*x + 12*B**2*
b**2*d**2*x**2)*log(e*(c + d*x)**2/(a + b*x)**2)/(9*a**5*b*d**2*g**4 - 18*a
**4*b**2*c*d*g**4 + 27*a**4*b**2*d**2*g**4*x + 9*a**3*b**3*c**2*g**4 - 54*a
**3*b**3*c*d*g**4*x + 27*a**3*b**3*d**2*g**4*x**2 + 27*a**2*b**4*c**2*g**4*
x - 54*a**2*b**4*c*d*g**4*x**2 + 9*a**2*b**4*d**2*g**4*x**3 + 27*a*b**5*c**
2*g**4*x**2 - 18*a*b**5*c*d*g**4*x**3 + 9*b**6*c**2*g**4*x**3) - (9*A**2*a*
*2*d**2 - 18*A**2*a*b*c*d + 9*A**2*b**2*c**2 - 66*A*B*a**2*d**2 + 42*A*B*a*
b*c*d - 12*A*B*b**2*c**2 + 170*B**2*a**2*d**2 - 46*B**2*a*b*c*d + 8*B**2*b*
*2*c**2 + x**2*(-36*A*B*b**2*d**2 + 132*B**2*b**2*d**2) + x*(-90*A*B*a*b*d*
*2 + 18*A*B*b**2*c*d + 294*B**2*a*b*d**2 - 30*B**2*b**2*c*d))/(27*a**5*b*d*
*2*g**4 - 54*a**4*b**2*c*d*g**4 + 27*a**3*b**3*c**2*g**4 + x**3*(27*a**2*b*
*4*d**2*g**4 - 54*a*b**5*c*d*g**4 + 27*b**6*c**2*g**4) + x**2*(81*a**3*b**3
*d**2*g**4 - 162*a**2*b**4*c*d*g**4 + 81*a*b**5*c**2*g**4) + x*(81*a**4*b**
2*d**2*g**4 - 162*a**3*b**3*c*d*g**4 + 81*a**2*b**4*c**2*g**4))
```

# 3.218 $\int \frac{\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2}{(ag+bgx)^5}\,dx$

**Optimal**. Leaf size=501

$$\frac{b^3B(c+dx)^4\left(B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)+A\right)}{4g^5(a+bx)^4(bc-ad)^4}-\frac{4b^2Bd(c+dx)^3\left(B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)+A\right)}{3g^5(a+bx)^3(bc-ad)^4}+\frac{Bd^4\log\left(\frac{c+dx}{a+bx}\right)\left(B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)+A\right)}{bg^5(bc-ad)^4}$$

```
[Out] 8*B^2*d^3*(d*x+c)/(-a*d+b*c)^4/g^5/(b*x+a)-3*b*B^2*d^2*(d*x+c)^2/(-a*d+b*c)
^4/g^5/(b*x+a)^2+8/9*b^2*B^2*d*(d*x+c)^3/(-a*d+b*c)^4/g^5/(b*x+a)^3-1/8*b^3
*B^2*(d*x+c)^4/(-a*d+b*c)^4/g^5/(b*x+a)^4-B^2*d^4*ln((d*x+c)/(b*x+a))^2/b/(
-a*d+b*c)^4/g^5-4*B*d^3*(d*x+c)*(A+B*ln(e*(d*x+c)^2/(b*x+a)^2))/(-a*d+b*c)^
4/g^5/(b*x+a)+3*b*B*d^2*(d*x+c)^2*(A+B*ln(e*(d*x+c)^2/(b*x+a)^2))/(-a*d+b*c
)^4/g^5/(b*x+a)^2-4/3*b^2*B*d*(d*x+c)^3*(A+B*ln(e*(d*x+c)^2/(b*x+a)^2))/(-a
*d+b*c)^4/g^5/(b*x+a)^3+1/4*b^3*B*(d*x+c)^4*(A+B*ln(e*(d*x+c)^2/(b*x+a)^2))
/(-a*d+b*c)^4/g^5/(b*x+a)^4+B*d^4*ln((d*x+c)/(b*x+a))*(A+B*ln(e*(d*x+c)^2/(
b*x+a)^2))/b/(-a*d+b*c)^4/g^5-1/4*(A+B*ln(e*(d*x+c)^2/(b*x+a)^2))^2/b/g^5/(
b*x+a)^4
```

**Rubi [C]** time = 1.43, antiderivative size = 758, normalized size of antiderivative = 1.51, number of steps used = 38, number of rules used = 11, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.324, Rules used = {2525, 12, 2528, 44, 2524, 2418, 2390, 2301, 2394, 2393, 2391}

$$\frac{2B^2d^4\text{PolyLog}\left(2,-\frac{d(a+bx)}{bc-ad}\right)}{bg^5(bc-ad)^4}+\frac{2B^2d^4\text{PolyLog}\left(2,\frac{b(c+dx)}{bc-ad}\right)}{bg^5(bc-ad)^4}-\frac{Bd^4\log(a+bx)\left(B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)+A\right)}{bg^5(bc-ad)^4}+\frac{Bd^4\log(c+dx)}{bg}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Log[(e*(c + d*x)^2)/(a + b*x)^2])^2/(a*g + b*g*x)^5,x]
```

```
[Out] -B^2/(8*b*g^5*(a + b*x)^4) + (7*B^2*d)/(18*b*(b*c - a*d)*g^5*(a + b*x)^3) -
 (13*B^2*d^2)/(12*b*(b*c - a*d)^2*g^5*(a + b*x)^2) + (25*B^2*d^3)/(6*b*(b*c
 - a*d)^3*g^5*(a + b*x)) + (25*B^2*d^4*Log[a + b*x])/(6*b*(b*c - a*d)^4*g^5
) - (B^2*d^4*Log[a + b*x]^2)/(b*(b*c - a*d)^4*g^5) - (25*B^2*d^4*Log[c + d*
x])/(6*b*(b*c - a*d)^4*g^5) + (2*B^2*d^4*Log[-((d*(a + b*x))/(b*c - a*d))]*
Log[c + d*x])/(b*(b*c - a*d)^4*g^5) - (B^2*d^4*Log[c + d*x]^2)/(b*(b*c - a*
d)^4*g^5) + (2*B^2*d^4*Log[a + b*x]*Log[(b*(c + d*x))/(b*c - a*d)])/(b*(b*c
 - a*d)^4*g^5) + (B*(A + B*Log[(e*(c + d*x)^2)/(a + b*x)^2]))/(4*b*g^5*(a +
 b*x)^4) - (B*d*(A + B*Log[(e*(c + d*x)^2)/(a + b*x)^2]))/(3*b*(b*c - a*d)*
g^5*(a + b*x)^3) + (B*d^2*(A + B*Log[(e*(c + d*x)^2)/(a + b*x)^2]))/(2*b*(b
*c - a*d)^2*g^5*(a + b*x)^2) - (B*d^3*(A + B*Log[(e*(c + d*x)^2)/(a + b*x)^
```

```
2]))/(b*(b*c - a*d)^3*g^5*(a + b*x)) - (B*d^4*Log[a + b*x]*(A + B*Log[(e*(c
 + d*x)^2)/(a + b*x)^2]))/(b*(b*c - a*d)^4*g^5) + (B*d^4*Log[c + d*x]*(A +
B*Log[(e*(c + d*x)^2)/(a + b*x)^2]))/(b*(b*c - a*d)^4*g^5) - (A + B*Log[(e*
(c + d*x)^2)/(a + b*x)^2])^2/(4*b*g^5*(a + b*x)^4) + (2*B^2*d^4*PolyLog[2,
-((d*(a + b*x))/(b*c - a*d))])/(b*(b*c - a*d)^4*g^5) + (2*B^2*d^4*PolyLog[2
, (b*(c + d*x))/(b*c - a*d)])/(b*(b*c - a*d)^4*g^5)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 2301

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

Rule 2390

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_))^(q_.), x_Symbol] :> Dist[1/e, Subst[Int[((f*x)/d)^q*(a + b*Log[c*x^
n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && E
qQ[e*f - d*g, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2393

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + (c*e*x)/g])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2394

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d + e*x
)^n]))/g, x] - Dist[(b*e*n)/g, Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2418

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*(RFx_), x_Sy
mbol] :> With[{u = ExpandIntegrand[(a + b*Log[c*(d + e*x)^n])^p, RFx, x]},
Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d, e, n}, x] && RationalFunctionQ[
RFx, x] && IntegerQ[p]
```

Rule 2524

```
Int[((a_.) + Log[(c_.)*(RFx_)^(p_.)]*(b_.))^(n_.)/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[(Log[d + e*x]*(a + b*Log[c*RFx^p])^n)/e, x] - Dist[(b*n*p)/e
, Int[(Log[d + e*x]*(a + b*Log[c*RFx^p])^(n - 1)*D[RFx, x])/RFx, x], x] /;
FreeQ[{a, b, c, d, e, p}, x] && RationalFunctionQ[RFx, x] && IGtQ[n, 0]
```

Rule 2525

```
Int[((a_.) + Log[(c_.)*(RFx_)^(p_.)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_))^(m_.
), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(a + b*Log[c*RFx^p])^n)/(e*(m + 1))
, x] - Dist[(b*n*p)/(e*(m + 1)), Int[SimplifyIntegrand[((d + e*x)^(m + 1)*(
a + b*Log[c*RFx^p])^(n - 1)*D[RFx, x])/RFx, x], x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && RationalFunctionQ[RFx, x] && IGtQ[n, 0] && (EqQ[n, 1] ||
IntegerQ[m]) && NeQ[m, -1]
```

Rule 2528

```
Int[((a_.) + Log[(c_.)*(RFx_)^(p_.)]*(b_.))^(n_.)*(RGx_), x_Symbol] :> With
[{u = ExpandIntegrand[(a + b*Log[c*RFx^p])^n, RGx, x]}, Int[u, x] /; SumQ[u
]] /; FreeQ[{a, b, c, p}, x] && RationalFunctionQ[RFx, x] && RationalFuncti
onQ[RGx, x] && IGtQ[n, 0]
```

Rubi steps

$$\int \frac{\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2}{(ag+bgx)^5}\,dx = -\frac{\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2}{4bg^5(a+bx)^4} + \frac{B\int \frac{2(bc-ad)\left(-A-B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)}{g^4(a+bx)^5(c+dx)}\,dx}{2bg}$$

$$= -\frac{\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2}{4bg^5(a+bx)^4} + \frac{(B(bc-ad))\int \frac{-A-B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)}{(a+bx)^5(c+dx)}\,dx}{bg^5}$$

$$= -\frac{\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2}{4bg^5(a+bx)^4} + \frac{(B(bc-ad))\int \left(\frac{b\left(-A-B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)}{(bc-ad)(a+bx)^5} - \frac{bd\left(-A-B\log\left(\frac{e(c+d}{(a+b}\right.\right.}{(bc-ad)^2(a+bx)}\right.}{}$$

$$= -\frac{\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2}{4bg^5(a+bx)^4} + \frac{B\int \frac{-A-B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)}{(a+bx)^5}\,dx}{g^5} + \frac{\left(Bd^4\right)\int \frac{-A-B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)}{a+bx}\,d}{(bc-ad)^4g^5}$$

$$= \frac{B\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)}{4bg^5(a+bx)^4} - \frac{Bd\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)}{3b(bc-ad)g^5(a+bx)^3} + \frac{Bd^2\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right.\right.}{2b(bc-ad)^2g^5(a+bx)}$$

$$= \frac{B\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)}{4bg^5(a+bx)^4} - \frac{Bd\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)}{3b(bc-ad)g^5(a+bx)^3} + \frac{Bd^2\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right.\right.}{2b(bc-ad)^2g^5(a+bx)}$$

$$= \frac{B\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)}{4bg^5(a+bx)^4} - \frac{Bd\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)}{3b(bc-ad)g^5(a+bx)^3} + \frac{Bd^2\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right.\right.}{2b(bc-ad)^2g^5(a+bx)}$$

$$= -\frac{B^2}{8bg^5(a+bx)^4} + \frac{7B^2d}{18b(bc-ad)g^5(a+bx)^3} - \frac{13B^2d^2}{12b(bc-ad)^2g^5(a+bx)^2} + \frac{}{6b(bc}$$

$$= -\frac{B^2}{8bg^5(a+bx)^4} + \frac{7B^2d}{18b(bc-ad)g^5(a+bx)^3} - \frac{13B^2d^2}{12b(bc-ad)^2g^5(a+bx)^2} + \frac{}{6b(bc}$$

$$= -\frac{B^2}{8bg^5(a+bx)^4} + \frac{7B^2d}{18b(bc-ad)g^5(a+bx)^3} - \frac{13B^2d^2}{12b(bc-ad)^2g^5(a+bx)^2} + \frac{}{6b(bc}$$

$$= -\frac{B^2}{8bg^5(a+bx)^4} + \frac{7B^2d}{18b(bc-ad)g^5(a+bx)^3} - \frac{13B^2d^2}{12b(bc-ad)^2g^5(a+bx)^2} + \frac{}{6b(bc}$$

**Mathematica [C]** time = 0.87, size = 762, normalized size = 1.52

$$\frac{B\left(-72d^4(a+bx)^4\log(a+bx)\left(B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)+A\right)+72d^4(a+bx)^4\log(c+dx)\left(B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)+A\right)+72d^3(a+bx)^3(ad-bc)\left(B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)+A\right)+36d^2(a+bx)^2\cdots\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Log[(e*(c + d*x)^2)/(a + b*x)^2])^2/(a*g + b*g*x)^5,x]
```

```
[Out] (-18*(A + B*Log[(e*(c + d*x)^2)/(a + b*x)^2])^2 + (B*(144*B*d^3*(a + b*x)^3
*(b*c - a*d + d*(a + b*x)*Log[a + b*x] - d*(a + b*x)*Log[c + d*x]) - 36*B*d
^2*(a + b*x)^2*((b*c - a*d)^2 + 2*d*(-(b*c) + a*d)*(a + b*x) - 2*d^2*(a + b
*x)^2*Log[a + b*x] + 2*d^2*(a + b*x)^2*Log[c + d*x]) + 8*B*d*(a + b*x)*(2*(
b*c - a*d)^3 - 3*d*(b*c - a*d)^2*(a + b*x) + 6*d^2*(b*c - a*d)*(a + b*x)^2
+ 6*d^3*(a + b*x)^3*Log[a + b*x] - 6*d^3*(a + b*x)^3*Log[c + d*x]) - 3*B*(3
*(b*c - a*d)^4 + 4*d*(-(b*c) + a*d)^3*(a + b*x) + 6*d^2*(b*c - a*d)^2*(a +
b*x)^2 + 12*d^3*(-(b*c) + a*d)*(a + b*x)^3 - 12*d^4*(a + b*x)^4*Log[a + b*x
] + 12*d^4*(a + b*x)^4*Log[c + d*x]) + 18*(b*c - a*d)^4*(A + B*Log[(e*(c +
d*x)^2)/(a + b*x)^2]) + 24*d*(-(b*c) + a*d)^3*(a + b*x)*(A + B*Log[(e*(c +
d*x)^2)/(a + b*x)^2]) + 36*d^2*(b*c - a*d)^2*(a + b*x)^2*(A + B*Log[(e*(c +
 d*x)^2)/(a + b*x)^2]) + 72*d^3*(-(b*c) + a*d)*(a + b*x)^3*(A + B*Log[(e*(c
 + d*x)^2)/(a + b*x)^2]) - 72*d^4*(a + b*x)^4*Log[a + b*x]*(A + B*Log[(e*(c
 + d*x)^2)/(a + b*x)^2]) + 72*d^4*(a + b*x)^4*Log[c + d*x]*(A + B*Log[(e*(c
 + d*x)^2)/(a + b*x)^2]) - 72*B*d^4*(a + b*x)^4*(Log[a + b*x]*(Log[a + b*x]
 - 2*Log[(b*(c + d*x))/(b*c - a*d)]) - 2*PolyLog[2, (d*(a + b*x))/(-(b*c) +
 a*d)]) + 72*B*d^4*(a + b*x)^4*((2*Log[(d*(a + b*x))/(-(b*c) + a*d)] - Log[
c + d*x])*Log[c + d*x] + 2*PolyLog[2, (b*(c + d*x))/(b*c - a*d)])))/(b*c -
a*d)^4)/(72*b*g^5*(a + b*x)^4)
```

**fricas [B]** time = 0.82, size = 1088, normalized size = 2.17

$$-\frac{9\left(2A^2-2AB+B^2\right)b^4c^4-8\left(9A^2-12AB+8B^2\right)ab^3c^3d+108\left(A^2-2AB+2B^2\right)a^2b^2c^2d^2-72\left(A^2-4AB\cdots\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*(d*x+c)^2/(b*x+a)^2))^2/(b*g*x+a*g)^5,x, algorithm="fr
icas")
```

```
[Out] -1/72*(9*(2*A^2 - 2*A*B + B^2)*b^4*c^4 - 8*(9*A^2 - 12*A*B + 8*B^2)*a*b^3*c
^3*d + 108*(A^2 - 2*A*B + 2*B^2)*a^2*b^2*c^2*d^2 - 72*(A^2 - 4*A*B + 8*B^2)
*a^3*b*c*d^3 + (18*A^2 - 150*A*B + 415*B^2)*a^4*d^4 + 12*((6*A*B - 25*B^2)*
b^4*c*d^3 - (6*A*B - 25*B^2)*a*b^3*d^4)*x^3 - 6*((6*A*B - 13*B^2)*b^4*c^2*d
```

```
^2 - 16*(3*A*B - 11*B^2)*a*b^3*c*d^3 + (42*A*B - 163*B^2)*a^2*b^2*d^4)*x^2
- 18*(B^2*b^4*d^4*x^4 + 4*B^2*a*b^3*d^4*x^3 + 6*B^2*a^2*b^2*d^4*x^2 + 4*B^2
*a^3*b*d^4*x - B^2*b^4*c^4 + 4*B^2*a*b^3*c^3*d - 6*B^2*a^2*b^2*c^2*d^2 + 4*
B^2*a^3*b*c*d^3)*log((d^2*e*x^2 + 2*c*d*e*x + c^2*e)/(b^2*x^2 + 2*a*b*x + a
^2))^2 + 4*((6*A*B - 7*B^2)*b^4*c^3*d - 12*(3*A*B - 5*B^2)*a*b^3*c^2*d^2 +
108*(A*B - 3*B^2)*a^2*b^2*c*d^3 - (78*A*B - 271*B^2)*a^3*b*d^4)*x - 6*((6*A
*B - 25*B^2)*b^4*d^4*x^4 - 3*(2*A*B - B^2)*b^4*c^4 + 8*(3*A*B - 2*B^2)*a*b^
3*c^3*d - 36*(A*B - B^2)*a^2*b^2*c^2*d^2 + 24*(A*B - 2*B^2)*a^3*b*c*d^3 - 4
*(3*B^2*b^4*c*d^3 - 2*(3*A*B - 11*B^2)*a*b^3*d^4)*x^3 + 6*(B^2*b^4*c^2*d^2
- 8*B^2*a*b^3*c*d^3 + 6*(A*B - 3*B^2)*a^2*b^2*d^4)*x^2 - 4*(B^2*b^4*c^3*d -
 6*B^2*a*b^3*c^2*d^2 + 18*B^2*a^2*b^2*c*d^3 - 6*(A*B - 2*B^2)*a^3*b*d^4)*x)
*log((d^2*e*x^2 + 2*c*d*e*x + c^2*e)/(b^2*x^2 + 2*a*b*x + a^2)))/((b^9*c^4
- 4*a*b^8*c^3*d + 6*a^2*b^7*c^2*d^2 - 4*a^3*b^6*c*d^3 + a^4*b^5*d^4)*g^5*x^
4 + 4*(a*b^8*c^4 - 4*a^2*b^7*c^3*d + 6*a^3*b^6*c^2*d^2 - 4*a^4*b^5*c*d^3 +
a^5*b^4*d^4)*g^5*x^3 + 6*(a^2*b^7*c^4 - 4*a^3*b^6*c^3*d + 6*a^4*b^5*c^2*d^2
 - 4*a^5*b^4*c*d^3 + a^6*b^3*d^4)*g^5*x^2 + 4*(a^3*b^6*c^4 - 4*a^4*b^5*c^3*
d + 6*a^5*b^4*c^2*d^2 - 4*a^6*b^3*c*d^3 + a^7*b^2*d^4)*g^5*x + (a^4*b^5*c^4
 - 4*a^5*b^4*c^3*d + 6*a^6*b^3*c^2*d^2 - 4*a^7*b^2*c*d^3 + a^8*b*d^4)*g^5)
```

**giac [A]** time = 2.22, size = 868, normalized size = 1.73

$$\frac{1}{4}\left(\frac{B^2d^4}{b^5c^4g^5-4\,ab^4c^3dg^5+6\,a^2b^3c^2d^2g^5-4\,a^3b^2cd^3g^5+a^4bd^4g^5}-\frac{B^2}{\left(bgx+ag\right)^4bg}\right)\log\left(\frac{\frac{b^2c^2g^2}{\left(bgx+ag\right)^2}-\frac{2\,abcdg^2}{\left(bgx+ag\right)^2}+\frac{\cdots}{(b\cdots}}{\cdots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*(d*x+c)^2/(b*x+a)^2))^2/(b*g*x+a*g)^5,x, algorithm="gi
ac")
```

```
[Out] 1/4*(B^2*d^4/(b^5*c^4*g^5 - 4*a*b^4*c^3*d*g^5 + 6*a^2*b^3*c^2*d^2*g^5 - 4*a
^3*b^2*c*d^3*g^5 + a^4*b*d^4*g^5) - B^2/((b*g*x + a*g)^4*b*g))*log((b^2*c^2
*g^2/(b*g*x + a*g)^2 - 2*a*b*c*d*g^2/(b*g*x + a*g)^2 + a^2*d^2*g^2/(b*g*x +
 a*g)^2 + 2*b*c*d*g/(b*g*x + a*g) - 2*a*d^2*g/(b*g*x + a*g) + d^2)/b^2)^2 -
 1/12*(12*B^2*d^3/((b^3*c^3*g^3 - 3*a*b^2*c^2*d*g^3 + 3*a^2*b*c*d^2*g^3 - a
^3*d^3*g^3)*(b*g*x + a*g)*b*g) - 6*B^2*d^2/((b^2*c^2*g - 2*a*b*c*d*g + a^2*
d^2*g)*(b*g*x + a*g)^2*b*g^2) + 4*B^2*d/((b*g*x + a*g)^3*(b*c - a*d)*b*g^2)
 + 3*(2*A*B*b^3*g^3 + B^2*b^3*g^3)/((b*g*x + a*g)^4*b^4*g^4))*log((b^2*c^2*
g^2/(b*g*x + a*g)^2 - 2*a*b*c*d*g^2/(b*g*x + a*g)^2 + a^2*d^2*g^2/(b*g*x +
a*g)^2 + 2*b*c*d*g/(b*g*x + a*g) - 2*a*d^2*g/(b*g*x + a*g) + d^2)/b^2) + 1/
6*(6*A*B*d^4 - 19*B^2*d^4)*log(-b*c*g/(b*g*x + a*g) + a*d*g/(b*g*x + a*g) -
 d)/(b^5*c^4*g^5 - 4*a*b^4*c^3*d*g^5 + 6*a^2*b^3*c^2*d^2*g^5 - 4*a^3*b^2*c*
d^3*g^5 + a^4*b*d^4*g^5) - 1/6*(6*A*B*d^3 - 19*B^2*d^3)/((b^3*c^3*g^3 - 3*a
*b^2*c^2*d*g^3 + 3*a^2*b*c*d^2*g^3 - a^3*d^3*g^3)*(b*g*x + a*g)*b*g) + 1/12
```

```
*(6*A*B*b*d^2 - 7*B^2*b*d^2)/((b^2*c^2*g - 2*a*b*c*d*g + a^2*d^2*g)*(b*g*x
+ a*g)^2*b^2*g^2) - 1/18*(6*A*B*b^2*d*g - B^2*b^2*d*g)/((b*g*x + a*g)^3*(b*
c - a*d)*b^3*g^3) - 1/8*(2*A^2*b^3*g^3 + 2*A*B*b^3*g^3 + B^2*b^3*g^3)/((b*g
*x + a*g)^4*b^4*g^4)
```

**maple [B]** time = 0.07, size = 1285, normalized size = 2.56

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*ln((d*x+c)^2/(b*x+a)^2*e)+A)^2/(b*g*x+a*g)^5,x)
```

```
[Out] 1/4/b/g^5*B^2/(b*x+a)^4*ln((1/(b*x+a)*a*d-1/(b*x+a)*b*c-d)^2/b^2*e)+1/b/g^5
*d^3*B^2/(a^3*d^3-3*a^2*b*c*d^2+3*a*b^2*c^2*d-b^3*c^3)/(b*x+a)*ln((1/(b*x+a
)*a*d-1/(b*x+a)*b*c-d)^2/b^2*e)+1/3/b/g^5*B^2*d/(a*d-b*c)/(b*x+a)^3*ln((1/(
b*x+a)*a*d-1/(b*x+a)*b*c-d)^2/b^2*e)+1/2/b/g^5*B^2*d^2/(a^2*d^2-2*a*b*c*d+b
^2*c^2)/(b*x+a)^2*ln((1/(b*x+a)*a*d-1/(b*x+a)*b*c-d)^2/b^2*e)+1/4*b^3/g^5*A
*B*c^4/(a*d-b*c)^4/(b*x+a)^4-1/g^5*A*B*c*d^4/(a*d-b*c)^5*ln(1/(b*x+a)*a*d-1
/(b*x+a)*b*c-d)-1/g^5*A*B*c/(a*d-b*c)^4/(b*x+a)*d^3-1/4/b/g^5*B^2/(b*x+a)^4
*ln((1/(b*x+a)*a*d-1/(b*x+a)*b*c-d)^2/b^2*e)^2-1/3*b^2/g^5*A*B*c^3/(a*d-b*c
)^4/(b*x+a)^3*d-1/g^5*A*B*a^3*d^3/(a*d-b*c)^4/(b*x+a)^4*c-1/g^5*A*B*a*d^3/(
a*d-b*c)^4/(b*x+a)^2*c-1/g^5*A*B*a^2*d^3/(a*d-b*c)^4/(b*x+a)^3*c+1/b/g^5*A*
B*a*d^5/(a*d-b*c)^5*ln(1/(b*x+a)*a*d-1/(b*x+a)*b*c-d)+1/b/g^5*A*B*a*d^4/(a*
d-b*c)^4/(b*x+a)+1/2/b/g^5*A*B*a^2*d^4/(a*d-b*c)^4/(b*x+a)^2+1/4/b/g^5*A*B*
a^4*d^4/(a*d-b*c)^4/(b*x+a)^4+1/3/b/g^5*A*B*a^3*d^4/(a*d-b*c)^4/(b*x+a)^3+1
/2*b/g^5*A*B*c^2/(a*d-b*c)^4/(b*x+a)^2*d^2-1/4/b/(b*x+a)^4/g^5*A^2-1/8/b/g^
5*B^2/(b*x+a)^4-1/2/b/g^5*A*B/(b*x+a)^4*ln((1/(b*x+a)*a*d-1/(b*x+a)*b*c-d)^
2/b^2*e)-7/18/b/g^5*B^2*d/(a*d-b*c)/(b*x+a)^3-13/12/b/g^5*B^2*d^2/(a^2*d^2-
2*a*b*c*d+b^2*c^2)/(b*x+a)^2-25/6/b/g^5*d^3*B^2/(a^3*d^3-3*a^2*b*c*d^2+3*a*
b^2*c^2*d-b^3*c^3)/(b*x+a)-25/12/b/g^5*d^4*B^2/(a^4*d^4-4*a^3*b*c*d^3+6*a^2
*b^2*c^2*d^2-4*a*b^3*c^3*d+b^4*c^4)*ln((1/(b*x+a)*a*d-1/(b*x+a)*b*c-d)^2/b^
2*e)+1/4/b/g^5*d^4*B^2/(a^4*d^4-4*a^3*b*c*d^3+6*a^2*b^2*c^2*d^2-4*a*b^3*c^3
*d+b^4*c^4)*ln((1/(b*x+a)*a*d-1/(b*x+a)*b*c-d)^2/b^2*e)^2+3/2*b/g^5*A*B*a^2
*d^2/(a*d-b*c)^4/(b*x+a)^4*c^2-b^2/g^5*A*B*a*d/(a*d-b*c)^4/(b*x+a)^4*c^3+b/
g^5*A*B*a*d^2/(a*d-b*c)^4/(b*x+a)^3*c^2
```

**maxima [B]** time = 2.51, size = 2278, normalized size = 4.55

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*(d*x+c)^2/(b*x+a)^2))^2/(b*g*x+a*g)^5,x, algorithm="ma
xima")
```

```
[Out] -1/72*(6*((12*b^3*d^3*x^3 - 3*b^3*c^3 + 13*a*b^2*c^2*d - 23*a^2*b*c*d^2 + 2
5*a^3*d^3 - 6*(b^3*c*d^2 - 7*a*b^2*d^3)*x^2 + 4*(b^3*c^2*d - 5*a*b^2*c*d^2
+ 13*a^2*b*d^3)*x)/((b^8*c^3 - 3*a*b^7*c^2*d + 3*a^2*b^6*c*d^2 - a^3*b^5*d^
3)*g^5*x^4 + 4*(a*b^7*c^3 - 3*a^2*b^6*c^2*d + 3*a^3*b^5*c*d^2 - a^4*b^4*d^3
)*g^5*x^3 + 6*(a^2*b^6*c^3 - 3*a^3*b^5*c^2*d + 3*a^4*b^4*c*d^2 - a^5*b^3*d^
3)*g^5*x^2 + 4*(a^3*b^5*c^3 - 3*a^4*b^4*c^2*d + 3*a^5*b^3*c*d^2 - a^6*b^2*d
^3)*g^5*x + (a^4*b^4*c^3 - 3*a^5*b^3*c^2*d + 3*a^6*b^2*c*d^2 - a^7*b*d^3)*g
^5) + 12*d^4*log(b*x + a)/((b^5*c^4 - 4*a*b^4*c^3*d + 6*a^2*b^3*c^2*d^2 - 4
*a^3*b^2*c*d^3 + a^4*b*d^4)*g^5) - 12*d^4*log(d*x + c)/((b^5*c^4 - 4*a*b^4*
c^3*d + 6*a^2*b^3*c^2*d^2 - 4*a^3*b^2*c*d^3 + a^4*b*d^4)*g^5))*log(d^2*e*x^
2/(b^2*x^2 + 2*a*b*x + a^2) + 2*c*d*e*x/(b^2*x^2 + 2*a*b*x + a^2) + c^2*e/(
b^2*x^2 + 2*a*b*x + a^2)) + (9*b^4*c^4 - 64*a*b^3*c^3*d + 216*a^2*b^2*c^2*d
^2 - 576*a^3*b*c*d^3 + 415*a^4*d^4 - 300*(b^4*c*d^3 - a*b^3*d^4)*x^3 + 6*(1
3*b^4*c^2*d^2 - 176*a*b^3*c*d^3 + 163*a^2*b^2*d^4)*x^2 + 72*(b^4*d^4*x^4 +
4*a*b^3*d^4*x^3 + 6*a^2*b^2*d^4*x^2 + 4*a^3*b*d^4*x + a^4*d^4)*log(b*x + a)
^2 + 72*(b^4*d^4*x^4 + 4*a*b^3*d^4*x^3 + 6*a^2*b^2*d^4*x^2 + 4*a^3*b*d^4*x
+ a^4*d^4)*log(d*x + c)^2 - 4*(7*b^4*c^3*d - 60*a*b^3*c^2*d^2 + 324*a^2*b^2
*c*d^3 - 271*a^3*b*d^4)*x - 300*(b^4*d^4*x^4 + 4*a*b^3*d^4*x^3 + 6*a^2*b^2*
d^4*x^2 + 4*a^3*b*d^4*x + a^4*d^4)*log(b*x + a) + 12*(25*b^4*d^4*x^4 + 100*
a*b^3*d^4*x^3 + 150*a^2*b^2*d^4*x^2 + 100*a^3*b*d^4*x + 25*a^4*d^4 - 12*(b^
4*d^4*x^4 + 4*a*b^3*d^4*x^3 + 6*a^2*b^2*d^4*x^2 + 4*a^3*b*d^4*x + a^4*d^4)*
log(b*x + a))*log(d*x + c))/(a^4*b^5*c^4*g^5 - 4*a^5*b^4*c^3*d*g^5 + 6*a^6*
b^3*c^2*d^2*g^5 - 4*a^7*b^2*c*d^3*g^5 + a^8*b*d^4*g^5 + (b^9*c^4*g^5 - 4*a*
b^8*c^3*d*g^5 + 6*a^2*b^7*c^2*d^2*g^5 - 4*a^3*b^6*c*d^3*g^5 + a^4*b^5*d^4*g
^5)*x^4 + 4*(a*b^8*c^4*g^5 - 4*a^2*b^7*c^3*d*g^5 + 6*a^3*b^6*c^2*d^2*g^5 -
4*a^4*b^5*c*d^3*g^5 + a^5*b^4*d^4*g^5)*x^3 + 6*(a^2*b^7*c^4*g^5 - 4*a^3*b^6
*c^3*d*g^5 + 6*a^4*b^5*c^2*d^2*g^5 - 4*a^5*b^4*c*d^3*g^5 + a^6*b^3*d^4*g^5)
*x^2 + 4*(a^3*b^6*c^4*g^5 - 4*a^4*b^5*c^3*d*g^5 + 6*a^5*b^4*c^2*d^2*g^5 - 4
*a^6*b^3*c*d^3*g^5 + a^7*b^2*d^4*g^5)*x))*B^2 - 1/12*A*B*((12*b^3*d^3*x^3 -
 3*b^3*c^3 + 13*a*b^2*c^2*d - 23*a^2*b*c*d^2 + 25*a^3*d^3 - 6*(b^3*c*d^2 -
7*a*b^2*d^3)*x^2 + 4*(b^3*c^2*d - 5*a*b^2*c*d^2 + 13*a^2*b*d^3)*x)/((b^8*c^
3 - 3*a*b^7*c^2*d + 3*a^2*b^6*c*d^2 - a^3*b^5*d^3)*g^5*x^4 + 4*(a*b^7*c^3 -
 3*a^2*b^6*c^2*d + 3*a^3*b^5*c*d^2 - a^4*b^4*d^3)*g^5*x^3 + 6*(a^2*b^6*c^3
- 3*a^3*b^5*c^2*d + 3*a^4*b^4*c*d^2 - a^5*b^3*d^3)*g^5*x^2 + 4*(a^3*b^5*c^3
 - 3*a^4*b^4*c^2*d + 3*a^5*b^3*c*d^2 - a^6*b^2*d^3)*g^5*x + (a^4*b^4*c^3 -
3*a^5*b^3*c^2*d + 3*a^6*b^2*c*d^2 - a^7*b*d^3)*g^5) + 6*log(d^2*e*x^2/(b^2*
x^2 + 2*a*b*x + a^2) + 2*c*d*e*x/(b^2*x^2 + 2*a*b*x + a^2) + c^2*e/(b^2*x^2
 + 2*a*b*x + a^2))/(b^5*g^5*x^4 + 4*a*b^4*g^5*x^3 + 6*a^2*b^3*g^5*x^2 + 4*a
^3*b^2*g^5*x + a^4*b*g^5) + 12*d^4*log(b*x + a)/((b^5*c^4 - 4*a*b^4*c^3*d +
 6*a^2*b^3*c^2*d^2 - 4*a^3*b^2*c*d^3 + a^4*b*d^4)*g^5) - 12*d^4*log(d*x + c
)/((b^5*c^4 - 4*a*b^4*c^3*d + 6*a^2*b^3*c^2*d^2 - 4*a^3*b^2*c*d^3 + a^4*b*d
^4)*g^5)) - 1/4*B^2*log(d^2*e*x^2/(b^2*x^2 + 2*a*b*x + a^2) + 2*c*d*e*x/(b^
2*x^2 + 2*a*b*x + a^2) + c^2*e/(b^2*x^2 + 2*a*b*x + a^2))^2/(b^5*g^5*x^4 +
4*a*b^4*g^5*x^3 + 6*a^2*b^3*g^5*x^2 + 4*a^3*b^2*g^5*x + a^4*b*g^5) - 1/4*A^
2/(b^5*g^5*x^4 + 4*a*b^4*g^5*x^3 + 6*a^2*b^3*g^5*x^2 + 4*a^3*b^2*g^5*x + a^
```

```
4*b*g^5)
```

**mupad [B]** time = 12.11, size = 1882, normalized size = 3.76

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*log((e*(c + d*x)^2)/(a + b*x)^2))^2/(a*g + b*g*x)^5,x)

[Out] (log((e*(c + d*x)^2)/(a + b*x)^2)*((B^2*d^4*(a*(a*((4*a^2*d^2 + b^2*c^2 - 5
*a*b*c*d)/(6*b*d^3) + (a*(a*d - b*c))/(2*b*d^2)) + (6*a^3*d^3 - b^3*c^3 + 5
*a*b^2*c^2*d - 10*a^2*b*c*d^2)/(6*b*d^4)) + (4*a^4*d^4 + b^4*c^4 + 10*a^2*b
^2*c^2*d^2 - 5*a*b^3*c^3*d - 10*a^3*b*c*d^3)/(2*b*d^5)))/(2*b*g^5*(a^4*d^4
+ b^4*c^4 + 6*a^2*b^2*c^2*d^2 - 4*a*b^3*c^3*d - 4*a^3*b*c*d^3)) - (A*B)/(2*
b^2*d*g^5) + (B^2*d^4*x^2*(b*(b*((4*a^2*d^2 + b^2*c^2 - 5*a*b*c*d)/(6*b*d^3
) + (a*(a*d - b*c))/(2*b*d^2)) + (4*a^2*d^2 + b^2*c^2 - 5*a*b*c*d)/(3*d^3)
+ (a*(a*d - b*c))/d^2) - a*((b^2*c - a*b*d)/(2*d^2) - (b*(a*d - b*c))/d^2)
+ (b^3*c^2 + 4*a^2*b*d^2 - 5*a*b^2*c*d)/(2*d^3)))/(2*b*g^5*(a^4*d^4 + b^4*c
^4 + 6*a^2*b^2*c^2*d^2 - 4*a*b^3*c^3*d - 4*a^3*b*c*d^3)) - (B^2*d^4*x^3*(b*
((b^2*c - a*b*d)/(2*d^2) - (b*(a*d - b*c))/d^2) + (b^3*c - a*b^2*d)/(2*d^2)
))/(2*b*g^5*(a^4*d^4 + b^4*c^4 + 6*a^2*b^2*c^2*d^2 - 4*a*b^3*c^3*d - 4*a^3*
b*c*d^3)) + (B^2*d^4*x*(b*(a*((4*a^2*d^2 + b^2*c^2 - 5*a*b*c*d)/(6*b*d^3) +
 (a*(a*d - b*c))/(2*b*d^2)) + (6*a^3*d^3 - b^3*c^3 + 5*a*b^2*c^2*d - 10*a^2
*b*c*d^2)/(6*b*d^4)) + a*(b*((4*a^2*d^2 + b^2*c^2 - 5*a*b*c*d)/(6*b*d^3) +
(a*(a*d - b*c))/(2*b*d^2)) + (4*a^2*d^2 + b^2*c^2 - 5*a*b*c*d)/(3*d^3) + (a
*(a*d - b*c))/d^2) + (6*a^3*d^3 - b^3*c^3 + 5*a*b^2*c^2*d - 10*a^2*b*c*d^2)
/(2*d^4)))/(2*b*g^5*(a^4*d^4 + b^4*c^4 + 6*a^2*b^2*c^2*d^2 - 4*a*b^3*c^3*d
- 4*a^3*b*c*d^3))))/((4*a^3*x)/d + a^4/(b*d) + (b^3*x^4)/d + (6*a^2*b*x^2)/
d + (4*a*b^2*x^3)/d) - log((e*(c + d*x)^2)/(a + b*x)^2)^2*(B^2/(4*b^2*g^5*(
4*a^3*x + a^4/b + b^3*x^4 + 6*a^2*b*x^2 + 4*a*b^2*x^3)) - (B^2*d^4)/(4*b*g^
5*(a^4*d^4 + b^4*c^4 + 6*a^2*b^2*c^2*d^2 - 4*a*b^3*c^3*d - 4*a^3*b*c*d^3)))
 - ((18*A^2*a^3*d^3 - 18*A^2*b^3*c^3 + 415*B^2*a^3*d^3 - 9*B^2*b^3*c^3 - 15
0*A*B*a^3*d^3 + 18*A*B*b^3*c^3 + 54*A^2*a*b^2*c^2*d - 54*A^2*a^2*b*c*d^2 +
55*B^2*a*b^2*c^2*d - 161*B^2*a^2*b*c*d^2 - 78*A*B*a*b^2*c^2*d + 138*A*B*a^2
*b*c*d^2)/(12*(a*d - b*c)) + (x^2*(163*B^2*a*b^2*d^3 - 13*B^2*b^3*c*d^2 - 4
2*A*B*a*b^2*d^3 + 6*A*B*b^3*c*d^2))/(2*(a*d - b*c)) + (x*(271*B^2*a^2*b*d^3
 + 7*B^2*b^3*c^2*d - 53*B^2*a*b^2*c*d^2 - 78*A*B*a^2*b*d^3 - 6*A*B*b^3*c^2*
d + 30*A*B*a*b^2*c*d^2))/(3*(a*d - b*c)) + (d*x^3*(25*B^2*b^3*d^2 - 6*A*B*b
^3*d^2))/(a*d - b*c))/(x*(24*a^3*b^4*c^2*g^5 + 24*a^5*b^2*d^2*g^5 - 48*a^4*
b^3*c*d*g^5) + x^3*(24*a*b^6*c^2*g^5 + 24*a^3*b^4*d^2*g^5 - 48*a^2*b^5*c*d*
g^5) + x^4*(6*b^7*c^2*g^5 + 6*a^2*b^5*d^2*g^5 - 12*a*b^6*c*d*g^5) + x^2*(36
*a^2*b^5*c^2*g^5 + 36*a^4*b^3*d^2*g^5 - 72*a^3*b^4*c*d*g^5) + 6*a^6*b*d^2*g
^5 + 6*a^4*b^3*c^2*g^5 - 12*a^5*b^2*c*d*g^5) + (B*d^4*atan((B*d^4*(6*A - 25
*B)*(6*b^5*c^4*g^5 - 6*a^4*b*d^4*g^5 - 12*a*b^4*c^3*d*g^5 + 12*a^3*b^2*c*d^
3*g^5)*1i)/(6*b*g^5*(a*d - b*c)^4*(25*B^2*d^4 - 6*A*B*d^4)) + (B*d^5*x*(6*A
```

```
 - 25*B)*(b^4*c^3*g^5 - a^3*b*d^3*g^5 - 3*a*b^3*c^2*d*g^5 + 3*a^2*b^2*c*d^2
*g^5)*2i)/(g^5*(a*d - b*c)^4*(25*B^2*d^4 - 6*A*B*d^4)))*(6*A - 25*B)*1i)/(3
*b*g^5*(a*d - b*c)^4)
```

**sympy [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*ln(e*(d*x+c)**2/(b*x+a)**2))**2/(b*g*x+a*g)**5,x)
```

```
[Out] Timed out
```

# 3.219 $\int \frac{(ag+bgx)^2}{A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)}\,dx$

**Optimal**. Leaf size=37

$$\text{Int}\left(\frac{(ag+bgx)^2}{B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)+A},x\right)$$

```
[Out] Unintegrable((b*g*x+a*g)^2/(A+B*ln(e*(d*x+c)^2/(b*x+a)^2)),x)
```

**Rubi [A]** time = 0.20, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{(ag+bgx)^2}{A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(a*g + b*g*x)^2/(A + B*Log[(e*(c + d*x)^2)/(a + b*x)^2]),x]
```

```
[Out] a^2*g^2*Defer[Int][(A + B*Log[(e*(c + d*x)^2)/(a + b*x)^2])^(-1), x] + 2*a*
b*g^2*Defer[Int][x/(A + B*Log[(e*(c + d*x)^2)/(a + b*x)^2]), x] + b^2*g^2*D
efer[Int][x^2/(A + B*Log[(e*(c + d*x)^2)/(a + b*x)^2]), x]
```

Rubi steps

$$\begin{aligned}\int \frac{(ag+bgx)^2}{A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)}\,dx &= \int \left(\frac{a^2g^2}{A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)}+\frac{2abg^2x}{A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)}+\frac{b^2g^2x^2}{A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)}\right)dx\\ &= \left(a^2g^2\right)\int \frac{1}{A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)}\,dx+\left(2abg^2\right)\int \frac{x}{A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)}\,dx+\left(b^2g^2\right)\int \frac{}{A}\end{aligned}$$

**Mathematica [A]** time = 0.16, size = 0, normalized size = 0.00

$$\int \frac{(ag+bgx)^2}{A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(a*g + b*g*x)^2/(A + B*Log[(e*(c + d*x)^2)/(a + b*x)^2]),x]
```

```
[Out] Integrate[(a*g + b*g*x)^2/(A + B*Log[(e*(c + d*x)^2)/(a + b*x)^2]), x]
```

**fricas [A]** time = 0.70, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{b^2g^2x^2 + 2\,abg^2x + a^2g^2}{B\log\left(\frac{d^2ex^2+2\,cdex+c^2e}{b^2x^2+2\,abx+a^2}\right) + A}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*g*x+a*g)^2/(A+B*log(e*(d*x+c)^2/(b*x+a)^2)),x, algorithm="fric
as")
```

```
[Out] integral((b^2*g^2*x^2 + 2*a*b*g^2*x + a^2*g^2)/(B*log((d^2*e*x^2 + 2*c*d*e*
x + c^2*e)/(b^2*x^2 + 2*a*b*x + a^2)) + A), x)
```

**giac [A]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(bgx + ag\right)^2}{B\log\left(\frac{(dx+c)^2e}{(bx+a)^2}\right) + A}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*g*x+a*g)^2/(A+B*log(e*(d*x+c)^2/(b*x+a)^2)),x, algorithm="giac
")
```

```
[Out] integrate((b*g*x + a*g)^2/(B*log((d*x + c)^2*e/(b*x + a)^2) + A), x)
```

**maple [A]** time = 0.80, size = 0, normalized size = 0.00

$$\int \frac{\left(bgx + ag\right)^2}{B\ln\left(\frac{(dx+c)^2e}{(bx+a)^2}\right) + A}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*g*x+a*g)^2/(B*ln((d*x+c)^2/(b*x+a)^2*e)+A),x)
```

```
[Out] int((b*g*x+a*g)^2/(B*ln((d*x+c)^2/(b*x+a)^2*e)+A),x)
```

**maxima [A]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(bgx + ag\right)^2}{B\log\left(\frac{(dx+c)^2e}{(bx+a)^2}\right) + A}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*g*x+a*g)^2/(A+B*log(e*(d*x+c)^2/(b*x+a)^2)),x, algorithm="maxi
ma")
```

```
[Out] integrate((b*g*x + a*g)^2/(B*log((d*x + c)^2*e/(b*x + a)^2) + A), x)
```

**mupad [A]** time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{\left(a\,g+b\,g\,x\right)^2}{A+B\,\ln\left(\frac{e\,\left(c+d\,x\right)^2}{\left(a+b\,x\right)^2}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*g + b*g*x)^2/(A + B*log((e*(c + d*x)^2)/(a + b*x)^2)),x)
```

```
[Out] int((a*g + b*g*x)^2/(A + B*log((e*(c + d*x)^2)/(a + b*x)^2)), x)
```

**sympy [A]** time = 0.00, size = 0, normalized size = 0.00

$$g^2\left(\int \frac{a^2}{A+B\log\left(\frac{c^2e}{a^2+2abx+b^2x^2}+\frac{2cdex}{a^2+2abx+b^2x^2}+\frac{d^2ex^2}{a^2+2abx+b^2x^2}\right)}\,dx+\int \frac{b^2x^2}{A+B\log\left(\frac{c^2e}{a^2+2abx+b^2x^2}+\frac{2cdex}{a^2+2abx+b^2x^2}+\frac{}{a^2+2}\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*g*x+a*g)**2/(A+B*ln(e*(d*x+c)**2/(b*x+a)**2)),x)
```

```
[Out] g**2*(Integral(a**2/(A + B*log(c**2*e/(a**2 + 2*a*b*x + b**2*x**2) + 2*c*d*
e*x/(a**2 + 2*a*b*x + b**2*x**2) + d**2*e*x**2/(a**2 + 2*a*b*x + b**2*x**2)
)), x) + Integral(b**2*x**2/(A + B*log(c**2*e/(a**2 + 2*a*b*x + b**2*x**2)
+ 2*c*d*e*x/(a**2 + 2*a*b*x + b**2*x**2) + d**2*e*x**2/(a**2 + 2*a*b*x + b*
*2*x**2))), x) + Integral(2*a*b*x/(A + B*log(c**2*e/(a**2 + 2*a*b*x + b**2*
x**2) + 2*c*d*e*x/(a**2 + 2*a*b*x + b**2*x**2) + d**2*e*x**2/(a**2 + 2*a*b*
x + b**2*x**2))), x))
```

# 3.220 $\int \frac{ag+bgx}{A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)}\,dx$

**Optimal**. Leaf size=35

$$\text{Int}\left(\frac{ag+bgx}{B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)+A},x\right)$$

```
[Out] Unintegrable((b*g*x+a*g)/(A+B*ln(e*(d*x+c)^2/(b*x+a)^2)),x)
```

**Rubi [A]** time = 0.10, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{ag+bgx}{A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(a*g + b*g*x)/(A + B*Log[(e*(c + d*x)^2)/(a + b*x)^2]),x]
```

```
[Out] a*g*Defer[Int][(A + B*Log[(e*(c + d*x)^2)/(a + b*x)^2])^(-1), x] + b*g*Defe
r[Int][x/(A + B*Log[(e*(c + d*x)^2)/(a + b*x)^2]), x]
```

Rubi steps

$$\begin{aligned}\int \frac{ag+bgx}{A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)}\,dx &= \int \left(\frac{ag}{A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)}+\frac{bgx}{A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)}\right)dx\\ &= (ag)\int \frac{1}{A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)}\,dx+(bg)\int \frac{x}{A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)}\,dx\end{aligned}$$

**Mathematica [A]** time = 0.12, size = 0, normalized size = 0.00

$$\int \frac{ag+bgx}{A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(a*g + b*g*x)/(A + B*Log[(e*(c + d*x)^2)/(a + b*x)^2]),x]
```

```
[Out] Integrate[(a*g + b*g*x)/(A + B*Log[(e*(c + d*x)^2)/(a + b*x)^2]), x]
```

**fricas [A]** time = 0.74, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{bgx+ag}{B\log\left(\frac{d^2ex^2+2\,cdex+c^2e}{b^2x^2+2\,abx+a^2}\right)+A},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*g*x+a*g)/(A+B*log(e*(d*x+c)^2/(b*x+a)^2)),x, algorithm="fricas
")
```

```
[Out] integral((b*g*x + a*g)/(B*log((d^2*e*x^2 + 2*c*d*e*x + c^2*e)/(b^2*x^2 + 2*
a*b*x + a^2)) + A), x)
```

**giac [A]** time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{bgx+ag}{B\log\left(\frac{(dx+c)^2e}{(bx+a)^2}\right)+A}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*g*x+a*g)/(A+B*log(e*(d*x+c)^2/(b*x+a)^2)),x, algorithm="giac")
```

```
[Out] integrate((b*g*x + a*g)/(B*log((d*x + c)^2*e/(b*x + a)^2) + A), x)
```

**maple [A]** time = 0.65, size = 0, normalized size = 0.00

$$\int\frac{bgx+ag}{B\ln\left(\frac{(dx+c)^2e}{(bx+a)^2}\right)+A}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*g*x+a*g)/(B*ln((d*x+c)^2/(b*x+a)^2*e)+A),x)
```

```
[Out] int((b*g*x+a*g)/(B*ln((d*x+c)^2/(b*x+a)^2*e)+A),x)
```

**maxima [A]** time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{bgx+ag}{B\log\left(\frac{(dx+c)^2e}{(bx+a)^2}\right)+A}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*g*x+a*g)/(A+B*log(e*(d*x+c)^2/(b*x+a)^2)),x, algorithm="maxima
")
```

```
[Out] integrate((b*g*x + a*g)/(B*log((d*x + c)^2*e/(b*x + a)^2) + A), x)
```

**mupad [A]** time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{a\,g+b\,g\,x}{A+B\,\ln\left(\frac{e\,{\left(c+d\,x\right)}^2}{{\left(a+b\,x\right)}^2}\right)} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*g + b*g*x)/(A + B*log((e*(c + d*x)^2)/(a + b*x)^2)),x)
```

```
[Out] int((a*g + b*g*x)/(A + B*log((e*(c + d*x)^2)/(a + b*x)^2)), x)
```

**sympy [A]** time = 0.00, size = 0, normalized size = 0.00

$$g\left(\int \frac{a}{A+B\log\left(\frac{c^2e}{a^2+2abx+b^2x^2}+\frac{2cdex}{a^2+2abx+b^2x^2}+\frac{d^2ex^2}{a^2+2abx+b^2x^2}\right)}\,dx+\int \frac{bx}{A+B\log\left(\frac{c^2e}{a^2+2abx+b^2x^2}+\frac{2cdex}{a^2+2abx+b^2x^2}+\frac{}{a^2+}\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*g*x+a*g)/(A+B*ln(e*(d*x+c)**2/(b*x+a)**2)),x)
```

```
[Out] g*(Integral(a/(A + B*log(c**2*e/(a**2 + 2*a*b*x + b**2*x**2) + 2*c*d*e*x/(a
**2 + 2*a*b*x + b**2*x**2) + d**2*e*x**2/(a**2 + 2*a*b*x + b**2*x**2))), x)
 + Integral(b*x/(A + B*log(c**2*e/(a**2 + 2*a*b*x + b**2*x**2) + 2*c*d*e*x/
(a**2 + 2*a*b*x + b**2*x**2) + d**2*e*x**2/(a**2 + 2*a*b*x + b**2*x**2))),
x))
```

# 3.221 $\int \frac{1}{(ag+bgx)\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)}\,dx$

**Optimal**. Leaf size=37

$$\text{Int}\left(\frac{1}{(ag+bgx)\left(B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)+A\right)},x\right)$$

```
[Out] Unintegrable(1/(b*g*x+a*g)/(A+B*ln(e*(d*x+c)^2/(b*x+a)^2)),x)
```

**Rubi [A]** time = 0.07, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{1}{(ag+bgx)\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[1/((a*g + b*g*x)*(A + B*Log[(e*(c + d*x)^2)/(a + b*x)^2])),x]
[Out] Defer[Int][1/((a*g + b*g*x)*(A + B*Log[(e*(c + d*x)^2)/(a + b*x)^2])), x]
```

Rubi steps

$$\int \frac{1}{(ag+bgx)\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)}\,dx=\int \frac{1}{(ag+bgx)\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)}\,dx$$

**Mathematica [A]** time = 0.07, size = 0, normalized size = 0.00

$$\int \frac{1}{(ag+bgx)\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[1/((a*g + b*g*x)*(A + B*Log[(e*(c + d*x)^2)/(a + b*x)^2])),x]
[Out] Integrate[1/((a*g + b*g*x)*(A + B*Log[(e*(c + d*x)^2)/(a + b*x)^2])), x]
```

**fricas** [A] time = 0.62, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{1}{Abgx + Aag + \left(Bbgx + Bag\right)\log\left(\frac{d^2ex^2+2\,cdex+c^2e}{b^2x^2+2\,abx+a^2}\right)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*g*x+a*g)/(A+B*log(e*(d*x+c)^2/(b*x+a)^2)),x, algorithm="fric
as")
```

```
[Out] integral(1/(A*b*g*x + A*a*g + (B*b*g*x + B*a*g)*log((d^2*e*x^2 + 2*c*d*e*x
+ c^2*e)/(b^2*x^2 + 2*a*b*x + a^2))), x)
```

**giac** [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(bgx + ag\right)\left(B\log\left(\frac{(dx+c)^2e}{(bx+a)^2}\right) + A\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*g*x+a*g)/(A+B*log(e*(d*x+c)^2/(b*x+a)^2)),x, algorithm="giac
")
```

```
[Out] integrate(1/((b*g*x + a*g)*(B*log((d*x + c)^2*e/(b*x + a)^2) + A)), x)
```

**maple** [A] time = 0.83, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(bgx + ag\right)\left(B\ln\left(\frac{(dx+c)^2e}{(bx+a)^2}\right) + A\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*g*x+a*g)/(B*ln((d*x+c)^2/(b*x+a)^2*e)+A),x)
```

```
[Out] int(1/(b*g*x+a*g)/(B*ln((d*x+c)^2/(b*x+a)^2*e)+A),x)
```

**maxima** [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(bgx + ag\right)\left(B\log\left(\frac{(dx+c)^2e}{(bx+a)^2}\right) + A\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*g*x+a*g)/(A+B*log(e*(d*x+c)^2/(b*x+a)^2)),x, algorithm="maxi
ma")
```

```
[Out] integrate(1/((b*g*x + a*g)*(B*log((d*x + c)^2*e/(b*x + a)^2) + A)), x)
```

**mupad [A]** time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{1}{\left(a\,g+b\,g\,x\right)\,\left(A+B\,\ln\left(\frac{e\,{\left(c+d\,x\right)}^2}{{\left(a+b\,x\right)}^2}\right)\right)} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*g + b*g*x)*(A + B*log((e*(c + d*x)^2)/(a + b*x)^2))),x)
```

```
[Out] int(1/((a*g + b*g*x)*(A + B*log((e*(c + d*x)^2)/(a + b*x)^2))), x)
```

**sympy [A]** time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{1}{Aa+Abx+Ba\log\left(\frac{c^2e}{a^2+2abx+b^2x^2}+\frac{2cdex}{a^2+2abx+b^2x^2}+\frac{d^2ex^2}{a^2+2abx+b^2x^2}\right)+Bbx\log\left(\frac{c^2e}{a^2+2abx+b^2x^2}+\frac{2cdex}{a^2+2abx+b^2x^2}+\frac{d^2ex^2}{a^2+2abx+b^2x^2}\right)}\,dx}{g}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*g*x+a*g)/(A+B*ln(e*(d*x+c)**2/(b*x+a)**2)),x)
```

```
[Out] Integral(1/(A*a + A*b*x + B*a*log(c**2*e/(a**2 + 2*a*b*x + b**2*x**2) + 2*c
*d*e*x/(a**2 + 2*a*b*x + b**2*x**2) + d**2*e*x**2/(a**2 + 2*a*b*x + b**2*x*
*2)) + B*b*x*log(c**2*e/(a**2 + 2*a*b*x + b**2*x**2) + 2*c*d*e*x/(a**2 + 2*
a*b*x + b**2*x**2) + d**2*e*x**2/(a**2 + 2*a*b*x + b**2*x**2))), x)/g
```

## 3.222 $\int \frac{1}{(ag+bgx)^2\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)}\,dx$

**Optimal**. Leaf size=91

$$-\frac{e^{-\frac{A}{2B}}(c+dx)\operatorname{Ei}\left(\frac{A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)}{2B}\right)}{2Bg^2(a+bx)(bc-ad)\sqrt{\frac{e(c+dx)^2}{(a+bx)^2}}}$$

```
[Out] -1/2*(d*x+c)*Ei(1/2*(A+B*ln(e*(d*x+c)^2/(b*x+a)^2))/B)/B/(-a*d+b*c)/exp(1/2
*A/B)/g^2/(b*x+a)/(e*(d*x+c)^2/(b*x+a)^2)^(1/2)
```

**Rubi [F]** time = 0.09, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{1}{(ag+bgx)^2\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[1/((a*g + b*g*x)^2*(A + B*Log[(e*(c + d*x)^2)/(a + b*x)^2])),x]
```

```
[Out] Defer[Int][1/((a*g + b*g*x)^2*(A + B*Log[(e*(c + d*x)^2)/(a + b*x)^2])), x]
```

Rubi steps

$$\int \frac{1}{(ag+bgx)^2\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)}\,dx=\int \frac{1}{(ag+bgx)^2\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)}\,dx$$

**Mathematica [F]** time = 0.08, size = 0, normalized size = 0.00

$$\int \frac{1}{(ag+bgx)^2\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[1/((a*g + b*g*x)^2*(A + B*Log[(e*(c + d*x)^2)/(a + b*x)^2])),x]
```

```
[Out] Integrate[1/((a*g + b*g*x)^2*(A + B*Log[(e*(c + d*x)^2)/(a + b*x)^2])), x]
```

**fricas [F]** time = 0.65, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{1}{Ab^2g^2x^2 + 2\,Aabg^2x + Aa^2g^2 + \left(Bb^2g^2x^2 + 2\,Babg^2x + Ba^2g^2\right)\log\left(\frac{d^2ex^2+2\,cdex+c^2e}{b^2x^2+2\,abx+a^2}\right)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*g*x+a*g)^2/(A+B*log(e*(d*x+c)^2/(b*x+a)^2)),x, algorithm="fr
icas")
```

```
[Out] integral(1/(A*b^2*g^2*x^2 + 2*A*a*b*g^2*x + A*a^2*g^2 + (B*b^2*g^2*x^2 + 2*
B*a*b*g^2*x + B*a^2*g^2)*log((d^2*e*x^2 + 2*c*d*e*x + c^2*e)/(b^2*x^2 + 2*a
*b*x + a^2))), x)
```

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(bgx + ag\right)^2\left(B\log\left(\frac{(dx+c)^2e}{(bx+a)^2}\right) + A\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*g*x+a*g)^2/(A+B*log(e*(d*x+c)^2/(b*x+a)^2)),x, algorithm="gi
ac")
```

```
[Out] integrate(1/((b*g*x + a*g)^2*(B*log((d*x + c)^2*e/(b*x + a)^2) + A)), x)
```

**maple [F]** time = 0.87, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(bgx + ag\right)^2\left(B\ln\left(\frac{(dx+c)^2e}{(bx+a)^2}\right) + A\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*g*x+a*g)^2/(B*ln((d*x+c)^2/(b*x+a)^2*e)+A),x)
```

```
[Out] int(1/(b*g*x+a*g)^2/(B*ln((d*x+c)^2/(b*x+a)^2*e)+A),x)
```

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(bgx + ag\right)^2\left(B\log\left(\frac{(dx+c)^2e}{(bx+a)^2}\right) + A\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*g*x+a*g)^2/(A+B*log(e*(d*x+c)^2/(b*x+a)^2)),x, algorithm="ma
xima")
```

```
[Out] integrate(1/((b*g*x + a*g)^2*(B*log((d*x + c)^2*e/(b*x + a)^2) + A)), x)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{{\left(a\,g+b\,g\,x\right)}^2\,\left(A+B\,\ln\left(\frac{e\,{\left(c+d\,x\right)}^2}{{\left(a+b\,x\right)}^2}\right)\right)} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*g + b*g*x)^2*(A + B*log((e*(c + d*x)^2)/(a + b*x)^2))),x)
```

```
[Out] int(1/((a*g + b*g*x)^2*(A + B*log((e*(c + d*x)^2)/(a + b*x)^2))), x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{1}{Aa^2+2Aabx+Ab^2x^2+Ba^2\log\left(\frac{c^2e}{a^2+2abx+b^2x^2}+\frac{2cdex}{a^2+2abx+b^2x^2}+\frac{d^2ex^2}{a^2+2abx+b^2x^2}\right)+2Babx\log\left(\frac{c^2e}{a^2+2abx+b^2x^2}+\frac{2cdex}{a^2+2abx+b^2x^2}+\frac{d^2ex^2}{a^2+2abx+b^2x^2}\right)+Bb^2x^2\log\left(\right.}}{g^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*g*x+a*g)**2/(A+B*ln(e*(d*x+c)**2/(b*x+a)**2)),x)
```

```
[Out] Integral(1/(A*a**2 + 2*A*a*b*x + A*b**2*x**2 + B*a**2*log(c**2*e/(a**2 + 2*
a*b*x + b**2*x**2) + 2*c*d*e*x/(a**2 + 2*a*b*x + b**2*x**2) + d**2*e*x**2/(
a**2 + 2*a*b*x + b**2*x**2)) + 2*B*a*b*x*log(c**2*e/(a**2 + 2*a*b*x + b**2*
x**2) + 2*c*d*e*x/(a**2 + 2*a*b*x + b**2*x**2) + d**2*e*x**2/(a**2 + 2*a*b*
x + b**2*x**2)) + B*b**2*x**2*log(c**2*e/(a**2 + 2*a*b*x + b**2*x**2) + 2*c
*d*e*x/(a**2 + 2*a*b*x + b**2*x**2) + d**2*e*x**2/(a**2 + 2*a*b*x + b**2*x*
*2))), x)/g**2
```

# 3.223 $\int \frac{1}{(ag+bgx)^3\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)}\,dx$

**Optimal**. Leaf size=151

$$\frac{de^{-\frac{A}{2B}}(c+dx)\text{Ei}\left(\frac{A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)}{2B}\right)}{2Bg^3(a+bx)(bc-ad)^2\sqrt{\frac{e(c+dx)^2}{(a+bx)^2}}}-\frac{be^{-\frac{A}{B}}\text{Ei}\left(\frac{A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)}{B}\right)}{2Beg^3(bc-ad)^2}$$

```
[Out] -1/2*b*Ei((A+B*ln(e*(d*x+c)^2/(b*x+a)^2))/B)/B/(-a*d+b*c)^2/e/exp(A/B)/g^3+
1/2*d*(d*x+c)*Ei(1/2*(A+B*ln(e*(d*x+c)^2/(b*x+a)^2))/B)/B/(-a*d+b*c)^2/exp(
1/2*A/B)/g^3/(b*x+a)/(e*(d*x+c)^2/(b*x+a)^2)^(1/2)
```

**Rubi [F]** time = 0.07, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{1}{(ag+bgx)^3\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[1/((a*g + b*g*x)^3*(A + B*Log[(e*(c + d*x)^2)/(a + b*x)^2])),x]
```

```
[Out] Defer[Int][1/((a*g + b*g*x)^3*(A + B*Log[(e*(c + d*x)^2)/(a + b*x)^2])), x]
```

Rubi steps

$$\int \frac{1}{(ag+bgx)^3\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)}\,dx=\int \frac{1}{(ag+bgx)^3\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)}\,dx$$

**Mathematica [F]** time = 0.08, size = 0, normalized size = 0.00

$$\int \frac{1}{(ag+bgx)^3\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[1/((a*g + b*g*x)^3*(A + B*Log[(e*(c + d*x)^2)/(a + b*x)^2])),x]
```

```
[Out] Integrate[1/((a*g + b*g*x)^3*(A + B*Log[(e*(c + d*x)^2)/(a + b*x)^2])), x]
```

**fricas [F]** time = 1.93, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{1}{Ab^3g^3x^3 + 3\,Aab^2g^3x^2 + 3\,Aa^2bg^3x + Aa^3g^3 + \left(Bb^3g^3x^3 + 3\,Bab^2g^3x^2 + 3\,Ba^2bg^3x + Ba^3g^3\right)\log\left(\frac{d^2 \cdots}{b \cdots}\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*g*x+a*g)^3/(A+B*log(e*(d*x+c)^2/(b*x+a)^2)),x, algorithm="fr
icas")
```

```
[Out] integral(1/(A*b^3*g^3*x^3 + 3*A*a*b^2*g^3*x^2 + 3*A*a^2*b*g^3*x + A*a^3*g^3
 + (B*b^3*g^3*x^3 + 3*B*a*b^2*g^3*x^2 + 3*B*a^2*b*g^3*x + B*a^3*g^3)*log((d
^2*e*x^2 + 2*c*d*e*x + c^2*e)/(b^2*x^2 + 2*a*b*x + a^2))), x)
```

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(bgx + ag\right)^3 \left(B\log\left(\frac{(dx+c)^2e}{(bx+a)^2}\right) + A\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*g*x+a*g)^3/(A+B*log(e*(d*x+c)^2/(b*x+a)^2)),x, algorithm="gi
ac")
```

```
[Out] integrate(1/((b*g*x + a*g)^3*(B*log((d*x + c)^2*e/(b*x + a)^2) + A)), x)
```

**maple [F]** time = 0.98, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(bgx + ag\right)^3 \left(B\ln\left(\frac{(dx+c)^2e}{(bx+a)^2}\right) + A\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*g*x+a*g)^3/(B*ln((d*x+c)^2/(b*x+a)^2*e)+A),x)
```

```
[Out] int(1/(b*g*x+a*g)^3/(B*ln((d*x+c)^2/(b*x+a)^2*e)+A),x)
```

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(bgx + ag\right)^3 \left(B\log\left(\frac{(dx+c)^2e}{(bx+a)^2}\right) + A\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*g*x+a*g)^3/(A+B*log(e*(d*x+c)^2/(b*x+a)^2)),x, algorithm="ma
xima")
```

```
[Out] integrate(1/((b*g*x + a*g)^3*(B*log((d*x + c)^2*e/(b*x + a)^2) + A)), x)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{{\left(a\,g+b\,g\,x\right)}^3\,\left(A+B\,\ln\left(\frac{e\,{\left(c+d\,x\right)}^2}{{\left(a+b\,x\right)}^2}\right)\right)} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*g + b*g*x)^3*(A + B*log((e*(c + d*x)^2)/(a + b*x)^2))),x)
```

```
[Out] int(1/((a*g + b*g*x)^3*(A + B*log((e*(c + d*x)^2)/(a + b*x)^2))), x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{1}{Aa^3+3Aa^2bx+3Aab^2x^2+Ab^3x^3+Ba^3\log\left(\frac{c^2e}{a^2+2abx+b^2x^2}+\frac{2cdex}{a^2+2abx+b^2x^2}+\frac{d^2ex^2}{a^2+2abx+b^2x^2}\right)+3Ba^2bx\log\left(\frac{c^2e}{a^2+2abx+b^2x^2}+\frac{2cdex}{a^2+2abx+b^2x^2}+\frac{d^2ex^2}{a^2+2abx+b^2x^2}\right)+3}}{g^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*g*x+a*g)**3/(A+B*ln(e*(d*x+c)**2/(b*x+a)**2)),x)
```

```
[Out] Integral(1/(A*a**3 + 3*A*a**2*b*x + 3*A*a*b**2*x**2 + A*b**3*x**3 + B*a**3*
log(c**2*e/(a**2 + 2*a*b*x + b**2*x**2) + 2*c*d*e*x/(a**2 + 2*a*b*x + b**2*
x**2) + d**2*e*x**2/(a**2 + 2*a*b*x + b**2*x**2)) + 3*B*a**2*b*x*log(c**2*e
/(a**2 + 2*a*b*x + b**2*x**2) + 2*c*d*e*x/(a**2 + 2*a*b*x + b**2*x**2) + d*
*2*e*x**2/(a**2 + 2*a*b*x + b**2*x**2)) + 3*B*a*b**2*x**2*log(c**2*e/(a**2
+ 2*a*b*x + b**2*x**2) + 2*c*d*e*x/(a**2 + 2*a*b*x + b**2*x**2) + d**2*e*x*
*2/(a**2 + 2*a*b*x + b**2*x**2)) + B*b**3*x**3*log(c**2*e/(a**2 + 2*a*b*x +
 b**2*x**2) + 2*c*d*e*x/(a**2 + 2*a*b*x + b**2*x**2) + d**2*e*x**2/(a**2 +
2*a*b*x + b**2*x**2))), x)/g**3
```

# 3.224 $\int \frac{(ag+bgx)^2}{\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2}\,dx$

**Optimal**. Leaf size=37

$$\text{Int}\left(\frac{(ag+bgx)^2}{\left(B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)+A\right)^2},x\right)$$

```
[Out] Unintegrable((b*g*x+a*g)^2/(A+B*ln(e*(d*x+c)^2/(b*x+a)^2))^2,x)
```

**Rubi [A]** time = 0.21, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{(ag+bgx)^2}{\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(a*g + b*g*x)^2/(A + B*Log[(e*(c + d*x)^2)/(a + b*x)^2])^2,x]
```

```
[Out] a^2*g^2*Defer[Int][(A + B*Log[(e*(c + d*x)^2)/(a + b*x)^2])^(-2), x] + 2*a*
b*g^2*Defer[Int][x/(A + B*Log[(e*(c + d*x)^2)/(a + b*x)^2])^2, x] + b^2*g^2
*Defer[Int][x^2/(A + B*Log[(e*(c + d*x)^2)/(a + b*x)^2])^2, x]
```

Rubi steps

$$\begin{aligned}\int \frac{(ag+bgx)^2}{\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2}\,dx &= \int \left(\frac{a^2g^2}{\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2}+\frac{2abg^2x}{\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2}+\frac{b^2g^2x^2}{\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2}\right.\\ &= \left(a^2g^2\right)\int \frac{1}{\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2}\,dx+\left(2abg^2\right)\int \frac{x}{\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2}\,dx+\end{aligned}$$

**Mathematica [A]** time = 0.47, size = 0, normalized size = 0.00

$$\int \frac{(ag+bgx)^2}{\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(a*g + b*g*x)^2/(A + B*Log[(e*(c + d*x)^2)/(a + b*x)^2])^2,x]
```

```
[Out] Integrate[(a*g + b*g*x)^2/(A + B*Log[(e*(c + d*x)^2)/(a + b*x)^2])^2, x]
```

**fricas [A]** time = 0.77, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{b^2g^2x^2 + 2\,abg^2x + a^2g^2}{B^2\log\left(\frac{d^2ex^2+2\,cdex+c^2e}{b^2x^2+2\,abx+a^2}\right)^2 + 2\,AB\log\left(\frac{d^2ex^2+2\,cdex+c^2e}{b^2x^2+2\,abx+a^2}\right) + A^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*g*x+a*g)^2/(A+B*log(e*(d*x+c)^2/(b*x+a)^2))^2,x, algorithm="fr
icas")
```

```
[Out] integral((b^2*g^2*x^2 + 2*a*b*g^2*x + a^2*g^2)/(B^2*log((d^2*e*x^2 + 2*c*d*
e*x + c^2*e)/(b^2*x^2 + 2*a*b*x + a^2))^2 + 2*A*B*log((d^2*e*x^2 + 2*c*d*e*
x + c^2*e)/(b^2*x^2 + 2*a*b*x + a^2)) + A^2), x)
```

**giac [A]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(bgx + ag)^2}{\left(B\log\left(\frac{(dx+c)^2e}{(bx+a)^2}\right) + A\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*g*x+a*g)^2/(A+B*log(e*(d*x+c)^2/(b*x+a)^2))^2,x, algorithm="gi
ac")
```

```
[Out] integrate((b*g*x + a*g)^2/(B*log((d*x + c)^2*e/(b*x + a)^2) + A)^2, x)
```

**maple [A]** time = 0.76, size = 0, normalized size = 0.00

$$\int \frac{(bgx + ag)^2}{\left(B\ln\left(\frac{(dx+c)^2e}{(bx+a)^2}\right) + A\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*g*x+a*g)^2/(B*ln((d*x+c)^2/(b*x+a)^2*e)+A)^2,x)
```

```
[Out] int((b*g*x+a*g)^2/(B*ln((d*x+c)^2/(b*x+a)^2*e)+A)^2,x)
```

**maxima [A]** time = 0.00, size = 0, normalized size = 0.00

$$-\frac{b^3dg^2x^4 + a^3cg^2 + \left(b^3cg^2 + 3\,ab^2dg^2\right)x^3 + 3\left(ab^2cg^2 + a^2bdg^2\right)x^2 + \left(3\,a^2bcg^2 + a^3dg^2\right)x}{2\left(2\,(bc - ad)B^2\log\left(bx + a\right) - 2\,(bc - ad)B^2\log\left(dx + c\right) - (bc - ad)AB - \left(bc\log(e) - ad\log(e)\right)B^2\right)} + \int \frac{\quad}{2\,(2\,(\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*g*x+a*g)^2/(A+B*log(e*(d*x+c)^2/(b*x+a)^2))^2,x, algorithm="ma
xima")
```

```
[Out] -1/2*(b^3*d*g^2*x^4 + a^3*c*g^2 + (b^3*c*g^2 + 3*a*b^2*d*g^2)*x^3 + 3*(a*b^
2*c*g^2 + a^2*b*d*g^2)*x^2 + (3*a^2*b*c*g^2 + a^3*d*g^2)*x)/(2*(b*c - a*d)*
B^2*log(b*x + a) - 2*(b*c - a*d)*B^2*log(d*x + c) - (b*c - a*d)*A*B - (b*c*
log(e) - a*d*log(e))*B^2) + integrate(1/2*(4*b^3*d*g^2*x^3 + 3*a^2*b*c*g^2
+ a^3*d*g^2 + 3*(b^3*c*g^2 + 3*a*b^2*d*g^2)*x^2 + 6*(a*b^2*c*g^2 + a^2*b*d*
g^2)*x)/(2*(b*c - a*d)*B^2*log(b*x + a) - 2*(b*c - a*d)*B^2*log(d*x + c) -
(b*c - a*d)*A*B - (b*c*log(e) - a*d*log(e))*B^2), x)
```

**mupad [A]** time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{{\left(a\,g+b\,g\,x\right)}^2}{{\left(A+B\,\ln\left(\frac{e\,{\left(c+d\,x\right)}^2}{{\left(a+b\,x\right)}^2}\right)\right)}^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*g + b*g*x)^2/(A + B*log((e*(c + d*x)^2)/(a + b*x)^2))^2,x)
```

```
[Out] int((a*g + b*g*x)^2/(A + B*log((e*(c + d*x)^2)/(a + b*x)^2))^2, x)
```

**sympy [A]** time = 0.00, size = 0, normalized size = 0.00

$$\frac{-a^3cg^2 - a^3dg^2x - 3a^2bcg^2x - 3a^2bdg^2x^2 - 3ab^2cg^2x^2 - 3ab^2dg^2x^3 - b^3cg^2x^3 - b^3dg^2x^4}{2ABad - 2ABbc + \left(2B^2ad - 2B^2bc\right)\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)} + \frac{g^2\left(\int \frac{\quad}{A+B\log\left(\frac{c^2e}{a^2+2abx+\cdots}\right)}\cdots\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*g*x+a*g)**2/(A+B*ln(e*(d*x+c)**2/(b*x+a)**2))**2,x)
```

```
[Out] (-a**3*c*g**2 - a**3*d*g**2*x - 3*a**2*b*c*g**2*x - 3*a**2*b*d*g**2*x**2 -
3*a*b**2*c*g**2*x**2 - 3*a*b**2*d*g**2*x**3 - b**3*c*g**2*x**3 - b**3*d*g**
2*x**4)/(2*A*B*a*d - 2*A*B*b*c + (2*B**2*a*d - 2*B**2*b*c)*log(e*(c + d*x)*
```

```
*2/(a + b*x)**2)) + g**2*(Integral(a**3*d/(A + B*log(c**2*e/(a**2 + 2*a*b*x
 + b**2*x**2) + 2*c*d*e*x/(a**2 + 2*a*b*x + b**2*x**2) + d**2*e*x**2/(a**2
+ 2*a*b*x + b**2*x**2))), x) + Integral(3*a**2*b*c/(A + B*log(c**2*e/(a**2
+ 2*a*b*x + b**2*x**2) + 2*c*d*e*x/(a**2 + 2*a*b*x + b**2*x**2) + d**2*e*x*
*2/(a**2 + 2*a*b*x + b**2*x**2))), x) + Integral(3*b**3*c*x**2/(A + B*log(c
**2*e/(a**2 + 2*a*b*x + b**2*x**2) + 2*c*d*e*x/(a**2 + 2*a*b*x + b**2*x**2)
 + d**2*e*x**2/(a**2 + 2*a*b*x + b**2*x**2))), x) + Integral(4*b**3*d*x**3/
(A + B*log(c**2*e/(a**2 + 2*a*b*x + b**2*x**2) + 2*c*d*e*x/(a**2 + 2*a*b*x
+ b**2*x**2) + d**2*e*x**2/(a**2 + 2*a*b*x + b**2*x**2))), x) + Integral(6*
a*b**2*c*x/(A + B*log(c**2*e/(a**2 + 2*a*b*x + b**2*x**2) + 2*c*d*e*x/(a**2
 + 2*a*b*x + b**2*x**2) + d**2*e*x**2/(a**2 + 2*a*b*x + b**2*x**2))), x) +
Integral(9*a*b**2*d*x**2/(A + B*log(c**2*e/(a**2 + 2*a*b*x + b**2*x**2) + 2
*c*d*e*x/(a**2 + 2*a*b*x + b**2*x**2) + d**2*e*x**2/(a**2 + 2*a*b*x + b**2*
x**2))), x) + Integral(6*a**2*b*d*x/(A + B*log(c**2*e/(a**2 + 2*a*b*x + b**
2*x**2) + 2*c*d*e*x/(a**2 + 2*a*b*x + b**2*x**2) + d**2*e*x**2/(a**2 + 2*a*
b*x + b**2*x**2))), x))/(2*B*(a*d - b*c))
```

# 3.225 $\int \frac{ag+bgx}{\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2}\,dx$

**Optimal**. Leaf size=35

$$\text{Int}\left(\frac{ag+bgx}{\left(B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)+A\right)^2},x\right)$$

```
[Out] Unintegrable((b*g*x+a*g)/(A+B*ln(e*(d*x+c)^2/(b*x+a)^2))^2,x)
```

**Rubi [A]** time = 0.11, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{ag+bgx}{\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(a*g + b*g*x)/(A + B*Log[(e*(c + d*x)^2)/(a + b*x)^2])^2,x]
```

```
[Out] a*g*Defer[Int][(A + B*Log[(e*(c + d*x)^2)/(a + b*x)^2])^(-2), x] + b*g*Defe
r[Int][x/(A + B*Log[(e*(c + d*x)^2)/(a + b*x)^2])^2, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{ag+bgx}{\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2}\,dx &= \int \left(\frac{ag}{\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2}+\frac{bgx}{\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2}\right)dx\\
&= (ag)\int \frac{1}{\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2}\,dx+(bg)\int \frac{x}{\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2}\,dx
\end{aligned}$$

**Mathematica [A]** time = 0.35, size = 0, normalized size = 0.00

$$\int \frac{ag+bgx}{\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(a*g + b*g*x)/(A + B*Log[(e*(c + d*x)^2)/(a + b*x)^2])^2,x]
```

```
[Out] Integrate[(a*g + b*g*x)/(A + B*Log[(e*(c + d*x)^2)/(a + b*x)^2])^2, x]
```

**fricas [A]** time = 0.53, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{bgx+ag}{B^2\log\left(\frac{d^2ex^2+2\,cdex+c^2e}{b^2x^2+2\,abx+a^2}\right)^2+2\,AB\log\left(\frac{d^2ex^2+2\,cdex+c^2e}{b^2x^2+2\,abx+a^2}\right)+A^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*g*x+a*g)/(A+B*log(e*(d*x+c)^2/(b*x+a)^2))^2,x, algorithm="fric
as")
```

```
[Out] integral((b*g*x + a*g)/(B^2*log((d^2*e*x^2 + 2*c*d*e*x + c^2*e)/(b^2*x^2 +
2*a*b*x + a^2))^2 + 2*A*B*log((d^2*e*x^2 + 2*c*d*e*x + c^2*e)/(b^2*x^2 + 2*
a*b*x + a^2)) + A^2), x)
```

**giac [A]** time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{bgx+ag}{\left(B\log\left(\frac{(dx+c)^2e}{(bx+a)^2}\right)+A\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*g*x+a*g)/(A+B*log(e*(d*x+c)^2/(b*x+a)^2))^2,x, algorithm="giac
")
```

```
[Out] integrate((b*g*x + a*g)/(B*log((d*x + c)^2*e/(b*x + a)^2) + A)^2, x)
```

**maple [A]** time = 0.76, size = 0, normalized size = 0.00

$$\int\frac{bgx+ag}{\left(B\ln\left(\frac{(dx+c)^2e}{(bx+a)^2}\right)+A\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*g*x+a*g)/(B*ln((d*x+c)^2/(b*x+a)^2*e)+A)^2,x)
```

```
[Out] int((b*g*x+a*g)/(B*ln((d*x+c)^2/(b*x+a)^2*e)+A)^2,x)
```

**maxima [A]** time = 0.00, size = 0, normalized size = 0.00

$$-\frac{b^2dgx^3+a^2cg+\left(b^2cg+2\,abdg\right)x^2+\left(2\,abcg+a^2dg\right)x}{2\left(2\,(bc-ad)B^2\log\left(bx+a\right)-2\,(bc-ad)B^2\log\left(dx+c\right)-(bc-ad)AB-\left(bc\log(e)-ad\log(e)\right)B^2\right)}+\int\frac{}{2\left(2\,(b\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*g*x+a*g)/(A+B*log(e*(d*x+c)^2/(b*x+a)^2))^2,x, algorithm="maxi
ma")
```

```
[Out] -1/2*(b^2*d*g*x^3 + a^2*c*g + (b^2*c*g + 2*a*b*d*g)*x^2 + (2*a*b*c*g + a^2*
d*g)*x)/(2*(b*c - a*d)*B^2*log(b*x + a) - 2*(b*c - a*d)*B^2*log(d*x + c) -
(b*c - a*d)*A*B - (b*c*log(e) - a*d*log(e))*B^2) + integrate(1/2*(3*b^2*d*g
*x^2 + 2*a*b*c*g + a^2*d*g + 2*(b^2*c*g + 2*a*b*d*g)*x)/(2*(b*c - a*d)*B^2*
log(b*x + a) - 2*(b*c - a*d)*B^2*log(d*x + c) - (b*c - a*d)*A*B - (b*c*log(
e) - a*d*log(e))*B^2), x)
```

**mupad [A]** time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{a\,g+b\,g\,x}{{\left(A+B\,\ln\left(\frac{e\,{\left(c+d\,x\right)}^2}{{\left(a+b\,x\right)}^2}\right)\right)}^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*g + b*g*x)/(A + B*log((e*(c + d*x)^2)/(a + b*x)^2))^2,x)
```

```
[Out] int((a*g + b*g*x)/(A + B*log((e*(c + d*x)^2)/(a + b*x)^2))^2, x)
```

**sympy [A]** time = 0.00, size = 0, normalized size = 0.00

$$\frac{-a^2cg - a^2dgx - 2abcgx - 2abdgx^2 - b^2cgx^2 - b^2dgx^3}{2ABad - 2ABbc + \left(2B^2ad - 2B^2bc\right)\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)} + \frac{g\left(\int \frac{a^2d}{A+B\log\left(\frac{c^2e}{a^2+2abx+b^2x^2}+\frac{2cdex}{a^2+2abx+b^2x^2}+\frac{d^2ex^2}{a^2+2abx+b^2x^2}\right)}\,dx + \int \cdots\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*g*x+a*g)/(A+B*ln(e*(d*x+c)**2/(b*x+a)**2))**2,x)
```

```
[Out] (-a**2*c*g - a**2*d*g*x - 2*a*b*c*g*x - 2*a*b*d*g*x**2 - b**2*c*g*x**2 - b*
*2*d*g*x**3)/(2*A*B*a*d - 2*A*B*b*c + (2*B**2*a*d - 2*B**2*b*c)*log(e*(c +
d*x)**2/(a + b*x)**2)) + g*(Integral(a**2*d/(A + B*log(c**2*e/(a**2 + 2*a*b
*x + b**2*x**2) + 2*c*d*e*x/(a**2 + 2*a*b*x + b**2*x**2) + d**2*e*x**2/(a**
2 + 2*a*b*x + b**2*x**2))), x) + Integral(2*a*b*c/(A + B*log(c**2*e/(a**2 +
 2*a*b*x + b**2*x**2) + 2*c*d*e*x/(a**2 + 2*a*b*x + b**2*x**2) + d**2*e*x**
2/(a**2 + 2*a*b*x + b**2*x**2))), x) + Integral(2*b**2*c*x/(A + B*log(c**2*
e/(a**2 + 2*a*b*x + b**2*x**2) + 2*c*d*e*x/(a**2 + 2*a*b*x + b**2*x**2) + d
**2*e*x**2/(a**2 + 2*a*b*x + b**2*x**2))), x) + Integral(3*b**2*d*x**2/(A +
 B*log(c**2*e/(a**2 + 2*a*b*x + b**2*x**2) + 2*c*d*e*x/(a**2 + 2*a*b*x + b*
```

```
*2*x**2) + d**2*e*x**2/(a**2 + 2*a*b*x + b**2*x**2))), x) + Integral(4*a*b*
d*x/(A + B*log(c**2*e/(a**2 + 2*a*b*x + b**2*x**2) + 2*c*d*e*x/(a**2 + 2*a*
b*x + b**2*x**2) + d**2*e*x**2/(a**2 + 2*a*b*x + b**2*x**2))), x))/(2*B*(a*
d - b*c))
```

# 3.226 $\int \frac{1}{(ag+bgx)\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2}\,dx$

**Optimal**. Leaf size=37

$$\mathrm{Int}\left(\frac{1}{(ag+bgx)\left(B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)+A\right)^2},x\right)$$

```
[Out] Unintegrable(1/(b*g*x+a*g)/(A+B*ln(e*(d*x+c)^2/(b*x+a)^2))^2,x)
```

**Rubi [A]** time = 0.08, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{1}{(ag+bgx)\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[1/((a*g + b*g*x)*(A + B*Log[(e*(c + d*x)^2)/(a + b*x)^2])^2),x]
[Out] Defer[Int][1/((a*g + b*g*x)*(A + B*Log[(e*(c + d*x)^2)/(a + b*x)^2])^2), x]
```

Rubi steps

$$\int \frac{1}{(ag+bgx)\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2}\,dx=\int \frac{1}{(ag+bgx)\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2}\,dx$$

**Mathematica [A]** time = 0.15, size = 0, normalized size = 0.00

$$\int \frac{1}{(ag+bgx)\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[1/((a*g + b*g*x)*(A + B*Log[(e*(c + d*x)^2)/(a + b*x)^2])^2),x]
```

```
[Out] Integrate[1/((a*g + b*g*x)*(A + B*Log[(e*(c + d*x)^2)/(a + b*x)^2])^2), x]
```

**fricas [A]** time = 0.81, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{1}{A^2bgx + A^2ag + \left(B^2bgx + B^2ag\right)\log\left(\frac{d^2ex^2+2\,cdex+c^2e}{b^2x^2+2\,abx+a^2}\right)^2 + 2\left(ABbgx + ABag\right)\log\left(\frac{d^2ex^2+2\,cdex+c^2e}{b^2x^2+2\,abx+a^2}\right)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*g*x+a*g)/(A+B*log(e*(d*x+c)^2/(b*x+a)^2))^2,x, algorithm="fr
icas")
```

```
[Out] integral(1/(A^2*b*g*x + A^2*a*g + (B^2*b*g*x + B^2*a*g)*log((d^2*e*x^2 + 2*
c*d*e*x + c^2*e)/(b^2*x^2 + 2*a*b*x + a^2))^2 + 2*(A*B*b*g*x + A*B*a*g)*log
((d^2*e*x^2 + 2*c*d*e*x + c^2*e)/(b^2*x^2 + 2*a*b*x + a^2))), x)
```

**giac [A]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(bgx + ag\right)\left(B\log\left(\frac{(dx+c)^2 e}{(bx+a)^2}\right) + A\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*g*x+a*g)/(A+B*log(e*(d*x+c)^2/(b*x+a)^2))^2,x, algorithm="gi
ac")
```

```
[Out] integrate(1/((b*g*x + a*g)*(B*log((d*x + c)^2*e/(b*x + a)^2) + A)^2), x)
```

**maple [A]** time = 0.76, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(bgx + ag\right)\left(B\ln\left(\frac{(dx+c)^2 e}{(bx+a)^2}\right) + A\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*g*x+a*g)/(B*ln((d*x+c)^2/(b*x+a)^2*e)+A)^2,x)
```

```
[Out] int(1/(b*g*x+a*g)/(B*ln((d*x+c)^2/(b*x+a)^2*e)+A)^2,x)
```

**maxima [A]** time = 0.00, size = 0, normalized size = 0.00

$$d\int \frac{1}{2\left(2\left(bcg - adg\right)B^2\log\left(bx + a\right) - 2\left(bcg - adg\right)B^2\log\left(dx + c\right) - \left(bcg - adg\right)AB - \left(bcg\log(e) - adg\log(e)\right)B}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*g*x+a*g)/(A+B*log(e*(d*x+c)^2/(b*x+a)^2))^2,x, algorithm="ma
xima")
```

```
[Out] d*integrate(1/2/(2*(b*c*g - a*d*g)*B^2*log(b*x + a) - 2*(b*c*g - a*d*g)*B^2
*log(d*x + c) - (b*c*g - a*d*g)*A*B - (b*c*g*log(e) - a*d*g*log(e))*B^2), x
) - 1/2*(d*x + c)/(2*(b*c*g - a*d*g)*B^2*log(b*x + a) - 2*(b*c*g - a*d*g)*B
^2*log(d*x + c) - (b*c*g - a*d*g)*A*B - (b*c*g*log(e) - a*d*g*log(e))*B^2)
```

**mupad [A]** time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{1}{\left(a\,g+b\,g\,x\right)\,{\left(A+B\,\ln\left(\frac{e\,{\left(c+d\,x\right)}^2}{{\left(a+b\,x\right)}^2}\right)\right)}^2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*g + b*g*x)*(A + B*log((e*(c + d*x)^2)/(a + b*x)^2))^2),x)
```

```
[Out] int(1/((a*g + b*g*x)*(A + B*log((e*(c + d*x)^2)/(a + b*x)^2))^2), x)
```

**sympy [A]** time = 0.00, size = 0, normalized size = 0.00

$$\frac{-c-dx}{2ABadg-2ABbcg+\left(2B^2adg-2B^2bcg\right)\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)}+\frac{d\int \frac{1}{A+B\log\left(\frac{c^2e}{a^2+2abx+b^2x^2}+\frac{2cdex}{a^2+2abx+b^2x^2}+\frac{d^2ex^2}{a^2+2abx+b^2x^2}\right)}\,dx}{2Bg\,(ad-bc)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*g*x+a*g)/(A+B*ln(e*(d*x+c)**2/(b*x+a)**2))**2,x)
```

```
[Out] (-c - d*x)/(2*A*B*a*d*g - 2*A*B*b*c*g + (2*B**2*a*d*g - 2*B**2*b*c*g)*log(e
*(c + d*x)**2/(a + b*x)**2)) + d*Integral(1/(A + B*log(c**2*e/(a**2 + 2*a*b
*x + b**2*x**2) + 2*c*d*e*x/(a**2 + 2*a*b*x + b**2*x**2) + d**2*e*x**2/(a**
2 + 2*a*b*x + b**2*x**2))), x)/(2*B*g*(a*d - b*c))
```

# 3.227 $\int \frac{1}{(ag+bgx)^2\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2}\,dx$

**Optimal**. Leaf size=147

$$\frac{c+dx}{2Bg^2(a+bx)(bc-ad)\left(B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)+A\right)}-\frac{e^{-\frac{A}{2B}}(c+dx)\mathrm{Ei}\left(\frac{A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)}{2B}\right)}{4B^2g^2(a+bx)(bc-ad)\sqrt{\frac{e(c+dx)^2}{(a+bx)^2}}}$$

```
[Out] 1/2*(d*x+c)/B/(-a*d+b*c)/g^2/(b*x+a)/(A+B*ln(e*(d*x+c)^2/(b*x+a)^2))-1/4*(d
*x+c)*Ei(1/2*(A+B*ln(e*(d*x+c)^2/(b*x+a)^2))/B)/B^2/(-a*d+b*c)/exp(1/2*A/B)
/g^2/(b*x+a)/(e*(d*x+c)^2/(b*x+a)^2)^(1/2)
```

**Rubi [F]** time = 0.09, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{1}{(ag+bgx)^2\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[1/((a*g + b*g*x)^2*(A + B*Log[(e*(c + d*x)^2)/(a + b*x)^2])^2),x]
```

```
[Out] Defer[Int][1/((a*g + b*g*x)^2*(A + B*Log[(e*(c + d*x)^2)/(a + b*x)^2])^2),
x]
```

Rubi steps

$$\int \frac{1}{(ag+bgx)^2\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2}\,dx=\int \frac{1}{(ag+bgx)^2\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2}\,dx$$

**Mathematica [F]** time = 0.19, size = 0, normalized size = 0.00

$$\int \frac{1}{(ag+bgx)^2\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[1/((a*g + b*g*x)^2*(A + B*Log[(e*(c + d*x)^2)/(a + b*x)^2])^2),x]
```

```
[Out] Integrate[1/((a*g + b*g*x)^2*(A + B*Log[(e*(c + d*x)^2)/(a + b*x)^2])^2), x
]
```

**fricas [F]** time = 0.72, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{1}{A^2b^2g^2x^2 + 2\,A^2abg^2x + A^2a^2g^2 + \left(B^2b^2g^2x^2 + 2\,B^2abg^2x + B^2a^2g^2\right)\log\left(\frac{d^2ex^2+2\,cdex+c^2e}{b^2x^2+2\,abx+a^2}\right)^2 + 2\left(ABb^2\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*g*x+a*g)^2/(A+B*log(e*(d*x+c)^2/(b*x+a)^2))^2,x, algorithm="
fricas")
```

```
[Out] integral(1/(A^2*b^2*g^2*x^2 + 2*A^2*a*b*g^2*x + A^2*a^2*g^2 + (B^2*b^2*g^2*
x^2 + 2*B^2*a*b*g^2*x + B^2*a^2*g^2)*log((d^2*e*x^2 + 2*c*d*e*x + c^2*e)/(b
^2*x^2 + 2*a*b*x + a^2))^2 + 2*(A*B*b^2*g^2*x^2 + 2*A*B*a*b*g^2*x + A*B*a^2
*g^2)*log((d^2*e*x^2 + 2*c*d*e*x + c^2*e)/(b^2*x^2 + 2*a*b*x + a^2))), x)
```

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(bgx + ag\right)^2\left(B\log\left(\frac{(dx+c)^2e}{(bx+a)^2}\right) + A\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*g*x+a*g)^2/(A+B*log(e*(d*x+c)^2/(b*x+a)^2))^2,x, algorithm="
giac")
```

```
[Out] integrate(1/((b*g*x + a*g)^2*(B*log((d*x + c)^2*e/(b*x + a)^2) + A)^2), x)
```

**maple [F]** time = 1.05, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(bgx + ag\right)^2\left(B\ln\left(\frac{(dx+c)^2e}{(bx+a)^2}\right) + A\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*g*x+a*g)^2/(B*ln((d*x+c)^2/(b*x+a)^2*e)+A)^2,x)
```

```
[Out] int(1/(b*g*x+a*g)^2/(B*ln((d*x+c)^2/(b*x+a)^2*e)+A)^2,x)
```

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$\frac{}{2\left(\left(abcg^2-a^2dg^2\right)AB+\left(abcg^2\log(e)-a^2dg^2\log(e)\right)B^2+\left(\left(b^2cg^2-abdg^2\right)AB+\left(b^2cg^2\log(e)-abdg^2\log(e)\right)B\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*g*x+a*g)^2/(A+B*log(e*(d*x+c)^2/(b*x+a)^2))^2,x, algorithm="
maxima")
```

```
[Out] 1/2*(d*x + c)/((a*b*c*g^2 - a^2*d*g^2)*A*B + (a*b*c*g^2*log(e) - a^2*d*g^2*
log(e))*B^2 + ((b^2*c*g^2 - a*b*d*g^2)*A*B + (b^2*c*g^2*log(e) - a*b*d*g^2*
log(e))*B^2)*x - 2*((b^2*c*g^2 - a*b*d*g^2)*B^2*x + (a*b*c*g^2 - a^2*d*g^2)
*B^2)*log(b*x + a) + 2*((b^2*c*g^2 - a*b*d*g^2)*B^2*x + (a*b*c*g^2 - a^2*d*
g^2)*B^2)*log(d*x + c)) + integrate(1/2/(B^2*a^2*g^2*log(e) + A*B*a^2*g^2 +
 (B^2*b^2*g^2*log(e) + A*B*b^2*g^2)*x^2 + 2*(B^2*a*b*g^2*log(e) + A*B*a*b*g
^2)*x - 2*(B^2*b^2*g^2*x^2 + 2*B^2*a*b*g^2*x + B^2*a^2*g^2)*log(b*x + a) +
2*(B^2*b^2*g^2*x^2 + 2*B^2*a*b*g^2*x + B^2*a^2*g^2)*log(d*x + c)), x)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{{\left(a\,g+b\,g\,x\right)}^2\,{\left(A+B\,\ln\left(\frac{e\,{\left(c+d\,x\right)}^2}{{\left(a+b\,x\right)}^2}\right)\right)}^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*g + b*g*x)^2*(A + B*log((e*(c + d*x)^2)/(a + b*x)^2))^2),x)
```

```
[Out] int(1/((a*g + b*g*x)^2*(A + B*log((e*(c + d*x)^2)/(a + b*x)^2))^2), x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\frac{-c-dx}{2ABa^2dg^2-2ABabcg^2+2ABabdg^2x-2ABb^2cg^2x+\left(2B^2a^2dg^2-2B^2abcg^2+2B^2abdg^2x-2B^2b^2cg^2x\right)\log\left(\frac{e(c}{(a}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*g*x+a*g)**2/(A+B*ln(e*(d*x+c)**2/(b*x+a)**2))**2,x)
```

```
[Out] (-c - d*x)/(2*A*B*a**2*d*g**2 - 2*A*B*a*b*c*g**2 + 2*A*B*a*b*d*g**2*x - 2*A
*B*b**2*c*g**2*x + (2*B**2*a**2*d*g**2 - 2*B**2*a*b*c*g**2 + 2*B**2*a*b*d*g
**2*x - 2*B**2*b**2*c*g**2*x)*log(e*(c + d*x)**2/(a + b*x)**2)) + Integral(
1/(A*a**2 + 2*A*a*b*x + A*b**2*x**2 + B*a**2*log(c**2*e/(a**2 + 2*a*b*x + b
```

```
**2*x**2) + 2*c*d*e*x/(a**2 + 2*a*b*x + b**2*x**2) + d**2*e*x**2/(a**2 + 2*
a*b*x + b**2*x**2)) + 2*B*a*b*x*log(c**2*e/(a**2 + 2*a*b*x + b**2*x**2) + 2
*c*d*e*x/(a**2 + 2*a*b*x + b**2*x**2) + d**2*e*x**2/(a**2 + 2*a*b*x + b**2*
x**2)) + B*b**2*x**2*log(c**2*e/(a**2 + 2*a*b*x + b**2*x**2) + 2*c*d*e*x/(a
**2 + 2*a*b*x + b**2*x**2) + d**2*e*x**2/(a**2 + 2*a*b*x + b**2*x**2))), x)
/(2*B*g**2)
```

## 3.228 $\int \frac{1}{(ag+bgx)^3\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2}\,dx$

**Optimal**. Leaf size=206

$$\frac{de^{-\frac{A}{2B}}(c+dx)\text{Ei}\left(\frac{A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)}{2B}\right)}{4B^2g^3(a+bx)(bc-ad)^2\sqrt{\frac{e(c+dx)^2}{(a+bx)^2}}}-\frac{be^{-\frac{A}{B}}\text{Ei}\left(\frac{A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)}{B}\right)}{2B^2eg^3(bc-ad)^2}+\frac{c+dx}{2Bg^3(a+bx)^2(bc-ad)\left(B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)+A\right)}$$

```
[Out] -1/2*b*Ei((A+B*ln(e*(d*x+c)^2/(b*x+a)^2))/B)/B^2/(-a*d+b*c)^2/e/exp(A/B)/g^
3+1/2*(d*x+c)/B/(-a*d+b*c)/g^3/(b*x+a)^2/(A+B*ln(e*(d*x+c)^2/(b*x+a)^2))+1/
4*d*(d*x+c)*Ei(1/2*(A+B*ln(e*(d*x+c)^2/(b*x+a)^2))/B)/B^2/(-a*d+b*c)^2/exp(
1/2*A/B)/g^3/(b*x+a)/(e*(d*x+c)^2/(b*x+a)^2)^(1/2)
```

**Rubi [F]** time = 0.09, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{1}{(ag+bgx)^3\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[1/((a*g + b*g*x)^3*(A + B*Log[(e*(c + d*x)^2)/(a + b*x)^2])^2),x]
```

```
[Out] Defer[Int][1/((a*g + b*g*x)^3*(A + B*Log[(e*(c + d*x)^2)/(a + b*x)^2])^2),
x]
```

Rubi steps

$$\int \frac{1}{(ag+bgx)^3\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2}\,dx=\int \frac{1}{(ag+bgx)^3\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2}\,dx$$

**Mathematica [F]** time = 0.35, size = 0, normalized size = 0.00

$$\int \frac{1}{(ag+bgx)^3\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[1/((a*g + b*g*x)^3*(A + B*Log[(e*(c + d*x)^2)/(a + b*x)^2])^2),x]
```

```
[Out] Integrate[1/((a*g + b*g*x)^3*(A + B*Log[(e*(c + d*x)^2)/(a + b*x)^2])^2), x
]
```

**fricas [F]** time = 0.71, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{1}{A^2b^3g^3x^3 + 3\,A^2ab^2g^3x^2 + 3\,A^2a^2bg^3x + A^2a^3g^3 + \left(B^2b^3g^3x^3 + 3\,B^2ab^2g^3x^2 + 3\,B^2a^2bg^3x + B^2a^3g^3\right)\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*g*x+a*g)^3/(A+B*log(e*(d*x+c)^2/(b*x+a)^2))^2,x, algorithm="
fricas")
```

```
[Out] integral(1/(A^2*b^3*g^3*x^3 + 3*A^2*a*b^2*g^3*x^2 + 3*A^2*a^2*b*g^3*x + A^2
*a^3*g^3 + (B^2*b^3*g^3*x^3 + 3*B^2*a*b^2*g^3*x^2 + 3*B^2*a^2*b*g^3*x + B^2
*a^3*g^3)*log((d^2*e*x^2 + 2*c*d*e*x + c^2*e)/(b^2*x^2 + 2*a*b*x + a^2))^2
+ 2*(A*B*b^3*g^3*x^3 + 3*A*B*a*b^2*g^3*x^2 + 3*A*B*a^2*b*g^3*x + A*B*a^3*g^
3)*log((d^2*e*x^2 + 2*c*d*e*x + c^2*e)/(b^2*x^2 + 2*a*b*x + a^2))), x)
```

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(bgx + ag\right)^3 \left(B\log\left(\frac{(dx+c)^2 e}{(bx+a)^2}\right) + A\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*g*x+a*g)^3/(A+B*log(e*(d*x+c)^2/(b*x+a)^2))^2,x, algorithm="
giac")
```

```
[Out] integrate(1/((b*g*x + a*g)^3*(B*log((d*x + c)^2*e/(b*x + a)^2) + A)^2), x)
```

**maple [F]** time = 1.32, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(bgx + ag\right)^3 \left(B\ln\left(\frac{(dx+c)^2 e}{(bx+a)^2}\right) + A\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*g*x+a*g)^3/(B*ln((d*x+c)^2/(b*x+a)^2*e)+A)^2,x)
```

[Out] int(1/(b*g*x+a*g)^3/(B*ln((d*x+c)^2/(b*x+a)^2*e)+A)^2,x)

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$\frac{}{2\left(\left(a^2bcg^3-a^3dg^3\right)AB+\left(a^2bcg^3\log(e)-a^3dg^3\log(e)\right)B^2+\left(\left(b^3cg^3-ab^2dg^3\right)AB+\left(b^3cg^3\log(e)-ab^2dg^3\log(\right.\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*g*x+a*g)^3/(A+B*log(e*(d*x+c)^2/(b*x+a)^2))^2,x, algorithm="
maxima")
```

```
[Out] 1/2*(d*x + c)/((a^2*b*c*g^3 - a^3*d*g^3)*A*B + (a^2*b*c*g^3*log(e) - a^3*d*
g^3*log(e))*B^2 + ((b^3*c*g^3 - a*b^2*d*g^3)*A*B + (b^3*c*g^3*log(e) - a*b^
2*d*g^3*log(e))*B^2)*x^2 + 2*((a*b^2*c*g^3 - a^2*b*d*g^3)*A*B + (a*b^2*c*g^
3*log(e) - a^2*b*d*g^3*log(e))*B^2)*x - 2*((b^3*c*g^3 - a*b^2*d*g^3)*B^2*x^
2 + 2*(a*b^2*c*g^3 - a^2*b*d*g^3)*B^2*x + (a^2*b*c*g^3 - a^3*d*g^3)*B^2)*lo
g(b*x + a) + 2*((b^3*c*g^3 - a*b^2*d*g^3)*B^2*x^2 + 2*(a*b^2*c*g^3 - a^2*b*
d*g^3)*B^2*x + (a^2*b*c*g^3 - a^3*d*g^3)*B^2)*log(d*x + c)) - integrate(-1/
2*(b*d*x + 2*b*c - a*d)/(((b^4*c*g^3 - a*b^3*d*g^3)*A*B + (b^4*c*g^3*log(e)
 - a*b^3*d*g^3*log(e))*B^2)*x^3 + (a^3*b*c*g^3 - a^4*d*g^3)*A*B + (a^3*b*c*
g^3*log(e) - a^4*d*g^3*log(e))*B^2 + 3*((a*b^3*c*g^3 - a^2*b^2*d*g^3)*A*B +
 (a*b^3*c*g^3*log(e) - a^2*b^2*d*g^3*log(e))*B^2)*x^2 + 3*((a^2*b^2*c*g^3 -
 a^3*b*d*g^3)*A*B + (a^2*b^2*c*g^3*log(e) - a^3*b*d*g^3*log(e))*B^2)*x - 2*
((b^4*c*g^3 - a*b^3*d*g^3)*B^2*x^3 + 3*(a*b^3*c*g^3 - a^2*b^2*d*g^3)*B^2*x^
2 + 3*(a^2*b^2*c*g^3 - a^3*b*d*g^3)*B^2*x + (a^3*b*c*g^3 - a^4*d*g^3)*B^2)*
log(b*x + a) + 2*((b^4*c*g^3 - a*b^3*d*g^3)*B^2*x^3 + 3*(a*b^3*c*g^3 - a^2*
b^2*d*g^3)*B^2*x^2 + 3*(a^2*b^2*c*g^3 - a^3*b*d*g^3)*B^2*x + (a^3*b*c*g^3 -
 a^4*d*g^3)*B^2)*log(d*x + c)), x)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{{\left(a\,g+b\,g\,x\right)}^3\,{\left(A+B\,\ln\left(\frac{e\,{\left(c+d\,x\right)}^2}{{\left(a+b\,x\right)}^2}\right)\right)}^2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*g + b*g*x)^3*(A + B*log((e*(c + d*x)^2)/(a + b*x)^2))^2),x)
```

```
[Out] int(1/((a*g + b*g*x)^3*(A + B*log((e*(c + d*x)^2)/(a + b*x)^2))^2), x)
```

**sympy [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*g*x+a*g)**3/(A+B*ln(e*(d*x+c)**2/(b*x+a)**2))**2,x)
[Out] Timed out
```

## 3.229 $\int \frac{1}{(ag+bgx)^2\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)}\,dx$

**Optimal**. Leaf size=96

$$\frac{e^{\frac{A}{Bn}}(c+dx)\left(e(a+bx)^n(c+dx)^{-n}\right)^{\frac{1}{n}}\text{Ei}\left(-\frac{A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)}{Bn}\right)}{Bg^2n(a+bx)(bc-ad)}$$

[Out] exp(A/B/n)*(d*x+c)*(e*(b*x+a)^n/((d*x+c)^n))^(1/n)*Ei((-A-B*ln(e*(b*x+a)^n/((d*x+c)^n)))/B/n)/B/(-a*d+b*c)/g^2/n/(b*x+a)

**Rubi [F]** time = 0.10, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{1}{(ag+bgx)^2\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)}\,dx$$

Verification is Not applicable to the result.

[In] Int[1/((a*g + b*g*x)^2*(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])),x]

[Out] Defer[Int][1/((a*g + b*g*x)^2*(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])), x]

Rubi steps

$$\int \frac{1}{(ag+bgx)^2\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)}\,dx=\int \frac{1}{(ag+bgx)^2\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)}\,dx$$

**Mathematica [F]** time = 0.06, size = 0, normalized size = 0.00

$$\int \frac{1}{(ag+bgx)^2\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)}\,dx$$

Verification is Not applicable to the result.

[In] Integrate[1/((a*g + b*g*x)^2*(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])),x]

[Out] Integrate[1/((a*g + b*g*x)^2*(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])), x]

**fricas [A]** time = 0.75, size = 62, normalized size = 0.65

$$\frac{e^{\left(\frac{B\log(e)+A}{Bn}\right)}\operatorname{log\_integral}\left(\frac{(dx+c)e^{\left(-\frac{B\log(e)+A}{Bn}\right)}}{bx+a}\right)}{(Bbc-Bad)g^2n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*g*x+a*g)^2/(A+B*log(e*(b*x+a)^n/((d*x+c)^n))),x, algorithm="
fricas")
```

```
[Out] e^((B*log(e) + A)/(B*n))*log_integral((d*x + c)*e^(-(B*log(e) + A)/(B*n))/(
b*x + a))/((B*b*c - B*a*d)*g^2*n)
```

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(bgx+ag)^2\left(B\log\left(\frac{(bx+a)^n e}{(dx+c)^n}\right)+A\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*g*x+a*g)^2/(A+B*log(e*(b*x+a)^n/((d*x+c)^n))),x, algorithm="
giac")
```

```
[Out] integrate(1/((b*g*x + a*g)^2*(B*log((b*x + a)^n*e/(d*x + c)^n) + A)), x)
```

**maple [F]** time = 0.04, size = 0, normalized size = 0.00

$$\int \frac{1}{(bgx+ag)^2\left(B\ln\left(e\,(bx+a)^n\,(dx+c)^{-n}\right)+A\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*g*x+a*g)^2/(A+B*ln(e*(b*x+a)^n/((d*x+c)^n))),x)
```

```
[Out] int(1/(b*g*x+a*g)^2/(A+B*ln(e*(b*x+a)^n/((d*x+c)^n))),x)
```

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(bgx+ag)^2\left(B\log\left(\frac{(bx+a)^n e}{(dx+c)^n}\right)+A\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*g*x+a*g)^2/(A+B*log(e*(b*x+a)^n/((d*x+c)^n))),x, algorithm="
maxima")
```

```
[Out] integrate(1/((b*g*x + a*g)^2*(B*log((b*x + a)^n*e/(d*x + c)^n) + A)), x)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\left(a\,g+b\,g\,x\right)^2\,\left(A+B\,\ln\left(\frac{e\,{\left(a+b\,x\right)}^n}{{\left(c+d\,x\right)}^n}\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*g + b*g*x)^2*(A + B*log((e*(a + b*x)^n)/(c + d*x)^n))),x)
```

```
[Out] int(1/((a*g + b*g*x)^2*(A + B*log((e*(a + b*x)^n)/(c + d*x)^n))), x)
```

**sympy [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*g*x+a*g)**2/(A+B*ln(e*(b*x+a)**n/((d*x+c)**n))),x)
```

```
[Out] Timed out
```

# 3.230 $\int (f+gx)^4 \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)\,dx$

**Optimal**. Leaf size=355

$$-\frac{Bg^2x^2(bc-ad)\left(a^2d^2g^2-abdg(5df-cg)+b^2\left(c^2g^2-5cdfg+10d^2f^2\right)\right)}{10b^3d^3}+\frac{Bgx(bc-ad)\left(a^3d^3g^3-a^2bd^2g^2(5d\right.}{}$$

```
[Out] 1/5*B*(-a*d+b*c)*g*(a^3*d^3*g^3-a^2*b*d^2*g^2*(-c*g+5*d*f)+a*b^2*d*g*(c^2*g
^2-5*c*d*f*g+10*d^2*f^2)-b^3*(-c^3*g^3+5*c^2*d*f*g^2-10*c*d^2*f^2*g+10*d^3*
f^3))*x/b^4/d^4-1/10*B*(-a*d+b*c)*g^2*(a^2*d^2*g^2-a*b*d*g*(-c*g+5*d*f)+b^2
*(c^2*g^2-5*c*d*f*g+10*d^2*f^2))*x^2/b^3/d^3-1/15*B*(-a*d+b*c)*g^3*(-a*d*g-
b*c*g+5*b*d*f)*x^3/b^2/d^2-1/20*B*(-a*d+b*c)*g^4*x^4/b/d-1/5*B*(-a*g+b*f)^5
*ln(b*x+a)/b^5/g+1/5*(g*x+f)^5*(A+B*ln(e*(b*x+a)/(d*x+c)))/g+1/5*B*(-c*g+d*
f)^5*ln(d*x+c)/d^5/g
```

**Rubi [A]** time = 0.56, antiderivative size = 339, normalized size of antiderivative = 0.95, number of steps used = 4, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {2525, 12, 72}

$$-\frac{Bg^2x^2(bc-ad)\left(a^2d^2g^2-abdg(5df-cg)+b^2\left(c^2g^2-5cdfg+10d^2f^2\right)\right)}{10b^3d^3}+\frac{Bgx\left(-10a^2b^2d^4f^2g+5a^3bd^4fg^2-\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(f + g*x)^4*(A + B*Log[(e*(a + b*x))/(c + d*x)]),x]
```

```
[Out] (B*g*(10*a*b^3*d^4*f^3 - 10*a^2*b^2*d^4*f^2*g + 5*a^3*b*d^4*f*g^2 - a^4*d^4
*g^3 - b^4*c*(10*d^3*f^3 - 10*c*d^2*f^2*g + 5*c^2*d*f*g^2 - c^3*g^3))*x)/(5
*b^4*d^4) - (B*(b*c - a*d)*g^2*(a^2*d^2*g^2 - a*b*d*g*(5*d*f - c*g) + b^2*(
10*d^2*f^2 - 5*c*d*f*g + c^2*g^2))*x^2)/(10*b^3*d^3) - (B*(b*c - a*d)*g^3*(
5*b*d*f - b*c*g - a*d*g)*x^3)/(15*b^2*d^2) - (B*(b*c - a*d)*g^4*x^4)/(20*b*
d) - (B*(b*f - a*g)^5*Log[a + b*x])/(5*b^5*g) + ((f + g*x)^5*(A + B*Log[(e*
(a + b*x))/(c + d*x)]))/(5*g) + (B*(d*f - c*g)^5*Log[c + d*x])/(5*d^5*g)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 72

```
Int[((e_.) + (f_.)*(x_))^(p_.)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
 x_Symbol] :> Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x]
```

```
/; FreeQ[{a, b, c, d, e, f}, x] && IntegerQ[p]
```

Rule 2525

```
Int[((a_.) + Log[(c_.)*(RFx_)^(p_.)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_))^(m_.
), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(a + b*Log[c*RFx^p])^n)/(e*(m + 1))
, x] - Dist[(b*n*p)/(e*(m + 1)), Int[SimplifyIntegrand[((d + e*x)^(m + 1)*(
a + b*Log[c*RFx^p])^(n - 1)*D[RFx, x])/RFx, x], x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && RationalFunctionQ[RFx, x] && IGtQ[n, 0] && (EqQ[n, 1] ||
IntegerQ[m]) && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int (f+gx)^4\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)dx &= \frac{(f+gx)^5\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{5g}-\frac{B\int\frac{(bc-ad)(f+gx)^5}{(a+bx)(c+dx)}\,dx}{5g}\\ &= \frac{(f+gx)^5\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{5g}-\frac{(B(bc-ad))\int\frac{(f+gx)^5}{(a+bx)(c+dx)}\,dx}{5g}\\ &= \frac{(f+gx)^5\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{5g}-\frac{(B(bc-ad))\int\left(\frac{g^2\left(-a^3d^3g^3+a^2bd^2g^2(5df\cdots\right.}{\cdots}\right.}{\cdots}\\ &= \frac{B(bc-ad)g\left(a^3d^3g^3-a^2bd^2g^2(5df-cg)+ab^2dg\left(10d^2f^2-5cdfg+\cdots\right.\right.}{5b^4d^4}\end{aligned}$$

**Mathematica [A]** time = 0.59, size = 279, normalized size = 0.79

$$\frac{Bg^2x(ad-bc)\left(-12a^3d^3g^3+6a^2bd^2g^2(-2cg+10df+dgx)-2ab^2dg\left(6c^2g^2-3cdg(10f+gx)+d^2\left(60f^2+15fgx+2g^2x^2\right)\right)+b^3\left(-12c^3g^3+6c^2dg^2(10f+gx)-2cd^2g\cdots\right.\right.}{12b^4d^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(f + g*x)^4*(A + B*Log[(e*(a + b*x))/(c + d*x)]),x]
```

```
[Out] ((B*(-(b*c) + a*d)*g^2*x*(-12*a^3*d^3*g^3 + 6*a^2*b*d^2*g^2*(10*d*f - 2*c*g
 + d*g*x) - 2*a*b^2*d*g*(6*c^2*g^2 - 3*c*d*g*(10*f + g*x) + d^2*(60*f^2 + 1
5*f*g*x + 2*g^2*x^2)) + b^3*(-12*c^3*g^3 + 6*c^2*d*g^2*(10*f + g*x) - 2*c*d
^2*g*(60*f^2 + 15*f*g*x + 2*g^2*x^2) + d^3*(120*f^3 + 60*f^2*g*x + 20*f*g^2
*x^2 + 3*g^3*x^3))))/(12*b^4*d^4) - (B*(b*f - a*g)^5*Log[a + b*x])/b^5 + (f
```

```
 + g*x)^5*(A + B*Log[(e*(a + b*x))/(c + d*x)]) + (B*(d*f - c*g)^5*Log[c + d
*x])/d^5)/(5*g)
```

**fricas [A]** time = 1.60, size = 636, normalized size = 1.79

$$\frac{12\,Ab^5d^5g^4x^5 + 3\left(20\,Ab^5d^5fg^3 - \left(Bb^5cd^4 - Bab^4d^5\right)g^4\right)x^4 + 4\left(30\,Ab^5d^5f^2g^2 - 5\left(Bb^5cd^4 - Bab^4d^5\right)fg^3 + \left(Bb\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^4*(A+B*log(e*(b*x+a)/(d*x+c))),x, algorithm="fricas")
```

```
[Out] 1/60*(12*A*b^5*d^5*g^4*x^5 + 3*(20*A*b^5*d^5*f*g^3 - (B*b^5*c*d^4 - B*a*b^4
*d^5)*g^4)*x^4 + 4*(30*A*b^5*d^5*f^2*g^2 - 5*(B*b^5*c*d^4 - B*a*b^4*d^5)*f*
g^3 + (B*b^5*c^2*d^3 - B*a^2*b^3*d^5)*g^4)*x^3 + 6*(20*A*b^5*d^5*f^3*g - 10
*(B*b^5*c*d^4 - B*a*b^4*d^5)*f^2*g^2 + 5*(B*b^5*c^2*d^3 - B*a^2*b^3*d^5)*f*
g^3 - (B*b^5*c^3*d^2 - B*a^3*b^2*d^5)*g^4)*x^2 + 12*(5*A*b^5*d^5*f^4 - 10*(
B*b^5*c*d^4 - B*a*b^4*d^5)*f^3*g + 10*(B*b^5*c^2*d^3 - B*a^2*b^3*d^5)*f^2*g
^2 - 5*(B*b^5*c^3*d^2 - B*a^3*b^2*d^5)*f*g^3 + (B*b^5*c^4*d - B*a^4*b*d^5)*
g^4)*x + 12*(5*B*a*b^4*d^5*f^4 - 10*B*a^2*b^3*d^5*f^3*g + 10*B*a^3*b^2*d^5*
f^2*g^2 - 5*B*a^4*b*d^5*f*g^3 + B*a^5*d^5*g^4)*log(b*x + a) - 12*(5*B*b^5*c
*d^4*f^4 - 10*B*b^5*c^2*d^3*f^3*g + 10*B*b^5*c^3*d^2*f^2*g^2 - 5*B*b^5*c^4*
d*f*g^3 + B*b^5*c^5*g^4)*log(d*x + c) + 12*(B*b^5*d^5*g^4*x^5 + 5*B*b^5*d^5
*f*g^3*x^4 + 10*B*b^5*d^5*f^2*g^2*x^3 + 10*B*b^5*d^5*f^3*g*x^2 + 5*B*b^5*d^
5*f^4*x)*log((b*e*x + a*e)/(d*x + c)))/(b^5*d^5)
```

**giac [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^4*(A+B*log(e*(b*x+a)/(d*x+c))),x, algorithm="giac")
```

```
[Out] Timed out
```

**maple [B]** time = 0.29, size = 14719, normalized size = 41.46

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x+f)^4*(B*ln((b*x+a)/(d*x+c)*e)+A),x)
```

```
[Out] result too large to display
```

**maxima [A]** time = 0.82, size = 593, normalized size = 1.67

$$\frac{1}{5}Ag^4x^5+Afg^3x^4+2Af^2g^2x^3+2Af^3gx^2+\left(x\log\left(\frac{bex}{dx+c}+\frac{ae}{dx+c}\right)+\frac{a\log(bx+a)}{b}-\frac{c\log(dx+c)}{d}\right)Bf^4+2\left(x^2\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^4*(A+B*log(e*(b*x+a)/(d*x+c))),x, algorithm="maxima")
```

```
[Out] 1/5*A*g^4*x^5 + A*f*g^3*x^4 + 2*A*f^2*g^2*x^3 + 2*A*f^3*g*x^2 + (x*log(b*e*
x/(d*x + c) + a*e/(d*x + c)) + a*log(b*x + a)/b - c*log(d*x + c)/d)*B*f^4 +
 2*(x^2*log(b*e*x/(d*x + c) + a*e/(d*x + c)) - a^2*log(b*x + a)/b^2 + c^2*l
og(d*x + c)/d^2 - (b*c - a*d)*x/(b*d))*B*f^3*g + (2*x^3*log(b*e*x/(d*x + c)
 + a*e/(d*x + c)) + 2*a^3*log(b*x + a)/b^3 - 2*c^3*log(d*x + c)/d^3 - ((b^2
*c*d - a*b*d^2)*x^2 - 2*(b^2*c^2 - a^2*d^2)*x)/(b^2*d^2))*B*f^2*g^2 + 1/6*(
6*x^4*log(b*e*x/(d*x + c) + a*e/(d*x + c)) - 6*a^4*log(b*x + a)/b^4 + 6*c^4
*log(d*x + c)/d^4 - (2*(b^3*c*d^2 - a*b^2*d^3)*x^3 - 3*(b^3*c^2*d - a^2*b*d
^3)*x^2 + 6*(b^3*c^3 - a^3*d^3)*x)/(b^3*d^3))*B*f*g^3 + 1/60*(12*x^5*log(b*
e*x/(d*x + c) + a*e/(d*x + c)) + 12*a^5*log(b*x + a)/b^5 - 12*c^5*log(d*x +
 c)/d^5 - (3*(b^4*c*d^3 - a*b^3*d^4)*x^4 - 4*(b^4*c^2*d^2 - a^2*b^2*d^4)*x^
3 + 6*(b^4*c^3*d - a^3*b*d^4)*x^2 - 12*(b^4*c^4 - a^4*d^4)*x)/(b^4*d^4))*B*
g^4 + A*f^4*x
```

**mupad [B]** time = 5.34, size = 1392, normalized size = 3.92

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f + g*x)^4*(A + B*log((e*(a + b*x))/(c + d*x))),x)
```

```
[Out] x^2*((20*A*a*c*f*g^3 + 20*A*b*d*f^3*g + 30*A*a*d*f^2*g^2 + 30*A*b*c*f^2*g^2
 + 10*B*a*d*f^2*g^2 - 10*B*b*c*f^2*g^2)/(10*b*d) + ((5*a*d + 5*b*c)*((((5*A
*a*d*g^4 + 5*A*b*c*g^4 + B*a*d*g^4 - B*b*c*g^4 + 20*A*b*d*f*g^3)/(5*b*d) -
(A*g^4*(5*a*d + 5*b*c))/(5*b*d))*(5*a*d + 5*b*c))/(5*b*d) - (5*A*a*c*g^4 +
20*A*a*d*f*g^3 + 20*A*b*c*f*g^3 + 5*B*a*d*f*g^3 - 5*B*b*c*f*g^3 + 30*A*b*d*
f^2*g^2)/(5*b*d) + (A*a*c*g^4)/(b*d)))/(10*b*d) - (a*c*((5*A*a*d*g^4 + 5*A*
b*c*g^4 + B*a*d*g^4 - B*b*c*g^4 + 20*A*b*d*f*g^3)/(5*b*d) - (A*g^4*(5*a*d +
 5*b*c))/(5*b*d)))/(2*b*d)) + x^4*((5*A*a*d*g^4 + 5*A*b*c*g^4 + B*a*d*g^4 -
 B*b*c*g^4 + 20*A*b*d*f*g^3)/(20*b*d) - (A*g^4*(5*a*d + 5*b*c))/(20*b*d)) +
 log((e*(a + b*x))/(c + d*x))*((B*g^4*x^5)/5 + B*f^4*x + 2*B*f^2*g^2*x^3 +
2*B*f^3*g*x^2 + B*f*g^3*x^4) + x*((5*A*b*d*f^4 + 20*A*a*d*f^3*g + 20*A*b*c*
f^3*g + 10*B*a*d*f^3*g - 10*B*b*c*f^3*g + 30*A*a*c*f^2*g^2)/(5*b*d) - ((5*a
*d + 5*b*c)*((20*A*a*c*f*g^3 + 20*A*b*d*f^3*g + 30*A*a*d*f^2*g^2 + 30*A*b*c
*f^2*g^2 + 10*B*a*d*f^2*g^2 - 10*B*b*c*f^2*g^2)/(5*b*d) + ((5*a*d + 5*b*c)*
((((5*A*a*d*g^4 + 5*A*b*c*g^4 + B*a*d*g^4 - B*b*c*g^4 + 20*A*b*d*f*g^3)/(5*
```

```
b*d) - (A*g^4*(5*a*d + 5*b*c))/(5*b*d))*(5*a*d + 5*b*c))/(5*b*d) - (5*A*a*c
*g^4 + 20*A*a*d*f*g^3 + 20*A*b*c*f*g^3 + 5*B*a*d*f*g^3 - 5*B*b*c*f*g^3 + 30
*A*b*d*f^2*g^2)/(5*b*d) + (A*a*c*g^4)/(b*d)))/(5*b*d) - (a*c*((5*A*a*d*g^4
+ 5*A*b*c*g^4 + B*a*d*g^4 - B*b*c*g^4 + 20*A*b*d*f*g^3)/(5*b*d) - (A*g^4*(5
*a*d + 5*b*c))/(5*b*d)))/(b*d)))/(5*b*d) + (a*c*((((5*A*a*d*g^4 + 5*A*b*c*g
^4 + B*a*d*g^4 - B*b*c*g^4 + 20*A*b*d*f*g^3)/(5*b*d) - (A*g^4*(5*a*d + 5*b*
c))/(5*b*d))*(5*a*d + 5*b*c))/(5*b*d) - (5*A*a*c*g^4 + 20*A*a*d*f*g^3 + 20*
A*b*c*f*g^3 + 5*B*a*d*f*g^3 - 5*B*b*c*f*g^3 + 30*A*b*d*f^2*g^2)/(5*b*d) + (
A*a*c*g^4)/(b*d)))/(b*d)) - x^3*((((5*A*a*d*g^4 + 5*A*b*c*g^4 + B*a*d*g^4 -
 B*b*c*g^4 + 20*A*b*d*f*g^3)/(5*b*d) - (A*g^4*(5*a*d + 5*b*c))/(5*b*d))*(5*
a*d + 5*b*c))/(15*b*d) - (5*A*a*c*g^4 + 20*A*a*d*f*g^3 + 20*A*b*c*f*g^3 + 5
*B*a*d*f*g^3 - 5*B*b*c*f*g^3 + 30*A*b*d*f^2*g^2)/(15*b*d) + (A*a*c*g^4)/(3*
b*d)) + (A*g^4*x^5)/5 + (log(a + b*x)*((B*a^5*g^4)/5 + B*a*b^4*f^4 - 2*B*a^
2*b^3*f^3*g + 2*B*a^3*b^2*f^2*g^2 - B*a^4*b*f*g^3))/b^5 - (log(c + d*x)*(B*
c^5*g^4 + 5*B*c*d^4*f^4 - 10*B*c^2*d^3*f^3*g + 10*B*c^3*d^2*f^2*g^2 - 5*B*c
^4*d*f*g^3))/(5*d^5)
```

**sympy [B]** time = 26.35, size = 1436, normalized size = 4.05

$$\frac{Ag^4x^5}{5}+\frac{Ba\left(a^4g^4-5a^3bfg^3+10a^2b^2f^2g^2-10ab^3f^3g+5b^4f^4\right)\log\left(x+\frac{Ba^5cd^4g^4-5Ba^4bcd^4fg^3+10Ba^3b^2cd^4f^2g^2-10Ba^2b\cdots}{\cdots}\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)**4*(A+B*ln(e*(b*x+a)/(d*x+c))),x)
```

```
[Out] A*g**4*x**5/5 + B*a*(a**4*g**4 - 5*a**3*b*f*g**3 + 10*a**2*b**2*f**2*g**2 -
 10*a*b**3*f**3*g + 5*b**4*f**4)*log(x + (B*a**5*c*d**4*g**4 - 5*B*a**4*b*c
*d**4*f*g**3 + 10*B*a**3*b**2*c*d**4*f**2*g**2 - 10*B*a**2*b**3*c*d**4*f**3
*g + B*a**2*d**5*(a**4*g**4 - 5*a**3*b*f*g**3 + 10*a**2*b**2*f**2*g**2 - 10
*a*b**3*f**3*g + 5*b**4*f**4)/b + B*a*b**4*c**5*g**4 - 5*B*a*b**4*c**4*d*f*
g**3 + 10*B*a*b**4*c**3*d**2*f**2*g**2 - 10*B*a*b**4*c**2*d**3*f**3*g + 10*
B*a*b**4*c*d**4*f**4 - B*a*c*d**4*(a**4*g**4 - 5*a**3*b*f*g**3 + 10*a**2*b*
*2*f**2*g**2 - 10*a*b**3*f**3*g + 5*b**4*f**4))/(B*a**5*d**5*g**4 - 5*B*a**
4*b*d**5*f*g**3 + 10*B*a**3*b**2*d**5*f**2*g**2 - 10*B*a**2*b**3*d**5*f**3*
g + 5*B*a*b**4*d**5*f**4 + B*b**5*c**5*g**4 - 5*B*b**5*c**4*d*f*g**3 + 10*B
*b**5*c**3*d**2*f**2*g**2 - 10*B*b**5*c**2*d**3*f**3*g + 5*B*b**5*c*d**4*f*
*4))/(5*b**5) - B*c*(c**4*g**4 - 5*c**3*d*f*g**3 + 10*c**2*d**2*f**2*g**2 -
 10*c*d**3*f**3*g + 5*d**4*f**4)*log(x + (B*a**5*c*d**4*g**4 - 5*B*a**4*b*c
*d**4*f*g**3 + 10*B*a**3*b**2*c*d**4*f**2*g**2 - 10*B*a**2*b**3*c*d**4*f**3
*g + B*a*b**4*c**5*g**4 - 5*B*a*b**4*c**4*d*f*g**3 + 10*B*a*b**4*c**3*d**2*
f**2*g**2 - 10*B*a*b**4*c**2*d**3*f**3*g + 10*B*a*b**4*c*d**4*f**4 - B*a*b*
*4*c*(c**4*g**4 - 5*c**3*d*f*g**3 + 10*c**2*d**2*f**2*g**2 - 10*c*d**3*f**3
*g + 5*d**4*f**4) + B*b**5*c**2*(c**4*g**4 - 5*c**3*d*f*g**3 + 10*c**2*d**2
```

```
*f**2*g**2 - 10*c*d**3*f**3*g + 5*d**4*f**4)/d)/(B*a**5*d**5*g**4 - 5*B*a**
4*b*d**5*f*g**3 + 10*B*a**3*b**2*d**5*f**2*g**2 - 10*B*a**2*b**3*d**5*f**3*
g + 5*B*a*b**4*d**5*f**4 + B*b**5*c**5*g**4 - 5*B*b**5*c**4*d*f*g**3 + 10*B
*b**5*c**3*d**2*f**2*g**2 - 10*B*b**5*c**2*d**3*f**3*g + 5*B*b**5*c*d**4*f*
*4))/(5*d**5) + x**4*(A*f*g**3 + B*a*g**4/(20*b) - B*c*g**4/(20*d)) + x**3*
(2*A*f**2*g**2 - B*a**2*g**4/(15*b**2) + B*a*f*g**3/(3*b) + B*c**2*g**4/(15
*d**2) - B*c*f*g**3/(3*d)) + x**2*(2*A*f**3*g + B*a**3*g**4/(10*b**3) - B*a
**2*f*g**3/(2*b**2) + B*a*f**2*g**2/b - B*c**3*g**4/(10*d**3) + B*c**2*f*g*
*3/(2*d**2) - B*c*f**2*g**2/d) + x*(A*f**4 - B*a**4*g**4/(5*b**4) + B*a**3*
f*g**3/b**3 - 2*B*a**2*f**2*g**2/b**2 + 2*B*a*f**3*g/b + B*c**4*g**4/(5*d**
4) - B*c**3*f*g**3/d**3 + 2*B*c**2*f**2*g**2/d**2 - 2*B*c*f**3*g/d) + (B*f*
*4*x + 2*B*f**3*g*x**2 + 2*B*f**2*g**2*x**3 + B*f*g**3*x**4 + B*g**4*x**5/5
)*log(e*(a + b*x)/(c + d*x))
```

# 3.231 $\int (f+gx)^3 \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)\,dx$

**Optimal**. Leaf size=227

$$-\frac{Bgx(bc-ad)\left(a^2d^2g^2-abdg(4df-cg)+b^2\left(c^2g^2-4cdfg+6d^2f^2\right)\right)}{4b^3d^3}+\frac{(f+gx)^4\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{4g}-\frac{B(bf$$

```
[Out] -1/4*B*(-a*d+b*c)*g*(a^2*d^2*g^2-a*b*d*g*(-c*g+4*d*f)+b^2*(c^2*g^2-4*c*d*f*
g+6*d^2*f^2))*x/b^3/d^3-1/8*B*(-a*d+b*c)*g^2*(-a*d*g-b*c*g+4*b*d*f)*x^2/b^2
/d^2-1/12*B*(-a*d+b*c)*g^3*x^3/b/d-1/4*B*(-a*g+b*f)^4*ln(b*x+a)/b^4/g+1/4*(
g*x+f)^4*(A+B*ln(e*(b*x+a)/(d*x+c)))/g+1/4*B*(-c*g+d*f)^4*ln(d*x+c)/d^4/g
```

**Rubi** [A] time = 0.34, antiderivative size = 227, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {2525, 12, 72}

$$-\frac{Bgx(bc-ad)\left(a^2d^2g^2-abdg(4df-cg)+b^2\left(c^2g^2-4cdfg+6d^2f^2\right)\right)}{4b^3d^3}+\frac{(f+gx)^4\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{4g}-\frac{Bg^2x^2$$

Antiderivative was successfully verified.

```
[In] Int[(f + g*x)^3*(A + B*Log[(e*(a + b*x))/(c + d*x)]),x]
```

```
[Out] -(B*(b*c - a*d)*g*(a^2*d^2*g^2 - a*b*d*g*(4*d*f - c*g) + b^2*(6*d^2*f^2 - 4
*c*d*f*g + c^2*g^2))*x)/(4*b^3*d^3) - (B*(b*c - a*d)*g^2*(4*b*d*f - b*c*g -
 a*d*g)*x^2)/(8*b^2*d^2) - (B*(b*c - a*d)*g^3*x^3)/(12*b*d) - (B*(b*f - a*g
)^4*Log[a + b*x])/(4*b^4*g) + ((f + g*x)^4*(A + B*Log[(e*(a + b*x))/(c + d*
x)]))/(4*g) + (B*(d*f - c*g)^4*Log[c + d*x])/(4*d^4*g)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 72

```
Int[((e_.) + (f_.)*(x_))^(p_.)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
 x_Symbol] :> Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x]
 /; FreeQ[{a, b, c, d, e, f}, x] && IntegerQ[p]
```

Rule 2525

```
Int[((a_.) + Log[(c_.)*(RFx_)^(p_.)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_))^(m_.
), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(a + b*Log[c*RFx^p])^n)/(e*(m + 1))
```

```
, x] - Dist[(b*n*p)/(e*(m + 1)), Int[SimplifyIntegrand[((d + e*x)^(m + 1)*(
a + b*Log[c*RFx^p])^(n - 1)*D[RFx, x])/RFx, x], x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && RationalFunctionQ[RFx, x] && IGtQ[n, 0] && (EqQ[n, 1] ||
IntegerQ[m]) && NeQ[m, -1]
```

Rubi steps

$$\int (f+gx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)dx = \frac{(f+gx)^4\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{4g} - \frac{B\int\frac{(bc-ad)(f+gx)^4}{(a+bx)(c+dx)}\,dx}{4g}$$
$$= \frac{(f+gx)^4\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{4g} - \frac{(B(bc-ad))\int\frac{(f+gx)^4}{(a+bx)(c+dx)}\,dx}{4g}$$
$$= \frac{(f+gx)^4\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{4g} - \frac{(B(bc-ad))\int\left(\frac{g^2\left(a^2d^2g^2-abdg(4df-cg)-\right.}{b^3}\right.}{}$$
$$= -\frac{B(bc-ad)g\left(a^2d^2g^2-abdg(4df-cg)+b^2\left(6d^2f^2-4cdfg+c^2g^2\right)\right)}{4b^3d^3}$$

**Mathematica [A]** time = 0.27, size = 215, normalized size = 0.95

$$\frac{(f+gx)^4\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)-\frac{B\left(6bdg^2x(bc-ad)\left(a^2d^2g^2+abdg(cg-4df)+b^2\left(c^2g^2-4cdfg+6d^2f^2\right)\right)+2b^3d^3g^4x^3(bc-ad)+3b^2d^2g^3x^2(bc-ad)\right.}{6b^4d^4}}{4g}$$

Antiderivative was successfully verified.

```
[In] Integrate[(f + g*x)^3*(A + B*Log[(e*(a + b*x))/(c + d*x)]),x]
```

```
[Out] ((f + g*x)^4*(A + B*Log[(e*(a + b*x))/(c + d*x)]) - (B*(6*b*d*(b*c - a*d)*g
^2*(a^2*d^2*g^2 + a*b*d*g*(-4*d*f + c*g) + b^2*(6*d^2*f^2 - 4*c*d*f*g + c^2
*g^2))*x + 3*b^2*d^2*(b*c - a*d)*g^3*(4*b*d*f - b*c*g - a*d*g)*x^2 + 2*b^3*
d^3*(b*c - a*d)*g^4*x^3 + 6*d^4*(b*f - a*g)^4*Log[a + b*x] - 6*b^4*(d*f - c
*g)^4*Log[c + d*x]))/(6*b^4*d^4))/(4*g)
```

**fricas [B]** time = 0.95, size = 445, normalized size = 1.96

$$\frac{6\,Ab^4d^4g^3x^4+2\left(12\,Ab^4d^4fg^2-\left(Bb^4cd^3-Bab^3d^4\right)g^3\right)x^3+3\left(12\,Ab^4d^4f^2g-4\left(Bb^4cd^3-Bab^3d^4\right)fg^2+\left(Bb^4c^2\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^3*(A+B*log(e*(b*x+a)/(d*x+c))),x, algorithm="fricas")
```

```
[Out] 1/24*(6*A*b^4*d^4*g^3*x^4 + 2*(12*A*b^4*d^4*f*g^2 - (B*b^4*c*d^3 - B*a*b^3*
d^4)*g^3)*x^3 + 3*(12*A*b^4*d^4*f^2*g - 4*(B*b^4*c*d^3 - B*a*b^3*d^4)*f*g^2
 + (B*b^4*c^2*d^2 - B*a^2*b^2*d^4)*g^3)*x^2 + 6*(4*A*b^4*d^4*f^3 - 6*(B*b^4
*c*d^3 - B*a*b^3*d^4)*f^2*g + 4*(B*b^4*c^2*d^2 - B*a^2*b^2*d^4)*f*g^2 - (B*
b^4*c^3*d - B*a^3*b*d^4)*g^3)*x + 6*(4*B*a*b^3*d^4*f^3 - 6*B*a^2*b^2*d^4*f^
2*g + 4*B*a^3*b*d^4*f*g^2 - B*a^4*d^4*g^3)*log(b*x + a) - 6*(4*B*b^4*c*d^3*
f^3 - 6*B*b^4*c^2*d^2*f^2*g + 4*B*b^4*c^3*d*f*g^2 - B*b^4*c^4*g^3)*log(d*x
+ c) + 6*(B*b^4*d^4*g^3*x^4 + 4*B*b^4*d^4*f*g^2*x^3 + 6*B*b^4*d^4*f^2*g*x^2
 + 4*B*b^4*d^4*f^3*x)*log((b*e*x + a*e)/(d*x + c)))/(b^4*d^4)
```

**giac [B]** time = 3.23, size = 11299, normalized size = 49.78

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^3*(A+B*log(e*(b*x+a)/(d*x+c))),x, algorithm="giac")
```

```
[Out] 1/24*(24*B*b^9*c^2*d^3*f^3*e^5*log(-b*e + (b*x*e + a*e)*d/(d*x + c)) - 48*B
*a*b^8*c*d^4*f^3*e^5*log(-b*e + (b*x*e + a*e)*d/(d*x + c)) + 24*B*a^2*b^7*d
^5*f^3*e^5*log(-b*e + (b*x*e + a*e)*d/(d*x + c)) - 36*B*b^9*c^3*d^2*f^2*g*e
^5*log(-b*e + (b*x*e + a*e)*d/(d*x + c)) + 36*B*a*b^8*c^2*d^3*f^2*g*e^5*log
(-b*e + (b*x*e + a*e)*d/(d*x + c)) + 36*B*a^2*b^7*c*d^4*f^2*g*e^5*log(-b*e
+ (b*x*e + a*e)*d/(d*x + c)) - 36*B*a^3*b^6*d^5*f^2*g*e^5*log(-b*e + (b*x*e
 + a*e)*d/(d*x + c)) + 24*B*b^9*c^4*d*f*g^2*e^5*log(-b*e + (b*x*e + a*e)*d/
(d*x + c)) - 24*B*a*b^8*c^3*d^2*f*g^2*e^5*log(-b*e + (b*x*e + a*e)*d/(d*x +
 c)) - 24*B*a^3*b^6*c*d^4*f*g^2*e^5*log(-b*e + (b*x*e + a*e)*d/(d*x + c)) +
 24*B*a^4*b^5*d^5*f*g^2*e^5*log(-b*e + (b*x*e + a*e)*d/(d*x + c)) - 6*B*b^9
*c^5*g^3*e^5*log(-b*e + (b*x*e + a*e)*d/(d*x + c)) + 6*B*a*b^8*c^4*d*g^3*e^
5*log(-b*e + (b*x*e + a*e)*d/(d*x + c)) + 6*B*a^4*b^5*c*d^4*g^3*e^5*log(-b*
e + (b*x*e + a*e)*d/(d*x + c)) - 6*B*a^5*b^4*d^5*g^3*e^5*log(-b*e + (b*x*e
+ a*e)*d/(d*x + c)) - 96*(b*x*e + a*e)*B*b^8*c^2*d^4*f^3*e^4*log(-b*e + (b*
x*e + a*e)*d/(d*x + c))/(d*x + c) + 192*(b*x*e + a*e)*B*a*b^7*c*d^5*f^3*e^4
*log(-b*e + (b*x*e + a*e)*d/(d*x + c))/(d*x + c) - 96*(b*x*e + a*e)*B*a^2*b
^6*d^6*f^3*e^4*log(-b*e + (b*x*e + a*e)*d/(d*x + c))/(d*x + c) + 144*(b*x*e
 + a*e)*B*b^8*c^3*d^3*f^2*g*e^4*log(-b*e + (b*x*e + a*e)*d/(d*x + c))/(d*x
+ c) - 144*(b*x*e + a*e)*B*a*b^7*c^2*d^4*f^2*g*e^4*log(-b*e + (b*x*e + a*e)
*d/(d*x + c))/(d*x + c) - 144*(b*x*e + a*e)*B*a^2*b^6*c*d^5*f^2*g*e^4*log(-
b*e + (b*x*e + a*e)*d/(d*x + c))/(d*x + c) + 144*(b*x*e + a*e)*B*a^3*b^5*d^
6*f^2*g*e^4*log(-b*e + (b*x*e + a*e)*d/(d*x + c))/(d*x + c) - 96*(b*x*e + a
*e)*B*b^8*c^4*d^2*f*g^2*e^4*log(-b*e + (b*x*e + a*e)*d/(d*x + c))/(d*x + c)
 + 96*(b*x*e + a*e)*B*a*b^7*c^3*d^3*f*g^2*e^4*log(-b*e + (b*x*e + a*e)*d/(d
*x + c))/(d*x + c) + 96*(b*x*e + a*e)*B*a^3*b^5*c*d^5*f*g^2*e^4*log(-b*e +
(b*x*e + a*e)*d/(d*x + c))/(d*x + c) - 96*(b*x*e + a*e)*B*a^4*b^4*d^6*f*g^2
```

*e^4*log(-b*e + (b*x*e + a*e)*d/(d*x + c))/(d*x + c) + 24*(b*x*e + a*e)*B*b^8*c^5*d*g^3*e^4*log(-b*e + (b*x*e + a*e)*d/(d*x + c))/(d*x + c) - 24*(b*x*e + a*e)*B*a*b^7*c^4*d^2*g^3*e^4*log(-b*e + (b*x*e + a*e)*d/(d*x + c))/(d*x + c) - 24*(b*x*e + a*e)*B*a^4*b^4*c*d^5*g^3*e^4*log(-b*e + (b*x*e + a*e)*d/(d*x + c))/(d*x + c) + 24*(b*x*e + a*e)*B*a^5*b^3*d^6*g^3*e^4*log(-b*e + (b*x*e + a*e)*d/(d*x + c))/(d*x + c) + 144*(b*x*e + a*e)^2*B*b^7*c^2*d^5*f^3*e^3*log(-b*e + (b*x*e + a*e)*d/(d*x + c))/(d*x + c)^2 - 288*(b*x*e + a*e)^2*B*a*b^6*c*d^6*f^3*e^3*log(-b*e + (b*x*e + a*e)*d/(d*x + c))/(d*x + c)^2 + 144*(b*x*e + a*e)^2*B*a^2*b^5*d^7*f^3*e^3*log(-b*e + (b*x*e + a*e)*d/(d*x + c))/(d*x + c)^2 - 216*(b*x*e + a*e)^2*B*b^7*c^3*d^4*f^2*g*e^3*log(-b*e + (b*x*e + a*e)*d/(d*x + c))/(d*x + c)^2 + 216*(b*x*e + a*e)^2*B*a*b^6*c^2*d^5*f^2*g*e^3*log(-b*e + (b*x*e + a*e)*d/(d*x + c))/(d*x + c)^2 + 216*(b*x*e + a*e)^2*B*a^2*b^5*c*d^6*f^2*g*e^3*log(-b*e + (b*x*e + a*e)*d/(d*x + c))/(d*x + c)^2 - 216*(b*x*e + a*e)^2*B*a^3*b^4*d^7*f^2*g*e^3*log(-b*e + (b*x*e + a*e)*d/(d*x + c))/(d*x + c)^2 + 144*(b*x*e + a*e)^2*B*b^7*c^4*d^3*f*g^2*e^3*log(-b*e + (b*x*e + a*e)*d/(d*x + c))/(d*x + c)^2 - 144*(b*x*e + a*e)^2*B*a*b^6*c^3*d^4*f*g^2*e^3*log(-b*e + (b*x*e + a*e)*d/(d*x + c))/(d*x + c)^2 - 144*(b*x*e + a*e)^2*B*a^3*b^4*c*d^6*f*g^2*e^3*log(-b*e + (b*x*e + a*e)*d/(d*x + c))/(d*x + c)^2 + 144*(b*x*e + a*e)^2*B*a^4*b^3*d^7*f*g^2*e^3*log(-b*e + (b*x*e + a*e)*d/(d*x + c))/(d*x + c)^2 - 36*(b*x*e + a*e)^2*B*b^7*c^5*d^2*g^3*e^3*log(-b*e + (b*x*e + a*e)*d/(d*x + c))/(d*x + c)^2 + 36*(b*x*e + a*e)^2*B*a*b^6*c^4*d^3*g^3*e^3*log(-b*e + (b*x*e + a*e)*d/(d*x + c))/(d*x + c)^2 + 36*(b*x*e + a*e)^2*B*a^4*b^3*c*d^6*g^3*e^3*log(-b*e + (b*x*e + a*e)*d/(d*x + c))/(d*x + c)^2 - 36*(b*x*e + a*e)^2*B*a^5*b^2*d^7*g^3*e^3*log(-b*e + (b*x*e + a*e)*d/(d*x + c))/(d*x + c)^2 - 96*(b*x*e + a*e)^3*B*b^6*c^2*d^6*f^3*e^2*log(-b*e + (b*x*e + a*e)*d/(d*x + c))/(d*x + c)^3 + 192*(b*x*e + a*e)^3*B*a*b^5*c*d^7*f^3*e^2*log(-b*e + (b*x*e + a*e)*d/(d*x + c))/(d*x + c)^3 - 96*(b*x*e + a*e)^3*B*a^2*b^4*d^8*f^3*e^2*log(-b*e + (b*x*e + a*e)*d/(d*x + c))/(d*x + c)^3 + 144*(b*x*e + a*e)^3*B*b^6*c^3*d^5*f^2*g*e^2*log(-b*e + (b*x*e + a*e)*d/(d*x + c))/(d*x + c)^3 - 144*(b*x*e + a*e)^3*B*a*b^5*c^2*d^6*f^2*g*e^2*log(-b*e + (b*x*e + a*e)*d/(d*x + c))/(d*x + c)^3 - 144*(b*x*e + a*e)^3*B*a^2*b^4*c*d^7*f^2*g*e^2*log(-b*e + (b*x*e + a*e)*d/(d*x + c))/(d*x + c)^3 + 144*(b*x*e + a*e)^3*B*a^3*b^3*d^8*f^2*g*e^2*log(-b*e + (b*x*e + a*e)*d/(d*x + c))/(d*x + c)^3 - 96*(b*x*e + a*e)^3*B*b^6*c^4*d^4*f*g^2*e^2*log(-b*e + (b*x*e + a*e)*d/(d*x + c))/(d*x + c)^3 + 96*(b*x*e + a*e)^3*B*a*b^5*c^3*d^5*f*g^2*e^2*log(-b*e + (b*x*e + a*e)*d/(d*x + c))/(d*x + c)^3 + 96*(b*x*e + a*e)^3*B*a^3*b^3*c*d^7*f*g^2*e^2*log(-b*e + (b*x*e + a*e)*d/(d*x + c))/(d*x + c)^3 - 96*(b*x*e + a*e)^3*B*a^4*b^2*d^8*f*g^2*e^2*log(-b*e + (b*x*e + a*e)*d/(d*x + c))/(d*x + c)^3 + 24*(b*x*e + a*e)^3*B*b^6*c^5*d^3*g^3*e^2*log(-b*e + (b*x*e + a*e)*d/(d*x + c))/(d*x + c)^3 - 24*(b*x*e + a*e)^3*B*a*b^5*c^4*d^4*g^3*e^2*log(-b*e + (b*x*e + a*e)*d/(d*x + c))/(d*x + c)^3 - 24*(b*x*e + a*e)^3*B*a^4*b^2*c*d^7*g^3*e^2*log(-b*e + (b*x*e + a*e)*d/(d*x + c))/(d*x + c)^3 + 24*(b*x*e + a*e)^3*B*a^5*b*d^8*g^3*e^2*log(-b*e + (b*x*e + a*e)*d/(d*x + c))/(d*x + c)^3 + 24*(b*x*e + a*e)^4*B*b^5*c^2*d^7*f^3*e*log(-b*e + (b*x*e + a*e)*d/(d*x + c))/(d*x + c)^4 - 48*(b*x*e +

```
a*e)^4*B*a*b^4*c*d^8*f^3*e*log(-b*e + (b*x*e + a*e)*d/(d*x + c))/(d*x + c)
^4 + 24*(b*x*e + a*e)^4*B*a^2*b^3*d^9*f^3*e*log(-b*e + (b*x*e + a*e)*d/(d*x
 + c))/(d*x + c)^4 - 36*(b*x*e + a*e)^4*B*b^5*c^3*d^6*f^2*g*e*log(-b*e + (b
*x*e + a*e)*d/(d*x + c))/(d*x + c)^4 + 36*(b*x*e + a*e)^4*B*a*b^4*c^2*d^7*f
^2*g*e*log(-b*e + (b*x*e + a*e)*d/(d*x + c))/(d*x + c)^4 + 36*(b*x*e + a*e)
^4*B*a^2*b^3*c*d^8*f^2*g*e*log(-b*e + (b*x*e + a*e)*d/(d*x + c))/(d*x + c)^
4 - 36*(b*x*e + a*e)^4*B*a^3*b^2*d^9*f^2*g*e*log(-b*e + (b*x*e + a*e)*d/(d*
x + c))/(d*x + c)^4 + 24*(b*x*e + a*e)^4*B*b^5*c^4*d^5*f*g^2*e*log(-b*e + (
b*x*e + a*e)*d/(d*x + c))/(d*x + c)^4 - 24*(b*x*e + a*e)^4*B*a*b^4*c^3*d^6*
f*g^2*e*log(-b*e + (b*x*e + a*e)*d/(d*x + c))/(d*x + c)^4 - 24*(b*x*e + a*e
)^4*B*a^3*b^2*c*d^8*f*g^2*e*log(-b*e + (b*x*e + a*e)*d/(d*x + c))/(d*x + c)
^4 + 24*(b*x*e + a*e)^4*B*a^4*b*d^9*f*g^2*e*log(-b*e + (b*x*e + a*e)*d/(d*x
 + c))/(d*x + c)^4 - 6*(b*x*e + a*e)^4*B*b^5*c^5*d^4*g^3*e*log(-b*e + (b*x*
e + a*e)*d/(d*x + c))/(d*x + c)^4 + 6*(b*x*e + a*e)^4*B*a*b^4*c^4*d^5*g^3*e
*log(-b*e + (b*x*e + a*e)*d/(d*x + c))/(d*x + c)^4 + 6*(b*x*e + a*e)^4*B*a^
4*b*c*d^8*g^3*e*log(-b*e + (b*x*e + a*e)*d/(d*x + c))/(d*x + c)^4 - 6*(b*x*
e + a*e)^4*B*a^5*d^9*g^3*e*log(-b*e + (b*x*e + a*e)*d/(d*x + c))/(d*x + c)^
4 + 24*(b*x*e + a*e)*B*b^8*c^2*d^4*f^3*e^4*log((b*x*e + a*e)/(d*x + c))/(d*
x + c) - 48*(b*x*e + a*e)*B*a*b^7*c*d^5*f^3*e^4*log((b*x*e + a*e)/(d*x + c)
)/(d*x + c) + 24*(b*x*e + a*e)*B*a^2*b^6*d^6*f^3*e^4*log((b*x*e + a*e)/(d*x
 + c))/(d*x + c) - 72*(b*x*e + a*e)*B*a*b^7*c^2*d^4*f^2*g*e^4*log((b*x*e +
a*e)/(d*x + c))/(d*x + c) + 144*(b*x*e + a*e)*B*a^2*b^6*c*d^5*f^2*g*e^4*log
((b*x*e + a*e)/(d*x + c))/(d*x + c) - 72*(b*x*e + a*e)*B*a^3*b^5*d^6*f^2*g*
e^4*log((b*x*e + a*e)/(d*x + c))/(d*x + c) + 72*(b*x*e + a*e)*B*a^2*b^6*c^2
*d^4*f*g^2*e^4*log((b*x*e + a*e)/(d*x + c))/(d*x + c) - 144*(b*x*e + a*e)*B
*a^3*b^5*c*d^5*f*g^2*e^4*log((b*x*e + a*e)/(d*x + c))/(d*x + c) + 72*(b*x*e
 + a*e)*B*a^4*b^4*d^6*f*g^2*e^4*log((b*x*e + a*e)/(d*x + c))/(d*x + c) - 24
*(b*x*e + a*e)*B*a^3*b^5*c^2*d^4*g^3*e^4*log((b*x*e + a*e)/(d*x + c))/(d*x
+ c) + 48*(b*x*e + a*e)*B*a^4*b^4*c*d^5*g^3*e^4*log((b*x*e + a*e)/(d*x + c)
)/(d*x + c) - 24*(b*x*e + a*e)*B*a^5*b^3*d^6*g^3*e^4*log((b*x*e + a*e)/(d*x
 + c))/(d*x + c) - 72*(b*x*e + a*e)^2*B*b^7*c^2*d^5*f^3*e^3*log((b*x*e + a*
e)/(d*x + c))/(d*x + c)^2 + 144*(b*x*e + a*e)^2*B*a*b^6*c*d^6*f^3*e^3*log((
b*x*e + a*e)/(d*x + c))/(d*x + c)^2 - 72*(b*x*e + a*e)^2*B*a^2*b^5*d^7*f^3*
e^3*log((b*x*e + a*e)/(d*x + c))/(d*x + c)^2 + 36*(b*x*e + a*e)^2*B*b^7*c^3
*d^4*f^2*g*e^3*log((b*x*e + a*e)/(d*x + c))/(d*x + c)^2 + 108*(b*x*e + a*e)
^2*B*a*b^6*c^2*d^5*f^2*g*e^3*log((b*x*e + a*e)/(d*x + c))/(d*x + c)^2 - 324
*(b*x*e + a*e)^2*B*a^2*b^5*c*d^6*f^2*g*e^3*log((b*x*e + a*e)/(d*x + c))/(d*
x + c)^2 + 180*(b*x*e + a*e)^2*B*a^3*b^4*d^7*f^2*g*e^3*log((b*x*e + a*e)/(d
*x + c))/(d*x + c)^2 - 72*(b*x*e + a*e)^2*B*a*b^6*c^3*d^4*f*g^2*e^3*log((b*
x*e + a*e)/(d*x + c))/(d*x + c)^2 + 216*(b*x*e + a*e)^2*B*a^3*b^4*c*d^6*f*g
^2*e^3*log((b*x*e + a*e)/(d*x + c))/(d*x + c)^2 - 144*(b*x*e + a*e)^2*B*a^4
*b^3*d^7*f*g^2*e^3*log((b*x*e + a*e)/(d*x + c))/(d*x + c)^2 + 36*(b*x*e + a
*e)^2*B*a^2*b^5*c^3*d^4*g^3*e^3*log((b*x*e + a*e)/(d*x + c))/(d*x + c)^2 -
36*(b*x*e + a*e)^2*B*a^3*b^4*c^2*d^5*g^3*e^3*log((b*x*e + a*e)/(d*x + c))/(
d*x + c)^2 - 36*(b*x*e + a*e)^2*B*a^4*b^3*c*d^6*g^3*e^3*log((b*x*e + a*e)/(
```

d*x + c))/(d*x + c)^2 + 36*(b*x*e + a*e)^2*B*a^5*b^2*d^7*g^3*e^3*log((b*x*e + a*e)/(d*x + c))/(d*x + c)^2 + 72*(b*x*e + a*e)^3*B*b^6*c^2*d^6*f^3*e^2*log((b*x*e + a*e)/(d*x + c))/(d*x + c)^3 - 144*(b*x*e + a*e)^3*B*a*b^5*c*d^7*f^3*e^2*log((b*x*e + a*e)/(d*x + c))/(d*x + c)^3 + 72*(b*x*e + a*e)^3*B*a^2*b^4*d^8*f^3*e^2*log((b*x*e + a*e)/(d*x + c))/(d*x + c)^3 - 72*(b*x*e + a*e)^3*B*b^6*c^3*d^5*f^2*g*e^2*log((b*x*e + a*e)/(d*x + c))/(d*x + c)^3 + 216*(b*x*e + a*e)^3*B*a^2*b^4*c*d^7*f^2*g*e^2*log((b*x*e + a*e)/(d*x + c))/(d*x + c)^3 - 144*(b*x*e + a*e)^3*B*a^3*b^3*d^8*f^2*g*e^2*log((b*x*e + a*e)/(d*x + c))/(d*x + c)^3 + 24*(b*x*e + a*e)^3*B*b^6*c^4*d^4*f*g^2*e^2*log((b*x*e + a*e)/(d*x + c))/(d*x + c)^3 + 48*(b*x*e + a*e)^3*B*a*b^5*c^3*d^5*f*g^2*e^2*log((b*x*e + a*e)/(d*x + c))/(d*x + c)^3 - 72*(b*x*e + a*e)^3*B*a^2*b^4*c^2*d^6*f*g^2*e^2*log((b*x*e + a*e)/(d*x + c))/(d*x + c)^3 - 96*(b*x*e + a*e)^3*B*a^3*b^3*c*d^7*f*g^2*e^2*log((b*x*e + a*e)/(d*x + c))/(d*x + c)^3 + 96*(b*x*e + a*e)^3*B*a^4*b^2*d^8*f*g^2*e^2*log((b*x*e + a*e)/(d*x + c))/(d*x + c)^3 - 24*(b*x*e + a*e)^3*B*a*b^5*c^4*d^4*g^3*e^2*log((b*x*e + a*e)/(d*x + c))/(d*x + c)^3 + 24*(b*x*e + a*e)^3*B*a^2*b^4*c^3*d^5*g^3*e^2*log((b*x*e + a*e)/(d*x + c))/(d*x + c)^3 + 24*(b*x*e + a*e)^3*B*a^4*b^2*c*d^7*g^3*e^2*log((b*x*e + a*e)/(d*x + c))/(d*x + c)^3 - 24*(b*x*e + a*e)^3*B*a^5*b*d^8*g^3*e^2*log((b*x*e + a*e)/(d*x + c))/(d*x + c)^3 - 24*(b*x*e + a*e)^4*B*b^5*c^2*d^7*f^3*e*log((b*x*e + a*e)/(d*x + c))/(d*x + c)^4 + 48*(b*x*e + a*e)^4*B*a*b^4*c*d^8*f^3*e*log((b*x*e + a*e)/(d*x + c))/(d*x + c)^4 - 24*(b*x*e + a*e)^4*B*a^2*b^3*d^9*f^3*e*log((b*x*e + a*e)/(d*x + c))/(d*x + c)^4 + 36*(b*x*e + a*e)^4*B*b^5*c^3*d^6*f^2*g*e*log((b*x*e + a*e)/(d*x + c))/(d*x + c)^4 - 36*(b*x*e + a*e)^4*B*a*b^4*c^2*d^7*f^2*g*e*log((b*x*e + a*e)/(d*x + c))/(d*x + c)^4 - 36*(b*x*e + a*e)^4*B*a^2*b^3*c*d^8*f^2*g*e*log((b*x*e + a*e)/(d*x + c))/(d*x + c)^4 + 36*(b*x*e + a*e)^4*B*a^3*b^2*d^9*f^2*g*e*log((b*x*e + a*e)/(d*x + c))/(d*x + c)^4 - 24*(b*x*e + a*e)^4*B*b^5*c^4*d^5*f*g^2*e*log((b*x*e + a*e)/(d*x + c))/(d*x + c)^4 + 24*(b*x*e + a*e)^4*B*a*b^4*c^3*d^6*f*g^2*e*log((b*x*e + a*e)/(d*x + c))/(d*x + c)^4 + 24*(b*x*e + a*e)^4*B*a^3*b^2*c*d^8*f*g^2*e*log((b*x*e + a*e)/(d*x + c))/(d*x + c)^4 - 24*(b*x*e + a*e)^4*B*a^4*b*d^9*f*g^2*e*log((b*x*e + a*e)/(d*x + c))/(d*x + c)^4 + 6*(b*x*e + a*e)^4*B*b^5*c^5*d^4*g^3*e*log((b*x*e + a*e)/(d*x + c))/(d*x + c)^4 - 6*(b*x*e + a*e)^4*B*a*b^4*c^4*d^5*g^3*e*log((b*x*e + a*e)/(d*x + c))/(d*x + c)^4 - 6*(b*x*e + a*e)^4*B*a^4*b*c*d^8*g^3*e*log((b*x*e + a*e)/(d*x + c))/(d*x + c)^4 + 6*(b*x*e + a*e)^4*B*a^5*d^9*g^3*e*log((b*x*e + a*e)/(d*x + c))/(d*x + c)^4 + 24*A*b^9*c^2*d^3*f^3*e^5 - 48*A*a*b^8*c*d^4*f^3*e^5 + 24*A*a^2*b^7*d^5*f^3*e^5 - 36*A*b^9*c^3*d^2*f^2*g*e^5 - 36*B*b^9*c^3*d^2*f^2*g*e^5 + 36*A*a*b^8*c^2*d^3*f^2*g*e^5 + 108*B*a*b^8*c^2*d^3*f^2*g*e^5 + 36*A*a^2*b^7*c*d^4*f^2*g*e^5 - 108*B*a^2*b^7*c*d^4*f^2*g*e^5 - 36*A*a^3*b^6*d^5*f^2*g*e^5 + 36*B*a^3*b^6*d^5*f^2*g*e^5 + 24*A*b^9*c^4*d*f*g^2*e^5 + 36*B*b^9*c^4*d*f*g^2*e^5 - 24*A*a*b^8*c^3*d^2*f*g^2*e^5 - 72*B*a*b^8*c^3*d^2*f*g^2*e^5 - 24*A*a^3*b^6*c*d^4*f*g^2*e^5 + 72*B*a^3*b^6*c*d^4*f*g^2*e^5 + 24*A*a^4*b^5*d^5*f*g^2*e^5 - 36*B*a^4*b^5*d^5*f*g^2*e^5 - 6*A*b^9*c^5*g^3*e^5 - 11*B*b^9*c^5*g^3*e^5 + 6*A*a*b^8*c^4*d*g^3*e^5 + 19*B*a*b^8*c^4*d*g^3*e^5 - 2*B*a^2*b^7*c^3*d^2*g^3*e^5 + 2*B*a^3*b^6*c^2*d^3*g^3*e^5 + 6*A*a^4*b

^5*c*d^4*g^3*e^5 - 19*B*a^4*b^5*c*d^4*g^3*e^5 - 6*A*a^5*b^4*d^5*g^3*e^5 + 1
1*B*a^5*b^4*d^5*g^3*e^5 - 72*(b*x*e + a*e)*A*b^8*c^2*d^4*f^3*e^4/(d*x + c)
+ 144*(b*x*e + a*e)*A*a*b^7*c*d^5*f^3*e^4/(d*x + c) - 72*(b*x*e + a*e)*A*a^
2*b^6*d^6*f^3*e^4/(d*x + c) + 144*(b*x*e + a*e)*A*b^8*c^3*d^3*f^2*g*e^4/(d*
x + c) + 108*(b*x*e + a*e)*B*b^8*c^3*d^3*f^2*g*e^4/(d*x + c) - 216*(b*x*e +
 a*e)*A*a*b^7*c^2*d^4*f^2*g*e^4/(d*x + c) - 324*(b*x*e + a*e)*B*a*b^7*c^2*d
^4*f^2*g*e^4/(d*x + c) + 324*(b*x*e + a*e)*B*a^2*b^6*c*d^5*f^2*g*e^4/(d*x +
 c) + 72*(b*x*e + a*e)*A*a^3*b^5*d^6*f^2*g*e^4/(d*x + c) - 108*(b*x*e + a*e
)*B*a^3*b^5*d^6*f^2*g*e^4/(d*x + c) - 96*(b*x*e + a*e)*A*b^8*c^4*d^2*f*g^2*
e^4/(d*x + c) - 120*(b*x*e + a*e)*B*b^8*c^4*d^2*f*g^2*e^4/(d*x + c) + 96*(b
*x*e + a*e)*A*a*b^7*c^3*d^3*f*g^2*e^4/(d*x + c) + 264*(b*x*e + a*e)*B*a*b^7
*c^3*d^3*f*g^2*e^4/(d*x + c) + 72*(b*x*e + a*e)*A*a^2*b^6*c^2*d^4*f*g^2*e^4
/(d*x + c) - 72*(b*x*e + a*e)*B*a^2*b^6*c^2*d^4*f*g^2*e^4/(d*x + c) - 48*(b
*x*e + a*e)*A*a^3*b^5*c*d^5*f*g^2*e^4/(d*x + c) - 168*(b*x*e + a*e)*B*a^3*b
^5*c*d^5*f*g^2*e^4/(d*x + c) - 24*(b*x*e + a*e)*A*a^4*b^4*d^6*f*g^2*e^4/(d*
x + c) + 96*(b*x*e + a*e)*B*a^4*b^4*d^6*f*g^2*e^4/(d*x + c) + 24*(b*x*e + a
*e)*A*b^8*c^5*d*g^3*e^4/(d*x + c) + 38*(b*x*e + a*e)*B*b^8*c^5*d*g^3*e^4/(d
*x + c) - 24*(b*x*e + a*e)*A*a*b^7*c^4*d^2*g^3*e^4/(d*x + c) - 70*(b*x*e +
a*e)*B*a*b^7*c^4*d^2*g^3*e^4/(d*x + c) + 8*(b*x*e + a*e)*B*a^2*b^6*c^3*d^3*
g^3*e^4/(d*x + c) - 24*(b*x*e + a*e)*A*a^3*b^5*c^2*d^4*g^3*e^4/(d*x + c) +
16*(b*x*e + a*e)*B*a^3*b^5*c^2*d^4*g^3*e^4/(d*x + c) + 24*(b*x*e + a*e)*A*a
^4*b^4*c*d^5*g^3*e^4/(d*x + c) + 34*(b*x*e + a*e)*B*a^4*b^4*c*d^5*g^3*e^4/(
d*x + c) - 26*(b*x*e + a*e)*B*a^5*b^3*d^6*g^3*e^4/(d*x + c) + 72*(b*x*e + a
*e)^2*A*b^7*c^2*d^5*f^3*e^3/(d*x + c)^2 - 144*(b*x*e + a*e)^2*A*a*b^6*c*d^6
*f^3*e^3/(d*x + c)^2 + 72*(b*x*e + a*e)^2*A*a^2*b^5*d^7*f^3*e^3/(d*x + c)^2
 - 180*(b*x*e + a*e)^2*A*b^7*c^3*d^4*f^2*g*e^3/(d*x + c)^2 - 108*(b*x*e + a
*e)^2*B*b^7*c^3*d^4*f^2*g*e^3/(d*x + c)^2 + 324*(b*x*e + a*e)^2*A*a*b^6*c^2
*d^5*f^2*g*e^3/(d*x + c)^2 + 324*(b*x*e + a*e)^2*B*a*b^6*c^2*d^5*f^2*g*e^3/
(d*x + c)^2 - 108*(b*x*e + a*e)^2*A*a^2*b^5*c*d^6*f^2*g*e^3/(d*x + c)^2 - 3
24*(b*x*e + a*e)^2*B*a^2*b^5*c*d^6*f^2*g*e^3/(d*x + c)^2 - 36*(b*x*e + a*e)
^2*A*a^3*b^4*d^7*f^2*g*e^3/(d*x + c)^2 + 108*(b*x*e + a*e)^2*B*a^3*b^4*d^7*
f^2*g*e^3/(d*x + c)^2 + 144*(b*x*e + a*e)^2*A*b^7*c^4*d^3*f*g^2*e^3/(d*x +
c)^2 + 132*(b*x*e + a*e)^2*B*b^7*c^4*d^3*f*g^2*e^3/(d*x + c)^2 - 216*(b*x*e
 + a*e)^2*A*a*b^6*c^3*d^4*f*g^2*e^3/(d*x + c)^2 - 312*(b*x*e + a*e)^2*B*a*b
^6*c^3*d^4*f*g^2*e^3/(d*x + c)^2 + 144*(b*x*e + a*e)^2*B*a^2*b^5*c^2*d^5*f*
g^2*e^3/(d*x + c)^2 + 72*(b*x*e + a*e)^2*A*a^3*b^4*c*d^6*f*g^2*e^3/(d*x + c
)^2 + 120*(b*x*e + a*e)^2*B*a^3*b^4*c*d^6*f*g^2*e^3/(d*x + c)^2 - 84*(b*x*e
 + a*e)^2*B*a^4*b^3*d^7*f*g^2*e^3/(d*x + c)^2 - 36*(b*x*e + a*e)^2*A*b^7*c^
5*d^2*g^3*e^3/(d*x + c)^2 - 45*(b*x*e + a*e)^2*B*b^7*c^5*d^2*g^3*e^3/(d*x +
 c)^2 + 36*(b*x*e + a*e)^2*A*a*b^6*c^4*d^3*g^3*e^3/(d*x + c)^2 + 93*(b*x*e
+ a*e)^2*B*a*b^6*c^4*d^3*g^3*e^3/(d*x + c)^2 + 36*(b*x*e + a*e)^2*A*a^2*b^5
*c^3*d^4*g^3*e^3/(d*x + c)^2 - 30*(b*x*e + a*e)^2*B*a^2*b^5*c^3*d^4*g^3*e^3
/(d*x + c)^2 - 36*(b*x*e + a*e)^2*A*a^3*b^4*c^2*d^5*g^3*e^3/(d*x + c)^2 - 1
8*(b*x*e + a*e)^2*B*a^3*b^4*c^2*d^5*g^3*e^3/(d*x + c)^2 - 21*(b*x*e + a*e)^
2*B*a^4*b^3*c*d^6*g^3*e^3/(d*x + c)^2 + 21*(b*x*e + a*e)^2*B*a^5*b^2*d^7*g^

```
3*e^3/(d*x + c)^2 - 24*(b*x*e + a*e)^3*A*b^6*c^2*d^6*f^3*e^2/(d*x + c)^3 +
48*(b*x*e + a*e)^3*A*a*b^5*c*d^7*f^3*e^2/(d*x + c)^3 - 24*(b*x*e + a*e)^3*A
*a^2*b^4*d^8*f^3*e^2/(d*x + c)^3 + 72*(b*x*e + a*e)^3*A*b^6*c^3*d^5*f^2*g*e
^2/(d*x + c)^3 + 36*(b*x*e + a*e)^3*B*b^6*c^3*d^5*f^2*g*e^2/(d*x + c)^3 - 1
44*(b*x*e + a*e)^3*A*a*b^5*c^2*d^6*f^2*g*e^2/(d*x + c)^3 - 108*(b*x*e + a*e
)^3*B*a*b^5*c^2*d^6*f^2*g*e^2/(d*x + c)^3 + 72*(b*x*e + a*e)^3*A*a^2*b^4*c*
d^7*f^2*g*e^2/(d*x + c)^3 + 108*(b*x*e + a*e)^3*B*a^2*b^4*c*d^7*f^2*g*e^2/(
d*x + c)^3 - 36*(b*x*e + a*e)^3*B*a^3*b^3*d^8*f^2*g*e^2/(d*x + c)^3 - 72*(b
*x*e + a*e)^3*A*b^6*c^4*d^4*f*g^2*e^2/(d*x + c)^3 - 48*(b*x*e + a*e)^3*B*b^
6*c^4*d^4*f*g^2*e^2/(d*x + c)^3 + 144*(b*x*e + a*e)^3*A*a*b^5*c^3*d^5*f*g^2
*e^2/(d*x + c)^3 + 120*(b*x*e + a*e)^3*B*a*b^5*c^3*d^5*f*g^2*e^2/(d*x + c)^
3 - 72*(b*x*e + a*e)^3*A*a^2*b^4*c^2*d^6*f*g^2*e^2/(d*x + c)^3 - 72*(b*x*e
+ a*e)^3*B*a^2*b^4*c^2*d^6*f*g^2*e^2/(d*x + c)^3 - 24*(b*x*e + a*e)^3*B*a^3
*b^3*c*d^7*f*g^2*e^2/(d*x + c)^3 + 24*(b*x*e + a*e)^3*B*a^4*b^2*d^8*f*g^2*e
^2/(d*x + c)^3 + 24*(b*x*e + a*e)^3*A*b^6*c^5*d^3*g^3*e^2/(d*x + c)^3 + 18*
(b*x*e + a*e)^3*B*b^6*c^5*d^3*g^3*e^2/(d*x + c)^3 - 48*(b*x*e + a*e)^3*A*a*
b^5*c^4*d^4*g^3*e^2/(d*x + c)^3 - 42*(b*x*e + a*e)^3*B*a*b^5*c^4*d^4*g^3*e^
2/(d*x + c)^3 + 24*(b*x*e + a*e)^3*A*a^2*b^4*c^3*d^5*g^3*e^2/(d*x + c)^3 +
24*(b*x*e + a*e)^3*B*a^2*b^4*c^3*d^5*g^3*e^2/(d*x + c)^3 + 6*(b*x*e + a*e)^
3*B*a^4*b^2*c*d^7*g^3*e^2/(d*x + c)^3 - 6*(b*x*e + a*e)^3*B*a^5*b*d^8*g^3*e
^2/(d*x + c)^3)*(b*c/((b*c*e - a*d*e)*(b*c - a*d)) - a*d/((b*c*e - a*d*e)*(
b*c - a*d)))/(b^8*d^4*e^4 - 4*(b*x*e + a*e)*b^7*d^5*e^3/(d*x + c) + 6*(b*x*
e + a*e)^2*b^6*d^6*e^2/(d*x + c)^2 - 4*(b*x*e + a*e)^3*b^5*d^7*e/(d*x + c)^
3 + (b*x*e + a*e)^4*b^4*d^8/(d*x + c)^4)
```

**maple [B]** time = 0.17, size = 8605, normalized size = 37.91

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x+f)^3*(B*ln((b*x+a)/(d*x+c)*e)+A),x)
```

```
[Out] result too large to display
```

**maxima [A]** time = 0.98, size = 415, normalized size = 1.83

$$\frac{1}{4}Ag^3x^4+Afg^2x^3+\frac{3}{2}Af^2gx^2+\left(x\log\left(\frac{bex}{dx+c}+\frac{ae}{dx+c}\right)+\frac{a\log(bx+a)}{b}-\frac{c\log(dx+c)}{d}\right)Bf^3+\frac{3}{2}\left(x^2\log\left(\frac{bex}{dx+\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^3*(A+B*log(e*(b*x+a)/(d*x+c))),x, algorithm="maxima")
```

```
[Out] 1/4*A*g^3*x^4 + A*f*g^2*x^3 + 3/2*A*f^2*g*x^2 + (x*log(b*e*x/(d*x + c) + a*
e/(d*x + c)) + a*log(b*x + a)/b - c*log(d*x + c)/d)*B*f^3 + 3/2*(x^2*log(b*
```

```
e*x/(d*x + c) + a*e/(d*x + c)) - a^2*log(b*x + a)/b^2 + c^2*log(d*x + c)/d^
2 - (b*c - a*d)*x/(b*d))*B*f^2*g + 1/2*(2*x^3*log(b*e*x/(d*x + c) + a*e/(d*
x + c)) + 2*a^3*log(b*x + a)/b^3 - 2*c^3*log(d*x + c)/d^3 - ((b^2*c*d - a*b
*d^2)*x^2 - 2*(b^2*c^2 - a^2*d^2)*x)/(b^2*d^2))*B*f*g^2 + 1/24*(6*x^4*log(b
*e*x/(d*x + c) + a*e/(d*x + c)) - 6*a^4*log(b*x + a)/b^4 + 6*c^4*log(d*x +
c)/d^4 - (2*(b^3*c*d^2 - a*b^2*d^3)*x^3 - 3*(b^3*c^2*d - a^2*b*d^3)*x^2 + 6
*(b^3*c^3 - a^3*d^3)*x)/(b^3*d^3))*B*g^3 + A*f^3*x
```

**mupad** [**B**] time = 4.69, size = 741, normalized size = 3.26

$$x\left(\frac{4\,A\,b\,d\,f^3+12\,A\,a\,c\,f\,g^2+12\,A\,a\,d\,f^2\,g+12\,A\,b\,c\,f^2\,g+6\,B\,a\,d\,f^2\,g-6\,B\,b\,c\,f^2\,g}{4\,b\,d}+\frac{(4\,a\,d+4\,b\,c)\left(\left(\frac{4\,A\,a\,d\,g^3+\cdots}{\cdots}\right.\right.}{\cdots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f + g*x)^3*(A + B*log((e*(a + b*x))/(c + d*x))),x)
```

```
[Out] x*((4*A*b*d*f^3 + 12*A*a*c*f*g^2 + 12*A*a*d*f^2*g + 12*A*b*c*f^2*g + 6*B*a*
d*f^2*g - 6*B*b*c*f^2*g)/(4*b*d) + ((4*a*d + 4*b*c)*((((4*A*a*d*g^3 + 4*A*b
*c*g^3 + B*a*d*g^3 - B*b*c*g^3 + 12*A*b*d*f*g^2)/(4*b*d) - (A*g^3*(4*a*d +
4*b*c))/(4*b*d))*(4*a*d + 4*b*c))/(4*b*d) - (4*A*a*c*g^3 + 12*A*a*d*f*g^2 +
 12*A*b*c*f*g^2 + 12*A*b*d*f^2*g + 4*B*a*d*f*g^2 - 4*B*b*c*f*g^2)/(4*b*d) +
 (A*a*c*g^3)/(b*d)))/(4*b*d) - (a*c*((4*A*a*d*g^3 + 4*A*b*c*g^3 + B*a*d*g^3
 - B*b*c*g^3 + 12*A*b*d*f*g^2)/(4*b*d) - (A*g^3*(4*a*d + 4*b*c))/(4*b*d)))/
(b*d)) - x^2*((((4*A*a*d*g^3 + 4*A*b*c*g^3 + B*a*d*g^3 - B*b*c*g^3 + 12*A*b
*d*f*g^2)/(4*b*d) - (A*g^3*(4*a*d + 4*b*c))/(4*b*d))*(4*a*d + 4*b*c))/(8*b*
d) - (4*A*a*c*g^3 + 12*A*a*d*f*g^2 + 12*A*b*c*f*g^2 + 12*A*b*d*f^2*g + 4*B*
a*d*f*g^2 - 4*B*b*c*f*g^2)/(8*b*d) + (A*a*c*g^3)/(2*b*d)) + log((e*(a + b*x
))/(c + d*x))*((B*g^3*x^4)/4 + B*f^3*x + (3*B*f^2*g*x^2)/2 + B*f*g^2*x^3) +
 x^3*((4*A*a*d*g^3 + 4*A*b*c*g^3 + B*a*d*g^3 - B*b*c*g^3 + 12*A*b*d*f*g^2)/
(12*b*d) - (A*g^3*(4*a*d + 4*b*c))/(12*b*d)) + (A*g^3*x^4)/4 - (log(a + b*x
)*(B*a^4*g^3 - 4*B*a*b^3*f^3 + 6*B*a^2*b^2*f^2*g - 4*B*a^3*b*f*g^2))/(4*b^4
) + (log(c + d*x)*(B*c^4*g^3 - 4*B*c*d^3*f^3 + 6*B*c^2*d^2*f^2*g - 4*B*c^3*
d*f*g^2))/(4*d^4)
```

**sympy** [**B**] time = 13.27, size = 998, normalized size = 4.40

$$\frac{Ag^3x^4}{4}-\frac{Ba\left(ag-2bf\right)\left(a^2g^2-2abfg+2b^2f^2\right)\log\left(x+\frac{Ba^4cd^3g^3-4Ba^3bcd^3fg^2+6Ba^2b^2cd^3f^2g+\frac{Ba^2d^4\left(ag-2bf\right)\left(a^2g^2-2abfg+2b^2f^2\right)}{b}\cdots}{Ba^4d^4g^3-4Ba^3bd^4fg^2+6Ba^2b^2d^4f^2g-4\cdots}\right.}{4b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)**3*(A+B*ln(e*(b*x+a)/(d*x+c))),x)
```

```
[Out] A*g**3*x**4/4 - B*a*(a*g - 2*b*f)*(a**2*g**2 - 2*a*b*f*g + 2*b**2*f**2)*log
(x + (B*a**4*c*d**3*g**3 - 4*B*a**3*b*c*d**3*f*g**2 + 6*B*a**2*b**2*c*d**3*
f**2*g + B*a**2*d**4*(a*g - 2*b*f)*(a**2*g**2 - 2*a*b*f*g + 2*b**2*f**2)/b
+ B*a*b**3*c**4*g**3 - 4*B*a*b**3*c**3*d*f*g**2 + 6*B*a*b**3*c**2*d**2*f**2
*g - 8*B*a*b**3*c*d**3*f**3 - B*a*c*d**3*(a*g - 2*b*f)*(a**2*g**2 - 2*a*b*f
*g + 2*b**2*f**2))/(B*a**4*d**4*g**3 - 4*B*a**3*b*d**4*f*g**2 + 6*B*a**2*b*
*2*d**4*f**2*g - 4*B*a*b**3*d**4*f**3 + B*b**4*c**4*g**3 - 4*B*b**4*c**3*d*
f*g**2 + 6*B*b**4*c**2*d**2*f**2*g - 4*B*b**4*c*d**3*f**3))/(4*b**4) + B*c*
(c*g - 2*d*f)*(c**2*g**2 - 2*c*d*f*g + 2*d**2*f**2)*log(x + (B*a**4*c*d**3*
g**3 - 4*B*a**3*b*c*d**3*f*g**2 + 6*B*a**2*b**2*c*d**3*f**2*g + B*a*b**3*c*
*4*g**3 - 4*B*a*b**3*c**3*d*f*g**2 + 6*B*a*b**3*c**2*d**2*f**2*g - 8*B*a*b*
*3*c*d**3*f**3 - B*a*b**3*c*(c*g - 2*d*f)*(c**2*g**2 - 2*c*d*f*g + 2*d**2*f
**2) + B*b**4*c**2*(c*g - 2*d*f)*(c**2*g**2 - 2*c*d*f*g + 2*d**2*f**2)/d)/(
B*a**4*d**4*g**3 - 4*B*a**3*b*d**4*f*g**2 + 6*B*a**2*b**2*d**4*f**2*g - 4*B
*a*b**3*d**4*f**3 + B*b**4*c**4*g**3 - 4*B*b**4*c**3*d*f*g**2 + 6*B*b**4*c*
*2*d**2*f**2*g - 4*B*b**4*c*d**3*f**3))/(4*d**4) + x**3*(A*f*g**2 + B*a*g**
3/(12*b) - B*c*g**3/(12*d)) + x**2*(3*A*f**2*g/2 - B*a**2*g**3/(8*b**2) + B
*a*f*g**2/(2*b) + B*c**2*g**3/(8*d**2) - B*c*f*g**2/(2*d)) + x*(A*f**3 + B*
a**3*g**3/(4*b**3) - B*a**2*f*g**2/b**2 + 3*B*a*f**2*g/(2*b) - B*c**3*g**3/
(4*d**3) + B*c**2*f*g**2/d**2 - 3*B*c*f**2*g/(2*d)) + (B*f**3*x + 3*B*f**2*
g*x**2/2 + B*f*g**2*x**3 + B*g**3*x**4/4)*log(e*(a + b*x)/(c + d*x))
```

# 3.232 $\int (f+gx)^2 \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right) dx$

**Optimal**. Leaf size=150

$$\frac{(f+gx)^3\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{3g}-\frac{B(bf-ag)^3\log(a+bx)}{3b^3g}-\frac{Bgx(bc-ad)(-adg-bcg+3bdf)}{3b^2d^2}-\frac{Bg^2x^2(bc-ad)}{6bd}+$$

```
[Out] -1/3*B*(-a*d+b*c)*g*(-a*d*g-b*c*g+3*b*d*f)*x/b^2/d^2-1/6*B*(-a*d+b*c)*g^2*x
^2/b/d-1/3*B*(-a*g+b*f)^3*ln(b*x+a)/b^3/g+1/3*(g*x+f)^3*(A+B*ln(e*(b*x+a)/(
d*x+c)))/g+1/3*B*(-c*g+d*f)^3*ln(d*x+c)/d^3/g
```

**Rubi** [A] time = 0.17, antiderivative size = 150, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {2525, 12, 72}

$$\frac{(f+gx)^3\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{3g}-\frac{Bgx(bc-ad)(-adg-bcg+3bdf)}{3b^2d^2}-\frac{B(bf-ag)^3\log(a+bx)}{3b^3g}-\frac{Bg^2x^2(bc-ad)}{6bd}+$$

Antiderivative was successfully verified.

```
[In] Int[(f + g*x)^2*(A + B*Log[(e*(a + b*x))/(c + d*x)]),x]
```

```
[Out] -(B*(b*c - a*d)*g*(3*b*d*f - b*c*g - a*d*g)*x)/(3*b^2*d^2) - (B*(b*c - a*d)
*g^2*x^2)/(6*b*d) - (B*(b*f - a*g)^3*Log[a + b*x])/(3*b^3*g) + ((f + g*x)^3
*(A + B*Log[(e*(a + b*x))/(c + d*x)]))/(3*g) + (B*(d*f - c*g)^3*Log[c + d*x
])/(3*d^3*g)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 72

```
Int[((e_.) + (f_.)*(x_))^(p_.)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
 x_Symbol] :> Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x]
 /; FreeQ[{a, b, c, d, e, f}, x] && IntegerQ[p]
```

Rule 2525

```
Int[((a_.) + Log[(c_.)*(RFx_)^(p_.)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_))^(m_.
), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(a + b*Log[c*RFx^p])^n)/(e*(m + 1))
, x] - Dist[(b*n*p)/(e*(m + 1)), Int[SimplifyIntegrand[((d + e*x)^(m + 1)*(
```

```
a + b*Log[c*RFx^p])^(n - 1)*D[RFx, x])/RFx, x], x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && RationalFunctionQ[RFx, x] && IGtQ[n, 0] && (EqQ[n, 1] ||
IntegerQ[m]) && NeQ[m, -1]
```

Rubi steps

$$\int (f+gx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)dx = \frac{(f+gx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{3g} - \frac{B\int\frac{(bc-ad)(f+gx)^3}{(a+bx)(c+dx)}\,dx}{3g}$$

$$= \frac{(f+gx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{3g} - \frac{(B(bc-ad))\int\frac{(f+gx)^3}{(a+bx)(c+dx)}\,dx}{3g}$$

$$= \frac{(f+gx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{3g} - \frac{(B(bc-ad))\int\left(\frac{g^2(3bdf-bcg-adg)}{b^2d^2} + \frac{g^3x}{bd}\right.}{3}$$

$$= -\frac{B(bc-ad)g(3bdf-bcg-adg)x}{3b^2d^2} - \frac{B(bc-ad)g^2x^2}{6bd} - \frac{B(bf-ag)^3\log}{3b^3g}$$

**Mathematica [A]** time = 0.13, size = 142, normalized size = 0.95

$$\frac{(f+gx)^3\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right) - \frac{B\left(b^2d^2g^3x^2(bc-ad)+2bdg^2x(bc-ad)(-adg-bcg+3bdf)+2d^3(bf-ag)^3\log(a+bx)-2b^3(df-cg)^3\log(c+dx)\right)}{2b^3d^3}}{3g}$$

Antiderivative was successfully verified.

```
[In] Integrate[(f + g*x)^2*(A + B*Log[(e*(a + b*x))/(c + d*x)]),x]
```

```
[Out] ((f + g*x)^3*(A + B*Log[(e*(a + b*x))/(c + d*x)]) - (B*(2*b*d*(b*c - a*d)*g
^2*(3*b*d*f - b*c*g - a*d*g)*x + b^2*d^2*(b*c - a*d)*g^3*x^2 + 2*d^3*(b*f -
 a*g)^3*Log[a + b*x] - 2*b^3*(d*f - c*g)^3*Log[c + d*x]))/(2*b^3*d^3))/(3*g
)
```

**fricas [A]** time = 0.88, size = 280, normalized size = 1.87

$$\frac{2\,Ab^3d^3g^2x^3 + \left(6\,Ab^3d^3fg - \left(Bb^3cd^2 - Bab^2d^3\right)g^2\right)x^2 + 2\left(3\,Ab^3d^3f^2 - 3\left(Bb^3cd^2 - Bab^2d^3\right)fg + \left(Bb^3c^2d - Ba^2\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^2*(A+B*log(e*(b*x+a)/(d*x+c))),x, algorithm="fricas")
```

```
[Out] 1/6*(2*A*b^3*d^3*g^2*x^3 + (6*A*b^3*d^3*f*g - (B*b^3*c*d^2 - B*a*b^2*d^3)*g
^2)*x^2 + 2*(3*A*b^3*d^3*f^2 - 3*(B*b^3*c*d^2 - B*a*b^2*d^3)*f*g + (B*b^3*c
^2*d - B*a^2*b*d^3)*g^2)*x + 2*(3*B*a*b^2*d^3*f^2 - 3*B*a^2*b*d^3*f*g + B*a
^3*d^3*g^2)*log(b*x + a) - 2*(3*B*b^3*c*d^2*f^2 - 3*B*b^3*c^2*d*f*g + B*b^3
*c^3*g^2)*log(d*x + c) + 2*(B*b^3*d^3*g^2*x^3 + 3*B*b^3*d^3*f*g*x^2 + 3*B*b
^3*d^3*f^2*x)*log((b*e*x + a*e)/(d*x + c)))/(b^3*d^3)
```

**giac [B]** time = 1.96, size = 5950, normalized size = 39.67

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^2*(A+B*log(e*(b*x+a)/(d*x+c))),x, algorithm="giac")
```

```
[Out] 1/6*(6*B*b^7*c^2*d^2*f^2*e^4*log(-b*e + (b*x*e + a*e)*d/(d*x + c)) - 12*B*a
*b^6*c*d^3*f^2*e^4*log(-b*e + (b*x*e + a*e)*d/(d*x + c)) + 6*B*a^2*b^5*d^4*
f^2*e^4*log(-b*e + (b*x*e + a*e)*d/(d*x + c)) - 6*B*b^7*c^3*d*f*g*e^4*log(-
b*e + (b*x*e + a*e)*d/(d*x + c)) + 6*B*a*b^6*c^2*d^2*f*g*e^4*log(-b*e + (b*
x*e + a*e)*d/(d*x + c)) + 6*B*a^2*b^5*c*d^3*f*g*e^4*log(-b*e + (b*x*e + a*e
)*d/(d*x + c)) - 6*B*a^3*b^4*d^4*f*g*e^4*log(-b*e + (b*x*e + a*e)*d/(d*x +
c)) + 2*B*b^7*c^4*g^2*e^4*log(-b*e + (b*x*e + a*e)*d/(d*x + c)) - 2*B*a*b^6
*c^3*d*g^2*e^4*log(-b*e + (b*x*e + a*e)*d/(d*x + c)) - 2*B*a^3*b^4*c*d^3*g^
2*e^4*log(-b*e + (b*x*e + a*e)*d/(d*x + c)) + 2*B*a^4*b^3*d^4*g^2*e^4*log(-
b*e + (b*x*e + a*e)*d/(d*x + c)) - 18*(b*x*e + a*e)*B*b^6*c^2*d^3*f^2*e^3*l
og(-b*e + (b*x*e + a*e)*d/(d*x + c))/(d*x + c) + 36*(b*x*e + a*e)*B*a*b^5*c
*d^4*f^2*e^3*log(-b*e + (b*x*e + a*e)*d/(d*x + c))/(d*x + c) - 18*(b*x*e +
a*e)*B*a^2*b^4*d^5*f^2*e^3*log(-b*e + (b*x*e + a*e)*d/(d*x + c))/(d*x + c)
+ 18*(b*x*e + a*e)*B*b^6*c^3*d^2*f*g*e^3*log(-b*e + (b*x*e + a*e)*d/(d*x +
c))/(d*x + c) - 18*(b*x*e + a*e)*B*a*b^5*c^2*d^3*f*g*e^3*log(-b*e + (b*x*e
+ a*e)*d/(d*x + c))/(d*x + c) - 18*(b*x*e + a*e)*B*a^2*b^4*c*d^4*f*g*e^3*lo
g(-b*e + (b*x*e + a*e)*d/(d*x + c))/(d*x + c) + 18*(b*x*e + a*e)*B*a^3*b^3*
d^5*f*g*e^3*log(-b*e + (b*x*e + a*e)*d/(d*x + c))/(d*x + c) - 6*(b*x*e + a*
e)*B*b^6*c^4*d*g^2*e^3*log(-b*e + (b*x*e + a*e)*d/(d*x + c))/(d*x + c) + 6*
(b*x*e + a*e)*B*a*b^5*c^3*d^2*g^2*e^3*log(-b*e + (b*x*e + a*e)*d/(d*x + c))
/(d*x + c) + 6*(b*x*e + a*e)*B*a^3*b^3*c*d^4*g^2*e^3*log(-b*e + (b*x*e + a*
e)*d/(d*x + c))/(d*x + c) - 6*(b*x*e + a*e)*B*a^4*b^2*d^5*g^2*e^3*log(-b*e
+ (b*x*e + a*e)*d/(d*x + c))/(d*x + c) + 18*(b*x*e + a*e)^2*B*b^5*c^2*d^4*f
^2*e^2*log(-b*e + (b*x*e + a*e)*d/(d*x + c))/(d*x + c)^2 - 36*(b*x*e + a*e)
^2*B*a*b^4*c*d^5*f^2*e^2*log(-b*e + (b*x*e + a*e)*d/(d*x + c))/(d*x + c)^2
+ 18*(b*x*e + a*e)^2*B*a^2*b^3*d^6*f^2*e^2*log(-b*e + (b*x*e + a*e)*d/(d*x
+ c))/(d*x + c)^2 - 18*(b*x*e + a*e)^2*B*b^5*c^3*d^3*f*g*e^2*log(-b*e + (b*
x*e + a*e)*d/(d*x + c))/(d*x + c)^2 + 18*(b*x*e + a*e)^2*B*a*b^4*c^2*d^4*f*
g*e^2*log(-b*e + (b*x*e + a*e)*d/(d*x + c))/(d*x + c)^2 + 18*(b*x*e + a*e)^
2*B*a^2*b^3*c*d^5*f*g*e^2*log(-b*e + (b*x*e + a*e)*d/(d*x + c))/(d*x + c)^2
 - 18*(b*x*e + a*e)^2*B*a^3*b^2*d^6*f*g*e^2*log(-b*e + (b*x*e + a*e)*d/(d*x
```

```
 + c))/(d*x + c)^2 + 6*(b*x*e + a*e)^2*B*b^5*c^4*d^2*g^2*e^2*log(-b*e + (b*
x*e + a*e)*d/(d*x + c))/(d*x + c)^2 - 6*(b*x*e + a*e)^2*B*a*b^4*c^3*d^3*g^2
*e^2*log(-b*e + (b*x*e + a*e)*d/(d*x + c))/(d*x + c)^2 - 6*(b*x*e + a*e)^2*
B*a^3*b^2*c*d^5*g^2*e^2*log(-b*e + (b*x*e + a*e)*d/(d*x + c))/(d*x + c)^2 +
 6*(b*x*e + a*e)^2*B*a^4*b*d^6*g^2*e^2*log(-b*e + (b*x*e + a*e)*d/(d*x + c)
)/(d*x + c)^2 - 6*(b*x*e + a*e)^3*B*b^4*c^2*d^5*f^2*e*log(-b*e + (b*x*e + a
*e)*d/(d*x + c))/(d*x + c)^3 + 12*(b*x*e + a*e)^3*B*a*b^3*c*d^6*f^2*e*log(-
b*e + (b*x*e + a*e)*d/(d*x + c))/(d*x + c)^3 - 6*(b*x*e + a*e)^3*B*a^2*b^2*
d^7*f^2*e*log(-b*e + (b*x*e + a*e)*d/(d*x + c))/(d*x + c)^3 + 6*(b*x*e + a*
e)^3*B*b^4*c^3*d^4*f*g*e*log(-b*e + (b*x*e + a*e)*d/(d*x + c))/(d*x + c)^3
- 6*(b*x*e + a*e)^3*B*a*b^3*c^2*d^5*f*g*e*log(-b*e + (b*x*e + a*e)*d/(d*x +
 c))/(d*x + c)^3 - 6*(b*x*e + a*e)^3*B*a^2*b^2*c*d^6*f*g*e*log(-b*e + (b*x*
e + a*e)*d/(d*x + c))/(d*x + c)^3 + 6*(b*x*e + a*e)^3*B*a^3*b*d^7*f*g*e*log
(-b*e + (b*x*e + a*e)*d/(d*x + c))/(d*x + c)^3 - 2*(b*x*e + a*e)^3*B*b^4*c^
4*d^3*g^2*e*log(-b*e + (b*x*e + a*e)*d/(d*x + c))/(d*x + c)^3 + 2*(b*x*e +
a*e)^3*B*a*b^3*c^3*d^4*g^2*e*log(-b*e + (b*x*e + a*e)*d/(d*x + c))/(d*x + c
)^3 + 2*(b*x*e + a*e)^3*B*a^3*b*c*d^6*g^2*e*log(-b*e + (b*x*e + a*e)*d/(d*x
 + c))/(d*x + c)^3 - 2*(b*x*e + a*e)^3*B*a^4*d^7*g^2*e*log(-b*e + (b*x*e +
a*e)*d/(d*x + c))/(d*x + c)^3 + 6*(b*x*e + a*e)*B*b^6*c^2*d^3*f^2*e^3*log((
b*x*e + a*e)/(d*x + c))/(d*x + c) - 12*(b*x*e + a*e)*B*a*b^5*c*d^4*f^2*e^3*
log((b*x*e + a*e)/(d*x + c))/(d*x + c) + 6*(b*x*e + a*e)*B*a^2*b^4*d^5*f^2*
e^3*log((b*x*e + a*e)/(d*x + c))/(d*x + c) - 12*(b*x*e + a*e)*B*a*b^5*c^2*d
^3*f*g*e^3*log((b*x*e + a*e)/(d*x + c))/(d*x + c) + 24*(b*x*e + a*e)*B*a^2*
b^4*c*d^4*f*g*e^3*log((b*x*e + a*e)/(d*x + c))/(d*x + c) - 12*(b*x*e + a*e)
*B*a^3*b^3*d^5*f*g*e^3*log((b*x*e + a*e)/(d*x + c))/(d*x + c) + 6*(b*x*e +
a*e)*B*a^2*b^4*c^2*d^3*g^2*e^3*log((b*x*e + a*e)/(d*x + c))/(d*x + c) - 12*
(b*x*e + a*e)*B*a^3*b^3*c*d^4*g^2*e^3*log((b*x*e + a*e)/(d*x + c))/(d*x + c
) + 6*(b*x*e + a*e)*B*a^4*b^2*d^5*g^2*e^3*log((b*x*e + a*e)/(d*x + c))/(d*x
 + c) - 12*(b*x*e + a*e)^2*B*b^5*c^2*d^4*f^2*e^2*log((b*x*e + a*e)/(d*x + c
))/(d*x + c)^2 + 24*(b*x*e + a*e)^2*B*a*b^4*c*d^5*f^2*e^2*log((b*x*e + a*e)
/(d*x + c))/(d*x + c)^2 - 12*(b*x*e + a*e)^2*B*a^2*b^3*d^6*f^2*e^2*log((b*x
*e + a*e)/(d*x + c))/(d*x + c)^2 + 6*(b*x*e + a*e)^2*B*b^5*c^3*d^3*f*g*e^2*
log((b*x*e + a*e)/(d*x + c))/(d*x + c)^2 + 6*(b*x*e + a*e)^2*B*a*b^4*c^2*d^
4*f*g*e^2*log((b*x*e + a*e)/(d*x + c))/(d*x + c)^2 - 30*(b*x*e + a*e)^2*B*a
^2*b^3*c*d^5*f*g*e^2*log((b*x*e + a*e)/(d*x + c))/(d*x + c)^2 + 18*(b*x*e +
 a*e)^2*B*a^3*b^2*d^6*f*g*e^2*log((b*x*e + a*e)/(d*x + c))/(d*x + c)^2 - 6*
(b*x*e + a*e)^2*B*a*b^4*c^3*d^3*g^2*e^2*log((b*x*e + a*e)/(d*x + c))/(d*x +
 c)^2 + 6*(b*x*e + a*e)^2*B*a^2*b^3*c^2*d^4*g^2*e^2*log((b*x*e + a*e)/(d*x
+ c))/(d*x + c)^2 + 6*(b*x*e + a*e)^2*B*a^3*b^2*c*d^5*g^2*e^2*log((b*x*e +
a*e)/(d*x + c))/(d*x + c)^2 - 6*(b*x*e + a*e)^2*B*a^4*b*d^6*g^2*e^2*log((b*
x*e + a*e)/(d*x + c))/(d*x + c)^2 + 6*(b*x*e + a*e)^3*B*b^4*c^2*d^5*f^2*e*l
og((b*x*e + a*e)/(d*x + c))/(d*x + c)^3 - 12*(b*x*e + a*e)^3*B*a*b^3*c*d^6*
f^2*e*log((b*x*e + a*e)/(d*x + c))/(d*x + c)^3 + 6*(b*x*e + a*e)^3*B*a^2*b^
2*d^7*f^2*e*log((b*x*e + a*e)/(d*x + c))/(d*x + c)^3 - 6*(b*x*e + a*e)^3*B*
b^4*c^3*d^4*f*g*e*log((b*x*e + a*e)/(d*x + c))/(d*x + c)^3 + 6*(b*x*e + a*e
```

)^3*B*a*b^3*c^2*d^5*f*g*e*log((b*x*e + a*e)/(d*x + c))/(d*x + c)^3 + 6*(b*x
*e + a*e)^3*B*a^2*b^2*c*d^6*f*g*e*log((b*x*e + a*e)/(d*x + c))/(d*x + c)^3
- 6*(b*x*e + a*e)^3*B*a^3*b*d^7*f*g*e*log((b*x*e + a*e)/(d*x + c))/(d*x + c
)^3 + 2*(b*x*e + a*e)^3*B*b^4*c^4*d^3*g^2*e*log((b*x*e + a*e)/(d*x + c))/(d
*x + c)^3 - 2*(b*x*e + a*e)^3*B*a*b^3*c^3*d^4*g^2*e*log((b*x*e + a*e)/(d*x
+ c))/(d*x + c)^3 - 2*(b*x*e + a*e)^3*B*a^3*b*c*d^6*g^2*e*log((b*x*e + a*e)
/(d*x + c))/(d*x + c)^3 + 2*(b*x*e + a*e)^3*B*a^4*d^7*g^2*e*log((b*x*e + a*
e)/(d*x + c))/(d*x + c)^3 + 6*A*b^7*c^2*d^2*f^2*e^4 - 12*A*a*b^6*c*d^3*f^2*
e^4 + 6*A*a^2*b^5*d^4*f^2*e^4 - 6*A*b^7*c^3*d*f*g*e^4 - 6*B*b^7*c^3*d*f*g*e
^4 + 6*A*a*b^6*c^2*d^2*f*g*e^4 + 18*B*a*b^6*c^2*d^2*f*g*e^4 + 6*A*a^2*b^5*c
*d^3*f*g*e^4 - 18*B*a^2*b^5*c*d^3*f*g*e^4 - 6*A*a^3*b^4*d^4*f*g*e^4 + 6*B*a
^3*b^4*d^4*f*g*e^4 + 2*A*b^7*c^4*g^2*e^4 + 3*B*b^7*c^4*g^2*e^4 - 2*A*a*b^6*
c^3*d*g^2*e^4 - 6*B*a*b^6*c^3*d*g^2*e^4 - 2*A*a^3*b^4*c*d^3*g^2*e^4 + 6*B*a
^3*b^4*c*d^3*g^2*e^4 + 2*A*a^4*b^3*d^4*g^2*e^4 - 3*B*a^4*b^3*d^4*g^2*e^4 -
12*(b*x*e + a*e)*A*b^6*c^2*d^3*f^2*e^3/(d*x + c) + 24*(b*x*e + a*e)*A*a*b^5
*c*d^4*f^2*e^3/(d*x + c) - 12*(b*x*e + a*e)*A*a^2*b^4*d^5*f^2*e^3/(d*x + c)
 + 18*(b*x*e + a*e)*A*b^6*c^3*d^2*f*g*e^3/(d*x + c) + 12*(b*x*e + a*e)*B*b^
6*c^3*d^2*f*g*e^3/(d*x + c) - 30*(b*x*e + a*e)*A*a*b^5*c^2*d^3*f*g*e^3/(d*x
 + c) - 36*(b*x*e + a*e)*B*a*b^5*c^2*d^3*f*g*e^3/(d*x + c) + 6*(b*x*e + a*e
)*A*a^2*b^4*c*d^4*f*g*e^3/(d*x + c) + 36*(b*x*e + a*e)*B*a^2*b^4*c*d^4*f*g*
e^3/(d*x + c) + 6*(b*x*e + a*e)*A*a^3*b^3*d^5*f*g*e^3/(d*x + c) - 12*(b*x*e
 + a*e)*B*a^3*b^3*d^5*f*g*e^3/(d*x + c) - 6*(b*x*e + a*e)*A*b^6*c^4*d*g^2*e
^3/(d*x + c) - 7*(b*x*e + a*e)*B*b^6*c^4*d*g^2*e^3/(d*x + c) + 6*(b*x*e + a
*e)*A*a*b^5*c^3*d^2*g^2*e^3/(d*x + c) + 16*(b*x*e + a*e)*B*a*b^5*c^3*d^2*g^
2*e^3/(d*x + c) + 6*(b*x*e + a*e)*A*a^2*b^4*c^2*d^3*g^2*e^3/(d*x + c) - 6*(
b*x*e + a*e)*B*a^2*b^4*c^2*d^3*g^2*e^3/(d*x + c) - 6*(b*x*e + a*e)*A*a^3*b^
3*c*d^4*g^2*e^3/(d*x + c) - 8*(b*x*e + a*e)*B*a^3*b^3*c*d^4*g^2*e^3/(d*x +
c) + 5*(b*x*e + a*e)*B*a^4*b^2*d^5*g^2*e^3/(d*x + c) + 6*(b*x*e + a*e)^2*A*
b^5*c^2*d^4*f^2*e^2/(d*x + c)^2 - 12*(b*x*e + a*e)^2*A*a*b^4*c*d^5*f^2*e^2/
(d*x + c)^2 + 6*(b*x*e + a*e)^2*A*a^2*b^3*d^6*f^2*e^2/(d*x + c)^2 - 12*(b*x
*e + a*e)^2*A*b^5*c^3*d^3*f*g*e^2/(d*x + c)^2 - 6*(b*x*e + a*e)^2*B*b^5*c^3
*d^3*f*g*e^2/(d*x + c)^2 + 24*(b*x*e + a*e)^2*A*a*b^4*c^2*d^4*f*g*e^2/(d*x
+ c)^2 + 18*(b*x*e + a*e)^2*B*a*b^4*c^2*d^4*f*g*e^2/(d*x + c)^2 - 12*(b*x*e
 + a*e)^2*A*a^2*b^3*c*d^5*f*g*e^2/(d*x + c)^2 - 18*(b*x*e + a*e)^2*B*a^2*b^
3*c*d^5*f*g*e^2/(d*x + c)^2 + 6*(b*x*e + a*e)^2*B*a^3*b^2*d^6*f*g*e^2/(d*x
+ c)^2 + 6*(b*x*e + a*e)^2*A*b^5*c^4*d^2*g^2*e^2/(d*x + c)^2 + 4*(b*x*e + a
*e)^2*B*b^5*c^4*d^2*g^2*e^2/(d*x + c)^2 - 12*(b*x*e + a*e)^2*A*a*b^4*c^3*d^
3*g^2*e^2/(d*x + c)^2 - 10*(b*x*e + a*e)^2*B*a*b^4*c^3*d^3*g^2*e^2/(d*x + c
)^2 + 6*(b*x*e + a*e)^2*A*a^2*b^3*c^2*d^4*g^2*e^2/(d*x + c)^2 + 6*(b*x*e +
a*e)^2*B*a^2*b^3*c^2*d^4*g^2*e^2/(d*x + c)^2 + 2*(b*x*e + a*e)^2*B*a^3*b^2*
c*d^5*g^2*e^2/(d*x + c)^2 - 2*(b*x*e + a*e)^2*B*a^4*b*d^6*g^2*e^2/(d*x + c)
^2)*(b*c/((b*c*e - a*d*e)*(b*c - a*d)) - a*d/((b*c*e - a*d*e)*(b*c - a*d)))
/(b^6*d^3*e^3 - 3*(b*x*e + a*e)*b^5*d^4*e^2/(d*x + c) + 3*(b*x*e + a*e)^2*b
^4*d^5*e/(d*x + c)^2 - (b*x*e + a*e)^3*b^3*d^6/(d*x + c)^3)

**maple [B]** time = 0.16, size = 4406, normalized size = 29.37

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x+f)^2*(B*ln((b*x+a)/(d*x+c)*e)+A),x)
```

```
[Out] 1/3/d^3*B*ln(-b*e+(b/d*e+(a*d-b*c)/(d*x+c)/d*e)*d)*c^3*g^2+2/d^2*e^2*A*g^2/
(1/(d*x+c)*a*d*e-1/(d*x+c)*b*c*e)^2*b*c^2*a-1/d*e^3*A*g^2/(1/(d*x+c)*a*d*e-
1/(d*x+c)*b*c*e)^3*a^2*b*c+1/d^2*e*B*g/(1/(d*x+c)*a*d*e-1/(d*x+c)*b*c*e)*c^
2*f*b-1/d^3*e^2*B*g^2*ln(b/d*e+(a*d-b*c)/(d*x+c)/d*e)/(1/(d*x+c)*a*d*e-1/(d
*x+c)*b*c*e)^2*c^3*b^2-1/d^3*e*B*ln(b/d*e+(a*d-b*c)/(d*x+c)/d*e)/(1/(d*x+c)
*a*d*e-1/(d*x+c)*b*c*e)*c^3*g^2*b-1/d*e*B*ln(b/d*e+(a*d-b*c)/(d*x+c)/d*e)/(
1/(d*x+c)*a*d*e-1/(d*x+c)*b*c*e)*f^2*b*c+1/d^2*e*B*ln(b/d*e+(a*d-b*c)/(d*x+
c)/d*e)/(1/(d*x+c)*a*d*e-1/(d*x+c)*b*c*e)*c^2*g^2*a-1/3/d^3*e^3*B*g^2*ln(b/
d*e+(a*d-b*c)/(d*x+c)/d*e)*b^3/(1/(d*x+c)*a*d*e-1/(d*x+c)*b*c*e)^3*c^3-1/d*
e^2*B*g^2*ln(b/d*e+(a*d-b*c)/(d*x+c)/d*e)/(1/(d*x+c)*a*d*e-1/(d*x+c)*b*c*e)
^2*a^2*c-20/3*e^3*B*g^2*ln(b/d*e+(a*d-b*c)/(d*x+c)/d*e)/(1/(d*x+c)*a*d*e-1/
(d*x+c)*b*c*e)^3*a^3/(d*x+c)^3*c^3-2*e*B*ln(b/d*e+(a*d-b*c)/(d*x+c)/d*e)/(1
/(d*x+c)*a*d*e-1/(d*x+c)*b*c*e)*f^2/(d*x+c)*a*c+1/d^2*e^3*A*g^2/(1/(d*x+c)*
a*d*e-1/(d*x+c)*b*c*e)^3*a*b^2*c^2+1/d^2*e^2*A*g/(1/(d*x+c)*a*d*e-1/(d*x+c)
*b*c*e)^2*b^2*c^2*f-2/d*e*B*g/(1/(d*x+c)*a*d*e-1/(d*x+c)*b*c*e)*c*f*a-2/d*e
*A/(1/(d*x+c)*a*d*e-1/(d*x+c)*b*c*e)*c*f*g*a+1/2/d^2*e^2*B*g^2/(1/(d*x+c)*a
*d*e-1/(d*x+c)*b*c*e)^2*a*c^2*b+2/d^2*e*A/(1/(d*x+c)*a*d*e-1/(d*x+c)*b*c*e)
*c^2*f*g*b+1/d*B*ln(-b*e+(b/d*e+(a*d-b*c)/(d*x+c)/d*e)*d)*f^2*c-1/3*B*g^2/b
^3*ln(-b*e+(b/d*e+(a*d-b*c)/(d*x+c)/d*e)*d)*a^3-B/b*ln(-b*e+(b/d*e+(a*d-b*c
)/(d*x+c)/d*e)*d)*f^2*a+e*A/(1/(d*x+c)*a*d*e-1/(d*x+c)*b*c*e)*f^2*a+1/3*e^3
*A*g^2/(1/(d*x+c)*a*d*e-1/(d*x+c)*b*c*e)^3*a^3+B*g/b^2*ln(-b*e+(b/d*e+(a*d-
b*c)/(d*x+c)/d*e)*d)*a^2*f+e*B*ln(b/d*e+(a*d-b*c)/(d*x+c)/d*e)/(1/(d*x+c)*a
*d*e-1/(d*x+c)*b*c*e)*f^2*a-1/d^2*B*ln(-b*e+(b/d*e+(a*d-b*c)/(d*x+c)/d*e)*d
)*c^2*f*g+1/3*e^3*B*g^2*ln(b/d*e+(a*d-b*c)/(d*x+c)/d*e)/(1/(d*x+c)*a*d*e-1/
(d*x+c)*b*c*e)^3*a^3-1/3*e*B*g^2/b^2/(1/(d*x+c)*a*d*e-1/(d*x+c)*b*c*e)*a^3+
1/6*e^2*B*g^2/b/(1/(d*x+c)*a*d*e-1/(d*x+c)*b*c*e)^2*a^3+e^2*A*g/(1/(d*x+c)*
a*d*e-1/(d*x+c)*b*c*e)^2*a^2*f-2/d*e^2*A*g/(1/(d*x+c)*a*d*e-1/(d*x+c)*b*c*e
)^2*b*c*f*a-1/d*e^3*B*g^2*ln(b/d*e+(a*d-b*c)/(d*x+c)/d*e)/(1/(d*x+c)*a*d*e-
1/(d*x+c)*b*c*e)^3*a^2*b*c+2/d^2*e^2*B*g^2*ln(b/d*e+(a*d-b*c)/(d*x+c)/d*e)/
(1/(d*x+c)*a*d*e-1/(d*x+c)*b*c*e)^2*c^2*b*a+1/d^2*e^3*B*g^2*ln(b/d*e+(a*d-b
*c)/(d*x+c)/d*e)*b^2/(1/(d*x+c)*a*d*e-1/(d*x+c)*b*c*e)^3*c^2*a+1/d^2*e^2*B*
g*ln(b/d*e+(a*d-b*c)/(d*x+c)/d*e)/(1/(d*x+c)*a*d*e-1/(d*x+c)*b*c*e)^2*c^2*f
*b^2+d*e*B*ln(b/d*e+(a*d-b*c)/(d*x+c)/d*e)/b/(1/(d*x+c)*a*d*e-1/(d*x+c)*b*c
*e)*f^2/(d*x+c)*a^2+1/d^3*e^2*B*g^2*ln(b/d*e+(a*d-b*c)/(d*x+c)/d*e)*b^2/(1/
(d*x+c)*a*d*e-1/(d*x+c)*b*c*e)^2*c^5/(d*x+c)^2-2/d^2*e*B*ln(b/d*e+(a*d-b*c)
/(d*x+c)/d*e)/(1/(d*x+c)*a*d*e-1/(d*x+c)*b*c*e)*c^3*g^2/(d*x+c)*a+4*d*e^2*B
*g*ln(b/d*e+(a*d-b*c)/(d*x+c)/d*e)/b/(1/(d*x+c)*a*d*e-1/(d*x+c)*b*c*e)^2*a^
```

```
3*f/(d*x+c)^2*c+1/d*e*B*ln(b/d*e+(a*d-b*c)/(d*x+c)/d*e)/b/(1/(d*x+c)*a*d*e-
1/(d*x+c)*b*c*e)*c^2*g^2/(d*x+c)*a^2-1/d^2*e^2*B*g*ln(b/d*e+(a*d-b*c)/(d*x+
c)/d*e)*b^2/(1/(d*x+c)*a*d*e-1/(d*x+c)*b*c*e)^2*c^4*f/(d*x+c)^2-4/d^2*e^2*B
*g^2*ln(b/d*e+(a*d-b*c)/(d*x+c)/d*e)*b/(1/(d*x+c)*a*d*e-1/(d*x+c)*b*c*e)^2*
c^4/(d*x+c)^2*a-2/d*e^2*B*g*ln(b/d*e+(a*d-b*c)/(d*x+c)/d*e)/(1/(d*x+c)*a*d*
e-1/(d*x+c)*b*c*e)^2*a*f*b*c-2*d^2*e^3*B*g^2*ln(b/d*e+(a*d-b*c)/(d*x+c)/d*e
)/b^2/(1/(d*x+c)*a*d*e-1/(d*x+c)*b*c*e)^3*a^5/(d*x+c)^3*c-d^2*e^2*B*g*ln(b/
d*e+(a*d-b*c)/(d*x+c)/d*e)/b^2/(1/(d*x+c)*a*d*e-1/(d*x+c)*b*c*e)^2*a^4*f/(d
*x+c)^2+d*e^2*B*g^2*ln(b/d*e+(a*d-b*c)/(d*x+c)/d*e)/b^2/(1/(d*x+c)*a*d*e-1/
(d*x+c)*b*c*e)^2*a^4*c/(d*x+c)^2+5/d*e^3*B*g^2*ln(b/d*e+(a*d-b*c)/(d*x+c)/d
*e)/(1/(d*x+c)*a*d*e-1/(d*x+c)*b*c*e)^3*a^2/(d*x+c)^3*c^4*b-2*e*B*ln(b/d*e+
(a*d-b*c)/(d*x+c)/d*e)/b/(1/(d*x+c)*a*d*e-1/(d*x+c)*b*c*e)*c*f*g/(d*x+c)*a^
2-2/d^2*e*B*ln(b/d*e+(a*d-b*c)/(d*x+c)/d*e)/(1/(d*x+c)*a*d*e-1/(d*x+c)*b*c*
e)*c^3*f*g/(d*x+c)*b-2/d^2*e^3*B*g^2*ln(b/d*e+(a*d-b*c)/(d*x+c)/d*e)*b^2/(1
/(d*x+c)*a*d*e-1/(d*x+c)*b*c*e)^3*c^5/(d*x+c)^3*a+4/d*e*B*ln(b/d*e+(a*d-b*c
)/(d*x+c)/d*e)/(1/(d*x+c)*a*d*e-1/(d*x+c)*b*c*e)*c^2*f*g/(d*x+c)*a+5*d*e^3*
B*g^2*ln(b/d*e+(a*d-b*c)/(d*x+c)/d*e)/b/(1/(d*x+c)*a*d*e-1/(d*x+c)*b*c*e)^3
*a^4/(d*x+c)^3*c^2+1/d*e*B*ln(b/d*e+(a*d-b*c)/(d*x+c)/d*e)/(1/(d*x+c)*a*d*e
-1/(d*x+c)*b*c*e)*f^2/(d*x+c)*c^2*b-6*e^2*B*g*ln(b/d*e+(a*d-b*c)/(d*x+c)/d*
e)/(1/(d*x+c)*a*d*e-1/(d*x+c)*b*c*e)^2*a^2*f/(d*x+c)^2*c^2+1/3/d^3*e^3*B*g^
2*ln(b/d*e+(a*d-b*c)/(d*x+c)/d*e)*b^3/(1/(d*x+c)*a*d*e-1/(d*x+c)*b*c*e)^3*c
^6/(d*x+c)^3+6/d*e^2*B*g^2*ln(b/d*e+(a*d-b*c)/(d*x+c)/d*e)/(1/(d*x+c)*a*d*e
-1/(d*x+c)*b*c*e)^2*c^3/(d*x+c)^2*a^2+2/d^2*e*B*ln(b/d*e+(a*d-b*c)/(d*x+c)/
d*e)/(1/(d*x+c)*a*d*e-1/(d*x+c)*b*c*e)*c^2*f*g*b+1/3*d^3*e^3*B*g^2*ln(b/d*e
+(a*d-b*c)/(d*x+c)/d*e)/b^3/(1/(d*x+c)*a*d*e-1/(d*x+c)*b*c*e)^3*a^6/(d*x+c)
^3-4*e^2*B*g^2*ln(b/d*e+(a*d-b*c)/(d*x+c)/d*e)/b/(1/(d*x+c)*a*d*e-1/(d*x+c)
*b*c*e)^2*c^2/(d*x+c)^2*a^3+1/d^3*e*B*ln(b/d*e+(a*d-b*c)/(d*x+c)/d*e)/(1/(d
*x+c)*a*d*e-1/(d*x+c)*b*c*e)*c^4*g^2/(d*x+c)*b-2/d*e*B*ln(b/d*e+(a*d-b*c)/(
d*x+c)/d*e)/(1/(d*x+c)*a*d*e-1/(d*x+c)*b*c*e)*c*f*g*a+4/d*e^2*B*g*ln(b/d*e+
(a*d-b*c)/(d*x+c)/d*e)/(1/(d*x+c)*a*d*e-1/(d*x+c)*b*c*e)^2*a*f/(d*x+c)^2*c^
3*b+e*B*g/b/(1/(d*x+c)*a*d*e-1/(d*x+c)*b*c*e)*a^2*f-1/d*e*A/(1/(d*x+c)*a*d*
e-1/(d*x+c)*b*c*e)*f^2*b*c+1/d^2*e*A/(1/(d*x+c)*a*d*e-1/(d*x+c)*b*c*e)*c^2*
g^2*a-1/d^3*e*A/(1/(d*x+c)*a*d*e-1/(d*x+c)*b*c*e)*c^3*g^2*b+1/d^2*e*B*g^2/(
1/(d*x+c)*a*d*e-1/(d*x+c)*b*c*e)*c^2*a-2/3/d^3*e*B*g^2/(1/(d*x+c)*a*d*e-1/(
d*x+c)*b*c*e)*c^3*b-1/d^3*e^2*A*g^2/(1/(d*x+c)*a*d*e-1/(d*x+c)*b*c*e)^2*b^2
*c^3-1/3/d^3*e^3*A*g^2/(1/(d*x+c)*a*d*e-1/(d*x+c)*b*c*e)^3*b^3*c^3-1/d*e^2*
A*g^2/(1/(d*x+c)*a*d*e-1/(d*x+c)*b*c*e)^2*a^2*c-1/6/d^3*e^2*B*g^2*b^2/(1/(d
*x+c)*a*d*e-1/(d*x+c)*b*c*e)^2*c^3-1/2/d*e^2*B*g^2/(1/(d*x+c)*a*d*e-1/(d*x+
c)*b*c*e)^2*a^2*c+e^2*B*g*ln(b/d*e+(a*d-b*c)/(d*x+c)/d*e)/(1/(d*x+c)*a*d*e-
1/(d*x+c)*b*c*e)^2*a^2*f
```

**maxima [A]** time = 0.64, size = 262, normalized size = 1.75

$$\frac{1}{3}Ag^2x^3+Afgx^2+\left(x\log\left(\frac{bex}{dx+c}+\frac{ae}{dx+c}\right)+\frac{a\log(bx+a)}{b}-\frac{c\log(dx+c)}{d}\right)Bf^2+\left(x^2\log\left(\frac{bex}{dx+c}+\frac{ae}{dx+c}\right)\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^2*(A+B*log(e*(b*x+a)/(d*x+c))),x, algorithm="maxima")
```

```
[Out] 1/3*A*g^2*x^3 + A*f*g*x^2 + (x*log(b*e*x/(d*x + c) + a*e/(d*x + c)) + a*log
(b*x + a)/b - c*log(d*x + c)/d)*B*f^2 + (x^2*log(b*e*x/(d*x + c) + a*e/(d*x
 + c)) - a^2*log(b*x + a)/b^2 + c^2*log(d*x + c)/d^2 - (b*c - a*d)*x/(b*d))
*B*f*g + 1/6*(2*x^3*log(b*e*x/(d*x + c) + a*e/(d*x + c)) + 2*a^3*log(b*x +
a)/b^3 - 2*c^3*log(d*x + c)/d^3 - ((b^2*c*d - a*b*d^2)*x^2 - 2*(b^2*c^2 - a
^2*d^2)*x)/(b^2*d^2))*B*g^2 + A*f^2*x
```

**mupad [B]** time = 4.73, size = 356, normalized size = 2.37

$$x^2\left(\frac{3\,A\,a\,d\,g^2+3\,A\,b\,c\,g^2+B\,a\,d\,g^2-B\,b\,c\,g^2+6\,A\,b\,d\,f\,g}{6\,b\,d}-\frac{A\,g^2\,\left(3\,a\,d+3\,b\,c\right)}{6\,b\,d}\right)+\ln\left(\frac{e\,\left(a+b\,x\right)}{c+d\,x}\right)\,\left(B\,f^2\,x+B\,f$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f + g*x)^2*(A + B*log((e*(a + b*x))/(c + d*x))),x)
```

```
[Out] x^2*((3*A*a*d*g^2 + 3*A*b*c*g^2 + B*a*d*g^2 - B*b*c*g^2 + 6*A*b*d*f*g)/(6*b
*d) - (A*g^2*(3*a*d + 3*b*c))/(6*b*d)) + log((e*(a + b*x))/(c + d*x))*((B*g
^2*x^3)/3 + B*f^2*x + B*f*g*x^2) - x*((((3*A*a*d*g^2 + 3*A*b*c*g^2 + B*a*d*
g^2 - B*b*c*g^2 + 6*A*b*d*f*g)/(3*b*d) - (A*g^2*(3*a*d + 3*b*c))/(3*b*d))*(
3*a*d + 3*b*c))/(3*b*d) - (3*A*a*c*g^2 + 3*A*b*d*f^2 + 6*A*a*d*f*g + 6*A*b*
c*f*g + 3*B*a*d*f*g - 3*B*b*c*f*g)/(3*b*d) + (A*a*c*g^2)/(b*d)) + (log(a +
b*x)*(B*a^3*g^2 + 3*B*a*b^2*f^2 - 3*B*a^2*b*f*g))/(3*b^3) - (log(c + d*x)*(
B*c^3*g^2 + 3*B*c*d^2*f^2 - 3*B*c^2*d*f*g))/(3*d^3) + (A*g^2*x^3)/3
```

**sympy [B]** time = 6.54, size = 658, normalized size = 4.39

$$\frac{Ag^2x^3}{3}+\frac{Ba\left(a^2g^2-3abfg+3b^2f^2\right)\log\left(x+\frac{Ba^3cd^2g^2-3Ba^2bcd^2fg+\frac{Ba^2d^3\left(a^2g^2-3abfg+3b^2f^2\right)}{b}+Bab^2c^3g^2-3Bab^2c^2dfg+6Bab^2cd^2f^2-B}{Ba^3d^3g^2-3Ba^2bd^3fg+3Bab^2d^3f^2+Bb^3c^3g^2-3Bb^3c^2dfg+3Bb^3cd^2}\right.}{3b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)**2*(A+B*ln(e*(b*x+a)/(d*x+c))),x)
```

```
[Out] A*g**2*x**3/3 + B*a*(a**2*g**2 - 3*a*b*f*g + 3*b**2*f**2)*log(x + (B*a**3*c
*d**2*g**2 - 3*B*a**2*b*c*d**2*f*g + B*a**2*d**3*(a**2*g**2 - 3*a*b*f*g + 3
*b**2*f**2)/b + B*a*b**2*c**3*g**2 - 3*B*a*b**2*c**2*d*f*g + 6*B*a*b**2*c*d
**2*f**2 - B*a*c*d**2*(a**2*g**2 - 3*a*b*f*g + 3*b**2*f**2))/(B*a**3*d**3*g
**2 - 3*B*a**2*b*d**3*f*g + 3*B*a*b**2*d**3*f**2 + B*b**3*c**3*g**2 - 3*B*b
```

```
**3*c**2*d*f*g + 3*B*b**3*c*d**2*f**2))/(3*b**3) - B*c*(c**2*g**2 - 3*c*d*f
*g + 3*d**2*f**2)*log(x + (B*a**3*c*d**2*g**2 - 3*B*a**2*b*c*d**2*f*g + B*a
*b**2*c**3*g**2 - 3*B*a*b**2*c**2*d*f*g + 6*B*a*b**2*c*d**2*f**2 - B*a*b**2
*c*(c**2*g**2 - 3*c*d*f*g + 3*d**2*f**2) + B*b**3*c**2*(c**2*g**2 - 3*c*d*f
*g + 3*d**2*f**2)/d)/(B*a**3*d**3*g**2 - 3*B*a**2*b*d**3*f*g + 3*B*a*b**2*d
**3*f**2 + B*b**3*c**3*g**2 - 3*B*b**3*c**2*d*f*g + 3*B*b**3*c*d**2*f**2))/
(3*d**3) + x**2*(A*f*g + B*a*g**2/(6*b) - B*c*g**2/(6*d)) + x*(A*f**2 - B*a
**2*g**2/(3*b**2) + B*a*f*g/b + B*c**2*g**2/(3*d**2) - B*c*f*g/d) + (B*f**2
*x + B*f*g*x**2 + B*g**2*x**3/3)*log(e*(a + b*x)/(c + d*x))
```

# 3.233 $\int (f+gx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)\,dx$

**Optimal**. Leaf size=109

$$\frac{(f+gx)^2\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{2g}-\frac{B(bf-ag)^2\log(a+bx)}{2b^2g}-\frac{Bgx(bc-ad)}{2bd}+\frac{B(df-cg)^2\log(c+dx)}{2d^2g}$$

```
[Out] -1/2*B*(-a*d+b*c)*g*x/b/d-1/2*B*(-a*g+b*f)^2*ln(b*x+a)/b^2/g+1/2*(g*x+f)^2*
(A+B*ln(e*(b*x+a)/(d*x+c)))/g+1/2*B*(-c*g+d*f)^2*ln(d*x+c)/d^2/g
```

**Rubi [A]** time = 0.10, antiderivative size = 109, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.120, Rules used = {2525, 12, 72}

$$\frac{(f+gx)^2\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{2g}-\frac{B(bf-ag)^2\log(a+bx)}{2b^2g}-\frac{Bgx(bc-ad)}{2bd}+\frac{B(df-cg)^2\log(c+dx)}{2d^2g}$$

Antiderivative was successfully verified.

```
[In] Int[(f + g*x)*(A + B*Log[(e*(a + b*x))/(c + d*x)]),x]
```

```
[Out] -(B*(b*c - a*d)*g*x)/(2*b*d) - (B*(b*f - a*g)^2*Log[a + b*x])/(2*b^2*g) + (
(f + g*x)^2*(A + B*Log[(e*(a + b*x))/(c + d*x)]))/(2*g) + (B*(d*f - c*g)^2*
Log[c + d*x])/(2*d^2*g)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 72

```
Int[((e_.) + (f_.)*(x_))^(p_.)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
 x_Symbol] :> Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x]
 /; FreeQ[{a, b, c, d, e, f}, x] && IntegerQ[p]
```

Rule 2525

```
Int[((a_.) + Log[(c_.)*(RFx_)^(p_.)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_))^(m_.
), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(a + b*Log[c*RFx^p])^n)/(e*(m + 1))
, x] - Dist[(b*n*p)/(e*(m + 1)), Int[SimplifyIntegrand[((d + e*x)^(m + 1)*(
a + b*Log[c*RFx^p])^(n - 1)*D[RFx, x])/RFx, x], x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && RationalFunctionQ[RFx, x] && IGtQ[n, 0] && (EqQ[n, 1] ||
```

```
IntegerQ[m]) && NeQ[m, -1]
```

Rubi steps

$$\int (f+gx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)dx = \frac{(f+gx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{2g} - \frac{B\int \frac{(bc-ad)(f+gx)^2}{(a+bx)(c+dx)}dx}{2g}$$
$$= \frac{(f+gx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{2g} - \frac{(B(bc-ad))\int \frac{(f+gx)^2}{(a+bx)(c+dx)}dx}{2g}$$
$$= \frac{(f+gx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{2g} - \frac{(B(bc-ad))\int \left(\frac{g^2}{bd}+\frac{(bf-ag)^2}{b(bc-ad)(a+bx)}+\right.}{2g}$$
$$= -\frac{B(bc-ad)gx}{2bd} - \frac{B(bf-ag)^2\log(a+bx)}{2b^2g} + \frac{(f+gx)^2\left(A+B\log\left(\frac{e(a}{c}\right.\right.}{2g}$$

**Mathematica [A]** time = 0.11, size = 114, normalized size = 1.05

$$\frac{b\left(d\left(Bg^2x(ad-bc)+Abd(f+gx)^2\right)+bBd^2(f+gx)^2\log\left(\frac{e(a+bx)}{c+dx}\right)+bB(df-cg)^2\log(c+dx)\right)-Bd^2(bf-ag)^2}{2b^2d^2g}$$

Antiderivative was successfully verified.

```
[In] Integrate[(f + g*x)*(A + B*Log[(e*(a + b*x))/(c + d*x)]),x]
```

```
[Out] (-(B*d^2*(b*f - a*g)^2*Log[a + b*x]) + b*(d*(B*(-(b*c) + a*d)*g^2*x + A*b*d
*(f + g*x)^2) + b*B*d^2*(f + g*x)^2*Log[(e*(a + b*x))/(c + d*x)] + b*B*(d*f
 - c*g)^2*Log[c + d*x]))/(2*b^2*d^2*g)
```

**fricas [A]** time = 0.69, size = 150, normalized size = 1.38

$$\frac{Ab^2d^2gx^2+\left(2\,Ab^2d^2f-\left(Bb^2cd-Babd^2\right)g\right)x+\left(2\,Babd^2f-Ba^2d^2g\right)\log\left(bx+a\right)-\left(2\,Bb^2cdf-Bb^2c^2g\right)\log\left(a\right.}{2\,b^2d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)*(A+B*log(e*(b*x+a)/(d*x+c))),x, algorithm="fricas")
```

```
[Out] 1/2*(A*b^2*d^2*g*x^2 + (2*A*b^2*d^2*f - (B*b^2*c*d - B*a*b*d^2)*g)*x + (2*B
*a*b*d^2*f - B*a^2*d^2*g)*log(b*x + a) - (2*B*b^2*c*d*f - B*b^2*c^2*g)*log(
```

```
d*x + c) + (B*b^2*d^2*g*x^2 + 2*B*b^2*d^2*f*x)*log((b*e*x + a*e)/(d*x + c))
)/(b^2*d^2)
```

**giac [B]** time = 1.06, size = 2355, normalized size = 21.61

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)*(A+B*log(e*(b*x+a)/(d*x+c))),x, algorithm="giac")
```

```
[Out] 1/2*(2*B*b^5*c^2*d*f*e^3*log(-b*e + (b*x*e + a*e)*d/(d*x + c)) - 4*B*a*b^4*
c*d^2*f*e^3*log(-b*e + (b*x*e + a*e)*d/(d*x + c)) + 2*B*a^2*b^3*d^3*f*e^3*l
og(-b*e + (b*x*e + a*e)*d/(d*x + c)) - B*b^5*c^3*g*e^3*log(-b*e + (b*x*e +
a*e)*d/(d*x + c)) + B*a*b^4*c^2*d*g*e^3*log(-b*e + (b*x*e + a*e)*d/(d*x + c
)) + B*a^2*b^3*c*d^2*g*e^3*log(-b*e + (b*x*e + a*e)*d/(d*x + c)) - B*a^3*b^
2*d^3*g*e^3*log(-b*e + (b*x*e + a*e)*d/(d*x + c)) - 4*(b*x*e + a*e)*B*b^4*c
^2*d^2*f*e^2*log(-b*e + (b*x*e + a*e)*d/(d*x + c))/(d*x + c) + 8*(b*x*e + a
*e)*B*a*b^3*c*d^3*f*e^2*log(-b*e + (b*x*e + a*e)*d/(d*x + c))/(d*x + c) - 4
*(b*x*e + a*e)*B*a^2*b^2*d^4*f*e^2*log(-b*e + (b*x*e + a*e)*d/(d*x + c))/(d
*x + c) + 2*(b*x*e + a*e)*B*b^4*c^3*d*g*e^2*log(-b*e + (b*x*e + a*e)*d/(d*x
 + c))/(d*x + c) - 2*(b*x*e + a*e)*B*a*b^3*c^2*d^2*g*e^2*log(-b*e + (b*x*e
+ a*e)*d/(d*x + c))/(d*x + c) - 2*(b*x*e + a*e)*B*a^2*b^2*c*d^3*g*e^2*log(-
b*e + (b*x*e + a*e)*d/(d*x + c))/(d*x + c) + 2*(b*x*e + a*e)*B*a^3*b*d^4*g*
e^2*log(-b*e + (b*x*e + a*e)*d/(d*x + c))/(d*x + c) + 2*(b*x*e + a*e)^2*B*b
^3*c^2*d^3*f*e*log(-b*e + (b*x*e + a*e)*d/(d*x + c))/(d*x + c)^2 - 4*(b*x*e
 + a*e)^2*B*a*b^2*c*d^4*f*e*log(-b*e + (b*x*e + a*e)*d/(d*x + c))/(d*x + c)
^2 + 2*(b*x*e + a*e)^2*B*a^2*b*d^5*f*e*log(-b*e + (b*x*e + a*e)*d/(d*x + c)
)/(d*x + c)^2 - (b*x*e + a*e)^2*B*b^3*c^3*d^2*g*e*log(-b*e + (b*x*e + a*e)*
d/(d*x + c))/(d*x + c)^2 + (b*x*e + a*e)^2*B*a*b^2*c^2*d^3*g*e*log(-b*e + (
b*x*e + a*e)*d/(d*x + c))/(d*x + c)^2 + (b*x*e + a*e)^2*B*a^2*b*c*d^4*g*e*l
og(-b*e + (b*x*e + a*e)*d/(d*x + c))/(d*x + c)^2 - (b*x*e + a*e)^2*B*a^3*d^
5*g*e*log(-b*e + (b*x*e + a*e)*d/(d*x + c))/(d*x + c)^2 + 2*(b*x*e + a*e)*B
*b^4*c^2*d^2*f*e^2*log((b*x*e + a*e)/(d*x + c))/(d*x + c) - 4*(b*x*e + a*e)
*B*a*b^3*c*d^3*f*e^2*log((b*x*e + a*e)/(d*x + c))/(d*x + c) + 2*(b*x*e + a*
e)*B*a^2*b^2*d^4*f*e^2*log((b*x*e + a*e)/(d*x + c))/(d*x + c) - 2*(b*x*e +
a*e)*B*a*b^3*c^2*d^2*g*e^2*log((b*x*e + a*e)/(d*x + c))/(d*x + c) + 4*(b*x*
e + a*e)*B*a^2*b^2*c*d^3*g*e^2*log((b*x*e + a*e)/(d*x + c))/(d*x + c) - 2*(
b*x*e + a*e)*B*a^3*b*d^4*g*e^2*log((b*x*e + a*e)/(d*x + c))/(d*x + c) - 2*(
b*x*e + a*e)^2*B*b^3*c^2*d^3*f*e*log((b*x*e + a*e)/(d*x + c))/(d*x + c)^2 +
 4*(b*x*e + a*e)^2*B*a*b^2*c*d^4*f*e*log((b*x*e + a*e)/(d*x + c))/(d*x + c)
^2 - 2*(b*x*e + a*e)^2*B*a^2*b*d^5*f*e*log((b*x*e + a*e)/(d*x + c))/(d*x +
c)^2 + (b*x*e + a*e)^2*B*b^3*c^3*d^2*g*e*log((b*x*e + a*e)/(d*x + c))/(d*x
+ c)^2 - (b*x*e + a*e)^2*B*a*b^2*c^2*d^3*g*e*log((b*x*e + a*e)/(d*x + c))/(
d*x + c)^2 - (b*x*e + a*e)^2*B*a^2*b*c*d^4*g*e*log((b*x*e + a*e)/(d*x + c))
/(d*x + c)^2 + (b*x*e + a*e)^2*B*a^3*d^5*g*e*log((b*x*e + a*e)/(d*x + c))/(
```

```
d*x + c)^2 + 2*A*b^5*c^2*d*f*e^3 - 4*A*a*b^4*c*d^2*f*e^3 + 2*A*a^2*b^3*d^3*
f*e^3 - A*b^5*c^3*g*e^3 - B*b^5*c^3*g*e^3 + A*a*b^4*c^2*d*g*e^3 + 3*B*a*b^4
*c^2*d*g*e^3 + A*a^2*b^3*c*d^2*g*e^3 - 3*B*a^2*b^3*c*d^2*g*e^3 - A*a^3*b^2*
d^3*g*e^3 + B*a^3*b^2*d^3*g*e^3 - 2*(b*x*e + a*e)*A*b^4*c^2*d^2*f*e^2/(d*x
+ c) + 4*(b*x*e + a*e)*A*a*b^3*c*d^3*f*e^2/(d*x + c) - 2*(b*x*e + a*e)*A*a^
2*b^2*d^4*f*e^2/(d*x + c) + 2*(b*x*e + a*e)*A*b^4*c^3*d*g*e^2/(d*x + c) + (
b*x*e + a*e)*B*b^4*c^3*d*g*e^2/(d*x + c) - 4*(b*x*e + a*e)*A*a*b^3*c^2*d^2*
g*e^2/(d*x + c) - 3*(b*x*e + a*e)*B*a*b^3*c^2*d^2*g*e^2/(d*x + c) + 2*(b*x*
e + a*e)*A*a^2*b^2*c*d^3*g*e^2/(d*x + c) + 3*(b*x*e + a*e)*B*a^2*b^2*c*d^3*
g*e^2/(d*x + c) - (b*x*e + a*e)*B*a^3*b*d^4*g*e^2/(d*x + c))*(b*c/((b*c*e -
 a*d*e)*(b*c - a*d)) - a*d/((b*c*e - a*d*e)*(b*c - a*d)))/(b^4*d^2*e^2 - 2*
(b*x*e + a*e)*b^3*d^3*e/(d*x + c) + (b*x*e + a*e)^2*b^2*d^4/(d*x + c)^2)
```

**maple [B]** time = 0.14, size = 1809, normalized size = 16.60

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x+f)*(B*ln((b*x+a)/(d*x+c)*e)+A),x)
```

```
[Out] -1/2*d^2*e^2*B*g*ln(b/d*e+(a*d-b*c)/(d*x+c)/d*e)/b^2/(1/(d*x+c)*a*d*e-1/(d*
x+c)*b*c*e)^2*a^4/(d*x+c)^2+1/2*B*g/b^2*ln(-b*e+(b/d*e+(a*d-b*c)/(d*x+c)/d*
e)*d)*a^2+e*A/(1/(d*x+c)*a*d*e-1/(d*x+c)*b*c*e)*f*a+1/2*e^2*A*g/(1/(d*x+c)*
a*d*e-1/(d*x+c)*b*c*e)^2*a^2-B/b*ln(-b*e+(b/d*e+(a*d-b*c)/(d*x+c)/d*e)*d)*f
*a-1/2/d^2*B*ln(-b*e+(b/d*e+(a*d-b*c)/(d*x+c)/d*e)*d)*c^2*g+1/d*B*ln(-b*e+(
b/d*e+(a*d-b*c)/(d*x+c)/d*e)*d)*f*c-1/d*e*B*ln(b/d*e+(a*d-b*c)/(d*x+c)/d*e)
/(1/(d*x+c)*a*d*e-1/(d*x+c)*b*c*e)*f*b*c-3*e^2*B*g*ln(b/d*e+(a*d-b*c)/(d*x+
c)/d*e)/(1/(d*x+c)*a*d*e-1/(d*x+c)*b*c*e)^2*a^2/(d*x+c)^2*c^2+2/d*e*B*ln(b/
d*e+(a*d-b*c)/(d*x+c)/d*e)/(1/(d*x+c)*a*d*e-1/(d*x+c)*b*c*e)*c^2*g/(d*x+c)*
a+d*e*B*ln(b/d*e+(a*d-b*c)/(d*x+c)/d*e)/b/(1/(d*x+c)*a*d*e-1/(d*x+c)*b*c*e)
*f/(d*x+c)*a^2-1/2/d^2*e^2*B*g*ln(b/d*e+(a*d-b*c)/(d*x+c)/d*e)*b^2/(1/(d*x+
c)*a*d*e-1/(d*x+c)*b*c*e)^2*c^4/(d*x+c)^2-1/d*e^2*B*g*ln(b/d*e+(a*d-b*c)/(d
*x+c)/d*e)/(1/(d*x+c)*a*d*e-1/(d*x+c)*b*c*e)^2*a*b*c+1/d*e*B*ln(b/d*e+(a*d-
b*c)/(d*x+c)/d*e)/(1/(d*x+c)*a*d*e-1/(d*x+c)*b*c*e)*f/(d*x+c)*c^2*b-e*B*ln(
b/d*e+(a*d-b*c)/(d*x+c)/d*e)/b/(1/(d*x+c)*a*d*e-1/(d*x+c)*b*c*e)*c*g/(d*x+c
)*a^2+1/2/d^2*e^2*B*g*ln(b/d*e+(a*d-b*c)/(d*x+c)/d*e)*b^2/(1/(d*x+c)*a*d*e-
1/(d*x+c)*b*c*e)^2*c^2+1/d^2*e*B*ln(b/d*e+(a*d-b*c)/(d*x+c)/d*e)/(1/(d*x+c)
*a*d*e-1/(d*x+c)*b*c*e)*c^2*g*b-1/d^2*e*B*ln(b/d*e+(a*d-b*c)/(d*x+c)/d*e)/(
1/(d*x+c)*a*d*e-1/(d*x+c)*b*c*e)*c^3*g/(d*x+c)*b+e*B*ln(b/d*e+(a*d-b*c)/(d*
x+c)/d*e)/(1/(d*x+c)*a*d*e-1/(d*x+c)*b*c*e)*f*a+1/2*e^2*B*g*ln(b/d*e+(a*d-b
*c)/(d*x+c)/d*e)/(1/(d*x+c)*a*d*e-1/(d*x+c)*b*c*e)^2*a^2+1/2*e*B*g/b/(1/(d*
x+c)*a*d*e-1/(d*x+c)*b*c*e)*a^2-1/d*e*B*ln(b/d*e+(a*d-b*c)/(d*x+c)/d*e)/(1/
(d*x+c)*a*d*e-1/(d*x+c)*b*c*e)*c*g*a-2*e*B*ln(b/d*e+(a*d-b*c)/(d*x+c)/d*e)/
(1/(d*x+c)*a*d*e-1/(d*x+c)*b*c*e)*f/(d*x+c)*a*c-1/d*e^2*A*g/(1/(d*x+c)*a*d*
e-1/(d*x+c)*b*c*e)^2*b*c*a+2*d*e^2*B*g*ln(b/d*e+(a*d-b*c)/(d*x+c)/d*e)/b/(1
```

```
/(d*x+c)*a*d*e-1/(d*x+c)*b*c*e)^2*a^3/(d*x+c)^2*c+2/d*e^2*B*g*ln(b/d*e+(a*d
-b*c)/(d*x+c)/d*e)/(1/(d*x+c)*a*d*e-1/(d*x+c)*b*c*e)^2*a/(d*x+c)^2*c^3*b+1/
2/d^2*e*B*g/(1/(d*x+c)*a*d*e-1/(d*x+c)*b*c*e)*c^2*b+1/d^2*e*A/(1/(d*x+c)*a*
d*e-1/(d*x+c)*b*c*e)*c^2*g*b-1/d*e*A/(1/(d*x+c)*a*d*e-1/(d*x+c)*b*c*e)*c*g*
a-1/d*e*A/(1/(d*x+c)*a*d*e-1/(d*x+c)*b*c*e)*f*b*c+1/2/d^2*e^2*A*g/(1/(d*x+c
)*a*d*e-1/(d*x+c)*b*c*e)^2*b^2*c^2-1/d*e*B*g/(1/(d*x+c)*a*d*e-1/(d*x+c)*b*c
*e)*c*a
```

**maxima [A]** time = 0.91, size = 140, normalized size = 1.28

$$\frac{1}{2}Agx^2+\left(x\log\left(\frac{bex}{dx+c}+\frac{ae}{dx+c}\right)+\frac{a\log(bx+a)}{b}-\frac{c\log(dx+c)}{d}\right)Bf+\frac{1}{2}\left(x^2\log\left(\frac{bex}{dx+c}+\frac{ae}{dx+c}\right)-\frac{a^2\log(b}{b^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)*(A+B*log(e*(b*x+a)/(d*x+c))),x, algorithm="maxima")
```

```
[Out] 1/2*A*g*x^2 + (x*log(b*e*x/(d*x + c) + a*e/(d*x + c)) + a*log(b*x + a)/b -
c*log(d*x + c)/d)*B*f + 1/2*(x^2*log(b*e*x/(d*x + c) + a*e/(d*x + c)) - a^2
*log(b*x + a)/b^2 + c^2*log(d*x + c)/d^2 - (b*c - a*d)*x/(b*d))*B*g + A*f*x
```

**mupad [B]** time = 4.24, size = 144, normalized size = 1.32

$$\ln\left(\frac{e\,(a+b\,x)}{c+d\,x}\right)\left(\frac{B\,g\,x^2}{2}+B\,f\,x\right)+x\,\left(\frac{2\,A\,a\,d\,g+2\,A\,b\,c\,g+2\,A\,b\,d\,f+B\,a\,d\,g-B\,b\,c\,g}{2\,b\,d}-\frac{A\,g\,(2\,a\,d+2\,b\,c)}{2\,b\,d}\right)-\frac{\ln}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f + g*x)*(A + B*log((e*(a + b*x))/(c + d*x))),x)
```

```
[Out] log((e*(a + b*x))/(c + d*x))*(B*f*x + (B*g*x^2)/2) + x*((2*A*a*d*g + 2*A*b*
c*g + 2*A*b*d*f + B*a*d*g - B*b*c*g)/(2*b*d) - (A*g*(2*a*d + 2*b*c))/(2*b*d
)) - (log(a + b*x)*(B*a^2*g - 2*B*a*b*f))/(2*b^2) + (log(c + d*x)*(B*c^2*g
- 2*B*c*d*f))/(2*d^2) + (A*g*x^2)/2
```

**sympy [B]** time = 3.01, size = 318, normalized size = 2.92

$$\frac{Agx^2}{2}-\frac{Ba\left(ag-2bf\right)\log\left(x+\frac{Ba^2cdg+\frac{Ba^2d^2\left(ag-2bf\right)}{b}+Babc^2g-4Babcdf-Bacd\left(ag-2bf\right)}{Ba^2d^2g-2Babd^2f+Bb^2c^2g-2Bb^2cdf}\right)}{2b^2}+\frac{Bc\left(cg-2df\right)\log\left(x+\frac{Ba^2cdg+Babc^2g}{Ba^2d^2}\right.}{2d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)*(A+B*ln(e*(b*x+a)/(d*x+c))),x)
```

```
[Out] A*g*x**2/2 - B*a*(a*g - 2*b*f)*log(x + (B*a**2*c*d*g + B*a**2*d**2*(a*g - 2
*b*f)/b + B*a*b*c**2*g - 4*B*a*b*c*d*f - B*a*c*d*(a*g - 2*b*f))/(B*a**2*d**
2*g - 2*B*a*b*d**2*f + B*b**2*c**2*g - 2*B*b**2*c*d*f))/(2*b**2) + B*c*(c*g
 - 2*d*f)*log(x + (B*a**2*c*d*g + B*a*b*c**2*g - 4*B*a*b*c*d*f - B*a*b*c*(c
*g - 2*d*f) + B*b**2*c**2*(c*g - 2*d*f)/d)/(B*a**2*d**2*g - 2*B*a*b*d**2*f
+ B*b**2*c**2*g - 2*B*b**2*c*d*f))/(2*d**2) + x*(A*f + B*a*g/(2*b) - B*c*g/
(2*d)) + (B*f*x + B*g*x**2/2)*log(e*(a + b*x)/(c + d*x))
```

# 3.234 $\int \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)\,dx$

**Optimal**. Leaf size=52

$$\frac{B(a+bx)\log\left(\frac{e(a+bx)}{c+dx}\right)}{b}-\frac{B(bc-ad)\log(c+dx)}{bd}+Ax$$

```
[Out] A*x+B*(b*x+a)*ln(e*(b*x+a)/(d*x+c))/b-B*(-a*d+b*c)*ln(d*x+c)/b/d
```

**Rubi [A]** time = 0.03, antiderivative size = 52, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {2486, 31}

$$\frac{B(a+bx)\log\left(\frac{e(a+bx)}{c+dx}\right)}{b}-\frac{B(bc-ad)\log(c+dx)}{bd}+Ax$$

Antiderivative was successfully verified.

```
[In] Int[A + B*Log[(e*(a + b*x))/(c + d*x)],x]
```

```
[Out] A*x + (B*(a + b*x)*Log[(e*(a + b*x))/(c + d*x)])/b - (B*(b*c - a*d)*Log[c +
 d*x])/(b*d)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 2486

```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))
^(r_.)]^(s_.), x_Symbol] :> Simp[((a + b*x)*Log[e*(f*(a + b*x)^p*(c + d*x)^
q)^r]^s)/b, x] + Dist[(q*r*s*(b*c - a*d))/b, Int[Log[e*(f*(a + b*x)^p*(c +
d*x)^q)^r]^(s - 1)/(c + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, p, q, r, s
}, x] && NeQ[b*c - a*d, 0] && EqQ[p + q, 0] && IGtQ[s, 0]
```

Rubi steps

$$\int \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)dx = Ax + B\int \log\left(\frac{e(a+bx)}{c+dx}\right)dx$$
$$= Ax + \frac{B(a+bx)\log\left(\frac{e(a+bx)}{c+dx}\right)}{b} - \frac{(B(bc-ad))\int \frac{1}{c+dx}\,dx}{b}$$
$$= Ax + \frac{B(a+bx)\log\left(\frac{e(a+bx)}{c+dx}\right)}{b} - \frac{B(bc-ad)\log(c+dx)}{bd}$$

**Mathematica [A]** time = 0.01, size = 52, normalized size = 1.00

$$\frac{B(a+bx)\log\left(\frac{e(a+bx)}{c+dx}\right)}{b} - \frac{B(bc-ad)\log(c+dx)}{bd} + Ax$$

Antiderivative was successfully verified.

```
[In] Integrate[A + B*Log[(e*(a + b*x))/(c + d*x)],x]
```

```
[Out] A*x + (B*(a + b*x)*Log[(e*(a + b*x))/(c + d*x)])/b - (B*(b*c - a*d)*Log[c +
 d*x])/(b*d)
```

**fricas [A]** time = 1.32, size = 56, normalized size = 1.08

$$\frac{Bbdx\log\left(\frac{bex+ae}{dx+c}\right) + Abdx + Bad\log\left(bx+a\right) - Bbc\log\left(dx+c\right)}{bd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(A+B*log(e*(b*x+a)/(d*x+c)),x, algorithm="fricas")
```

```
[Out] (B*b*d*x*log((b*e*x + a*e)/(d*x + c)) + A*b*d*x + B*a*d*log(b*x + a) - B*b*
c*log(d*x + c))/(b*d)
```

**giac [B]** time = 0.52, size = 427, normalized size = 8.21

$$-\left(\left(b^2c^2e^2 - 2\,abcde^2 + a^2d^2e^2\right)\left(\frac{e^{(-1)}\log\left(\frac{|bxe+ae|}{|dx+c|}\right)}{bd} - \frac{e^{(-1)}\log\left(\left|-be+\frac{(bxe+ae)d}{dx+c}\right|\right)}{bd}\right) - \frac{\left(b^2c^2e^2 - 2\,abcde^2 + a^2d^2e^2\right)}{\left(be - \frac{(b}{}\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(A+B*log(e*(b*x+a)/(d*x+c)),x, algorithm="giac")
```

```
[Out] -((b^2*c^2*e^2 - 2*a*b*c*d*e^2 + a^2*d^2*e^2)*(e^(-1)*log(abs(b*x*e + a*e)/
abs(d*x + c))/(b*d) - e^(-1)*log(abs(-b*e + (b*x*e + a*e)*d/(d*x + c)))/(b*
d)) - (b^2*c^2*e^2 - 2*a*b*c*d*e^2 + a^2*d^2*e^2)*log((a - b*(a/(b*c - a*d)
 - (b*x*e + a*e)*c/((b*c*e - a*d*e)*(d*x + c)))/(b/(b*c - a*d) - (b*x*e + a
*e)*d/((b*c*e - a*d*e)*(d*x + c))))*e/(c - d*(a/(b*c - a*d) - (b*x*e + a*e)
*c/((b*c*e - a*d*e)*(d*x + c)))/(b/(b*c - a*d) - (b*x*e + a*e)*d/((b*c*e -
a*d*e)*(d*x + c)))))/((b*e - (b*x*e + a*e)*d/(d*x + c))*d))*B*(b*c/((b*c*e
- a*d*e)*(b*c - a*d)) - a*d/((b*c*e - a*d*e)*(b*c - a*d))) + A*x
```

**maple [B]** time = 0.13, size = 418, normalized size = 8.04

$$\frac{B a^2 d e \ln\left(\frac{be}{d}+\frac{(ad-bc)e}{(dx+c)d}\right)}{\left(\frac{ade}{dx+c}-\frac{bce}{dx+c}\right)(dx+c)\,b}-\frac{2Bace\ln\left(\frac{be}{d}+\frac{(ad-bc)e}{(dx+c)d}\right)}{\left(\frac{ade}{dx+c}-\frac{bce}{dx+c}\right)(dx+c)}+\frac{Bb\,c^2e\ln\left(\frac{be}{d}+\frac{(ad-bc)e}{(dx+c)d}\right)}{\left(\frac{ade}{dx+c}-\frac{bce}{dx+c}\right)(dx+c)\,d}+\frac{Bae\ln\left(\frac{be}{d}+\frac{(ad-bc)e}{(dx+c)d}\right)}{\frac{ade}{dx+c}-\frac{bce}{dx+c}}-\frac{Bbce\ln\left(\frac{be}{d}+\cdots\right.}{\left(\frac{ade}{dx+c}-\frac{bce}{dx+\cdots}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(B*ln((b*x+a)/(d*x+c)*e)+A,x)
```

```
[Out] A*x-B/b*ln(-b*e+(b/d*e+(a*d-b*c)/(d*x+c)/d*e)*d)*a+B*ln(-b*e+(b/d*e+(a*d-b*
c)/(d*x+c)/d*e)*d)/d*c+e*B*ln(b/d*e+(a*d-b*c)/(d*x+c)/d*e)/(1/(d*x+c)*a*d*e
-1/(d*x+c)*b*c*e)*a-e*B*ln(b/d*e+(a*d-b*c)/(d*x+c)/d*e)/(1/(d*x+c)*a*d*e-1/
(d*x+c)*b*c*e)/d*b*c+e*B*ln(b/d*e+(a*d-b*c)/(d*x+c)/d*e)/b/(1/(d*x+c)*a*d*e
-1/(d*x+c)*b*c*e)/(d*x+c)*a^2*d-2*e*B*ln(b/d*e+(a*d-b*c)/(d*x+c)/d*e)/(1/(d
*x+c)*a*d*e-1/(d*x+c)*b*c*e)/(d*x+c)*a*c+e*B*ln(b/d*e+(a*d-b*c)/(d*x+c)/d*e
)/(1/(d*x+c)*a*d*e-1/(d*x+c)*b*c*e)/d/(d*x+c)*c^2*b
```

**maxima [A]** time = 0.62, size = 54, normalized size = 1.04

$$\left(x\log\left(\frac{(bx+a)e}{dx+c}\right)+\frac{\frac{ae\log(bx+a)}{b}-\frac{ce\log(dx+c)}{d}}{e}\right)B+Ax$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(A+B*log(e*(b*x+a)/(d*x+c)),x, algorithm="maxima")
```

```
[Out] (x*log((b*x + a)*e/(d*x + c)) + (a*e*log(b*x + a)/b - c*e*log(d*x + c)/d)/e
)*B + A*x
```

**mupad [B]** time = 4.12, size = 47, normalized size = 0.90

$$A\,x+B\,x\,\ln\left(\frac{e\,\left(a+b\,x\right)}{c+d\,x}\right)+\frac{B\,a\,\ln\left(a+b\,x\right)}{b}-\frac{B\,c\,\ln\left(c+d\,x\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(A + B*log((e*(a + b*x))/(c + d*x)),x)
```

```
[Out] A*x + B*x*log((e*(a + b*x))/(c + d*x)) + (B*a*log(a + b*x))/b - (B*c*log(c
+ d*x))/d
```

**sympy [A]** time = 1.00, size = 83, normalized size = 1.60

$$Ax + \frac{Ba\log\left(x + \frac{\frac{Ba^2d}{b} + Bac}{Bad + Bbc}\right)}{b} - \frac{Bc\log\left(x + \frac{Bac + \frac{Bbc^2}{d}}{Bad + Bbc}\right)}{d} + Bx\log\left(\frac{e\,(a + bx)}{c + dx}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(A+B*ln(e*(b*x+a)/(d*x+c)),x)
```

```
[Out] A*x + B*a*log(x + (B*a**2*d/b + B*a*c)/(B*a*d + B*b*c))/b - B*c*log(x + (B*
a*c + B*b*c**2/d)/(B*a*d + B*b*c))/d + B*x*log(e*(a + b*x)/(c + d*x))
```

# 3.235 $\int \frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{f+gx}\,dx$

**Optimal**. Leaf size=140

$$\frac{\log(f+gx)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{g}-\frac{B\text{Li}_2\left(\frac{b(f+gx)}{bf-ag}\right)}{g}-\frac{B\log(f+gx)\log\left(-\frac{g(a+bx)}{bf-ag}\right)}{g}+\frac{B\text{Li}_2\left(\frac{d(f+gx)}{df-cg}\right)}{g}+\frac{B\log(f+gx)\,\text{l}}{g}$$

```
[Out] -B*ln(-g*(b*x+a)/(-a*g+b*f))*ln(g*x+f)/g+(A+B*ln(e*(b*x+a)/(d*x+c)))*ln(g*x
+f)/g+B*ln(-g*(d*x+c)/(-c*g+d*f))*ln(g*x+f)/g-B*polylog(2,b*(g*x+f)/(-a*g+b
*f))/g+B*polylog(2,d*(g*x+f)/(-c*g+d*f))/g
```

**Rubi [A]** time = 0.25, antiderivative size = 140, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {2524, 12, 2418, 2394, 2393, 2391}

$$-\frac{B\text{PolyLog}\left(2,\frac{b(f+gx)}{bf-ag}\right)}{g}+\frac{B\text{PolyLog}\left(2,\frac{d(f+gx)}{df-cg}\right)}{g}+\frac{\log(f+gx)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{g}-\frac{B\log(f+gx)\log\left(-\frac{g(a+bx)}{bf-ag}\right.}{g}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Log[(e*(a + b*x))/(c + d*x)])/(f + g*x),x]
```

```
[Out] -((B*Log[-((g*(a + b*x))/(b*f - a*g))]*Log[f + g*x])/g) + ((A + B*Log[(e*(a
 + b*x))/(c + d*x)])*Log[f + g*x])/g + (B*Log[-((g*(c + d*x))/(d*f - c*g))]
*Log[f + g*x])/g - (B*PolyLog[2, (b*(f + g*x))/(b*f - a*g)])/g + (B*PolyLog
[2, (d*(f + g*x))/(d*f - c*g)])/g
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2393

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + (c*e*x)/g])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
```

```
(e*f - d*g), 0]
```

Rule 2394

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d + e*x
)^n]))/g, x] - Dist[(b*e*n)/g, Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2418

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*(RFx_), x_Sy
mbol] :> With[{u = ExpandIntegrand[(a + b*Log[c*(d + e*x)^n])^p, RFx, x]},
Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d, e, n}, x] && RationalFunctionQ[
RFx, x] && IntegerQ[p]
```

Rule 2524

```
Int[((a_.) + Log[(c_.)*(RFx_)^(p_.)]*(b_.))^(n_.)/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[(Log[d + e*x]*(a + b*Log[c*RFx^p])^n)/e, x] - Dist[(b*n*p)/e
, Int[(Log[d + e*x]*(a + b*Log[c*RFx^p])^(n - 1)*D[RFx, x])/RFx, x], x] /;
FreeQ[{a, b, c, d, e, p}, x] && RationalFunctionQ[RFx, x] && IGtQ[n, 0]
```

Rubi steps

$$\int \frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{f+gx}\,dx = \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)\log(f+gx)}{g} - \frac{B\int \frac{(c+dx)\left(-\frac{de(a+bx)}{(c+dx)^2}+\frac{be}{c+dx}\right)\log(f+gx)}{e(a+bx)}\,dx}{g}$$

$$= \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)\log(f+gx)}{g} - \frac{B\int \frac{(c+dx)\left(-\frac{de(a+bx)}{(c+dx)^2}+\frac{be}{c+dx}\right)\log(f+gx)}{a+bx}\,dx}{eg}$$

$$= \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)\log(f+gx)}{g} - \frac{B\int \left(\frac{be\log(f+gx)}{a+bx}-\frac{de\log(f+gx)}{c+dx}\right)dx}{eg}$$

$$= \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)\log(f+gx)}{g} - \frac{(bB)\int \frac{\log(f+gx)}{a+bx}\,dx}{g} + \frac{(Bd)\int \frac{\log(f+gx)}{c+dx}\,dx}{g}$$

$$= -\frac{B\log\left(-\frac{g(a+bx)}{bf-ag}\right)\log(f+gx)}{g} + \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)\log(f+gx)}{g} + \frac{B\log\left(-\frac{g(c+dx)}{df-cg}\right.}{g}$$

$$= -\frac{B\log\left(-\frac{g(a+bx)}{bf-ag}\right)\log(f+gx)}{g} + \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)\log(f+gx)}{g} + \frac{B\log\left(-\frac{g(c+dx)}{df-cg}\right.}{g}$$

$$= -\frac{B\log\left(-\frac{g(a+bx)}{bf-ag}\right)\log(f+gx)}{g} + \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)\log(f+gx)}{g} + \frac{B\log\left(-\frac{g(c+dx)}{df-cg}\right.}{g}$$

**Mathematica [A]** time = 0.06, size = 115, normalized size = 0.82

$$\frac{\log(f+gx)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)-B\log\left(\frac{g(a+bx)}{ag-bf}\right)+A+B\log\left(\frac{g(c+dx)}{cg-df}\right)\right)-B\text{Li}_2\left(\frac{b(f+gx)}{bf-ag}\right)+B\text{Li}_2\left(\frac{d(f+gx)}{df-cg}\right)}{g}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Log[(e*(a + b*x))/(c + d*x)])/(f + g*x),x]
```

```
[Out] ((A - B*Log[(g*(a + b*x))/(-(b*f) + a*g)] + B*Log[(e*(a + b*x))/(c + d*x)]
+ B*Log[(g*(c + d*x))/(-(d*f) + c*g)])*Log[f + g*x] - B*PolyLog[2, (b*(f +
g*x))/(b*f - a*g)] + B*PolyLog[2, (d*(f + g*x))/(d*f - c*g)])/g
```

**fricas [F]** time = 1.88, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{B\log\left(\frac{bex+ae}{dx+c}\right)+A}{gx+f}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*(b*x+a)/(d*x+c)))/(g*x+f),x, algorithm="fricas")
```

```
[Out] integral((B*log((b*e*x + a*e)/(d*x + c)) + A)/(g*x + f), x)
```

**giac [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*(b*x+a)/(d*x+c)))/(g*x+f),x, algorithm="giac")
```

```
[Out] Timed out
```

**maple [B]** time = 0.20, size = 1400, normalized size = 10.00

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*ln((b*x+a)/(d*x+c)*e)+A)/(g*x+f),x)
```

```
[Out] -d*A/g/(a*d-b*c)*ln(-b*e+(b/d*e+(a*d-b*c)/(d*x+c)/d*e)*d)*a+A/g/(a*d-b*c)*l
n(-b*e+(b/d*e+(a*d-b*c)/(d*x+c)/d*e)*d)*b*c+d*A/g/(a*d-b*c)*ln((b/d*e+(a*d-
b*c)/(d*x+c)/d*e)*c*g-d*(b/d*e+(a*d-b*c)/(d*x+c)/d*e)*f-a*e*g+b*e*f)*a-A/g/
(a*d-b*c)*ln((b/d*e+(a*d-b*c)/(d*x+c)/d*e)*c*g-d*(b/d*e+(a*d-b*c)/(d*x+c)/d
*e)*f-a*e*g+b*e*f)*b*c-d*B/g/(a*d-b*c)*dilog(-(-b*e+(b/d*e+(a*d-b*c)/(d*x+c
)/d*e)*d)/b/e)*a+B/g/(a*d-b*c)*dilog(-(-b*e+(b/d*e+(a*d-b*c)/(d*x+c)/d*e)*d
)/b/e)*b*c-d*B/g/(a*d-b*c)*ln(b/d*e+(a*d-b*c)/(d*x+c)/d*e)*ln(-(-b*e+(b/d*e
+(a*d-b*c)/(d*x+c)/d*e)*d)/b/e)*a+B/g/(a*d-b*c)*ln(b/d*e+(a*d-b*c)/(d*x+c)/
d*e)*ln(-(-b*e+(b/d*e+(a*d-b*c)/(d*x+c)/d*e)*d)/b/e)*b*c+d*B/(a*d-b*c)*dilo
g(((c*g-d*f)*(b/d*e+(a*d-b*c)/(d*x+c)/d*e)-a*e*g+b*e*f)/(-a*e*g+b*e*f))/(c*
g-d*f)*c*a-B/(a*d-b*c)*dilog(((c*g-d*f)*(b/d*e+(a*d-b*c)/(d*x+c)/d*e)-a*e*g
+b*e*f)/(-a*e*g+b*e*f))/(c*g-d*f)*c^2*b-d^2*B/g/(a*d-b*c)*dilog(((c*g-d*f)*
(b/d*e+(a*d-b*c)/(d*x+c)/d*e)-a*e*g+b*e*f)/(-a*e*g+b*e*f))/(c*g-d*f)*f*a+d*
B/g/(a*d-b*c)*dilog(((c*g-d*f)*(b/d*e+(a*d-b*c)/(d*x+c)/d*e)-a*e*g+b*e*f)/(
-a*e*g+b*e*f))/(c*g-d*f)*f*b*c+d*B/(a*d-b*c)*ln(b/d*e+(a*d-b*c)/(d*x+c)/d*e
)*ln(((c*g-d*f)*(b/d*e+(a*d-b*c)/(d*x+c)/d*e)-a*e*g+b*e*f)/(-a*e*g+b*e*f))/
(c*g-d*f)*c*a-B/(a*d-b*c)*ln(b/d*e+(a*d-b*c)/(d*x+c)/d*e)*ln(((c*g-d*f)*(b/
d*e+(a*d-b*c)/(d*x+c)/d*e)-a*e*g+b*e*f)/(-a*e*g+b*e*f))/(c*g-d*f)*c^2*b-d^2
*B/g/(a*d-b*c)*ln(b/d*e+(a*d-b*c)/(d*x+c)/d*e)*ln(((c*g-d*f)*(b/d*e+(a*d-b*
c)/(d*x+c)/d*e)-a*e*g+b*e*f)/(-a*e*g+b*e*f))/(c*g-d*f)*f*a+d*B/g/(a*d-b*c)*
ln(b/d*e+(a*d-b*c)/(d*x+c)/d*e)*ln(((c*g-d*f)*(b/d*e+(a*d-b*c)/(d*x+c)/d*e)
-a*e*g+b*e*f)/(-a*e*g+b*e*f))/(c*g-d*f)*f*b*c
```

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$-B\int -\frac{\log(bx+a)-\log(dx+c)+\log(e)}{gx+f}\,dx+\frac{A\log\left(gx+f\right)}{g}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*(b*x+a)/(d*x+c)))/(g*x+f),x, algorithm="maxima")
```

```
[Out] -B*integrate(-(log(b*x + a) - log(d*x + c) + log(e))/(g*x + f), x) + A*log(
g*x + f)/g
```

**mupad** [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{A+B\,\ln\left(\frac{e\,(a+b\,x)}{c+d\,x}\right)}{f+g\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*log((e*(a + b*x))/(c + d*x)))/(f + g*x),x)
```

```
[Out] int((A + B*log((e*(a + b*x))/(c + d*x)))/(f + g*x), x)
```

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{A+B\log\left(\frac{ae}{c+dx}+\frac{bex}{c+dx}\right)}{f+gx}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*ln(e*(b*x+a)/(d*x+c)))/(g*x+f),x)
```

```
[Out] Integral((A + B*log(a*e/(c + d*x) + b*e*x/(c + d*x)))/(f + g*x), x)
```

# 3.236 $\int \frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{(f+gx)^2}\,dx$

**Optimal**. Leaf size=87

$$\frac{(a+bx)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{(f+gx)(bf-ag)}+\frac{B(bc-ad)\log\left(\frac{f+gx}{c+dx}\right)}{(bf-ag)(df-cg)}$$

```
[Out] (b*x+a)*(A+B*ln(e*(b*x+a)/(d*x+c)))/(-a*g+b*f)/(g*x+f)+B*(-a*d+b*c)*ln((g*x
+f)/(d*x+c))/(-a*g+b*f)/(-c*g+d*f)
```

**Rubi [A]** time = 0.11, antiderivative size = 113, normalized size of antiderivative = 1.30, number of steps used = 4, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {2525, 12, 72}

$$-\frac{B\log\left(\frac{e(a+bx)}{c+dx}\right)+A}{g(f+gx)}+\frac{B(bc-ad)\log(f+gx)}{(bf-ag)(df-cg)}+\frac{bB\log(a+bx)}{g(bf-ag)}-\frac{Bd\log(c+dx)}{g(df-cg)}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Log[(e*(a + b*x))/(c + d*x)])/(f + g*x)^2,x]
```

```
[Out] (b*B*Log[a + b*x])/(g*(b*f - a*g)) - (A + B*Log[(e*(a + b*x))/(c + d*x)])/(
g*(f + g*x)) - (B*d*Log[c + d*x])/(g*(d*f - c*g)) + (B*(b*c - a*d)*Log[f +
g*x])/((b*f - a*g)*(d*f - c*g))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 72

```
Int[((e_.) + (f_.)*(x_))^(p_.)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
 x_Symbol] :> Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x]
 /; FreeQ[{a, b, c, d, e, f}, x] && IntegerQ[p]
```

Rule 2525

```
Int[((a_.) + Log[(c_.)*(RFx_)^(p_.)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_))^(m_.
), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(a + b*Log[c*RFx^p])^n)/(e*(m + 1))
, x] - Dist[(b*n*p)/(e*(m + 1)), Int[SimplifyIntegrand[((d + e*x)^(m + 1)*(
a + b*Log[c*RFx^p])^(n - 1)*D[RFx, x])/RFx, x], x], x] /; FreeQ[{a, b, c, d
```

```
, e, m, p}, x] && RationalFunctionQ[RFx, x] && IGtQ[n, 0] && (EqQ[n, 1] ||
IntegerQ[m]) && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{(f+gx)^2}\,dx &= -\frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{g(f+gx)}+\frac{B\int \frac{bc-ad}{(a+bx)(c+dx)(f+gx)}\,dx}{g}\\ &= -\frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{g(f+gx)}+\frac{(B(bc-ad))\int \frac{1}{(a+bx)(c+dx)(f+gx)}\,dx}{g}\\ &= -\frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{g(f+gx)}+\frac{(B(bc-ad))\int \left(\frac{b^2}{(bc-ad)(bf-ag)(a+bx)}+\frac{d^2}{(bc-ad)(-df+cg)(c+dx)}+\frac{}{(bf-}\right.}{g}\\ &= \frac{bB\log(a+bx)}{g(bf-ag)}-\frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{g(f+gx)}-\frac{Bd\log(c+dx)}{g(df-cg)}+\frac{B(bc-ad)\log(f+gx)}{(bf-ag)(df-cg)}\end{aligned}$$

**Mathematica [A]** time = 0.13, size = 105, normalized size = 1.21

$$\frac{\frac{B(b\log(a+bx)(df-cg)+\log(c+dx)(adg-bdf)+g(bc-ad)\log(f+gx))}{(bf-ag)(df-cg)}-\frac{B\log\left(\frac{e(a+bx)}{c+dx}\right)+A}{f+gx}}{g}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Log[(e*(a + b*x))/(c + d*x)])/(f + g*x)^2,x]
```

```
[Out] (-((A + B*Log[(e*(a + b*x))/(c + d*x)])/(f + g*x)) + (B*(b*(d*f - c*g)*Log[
a + b*x] + (-(b*d*f) + a*d*g)*Log[c + d*x] + (b*c - a*d)*g*Log[f + g*x]))/(
(b*f - a*g)*(d*f - c*g)))/g
```

**fricas [B]** time = 10.90, size = 255, normalized size = 2.93

$$-\frac{Abdf^2+Aacg^2-(Abc+Aad)fg-\left(Bbdf^2-Bbcfg+\left(Bbdfg-Bbcg^2\right)x\right)\log(bx+a)+\left(Bbdf^2-Badfg+\left(\right.\right.}{bdf^3g+acfg^3-(bc+a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*(b*x+a)/(d*x+c)))/(g*x+f)^2,x, algorithm="fricas")
```

```
[Out] -(A*b*d*f^2 + A*a*c*g^2 - (A*b*c + A*a*d)*f*g - (B*b*d*f^2 - B*b*c*f*g + (B
*b*d*f*g - B*b*c*g^2)*x)*log(b*x + a) + (B*b*d*f^2 - B*a*d*f*g + (B*b*d*f*g
 - B*a*d*g^2)*x)*log(d*x + c) - ((B*b*c - B*a*d)*g^2*x + (B*b*c - B*a*d)*f*
g)*log(g*x + f) + (B*b*d*f^2 + B*a*c*g^2 - (B*b*c + B*a*d)*f*g)*log((b*e*x
+ a*e)/(d*x + c)))/(b*d*f^3*g + a*c*f*g^3 - (b*c + a*d)*f^2*g^2 + (b*d*f^2*
g^2 + a*c*g^4 - (b*c + a*d)*f*g^3)*x)
```

**giac [B]** time = 1.11, size = 1537, normalized size = 17.67

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*(b*x+a)/(d*x+c)))/(g*x+f)^2,x, algorithm="giac")
```

```
[Out] (B*b^3*c^2*f*e^2*log(-b*f*e + a*g*e + (b*x*e + a*e)*d*f/(d*x + c) - (b*x*e
+ a*e)*c*g/(d*x + c)) - 2*B*a*b^2*c*d*f*e^2*log(-b*f*e + a*g*e + (b*x*e + a
*e)*d*f/(d*x + c) - (b*x*e + a*e)*c*g/(d*x + c)) + B*a^2*b*d^2*f*e^2*log(-b
*f*e + a*g*e + (b*x*e + a*e)*d*f/(d*x + c) - (b*x*e + a*e)*c*g/(d*x + c)) -
 B*a*b^2*c^2*g*e^2*log(-b*f*e + a*g*e + (b*x*e + a*e)*d*f/(d*x + c) - (b*x*
e + a*e)*c*g/(d*x + c)) + 2*B*a^2*b*c*d*g*e^2*log(-b*f*e + a*g*e + (b*x*e +
 a*e)*d*f/(d*x + c) - (b*x*e + a*e)*c*g/(d*x + c)) - B*a^3*d^2*g*e^2*log(-b
*f*e + a*g*e + (b*x*e + a*e)*d*f/(d*x + c) - (b*x*e + a*e)*c*g/(d*x + c)) -
 (b*x*e + a*e)*B*b^2*c^2*d*f*e*log(-b*f*e + a*g*e + (b*x*e + a*e)*d*f/(d*x
+ c) - (b*x*e + a*e)*c*g/(d*x + c))/(d*x + c) + 2*(b*x*e + a*e)*B*a*b*c*d^2
*f*e*log(-b*f*e + a*g*e + (b*x*e + a*e)*d*f/(d*x + c) - (b*x*e + a*e)*c*g/(
d*x + c))/(d*x + c) - (b*x*e + a*e)*B*a^2*d^3*f*e*log(-b*f*e + a*g*e + (b*x
*e + a*e)*d*f/(d*x + c) - (b*x*e + a*e)*c*g/(d*x + c))/(d*x + c) + (b*x*e +
 a*e)*B*b^2*c^3*g*e*log(-b*f*e + a*g*e + (b*x*e + a*e)*d*f/(d*x + c) - (b*x
*e + a*e)*c*g/(d*x + c))/(d*x + c) - 2*(b*x*e + a*e)*B*a*b*c^2*d*g*e*log(-b
*f*e + a*g*e + (b*x*e + a*e)*d*f/(d*x + c) - (b*x*e + a*e)*c*g/(d*x + c))/(
d*x + c) + (b*x*e + a*e)*B*a^2*c*d^2*g*e*log(-b*f*e + a*g*e + (b*x*e + a*e)
*d*f/(d*x + c) - (b*x*e + a*e)*c*g/(d*x + c))/(d*x + c) + (b*x*e + a*e)*B*b
^2*c^2*d*f*e*log((b*x*e + a*e)/(d*x + c))/(d*x + c) - 2*(b*x*e + a*e)*B*a*b
*c*d^2*f*e*log((b*x*e + a*e)/(d*x + c))/(d*x + c) + (b*x*e + a*e)*B*a^2*d^3
*f*e*log((b*x*e + a*e)/(d*x + c))/(d*x + c) - (b*x*e + a*e)*B*b^2*c^3*g*e*l
og((b*x*e + a*e)/(d*x + c))/(d*x + c) + 2*(b*x*e + a*e)*B*a*b*c^2*d*g*e*log
((b*x*e + a*e)/(d*x + c))/(d*x + c) - (b*x*e + a*e)*B*a^2*c*d^2*g*e*log((b*
x*e + a*e)/(d*x + c))/(d*x + c) + A*b^3*c^2*f*e^2 - 2*A*a*b^2*c*d*f*e^2 + A
*a^2*b*d^2*f*e^2 - A*a*b^2*c^2*g*e^2 + 2*A*a^2*b*c*d*g*e^2 - A*a^3*d^2*g*e^
2)*(b*c/((b*c*e - a*d*e)*(b*c - a*d)) - a*d/((b*c*e - a*d*e)*(b*c - a*d)))/
(b^2*d*f^3*e - b^2*c*f^2*g*e - 2*a*b*d*f^2*g*e + 2*a*b*c*f*g^2*e + a^2*d*f*
g^2*e - a^2*c*g^3*e - (b*x*e + a*e)*b*d^2*f^3/(d*x + c) + 2*(b*x*e + a*e)*b
*c*d*f^2*g/(d*x + c) + (b*x*e + a*e)*a*d^2*f^2*g/(d*x + c) - (b*x*e + a*e)*
b*c^2*f*g^2/(d*x + c) - 2*(b*x*e + a*e)*a*c*d*f*g^2/(d*x + c) + (b*x*e + a*
e)*a*c^2*g^3/(d*x + c))
```

**maple [B]** time = 0.13, size = 926, normalized size = 10.64

$$\frac{B\,a^{2}de\ln\left(\frac{be}{d}+\frac{(ad-bc)e}{(dx+c)d}\right)}{\left(ag-bf\right)\left(\frac{aceg}{dx+c}-\frac{adef}{dx+c}-\frac{b\,c^{2}eg}{(dx+c)d}+\frac{bcef}{dx+c}-aeg+\frac{bceg}{d}\right)(dx+c)}-\frac{2Babce\ln\left(\frac{be}{d}+\frac{(ad-bc)e}{(dx+c)d}\right)}{\left(ag-bf\right)\left(\frac{aceg}{dx+c}-\frac{adef}{dx+c}-\frac{b\,c^{2}eg}{(dx+c)d}+\frac{bcef}{dx+c}-aeg+\frac{bceg}{d}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*ln((b*x+a)/(d*x+c)*e)+A)/(g*x+f)^2,x)
```

```
[Out] d*e*A/(b/d*e*c*g+e/(d*x+c)*a*c*g-e/(d*x+c)*a*d*f-e/d/(d*x+c)*b*c^2*g+e/(d*x
+c)*b*c*f-a*e*g)/(c*g-d*f)*a-e*A/(b/d*e*c*g+e/(d*x+c)*a*c*g-e/(d*x+c)*a*d*f
-e/d/(d*x+c)*b*c^2*g+e/(d*x+c)*b*c*f-a*e*g)/(c*g-d*f)*b*c-d*B/(a*g-b*f)*ln(
-a*e*g+b*e*f+(c*g-d*f)*(b/d*e+(a*d-b*c)/(d*x+c)/d*e))/(c*g-d*f)*a+B/(a*g-b*
f)*ln(-a*e*g+b*e*f+(c*g-d*f)*(b/d*e+(a*d-b*c)/(d*x+c)/d*e))/(c*g-d*f)*b*c+e
*B*ln(b/d*e+(a*d-b*c)/(d*x+c)/d*e)/(a*g-b*f)/(b/d*e*c*g+e/(d*x+c)*a*c*g-e/(
d*x+c)*a*d*f-e/d/(d*x+c)*b*c^2*g+e/(d*x+c)*b*c*f-a*e*g)*b*a-1/d*e*B*ln(b/d*
e+(a*d-b*c)/(d*x+c)/d*e)/(a*g-b*f)/(b/d*e*c*g+e/(d*x+c)*a*c*g-e/(d*x+c)*a*d
*f-e/d/(d*x+c)*b*c^2*g+e/(d*x+c)*b*c*f-a*e*g)*b^2*c+d*e*B*ln(b/d*e+(a*d-b*c
)/(d*x+c)/d*e)/(a*g-b*f)/(b/d*e*c*g+e/(d*x+c)*a*c*g-e/(d*x+c)*a*d*f-e/d/(d*
x+c)*b*c^2*g+e/(d*x+c)*b*c*f-a*e*g)/(d*x+c)*a^2-2*e*B*ln(b/d*e+(a*d-b*c)/(d
*x+c)/d*e)/(a*g-b*f)/(b/d*e*c*g+e/(d*x+c)*a*c*g-e/(d*x+c)*a*d*f-e/d/(d*x+c)
*b*c^2*g+e/(d*x+c)*b*c*f-a*e*g)/(d*x+c)*a*b*c+1/d*e*B*ln(b/d*e+(a*d-b*c)/(d
*x+c)/d*e)/(a*g-b*f)/(b/d*e*c*g+e/(d*x+c)*a*c*g-e/(d*x+c)*a*d*f-e/d/(d*x+c)
*b*c^2*g+e/(d*x+c)*b*c*f-a*e*g)/(d*x+c)*b^2*c^2
```

**maxima [A]** time = 0.77, size = 138, normalized size = 1.59

$$B\left(\frac{b\log\left(bx+a\right)}{bfg-ag^{2}}-\frac{d\log\left(dx+c\right)}{dfg-cg^{2}}+\frac{(bc-ad)\log\left(gx+f\right)}{bdf^{2}+acg^{2}-(bc+ad)fg}-\frac{\log\left(\frac{bex}{dx+c}+\frac{ae}{dx+c}\right)}{g^{2}x+fg}\right)-\frac{A}{g^{2}x+fg}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*(b*x+a)/(d*x+c)))/(g*x+f)^2,x, algorithm="maxima")
```

```
[Out] B*(b*log(b*x + a)/(b*f*g - a*g^2) - d*log(d*x + c)/(d*f*g - c*g^2) + (b*c -
 a*d)*log(g*x + f)/(b*d*f^2 + a*c*g^2 - (b*c + a*d)*f*g) - log(b*e*x/(d*x +
 c) + a*e/(d*x + c))/(g^2*x + f*g)) - A/(g^2*x + f*g)
```

**mupad [B]** time = 5.17, size = 166, normalized size = 1.91

$$\frac{B\,d\,\ln\left(c+d\,x\right)}{c\,g^2-d\,f\,g}-\frac{B\,\ln\left(\frac{a\,e+b\,e\,x}{c+d\,x}\right)}{x\,g^2+f\,g}-\frac{B\,b\,\ln\left(a+b\,x\right)}{a\,g^2-b\,f\,g}-\frac{A}{x\,g^2+f\,g}-\frac{B\,a\,d\,\ln\left(f+g\,x\right)}{a\,c\,g^2+b\,d\,f^2-a\,d\,f\,g-b\,c\,f\,g}+\frac{B\,b\,c\,\ln\left(f\right.}{a\,c\,g^2+b\,d\,f^2-a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*log((e*(a + b*x))/(c + d*x)))/(f + g*x)^2,x)
```

```
[Out] (B*d*log(c + d*x))/(c*g^2 - d*f*g) - (B*log((a*e + b*e*x)/(c + d*x)))/(f*g
+ g^2*x) - (B*b*log(a + b*x))/(a*g^2 - b*f*g) - A/(f*g + g^2*x) - (B*a*d*lo
g(f + g*x))/(a*c*g^2 + b*d*f^2 - a*d*f*g - b*c*f*g) + (B*b*c*log(f + g*x))/
(a*c*g^2 + b*d*f^2 - a*d*f*g - b*c*f*g)
```

**sympy [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*ln(e*(b*x+a)/(d*x+c)))/(g*x+f)**2,x)
```

```
[Out] Timed out
```

## 3.237 $\int \frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{(f+gx)^3}\,dx$

**Optimal**. Leaf size=183

$$-\frac{B\log\left(\frac{e(a+bx)}{c+dx}\right)+A}{2g(f+gx)^2}+\frac{b^2B\log(a+bx)}{2g(bf-ag)^2}-\frac{B(bc-ad)}{2(f+gx)(bf-ag)(df-cg)}+\frac{B(bc-ad)\log(f+gx)(-adg-bcg+2bdf}{2(bf-ag)^2(df-cg)^2}$$

```
[Out] -1/2*B*(-a*d+b*c)/(-a*g+b*f)/(-c*g+d*f)/(g*x+f)+1/2*b^2*B*ln(b*x+a)/g/(-a*g
+b*f)^2+1/2*(-A-B*ln(e*(b*x+a)/(d*x+c)))/g/(g*x+f)^2-1/2*B*d^2*ln(d*x+c)/g/
(-c*g+d*f)^2+1/2*B*(-a*d+b*c)*(-a*d*g-b*c*g+2*b*d*f)*ln(g*x+f)/(-a*g+b*f)^2
/(-c*g+d*f)^2
```

**Rubi [A]** time = 0.19, antiderivative size = 183, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {2525, 12, 72}

$$-\frac{B\log\left(\frac{e(a+bx)}{c+dx}\right)+A}{2g(f+gx)^2}+\frac{b^2B\log(a+bx)}{2g(bf-ag)^2}-\frac{B(bc-ad)}{2(f+gx)(bf-ag)(df-cg)}+\frac{B(bc-ad)\log(f+gx)(-adg-bcg+2bdf}{2(bf-ag)^2(df-cg)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Log[(e*(a + b*x))/(c + d*x)])/(f + g*x)^3,x]
```

```
[Out] -(B*(b*c - a*d))/(2*(b*f - a*g)*(d*f - c*g)*(f + g*x)) + (b^2*B*Log[a + b*x
])/(2*g*(b*f - a*g)^2) - (A + B*Log[(e*(a + b*x))/(c + d*x)])/(2*g*(f + g*x
)^2) - (B*d^2*Log[c + d*x])/(2*g*(d*f - c*g)^2) + (B*(b*c - a*d)*(2*b*d*f -
 b*c*g - a*d*g)*Log[f + g*x])/(2*(b*f - a*g)^2*(d*f - c*g)^2)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 72

```
Int[((e_.) + (f_.)*(x_))^(p_.)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
 x_Symbol] :> Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x]
 /; FreeQ[{a, b, c, d, e, f}, x] && IntegerQ[p]
```

Rule 2525

```
Int[((a_.) + Log[(c_.)*(RFx_)^(p_.)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_))^(m_.
), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(a + b*Log[c*RFx^p])^n)/(e*(m + 1))
```

```
, x] - Dist[(b*n*p)/(e*(m + 1)), Int[SimplifyIntegrand[((d + e*x)^(m + 1)*(
a + b*Log[c*RFx^p])^(n - 1)*D[RFx, x])/RFx, x], x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && RationalFunctionQ[RFx, x] && IGtQ[n, 0] && (EqQ[n, 1] ||
IntegerQ[m]) && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\int \frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{(f+gx)^3}\,dx &= -\frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{2g(f+gx)^2}+\frac{B\int\frac{bc-ad}{(a+bx)(c+dx)(f+gx)^2}\,dx}{2g}\\
&= -\frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{2g(f+gx)^2}+\frac{(B(bc-ad))\int\frac{1}{(a+bx)(c+dx)(f+gx)^2}\,dx}{2g}\\
&= -\frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{2g(f+gx)^2}+\frac{(B(bc-ad))\int\left(\frac{b^3}{(bc-ad)(bf-ag)^2(a+bx)}-\frac{d^3}{(bc-ad)(-df+cg)^2(c+dx)}+\right.}{2g}\\
&= -\frac{B(bc-ad)}{2(bf-ag)(df-cg)(f+gx)}+\frac{b^2B\log(a+bx)}{2g(bf-ag)^2}-\frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{2g(f+gx)^2}-\frac{Bd^2\log(c}{2g(df-}
\end{aligned}$$

**Mathematica [A]** time = 0.50, size = 169, normalized size = 0.92

$$\frac{B(bc-ad)\left(\frac{b^2\log(a+bx)}{(bc-ad)(bf-ag)^2}+\frac{\frac{d^2\log(c+dx)}{ad-bc}+\frac{g(cg-df)}{(f+gx)(bf-ag)}-\frac{g\log(f+gx)(adg+bcg-2bdf)}{(bf-ag)^2}}{(df-cg)^2}\right)-\frac{B\log\left(\frac{e(a+bx)}{c+dx}\right)+A}{(f+gx)^2}}{2g}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Log[(e*(a + b*x))/(c + d*x)])/(f + g*x)^3,x]
```

```
[Out] (-((A + B*Log[(e*(a + b*x))/(c + d*x)])/(f + g*x)^2) + B*(b*c - a*d)*((b^2*
Log[a + b*x])/((b*c - a*d)*(b*f - a*g)^2) + ((g*(-(d*f) + c*g))/((b*f - a*g
)*(f + g*x)) + (d^2*Log[c + d*x])/(-(b*c) + a*d) - (g*(-2*b*d*f + b*c*g + a
*d*g)*Log[f + g*x])/(b*f - a*g)^2)/(d*f - c*g)^2))/(2*g)
```

**fricas [B]** time = 139.54, size = 1017, normalized size = 5.56

$$-\frac{Ab^2d^2f^4+Aa^2c^2g^4-\left((2\,A-B)b^2cd+(2\,A+B)abd^2\right)f^3g+\left((A-B)b^2c^2+4\,Aabcd+(A+B)a^2d^2\right)f^2g^2-(}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*(b*x+a)/(d*x+c)))/(g*x+f)^3,x, algorithm="fricas")
```

```
[Out] -1/2*(A*b^2*d^2*f^4 + A*a^2*c^2*g^4 - ((2*A - B)*b^2*c*d + (2*A + B)*a*b*d^
2)*f^3*g + ((A - B)*b^2*c^2 + 4*A*a*b*c*d + (A + B)*a^2*d^2)*f^2*g^2 - ((2*
A - B)*a*b*c^2 + (2*A + B)*a^2*c*d)*f*g^3 + ((B*b^2*c*d - B*a*b*d^2)*f^2*g^
2 - (B*b^2*c^2 - B*a^2*d^2)*f*g^3 + (B*a*b*c^2 - B*a^2*c*d)*g^4)*x - (B*b^2
*d^2*f^4 - 2*B*b^2*c*d*f^3*g + B*b^2*c^2*f^2*g^2 + (B*b^2*d^2*f^2*g^2 - 2*B
*b^2*c*d*f*g^3 + B*b^2*c^2*g^4)*x^2 + 2*(B*b^2*d^2*f^3*g - 2*B*b^2*c*d*f^2*
g^2 + B*b^2*c^2*f*g^3)*x)*log(b*x + a) + (B*b^2*d^2*f^4 - 2*B*a*b*d^2*f^3*g
 + B*a^2*d^2*f^2*g^2 + (B*b^2*d^2*f^2*g^2 - 2*B*a*b*d^2*f*g^3 + B*a^2*d^2*g
^4)*x^2 + 2*(B*b^2*d^2*f^3*g - 2*B*a*b*d^2*f^2*g^2 + B*a^2*d^2*f*g^3)*x)*lo
g(d*x + c) - (2*(B*b^2*c*d - B*a*b*d^2)*f^3*g - (B*b^2*c^2 - B*a^2*d^2)*f^2
*g^2 + (2*(B*b^2*c*d - B*a*b*d^2)*f*g^3 - (B*b^2*c^2 - B*a^2*d^2)*g^4)*x^2
+ 2*(2*(B*b^2*c*d - B*a*b*d^2)*f^2*g^2 - (B*b^2*c^2 - B*a^2*d^2)*f*g^3)*x)*
log(g*x + f) + (B*b^2*d^2*f^4 + B*a^2*c^2*g^4 - 2*(B*b^2*c*d + B*a*b*d^2)*f
^3*g + (B*b^2*c^2 + 4*B*a*b*c*d + B*a^2*d^2)*f^2*g^2 - 2*(B*a*b*c^2 + B*a^2
*c*d)*f*g^3)*log((b*e*x + a*e)/(d*x + c)))/(b^2*d^2*f^6*g + a^2*c^2*f^2*g^5
 - 2*(b^2*c*d + a*b*d^2)*f^5*g^2 + (b^2*c^2 + 4*a*b*c*d + a^2*d^2)*f^4*g^3
- 2*(a*b*c^2 + a^2*c*d)*f^3*g^4 + (b^2*d^2*f^4*g^3 + a^2*c^2*g^7 - 2*(b^2*c
*d + a*b*d^2)*f^3*g^4 + (b^2*c^2 + 4*a*b*c*d + a^2*d^2)*f^2*g^5 - 2*(a*b*c^
2 + a^2*c*d)*f*g^6)*x^2 + 2*(b^2*d^2*f^5*g^2 + a^2*c^2*f*g^6 - 2*(b^2*c*d +
 a*b*d^2)*f^4*g^3 + (b^2*c^2 + 4*a*b*c*d + a^2*d^2)*f^3*g^4 - 2*(a*b*c^2 +
a^2*c*d)*f^2*g^5)*x)
```

**giac [B]** time = 2.09, size = 7600, normalized size = 41.53

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*(b*x+a)/(d*x+c)))/(g*x+f)^3,x, algorithm="giac")
```

```
[Out] 1/2*(2*B*b^5*c^2*d*f^3*e^3*log(-b*f*e + a*g*e + (b*x*e + a*e)*d*f/(d*x + c)
 - (b*x*e + a*e)*c*g/(d*x + c)) - 4*B*a*b^4*c*d^2*f^3*e^3*log(-b*f*e + a*g*
e + (b*x*e + a*e)*d*f/(d*x + c) - (b*x*e + a*e)*c*g/(d*x + c)) + 2*B*a^2*b^
3*d^3*f^3*e^3*log(-b*f*e + a*g*e + (b*x*e + a*e)*d*f/(d*x + c) - (b*x*e + a
*e)*c*g/(d*x + c)) - B*b^5*c^3*f^2*g*e^3*log(-b*f*e + a*g*e + (b*x*e + a*e)
*d*f/(d*x + c) - (b*x*e + a*e)*c*g/(d*x + c)) - 3*B*a*b^4*c^2*d*f^2*g*e^3*l
og(-b*f*e + a*g*e + (b*x*e + a*e)*d*f/(d*x + c) - (b*x*e + a*e)*c*g/(d*x +
c)) + 9*B*a^2*b^3*c*d^2*f^2*g*e^3*log(-b*f*e + a*g*e + (b*x*e + a*e)*d*f/(d
*x + c) - (b*x*e + a*e)*c*g/(d*x + c)) - 5*B*a^3*b^2*d^3*f^2*g*e^3*log(-b*f
*e + a*g*e + (b*x*e + a*e)*d*f/(d*x + c) - (b*x*e + a*e)*c*g/(d*x + c)) + 2
*B*a*b^4*c^3*f*g^2*e^3*log(-b*f*e + a*g*e + (b*x*e + a*e)*d*f/(d*x + c) - (
b*x*e + a*e)*c*g/(d*x + c)) - 6*B*a^3*b^2*c*d^2*f*g^2*e^3*log(-b*f*e + a*g*
e + (b*x*e + a*e)*d*f/(d*x + c) - (b*x*e + a*e)*c*g/(d*x + c)) + 4*B*a^4*b*
d^3*f*g^2*e^3*log(-b*f*e + a*g*e + (b*x*e + a*e)*d*f/(d*x + c) - (b*x*e + a
```

*e)*c*g/(d*x + c)) - B*a^2*b^3*c^3*g^3*e^3*log(-b*f*e + a*g*e + (b*x*e + a*e)*d*f/(d*x + c) - (b*x*e + a*e)*c*g/(d*x + c)) + B*a^3*b^2*c^2*d*g^3*e^3*log(-b*f*e + a*g*e + (b*x*e + a*e)*d*f/(d*x + c) - (b*x*e + a*e)*c*g/(d*x + c)) + B*a^4*b*c*d^2*g^3*e^3*log(-b*f*e + a*g*e + (b*x*e + a*e)*d*f/(d*x + c) - (b*x*e + a*e)*c*g/(d*x + c)) - B*a^5*d^3*g^3*e^3*log(-b*f*e + a*g*e + (b*x*e + a*e)*d*f/(d*x + c) - (b*x*e + a*e)*c*g/(d*x + c)) - 4*(b*x*e + a*e)*B*b^4*c^2*d^2*f^3*e^2*log(-b*f*e + a*g*e + (b*x*e + a*e)*d*f/(d*x + c) - (b*x*e + a*e)*c*g/(d*x + c))/(d*x + c) + 8*(b*x*e + a*e)*B*a*b^3*c*d^3*f^3*e^2*log(-b*f*e + a*g*e + (b*x*e + a*e)*d*f/(d*x + c) - (b*x*e + a*e)*c*g/(d*x + c))/(d*x + c) - 4*(b*x*e + a*e)*B*a^2*b^2*d^4*f^3*e^2*log(-b*f*e + a*g*e + (b*x*e + a*e)*d*f/(d*x + c) - (b*x*e + a*e)*c*g/(d*x + c))/(d*x + c) + 6*(b*x*e + a*e)*B*b^4*c^3*d*f^2*g*e^2*log(-b*f*e + a*g*e + (b*x*e + a*e)*d*f/(d*x + c) - (b*x*e + a*e)*c*g/(d*x + c))/(d*x + c) - 6*(b*x*e + a*e)*B*a*b^3*c^2*d^2*f^2*g*e^2*log(-b*f*e + a*g*e + (b*x*e + a*e)*d*f/(d*x + c) - (b*x*e + a*e)*c*g/(d*x + c))/(d*x + c) - 6*(b*x*e + a*e)*B*a^2*b^2*c*d^3*f^2*g*e^2*log(-b*f*e + a*g*e + (b*x*e + a*e)*d*f/(d*x + c) - (b*x*e + a*e)*c*g/(d*x + c))/(d*x + c) + 6*(b*x*e + a*e)*B*a^3*b*d^4*f^2*g*e^2*log(-b*f*e + a*g*e + (b*x*e + a*e)*d*f/(d*x + c) - (b*x*e + a*e)*c*g/(d*x + c))/(d*x + c) - 2*(b*x*e + a*e)*B*b^4*c^4*f*g^2*e^2*log(-b*f*e + a*g*e + (b*x*e + a*e)*d*f/(d*x + c) - (b*x*e + a*e)*c*g/(d*x + c))/(d*x + c) - 4*(b*x*e + a*e)*B*a*b^3*c^3*d*f*g^2*e^2*log(-b*f*e + a*g*e + (b*x*e + a*e)*d*f/(d*x + c) - (b*x*e + a*e)*c*g/(d*x + c))/(d*x + c) + 12*(b*x*e + a*e)*B*a^2*b^2*c^2*d^2*f*g^2*e^2*log(-b*f*e + a*g*e + (b*x*e + a*e)*d*f/(d*x + c) - (b*x*e + a*e)*c*g/(d*x + c))/(d*x + c) - 4*(b*x*e + a*e)*B*a^3*b*c*d^3*f*g^2*e^2*log(-b*f*e + a*g*e + (b*x*e + a*e)*d*f/(d*x + c) - (b*x*e + a*e)*c*g/(d*x + c))/(d*x + c) - 2*(b*x*e + a*e)*B*a^4*d^4*f*g^2*e^2*log(-b*f*e + a*g*e + (b*x*e + a*e)*d*f/(d*x + c) - (b*x*e + a*e)*c*g/(d*x + c))/(d*x + c) + 2*(b*x*e + a*e)*B*a*b^3*c^4*g^3*e^2*log(-b*f*e + a*g*e + (b*x*e + a*e)*d*f/(d*x + c) - (b*x*e + a*e)*c*g/(d*x + c))/(d*x + c) - 2*(b*x*e + a*e)*B*a^2*b^2*c^3*d*g^3*e^2*log(-b*f*e + a*g*e + (b*x*e + a*e)*d*f/(d*x + c) - (b*x*e + a*e)*c*g/(d*x + c))/(d*x + c) - 2*(b*x*e + a*e)*B*a^3*b*c^2*d^2*g^3*e^2*log(-b*f*e + a*g*e + (b*x*e + a*e)*d*f/(d*x + c) - (b*x*e + a*e)*c*g/(d*x + c))/(d*x + c) + 2*(b*x*e + a*e)*B*a^4*c*d^3*g^3*e^2*log(-b*f*e + a*g*e + (b*x*e + a*e)*d*f/(d*x + c) - (b*x*e + a*e)*c*g/(d*x + c))/(d*x + c) + 2*(b*x*e + a*e)^2*B*b^3*c^2*d^3*f^3*e*log(-b*f*e + a*g*e + (b*x*e + a*e)*d*f/(d*x + c) - (b*x*e + a*e)*c*g/(d*x + c))/(d*x + c)^2 - 4*(b*x*e + a*e)^2*B*a*b^2*c*d^4*f^3*e*log(-b*f*e + a*g*e + (b*x*e + a*e)*d*f/(d*x + c) - (b*x*e + a*e)*c*g/(d*x + c))/(d*x + c)^2 + 2*(b*x*e + a*e)^2*B*a^2*b*d^5*f^3*e*log(-b*f*e + a*g*e + (b*x*e + a*e)*d*f/(d*x + c) - (b*x*e + a*e)*c*g/(d*x + c))/(d*x + c)^2 - 5*(b*x*e + a*e)^2*B*b^3*c^3*d^2*f^2*g*e*log(-b*f*e + a*g*e + (b*x*e + a*e)*d*f/(d*x + c) - (b*x*e + a*e)*c*g/(d*x + c))/(d*x + c)^2 + 9*(b*x*e + a*e)^2*B*a*b^2*c^2*d^3*f^2*g*e*log(-b*f*e + a*g*e + (b*x*e + a*e)*d*f/(d*x + c) - (b*x*e + a*e)*c*g/(d*x + c))/(d*x + c)^2 - 3*(b*x*e + a*e)^2*B*a^2*b*c*d^4*f^2*g*e*log(-b*f*e + a*g*e + (b*x*e + a*e)*d*f/(d*x + c) - (b*x*e + a*e)*c*g/(d*x + c))/(d*x + c)^2 - (b*x*e + a*e)^2*B*a^3*d^5*f^2*g*e*log(-b*f*e +

a*g*e + (b*x*e + a*e)*d*f/(d*x + c) - (b*x*e + a*e)*c*g/(d*x + c))/(d*x + c
)^2 + 4*(b*x*e + a*e)^2*B*b^3*c^4*d*f*g^2*e*log(-b*f*e + a*g*e + (b*x*e + a
*e)*d*f/(d*x + c) - (b*x*e + a*e)*c*g/(d*x + c))/(d*x + c)^2 - 6*(b*x*e + a
*e)^2*B*a*b^2*c^3*d^2*f*g^2*e*log(-b*f*e + a*g*e + (b*x*e + a*e)*d*f/(d*x +
 c) - (b*x*e + a*e)*c*g/(d*x + c))/(d*x + c)^2 + 2*(b*x*e + a*e)^2*B*a^3*c*
d^4*f*g^2*e*log(-b*f*e + a*g*e + (b*x*e + a*e)*d*f/(d*x + c) - (b*x*e + a*e
)*c*g/(d*x + c))/(d*x + c)^2 - (b*x*e + a*e)^2*B*b^3*c^5*g^3*e*log(-b*f*e +
 a*g*e + (b*x*e + a*e)*d*f/(d*x + c) - (b*x*e + a*e)*c*g/(d*x + c))/(d*x +
c)^2 + (b*x*e + a*e)^2*B*a*b^2*c^4*d*g^3*e*log(-b*f*e + a*g*e + (b*x*e + a*
e)*d*f/(d*x + c) - (b*x*e + a*e)*c*g/(d*x + c))/(d*x + c)^2 + (b*x*e + a*e)
^2*B*a^2*b*c^3*d^2*g^3*e*log(-b*f*e + a*g*e + (b*x*e + a*e)*d*f/(d*x + c) -
 (b*x*e + a*e)*c*g/(d*x + c))/(d*x + c)^2 - (b*x*e + a*e)^2*B*a^3*c^2*d^3*g
^3*e*log(-b*f*e + a*g*e + (b*x*e + a*e)*d*f/(d*x + c) - (b*x*e + a*e)*c*g/(
d*x + c))/(d*x + c)^2 + 2*(b*x*e + a*e)*B*b^4*c^2*d^2*f^3*e^2*log((b*x*e +
a*e)/(d*x + c))/(d*x + c) - 4*(b*x*e + a*e)*B*a*b^3*c*d^3*f^3*e^2*log((b*x*
e + a*e)/(d*x + c))/(d*x + c) + 2*(b*x*e + a*e)*B*a^2*b^2*d^4*f^3*e^2*log((
b*x*e + a*e)/(d*x + c))/(d*x + c) - 4*(b*x*e + a*e)*B*b^4*c^3*d*f^2*g*e^2*l
og((b*x*e + a*e)/(d*x + c))/(d*x + c) + 6*(b*x*e + a*e)*B*a*b^3*c^2*d^2*f^2
*g*e^2*log((b*x*e + a*e)/(d*x + c))/(d*x + c) - 2*(b*x*e + a*e)*B*a^3*b*d^4
*f^2*g*e^2*log((b*x*e + a*e)/(d*x + c))/(d*x + c) + 2*(b*x*e + a*e)*B*b^4*c
^4*f*g^2*e^2*log((b*x*e + a*e)/(d*x + c))/(d*x + c) - 6*(b*x*e + a*e)*B*a^2
*b^2*c^2*d^2*f*g^2*e^2*log((b*x*e + a*e)/(d*x + c))/(d*x + c) + 4*(b*x*e +
a*e)*B*a^3*b*c*d^3*f*g^2*e^2*log((b*x*e + a*e)/(d*x + c))/(d*x + c) - 2*(b*
x*e + a*e)*B*a*b^3*c^4*g^3*e^2*log((b*x*e + a*e)/(d*x + c))/(d*x + c) + 4*(
b*x*e + a*e)*B*a^2*b^2*c^3*d*g^3*e^2*log((b*x*e + a*e)/(d*x + c))/(d*x + c)
 - 2*(b*x*e + a*e)*B*a^3*b*c^2*d^2*g^3*e^2*log((b*x*e + a*e)/(d*x + c))/(d*
x + c) - 2*(b*x*e + a*e)^2*B*b^3*c^2*d^3*f^3*e*log((b*x*e + a*e)/(d*x + c))
/(d*x + c)^2 + 4*(b*x*e + a*e)^2*B*a*b^2*c*d^4*f^3*e*log((b*x*e + a*e)/(d*x
 + c))/(d*x + c)^2 - 2*(b*x*e + a*e)^2*B*a^2*b*d^5*f^3*e*log((b*x*e + a*e)/
(d*x + c))/(d*x + c)^2 + 5*(b*x*e + a*e)^2*B*b^3*c^3*d^2*f^2*g*e*log((b*x*e
 + a*e)/(d*x + c))/(d*x + c)^2 - 9*(b*x*e + a*e)^2*B*a*b^2*c^2*d^3*f^2*g*e*
log((b*x*e + a*e)/(d*x + c))/(d*x + c)^2 + 3*(b*x*e + a*e)^2*B*a^2*b*c*d^4*
f^2*g*e*log((b*x*e + a*e)/(d*x + c))/(d*x + c)^2 + (b*x*e + a*e)^2*B*a^3*d^
5*f^2*g*e*log((b*x*e + a*e)/(d*x + c))/(d*x + c)^2 - 4*(b*x*e + a*e)^2*B*b^
3*c^4*d*f*g^2*e*log((b*x*e + a*e)/(d*x + c))/(d*x + c)^2 + 6*(b*x*e + a*e)^
2*B*a*b^2*c^3*d^2*f*g^2*e*log((b*x*e + a*e)/(d*x + c))/(d*x + c)^2 - 2*(b*x
*e + a*e)^2*B*a^3*c*d^4*f*g^2*e*log((b*x*e + a*e)/(d*x + c))/(d*x + c)^2 +
(b*x*e + a*e)^2*B*b^3*c^5*g^3*e*log((b*x*e + a*e)/(d*x + c))/(d*x + c)^2 -
(b*x*e + a*e)^2*B*a*b^2*c^4*d*g^3*e*log((b*x*e + a*e)/(d*x + c))/(d*x + c)^
2 - (b*x*e + a*e)^2*B*a^2*b*c^3*d^2*g^3*e*log((b*x*e + a*e)/(d*x + c))/(d*x
 + c)^2 + (b*x*e + a*e)^2*B*a^3*c^2*d^3*g^3*e*log((b*x*e + a*e)/(d*x + c))/
(d*x + c)^2 + 2*A*b^5*c^2*d*f^3*e^3 - 4*A*a*b^4*c*d^2*f^3*e^3 + 2*A*a^2*b^3
*d^3*f^3*e^3 - A*b^5*c^3*f^2*g*e^3 + B*b^5*c^3*f^2*g*e^3 - 3*A*a*b^4*c^2*d*
f^2*g*e^3 - 3*B*a*b^4*c^2*d*f^2*g*e^3 + 9*A*a^2*b^3*c*d^2*f^2*g*e^3 + 3*B*a
^2*b^3*c*d^2*f^2*g*e^3 - 5*A*a^3*b^2*d^3*f^2*g*e^3 - B*a^3*b^2*d^3*f^2*g*e^

3 + 2*A*a*b^4*c^3*f*g^2*e^3 - 2*B*a*b^4*c^3*f*g^2*e^3 + 6*B*a^2*b^3*c^2*d*f
*g^2*e^3 - 6*A*a^3*b^2*c*d^2*f*g^2*e^3 - 6*B*a^3*b^2*c*d^2*f*g^2*e^3 + 4*A*
a^4*b*d^3*f*g^2*e^3 + 2*B*a^4*b*d^3*f*g^2*e^3 - A*a^2*b^3*c^3*g^3*e^3 + B*a
^2*b^3*c^3*g^3*e^3 + A*a^3*b^2*c^2*d*g^3*e^3 - 3*B*a^3*b^2*c^2*d*g^3*e^3 +
A*a^4*b*c*d^2*g^3*e^3 + 3*B*a^4*b*c*d^2*g^3*e^3 - A*a^5*d^3*g^3*e^3 - B*a^5
*d^3*g^3*e^3 - 2*(b*x*e + a*e)*A*b^4*c^2*d^2*f^3*e^2/(d*x + c) + 4*(b*x*e +
 a*e)*A*a*b^3*c*d^3*f^3*e^2/(d*x + c) - 2*(b*x*e + a*e)*A*a^2*b^2*d^4*f^3*e
^2/(d*x + c) + 2*(b*x*e + a*e)*A*b^4*c^3*d*f^2*g*e^2/(d*x + c) - (b*x*e + a
*e)*B*b^4*c^3*d*f^2*g*e^2/(d*x + c) + 3*(b*x*e + a*e)*B*a*b^3*c^2*d^2*f^2*g
*e^2/(d*x + c) - 6*(b*x*e + a*e)*A*a^2*b^2*c*d^3*f^2*g*e^2/(d*x + c) - 3*(b
*x*e + a*e)*B*a^2*b^2*c*d^3*f^2*g*e^2/(d*x + c) + 4*(b*x*e + a*e)*A*a^3*b*d
^4*f^2*g*e^2/(d*x + c) + (b*x*e + a*e)*B*a^3*b*d^4*f^2*g*e^2/(d*x + c) + (b
*x*e + a*e)*B*b^4*c^4*f*g^2*e^2/(d*x + c) - 4*(b*x*e + a*e)*A*a*b^3*c^3*d*f
*g^2*e^2/(d*x + c) - 2*(b*x*e + a*e)*B*a*b^3*c^3*d*f*g^2*e^2/(d*x + c) + 6*
(b*x*e + a*e)*A*a^2*b^2*c^2*d^2*f*g^2*e^2/(d*x + c) + 2*(b*x*e + a*e)*B*a^3
*b*c*d^3*f*g^2*e^2/(d*x + c) - 2*(b*x*e + a*e)*A*a^4*d^4*f*g^2*e^2/(d*x + c
) - (b*x*e + a*e)*B*a^4*d^4*f*g^2*e^2/(d*x + c) - (b*x*e + a*e)*B*a*b^3*c^4
*g^3*e^2/(d*x + c) + 2*(b*x*e + a*e)*A*a^2*b^2*c^3*d*g^3*e^2/(d*x + c) + 3*
(b*x*e + a*e)*B*a^2*b^2*c^3*d*g^3*e^2/(d*x + c) - 4*(b*x*e + a*e)*A*a^3*b*c
^2*d^2*g^3*e^2/(d*x + c) - 3*(b*x*e + a*e)*B*a^3*b*c^2*d^2*g^3*e^2/(d*x + c
) + 2*(b*x*e + a*e)*A*a^4*c*d^3*g^3*e^2/(d*x + c) + (b*x*e + a*e)*B*a^4*c*d
^3*g^3*e^2/(d*x + c))*(b*c/((b*c*e - a*d*e)*(b*c - a*d)) - a*d/((b*c*e - a*
d*e)*(b*c - a*d)))/(b^4*d^2*f^6*e^2 - 2*b^4*c*d*f^5*g*e^2 - 4*a*b^3*d^2*f^5
*g*e^2 + b^4*c^2*f^4*g^2*e^2 + 8*a*b^3*c*d*f^4*g^2*e^2 + 6*a^2*b^2*d^2*f^4*
g^2*e^2 - 4*a*b^3*c^2*f^3*g^3*e^2 - 12*a^2*b^2*c*d*f^3*g^3*e^2 - 4*a^3*b*d^
2*f^3*g^3*e^2 + 6*a^2*b^2*c^2*f^2*g^4*e^2 + 8*a^3*b*c*d*f^2*g^4*e^2 + a^4*d
^2*f^2*g^4*e^2 - 4*a^3*b*c^2*f*g^5*e^2 - 2*a^4*c*d*f*g^5*e^2 + a^4*c^2*g^6*
e^2 - 2*(b*x*e + a*e)*b^3*d^3*f^6*e/(d*x + c) + 6*(b*x*e + a*e)*b^3*c*d^2*f
^5*g*e/(d*x + c) + 6*(b*x*e + a*e)*a*b^2*d^3*f^5*g*e/(d*x + c) - 6*(b*x*e +
 a*e)*b^3*c^2*d*f^4*g^2*e/(d*x + c) - 18*(b*x*e + a*e)*a*b^2*c*d^2*f^4*g^2*
e/(d*x + c) - 6*(b*x*e + a*e)*a^2*b*d^3*f^4*g^2*e/(d*x + c) + 2*(b*x*e + a*
e)*b^3*c^3*f^3*g^3*e/(d*x + c) + 18*(b*x*e + a*e)*a*b^2*c^2*d*f^3*g^3*e/(d*
x + c) + 18*(b*x*e + a*e)*a^2*b*c*d^2*f^3*g^3*e/(d*x + c) + 2*(b*x*e + a*e)
*a^3*d^3*f^3*g^3*e/(d*x + c) - 6*(b*x*e + a*e)*a*b^2*c^3*f^2*g^4*e/(d*x + c
) - 18*(b*x*e + a*e)*a^2*b*c^2*d*f^2*g^4*e/(d*x + c) - 6*(b*x*e + a*e)*a^3*
c*d^2*f^2*g^4*e/(d*x + c) + 6*(b*x*e + a*e)*a^2*b*c^3*f*g^5*e/(d*x + c) + 6
*(b*x*e + a*e)*a^3*c^2*d*f*g^5*e/(d*x + c) - 2*(b*x*e + a*e)*a^3*c^3*g^6*e/
(d*x + c) + (b*x*e + a*e)^2*b^2*d^4*f^6/(d*x + c)^2 - 4*(b*x*e + a*e)^2*b^2
*c*d^3*f^5*g/(d*x + c)^2 - 2*(b*x*e + a*e)^2*a*b*d^4*f^5*g/(d*x + c)^2 + 6*
(b*x*e + a*e)^2*b^2*c^2*d^2*f^4*g^2/(d*x + c)^2 + 8*(b*x*e + a*e)^2*a*b*c*d
^3*f^4*g^2/(d*x + c)^2 + (b*x*e + a*e)^2*a^2*d^4*f^4*g^2/(d*x + c)^2 - 4*(b
*x*e + a*e)^2*b^2*c^3*d*f^3*g^3/(d*x + c)^2 - 12*(b*x*e + a*e)^2*a*b*c^2*d^
2*f^3*g^3/(d*x + c)^2 - 4*(b*x*e + a*e)^2*a^2*c*d^3*f^3*g^3/(d*x + c)^2 + (
b*x*e + a*e)^2*b^2*c^4*f^2*g^4/(d*x + c)^2 + 8*(b*x*e + a*e)^2*a*b*c^3*d*f^
2*g^4/(d*x + c)^2 + 6*(b*x*e + a*e)^2*a^2*c^2*d^2*f^2*g^4/(d*x + c)^2 - 2*(

b*x*e + a*e)^2*a*b*c^4*f*g^5/(d*x + c)^2 - 4*(b*x*e + a*e)^2*a^2*c^3*d*f*g^5/(d*x + c)^2 + (b*x*e + a*e)^2*a^2*c^4*g^6/(d*x + c)^2)

**maple [B]** time = 0.16, size = 5274, normalized size = 28.82

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*ln((b*x+a)/(d*x+c)*e)+A)/(g*x+f)^3,x)
```

```
[Out] result too large to display
```

**maxima [B]** time = 0.99, size = 351, normalized size = 1.92

$$\frac{1}{2}\left(\frac{b^2\log(bx+a)}{b^2f^2g-2\,abfg^2+a^2g^3}-\frac{d^2\log(dx+c)}{d^2f^2g-2\,cdfg^2+c^2g^3}+\frac{\left(2\left(b^2cd-abd^2\right)f-\left(b^2c^2-a^2d^2\right)g\right)}{b^2d^2f^4+a^2c^2g^4-2\left(b^2cd+abd^2\right)f^3g+\left(b^2c^2+4\,abcd+\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*(b*x+a)/(d*x+c)))/(g*x+f)^3,x, algorithm="maxima")
```

```
[Out] 1/2*(b^2*log(b*x + a)/(b^2*f^2*g - 2*a*b*f*g^2 + a^2*g^3) - d^2*log(d*x + c
)/(d^2*f^2*g - 2*c*d*f*g^2 + c^2*g^3) + (2*(b^2*c*d - a*b*d^2)*f - (b^2*c^2
 - a^2*d^2)*g)*log(g*x + f)/(b^2*d^2*f^4 + a^2*c^2*g^4 - 2*(b^2*c*d + a*b*d
^2)*f^3*g + (b^2*c^2 + 4*a*b*c*d + a^2*d^2)*f^2*g^2 - 2*(a*b*c^2 + a^2*c*d)
*f*g^3) - (b*c - a*d)/(b*d*f^3 + a*c*f*g^2 - (b*c + a*d)*f^2*g + (b*d*f^2*g
 + a*c*g^3 - (b*c + a*d)*f*g^2)*x) - log(b*e*x/(d*x + c) + a*e/(d*x + c))/(
g^3*x^2 + 2*f*g^2*x + f^2*g))*B - 1/2*A/(g^3*x^2 + 2*f*g^2*x + f^2*g)
```

**mupad [B]** time = 7.21, size = 417, normalized size = 2.28

$$\frac{\ln\left(f+g\,x\right)\,\left(g\,\left(B\,a^2\,d^2-B\,b^2\,c^2\right)-2\,B\,a\,b\,d^2\,f+2\,B\,b^2\,c\,d\,f\right)}{2\,a^2\,c^2\,g^4-4\,a^2\,c\,d\,f\,g^3+2\,a^2\,d^2\,f^2\,g^2-4\,a\,b\,c^2\,f\,g^3+8\,a\,b\,c\,d\,f^2\,g^2-4\,a\,b\,d^2\,f^3\,g+2\,b^2\,c^2\,f^2\,g^2-4\,b^2\,c\,d\,f^3\,g+}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*log((e*(a + b*x))/(c + d*x)))/(f + g*x)^3,x)
```

```
[Out] (log(f + g*x)*(g*(B*a^2*d^2 - B*b^2*c^2) - 2*B*a*b*d^2*f + 2*B*b^2*c*d*f))/
(2*a^2*c^2*g^4 + 2*b^2*d^2*f^4 + 2*a^2*d^2*f^2*g^2 + 2*b^2*c^2*f^2*g^2 - 4*
a*b*c^2*f*g^3 - 4*a*b*d^2*f^3*g - 4*a^2*c*d*f*g^3 - 4*b^2*c*d*f^3*g + 8*a*b
*c*d*f^2*g^2) - ((A*a*c*g^2 + A*b*d*f^2 - A*a*d*f*g - A*b*c*f*g - B*a*d*f*g
 + B*b*c*f*g)/(a*c*g^2 + b*d*f^2 - a*d*f*g - b*c*f*g) - (x*(B*a*d*g^2 - B*b
```

```
*c*g^2))/(a*c*g^2 + b*d*f^2 - a*d*f*g - b*c*f*g))/(2*f^2*g + 2*g^3*x^2 + 4*
f*g^2*x) + (B*b^2*log(a + b*x))/(2*a^2*g^3 + 2*b^2*f^2*g - 4*a*b*f*g^2) - (
B*log((e*(a + b*x))/(c + d*x)))/(2*g*(f^2 + g^2*x^2 + 2*f*g*x)) - (B*d^2*lo
g(c + d*x))/(2*c^2*g^3 + 2*d^2*f^2*g - 4*c*d*f*g^2)
```

**sympy [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*ln(e*(b*x+a)/(d*x+c)))/(g*x+f)**3,x)
```

```
[Out] Timed out
```

# 3.238 $\int \frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{(f+gx)^4}\,dx$

**Optimal**. Leaf size=275

$$\frac{B(bc-ad)\log(f+gx)\left(a^2d^2g^2-abdg(3df-cg)+b^2\left(c^2g^2-3cdfg+3d^2f^2\right)\right)}{3(bf-ag)^3(df-cg)^3}-\frac{B\log\left(\frac{e(a+bx)}{c+dx}\right)+A}{3g(f+gx)^3}+\frac{b^3B\log(a}{3g(bf-a}$$

```
[Out] -1/6*B*(-a*d+b*c)/(-a*g+b*f)/(-c*g+d*f)/(g*x+f)^2-1/3*B*(-a*d+b*c)*(-a*d*g-
b*c*g+2*b*d*f)/(-a*g+b*f)^2/(-c*g+d*f)^2/(g*x+f)+1/3*b^3*B*ln(b*x+a)/g/(-a*
g+b*f)^3+1/3*(-A-B*ln(e*(b*x+a)/(d*x+c)))/g/(g*x+f)^3-1/3*B*d^3*ln(d*x+c)/g
/(-c*g+d*f)^3+1/3*B*(-a*d+b*c)*(a^2*d^2*g^2-a*b*d*g*(-c*g+3*d*f)+b^2*(c^2*g
^2-3*c*d*f*g+3*d^2*f^2))*ln(g*x+f)/(-a*g+b*f)^3/(-c*g+d*f)^3
```

**Rubi [A]** time = 0.40, antiderivative size = 275, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {2525, 12, 72}

$$\frac{B(bc-ad)\log(f+gx)\left(a^2d^2g^2-abdg(3df-cg)+b^2\left(c^2g^2-3cdfg+3d^2f^2\right)\right)}{3(bf-ag)^3(df-cg)^3}-\frac{B\log\left(\frac{e(a+bx)}{c+dx}\right)+A}{3g(f+gx)^3}+\frac{b^3B\log(a}{3g(bf-a}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Log[(e*(a + b*x))/(c + d*x)])/(f + g*x)^4,x]
```

```
[Out] -(B*(b*c - a*d))/(6*(b*f - a*g)*(d*f - c*g)*(f + g*x)^2) - (B*(b*c - a*d)*(
2*b*d*f - b*c*g - a*d*g))/(3*(b*f - a*g)^2*(d*f - c*g)^2*(f + g*x)) + (b^3*
B*Log[a + b*x])/(3*g*(b*f - a*g)^3) - (A + B*Log[(e*(a + b*x))/(c + d*x)])/
(3*g*(f + g*x)^3) - (B*d^3*Log[c + d*x])/(3*g*(d*f - c*g)^3) + (B*(b*c - a*
d)*(a^2*d^2*g^2 - a*b*d*g*(3*d*f - c*g) + b^2*(3*d^2*f^2 - 3*c*d*f*g + c^2*
g^2))*Log[f + g*x])/(3*(b*f - a*g)^3*(d*f - c*g)^3)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 72

```
Int[((e_.) + (f_.)*(x_))^(p_.)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
 x_Symbol] :> Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x]
 /; FreeQ[{a, b, c, d, e, f}, x] && IntegerQ[p]
```

Rule 2525

```
Int[((a_.) + Log[(c_.)*(RFx_)^(p_.)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_))^(m_.
), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(a + b*Log[c*RFx^p])^n)/(e*(m + 1))
, x] - Dist[(b*n*p)/(e*(m + 1)), Int[SimplifyIntegrand[((d + e*x)^(m + 1)*(
a + b*Log[c*RFx^p])^(n - 1)*D[RFx, x])/RFx, x], x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && RationalFunctionQ[RFx, x] && IGtQ[n, 0] && (EqQ[n, 1] ||
IntegerQ[m]) && NeQ[m, -1]
```

Rubi steps

$$\int \frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{(f+gx)^4}\,dx = -\frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{3g(f+gx)^3}+\frac{B\int\frac{bc-ad}{(a+bx)(c+dx)(f+gx)^3}\,dx}{3g}$$

$$= -\frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{3g(f+gx)^3}+\frac{(B(bc-ad))\int\frac{1}{(a+bx)(c+dx)(f+gx)^3}\,dx}{3g}$$

$$= -\frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{3g(f+gx)^3}+\frac{(B(bc-ad))\int\left(\frac{b^4}{(bc-ad)(bf-ag)^3(a+bx)}+\frac{d^4}{(bc-ad)(-df+cg)^3(c+dx)}+\cdots\right.}{}$$

$$= -\frac{B(bc-ad)}{6(bf-ag)(df-cg)(f+gx)^2}-\frac{B(bc-ad)(2bdf-bcg-adg)}{3(bf-ag)^2(df-cg)^2(f+gx)}+\frac{b^3B\log(a+bx)}{3g(bf-ag)^3}$$

**Mathematica [A]** time = 0.72, size = 260, normalized size = 0.95

$$\frac{B(bc-ad)\left(\frac{g\log(f+gx)\left(a^2d^2g^2+abdg(cg-3df)+b^2\left(c^2g^2-3cdfg+3d^2f^2\right)\right)}{(bf-ag)^3(df-cg)^3}+\frac{b^3\log(a+bx)}{(bc-ad)(bf-ag)^3}+\frac{d^3\log(c+dx)}{(bc-ad)(cg-df)^3}+\frac{g(adg+bcg-2bdf)}{(f+gx)(bf-ag)^2(df-cg)}\cdots\right.}{3g}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Log[(e*(a + b*x))/(c + d*x)])/(f + g*x)^4,x]
```

```
[Out] (-((A + B*Log[(e*(a + b*x))/(c + d*x)])/(f + g*x)^3) + B*(b*c - a*d)*(-1/2*
g/((b*f - a*g)*(d*f - c*g)*(f + g*x)^2) + (g*(-2*b*d*f + b*c*g + a*d*g))/((
b*f - a*g)^2*(d*f - c*g)^2*(f + g*x)) + (b^3*Log[a + b*x])/((b*c - a*d)*(b*
f - a*g)^3) + (d^3*Log[c + d*x])/((b*c - a*d)*(-(d*f) + c*g)^3) + (g*(a^2*d
^2*g^2 + a*b*d*g*(-3*d*f + c*g) + b^2*(3*d^2*f^2 - 3*c*d*f*g + c^2*g^2))*Lo
g[f + g*x])/((b*f - a*g)^3*(d*f - c*g)^3)))/(3*g)
```

**fricas [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*(b*x+a)/(d*x+c)))/(g*x+f)^4,x, algorithm="fricas")
[Out] Timed out
```

**giac [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*(b*x+a)/(d*x+c)))/(g*x+f)^4,x, algorithm="giac")
[Out] Timed out
```

**maple [B]** time = 0.18, size = 18285, normalized size = 66.49

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*ln((b*x+a)/(d*x+c)*e)+A)/(g*x+f)^4,x)
[Out] result too large to display
```

**maxima [B]** time = 1.29, size = 848, normalized size = 3.08

$$\frac{1}{6}\left(\frac{2\,b^{3}\log\left(bx+a\right)}{b^{3}f^{3}g-3\,ab^{2}f^{2}g^{2}+3\,a^{2}bfg^{3}-a^{3}g^{4}}-\frac{2\,d^{3}\log\left(dx+c\right)}{d^{3}f^{3}g-3\,cd^{2}f^{2}g^{2}+3\,c^{2}dfg^{3}-c^{3}g^{4}}+\frac{}{b^{3}d^{3}f^{6}+a^{3}c^{3}g^{6}-3\left(b^{3}cd^{2}+\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*(b*x+a)/(d*x+c)))/(g*x+f)^4,x, algorithm="maxima")
[Out] 1/6*(2*b^3*log(b*x + a)/(b^3*f^3*g - 3*a*b^2*f^2*g^2 + 3*a^2*b*f*g^3 - a^3*
g^4) - 2*d^3*log(d*x + c)/(d^3*f^3*g - 3*c*d^2*f^2*g^2 + 3*c^2*d*f*g^3 - c^
3*g^4) + 2*(3*(b^3*c*d^2 - a*b^2*d^3)*f^2 - 3*(b^3*c^2*d - a^2*b*d^3)*f*g +
 (b^3*c^3 - a^3*d^3)*g^2)*log(g*x + f)/(b^3*d^3*f^6 + a^3*c^3*g^6 - 3*(b^3*
c*d^2 + a*b^2*d^3)*f^5*g + 3*(b^3*c^2*d + 3*a*b^2*c*d^2 + a^2*b*d^3)*f^4*g^
2 - (b^3*c^3 + 9*a*b^2*c^2*d + 9*a^2*b*c*d^2 + a^3*d^3)*f^3*g^3 + 3*(a*b^2*
c^3 + 3*a^2*b*c^2*d + a^3*c*d^2)*f^2*g^4 - 3*(a^2*b*c^3 + a^3*c^2*d)*f*g^5)
```

```
 - (5*(b^2*c*d - a*b*d^2)*f^2 - 3*(b^2*c^2 - a^2*d^2)*f*g + (a*b*c^2 - a^2*
c*d)*g^2 + 2*(2*(b^2*c*d - a*b*d^2)*f*g - (b^2*c^2 - a^2*d^2)*g^2)*x)/(b^2*
d^2*f^6 + a^2*c^2*f^2*g^4 - 2*(b^2*c*d + a*b*d^2)*f^5*g + (b^2*c^2 + 4*a*b*
c*d + a^2*d^2)*f^4*g^2 - 2*(a*b*c^2 + a^2*c*d)*f^3*g^3 + (b^2*d^2*f^4*g^2 +
 a^2*c^2*g^6 - 2*(b^2*c*d + a*b*d^2)*f^3*g^3 + (b^2*c^2 + 4*a*b*c*d + a^2*d
^2)*f^2*g^4 - 2*(a*b*c^2 + a^2*c*d)*f*g^5)*x^2 + 2*(b^2*d^2*f^5*g + a^2*c^2
*f*g^5 - 2*(b^2*c*d + a*b*d^2)*f^4*g^2 + (b^2*c^2 + 4*a*b*c*d + a^2*d^2)*f^
3*g^3 - 2*(a*b*c^2 + a^2*c*d)*f^2*g^4)*x) - 2*log(b*e*x/(d*x + c) + a*e/(d*
x + c))/(g^4*x^3 + 3*f*g^3*x^2 + 3*f^2*g^2*x + f^3*g))*B - 1/3*A/(g^4*x^3 +
 3*f*g^3*x^2 + 3*f^2*g^2*x + f^3*g)
```

**mupad** [B] time = 10.67, size = 1154, normalized size = 4.20

$$\frac{\ln\left(f+g\,x\right)\,\left(g\,\left(3\,B\,a^2\,b\,d^3\,f-3\,B\right.\right.}{3\,a^3\,c^3\,g^6-9\,a^3\,c^2\,d\,f\,g^5+9\,a^3\,c\,d^2\,f^2\,g^4-3\,a^3\,d^3\,f^3\,g^3-9\,a^2\,b\,c^3\,f\,g^5+27\,a^2\,b\,c^2\,d\,f^2\,g^4-27\,a^2\,b\,c\,d^2\,f^3\,g^3+9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*log((e*(a + b*x))/(c + d*x)))/(f + g*x)^4,x)
```

```
[Out] (log(f + g*x)*(g*(3*B*a^2*b*d^3*f - 3*B*b^3*c^2*d*f) - g^2*(B*a^3*d^3 - B*b
^3*c^3) - 3*B*a*b^2*d^3*f^2 + 3*B*b^3*c*d^2*f^2))/(3*a^3*c^3*g^6 + 3*b^3*d^
3*f^6 - 3*a^3*d^3*f^3*g^3 - 3*b^3*c^3*f^3*g^3 - 9*a^2*b*c^3*f*g^5 - 9*a*b^2
*d^3*f^5*g - 9*a^3*c^2*d*f*g^5 - 9*b^3*c*d^2*f^5*g + 9*a*b^2*c^3*f^2*g^4 +
9*a^2*b*d^3*f^4*g^2 + 9*a^3*c*d^2*f^2*g^4 + 9*b^3*c^2*d*f^4*g^2 + 27*a*b^2*
c*d^2*f^4*g^2 - 27*a*b^2*c^2*d*f^3*g^3 - 27*a^2*b*c*d^2*f^3*g^3 + 27*a^2*b*
c^2*d*f^2*g^4) - ((2*A*a^2*c^2*g^4 + 2*A*b^2*d^2*f^4 + 2*A*a^2*d^2*f^2*g^2
+ 2*A*b^2*c^2*f^2*g^2 + 3*B*a^2*d^2*f^2*g^2 - 3*B*b^2*c^2*f^2*g^2 - 4*A*a*b
*c^2*f*g^3 - 4*A*a*b*d^2*f^3*g + B*a*b*c^2*f*g^3 - 4*A*a^2*c*d*f*g^3 - 5*B*
a*b*d^2*f^3*g - 4*A*b^2*c*d*f^3*g - B*a^2*c*d*f*g^3 + 5*B*b^2*c*d*f^3*g + 8
*A*a*b*c*d*f^2*g^2)/(2*(a^2*c^2*g^4 + b^2*d^2*f^4 + a^2*d^2*f^2*g^2 + b^2*c
^2*f^2*g^2 - 2*a*b*c^2*f*g^3 - 2*a*b*d^2*f^3*g - 2*a^2*c*d*f*g^3 - 2*b^2*c*
d*f^3*g + 4*a*b*c*d*f^2*g^2)) + (x^2*(B*a^2*d^2*g^4 - B*b^2*c^2*g^4 - 2*B*a
*b*d^2*f*g^3 + 2*B*b^2*c*d*f*g^3))/(a^2*c^2*g^4 + b^2*d^2*f^4 + a^2*d^2*f^2
*g^2 + b^2*c^2*f^2*g^2 - 2*a*b*c^2*f*g^3 - 2*a*b*d^2*f^3*g - 2*a^2*c*d*f*g^
3 - 2*b^2*c*d*f^3*g + 4*a*b*c*d*f^2*g^2) + (x*(5*B*a^2*d^2*f*g^3 - 5*B*b^2*
c^2*f*g^3 + B*a*b*c^2*g^4 - B*a^2*c*d*g^4 - 9*B*a*b*d^2*f^2*g^2 + 9*B*b^2*c
*d*f^2*g^2))/(2*(a^2*c^2*g^4 + b^2*d^2*f^4 + a^2*d^2*f^2*g^2 + b^2*c^2*f^2*
g^2 - 2*a*b*c^2*f*g^3 - 2*a*b*d^2*f^3*g - 2*a^2*c*d*f*g^3 - 2*b^2*c*d*f^3*g
 + 4*a*b*c*d*f^2*g^2)))/(3*f^3*g + 3*g^4*x^3 + 9*f^2*g^2*x + 9*f*g^3*x^2) -
 (B*b^3*log(a + b*x))/(3*a^3*g^4 - 3*b^3*f^3*g + 9*a*b^2*f^2*g^2 - 9*a^2*b*
f*g^3) + (B*d^3*log(c + d*x))/(3*c^3*g^4 - 3*d^3*f^3*g + 9*c*d^2*f^2*g^2 -
9*c^2*d*f*g^3) - (B*log((e*(a + b*x))/(c + d*x)))/(3*g*(f^3 + g^3*x^3 + 3*f
^2*g*x + 3*f*g^2*x^2))
```

**sympy [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*ln(e*(b*x+a)/(d*x+c)))/(g*x+f)**4,x)
[Out] Timed out
```

# 3.239 $\int \frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{(f+gx)^5}\,dx$

**Optimal**. Leaf size=379

$$-\frac{B(bc-ad)\left(a^2d^2g^2-abdg(3df-cg)+b^2\left(c^2g^2-3cdfg+3d^2f^2\right)\right)}{4(f+gx)(bf-ag)^3(df-cg)^3}-\frac{B(bc-ad)\log(f+gx)(-adg-bcg+2bd}{4(b}$$

```
[Out] -1/12*B*(-a*d+b*c)/(-a*g+b*f)/(-c*g+d*f)/(g*x+f)^3-1/8*B*(-a*d+b*c)*(-a*d*g
-b*c*g+2*b*d*f)/(-a*g+b*f)^2/(-c*g+d*f)^2/(g*x+f)^2-1/4*B*(-a*d+b*c)*(a^2*d
^2*g^2-a*b*d*g*(-c*g+3*d*f)+b^2*(c^2*g^2-3*c*d*f*g+3*d^2*f^2))/(-a*g+b*f)^3
/(-c*g+d*f)^3/(g*x+f)+1/4*b^4*B*ln(b*x+a)/g/(-a*g+b*f)^4+1/4*(-A-B*ln(e*(b*
x+a)/(d*x+c)))/g/(g*x+f)^4-1/4*B*d^4*ln(d*x+c)/g/(-c*g+d*f)^4-1/4*B*(-a*d+b
*c)*(-a*d*g-b*c*g+2*b*d*f)*(2*a*b*d^2*f*g-a^2*d^2*g^2-b^2*(c^2*g^2-2*c*d*f*
g+2*d^2*f^2))*ln(g*x+f)/(-a*g+b*f)^4/(-c*g+d*f)^4
```

**Rubi [A]** time = 0.62, antiderivative size = 379, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {2525, 12, 72}

$$-\frac{B(bc-ad)\left(a^2d^2g^2-abdg(3df-cg)+b^2\left(c^2g^2-3cdfg+3d^2f^2\right)\right)}{4(f+gx)(bf-ag)^3(df-cg)^3}-\frac{B(bc-ad)\log(f+gx)(-adg-bcg+2bd}{4(b}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Log[(e*(a + b*x))/(c + d*x)])/(f + g*x)^5,x]
```

```
[Out] -(B*(b*c - a*d))/(12*(b*f - a*g)*(d*f - c*g)*(f + g*x)^3) - (B*(b*c - a*d)*
(2*b*d*f - b*c*g - a*d*g))/(8*(b*f - a*g)^2*(d*f - c*g)^2*(f + g*x)^2) - (B
*(b*c - a*d)*(a^2*d^2*g^2 - a*b*d*g*(3*d*f - c*g) + b^2*(3*d^2*f^2 - 3*c*d*
f*g + c^2*g^2)))/(4*(b*f - a*g)^3*(d*f - c*g)^3*(f + g*x)) + (b^4*B*Log[a +
 b*x])/(4*g*(b*f - a*g)^4) - (A + B*Log[(e*(a + b*x))/(c + d*x)])/(4*g*(f +
 g*x)^4) - (B*d^4*Log[c + d*x])/(4*g*(d*f - c*g)^4) - (B*(b*c - a*d)*(2*b*d
*f - b*c*g - a*d*g)*(2*a*b*d^2*f*g - a^2*d^2*g^2 - b^2*(2*d^2*f^2 - 2*c*d*f
*g + c^2*g^2))*Log[f + g*x])/(4*(b*f - a*g)^4*(d*f - c*g)^4)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 72

```
Int[((e_.) + (f_.)*(x_))^(p_.)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
 x_Symbol] :> Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x]
 /; FreeQ[{a, b, c, d, e, f}, x] && IntegerQ[p]
```

Rule 2525

```
Int[((a_.) + Log[(c_.)*(RFx_)^(p_.)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_))^(m_.
), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(a + b*Log[c*RFx^p])^n)/(e*(m + 1))
, x] - Dist[(b*n*p)/(e*(m + 1)), Int[SimplifyIntegrand[((d + e*x)^(m + 1)*(
a + b*Log[c*RFx^p])^(n - 1)*D[RFx, x])/RFx, x], x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && RationalFunctionQ[RFx, x] && IGtQ[n, 0] && (EqQ[n, 1] ||
IntegerQ[m]) && NeQ[m, -1]
```

Rubi steps

$$\int \frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{(f+gx)^5}\,dx = -\frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{4g(f+gx)^4}+\frac{B\int \frac{bc-ad}{(a+bx)(c+dx)(f+gx)^4}\,dx}{4g}$$

$$= -\frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{4g(f+gx)^4}+\frac{(B(bc-ad))\int \frac{1}{(a+bx)(c+dx)(f+gx)^4}\,dx}{4g}$$

$$= -\frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{4g(f+gx)^4}+\frac{(B(bc-ad))\int \left(\frac{b^5}{(bc-ad)(bf-ag)^4(a+bx)}-\frac{d^5}{(bc-ad)(-df+cg)^4(c+dx)}+\frac{}{(bf}\right.}{}$$

$$= -\frac{B(bc-ad)}{12(bf-ag)(df-cg)(f+gx)^3}-\frac{B(bc-ad)(2bdf-bcg-adg)}{8(bf-ag)^2(df-cg)^2(f+gx)^2}-\frac{B(bc-ad)\left(a^2d\right.}{}$$

**Mathematica [A]** time = 0.93, size = 355, normalized size = 0.94

$$\frac{B(bc-ad)\left(-\frac{g\left(a^2d^2g^2+abdg(cg-3df)+b^2\left(c^2g^2-3cdfg+3d^2f^2\right)\right)}{(f+gx)(bf-ag)^3(df-cg)^3}-\frac{g\log(f+gx)(adg+bcg-2bdf)\left(a^2d^2g^2-2abd^2fg+b^2\left(c^2g^2-2cdfg+2d^2f^2\right)\right)}{(bf-ag)^4(df-cg)^4}+\right.}{4g}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Log[(e*(a + b*x))/(c + d*x)])/(f + g*x)^5,x]
```

```
[Out] (-((A + B*Log[(e*(a + b*x))/(c + d*x)])/(f + g*x)^4) + B*(b*c - a*d)*(-1/3*
g/((b*f - a*g)*(d*f - c*g)*(f + g*x)^3) + (g*(-2*b*d*f + b*c*g + a*d*g))/(2
*(b*f - a*g)^2*(d*f - c*g)^2*(f + g*x)^2) - (g*(a^2*d^2*g^2 + a*b*d*g*(-3*d
```

```
*f + c*g) + b^2*(3*d^2*f^2 - 3*c*d*f*g + c^2*g^2)))/((b*f - a*g)^3*(d*f - c
*g)^3*(f + g*x)) + (b^4*Log[a + b*x])/((b*c - a*d)*(b*f - a*g)^4) - (d^4*Lo
g[c + d*x])/((b*c - a*d)*(d*f - c*g)^4) - (g*(-2*b*d*f + b*c*g + a*d*g)*(-2
*a*b*d^2*f*g + a^2*d^2*g^2 + b^2*(2*d^2*f^2 - 2*c*d*f*g + c^2*g^2))*Log[f +
 g*x])/((b*f - a*g)^4*(d*f - c*g)^4)))/(4*g)
```

**fricas [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*(b*x+a)/(d*x+c)))/(g*x+f)^5,x, algorithm="fricas")
```

```
[Out] Timed out
```

**giac [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*(b*x+a)/(d*x+c)))/(g*x+f)^5,x, algorithm="giac")
```

```
[Out] Timed out
```

**maple [B]** time = 0.39, size = 44893, normalized size = 118.45

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*ln((b*x+a)/(d*x+c)*e)+A)/(g*x+f)^5,x)
```

```
[Out] result too large to display
```

**maxima [B]** time = 1.99, size = 1757, normalized size = 4.64

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*(b*x+a)/(d*x+c)))/(g*x+f)^5,x, algorithm="maxima")
```

```
[Out] 1/24*(6*b^4*log(b*x + a)/(b^4*f^4*g - 4*a*b^3*f^3*g^2 + 6*a^2*b^2*f^2*g^3 -
 4*a^3*b*f*g^4 + a^4*g^5) - 6*d^4*log(d*x + c)/(d^4*f^4*g - 4*c*d^3*f^3*g^2
 + 6*c^2*d^2*f^2*g^3 - 4*c^3*d*f*g^4 + c^4*g^5) + 6*(4*(b^4*c*d^3 - a*b^3*d
^4)*f^3 - 6*(b^4*c^2*d^2 - a^2*b^2*d^4)*f^2*g + 4*(b^4*c^3*d - a^3*b*d^4)*f
```

```
*g^2 - (b^4*c^4 - a^4*d^4)*g^3)*log(g*x + f)/(b^4*d^4*f^8 + a^4*c^4*g^8 - 4
*(b^4*c*d^3 + a*b^3*d^4)*f^7*g + 2*(3*b^4*c^2*d^2 + 8*a*b^3*c*d^3 + 3*a^2*b
^2*d^4)*f^6*g^2 - 4*(b^4*c^3*d + 6*a*b^3*c^2*d^2 + 6*a^2*b^2*c*d^3 + a^3*b*
d^4)*f^5*g^3 + (b^4*c^4 + 16*a*b^3*c^3*d + 36*a^2*b^2*c^2*d^2 + 16*a^3*b*c*
d^3 + a^4*d^4)*f^4*g^4 - 4*(a*b^3*c^4 + 6*a^2*b^2*c^3*d + 6*a^3*b*c^2*d^2 +
 a^4*c*d^3)*f^3*g^5 + 2*(3*a^2*b^2*c^4 + 8*a^3*b*c^3*d + 3*a^4*c^2*d^2)*f^2
*g^6 - 4*(a^3*b*c^4 + a^4*c^3*d)*f*g^7) - (26*(b^3*c*d^2 - a*b^2*d^3)*f^4 -
 31*(b^3*c^2*d - a^2*b*d^3)*f^3*g + (11*b^3*c^3 + 15*a*b^2*c^2*d - 15*a^2*b
*c*d^2 - 11*a^3*d^3)*f^2*g^2 - 7*(a*b^2*c^3 - a^3*c*d^2)*f*g^3 + 2*(a^2*b*c
^3 - a^3*c^2*d)*g^4 + 6*(3*(b^3*c*d^2 - a*b^2*d^3)*f^2*g^2 - 3*(b^3*c^2*d -
 a^2*b*d^3)*f*g^3 + (b^3*c^3 - a^3*d^3)*g^4)*x^2 + 3*(14*(b^3*c*d^2 - a*b^2
*d^3)*f^3*g - 15*(b^3*c^2*d - a^2*b*d^3)*f^2*g^2 + (5*b^3*c^3 + 3*a*b^2*c^2
*d - 3*a^2*b*c*d^2 - 5*a^3*d^3)*f*g^3 - (a*b^2*c^3 - a^3*c*d^2)*g^4)*x)/(b^
3*d^3*f^9 + a^3*c^3*f^3*g^6 - 3*(b^3*c*d^2 + a*b^2*d^3)*f^8*g + 3*(b^3*c^2*
d + 3*a*b^2*c*d^2 + a^2*b*d^3)*f^7*g^2 - (b^3*c^3 + 9*a*b^2*c^2*d + 9*a^2*b
*c*d^2 + a^3*d^3)*f^6*g^3 + 3*(a*b^2*c^3 + 3*a^2*b*c^2*d + a^3*c*d^2)*f^5*g
^4 - 3*(a^2*b*c^3 + a^3*c^2*d)*f^4*g^5 + (b^3*d^3*f^6*g^3 + a^3*c^3*g^9 - 3
*(b^3*c*d^2 + a*b^2*d^3)*f^5*g^4 + 3*(b^3*c^2*d + 3*a*b^2*c*d^2 + a^2*b*d^3
)*f^4*g^5 - (b^3*c^3 + 9*a*b^2*c^2*d + 9*a^2*b*c*d^2 + a^3*d^3)*f^3*g^6 + 3
*(a*b^2*c^3 + 3*a^2*b*c^2*d + a^3*c*d^2)*f^2*g^7 - 3*(a^2*b*c^3 + a^3*c^2*d
)*f*g^8)*x^3 + 3*(b^3*d^3*f^7*g^2 + a^3*c^3*f*g^8 - 3*(b^3*c*d^2 + a*b^2*d^
3)*f^6*g^3 + 3*(b^3*c^2*d + 3*a*b^2*c*d^2 + a^2*b*d^3)*f^5*g^4 - (b^3*c^3 +
 9*a*b^2*c^2*d + 9*a^2*b*c*d^2 + a^3*d^3)*f^4*g^5 + 3*(a*b^2*c^3 + 3*a^2*b*
c^2*d + a^3*c*d^2)*f^3*g^6 - 3*(a^2*b*c^3 + a^3*c^2*d)*f^2*g^7)*x^2 + 3*(b^
3*d^3*f^8*g + a^3*c^3*f^2*g^7 - 3*(b^3*c*d^2 + a*b^2*d^3)*f^7*g^2 + 3*(b^3*
c^2*d + 3*a*b^2*c*d^2 + a^2*b*d^3)*f^6*g^3 - (b^3*c^3 + 9*a*b^2*c^2*d + 9*a
^2*b*c*d^2 + a^3*d^3)*f^5*g^4 + 3*(a*b^2*c^3 + 3*a^2*b*c^2*d + a^3*c*d^2)*f
^4*g^5 - 3*(a^2*b*c^3 + a^3*c^2*d)*f^3*g^6)*x) - 6*log(b*e*x/(d*x + c) + a*
e/(d*x + c))/(g^5*x^4 + 4*f*g^4*x^3 + 6*f^2*g^3*x^2 + 4*f^3*g^2*x + f^4*g))
*B - 1/4*A/(g^5*x^4 + 4*f*g^4*x^3 + 6*f^2*g^3*x^2 + 4*f^3*g^2*x + f^4*g)
```

**mupad [B]** time = 16.22, size = 2518, normalized size = 6.64

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*log((e*(a + b*x))/(c + d*x)))/(f + g*x)^5,x)
```

```
[Out] (log(f + g*x)*(g*(6*B*a^2*b^2*d^4*f^2 - 6*B*b^4*c^2*d^2*f^2) - g^2*(4*B*a^3
*b*d^4*f - 4*B*b^4*c^3*d*f) + g^3*(B*a^4*d^4 - B*b^4*c^4) - 4*B*a*b^3*d^4*f
^3 + 4*B*b^4*c*d^3*f^3))/(4*a^4*c^4*g^8 + 4*b^4*d^4*f^8 + 4*a^4*d^4*f^4*g^4
 + 4*b^4*c^4*f^4*g^4 + 24*a^2*b^2*c^4*f^2*g^6 + 24*a^2*b^2*d^4*f^6*g^2 + 24
*a^4*c^2*d^2*f^2*g^6 + 24*b^4*c^2*d^2*f^6*g^2 - 16*a^3*b*c^4*f*g^7 - 16*a*b
^3*d^4*f^7*g - 16*a^4*c^3*d*f*g^7 - 16*b^4*c*d^3*f^7*g - 16*a*b^3*c^4*f^3*g
^5 - 16*a^3*b*d^4*f^5*g^3 - 16*a^4*c*d^3*f^3*g^5 - 16*b^4*c^3*d*f^5*g^3 + 6
```

4*a*b^3*c*d^3*f^6*g^2 + 64*a*b^3*c^3*d*f^4*g^4 + 64*a^3*b*c*d^3*f^4*g^4 + 6
4*a^3*b*c^3*d*f^2*g^6 - 96*a*b^3*c^2*d^2*f^5*g^3 - 96*a^2*b^2*c*d^3*f^5*g^3
 - 96*a^2*b^2*c^3*d*f^3*g^5 - 96*a^3*b*c^2*d^2*f^3*g^5 + 144*a^2*b^2*c^2*d^
2*f^4*g^4) - ((6*A*a^3*c^3*g^6 + 6*A*b^3*d^3*f^6 - 6*A*a^3*d^3*f^3*g^3 - 6*
A*b^3*c^3*f^3*g^3 - 11*B*a^3*d^3*f^3*g^3 + 11*B*b^3*c^3*f^3*g^3 + 18*A*a*b^
2*c^3*f^2*g^4 + 18*A*a^2*b*d^3*f^4*g^2 - 7*B*a*b^2*c^3*f^2*g^4 + 18*A*a^3*c
*d^2*f^2*g^4 + 31*B*a^2*b*d^3*f^4*g^2 + 18*A*b^3*c^2*d*f^4*g^2 + 7*B*a^3*c*
d^2*f^2*g^4 - 31*B*b^3*c^2*d*f^4*g^2 - 18*A*a^2*b*c^3*f*g^5 - 18*A*a*b^2*d^
3*f^5*g + 2*B*a^2*b*c^3*f*g^5 - 18*A*a^3*c^2*d*f*g^5 - 26*B*a*b^2*d^3*f^5*g
 - 18*A*b^3*c*d^2*f^5*g - 2*B*a^3*c^2*d*f*g^5 + 26*B*b^3*c*d^2*f^5*g + 54*A
*a*b^2*c*d^2*f^4*g^2 - 54*A*a*b^2*c^2*d*f^3*g^3 - 54*A*a^2*b*c*d^2*f^3*g^3
+ 54*A*a^2*b*c^2*d*f^2*g^4 + 15*B*a*b^2*c^2*d*f^3*g^3 - 15*B*a^2*b*c*d^2*f^
3*g^3)/(6*(a^3*c^3*g^6 + b^3*d^3*f^6 - a^3*d^3*f^3*g^3 - b^3*c^3*f^3*g^3 -
3*a^2*b*c^3*f*g^5 - 3*a*b^2*d^3*f^5*g - 3*a^3*c^2*d*f*g^5 - 3*b^3*c*d^2*f^5
*g + 3*a*b^2*c^3*f^2*g^4 + 3*a^2*b*d^3*f^4*g^2 + 3*a^3*c*d^2*f^2*g^4 + 3*b^
3*c^2*d*f^4*g^2 + 9*a*b^2*c*d^2*f^4*g^2 - 9*a*b^2*c^2*d*f^3*g^3 - 9*a^2*b*c
*d^2*f^3*g^3 + 9*a^2*b*c^2*d*f^2*g^4)) - (x^2*(B*a*b^2*c^3*g^6 - B*a^3*c*d^
2*g^6 + 7*B*a^3*d^3*f*g^5 - 7*B*b^3*c^3*f*g^5 + 20*B*a*b^2*d^3*f^3*g^3 - 21
*B*a^2*b*d^3*f^2*g^4 - 20*B*b^3*c*d^2*f^3*g^3 + 21*B*b^3*c^2*d*f^2*g^4 - 3*
B*a*b^2*c^2*d*f*g^5 + 3*B*a^2*b*c*d^2*f*g^5))/(2*(a^3*c^3*g^6 + b^3*d^3*f^6
 - a^3*d^3*f^3*g^3 - b^3*c^3*f^3*g^3 - 3*a^2*b*c^3*f*g^5 - 3*a*b^2*d^3*f^5*
g - 3*a^3*c^2*d*f*g^5 - 3*b^3*c*d^2*f^5*g + 3*a*b^2*c^3*f^2*g^4 + 3*a^2*b*d
^3*f^4*g^2 + 3*a^3*c*d^2*f^2*g^4 + 3*b^3*c^2*d*f^4*g^2 + 9*a*b^2*c*d^2*f^4*
g^2 - 9*a*b^2*c^2*d*f^3*g^3 - 9*a^2*b*c*d^2*f^3*g^3 + 9*a^2*b*c^2*d*f^2*g^4
)) + (x*(B*a^2*b*c^3*g^6 - B*a^3*c^2*d*g^6 - 13*B*a^3*d^3*f^2*g^4 + 13*B*b^
3*c^3*f^2*g^4 - 34*B*a*b^2*d^3*f^4*g^2 + 38*B*a^2*b*d^3*f^3*g^3 + 34*B*b^3*
c*d^2*f^4*g^2 - 38*B*b^3*c^2*d*f^3*g^3 - 5*B*a*b^2*c^3*f*g^5 + 5*B*a^3*c*d^
2*f*g^5 + 12*B*a*b^2*c^2*d*f^2*g^4 - 12*B*a^2*b*c*d^2*f^2*g^4))/(3*(a^3*c^3
*g^6 + b^3*d^3*f^6 - a^3*d^3*f^3*g^3 - b^3*c^3*f^3*g^3 - 3*a^2*b*c^3*f*g^5
- 3*a*b^2*d^3*f^5*g - 3*a^3*c^2*d*f*g^5 - 3*b^3*c*d^2*f^5*g + 3*a*b^2*c^3*f
^2*g^4 + 3*a^2*b*d^3*f^4*g^2 + 3*a^3*c*d^2*f^2*g^4 + 3*b^3*c^2*d*f^4*g^2 +
9*a*b^2*c*d^2*f^4*g^2 - 9*a*b^2*c^2*d*f^3*g^3 - 9*a^2*b*c*d^2*f^3*g^3 + 9*a
^2*b*c^2*d*f^2*g^4)) - (x^3*(B*a^3*d^3*g^6 - B*b^3*c^3*g^6 + 3*B*a*b^2*d^3*
f^2*g^4 - 3*B*b^3*c*d^2*f^2*g^4 - 3*B*a^2*b*d^3*f*g^5 + 3*B*b^3*c^2*d*f*g^5
))/(a^3*c^3*g^6 + b^3*d^3*f^6 - a^3*d^3*f^3*g^3 - b^3*c^3*f^3*g^3 - 3*a^2*b
*c^3*f*g^5 - 3*a*b^2*d^3*f^5*g - 3*a^3*c^2*d*f*g^5 - 3*b^3*c*d^2*f^5*g + 3*
a*b^2*c^3*f^2*g^4 + 3*a^2*b*d^3*f^4*g^2 + 3*a^3*c*d^2*f^2*g^4 + 3*b^3*c^2*d
*f^4*g^2 + 9*a*b^2*c*d^2*f^4*g^2 - 9*a*b^2*c^2*d*f^3*g^3 - 9*a^2*b*c*d^2*f^
3*g^3 + 9*a^2*b*c^2*d*f^2*g^4))/(4*f^4*g + 4*g^5*x^4 + 16*f^3*g^2*x + 16*f*
g^4*x^3 + 24*f^2*g^3*x^2) - (B*log((e*(a + b*x))/(c + d*x)))/(4*g*(f^4 + g^
4*x^4 + 4*f^3*g*x + 4*f*g^3*x^3 + 6*f^2*g^2*x^2)) + (B*b^4*log(a + b*x))/(4
*a^4*g^5 + 4*b^4*f^4*g - 16*a*b^3*f^3*g^2 + 24*a^2*b^2*f^2*g^3 - 16*a^3*b*f
*g^4) - (B*d^4*log(c + d*x))/(4*c^4*g^5 + 4*d^4*f^4*g - 16*c*d^3*f^3*g^2 +
24*c^2*d^2*f^2*g^3 - 16*c^3*d*f*g^4)

**sympy [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*ln(e*(b*x+a)/(d*x+c)))/(g*x+f)**5,x)
```

```
[Out] Timed out
```

# 3.240 $\int (f+gx)^3 \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2 dx$

**Optimal**. Leaf size=874

$$\frac{B^2g^3\log\left(\frac{a+bx}{c+dx}\right)(bc-ad)^4}{6b^4d^4}+\frac{B^2g^3\log(c+dx)(bc-ad)^4}{6b^4d^4}+\frac{B^2g^3x(bc-ad)^3}{6b^3d^3}+\frac{B^2g^2(4bdf-3bcg-adg)\log\left(\frac{a+bx}{c+dx}\right)}{4b^4d^4}$$

```
[Out] 1/6*B^2*(-a*d+b*c)^3*g^3*x/b^3/d^3+1/4*B^2*(-a*d+b*c)^2*g^2*(-a*d*g-3*b*c*g
+4*b*d*f)*x/b^3/d^3+1/12*B^2*(-a*d+b*c)^2*g^3*(d*x+c)^2/b^2/d^4+1/6*B^2*(-a
*d+b*c)^4*g^3*ln((b*x+a)/(d*x+c))/b^4/d^4+1/4*B^2*(-a*d+b*c)^3*g^2*(-a*d*g-
3*b*c*g+4*b*d*f)*ln((b*x+a)/(d*x+c))/b^4/d^4-1/2*B*(-a*d+b*c)*g*(a^2*d^2*g^
2-2*a*b*d*g*(-c*g+2*d*f)+b^2*(3*c^2*g^2-8*c*d*f*g+6*d^2*f^2))*(b*x+a)*(A+B*
ln(e*(b*x+a)/(d*x+c)))/b^4/d^3-1/4*B*(-a*d+b*c)*g^2*(-a*d*g-3*b*c*g+4*b*d*f
)*(d*x+c)^2*(A+B*ln(e*(b*x+a)/(d*x+c)))/b^2/d^4-1/6*B*(-a*d+b*c)*g^3*(d*x+c
)^3*(A+B*ln(e*(b*x+a)/(d*x+c)))/b/d^4-1/2*B*(-a*d+b*c)*(-a*d*g-b*c*g+2*b*d*
f)*(2*a*b*d^2*f*g-a^2*d^2*g^2-b^2*(c^2*g^2-2*c*d*f*g+2*d^2*f^2))*ln((-a*d+b
*c)/b/(d*x+c))*(A+B*ln(e*(b*x+a)/(d*x+c)))/b^4/d^4-1/4*(-a*g+b*f)^4*(A+B*ln
(e*(b*x+a)/(d*x+c)))^2/b^4/g+1/4*(g*x+f)^4*(A+B*ln(e*(b*x+a)/(d*x+c)))^2/g+
1/6*B^2*(-a*d+b*c)^4*g^3*ln(d*x+c)/b^4/d^4+1/4*B^2*(-a*d+b*c)^3*g^2*(-a*d*g
-3*b*c*g+4*b*d*f)*ln(d*x+c)/b^4/d^4+1/2*B^2*(-a*d+b*c)^2*g*(a^2*d^2*g^2-2*a
*b*d*g*(-c*g+2*d*f)+b^2*(3*c^2*g^2-8*c*d*f*g+6*d^2*f^2))*ln(d*x+c)/b^4/d^4-
1/2*B^2*(-a*d+b*c)*(-a*d*g-b*c*g+2*b*d*f)*(2*a*b*d^2*f*g-a^2*d^2*g^2-b^2*(c
^2*g^2-2*c*d*f*g+2*d^2*f^2))*polylog(2,d*(b*x+a)/b/(d*x+c))/b^4/d^4
```

**Rubi** [A] time = 1.74, antiderivative size = 994, normalized size of antiderivative = 1.14, number of steps used = 33, number of rules used = 13, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.448, Rules used = {2525, 12, 2528, 2486, 31, 72, 2524, 2418, 2390, 2301, 2394, 2393, 2391}

$$\frac{B^2\log^2(a+bx)(bf-ag)^4}{4b^4g}-\frac{B\log(a+bx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)(bf-ag)^4}{2b^4g}-\frac{B^2\log(a+bx)\log\left(\frac{b(c+dx)}{bc-ad}\right)(bf-ag}{2b^4g}$$

Antiderivative was successfully verified.

```
[In] Int[(f + g*x)^3*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2,x]
```

```
[Out] -(B^2*(b*c - a*d)^2*(b*c + a*d)*g^3*x)/(6*b^3*d^3) + (B^2*(b*c - a*d)^2*g^2
*(4*b*d*f - b*c*g - a*d*g)*x)/(4*b^3*d^3) - (A*B*(b*c - a*d)*g*(a^2*d^2*g^2
 - a*b*d*g*(4*d*f - c*g) + b^2*(6*d^2*f^2 - 4*c*d*f*g + c^2*g^2))*x)/(2*b^3
*d^3) + (B^2*(b*c - a*d)^2*g^3*x^2)/(12*b^2*d^2) - (a^3*B^2*(b*c - a*d)*g^3
*Log[a + b*x])/(6*b^4*d) + (a^2*B^2*(b*c - a*d)*g^2*(4*b*d*f - b*c*g - a*d*
g)*Log[a + b*x])/(4*b^4*d^2) + (B^2*(b*f - a*g)^4*Log[a + b*x]^2)/(4*b^4*g)
```

```
 - (B^2*(b*c - a*d)*g*(a^2*d^2*g^2 - a*b*d*g*(4*d*f - c*g) + b^2*(6*d^2*f^2
 - 4*c*d*f*g + c^2*g^2))*(a + b*x)*Log[(e*(a + b*x))/(c + d*x)])/(2*b^4*d^3
) - (B*(b*c - a*d)*g^2*(4*b*d*f - b*c*g - a*d*g)*x^2*(A + B*Log[(e*(a + b*x
))/(c + d*x)]))/(4*b^2*d^2) - (B*(b*c - a*d)*g^3*x^3*(A + B*Log[(e*(a + b*x
))/(c + d*x)]))/(6*b*d) - (B*(b*f - a*g)^4*Log[a + b*x]*(A + B*Log[(e*(a +
b*x))/(c + d*x)]))/(2*b^4*g) + ((f + g*x)^4*(A + B*Log[(e*(a + b*x))/(c + d
*x)])^2)/(4*g) + (B^2*c^3*(b*c - a*d)*g^3*Log[c + d*x])/(6*b*d^4) - (B^2*c^
2*(b*c - a*d)*g^2*(4*b*d*f - b*c*g - a*d*g)*Log[c + d*x])/(4*b^2*d^4) + (B^
2*(b*c - a*d)^2*g*(a^2*d^2*g^2 - a*b*d*g*(4*d*f - c*g) + b^2*(6*d^2*f^2 - 4
*c*d*f*g + c^2*g^2))*Log[c + d*x])/(2*b^4*d^4) - (B^2*(d*f - c*g)^4*Log[-((
d*(a + b*x))/(b*c - a*d))]*Log[c + d*x])/(2*d^4*g) + (B*(d*f - c*g)^4*(A +
B*Log[(e*(a + b*x))/(c + d*x)])*Log[c + d*x])/(2*d^4*g) + (B^2*(d*f - c*g)^
4*Log[c + d*x]^2)/(4*d^4*g) - (B^2*(b*f - a*g)^4*Log[a + b*x]*Log[(b*(c + d
*x))/(b*c - a*d)])/(2*b^4*g) - (B^2*(b*f - a*g)^4*PolyLog[2, -((d*(a + b*x)
)/(b*c - a*d))])/(2*b^4*g) - (B^2*(d*f - c*g)^4*PolyLog[2, (b*(c + d*x))/(b
*c - a*d)])/(2*d^4*g)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 72

```
Int[((e_.) + (f_.)*(x_))^(p_.)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
 x_Symbol] :> Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x]
 /; FreeQ[{a, b, c, d, e, f}, x] && IntegerQ[p]
```

Rule 2301

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

Rule 2390

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_))^(q_.), x_Symbol] :> Dist[1/e, Subst[Int[((f*x)/d)^q*(a + b*Log[c*x^
n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && E
qQ[e*f - d*g, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2393

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + (c*e*x)/g])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2394

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d + e*x
)^n]))/g, x] - Dist[(b*e*n)/g, Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2418

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*(RFx_), x_Sy
mbol] :> With[{u = ExpandIntegrand[(a + b*Log[c*(d + e*x)^n])^p, RFx, x]},
Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d, e, n}, x] && RationalFunctionQ[
RFx, x] && IntegerQ[p]
```

Rule 2486

```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))
^(r_.)]^(s_.), x_Symbol] :> Simp[((a + b*x)*Log[e*(f*(a + b*x)^p*(c + d*x)^
q)^r]^s)/b, x] + Dist[(q*r*s*(b*c - a*d))/b, Int[Log[e*(f*(a + b*x)^p*(c +
d*x)^q)^r]^(s - 1)/(c + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, p, q, r, s
}, x] && NeQ[b*c - a*d, 0] && EqQ[p + q, 0] && IGtQ[s, 0]
```

Rule 2524

```
Int[((a_.) + Log[(c_.)*(RFx_)^(p_.)]*(b_.))^(n_.)/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[(Log[d + e*x]*(a + b*Log[c*RFx^p])^n)/e, x] - Dist[(b*n*p)/e
, Int[(Log[d + e*x]*(a + b*Log[c*RFx^p])^(n - 1)*D[RFx, x])/RFx, x], x] /;
FreeQ[{a, b, c, d, e, p}, x] && RationalFunctionQ[RFx, x] && IGtQ[n, 0]
```

Rule 2525

```
Int[((a_.) + Log[(c_.)*(RFx_)^(p_.)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_))^(m_.
), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(a + b*Log[c*RFx^p])^n)/(e*(m + 1))
```

```
, x] - Dist[(b*n*p)/(e*(m + 1)), Int[SimplifyIntegrand[((d + e*x)^(m + 1)*(
a + b*Log[c*RFx^p])^(n - 1)*D[RFx, x])/RFx, x], x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && RationalFunctionQ[RFx, x] && IGtQ[n, 0] && (EqQ[n, 1] ||
IntegerQ[m]) && NeQ[m, -1]
```

Rule 2528

```
Int[((a_.) + Log[(c_.)*(RFx_)^(p_.)]*(b_.))^(n_.)*(RGx_), x_Symbol] :> With
[{u = ExpandIntegrand[(a + b*Log[c*RFx^p])^n, RGx, x]}, Int[u, x] /; SumQ[u
]] /; FreeQ[{a, b, c, p}, x] && RationalFunctionQ[RFx, x] && RationalFuncti
onQ[RGx, x] && IGtQ[n, 0]
```

Rubi steps

$$\int (f+gx)^3 \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2 dx = \frac{(f+gx)^4\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{4g} - \frac{B\int \frac{(bc-ad)(f+gx)^4\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(a+bx)(c+dx)}}{2g}$$

$$= \frac{(f+gx)^4\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{4g} - \frac{(B(bc-ad))\int \frac{(f+gx)^4\left(A+B\log\left(\frac{e(a+}{c+}\right)\right)}{(a+bx)(c+dx)}}{2g}$$

$$= \frac{(f+gx)^4\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{4g} - \frac{(B(bc-ad))\int\left(\frac{g^2\left(a^2d^2g^2-abdg(4df-\right.}{}\right.}{}$$

$$= \frac{(f+gx)^4\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{4g} - \frac{\left(B(bc-ad)g^3\right)\int x^2\left(A+B\log\left(\right.\right.}{2bd}$$

$$= -\frac{AB(bc-ad)g\left(a^2d^2g^2-abdg(4df-cg)+b^2\left(6d^2f^2-4cdfg+c^2\right.\right.}{2b^3d^3}$$

$$= -\frac{AB(bc-ad)g\left(a^2d^2g^2-abdg(4df-cg)+b^2\left(6d^2f^2-4cdfg+c^2\right.\right.}{2b^3d^3}$$

$$= -\frac{AB(bc-ad)g\left(a^2d^2g^2-abdg(4df-cg)+b^2\left(6d^2f^2-4cdfg+c^2\right.\right.}{2b^3d^3}$$

$$= -\frac{B^2(bc-ad)^2(bc+ad)g^3x}{6b^3d^3} + \frac{B^2(bc-ad)^2g^2(4bdf-bcg-adg)x}{4b^3d^3} -$$

$$= -\frac{B^2(bc-ad)^2(bc+ad)g^3x}{6b^3d^3} + \frac{B^2(bc-ad)^2g^2(4bdf-bcg-adg)x}{4b^3d^3} -$$

$$= -\frac{B^2(bc-ad)^2(bc+ad)g^3x}{6b^3d^3} + \frac{B^2(bc-ad)^2g^2(4bdf-bcg-adg)x}{4b^3d^3} -$$

$$= -\frac{B^2(bc-ad)^2(bc+ad)g^3x}{6b^3d^3} + \frac{B^2(bc-ad)^2g^2(4bdf-bcg-adg)x}{4b^3d^3} -$$

**Mathematica [A]** time = 0.98, size = 733, normalized size = 0.84

$$\frac{(f+gx)^4\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2-\frac{B\left(Bg^4(bc-ad)\left(2a^3d^3\log(a+bx)+bdx(bc-ad)(2ad+2bc-bdx)-2b^3c^3\log(c+dx)\right)+6Abdg^2x(bc-ad)\left(a^2d^2g^2\cdots\right.}{\cdots}}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Integrate[(f + g*x)^3*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2,x]
```

```
[Out] ((f + g*x)^4*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2 - (B*(6*A*b*d*(b*c - a*
d)*g^2*(a^2*d^2*g^2 + a*b*d*g*(-4*d*f + c*g) + b^2*(6*d^2*f^2 - 4*c*d*f*g +
 c^2*g^2))*x + 6*B*d*(b*c - a*d)*g^2*(a^2*d^2*g^2 + a*b*d*g*(-4*d*f + c*g)
+ b^2*(6*d^2*f^2 - 4*c*d*f*g + c^2*g^2))*(a + b*x)*Log[(e*(a + b*x))/(c + d
*x)] + 3*b^2*d^2*(b*c - a*d)*g^3*(4*b*d*f - b*c*g - a*d*g)*x^2*(A + B*Log[(
e*(a + b*x))/(c + d*x)]) + 2*b^3*d^3*(b*c - a*d)*g^4*x^3*(A + B*Log[(e*(a +
 b*x))/(c + d*x)]) + 6*d^4*(b*f - a*g)^4*Log[a + b*x]*(A + B*Log[(e*(a + b*
x))/(c + d*x)]) - 6*B*(b*c - a*d)^2*g^2*(a^2*d^2*g^2 + a*b*d*g*(-4*d*f + c*
g) + b^2*(6*d^2*f^2 - 4*c*d*f*g + c^2*g^2))*Log[c + d*x] - 6*b^4*(d*f - c*g
)^4*(A + B*Log[(e*(a + b*x))/(c + d*x)])*Log[c + d*x] + B*(b*c - a*d)*g^4*(
b*d*(b*c - a*d)*x*(2*b*c + 2*a*d - b*d*x) + 2*a^3*d^3*Log[a + b*x] - 2*b^3*
c^3*Log[c + d*x]) - 3*B*(b*c - a*d)*g^3*(-4*b*d*f + b*c*g + a*d*g)*(-(a^2*d
^2*Log[a + b*x]) + b*(d*(-(b*c) + a*d)*x + b*c^2*Log[c + d*x])) - 3*B*d^4*(
b*f - a*g)^4*(Log[a + b*x]*(Log[a + b*x] - 2*Log[(b*(c + d*x))/(b*c - a*d)]
) - 2*PolyLog[2, (d*(a + b*x))/(-(b*c) + a*d)]) + 3*b^4*B*(d*f - c*g)^4*((2
*Log[(d*(a + b*x))/(-(b*c) + a*d)] - Log[c + d*x])*Log[c + d*x] + 2*PolyLog
[2, (b*(c + d*x))/(b*c - a*d)])))/(3*b^4*d^4))/(4*g)
```

**fricas [F]** time = 1.87, size = 0, normalized size = 0.00

$$\text{integral}\left(A^2g^3x^3+3A^2fg^2x^2+3A^2f^2gx+A^2f^3+\left(B^2g^3x^3+3B^2fg^2x^2+3B^2f^2gx+B^2f^3\right)\log\left(\frac{bex+ae}{dx+c}\right)^2+\cdots\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^3*(A+B*log(e*(b*x+a)/(d*x+c)))^2,x, algorithm="fricas")
```

```
[Out] integral(A^2*g^3*x^3 + 3*A^2*f*g^2*x^2 + 3*A^2*f^2*g*x + A^2*f^3 + (B^2*g^3
*x^3 + 3*B^2*f*g^2*x^2 + 3*B^2*f^2*g*x + B^2*f^3)*log((b*e*x + a*e)/(d*x +
c))^2 + 2*(A*B*g^3*x^3 + 3*A*B*f*g^2*x^2 + 3*A*B*f^2*g*x + A*B*f^3)*log((b*
e*x + a*e)/(d*x + c)), x)
```

**giac [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^3*(A+B*log(e*(b*x+a)/(d*x+c)))^2,x, algorithm="giac")
```

```
[Out] Timed out
```

**maple [F]** time = 2.58, size = 0, normalized size = 0.00

$$\int \left(gx+f\right)^{3} \left(B\ln\left(\frac{(bx+a)\,e}{dx+c}\right)+A\right)^{2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x+f)^3*(B*ln((b*x+a)/(d*x+c)*e)+A)^2,x)
```

```
[Out] int((g*x+f)^3*(B*ln((b*x+a)/(d*x+c)*e)+A)^2,x)
```

**maxima [B]** time = 1.91, size = 2140, normalized size = 2.45

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^3*(A+B*log(e*(b*x+a)/(d*x+c)))^2,x, algorithm="maxima")
```

```
[Out] 1/4*A^2*g^3*x^4 + A^2*f*g^2*x^3 + 3/2*A^2*f^2*g*x^2 + 2*(x*log(b*e*x/(d*x +
 c) + a*e/(d*x + c)) + a*log(b*x + a)/b - c*log(d*x + c)/d)*A*B*f^3 + 3*(x^
2*log(b*e*x/(d*x + c) + a*e/(d*x + c)) - a^2*log(b*x + a)/b^2 + c^2*log(d*x
 + c)/d^2 - (b*c - a*d)*x/(b*d))*A*B*f^2*g + (2*x^3*log(b*e*x/(d*x + c) + a
*e/(d*x + c)) + 2*a^3*log(b*x + a)/b^3 - 2*c^3*log(d*x + c)/d^3 - ((b^2*c*d
 - a*b*d^2)*x^2 - 2*(b^2*c^2 - a^2*d^2)*x)/(b^2*d^2))*A*B*f*g^2 + 1/12*(6*x
^4*log(b*e*x/(d*x + c) + a*e/(d*x + c)) - 6*a^4*log(b*x + a)/b^4 + 6*c^4*lo
g(d*x + c)/d^4 - (2*(b^3*c*d^2 - a*b^2*d^3)*x^3 - 3*(b^3*c^2*d - a^2*b*d^3)
*x^2 + 6*(b^3*c^3 - a^3*d^3)*x)/(b^3*d^3))*A*B*g^3 + A^2*f^3*x - 1/12*(6*a^
3*c*d^3*g^3 - 3*(8*c*d^3*f*g^2 - c^2*d^2*g^3)*a^2*b + 2*(18*c*d^3*f^2*g - 6
*c^2*d^2*f*g^2 + c^3*d*g^3)*a*b^2 + (24*c*d^3*f^3*log(e) - (6*g^3*log(e) +
11*g^3)*c^4 + 12*(2*f*g^2*log(e) + 3*f*g^2)*c^3*d - 36*(f^2*g*log(e) + f^2*
g)*c^2*d^2)*b^3)*B^2*log(d*x + c)/(b^3*d^4) + 1/2*(4*a*b^3*d^4*f^3 - 6*a^2*
b^2*d^4*f^2*g + 4*a^3*b*d^4*f*g^2 - a^4*d^4*g^3 - (4*c*d^3*f^3 - 6*c^2*d^2*
f^2*g + 4*c^3*d*f*g^2 - c^4*g^3)*b^4)*(log(b*x + a)*log((b*d*x + a*d)/(b*c
- a*d) + 1) + dilog(-(b*d*x + a*d)/(b*c - a*d)))*B^2/(b^4*d^4) + 1/12*(3*B^
2*b^4*d^4*g^3*x^4*log(e)^2 + 2*(a*b^3*d^4*g^3*log(e) + (6*d^4*f*g^2*log(e)^
2 - c*d^3*g^3*log(e))*b^4)*B^2*x^3 - ((3*g^3*log(e) - g^3)*a^2*b^2*d^4 - 2*
(6*d^4*f*g^2*log(e) - c*d^3*g^3)*a*b^3 - (18*d^4*f^2*g*log(e)^2 - 12*c*d^3*
f*g^2*log(e) + (3*g^3*log(e) + g^3)*c^2*d^2)*b^4)*B^2*x^2 + ((6*g^3*log(e)
- 5*g^3)*a^3*b*d^4 + (5*c*d^3*g^3 - 12*(2*f*g^2*log(e) - f*g^2)*d^4)*a^2*b^
2 + (36*d^4*f^2*g*log(e) - 24*c*d^3*f*g^2 + 5*c^2*d^2*g^3)*a*b^3 + (12*d^4*
```

```
f^3*log(e)^2 - 36*c*d^3*f^2*g*log(e) - (6*g^3*log(e) + 5*g^3)*c^3*d + 12*(2
*f*g^2*log(e) + f*g^2)*c^2*d^2)*b^4)*B^2*x + 3*(B^2*b^4*d^4*g^3*x^4 + 4*B^2
*b^4*d^4*f*g^2*x^3 + 6*B^2*b^4*d^4*f^2*g*x^2 + 4*B^2*b^4*d^4*f^3*x + (4*a*b
^3*d^4*f^3 - 6*a^2*b^2*d^4*f^2*g + 4*a^3*b*d^4*f*g^2 - a^4*d^4*g^3)*B^2)*lo
g(b*x + a)^2 + 3*(B^2*b^4*d^4*g^3*x^4 + 4*B^2*b^4*d^4*f*g^2*x^3 + 6*B^2*b^4
*d^4*f^2*g*x^2 + 4*B^2*b^4*d^4*f^3*x + (4*c*d^3*f^3 - 6*c^2*d^2*f^2*g + 4*c
^3*d*f*g^2 - c^4*g^3)*B^2*b^4)*log(d*x + c)^2 + (6*B^2*b^4*d^4*g^3*x^4*log(
e) + 2*(a*b^3*d^4*g^3 + (12*d^4*f*g^2*log(e) - c*d^3*g^3)*b^4)*B^2*x^3 + 3*
(4*a*b^3*d^4*f*g^2 - a^2*b^2*d^4*g^3 + (12*d^4*f^2*g*log(e) - 4*c*d^3*f*g^2
 + c^2*d^2*g^3)*b^4)*B^2*x^2 + 6*(6*a*b^3*d^4*f^2*g - 4*a^2*b^2*d^4*f*g^2 +
 a^3*b*d^4*g^3 + (4*d^4*f^3*log(e) - 6*c*d^3*f^2*g + 4*c^2*d^2*f*g^2 - c^3*
d*g^3)*b^4)*B^2*x - ((6*g^3*log(e) - 11*g^3)*a^4*d^4 + 2*(c*d^3*g^3 - 6*(2*
f*g^2*log(e) - 3*f*g^2)*d^4)*a^3*b - 3*(4*c*d^3*f*g^2 - c^2*d^2*g^3 - 12*(f
^2*g*log(e) - f^2*g)*d^4)*a^2*b^2 - 6*(4*d^4*f^3*log(e) - 6*c*d^3*f^2*g + 4
*c^2*d^2*f*g^2 - c^3*d*g^3)*a*b^3)*B^2)*log(b*x + a) - (6*B^2*b^4*d^4*g^3*x
^4*log(e) + 2*(a*b^3*d^4*g^3 + (12*d^4*f*g^2*log(e) - c*d^3*g^3)*b^4)*B^2*x
^3 + 3*(4*a*b^3*d^4*f*g^2 - a^2*b^2*d^4*g^3 + (12*d^4*f^2*g*log(e) - 4*c*d^
3*f*g^2 + c^2*d^2*g^3)*b^4)*B^2*x^2 + 6*(6*a*b^3*d^4*f^2*g - 4*a^2*b^2*d^4*
f*g^2 + a^3*b*d^4*g^3 + (4*d^4*f^3*log(e) - 6*c*d^3*f^2*g + 4*c^2*d^2*f*g^2
 - c^3*d*g^3)*b^4)*B^2*x + 6*(B^2*b^4*d^4*g^3*x^4 + 4*B^2*b^4*d^4*f*g^2*x^3
 + 6*B^2*b^4*d^4*f^2*g*x^2 + 4*B^2*b^4*d^4*f^3*x + (4*a*b^3*d^4*f^3 - 6*a^2
*b^2*d^4*f^2*g + 4*a^3*b*d^4*f*g^2 - a^4*d^4*g^3)*B^2)*log(b*x + a))*log(d*
x + c))/(b^4*d^4)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.00

$$\int \left(f+g\,x\right)^3\,{\left(A+B\,\ln\left(\frac{e\,\left(a+b\,x\right)}{c+d\,x}\right)\right)}^2 \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f + g*x)^3*(A + B*log((e*(a + b*x))/(c + d*x)))^2,x)
```

```
[Out] int((f + g*x)^3*(A + B*log((e*(a + b*x))/(c + d*x)))^2, x)
```

**sympy [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)**3*(A+B*ln(e*(b*x+a)/(d*x+c)))**2,x)
```

```
[Out] Timed out
```

# 3.241 $\int (f+gx)^2 \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2 dx$

**Optimal**. Leaf size=532

$$\frac{2B(bc-ad)\left(a^2d^2g^2-abdg(3df-cg)+b^2\left(c^2g^2-3cdfg+3d^2f^2\right)\right)\log\left(\frac{bc-ad}{b(c+dx)}\right)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{3b^3d^3}+\frac{2B^2(bc$$

```
[Out] 1/3*B^2*(-a*d+b*c)^2*g^2*x/b^2/d^2+1/3*B^2*(-a*d+b*c)^3*g^2*ln((b*x+a)/(d*x
+c))/b^3/d^3-2/3*B*(-a*d+b*c)*g*(-a*d*g-2*b*c*g+3*b*d*f)*(b*x+a)*(A+B*ln(e*
(b*x+a)/(d*x+c)))/b^3/d^2-1/3*B*(-a*d+b*c)*g^2*(d*x+c)^2*(A+B*ln(e*(b*x+a)/
(d*x+c)))/b/d^3+2/3*B*(-a*d+b*c)*(a^2*d^2*g^2-a*b*d*g*(-c*g+3*d*f)+b^2*(c^2
*g^2-3*c*d*f*g+3*d^2*f^2))*ln((-a*d+b*c)/b/(d*x+c))*(A+B*ln(e*(b*x+a)/(d*x+
c)))/b^3/d^3-1/3*(-a*g+b*f)^3*(A+B*ln(e*(b*x+a)/(d*x+c)))^2/b^3/g+1/3*(g*x+
f)^3*(A+B*ln(e*(b*x+a)/(d*x+c)))^2/g+1/3*B^2*(-a*d+b*c)^3*g^2*ln(d*x+c)/b^3
/d^3+2/3*B^2*(-a*d+b*c)^2*g*(-a*d*g-2*b*c*g+3*b*d*f)*ln(d*x+c)/b^3/d^3+2/3*
B^2*(-a*d+b*c)*(a^2*d^2*g^2-a*b*d*g*(-c*g+3*d*f)+b^2*(c^2*g^2-3*c*d*f*g+3*d
^2*f^2))*polylog(2,d*(b*x+a)/b/(d*x+c))/b^3/d^3
```

**Rubi [A]** time = 1.09, antiderivative size = 649, normalized size of antiderivative = 1.22, number of steps used = 29, number of rules used = 13, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.448$, Rules used = {2525, 12, 2528, 2486, 31, 72, 2524, 2418, 2390, 2301, 2394, 2393, 2391}

$$-\frac{2B^2(bf-ag)^3\text{PolyLog}\left(2,-\frac{d(a+bx)}{bc-ad}\right)}{3b^3g}-\frac{2B^2(df-cg)^3\text{PolyLog}\left(2,\frac{b(c+dx)}{bc-ad}\right)}{3d^3g}+\frac{a^2B^2g^2(bc-ad)\log(a+bx)}{3b^3d}-\frac{2ABg$$

Antiderivative was successfully verified.

```
[In] Int[(f + g*x)^2*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2,x]
```

```
[Out] (B^2*(b*c - a*d)^2*g^2*x)/(3*b^2*d^2) - (2*A*B*(b*c - a*d)*g*(3*b*d*f - b*c
*g - a*d*g)*x)/(3*b^2*d^2) + (a^2*B^2*(b*c - a*d)*g^2*Log[a + b*x])/(3*b^3*
d) + (B^2*(b*f - a*g)^3*Log[a + b*x]^2)/(3*b^3*g) - (2*B^2*(b*c - a*d)*g*(3
*b*d*f - b*c*g - a*d*g)*(a + b*x)*Log[(e*(a + b*x))/(c + d*x)])/(3*b^3*d^2)
 - (B*(b*c - a*d)*g^2*x^2*(A + B*Log[(e*(a + b*x))/(c + d*x)]))/(3*b*d) - (
2*B*(b*f - a*g)^3*Log[a + b*x]*(A + B*Log[(e*(a + b*x))/(c + d*x)]))/(3*b^3
*g) + ((f + g*x)^3*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2)/(3*g) - (B^2*c^2
*(b*c - a*d)*g^2*Log[c + d*x])/(3*b*d^3) + (2*B^2*(b*c - a*d)^2*g*(3*b*d*f
- b*c*g - a*d*g)*Log[c + d*x])/(3*b^3*d^3) - (2*B^2*(d*f - c*g)^3*Log[-((d*
(a + b*x))/(b*c - a*d))]*Log[c + d*x])/(3*d^3*g) + (2*B*(d*f - c*g)^3*(A +
B*Log[(e*(a + b*x))/(c + d*x)])*Log[c + d*x])/(3*d^3*g) + (B^2*(d*f - c*g)^
3*Log[c + d*x]^2)/(3*d^3*g) - (2*B^2*(b*f - a*g)^3*Log[a + b*x]*Log[(b*(c +
```

```
 d*x))/(b*c - a*d)])/(3*b^3*g) - (2*B^2*(b*f - a*g)^3*PolyLog[2, -((d*(a +
b*x))/(b*c - a*d))])/(3*b^3*g) - (2*B^2*(d*f - c*g)^3*PolyLog[2, (b*(c + d*
x))/(b*c - a*d)])/(3*d^3*g)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 72

```
Int[((e_.) + (f_.)*(x_))^(p_.)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
 x_Symbol] :> Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x]
 /; FreeQ[{a, b, c, d, e, f}, x] && IntegerQ[p]
```

Rule 2301

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

Rule 2390

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_))^(q_.), x_Symbol] :> Dist[1/e, Subst[Int[((f*x)/d)^q*(a + b*Log[c*x^
n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && E
qQ[e*f - d*g, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2393

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + (c*e*x)/g])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2394

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d + e*x
)^n]))/g, x] - Dist[(b*e*n)/g, Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2418

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*(RFx_), x_Sy
mbol] :> With[{u = ExpandIntegrand[(a + b*Log[c*(d + e*x)^n])^p, RFx, x]},
Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d, e, n}, x] && RationalFunctionQ[
RFx, x] && IntegerQ[p]
```

Rule 2486

```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))
^(r_.)]^(s_.), x_Symbol] :> Simp[((a + b*x)*Log[e*(f*(a + b*x)^p*(c + d*x)^
q)^r]^s)/b, x] + Dist[(q*r*s*(b*c - a*d))/b, Int[Log[e*(f*(a + b*x)^p*(c +
d*x)^q)^r]^(s - 1)/(c + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, p, q, r, s
}, x] && NeQ[b*c - a*d, 0] && EqQ[p + q, 0] && IGtQ[s, 0]
```

Rule 2524

```
Int[((a_.) + Log[(c_.)*(RFx_)^(p_.)]*(b_.))^(n_.)/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[(Log[d + e*x]*(a + b*Log[c*RFx^p])^n)/e, x] - Dist[(b*n*p)/e
, Int[(Log[d + e*x]*(a + b*Log[c*RFx^p])^(n - 1)*D[RFx, x])/RFx, x], x] /;
FreeQ[{a, b, c, d, e, p}, x] && RationalFunctionQ[RFx, x] && IGtQ[n, 0]
```

Rule 2525

```
Int[((a_.) + Log[(c_.)*(RFx_)^(p_.)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_))^(m_.
), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(a + b*Log[c*RFx^p])^n)/(e*(m + 1))
, x] - Dist[(b*n*p)/(e*(m + 1)), Int[SimplifyIntegrand[((d + e*x)^(m + 1)*(
a + b*Log[c*RFx^p])^(n - 1)*D[RFx, x])/RFx, x], x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && RationalFunctionQ[RFx, x] && IGtQ[n, 0] && (EqQ[n, 1] ||
IntegerQ[m]) && NeQ[m, -1]
```

Rule 2528

```
Int[((a_.) + Log[(c_.)*(RFx_)^(p_.)]*(b_.))^(n_.)*(RGx_), x_Symbol] :> With
[{u = ExpandIntegrand[(a + b*Log[c*RFx^p])^n, RGx, x]}, Int[u, x] /; SumQ[u
]] /; FreeQ[{a, b, c, p}, x] && RationalFunctionQ[RFx, x] && RationalFuncti
onQ[RGx, x] && IGtQ[n, 0]
```

Rubi steps

$$\int (f+gx)^2 \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2 dx = \frac{(f+gx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{3g} - \frac{(2B)\int \frac{(bc-ad)(f+gx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(a+bx)(c+dx)}}{3g}$$

$$= \frac{(f+gx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{3g} - \frac{(2B(bc-ad))\int \frac{(f+gx)^3\left(A+B\log\left(\frac{e(a+}{c+d}\right.\right.}{(a+bx)(c+dx)}}{3g}$$

$$= \frac{(f+gx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{3g} - \frac{(2B(bc-ad))\int \left(\frac{g^2(3bdf-bcg-adg)\left(A\right.}{b^2d^2}\right.}{}$$

$$= \frac{(f+gx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{3g} - \frac{\left(2B(bc-ad)g^2\right)\int x\left(A+B\log\left(\frac{e}{}\right.\right.}{3bd}$$

$$= -\frac{2AB(bc-ad)g(3bdf-bcg-adg)x}{3b^2d^2} - \frac{B(bc-ad)g^2x^2\left(A+B\log\left(\frac{e}{}\right.\right.}{3bd}$$

$$= -\frac{2AB(bc-ad)g(3bdf-bcg-adg)x}{3b^2d^2} - \frac{2B^2(bc-ad)g(3bdf-bcg-}{3b^3d}$$

$$= -\frac{2AB(bc-ad)g(3bdf-bcg-adg)x}{3b^2d^2} - \frac{2B^2(bc-ad)g(3bdf-bcg-}{3b^3d}$$

$$= \frac{B^2(bc-ad)^2g^2x}{3b^2d^2} - \frac{2AB(bc-ad)g(3bdf-bcg-adg)x}{3b^2d^2} + \frac{a^2B^2(bc-a}{}$$

$$= \frac{B^2(bc-ad)^2g^2x}{3b^2d^2} - \frac{2AB(bc-ad)g(3bdf-bcg-adg)x}{3b^2d^2} + \frac{a^2B^2(bc-a}{}$$

$$= \frac{B^2(bc-ad)^2g^2x}{3b^2d^2} - \frac{2AB(bc-ad)g(3bdf-bcg-adg)x}{3b^2d^2} + \frac{a^2B^2(bc-a}{}$$

$$= \frac{B^2(bc-ad)^2g^2x}{3b^2d^2} - \frac{2AB(bc-ad)g(3bdf-bcg-adg)x}{3b^2d^2} + \frac{a^2B^2(bc-a}{}$$

**Mathematica [A]** time = 0.53, size = 486, normalized size = 0.91

$$\frac{(f+gx)^3\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2-\frac{B\left(-Bg^3(bc-ad)\left(a^2d^2\log(a+bx)-b\left(dx(ad-bc)+bc^2\log(c+dx)\right)\right)-2b^3(df-cg)^3\log(c+dx)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)\right.\right.}{}}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(f + g*x)^2*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2,x]
```

```
[Out] ((f + g*x)^3*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2 - (B*(2*A*b*d*(b*c - a*
d)*g^2*(3*b*d*f - b*c*g - a*d*g)*x + 2*B*d*(b*c - a*d)*g^2*(3*b*d*f - b*c*g
 - a*d*g)*(a + b*x)*Log[(e*(a + b*x))/(c + d*x)] + b^2*d^2*(b*c - a*d)*g^3*
x^2*(A + B*Log[(e*(a + b*x))/(c + d*x)]) + 2*d^3*(b*f - a*g)^3*Log[a + b*x]
*(A + B*Log[(e*(a + b*x))/(c + d*x)]) + 2*B*(b*c - a*d)^2*g^2*(-3*b*d*f + b
*c*g + a*d*g)*Log[c + d*x] - 2*b^3*(d*f - c*g)^3*(A + B*Log[(e*(a + b*x))/(
c + d*x)])*Log[c + d*x] - B*(b*c - a*d)*g^3*(a^2*d^2*Log[a + b*x] - b*(d*(-
(b*c) + a*d)*x + b*c^2*Log[c + d*x])) - B*d^3*(b*f - a*g)^3*(Log[a + b*x]*(
Log[a + b*x] - 2*Log[(b*(c + d*x))/(b*c - a*d)]) - 2*PolyLog[2, (d*(a + b*x
))/(-(b*c) + a*d)]) + b^3*B*(d*f - c*g)^3*((2*Log[(d*(a + b*x))/(-(b*c) + a
*d)] - Log[c + d*x])*Log[c + d*x] + 2*PolyLog[2, (b*(c + d*x))/(b*c - a*d)]
)))/(b^3*d^3))/(3*g)
```

**fricas [F]** time = 0.83, size = 0, normalized size = 0.00

$$\text{integral}\left(A^2g^2x^2+2\,A^2fgx+A^2f^2+\left(B^2g^2x^2+2\,B^2fgx+B^2f^2\right)\log\left(\frac{bex+ae}{dx+c}\right)^2+2\left(ABg^2x^2+2\,ABfgx+\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^2*(A+B*log(e*(b*x+a)/(d*x+c)))^2,x, algorithm="fricas")
```

```
[Out] integral(A^2*g^2*x^2 + 2*A^2*f*g*x + A^2*f^2 + (B^2*g^2*x^2 + 2*B^2*f*g*x +
 B^2*f^2)*log((b*e*x + a*e)/(d*x + c))^2 + 2*(A*B*g^2*x^2 + 2*A*B*f*g*x + A
*B*f^2)*log((b*e*x + a*e)/(d*x + c)), x)
```

**giac [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^2*(A+B*log(e*(b*x+a)/(d*x+c)))^2,x, algorithm="giac")
```

```
[Out] Timed out
```

**maple [F]** time = 2.10, size = 0, normalized size = 0.00

$$\int \left(gx+f\right)^{2} \left(B\ln\left(\frac{(bx+a)\,e}{dx+c}\right)+A\right)^{2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x+f)^2*(B*ln((b*x+a)/(d*x+c)*e)+A)^2,x)
```

```
[Out] int((g*x+f)^2*(B*ln((b*x+a)/(d*x+c)*e)+A)^2,x)
```

**maxima [B]** time = 2.04, size = 1300, normalized size = 2.44

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^2*(A+B*log(e*(b*x+a)/(d*x+c)))^2,x, algorithm="maxima")
```

```
[Out] 1/3*A^2*g^2*x^3 + A^2*f*g*x^2 + 2*(x*log(b*e*x/(d*x + c) + a*e/(d*x + c)) +
 a*log(b*x + a)/b - c*log(d*x + c)/d)*A*B*f^2 + 2*(x^2*log(b*e*x/(d*x + c)
+ a*e/(d*x + c)) - a^2*log(b*x + a)/b^2 + c^2*log(d*x + c)/d^2 - (b*c - a*d
)*x/(b*d))*A*B*f*g + 1/3*(2*x^3*log(b*e*x/(d*x + c) + a*e/(d*x + c)) + 2*a^
3*log(b*x + a)/b^3 - 2*c^3*log(d*x + c)/d^3 - ((b^2*c*d - a*b*d^2)*x^2 - 2*
(b^2*c^2 - a^2*d^2)*x)/(b^2*d^2))*A*B*g^2 + A^2*f^2*x + 1/3*(2*a^2*c*d^2*g^
2 - (6*c*d^2*f*g - c^2*d*g^2)*a*b - (6*c*d^2*f^2*log(e) + (2*g^2*log(e) + 3
*g^2)*c^3 - 6*(f*g*log(e) + f*g)*c^2*d)*b^2)*B^2*log(d*x + c)/(b^2*d^3) + 2
/3*(3*a*b^2*d^3*f^2 - 3*a^2*b*d^3*f*g + a^3*d^3*g^2 - (3*c*d^2*f^2 - 3*c^2*
d*f*g + c^3*g^2)*b^3)*(log(b*x + a)*log((b*d*x + a*d)/(b*c - a*d) + 1) + di
log(-(b*d*x + a*d)/(b*c - a*d)))*B^2/(b^3*d^3) + 1/3*(B^2*b^3*d^3*g^2*x^3*l
og(e)^2 + (a*b^2*d^3*g^2*log(e) + (3*d^3*f*g*log(e)^2 - c*d^2*g^2*log(e))*b
^3)*B^2*x^2 - ((2*g^2*log(e) - g^2)*a^2*b*d^3 - 2*(3*d^3*f*g*log(e) - c*d^2
*g^2)*a*b^2 - (3*d^3*f^2*log(e)^2 - 6*c*d^2*f*g*log(e) + (2*g^2*log(e) + g^
2)*c^2*d)*b^3)*B^2*x + (B^2*b^3*d^3*g^2*x^3 + 3*B^2*b^3*d^3*f*g*x^2 + 3*B^2
*b^3*d^3*f^2*x + (3*a*b^2*d^3*f^2 - 3*a^2*b*d^3*f*g + a^3*d^3*g^2)*B^2)*log
(b*x + a)^2 + (B^2*b^3*d^3*g^2*x^3 + 3*B^2*b^3*d^3*f*g*x^2 + 3*B^2*b^3*d^3*
f^2*x + (3*c*d^2*f^2 - 3*c^2*d*f*g + c^3*g^2)*B^2*b^3)*log(d*x + c)^2 + (2*
B^2*b^3*d^3*g^2*x^3*log(e) + (a*b^2*d^3*g^2 + (6*d^3*f*g*log(e) - c*d^2*g^2
)*b^3)*B^2*x^2 + 2*(3*a*b^2*d^3*f*g - a^2*b*d^3*g^2 + (3*d^3*f^2*log(e) - 3
*c*d^2*f*g + c^2*d*g^2)*b^3)*B^2*x + ((2*g^2*log(e) - 3*g^2)*a^3*d^3 + (c*d
^2*g^2 - 6*(f*g*log(e) - f*g)*d^3)*a^2*b + 2*(3*d^3*f^2*log(e) - 3*c*d^2*f*
g + c^2*d*g^2)*a*b^2)*B^2)*log(b*x + a) - (2*B^2*b^3*d^3*g^2*x^3*log(e) + (
a*b^2*d^3*g^2 + (6*d^3*f*g*log(e) - c*d^2*g^2)*b^3)*B^2*x^2 + 2*(3*a*b^2*d^
3*f*g - a^2*b*d^3*g^2 + (3*d^3*f^2*log(e) - 3*c*d^2*f*g + c^2*d*g^2)*b^3)*B
^2*x + 2*(B^2*b^3*d^3*g^2*x^3 + 3*B^2*b^3*d^3*f*g*x^2 + 3*B^2*b^3*d^3*f^2*x
```

```
 + (3*a*b^2*d^3*f^2 - 3*a^2*b*d^3*f*g + a^3*d^3*g^2)*B^2)*log(b*x + a))*log
(d*x + c))/(b^3*d^3)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.00

$$\int (f + g\,x)^2\,\left(A + B\,\ln\left(\frac{e\,(a + b\,x)}{c + d\,x}\right)\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f + g*x)^2*(A + B*log((e*(a + b*x))/(c + d*x)))^2,x)
[Out] int((f + g*x)^2*(A + B*log((e*(a + b*x))/(c + d*x)))^2, x)
```

**sympy [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)**2*(A+B*ln(e*(b*x+a)/(d*x+c)))**2,x)
[Out] Timed out
```

# 3.242 $\int (f+gx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2\,dx$

**Optimal**. Leaf size=270

$$\frac{B(bc-ad)(-adg-bcg+2bdf)\log\left(\frac{bc-ad}{b(c+dx)}\right)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{b^2d^2}-\frac{(bf-ag)^2\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2}{2b^2g}-\frac{Bg(a+bx)}{}$$

```
[Out] -B*(-a*d+b*c)*g*(b*x+a)*(A+B*ln(e*(b*x+a)/(d*x+c)))/b^2/d+B*(-a*d+b*c)*(-a*
d*g-b*c*g+2*b*d*f)*ln((-a*d+b*c)/b/(d*x+c))*(A+B*ln(e*(b*x+a)/(d*x+c)))/b^2
/d^2-1/2*(-a*g+b*f)^2*(A+B*ln(e*(b*x+a)/(d*x+c)))^2/b^2/g+1/2*(g*x+f)^2*(A+
B*ln(e*(b*x+a)/(d*x+c)))^2/g+B^2*(-a*d+b*c)^2*g*ln(d*x+c)/b^2/d^2+B^2*(-a*d
+b*c)*(-a*d*g-b*c*g+2*b*d*f)*polylog(2,d*(b*x+a)/b/(d*x+c))/b^2/d^2
```

**Rubi [A]** time = 0.82, antiderivative size = 444, normalized size of antiderivative = 1.64, number of steps used = 25, number of rules used = 12, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.444, Rules used = {2525, 12, 2528, 2486, 31, 2524, 2418, 2390, 2301, 2394, 2393, 2391}

$$-\frac{B^2(bf-ag)^2\text{PolyLog}\left(2,-\frac{d(a+bx)}{bc-ad}\right)}{b^2g}-\frac{B^2(df-cg)^2\text{PolyLog}\left(2,\frac{b(c+dx)}{bc-ad}\right)}{d^2g}-\frac{B(bf-ag)^2\log(a+bx)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)\right.}{b^2g}$$

Antiderivative was successfully verified.

```
[In] Int[(f + g*x)*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2,x]
```

```
[Out] -((A*B*(b*c - a*d)*g*x)/(b*d)) + (B^2*(b*f - a*g)^2*Log[a + b*x]^2)/(2*b^2*
g) - (B^2*(b*c - a*d)*g*(a + b*x)*Log[(e*(a + b*x))/(c + d*x)])/(b^2*d) - (
B*(b*f - a*g)^2*Log[a + b*x]*(A + B*Log[(e*(a + b*x))/(c + d*x)]))/(b^2*g)
+ ((f + g*x)^2*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2)/(2*g) + (B^2*(b*c -
a*d)^2*g*Log[c + d*x])/(b^2*d^2) - (B^2*(d*f - c*g)^2*Log[-((d*(a + b*x))/(
b*c - a*d))]*Log[c + d*x])/(d^2*g) + (B*(d*f - c*g)^2*(A + B*Log[(e*(a + b*
x))/(c + d*x)])*Log[c + d*x])/(d^2*g) + (B^2*(d*f - c*g)^2*Log[c + d*x]^2)/
(2*d^2*g) - (B^2*(b*f - a*g)^2*Log[a + b*x]*Log[(b*(c + d*x))/(b*c - a*d)])
/(b^2*g) - (B^2*(b*f - a*g)^2*PolyLog[2, -((d*(a + b*x))/(b*c - a*d))])/(b^
2*g) - (B^2*(d*f - c*g)^2*PolyLog[2, (b*(c + d*x))/(b*c - a*d)])/(d^2*g)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 2301

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

Rule 2390

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_))^(q_.), x_Symbol] :> Dist[1/e, Subst[Int[((f*x)/d)^q*(a + b*Log[c*x^
n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && E
qQ[e*f - d*g, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2393

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + (c*e*x)/g])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2394

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d + e*x
)^n]))/g, x] - Dist[(b*e*n)/g, Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2418

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*(RFx_), x_Sy
mbol] :> With[{u = ExpandIntegrand[(a + b*Log[c*(d + e*x)^n])^p, RFx, x]},
Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d, e, n}, x] && RationalFunctionQ[
RFx, x] && IntegerQ[p]
```

Rule 2486

```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))
^(r_.)]^(s_.), x_Symbol] :> Simp[((a + b*x)*Log[e*(f*(a + b*x)^p*(c + d*x)^
```

```
q)^r]^s)/b, x] + Dist[(q*r*s*(b*c - a*d))/b, Int[Log[e*(f*(a + b*x)^p*(c +
d*x)^q)^r]^(s - 1)/(c + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, p, q, r, s
}, x] && NeQ[b*c - a*d, 0] && EqQ[p + q, 0] && IGtQ[s, 0]
```

Rule 2524

```
Int[((a_.) + Log[(c_.)*(RFx_)^(p_.)]*(b_.))^(n_.)/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[(Log[d + e*x]*(a + b*Log[c*RFx^p])^n)/e, x] - Dist[(b*n*p)/e
, Int[(Log[d + e*x]*(a + b*Log[c*RFx^p])^(n - 1)*D[RFx, x])/RFx, x], x] /;
FreeQ[{a, b, c, d, e, p}, x] && RationalFunctionQ[RFx, x] && IGtQ[n, 0]
```

Rule 2525

```
Int[((a_.) + Log[(c_.)*(RFx_)^(p_.)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_))^(m_.
), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(a + b*Log[c*RFx^p])^n)/(e*(m + 1))
, x] - Dist[(b*n*p)/(e*(m + 1)), Int[SimplifyIntegrand[((d + e*x)^(m + 1)*(
a + b*Log[c*RFx^p])^(n - 1)*D[RFx, x])/RFx, x], x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && RationalFunctionQ[RFx, x] && IGtQ[n, 0] && (EqQ[n, 1] ||
IntegerQ[m]) && NeQ[m, -1]
```

Rule 2528

```
Int[((a_.) + Log[(c_.)*(RFx_)^(p_.)]*(b_.))^(n_.)*(RGx_), x_Symbol] :> With
[{u = ExpandIntegrand[(a + b*Log[c*RFx^p])^n, RGx, x]}, Int[u, x] /; SumQ[u
]] /; FreeQ[{a, b, c, p}, x] && RationalFunctionQ[RFx, x] && RationalFuncti
onQ[RGx, x] && IGtQ[n, 0]
```

Rubi steps

$$\int (f+gx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2 dx = \frac{(f+gx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{2g} - \frac{B\int \frac{(bc-ad)(f+gx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(a+bx)(c+dx)}\,d\ldots}{g}$$

$$= \frac{(f+gx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{2g} - \frac{(B(bc-ad))\int \frac{(f+gx)^2\left(A+B\log\left(\frac{e(a+b\ldots}{c+dx\ldots}\right)\right)}{(a+bx)(c+dx)}}{g}$$

$$= \frac{(f+gx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{2g} - \frac{(B(bc-ad))\int\left(\frac{g^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{bd}\ldots\right)}{\ldots}$$

$$= \frac{(f+gx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{2g} - \frac{(B(bc-ad)g)\int\left(A+B\log\left(\frac{e(a+b\ldots}{c+d\ldots}\right)\right)\ldots}{bd}$$

$$= -\frac{AB(bc-ad)gx}{bd} - \frac{B(bf-ag)^2\log(a+bx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{b^2g} + \ldots$$

$$= -\frac{AB(bc-ad)gx}{bd} - \frac{B^2(bc-ad)g(a+bx)\log\left(\frac{e(a+bx)}{c+dx}\right)}{b^2d} - \frac{B(bf-ag)^2\ldots}{\ldots}$$

$$= -\frac{AB(bc-ad)gx}{bd} - \frac{B^2(bc-ad)g(a+bx)\log\left(\frac{e(a+bx)}{c+dx}\right)}{b^2d} - \frac{B(bf-ag)^2\ldots}{\ldots}$$

$$= -\frac{AB(bc-ad)gx}{bd} - \frac{B^2(bc-ad)g(a+bx)\log\left(\frac{e(a+bx)}{c+dx}\right)}{b^2d} - \frac{B(bf-ag)^2\ldots}{\ldots}$$

$$= -\frac{AB(bc-ad)gx}{bd} - \frac{B^2(bc-ad)g(a+bx)\log\left(\frac{e(a+bx)}{c+dx}\right)}{b^2d} - \frac{B(bf-ag)^2\ldots}{\ldots}$$

$$= -\frac{AB(bc-ad)gx}{bd} + \frac{B^2(bf-ag)^2\log^2(a+bx)}{2b^2g} - \frac{B^2(bc-ad)g(a+bx\ldots}{b^2d}$$

$$= -\frac{AB(bc-ad)gx}{bd} + \frac{B^2(bf-ag)^2\log^2(a+bx)}{2b^2g} - \frac{B^2(bc-ad)g(a+bx\ldots}{b^2d}$$

**Mathematica** [A] time = 0.35, size = 346, normalized size = 1.28

$$(f+gx)^2\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2-\frac{B\left(-2b^2(df-cg)^2\log(c+dx)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)+2d^2(bf-ag)^2\log(a+bx)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)+2Abdg^2x(b\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(f + g*x)*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2,x]
```

```
[Out] ((f + g*x)^2*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2 - (B*(2*A*b*d*(b*c - a*
d)*g^2*x + 2*B*d*(b*c - a*d)*g^2*(a + b*x)*Log[(e*(a + b*x))/(c + d*x)] + 2
*d^2*(b*f - a*g)^2*Log[a + b*x]*(A + B*Log[(e*(a + b*x))/(c + d*x)]) - 2*B*
(b*c - a*d)^2*g^2*Log[c + d*x] - 2*b^2*(d*f - c*g)^2*(A + B*Log[(e*(a + b*x
))/(c + d*x)])*Log[c + d*x] - B*d^2*(b*f - a*g)^2*(Log[a + b*x]*(Log[a + b*
x] - 2*Log[(b*(c + d*x))/(b*c - a*d)]) - 2*PolyLog[2, (d*(a + b*x))/(-(b*c)
 + a*d)]) + b^2*B*(d*f - c*g)^2*((2*Log[(d*(a + b*x))/(-(b*c) + a*d)] - Log
[c + d*x])*Log[c + d*x] + 2*PolyLog[2, (b*(c + d*x))/(b*c - a*d)])))/(b^2*d
^2))/(2*g)
```

**fricas** [F] time = 1.12, size = 0, normalized size = 0.00

$$\text{integral}\left(A^2gx+A^2f+\left(B^2gx+B^2f\right)\log\left(\frac{bex+ae}{dx+c}\right)^2+2\left(ABgx+ABf\right)\log\left(\frac{bex+ae}{dx+c}\right),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)*(A+B*log(e*(b*x+a)/(d*x+c)))^2,x, algorithm="fricas")
```

```
[Out] integral(A^2*g*x + A^2*f + (B^2*g*x + B^2*f)*log((b*e*x + a*e)/(d*x + c))^2
 + 2*(A*B*g*x + A*B*f)*log((b*e*x + a*e)/(d*x + c)), x)
```

**giac** [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)*(A+B*log(e*(b*x+a)/(d*x+c)))^2,x, algorithm="giac")
```

```
[Out] Timed out
```

**maple** [F] time = 1.67, size = 0, normalized size = 0.00

$$\int\left(gx+f\right)\left(B\ln\left(\frac{(bx+a)e}{dx+c}\right)+A\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((g*x+f)*(B*ln((b*x+a)/(d*x+c)*e)+A)^2,x)

[Out] int((g*x+f)*(B*ln((b*x+a)/(d*x+c)*e)+A)^2,x)

**maxima [B]** time = 1.92, size = 673, normalized size = 2.49

$$\frac{1}{2}A^2gx^2+2\left(x\log\left(\frac{bex}{dx+c}+\frac{ae}{dx+c}\right)+\frac{a\log(bx+a)}{b}-\frac{c\log(dx+c)}{d}\right)ABf+\left(x^2\log\left(\frac{bex}{dx+c}+\frac{ae}{dx+c}\right)-\frac{a^2\log}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((g*x+f)*(A+B*log(e*(b*x+a)/(d*x+c)))^2,x, algorithm="maxima")

```
[Out] 1/2*A^2*g*x^2 + 2*(x*log(b*e*x/(d*x + c) + a*e/(d*x + c)) + a*log(b*x + a)/
b - c*log(d*x + c)/d)*A*B*f + (x^2*log(b*e*x/(d*x + c) + a*e/(d*x + c)) - a
^2*log(b*x + a)/b^2 + c^2*log(d*x + c)/d^2 - (b*c - a*d)*x/(b*d))*A*B*g + A
^2*f*x - (a*c*d*g + (2*c*d*f*log(e) - (g*log(e) + g)*c^2)*b)*B^2*log(d*x +
c)/(b*d^2) + (2*a*b*d^2*f - a^2*d^2*g - (2*c*d*f - c^2*g)*b^2)*(log(b*x + a
)*log((b*d*x + a*d)/(b*c - a*d) + 1) + dilog(-(b*d*x + a*d)/(b*c - a*d)))*B
^2/(b^2*d^2) + 1/2*(B^2*b^2*d^2*g*x^2*log(e)^2 + 2*(a*b*d^2*g*log(e) + (d^2
*f*log(e)^2 - c*d*g*log(e))*b^2)*B^2*x + (B^2*b^2*d^2*g*x^2 + 2*B^2*b^2*d^2
*f*x + (2*a*b*d^2*f - a^2*d^2*g)*B^2)*log(b*x + a)^2 + (B^2*b^2*d^2*g*x^2 +
 2*B^2*b^2*d^2*f*x + (2*c*d*f - c^2*g)*B^2*b^2)*log(d*x + c)^2 + 2*(B^2*b^2
*d^2*g*x^2*log(e) + (a*b*d^2*g + (2*d^2*f*log(e) - c*d*g)*b^2)*B^2*x - ((g*
log(e) - g)*a^2*d^2 - (2*d^2*f*log(e) - c*d*g)*a*b)*B^2)*log(b*x + a) - 2*(
B^2*b^2*d^2*g*x^2*log(e) + (a*b*d^2*g + (2*d^2*f*log(e) - c*d*g)*b^2)*B^2*x
 + (B^2*b^2*d^2*g*x^2 + 2*B^2*b^2*d^2*f*x + (2*a*b*d^2*f - a^2*d^2*g)*B^2)*
log(b*x + a))*log(d*x + c))/(b^2*d^2)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.00

$$\int (f+g\,x)\,\left(A+B\,\ln\left(\frac{e\,(a+b\,x)}{c+d\,x}\right)\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((f + g*x)*(A + B*log((e*(a + b*x))/(c + d*x)))^2,x)

[Out] int((f + g*x)*(A + B*log((e*(a + b*x))/(c + d*x)))^2, x)

**sympy [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)*(A+B*ln(e*(b*x+a)/(d*x+c)))**2,x)
[Out] Timed out
```

# 3.243 $\int \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2\,dx$

**Optimal**. Leaf size=125

$$\frac{2B(bc-ad)\log\left(\frac{bc-ad}{b(c+dx)}\right)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{bd}+\frac{(a+bx)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2}{b}+\frac{2B^2(bc-ad)\text{Li}_2\left(\frac{d(a+bx)}{b(c+dx)}\right)}{bd}$$

```
[Out] 2*B*(-a*d+b*c)*ln((-a*d+b*c)/b/(d*x+c))*(A+B*ln(e*(b*x+a)/(d*x+c)))/b/d+(b*
x+a)*(A+B*ln(e*(b*x+a)/(d*x+c)))^2/b+2*B^2*(-a*d+b*c)*polylog(2,d*(b*x+a)/b
/(d*x+c))/b/d
```

**Rubi [A]** time = 0.64, antiderivative size = 246, normalized size of antiderivative = 1.97, number of steps used = 22, number of rules used = 10, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.476, Rules used = {2523, 12, 2528, 2524, 2418, 2390, 2301, 2394, 2393, 2391}

$$\frac{2aB^2\text{PolyLog}\left(2,-\frac{d(a+bx)}{bc-ad}\right)}{b}+\frac{2B^2c\text{PolyLog}\left(2,\frac{b(c+dx)}{bc-ad}\right)}{d}+\frac{2aB\log(a+bx)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{b}-\frac{2Bc\log(c+dx)}{}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Log[(e*(a + b*x))/(c + d*x)])^2,x]
```

```
[Out] -((a*B^2*Log[a + b*x]^2)/b) + (2*a*B*Log[a + b*x]*(A + B*Log[(e*(a + b*x))/
(c + d*x)]))/b + x*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2 + (2*B^2*c*Log[-(
(d*(a + b*x))/(b*c - a*d))]*Log[c + d*x])/d - (2*B*c*(A + B*Log[(e*(a + b*x
))/(c + d*x)])*Log[c + d*x])/d - (B^2*c*Log[c + d*x]^2)/d + (2*a*B^2*Log[a
+ b*x]*Log[(b*(c + d*x))/(b*c - a*d)])/b + (2*a*B^2*PolyLog[2, -((d*(a + b*
x))/(b*c - a*d))])/b + (2*B^2*c*PolyLog[2, (b*(c + d*x))/(b*c - a*d)])/d
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2301

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

Rule 2390

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_))^(q_.), x_Symbol] :> Dist[1/e, Subst[Int[((f*x)/d)^q*(a + b*Log[c*x^
```

```
n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && E
qQ[e*f - d*g, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2393

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + (c*e*x)/g])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2394

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d + e*x
)^n]))/g, x] - Dist[(b*e*n)/g, Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2418

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*(RFx_), x_Sy
mbol] :> With[{u = ExpandIntegrand[(a + b*Log[c*(d + e*x)^n])^p, RFx, x]},
Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d, e, n}, x] && RationalFunctionQ[
RFx, x] && IntegerQ[p]
```

Rule 2523

```
Int[((a_.) + Log[(c_.)*(RFx_)^(p_.)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a +
 b*Log[c*RFx^p])^n, x] - Dist[b*n*p, Int[SimplifyIntegrand[(x*(a + b*Log[c*
RFx^p])^(n - 1)*D[RFx, x])/RFx, x], x], x] /; FreeQ[{a, b, c, p}, x] && Rat
ionalFunctionQ[RFx, x] && IGtQ[n, 0]
```

Rule 2524

```
Int[((a_.) + Log[(c_.)*(RFx_)^(p_.)]*(b_.))^(n_.)/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[(Log[d + e*x]*(a + b*Log[c*RFx^p])^n)/e, x] - Dist[(b*n*p)/e
, Int[(Log[d + e*x]*(a + b*Log[c*RFx^p])^(n - 1)*D[RFx, x])/RFx, x], x] /;
FreeQ[{a, b, c, d, e, p}, x] && RationalFunctionQ[RFx, x] && IGtQ[n, 0]
```

Rule 2528

```
Int[((a_.) + Log[(c_.)*(RFx_)^(p_.)]*(b_.))^(n_.)*(RGx_), x_Symbol] :> With
[{u = ExpandIntegrand[(a + b*Log[c*RFx^p])^n, RGx, x]}, Int[u, x] /; SumQ[u
]] /; FreeQ[{a, b, c, p}, x] && RationalFunctionQ[RFx, x] && RationalFuncti
onQ[RGx, x] && IGtQ[n, 0]
```

Rubi steps

$$\int \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2 dx = x\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2-(2B)\int \frac{(bc-ad)x\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(a+bx)(c+dx)}dx$$

$$= x\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2-(2B(bc-ad))\int \frac{x\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(a+bx)(c+dx)}dx$$

$$= x\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2-(2B(bc-ad))\int \left(-\frac{a\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(bc-ad)(a+bx)}+\frac{c\left(A\right.}{}\right.$$

$$= x\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2+(2aB)\int \frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{a+bx}dx-(2Bc)\int \frac{A+B}{}$$

$$= \frac{2aB\log(a+bx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{b}+x\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2-\frac{2Bc\left(A\right.}{}$$

$$= \frac{2aB\log(a+bx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{b}+x\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2-\frac{2Bc\left(A\right.}{}$$

$$= \frac{2aB\log(a+bx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{b}+x\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2-\frac{2Bc\left(A\right.}{}$$

$$= \frac{2aB\log(a+bx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{b}+x\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2-\frac{2Bc\left(A\right.}{}$$

$$= \frac{2aB\log(a+bx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{b}+x\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2+\frac{2B^2c\,\mathrm{lo}}{}$$

$$= -\frac{aB^2\log^2(a+bx)}{b}+\frac{2aB\log(a+bx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{b}+x\left(A+B\log\left(\frac{e(\right.}{}\right.$$

$$= -\frac{aB^2\log^2(a+bx)}{b}+\frac{2aB\log(a+bx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{b}+x\left(A+B\log\left(\frac{e(\right.}{}\right.$$

**Mathematica [A]** time = 0.21, size = 214, normalized size = 1.71

$$\frac{B\left(2ad\log(a+bx)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)-2bc\log(c+dx)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)-aBd\left(\log(a+bx)\left(\log(a+bx)\right.\right.\right.}{bd}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Log[(e*(a + b*x))/(c + d*x)])^2,x]
```

```
[Out] x*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2 + (B*(2*a*d*Log[a + b*x]*(A + B*Lo
g[(e*(a + b*x))/(c + d*x)]) - 2*b*c*(A + B*Log[(e*(a + b*x))/(c + d*x)])*Lo
g[c + d*x] - a*B*d*(Log[a + b*x]*(Log[a + b*x] - 2*Log[(b*(c + d*x))/(b*c -
 a*d)]) - 2*PolyLog[2, (d*(a + b*x))/(-(b*c) + a*d)]) + b*B*c*((2*Log[(d*(a
 + b*x))/(-(b*c) + a*d)] - Log[c + d*x])*Log[c + d*x] + 2*PolyLog[2, (b*(c
+ d*x))/(b*c - a*d)])))/(b*d)
```

**fricas [F]** time = 1.17, size = 0, normalized size = 0.00

$$\text{integral}\left(B^2\log\left(\frac{bex+ae}{dx+c}\right)^2+2\,AB\log\left(\frac{bex+ae}{dx+c}\right)+A^2,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*(b*x+a)/(d*x+c)))^2,x, algorithm="fricas")
```

```
[Out] integral(B^2*log((b*e*x + a*e)/(d*x + c))^2 + 2*A*B*log((b*e*x + a*e)/(d*x
+ c)) + A^2, x)
```

**giac [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*(b*x+a)/(d*x+c)))^2,x, algorithm="giac")
```

```
[Out] Timed out
```

**maple [F]** time = 1.52, size = 0, normalized size = 0.00

$$\int\left(B\ln\left(\frac{(bx+a)\,e}{dx+c}\right)+A\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*ln((b*x+a)/(d*x+c)*e)+A)^2,x)
```

```
[Out] int((B*ln((b*x+a)/(d*x+c)*e)+A)^2,x)
```

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$2\left(x\log\left(\frac{(bx+a)e}{dx+c}\right)+\frac{\frac{ae\log(bx+a)}{b}-\frac{ce\log(dx+c)}{d}}{e}\right)AB+A^2x+B^2\left(\frac{bdx\log(bx+a)^2+(bdx+bc)\log(dx+c)^2-2\,(b\cdots}{bd\cdots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*(b*x+a)/(d*x+c)))^2,x, algorithm="maxima")
```

```
[Out] 2*(x*log((b*x + a)*e/(d*x + c)) + (a*e*log(b*x + a)/b - c*e*log(d*x + c)/d)
/e)*A*B + A^2*x + B^2*((b*d*x*log(b*x + a)^2 + (b*d*x + b*c)*log(d*x + c)^2
 - 2*(b*d*x*log(e) + (b*d*x + a*d)*log(b*x + a))*log(d*x + c))/(b*d) + inte
grate(((log(e)^2 + 2*log(e))*b^2*d*x^2 + a*b*c*log(e)^2 + (b^2*c*log(e)^2 +
 (log(e)^2 + 2*log(e))*a*b*d)*x + 2*(b^2*d*x^2*log(e) + a*b*c*log(e) + a^2*
d + (a*b*d*(log(e) + 2) + b^2*c*(log(e) - 1))*x)*log(b*x + a))/(b^2*d*x^2 +
 a*b*c + (b^2*c + a*b*d)*x), x))
```

**mupad** **[F]** time = 0.00, size = -1, normalized size = -0.01

$$\int \left(A + B\,\ln\left(\frac{e\,\left(a + b\,x\right)}{c + d\,x}\right)\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*log((e*(a + b*x))/(c + d*x)))^2,x)
```

```
[Out] int((A + B*log((e*(a + b*x))/(c + d*x)))^2, x)
```

**sympy** **[F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*ln(e*(b*x+a)/(d*x+c)))**2,x)
```

```
[Out] Timed out
```

# 3.244 $\int \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{f+gx}\,dx$

**Optimal**. Leaf size=277

$$\frac{2B\mathrm{Li}_2\left(\frac{(df-cg)(a+bx)}{(bf-ag)(c+dx)}\right)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{g}+\frac{\log\left(1-\frac{(a+bx)(df-cg)}{(c+dx)(bf-ag)}\right)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2}{g}-\frac{2B\mathrm{Li}_2\left(\frac{d(a+bx)}{b(c+dx)}\right)\left(B\log\right.}{g}$$

```
[Out] -ln((-a*d+b*c)/b/(d*x+c))*(A+B*ln(e*(b*x+a)/(d*x+c)))^2/g+(A+B*ln(e*(b*x+a)
/(d*x+c)))^2*ln(1-(-c*g+d*f)*(b*x+a)/(-a*g+b*f)/(d*x+c))/g-2*B*(A+B*ln(e*(b
*x+a)/(d*x+c)))*polylog(2,d*(b*x+a)/b/(d*x+c))/g+2*B*(A+B*ln(e*(b*x+a)/(d*x
+c)))*polylog(2,(-c*g+d*f)*(b*x+a)/(-a*g+b*f)/(d*x+c))/g+2*B^2*polylog(3,d*
(b*x+a)/b/(d*x+c))/g-2*B^2*polylog(3,(-c*g+d*f)*(b*x+a)/(-a*g+b*f)/(d*x+c))
/g
```

**Rubi [B]** time = 4.90, antiderivative size = 1998, normalized size of antiderivative = 7.21, number of steps used = 41, number of rules used = 21, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.724, Rules used = {2524, 12, 2528, 2418, 2390, 2301, 2394, 2393, 2391, 6688, 6742, 2500, 2433, 2375, 2317, 2374, 6589, 2440, 2437, 2435, 2315}

result too large to display

Antiderivative was successfully verified.

```
[In] Int[(A + B*Log[(e*(a + b*x))/(c + d*x)])^2/(f + g*x),x]
```

```
[Out] -((B^2*Log[a + b*x]^2*Log[f + g*x])/g) - (2*A*B*Log[-((g*(a + b*x))/(b*f -
a*g))]*Log[f + g*x])/g - (B^2*Log[(c + d*x)^(-1)]^2*Log[f + g*x])/g + (2*B^
2*Log[-((g*(a + b*x))/(b*f - a*g))]*(Log[a + b*x] + Log[(c + d*x)^(-1)] - L
og[(e*(a + b*x))/(c + d*x)])*Log[f + g*x])/g + ((A + B*Log[(e*(a + b*x))/(c
 + d*x)])^2*Log[f + g*x])/g + (2*B^2*Log[-((d*(a + b*x))/(b*c - a*d))]*Log[
c + d*x]*Log[f + g*x])/g - (2*B^2*Log[-((g*(a + b*x))/(b*f - a*g))]*(Log[(c
 + d*x)^(-1)] + Log[c + d*x])*Log[f + g*x])/g + (2*B^2*Log[a + b*x]*Log[(b*
(c + d*x))/(b*c - a*d)]*Log[f + g*x])/g + (2*A*B*Log[-((g*(c + d*x))/(d*f -
 c*g))]*Log[f + g*x])/g - (2*B^2*(Log[a + b*x] + Log[(c + d*x)^(-1)] - Log[
(e*(a + b*x))/(c + d*x)])*Log[-((g*(c + d*x))/(d*f - c*g))]*Log[f + g*x])/g
 + (B^2*Log[a + b*x]^2*Log[(b*(f + g*x))/(b*f - a*g)])/g + (B^2*Log[(c + d*
x)^(-1)]^2*Log[(d*(f + g*x))/(d*f - c*g)])/g + (B^2*(Log[(b*(c + d*x))/(b*c
 - a*d)] + Log[(b*f - a*g)/(b*(f + g*x))] - Log[((b*f - a*g)*(c + d*x))/((b
*c - a*d)*(f + g*x))])*Log[-(((b*c - a*d)*(f + g*x))/((d*f - c*g)*(a + b*x)
))]^2)/g - (B^2*(Log[(b*(c + d*x))/(b*c - a*d)] - Log[-((g*(c + d*x))/(d*f
- c*g))])*(Log[a + b*x] + Log[-(((b*c - a*d)*(f + g*x))/((d*f - c*g)*(a + b
*x)))])^2)/g + (B^2*(Log[-((d*(a + b*x))/(b*c - a*d))] + Log[(d*f - c*g)/(d
```

```
*(f + g*x))] - Log[-(((d*f - c*g)*(a + b*x))/((b*c - a*d)*(f + g*x)))])*Log
[((b*c - a*d)*(f + g*x))/((b*f - a*g)*(c + d*x))]^2)/g - (B^2*(Log[-((d*(a
+ b*x))/(b*c - a*d))] - Log[-((g*(a + b*x))/(b*f - a*g))])*(Log[c + d*x] +
Log[((b*c - a*d)*(f + g*x))/((b*f - a*g)*(c + d*x))])^2)/g + (2*B^2*(Log[f
+ g*x] - Log[-(((b*c - a*d)*(f + g*x))/((d*f - c*g)*(a + b*x)))])*PolyLog[2
, -((d*(a + b*x))/(b*c - a*d))])/g + (2*B^2*Log[a + b*x]*PolyLog[2, -((g*(a
 + b*x))/(b*f - a*g))])/g + (2*B^2*(Log[f + g*x] - Log[((b*c - a*d)*(f + g*
x))/((b*f - a*g)*(c + d*x))])*PolyLog[2, (b*(c + d*x))/(b*c - a*d)])/g - (2
*B^2*Log[(c + d*x)^(-1)]*PolyLog[2, -((g*(c + d*x))/(d*f - c*g))])/g - (2*B
^2*Log[-(((b*c - a*d)*(f + g*x))/((d*f - c*g)*(a + b*x)))]*PolyLog[2, (g*(a
 + b*x))/(b*(f + g*x))])/g + (2*B^2*Log[-(((b*c - a*d)*(f + g*x))/((d*f - c
*g)*(a + b*x)))]*PolyLog[2, -(((d*f - c*g)*(a + b*x))/((b*c - a*d)*(f + g*x
)))])/g - (2*B^2*Log[((b*c - a*d)*(f + g*x))/((b*f - a*g)*(c + d*x))]*PolyL
og[2, (g*(c + d*x))/(d*(f + g*x))])/g + (2*B^2*Log[((b*c - a*d)*(f + g*x))/
((b*f - a*g)*(c + d*x))]*PolyLog[2, ((b*f - a*g)*(c + d*x))/((b*c - a*d)*(f
 + g*x))])/g - (2*A*B*PolyLog[2, (b*(f + g*x))/(b*f - a*g)])/g + (2*B^2*(Lo
g[a + b*x] + Log[(c + d*x)^(-1)] - Log[(e*(a + b*x))/(c + d*x)])*PolyLog[2,
 (b*(f + g*x))/(b*f - a*g)])/g - (2*B^2*(Log[(c + d*x)^(-1)] + Log[c + d*x]
)*PolyLog[2, (b*(f + g*x))/(b*f - a*g)])/g + (2*B^2*(Log[c + d*x] + Log[((b
*c - a*d)*(f + g*x))/((b*f - a*g)*(c + d*x))])*PolyLog[2, (b*(f + g*x))/(b*
f - a*g)])/g + (2*A*B*PolyLog[2, (d*(f + g*x))/(d*f - c*g)])/g - (2*B^2*(Lo
g[a + b*x] + Log[(c + d*x)^(-1)] - Log[(e*(a + b*x))/(c + d*x)])*PolyLog[2,
 (d*(f + g*x))/(d*f - c*g)])/g + (2*B^2*(Log[a + b*x] + Log[-(((b*c - a*d)*
(f + g*x))/((d*f - c*g)*(a + b*x)))])*PolyLog[2, (d*(f + g*x))/(d*f - c*g)]
)/g - (2*B^2*PolyLog[3, -((d*(a + b*x))/(b*c - a*d))])/g - (2*B^2*PolyLog[3
, -((g*(a + b*x))/(b*f - a*g))])/g - (2*B^2*PolyLog[3, (b*(c + d*x))/(b*c -
 a*d)])/g - (2*B^2*PolyLog[3, -((g*(c + d*x))/(d*f - c*g))])/g - (2*B^2*Pol
yLog[3, (g*(a + b*x))/(b*(f + g*x))])/g + (2*B^2*PolyLog[3, -(((d*f - c*g)*
(a + b*x))/((b*c - a*d)*(f + g*x)))])/g - (2*B^2*PolyLog[3, (g*(c + d*x))/(
d*(f + g*x))])/g + (2*B^2*PolyLog[3, ((b*f - a*g)*(c + d*x))/((b*c - a*d)*(
f + g*x))])/g - (2*B^2*PolyLog[3, (b*(f + g*x))/(b*f - a*g)])/g - (2*B^2*Po
lyLog[3, (d*(f + g*x))/(d*f - c*g)])/g
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2301

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
```

```
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2317

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symb
ol] :> Simp[(Log[1 + (e*x)/d]*(a + b*Log[c*x^n])^p)/e, x] - Dist[(b*n*p)/e,
 Int[(Log[1 + (e*x)/d]*(a + b*Log[c*x^n])^(p - 1))/x, x], x] /; FreeQ[{a, b
, c, d, e, n}, x] && IGtQ[p, 0]
```

Rule 2374

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> -Simp[(PolyLog[2, -(d*f*x^m)]*(a + b*Log[c*x
^n])^p)/m, x] + Dist[(b*n*p)/m, Int[(PolyLog[2, -(d*f*x^m)]*(a + b*Log[c*x^
n])^(p - 1))/x, x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p, 0]
&& EqQ[d*e, 1]
```

Rule 2375

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))^(r_.)]*((a_.) + Log[(c_.)*(x_)^(n_
.)]*(b_.))^(p_.))/(x_), x_Symbol] :> Simp[(Log[d*(e + f*x^m)^r]*(a + b*Log[
c*x^n])^(p + 1))/(b*n*(p + 1)), x] - Dist[(f*m*r)/(b*n*(p + 1)), Int[(x^(m
- 1)*(a + b*Log[c*x^n])^(p + 1))/(e + f*x^m), x], x] /; FreeQ[{a, b, c, d,
e, f, r, m, n}, x] && IGtQ[p, 0] && NeQ[d*e, 1]
```

Rule 2390

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_))^(q_.), x_Symbol] :> Dist[1/e, Subst[Int[((f*x)/d)^q*(a + b*Log[c*x^
n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && E
qQ[e*f - d*g, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2393

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + (c*e*x)/g])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2394

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d + e*x
)^n]))/g, x] - Dist[(b*e*n)/g, Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2418

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*(RFx_), x_Sy
mbol] :> With[{u = ExpandIntegrand[(a + b*Log[c*(d + e*x)^n])^p, RFx, x]},
Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d, e, n}, x] && RationalFunctionQ[
RFx, x] && IntegerQ[p]
```

Rule 2433

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + Log
[(h_.)*((i_.) + (j_.)*(x_))^(m_.)]*(g_.))*((k_.) + (l_.)*(x_))^(r_.), x_Sym
bol] :> Dist[1/e, Subst[Int[((k*x)/d)^r*(a + b*Log[c*x^n])^p*(f + g*Log[h*(
(e*i - d*j)/e + (j*x)/e)^m]), x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e,
f, g, h, i, j, k, l, n, p, r}, x] && EqQ[e*k - d*l, 0]
```

Rule 2435

```
Int[(Log[(a_) + (b_.)*(x_)]*Log[(c_) + (d_.)*(x_)])/(x_), x_Symbol] :> Simp
[Log[-((b*x)/a)]*Log[a + b*x]*Log[c + d*x], x] + (Simp[(1*(Log[-((b*x)/a)]
- Log[-(((b*c - a*d)*x)/(a*(c + d*x)))] + Log[(b*c - a*d)/(b*(c + d*x))])*L
og[(a*(c + d*x))/(c*(a + b*x))]^2)/2, x] - Simp[(1*(Log[-((b*x)/a)] - Log[-
((d*x)/c)])*(Log[a + b*x] + Log[(a*(c + d*x))/(c*(a + b*x))])^2)/2, x] + Si
mp[(Log[c + d*x] - Log[(a*(c + d*x))/(c*(a + b*x))])*PolyLog[2, 1 + (b*x)/a
], x] + Simp[(Log[a + b*x] + Log[(a*(c + d*x))/(c*(a + b*x))])*PolyLog[2, 1
 + (d*x)/c], x] + Simp[Log[(a*(c + d*x))/(c*(a + b*x))]*PolyLog[2, (c*(a +
b*x))/(a*(c + d*x))], x] - Simp[Log[(a*(c + d*x))/(c*(a + b*x))]*PolyLog[2,
 (d*(a + b*x))/(b*(c + d*x))], x] - Simp[PolyLog[3, 1 + (b*x)/a], x] - Simp
[PolyLog[3, 1 + (d*x)/c], x] + Simp[PolyLog[3, (c*(a + b*x))/(a*(c + d*x))]
, x] - Simp[PolyLog[3, (d*(a + b*x))/(b*(c + d*x))], x]) /; FreeQ[{a, b, c,
 d}, x] && NeQ[b*c - a*d, 0]
```

Rule 2437

```
Int[(Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*Log[(h_.)*((i_.) + (j_.)*(x_))^(m
_.)])/(x_), x_Symbol] :> Dist[m, Int[(Log[i + j*x]*Log[c*(d + e*x)^n])/x, x
], x] - Dist[m*Log[i + j*x] - Log[h*(i + j*x)^m], Int[Log[c*(d + e*x)^n]/x,
 x], x] /; FreeQ[{c, d, e, h, i, j, m, n}, x] && NeQ[e*i - d*j, 0] && NeQ[i
 + j*x, h*(i + j*x)^m]
```

Rule 2440

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + Log[(h_.)
*((i_.) + (j_.)*(x_))^(m_.)]*(g_.))*((k_) + (l_.)*(x_))^(r_.), x_Symbol] :>
 Dist[1/l, Subst[Int[x^r*(a + b*Log[c*(-((e*k - d*l)/l) + (e*x)/l)^n])*(f +
 g*Log[h*(-((j*k - i*l)/l) + (j*x)/l)^m]), x], x, k + l*x], x] /; FreeQ[{a,
 b, c, d, e, f, g, h, i, j, k, l, m, n}, x] && IntegerQ[r]
```

Rule 2500

```
Int[(Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.)
)^(r_.)]*((s_.) + Log[(i_.)*((g_.) + (h_.)*(x_))^(n_.)]*(t_.)))/((j_.) + (k
_.)*(x_)), x_Symbol] :> Dist[Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r] - Log[(a
+ b*x)^(p*r)] - Log[(c + d*x)^(q*r)], Int[(s + t*Log[i*(g + h*x)^n])/(j + k
*x), x], x] + (Int[(Log[(a + b*x)^(p*r)]*(s + t*Log[i*(g + h*x)^n]))/(j + k
*x), x] + Int[(Log[(c + d*x)^(q*r)]*(s + t*Log[i*(g + h*x)^n]))/(j + k*x),
x]) /; FreeQ[{a, b, c, d, e, f, g, h, i, j, k, s, t, n, p, q, r}, x] && NeQ
[b*c - a*d, 0]
```

Rule 2524

```
Int[((a_.) + Log[(c_.)*(RFx_)^(p_.)]*(b_.))^(n_.)/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[(Log[d + e*x]*(a + b*Log[c*RFx^p])^n)/e, x] - Dist[(b*n*p)/e
, Int[(Log[d + e*x]*(a + b*Log[c*RFx^p])^(n - 1)*D[RFx, x])/RFx, x], x] /;
FreeQ[{a, b, c, d, e, p}, x] && RationalFunctionQ[RFx, x] && IGtQ[n, 0]
```

Rule 2528

```
Int[((a_.) + Log[(c_.)*(RFx_)^(p_.)]*(b_.))^(n_.)*(RGx_), x_Symbol] :> With
[{u = ExpandIntegrand[(a + b*Log[c*RFx^p])^n, RGx, x]}, Int[u, x] /; SumQ[u
]] /; FreeQ[{a, b, c, p}, x] && RationalFunctionQ[RFx, x] && RationalFuncti
onQ[RGx, x] && IGtQ[n, 0]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6688

```
Int[u_, x_Symbol] :> With[{v = SimplifyIntegrand[u, x]}, Int[v, x] /; Simpl
erIntegrandQ[v, u, x]]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\int \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{f+gx}\,dx = \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2\log(f+gx)}{g} - \frac{(2B)\int \frac{(c+dx)\left(-\frac{de(a+bx)}{(c+dx)^2}+\frac{be}{c+dx}\right)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)\log}{e(a+bx)}}{g}$$

$$= \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2\log(f+gx)}{g} - \frac{(2B)\int \frac{(c+dx)\left(-\frac{de(a+bx)}{(c+dx)^2}+\frac{be}{c+dx}\right)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)\log}{a+bx}}{eg}$$

$$= \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2\log(f+gx)}{g} - \frac{(2B)\int \frac{(bc-ad)e\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)\log(f+gx)}{(a+bx)(c+dx)}\,dx}{eg}$$

$$= \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2\log(f+gx)}{g} - \frac{(2B(bc-ad))\int \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)\log(f+gx)}{(a+bx)(c+dx)}\,dx}{g}$$

$$= \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2\log(f+gx)}{g} - \frac{(2B(bc-ad))\int \left(\frac{b\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)\log(f+gx)}{(bc-ad)(a+bx)} -\right.}{g}$$

$$= \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2\log(f+gx)}{g} - \frac{(2bB)\int \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)\log(f+gx)}{a+bx}\,dx}{g} + \frac{(2Ba}{}$$

$$= \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2\log(f+gx)}{g} - \frac{(2bB)\int \left(\frac{A\log(f+gx)}{a+bx} + \frac{B\log\left(\frac{e(a+bx)}{c+dx}\right)\log(f+gx)}{a+bx}\right)}{g}$$

$$= \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2\log(f+gx)}{g} - \frac{(2AbB)\int \frac{\log(f+gx)}{a+bx}\,dx}{g} - \frac{\left(2bB^2\right)\int \frac{\log\left(\frac{e(a+bx)}{c+dx}\right)}{a}}{g}$$

$$= -\frac{2AB\log\left(-\frac{g(a+bx)}{bf-ag}\right)\log(f+gx)}{g} + \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2\log(f+gx)}{g} + \frac{2AB\,\mathrm{l}}{}$$

$$= -\frac{2AB\log\left(-\frac{g(a+bx)}{bf-ag}\right)\log(f+gx)}{g} + \frac{2B^2\log\left(-\frac{g(a+bx)}{bf-ag}\right)\left(\log(a+bx)+\log\left(\frac{1}{c+dx}\right)\right.}{g}$$

$$= -\frac{B^2\log^2(a+bx)\log(f+gx)}{g} - \frac{2AB\log\left(-\frac{g(a+bx)}{bf-ag}\right)\log(f+gx)}{g} - \frac{B^2\log^2\left(\frac{1}{c+dx}\right.}{g}$$

$$= -\frac{B^2\log^2(a+bx)\log(f+gx)}{g} - \frac{2AB\log\left(-\frac{g(a+bx)}{bf-ag}\right)\log(f+gx)}{g} - \frac{B^2\log^2\left(\frac{1}{c+dx}\right.}{g}$$

$$= -\frac{B^2\log^2(a+bx)\log(f+gx)}{g} - \frac{2AB\log\left(-\frac{g(a+bx)}{bf-ag}\right)\log(f+gx)}{g} - \frac{B^2\log^2\left(\frac{1}{c+dx}\right.}{}$$

**Mathematica [A]** time = 0.85, size = 431, normalized size = 1.56

$$2AB\log(f+gx)\log\left(\frac{e(a+bx)}{c+dx}\right)+2AB\text{Li}_2\left(\frac{g(a+bx)}{ag-bf}\right)-2AB\log\left(\frac{a}{b}+x\right)\log(f+gx)+2AB\log\left(\frac{a}{b}+x\right)\log\left(\frac{b(f+gx)}{bf-ag}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Log[(e*(a + b*x))/(c + d*x)])^2/(f + g*x),x]
```

```
[Out] (-(B^2*Log[(e*(a + b*x))/(c + d*x)]^2*Log[(b*c - a*d)/(b*c + b*d*x)]) + A^2
*Log[f + g*x] - 2*A*B*Log[a/b + x]*Log[f + g*x] + 2*A*B*Log[c/d + x]*Log[f
+ g*x] + 2*A*B*Log[(e*(a + b*x))/(c + d*x)]*Log[f + g*x] + 2*A*B*Log[a/b +
x]*Log[(b*(f + g*x))/(b*f - a*g)] - 2*A*B*Log[c/d + x]*Log[(d*(f + g*x))/(d
*f - c*g)] + B^2*Log[(e*(a + b*x))/(c + d*x)]^2*Log[((b*c - a*d)*(f + g*x))
/((b*f - a*g)*(c + d*x))] + 2*A*B*PolyLog[2, (g*(a + b*x))/(-(b*f) + a*g)]
- 2*B^2*Log[(e*(a + b*x))/(c + d*x)]*PolyLog[2, (d*(a + b*x))/(b*(c + d*x))
] + 2*B^2*Log[(e*(a + b*x))/(c + d*x)]*PolyLog[2, ((d*f - c*g)*(a + b*x))/(
(b*f - a*g)*(c + d*x))] - 2*A*B*PolyLog[2, (g*(c + d*x))/(-(d*f) + c*g)] +
2*B^2*PolyLog[3, (d*(a + b*x))/(b*(c + d*x))] - 2*B^2*PolyLog[3, ((d*f - c*
g)*(a + b*x))/((b*f - a*g)*(c + d*x))])/g
```

**fricas [F]** time = 1.08, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{B^2\log\left(\frac{bex+ae}{dx+c}\right)^2+2\,AB\log\left(\frac{bex+ae}{dx+c}\right)+A^2}{gx+f},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*(b*x+a)/(d*x+c)))^2/(g*x+f),x, algorithm="fricas")
```

```
[Out] integral((B^2*log((b*e*x + a*e)/(d*x + c))^2 + 2*A*B*log((b*e*x + a*e)/(d*x
 + c)) + A^2)/(g*x + f), x)
```

**giac [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*(b*x+a)/(d*x+c)))^2/(g*x+f),x, algorithm="giac")
```

```
[Out] Timed out
```

**maple [B]** time = 0.08, size = 2428, normalized size = 8.77

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*ln((b*x+a)/(d*x+c)*e)+A)^2/(g*x+f),x)
```

```
[Out] 2*d*A*B/g/(a*d-b*c)*ln(b/d*e+(a*d-b*c)/(d*x+c)/d*e)*ln((-a*e*g+b*e*f+(c*g-d
*f)*(b/d*e+(a*d-b*c)/(d*x+c)/d*e))/(-a*e*g+b*e*f))/(c*g-d*f)*f*b*c+2*A*B/g/
(a*d-b*c)*ln(b/d*e+(a*d-b*c)/(d*x+c)/d*e)*ln(-(-b*e+(b/d*e+(a*d-b*c)/(d*x+c
)/d*e)*d)/b/e)*b*c-2*d*A*B/g/(a*d-b*c)*ln(b/d*e+(a*d-b*c)/(d*x+c)/d*e)*ln(-
(-b*e+(b/d*e+(a*d-b*c)/(d*x+c)/d*e)*d)/b/e)*a+2*d*A*B/(a*d-b*c)*dilog((-a*e
*g+b*e*f+(c*g-d*f)*(b/d*e+(a*d-b*c)/(d*x+c)/d*e))/(-a*e*g+b*e*f))/(c*g-d*f)
*c*a-2*B^2/g/(a*d-b*c)*ln(b/d*e+(a*d-b*c)/(d*x+c)/d*e)*polylog(2,-(c*g-d*f)
/(-a*e*g+b*e*f)*(b/d*e+(a*d-b*c)/(d*x+c)/d*e))*b*c+2*A*B/g/(a*d-b*c)*dilog(
-(-b*e+(b/d*e+(a*d-b*c)/(d*x+c)/d*e)*d)/b/e)*b*c+B^2/g/(a*d-b*c)*ln(b/d*e+(
a*d-b*c)/(d*x+c)/d*e)^2*ln(-(b/d*e+(a*d-b*c)/(d*x+c)/d*e)/b*d/e+1)*b*c+2*B^
2/g/(a*d-b*c)*ln(b/d*e+(a*d-b*c)/(d*x+c)/d*e)*polylog(2,(b/d*e+(a*d-b*c)/(d
*x+c)/d*e)/b*d/e)*b*c+d*B^2/g/(a*d-b*c)*ln(b/d*e+(a*d-b*c)/(d*x+c)/d*e)^2*l
n(1+(c*g-d*f)/(-a*e*g+b*e*f)*(b/d*e+(a*d-b*c)/(d*x+c)/d*e))*a-2*d*B^2/g/(a*
d-b*c)*ln(b/d*e+(a*d-b*c)/(d*x+c)/d*e)*polylog(2,(b/d*e+(a*d-b*c)/(d*x+c)/d
*e)/b*d/e)*a-d*B^2/g/(a*d-b*c)*ln(b/d*e+(a*d-b*c)/(d*x+c)/d*e)^2*ln(-(b/d*e
+(a*d-b*c)/(d*x+c)/d*e)/b*d/e+1)*a-2*d^2*A*B/g/(a*d-b*c)*dilog((-a*e*g+b*e*
f+(c*g-d*f)*(b/d*e+(a*d-b*c)/(d*x+c)/d*e))/(-a*e*g+b*e*f))/(c*g-d*f)*f*a+2*
d*A*B/(a*d-b*c)*ln(b/d*e+(a*d-b*c)/(d*x+c)/d*e)*ln((-a*e*g+b*e*f+(c*g-d*f)*
(b/d*e+(a*d-b*c)/(d*x+c)/d*e))/(-a*e*g+b*e*f))/(c*g-d*f)*c*a-2*A*B/(a*d-b*c
)*ln(b/d*e+(a*d-b*c)/(d*x+c)/d*e)*ln((-a*e*g+b*e*f+(c*g-d*f)*(b/d*e+(a*d-b*
c)/(d*x+c)/d*e))/(-a*e*g+b*e*f))/(c*g-d*f)*c^2*b-2*d*B^2/g/(a*d-b*c)*polylo
g(3,-(c*g-d*f)/(-a*e*g+b*e*f)*(b/d*e+(a*d-b*c)/(d*x+c)/d*e))*a-d*A^2/g/(a*d
-b*c)*ln(-b*e+(b/d*e+(a*d-b*c)/(d*x+c)/d*e)*d)*a+d*A^2/g/(a*d-b*c)*ln(-a*e*
g+b*e*f+(b/d*e+(a*d-b*c)/(d*x+c)/d*e)*c*g-(b/d*e+(a*d-b*c)/(d*x+c)/d*e)*d*f
)*a+2*d*B^2/g/(a*d-b*c)*polylog(3,(b/d*e+(a*d-b*c)/(d*x+c)/d*e)/b*d/e)*a-A^
2/g/(a*d-b*c)*ln(-a*e*g+b*e*f+(b/d*e+(a*d-b*c)/(d*x+c)/d*e)*c*g-(b/d*e+(a*d
-b*c)/(d*x+c)/d*e)*d*f)*b*c+A^2/g/(a*d-b*c)*ln(-b*e+(b/d*e+(a*d-b*c)/(d*x+c
)/d*e)*d)*b*c+2*d*B^2/g/(a*d-b*c)*ln(b/d*e+(a*d-b*c)/(d*x+c)/d*e)*polylog(2
,-(c*g-d*f)/(-a*e*g+b*e*f)*(b/d*e+(a*d-b*c)/(d*x+c)/d*e))*a-2*A*B/(a*d-b*c)
*dilog((-a*e*g+b*e*f+(c*g-d*f)*(b/d*e+(a*d-b*c)/(d*x+c)/d*e))/(-a*e*g+b*e*f
))/(c*g-d*f)*c^2*b-2*B^2/g/(a*d-b*c)*polylog(3,(b/d*e+(a*d-b*c)/(d*x+c)/d*e
)/b*d/e)*b*c+2*B^2/g/(a*d-b*c)*polylog(3,-(c*g-d*f)/(-a*e*g+b*e*f)*(b/d*e+(
a*d-b*c)/(d*x+c)/d*e))*b*c-B^2/g/(a*d-b*c)*ln(b/d*e+(a*d-b*c)/(d*x+c)/d*e)^
2*ln(1+(c*g-d*f)/(-a*e*g+b*e*f)*(b/d*e+(a*d-b*c)/(d*x+c)/d*e))*b*c-2*d*A*B/
g/(a*d-b*c)*dilog(-(-b*e+(b/d*e+(a*d-b*c)/(d*x+c)/d*e)*d)/b/e)*a-2*d^2*A*B/
g/(a*d-b*c)*ln(b/d*e+(a*d-b*c)/(d*x+c)/d*e)*ln((-a*e*g+b*e*f+(c*g-d*f)*(b/d
*e+(a*d-b*c)/(d*x+c)/d*e))/(-a*e*g+b*e*f))/(c*g-d*f)*f*a+2*d*A*B/g/(a*d-b*c
)*dilog((-a*e*g+b*e*f+(c*g-d*f)*(b/d*e+(a*d-b*c)/(d*x+c)/d*e))/(-a*e*g+b*e*
f))/(c*g-d*f)*f*b*c
```

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$\frac{A^2 \log\left(gx+f\right)}{g} - \int -\frac{B^2 \log\left(bx+a\right)^2 + B^2 \log(e)^2 + 2\,AB \log(e) + 2\left(B^2 \log(e) + AB\right) \log\left(bx+a\right) - 2\left(B^2 \log\left(\right.\right.}{gx+f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*(b*x+a)/(d*x+c)))^2/(g*x+f),x, algorithm="maxima")
```

```
[Out] A^2*log(g*x + f)/g - integrate(-(B^2*log(b*x + a)^2 + B^2*log(e)^2 + 2*A*B*
log(e) + 2*(B^2*log(e) + A*B)*log(b*x + a) - 2*(B^2*log(b*x + a) + B^2*log(
e) + A*B)*log(d*x + c))/(g*x + f), x)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(A + B\,\ln\left(\frac{e\,(a+b\,x)}{c+d\,x}\right)\right)^2}{f + g\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*log((e*(a + b*x))/(c + d*x)))^2/(f + g*x),x)
```

```
[Out] int((A + B*log((e*(a + b*x))/(c + d*x)))^2/(f + g*x), x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(A + B \log\left(\frac{ae}{c+dx} + \frac{bex}{c+dx}\right)\right)^2}{f + gx}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*ln(e*(b*x+a)/(d*x+c)))**2/(g*x+f),x)
```

```
[Out] Integral((A + B*log(a*e/(c + d*x) + b*e*x/(c + d*x)))**2/(f + g*x), x)
```

# 3.245 $\int \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(f+gx)^2}\,dx$

**Optimal**. Leaf size=196

$$\frac{2B(bc-ad)\log\left(1-\frac{(a+bx)(df-cg)}{(c+dx)(bf-ag)}\right)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{(bf-ag)(df-cg)}+\frac{(a+bx)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2}{(f+gx)(bf-ag)}+\frac{2B^2(bc-ad)\mathrm{Li}_2\left(\frac{(df-\cdots}{(bf-\cdots}\right.}{(bf-ag)(df-\cdots}$$

```
[Out] (b*x+a)*(A+B*ln(e*(b*x+a)/(d*x+c)))^2/(-a*g+b*f)/(g*x+f)+2*B*(-a*d+b*c)*(A+
B*ln(e*(b*x+a)/(d*x+c)))*ln(1-(-c*g+d*f)*(b*x+a)/(-a*g+b*f)/(d*x+c))/(-a*g+
b*f)/(-c*g+d*f)+2*B^2*(-a*d+b*c)*polylog(2,(-c*g+d*f)*(b*x+a)/(-a*g+b*f)/(d
*x+c))/(-a*g+b*f)/(-c*g+d*f)
```

**Rubi [B]** time = 1.12, antiderivative size = 612, normalized size of antiderivative = 3.12, number of steps used = 32, number of rules used = 10, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.345, Rules used = {2525, 12, 2528, 2524, 2418, 2390, 2301, 2394, 2393, 2391}

$$\frac{2bB^2\mathrm{PolyLog}\left(2,-\frac{d(a+bx)}{bc-ad}\right)}{g(bf-ag)}+\frac{2B^2d\mathrm{PolyLog}\left(2,\frac{b(c+dx)}{bc-ad}\right)}{g(df-cg)}-\frac{2B^2(bc-ad)\mathrm{PolyLog}\left(2,\frac{b(f+gx)}{bf-ag}\right)}{(bf-ag)(df-cg)}+\frac{2B^2(bc-ad)\mathrm{PolyL\cdots}}{(bf-ag)(d\cdots}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Log[(e*(a + b*x))/(c + d*x)])^2/(f + g*x)^2,x]
```

```
[Out] -((b*B^2*Log[a + b*x]^2)/(g*(b*f - a*g))) + (2*b*B*Log[a + b*x]*(A + B*Log[
(e*(a + b*x))/(c + d*x)]))/(g*(b*f - a*g)) - (A + B*Log[(e*(a + b*x))/(c +
d*x)])^2/(g*(f + g*x)) + (2*B^2*d*Log[-((d*(a + b*x))/(b*c - a*d))]*Log[c +
 d*x])/(g*(d*f - c*g)) - (2*B*d*(A + B*Log[(e*(a + b*x))/(c + d*x)])*Log[c
+ d*x])/(g*(d*f - c*g)) - (B^2*d*Log[c + d*x]^2)/(g*(d*f - c*g)) + (2*b*B^2
*Log[a + b*x]*Log[(b*(c + d*x))/(b*c - a*d)])/(g*(b*f - a*g)) - (2*B^2*(b*c
 - a*d)*Log[-((g*(a + b*x))/(b*f - a*g))]*Log[f + g*x])/((b*f - a*g)*(d*f -
 c*g)) + (2*B*(b*c - a*d)*(A + B*Log[(e*(a + b*x))/(c + d*x)])*Log[f + g*x]
)/((b*f - a*g)*(d*f - c*g)) + (2*B^2*(b*c - a*d)*Log[-((g*(c + d*x))/(d*f -
 c*g))]*Log[f + g*x])/((b*f - a*g)*(d*f - c*g)) + (2*b*B^2*PolyLog[2, -((d*
(a + b*x))/(b*c - a*d))])/(g*(b*f - a*g)) + (2*B^2*d*PolyLog[2, (b*(c + d*x
))/(b*c - a*d)])/(g*(d*f - c*g)) - (2*B^2*(b*c - a*d)*PolyLog[2, (b*(f + g*
x))/(b*f - a*g)])/((b*f - a*g)*(d*f - c*g)) + (2*B^2*(b*c - a*d)*PolyLog[2,
 (d*(f + g*x))/(d*f - c*g)])/((b*f - a*g)*(d*f - c*g))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2301

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

Rule 2390

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_))^(q_.), x_Symbol] :> Dist[1/e, Subst[Int[((f*x)/d)^q*(a + b*Log[c*x^
n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && E
qQ[e*f - d*g, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2393

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + (c*e*x)/g])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2394

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d + e*x
)^n]))/g, x] - Dist[(b*e*n)/g, Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2418

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*(RFx_), x_Sy
mbol] :> With[{u = ExpandIntegrand[(a + b*Log[c*(d + e*x)^n])^p, RFx, x]},
Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d, e, n}, x] && RationalFunctionQ[
RFx, x] && IntegerQ[p]
```

Rule 2524

```
Int[((a_.) + Log[(c_.)*(RFx_)^(p_.)]*(b_.))^(n_.)/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[(Log[d + e*x]*(a + b*Log[c*RFx^p])^n)/e, x] - Dist[(b*n*p)/e
, Int[(Log[d + e*x]*(a + b*Log[c*RFx^p])^(n - 1)*D[RFx, x])/RFx, x], x] /;
FreeQ[{a, b, c, d, e, p}, x] && RationalFunctionQ[RFx, x] && IGtQ[n, 0]
```

Rule 2525

```
Int[((a_.) + Log[(c_.)*(RFx_)^(p_.)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_))^(m_.
), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(a + b*Log[c*RFx^p])^n)/(e*(m + 1))
, x] - Dist[(b*n*p)/(e*(m + 1)), Int[SimplifyIntegrand[((d + e*x)^(m + 1)*(
a + b*Log[c*RFx^p])^(n - 1)*D[RFx, x])/RFx, x], x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && RationalFunctionQ[RFx, x] && IGtQ[n, 0] && (EqQ[n, 1] ||
IntegerQ[m]) && NeQ[m, -1]
```

Rule 2528

```
Int[((a_.) + Log[(c_.)*(RFx_)^(p_.)]*(b_.))^(n_.)*(RGx_), x_Symbol] :> With
[{u = ExpandIntegrand[(a + b*Log[c*RFx^p])^n, RGx, x]}, Int[u, x] /; SumQ[u
]] /; FreeQ[{a, b, c, p}, x] && RationalFunctionQ[RFx, x] && RationalFuncti
onQ[RGx, x] && IGtQ[n, 0]
```

Rubi steps

$$\int \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(f+gx)^2}\,dx = -\frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{g(f+gx)} + \frac{(2B)\int \frac{(bc-ad)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(a+bx)(c+dx)(f+gx)}\,dx}{g}$$

$$= -\frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{g(f+gx)} + \frac{(2B(bc-ad))\int \frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{(a+bx)(c+dx)(f+gx)}\,dx}{g}$$

$$= -\frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{g(f+gx)} + \frac{(2B(bc-ad))\int \left(\frac{b^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(bc-ad)(bf-ag)(a+bx)} + \frac{d^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(bc-ad)(-df+cg)(c+dx}\right.}{g}$$

$$= -\frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{g(f+gx)} + \frac{\left(2b^2B\right)\int \frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{a+bx}\,dx}{g(bf-ag)} - \frac{\left(2Bd^2\right)\int \frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{c+dx}\,d}{g(df-cg)}$$

$$= \frac{2bB\log(a+bx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{g(bf-ag)} - \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{g(f+gx)} - \frac{2Bd\left(A+B\log\right.}{g(}$$

$$= \frac{2bB\log(a+bx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{g(bf-ag)} - \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{g(f+gx)} - \frac{2Bd\left(A+B\log\right.}{g(}$$

$$= \frac{2bB\log(a+bx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{g(bf-ag)} - \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{g(f+gx)} - \frac{2Bd\left(A+B\log\right.}{g(}$$

$$= \frac{2bB\log(a+bx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{g(bf-ag)} - \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{g(f+gx)} - \frac{2Bd\left(A+B\log\right.}{g(}$$

$$= \frac{2bB\log(a+bx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{g(bf-ag)} - \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{g(f+gx)} + \frac{2B^2d\log\left(-\frac{d(a+}{bc-}\right.}{g(df}$$

$$= -\frac{bB^2\log^2(a+bx)}{g(bf-ag)} + \frac{2bB\log(a+bx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{g(bf-ag)} - \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right.\right.}{g(f+gx)}$$

$$= -\frac{bB^2\log^2(a+bx)}{g(bf-ag)} + \frac{2bB\log(a+bx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{g(bf-ag)} - \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right.\right.}{g(f+gx)}$$

**Mathematica [B]** time = 0.58, size = 402, normalized size = 2.05

$$\frac{B\left(2b\log(a+bx)(df-cg)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)-2d(bf-ag)\log(c+dx)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)+2g(bc-ad)\log(f+gx)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)-bB(df-cg)\left(\log(a+\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Log[(e*(a + b*x))/(c + d*x)])^2/(f + g*x)^2,x]
```

```
[Out] (-((A + B*Log[(e*(a + b*x))/(c + d*x)])^2/(f + g*x)) + (B*(2*b*(d*f - c*g)*
Log[a + b*x]*(A + B*Log[(e*(a + b*x))/(c + d*x)]) - 2*d*(b*f - a*g)*(A + B*
Log[(e*(a + b*x))/(c + d*x)])*Log[c + d*x] + 2*(b*c - a*d)*g*(A + B*Log[(e*
(a + b*x))/(c + d*x)])*Log[f + g*x] - b*B*(d*f - c*g)*(Log[a + b*x]*(Log[a
+ b*x] - 2*Log[(b*(c + d*x))/(b*c - a*d)]) - 2*PolyLog[2, (d*(a + b*x))/(-(
b*c) + a*d)]) + B*d*(b*f - a*g)*((2*Log[(d*(a + b*x))/(-(b*c) + a*d)] - Log
[c + d*x])*Log[c + d*x] + 2*PolyLog[2, (b*(c + d*x))/(b*c - a*d)]) - 2*B*(b
*c - a*d)*g*((Log[(g*(a + b*x))/(-(b*f) + a*g)] - Log[(g*(c + d*x))/(-(d*f)
 + c*g)])*Log[f + g*x] + PolyLog[2, (b*(f + g*x))/(b*f - a*g)] - PolyLog[2,
 (d*(f + g*x))/(d*f - c*g)])))/((b*f - a*g)*(d*f - c*g)))/g
```

**fricas [F]** time = 0.81, size = 0, normalized size = 0.00

$$\operatorname{integral}\left(\frac{B^{2}\log\left(\frac{bex+ae}{dx+c}\right)^{2}+2\,AB\log\left(\frac{bex+ae}{dx+c}\right)+A^{2}}{g^{2}x^{2}+2\,fgx+f^{2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*(b*x+a)/(d*x+c)))^2/(g*x+f)^2,x, algorithm="fricas")
```

```
[Out] integral((B^2*log((b*e*x + a*e)/(d*x + c))^2 + 2*A*B*log((b*e*x + a*e)/(d*x
 + c)) + A^2)/(g^2*x^2 + 2*f*g*x + f^2), x)
```

**giac [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*(b*x+a)/(d*x+c)))^2/(g*x+f)^2,x, algorithm="giac")
```

```
[Out] Timed out
```

**maple [F]** time = 1.80, size = 0, normalized size = 0.00

$$\int \frac{\left(B \ln \left(\frac{(bx+a)e}{dx+c}\right)+A\right)^{2}}{\left(gx+f\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*ln((b*x+a)/(d*x+c)*e)+A)^2/(g*x+f)^2,x)
```

```
[Out] int((B*ln((b*x+a)/(d*x+c)*e)+A)^2/(g*x+f)^2,x)
```

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$2\,AB\left(\frac{b\log\left(bx+a\right)}{bfg-ag^{2}}-\frac{d\log\left(dx+c\right)}{dfg-cg^{2}}+\frac{(bc-ad)\log\left(gx+f\right)}{bdf^{2}+acg^{2}-(bc+ad)fg}-\frac{\log\left(\frac{bex}{dx+c}+\frac{ae}{dx+c}\right)}{g^{2}x+fg}\right)-B^{2}\left(\frac{\log\left(dx+c\right)^{2}}{g^{2}x+fg}+\int -$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*(b*x+a)/(d*x+c)))^2/(g*x+f)^2,x, algorithm="maxima")
```

```
[Out] 2*A*B*(b*log(b*x + a)/(b*f*g - a*g^2) - d*log(d*x + c)/(d*f*g - c*g^2) + (b
*c - a*d)*log(g*x + f)/(b*d*f^2 + a*c*g^2 - (b*c + a*d)*f*g) - log(b*e*x/(d
*x + c) + a*e/(d*x + c))/(g^2*x + f*g)) - B^2*(log(d*x + c)^2/(g^2*x + f*g)
 + integrate(-(d*g*x*log(e)^2 + c*g*log(e)^2 + (d*g*x + c*g)*log(b*x + a)^2
 + 2*(d*g*x*log(e) + c*g*log(e))*log(b*x + a) - 2*((g*log(e) - g)*d*x + c*g
*log(e) - d*f + (d*g*x + c*g)*log(b*x + a))*log(d*x + c))/(d*g^3*x^3 + c*f^
2*g + (2*d*f*g^2 + c*g^3)*x^2 + (d*f^2*g + 2*c*f*g^2)*x), x)) - A^2/(g^2*x
+ f*g)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\left(A+B\,\ln\left(\frac{e\,\left(a+b\,x\right)}{c+d\,x}\right)\right)}^2}{{\left(f+g\,x\right)}^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*log((e*(a + b*x))/(c + d*x)))^2/(f + g*x)^2,x)
```

```
[Out] int((A + B*log((e*(a + b*x))/(c + d*x)))^2/(f + g*x)^2, x)
```

**sympy [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*ln(e*(b*x+a)/(d*x+c)))**2/(g*x+f)**2,x)
```

```
[Out] Timed out
```

## 3.246 $\int \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(f+gx)^3}\,dx$

**Optimal**. Leaf size=369

$$\frac{b^2\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2}{2g(bf-ag)^2}+\frac{Bg(a+bx)(bc-ad)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{(f+gx)(bf-ag)^2(df-cg)}+\frac{B(bc-ad)(-adg-bcg+2bdf)\log\left(1-\frac{(a\cdots}{(c+\cdots}\right.}{(bf-ag)^2(df-\cdots}$$

```
[Out] B*(-a*d+b*c)*g*(b*x+a)*(A+B*ln(e*(b*x+a)/(d*x+c)))/(-a*g+b*f)^2/(-c*g+d*f)/
(g*x+f)+1/2*b^2*(A+B*ln(e*(b*x+a)/(d*x+c)))^2/g/(-a*g+b*f)^2-1/2*(A+B*ln(e*
(b*x+a)/(d*x+c)))^2/g/(g*x+f)^2+B^2*(-a*d+b*c)^2*g*ln((g*x+f)/(d*x+c))/(-a*
g+b*f)^2/(-c*g+d*f)^2+B*(-a*d+b*c)*(-a*d*g-b*c*g+2*b*d*f)*(A+B*ln(e*(b*x+a)
/(d*x+c)))*ln(1-(-c*g+d*f)*(b*x+a)/(-a*g+b*f)/(d*x+c))/(-a*g+b*f)^2/(-c*g+d
*f)^2+B^2*(-a*d+b*c)*(-a*d*g-b*c*g+2*b*d*f)*polylog(2,(-c*g+d*f)*(b*x+a)/(-
a*g+b*f)/(d*x+c))/(-a*g+b*f)^2/(-c*g+d*f)^2
```

**Rubi [B]** time = 1.48, antiderivative size = 883, normalized size of antiderivative = 2.39, number of steps used = 36, number of rules used = 11, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.379, Rules used = {2525, 12, 2528, 2524, 2418, 2390, 2301, 2394, 2393, 2391, 72}

$$-\frac{B^2\log^2(a+bx)b^2}{2g(bf-ag)^2}+\frac{B\log(a+bx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)b^2}{g(bf-ag)^2}+\frac{B^2\log(a+bx)\log\left(\frac{b(c+dx)}{bc-ad}\right)b^2}{g(bf-ag)^2}+\frac{B^2\text{PolyLog}\left(2,-\frac{d(a\cdots}{bc\cdots}\right)}{g(bf-ag)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Log[(e*(a + b*x))/(c + d*x)])^2/(f + g*x)^3,x]
```

```
[Out] (b*B^2*(b*c - a*d)*Log[a + b*x])/((b*f - a*g)^2*(d*f - c*g)) - (b^2*B^2*Log
[a + b*x]^2)/(2*g*(b*f - a*g)^2) - (B*(b*c - a*d)*(A + B*Log[(e*(a + b*x))/
(c + d*x)]))/((b*f - a*g)*(d*f - c*g)*(f + g*x)) + (b^2*B*Log[a + b*x]*(A +
 B*Log[(e*(a + b*x))/(c + d*x)]))/(g*(b*f - a*g)^2) - (A + B*Log[(e*(a + b*
x))/(c + d*x)])^2/(2*g*(f + g*x)^2) - (B^2*d*(b*c - a*d)*Log[c + d*x])/((b*
f - a*g)*(d*f - c*g)^2) + (B^2*d^2*Log[-((d*(a + b*x))/(b*c - a*d))]*Log[c
+ d*x])/(g*(d*f - c*g)^2) - (B*d^2*(A + B*Log[(e*(a + b*x))/(c + d*x)])*Log
[c + d*x])/(g*(d*f - c*g)^2) - (B^2*d^2*Log[c + d*x]^2)/(2*g*(d*f - c*g)^2)
 + (b^2*B^2*Log[a + b*x]*Log[(b*(c + d*x))/(b*c - a*d)])/(g*(b*f - a*g)^2)
+ (B^2*(b*c - a*d)^2*g*Log[f + g*x])/((b*f - a*g)^2*(d*f - c*g)^2) - (B^2*(
b*c - a*d)*(2*b*d*f - b*c*g - a*d*g)*Log[-((g*(a + b*x))/(b*f - a*g))]*Log[
f + g*x])/((b*f - a*g)^2*(d*f - c*g)^2) + (B*(b*c - a*d)*(2*b*d*f - b*c*g -
 a*d*g)*(A + B*Log[(e*(a + b*x))/(c + d*x)])*Log[f + g*x])/((b*f - a*g)^2*(
d*f - c*g)^2) + (B^2*(b*c - a*d)*(2*b*d*f - b*c*g - a*d*g)*Log[-((g*(c + d*
x))/(d*f - c*g))]*Log[f + g*x])/((b*f - a*g)^2*(d*f - c*g)^2) + (b^2*B^2*Po
```

```
lyLog[2, -((d*(a + b*x))/(b*c - a*d))])/(g*(b*f - a*g)^2) + (B^2*d^2*PolyLo
g[2, (b*(c + d*x))/(b*c - a*d)])/(g*(d*f - c*g)^2) - (B^2*(b*c - a*d)*(2*b*
d*f - b*c*g - a*d*g)*PolyLog[2, (b*(f + g*x))/(b*f - a*g)])/((b*f - a*g)^2*
(d*f - c*g)^2) + (B^2*(b*c - a*d)*(2*b*d*f - b*c*g - a*d*g)*PolyLog[2, (d*(
f + g*x))/(d*f - c*g)])/((b*f - a*g)^2*(d*f - c*g)^2)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 72

```
Int[((e_.) + (f_.)*(x_))^(p_.)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
 x_Symbol] :> Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x]
 /; FreeQ[{a, b, c, d, e, f}, x] && IntegerQ[p]
```

Rule 2301

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

Rule 2390

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_))^(q_.), x_Symbol] :> Dist[1/e, Subst[Int[((f*x)/d)^q*(a + b*Log[c*x^
n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && E
qQ[e*f - d*g, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2393

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + (c*e*x)/g])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2394

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d + e*x
)^n]))/g, x] - Dist[(b*e*n)/g, Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
```

```
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2418

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*(RFx_), x_Sy
mbol] :> With[{u = ExpandIntegrand[(a + b*Log[c*(d + e*x)^n])^p, RFx, x]},
Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d, e, n}, x] && RationalFunctionQ[
RFx, x] && IntegerQ[p]
```

Rule 2524

```
Int[((a_.) + Log[(c_.)*(RFx_)^(p_.)]*(b_.))^(n_.)/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[(Log[d + e*x]*(a + b*Log[c*RFx^p])^n)/e, x] - Dist[(b*n*p)/e
, Int[(Log[d + e*x]*(a + b*Log[c*RFx^p])^(n - 1)*D[RFx, x])/RFx, x], x] /;
FreeQ[{a, b, c, d, e, p}, x] && RationalFunctionQ[RFx, x] && IGtQ[n, 0]
```

Rule 2525

```
Int[((a_.) + Log[(c_.)*(RFx_)^(p_.)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_))^(m_.
), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(a + b*Log[c*RFx^p])^n)/(e*(m + 1))
, x] - Dist[(b*n*p)/(e*(m + 1)), Int[SimplifyIntegrand[((d + e*x)^(m + 1)*(
a + b*Log[c*RFx^p])^(n - 1)*D[RFx, x])/RFx, x], x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && RationalFunctionQ[RFx, x] && IGtQ[n, 0] && (EqQ[n, 1] ||
IntegerQ[m]) && NeQ[m, -1]
```

Rule 2528

```
Int[((a_.) + Log[(c_.)*(RFx_)^(p_.)]*(b_.))^(n_.)*(RGx_), x_Symbol] :> With
[{u = ExpandIntegrand[(a + b*Log[c*RFx^p])^n, RGx, x]}, Int[u, x] /; SumQ[u
]] /; FreeQ[{a, b, c, p}, x] && RationalFunctionQ[RFx, x] && RationalFuncti
onQ[RGx, x] && IGtQ[n, 0]
```

Rubi steps

$$\int \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(f+gx)^3}\,dx = -\frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{2g(f+gx)^2} + \frac{B\int \frac{(bc-ad)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(a+bx)(c+dx)(f+gx)^2}\,dx}{g}$$

$$= -\frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{2g(f+gx)^2} + \frac{(B(bc-ad))\int \frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{(a+bx)(c+dx)(f+gx)^2}\,dx}{g}$$

$$= -\frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{2g(f+gx)^2} + \frac{(B(bc-ad))\int \left(\frac{b^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(bc-ad)(bf-ag)^2(a+bx)} - \frac{d^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right.}{(bc-ad)(-df+cg)^2(c+}\right.}{}$$

$$= -\frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{2g(f+gx)^2} + \frac{\left(b^3B\right)\int \frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{a+bx}\,dx}{g(bf-ag)^2} - \frac{\left(Bd^3\right)\int \frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{c+dx}\,dx}{g(df-cg)^2}$$

$$= -\frac{B(bc-ad)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(bf-ag)(df-cg)(f+gx)} + \frac{b^2B\log(a+bx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{g(bf-ag)^2} - \frac{\left(A\right.}{}$$

$$= -\frac{B(bc-ad)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(bf-ag)(df-cg)(f+gx)} + \frac{b^2B\log(a+bx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{g(bf-ag)^2} - \frac{\left(A\right.}{}$$

$$= -\frac{B(bc-ad)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(bf-ag)(df-cg)(f+gx)} + \frac{b^2B\log(a+bx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{g(bf-ag)^2} - \frac{\left(A\right.}{}$$

$$= \frac{bB^2(bc-ad)\log(a+bx)}{(bf-ag)^2(df-cg)} - \frac{B(bc-ad)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(bf-ag)(df-cg)(f+gx)} + \frac{b^2B\log(a+bx)\left(\right.}{g(bf}$$

$$= \frac{bB^2(bc-ad)\log(a+bx)}{(bf-ag)^2(df-cg)} - \frac{B(bc-ad)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(bf-ag)(df-cg)(f+gx)} + \frac{b^2B\log(a+bx)\left(\right.}{g(bf}$$

$$= \frac{bB^2(bc-ad)\log(a+bx)}{(bf-ag)^2(df-cg)} - \frac{b^2B^2\log^2(a+bx)}{2g(bf-ag)^2} - \frac{B(bc-ad)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right.}{(bf-ag)(df-cg)(f+gx)}$$

$$= \frac{bB^2(bc-ad)\log(a+bx)}{(bf-ag)^2(df-cg)} - \frac{b^2B^2\log^2(a+bx)}{2g(bf-ag)^2} - \frac{B(bc-ad)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right.}{(bf-ag)(df-cg)(f+gx)}$$

**Mathematica [A]** time = 1.54, size = 595, normalized size = 1.61

$$-\frac{B(f+gx)\left(-2b^2(f+gx)\log(a+bx)(df-cg)^2\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)+2d^2(f+gx)(bf-ag)^2\log(c+dx)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)+2g(bc-ad)(bf-ag)(df-cg)\left(B\log\left(\right.\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Log[(e*(a + b*x))/(c + d*x)])^2/(f + g*x)^3,x]
```

```
[Out] -1/2*((A + B*Log[(e*(a + b*x))/(c + d*x)])^2 + (B*(f + g*x)*(2*(b*c - a*d)*
g*(b*f - a*g)*(d*f - c*g)*(A + B*Log[(e*(a + b*x))/(c + d*x)]) - 2*b^2*(d*f
 - c*g)^2*(f + g*x)*Log[a + b*x]*(A + B*Log[(e*(a + b*x))/(c + d*x)]) + 2*d
^2*(b*f - a*g)^2*(f + g*x)*(A + B*Log[(e*(a + b*x))/(c + d*x)])*Log[c + d*x
] + 2*(b*c - a*d)*g*(-2*b*d*f + b*c*g + a*d*g)*(f + g*x)*(A + B*Log[(e*(a +
 b*x))/(c + d*x)])*Log[f + g*x] - 2*B*(b*c - a*d)*g*(f + g*x)*(b*(d*f - c*g
)*Log[a + b*x] + (-(b*d*f) + a*d*g)*Log[c + d*x] + (b*c - a*d)*g*Log[f + g*
x]) + b^2*B*(d*f - c*g)^2*(f + g*x)*(Log[a + b*x]*(Log[a + b*x] - 2*Log[(b*
(c + d*x))/(b*c - a*d)]) - 2*PolyLog[2, (d*(a + b*x))/(-(b*c) + a*d)]) - B*
d^2*(b*f - a*g)^2*(f + g*x)*((2*Log[(d*(a + b*x))/(-(b*c) + a*d)] - Log[c +
 d*x])*Log[c + d*x] + 2*PolyLog[2, (b*(c + d*x))/(b*c - a*d)]) - 2*B*(b*c -
 a*d)*g*(-2*b*d*f + b*c*g + a*d*g)*(f + g*x)*((Log[(g*(a + b*x))/(-(b*f) +
a*g)] - Log[(g*(c + d*x))/(-(d*f) + c*g)])*Log[f + g*x] + PolyLog[2, (b*(f
+ g*x))/(b*f - a*g)] - PolyLog[2, (d*(f + g*x))/(d*f - c*g)])))/((b*f - a*g
)^2*(d*f - c*g)^2))/(g*(f + g*x)^2)
```

**fricas [F]** time = 0.90, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{B^2\log\left(\frac{bex+ae}{dx+c}\right)^2+2\,AB\log\left(\frac{bex+ae}{dx+c}\right)+A^2}{g^3x^3+3\,fg^2x^2+3\,f^2gx+f^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*(b*x+a)/(d*x+c)))^2/(g*x+f)^3,x, algorithm="fricas")
```

```
[Out] integral((B^2*log((b*e*x + a*e)/(d*x + c))^2 + 2*A*B*log((b*e*x + a*e)/(d*x
 + c)) + A^2)/(g^3*x^3 + 3*f*g^2*x^2 + 3*f^2*g*x + f^3), x)
```

**giac [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*(b*x+a)/(d*x+c)))^2/(g*x+f)^3,x, algorithm="giac")
```

```
[Out] Timed out
```

**maple [F]** time = 2.73, size = 0, normalized size = 0.00

$$\int \frac{\left(B\ln\left(\frac{(bx+a)e}{dx+c}\right)+A\right)^{2}}{\left(gx+f\right)^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*ln((b*x+a)/(d*x+c)*e)+A)^2/(g*x+f)^3,x)
[Out] int((B*ln((b*x+a)/(d*x+c)*e)+A)^2/(g*x+f)^3,x)
```

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$\left(\frac{b^2\log(bx+a)}{b^2f^2g-2\,abfg^2+a^2g^3}-\frac{d^2\log(dx+c)}{d^2f^2g-2\,cdfg^2+c^2g^3}+\frac{\left(2\left(b^2cd-abd^2\right)f-\left(b^2c^2-a^2d^2\right)g\right)}{b^2d^2f^4+a^2c^2g^4-2\left(b^2cd+abd^2\right)f^3g+\left(b^2c^2+4\,abcd+\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*(b*x+a)/(d*x+c)))^2/(g*x+f)^3,x, algorithm="maxima")
[Out] (b^2*log(b*x + a)/(b^2*f^2*g - 2*a*b*f*g^2 + a^2*g^3) - d^2*log(d*x + c)/(d
^2*f^2*g - 2*c*d*f*g^2 + c^2*g^3) + (2*(b^2*c*d - a*b*d^2)*f - (b^2*c^2 - a
^2*d^2)*g)*log(g*x + f)/(b^2*d^2*f^4 + a^2*c^2*g^4 - 2*(b^2*c*d + a*b*d^2)*
f^3*g + (b^2*c^2 + 4*a*b*c*d + a^2*d^2)*f^2*g^2 - 2*(a*b*c^2 + a^2*c*d)*f*g
^3) - (b*c - a*d)/(b*d*f^3 + a*c*f*g^2 - (b*c + a*d)*f^2*g + (b*d*f^2*g + a
*c*g^3 - (b*c + a*d)*f*g^2)*x) - log(b*e*x/(d*x + c) + a*e/(d*x + c))/(g^3*
x^2 + 2*f*g^2*x + f^2*g))*A*B - 1/2*B^2*(log(d*x + c)^2/(g^3*x^2 + 2*f*g^2*
x + f^2*g) + 2*integrate(-(d*g*x*log(e)^2 + c*g*log(e)^2 + (d*g*x + c*g)*lo
g(b*x + a)^2 + 2*(d*g*x*log(e) + c*g*log(e))*log(b*x + a) - ((2*g*log(e) -
g)*d*x + 2*c*g*log(e) - d*f + 2*(d*g*x + c*g)*log(b*x + a))*log(d*x + c))/(
d*g^4*x^4 + c*f^3*g + (3*d*f*g^3 + c*g^4)*x^3 + 3*(d*f^2*g^2 + c*f*g^3)*x^2
 + (d*f^3*g + 3*c*f^2*g^2)*x), x)) - 1/2*A^2/(g^3*x^2 + 2*f*g^2*x + f^2*g)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(A+B\,\ln\left(\frac{e\,(a+b\,x)}{c+d\,x}\right)\right)^2}{\left(f+g\,x\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*log((e*(a + b*x))/(c + d*x)))^2/(f + g*x)^3,x)
```

```
[Out] int((A + B*log((e*(a + b*x))/(c + d*x)))^2/(f + g*x)^3, x)
```

**sympy [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*ln(e*(b*x+a)/(d*x+c)))**2/(g*x+f)**3,x)
```

```
[Out] Timed out
```

## 3.247 $\int \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(f+gx)^4}\,dx$

**Optimal**. Leaf size=714

$$\frac{2B(bc-ad)\left(a^2d^2g^2-abdg(3df-cg)+b^2\left(c^2g^2-3cdfg+3d^2f^2\right)\right)\log\left(1-\frac{(a+bx)(df-cg)}{(c+dx)(bf-ag)}\right)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{3(bf-ag)^3(df-cg)^3}$$

```
[Out] 1/3*B^2*(-a*d+b*c)^2*g^2*(d*x+c)/(-a*g+b*f)^2/(-c*g+d*f)^3/(g*x+f)+1/3*B^2*
(-a*d+b*c)^3*g^2*ln((b*x+a)/(d*x+c))/(-a*g+b*f)^3/(-c*g+d*f)^3-1/3*B*(-a*d+
b*c)*g^2*(d*x+c)^2*(A+B*ln(e*(b*x+a)/(d*x+c)))/(-a*g+b*f)/(-c*g+d*f)^3/(g*x
+f)^2+2/3*B*(-a*d+b*c)*g*(-2*a*d*g-b*c*g+3*b*d*f)*(b*x+a)*(A+B*ln(e*(b*x+a)
/(d*x+c)))/(-a*g+b*f)^3/(-c*g+d*f)^2/(g*x+f)+1/3*b^3*(A+B*ln(e*(b*x+a)/(d*x
+c)))^2/g/(-a*g+b*f)^3-1/3*(A+B*ln(e*(b*x+a)/(d*x+c)))^2/g/(g*x+f)^3-1/3*B^
2*(-a*d+b*c)^3*g^2*ln((g*x+f)/(d*x+c))/(-a*g+b*f)^3/(-c*g+d*f)^3+2/3*B^2*(-
a*d+b*c)^2*g*(-2*a*d*g-b*c*g+3*b*d*f)*ln((g*x+f)/(d*x+c))/(-a*g+b*f)^3/(-c*
g+d*f)^3+2/3*B*(-a*d+b*c)*(a^2*d^2*g^2-a*b*d*g*(-c*g+3*d*f)+b^2*(c^2*g^2-3*
c*d*f*g+3*d^2*f^2))*(A+B*ln(e*(b*x+a)/(d*x+c)))*ln(1-(-c*g+d*f)*(b*x+a)/(-a
*g+b*f)/(d*x+c))/(-a*g+b*f)^3/(-c*g+d*f)^3+2/3*B^2*(-a*d+b*c)*(a^2*d^2*g^2-
a*b*d*g*(-c*g+3*d*f)+b^2*(c^2*g^2-3*c*d*f*g+3*d^2*f^2))*polylog(2,(-c*g+d*f
)*(b*x+a)/(-a*g+b*f)/(d*x+c))/(-a*g+b*f)^3/(-c*g+d*f)^3
```

**Rubi** [A] time = 2.38, antiderivative size = 1356, normalized size of antiderivative = 1.90, number of steps used = 40, number of rules used = 11, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.379, Rules used = {2525, 12, 2528, 2524, 2418, 2390, 2301, 2394, 2393, 2391, 72}

$$-\frac{B^2\log^2(a+bx)b^3}{3g(bf-ag)^3}+\frac{2B\log(a+bx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)b^3}{3g(bf-ag)^3}+\frac{2B^2\log(a+bx)\log\left(\frac{b(c+dx)}{bc-ad}\right)b^3}{3g(bf-ag)^3}+\frac{2B^2\text{PolyLog}\left(2\right.}{3g(bf-}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Log[(e*(a + b*x))/(c + d*x)])^2/(f + g*x)^4,x]
```

```
[Out] -(B^2*(b*c - a*d)^2*g)/(3*(b*f - a*g)^2*(d*f - c*g)^2*(f + g*x)) + (b^2*B^2
*(b*c - a*d)*Log[a + b*x])/(3*(b*f - a*g)^3*(d*f - c*g)) + (2*b*B^2*(b*c -
a*d)*(2*b*d*f - b*c*g - a*d*g)*Log[a + b*x])/(3*(b*f - a*g)^3*(d*f - c*g)^2
) - (b^3*B^2*Log[a + b*x]^2)/(3*g*(b*f - a*g)^3) - (B*(b*c - a*d)*(A + B*Lo
g[(e*(a + b*x))/(c + d*x)]))/(3*(b*f - a*g)*(d*f - c*g)*(f + g*x)^2) - (2*B
*(b*c - a*d)*(2*b*d*f - b*c*g - a*d*g)*(A + B*Log[(e*(a + b*x))/(c + d*x)])
)/(3*(b*f - a*g)^2*(d*f - c*g)^2*(f + g*x)) + (2*b^3*B*Log[a + b*x]*(A + B*
Log[(e*(a + b*x))/(c + d*x)]))/(3*g*(b*f - a*g)^3) - (A + B*Log[(e*(a + b*x
```

```
))/(c + d*x)])^2/(3*g*(f + g*x)^3) - (B^2*d^2*(b*c - a*d)*Log[c + d*x])/(3*
(b*f - a*g)*(d*f - c*g)^3) - (2*B^2*d*(b*c - a*d)*(2*b*d*f - b*c*g - a*d*g)
*Log[c + d*x])/(3*(b*f - a*g)^2*(d*f - c*g)^3) + (2*B^2*d^3*Log[-((d*(a + b
*x))/(b*c - a*d))]*Log[c + d*x])/(3*g*(d*f - c*g)^3) - (2*B*d^3*(A + B*Log[
(e*(a + b*x))/(c + d*x)])*Log[c + d*x])/(3*g*(d*f - c*g)^3) - (B^2*d^3*Log[
c + d*x]^2)/(3*g*(d*f - c*g)^3) + (2*b^3*B^2*Log[a + b*x]*Log[(b*(c + d*x))
/(b*c - a*d)])/(3*g*(b*f - a*g)^3) + (B^2*(b*c - a*d)^2*g*(2*b*d*f - b*c*g
- a*d*g)*Log[f + g*x])/((b*f - a*g)^3*(d*f - c*g)^3) - (2*B^2*(b*c - a*d)*(
a^2*d^2*g^2 - a*b*d*g*(3*d*f - c*g) + b^2*(3*d^2*f^2 - 3*c*d*f*g + c^2*g^2)
)*Log[-((g*(a + b*x))/(b*f - a*g))]*Log[f + g*x])/(3*(b*f - a*g)^3*(d*f - c
*g)^3) + (2*B*(b*c - a*d)*(a^2*d^2*g^2 - a*b*d*g*(3*d*f - c*g) + b^2*(3*d^2
*f^2 - 3*c*d*f*g + c^2*g^2))*(A + B*Log[(e*(a + b*x))/(c + d*x)])*Log[f + g
*x])/(3*(b*f - a*g)^3*(d*f - c*g)^3) + (2*B^2*(b*c - a*d)*(a^2*d^2*g^2 - a*
b*d*g*(3*d*f - c*g) + b^2*(3*d^2*f^2 - 3*c*d*f*g + c^2*g^2))*Log[-((g*(c +
d*x))/(d*f - c*g))]*Log[f + g*x])/(3*(b*f - a*g)^3*(d*f - c*g)^3) + (2*b^3*
B^2*PolyLog[2, -((d*(a + b*x))/(b*c - a*d))])/(3*g*(b*f - a*g)^3) + (2*B^2*
d^3*PolyLog[2, (b*(c + d*x))/(b*c - a*d)])/(3*g*(d*f - c*g)^3) - (2*B^2*(b*
c - a*d)*(a^2*d^2*g^2 - a*b*d*g*(3*d*f - c*g) + b^2*(3*d^2*f^2 - 3*c*d*f*g
+ c^2*g^2))*PolyLog[2, (b*(f + g*x))/(b*f - a*g)])/(3*(b*f - a*g)^3*(d*f -
c*g)^3) + (2*B^2*(b*c - a*d)*(a^2*d^2*g^2 - a*b*d*g*(3*d*f - c*g) + b^2*(3*
d^2*f^2 - 3*c*d*f*g + c^2*g^2))*PolyLog[2, (d*(f + g*x))/(d*f - c*g)])/(3*(
b*f - a*g)^3*(d*f - c*g)^3)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 72

```
Int[((e_.) + (f_.)*(x_))^(p_.)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
 x_Symbol] :> Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x]
 /; FreeQ[{a, b, c, d, e, f}, x] && IntegerQ[p]
```

Rule 2301

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

Rule 2390

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_))^(q_.), x_Symbol] :> Dist[1/e, Subst[Int[((f*x)/d)^q*(a + b*Log[c*x^
n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && E
qQ[e*f - d*g, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2393

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + (c*e*x)/g])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2394

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d + e*x
)^n]))/g, x] - Dist[(b*e*n)/g, Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2418

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*(RFx_), x_Sy
mbol] :> With[{u = ExpandIntegrand[(a + b*Log[c*(d + e*x)^n])^p, RFx, x]},
Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d, e, n}, x] && RationalFunctionQ[
RFx, x] && IntegerQ[p]
```

Rule 2524

```
Int[((a_.) + Log[(c_.)*(RFx_)^(p_.)]*(b_.))^(n_.)/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[(Log[d + e*x]*(a + b*Log[c*RFx^p])^n)/e, x] - Dist[(b*n*p)/e
, Int[(Log[d + e*x]*(a + b*Log[c*RFx^p])^(n - 1)*D[RFx, x])/RFx, x], x] /;
FreeQ[{a, b, c, d, e, p}, x] && RationalFunctionQ[RFx, x] && IGtQ[n, 0]
```

Rule 2525

```
Int[((a_.) + Log[(c_.)*(RFx_)^(p_.)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_))^(m_.
), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(a + b*Log[c*RFx^p])^n)/(e*(m + 1))
, x] - Dist[(b*n*p)/(e*(m + 1)), Int[SimplifyIntegrand[((d + e*x)^(m + 1)*(
a + b*Log[c*RFx^p])^(n - 1)*D[RFx, x])/RFx, x], x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && RationalFunctionQ[RFx, x] && IGtQ[n, 0] && (EqQ[n, 1] ||
IntegerQ[m]) && NeQ[m, -1]
```

Rule 2528

```
Int[((a_.) + Log[(c_.)*(RFx_)^(p_.)]*(b_.))^(n_.)*(RGx_), x_Symbol] :> With
```

```
[{u = ExpandIntegrand[(a + b*Log[c*RFx^p])^n, RGx, x]}, Int[u, x] /; SumQ[u
]] /; FreeQ[{a, b, c, p}, x] && RationalFunctionQ[RFx, x] && RationalFuncti
onQ[RGx, x] && IGtQ[n, 0]
```

Rubi steps

$$\int \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(f+gx)^4}\,dx = -\frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{3g(f+gx)^3} + \frac{(2B)\int \frac{(bc-ad)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(a+bx)(c+dx)(f+gx)^3}\,dx}{3g}$$

$$= -\frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{3g(f+gx)^3} + \frac{(2B(bc-ad))\int \frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{(a+bx)(c+dx)(f+gx)^3}\,dx}{3g}$$

$$= -\frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{3g(f+gx)^3} + \frac{(2B(bc-ad))\int \left(\frac{b^4\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(bc-ad)(bf-ag)^3(a+bx)} + \frac{d^4\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right.\right.}{(bc-ad)(-df+cg)^3(c}\right.}{}$$

$$= -\frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{3g(f+gx)^3} + \frac{\left(2b^4B\right)\int \frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{a+bx}\,dx}{3g(bf-ag)^3} - \frac{\left(2Bd^4\right)\int \frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{c+dx}}{3g(dg-cg)^3}$$

$$= -\frac{B(bc-ad)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{3(bf-ag)(df-cg)(f+gx)^2} - \frac{2B(bc-ad)(2bdf-bcg-adg)\left(A+B\log\left(\right.\right.}{3(bf-ag)^2(df-cg)^2(f+gx)}$$

$$= -\frac{B(bc-ad)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{3(bf-ag)(df-cg)(f+gx)^2} - \frac{2B(bc-ad)(2bdf-bcg-adg)\left(A+B\log\left(\right.\right.}{3(bf-ag)^2(df-cg)^2(f+gx)}$$

$$= -\frac{B(bc-ad)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{3(bf-ag)(df-cg)(f+gx)^2} - \frac{2B(bc-ad)(2bdf-bcg-adg)\left(A+B\log\left(\right.\right.}{3(bf-ag)^2(df-cg)^2(f+gx)}$$

$$= -\frac{B^2(bc-ad)^2g}{3(bf-ag)^2(df-cg)^2(f+gx)} + \frac{b^2B^2(bc-ad)\log(a+bx)}{3(bf-ag)^3(df-cg)} + \frac{2bB^2(bc-ad)(2}{3(bf}$$

$$= -\frac{B^2(bc-ad)^2g}{3(bf-ag)^2(df-cg)^2(f+gx)} + \frac{b^2B^2(bc-ad)\log(a+bx)}{3(bf-ag)^3(df-cg)} + \frac{2bB^2(bc-ad)(2}{3(bf}$$

$$= -\frac{B^2(bc-ad)^2g}{3(bf-ag)^2(df-cg)^2(f+gx)} + \frac{b^2B^2(bc-ad)\log(a+bx)}{3(bf-ag)^3(df-cg)} + \frac{2bB^2(bc-ad)(2}{3(bf}$$

$$= -\frac{B^2(bc-ad)^2g}{3(bf-ag)^2(df-cg)^2(f+gx)} + \frac{b^2B^2(bc-ad)\log(a+bx)}{3(bf-ag)^3(df-cg)} + \frac{2bB^2(bc-ad)(2}{3(bf}$$

**Mathematica [A]** time = 3.18, size = 894, normalized size = 1.25

$$-\frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2+\frac{B(f+gx)\left(2d^3(f+gx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)\log(c+dx)(bf-ag)^3-Bd^3(f+gx)^2\left(\left(2\log\left(\frac{d(a+bx)}{ad-bc}\right)-\log(c+dx)\right)\log(c+dx)+\cdots\right.\right.}{\cdots}}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Log[(e*(a + b*x))/(c + d*x)])^2/(f + g*x)^4,x]
```

```
[Out] -1/3*((A + B*Log[(e*(a + b*x))/(c + d*x)])^2 + (B*(f + g*x)*((b*c - a*d)*g*
(b*f - a*g)^2*(d*f - c*g)^2*(A + B*Log[(e*(a + b*x))/(c + d*x)]) + 2*(b*c -
 a*d)*g*(b*f - a*g)*(-(d*f) + c*g)*(-2*b*d*f + b*c*g + a*d*g)*(f + g*x)*(A
+ B*Log[(e*(a + b*x))/(c + d*x)]) - 2*b^3*(d*f - c*g)^3*(f + g*x)^2*Log[a +
 b*x]*(A + B*Log[(e*(a + b*x))/(c + d*x)]) + 2*d^3*(b*f - a*g)^3*(f + g*x)^
2*(A + B*Log[(e*(a + b*x))/(c + d*x)])*Log[c + d*x] - 2*(b*c - a*d)*g*(a^2*
d^2*g^2 + a*b*d*g*(-3*d*f + c*g) + b^2*(3*d^2*f^2 - 3*c*d*f*g + c^2*g^2))*(
f + g*x)^2*(A + B*Log[(e*(a + b*x))/(c + d*x)])*Log[f + g*x] - 2*B*(b*c - a
*d)*g*(2*b*d*f - b*c*g - a*d*g)*(f + g*x)^2*(b*(d*f - c*g)*Log[a + b*x] + (
-(b*d*f) + a*d*g)*Log[c + d*x] + (b*c - a*d)*g*Log[f + g*x]) + B*(b*c - a*d
)*g*(f + g*x)*((b*c - a*d)*g*(b*f - a*g)*(d*f - c*g) - b^2*(d*f - c*g)^2*(f
 + g*x)*Log[a + b*x] + d^2*(b*f - a*g)^2*(f + g*x)*Log[c + d*x] + (b*c - a*
d)*g*(-2*b*d*f + b*c*g + a*d*g)*(f + g*x)*Log[f + g*x]) + b^3*B*(d*f - c*g)
^3*(f + g*x)^2*(Log[a + b*x]*(Log[a + b*x] - 2*Log[(b*(c + d*x))/(b*c - a*d
)]) - 2*PolyLog[2, (d*(a + b*x))/(-(b*c) + a*d)]) - B*d^3*(b*f - a*g)^3*(f
+ g*x)^2*((2*Log[(d*(a + b*x))/(-(b*c) + a*d)] - Log[c + d*x])*Log[c + d*x]
 + 2*PolyLog[2, (b*(c + d*x))/(b*c - a*d)]) + 2*B*(b*c - a*d)*g*(a^2*d^2*g^
2 + a*b*d*g*(-3*d*f + c*g) + b^2*(3*d^2*f^2 - 3*c*d*f*g + c^2*g^2))*(f + g*
x)^2*((Log[(g*(a + b*x))/(-(b*f) + a*g)] - Log[(g*(c + d*x))/(-(d*f) + c*g)
])*Log[f + g*x] + PolyLog[2, (b*(f + g*x))/(b*f - a*g)] - PolyLog[2, (d*(f
+ g*x))/(d*f - c*g)])))/((b*f - a*g)^3*(d*f - c*g)^3))/(g*(f + g*x)^3)
```

**fricas [F]** time = 1.16, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{B^2\log\left(\frac{bex+ae}{dx+c}\right)^2+2\,AB\log\left(\frac{bex+ae}{dx+c}\right)+A^2}{g^4x^4+4\,fg^3x^3+6\,f^2g^2x^2+4\,f^3gx+f^4},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*(b*x+a)/(d*x+c)))^2/(g*x+f)^4,x, algorithm="fricas")
```

```
[Out] integral((B^2*log((b*e*x + a*e)/(d*x + c))^2 + 2*A*B*log((b*e*x + a*e)/(d*x
 + c)) + A^2)/(g^4*x^4 + 4*f*g^3*x^3 + 6*f^2*g^2*x^2 + 4*f^3*g*x + f^4), x)
```

**giac [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*(b*x+a)/(d*x+c)))^2/(g*x+f)^4,x, algorithm="giac")
[Out] Timed out
```

**maple [F]** time = 4.24, size = 0, normalized size = 0.00

$$\int \frac{\left(B\ln\left(\frac{(bx+a)e}{dx+c}\right)+A\right)^2}{\left(gx+f\right)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*ln((b*x+a)/(d*x+c)*e)+A)^2/(g*x+f)^4,x)
[Out] int((B*ln((b*x+a)/(d*x+c)*e)+A)^2/(g*x+f)^4,x)
```

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*(b*x+a)/(d*x+c)))^2/(g*x+f)^4,x, algorithm="maxima")
[Out] 1/3*(2*b^3*log(b*x + a)/(b^3*f^3*g - 3*a*b^2*f^2*g^2 + 3*a^2*b*f*g^3 - a^3*
g^4) - 2*d^3*log(d*x + c)/(d^3*f^3*g - 3*c*d^2*f^2*g^2 + 3*c^2*d*f*g^3 - c^
3*g^4) + 2*(3*(b^3*c*d^2 - a*b^2*d^3)*f^2 - 3*(b^3*c^2*d - a^2*b*d^3)*f*g +
 (b^3*c^3 - a^3*d^3)*g^2)*log(g*x + f)/(b^3*d^3*f^6 + a^3*c^3*g^6 - 3*(b^3*
c*d^2 + a*b^2*d^3)*f^5*g + 3*(b^3*c^2*d + 3*a*b^2*c*d^2 + a^2*b*d^3)*f^4*g^
2 - (b^3*c^3 + 9*a*b^2*c^2*d + 9*a^2*b*c*d^2 + a^3*d^3)*f^3*g^3 + 3*(a*b^2*
c^3 + 3*a^2*b*c^2*d + a^3*c*d^2)*f^2*g^4 - 3*(a^2*b*c^3 + a^3*c^2*d)*f*g^5)
 - (5*(b^2*c*d - a*b*d^2)*f^2 - 3*(b^2*c^2 - a^2*d^2)*f*g + (a*b*c^2 - a^2*
c*d)*g^2 + 2*(2*(b^2*c*d - a*b*d^2)*f*g - (b^2*c^2 - a^2*d^2)*g^2)*x)/(b^2*
d^2*f^6 + a^2*c^2*f^2*g^4 - 2*(b^2*c*d + a*b*d^2)*f^5*g + (b^2*c^2 + 4*a*b*
c*d + a^2*d^2)*f^4*g^2 - 2*(a*b*c^2 + a^2*c*d)*f^3*g^3 + (b^2*d^2*f^4*g^2 +
 a^2*c^2*g^6 - 2*(b^2*c*d + a*b*d^2)*f^3*g^3 + (b^2*c^2 + 4*a*b*c*d + a^2*d
^2)*f^2*g^4 - 2*(a*b*c^2 + a^2*c*d)*f*g^5)*x^2 + 2*(b^2*d^2*f^5*g + a^2*c^2
*f*g^5 - 2*(b^2*c*d + a*b*d^2)*f^4*g^2 + (b^2*c^2 + 4*a*b*c*d + a^2*d^2)*f^
3*g^3 - 2*(a*b*c^2 + a^2*c*d)*f^2*g^4)*x) - 2*log(b*e*x/(d*x + c) + a*e/(d*
x + c))/(g^4*x^3 + 3*f*g^3*x^2 + 3*f^2*g^2*x + f^3*g))*A*B - 1/3*B^2*(log(d
```

```
*x + c)^2/(g^4*x^3 + 3*f*g^3*x^2 + 3*f^2*g^2*x + f^3*g) + 3*integrate(-1/3*
(3*d*g*x*log(e)^2 + 3*c*g*log(e)^2 + 3*(d*g*x + c*g)*log(b*x + a)^2 + 6*(d*
g*x*log(e) + c*g*log(e))*log(b*x + a) - 2*((3*g*log(e) - g)*d*x + 3*c*g*log
(e) - d*f + 3*(d*g*x + c*g)*log(b*x + a))*log(d*x + c))/(d*g^5*x^5 + c*f^4*
g + (4*d*f*g^4 + c*g^5)*x^4 + 2*(3*d*f^2*g^3 + 2*c*f*g^4)*x^3 + 2*(2*d*f^3*
g^2 + 3*c*f^2*g^3)*x^2 + (d*f^4*g + 4*c*f^3*g^2)*x), x)) - 1/3*A^2/(g^4*x^3
 + 3*f*g^3*x^2 + 3*f^2*g^2*x + f^3*g)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(A+B\,\ln\left(\frac{e\,\left(a+b\,x\right)}{c+d\,x}\right)\right)^2}{\left(f+g\,x\right)^4} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*log((e*(a + b*x))/(c + d*x)))^2/(f + g*x)^4,x)
[Out] int((A + B*log((e*(a + b*x))/(c + d*x)))^2/(f + g*x)^4, x)
```

**sympy [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*ln(e*(b*x+a)/(d*x+c)))**2/(g*x+f)**4,x)
[Out] Timed out
```

# 3.248 $\int \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(f+gx)^5}\,dx$

**Optimal**. Leaf size=1159

$$\frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2 b^4}{4g(bf-ag)^4}-\frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{4g(f+gx)^4}+\frac{B^2(bc-ad)^3g^2(4bdf-bcg-3adg)\log\left(\frac{a+bx}{c+dx}\right)}{4(bf-ag)^4(df-cg)^4}-\frac{B^2(bc-ad)^4}{6(bf-ag)}$$

```
[Out] -1/12*B^2*(-a*d+b*c)^2*g^3*(d*x+c)^2/(-a*g+b*f)^2/(-c*g+d*f)^4/(g*x+f)^2-1/
6*B^2*(-a*d+b*c)^3*g^3*(d*x+c)/(-a*g+b*f)^3/(-c*g+d*f)^4/(g*x+f)+1/4*B^2*(-
a*d+b*c)^2*g^2*(-3*a*d*g-b*c*g+4*b*d*f)*(d*x+c)/(-a*g+b*f)^3/(-c*g+d*f)^4/(
g*x+f)-1/6*B^2*(-a*d+b*c)^4*g^3*ln((b*x+a)/(d*x+c))/(-a*g+b*f)^4/(-c*g+d*f)
^4+1/4*B^2*(-a*d+b*c)^3*g^2*(-3*a*d*g-b*c*g+4*b*d*f)*ln((b*x+a)/(d*x+c))/(-
a*g+b*f)^4/(-c*g+d*f)^4+1/6*B*(-a*d+b*c)*g^3*(d*x+c)^3*(A+B*ln(e*(b*x+a)/(d
*x+c)))/(-a*g+b*f)/(-c*g+d*f)^4/(g*x+f)^3-1/4*B*(-a*d+b*c)*g^2*(-3*a*d*g-b*
c*g+4*b*d*f)*(d*x+c)^2*(A+B*ln(e*(b*x+a)/(d*x+c)))/(-a*g+b*f)^2/(-c*g+d*f)^
4/(g*x+f)^2+1/2*B*(-a*d+b*c)*g*(3*a^2*d^2*g^2-2*a*b*d*g*(-c*g+4*d*f)+b^2*(c
^2*g^2-4*c*d*f*g+6*d^2*f^2))*(b*x+a)*(A+B*ln(e*(b*x+a)/(d*x+c)))/(-a*g+b*f)
^4/(-c*g+d*f)^3/(g*x+f)+1/4*b^4*(A+B*ln(e*(b*x+a)/(d*x+c)))^2/g/(-a*g+b*f)^
4-1/4*(A+B*ln(e*(b*x+a)/(d*x+c)))^2/g/(g*x+f)^4+1/6*B^2*(-a*d+b*c)^4*g^3*ln
((g*x+f)/(d*x+c))/(-a*g+b*f)^4/(-c*g+d*f)^4-1/4*B^2*(-a*d+b*c)^3*g^2*(-3*a*
d*g-b*c*g+4*b*d*f)*ln((g*x+f)/(d*x+c))/(-a*g+b*f)^4/(-c*g+d*f)^4+1/2*B^2*(-
a*d+b*c)^2*g*(3*a^2*d^2*g^2-2*a*b*d*g*(-c*g+4*d*f)+b^2*(c^2*g^2-4*c*d*f*g+6
*d^2*f^2))*ln((g*x+f)/(d*x+c))/(-a*g+b*f)^4/(-c*g+d*f)^4-1/2*B*(-a*d+b*c)*(
-a*d*g-b*c*g+2*b*d*f)*(2*a*b*d^2*f*g-a^2*d^2*g^2-b^2*(c^2*g^2-2*c*d*f*g+2*d
^2*f^2))*(A+B*ln(e*(b*x+a)/(d*x+c)))*ln(1-(-c*g+d*f)*(b*x+a)/(-a*g+b*f)/(d*
x+c))/(-a*g+b*f)^4/(-c*g+d*f)^4-1/2*B^2*(-a*d+b*c)*(-a*d*g-b*c*g+2*b*d*f)*(
2*a*b*d^2*f*g-a^2*d^2*g^2-b^2*(c^2*g^2-2*c*d*f*g+2*d^2*f^2))*polylog(2,(-c*
g+d*f)*(b*x+a)/(-a*g+b*f)/(d*x+c))/(-a*g+b*f)^4/(-c*g+d*f)^4
```

**Rubi** [A] time = 3.40, antiderivative size = 1881, normalized size of antiderivative = 1.62, number of steps used = 44, number of rules used = 11, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.379, Rules used = {2525, 12, 2528, 2524, 2418, 2390, 2301, 2394, 2393, 2391, 72}

result too large to display

Antiderivative was successfully verified.

```
[In] Int[(A + B*Log[(e*(a + b*x))/(c + d*x)])^2/(f + g*x)^5,x]
```

```
[Out] -(B^2*(b*c - a*d)^2*g)/(12*(b*f - a*g)^2*(d*f - c*g)^2*(f + g*x)^2) - (5*B^
2*(b*c - a*d)^2*g*(2*b*d*f - b*c*g - a*d*g))/(12*(b*f - a*g)^3*(d*f - c*g)^
3*(f + g*x)) + (b^3*B^2*(b*c - a*d)*Log[a + b*x])/(6*(b*f - a*g)^4*(d*f - c
```

```
*g)) + (b^2*B^2*(b*c - a*d)*(2*b*d*f - b*c*g - a*d*g)*Log[a + b*x])/(4*(b*f
 - a*g)^4*(d*f - c*g)^2) + (b*B^2*(b*c - a*d)*(a^2*d^2*g^2 - a*b*d*g*(3*d*f
 - c*g) + b^2*(3*d^2*f^2 - 3*c*d*f*g + c^2*g^2))*Log[a + b*x])/(2*(b*f - a*
g)^4*(d*f - c*g)^3) - (b^4*B^2*Log[a + b*x]^2)/(4*g*(b*f - a*g)^4) - (B*(b*
c - a*d)*(A + B*Log[(e*(a + b*x))/(c + d*x)]))/(6*(b*f - a*g)*(d*f - c*g)*(
f + g*x)^3) - (B*(b*c - a*d)*(2*b*d*f - b*c*g - a*d*g)*(A + B*Log[(e*(a + b
*x))/(c + d*x)]))/(4*(b*f - a*g)^2*(d*f - c*g)^2*(f + g*x)^2) - (B*(b*c - a
*d)*(a^2*d^2*g^2 - a*b*d*g*(3*d*f - c*g) + b^2*(3*d^2*f^2 - 3*c*d*f*g + c^2
*g^2))*(A + B*Log[(e*(a + b*x))/(c + d*x)]))/(2*(b*f - a*g)^3*(d*f - c*g)^3
*(f + g*x)) + (b^4*B*Log[a + b*x]*(A + B*Log[(e*(a + b*x))/(c + d*x)]))/(2*
g*(b*f - a*g)^4) - (A + B*Log[(e*(a + b*x))/(c + d*x)])^2/(4*g*(f + g*x)^4)
 - (B^2*d^3*(b*c - a*d)*Log[c + d*x])/(6*(b*f - a*g)*(d*f - c*g)^4) - (B^2*
d^2*(b*c - a*d)*(2*b*d*f - b*c*g - a*d*g)*Log[c + d*x])/(4*(b*f - a*g)^2*(d
*f - c*g)^4) - (B^2*d*(b*c - a*d)*(a^2*d^2*g^2 - a*b*d*g*(3*d*f - c*g) + b^
2*(3*d^2*f^2 - 3*c*d*f*g + c^2*g^2))*Log[c + d*x])/(2*(b*f - a*g)^3*(d*f -
c*g)^4) + (B^2*d^4*Log[-((d*(a + b*x))/(b*c - a*d))]*Log[c + d*x])/(2*g*(d*
f - c*g)^4) - (B*d^4*(A + B*Log[(e*(a + b*x))/(c + d*x)])*Log[c + d*x])/(2*
g*(d*f - c*g)^4) - (B^2*d^4*Log[c + d*x]^2)/(4*g*(d*f - c*g)^4) + (b^4*B^2*
Log[a + b*x]*Log[(b*(c + d*x))/(b*c - a*d)])/(2*g*(b*f - a*g)^4) + (B^2*(b*
c - a*d)^2*g*(2*b*d*f - b*c*g - a*d*g)^2*Log[f + g*x])/(4*(b*f - a*g)^4*(d*
f - c*g)^4) + (2*B^2*(b*c - a*d)^2*g*(a^2*d^2*g^2 - a*b*d*g*(3*d*f - c*g) +
 b^2*(3*d^2*f^2 - 3*c*d*f*g + c^2*g^2))*Log[f + g*x])/(3*(b*f - a*g)^4*(d*f
 - c*g)^4) + (B^2*(b*c - a*d)*(2*b*d*f - b*c*g - a*d*g)*(2*a*b*d^2*f*g - a^
2*d^2*g^2 - b^2*(2*d^2*f^2 - 2*c*d*f*g + c^2*g^2))*Log[-((g*(a + b*x))/(b*f
 - a*g))]*Log[f + g*x])/(2*(b*f - a*g)^4*(d*f - c*g)^4) - (B*(b*c - a*d)*(2
*b*d*f - b*c*g - a*d*g)*(2*a*b*d^2*f*g - a^2*d^2*g^2 - b^2*(2*d^2*f^2 - 2*c
*d*f*g + c^2*g^2))*(A + B*Log[(e*(a + b*x))/(c + d*x)])*Log[f + g*x])/(2*(b
*f - a*g)^4*(d*f - c*g)^4) - (B^2*(b*c - a*d)*(2*b*d*f - b*c*g - a*d*g)*(2*
a*b*d^2*f*g - a^2*d^2*g^2 - b^2*(2*d^2*f^2 - 2*c*d*f*g + c^2*g^2))*Log[-((g
*(c + d*x))/(d*f - c*g))]*Log[f + g*x])/(2*(b*f - a*g)^4*(d*f - c*g)^4) + (
b^4*B^2*PolyLog[2, -((d*(a + b*x))/(b*c - a*d))])/(2*g*(b*f - a*g)^4) + (B^
2*d^4*PolyLog[2, (b*(c + d*x))/(b*c - a*d)])/(2*g*(d*f - c*g)^4) + (B^2*(b*
c - a*d)*(2*b*d*f - b*c*g - a*d*g)*(2*a*b*d^2*f*g - a^2*d^2*g^2 - b^2*(2*d^
2*f^2 - 2*c*d*f*g + c^2*g^2))*PolyLog[2, (b*(f + g*x))/(b*f - a*g)])/(2*(b*
f - a*g)^4*(d*f - c*g)^4) - (B^2*(b*c - a*d)*(2*b*d*f - b*c*g - a*d*g)*(2*a
*b*d^2*f*g - a^2*d^2*g^2 - b^2*(2*d^2*f^2 - 2*c*d*f*g + c^2*g^2))*PolyLog[2
, (d*(f + g*x))/(d*f - c*g)])/(2*(b*f - a*g)^4*(d*f - c*g)^4)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 72

```
Int[((e_.) + (f_.)*(x_))^(p_.)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
```

```
 x_Symbol] :> Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x]
 /; FreeQ[{a, b, c, d, e, f}, x] && IntegerQ[p]
```

Rule 2301

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

Rule 2390

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_))^(q_.), x_Symbol] :> Dist[1/e, Subst[Int[((f*x)/d)^q*(a + b*Log[c*x^
n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && E
qQ[e*f - d*g, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2393

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + (c*e*x)/g])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2394

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d + e*x
)^n]))/g, x] - Dist[(b*e*n)/g, Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2418

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*(RFx_), x_Sy
mbol] :> With[{u = ExpandIntegrand[(a + b*Log[c*(d + e*x)^n])^p, RFx, x]},
Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d, e, n}, x] && RationalFunctionQ[
RFx, x] && IntegerQ[p]
```

Rule 2524

```
Int[((a_.) + Log[(c_.)*(RFx_)^(p_.)]*(b_.))^(n_.)/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[(Log[d + e*x]*(a + b*Log[c*RFx^p])^n)/e, x] - Dist[(b*n*p)/e
```

```
, Int[(Log[d + e*x]*(a + b*Log[c*RFx^p])^(n - 1)*D[RFx, x])/RFx, x], x] /;
FreeQ[{a, b, c, d, e, p}, x] && RationalFunctionQ[RFx, x] && IGtQ[n, 0]
```

Rule 2525

```
Int[((a_.) + Log[(c_.)*(RFx_)^(p_.)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_))^(m_.
), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(a + b*Log[c*RFx^p])^n)/(e*(m + 1))
, x] - Dist[(b*n*p)/(e*(m + 1)), Int[SimplifyIntegrand[((d + e*x)^(m + 1)*(
a + b*Log[c*RFx^p])^(n - 1)*D[RFx, x])/RFx, x], x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && RationalFunctionQ[RFx, x] && IGtQ[n, 0] && (EqQ[n, 1] ||
IntegerQ[m]) && NeQ[m, -1]
```

Rule 2528

```
Int[((a_.) + Log[(c_.)*(RFx_)^(p_.)]*(b_.))^(n_.)*(RGx_), x_Symbol] :> With
[{u = ExpandIntegrand[(a + b*Log[c*RFx^p])^n, RGx, x]}, Int[u, x] /; SumQ[u
]] /; FreeQ[{a, b, c, p}, x] && RationalFunctionQ[RFx, x] && RationalFuncti
onQ[RGx, x] && IGtQ[n, 0]
```

Rubi steps

$$\int \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(f+gx)^5}\,dx = -\frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{4g(f+gx)^4} + \frac{B\int \frac{(bc-ad)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(a+bx)(c+dx)(f+gx)^4}\,dx}{2g}$$

$$= -\frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{4g(f+gx)^4} + \frac{(B(bc-ad))\int \frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{(a+bx)(c+dx)(f+gx)^4}\,dx}{2g}$$

$$= -\frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{4g(f+gx)^4} + \frac{(B(bc-ad))\int \left(\frac{b^5\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(bc-ad)(bf-ag)^4(a+bx)} - \frac{d^5\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right.\right.}{(bc-ad)(-df+cg)^4(c+\cdots}\right.}{\cdots}$$

$$= -\frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{4g(f+gx)^4} + \frac{\left(b^5B\right)\int \frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{a+bx}\,dx}{2g(bf-ag)^4} - \frac{\left(Bd^5\right)\int \frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{c+dx}\,dx}{2g(df-cg)^4}\cdots$$

$$= -\frac{B(bc-ad)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{6(bf-ag)(df-cg)(f+gx)^3} - \frac{B(bc-ad)(2bdf-bcg-adg)\left(A+B\log\left(\cdots\right.\right.}{4(bf-ag)^2(df-cg)^2(f+gx)^2}\cdots$$

$$= -\frac{B(bc-ad)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{6(bf-ag)(df-cg)(f+gx)^3} - \frac{B(bc-ad)(2bdf-bcg-adg)\left(A+B\log\left(\cdots\right.\right.}{4(bf-ag)^2(df-cg)^2(f+gx)^2}\cdots$$

$$= -\frac{B(bc-ad)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{6(bf-ag)(df-cg)(f+gx)^3} - \frac{B(bc-ad)(2bdf-bcg-adg)\left(A+B\log\left(\cdots\right.\right.}{4(bf-ag)^2(df-cg)^2(f+gx)^2}\cdots$$

$$= -\frac{B^2(bc-ad)^2g}{12(bf-ag)^2(df-cg)^2(f+gx)^2} - \frac{5B^2(bc-ad)^2g(2bdf-bcg-adg)}{12(bf-ag)^3(df-cg)^3(f+gx)} + \frac{b^3B^2(\cdots}{6(b\cdots}$$

$$= -\frac{B^2(bc-ad)^2g}{12(bf-ag)^2(df-cg)^2(f+gx)^2} - \frac{5B^2(bc-ad)^2g(2bdf-bcg-adg)}{12(bf-ag)^3(df-cg)^3(f+gx)} + \frac{b^3B^2(\cdots}{6(b\cdots}$$

$$= -\frac{B^2(bc-ad)^2g}{12(bf-ag)^2(df-cg)^2(f+gx)^2} - \frac{5B^2(bc-ad)^2g(2bdf-bcg-adg)}{12(bf-ag)^3(df-cg)^3(f+gx)} + \frac{b^3B^2(\cdots}{6(b\cdots}$$

$$= -\frac{B^2(bc-ad)^2g}{12(bf-ag)^2(df-cg)^2(f+gx)^2} - \frac{5B^2(bc-ad)^2g(2bdf-bcg-adg)}{12(bf-ag)^3(df-cg)^3(f+gx)} + \frac{b^3B^2(\cdots}{6(b\cdots}$$

**Mathematica [A]** time = 7.42, size = 1448, normalized size = 1.25

$$B(bc-ad)\left(\frac{\log(a+bx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)b^4}{(bc-ad)(bf-ag)^4}-\frac{B\left(\log^2(a+bx)-2\log\left(\frac{b(c+dx)}{bc-ad}\right)\log(a+bx)-2\text{Li}_2\left(-\frac{d(a+bx)}{bc-ad}\right)\right)b^4}{2(bc-ad)(bf-ag)^4}-\frac{g\left(\left(3d^2f^2-3cdgf+c^2g^2\right)b^2-adg(3d\cdots}{(bf-ag)^3(d\cdots}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Log[(e*(a + b*x))/(c + d*x)])^2/(f + g*x)^5,x]
```

```
[Out] -1/4*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2/(g*(f + g*x)^4) + (B*(b*c - a*d
)*(-1/3*(g*(A + B*Log[(e*(a + b*x))/(c + d*x)]))/((b*f - a*g)*(d*f - c*g)*(
f + g*x)^3) - (g*(2*b*d*f - b*c*g - a*d*g)*(A + B*Log[(e*(a + b*x))/(c + d*
x)]))/(2*(b*f - a*g)^2*(d*f - c*g)^2*(f + g*x)^2) - (g*(a^2*d^2*g^2 - a*b*d
*g*(3*d*f - c*g) + b^2*(3*d^2*f^2 - 3*c*d*f*g + c^2*g^2))*(A + B*Log[(e*(a
+ b*x))/(c + d*x)]))/((b*f - a*g)^3*(d*f - c*g)^3*(f + g*x)) + (b^4*Log[a +
 b*x]*(A + B*Log[(e*(a + b*x))/(c + d*x)]))/((b*c - a*d)*(b*f - a*g)^4) - (
d^4*(A + B*Log[(e*(a + b*x))/(c + d*x)])*Log[c + d*x])/((b*c - a*d)*(d*f -
c*g)^4) + (g*(2*b*d*f - b*c*g - a*d*g)*(2*b^2*d^2*f^2 - 2*b^2*c*d*f*g - 2*a
*b*d^2*f*g + b^2*c^2*g^2 + a^2*d^2*g^2)*(A + B*Log[(e*(a + b*x))/(c + d*x)]
)*Log[f + g*x])/((b*f - a*g)^4*(d*f - c*g)^4) + (B*(b*c - a*d)*g*(a^2*d^2*g
^2 - a*b*d*g*(3*d*f - c*g) + b^2*(3*d^2*f^2 - 3*c*d*f*g + c^2*g^2))*((b*Log
[a + b*x])/((b*c - a*d)*(b*f - a*g)) - (d*Log[c + d*x])/((b*c - a*d)*(d*f -
 c*g)) + (g*Log[f + g*x])/((b*f - a*g)*(d*f - c*g))))/((b*f - a*g)^3*(d*f -
 c*g)^3) - (B*(b*c - a*d)*g*(2*b*d*f - b*c*g - a*d*g)*(g/((b*f - a*g)*(d*f
- c*g)*(f + g*x)) - (b^2*Log[a + b*x])/((b*c - a*d)*(b*f - a*g)^2) + (d^2*L
og[c + d*x])/((b*c - a*d)*(d*f - c*g)^2) - (g*(2*b*d*f - b*c*g - a*d*g)*Log
[f + g*x])/((b*f - a*g)^2*(d*f - c*g)^2)))/(2*(b*f - a*g)^2*(d*f - c*g)^2)
- (B*(b*c - a*d)*g*(g/((b*f - a*g)*(d*f - c*g)*(f + g*x)^2) + (2*g*(2*b*d*f
 - b*c*g - a*d*g))/((b*f - a*g)^2*(d*f - c*g)^2*(f + g*x)) - (2*b^3*Log[a +
 b*x])/((b*c - a*d)*(b*f - a*g)^3) + (2*d^3*Log[c + d*x])/((b*c - a*d)*(d*f
 - c*g)^3) - (2*g*(a^2*d^2*g^2 - a*b*d*g*(3*d*f - c*g) + b^2*(3*d^2*f^2 - 3
*c*d*f*g + c^2*g^2))*Log[f + g*x])/((b*f - a*g)^3*(d*f - c*g)^3)))/(6*(b*f
- a*g)*(d*f - c*g)) - (b^4*B*(Log[a + b*x]^2 - 2*Log[a + b*x]*Log[(b*(c + d
*x))/(b*c - a*d)] - 2*PolyLog[2, -((d*(a + b*x))/(b*c - a*d))]))/(2*(b*c -
a*d)*(b*f - a*g)^4) + (B*d^4*(2*Log[-((d*(a + b*x))/(b*c - a*d))]*Log[c + d
*x] - Log[c + d*x]^2 + 2*PolyLog[2, (b*(c + d*x))/(b*c - a*d)]))/(2*(b*c -
a*d)*(d*f - c*g)^4) - (B*g*(2*b*d*f - b*c*g - a*d*g)*(2*b^2*d^2*f^2 - 2*b^2
*c*d*f*g - 2*a*b*d^2*f*g + b^2*c^2*g^2 + a^2*d^2*g^2)*(Log[-((g*(a + b*x))/
(b*f - a*g))]*Log[f + g*x] - Log[-((g*(c + d*x))/(d*f - c*g))]*Log[f + g*x]
 + PolyLog[2, (b*(f + g*x))/(b*f - a*g)] - PolyLog[2, (d*(f + g*x))/(d*f -
c*g)]))/((b*f - a*g)^4*(d*f - c*g)^4)))/(2*g)
```

**fricas [F]** time = 1.21, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{B^2\log\left(\frac{bex+ae}{dx+c}\right)^2+2\,AB\log\left(\frac{bex+ae}{dx+c}\right)+A^2}{g^5x^5+5\,fg^4x^4+10\,f^2g^3x^3+10\,f^3g^2x^2+5\,f^4gx+f^5},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*(b*x+a)/(d*x+c)))^2/(g*x+f)^5,x, algorithm="fricas")
```

```
[Out] integral((B^2*log((b*e*x + a*e)/(d*x + c))^2 + 2*A*B*log((b*e*x + a*e)/(d*x
 + c)) + A^2)/(g^5*x^5 + 5*f*g^4*x^4 + 10*f^2*g^3*x^3 + 10*f^3*g^2*x^2 + 5*
f^4*g*x + f^5), x)
```

**giac [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*(b*x+a)/(d*x+c)))^2/(g*x+f)^5,x, algorithm="giac")
```

```
[Out] Timed out
```

**maple [F]** time = 6.98, size = 0, normalized size = 0.00

$$\int \frac{\left(B\ln\left(\frac{(bx+a)e}{dx+c}\right)+A\right)^2}{\left(gx+f\right)^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*ln((b*x+a)/(d*x+c)*e)+A)^2/(g*x+f)^5,x)
```

```
[Out] int((B*ln((b*x+a)/(d*x+c)*e)+A)^2/(g*x+f)^5,x)
```

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*(b*x+a)/(d*x+c)))^2/(g*x+f)^5,x, algorithm="maxima")
```

```
[Out] 1/12*(6*b^4*log(b*x + a)/(b^4*f^4*g - 4*a*b^3*f^3*g^2 + 6*a^2*b^2*f^2*g^3 -
 4*a^3*b*f*g^4 + a^4*g^5) - 6*d^4*log(d*x + c)/(d^4*f^4*g - 4*c*d^3*f^3*g^2
```

```
 + 6*c^2*d^2*f^2*g^3 - 4*c^3*d*f*g^4 + c^4*g^5) + 6*(4*(b^4*c*d^3 - a*b^3*d
^4)*f^3 - 6*(b^4*c^2*d^2 - a^2*b^2*d^4)*f^2*g + 4*(b^4*c^3*d - a^3*b*d^4)*f
*g^2 - (b^4*c^4 - a^4*d^4)*g^3)*log(g*x + f)/(b^4*d^4*f^8 + a^4*c^4*g^8 - 4
*(b^4*c*d^3 + a*b^3*d^4)*f^7*g + 2*(3*b^4*c^2*d^2 + 8*a*b^3*c*d^3 + 3*a^2*b
^2*d^4)*f^6*g^2 - 4*(b^4*c^3*d + 6*a*b^3*c^2*d^2 + 6*a^2*b^2*c*d^3 + a^3*b*
d^4)*f^5*g^3 + (b^4*c^4 + 16*a*b^3*c^3*d + 36*a^2*b^2*c^2*d^2 + 16*a^3*b*c*
d^3 + a^4*d^4)*f^4*g^4 - 4*(a*b^3*c^4 + 6*a^2*b^2*c^3*d + 6*a^3*b*c^2*d^2 +
 a^4*c*d^3)*f^3*g^5 + 2*(3*a^2*b^2*c^4 + 8*a^3*b*c^3*d + 3*a^4*c^2*d^2)*f^2
*g^6 - 4*(a^3*b*c^4 + a^4*c^3*d)*f*g^7) - (26*(b^3*c*d^2 - a*b^2*d^3)*f^4 -
 31*(b^3*c^2*d - a^2*b*d^3)*f^3*g + (11*b^3*c^3 + 15*a*b^2*c^2*d - 15*a^2*b
*c*d^2 - 11*a^3*d^3)*f^2*g^2 - 7*(a*b^2*c^3 - a^3*c*d^2)*f*g^3 + 2*(a^2*b*c
^3 - a^3*c^2*d)*g^4 + 6*(3*(b^3*c*d^2 - a*b^2*d^3)*f^2*g^2 - 3*(b^3*c^2*d -
 a^2*b*d^3)*f*g^3 + (b^3*c^3 - a^3*d^3)*g^4)*x^2 + 3*(14*(b^3*c*d^2 - a*b^2
*d^3)*f^3*g - 15*(b^3*c^2*d - a^2*b*d^3)*f^2*g^2 + (5*b^3*c^3 + 3*a*b^2*c^2
*d - 3*a^2*b*c*d^2 - 5*a^3*d^3)*f*g^3 - (a*b^2*c^3 - a^3*c*d^2)*g^4)*x)/(b^
3*d^3*f^9 + a^3*c^3*f^3*g^6 - 3*(b^3*c*d^2 + a*b^2*d^3)*f^8*g + 3*(b^3*c^2*
d + 3*a*b^2*c*d^2 + a^2*b*d^3)*f^7*g^2 - (b^3*c^3 + 9*a*b^2*c^2*d + 9*a^2*b
*c*d^2 + a^3*d^3)*f^6*g^3 + 3*(a*b^2*c^3 + 3*a^2*b*c^2*d + a^3*c*d^2)*f^5*g
^4 - 3*(a^2*b*c^3 + a^3*c^2*d)*f^4*g^5 + (b^3*d^3*f^6*g^3 + a^3*c^3*g^9 - 3
*(b^3*c*d^2 + a*b^2*d^3)*f^5*g^4 + 3*(b^3*c^2*d + 3*a*b^2*c*d^2 + a^2*b*d^3
)*f^4*g^5 - (b^3*c^3 + 9*a*b^2*c^2*d + 9*a^2*b*c*d^2 + a^3*d^3)*f^3*g^6 + 3
*(a*b^2*c^3 + 3*a^2*b*c^2*d + a^3*c*d^2)*f^2*g^7 - 3*(a^2*b*c^3 + a^3*c^2*d
)*f*g^8)*x^3 + 3*(b^3*d^3*f^7*g^2 + a^3*c^3*f*g^8 - 3*(b^3*c*d^2 + a*b^2*d^
3)*f^6*g^3 + 3*(b^3*c^2*d + 3*a*b^2*c*d^2 + a^2*b*d^3)*f^5*g^4 - (b^3*c^3 +
 9*a*b^2*c^2*d + 9*a^2*b*c*d^2 + a^3*d^3)*f^4*g^5 + 3*(a*b^2*c^3 + 3*a^2*b*
c^2*d + a^3*c*d^2)*f^3*g^6 - 3*(a^2*b*c^3 + a^3*c^2*d)*f^2*g^7)*x^2 + 3*(b^
3*d^3*f^8*g + a^3*c^3*f^2*g^7 - 3*(b^3*c*d^2 + a*b^2*d^3)*f^7*g^2 + 3*(b^3*
c^2*d + 3*a*b^2*c*d^2 + a^2*b*d^3)*f^6*g^3 - (b^3*c^3 + 9*a*b^2*c^2*d + 9*a
^2*b*c*d^2 + a^3*d^3)*f^5*g^4 + 3*(a*b^2*c^3 + 3*a^2*b*c^2*d + a^3*c*d^2)*f
^4*g^5 - 3*(a^2*b*c^3 + a^3*c^2*d)*f^3*g^6)*x) - 6*log(b*e*x/(d*x + c) + a*
e/(d*x + c))/(g^5*x^4 + 4*f*g^4*x^3 + 6*f^2*g^3*x^2 + 4*f^3*g^2*x + f^4*g))
*A*B - 1/4*B^2*(log(d*x + c)^2/(g^5*x^4 + 4*f*g^4*x^3 + 6*f^2*g^3*x^2 + 4*f
^3*g^2*x + f^4*g) + 4*integrate(-1/2*(2*d*g*x*log(e)^2 + 2*c*g*log(e)^2 + 2
*(d*g*x + c*g)*log(b*x + a)^2 + 4*(d*g*x*log(e) + c*g*log(e))*log(b*x + a)
- ((4*g*log(e) - g)*d*x + 4*c*g*log(e) - d*f + 4*(d*g*x + c*g)*log(b*x + a)
)*log(d*x + c))/(d*g^6*x^6 + c*f^5*g + (5*d*f*g^5 + c*g^6)*x^5 + 5*(2*d*f^2
*g^4 + c*f*g^5)*x^4 + 10*(d*f^3*g^3 + c*f^2*g^4)*x^3 + 5*(d*f^4*g^2 + 2*c*f
^3*g^3)*x^2 + (d*f^5*g + 5*c*f^4*g^2)*x), x)) - 1/4*A^2/(g^5*x^4 + 4*f*g^4*
x^3 + 6*f^2*g^3*x^2 + 4*f^3*g^2*x + f^4*g)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(A+B\,\ln\left(\frac{e\,(a+b\,x)}{c+d\,x}\right)\right)^2}{\left(f+g\,x\right)^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*log((e*(a + b*x))/(c + d*x)))^2/(f + g*x)^5,x)
[Out] int((A + B*log((e*(a + b*x))/(c + d*x)))^2/(f + g*x)^5, x)
```

**sympy [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*ln(e*(b*x+a)/(d*x+c)))**2/(g*x+f)**5,x)
[Out] Timed out
```

# 3.249 $\int \frac{\log\left(\frac{1+x}{-1+x}\right)}{x^2} dx$

**Optimal**. Leaf size=35

$$2\log\left(-\frac{x}{1-x}\right) - \frac{(x+1)\log\left(-\frac{x+1}{1-x}\right)}{x}$$

```
[Out] 2*ln(-x/(1-x))-(1+x)*ln((-1-x)/(1-x))/x
```

**Rubi [A]** time = 0.01, antiderivative size = 34, normalized size of antiderivative = 0.97, number of steps used = 4, number of rules used = 4, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {2490, 36, 29, 31}

$$2\log(x) - 2\log(x+1) - \frac{(1-x)\log\left(-\frac{x+1}{1-x}\right)}{x}$$

Antiderivative was successfully verified.

```
[In] Int[Log[(1 + x)/(-1 + x)]/x^2,x]
```

```
[Out] 2*Log[x] - 2*Log[1 + x] - ((1 - x)*Log[-((1 + x)/(1 - x))])/x
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 2490

```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))
^(r_.)]^(s_.)/((g_.) + (h_.)*(x_))^2, x_Symbol] :> Simp[((a + b*x)*Log[e*(f
*(a + b*x)^p*(c + d*x)^q)^r]^s)/((b*g - a*h)*(g + h*x)), x] - Dist[(p*r*s*(
b*c - a*d))/(b*g - a*h), Int[Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^(s - 1)/(
(c + d*x)*(g + h*x)), x], x] /; FreeQ[{a, b, c, d, e, f, g, h, p, q, r, s},
```

```
x] && NeQ[b*c - a*d, 0] && EqQ[p + q, 0] && NeQ[b*g - a*h, 0] && IGtQ[s, 0
]
```

Rubi steps

$$\begin{aligned}
\int \frac{\log\left(\frac{1+x}{-1+x}\right)}{x^2}\,dx &= -\frac{(1-x)\log\left(-\frac{1+x}{1-x}\right)}{x} + 2\int \frac{1}{x(1+x)}\,dx \\
&= -\frac{(1-x)\log\left(-\frac{1+x}{1-x}\right)}{x} + 2\int \frac{1}{x}\,dx - 2\int \frac{1}{1+x}\,dx \\
&= 2\log(x) - 2\log(1+x) - \frac{(1-x)\log\left(-\frac{1+x}{1-x}\right)}{x}
\end{aligned}$$

**Mathematica [A]** time = 0.01, size = 30, normalized size = 0.86

$$-\log\left(1-x^2\right) + 2\log(x) - \frac{\log\left(\frac{x+1}{x-1}\right)}{x}$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[(1 + x)/(-1 + x)]/x^2,x]
[Out] 2*Log[x] - Log[(1 + x)/(-1 + x)]/x - Log[1 - x^2]
```

**fricas [A]** time = 1.00, size = 29, normalized size = 0.83

$$-\frac{x\log\left(x^2-1\right) - 2\,x\log(x) + \log\left(\frac{x+1}{x-1}\right)}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log((1+x)/(-1+x))/x^2,x, algorithm="fricas")
[Out] -(x*log(x^2 - 1) - 2*x*log(x) + log((x + 1)/(x - 1)))/x
```

**giac [B]** time = 0.42, size = 103, normalized size = 2.94

$$\frac{2\,\log\left(\frac{\frac{\frac{x+1}{x-1}+1}{\frac{x+1}{x-1}-1}+1}{\frac{\frac{x+1}{x-1}+1}{\frac{x+1}{x-1}-1}-1}\right)}{\frac{x+1}{x-1}+1} - 2\,\log\left(\frac{|x+1|}{|x-1|}\right) + 2\,\log\left(\left|\frac{x+1}{x-1}+1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log((1+x)/(-1+x))/x^2,x, algorithm="giac")
```

```
[Out] 2*log((((x + 1)/(x - 1) + 1)/((x + 1)/(x - 1) - 1) + 1)/(((x + 1)/(x - 1) +
 1)/((x + 1)/(x - 1) - 1) - 1))/((x + 1)/(x - 1) + 1) - 2*log(abs(x + 1)/ab
s(x - 1)) + 2*log(abs((x + 1)/(x - 1) + 1))
```

**maple [A]** time = 0.10, size = 46, normalized size = 1.31

$$-\frac{2\left(1+\frac{2}{x-1}\right)\ln\left(1+\frac{2}{x-1}\right)}{\frac{2}{x-1}+2}+2\ln\left(\frac{2}{x-1}+2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln((x+1)/(x-1))/x^2,x)
```

```
[Out] 2*ln(2/(x-1)+2)-2*ln(1+2/(x-1))*(1+2/(x-1))/(2/(x-1)+2)
```

**maxima [A]** time = 0.48, size = 32, normalized size = 0.91

$$-\frac{\log\left(\frac{x+1}{x-1}\right)}{x}-\log\left(x+1\right)-\log\left(x-1\right)+2\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log((1+x)/(-1+x))/x^2,x, algorithm="maxima")
```

```
[Out] -log((x + 1)/(x - 1))/x - log(x + 1) - log(x - 1) + 2*log(x)
```

**mupad [B]** time = 0.19, size = 28, normalized size = 0.80

$$2\ln(x)-\ln\left(x^2-1\right)-\frac{\ln\left(\frac{x+1}{x-1}\right)}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log((x + 1)/(x - 1))/x^2,x)
```

```
[Out] 2*log(x) - log(x^2 - 1) - log((x + 1)/(x - 1))/x
```

**sympy [A]** time = 0.15, size = 20, normalized size = 0.57

$$2\log\left(x\right)-\log\left(x^2-1\right)-\frac{\log\left(\frac{x+1}{x-1}\right)}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln((1+x)/(-1+x))/x**2,x)
```

```
[Out] 2*log(x) - log(x**2 - 1) - log((x + 1)/(x - 1))/x
```

# 3.250 $\int \frac{(f+gx)^2}{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}\,dx$

**Optimal**. Leaf size=32

$$\text{Int}\left(\frac{(f+gx)^2}{B\log\left(\frac{e(a+bx)}{c+dx}\right)+A},x\right)$$

```
[Out] Unintegrable((g*x+f)^2/(A+B*ln(e*(b*x+a)/(d*x+c))),x)
```

**Rubi [A]** time = 0.17, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{(f+gx)^2}{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(f + g*x)^2/(A + B*Log[(e*(a + b*x))/(c + d*x)]),x]
```

```
[Out] f^2*Defer[Int][(A + B*Log[(e*(a + b*x))/(c + d*x)])^(-1), x] + 2*f*g*Defer[
Int][x/(A + B*Log[(e*(a + b*x))/(c + d*x)]), x] + g^2*Defer[Int][x^2/(A + B
*Log[(e*(a + b*x))/(c + d*x)]), x]
```

Rubi steps

$$\begin{aligned}\int \frac{(f+gx)^2}{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}\,dx &= \int \left(\frac{f^2}{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}+\frac{2fgx}{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}+\frac{g^2x^2}{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}\right)dx\\ &= f^2\int \frac{1}{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}\,dx+(2fg)\int \frac{x}{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}\,dx+g^2\int \frac{x^2}{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}\end{aligned}$$

**Mathematica [A]** time = 0.43, size = 0, normalized size = 0.00

$$\int \frac{(f+gx)^2}{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(f + g*x)^2/(A + B*Log[(e*(a + b*x))/(c + d*x)]),x]
[Out] Integrate[(f + g*x)^2/(A + B*Log[(e*(a + b*x))/(c + d*x)]), x]
```

**fricas [A]** time = 0.75, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{g^2x^2 + 2\,fgx + f^2}{B\log\left(\frac{bex+ae}{dx+c}\right) + A}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^2/(A+B*log(e*(b*x+a)/(d*x+c))),x, algorithm="fricas")
[Out] integral((g^2*x^2 + 2*f*g*x + f^2)/(B*log((b*e*x + a*e)/(d*x + c)) + A), x)
```

**giac [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^2/(A+B*log(e*(b*x+a)/(d*x+c))),x, algorithm="giac")
[Out] Timed out
```

**maple [A]** time = 1.15, size = 0, normalized size = 0.00

$$\int \frac{\left(gx+f\right)^2}{B\ln\left(\frac{(bx+a)e}{dx+c}\right) + A}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x+f)^2/(B*ln((b*x+a)/(d*x+c)*e)+A),x)
[Out] int((g*x+f)^2/(B*ln((b*x+a)/(d*x+c)*e)+A),x)
```

**maxima [A]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(gx+f\right)^2}{B\log\left(\frac{(bx+a)e}{dx+c}\right) + A}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^2/(A+B*log(e*(b*x+a)/(d*x+c))),x, algorithm="maxima")
```

```
[Out] integrate((g*x + f)^2/(B*log((b*x + a)*e/(d*x + c)) + A), x)
```

**mupad** [A] time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{\left(f+g\,x\right)^2}{A+B\,\ln\left(\frac{e\,\left(a+b\,x\right)}{c+d\,x}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f + g*x)^2/(A + B*log((e*(a + b*x))/(c + d*x))),x)
[Out] int((f + g*x)^2/(A + B*log((e*(a + b*x))/(c + d*x))), x)
```

**sympy** [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(f+gx\right)^2}{A+B\log\left(\frac{ae}{c+dx}+\frac{bex}{c+dx}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)**2/(A+B*ln(e*(b*x+a)/(d*x+c))),x)
[Out] Integral((f + g*x)**2/(A + B*log(a*e/(c + d*x) + b*e*x/(c + d*x))), x)
```

# 3.251 $\int \frac{f+gx}{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}\,dx$

**Optimal**. Leaf size=30

$$\text{Int}\left(\frac{f+gx}{B\log\left(\frac{e(a+bx)}{c+dx}\right)+A},x\right)$$

```
[Out] Unintegrable((g*x+f)/(A+B*ln(e*(b*x+a)/(d*x+c))),x)
```

**Rubi [A]** time = 0.09, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{f+gx}{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(f + g*x)/(A + B*Log[(e*(a + b*x))/(c + d*x)]),x]
```

```
[Out] f*Defer[Int][(A + B*Log[(e*(a + b*x))/(c + d*x)])^(-1), x] + g*Defer[Int][x
/(A + B*Log[(e*(a + b*x))/(c + d*x)]), x]
```

Rubi steps

$$\begin{aligned}\int \frac{f+gx}{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}\,dx &= \int \left(\frac{f}{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}+\frac{gx}{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}\right)dx\\ &= f\int \frac{1}{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}\,dx+g\int \frac{x}{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}\,dx\end{aligned}$$

**Mathematica [A]** time = 0.25, size = 0, normalized size = 0.00

$$\int \frac{f+gx}{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(f + g*x)/(A + B*Log[(e*(a + b*x))/(c + d*x)]),x]
```

```
[Out] Integrate[(f + g*x)/(A + B*Log[(e*(a + b*x))/(c + d*x)]), x]
```

**fricas [A]** time = 0.90, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{gx+f}{B\log\left(\frac{bex+ae}{dx+c}\right)+A},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)/(A+B*log(e*(b*x+a)/(d*x+c))),x, algorithm="fricas")
[Out] integral((g*x + f)/(B*log((b*e*x + a*e)/(d*x + c)) + A), x)
```

**giac [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)/(A+B*log(e*(b*x+a)/(d*x+c))),x, algorithm="giac")
[Out] Timed out
```

**maple [A]** time = 0.99, size = 0, normalized size = 0.00

$$\int \frac{gx+f}{B\ln\left(\frac{(bx+a)e}{dx+c}\right)+A}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x+f)/(B*ln((b*x+a)/(d*x+c)*e)+A),x)
[Out] int((g*x+f)/(B*ln((b*x+a)/(d*x+c)*e)+A),x)
```

**maxima [A]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{gx+f}{B\log\left(\frac{(bx+a)e}{dx+c}\right)+A}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)/(A+B*log(e*(b*x+a)/(d*x+c))),x, algorithm="maxima")
[Out] integrate((g*x + f)/(B*log((b*x + a)*e/(d*x + c)) + A), x)
```

**mupad** [A] time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{f+g\,x}{A+B\,\ln\left(\frac{e\,(a+b\,x)}{c+d\,x}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f + g*x)/(A + B*log((e*(a + b*x))/(c + d*x))),x)
[Out] int((f + g*x)/(A + B*log((e*(a + b*x))/(c + d*x))), x)
```

**sympy** [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{f+gx}{A+B\log\left(\frac{ae}{c+dx}+\frac{bex}{c+dx}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)/(A+B*ln(e*(b*x+a)/(d*x+c))),x)
[Out] Integral((f + g*x)/(A + B*log(a*e/(c + d*x) + b*e*x/(c + d*x))), x)
```

# 3.252 $\int \frac{1}{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}\,dx$

**Optimal**. Leaf size=24

$$\text{Int}\left(\frac{1}{B\log\left(\frac{e(a+bx)}{c+dx}\right)+A},x\right)$$

```
[Out] Unintegrable(1/(A+B*ln(e*(b*x+a)/(d*x+c))),x)
```

**Rubi [A]** time = 0.01, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{1}{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(A + B*Log[(e*(a + b*x))/(c + d*x)])^(-1),x]
```

```
[Out] Defer[Int][(A + B*Log[(e*(a + b*x))/(c + d*x)])^(-1), x]
```

Rubi steps

$$\int \frac{1}{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}\,dx=\int \frac{1}{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}\,dx$$

**Mathematica [A]** time = 0.02, size = 0, normalized size = 0.00

$$\int \frac{1}{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(A + B*Log[(e*(a + b*x))/(c + d*x)])^(-1),x]
```

```
[Out] Integrate[(A + B*Log[(e*(a + b*x))/(c + d*x)])^(-1), x]
```

**fricas** [A] time = 0.98, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{1}{B\log\left(\frac{bex+ae}{dx+c}\right)+A},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(A+B*log(e*(b*x+a)/(d*x+c))),x, algorithm="fricas")
[Out] integral(1/(B*log((b*e*x + a*e)/(d*x + c)) + A), x)
```

**giac** [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(A+B*log(e*(b*x+a)/(d*x+c))),x, algorithm="giac")
[Out] Timed out
```

**maple** [A] time = 0.92, size = 0, normalized size = 0.00

$$\int \frac{1}{B\ln\left(\frac{(bx+a)e}{dx+c}\right)+A}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(B*ln((b*x+a)/(d*x+c)*e)+A),x)
[Out] int(1/(B*ln((b*x+a)/(d*x+c)*e)+A),x)
```

**maxima** [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{B\log\left(\frac{(bx+a)e}{dx+c}\right)+A}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(A+B*log(e*(b*x+a)/(d*x+c))),x, algorithm="maxima")
[Out] integrate(1/(B*log((b*x + a)*e/(d*x + c)) + A), x)
```

**mupad** [A] time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{1}{A+B\,\ln\left(\frac{e\,(a+b\,x)}{c+d\,x}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(A + B*log((e*(a + b*x))/(c + d*x))),x)
```

```
[Out] int(1/(A + B*log((e*(a + b*x))/(c + d*x))), x)
```

**sympy [A]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{A + B\log\left(\frac{e(a+bx)}{c+dx}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(A+B*ln(e*(b*x+a)/(d*x+c))),x)
```

```
[Out] Integral(1/(A + B*log(e*(a + b*x)/(c + d*x))), x)
```

## 3.253 $\int \frac{1}{(f+gx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}\,dx$

**Optimal**. Leaf size=32

$$\text{Int}\left(\frac{1}{(f+gx)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)},x\right)$$

```
[Out] Unintegrable(1/(g*x+f)/(A+B*ln(e*(b*x+a)/(d*x+c))),x)
```

**Rubi** [A] time = 0.07, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{1}{(f+gx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[1/((f + g*x)*(A + B*Log[(e*(a + b*x))/(c + d*x)])),x]
[Out] Defer[Int][1/((f + g*x)*(A + B*Log[(e*(a + b*x))/(c + d*x)])), x]
```

Rubi steps

$$\int \frac{1}{(f+gx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}\,dx=\int \frac{1}{(f+gx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}\,dx$$

**Mathematica** [A] time = 0.90, size = 0, normalized size = 0.00

$$\int \frac{1}{(f+gx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[1/((f + g*x)*(A + B*Log[(e*(a + b*x))/(c + d*x)])),x]
[Out] Integrate[1/((f + g*x)*(A + B*Log[(e*(a + b*x))/(c + d*x)])), x]
```

**fricas** [A] time = 0.89, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{1}{Agx + Af + \left(Bgx + Bf\right)\log\left(\frac{bex+ae}{dx+c}\right)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(g*x+f)/(A+B*log(e*(b*x+a)/(d*x+c))),x, algorithm="fricas")
[Out] integral(1/(A*g*x + A*f + (B*g*x + B*f)*log((b*e*x + a*e)/(d*x + c))), x)
```

**giac** [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(g*x+f)/(A+B*log(e*(b*x+a)/(d*x+c))),x, algorithm="giac")
[Out] Timed out
```

**maple** [A] time = 1.31, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(gx+f\right)\left(B\ln\left(\frac{(bx+a)e}{dx+c}\right)+A\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(g*x+f)/(B*ln((b*x+a)/(d*x+c)*e)+A),x)
[Out] int(1/(g*x+f)/(B*ln((b*x+a)/(d*x+c)*e)+A),x)
```

**maxima** [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(gx+f\right)\left(B\log\left(\frac{(bx+a)e}{dx+c}\right)+A\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(g*x+f)/(A+B*log(e*(b*x+a)/(d*x+c))),x, algorithm="maxima")
[Out] integrate(1/((g*x + f)*(B*log((b*x + a)*e/(d*x + c)) + A)), x)
```

**mupad** [A] time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{1}{\left(f+g\,x\right)\,\left(A+B\,\ln\left(\frac{e\,(a+b\,x)}{c+d\,x}\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((f + g*x)*(A + B*log((e*(a + b*x))/(c + d*x)))),x)
```

```
[Out] int(1/((f + g*x)*(A + B*log((e*(a + b*x))/(c + d*x)))), x)
```

**sympy [A]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(A + B\log\left(\frac{ae}{c+dx} + \frac{bex}{c+dx}\right)\right)\left(f + gx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(g*x+f)/(A+B*ln(e*(b*x+a)/(d*x+c))),x)
```

```
[Out] Integral(1/((A + B*log(a*e/(c + d*x) + b*e*x/(c + d*x)))*(f + g*x)), x)
```

# 3.254 $\int \frac{1}{(f+gx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}\,dx$

**Optimal**. Leaf size=32

$$\operatorname{Int}\left(\frac{1}{(f+gx)^2\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)},x\right)$$

```
[Out] Unintegrable(1/(g*x+f)^2/(A+B*ln(e*(b*x+a)/(d*x+c))),x)
```

**Rubi [A]** time = 0.07, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{1}{(f+gx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[1/((f + g*x)^2*(A + B*Log[(e*(a + b*x))/(c + d*x)])),x]
[Out] Defer[Int][1/((f + g*x)^2*(A + B*Log[(e*(a + b*x))/(c + d*x)])), x]
```

Rubi steps

$$\int \frac{1}{(f+gx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}\,dx=\int \frac{1}{(f+gx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}\,dx$$

**Mathematica [A]** time = 0.88, size = 0, normalized size = 0.00

$$\int \frac{1}{(f+gx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[1/((f + g*x)^2*(A + B*Log[(e*(a + b*x))/(c + d*x)])),x]
[Out] Integrate[1/((f + g*x)^2*(A + B*Log[(e*(a + b*x))/(c + d*x)])), x]
```

**fricas** [A] time = 0.75, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{1}{Ag^2x^2 + 2\,Afgx + Af^2 + \left(Bg^2x^2 + 2\,Bfgx + Bf^2\right)\log\left(\frac{bex+ae}{dx+c}\right)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(g*x+f)^2/(A+B*log(e*(b*x+a)/(d*x+c))),x, algorithm="fricas")
[Out] integral(1/(A*g^2*x^2 + 2*A*f*g*x + A*f^2 + (B*g^2*x^2 + 2*B*f*g*x + B*f^2)
*log((b*e*x + a*e)/(d*x + c))), x)
```

**giac** [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(g*x+f)^2/(A+B*log(e*(b*x+a)/(d*x+c))),x, algorithm="giac")
[Out] Timed out
```

**maple** [A] time = 1.19, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(gx + f\right)^2 \left(B\ln\left(\frac{(bx+a)e}{dx+c}\right) + A\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(g*x+f)^2/(B*ln((b*x+a)/(d*x+c)*e)+A),x)
[Out] int(1/(g*x+f)^2/(B*ln((b*x+a)/(d*x+c)*e)+A),x)
```

**maxima** [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(gx + f\right)^2\left(B\log\left(\frac{(bx+a)e}{dx+c}\right) + A\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(g*x+f)^2/(A+B*log(e*(b*x+a)/(d*x+c))),x, algorithm="maxima")
[Out] integrate(1/((g*x + f)^2*(B*log((b*x + a)*e/(d*x + c)) + A)), x)
```

**mupad** [A] time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{1}{\left(f + g\,x\right)^{2}\,\left(A + B\,\ln\left(\frac{e\,\left(a+b\,x\right)}{c+d\,x}\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((f + g*x)^2*(A + B*log((e*(a + b*x))/(c + d*x)))),x)
[Out] int(1/((f + g*x)^2*(A + B*log((e*(a + b*x))/(c + d*x)))), x)
```

**sympy** [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(A + B\log\left(\frac{ae}{c+dx} + \frac{bex}{c+dx}\right)\right)\left(f + gx\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(g*x+f)**2/(A+B*ln(e*(b*x+a)/(d*x+c))),x)
[Out] Integral(1/((A + B*log(a*e/(c + d*x) + b*e*x/(c + d*x)))*(f + g*x)**2), x)
```

## 3.255 $\int \frac{1}{(f+gx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}\,dx$

**Optimal**. Leaf size=32

$$\mathrm{Int}\left(\frac{1}{(f+gx)^3\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)},x\right)$$

```
[Out] Unintegrable(1/(g*x+f)^3/(A+B*ln(e*(b*x+a)/(d*x+c))),x)
```

**Rubi [A]** time = 0.06, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{1}{(f+gx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[1/((f + g*x)^3*(A + B*Log[(e*(a + b*x))/(c + d*x)])),x]
[Out] Defer[Int][1/((f + g*x)^3*(A + B*Log[(e*(a + b*x))/(c + d*x)])), x]
```

Rubi steps

$$\int \frac{1}{(f+gx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}\,dx=\int \frac{1}{(f+gx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}\,dx$$

**Mathematica [A]** time = 8.15, size = 0, normalized size = 0.00

$$\int \frac{1}{(f+gx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[1/((f + g*x)^3*(A + B*Log[(e*(a + b*x))/(c + d*x)])),x]
[Out] Integrate[1/((f + g*x)^3*(A + B*Log[(e*(a + b*x))/(c + d*x)])), x]
```

**fricas [A]** time = 0.77, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{1}{Ag^3x^3 + 3\,Afg^2x^2 + 3\,Af^2gx + Af^3 + \left(Bg^3x^3 + 3\,Bfg^2x^2 + 3\,Bf^2gx + Bf^3\right)\log\left(\frac{bex+ae}{dx+c}\right)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(g*x+f)^3/(A+B*log(e*(b*x+a)/(d*x+c))),x, algorithm="fricas")
```

```
[Out] integral(1/(A*g^3*x^3 + 3*A*f*g^2*x^2 + 3*A*f^2*g*x + A*f^3 + (B*g^3*x^3 +
3*B*f*g^2*x^2 + 3*B*f^2*g*x + B*f^3)*log((b*e*x + a*e)/(d*x + c))), x)
```

**giac [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(g*x+f)^3/(A+B*log(e*(b*x+a)/(d*x+c))),x, algorithm="giac")
```

```
[Out] Timed out
```

**maple [A]** time = 1.26, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(gx+f\right)^3 \left(B\ln\left(\frac{(bx+a)e}{dx+c}\right)+A\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(g*x+f)^3/(B*ln((b*x+a)/(d*x+c)*e)+A),x)
```

```
[Out] int(1/(g*x+f)^3/(B*ln((b*x+a)/(d*x+c)*e)+A),x)
```

**maxima [A]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(gx+f\right)^3 \left(B\log\left(\frac{(bx+a)e}{dx+c}\right)+A\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(g*x+f)^3/(A+B*log(e*(b*x+a)/(d*x+c))),x, algorithm="maxima")
```

```
[Out] integrate(1/((g*x + f)^3*(B*log((b*x + a)*e/(d*x + c)) + A)), x)
```

**mupad** [**A**] time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{1}{\left(f+g\,x\right)^3\,\left(A+B\,\ln\left(\frac{e\,\left(a+b\,x\right)}{c+d\,x}\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((f + g*x)^3*(A + B*log((e*(a + b*x))/(c + d*x)))),x)
[Out] int(1/((f + g*x)^3*(A + B*log((e*(a + b*x))/(c + d*x)))), x)
```

**sympy** [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(g*x+f)**3/(A+B*ln(e*(b*x+a)/(d*x+c))),x)
[Out] Timed out
```

## 3.256 $\int \frac{(f+gx)^2}{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}\,dx$

**Optimal**. Leaf size=32

$$\text{Int}\left(\frac{(f+gx)^2}{\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2},x\right)$$

```
[Out] Unintegrable((g*x+f)^2/(A+B*ln(e*(b*x+a)/(d*x+c)))^2,x)
```

**Rubi [A]** time = 0.18, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{(f+gx)^2}{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(f + g*x)^2/(A + B*Log[(e*(a + b*x))/(c + d*x)])^2,x]
```

```
[Out] f^2*Defer[Int][(A + B*Log[(e*(a + b*x))/(c + d*x)])^(-2), x] + 2*f*g*Defer[
Int][x/(A + B*Log[(e*(a + b*x))/(c + d*x)])^2, x] + g^2*Defer[Int][x^2/(A +
 B*Log[(e*(a + b*x))/(c + d*x)])^2, x]
```

Rubi steps

$$\int \frac{(f+gx)^2}{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}\,dx = \int \left(\frac{f^2}{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}+\frac{2fgx}{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}+\frac{g^2x^2}{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}\right) dx$$
$$= f^2\int \frac{1}{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}\,dx+(2fg)\int \frac{x}{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}\,dx+g^2\int \frac{}{\left(A+\right.}$$

**Mathematica [A]** time = 1.37, size = 0, normalized size = 0.00

$$\int \frac{(f+gx)^2}{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(f + g*x)^2/(A + B*Log[(e*(a + b*x))/(c + d*x)])^2,x]
[Out] Integrate[(f + g*x)^2/(A + B*Log[(e*(a + b*x))/(c + d*x)])^2, x]
```

**fricas [A]** time = 0.89, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{g^2x^2 + 2\,fgx + f^2}{B^2\log\left(\frac{bex+ae}{dx+c}\right)^2 + 2\,AB\log\left(\frac{bex+ae}{dx+c}\right) + A^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^2/(A+B*log(e*(b*x+a)/(d*x+c)))^2,x, algorithm="fricas")
[Out] integral((g^2*x^2 + 2*f*g*x + f^2)/(B^2*log((b*e*x + a*e)/(d*x + c))^2 + 2*
A*B*log((b*e*x + a*e)/(d*x + c)) + A^2), x)
```

**giac [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^2/(A+B*log(e*(b*x+a)/(d*x+c)))^2,x, algorithm="giac")
[Out] Timed out
```

**maple [A]** time = 1.11, size = 0, normalized size = 0.00

$$\int \frac{\left(gx + f\right)^2}{\left(B\ln\left(\frac{(bx+a)e}{dx+c}\right) + A\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x+f)^2/(B*ln((b*x+a)/(d*x+c)*e)+A)^2,x)
[Out] int((g*x+f)^2/(B*ln((b*x+a)/(d*x+c)*e)+A)^2,x)
```

**maxima [A]** time = 0.00, size = 0, normalized size = 0.00

$$-\frac{bdg^2x^4 + acf^2 + \left(adg^2 + \left(2\,dfg + cg^2\right)b\right)x^3 + \left(\left(2\,dfg + cg^2\right)a + \left(df^2 + 2\,cfg\right)b\right)x^2 + \left(bcf^2 + \left(df^2 + 2\,cfg\right)a}{(bc - ad)B^2\log\left(bx + a\right) - (bc - ad)B^2\log\left(dx + c\right) + (bc - ad)AB + \left(bc\log(e) - ad\log(e)\right)B^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^2/(A+B*log(e*(b*x+a)/(d*x+c)))^2,x, algorithm="maxima")
```

```
[Out] -(b*d*g^2*x^4 + a*c*f^2 + (a*d*g^2 + (2*d*f*g + c*g^2)*b)*x^3 + ((2*d*f*g +
 c*g^2)*a + (d*f^2 + 2*c*f*g)*b)*x^2 + (b*c*f^2 + (d*f^2 + 2*c*f*g)*a)*x)/(
(b*c - a*d)*B^2*log(b*x + a) - (b*c - a*d)*B^2*log(d*x + c) + (b*c - a*d)*A
*B + (b*c*log(e) - a*d*log(e))*B^2) + integrate((4*b*d*g^2*x^3 + b*c*f^2 +
3*(a*d*g^2 + (2*d*f*g + c*g^2)*b)*x^2 + (d*f^2 + 2*c*f*g)*a + 2*((2*d*f*g +
 c*g^2)*a + (d*f^2 + 2*c*f*g)*b)*x)/((b*c - a*d)*B^2*log(b*x + a) - (b*c -
a*d)*B^2*log(d*x + c) + (b*c - a*d)*A*B + (b*c*log(e) - a*d*log(e))*B^2), x
)
```

**mupad** [**A**]   time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{\left(f+g\,x\right)^2}{\left(A+B\,\ln\left(\frac{e\,\left(a+b\,x\right)}{c+d\,x}\right)\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f + g*x)^2/(A + B*log((e*(a + b*x))/(c + d*x)))^2,x)
```

```
[Out] int((f + g*x)^2/(A + B*log((e*(a + b*x))/(c + d*x)))^2, x)
```

**sympy** [**A**]   time = 0.00, size = 0, normalized size = 0.00

$$\frac{acf^2 + 2acfgx + acg^2x^2 + adf^2x + 2adfgx^2 + adg^2x^3 + bcf^2x + 2bcfgx^2 + bcg^2x^3 + bdf^2x^2 + 2bdfgx^3 + bdg^2}{ABad - ABbc + \left(B^2ad - B^2bc\right)\log\left(\frac{e(a+bx)}{c+dx}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)**2/(A+B*ln(e*(b*x+a)/(d*x+c)))**2,x)
```

```
[Out] (a*c*f**2 + 2*a*c*f*g*x + a*c*g**2*x**2 + a*d*f**2*x + 2*a*d*f*g*x**2 + a*d
*g**2*x**3 + b*c*f**2*x + 2*b*c*f*g*x**2 + b*c*g**2*x**3 + b*d*f**2*x**2 +
2*b*d*f*g*x**3 + b*d*g**2*x**4)/(A*B*a*d - A*B*b*c + (B**2*a*d - B**2*b*c)*
log(e*(a + b*x)/(c + d*x))) - (Integral(a*d*f**2/(A + B*log(a*e/(c + d*x) +
 b*e*x/(c + d*x))), x) + Integral(b*c*f**2/(A + B*log(a*e/(c + d*x) + b*e*x
/(c + d*x))), x) + Integral(2*a*c*f*g/(A + B*log(a*e/(c + d*x) + b*e*x/(c +
 d*x))), x) + Integral(2*a*c*g**2*x/(A + B*log(a*e/(c + d*x) + b*e*x/(c + d
*x))), x) + Integral(3*a*d*g**2*x**2/(A + B*log(a*e/(c + d*x) + b*e*x/(c +
d*x))), x) + Integral(3*b*c*g**2*x**2/(A + B*log(a*e/(c + d*x) + b*e*x/(c +
 d*x))), x) + Integral(2*b*d*f**2*x/(A + B*log(a*e/(c + d*x) + b*e*x/(c + d
*x))), x) + Integral(4*b*d*g**2*x**3/(A + B*log(a*e/(c + d*x) + b*e*x/(c +
d*x))), x) + Integral(4*a*d*f*g*x/(A + B*log(a*e/(c + d*x) + b*e*x/(c + d*x
```

```
))), x) + Integral(4*b*c*f*g*x/(A + B*log(a*e/(c + d*x) + b*e*x/(c + d*x)))
, x) + Integral(6*b*d*f*g*x**2/(A + B*log(a*e/(c + d*x) + b*e*x/(c + d*x)))
, x))/(B*(a*d - b*c))
```

# 3.257 $\int \frac{f+gx}{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}\,dx$

**Optimal**. Leaf size=30

$$\mathrm{Int}\left(\frac{f+gx}{\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2},x\right)$$

```
[Out] Unintegrable((g*x+f)/(A+B*ln(e*(b*x+a)/(d*x+c)))^2,x)
```

**Rubi [A]** time = 0.10, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{f+gx}{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(f + g*x)/(A + B*Log[(e*(a + b*x))/(c + d*x)])^2,x]
```

```
[Out] f*Defer[Int][(A + B*Log[(e*(a + b*x))/(c + d*x)])^(-2), x] + g*Defer[Int][x
/(A + B*Log[(e*(a + b*x))/(c + d*x)])^2, x]
```

Rubi steps

$$\begin{aligned}\int \frac{f+gx}{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}\,dx &= \int \left(\frac{f}{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}+\frac{gx}{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}\right)dx\\ &= f\int \frac{1}{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}\,dx+g\int \frac{x}{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}\,dx\end{aligned}$$

**Mathematica [A]** time = 0.95, size = 0, normalized size = 0.00

$$\int \frac{f+gx}{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(f + g*x)/(A + B*Log[(e*(a + b*x))/(c + d*x)])^2,x]
```

```
[Out] Integrate[(f + g*x)/(A + B*Log[(e*(a + b*x))/(c + d*x)])^2, x]
```

**fricas [A]** time = 1.00, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{gx+f}{B^2\log\left(\frac{bex+ae}{dx+c}\right)^2+2\,AB\log\left(\frac{bex+ae}{dx+c}\right)+A^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)/(A+B*log(e*(b*x+a)/(d*x+c)))^2,x, algorithm="fricas")
```

```
[Out] integral((g*x + f)/(B^2*log((b*e*x + a*e)/(d*x + c))^2 + 2*A*B*log((b*e*x +
 a*e)/(d*x + c)) + A^2), x)
```

**giac [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)/(A+B*log(e*(b*x+a)/(d*x+c)))^2,x, algorithm="giac")
```

```
[Out] Timed out
```

**maple [A]** time = 1.04, size = 0, normalized size = 0.00

$$\int \frac{gx+f}{\left(B\ln\left(\frac{(bx+a)e}{dx+c}\right)+A\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x+f)/(B*ln((b*x+a)/(d*x+c)*e)+A)^2,x)
```

```
[Out] int((g*x+f)/(B*ln((b*x+a)/(d*x+c)*e)+A)^2,x)
```

**maxima [A]** time = 0.00, size = 0, normalized size = 0.00

$$-\frac{bdgx^3+acf+\left(adg+\left(df+cg\right)b\right)x^2+\left(bcf+\left(df+cg\right)a\right)x}{(bc-ad)B^2\log\left(bx+a\right)-(bc-ad)B^2\log\left(dx+c\right)+(bc-ad)AB+\left(bc\log(e)-ad\log(e)\right)B^2}+\int\frac{}{(bc-ad)B^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)/(A+B*log(e*(b*x+a)/(d*x+c)))^2,x, algorithm="maxima")
```

```
[Out] -(b*d*g*x^3 + a*c*f + (a*d*g + (d*f + c*g)*b)*x^2 + (b*c*f + (d*f + c*g)*a)
*x)/((b*c - a*d)*B^2*log(b*x + a) - (b*c - a*d)*B^2*log(d*x + c) + (b*c - a
*d)*A*B + (b*c*log(e) - a*d*log(e))*B^2) + integrate((3*b*d*g*x^2 + b*c*f +
 (d*f + c*g)*a + 2*(a*d*g + (d*f + c*g)*b)*x)/((b*c - a*d)*B^2*log(b*x + a)
 - (b*c - a*d)*B^2*log(d*x + c) + (b*c - a*d)*A*B + (b*c*log(e) - a*d*log(e
))*B^2), x)
```

**mupad [A]** time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{f+g\,x}{{\left(A+B\,\ln\left(\frac{e\,\left(a+b\,x\right)}{c+d\,x}\right)\right)}^2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f + g*x)/(A + B*log((e*(a + b*x))/(c + d*x)))^2,x)
```

```
[Out] int((f + g*x)/(A + B*log((e*(a + b*x))/(c + d*x)))^2, x)
```

**sympy [A]** time = 0.00, size = 0, normalized size = 0.00

$$\frac{acf + acgx + adfx + adgx^2 + bcfx + bcgx^2 + bdfx^2 + bdgx^3}{ABad - ABbc + \left(B^2ad - B^2bc\right)\log\left(\frac{e(a+bx)}{c+dx}\right)} - \frac{\int \frac{acg}{A+B\log\left(\frac{ae}{c+dx}+\frac{bex}{c+dx}\right)}\,dx + \int \frac{adf}{A+B\log\left(\frac{ae}{c+dx}+\frac{bex}{c+dx}\right)}\,dx + \cdots}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)/(A+B*ln(e*(b*x+a)/(d*x+c)))**2,x)
```

```
[Out] (a*c*f + a*c*g*x + a*d*f*x + a*d*g*x**2 + b*c*f*x + b*c*g*x**2 + b*d*f*x**2
 + b*d*g*x**3)/(A*B*a*d - A*B*b*c + (B**2*a*d - B**2*b*c)*log(e*(a + b*x)/(
c + d*x))) - (Integral(a*c*g/(A + B*log(a*e/(c + d*x) + b*e*x/(c + d*x))),
x) + Integral(a*d*f/(A + B*log(a*e/(c + d*x) + b*e*x/(c + d*x))), x) + Inte
gral(b*c*f/(A + B*log(a*e/(c + d*x) + b*e*x/(c + d*x))), x) + Integral(2*a*
d*g*x/(A + B*log(a*e/(c + d*x) + b*e*x/(c + d*x))), x) + Integral(2*b*c*g*x
/(A + B*log(a*e/(c + d*x) + b*e*x/(c + d*x))), x) + Integral(2*b*d*f*x/(A +
 B*log(a*e/(c + d*x) + b*e*x/(c + d*x))), x) + Integral(3*b*d*g*x**2/(A + B
*log(a*e/(c + d*x) + b*e*x/(c + d*x))), x))/(B*(a*d - b*c))
```

# 3.258 $\int \frac{1}{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}\,dx$

**Optimal**. Leaf size=24

$$\text{Int}\left(\frac{1}{\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2},x\right)$$

```
[Out] Unintegrable(1/(A+B*ln(e*(b*x+a)/(d*x+c)))^2,x)
```

**Rubi** [A] time = 0.01, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{1}{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(A + B*Log[(e*(a + b*x))/(c + d*x)])^(-2),x]
```

```
[Out] Defer[Int][(A + B*Log[(e*(a + b*x))/(c + d*x)])^(-2), x]
```

Rubi steps

$$\int \frac{1}{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}\,dx=\int \frac{1}{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}\,dx$$

**Mathematica** [A] time = 0.59, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(A + B*Log[(e*(a + b*x))/(c + d*x)])^(-2),x]
```

```
[Out] Integrate[(A + B*Log[(e*(a + b*x))/(c + d*x)])^(-2), x]
```

**fricas [A]** time = 0.58, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{1}{B^2\log\left(\frac{bex+ae}{dx+c}\right)^2+2\,AB\log\left(\frac{bex+ae}{dx+c}\right)+A^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(A+B*log(e*(b*x+a)/(d*x+c)))^2,x, algorithm="fricas")
```

```
[Out] integral(1/(B^2*log((b*e*x + a*e)/(d*x + c))^2 + 2*A*B*log((b*e*x + a*e)/(d
*x + c)) + A^2), x)
```

**giac [A]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(B\log\left(\frac{(bx+a)e}{dx+c}\right)+A\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(A+B*log(e*(b*x+a)/(d*x+c)))^2,x, algorithm="giac")
```

```
[Out] integrate((B*log((b*x + a)*e/(d*x + c)) + A)^(-2), x)
```

**maple [A]** time = 1.08, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(B\ln\left(\frac{(bx+a)e}{dx+c}\right)+A\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(B*ln((b*x+a)/(d*x+c)*e)+A)^2,x)
```

```
[Out] int(1/(B*ln((b*x+a)/(d*x+c)*e)+A)^2,x)
```

**maxima [A]** time = 0.00, size = 0, normalized size = 0.00

$$-\frac{bdx^2+ac+(bc+ad)x}{(bc-ad)B^2\log(bx+a)-(bc-ad)B^2\log(dx+c)+(bc-ad)AB+\left(bc\log(e)-ad\log(e)\right)B^2}+\int \frac{}{(bc-ad)B^2\,\mathrm{lo}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(A+B*log(e*(b*x+a)/(d*x+c)))^2,x, algorithm="maxima")
```

```
[Out] -(b*d*x^2 + a*c + (b*c + a*d)*x)/((b*c - a*d)*B^2*log(b*x + a) - (b*c - a*d
)*B^2*log(d*x + c) + (b*c - a*d)*A*B + (b*c*log(e) - a*d*log(e))*B^2) + int
egrate((2*b*d*x + b*c + a*d)/((b*c - a*d)*B^2*log(b*x + a) - (b*c - a*d)*B^
2*log(d*x + c) + (b*c - a*d)*A*B + (b*c*log(e) - a*d*log(e))*B^2), x)
```

**mupad [A]** time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{1}{\left(A + B \ln\left(\frac{e\,(a+b\,x)}{c+d\,x}\right)\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(A + B*log((e*(a + b*x))/(c + d*x)))^2,x)
```

```
[Out] int(1/(A + B*log((e*(a + b*x))/(c + d*x)))^2, x)
```

**sympy [A]** time = 0.00, size = 0, normalized size = 0.00

$$\frac{ac + adx + bcx + bdx^2}{ABad - ABbc + \left(B^2ad - B^2bc\right)\log\left(\frac{e(a+bx)}{c+dx}\right)} - \frac{\int \frac{ad}{A+B\log\left(\frac{ae}{c+dx}+\frac{bex}{c+dx}\right)}\,dx + \int \frac{bc}{A+B\log\left(\frac{ae}{c+dx}+\frac{bex}{c+dx}\right)}\,dx + \int \frac{2bdx}{A+B\log\left(\frac{ae}{c+dx}+\right.}}{B\,(ad-bc)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(A+B*ln(e*(b*x+a)/(d*x+c)))**2,x)
```

```
[Out] (a*c + a*d*x + b*c*x + b*d*x**2)/(A*B*a*d - A*B*b*c + (B**2*a*d - B**2*b*c)
*log(e*(a + b*x)/(c + d*x))) - (Integral(a*d/(A + B*log(a*e/(c + d*x) + b*e
*x/(c + d*x))), x) + Integral(b*c/(A + B*log(a*e/(c + d*x) + b*e*x/(c + d*x
))), x) + Integral(2*b*d*x/(A + B*log(a*e/(c + d*x) + b*e*x/(c + d*x))), x)
)/(B*(a*d - b*c))
```

## 3.259 $\int \frac{1}{(f+gx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}\,dx$

**Optimal**. Leaf size=32

$$\mathrm{Int}\left(\frac{1}{(f+gx)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2},x\right)$$

```
[Out] Unintegrable(1/(g*x+f)/(A+B*ln(e*(b*x+a)/(d*x+c)))^2,x)
```

**Rubi [A]** time = 0.07, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{1}{(f+gx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[1/((f + g*x)*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2),x]
[Out] Defer[Int][1/((f + g*x)*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2), x]
```

Rubi steps

$$\int \frac{1}{(f+gx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}\,dx=\int \frac{1}{(f+gx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}\,dx$$

**Mathematica [A]** time = 2.60, size = 0, normalized size = 0.00

$$\int \frac{1}{(f+gx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[1/((f + g*x)*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2),x]
[Out] Integrate[1/((f + g*x)*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2), x]
```

**fricas** [A] time = 0.82, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{1}{A^2gx + A^2f + \left(B^2gx + B^2f\right)\log\left(\frac{bex+ae}{dx+c}\right)^2 + 2\left(ABgx + ABf\right)\log\left(\frac{bex+ae}{dx+c}\right)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(g*x+f)/(A+B*log(e*(b*x+a)/(d*x+c)))^2,x, algorithm="fricas")
```

```
[Out] integral(1/(A^2*g*x + A^2*f + (B^2*g*x + B^2*f)*log((b*e*x + a*e)/(d*x + c)
)^2 + 2*(A*B*g*x + A*B*f)*log((b*e*x + a*e)/(d*x + c))), x)
```

**giac** [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(gx+f\right)\left(B\log\left(\frac{(bx+a)e}{dx+c}\right)+A\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(g*x+f)/(A+B*log(e*(b*x+a)/(d*x+c)))^2,x, algorithm="giac")
```

```
[Out] integrate(1/((g*x + f)*(B*log((b*x + a)*e/(d*x + c)) + A)^2), x)
```

**maple** [A] time = 1.37, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(gx+f\right)\left(B\ln\left(\frac{(bx+a)e}{dx+c}\right)+A\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(g*x+f)/(B*ln((b*x+a)/(d*x+c)*e)+A)^2,x)
```

```
[Out] int(1/(g*x+f)/(B*ln((b*x+a)/(d*x+c)*e)+A)^2,x)
```

**maxima** [A] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{bdx^2 + ac + (bc+ad)x}{\left(bcf - adf\right)AB + \left(bcf\log(e) - adf\log(e)\right)B^2 + \left(\left(bcg - adg\right)AB + \left(bcg\log(e) - adg\log(e)\right)B^2\right)x + \left(\left(bcg - ad\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(g*x+f)/(A+B*log(e*(b*x+a)/(d*x+c)))^2,x, algorithm="maxima")
```

```
[Out] -(b*d*x^2 + a*c + (b*c + a*d)*x)/((b*c*f - a*d*f)*A*B + (b*c*f*log(e) - a*d
*f*log(e))*B^2 + ((b*c*g - a*d*g)*A*B + (b*c*g*log(e) - a*d*g*log(e))*B^2)*
x + ((b*c*g - a*d*g)*B^2*x + (b*c*f - a*d*f)*B^2)*log(b*x + a) - ((b*c*g -
a*d*g)*B^2*x + (b*c*f - a*d*f)*B^2)*log(d*x + c)) + integrate((b*d*g*x^2 +
2*b*d*f*x + b*c*f + (d*f - c*g)*a)/((b*c*f^2 - a*d*f^2)*A*B + (b*c*f^2*log(
e) - a*d*f^2*log(e))*B^2 + ((b*c*g^2 - a*d*g^2)*A*B + (b*c*g^2*log(e) - a*d
*g^2*log(e))*B^2)*x^2 + 2*((b*c*f*g - a*d*f*g)*A*B + (b*c*f*g*log(e) - a*d*
f*g*log(e))*B^2)*x + ((b*c*g^2 - a*d*g^2)*B^2*x^2 + 2*(b*c*f*g - a*d*f*g)*B
^2*x + (b*c*f^2 - a*d*f^2)*B^2)*log(b*x + a) - ((b*c*g^2 - a*d*g^2)*B^2*x^2
 + 2*(b*c*f*g - a*d*f*g)*B^2*x + (b*c*f^2 - a*d*f^2)*B^2)*log(d*x + c)), x)
```

**mupad [A]** time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{1}{\left(f+g\,x\right)\,{\left(A+B\,\ln\left(\frac{e\,\left(a+b\,x\right)}{c+d\,x}\right)\right)}^2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((f + g*x)*(A + B*log((e*(a + b*x))/(c + d*x)))^2),x)
```

```
[Out] int(1/((f + g*x)*(A + B*log((e*(a + b*x))/(c + d*x)))^2), x)
```

**sympy [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(g*x+f)/(A+B*ln(e*(b*x+a)/(d*x+c)))**2,x)
```

```
[Out] Timed out
```

# 3.260 $\int \frac{1}{(f+gx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}\,dx$

**Optimal**. Leaf size=32

$$\text{Int}\left(\frac{1}{(f+gx)^2\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2},x\right)$$

```
[Out] Unintegrable(1/(g*x+f)^2/(A+B*ln(e*(b*x+a)/(d*x+c)))^2,x)
```

**Rubi** [A] time = 0.07, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{1}{(f+gx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[1/((f + g*x)^2*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2),x]
[Out] Defer[Int][1/((f + g*x)^2*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2), x]
```

Rubi steps

$$\int \frac{1}{(f+gx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}\,dx = \int \frac{1}{(f+gx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}\,dx$$

**Mathematica** [A] time = 3.93, size = 0, normalized size = 0.00

$$\int \frac{1}{(f+gx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[1/((f + g*x)^2*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2),x]
[Out] Integrate[1/((f + g*x)^2*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2), x]
```

**fricas [A]** time = 0.86, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{1}{A^2g^2x^2 + 2\,A^2fgx + A^2f^2 + \left(B^2g^2x^2 + 2\,B^2fgx + B^2f^2\right)\log\left(\frac{bex+ae}{dx+c}\right)^2 + 2\,\left(ABg^2x^2 + 2\,ABfgx + ABf\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(g*x+f)^2/(A+B*log(e*(b*x+a)/(d*x+c)))^2,x, algorithm="fricas")
```

```
[Out] integral(1/(A^2*g^2*x^2 + 2*A^2*f*g*x + A^2*f^2 + (B^2*g^2*x^2 + 2*B^2*f*g*
x + B^2*f^2)*log((b*e*x + a*e)/(d*x + c))^2 + 2*(A*B*g^2*x^2 + 2*A*B*f*g*x
+ A*B*f^2)*log((b*e*x + a*e)/(d*x + c))), x)
```

**giac [A]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(gx+f\right)^2\left(B\log\left(\frac{(bx+a)e}{dx+c}\right)+A\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(g*x+f)^2/(A+B*log(e*(b*x+a)/(d*x+c)))^2,x, algorithm="giac")
```

```
[Out] integrate(1/((g*x + f)^2*(B*log((b*x + a)*e/(d*x + c)) + A)^2), x)
```

**maple [A]** time = 1.50, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(gx+f\right)^2\left(B\ln\left(\frac{(bx+a)e}{dx+c}\right)+A\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(g*x+f)^2/(B*ln((b*x+a)/(d*x+c)*e)+A)^2,x)
```

```
[Out] int(1/(g*x+f)^2/(B*ln((b*x+a)/(d*x+c)*e)+A)^2,x)
```

**maxima [A]** time = 0.00, size = 0, normalized size = 0.00

$$-\frac{}{\left(bcf^2-adf^2\right)AB+\left(bcf^2\log(e)-adf^2\log(e)\right)B^2+\left(\left(bcg^2-adg^2\right)AB+\left(bcg^2\log(e)-adg^2\log(e)\right)B^2\right)x^2+2\,(}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(g*x+f)^2/(A+B*log(e*(b*x+a)/(d*x+c)))^2,x, algorithm="maxima")
```

```
[Out] -(b*d*x^2 + a*c + (b*c + a*d)*x)/((b*c*f^2 - a*d*f^2)*A*B + (b*c*f^2*log(e)
 - a*d*f^2*log(e))*B^2 + ((b*c*g^2 - a*d*g^2)*A*B + (b*c*g^2*log(e) - a*d*g
^2*log(e))*B^2)*x^2 + 2*((b*c*f*g - a*d*f*g)*A*B + (b*c*f*g*log(e) - a*d*f*
g*log(e))*B^2)*x + ((b*c*g^2 - a*d*g^2)*B^2*x^2 + 2*(b*c*f*g - a*d*f*g)*B^2
*x + (b*c*f^2 - a*d*f^2)*B^2)*log(b*x + a) - ((b*c*g^2 - a*d*g^2)*B^2*x^2 +
 2*(b*c*f*g - a*d*f*g)*B^2*x + (b*c*f^2 - a*d*f^2)*B^2)*log(d*x + c)) - int
egrate(-(b*c*f + (d*f - 2*c*g)*a - (a*d*g - (2*d*f - c*g)*b)*x)/(((b*c*g^3
- a*d*g^3)*A*B + (b*c*g^3*log(e) - a*d*g^3*log(e))*B^2)*x^3 + (b*c*f^3 - a*
d*f^3)*A*B + (b*c*f^3*log(e) - a*d*f^3*log(e))*B^2 + 3*((b*c*f*g^2 - a*d*f*
g^2)*A*B + (b*c*f*g^2*log(e) - a*d*f*g^2*log(e))*B^2)*x^2 + 3*((b*c*f^2*g -
 a*d*f^2*g)*A*B + (b*c*f^2*g*log(e) - a*d*f^2*g*log(e))*B^2)*x + ((b*c*g^3
- a*d*g^3)*B^2*x^3 + 3*(b*c*f*g^2 - a*d*f*g^2)*B^2*x^2 + 3*(b*c*f^2*g - a*d
*f^2*g)*B^2*x + (b*c*f^3 - a*d*f^3)*B^2)*log(b*x + a) - ((b*c*g^3 - a*d*g^3
)*B^2*x^3 + 3*(b*c*f*g^2 - a*d*f*g^2)*B^2*x^2 + 3*(b*c*f^2*g - a*d*f^2*g)*B
^2*x + (b*c*f^3 - a*d*f^3)*B^2)*log(d*x + c)), x)
```

**mupad** [**A**] time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{1}{{\left (f+g\,x\right )}^2\,{\left (A+B\,\ln \left (\frac{e\,\left (a+b\,x\right )}{c+d\,x}\right )\right )}^2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((f + g*x)^2*(A + B*log((e*(a + b*x))/(c + d*x)))^2),x)
```

```
[Out] int(1/((f + g*x)^2*(A + B*log((e*(a + b*x))/(c + d*x)))^2), x)
```

**sympy** [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(g*x+f)**2/(A+B*ln(e*(b*x+a)/(d*x+c)))**2,x)
```

```
[Out] Timed out
```

# 3.261 $\int \frac{1}{(f+gx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}\,dx$

**Optimal**. Leaf size=32

$$\text{Int}\left(\frac{1}{(f+gx)^3\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2},x\right)$$

```
[Out] Unintegrable(1/(g*x+f)^3/(A+B*ln(e*(b*x+a)/(d*x+c)))^2,x)
```

**Rubi [A]** time = 0.07, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{1}{(f+gx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[1/((f + g*x)^3*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2),x]
[Out] Defer[Int][1/((f + g*x)^3*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2), x]
```

Rubi steps

$$\int \frac{1}{(f+gx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}\,dx=\int \frac{1}{(f+gx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}\,dx$$

**Mathematica [A]** time = 30.01, size = 0, normalized size = 0.00

$$\int \frac{1}{(f+gx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[1/((f + g*x)^3*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2),x]
[Out] Integrate[1/((f + g*x)^3*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2), x]
```

**fricas [A]** time = 0.89, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{1}{A^2g^3x^3 + 3\,A^2fg^2x^2 + 3\,A^2f^2gx + A^2f^3 + \left(B^2g^3x^3 + 3\,B^2fg^2x^2 + 3\,B^2f^2gx + B^2f^3\right)\log\left(\frac{bex+ae}{dx+c}\right)^2 + \cdots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(g*x+f)^3/(A+B*log(e*(b*x+a)/(d*x+c)))^2,x, algorithm="fricas")
```

```
[Out] integral(1/(A^2*g^3*x^3 + 3*A^2*f*g^2*x^2 + 3*A^2*f^2*g*x + A^2*f^3 + (B^2*
g^3*x^3 + 3*B^2*f*g^2*x^2 + 3*B^2*f^2*g*x + B^2*f^3)*log((b*e*x + a*e)/(d*x
 + c))^2 + 2*(A*B*g^3*x^3 + 3*A*B*f*g^2*x^2 + 3*A*B*f^2*g*x + A*B*f^3)*log(
(b*e*x + a*e)/(d*x + c))), x)
```

**giac [A]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(gx+f\right)^3\left(B\log\left(\frac{(bx+a)e}{dx+c}\right)+A\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(g*x+f)^3/(A+B*log(e*(b*x+a)/(d*x+c)))^2,x, algorithm="giac")
```

```
[Out] integrate(1/((g*x + f)^3*(B*log((b*x + a)*e/(d*x + c)) + A)^2), x)
```

**maple [A]** time = 1.80, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(gx+f\right)^3\left(B\ln\left(\frac{(bx+a)e}{dx+c}\right)+A\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(g*x+f)^3/(B*ln((b*x+a)/(d*x+c)*e)+A)^2,x)
```

```
[Out] int(1/(g*x+f)^3/(B*ln((b*x+a)/(d*x+c)*e)+A)^2,x)
```

**maxima [A]** time = 0.00, size = 0, normalized size = 0.00

$$-\frac{\cdots}{\left(\left(bcg^3-adg^3\right)AB+\left(bcg^3\log(e)-adg^3\log(e)\right)B^2\right)x^3+\left(bcf^3-adf^3\right)AB+\left(bcf^3\log(e)-adf^3\log(e)\right)B^2+3\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(g*x+f)^3/(A+B*log(e*(b*x+a)/(d*x+c)))^2,x, algorithm="maxima")
```

```
[Out] -(b*d*x^2 + a*c + (b*c + a*d)*x)/(((b*c*g^3 - a*d*g^3)*A*B + (b*c*g^3*log(e
) - a*d*g^3*log(e))*B^2)*x^3 + (b*c*f^3 - a*d*f^3)*A*B + (b*c*f^3*log(e) -
a*d*f^3*log(e))*B^2 + 3*((b*c*f*g^2 - a*d*f*g^2)*A*B + (b*c*f*g^2*log(e) -
a*d*f*g^2*log(e))*B^2)*x^2 + 3*((b*c*f^2*g - a*d*f^2*g)*A*B + (b*c*f^2*g*lo
g(e) - a*d*f^2*g*log(e))*B^2)*x + ((b*c*g^3 - a*d*g^3)*B^2*x^3 + 3*(b*c*f*g
^2 - a*d*f*g^2)*B^2*x^2 + 3*(b*c*f^2*g - a*d*f^2*g)*B^2*x + (b*c*f^3 - a*d*
f^3)*B^2)*log(b*x + a) - ((b*c*g^3 - a*d*g^3)*B^2*x^3 + 3*(b*c*f*g^2 - a*d*
f*g^2)*B^2*x^2 + 3*(b*c*f^2*g - a*d*f^2*g)*B^2*x + (b*c*f^3 - a*d*f^3)*B^2)
*log(d*x + c)) - integrate((b*d*g*x^2 - b*c*f - (d*f - 3*c*g)*a + 2*(a*d*g
- (d*f - c*g)*b)*x)/(((b*c*g^4 - a*d*g^4)*A*B + (b*c*g^4*log(e) - a*d*g^4*l
og(e))*B^2)*x^4 + 4*((b*c*f*g^3 - a*d*f*g^3)*A*B + (b*c*f*g^3*log(e) - a*d*
f*g^3*log(e))*B^2)*x^3 + (b*c*f^4 - a*d*f^4)*A*B + (b*c*f^4*log(e) - a*d*f^
4*log(e))*B^2 + 6*((b*c*f^2*g^2 - a*d*f^2*g^2)*A*B + (b*c*f^2*g^2*log(e) -
a*d*f^2*g^2*log(e))*B^2)*x^2 + 4*((b*c*f^3*g - a*d*f^3*g)*A*B + (b*c*f^3*g*
log(e) - a*d*f^3*g*log(e))*B^2)*x + ((b*c*g^4 - a*d*g^4)*B^2*x^4 + 4*(b*c*f
*g^3 - a*d*f*g^3)*B^2*x^3 + 6*(b*c*f^2*g^2 - a*d*f^2*g^2)*B^2*x^2 + 4*(b*c*
f^3*g - a*d*f^3*g)*B^2*x + (b*c*f^4 - a*d*f^4)*B^2)*log(b*x + a) - ((b*c*g^
4 - a*d*g^4)*B^2*x^4 + 4*(b*c*f*g^3 - a*d*f*g^3)*B^2*x^3 + 6*(b*c*f^2*g^2 -
 a*d*f^2*g^2)*B^2*x^2 + 4*(b*c*f^3*g - a*d*f^3*g)*B^2*x + (b*c*f^4 - a*d*f^
4)*B^2)*log(d*x + c)), x)
```

**mupad [A]** time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{1}{{\left(f+g\,x\right)}^3\,{\left(A+B\,\ln\left(\frac{e\,\left(a+b\,x\right)}{c+d\,x}\right)\right)}^2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((f + g*x)^3*(A + B*log((e*(a + b*x))/(c + d*x)))^2),x)
```

```
[Out] int(1/((f + g*x)^3*(A + B*log((e*(a + b*x))/(c + d*x)))^2), x)
```

**sympy [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(g*x+f)**3/(A+B*ln(e*(b*x+a)/(d*x+c)))**2,x)
```

```
[Out] Timed out
```

# 3.262 $\int (f+gx)^4 \left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)\,dx$

**Optimal**. Leaf size=357

$$-\frac{Bg^2x^2(bc-ad)\left(a^2d^2g^2-abdg(5df-cg)+b^2\left(c^2g^2-5cdfg+10d^2f^2\right)\right)}{5b^3d^3}+\frac{2Bgx(bc-ad)\left(a^3d^3g^3-a^2bd^2g^2(5c$$

```
[Out] 2/5*B*(-a*d+b*c)*g*(a^3*d^3*g^3-a^2*b*d^2*g^2*(-c*g+5*d*f)+a*b^2*d*g*(c^2*g
^2-5*c*d*f*g+10*d^2*f^2)-b^3*(-c^3*g^3+5*c^2*d*f*g^2-10*c*d^2*f^2*g+10*d^3*
f^3))*x/b^4/d^4-1/5*B*(-a*d+b*c)*g^2*(a^2*d^2*g^2-a*b*d*g*(-c*g+5*d*f)+b^2*
(c^2*g^2-5*c*d*f*g+10*d^2*f^2))*x^2/b^3/d^3-2/15*B*(-a*d+b*c)*g^3*(-a*d*g-b
*c*g+5*b*d*f)*x^3/b^2/d^2-1/10*B*(-a*d+b*c)*g^4*x^4/b/d-2/5*B*(-a*g+b*f)^5*
ln(b*x+a)/b^5/g+1/5*(g*x+f)^5*(A+B*ln(e*(b*x+a)^2/(d*x+c)^2))/g+2/5*B*(-c*g
+d*f)^5*ln(d*x+c)/d^5/g
```

**Rubi** [A] time = 0.50, antiderivative size = 341, normalized size of antiderivative = 0.96, number of steps used = 4, number of rules used = 3, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.103, Rules used = {2525, 12, 72}

$$-\frac{Bg^2x^2(bc-ad)\left(a^2d^2g^2-abdg(5df-cg)+b^2\left(c^2g^2-5cdfg+10d^2f^2\right)\right)}{5b^3d^3}+\frac{2Bgx\left(-10a^2b^2d^4f^2g+5a^3bd^4fg^2\right.$$

Antiderivative was successfully verified.

```
[In] Int[(f + g*x)^4*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2]),x]
```

```
[Out] (2*B*g*(10*a*b^3*d^4*f^3 - 10*a^2*b^2*d^4*f^2*g + 5*a^3*b*d^4*f*g^2 - a^4*d
^4*g^3 - b^4*c*(10*d^3*f^3 - 10*c*d^2*f^2*g + 5*c^2*d*f*g^2 - c^3*g^3))*x)/
(5*b^4*d^4) - (B*(b*c - a*d)*g^2*(a^2*d^2*g^2 - a*b*d*g*(5*d*f - c*g) + b^2
*(10*d^2*f^2 - 5*c*d*f*g + c^2*g^2))*x^2)/(5*b^3*d^3) - (2*B*(b*c - a*d)*g^
3*(5*b*d*f - b*c*g - a*d*g)*x^3)/(15*b^2*d^2) - (B*(b*c - a*d)*g^4*x^4)/(10
*b*d) - (2*B*(b*f - a*g)^5*Log[a + b*x])/(5*b^5*g) + ((f + g*x)^5*(A + B*Lo
g[(e*(a + b*x)^2)/(c + d*x)^2]))/(5*g) + (2*B*(d*f - c*g)^5*Log[c + d*x])/(
5*d^5*g)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 72

```
Int[((e_.) + (f_.)*(x_))^(p_.)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
 x_Symbol] :> Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x]
```

```
/; FreeQ[{a, b, c, d, e, f}, x] && IntegerQ[p]
```

Rule 2525

```
Int[((a_.) + Log[(c_.)*(RFx_)^(p_.)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_))^(m_.
), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(a + b*Log[c*RFx^p])^n)/(e*(m + 1))
, x] - Dist[(b*n*p)/(e*(m + 1)), Int[SimplifyIntegrand[((d + e*x)^(m + 1)*(
a + b*Log[c*RFx^p])^(n - 1)*D[RFx, x])/RFx, x], x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && RationalFunctionQ[RFx, x] && IGtQ[n, 0] && (EqQ[n, 1] ||
IntegerQ[m]) && NeQ[m, -1]
```

Rubi steps

$$\int (f+gx)^4\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)dx=\frac{(f+gx)^5\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}{5g}-\frac{B\int\frac{2(bc-ad)(f+gx)^5}{(a+bx)(c+dx)}dx}{5g}$$

$$=\frac{(f+gx)^5\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}{5g}-\frac{(2B(bc-ad))\int\frac{(f+gx)^5}{(a+bx)(c+dx)}dx}{5g}$$

$$=\frac{(f+gx)^5\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}{5g}-\frac{(2B(bc-ad))\int\left(\frac{g^2\left(-a^3d^3g^3+a^2bd^2g^2(\right.}{}\right.}{}$$

$$=\frac{2B(bc-ad)g\left(a^3d^3g^3-a^2bd^2g^2(5df-cg)+ab^2dg\left(10d^2f^2-5cdfg\right.\right.}{5b^4d^4}$$

**Mathematica [A]** time = 0.60, size = 282, normalized size = 0.79

$$\frac{Bg^2x(ad-bc)\left(-12a^3d^3g^3+6a^2bd^2g^2(-2cg+10df+dgx)-2ab^2dg\left(6c^2g^2-3cdg(10f+gx)+d^2\left(60f^2+15fgx+2g^2x^2\right)\right)+b^3\left(-12c^3g^3+6c^2dg^2(10f+gx)-2cd^2g\right.\right.}{6b^4d^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(f + g*x)^4*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2]),x]
```

```
[Out] ((B*(-(b*c) + a*d)*g^2*x*(-12*a^3*d^3*g^3 + 6*a^2*b*d^2*g^2*(10*d*f - 2*c*g
 + d*g*x) - 2*a*b^2*d*g*(6*c^2*g^2 - 3*c*d*g*(10*f + g*x) + d^2*(60*f^2 + 1
5*f*g*x + 2*g^2*x^2)) + b^3*(-12*c^3*g^3 + 6*c^2*d*g^2*(10*f + g*x) - 2*c*d
^2*g*(60*f^2 + 15*f*g*x + 2*g^2*x^2) + d^3*(120*f^3 + 60*f^2*g*x + 20*f*g^2
*x^2 + 3*g^3*x^3))))/(6*b^4*d^4) - (2*B*(b*f - a*g)^5*Log[a + b*x])/b^5 + (
```

```
f + g*x)^5*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2]) + (2*B*(d*f - c*g)^5*Lo
g[c + d*x])/d^5)/(5*g)
```

**fricas [A]** time = 2.35, size = 660, normalized size = 1.85

$$\frac{6\,Ab^5d^5g^4x^5 + 3\left(10\,Ab^5d^5fg^3 - \left(Bb^5cd^4 - Bab^4d^5\right)g^4\right)x^4 + 4\left(15\,Ab^5d^5f^2g^2 - 5\left(Bb^5cd^4 - Bab^4d^5\right)fg^3 + \left(Bb^5\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^4*(A+B*log(e*(b*x+a)^2/(d*x+c)^2)),x, algorithm="fricas")
```

```
[Out] 1/30*(6*A*b^5*d^5*g^4*x^5 + 3*(10*A*b^5*d^5*f*g^3 - (B*b^5*c*d^4 - B*a*b^4*
d^5)*g^4)*x^4 + 4*(15*A*b^5*d^5*f^2*g^2 - 5*(B*b^5*c*d^4 - B*a*b^4*d^5)*f*g
^3 + (B*b^5*c^2*d^3 - B*a^2*b^3*d^5)*g^4)*x^3 + 6*(10*A*b^5*d^5*f^3*g - 10*
(B*b^5*c*d^4 - B*a*b^4*d^5)*f^2*g^2 + 5*(B*b^5*c^2*d^3 - B*a^2*b^3*d^5)*f*g
^3 - (B*b^5*c^3*d^2 - B*a^3*b^2*d^5)*g^4)*x^2 + 6*(5*A*b^5*d^5*f^4 - 20*(B*
b^5*c*d^4 - B*a*b^4*d^5)*f^3*g + 20*(B*b^5*c^2*d^3 - B*a^2*b^3*d^5)*f^2*g^2
 - 10*(B*b^5*c^3*d^2 - B*a^3*b^2*d^5)*f*g^3 + 2*(B*b^5*c^4*d - B*a^4*b*d^5)
*g^4)*x + 12*(5*B*a*b^4*d^5*f^4 - 10*B*a^2*b^3*d^5*f^3*g + 10*B*a^3*b^2*d^5
*f^2*g^2 - 5*B*a^4*b*d^5*f*g^3 + B*a^5*d^5*g^4)*log(b*x + a) - 12*(5*B*b^5*
c*d^4*f^4 - 10*B*b^5*c^2*d^3*f^3*g + 10*B*b^5*c^3*d^2*f^2*g^2 - 5*B*b^5*c^4
*d*f*g^3 + B*b^5*c^5*g^4)*log(d*x + c) + 6*(B*b^5*d^5*g^4*x^5 + 5*B*b^5*d^5
*f*g^3*x^4 + 10*B*b^5*d^5*f^2*g^2*x^3 + 10*B*b^5*d^5*f^3*g*x^2 + 5*B*b^5*d^
5*f^4*x)*log((b^2*e*x^2 + 2*a*b*e*x + a^2*e)/(d^2*x^2 + 2*c*d*x + c^2)))/(b
^5*d^5)
```

**giac [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^4*(A+B*log(e*(b*x+a)^2/(d*x+c)^2)),x, algorithm="giac")
```

```
[Out] Timed out
```

**maple [B]** time = 0.15, size = 2438, normalized size = 6.83

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x+f)^4*(B*ln((b*x+a)^2/(d*x+c)^2*e)+A),x)
```

```
[Out] A*x*f^4+1/5*A*x^5*g^4-8*B/b/(a*d-b*c)*ln(1/(d*x+c)*a*d-1/(d*x+c)*b*c+b)*a^2
*c*f^3*g-12/d^2*B/b*ln(1/(d*x+c)*a*d-1/(d*x+c)*b*c+b)*a*c^2*f^2*g^2+16/d^3*
```

```
B/(a*d-b*c)*ln(1/(d*x+c)*a*d-1/(d*x+c)*b*c+b)*a*c^4*f*g^3+2/d^3*B/b/(a*d-b*
c)*ln(1/(d*x+c)*a*d-1/(d*x+c)*b*c+b)*a^2*c^4*g^4-8/d^2*B/(a*d-b*c)*ln(1/(d*
x+c)*a*d-1/(d*x+c)*b*c+b)*c^3*b*f^3*g+12/d^3*B/(a*d-b*c)*ln(1/(d*x+c)*a*d-1
/(d*x+c)*b*c+b)*c^4*b*f^2*g^2+8/d^3*B*g^3*a/b*ln(1/(d*x+c)*a*d-1/(d*x+c)*b*
c+b)*c^3*f+8/d*B*g/b*ln(1/(d*x+c)*a*d-1/(d*x+c)*b*c+b)*a*c*f^3-24/d^2*B/(a*
d-b*c)*ln(1/(d*x+c)*a*d-1/(d*x+c)*b*c+b)*a*c^3*f^2*g^2+16/d*B/(a*d-b*c)*ln(
1/(d*x+c)*a*d-1/(d*x+c)*b*c+b)*a*c^2*f^3*g-8/d^4*B/(a*d-b*c)*ln(1/(d*x+c)*a
*d-1/(d*x+c)*b*c+b)*c^5*b*f*g^3+5/6/d^5*B*g^4*c^5+B*ln((1/(d*x+c)*a*d-1/(d*
x+c)*b*c+b)^2/d^2*e)*x*f^4+1/5*B*g^4*ln((1/(d*x+c)*a*d-1/(d*x+c)*b*c+b)^2/d
^2*e)*x^5+1/d*A*c*f^4+1/5/d^5*A*c^5*g^4+A*x^4*f*g^3+2*A*x^2*f^3*g+2*A*x^3*f
^2*g^2-8/d^2*B/b/(a*d-b*c)*ln(1/(d*x+c)*a*d-1/(d*x+c)*b*c+b)*a^2*c^3*f*g^3+
12/d*B/b/(a*d-b*c)*ln(1/(d*x+c)*a*d-1/(d*x+c)*b*c+b)*a^2*c^2*f^2*g^2-2/d^2*
B/b*a*c^2*f^2*g^2+2/3/d^3*B*g^3*a/b*c^3*f+1/d^2*B*g^3*a^2/b^2*c^2*f+4/d*B*g
/b*a*f^3*c+2/d*B*g^3*a^3/b^3*f*c-4/d*B*g^2*a^2/b^2*f^2*c+4*B*g/b^2*ln(1/(d*
x+c))*a^2*f^3+2*B*g^3*a^4/b^4*ln(1/(d*x+c))*f-2*B*g^3*a^4/b^4*ln(1/(d*x+c)*
a*d-1/(d*x+c)*b*c+b)*f+2/3*B*g^3*a/b*f*x^3-1/10/d*B*g^4*c*x^4-2/5*B*g^4*a^4
/b^4*x+1/10*B*g^4*a/b*x^4+1/5*B*g^4*a^3/b^3*x^2-2/15*B*g^4*a^2/b^2*x^3-4/d^
4*B/(a*d-b*c)*ln(1/(d*x+c)*a*d-1/(d*x+c)*b*c+b)*a*c^5*g^4+2/d*B/(a*d-b*c)*l
n(1/(d*x+c)*a*d-1/(d*x+c)*b*c+b)*c^2*b*f^4+2*d*B/b/(a*d-b*c)*ln(1/(d*x+c)*a
*d-1/(d*x+c)*b*c+b)*a^2*f^4-2/d^4*B*g^4*a/b*ln(1/(d*x+c)*a*d-1/(d*x+c)*b*c+
b)*c^4+2/d^5*B/(a*d-b*c)*ln(1/(d*x+c)*a*d-1/(d*x+c)*b*c+b)*c^6*b*g^4+2/5/d^
5*B*ln(1/(d*x+c))*c^5*g^4+8/5/d^5*B*g^4*c^5*ln(1/(d*x+c)*a*d-1/(d*x+c)*b*c+
b)+2*B*g*ln((1/(d*x+c)*a*d-1/(d*x+c)*b*c+b)^2/d^2*e)*f^3*x^2+B*g^3*ln((1/(d
*x+c)*a*d-1/(d*x+c)*b*c+b)^2/d^2*e)*f*x^4+2*B*g^2*ln((1/(d*x+c)*a*d-1/(d*x+
c)*b*c+b)^2/d^2*e)*f^2*x^3-2/5*B*g^4*a^5/b^5*ln(1/(d*x+c))+2/5*B*g^4*a^5/b^
5*ln(1/(d*x+c)*a*d-1/(d*x+c)*b*c+b)-2*B/b*ln(1/(d*x+c))*a*f^4+2/15/d^2*B*g^
4*c^2*x^3-1/5/d^3*B*g^4*c^3*x^2+2/5/d^4*B*g^4*c^4*x+1/d*B*ln((1/(d*x+c)*a*d
-1/(d*x+c)*b*c+b)^2/d^2*e)*c*f^4+2/d*B*ln(1/(d*x+c))*c*f^4+1/5/d^5*B*g^4*ln
((1/(d*x+c)*a*d-1/(d*x+c)*b*c+b)^2/d^2*e)*c^5-11/3/d^4*B*g^3*c^4*f+6/d^3*B*
c^3*f^2*g^2-4/d^2*B*c^2*f^3*g-1/d^4*A*c^4*f*g^3+2/d^3*A*c^3*f^2*g^2-2/d^2*A
*c^2*f^3*g-1/10/d^4*B*g^4*a/b*c^4-1/5/d^2*B*g^4*a^3/b^3*c^2-2/15/d^3*B*g^4*
a^2/b^2*c^3-2/5/d*B*g^4*a^4/b^4*c-4/d*B*g*c*f^3*x-4*B*g^2*a^3/b^3*ln(1/(d*x
+c))*f^2-4*B/(a*d-b*c)*ln(1/(d*x+c)*a*d-1/(d*x+c)*b*c+b)*a*c*f^4+4*B*g^2*a^
3/b^3*ln(1/(d*x+c)*a*d-1/(d*x+c)*b*c+b)*f^2-4*B*g/b^2*ln(1/(d*x+c)*a*d-1/(d
*x+c)*b*c+b)*a^2*f^3+4/d^3*B*ln(1/(d*x+c))*c^3*f^2*g^2-4/d^2*B*ln(1/(d*x+c)
)*c^2*f^3*g-6/d^4*B*g^3*c^4*ln(1/(d*x+c)*a*d-1/(d*x+c)*b*c+b)*f+8/d^3*B*ln(
1/(d*x+c)*a*d-1/(d*x+c)*b*c+b)*c^3*f^2*g^2-4/d^2*B*g*ln(1/(d*x+c)*a*d-1/(d*
x+c)*b*c+b)*c^2*f^3-1/d^4*B*g^3*ln((1/(d*x+c)*a*d-1/(d*x+c)*b*c+b)^2/d^2*e)
*c^4*f+2/d^3*B*ln((1/(d*x+c)*a*d-1/(d*x+c)*b*c+b)^2/d^2*e)*c^3*f^2*g^2-2/d^
2*B*ln((1/(d*x+c)*a*d-1/(d*x+c)*b*c+b)^2/d^2*e)*c^2*f^3*g+4*B*g/b*a*f^3*x+2
*B*g^3*a^3/b^3*f*x-4*B*g^2*a^2/b^2*f^2*x+2*B*g^2*a/b*f^2*x^2-2/d^4*B*ln(1/(
d*x+c))*c^4*f*g^3-B*g^3*a^2/b^2*f*x^2+4/d^2*B*c^2*f^2*g^2*x+1/d^2*B*g^3*c^2
*f*x^2-2/d^3*B*g^3*c^3*f*x-2/3/d*B*g^3*c*f*x^3-2/d*B*g^2*c*f^2*x^2
```

**maxima [B]** time = 1.17, size = 855, normalized size = 2.39

$$\frac{1}{5}\,Ag^4x^5+Afg^3x^4+2\,Af^2g^2x^3+2\,Af^3gx^2+\left(x\log\left(\frac{b^2ex^2}{d^2x^2+2\,cdx+c^2}+\frac{2\,abex}{d^2x^2+2\,cdx+c^2}+\frac{a^2e}{d^2x^2+2\,cdx+c^2}\right)\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^4*(A+B*log(e*(b*x+a)^2/(d*x+c)^2)),x, algorithm="maxima")
```

```
[Out] 1/5*A*g^4*x^5 + A*f*g^3*x^4 + 2*A*f^2*g^2*x^3 + 2*A*f^3*g*x^2 + (x*log(b^2*
e*x^2/(d^2*x^2 + 2*c*d*x + c^2) + 2*a*b*e*x/(d^2*x^2 + 2*c*d*x + c^2) + a^2
*e/(d^2*x^2 + 2*c*d*x + c^2)) + 2*a*log(b*x + a)/b - 2*c*log(d*x + c)/d)*B*
f^4 + 2*(x^2*log(b^2*e*x^2/(d^2*x^2 + 2*c*d*x + c^2) + 2*a*b*e*x/(d^2*x^2 +
 2*c*d*x + c^2) + a^2*e/(d^2*x^2 + 2*c*d*x + c^2)) - 2*a^2*log(b*x + a)/b^2
 + 2*c^2*log(d*x + c)/d^2 - 2*(b*c - a*d)*x/(b*d))*B*f^3*g + 2*(x^3*log(b^2
*e*x^2/(d^2*x^2 + 2*c*d*x + c^2) + 2*a*b*e*x/(d^2*x^2 + 2*c*d*x + c^2) + a^
2*e/(d^2*x^2 + 2*c*d*x + c^2)) + 2*a^3*log(b*x + a)/b^3 - 2*c^3*log(d*x + c
)/d^3 - ((b^2*c*d - a*b*d^2)*x^2 - 2*(b^2*c^2 - a^2*d^2)*x)/(b^2*d^2))*B*f^
2*g^2 + 1/3*(3*x^4*log(b^2*e*x^2/(d^2*x^2 + 2*c*d*x + c^2) + 2*a*b*e*x/(d^2
*x^2 + 2*c*d*x + c^2) + a^2*e/(d^2*x^2 + 2*c*d*x + c^2)) - 6*a^4*log(b*x +
a)/b^4 + 6*c^4*log(d*x + c)/d^4 - (2*(b^3*c*d^2 - a*b^2*d^3)*x^3 - 3*(b^3*c
^2*d - a^2*b*d^3)*x^2 + 6*(b^3*c^3 - a^3*d^3)*x)/(b^3*d^3))*B*f*g^3 + 1/30*
(6*x^5*log(b^2*e*x^2/(d^2*x^2 + 2*c*d*x + c^2) + 2*a*b*e*x/(d^2*x^2 + 2*c*d
*x + c^2) + a^2*e/(d^2*x^2 + 2*c*d*x + c^2)) + 12*a^5*log(b*x + a)/b^5 - 12
*c^5*log(d*x + c)/d^5 - (3*(b^4*c*d^3 - a*b^3*d^4)*x^4 - 4*(b^4*c^2*d^2 - a
^2*b^2*d^4)*x^3 + 6*(b^4*c^3*d - a^3*b*d^4)*x^2 - 12*(b^4*c^4 - a^4*d^4)*x)
/(b^4*d^4))*B*g^4 + A*f^4*x
```

**mupad [B]** time = 5.33, size = 1403, normalized size = 3.93

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f + g*x)^4*(A + B*log((e*(a + b*x)^2)/(c + d*x)^2)),x)
```

```
[Out] log((e*(a + b*x)^2)/(c + d*x)^2)*((B*g^4*x^5)/5 + B*f^4*x + 2*B*f^2*g^2*x^3
 + 2*B*f^3*g*x^2 + B*f*g^3*x^4) + x^2*((20*A*a*c*f*g^3 + 20*A*b*d*f^3*g + 3
0*A*a*d*f^2*g^2 + 30*A*b*c*f^2*g^2 + 20*B*a*d*f^2*g^2 - 20*B*b*c*f^2*g^2)/(
10*b*d) + ((5*a*d + 5*b*c)*((((5*A*a*d*g^4 + 5*A*b*c*g^4 + 2*B*a*d*g^4 - 2*
B*b*c*g^4 + 20*A*b*d*f*g^3)/(5*b*d) - (A*g^4*(5*a*d + 5*b*c))/(5*b*d))*(5*a
*d + 5*b*c))/(5*b*d) - (5*A*a*c*g^4 + 20*A*a*d*f*g^3 + 20*A*b*c*f*g^3 + 10*
B*a*d*f*g^3 - 10*B*b*c*f*g^3 + 30*A*b*d*f^2*g^2)/(5*b*d) + (A*a*c*g^4)/(b*d
)))/(10*b*d) - (a*c*((5*A*a*d*g^4 + 5*A*b*c*g^4 + 2*B*a*d*g^4 - 2*B*b*c*g^4
 + 20*A*b*d*f*g^3)/(5*b*d) - (A*g^4*(5*a*d + 5*b*c))/(5*b*d)))/(2*b*d)) + x
^4*((5*A*a*d*g^4 + 5*A*b*c*g^4 + 2*B*a*d*g^4 - 2*B*b*c*g^4 + 20*A*b*d*f*g^3
```

```
)/(20*b*d) - (A*g^4*(5*a*d + 5*b*c))/(20*b*d)) + x*((5*A*b*d*f^4 + 20*A*a*d
*f^3*g + 20*A*b*c*f^3*g + 20*B*a*d*f^3*g - 20*B*b*c*f^3*g + 30*A*a*c*f^2*g^
2)/(5*b*d) - ((5*a*d + 5*b*c)*((20*A*a*c*f*g^3 + 20*A*b*d*f^3*g + 30*A*a*d*
f^2*g^2 + 30*A*b*c*f^2*g^2 + 20*B*a*d*f^2*g^2 - 20*B*b*c*f^2*g^2)/(5*b*d) +
 ((5*a*d + 5*b*c)*((((5*A*a*d*g^4 + 5*A*b*c*g^4 + 2*B*a*d*g^4 - 2*B*b*c*g^4
 + 20*A*b*d*f*g^3)/(5*b*d) - (A*g^4*(5*a*d + 5*b*c))/(5*b*d))*(5*a*d + 5*b*
c))/(5*b*d) - (5*A*a*c*g^4 + 20*A*a*d*f*g^3 + 20*A*b*c*f*g^3 + 10*B*a*d*f*g
^3 - 10*B*b*c*f*g^3 + 30*A*b*d*f^2*g^2)/(5*b*d) + (A*a*c*g^4)/(b*d)))/(5*b*
d) - (a*c*((5*A*a*d*g^4 + 5*A*b*c*g^4 + 2*B*a*d*g^4 - 2*B*b*c*g^4 + 20*A*b*
d*f*g^3)/(5*b*d) - (A*g^4*(5*a*d + 5*b*c))/(5*b*d)))/(b*d)))/(5*b*d) + (a*c
*((((5*A*a*d*g^4 + 5*A*b*c*g^4 + 2*B*a*d*g^4 - 2*B*b*c*g^4 + 20*A*b*d*f*g^3
)/(5*b*d) - (A*g^4*(5*a*d + 5*b*c))/(5*b*d))*(5*a*d + 5*b*c))/(5*b*d) - (5*
A*a*c*g^4 + 20*A*a*d*f*g^3 + 20*A*b*c*f*g^3 + 10*B*a*d*f*g^3 - 10*B*b*c*f*g
^3 + 30*A*b*d*f^2*g^2)/(5*b*d) + (A*a*c*g^4)/(b*d)))/(b*d)) - x^3*((((5*A*a
*d*g^4 + 5*A*b*c*g^4 + 2*B*a*d*g^4 - 2*B*b*c*g^4 + 20*A*b*d*f*g^3)/(5*b*d)
- (A*g^4*(5*a*d + 5*b*c))/(5*b*d))*(5*a*d + 5*b*c))/(15*b*d) - (5*A*a*c*g^4
 + 20*A*a*d*f*g^3 + 20*A*b*c*f*g^3 + 10*B*a*d*f*g^3 - 10*B*b*c*f*g^3 + 30*A
*b*d*f^2*g^2)/(15*b*d) + (A*a*c*g^4)/(3*b*d)) + (A*g^4*x^5)/5 + (log(a + b*
x)*((2*B*a^5*g^4)/5 + 2*B*a*b^4*f^4 - 4*B*a^2*b^3*f^3*g + 4*B*a^3*b^2*f^2*g
^2 - 2*B*a^4*b*f*g^3))/b^5 - (log(c + d*x)*(2*B*c^5*g^4 + 10*B*c*d^4*f^4 -
20*B*c^2*d^3*f^3*g + 20*B*c^3*d^2*f^2*g^2 - 10*B*c^4*d*f*g^3))/(5*d^5)
```

**sympy [B]** time = 26.59, size = 1477, normalized size = 4.14

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)**4*(A+B*ln(e*(b*x+a)**2/(d*x+c)**2)),x)
```

```
[Out] A*g**4*x**5/5 + 2*B*a*(a**4*g**4 - 5*a**3*b*f*g**3 + 10*a**2*b**2*f**2*g**2
 - 10*a*b**3*f**3*g + 5*b**4*f**4)*log(x + (2*B*a**5*c*d**4*g**4 - 10*B*a**
4*b*c*d**4*f*g**3 + 20*B*a**3*b**2*c*d**4*f**2*g**2 - 20*B*a**2*b**3*c*d**4
*f**3*g + 2*B*a**2*d**5*(a**4*g**4 - 5*a**3*b*f*g**3 + 10*a**2*b**2*f**2*g*
*2 - 10*a*b**3*f**3*g + 5*b**4*f**4)/b + 2*B*a*b**4*c**5*g**4 - 10*B*a*b**4
*c**4*d*f*g**3 + 20*B*a*b**4*c**3*d**2*f**2*g**2 - 20*B*a*b**4*c**2*d**3*f*
*3*g + 20*B*a*b**4*c*d**4*f**4 - 2*B*a*c*d**4*(a**4*g**4 - 5*a**3*b*f*g**3
+ 10*a**2*b**2*f**2*g**2 - 10*a*b**3*f**3*g + 5*b**4*f**4))/(2*B*a**5*d**5*
g**4 - 10*B*a**4*b*d**5*f*g**3 + 20*B*a**3*b**2*d**5*f**2*g**2 - 20*B*a**2*
b**3*d**5*f**3*g + 10*B*a*b**4*d**5*f**4 + 2*B*b**5*c**5*g**4 - 10*B*b**5*c
**4*d*f*g**3 + 20*B*b**5*c**3*d**2*f**2*g**2 - 20*B*b**5*c**2*d**3*f**3*g +
 10*B*b**5*c*d**4*f**4))/(5*b**5) - 2*B*c*(c**4*g**4 - 5*c**3*d*f*g**3 + 10
*c**2*d**2*f**2*g**2 - 10*c*d**3*f**3*g + 5*d**4*f**4)*log(x + (2*B*a**5*c*
d**4*g**4 - 10*B*a**4*b*c*d**4*f*g**3 + 20*B*a**3*b**2*c*d**4*f**2*g**2 - 2
0*B*a**2*b**3*c*d**4*f**3*g + 2*B*a*b**4*c**5*g**4 - 10*B*a*b**4*c**4*d*f*g
**3 + 20*B*a*b**4*c**3*d**2*f**2*g**2 - 20*B*a*b**4*c**2*d**3*f**3*g + 20*B
```

```
*a*b**4*c*d**4*f**4 - 2*B*a*b**4*c*(c**4*g**4 - 5*c**3*d*f*g**3 + 10*c**2*d
**2*f**2*g**2 - 10*c*d**3*f**3*g + 5*d**4*f**4) + 2*B*b**5*c**2*(c**4*g**4
- 5*c**3*d*f*g**3 + 10*c**2*d**2*f**2*g**2 - 10*c*d**3*f**3*g + 5*d**4*f**4
)/d)/(2*B*a**5*d**5*g**4 - 10*B*a**4*b*d**5*f*g**3 + 20*B*a**3*b**2*d**5*f*
*2*g**2 - 20*B*a**2*b**3*d**5*f**3*g + 10*B*a*b**4*d**5*f**4 + 2*B*b**5*c**
5*g**4 - 10*B*b**5*c**4*d*f*g**3 + 20*B*b**5*c**3*d**2*f**2*g**2 - 20*B*b**
5*c**2*d**3*f**3*g + 10*B*b**5*c*d**4*f**4))/(5*d**5) + x**4*(A*f*g**3 + B*
a*g**4/(10*b) - B*c*g**4/(10*d)) + x**3*(2*A*f**2*g**2 - 2*B*a**2*g**4/(15*
b**2) + 2*B*a*f*g**3/(3*b) + 2*B*c**2*g**4/(15*d**2) - 2*B*c*f*g**3/(3*d))
+ x**2*(2*A*f**3*g + B*a**3*g**4/(5*b**3) - B*a**2*f*g**3/b**2 + 2*B*a*f**2
*g**2/b - B*c**3*g**4/(5*d**3) + B*c**2*f*g**3/d**2 - 2*B*c*f**2*g**2/d) +
x*(A*f**4 - 2*B*a**4*g**4/(5*b**4) + 2*B*a**3*f*g**3/b**3 - 4*B*a**2*f**2*g
**2/b**2 + 4*B*a*f**3*g/b + 2*B*c**4*g**4/(5*d**4) - 2*B*c**3*f*g**3/d**3 +
 4*B*c**2*f**2*g**2/d**2 - 4*B*c*f**3*g/d) + (B*f**4*x + 2*B*f**3*g*x**2 +
2*B*f**2*g**2*x**3 + B*f*g**3*x**4 + B*g**4*x**5/5)*log(e*(a + b*x)**2/(c +
 d*x)**2)
```

# 3.263 $\int (f+gx)^3 \left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right) dx$

**Optimal**. Leaf size=229

$$-\frac{Bgx(bc-ad)\left(a^2d^2g^2-abdg(4df-cg)+b^2\left(c^2g^2-4cdfg+6d^2f^2\right)\right)}{2b^3d^3}+\frac{(f+gx)^4\left(B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+A\right)}{4g}-\frac{B(bf-$$

```
[Out] -1/2*B*(-a*d+b*c)*g*(a^2*d^2*g^2-a*b*d*g*(-c*g+4*d*f)+b^2*(c^2*g^2-4*c*d*f*
g+6*d^2*f^2))*x/b^3/d^3-1/4*B*(-a*d+b*c)*g^2*(-a*d*g-b*c*g+4*b*d*f)*x^2/b^2
/d^2-1/6*B*(-a*d+b*c)*g^3*x^3/b/d-1/2*B*(-a*g+b*f)^4*ln(b*x+a)/b^4/g+1/4*(g
*x+f)^4*(A+B*ln(e*(b*x+a)^2/(d*x+c)^2))/g+1/2*B*(-c*g+d*f)^4*ln(d*x+c)/d^4/
g
```

**Rubi [A]** time = 0.32, antiderivative size = 229, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.103, Rules used = {2525, 12, 72}

$$-\frac{Bgx(bc-ad)\left(a^2d^2g^2-abdg(4df-cg)+b^2\left(c^2g^2-4cdfg+6d^2f^2\right)\right)}{2b^3d^3}+\frac{(f+gx)^4\left(B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+A\right)}{4g}-\frac{Bg^2x^2$$

Antiderivative was successfully verified.

```
[In] Int[(f + g*x)^3*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2]),x]
```

```
[Out] -(B*(b*c - a*d)*g*(a^2*d^2*g^2 - a*b*d*g*(4*d*f - c*g) + b^2*(6*d^2*f^2 - 4
*c*d*f*g + c^2*g^2))*x)/(2*b^3*d^3) - (B*(b*c - a*d)*g^2*(4*b*d*f - b*c*g -
 a*d*g)*x^2)/(4*b^2*d^2) - (B*(b*c - a*d)*g^3*x^3)/(6*b*d) - (B*(b*f - a*g)
^4*Log[a + b*x])/(2*b^4*g) + ((f + g*x)^4*(A + B*Log[(e*(a + b*x)^2)/(c + d
*x)^2]))/(4*g) + (B*(d*f - c*g)^4*Log[c + d*x])/(2*d^4*g)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 72

```
Int[((e_.) + (f_.)*(x_))^(p_.)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
 x_Symbol] :> Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x]
 /; FreeQ[{a, b, c, d, e, f}, x] && IntegerQ[p]
```

Rule 2525

```
Int[((a_.) + Log[(c_.)*(RFx_)^(p_.)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_))^(m_.
), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(a + b*Log[c*RFx^p])^n)/(e*(m + 1))
, x] - Dist[(b*n*p)/(e*(m + 1)), Int[SimplifyIntegrand[((d + e*x)^(m + 1)*(
a + b*Log[c*RFx^p])^(n - 1)*D[RFx, x])/RFx, x], x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && RationalFunctionQ[RFx, x] && IGtQ[n, 0] && (EqQ[n, 1] ||
IntegerQ[m]) && NeQ[m, -1]
```

Rubi steps

$$\int (f+gx)^3\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)dx = \frac{(f+gx)^4\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}{4g}-\frac{B\int\frac{2(bc-ad)(f+gx)^4}{(a+bx)(c+dx)}\,dx}{4g}$$

$$=\frac{(f+gx)^4\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}{4g}-\frac{(B(bc-ad))\int\frac{(f+gx)^4}{(a+bx)(c+dx)}\,dx}{2g}$$

$$=\frac{(f+gx)^4\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}{4g}-\frac{(B(bc-ad))\int\left(\frac{g^2\left(a^2d^2g^2-abdg(4df-\cdots\right.}{\cdots}\right.}{\cdots}$$

$$=-\frac{B(bc-ad)g\left(a^2d^2g^2-abdg(4df-cg)+b^2\left(6d^2f^2-4cdfg+c^2g^2\cdots\right.\right.}{2b^3d^3}$$

**Mathematica [A]** time = 0.26, size = 217, normalized size = 0.95

$$\frac{(f+gx)^4\left(B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+A\right)-\frac{B\left(6bdg^2x(bc-ad)\left(a^2d^2g^2+abdg(cg-4df)+b^2\left(c^2g^2-4cdfg+6d^2f^2\right)\right)+2b^3d^3g^4x^3(bc-ad)+3b^2d^2g^3x^2(bc-\cdots\right.}{3b^4d^4}}{4g}$$

Antiderivative was successfully verified.

```
[In] Integrate[(f + g*x)^3*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2]),x]
```

```
[Out] ((f + g*x)^4*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2]) - (B*(6*b*d*(b*c - a*
d)*g^2*(a^2*d^2*g^2 + a*b*d*g*(-4*d*f + c*g) + b^2*(6*d^2*f^2 - 4*c*d*f*g +
 c^2*g^2))*x + 3*b^2*d^2*(b*c - a*d)*g^3*(4*b*d*f - b*c*g - a*d*g)*x^2 + 2*
b^3*d^3*(b*c - a*d)*g^4*x^3 + 6*d^4*(b*f - a*g)^4*Log[a + b*x] - 6*b^4*(d*f
 - c*g)^4*Log[c + d*x]))/(3*b^4*d^4))/(4*g)
```

**fricas [B]** time = 1.31, size = 468, normalized size = 2.04

$$\frac{3\,Ab^4d^4g^3x^4+2\left(6\,Ab^4d^4fg^2-\left(Bb^4cd^3-Bab^3d^4\right)g^3\right)x^3+3\left(6\,Ab^4d^4f^2g-4\left(Bb^4cd^3-Bab^3d^4\right)fg^2+\left(Bb^4c^2d\cdots\right.\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^3*(A+B*log(e*(b*x+a)^2/(d*x+c)^2)),x, algorithm="fricas")
```

```
[Out] 1/12*(3*A*b^4*d^4*g^3*x^4 + 2*(6*A*b^4*d^4*f*g^2 - (B*b^4*c*d^3 - B*a*b^3*d
^4)*g^3)*x^3 + 3*(6*A*b^4*d^4*f^2*g - 4*(B*b^4*c*d^3 - B*a*b^3*d^4)*f*g^2 +
 (B*b^4*c^2*d^2 - B*a^2*b^2*d^4)*g^3)*x^2 + 6*(2*A*b^4*d^4*f^3 - 6*(B*b^4*c
*d^3 - B*a*b^3*d^4)*f^2*g + 4*(B*b^4*c^2*d^2 - B*a^2*b^2*d^4)*f*g^2 - (B*b^
4*c^3*d - B*a^3*b*d^4)*g^3)*x + 6*(4*B*a*b^3*d^4*f^3 - 6*B*a^2*b^2*d^4*f^2*
g + 4*B*a^3*b*d^4*f*g^2 - B*a^4*d^4*g^3)*log(b*x + a) - 6*(4*B*b^4*c*d^3*f^
3 - 6*B*b^4*c^2*d^2*f^2*g + 4*B*b^4*c^3*d*f*g^2 - B*b^4*c^4*g^3)*log(d*x +
c) + 3*(B*b^4*d^4*g^3*x^4 + 4*B*b^4*d^4*f*g^2*x^3 + 6*B*b^4*d^4*f^2*g*x^2 +
 4*B*b^4*d^4*f^3*x)*log((b^2*e*x^2 + 2*a*b*e*x + a^2*e)/(d^2*x^2 + 2*c*d*x
+ c^2)))/(b^4*d^4)
```

**giac [B]** time = 167.45, size = 447, normalized size = 1.95

$$\frac{1}{4}\left(Ag^3+Bg^3\right)x^4+\frac{\left(6\,Abdfg^2+6\,Bbdfg^2-Bbcg^3+Badg^3\right)x^3}{6\,bd}+\frac{1}{4}\left(Bg^3x^4+4\,Bfg^2x^3+6\,Bf^2gx^2+4\,Bf^3x\right)\log\left(\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^3*(A+B*log(e*(b*x+a)^2/(d*x+c)^2)),x, algorithm="giac")
```

```
[Out] 1/4*(A*g^3 + B*g^3)*x^4 + 1/6*(6*A*b*d*f*g^2 + 6*B*b*d*f*g^2 - B*b*c*g^3 +
B*a*d*g^3)*x^3/(b*d) + 1/4*(B*g^3*x^4 + 4*B*f*g^2*x^3 + 6*B*f^2*g*x^2 + 4*B
*f^3*x)*log((b^2*x^2 + 2*a*b*x + a^2)/(d^2*x^2 + 2*c*d*x + c^2)) + 1/4*(6*A
*b^2*d^2*f^2*g + 6*B*b^2*d^2*f^2*g - 4*B*b^2*c*d*f*g^2 + 4*B*a*b*d^2*f*g^2
+ B*b^2*c^2*g^3 - B*a^2*d^2*g^3)*x^2/(b^2*d^2) + 1/2*(4*B*a*b^3*f^3 - 6*B*a
^2*b^2*f^2*g + 4*B*a^3*b*f*g^2 - B*a^4*g^3)*log(b*x + a)/b^4 - 1/2*(4*B*c*d
^3*f^3 - 6*B*c^2*d^2*f^2*g + 4*B*c^3*d*f*g^2 - B*c^4*g^3)*log(-d*x - c)/d^4
 + 1/2*(2*A*b^3*d^3*f^3 + 2*B*b^3*d^3*f^3 - 6*B*b^3*c*d^2*f^2*g + 6*B*a*b^2
*d^3*f^2*g + 4*B*b^3*c^2*d*f*g^2 - 4*B*a^2*b*d^3*f*g^2 - B*b^3*c^3*g^3 + B*
a^3*d^3*g^3)*x/(b^3*d^3)
```

**maple [B]** time = 0.09, size = 1783, normalized size = 7.79

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x+f)^3*(B*ln((b*x+a)^2/(d*x+c)^2*e)+A),x)
```

```
[Out] -3/2/d^2*A*c^2*f^2*g-3/d^2*B*c^2*f^2*g+1/d^3*A*c^3*f*g^2+3/d^3*B*c^3*f*g^2-
1/2/d^3*B*c^3*g^3*x+1/4/d^2*B*c^2*g^3*x^2+B*g^2*ln((1/(d*x+c)*a*d-1/(d*x+c)
*b*c+b)^2/d^2*e)*f*x^3+3/2*B*g*ln((1/(d*x+c)*a*d-1/(d*x+c)*b*c+b)^2/d^2*e)*
```

```
f^2*x^2-2*B/b*ln(1/(d*x+c))*a*f^3+2/d*B/(a*d-b*c)*ln(1/(d*x+c)*a*d-1/(d*x+c
)*b*c+b)*c^2*b*f^3+2/d^3*B*g^3*a/b*ln(1/(d*x+c)*a*d-1/(d*x+c)*b*c+b)*c^3+2*
d*B/b/(a*d-b*c)*ln(1/(d*x+c)*a*d-1/(d*x+c)*b*c+b)*a^2*f^3-11/12/d^4*B*g^3*c
^4-1/4/d^4*A*c^4*g^3+1/d*A*c*f^3+A*x^3*f*g^2+3/2*A*x^2*f^2*g+1/4*B*g^3*ln((
1/(d*x+c)*a*d-1/(d*x+c)*b*c+b)^2/d^2*e)*x^4+B*ln((1/(d*x+c)*a*d-1/(d*x+c)*b
*c+b)^2/d^2*e)*x*f^3-2/d^2*B/b/(a*d-b*c)*ln(1/(d*x+c)*a*d-1/(d*x+c)*b*c+b)*
a^2*c^3*g^3+6/d^3*B/(a*d-b*c)*ln(1/(d*x+c)*a*d-1/(d*x+c)*b*c+b)*c^4*b*f*g^2
-12/d^2*B/(a*d-b*c)*ln(1/(d*x+c)*a*d-1/(d*x+c)*b*c+b)*a*c^3*f*g^2-6/d^2*B/(
a*d-b*c)*ln(1/(d*x+c)*a*d-1/(d*x+c)*b*c+b)*c^3*b*f^2*g-6/d^2*B/b*ln(1/(d*x+
c)*a*d-1/(d*x+c)*b*c+b)*a*c^2*f*g^2+6/d*B*g/b*ln(1/(d*x+c)*a*d-1/(d*x+c)*b*
c+b)*a*c*f^2+12/d*B/(a*d-b*c)*ln(1/(d*x+c)*a*d-1/(d*x+c)*b*c+b)*a*c^2*f^2*g
-6*B/b/(a*d-b*c)*ln(1/(d*x+c)*a*d-1/(d*x+c)*b*c+b)*a^2*c*f^2*g+1/4*A*x^4*g^
3+A*x*f^3+6/d*B/b/(a*d-b*c)*ln(1/(d*x+c)*a*d-1/(d*x+c)*b*c+b)*a^2*c^2*f*g^2
+1/2/d*B*g^3*a^3/b^3*c+1/4/d^2*B*g^3*a^2/b^2*c^2+1/6/d^3*B*g^3*a/b*c^3+1/d^
3*B*g^2*ln((1/(d*x+c)*a*d-1/(d*x+c)*b*c+b)^2/d^2*e)*f*c^3-3/d^2*B*g*ln(1/(d
*x+c)*a*d-1/(d*x+c)*b*c+b)*c^2*f^2-3/2/d^2*B*g*ln((1/(d*x+c)*a*d-1/(d*x+c)*
b*c+b)^2/d^2*e)*f^2*c^2+4/d^3*B*ln(1/(d*x+c)*a*d-1/(d*x+c)*b*c+b)*c^3*f*g^2
-1/2/d^4*B*ln(1/(d*x+c))*c^4*g^3-1/4/d^4*B*ln((1/(d*x+c)*a*d-1/(d*x+c)*b*c+
b)^2/d^2*e)*c^4*g^3+1/d*B*ln((1/(d*x+c)*a*d-1/(d*x+c)*b*c+b)^2/d^2*e)*c*f^3
+2/d*B*ln(1/(d*x+c))*c*f^3-3/2/d^4*B*g^3*c^4*ln(1/(d*x+c)*a*d-1/(d*x+c)*b*c
+b)-1/2*B*g^3*a^4/b^4*ln(1/(d*x+c)*a*d-1/(d*x+c)*b*c+b)+1/2*B*g^3*a^4/b^4*l
n(1/(d*x+c))+1/6*B*g^3*a/b*x^3-1/4*B*g^3*a^2/b^2*x^2+1/2*B*g^3*a^3/b^3*x-1/
6/d*B*g^3*c*x^3+2*B*g^2*a^3/b^3*ln(1/(d*x+c)*a*d-1/(d*x+c)*b*c+b)*f-2*B*g^2
*a^3/b^3*ln(1/(d*x+c))*f+3*B*g/b*a*f^2*x-2*B*g^2*a^2/b^2*f*x+2/d^2*B*c^2*f*
g^2*x-1/d*B*g^2*c*f*x^2-3/d*B*g*c*f^2*x+2/d^3*B*ln(1/(d*x+c))*c^3*f*g^2-3*B
*g/b^2*ln(1/(d*x+c)*a*d-1/(d*x+c)*b*c+b)*a^2*f^2+3*B*g/b^2*ln(1/(d*x+c))*a^
2*f^2+B*g^2*a/b*f*x^2-3/d^2*B*ln(1/(d*x+c))*c^2*f^2*g-4*B/(a*d-b*c)*ln(1/(d
*x+c)*a*d-1/(d*x+c)*b*c+b)*a*c*f^3+4/d^3*B/(a*d-b*c)*ln(1/(d*x+c)*a*d-1/(d*
x+c)*b*c+b)*a*c^4*g^3-2/d^4*B/(a*d-b*c)*ln(1/(d*x+c)*a*d-1/(d*x+c)*b*c+b)*c
^5*b*g^3+3/d*B*g/b*a*f^2*c-1/d^2*B/b*a*c^2*f*g^2-2/d*B*g^2*a^2/b^2*f*c
```

**maxima [B]** time = 1.06, size = 623, normalized size = 2.72

$$\frac{1}{4}Ag^3x^4+Afg^2x^3+\frac{3}{2}Af^2gx^2+\left(x\log\left(\frac{b^2ex^2}{d^2x^2+2\,cdx+c^2}+\frac{2\,abex}{d^2x^2+2\,cdx+c^2}+\frac{a^2e}{d^2x^2+2\,cdx+c^2}\right)+\frac{2\,a\log(b}{b}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^3*(A+B*log(e*(b*x+a)^2/(d*x+c)^2)),x, algorithm="maxima")
```

```
[Out] 1/4*A*g^3*x^4 + A*f*g^2*x^3 + 3/2*A*f^2*g*x^2 + (x*log(b^2*e*x^2/(d^2*x^2 +
 2*c*d*x + c^2) + 2*a*b*e*x/(d^2*x^2 + 2*c*d*x + c^2) + a^2*e/(d^2*x^2 + 2*
c*d*x + c^2)) + 2*a*log(b*x + a)/b - 2*c*log(d*x + c)/d)*B*f^3 + 3/2*(x^2*l
og(b^2*e*x^2/(d^2*x^2 + 2*c*d*x + c^2) + 2*a*b*e*x/(d^2*x^2 + 2*c*d*x + c^2
) + a^2*e/(d^2*x^2 + 2*c*d*x + c^2)) - 2*a^2*log(b*x + a)/b^2 + 2*c^2*log(d
```

```
*x + c)/d^2 - 2*(b*c - a*d)*x/(b*d))*B*f^2*g + (x^3*log(b^2*e*x^2/(d^2*x^2
+ 2*c*d*x + c^2) + 2*a*b*e*x/(d^2*x^2 + 2*c*d*x + c^2) + a^2*e/(d^2*x^2 + 2
*c*d*x + c^2)) + 2*a^3*log(b*x + a)/b^3 - 2*c^3*log(d*x + c)/d^3 - ((b^2*c*
d - a*b*d^2)*x^2 - 2*(b^2*c^2 - a^2*d^2)*x)/(b^2*d^2))*B*f*g^2 + 1/12*(3*x^
4*log(b^2*e*x^2/(d^2*x^2 + 2*c*d*x + c^2) + 2*a*b*e*x/(d^2*x^2 + 2*c*d*x +
c^2) + a^2*e/(d^2*x^2 + 2*c*d*x + c^2)) - 6*a^4*log(b*x + a)/b^4 + 6*c^4*lo
g(d*x + c)/d^4 - (2*(b^3*c*d^2 - a*b^2*d^3)*x^3 - 3*(b^3*c^2*d - a^2*b*d^3)
*x^2 + 6*(b^3*c^3 - a^3*d^3)*x)/(b^3*d^3))*B*g^3 + A*f^3*x
```

**mupad [B]** time = 5.03, size = 743, normalized size = 3.24

$$\ln\left(\frac{e\,(a+b\,x)^2}{(c+d\,x)^2}\right)\left(B\,f^3\,x+\frac{3\,B\,f^2\,g\,x^2}{2}+B\,f\,g^2\,x^3+\frac{B\,g^3\,x^4}{4}\right)+x\left(\frac{2\,A\,b\,d\,f^3+6\,A\,a\,c\,f\,g^2+6\,A\,a\,d\,f^2\,g+6\,A\,b\,c\,}{2\,b\,d}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f + g*x)^3*(A + B*log((e*(a + b*x)^2)/(c + d*x)^2)),x)
```

```
[Out] log((e*(a + b*x)^2)/(c + d*x)^2)*((B*g^3*x^4)/4 + B*f^3*x + (3*B*f^2*g*x^2)
/2 + B*f*g^2*x^3) + x*((2*A*b*d*f^3 + 6*A*a*c*f*g^2 + 6*A*a*d*f^2*g + 6*A*b
*c*f^2*g + 6*B*a*d*f^2*g - 6*B*b*c*f^2*g)/(2*b*d) + ((2*a*d + 2*b*c)*((((2*
A*a*d*g^3 + 2*A*b*c*g^3 + B*a*d*g^3 - B*b*c*g^3 + 6*A*b*d*f*g^2)/(2*b*d) -
(A*g^3*(2*a*d + 2*b*c))/(2*b*d))*(2*a*d + 2*b*c))/(2*b*d) - (2*A*a*c*g^3 +
6*A*a*d*f*g^2 + 6*A*b*c*f*g^2 + 6*A*b*d*f^2*g + 4*B*a*d*f*g^2 - 4*B*b*c*f*g
^2)/(2*b*d) + (A*a*c*g^3)/(b*d)))/(2*b*d) - (a*c*((2*A*a*d*g^3 + 2*A*b*c*g^
3 + B*a*d*g^3 - B*b*c*g^3 + 6*A*b*d*f*g^2)/(2*b*d) - (A*g^3*(2*a*d + 2*b*c)
)/(2*b*d)))/(b*d)) - x^2*((((2*A*a*d*g^3 + 2*A*b*c*g^3 + B*a*d*g^3 - B*b*c*
g^3 + 6*A*b*d*f*g^2)/(2*b*d) - (A*g^3*(2*a*d + 2*b*c))/(2*b*d))*(2*a*d + 2*
b*c))/(4*b*d) - (2*A*a*c*g^3 + 6*A*a*d*f*g^2 + 6*A*b*c*f*g^2 + 6*A*b*d*f^2*
g + 4*B*a*d*f*g^2 - 4*B*b*c*f*g^2)/(4*b*d) + (A*a*c*g^3)/(2*b*d)) + x^3*((2
*A*a*d*g^3 + 2*A*b*c*g^3 + B*a*d*g^3 - B*b*c*g^3 + 6*A*b*d*f*g^2)/(6*b*d) -
 (A*g^3*(2*a*d + 2*b*c))/(6*b*d)) + (A*g^3*x^4)/4 - (log(a + b*x)*(B*a^4*g^
3 - 4*B*a*b^3*f^3 + 6*B*a^2*b^2*f^2*g - 4*B*a^3*b*f*g^2))/(2*b^4) + (log(c
+ d*x)*(B*c^4*g^3 - 4*B*c*d^3*f^3 + 6*B*c^2*d^2*f^2*g - 4*B*c^3*d*f*g^2))/(
2*d^4)
```

**sympy [B]** time = 12.64, size = 998, normalized size = 4.36

$$\frac{Ag^3x^4}{4}-\frac{Ba\left(ag-2bf\right)\left(a^2g^2-2abfg+2b^2f^2\right)\log\left(x+\frac{Ba^4cd^3g^3-4Ba^3bcd^3fg^2+6Ba^2b^2cd^3f^2g+\frac{Ba^2d^4\left(ag-2bf\right)\left(a^2g^2-2abfg+2b^2f^2\right)}{b}+}{Ba^4d^4g^3-4Ba^3bd^4fg^2+6Ba^2b^2d^4f^2g-4B}\right.}{2b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)**3*(A+B*ln(e*(b*x+a)**2/(d*x+c)**2)),x)
```

```
[Out] A*g**3*x**4/4 - B*a*(a*g - 2*b*f)*(a**2*g**2 - 2*a*b*f*g + 2*b**2*f**2)*log
(x + (B*a**4*c*d**3*g**3 - 4*B*a**3*b*c*d**3*f*g**2 + 6*B*a**2*b**2*c*d**3*
f**2*g + B*a**2*d**4*(a*g - 2*b*f)*(a**2*g**2 - 2*a*b*f*g + 2*b**2*f**2)/b
+ B*a*b**3*c**4*g**3 - 4*B*a*b**3*c**3*d*f*g**2 + 6*B*a*b**3*c**2*d**2*f**2
*g - 8*B*a*b**3*c*d**3*f**3 - B*a*c*d**3*(a*g - 2*b*f)*(a**2*g**2 - 2*a*b*f
*g + 2*b**2*f**2))/(B*a**4*d**4*g**3 - 4*B*a**3*b*d**4*f*g**2 + 6*B*a**2*b*
*2*d**4*f**2*g - 4*B*a*b**3*d**4*f**3 + B*b**4*c**4*g**3 - 4*B*b**4*c**3*d*
f*g**2 + 6*B*b**4*c**2*d**2*f**2*g - 4*B*b**4*c*d**3*f**3))/(2*b**4) + B*c*
(c*g - 2*d*f)*(c**2*g**2 - 2*c*d*f*g + 2*d**2*f**2)*log(x + (B*a**4*c*d**3*
g**3 - 4*B*a**3*b*c*d**3*f*g**2 + 6*B*a**2*b**2*c*d**3*f**2*g + B*a*b**3*c*
*4*g**3 - 4*B*a*b**3*c**3*d*f*g**2 + 6*B*a*b**3*c**2*d**2*f**2*g - 8*B*a*b*
*3*c*d**3*f**3 - B*a*b**3*c*(c*g - 2*d*f)*(c**2*g**2 - 2*c*d*f*g + 2*d**2*f
**2) + B*b**4*c**2*(c*g - 2*d*f)*(c**2*g**2 - 2*c*d*f*g + 2*d**2*f**2)/d)/(
B*a**4*d**4*g**3 - 4*B*a**3*b*d**4*f*g**2 + 6*B*a**2*b**2*d**4*f**2*g - 4*B
*a*b**3*d**4*f**3 + B*b**4*c**4*g**3 - 4*B*b**4*c**3*d*f*g**2 + 6*B*b**4*c*
*2*d**2*f**2*g - 4*B*b**4*c*d**3*f**3))/(2*d**4) + x**3*(A*f*g**2 + B*a*g**
3/(6*b) - B*c*g**3/(6*d)) + x**2*(3*A*f**2*g/2 - B*a**2*g**3/(4*b**2) + B*a
*f*g**2/b + B*c**2*g**3/(4*d**2) - B*c*f*g**2/d) + x*(A*f**3 + B*a**3*g**3/
(2*b**3) - 2*B*a**2*f*g**2/b**2 + 3*B*a*f**2*g/b - B*c**3*g**3/(2*d**3) + 2
*B*c**2*f*g**2/d**2 - 3*B*c*f**2*g/d) + (B*f**3*x + 3*B*f**2*g*x**2/2 + B*f
*g**2*x**3 + B*g**3*x**4/4)*log(e*(a + b*x)**2/(c + d*x)**2)
```

# 3.264 $\int (f+gx)^2 \left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)\,dx$

**Optimal**. Leaf size=152

$$\frac{(f+gx)^3\left(B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+A\right)}{3g}-\frac{2B(bf-ag)^3\log(a+bx)}{3b^3g}-\frac{2Bgx(bc-ad)(-adg-bcg+3bdf)}{3b^2d^2}-\frac{Bg^2x^2(bc-ad)}{3bd}$$

```
[Out] -2/3*B*(-a*d+b*c)*g*(-a*d*g-b*c*g+3*b*d*f)*x/b^2/d^2-1/3*B*(-a*d+b*c)*g^2*x
^2/b/d-2/3*B*(-a*g+b*f)^3*ln(b*x+a)/b^3/g+1/3*(g*x+f)^3*(A+B*ln(e*(b*x+a)^2
/(d*x+c)^2))/g+2/3*B*(-c*g+d*f)^3*ln(d*x+c)/d^3/g
```

**Rubi [A]** time = 0.16, antiderivative size = 152, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.103, Rules used = {2525, 12, 72}

$$\frac{(f+gx)^3\left(B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+A\right)}{3g}-\frac{2Bgx(bc-ad)(-adg-bcg+3bdf)}{3b^2d^2}-\frac{2B(bf-ag)^3\log(a+bx)}{3b^3g}-\frac{Bg^2x^2(bc-ad)}{3bd}$$

Antiderivative was successfully verified.

```
[In] Int[(f + g*x)^2*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2]),x]
```

```
[Out] (-2*B*(b*c - a*d)*g*(3*b*d*f - b*c*g - a*d*g)*x)/(3*b^2*d^2) - (B*(b*c - a*
d)*g^2*x^2)/(3*b*d) - (2*B*(b*f - a*g)^3*Log[a + b*x])/(3*b^3*g) + ((f + g*
x)^3*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2]))/(3*g) + (2*B*(d*f - c*g)^3*L
og[c + d*x])/(3*d^3*g)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 72

```
Int[((e_.) + (f_.)*(x_))^(p_.)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
 x_Symbol] :> Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x]
 /; FreeQ[{a, b, c, d, e, f}, x] && IntegerQ[p]
```

Rule 2525

```
Int[((a_.) + Log[(c_.)*(RFx_)^(p_.)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_))^(m_.
), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(a + b*Log[c*RFx^p])^n)/(e*(m + 1))
, x] - Dist[(b*n*p)/(e*(m + 1)), Int[SimplifyIntegrand[((d + e*x)^(m + 1)*(
```

```
a + b*Log[c*RFx^p])^(n - 1)*D[RFx, x])/RFx, x], x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && RationalFunctionQ[RFx, x] && IGtQ[n, 0] && (EqQ[n, 1] ||
IntegerQ[m]) && NeQ[m, -1]
```

Rubi steps

$$\int (f+gx)^2 \left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right) dx = \frac{(f+gx)^3\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}{3g} - \frac{B\int \frac{2(bc-ad)(f+gx)^3}{(a+bx)(c+dx)}\,dx}{3g}$$

$$= \frac{(f+gx)^3\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}{3g} - \frac{(2B(bc-ad))\int \frac{(f+gx)^3}{(a+bx)(c+dx)}\,dx}{3g}$$

$$= \frac{(f+gx)^3\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}{3g} - \frac{(2B(bc-ad))\int \left(\frac{g^2(3bdf-bcg-adg)}{b^2d^2} \ldots\right.}{}$$

$$= -\frac{2B(bc-ad)g(3bdf-bcg-adg)x}{3b^2d^2} - \frac{B(bc-ad)g^2x^2}{3bd} - \frac{2B(bf-ag \ldots}{3 \ldots}$$

**Mathematica [A]** time = 0.14, size = 142, normalized size = 0.93

$$\frac{(f+gx)^3\left(B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+A\right) - \frac{B\left(b^2d^2g^3x^2(bc-ad)+2bdg^2x(bc-ad)(-adg-bcg+3bdf)+2d^3(bf-ag)^3\log(a+bx)-2b^3(df-cg)^3\log(c+dx)\right)}{b^3d^3}}{3g}$$

Antiderivative was successfully verified.

```
[In] Integrate[(f + g*x)^2*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2]),x]
```

```
[Out] ((f + g*x)^3*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2]) - (B*(2*b*d*(b*c - a*
d)*g^2*(3*b*d*f - b*c*g - a*d*g)*x + b^2*d^2*(b*c - a*d)*g^3*x^2 + 2*d^3*(b
*f - a*g)^3*Log[a + b*x] - 2*b^3*(d*f - c*g)^3*Log[c + d*x]))/(b^3*d^3))/(3
*g)
```

**fricas [B]** time = 0.99, size = 301, normalized size = 1.98

$$\frac{Ab^3d^3g^2x^3 + \left(3\,Ab^3d^3fg - \left(Bb^3cd^2 - Bab^2d^3\right)g^2\right)x^2 + \left(3\,Ab^3d^3f^2 - 6\left(Bb^3cd^2 - Bab^2d^3\right)fg + 2\left(Bb^3c^2d - Ba^2b \ldots\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^2*(A+B*log(e*(b*x+a)^2/(d*x+c)^2)),x, algorithm="fricas")
```

```
[Out] 1/3*(A*b^3*d^3*g^2*x^3 + (3*A*b^3*d^3*f*g - (B*b^3*c*d^2 - B*a*b^2*d^3)*g^2
)*x^2 + (3*A*b^3*d^3*f^2 - 6*(B*b^3*c*d^2 - B*a*b^2*d^3)*f*g + 2*(B*b^3*c^2
*d - B*a^2*b*d^3)*g^2)*x + 2*(3*B*a*b^2*d^3*f^2 - 3*B*a^2*b*d^3*f*g + B*a^3
*d^3*g^2)*log(b*x + a) - 2*(3*B*b^3*c*d^2*f^2 - 3*B*b^3*c^2*d*f*g + B*b^3*c
^3*g^2)*log(d*x + c) + (B*b^3*d^3*g^2*x^3 + 3*B*b^3*d^3*f*g*x^2 + 3*B*b^3*d
^3*f^2*x)*log((b^2*e*x^2 + 2*a*b*e*x + a^2*e)/(d^2*x^2 + 2*c*d*x + c^2)))/(
b^3*d^3)
```

**giac [A]** time = 12.58, size = 279, normalized size = 1.84

$$\frac{1}{3}\left(Ag^2+Bg^2\right)x^3+\frac{1}{3}\left(Bg^2x^3+3\,Bfgx^2+3\,Bf^2x\right)\log\left(\frac{b^2x^2+2\,abx+a^2}{d^2x^2+2\,cdx+c^2}\right)+\frac{\left(3\,Abdfg+3\,Bbdfg-Bbcg^2+Badg\right.}{3\,bd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^2*(A+B*log(e*(b*x+a)^2/(d*x+c)^2)),x, algorithm="giac")
```

```
[Out] 1/3*(A*g^2 + B*g^2)*x^3 + 1/3*(B*g^2*x^3 + 3*B*f*g*x^2 + 3*B*f^2*x)*log((b^
2*x^2 + 2*a*b*x + a^2)/(d^2*x^2 + 2*c*d*x + c^2)) + 1/3*(3*A*b*d*f*g + 3*B*
b*d*f*g - B*b*c*g^2 + B*a*d*g^2)*x^2/(b*d) + 2/3*(3*B*a*b^2*f^2 - 3*B*a^2*b
*f*g + B*a^3*g^2)*log(b*x + a)/b^3 - 2/3*(3*B*c*d^2*f^2 - 3*B*c^2*d*f*g + B
*c^3*g^2)*log(-d*x - c)/d^3 + 1/3*(3*A*b^2*d^2*f^2 + 3*B*b^2*d^2*f^2 - 6*B*
b^2*c*d*f*g + 6*B*a*b*d^2*f*g + 2*B*b^2*c^2*g^2 - 2*B*a^2*d^2*g^2)*x/(b^2*d
^2)
```

**maple [B]** time = 0.08, size = 1188, normalized size = 7.82

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x+f)^2*(B*ln((b*x+a)^2/(d*x+c)^2*e)+A),x)
```

```
[Out] A*x*f^2+1/3*A*x^3*g^2+1/3/d^3*B*ln((1/(d*x+c)*a*d-1/(d*x+c)*b*c+b)^2/d^2*e)
*c^3*g^2+1/3/d^3*A*c^3*g^2+1/d*A*c*f^2+1/3*B*g^2*ln((1/(d*x+c)*a*d-1/(d*x+c
)*b*c+b)^2/d^2*e)*x^3+B*ln((1/(d*x+c)*a*d-1/(d*x+c)*b*c+b)^2/d^2*e)*x*f^2+A
*x^2*f*g+1/d^3*B*c^3*g^2-4*B/b/(a*d-b*c)*ln(1/(d*x+c)*a*d-1/(d*x+c)*b*c+b)*
a^2*c*f*g-4/d^2*B/(a*d-b*c)*ln(1/(d*x+c)*a*d-1/(d*x+c)*b*c+b)*c^3*b*f*g+4/d
*B*g/b*ln(1/(d*x+c)*a*d-1/(d*x+c)*b*c+b)*a*c*f+8/d*B/(a*d-b*c)*ln(1/(d*x+c)
*a*d-1/(d*x+c)*b*c+b)*a*c^2*f*g+2/d*B/b/(a*d-b*c)*ln(1/(d*x+c)*a*d-1/(d*x+c
)*b*c+b)*a^2*c^2*g^2-2/3*B*g^2*a^2/b^2*x+1/3*B*g^2*a/b*x^2+2/d*B*ln(1/(d*x+
c))*c*f^2+2/3*B*g^2*a^3/b^3*ln(1/(d*x+c)*a*d-1/(d*x+c)*b*c+b)-2/3*B*g^2*a^3
/b^3*ln(1/(d*x+c))+B*g*ln((1/(d*x+c)*a*d-1/(d*x+c)*b*c+b)^2/d^2*e)*f*x^2+1/
d*B*ln((1/(d*x+c)*a*d-1/(d*x+c)*b*c+b)^2/d^2*e)*c*f^2+4/3/d^3*B*ln(1/(d*x+c
)*a*d-1/(d*x+c)*b*c+b)*c^3*g^2-2*B/b*ln(1/(d*x+c))*a*f^2-1/3/d*B*g^2*c*x^2+
2/3/d^2*B*c^2*g^2*x+2/3/d^3*B*ln(1/(d*x+c))*c^3*g^2-1/d^2*A*c^2*f*g-2/d^2*B
```

```
*g*c^2*f-2*B*g/b^2*ln(1/(d*x+c)*a*d-1/(d*x+c)*b*c+b)*a^2*f-2/d*B*g*c*f*x-2/
d^2*B*ln(1/(d*x+c))*c^2*f*g-1/d^2*B*g*ln((1/(d*x+c)*a*d-1/(d*x+c)*b*c+b)^2/
d^2*e)*f*c^2-2/d^2*B*g*ln(1/(d*x+c)*a*d-1/(d*x+c)*b*c+b)*c^2*f+2*B*g/b^2*ln
(1/(d*x+c))*a^2*f-4*B/(a*d-b*c)*ln(1/(d*x+c)*a*d-1/(d*x+c)*b*c+b)*a*c*f^2+2
*B*g/b*a*f*x-2/3/d*B*g^2*a^2/b^2*c-1/3/d^2*B*g^2*a/b*c^2-4/d^2*B/(a*d-b*c)*
ln(1/(d*x+c)*a*d-1/(d*x+c)*b*c+b)*a*c^3*g^2+2*d*B/b/(a*d-b*c)*ln(1/(d*x+c)*
a*d-1/(d*x+c)*b*c+b)*a^2*f^2-2/d^2*B/b*ln(1/(d*x+c)*a*d-1/(d*x+c)*b*c+b)*a*
c^2*g^2+2/d*B/(a*d-b*c)*ln(1/(d*x+c)*a*d-1/(d*x+c)*b*c+b)*c^2*b*f^2+2/d^3*B
/(a*d-b*c)*ln(1/(d*x+c)*a*d-1/(d*x+c)*b*c+b)*c^4*b*g^2+2/d*B*g/b*a*c*f
```

**maxima [B]** time = 0.79, size = 419, normalized size = 2.76

$$\frac{1}{3}\,Ag^2x^3+Afgx^2+\left(x\log\left(\frac{b^2ex^2}{d^2x^2+2\,cdx+c^2}+\frac{2\,abex}{d^2x^2+2\,cdx+c^2}+\frac{a^2e}{d^2x^2+2\,cdx+c^2}\right)+\frac{2\,a\log\left(bx+a\right)}{b}-\cdots$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^2*(A+B*log(e*(b*x+a)^2/(d*x+c)^2)),x, algorithm="maxima")
```

```
[Out] 1/3*A*g^2*x^3 + A*f*g*x^2 + (x*log(b^2*e*x^2/(d^2*x^2 + 2*c*d*x + c^2) + 2*
a*b*e*x/(d^2*x^2 + 2*c*d*x + c^2) + a^2*e/(d^2*x^2 + 2*c*d*x + c^2)) + 2*a*
log(b*x + a)/b - 2*c*log(d*x + c)/d)*B*f^2 + (x^2*log(b^2*e*x^2/(d^2*x^2 +
2*c*d*x + c^2) + 2*a*b*e*x/(d^2*x^2 + 2*c*d*x + c^2) + a^2*e/(d^2*x^2 + 2*c
*d*x + c^2)) - 2*a^2*log(b*x + a)/b^2 + 2*c^2*log(d*x + c)/d^2 - 2*(b*c - a
*d)*x/(b*d))*B*f*g + 1/3*(x^3*log(b^2*e*x^2/(d^2*x^2 + 2*c*d*x + c^2) + 2*a
*b*e*x/(d^2*x^2 + 2*c*d*x + c^2) + a^2*e/(d^2*x^2 + 2*c*d*x + c^2)) + 2*a^3
*log(b*x + a)/b^3 - 2*c^3*log(d*x + c)/d^3 - ((b^2*c*d - a*b*d^2)*x^2 - 2*(
b^2*c^2 - a^2*d^2)*x)/(b^2*d^2))*B*g^2 + A*f^2*x
```

**mupad [B]** time = 4.79, size = 362, normalized size = 2.38

$$\ln\left(\frac{e\,{\left(a+b\,x\right)}^2}{{\left(c+d\,x\right)}^2}\right)\,\left(B\,f^2\,x+B\,f\,g\,x^2+\frac{B\,g^2\,x^3}{3}\right)+x^2\,\left(\frac{3\,A\,a\,d\,g^2+3\,A\,b\,c\,g^2+2\,B\,a\,d\,g^2-2\,B\,b\,c\,g^2+6\,A\,b\,d\,f\,g}{6\,b\,d}-\cdots\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f + g*x)^2*(A + B*log((e*(a + b*x)^2)/(c + d*x)^2)),x)
```

```
[Out] log((e*(a + b*x)^2)/(c + d*x)^2)*((B*g^2*x^3)/3 + B*f^2*x + B*f*g*x^2) + x^
2*((3*A*a*d*g^2 + 3*A*b*c*g^2 + 2*B*a*d*g^2 - 2*B*b*c*g^2 + 6*A*b*d*f*g)/(6
*b*d) - (A*g^2*(3*a*d + 3*b*c))/(6*b*d)) - x*((((3*A*a*d*g^2 + 3*A*b*c*g^2
+ 2*B*a*d*g^2 - 2*B*b*c*g^2 + 6*A*b*d*f*g)/(3*b*d) - (A*g^2*(3*a*d + 3*b*c)
)/(3*b*d))*(3*a*d + 3*b*c))/(3*b*d) - (3*A*a*c*g^2 + 3*A*b*d*f^2 + 6*A*a*d*
f*g + 6*A*b*c*f*g + 6*B*a*d*f*g - 6*B*b*c*f*g)/(3*b*d) + (A*a*c*g^2)/(b*d))
```

```
 + (log(a + b*x)*(2*B*a^3*g^2 + 6*B*a*b^2*f^2 - 6*B*a^2*b*f*g))/(3*b^3) - (
log(c + d*x)*(2*B*c^3*g^2 + 6*B*c*d^2*f^2 - 6*B*c^2*d*f*g))/(3*d^3) + (A*g^
2*x^3)/3
```

**sympy [B]** time = 6.81, size = 692, normalized size = 4.55

$$\frac{Ag^2x^3}{3}+\frac{2Ba\left(a^2g^2-3abfg+3b^2f^2\right)\log\left(x+\frac{2Ba^3cd^2g^2-6Ba^2bcd^2fg+\frac{2Ba^2d^3\left(a^2g^2-3abfg+3b^2f^2\right)}{b}+2Bab^2c^3g^2-6Bab^2c^2dfg+12Bab^2cd}{2Ba^3d^3g^2-6Ba^2bd^3fg+6Bab^2d^3f^2+2Bb^3c^3g^2-6Bb^3c^2dfg+6B}\right.}{3b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)**2*(A+B*ln(e*(b*x+a)**2/(d*x+c)**2)),x)
```

```
[Out] A*g**2*x**3/3 + 2*B*a*(a**2*g**2 - 3*a*b*f*g + 3*b**2*f**2)*log(x + (2*B*a*
*3*c*d**2*g**2 - 6*B*a**2*b*c*d**2*f*g + 2*B*a**2*d**3*(a**2*g**2 - 3*a*b*f
*g + 3*b**2*f**2)/b + 2*B*a*b**2*c**3*g**2 - 6*B*a*b**2*c**2*d*f*g + 12*B*a
*b**2*c*d**2*f**2 - 2*B*a*c*d**2*(a**2*g**2 - 3*a*b*f*g + 3*b**2*f**2))/(2*
B*a**3*d**3*g**2 - 6*B*a**2*b*d**3*f*g + 6*B*a*b**2*d**3*f**2 + 2*B*b**3*c*
*3*g**2 - 6*B*b**3*c**2*d*f*g + 6*B*b**3*c*d**2*f**2))/(3*b**3) - 2*B*c*(c*
*2*g**2 - 3*c*d*f*g + 3*d**2*f**2)*log(x + (2*B*a**3*c*d**2*g**2 - 6*B*a**2
*b*c*d**2*f*g + 2*B*a*b**2*c**3*g**2 - 6*B*a*b**2*c**2*d*f*g + 12*B*a*b**2*
c*d**2*f**2 - 2*B*a*b**2*c*(c**2*g**2 - 3*c*d*f*g + 3*d**2*f**2) + 2*B*b**3
*c**2*(c**2*g**2 - 3*c*d*f*g + 3*d**2*f**2)/d)/(2*B*a**3*d**3*g**2 - 6*B*a*
*2*b*d**3*f*g + 6*B*a*b**2*d**3*f**2 + 2*B*b**3*c**3*g**2 - 6*B*b**3*c**2*d
*f*g + 6*B*b**3*c*d**2*f**2))/(3*d**3) + x**2*(A*f*g + B*a*g**2/(3*b) - B*c
*g**2/(3*d)) + x*(A*f**2 - 2*B*a**2*g**2/(3*b**2) + 2*B*a*f*g/b + 2*B*c**2*
g**2/(3*d**2) - 2*B*c*f*g/d) + (B*f**2*x + B*f*g*x**2 + B*g**2*x**3/3)*log(
e*(a + b*x)**2/(c + d*x)**2)
```

# 3.265 $\int (f+gx)\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)dx$

**Optimal**. Leaf size=104

$$\frac{(f+gx)^2\left(B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+A\right)}{2g}-\frac{B(bf-ag)^2\log(a+bx)}{b^2g}-\frac{Bgx(bc-ad)}{bd}+\frac{B(df-cg)^2\log(c+dx)}{d^2g}$$

```
[Out] -B*(-a*d+b*c)*g*x/b/d-B*(-a*g+b*f)^2*ln(b*x+a)/b^2/g+1/2*(g*x+f)^2*(A+B*ln(
e*(b*x+a)^2/(d*x+c)^2))/g+B*(-c*g+d*f)^2*ln(d*x+c)/d^2/g
```

**Rubi [A]** time = 0.09, antiderivative size = 104, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {2525, 12, 72}

$$\frac{(f+gx)^2\left(B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+A\right)}{2g}-\frac{B(bf-ag)^2\log(a+bx)}{b^2g}-\frac{Bgx(bc-ad)}{bd}+\frac{B(df-cg)^2\log(c+dx)}{d^2g}$$

Antiderivative was successfully verified.

```
[In] Int[(f + g*x)*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2]),x]
```

```
[Out] -((B*(b*c - a*d)*g*x)/(b*d)) - (B*(b*f - a*g)^2*Log[a + b*x])/(b^2*g) + ((f
 + g*x)^2*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2]))/(2*g) + (B*(d*f - c*g)^
2*Log[c + d*x])/(d^2*g)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 72

```
Int[((e_.) + (f_.)*(x_))^(p_.)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
 x_Symbol] :> Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x]
 /; FreeQ[{a, b, c, d, e, f}, x] && IntegerQ[p]
```

Rule 2525

```
Int[((a_.) + Log[(c_.)*(RFx_)^(p_.)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_))^(m_.
), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(a + b*Log[c*RFx^p])^n)/(e*(m + 1))
, x] - Dist[(b*n*p)/(e*(m + 1)), Int[SimplifyIntegrand[((d + e*x)^(m + 1)*(
a + b*Log[c*RFx^p])^(n - 1)*D[RFx, x])/RFx, x], x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && RationalFunctionQ[RFx, x] && IGtQ[n, 0] && (EqQ[n, 1] ||
```

```
IntegerQ[m]) && NeQ[m, -1]
```

Rubi steps

$$\int (f+gx)\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)dx = \frac{(f+gx)^2\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}{2g} - \frac{B\int \frac{2(bc-ad)(f+gx)^2}{(a+bx)(c+dx)}\,dx}{2g}$$

$$= \frac{(f+gx)^2\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}{2g} - \frac{(B(bc-ad))\int \frac{(f+gx)^2}{(a+bx)(c+dx)}\,dx}{g}$$

$$= \frac{(f+gx)^2\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}{2g} - \frac{(B(bc-ad))\int \left(\frac{g^2}{bd} + \frac{(bf-ag)^2}{b(bc-ad)(a+bx)} + \right.}{g}$$

$$= -\frac{B(bc-ad)gx}{bd} - \frac{B(bf-ag)^2\log(a+bx)}{b^2g} + \frac{(f+gx)^2\left(A+B\log\left(\frac{e(a}{(c-}\right.\right.}{2g}$$

**Mathematica [A]** time = 0.11, size = 118, normalized size = 1.13

$$\frac{b\left(d\left(2Bg^2x(ad-bc)+Abd(f+gx)^2\right)+bBd^2(f+gx)^2\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+2bB(df-cg)^2\log(c+dx)\right)-2Bd^2(bf-a}{2b^2d^2g}$$

Antiderivative was successfully verified.

```
[In] Integrate[(f + g*x)*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2]),x]
```

```
[Out] (-2*B*d^2*(b*f - a*g)^2*Log[a + b*x] + b*(d*(2*B*(-(b*c) + a*d)*g^2*x + A*b
*d*(f + g*x)^2) + b*B*d^2*(f + g*x)^2*Log[(e*(a + b*x)^2)/(c + d*x)^2] + 2*
b*B*(d*f - c*g)^2*Log[c + d*x]))/(2*b^2*d^2*g)
```

**fricas [A]** time = 0.87, size = 174, normalized size = 1.67

$$\frac{Ab^2d^2gx^2+2\left(Ab^2d^2f-\left(Bb^2cd-Babd^2\right)g\right)x+2\left(2\,Babd^2f-Ba^2d^2g\right)\log\left(bx+a\right)-2\left(2\,Bb^2cdf-Bb^2c^2g\right)\log}{2\,b^2d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)*(A+B*log(e*(b*x+a)^2/(d*x+c)^2)),x, algorithm="fricas")
```

```
[Out] 1/2*(A*b^2*d^2*g*x^2 + 2*(A*b^2*d^2*f - (B*b^2*c*d - B*a*b*d^2)*g)*x + 2*(2
*B*a*b*d^2*f - B*a^2*d^2*g)*log(b*x + a) - 2*(2*B*b^2*c*d*f - B*b^2*c^2*g)*
```

```
log(d*x + c) + (B*b^2*d^2*g*x^2 + 2*B*b^2*d^2*f*x)*log((b^2*e*x^2 + 2*a*b*e
*x + a^2*e)/(d^2*x^2 + 2*c*d*x + c^2)))/(b^2*d^2)
```

**giac [A]** time = 1.11, size = 145, normalized size = 1.39

$$\frac{1}{2}\left(Ag+Bg\right)x^2+\frac{1}{2}\left(Bgx^2+2\,Bfx\right)\log\left(\frac{b^2x^2+2\,abx+a^2}{d^2x^2+2\,cdx+c^2}\right)+\frac{\left(Abdf+Bbdf-Bbcg+Badg\right)x}{bd}+\frac{\left(2\,Babf-Ba^2g\right.}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)*(A+B*log(e*(b*x+a)^2/(d*x+c)^2)),x, algorithm="giac")
```

```
[Out] 1/2*(A*g + B*g)*x^2 + 1/2*(B*g*x^2 + 2*B*f*x)*log((b^2*x^2 + 2*a*b*x + a^2)
/(d^2*x^2 + 2*c*d*x + c^2)) + (A*b*d*f + B*b*d*f - B*b*c*g + B*a*d*g)*x/(b*
d) + (2*B*a*b*f - B*a^2*g)*log(b*x + a)/b^2 - (2*B*c*d*f - B*c^2*g)*log(-d*
x - c)/d^2
```

**maple [B]** time = 0.08, size = 656, normalized size = 6.31

$$-\frac{2B\,a^2cg\ln\left(\frac{ad}{dx+c}-\frac{bc}{dx+c}+b\right)}{(ad-bc)\,b}+\frac{2B\,a^2df\ln\left(\frac{ad}{dx+c}-\frac{bc}{dx+c}+b\right)}{(ad-bc)\,b}+\frac{4Ba\,c^2g\ln\left(\frac{ad}{dx+c}-\frac{bc}{dx+c}+b\right)}{(ad-bc)\,d}-\frac{4Bacf\ln\left(\frac{ad}{dx+c}-\frac{b}{dx}\right.}{ad-bc}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x+f)*(B*ln((b*x+a)^2/(d*x+c)^2*e)+A),x)
```

```
[Out] -2/d^2*B/(a*d-b*c)*ln(1/(d*x+c)*a*d-1/(d*x+c)*b*c+b)*c^3*b*g+1/2*B*g*ln((1/
(d*x+c)*a*d-1/(d*x+c)*b*c+b)^2/d^2*e)*x^2+B*ln((1/(d*x+c)*a*d-1/(d*x+c)*b*c
+b)^2/d^2*e)*x*f-1/2/d^2*B*ln((1/(d*x+c)*a*d-1/(d*x+c)*b*c+b)^2/d^2*e)*c^2*
g-1/d^2*B*g*ln(1/(d*x+c)*a*d-1/(d*x+c)*b*c+b)*c^2-1/d^2*B*ln(1/(d*x+c))*c^2
*g+1/d*B*ln((1/(d*x+c)*a*d-1/(d*x+c)*b*c+b)^2/d^2*e)*c*f-B*g/b^2*ln(1/(d*x+
c)*a*d-1/(d*x+c)*b*c+b)*a^2+2/d*B*ln(1/(d*x+c))*c*f-1/d*B*c*g*x+B*g/b^2*ln(
1/(d*x+c))*a^2-2*B/b*ln(1/(d*x+c))*a*f+B*g/b*a*x-1/d^2*B*c^2*g-1/2/d^2*A*c^
2*g+1/d*A*c*f+1/2*A*x^2*g+A*x*f-4*B/(a*d-b*c)*ln(1/(d*x+c)*a*d-1/(d*x+c)*b*
c+b)*a*c*f+1/d*B*g/b*a*c+2/d*B*g/b*ln(1/(d*x+c)*a*d-1/(d*x+c)*b*c+b)*a*c+2*
d*B/b/(a*d-b*c)*ln(1/(d*x+c)*a*d-1/(d*x+c)*b*c+b)*a^2*f+2/d*B/(a*d-b*c)*ln(
1/(d*x+c)*a*d-1/(d*x+c)*b*c+b)*c^2*b*f+4/d*B/(a*d-b*c)*ln(1/(d*x+c)*a*d-1/(
d*x+c)*b*c+b)*a*c^2*g-2*B/b/(a*d-b*c)*ln(1/(d*x+c)*a*d-1/(d*x+c)*b*c+b)*a^2
*c*g
```

**maxima [B]** time = 0.76, size = 246, normalized size = 2.37

$$\frac{1}{2}\,Agx^2+\left(x\log\left(\frac{b^2ex^2}{d^2x^2+2\,cdx+c^2}+\frac{2\,abex}{d^2x^2+2\,cdx+c^2}+\frac{a^2e}{d^2x^2+2\,cdx+c^2}\right)+\frac{2\,a\log\left(bx+a\right)}{b}-\frac{2\,c\log\left(dx+\right.}{d}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)*(A+B*log(e*(b*x+a)^2/(d*x+c)^2)),x, algorithm="maxima")
```

```
[Out] 1/2*A*g*x^2 + (x*log(b^2*e*x^2/(d^2*x^2 + 2*c*d*x + c^2) + 2*a*b*e*x/(d^2*x
^2 + 2*c*d*x + c^2) + a^2*e/(d^2*x^2 + 2*c*d*x + c^2)) + 2*a*log(b*x + a)/b
 - 2*c*log(d*x + c)/d)*B*f + 1/2*(x^2*log(b^2*e*x^2/(d^2*x^2 + 2*c*d*x + c^
2) + 2*a*b*e*x/(d^2*x^2 + 2*c*d*x + c^2) + a^2*e/(d^2*x^2 + 2*c*d*x + c^2))
 - 2*a^2*log(b*x + a)/b^2 + 2*c^2*log(d*x + c)/d^2 - 2*(b*c - a*d)*x/(b*d))
*B*g + A*f*x
```

**mupad [B]** time = 4.50, size = 133, normalized size = 1.28

$$\ln\left(\frac{e\,(a+b\,x)^2}{(c+d\,x)^2}\right)\left(\frac{B\,g\,x^2}{2}+B\,f\,x\right)+x\,\left(\frac{A\,a\,d\,g+A\,b\,c\,g+A\,b\,d\,f+B\,a\,d\,g-B\,b\,c\,g}{b\,d}-\frac{A\,g\,(a\,d+b\,c)}{b\,d}\right)+\frac{A\,g\,x^2}{2}-\frac{B}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f + g*x)*(A + B*log((e*(a + b*x)^2)/(c + d*x)^2)),x)
```

```
[Out] log((e*(a + b*x)^2)/(c + d*x)^2)*(B*f*x + (B*g*x^2)/2) + x*((A*a*d*g + A*b*
c*g + A*b*d*f + B*a*d*g - B*b*c*g)/(b*d) - (A*g*(a*d + b*c))/(b*d)) + (A*g*
x^2)/2 - (B*a*log(a + b*x)*(a*g - 2*b*f))/b^2 + (B*c*log(c + d*x)*(c*g - 2*
d*f))/d^2
```

**sympy [B]** time = 2.79, size = 314, normalized size = 3.02

$$\frac{Agx^2}{2}-\frac{Ba\left(ag-2bf\right)\log\left(x+\frac{Ba^2cdg+\frac{Ba^2d^2\left(ag-2bf\right)}{b}+Babc^2g-4Babcdf-Bacd\left(ag-2bf\right)}{Ba^2d^2g-2Babd^2f+Bb^2c^2g-2Bb^2cdf}\right)}{b^2}+\frac{Bc\left(cg-2df\right)\log\left(x+\frac{Ba^2cdg+Babc^2g}{Ba^2d^2}\right)}{d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)*(A+B*ln(e*(b*x+a)**2/(d*x+c)**2)),x)
```

```
[Out] A*g*x**2/2 - B*a*(a*g - 2*b*f)*log(x + (B*a**2*c*d*g + B*a**2*d**2*(a*g - 2
*b*f)/b + B*a*b*c**2*g - 4*B*a*b*c*d*f - B*a*c*d*(a*g - 2*b*f))/(B*a**2*d**
2*g - 2*B*a*b*d**2*f + B*b**2*c**2*g - 2*B*b**2*c*d*f))/b**2 + B*c*(c*g - 2
*d*f)*log(x + (B*a**2*c*d*g + B*a*b*c**2*g - 4*B*a*b*c*d*f - B*a*b*c*(c*g -
 2*d*f) + B*b**2*c**2*(c*g - 2*d*f)/d)/(B*a**2*d**2*g - 2*B*a*b*d**2*f + B*
b**2*c**2*g - 2*B*b**2*c*d*f))/d**2 + x*(A*f + B*a*g/b - B*c*g/d) + (B*f*x
 + B*g*x**2/2)*log(e*(a + b*x)**2/(c + d*x)**2)
```

# 3.266 $\int \left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)\,dx$

**Optimal**. Leaf size=54

$$\frac{B(a+bx)\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}{b}-\frac{2B(bc-ad)\log(c+dx)}{bd}+Ax$$

```
[Out] A*x+B*(b*x+a)*ln(e*(b*x+a)^2/(d*x+c)^2)/b-2*B*(-a*d+b*c)*ln(d*x+c)/b/d
```

**Rubi [A]** time = 0.03, antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {2486, 31}

$$\frac{B(a+bx)\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}{b}-\frac{2B(bc-ad)\log(c+dx)}{bd}+Ax$$

Antiderivative was successfully verified.

```
[In] Int[A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2],x]
```

```
[Out] A*x + (B*(a + b*x)*Log[(e*(a + b*x)^2)/(c + d*x)^2])/b - (2*B*(b*c - a*d)*L
og[c + d*x])/(b*d)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 2486

```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))
^(r_.)]^(s_.), x_Symbol] :> Simp[((a + b*x)*Log[e*(f*(a + b*x)^p*(c + d*x)^
q)^r]^s)/b, x] + Dist[(q*r*s*(b*c - a*d))/b, Int[Log[e*(f*(a + b*x)^p*(c +
d*x)^q)^r]^(s - 1)/(c + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, p, q, r, s
}, x] && NeQ[b*c - a*d, 0] && EqQ[p + q, 0] && IGtQ[s, 0]
```

Rubi steps

$$\int \left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)dx = Ax+B\int \log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)dx$$
$$= Ax+\frac{B(a+bx)\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}{b}-\frac{(2B(bc-ad))\int \frac{1}{c+dx}\,dx}{b}$$
$$= Ax+\frac{B(a+bx)\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}{b}-\frac{2B(bc-ad)\log(c+dx)}{bd}$$

**Mathematica [A]** time = 0.02, size = 54, normalized size = 1.00

$$\frac{B(a+bx)\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}{b}-\frac{2B(bc-ad)\log(c+dx)}{bd}+Ax$$

Antiderivative was successfully verified.

```
[In] Integrate[A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2],x]
```

```
[Out] A*x + (B*(a + b*x)*Log[(e*(a + b*x)^2)/(c + d*x)^2])/b - (2*B*(b*c - a*d)*L
og[c + d*x])/(b*d)
```

**fricas [A]** time = 0.87, size = 80, normalized size = 1.48

$$\frac{Bbdx\log\left(\frac{b^2ex^2+2\,abex+a^2e}{d^2x^2+2\,cdx+c^2}\right)+Abdx+2\,Bad\log\left(bx+a\right)-2\,Bbc\log\left(dx+c\right)}{bd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(A+B*log(e*(b*x+a)^2/(d*x+c)^2),x, algorithm="fricas")
```

```
[Out] (B*b*d*x*log((b^2*e*x^2 + 2*a*b*e*x + a^2*e)/(d^2*x^2 + 2*c*d*x + c^2)) + A
*b*d*x + 2*B*a*d*log(b*x + a) - 2*B*b*c*log(d*x + c))/(b*d)
```

**giac [A]** time = 0.26, size = 83, normalized size = 1.54

$$\left(2\left(bc-ad\right)\left(\frac{a\log\left(|bx+a|\right)}{b^2c-abd}-\frac{c\log\left(|dx+c|\right)}{bcd-ad^2}\right)+x\log\left(\frac{\left(bx+a\right)^2e}{\left(dx+c\right)^2}\right)\right)B+Ax$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(A+B*log(e*(b*x+a)^2/(d*x+c)^2),x, algorithm="giac")
```

```
[Out] (2*(b*c - a*d)*(a*log(abs(b*x + a))/(b^2*c - a*b*d) - c*log(abs(d*x + c))/(
b*c*d - a*d^2)) + x*log((b*x + a)^2*e/(d*x + c)^2))*B + A*x
```

**maple [B]** time = 0.06, size = 233, normalized size = 4.31

$$\frac{2Ba^2d\ln\left(\frac{ad}{dx+c}-\frac{bc}{dx+c}+b\right)}{(ad-bc)\,b}-\frac{4Bac\ln\left(\frac{ad}{dx+c}-\frac{bc}{dx+c}+b\right)}{ad-bc}+\frac{2Bbc^2\ln\left(\frac{ad}{dx+c}-\frac{bc}{dx+c}+b\right)}{(ad-bc)\,d}+Bx\ln\left(\frac{\left(\frac{ad}{dx+c}-\frac{bc}{dx+c}+b\right)^2 e}{d^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(B*ln((b*x+a)^2/(d*x+c)^2*e)+A,x)
```

```
[Out] A*x+B*ln((1/(d*x+c)*a*d-1/(d*x+c)*b*c+b)^2/d^2*e)*x+B/d*ln((1/(d*x+c)*a*d-1
/(d*x+c)*b*c+b)^2/d^2*e)*c+2*B*d/b/(a*d-b*c)*ln(1/(d*x+c)*a*d-1/(d*x+c)*b*c
+b)*a^2-4*B/(a*d-b*c)*ln(1/(d*x+c)*a*d-1/(d*x+c)*b*c+b)*a*c+2*B/d/(a*d-b*c)
*ln(1/(d*x+c)*a*d-1/(d*x+c)*b*c+b)*c^2*b-2*B/b*ln(1/(d*x+c))*a+2*B/d*ln(1/(
d*x+c))*c
```

**maxima [A]** time = 0.63, size = 57, normalized size = 1.06

$$\left(x\log\left(\frac{(bx+a)^2 e}{(dx+c)^2}\right)+\frac{2\left(\frac{ae\log(bx+a)}{b}-\frac{ce\log(dx+c)}{d}\right)}{e}\right)B+Ax$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(A+B*log(e*(b*x+a)^2/(d*x+c)^2),x, algorithm="maxima")
```

```
[Out] (x*log((b*x + a)^2*e/(d*x + c)^2) + 2*(a*e*log(b*x + a)/b - c*e*log(d*x + c
)/d)/e)*B + A*x
```

**mupad [B]** time = 4.29, size = 50, normalized size = 0.93

$$Ax+Bx\ln\left(\frac{e\,(a+b\,x)^2}{(c+d\,x)^2}\right)+\frac{2\,B\,a\,\ln(a+b\,x)}{b}-\frac{2\,B\,c\,\ln(c+d\,x)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(A + B*log((e*(a + b*x)^2)/(c + d*x)^2),x)
```

```
[Out] A*x + B*x*log((e*(a + b*x)^2)/(c + d*x)^2) + (2*B*a*log(a + b*x))/b - (2*B*
c*log(c + d*x))/d
```

**sympy [B]** time = 1.07, size = 104, normalized size = 1.93

$$Ax+\frac{2Ba\log\left(x+\frac{\frac{2Ba^2d}{b}+2Bac}{2Bad+2Bbc}\right)}{b}-\frac{2Bc\log\left(x+\frac{2Bac+\frac{2Bbc^2}{d}}{2Bad+2Bbc}\right)}{d}+Bx\log\left(\frac{e\,(a+bx)^2}{(c+dx)^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(A+B*ln(e*(b*x+a)**2/(d*x+c)**2),x)
```

```
[Out] A*x + 2*B*a*log(x + (2*B*a**2*d/b + 2*B*a*c)/(2*B*a*d + 2*B*b*c))/b - 2*B*c
*log(x + (2*B*a*c + 2*B*b*c**2/d)/(2*B*a*d + 2*B*b*c))/d + B*x*log(e*(a + b
*x)**2/(c + d*x)**2)
```

# 3.267 $\int \frac{A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}{f+gx}\,dx$

**Optimal**. Leaf size=144

$$\frac{\log(f+gx)\left(B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+A\right)}{g}-\frac{2B\text{Li}_2\left(\frac{b(f+gx)}{bf-ag}\right)}{g}-\frac{2B\log(f+gx)\log\left(-\frac{g(a+bx)}{bf-ag}\right)}{g}+\frac{2B\text{Li}_2\left(\frac{d(f+gx)}{df-cg}\right)}{g}+\frac{2B\log(f}{}$$

```
[Out] -2*B*ln(-g*(b*x+a)/(-a*g+b*f))*ln(g*x+f)/g+(A+B*ln(e*(b*x+a)^2/(d*x+c)^2))*
ln(g*x+f)/g+2*B*ln(-g*(d*x+c)/(-c*g+d*f))*ln(g*x+f)/g-2*B*polylog(2,b*(g*x+
f)/(-a*g+b*f))/g+2*B*polylog(2,d*(g*x+f)/(-c*g+d*f))/g
```

**Rubi [A]** time = 0.31, antiderivative size = 144, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 6, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.207, Rules used = {2524, 12, 2418, 2394, 2393, 2391}

$$-\frac{2B\text{PolyLog}\left(2,\frac{b(f+gx)}{bf-ag}\right)}{g}+\frac{2B\text{PolyLog}\left(2,\frac{d(f+gx)}{df-cg}\right)}{g}+\frac{\log(f+gx)\left(B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+A\right)}{g}-\frac{2B\log(f+gx)\log\left(-\frac{g}{}\right.}{g}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])/(f + g*x),x]
```

```
[Out] (-2*B*Log[-((g*(a + b*x))/(b*f - a*g))]*Log[f + g*x])/g + ((A + B*Log[(e*(a
 + b*x)^2)/(c + d*x)^2])*Log[f + g*x])/g + (2*B*Log[-((g*(c + d*x))/(d*f -
c*g))]*Log[f + g*x])/g - (2*B*PolyLog[2, (b*(f + g*x))/(b*f - a*g)])/g + (2
*B*PolyLog[2, (d*(f + g*x))/(d*f - c*g)])/g
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2393

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + (c*e*x)/g])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
```

```
(e*f - d*g), 0]
```

Rule 2394

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d + e*x
)^n]))/g, x] - Dist[(b*e*n)/g, Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2418

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*(RFx_), x_Sy
mbol] :> With[{u = ExpandIntegrand[(a + b*Log[c*(d + e*x)^n])^p, RFx, x]},
Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d, e, n}, x] && RationalFunctionQ[
RFx, x] && IntegerQ[p]
```

Rule 2524

```
Int[((a_.) + Log[(c_.)*(RFx_)^(p_.)]*(b_.))^(n_.)/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[(Log[d + e*x]*(a + b*Log[c*RFx^p])^n)/e, x] - Dist[(b*n*p)/e
, Int[(Log[d + e*x]*(a + b*Log[c*RFx^p])^(n - 1)*D[RFx, x])/RFx, x], x] /;
FreeQ[{a, b, c, d, e, p}, x] && RationalFunctionQ[RFx, x] && IGtQ[n, 0]
```

Rubi steps

$$\int \frac{A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}{f+gx}\,dx = \frac{\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)\log(f+gx)}{g} - \frac{B\int\frac{(c+dx)^2\left(-\frac{2de(a+bx)^2}{(c+dx)^3}+\frac{2be(a+bx)}{(c+dx)^2}\right)\log(f+gx)}{e(a+bx)^2}dx}{g}$$

$$= \frac{\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)\log(f+gx)}{g} - \frac{B\int\frac{(c+dx)^2\left(-\frac{2de(a+bx)^2}{(c+dx)^3}+\frac{2be(a+bx)}{(c+dx)^2}\right)\log(f+gx)}{(a+bx)^2}dx}{eg}$$

$$= \frac{\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)\log(f+gx)}{g} - \frac{B\int\left(\frac{2be\log(f+gx)}{a+bx}-\frac{2de\log(f+gx)}{c+dx}\right)dx}{eg}$$

$$= \frac{\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)\log(f+gx)}{g} - \frac{(2bB)\int\frac{\log(f+gx)}{a+bx}dx}{g} + \frac{(2Bd)\int\frac{\log(f+gx)}{c+dx}dx}{g}$$

$$= -\frac{2B\log\left(-\frac{g(a+bx)}{bf-ag}\right)\log(f+gx)}{g} + \frac{\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)\log(f+gx)}{g} + \frac{2B\log\left(-\right.}{}$$

$$= -\frac{2B\log\left(-\frac{g(a+bx)}{bf-ag}\right)\log(f+gx)}{g} + \frac{\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)\log(f+gx)}{g} + \frac{2B\log\left(-\right.}{}$$

$$= -\frac{2B\log\left(-\frac{g(a+bx)}{bf-ag}\right)\log(f+gx)}{g} + \frac{\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)\log(f+gx)}{g} + \frac{2B\log\left(-\right.}{}$$

**Mathematica [A]** time = 0.06, size = 119, normalized size = 0.83

$$\frac{\log(f+gx)\left(B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)-2B\log\left(\frac{g(a+bx)}{ag-bf}\right)+A+2B\log\left(\frac{g(c+dx)}{cg-df}\right)\right)-2B\text{Li}_2\left(\frac{b(f+gx)}{bf-ag}\right)+2B\text{Li}_2\left(\frac{d(f+gx)}{df-cg}\right)}{g}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])/(f + g*x),x]
```

```
[Out] ((A - 2*B*Log[(g*(a + b*x))/(-(b*f) + a*g)] + B*Log[(e*(a + b*x)^2)/(c + d*
x)^2] + 2*B*Log[(g*(c + d*x))/(-(d*f) + c*g)])*Log[f + g*x] - 2*B*PolyLog[2
, (b*(f + g*x))/(b*f - a*g)] + 2*B*PolyLog[2, (d*(f + g*x))/(d*f - c*g)])/g
```

**fricas [F]** time = 0.92, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{B\log\left(\frac{b^2ex^2+2\,abex+a^2e}{d^2x^2+2\,cdx+c^2}\right)+A}{gx+f}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*(b*x+a)^2/(d*x+c)^2))/(g*x+f),x, algorithm="fricas")
```

```
[Out] integral((B*log((b^2*e*x^2 + 2*a*b*e*x + a^2*e)/(d^2*x^2 + 2*c*d*x + c^2))
+ A)/(g*x + f), x)
```

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{B\log\left(\frac{(bx+a)^2e}{(dx+c)^2}\right)+A}{gx+f}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*(b*x+a)^2/(d*x+c)^2))/(g*x+f),x, algorithm="giac")
```

```
[Out] integrate((B*log((b*x + a)^2*e/(d*x + c)^2) + A)/(g*x + f), x)
```

**maple [B]** time = 0.15, size = 1143, normalized size = 7.94

$$-\frac{2Bacd\ln\left(\frac{adg-bdf+\left(-g+\frac{cg-df}{dx+c}\right)(ad-bc)}{adg-bdf}\right)\ln\left(-g+\frac{cg-df}{dx+c}\right)}{\left(cg-df\right)(ad-bc)}+\frac{2Ba\,d^2f\ln\left(\frac{adg-bdf+\left(-g+\frac{cg-df}{dx+c}\right)(ad-bc)}{adg-bdf}\right)\ln\left(-g+\frac{cg-df}{dx+c}\right)}{\left(cg-df\right)(ad-bc)\,g}+\frac{2Bb}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*ln((b*x+a)^2/(d*x+c)^2*e)+A)/(g*x+f),x)
```

```
[Out] A/g*ln(1/(d*x+c)*c*g-d/(d*x+c)*f-g)-A/g*ln(1/(d*x+c))+B*ln((c*g-d*f)/(d*x+c
)-g)/(c*g-d*f)*ln((1/(d*x+c)*a*d-1/(d*x+c)*b*c+b)^2/d^2*e)*c-d*B/g*ln((c*g-
d*f)/(d*x+c)-g)/(c*g-d*f)*ln((1/(d*x+c)*a*d-1/(d*x+c)*b*c+b)^2/d^2*e)*f-2*d
*B/(c*g-d*f)*dilog((((c*g-d*f)/(d*x+c)-g)*(a*d-b*c)+a*d*g-b*d*f)/(a*d*g-b*d
*f))/(a*d-b*c)*a*c+2*d^2*B/g/(c*g-d*f)*dilog((((c*g-d*f)/(d*x+c)-g)*(a*d-b*
c)+a*d*g-b*d*f)/(a*d*g-b*d*f))/(a*d-b*c)*a*f+2*B/(c*g-d*f)*dilog((((c*g-d*f
)/(d*x+c)-g)*(a*d-b*c)+a*d*g-b*d*f)/(a*d*g-b*d*f))/(a*d-b*c)*b*c^2-2*d*B/g/
(c*g-d*f)*dilog((((c*g-d*f)/(d*x+c)-g)*(a*d-b*c)+a*d*g-b*d*f)/(a*d*g-b*d*f)
)/(a*d-b*c)*b*c*f-2*d*B/(c*g-d*f)*ln((c*g-d*f)/(d*x+c)-g)*ln((((c*g-d*f)/(d
*x+c)-g)*(a*d-b*c)+a*d*g-b*d*f)/(a*d*g-b*d*f))/(a*d-b*c)*a*c+2*d^2*B/g/(c*g
-d*f)*ln((c*g-d*f)/(d*x+c)-g)*ln((((c*g-d*f)/(d*x+c)-g)*(a*d-b*c)+a*d*g-b*d
*f)/(a*d*g-b*d*f))/(a*d-b*c)*a*f+2*B/(c*g-d*f)*ln((c*g-d*f)/(d*x+c)-g)*ln((
((c*g-d*f)/(d*x+c)-g)*(a*d-b*c)+a*d*g-b*d*f)/(a*d*g-b*d*f))/(a*d-b*c)*b*c^2
-2*d*B/g/(c*g-d*f)*ln((c*g-d*f)/(d*x+c)-g)*ln((((c*g-d*f)/(d*x+c)-g)*(a*d-b
*c)+a*d*g-b*d*f)/(a*d*g-b*d*f))/(a*d-b*c)*b*c*f-B/g*ln(1/(d*x+c))*ln((1/(d*
x+c)*a*d-1/(d*x+c)*b*c+b)^2/d^2*e)+2*d*B/g*dilog((b+1/(d*x+c)*(a*d-b*c))/b)
```

```
/(a*d-b*c)*a-2*B/g*dilog((b+1/(d*x+c)*(a*d-b*c))/b)/(a*d-b*c)*b*c+2*d*B/g*l
n(1/(d*x+c))*ln((b+1/(d*x+c)*(a*d-b*c))/b)/(a*d-b*c)*a-2*B/g*ln(1/(d*x+c))*
ln((b+1/(d*x+c)*(a*d-b*c))/b)/(a*d-b*c)*b*c
```

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$-B\int -\frac{2\log(bx+a)-2\log(dx+c)+\log(e)}{gx+f}\,dx+\frac{A\log\left(gx+f\right)}{g}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*(b*x+a)^2/(d*x+c)^2))/(g*x+f),x, algorithm="maxima")
```

```
[Out] -B*integrate(-(2*log(b*x + a) - 2*log(d*x + c) + log(e))/(g*x + f), x) + A*
log(g*x + f)/g
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{A+B\,\ln\left(\frac{e\,(a+b\,x)^2}{(c+d\,x)^2}\right)}{f+g\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*log((e*(a + b*x)^2)/(c + d*x)^2))/(f + g*x),x)
```

```
[Out] int((A + B*log((e*(a + b*x)^2)/(c + d*x)^2))/(f + g*x), x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{A+B\log\left(\frac{a^2e}{c^2+2cdx+d^2x^2}+\frac{2abex}{c^2+2cdx+d^2x^2}+\frac{b^2ex^2}{c^2+2cdx+d^2x^2}\right)}{f+gx}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*ln(e*(b*x+a)**2/(d*x+c)**2))/(g*x+f),x)
```

```
[Out] Integral((A + B*log(a**2*e/(c**2 + 2*c*d*x + d**2*x**2) + 2*a*b*e*x/(c**2 +
 2*c*d*x + d**2*x**2) + b**2*e*x**2/(c**2 + 2*c*d*x + d**2*x**2)))/(f + g*x
), x)
```

# 3.268 $\int \frac{A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}{(f+gx)^2}\,dx$

**Optimal**. Leaf size=90

$$\frac{(a+bx)\left(B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+A\right)}{(f+gx)(bf-ag)}+\frac{2B(bc-ad)\log\left(\frac{f+gx}{c+dx}\right)}{(bf-ag)(df-cg)}$$

```
[Out] (b*x+a)*(A+B*ln(e*(b*x+a)^2/(d*x+c)^2))/(-a*g+b*f)/(g*x+f)+2*B*(-a*d+b*c)*l
n((g*x+f)/(d*x+c))/(-a*g+b*f)/(-c*g+d*f)
```

**Rubi [A]** time = 0.09, antiderivative size = 117, normalized size of antiderivative = 1.30, number of steps used = 4, number of rules used = 3, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.103, Rules used = {2525, 12, 72}

$$-\frac{B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+A}{g(f+gx)}+\frac{2B(bc-ad)\log(f+gx)}{(bf-ag)(df-cg)}+\frac{2bB\log(a+bx)}{g(bf-ag)}-\frac{2Bd\log(c+dx)}{g(df-cg)}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])/(f + g*x)^2,x]
```

```
[Out] (2*b*B*Log[a + b*x])/(g*(b*f - a*g)) - (A + B*Log[(e*(a + b*x)^2)/(c + d*x)
^2])/(g*(f + g*x)) - (2*B*d*Log[c + d*x])/(g*(d*f - c*g)) + (2*B*(b*c - a*d
)*Log[f + g*x])/((b*f - a*g)*(d*f - c*g))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] && !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 72

```
Int[((e_.) + (f_.)*(x_))^(p_.)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
 x_Symbol] :> Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x]
 /; FreeQ[{a, b, c, d, e, f}, x] && IntegerQ[p]
```

Rule 2525

```
Int[((a_.) + Log[(c_.)*(RFx_)^(p_.)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_))^(m_.
), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(a + b*Log[c*RFx^p])^n)/(e*(m + 1))
, x] - Dist[(b*n*p)/(e*(m + 1)), Int[SimplifyIntegrand[((d + e*x)^(m + 1)*(
a + b*Log[c*RFx^p])^(n - 1)*D[RFx, x])/RFx, x], x], x] /; FreeQ[{a, b, c, d
```

```
, e, m, p}, x] && RationalFunctionQ[RFx, x] && IGtQ[n, 0] && (EqQ[n, 1] ||
IntegerQ[m]) && NeQ[m, -1]
```

Rubi steps

$$\int \frac{A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}{(f+gx)^2}\,dx = -\frac{A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}{g(f+gx)} + \frac{B\int \frac{2(bc-ad)}{(a+bx)(c+dx)(f+gx)}\,dx}{g}$$

$$= -\frac{A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}{g(f+gx)} + \frac{(2B(bc-ad))\int \frac{1}{(a+bx)(c+dx)(f+gx)}\,dx}{g}$$

$$= -\frac{A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}{g(f+gx)} + \frac{(2B(bc-ad))\int \left(\frac{b^2}{(bc-ad)(bf-ag)(a+bx)} + \frac{d^2}{(bc-ad)(-df+cg)(c+dx)} + \cdots\right.}{g}$$

$$= \frac{2bB\log(a+bx)}{g(bf-ag)} - \frac{A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}{g(f+gx)} - \frac{2Bd\log(c+dx)}{g(df-cg)} + \frac{2B(bc-ad)\log(f+g\cdots}{(bf-ag)(df-cg)\cdots}$$

**Mathematica [A]** time = 0.13, size = 108, normalized size = 1.20

$$\frac{\frac{2B(b\log(a+bx)(df-cg)+\log(c+dx)(adg-bdf)+g(bc-ad)\log(f+gx))}{(bf-ag)(df-cg)} - \frac{B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+A}{f+gx}}{g}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])/(f + g*x)^2,x]
```

```
[Out] (-((A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])/(f + g*x)) + (2*B*(b*(d*f - c*g
)*Log[a + b*x] + (-(b*d*f) + a*d*g)*Log[c + d*x] + (b*c - a*d)*g*Log[f + g*
x]))/((b*f - a*g)*(d*f - c*g)))/g
```

**fricas [B]** time = 10.61, size = 279, normalized size = 3.10

$$-\frac{Abdf^2 + Aacg^2 - (Abc + Aad)fg - 2\left(Bbdf^2 - Bbcfg + \left(Bbdfg - Bbcg^2\right)x\right)\log(bx+a) + 2\left(Bbdf^2 - Badfg\cdots\right.}{bdf^3g + acfg^3\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*(b*x+a)^2/(d*x+c)^2))/(g*x+f)^2,x, algorithm="fricas")
```

```
[Out] -(A*b*d*f^2 + A*a*c*g^2 - (A*b*c + A*a*d)*f*g - 2*(B*b*d*f^2 - B*b*c*f*g +
(B*b*d*f*g - B*b*c*g^2)*x)*log(b*x + a) + 2*(B*b*d*f^2 - B*a*d*f*g + (B*b*d
*f*g - B*a*d*g^2)*x)*log(d*x + c) - 2*((B*b*c - B*a*d)*g^2*x + (B*b*c - B*a
*d)*f*g)*log(g*x + f) + (B*b*d*f^2 + B*a*c*g^2 - (B*b*c + B*a*d)*f*g)*log((
b^2*e*x^2 + 2*a*b*e*x + a^2*e)/(d^2*x^2 + 2*c*d*x + c^2)))/(b*d*f^3*g + a*c
*f*g^3 - (b*c + a*d)*f^2*g^2 + (b*d*f^2*g^2 + a*c*g^4 - (b*c + a*d)*f*g^3)*
x)
```

**giac [F(-2)]** time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*(b*x+a)^2/(d*x+c)^2))/(g*x+f)^2,x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Undef
/Unsigned Inf encountered in limitUndef/Unsigned Inf encountered in limitB*
(-(g*x+f)^-1/g*ln((b*(-f+1/g/(g*x+f)^-1*g)/g+a)^2*exp(1)/(d*(-f+1/g/(g*x+f)
^-1*g)/g+c)^2)-(-2*a*d*g^2+2*b*c*g^2)*(1/(2*a*c*g^4-2*a*g^3*d*f-2*c*g^3*f*b
+2*g^2*d*f^2*b)*ln(abs((-(g*x+f)^-1/g)^2*a*c*g^4-(-(g*x+f)^-1/g)^2*a*g^3*d*
f-(-(g*x+f)^-1/g)^2*c*g^3*f*b+(-(g*x+f)^-1/g)^2*g^2*d*f^2*b+(g*x+f)^-1/g*a*
g^2*d+(g*x+f)^-1/g*c*g^2*b-2*(g*x+f)^-1/g*g*d*f*b+d*b))+(a*g*d+c*g*b-2*d*f*
b)/(2*a*c*g^3-2*a*g^2*d*f-2*c*g^2*f*b+2*g*d*f^2*b)/abs(a*g^2*d-g^2*c*b)*ln(
abs(-2*(g*x+f)^-1/g*a*c*g^4+2*(g*x+f)^-1/g*a*g^3*d*f+2*(g*x+f)^-1/g*c*g^3*f
*b-2*(g*x+f)^-1/g*g^2*d*f^2*b-a*g^2*d-c*g^2*b+2*g*d*f*b-abs(a*g^2*d-g^2*c*b
))/abs(-2*(g*x+f)^-1/g*a*c*g^4+2*(g*x+f)^-1/g*a*g^3*d*f+2*(g*x+f)^-1/g*c*g^
3*f*b-2*(g*x+f)^-1/g*g^2*d*f^2*b-a*g^2*d-c*g^2*b+2*g*d*f*b+abs(a*g^2*d-g^2*
c*b)))))-A*(g*x+f)^-1/g
```

**maple [B]** time = 0.09, size = 388, normalized size = 4.31

$$\frac{B a d \ln\left(\frac{\left(\frac{a d}{d x+c}-\frac{b c}{d x+c}+b\right)^{2} e}{d^{2}}\right)}{\left(\frac{c g}{d x+c}-\frac{d f}{d x+c}-g\right)\left(a g-b f\right)\left(d x+c\right)}-\frac{2 B a d \ln\left(\frac{c g}{d x+c}-\frac{d f}{d x+c}-g\right)}{a c g^{2}-a d f g-b c f g+b d f^{2}}-\frac{B b c \ln\left(\frac{\left(\frac{a d}{d x+c}-\frac{b c}{d x+c}+b\right)^{2} e}{d^{2}}\right)}{\left(\frac{c g}{d x+c}-\frac{d f}{d x+c}-g\right)\left(a g-b f\right)\left(d x+c\right)}+\frac{2 B b c \ln}{a c g^{2}-}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*ln((b*x+a)^2/(d*x+c)^2*e)+A)/(g*x+f)^2,x)
```

```
[Out] d*A/(1/(d*x+c)*c*g-1/(d*x+c)*d*f-g)/(c*g-d*f)+1/(1/(d*x+c)*c*g-1/(d*x+c)*d*
f-g)*b*B/(a*g-b*f)*ln((1/(d*x+c)*a*d-1/(d*x+c)*b*c+b)^2/d^2*e)+d/(1/(d*x+c)
*c*g-1/(d*x+c)*d*f-g)*B/(a*g-b*f)/(d*x+c)*ln((1/(d*x+c)*a*d-1/(d*x+c)*b*c+b
)^2/d^2*e)*a-1/(1/(d*x+c)*c*g-1/(d*x+c)*d*f-g)*B/(a*g-b*f)/(d*x+c)*ln((1/(d
```

```
*x+c)*a*d-1/(d*x+c)*b*c+b)^2/d^2*e)*b*c-2*d*B/(a*c*g^2-a*d*f*g-b*c*f*g+b*d*
f^2)*ln(1/(d*x+c)*c*g-1/(d*x+c)*d*f-g)*a+2*B/(a*c*g^2-a*d*f*g-b*c*f*g+b*d*f
^2)*ln(1/(d*x+c)*c*g-1/(d*x+c)*d*f-g)*b*c
```

**maxima [B]** time = 0.70, size = 192, normalized size = 2.13

$$B\left(\frac{2\,b\log\left(bx+a\right)}{bfg-ag^2}-\frac{2\,d\log\left(dx+c\right)}{dfg-cg^2}+\frac{2\,(bc-ad)\log\left(gx+f\right)}{bdf^2+acg^2-(bc+ad)fg}-\frac{\log\left(\frac{b^2ex^2}{d^2x^2+2\,cdx+c^2}+\frac{2\,abex}{d^2x^2+2\,cdx+c^2}+\frac{a^2e}{d^2x^2+2\,cdx}\right.}{g^2x+fg}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*(b*x+a)^2/(d*x+c)^2))/(g*x+f)^2,x, algorithm="maxima")
```

```
[Out] B*(2*b*log(b*x + a)/(b*f*g - a*g^2) - 2*d*log(d*x + c)/(d*f*g - c*g^2) + 2*
(b*c - a*d)*log(g*x + f)/(b*d*f^2 + a*c*g^2 - (b*c + a*d)*f*g) - log(b^2*e*
x^2/(d^2*x^2 + 2*c*d*x + c^2) + 2*a*b*e*x/(d^2*x^2 + 2*c*d*x + c^2) + a^2*e
/(d^2*x^2 + 2*c*d*x + c^2))/(g^2*x + f*g)) - A/(g^2*x + f*g)
```

**mupad [B]** time = 5.34, size = 191, normalized size = 2.12

$$\frac{2\,B\,d\,\ln\left(c+d\,x\right)}{c\,g^2-d\,f\,g}-\frac{B\,\ln\left(\frac{e\,a^2+2\,e\,a\,b\,x+e\,b^2\,x^2}{c^2+2\,c\,d\,x+d^2\,x^2}\right)}{x\,g^2+f\,g}-\frac{2\,B\,b\,\ln\left(a+b\,x\right)}{a\,g^2-b\,f\,g}-\frac{A}{x\,g^2+f\,g}-\frac{2\,B\,a\,d\,\ln\left(f+g\,x\right)}{a\,c\,g^2+b\,d\,f^2-a\,d\,f\,g-b\,c\,f\,g}+\frac{}{a\,c\,g}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*log((e*(a + b*x)^2)/(c + d*x)^2))/(f + g*x)^2,x)
```

```
[Out] (2*B*d*log(c + d*x))/(c*g^2 - d*f*g) - (B*log((a^2*e + b^2*e*x^2 + 2*a*b*e*
x)/(c^2 + d^2*x^2 + 2*c*d*x)))/(f*g + g^2*x) - (2*B*b*log(a + b*x))/(a*g^2
- b*f*g) - A/(f*g + g^2*x) - (2*B*a*d*log(f + g*x))/(a*c*g^2 + b*d*f^2 - a*
d*f*g - b*c*f*g) + (2*B*b*c*log(f + g*x))/(a*c*g^2 + b*d*f^2 - a*d*f*g - b*
c*f*g)
```

**sympy [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*ln(e*(b*x+a)**2/(d*x+c)**2))/(g*x+f)**2,x)
```

```
[Out] Timed out
```

# 3.269 $\int \frac{A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}{(f+gx)^3}\,dx$

**Optimal**. Leaf size=175

$$-\frac{B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+A}{2g(f+gx)^2}+\frac{b^2B\log(a+bx)}{g(bf-ag)^2}-\frac{B(bc-ad)}{(f+gx)(bf-ag)(df-cg)}+\frac{B(bc-ad)\log(f+gx)(-adg-bcg+2bdf}{(bf-ag)^2(df-cg)^2}$$

```
[Out] -B*(-a*d+b*c)/(-a*g+b*f)/(-c*g+d*f)/(g*x+f)+b^2*B*ln(b*x+a)/g/(-a*g+b*f)^2+
1/2*(-A-B*ln(e*(b*x+a)^2/(d*x+c)^2))/g/(g*x+f)^2-B*d^2*ln(d*x+c)/g/(-c*g+d*
f)^2+B*(-a*d+b*c)*(-a*d*g-b*c*g+2*b*d*f)*ln(g*x+f)/(-a*g+b*f)^2/(-c*g+d*f)^
2
```

**Rubi [A]** time = 0.17, antiderivative size = 175, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.103, Rules used = {2525, 12, 72}

$$-\frac{B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+A}{2g(f+gx)^2}+\frac{b^2B\log(a+bx)}{g(bf-ag)^2}-\frac{B(bc-ad)}{(f+gx)(bf-ag)(df-cg)}+\frac{B(bc-ad)\log(f+gx)(-adg-bcg+2bdf}{(bf-ag)^2(df-cg)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])/(f + g*x)^3,x]
```

```
[Out] -((B*(b*c - a*d))/((b*f - a*g)*(d*f - c*g)*(f + g*x))) + (b^2*B*Log[a + b*x
])/(g*(b*f - a*g)^2) - (A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])/(2*g*(f + g
*x)^2) - (B*d^2*Log[c + d*x])/(g*(d*f - c*g)^2) + (B*(b*c - a*d)*(2*b*d*f -
 b*c*g - a*d*g)*Log[f + g*x])/((b*f - a*g)^2*(d*f - c*g)^2)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 72

```
Int[((e_.) + (f_.)*(x_))^(p_.)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
 x_Symbol] :> Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x]
 /; FreeQ[{a, b, c, d, e, f}, x] && IntegerQ[p]
```

Rule 2525

```
Int[((a_.) + Log[(c_.)*(RFx_)^(p_.)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_))^(m_.
), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(a + b*Log[c*RFx^p])^n)/(e*(m + 1))
```

```
, x] - Dist[(b*n*p)/(e*(m + 1)), Int[SimplifyIntegrand[((d + e*x)^(m + 1)*(
a + b*Log[c*RFx^p])^(n - 1)*D[RFx, x])/RFx, x], x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && RationalFunctionQ[RFx, x] && IGtQ[n, 0] && (EqQ[n, 1] ||
IntegerQ[m]) && NeQ[m, -1]
```

Rubi steps

$$\int \frac{A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}{(f+gx)^3}\,dx = -\frac{A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}{2g(f+gx)^2}+\frac{B\int\frac{2(bc-ad)}{(a+bx)(c+dx)(f+gx)^2}\,dx}{2g}$$
$$= -\frac{A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}{2g(f+gx)^2}+\frac{(B(bc-ad))\int\frac{1}{(a+bx)(c+dx)(f+gx)^2}\,dx}{g}$$
$$= -\frac{A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}{2g(f+gx)^2}+\frac{(B(bc-ad))\int\left(\frac{b^3}{(bc-ad)(bf-ag)^2(a+bx)}-\frac{d^3}{(bc-ad)(-df+cg)^2(c+dx)}+\right.}{g}$$
$$= -\frac{B(bc-ad)}{(bf-ag)(df-cg)(f+gx)}+\frac{b^2B\log(a+bx)}{g(bf-ag)^2}-\frac{A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}{2g(f+gx)^2}-\frac{Bd^2\log(}{g(df-}$$

**Mathematica [A]** time = 0.55, size = 172, normalized size = 0.98

$$\frac{2B(bc-ad)\left(\frac{b^2\log(a+bx)}{(bc-ad)(bf-ag)^2}+\frac{\frac{d^2\log(c+dx)}{ad-bc}+\frac{g(cg-df)}{(f+gx)(bf-ag)}-\frac{g\log(f+gx)(adg+bcg-2bdf)}{(bf-ag)^2}}{(df-cg)^2}\right)-\frac{B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+A}{(f+gx)^2}}{2g}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])/(f + g*x)^3,x]
```

```
[Out] (-((A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])/(f + g*x)^2) + 2*B*(b*c - a*d)*
((b^2*Log[a + b*x])/((b*c - a*d)*(b*f - a*g)^2) + ((g*(-(d*f) + c*g))/((b*f
 - a*g)*(f + g*x)) + (d^2*Log[c + d*x])/(-(b*c) + a*d) - (g*(-2*b*d*f + b*c
*g + a*d*g)*Log[f + g*x])/(b*f - a*g)^2)/(d*f - c*g)^2))/(2*g)
```

**fricas [B]** time = 151.72, size = 1036, normalized size = 5.92

$$-\frac{Ab^2d^2f^4+Aa^2c^2g^4-2\left((A-B)b^2cd+(A+B)abd^2\right)f^3g+\left((A-2B)b^2c^2+4Aabcd+(A+2B)a^2d^2\right)f^2g^2-}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*(b*x+a)^2/(d*x+c)^2))/(g*x+f)^3,x, algorithm="fricas")
```

```
[Out] -1/2*(A*b^2*d^2*f^4 + A*a^2*c^2*g^4 - 2*((A - B)*b^2*c*d + (A + B)*a*b*d^2)
*f^3*g + ((A - 2*B)*b^2*c^2 + 4*A*a*b*c*d + (A + 2*B)*a^2*d^2)*f^2*g^2 - 2*
((A - B)*a*b*c^2 + (A + B)*a^2*c*d)*f*g^3 + 2*((B*b^2*c*d - B*a*b*d^2)*f^2*
g^2 - (B*b^2*c^2 - B*a^2*d^2)*f*g^3 + (B*a*b*c^2 - B*a^2*c*d)*g^4)*x - 2*(B
*b^2*d^2*f^4 - 2*B*b^2*c*d*f^3*g + B*b^2*c^2*f^2*g^2 + (B*b^2*d^2*f^2*g^2 -
 2*B*b^2*c*d*f*g^3 + B*b^2*c^2*g^4)*x^2 + 2*(B*b^2*d^2*f^3*g - 2*B*b^2*c*d*
f^2*g^2 + B*b^2*c^2*f*g^3)*x)*log(b*x + a) + 2*(B*b^2*d^2*f^4 - 2*B*a*b*d^2
*f^3*g + B*a^2*d^2*f^2*g^2 + (B*b^2*d^2*f^2*g^2 - 2*B*a*b*d^2*f*g^3 + B*a^2
*d^2*g^4)*x^2 + 2*(B*b^2*d^2*f^3*g - 2*B*a*b*d^2*f^2*g^2 + B*a^2*d^2*f*g^3)
*x)*log(d*x + c) - 2*(2*(B*b^2*c*d - B*a*b*d^2)*f^3*g - (B*b^2*c^2 - B*a^2*
d^2)*f^2*g^2 + (2*(B*b^2*c*d - B*a*b*d^2)*f*g^3 - (B*b^2*c^2 - B*a^2*d^2)*g
^4)*x^2 + 2*(2*(B*b^2*c*d - B*a*b*d^2)*f^2*g^2 - (B*b^2*c^2 - B*a^2*d^2)*f*
g^3)*x)*log(g*x + f) + (B*b^2*d^2*f^4 + B*a^2*c^2*g^4 - 2*(B*b^2*c*d + B*a*
b*d^2)*f^3*g + (B*b^2*c^2 + 4*B*a*b*c*d + B*a^2*d^2)*f^2*g^2 - 2*(B*a*b*c^2
 + B*a^2*c*d)*f*g^3)*log((b^2*e*x^2 + 2*a*b*e*x + a^2*e)/(d^2*x^2 + 2*c*d*x
 + c^2)))/(b^2*d^2*f^6*g + a^2*c^2*f^2*g^5 - 2*(b^2*c*d + a*b*d^2)*f^5*g^2
+ (b^2*c^2 + 4*a*b*c*d + a^2*d^2)*f^4*g^3 - 2*(a*b*c^2 + a^2*c*d)*f^3*g^4 +
 (b^2*d^2*f^4*g^3 + a^2*c^2*g^7 - 2*(b^2*c*d + a*b*d^2)*f^3*g^4 + (b^2*c^2
+ 4*a*b*c*d + a^2*d^2)*f^2*g^5 - 2*(a*b*c^2 + a^2*c*d)*f*g^6)*x^2 + 2*(b^2*
d^2*f^5*g^2 + a^2*c^2*f*g^6 - 2*(b^2*c*d + a*b*d^2)*f^4*g^3 + (b^2*c^2 + 4*
a*b*c*d + a^2*d^2)*f^3*g^4 - 2*(a*b*c^2 + a^2*c*d)*f^2*g^5)*x)
```

**giac [B]** time = 0.77, size = 495, normalized size = 2.83

$$\frac{Bb^3\log(|bx+a|)}{b^3f^2g-2\,ab^2fg^2+a^2bg^3}-\frac{Bd^3\log(|dx+c|)}{d^3f^2g-2\,cd^2fg^2+c^2dg^3}+\frac{\left(2\,Bb^2cdf-2\,Babd^2f-Bb^2c^2g+\right.}{b^2d^2f^4-2\,b^2cdf^3g-2\,abd^2f^3g+b^2c^2f^2g^2+4\,abcdf^2g^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*(b*x+a)^2/(d*x+c)^2))/(g*x+f)^3,x, algorithm="giac")
```

```
[Out] B*b^3*log(abs(b*x + a))/(b^3*f^2*g - 2*a*b^2*f*g^2 + a^2*b*g^3) - B*d^3*log
(abs(d*x + c))/(d^3*f^2*g - 2*c*d^2*f*g^2 + c^2*d*g^3) + (2*B*b^2*c*d*f - 2
*B*a*b*d^2*f - B*b^2*c^2*g + B*a^2*d^2*g)*log(g*x + f)/(b^2*d^2*f^4 - 2*b^2
*c*d*f^3*g - 2*a*b*d^2*f^3*g + b^2*c^2*f^2*g^2 + 4*a*b*c*d*f^2*g^2 + a^2*d^
2*f^2*g^2 - 2*a*b*c^2*f*g^3 - 2*a^2*c*d*f*g^3 + a^2*c^2*g^4) - 1/2*B*log((b
^2*x^2 + 2*a*b*x + a^2)/(d^2*x^2 + 2*c*d*x + c^2))/(g^3*x^2 + 2*f*g^2*x + f
^2*g) - 1/2*(2*B*b*c*g^2*x - 2*B*a*d*g^2*x + A*b*d*f^2 + B*b*d*f^2 - A*b*c*
f*g + B*b*c*f*g - A*a*d*f*g - 3*B*a*d*f*g + A*a*c*g^2 + B*a*c*g^2)/(b*d*f^2
*g^3*x^2 - b*c*f*g^4*x^2 - a*d*f*g^4*x^2 + a*c*g^5*x^2 + 2*b*d*f^3*g^2*x -
2*b*c*f^2*g^3*x - 2*a*d*f^2*g^3*x + 2*a*c*f*g^4*x + b*d*f^4*g - b*c*f^3*g^2
 - a*d*f^3*g^2 + a*c*f^2*g^3)
```

**maple [B]** time = 0.19, size = 1554, normalized size = 8.88

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*ln((b*x+a)^2/(d*x+c)^2*e)+A)/(g*x+f)^3,x)
```

```
[Out] -d^2*A/(c*g-d*f)^2/(1/(d*x+c)*c*g-1/(d*x+c)*d*f-g)-1/2*d^2*A*g/(c*g-d*f)^2/
(1/(d*x+c)*c*g-1/(d*x+c)*d*f-g)^2-d^2/(1/(d*x+c)*c*g-1/(d*x+c)*d*f-g)^2/(a*
g-b*f)/(d*x+c)^2*B*a+d/(1/(d*x+c)*c*g-1/(d*x+c)*d*f-g)^2/(a*g-b*f)/(d*x+c)^
2*B*b*c-1/(1/(d*x+c)*c*g-1/(d*x+c)*d*f-g)^2*b^2*B/(a^2*g^2-2*a*b*f*g+b^2*f^
2)/(d*x+c)*ln((1/(d*x+c)*a*d-1/(d*x+c)*b*c+b)^2/d^2*e)*c*g+d/(1/(d*x+c)*c*g
-1/(d*x+c)*d*f-g)^2*b^2*B/(a^2*g^2-2*a*b*f*g+b^2*f^2)/(d*x+c)*ln((1/(d*x+c)
*a*d-1/(d*x+c)*b*c+b)^2/d^2*e)*f+d^2/(1/(d*x+c)*c*g-1/(d*x+c)*d*f-g)^2*g/(a
*c*g^2-a*d*f*g-b*c*f*g+b*d*f^2)/(d*x+c)*B*a-d/(1/(d*x+c)*c*g-1/(d*x+c)*d*f-
g)^2*g/(a*c*g^2-a*d*f*g-b*c*f*g+b*d*f^2)/(d*x+c)*B*b*c-1/2*d^2/(1/(d*x+c)*c
*g-1/(d*x+c)*d*f-g)^2*B/(a^2*g^2-2*a*b*f*g+b^2*f^2)/(d*x+c)^2*ln((1/(d*x+c)
*a*d-1/(d*x+c)*b*c+b)^2/d^2*e)*a^2*g+d^2/(1/(d*x+c)*c*g-1/(d*x+c)*d*f-g)^2*
B/(a^2*g^2-2*a*b*f*g+b^2*f^2)/(d*x+c)^2*ln((1/(d*x+c)*a*d-1/(d*x+c)*b*c+b)^
2/d^2*e)*a*b*f+1/2/(1/(d*x+c)*c*g-1/(d*x+c)*d*f-g)^2*B/(a^2*g^2-2*a*b*f*g+b
^2*f^2)/(d*x+c)^2*ln((1/(d*x+c)*a*d-1/(d*x+c)*b*c+b)^2/d^2*e)*b^2*c^2*g-d/(
1/(d*x+c)*c*g-1/(d*x+c)*d*f-g)^2*B/(a^2*g^2-2*a*b*f*g+b^2*f^2)/(d*x+c)^2*ln
((1/(d*x+c)*a*d-1/(d*x+c)*b*c+b)^2/d^2*e)*b^2*c*f+1/2/(1/(d*x+c)*c*g-1/(d*x
+c)*d*f-g)^2*b^2*g*B/(a^2*g^2-2*a*b*f*g+b^2*f^2)*ln((1/(d*x+c)*a*d-1/(d*x+c
)*b*c+b)^2/d^2*e)+d^2*B/(a^2*c^2*g^4-2*a^2*c*d*f*g^3+a^2*d^2*f^2*g^2-2*a*b*
c^2*f*g^3+4*a*b*c*d*f^2*g^2-2*a*b*d^2*f^3*g+b^2*c^2*f^2*g^2-2*b^2*c*d*f^3*g
+b^2*d^2*f^4)*ln(1/(d*x+c)*c*g-1/(d*x+c)*d*f-g)*a^2*g-2*d^2*B/(a^2*c^2*g^4-
2*a^2*c*d*f*g^3+a^2*d^2*f^2*g^2-2*a*b*c^2*f*g^3+4*a*b*c*d*f^2*g^2-2*a*b*d^2
*f^3*g+b^2*c^2*f^2*g^2-2*b^2*c*d*f^3*g+b^2*d^2*f^4)*ln(1/(d*x+c)*c*g-1/(d*x
+c)*d*f-g)*a*b*f-B/(a^2*c^2*g^4-2*a^2*c*d*f*g^3+a^2*d^2*f^2*g^2-2*a*b*c^2*f
*g^3+4*a*b*c*d*f^2*g^2-2*a*b*d^2*f^3*g+b^2*c^2*f^2*g^2-2*b^2*c*d*f^3*g+b^2*
d^2*f^4)*ln(1/(d*x+c)*c*g-1/(d*x+c)*d*f-g)*b^2*c^2*g+2*d*B/(a^2*c^2*g^4-2*a
^2*c*d*f*g^3+a^2*d^2*f^2*g^2-2*a*b*c^2*f*g^3+4*a*b*c*d*f^2*g^2-2*a*b*d^2*f^
3*g+b^2*c^2*f^2*g^2-2*b^2*c*d*f^3*g+b^2*d^2*f^4)*ln(1/(d*x+c)*c*g-1/(d*x+c)
*d*f-g)*b^2*c*f
```

**maxima [B]** time = 1.05, size = 405, normalized size = 2.31

$$\frac{1}{2}\left(\frac{2\,b^2\log\left(bx+a\right)}{b^2f^2g-2\,abfg^2+a^2g^3}-\frac{2\,d^2\log\left(dx+c\right)}{d^2f^2g-2\,cdfg^2+c^2g^3}+\frac{2\left(2\left(b^2cd-abd^2\right)f-\left(b^2c^2-a^2d^2\right)\cdots\right.}{b^2d^2f^4+a^2c^2g^4-2\left(b^2cd+abd^2\right)f^3g+\left(b^2c^2+4\,abcd\cdots\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*(b*x+a)^2/(d*x+c)^2))/(g*x+f)^3,x, algorithm="maxima")
```

```
[Out] 1/2*(2*b^2*log(b*x + a)/(b^2*f^2*g - 2*a*b*f*g^2 + a^2*g^3) - 2*d^2*log(d*x
 + c)/(d^2*f^2*g - 2*c*d*f*g^2 + c^2*g^3) + 2*(2*(b^2*c*d - a*b*d^2)*f - (b
^2*c^2 - a^2*d^2)*g)*log(g*x + f)/(b^2*d^2*f^4 + a^2*c^2*g^4 - 2*(b^2*c*d +
 a*b*d^2)*f^3*g + (b^2*c^2 + 4*a*b*c*d + a^2*d^2)*f^2*g^2 - 2*(a*b*c^2 + a^
2*c*d)*f*g^3) - 2*(b*c - a*d)/(b*d*f^3 + a*c*f*g^2 - (b*c + a*d)*f^2*g + (b
*d*f^2*g + a*c*g^3 - (b*c + a*d)*f*g^2)*x) - log(b^2*e*x^2/(d^2*x^2 + 2*c*d
*x + c^2) + 2*a*b*e*x/(d^2*x^2 + 2*c*d*x + c^2) + a^2*e/(d^2*x^2 + 2*c*d*x
+ c^2))/(g^3*x^2 + 2*f*g^2*x + f^2*g))*B - 1/2*A/(g^3*x^2 + 2*f*g^2*x + f^2
*g)
```

**mupad [B]** time = 7.45, size = 412, normalized size = 2.35

$$\frac{\ln\left(f+g\,x\right)\,\left(g\,\left(B\,a^2\,d^2-B\,b^2\,c^2\right)-2\,B\,a\,b\,d^2\,f+2\,B\,b^2\,c\,d\,f\right)}{a^2\,c^2\,g^4-2\,a^2\,c\,d\,f\,g^3+a^2\,d^2\,f^2\,g^2-2\,a\,b\,c^2\,f\,g^3+4\,a\,b\,c\,d\,f^2\,g^2-2\,a\,b\,d^2\,f^3\,g+b^2\,c^2\,f^2\,g^2-2\,b^2\,c\,d\,f^3\,g+b^2\,d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*log((e*(a + b*x)^2)/(c + d*x)^2))/(f + g*x)^3,x)
```

```
[Out] (log(f + g*x)*(g*(B*a^2*d^2 - B*b^2*c^2) - 2*B*a*b*d^2*f + 2*B*b^2*c*d*f))/
(a^2*c^2*g^4 + b^2*d^2*f^4 + a^2*d^2*f^2*g^2 + b^2*c^2*f^2*g^2 - 2*a*b*c^2*
f*g^3 - 2*a*b*d^2*f^3*g - 2*a^2*c*d*f*g^3 - 2*b^2*c*d*f^3*g + 4*a*b*c*d*f^2
*g^2) - ((A*a*c*g^2 + A*b*d*f^2 - A*a*d*f*g - A*b*c*f*g - 2*B*a*d*f*g + 2*B
*b*c*f*g)/(2*(a*c*g^2 + b*d*f^2 - a*d*f*g - b*c*f*g)) - (x*(B*a*d*g^2 - B*b
*c*g^2))/(a*c*g^2 + b*d*f^2 - a*d*f*g - b*c*f*g))/(f^2*g + g^3*x^2 + 2*f*g^
2*x) + (B*b^2*log(a + b*x))/(a^2*g^3 + b^2*f^2*g - 2*a*b*f*g^2) - (B*d^2*lo
g(c + d*x))/(c^2*g^3 + d^2*f^2*g - 2*c*d*f*g^2) - (B*log((e*(a + b*x)^2)/(c
 + d*x)^2))/(2*g*(f^2 + g^2*x^2 + 2*f*g*x))
```

**sympy [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*ln(e*(b*x+a)**2/(d*x+c)**2))/(g*x+f)**3,x)
```

```
[Out] Timed out
```

# 3.270 $\int \frac{A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}{(f+gx)^4}\,dx$

**Optimal**. Leaf size=277

$$\frac{2B(bc-ad)\log(f+gx)\left(a^2d^2g^2-abdg(3df-cg)+b^2\left(c^2g^2-3cdfg+3d^2f^2\right)\right)}{3(bf-ag)^3(df-cg)^3}-\frac{B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+A}{3g(f+gx)^3}+\frac{2b^3B\,\mathrm{lc}}{3g(b}$$

```
[Out] -1/3*B*(-a*d+b*c)/(-a*g+b*f)/(-c*g+d*f)/(g*x+f)^2-2/3*B*(-a*d+b*c)*(-a*d*g-
b*c*g+2*b*d*f)/(-a*g+b*f)^2/(-c*g+d*f)^2/(g*x+f)+2/3*b^3*B*ln(b*x+a)/g/(-a*
g+b*f)^3+1/3*(-A-B*ln(e*(b*x+a)^2/(d*x+c)^2))/g/(g*x+f)^3-2/3*B*d^3*ln(d*x+
c)/g/(-c*g+d*f)^3+2/3*B*(-a*d+b*c)*(a^2*d^2*g^2-a*b*d*g*(-c*g+3*d*f)+b^2*(c
^2*g^2-3*c*d*f*g+3*d^2*f^2))*ln(g*x+f)/(-a*g+b*f)^3/(-c*g+d*f)^3
```

**Rubi [A]** time = 0.33, antiderivative size = 277, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.103, Rules used = {2525, 12, 72}

$$\frac{2B(bc-ad)\log(f+gx)\left(a^2d^2g^2-abdg(3df-cg)+b^2\left(c^2g^2-3cdfg+3d^2f^2\right)\right)}{3(bf-ag)^3(df-cg)^3}-\frac{B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+A}{3g(f+gx)^3}+\frac{2b^3B\,\mathrm{lc}}{3g(b}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])/(f + g*x)^4,x]
```

```
[Out] -(B*(b*c - a*d))/(3*(b*f - a*g)*(d*f - c*g)*(f + g*x)^2) - (2*B*(b*c - a*d)
*(2*b*d*f - b*c*g - a*d*g))/(3*(b*f - a*g)^2*(d*f - c*g)^2*(f + g*x)) + (2*
b^3*B*Log[a + b*x])/(3*g*(b*f - a*g)^3) - (A + B*Log[(e*(a + b*x)^2)/(c + d
*x)^2])/(3*g*(f + g*x)^3) - (2*B*d^3*Log[c + d*x])/(3*g*(d*f - c*g)^3) + (2
*B*(b*c - a*d)*(a^2*d^2*g^2 - a*b*d*g*(3*d*f - c*g) + b^2*(3*d^2*f^2 - 3*c*
d*f*g + c^2*g^2))*Log[f + g*x])/(3*(b*f - a*g)^3*(d*f - c*g)^3)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 72

```
Int[((e_.) + (f_.)*(x_))^(p_.)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
 x_Symbol] :> Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x]
 /; FreeQ[{a, b, c, d, e, f}, x] && IntegerQ[p]
```

Rule 2525

```
Int[((a_.) + Log[(c_.)*(RFx_)^(p_.)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_))^(m_.
), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(a + b*Log[c*RFx^p])^n)/(e*(m + 1))
, x] - Dist[(b*n*p)/(e*(m + 1)), Int[SimplifyIntegrand[((d + e*x)^(m + 1)*(
a + b*Log[c*RFx^p])^(n - 1)*D[RFx, x])/RFx, x], x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && RationalFunctionQ[RFx, x] && IGtQ[n, 0] && (EqQ[n, 1] ||
IntegerQ[m]) && NeQ[m, -1]
```

Rubi steps

$$\int \frac{A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}{(f+gx)^4}\,dx = -\frac{A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}{3g(f+gx)^3}+\frac{B\int \frac{2(bc-ad)}{(a+bx)(c+dx)(f+gx)^3}\,dx}{3g}$$

$$= -\frac{A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}{3g(f+gx)^3}+\frac{(2B(bc-ad))\int \frac{1}{(a+bx)(c+dx)(f+gx)^3}\,dx}{3g}$$

$$= -\frac{A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}{3g(f+gx)^3}+\frac{(2B(bc-ad))\int \left(\frac{b^4}{(bc-ad)(bf-ag)^3(a+bx)}+\frac{d^4}{(bc-ad)(-df+cg)^3(c+dx)}+\right.}{}$$

$$= -\frac{B(bc-ad)}{3(bf-ag)(df-cg)(f+gx)^2}-\frac{2B(bc-ad)(2bdf-bcg-adg)}{3(bf-ag)^2(df-cg)^2(f+gx)}+\frac{2b^3B\log(a+bx}{3g(bf-ag)^3}$$

**Mathematica [A]** time = 0.74, size = 263, normalized size = 0.95

$$\frac{2B(bc-ad)\left(\frac{g\log(f+gx)\left(a^2d^2g^2+abdg(cg-3df)+b^2\left(c^2g^2-3cdfg+3d^2f^2\right)\right)}{(bf-ag)^3(df-cg)^3}+\frac{b^3\log(a+bx)}{(bc-ad)(bf-ag)^3}+\frac{d^3\log(c+dx)}{(bc-ad)(cg-df)^3}+\frac{g(adg+bcg-2bdf)}{(f+gx)(bf-ag)^2(df-cg)^2}\right.}{3g}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])/(f + g*x)^4,x]
```

```
[Out] (-((A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])/(f + g*x)^3) + 2*B*(b*c - a*d)*
(-1/2*g/((b*f - a*g)*(d*f - c*g)*(f + g*x)^2) + (g*(-2*b*d*f + b*c*g + a*d*
g))/((b*f - a*g)^2*(d*f - c*g)^2*(f + g*x)) + (b^3*Log[a + b*x])/((b*c - a*
d)*(b*f - a*g)^3) + (d^3*Log[c + d*x])/((b*c - a*d)*(-(d*f) + c*g)^3) + (g*
(a^2*d^2*g^2 + a*b*d*g*(-3*d*f + c*g) + b^2*(3*d^2*f^2 - 3*c*d*f*g + c^2*g^
2))*Log[f + g*x])/((b*f - a*g)^3*(d*f - c*g)^3)))/(3*g)
```

**fricas [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*(b*x+a)^2/(d*x+c)^2))/(g*x+f)^4,x, algorithm="fricas")
```

```
[Out] Timed out
```

**giac [B]** time = 3.67, size = 1391, normalized size = 5.02

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*(b*x+a)^2/(d*x+c)^2))/(g*x+f)^4,x, algorithm="giac")
```

```
[Out] 2/3*B*b^4*log(abs(b*x + a))/(b^4*f^3*g - 3*a*b^3*f^2*g^2 + 3*a^2*b^2*f*g^3
- a^3*b*g^4) - 2/3*B*d^4*log(abs(d*x + c))/(d^4*f^3*g - 3*c*d^3*f^2*g^2 + 3
*c^2*d^2*f*g^3 - c^3*d*g^4) + 2/3*(3*B*b^3*c*d^2*f^2 - 3*B*a*b^2*d^3*f^2 -
3*B*b^3*c^2*d*f*g + 3*B*a^2*b*d^3*f*g + B*b^3*c^3*g^2 - B*a^3*d^3*g^2)*log(
g*x + f)/(b^3*d^3*f^6 - 3*b^3*c*d^2*f^5*g - 3*a*b^2*d^3*f^5*g + 3*b^3*c^2*d
*f^4*g^2 + 9*a*b^2*c*d^2*f^4*g^2 + 3*a^2*b*d^3*f^4*g^2 - b^3*c^3*f^3*g^3 -
9*a*b^2*c^2*d*f^3*g^3 - 9*a^2*b*c*d^2*f^3*g^3 - a^3*d^3*f^3*g^3 + 3*a*b^2*c
^3*f^2*g^4 + 9*a^2*b*c^2*d*f^2*g^4 + 3*a^3*c*d^2*f^2*g^4 - 3*a^2*b*c^3*f*g^
5 - 3*a^3*c^2*d*f*g^5 + a^3*c^3*g^6) - 1/3*B*log((b^2*x^2 + 2*a*b*x + a^2)/
(d^2*x^2 + 2*c*d*x + c^2))/(g^4*x^3 + 3*f*g^3*x^2 + 3*f^2*g^2*x + f^3*g) -
1/3*(4*B*b^2*c*d*f*g^3*x^2 - 4*B*a*b*d^2*f*g^3*x^2 - 2*B*b^2*c^2*g^4*x^2 +
2*B*a^2*d^2*g^4*x^2 + 9*B*b^2*c*d*f^2*g^2*x - 9*B*a*b*d^2*f^2*g^2*x - 5*B*b
^2*c^2*f*g^3*x + 5*B*a^2*d^2*f*g^3*x + B*a*b*c^2*g^4*x - B*a^2*c*d*g^4*x +
A*b^2*d^2*f^4 + B*b^2*d^2*f^4 - 2*A*b^2*c*d*f^3*g + 3*B*b^2*c*d*f^3*g - 2*A
*a*b*d^2*f^3*g - 7*B*a*b*d^2*f^3*g + A*b^2*c^2*f^2*g^2 - 2*B*b^2*c^2*f^2*g^
2 + 4*A*a*b*c*d*f^2*g^2 + 4*B*a*b*c*d*f^2*g^2 + A*a^2*d^2*f^2*g^2 + 4*B*a^2
*d^2*f^2*g^2 - 2*A*a*b*c^2*f*g^3 - B*a*b*c^2*f*g^3 - 2*A*a^2*c*d*f*g^3 - 3*
B*a^2*c*d*f*g^3 + A*a^2*c^2*g^4 + B*a^2*c^2*g^4)/(b^2*d^2*f^4*g^4*x^3 - 2*b
^2*c*d*f^3*g^5*x^3 - 2*a*b*d^2*f^3*g^5*x^3 + b^2*c^2*f^2*g^6*x^3 + 4*a*b*c*
d*f^2*g^6*x^3 + a^2*d^2*f^2*g^6*x^3 - 2*a*b*c^2*f*g^7*x^3 - 2*a^2*c*d*f*g^7
*x^3 + a^2*c^2*g^8*x^3 + 3*b^2*d^2*f^5*g^3*x^2 - 6*b^2*c*d*f^4*g^4*x^2 - 6*
a*b*d^2*f^4*g^4*x^2 + 3*b^2*c^2*f^3*g^5*x^2 + 12*a*b*c*d*f^3*g^5*x^2 + 3*a^
2*d^2*f^3*g^5*x^2 - 6*a*b*c^2*f^2*g^6*x^2 - 6*a^2*c*d*f^2*g^6*x^2 + 3*a^2*c
^2*f*g^7*x^2 + 3*b^2*d^2*f^6*g^2*x - 6*b^2*c*d*f^5*g^3*x - 6*a*b*d^2*f^5*g^
3*x + 3*b^2*c^2*f^4*g^4*x + 12*a*b*c*d*f^4*g^4*x + 3*a^2*d^2*f^4*g^4*x - 6*
a*b*c^2*f^3*g^5*x - 6*a^2*c*d*f^3*g^5*x + 3*a^2*c^2*f^2*g^6*x + b^2*d^2*f^7
*g - 2*b^2*c*d*f^6*g^2 - 2*a*b*d^2*f^6*g^2 + b^2*c^2*f^5*g^3 + 4*a*b*c*d*f^
5*g^3 + a^2*d^2*f^5*g^3 - 2*a*b*c^2*f^4*g^4 - 2*a^2*c*d*f^4*g^4 + a^2*c^2*f
^3*g^5)
```

**maple [B]** time = 0.30, size = 4421, normalized size = 15.96

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*ln((b*x+a)^2/(d*x+c)^2*e)+A)/(g*x+f)^4,x)
```

```
[Out] 1/3*d^2/(1/(d*x+c)*c*g-1/(d*x+c)*d*f-g)^3/(c*g-d*f)*g^2/(a^2*g^2-2*a*b*f*g+
b^2*f^2)/(d*x+c)^2*B*a*b*c+d/(1/(d*x+c)*c*g-1/(d*x+c)*d*f-g)^3*B/(a^3*g^3-3
*a^2*b*f*g^2+3*a*b^2*f^2*g-b^3*f^3)/(d*x+c)^3*ln((1/(d*x+c)*a*d-1/(d*x+c)*b
*c+b)^2/d^2*e)*b^3*c^2*f*g-2*d/(1/(d*x+c)*c*g-1/(d*x+c)*d*f-g)^3*b^3*B/(a^3
*g^3-3*a^2*b*f*g^2+3*a*b^2*f^2*g-b^3*f^3)/(d*x+c)^2*ln((1/(d*x+c)*a*d-1/(d*
x+c)*b*c+b)^2/d^2*e)*c*f*g-d^3/(1/(d*x+c)*c*g-1/(d*x+c)*d*f-g)^3*B/(a^3*g^3
-3*a^2*b*f*g^2+3*a*b^2*f^2*g-b^3*f^3)/(d*x+c)^3*ln((1/(d*x+c)*a*d-1/(d*x+c)
*b*c+b)^2/d^2*e)*a^2*b*f*g+3*d^3/(1/(d*x+c)*c*g-1/(d*x+c)*d*f-g)^3/(c*g-d*f
)*g/(a^2*g^2-2*a*b*f*g+b^2*f^2)/(d*x+c)^2*B*a*b*f-3*d^2/(1/(d*x+c)*c*g-1/(d
*x+c)*d*f-g)^3/(c*g-d*f)*g/(a^2*g^2-2*a*b*f*g+b^2*f^2)/(d*x+c)^2*B*b^2*c*f-
1/(1/(d*x+c)*c*g-1/(d*x+c)*d*f-g)^3*b^3*g^2*B/(a^3*g^3-3*a^2*b*f*g^2+3*a*b^
2*f^2*g-b^3*f^3)/(d*x+c)*ln((1/(d*x+c)*a*d-1/(d*x+c)*b*c+b)^2/d^2*e)*c-5/3*
d^3/(1/(d*x+c)*c*g-1/(d*x+c)*d*f-g)^3/(a^2*g^2-2*a*b*f*g+b^2*f^2)/(d*x+c)^3
*B*a*b*f+5/3*d^2/(1/(d*x+c)*c*g-1/(d*x+c)*d*f-g)^3/(a^2*g^2-2*a*b*f*g+b^2*f
^2)/(d*x+c)^3*B*b^2*c*f-2/3*d/(1/(d*x+c)*c*g-1/(d*x+c)*d*f-g)^3*g^3/(a^2*c^
2*g^4-2*a^2*c*d*f*g^3+a^2*d^2*f^2*g^2-2*a*b*c^2*f*g^3+4*a*b*c*d*f^2*g^2-2*a
*b*d^2*f^3*g+b^2*c^2*f^2*g^2-2*b^2*c*d*f^3*g+b^2*d^2*f^4)/(d*x+c)*B*b^2*c^2
-2/3*d/(1/(d*x+c)*c*g-1/(d*x+c)*d*f-g)^3/(a^2*g^2-2*a*b*f*g+b^2*f^2)*g/(d*x
+c)^3*B*b^2*c^2-5/3*d^3/(1/(d*x+c)*c*g-1/(d*x+c)*d*f-g)^3/(c*g-d*f)*g^2/(a^
2*g^2-2*a*b*f*g+b^2*f^2)/(d*x+c)^2*B*a^2+1/3*d^3*A*g^2/(c*g-d*f)^3/(1/(d*x+
c)*c*g-1/(d*x+c)*d*f-g)^3+d^2/(1/(d*x+c)*c*g-1/(d*x+c)*d*f-g)^3*b^3*B/(a^3*
g^3-3*a^2*b*f*g^2+3*a*b^2*f^2*g-b^3*f^3)/(d*x+c)^2*ln((1/(d*x+c)*a*d-1/(d*x
+c)*b*c+b)^2/d^2*e)*f^2-2*d*B/(a^3*c^3*g^6-3*a^3*c^2*d*f*g^5+3*a^3*c*d^2*f^
2*g^4-a^3*d^3*f^3*g^3-3*a^2*b*c^3*f*g^5+9*a^2*b*c^2*d*f^2*g^4-9*a^2*b*c*d^2
*f^3*g^3+3*a^2*b*d^3*f^4*g^2+3*a*b^2*c^3*f^2*g^4-9*a*b^2*c^2*d*f^3*g^3+9*a*
b^2*c*d^2*f^4*g^2-3*a*b^2*d^3*f^5*g-b^3*c^3*f^3*g^3+3*b^3*c^2*d*f^4*g^2-3*b
^3*c*d^2*f^5*g+b^3*d^3*f^6)*ln(1/(d*x+c)*c*g-1/(d*x+c)*d*f-g)*b^3*c^2*f*g+2
*d^3*B/(a^3*c^3*g^6-3*a^3*c^2*d*f*g^5+3*a^3*c*d^2*f^2*g^4-a^3*d^3*f^3*g^3-3
*a^2*b*c^3*f*g^5+9*a^2*b*c^2*d*f^2*g^4-9*a^2*b*c*d^2*f^3*g^3+3*a^2*b*d^3*f^
4*g^2+3*a*b^2*c^3*f^2*g^4-9*a*b^2*c^2*d*f^3*g^3+9*a*b^2*c*d^2*f^4*g^2-3*a*b
^2*d^3*f^5*g-b^3*c^3*f^3*g^3+3*b^3*c^2*d*f^4*g^2-3*b^3*c*d^2*f^5*g+b^3*d^3*
f^6)*ln(1/(d*x+c)*c*g-1/(d*x+c)*d*f-g)*a^2*b*f*g+1/3*d^3/(1/(d*x+c)*c*g-1/(
d*x+c)*d*f-g)^3*B/(a^3*g^3-3*a^2*b*f*g^2+3*a*b^2*f^2*g-b^3*f^3)/(d*x+c)^3*l
n((1/(d*x+c)*a*d-1/(d*x+c)*b*c+b)^2/d^2*e)*a^3*g^2-1/3/(1/(d*x+c)*c*g-1/(d*
x+c)*d*f-g)^3*B/(a^3*g^3-3*a^2*b*f*g^2+3*a*b^2*f^2*g-b^3*f^3)/(d*x+c)^3*ln(
(1/(d*x+c)*a*d-1/(d*x+c)*b*c+b)^2/d^2*e)*b^3*c^3*g^2+1/(1/(d*x+c)*c*g-1/(d*
x+c)*d*f-g)^3*b^3*B/(a^3*g^3-3*a^2*b*f*g^2+3*a*b^2*f^2*g-b^3*f^3)/(d*x+c)^2
```

```
*ln((1/(d*x+c)*a*d-1/(d*x+c)*b*c+b)^2/d^2*e)*c^2*g^2+d^3*A/(c*g-d*f)^3/(1/(
d*x+c)*c*g-1/(d*x+c)*d*f-g)-1/3*d^2/(1/(d*x+c)*c*g-1/(d*x+c)*d*f-g)^3/(a^2*
g^2-2*a*b*f*g+b^2*f^2)*g/(d*x+c)^3*B*a*b*c+d^3/(1/(d*x+c)*c*g-1/(d*x+c)*d*f
-g)^3*B/(a^3*g^3-3*a^2*b*f*g^2+3*a*b^2*f^2*g-b^3*f^3)/(d*x+c)^3*ln((1/(d*x+
c)*a*d-1/(d*x+c)*b*c+b)^2/d^2*e)*a*b^2*f^2+d/(1/(d*x+c)*c*g-1/(d*x+c)*d*f-g
)^3*b^3*g*B/(a^3*g^3-3*a^2*b*f*g^2+3*a*b^2*f^2*g-b^3*f^3)/(d*x+c)*ln((1/(d*
x+c)*a*d-1/(d*x+c)*b*c+b)^2/d^2*e)*f-d^2/(1/(d*x+c)*c*g-1/(d*x+c)*d*f-g)^3*
B/(a^3*g^3-3*a^2*b*f*g^2+3*a*b^2*f^2*g-b^3*f^3)/(d*x+c)^3*ln((1/(d*x+c)*a*d
-1/(d*x+c)*b*c+b)^2/d^2*e)*b^3*c*f^2+d^3*A*g/(c*g-d*f)^3/(1/(d*x+c)*c*g-1/(
d*x+c)*d*f-g)^2-4/3*d^3/(1/(d*x+c)*c*g-1/(d*x+c)*d*f-g)^3*g^2/(a^2*c^2*g^4-
2*a^2*c*d*f*g^3+a^2*d^2*f^2*g^2-2*a*b*c^2*f*g^3+4*a*b*c*d*f^2*g^2-2*a*b*d^2
*f^3*g+b^2*c^2*f^2*g^2-2*b^2*c*d*f^3*g+b^2*d^2*f^4)/(d*x+c)*B*a*b*f+4/3*d^2
/(1/(d*x+c)*c*g-1/(d*x+c)*d*f-g)^3*g^2/(a^2*c^2*g^4-2*a^2*c*d*f*g^3+a^2*d^2
*f^2*g^2-2*a*b*c^2*f*g^3+4*a*b*c*d*f^2*g^2-2*a*b*d^2*f^3*g+b^2*c^2*f^2*g^2-
2*b^2*c*d*f^3*g+b^2*d^2*f^4)/(d*x+c)*B*b^2*c*f+4/3*d/(1/(d*x+c)*c*g-1/(d*x+
c)*d*f-g)^3/(c*g-d*f)*g^2/(a^2*g^2-2*a*b*f*g+b^2*f^2)/(d*x+c)^2*B*b^2*c^2-2
/3*d^3*B/(a^3*c^3*g^6-3*a^3*c^2*d*f*g^5+3*a^3*c*d^2*f^2*g^4-a^3*d^3*f^3*g^3
-3*a^2*b*c^3*f*g^5+9*a^2*b*c^2*d*f^2*g^4-9*a^2*b*c*d^2*f^3*g^3+3*a^2*b*d^3*
f^4*g^2+3*a*b^2*c^3*f^2*g^4-9*a*b^2*c^2*d*f^3*g^3+9*a*b^2*c*d^2*f^4*g^2-3*a
*b^2*d^3*f^5*g-b^3*c^3*f^3*g^3+3*b^3*c^2*d*f^4*g^2-3*b^3*c*d^2*f^5*g+b^3*d^
3*f^6)*ln(1/(d*x+c)*c*g-1/(d*x+c)*d*f-g)*a^3*g^2+2/3*B/(a^3*c^3*g^6-3*a^3*c
^2*d*f*g^5+3*a^3*c*d^2*f^2*g^4-a^3*d^3*f^3*g^3-3*a^2*b*c^3*f*g^5+9*a^2*b*c^
2*d*f^2*g^4-9*a^2*b*c*d^2*f^3*g^3+3*a^2*b*d^3*f^4*g^2+3*a*b^2*c^3*f^2*g^4-9
*a*b^2*c^2*d*f^3*g^3+9*a*b^2*c*d^2*f^4*g^2-3*a*b^2*d^3*f^5*g-b^3*c^3*f^3*g^
3+3*b^3*c^2*d*f^4*g^2-3*b^3*c*d^2*f^5*g+b^3*d^3*f^6)*ln(1/(d*x+c)*c*g-1/(d*
x+c)*d*f-g)*b^3*c^3*g^2+1/3/(1/(d*x+c)*c*g-1/(d*x+c)*d*f-g)^3*b^3*g^2*B/(a^
3*g^3-3*a^2*b*f*g^2+3*a*b^2*f^2*g-b^3*f^3)*ln((1/(d*x+c)*a*d-1/(d*x+c)*b*c+
b)^2/d^2*e)+2/3*d^3/(1/(d*x+c)*c*g-1/(d*x+c)*d*f-g)^3*g^3/(a^2*c^2*g^4-2*a^
2*c*d*f*g^3+a^2*d^2*f^2*g^2-2*a*b*c^2*f*g^3+4*a*b*c*d*f^2*g^2-2*a*b*d^2*f^3
*g+b^2*c^2*f^2*g^2-2*b^2*c*d*f^3*g+b^2*d^2*f^4)/(d*x+c)*B*a^2+d^3/(1/(d*x+c
)*c*g-1/(d*x+c)*d*f-g)^3/(a^2*g^2-2*a*b*f*g+b^2*f^2)*g/(d*x+c)^3*B*a^2-2*d^
3*B/(a^3*c^3*g^6-3*a^3*c^2*d*f*g^5+3*a^3*c*d^2*f^2*g^4-a^3*d^3*f^3*g^3-3*a^
2*b*c^3*f*g^5+9*a^2*b*c^2*d*f^2*g^4-9*a^2*b*c*d^2*f^3*g^3+3*a^2*b*d^3*f^4*g
^2+3*a*b^2*c^3*f^2*g^4-9*a*b^2*c^2*d*f^3*g^3+9*a*b^2*c*d^2*f^4*g^2-3*a*b^2*
d^3*f^5*g-b^3*c^3*f^3*g^3+3*b^3*c^2*d*f^4*g^2-3*b^3*c*d^2*f^5*g+b^3*d^3*f^6
)*ln(1/(d*x+c)*c*g-1/(d*x+c)*d*f-g)*a*b^2*f^2+2*d^2*B/(a^3*c^3*g^6-3*a^3*c^
2*d*f*g^5+3*a^3*c*d^2*f^2*g^4-a^3*d^3*f^3*g^3-3*a^2*b*c^3*f*g^5+9*a^2*b*c^2
*d*f^2*g^4-9*a^2*b*c*d^2*f^3*g^3+3*a^2*b*d^3*f^4*g^2+3*a*b^2*c^3*f^2*g^4-9*
a*b^2*c^2*d*f^3*g^3+9*a*b^2*c*d^2*f^4*g^2-3*a*b^2*d^3*f^5*g-b^3*c^3*f^3*g^3
+3*b^3*c^2*d*f^4*g^2-3*b^3*c*d^2*f^5*g+b^3*d^3*f^6)*ln(1/(d*x+c)*c*g-1/(d*x
+c)*d*f-g)*b^3*c*f^2
```

**maxima [B]** time = 1.73, size = 900, normalized size = 3.25

$$\frac{1}{3}\left(\frac{2\,b^3\log\left(bx+a\right)}{b^3f^3g-3\,ab^2f^2g^2+3\,a^2bfg^3-a^3g^4}-\frac{2\,d^3\log\left(dx+c\right)}{d^3f^3g-3\,cd^2f^2g^2+3\,c^2dfg^3-c^3g^4}+\frac{}{b^3d^3f^6+a^3c^3g^6-3\left(b^3cd^2+\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*(b*x+a)^2/(d*x+c)^2))/(g*x+f)^4,x, algorithm="maxima")
```

```
[Out] 1/3*(2*b^3*log(b*x + a)/(b^3*f^3*g - 3*a*b^2*f^2*g^2 + 3*a^2*b*f*g^3 - a^3*
g^4) - 2*d^3*log(d*x + c)/(d^3*f^3*g - 3*c*d^2*f^2*g^2 + 3*c^2*d*f*g^3 - c^
3*g^4) + 2*(3*(b^3*c*d^2 - a*b^2*d^3)*f^2 - 3*(b^3*c^2*d - a^2*b*d^3)*f*g +
 (b^3*c^3 - a^3*d^3)*g^2)*log(g*x + f)/(b^3*d^3*f^6 + a^3*c^3*g^6 - 3*(b^3*
c*d^2 + a*b^2*d^3)*f^5*g + 3*(b^3*c^2*d + 3*a*b^2*c*d^2 + a^2*b*d^3)*f^4*g^
2 - (b^3*c^3 + 9*a*b^2*c^2*d + 9*a^2*b*c*d^2 + a^3*d^3)*f^3*g^3 + 3*(a*b^2*
c^3 + 3*a^2*b*c^2*d + a^3*c*d^2)*f^2*g^4 - 3*(a^2*b*c^3 + a^3*c^2*d)*f*g^5)
 - (5*(b^2*c*d - a*b*d^2)*f^2 - 3*(b^2*c^2 - a^2*d^2)*f*g + (a*b*c^2 - a^2*
c*d)*g^2 + 2*(2*(b^2*c*d - a*b*d^2)*f*g - (b^2*c^2 - a^2*d^2)*g^2)*x)/(b^2*
d^2*f^6 + a^2*c^2*f^2*g^4 - 2*(b^2*c*d + a*b*d^2)*f^5*g + (b^2*c^2 + 4*a*b*
c*d + a^2*d^2)*f^4*g^2 - 2*(a*b*c^2 + a^2*c*d)*f^3*g^3 + (b^2*d^2*f^4*g^2 +
 a^2*c^2*g^6 - 2*(b^2*c*d + a*b*d^2)*f^3*g^3 + (b^2*c^2 + 4*a*b*c*d + a^2*d
^2)*f^2*g^4 - 2*(a*b*c^2 + a^2*c*d)*f*g^5)*x^2 + 2*(b^2*d^2*f^5*g + a^2*c^2
*f*g^5 - 2*(b^2*c*d + a*b*d^2)*f^4*g^2 + (b^2*c^2 + 4*a*b*c*d + a^2*d^2)*f^
3*g^3 - 2*(a*b*c^2 + a^2*c*d)*f^2*g^4)*x) - log(b^2*e*x^2/(d^2*x^2 + 2*c*d*
x + c^2) + 2*a*b*e*x/(d^2*x^2 + 2*c*d*x + c^2) + a^2*e/(d^2*x^2 + 2*c*d*x +
 c^2))/(g^4*x^3 + 3*f*g^3*x^2 + 3*f^2*g^2*x + f^3*g))*B - 1/3*A/(g^4*x^3 +
3*f*g^3*x^2 + 3*f^2*g^2*x + f^3*g)
```

**mupad [B]** time = 11.58, size = 1147, normalized size = 4.14

$$\frac{\ln\left(f+g\,x\right)\,\left(g\,\left(6\,B\,a^2\,b\,d^3\,f-6\,B\,b^3\,c\right.\right.}{3\,a^3\,c^3\,g^6-9\,a^3\,c^2\,d\,f\,g^5+9\,a^3\,c\,d^2\,f^2\,g^4-3\,a^3\,d^3\,f^3\,g^3-9\,a^2\,b\,c^3\,f\,g^5+27\,a^2\,b\,c^2\,d\,f^2\,g^4-27\,a^2\,b\,c\,d^2\,f^3\,g^3+9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*log((e*(a + b*x)^2)/(c + d*x)^2))/(f + g*x)^4,x)
```

```
[Out] (log(f + g*x)*(g*(6*B*a^2*b*d^3*f - 6*B*b^3*c^2*d*f) - g^2*(2*B*a^3*d^3 - 2
*B*b^3*c^3) - 6*B*a*b^2*d^3*f^2 + 6*B*b^3*c*d^2*f^2))/(3*a^3*c^3*g^6 + 3*b^
3*d^3*f^6 - 3*a^3*d^3*f^3*g^3 - 3*b^3*c^3*f^3*g^3 - 9*a^2*b*c^3*f*g^5 - 9*a
*b^2*d^3*f^5*g - 9*a^3*c^2*d*f*g^5 - 9*b^3*c*d^2*f^5*g + 9*a*b^2*c^3*f^2*g^
4 + 9*a^2*b*d^3*f^4*g^2 + 9*a^3*c*d^2*f^2*g^4 + 9*b^3*c^2*d*f^4*g^2 + 27*a*
b^2*c*d^2*f^4*g^2 - 27*a*b^2*c^2*d*f^3*g^3 - 27*a^2*b*c*d^2*f^3*g^3 + 27*a^
```

```
2*b*c^2*d*f^2*g^4) - ((A*a^2*c^2*g^4 + A*b^2*d^2*f^4 + A*a^2*d^2*f^2*g^2 +
A*b^2*c^2*f^2*g^2 + 3*B*a^2*d^2*f^2*g^2 - 3*B*b^2*c^2*f^2*g^2 - 2*A*a*b*c^2
*f*g^3 - 2*A*a*b*d^2*f^3*g + B*a*b*c^2*f*g^3 - 2*A*a^2*c*d*f*g^3 - 5*B*a*b*
d^2*f^3*g - 2*A*b^2*c*d*f^3*g - B*a^2*c*d*f*g^3 + 5*B*b^2*c*d*f^3*g + 4*A*a
*b*c*d*f^2*g^2)/(a^2*c^2*g^4 + b^2*d^2*f^4 + a^2*d^2*f^2*g^2 + b^2*c^2*f^2*
g^2 - 2*a*b*c^2*f*g^3 - 2*a*b*d^2*f^3*g - 2*a^2*c*d*f*g^3 - 2*b^2*c*d*f^3*g
 + 4*a*b*c*d*f^2*g^2) + (2*x^2*(B*a^2*d^2*g^4 - B*b^2*c^2*g^4 - 2*B*a*b*d^2
*f*g^3 + 2*B*b^2*c*d*f*g^3))/(a^2*c^2*g^4 + b^2*d^2*f^4 + a^2*d^2*f^2*g^2 +
 b^2*c^2*f^2*g^2 - 2*a*b*c^2*f*g^3 - 2*a*b*d^2*f^3*g - 2*a^2*c*d*f*g^3 - 2*
b^2*c*d*f^3*g + 4*a*b*c*d*f^2*g^2) + (x*(5*B*a^2*d^2*f*g^3 - 5*B*b^2*c^2*f*
g^3 + B*a*b*c^2*g^4 - B*a^2*c*d*g^4 - 9*B*a*b*d^2*f^2*g^2 + 9*B*b^2*c*d*f^2
*g^2))/(a^2*c^2*g^4 + b^2*d^2*f^4 + a^2*d^2*f^2*g^2 + b^2*c^2*f^2*g^2 - 2*a
*b*c^2*f*g^3 - 2*a*b*d^2*f^3*g - 2*a^2*c*d*f*g^3 - 2*b^2*c*d*f^3*g + 4*a*b*
c*d*f^2*g^2))/(3*f^3*g + 3*g^4*x^3 + 9*f^2*g^2*x + 9*f*g^3*x^2) - (B*log((e
*(a + b*x)^2)/(c + d*x)^2))/(3*g*(f^3 + g^3*x^3 + 3*f^2*g*x + 3*f*g^2*x^2))
 - (2*B*b^3*log(a + b*x))/(3*a^3*g^4 - 3*b^3*f^3*g + 9*a*b^2*f^2*g^2 - 9*a^
2*b*f*g^3) + (2*B*d^3*log(c + d*x))/(3*c^3*g^4 - 3*d^3*f^3*g + 9*c*d^2*f^2*
g^2 - 9*c^2*d*f*g^3)
```

**sympy [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*ln(e*(b*x+a)**2/(d*x+c)**2))/(g*x+f)**4,x)
```

```
[Out] Timed out
```

# 3.271 $\int \frac{A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}{(f+gx)^5}\,dx$

**Optimal**. Leaf size=381

$$-\frac{B(bc-ad)\left(a^2d^2g^2-abdg(3df-cg)+b^2\left(c^2g^2-3cdfg+3d^2f^2\right)\right)}{2(f+gx)(bf-ag)^3(df-cg)^3}-\frac{B(bc-ad)\log(f+gx)(-adg-bcg+2bdf}{2(bf}$$

```
[Out] -1/6*B*(-a*d+b*c)/(-a*g+b*f)/(-c*g+d*f)/(g*x+f)^3-1/4*B*(-a*d+b*c)*(-a*d*g-
b*c*g+2*b*d*f)/(-a*g+b*f)^2/(-c*g+d*f)^2/(g*x+f)^2-1/2*B*(-a*d+b*c)*(a^2*d^
2*g^2-a*b*d*g*(-c*g+3*d*f)+b^2*(c^2*g^2-3*c*d*f*g+3*d^2*f^2))/(-a*g+b*f)^3/
(-c*g+d*f)^3/(g*x+f)+1/2*b^4*B*ln(b*x+a)/g/(-a*g+b*f)^4+1/4*(-A-B*ln(e*(b*x
+a)^2/(d*x+c)^2))/g/(g*x+f)^4-1/2*B*d^4*ln(d*x+c)/g/(-c*g+d*f)^4-1/2*B*(-a*
d+b*c)*(-a*d*g-b*c*g+2*b*d*f)*(2*a*b*d^2*f*g-a^2*d^2*g^2-b^2*(c^2*g^2-2*c*d
*f*g+2*d^2*f^2))*ln(g*x+f)/(-a*g+b*f)^4/(-c*g+d*f)^4
```

**Rubi [A]** time = 0.55, antiderivative size = 381, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.103, Rules used = {2525, 12, 72}

$$-\frac{B(bc-ad)\left(a^2d^2g^2-abdg(3df-cg)+b^2\left(c^2g^2-3cdfg+3d^2f^2\right)\right)}{2(f+gx)(bf-ag)^3(df-cg)^3}-\frac{B(bc-ad)\log(f+gx)(-adg-bcg+2bdf}{2(bf}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])/(f + g*x)^5,x]
```

```
[Out] -(B*(b*c - a*d))/(6*(b*f - a*g)*(d*f - c*g)*(f + g*x)^3) - (B*(b*c - a*d)*(
2*b*d*f - b*c*g - a*d*g))/(4*(b*f - a*g)^2*(d*f - c*g)^2*(f + g*x)^2) - (B*
(b*c - a*d)*(a^2*d^2*g^2 - a*b*d*g*(3*d*f - c*g) + b^2*(3*d^2*f^2 - 3*c*d*f
*g + c^2*g^2)))/(2*(b*f - a*g)^3*(d*f - c*g)^3*(f + g*x)) + (b^4*B*Log[a +
b*x])/(2*g*(b*f - a*g)^4) - (A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])/(4*g*(
f + g*x)^4) - (B*d^4*Log[c + d*x])/(2*g*(d*f - c*g)^4) - (B*(b*c - a*d)*(2*
b*d*f - b*c*g - a*d*g)*(2*a*b*d^2*f*g - a^2*d^2*g^2 - b^2*(2*d^2*f^2 - 2*c*
d*f*g + c^2*g^2))*Log[f + g*x])/(2*(b*f - a*g)^4*(d*f - c*g)^4)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 72

```
Int[((e_.) + (f_.)*(x_))^(p_.)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
 x_Symbol] :> Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x]
 /; FreeQ[{a, b, c, d, e, f}, x] && IntegerQ[p]
```

Rule 2525

```
Int[((a_.) + Log[(c_.)*(RFx_)^(p_.)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_))^(m_.
), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(a + b*Log[c*RFx^p])^n)/(e*(m + 1))
, x] - Dist[(b*n*p)/(e*(m + 1)), Int[SimplifyIntegrand[((d + e*x)^(m + 1)*(
a + b*Log[c*RFx^p])^(n - 1)*D[RFx, x])/RFx, x], x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && RationalFunctionQ[RFx, x] && IGtQ[n, 0] && (EqQ[n, 1] ||
IntegerQ[m]) && NeQ[m, -1]
```

Rubi steps

$$\int \frac{A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}{(f+gx)^5}\,dx = -\frac{A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}{4g(f+gx)^4}+\frac{B\int \frac{2(bc-ad)}{(a+bx)(c+dx)(f+gx)^4}\,dx}{4g}$$

$$= -\frac{A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}{4g(f+gx)^4}+\frac{(B(bc-ad))\int \frac{1}{(a+bx)(c+dx)(f+gx)^4}\,dx}{2g}$$

$$= -\frac{A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}{4g(f+gx)^4}+\frac{(B(bc-ad))\int \left(\frac{b^5}{(bc-ad)(bf-ag)^4(a+bx)}-\frac{d^5}{(bc-ad)(-df+cg)^4(c+dx)}+\right.}{}$$

$$= -\frac{B(bc-ad)}{6(bf-ag)(df-cg)(f+gx)^3}-\frac{B(bc-ad)(2bdf-bcg-adg)}{4(bf-ag)^2(df-cg)^2(f+gx)^2}-\frac{B(bc-ad)\left(a^2\right.}{}$$

**Mathematica [A]** time = 0.97, size = 358, normalized size = 0.94

$$\frac{2B(bc-ad)\left(-\frac{g\left(a^2d^2g^2+abdg(cg-3df)+b^2\left(c^2g^2-3cdfg+3d^2f^2\right)\right)}{(f+gx)(bf-ag)^3(df-cg)^3}-\frac{g\log(f+gx)(adg+bcg-2bdf)\left(a^2d^2g^2-2abd^2fg+b^2\left(c^2g^2-2cdfg+2d^2f^2\right)\right)}{(bf-ag)^4(df-cg)^4}\right.}{4g}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])/(f + g*x)^5,x]
```

```
[Out] (-((A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])/(f + g*x)^4) + 2*B*(b*c - a*d)*
(-1/3*g/((b*f - a*g)*(d*f - c*g)*(f + g*x)^3) + (g*(-2*b*d*f + b*c*g + a*d*
g))/(2*(b*f - a*g)^2*(d*f - c*g)^2*(f + g*x)^2) - (g*(a^2*d^2*g^2 + a*b*d*g
```

```
*(-3*d*f + c*g) + b^2*(3*d^2*f^2 - 3*c*d*f*g + c^2*g^2)))/((b*f - a*g)^3*(d
*f - c*g)^3*(f + g*x)) + (b^4*Log[a + b*x])/((b*c - a*d)*(b*f - a*g)^4) - (
d^4*Log[c + d*x])/((b*c - a*d)*(d*f - c*g)^4) - (g*(-2*b*d*f + b*c*g + a*d*
g)*(-2*a*b*d^2*f*g + a^2*d^2*g^2 + b^2*(2*d^2*f^2 - 2*c*d*f*g + c^2*g^2))*L
og[f + g*x])/((b*f - a*g)^4*(d*f - c*g)^4)))/(4*g)
```

**fricas [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*(b*x+a)^2/(d*x+c)^2))/(g*x+f)^5,x, algorithm="fricas")
```

```
[Out] Timed out
```

**giac [F(-2)]** time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*(b*x+a)^2/(d*x+c)^2))/(g*x+f)^5,x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Undef
/Unsigned Inf encountered in limitUndef/Unsigned Inf encountered in limit(-
1/4*A*g^3-1/4*B*g^3)*(-(g*x+f)^-1/g)^4+(-B*a*d*g^3+B*b*c*g^3)/(6*a*c*g^2-6*
a*d*f*g-6*b*c*f*g+6*b*d*f^2)*(-(g*x+f)^-1/g)^3+(-B*a^2*d^2*g^3+2*B*a*b*d^2*
f*g^2+B*b^2*c^2*g^3-2*B*b^2*c*d*f*g^2)/(4*a^2*c^2*g^4-8*a^2*c*d*f*g^3+4*a^2
*d^2*f^2*g^2-8*a*b*c^2*f*g^3+16*a*b*c*d*f^2*g^2-8*a*b*d^2*f^3*g+4*b^2*c^2*f
^2*g^2-8*b^2*c*d*f^3*g+4*b^2*d^2*f^4)*(-(g*x+f)^-1/g)^2-(-B*a^3*d^3*g^3+3*B
*a^2*b*d^3*f*g^2-3*B*a*b^2*d^3*f^2*g+B*b^3*c^3*g^3-3*B*b^3*c^2*d*f*g^2+3*B*
b^3*c*d^2*f^2*g)/(2*a^3*c^3*g^6-6*a^3*c^2*d*f*g^5+6*a^3*c*d^2*f^2*g^4-2*a^3
*d^3*f^3*g^3-6*a^2*b*c^3*f*g^5+18*a^2*b*c^2*d*f^2*g^4-18*a^2*b*c*d^2*f^3*g^
3+6*a^2*b*d^3*f^4*g^2+6*a*b^2*c^3*f^2*g^4-18*a*b^2*c^2*d*f^3*g^3+18*a*b^2*c
*d^2*f^4*g^2-6*a*b^2*d^3*f^5*g-2*b^3*c^3*f^3*g^3+6*b^3*c^2*d*f^4*g^2-6*b^3*
c*d^2*f^5*g+2*b^3*d^3*f^6)*(g*x+f)^-1/g-1/4*B*g^3*(-(g*x+f)^-1/g)^4*ln((a^2
*g^4*(-(g*x+f)^-1/g)^2-2*a*b*f*g^3*(-(g*x+f)^-1/g)^2+2*a*b*g^2*(g*x+f)^-1/g
+b^2*f^2*g^2*(-(g*x+f)^-1/g)^2-2*b^2*f*g*(g*x+f)^-1/g+b^2)/(c^2*g^4*(-(g*x+
f)^-1/g)^2-2*c*d*f*g^3*(-(g*x+f)^-1/g)^2+2*c*d*g^2*(g*x+f)^-1/g+d^2*f^2*g^2
*(-(g*x+f)^-1/g)^2-2*d^2*f*g*(g*x+f)^-1/g+d^2))+g/g*((-B*a^4*d^4*g^3+4*B*a^
3*d^4*g^2*b*f-6*B*a^2*d^4*g*b^2*f^2+4*B*a*d^4*b^3*f^3-4*B*d^3*b^4*c*f^3+6*B
*d^2*g*b^4*c^2*f^2-4*B*d*g^2*b^4*c^3*f+B*g^3*b^4*c^4)/(4*a^4*d^4*g^4*f^4-16
*a^4*d^3*g^5*c*f^3+24*a^4*d^2*g^6*c^2*f^2-16*a^4*d*g^7*c^3*f+4*a^4*g^8*c^4-
16*a^3*d^4*g^3*b*f^5+64*a^3*d^3*g^4*b*c*f^4-96*a^3*d^2*g^5*b*c^2*f^3+64*a^3
*d*g^6*b*c^3*f^2-16*a^3*g^7*b*c^4*f+24*a^2*d^4*g^2*b^2*f^6-96*a^2*d^3*g^3*b
^2*c*f^5+144*a^2*d^2*g^4*b^2*c^2*f^4-96*a^2*d*g^5*b^2*c^3*f^3+24*a^2*g^6*b^
```

```
2*c^4*f^2-16*a*d^4*g*b^3*f^7+64*a*d^3*g^2*b^3*c*f^6-96*a*d^2*g^3*b^3*c^2*f^
5+64*a*d*g^4*b^3*c^3*f^4-16*a*g^5*b^3*c^4*f^3+4*d^4*b^4*f^8-16*d^3*g*b^4*c*
f^7+24*d^2*g^2*b^4*c^2*f^6-16*d*g^3*b^4*c^3*f^5+4*g^4*b^4*c^4*f^4)*ln(abs(-
(-(g*x+f)^-1/g)^2*a*d*g^3*f+(-(g*x+f)^-1/g)^2*a*g^4*c+(-(g*x+f)^-1/g)^2*d*g
^2*b*f^2-(-(g*x+f)^-1/g)^2*g^3*b*c*f+(g*x+f)^-1/g*a*d*g^2-2*(g*x+f)^-1/g*d*
g*b*f+(g*x+f)^-1/g*g^2*b*c+d*b))+(-B*a^5*d^5*g^5+4*B*a^4*d^5*g^4*b*f+B*a^4*
d^4*g^5*b*c-6*B*a^3*d^5*g^3*b^2*f^2-4*B*a^3*d^4*g^4*b^2*c*f+4*B*a^2*d^5*g^2
*b^3*f^3+6*B*a^2*d^4*g^3*b^3*c*f^2-2*B*a*d^5*g*b^4*f^4-6*B*a*d^3*g^3*b^4*c^
2*f^2+4*B*a*d^2*g^4*b^4*c^3*f-B*a*d*g^5*b^4*c^4+2*B*d^4*g*b^5*c*f^4-4*B*d^3
*g^2*b^5*c^2*f^3+6*B*d^2*g^3*b^5*c^3*f^2-4*B*d*g^4*b^5*c^4*f+B*g^5*b^5*c^5)
/(4*a^4*d^4*g^4*f^4-16*a^4*d^3*g^5*c*f^3+24*a^4*d^2*g^6*c^2*f^2-16*a^4*d*g^
7*c^3*f+4*a^4*g^8*c^4-16*a^3*d^4*g^3*b*f^5+64*a^3*d^3*g^4*b*c*f^4-96*a^3*d^
2*g^5*b*c^2*f^3+64*a^3*d*g^6*b*c^3*f^2-16*a^3*g^7*b*c^4*f+24*a^2*d^4*g^2*b^
2*f^6-96*a^2*d^3*g^3*b^2*c*f^5+144*a^2*d^2*g^4*b^2*c^2*f^4-96*a^2*d*g^5*b^2
*c^3*f^3+24*a^2*g^6*b^2*c^4*f^2-16*a*d^4*g*b^3*f^7+64*a*d^3*g^2*b^3*c*f^6-9
6*a*d^2*g^3*b^3*c^2*f^5+64*a*d*g^4*b^3*c^3*f^4-16*a*g^5*b^3*c^4*f^3+4*d^4*b
^4*f^8-16*d^3*g*b^4*c*f^7+24*d^2*g^2*b^4*c^2*f^6-16*d*g^3*b^4*c^3*f^5+4*g^4
*b^4*c^4*f^4)/abs(a*d*g^2-g^2*b*c)*ln(abs(2*(g*x+f)^-1/g*a*d*g^3*f-2*(g*x+f
)^-1/g*a*g^4*c-2*(g*x+f)^-1/g*d*g^2*b*f^2+2*(g*x+f)^-1/g*g^3*b*c*f-a*d*g^2+
2*d*g*b*f-g^2*b*c-abs(a*d*g^2-g^2*b*c))/abs(2*(g*x+f)^-1/g*a*d*g^3*f-2*(g*x
+f)^-1/g*a*g^4*c-2*(g*x+f)^-1/g*d*g^2*b*f^2+2*(g*x+f)^-1/g*g^3*b*c*f-a*d*g^
2+2*d*g*b*f-g^2*b*c+abs(a*d*g^2-g^2*b*c))))
```

**maple [B]** time = 0.45, size = 10401, normalized size = 27.30

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*ln((b*x+a)^2/(d*x+c)^2*e)+A)/(g*x+f)^5,x)

[Out] result too large to display
```

**maxima [B]** time = 2.00, size = 1809, normalized size = 4.75

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*(b*x+a)^2/(d*x+c)^2))/(g*x+f)^5,x, algorithm="maxima")

[Out] 1/12*(6*b^4*log(b*x + a)/(b^4*f^4*g - 4*a*b^3*f^3*g^2 + 6*a^2*b^2*f^2*g^3 -
 4*a^3*b*f*g^4 + a^4*g^5) - 6*d^4*log(d*x + c)/(d^4*f^4*g - 4*c*d^3*f^3*g^2
 + 6*c^2*d^2*f^2*g^3 - 4*c^3*d*f*g^4 + c^4*g^5) + 6*(4*(b^4*c*d^3 - a*b^3*d
^4)*f^3 - 6*(b^4*c^2*d^2 - a^2*b^2*d^4)*f^2*g + 4*(b^4*c^3*d - a^3*b*d^4)*f
*g^2 - (b^4*c^4 - a^4*d^4)*g^3)*log(g*x + f)/(b^4*d^4*f^8 + a^4*c^4*g^8 - 4
*(b^4*c*d^3 + a*b^3*d^4)*f^7*g + 2*(3*b^4*c^2*d^2 + 8*a*b^3*c*d^3 + 3*a^2*b
```

```
^2*d^4)*f^6*g^2 - 4*(b^4*c^3*d + 6*a*b^3*c^2*d^2 + 6*a^2*b^2*c*d^3 + a^3*b*
d^4)*f^5*g^3 + (b^4*c^4 + 16*a*b^3*c^3*d + 36*a^2*b^2*c^2*d^2 + 16*a^3*b*c*
d^3 + a^4*d^4)*f^4*g^4 - 4*(a*b^3*c^4 + 6*a^2*b^2*c^3*d + 6*a^3*b*c^2*d^2 +
 a^4*c*d^3)*f^3*g^5 + 2*(3*a^2*b^2*c^4 + 8*a^3*b*c^3*d + 3*a^4*c^2*d^2)*f^2
*g^6 - 4*(a^3*b*c^4 + a^4*c^3*d)*f*g^7) - (26*(b^3*c*d^2 - a*b^2*d^3)*f^4 -
 31*(b^3*c^2*d - a^2*b*d^3)*f^3*g + (11*b^3*c^3 + 15*a*b^2*c^2*d - 15*a^2*b
*c*d^2 - 11*a^3*d^3)*f^2*g^2 - 7*(a*b^2*c^3 - a^3*c*d^2)*f*g^3 + 2*(a^2*b*c
^3 - a^3*c^2*d)*g^4 + 6*(3*(b^3*c*d^2 - a*b^2*d^3)*f^2*g^2 - 3*(b^3*c^2*d -
 a^2*b*d^3)*f*g^3 + (b^3*c^3 - a^3*d^3)*g^4)*x^2 + 3*(14*(b^3*c*d^2 - a*b^2
*d^3)*f^3*g - 15*(b^3*c^2*d - a^2*b*d^3)*f^2*g^2 + (5*b^3*c^3 + 3*a*b^2*c^2
*d - 3*a^2*b*c*d^2 - 5*a^3*d^3)*f*g^3 - (a*b^2*c^3 - a^3*c*d^2)*g^4)*x)/(b^
3*d^3*f^9 + a^3*c^3*f^3*g^6 - 3*(b^3*c*d^2 + a*b^2*d^3)*f^8*g + 3*(b^3*c^2*
d + 3*a*b^2*c*d^2 + a^2*b*d^3)*f^7*g^2 - (b^3*c^3 + 9*a*b^2*c^2*d + 9*a^2*b
*c*d^2 + a^3*d^3)*f^6*g^3 + 3*(a*b^2*c^3 + 3*a^2*b*c^2*d + a^3*c*d^2)*f^5*g
^4 - 3*(a^2*b*c^3 + a^3*c^2*d)*f^4*g^5 + (b^3*d^3*f^6*g^3 + a^3*c^3*g^9 - 3
*(b^3*c*d^2 + a*b^2*d^3)*f^5*g^4 + 3*(b^3*c^2*d + 3*a*b^2*c*d^2 + a^2*b*d^3
)*f^4*g^5 - (b^3*c^3 + 9*a*b^2*c^2*d + 9*a^2*b*c*d^2 + a^3*d^3)*f^3*g^6 + 3
*(a*b^2*c^3 + 3*a^2*b*c^2*d + a^3*c*d^2)*f^2*g^7 - 3*(a^2*b*c^3 + a^3*c^2*d
)*f*g^8)*x^3 + 3*(b^3*d^3*f^7*g^2 + a^3*c^3*f*g^8 - 3*(b^3*c*d^2 + a*b^2*d^
3)*f^6*g^3 + 3*(b^3*c^2*d + 3*a*b^2*c*d^2 + a^2*b*d^3)*f^5*g^4 - (b^3*c^3 +
 9*a*b^2*c^2*d + 9*a^2*b*c*d^2 + a^3*d^3)*f^4*g^5 + 3*(a*b^2*c^3 + 3*a^2*b*
c^2*d + a^3*c*d^2)*f^3*g^6 - 3*(a^2*b*c^3 + a^3*c^2*d)*f^2*g^7)*x^2 + 3*(b^
3*d^3*f^8*g + a^3*c^3*f^2*g^7 - 3*(b^3*c*d^2 + a*b^2*d^3)*f^7*g^2 + 3*(b^3*
c^2*d + 3*a*b^2*c*d^2 + a^2*b*d^3)*f^6*g^3 - (b^3*c^3 + 9*a*b^2*c^2*d + 9*a
^2*b*c*d^2 + a^3*d^3)*f^5*g^4 + 3*(a*b^2*c^3 + 3*a^2*b*c^2*d + a^3*c*d^2)*f
^4*g^5 - 3*(a^2*b*c^3 + a^3*c^2*d)*f^3*g^6)*x) - 3*log(b^2*e*x^2/(d^2*x^2 +
 2*c*d*x + c^2) + 2*a*b*e*x/(d^2*x^2 + 2*c*d*x + c^2) + a^2*e/(d^2*x^2 + 2*
c*d*x + c^2))/(g^5*x^4 + 4*f*g^4*x^3 + 6*f^2*g^3*x^2 + 4*f^3*g^2*x + f^4*g)
)*B - 1/4*A/(g^5*x^4 + 4*f*g^4*x^3 + 6*f^2*g^3*x^2 + 4*f^3*g^2*x + f^4*g)
```

**mupad [B]** time = 17.43, size = 2520, normalized size = 6.61

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*log((e*(a + b*x)^2)/(c + d*x)^2))/(f + g*x)^5,x)
```

```
[Out] (log(f + g*x)*(g*(6*B*a^2*b^2*d^4*f^2 - 6*B*b^4*c^2*d^2*f^2) - g^2*(4*B*a^3
*b*d^4*f - 4*B*b^4*c^3*d*f) + g^3*(B*a^4*d^4 - B*b^4*c^4) - 4*B*a*b^3*d^4*f
^3 + 4*B*b^4*c*d^3*f^3))/(2*a^4*c^4*g^8 + 2*b^4*d^4*f^8 + 2*a^4*d^4*f^4*g^4
 + 2*b^4*c^4*f^4*g^4 + 12*a^2*b^2*c^4*f^2*g^6 + 12*a^2*b^2*d^4*f^6*g^2 + 12
*a^4*c^2*d^2*f^2*g^6 + 12*b^4*c^2*d^2*f^6*g^2 - 8*a^3*b*c^4*f*g^7 - 8*a*b^3
*d^4*f^7*g - 8*a^4*c^3*d*f*g^7 - 8*b^4*c*d^3*f^7*g - 8*a*b^3*c^4*f^3*g^5 -
8*a^3*b*d^4*f^5*g^3 - 8*a^4*c*d^3*f^3*g^5 - 8*b^4*c^3*d*f^5*g^3 + 32*a*b^3*
c*d^3*f^6*g^2 + 32*a*b^3*c^3*d*f^4*g^4 + 32*a^3*b*c*d^3*f^4*g^4 + 32*a^3*b*
```

```
c^3*d*f^2*g^6 - 48*a*b^3*c^2*d^2*f^5*g^3 - 48*a^2*b^2*c*d^3*f^5*g^3 - 48*a^
2*b^2*c^3*d*f^3*g^5 - 48*a^3*b*c^2*d^2*f^3*g^5 + 72*a^2*b^2*c^2*d^2*f^4*g^4
) - ((3*A*a^3*c^3*g^6 + 3*A*b^3*d^3*f^6 - 3*A*a^3*d^3*f^3*g^3 - 3*A*b^3*c^3
*f^3*g^3 - 11*B*a^3*d^3*f^3*g^3 + 11*B*b^3*c^3*f^3*g^3 + 9*A*a*b^2*c^3*f^2*
g^4 + 9*A*a^2*b*d^3*f^4*g^2 - 7*B*a*b^2*c^3*f^2*g^4 + 9*A*a^3*c*d^2*f^2*g^4
 + 31*B*a^2*b*d^3*f^4*g^2 + 9*A*b^3*c^2*d*f^4*g^2 + 7*B*a^3*c*d^2*f^2*g^4 -
 31*B*b^3*c^2*d*f^4*g^2 - 9*A*a^2*b*c^3*f*g^5 - 9*A*a*b^2*d^3*f^5*g + 2*B*a
^2*b*c^3*f*g^5 - 9*A*a^3*c^2*d*f*g^5 - 26*B*a*b^2*d^3*f^5*g - 9*A*b^3*c*d^2
*f^5*g - 2*B*a^3*c^2*d*f*g^5 + 26*B*b^3*c*d^2*f^5*g + 27*A*a*b^2*c*d^2*f^4*
g^2 - 27*A*a*b^2*c^2*d*f^3*g^3 - 27*A*a^2*b*c*d^2*f^3*g^3 + 27*A*a^2*b*c^2*
d*f^2*g^4 + 15*B*a*b^2*c^2*d*f^3*g^3 - 15*B*a^2*b*c*d^2*f^3*g^3)/(6*(a^3*c^
3*g^6 + b^3*d^3*f^6 - a^3*d^3*f^3*g^3 - b^3*c^3*f^3*g^3 - 3*a^2*b*c^3*f*g^5
 - 3*a*b^2*d^3*f^5*g - 3*a^3*c^2*d*f*g^5 - 3*b^3*c*d^2*f^5*g + 3*a*b^2*c^3*
f^2*g^4 + 3*a^2*b*d^3*f^4*g^2 + 3*a^3*c*d^2*f^2*g^4 + 3*b^3*c^2*d*f^4*g^2 +
 9*a*b^2*c*d^2*f^4*g^2 - 9*a*b^2*c^2*d*f^3*g^3 - 9*a^2*b*c*d^2*f^3*g^3 + 9*
a^2*b*c^2*d*f^2*g^4)) - (x^2*(B*a*b^2*c^3*g^6 - B*a^3*c*d^2*g^6 + 7*B*a^3*d
^3*f*g^5 - 7*B*b^3*c^3*f*g^5 + 20*B*a*b^2*d^3*f^3*g^3 - 21*B*a^2*b*d^3*f^2*
g^4 - 20*B*b^3*c*d^2*f^3*g^3 + 21*B*b^3*c^2*d*f^2*g^4 - 3*B*a*b^2*c^2*d*f*g
^5 + 3*B*a^2*b*c*d^2*f*g^5))/(2*(a^3*c^3*g^6 + b^3*d^3*f^6 - a^3*d^3*f^3*g^
3 - b^3*c^3*f^3*g^3 - 3*a^2*b*c^3*f*g^5 - 3*a*b^2*d^3*f^5*g - 3*a^3*c^2*d*f
*g^5 - 3*b^3*c*d^2*f^5*g + 3*a*b^2*c^3*f^2*g^4 + 3*a^2*b*d^3*f^4*g^2 + 3*a^
3*c*d^2*f^2*g^4 + 3*b^3*c^2*d*f^4*g^2 + 9*a*b^2*c*d^2*f^4*g^2 - 9*a*b^2*c^2
*d*f^3*g^3 - 9*a^2*b*c*d^2*f^3*g^3 + 9*a^2*b*c^2*d*f^2*g^4)) + (x*(B*a^2*b*
c^3*g^6 - B*a^3*c^2*d*g^6 - 13*B*a^3*d^3*f^2*g^4 + 13*B*b^3*c^3*f^2*g^4 - 3
4*B*a*b^2*d^3*f^4*g^2 + 38*B*a^2*b*d^3*f^3*g^3 + 34*B*b^3*c*d^2*f^4*g^2 - 3
8*B*b^3*c^2*d*f^3*g^3 - 5*B*a*b^2*c^3*f*g^5 + 5*B*a^3*c*d^2*f*g^5 + 12*B*a*
b^2*c^2*d*f^2*g^4 - 12*B*a^2*b*c*d^2*f^2*g^4))/(3*(a^3*c^3*g^6 + b^3*d^3*f^
6 - a^3*d^3*f^3*g^3 - b^3*c^3*f^3*g^3 - 3*a^2*b*c^3*f*g^5 - 3*a*b^2*d^3*f^5
*g - 3*a^3*c^2*d*f*g^5 - 3*b^3*c*d^2*f^5*g + 3*a*b^2*c^3*f^2*g^4 + 3*a^2*b*
d^3*f^4*g^2 + 3*a^3*c*d^2*f^2*g^4 + 3*b^3*c^2*d*f^4*g^2 + 9*a*b^2*c*d^2*f^4
*g^2 - 9*a*b^2*c^2*d*f^3*g^3 - 9*a^2*b*c*d^2*f^3*g^3 + 9*a^2*b*c^2*d*f^2*g^
4)) - (x^3*(B*a^3*d^3*g^6 - B*b^3*c^3*g^6 + 3*B*a*b^2*d^3*f^2*g^4 - 3*B*b^3
*c*d^2*f^2*g^4 - 3*B*a^2*b*d^3*f*g^5 + 3*B*b^3*c^2*d*f*g^5))/(a^3*c^3*g^6 +
 b^3*d^3*f^6 - a^3*d^3*f^3*g^3 - b^3*c^3*f^3*g^3 - 3*a^2*b*c^3*f*g^5 - 3*a*
b^2*d^3*f^5*g - 3*a^3*c^2*d*f*g^5 - 3*b^3*c*d^2*f^5*g + 3*a*b^2*c^3*f^2*g^4
 + 3*a^2*b*d^3*f^4*g^2 + 3*a^3*c*d^2*f^2*g^4 + 3*b^3*c^2*d*f^4*g^2 + 9*a*b^
2*c*d^2*f^4*g^2 - 9*a*b^2*c^2*d*f^3*g^3 - 9*a^2*b*c*d^2*f^3*g^3 + 9*a^2*b*c
^2*d*f^2*g^4))/(2*f^4*g + 2*g^5*x^4 + 8*f^3*g^2*x + 8*f*g^4*x^3 + 12*f^2*g^
3*x^2) + (B*b^4*log(a + b*x))/(2*a^4*g^5 + 2*b^4*f^4*g - 8*a*b^3*f^3*g^2 +
12*a^2*b^2*f^2*g^3 - 8*a^3*b*f*g^4) - (B*log((e*(a + b*x)^2)/(c + d*x)^2))/
(4*g*(f^4 + g^4*x^4 + 4*f^3*g*x + 4*f*g^3*x^3 + 6*f^2*g^2*x^2)) - (B*d^4*lo
g(c + d*x))/(2*c^4*g^5 + 2*d^4*f^4*g - 8*c*d^3*f^3*g^2 + 12*c^2*d^2*f^2*g^3
 - 8*c^3*d*f*g^4)
```

**sympy [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*ln(e*(b*x+a)**2/(d*x+c)**2))/(g*x+f)**5,x)
[Out] Timed out
```

## 3.272 $\int (f+gx)^3 \left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2 dx$

**Optimal**. Leaf size=869

$$\frac{2B^2g^3\log\left(\frac{a+bx}{c+dx}\right)(bc-ad)^4}{3b^4d^4}+\frac{2B^2g^3\log(c+dx)(bc-ad)^4}{3b^4d^4}+\frac{2B^2g^3x(bc-ad)^3}{3b^3d^3}+\frac{B^2g^2(4bdf-3bcg-adg)\log\left(\frac{a+}{c+}\right.}{b^4d^4}$$

```
[Out] 2/3*B^2*(-a*d+b*c)^3*g^3*x/b^3/d^3+B^2*(-a*d+b*c)^2*g^2*(-a*d*g-3*b*c*g+4*b
*d*f)*x/b^3/d^3+1/3*B^2*(-a*d+b*c)^2*g^3*(d*x+c)^2/b^2/d^4-B*(-a*d+b*c)*g*(
a^2*d^2*g^2-2*a*b*d*g*(-c*g+2*d*f)+b^2*(3*c^2*g^2-8*c*d*f*g+6*d^2*f^2))*(b*
x+a)*(A+B*ln(e*(b*x+a)^2/(d*x+c)^2))/b^4/d^3-1/2*B*(-a*d+b*c)*g^2*(-a*d*g-3
*b*c*g+4*b*d*f)*(d*x+c)^2*(A+B*ln(e*(b*x+a)^2/(d*x+c)^2))/b^2/d^4-1/3*B*(-a
*d+b*c)*g^3*(d*x+c)^3*(A+B*ln(e*(b*x+a)^2/(d*x+c)^2))/b/d^4-1/4*(-a*g+b*f)^
4*(A+B*ln(e*(b*x+a)^2/(d*x+c)^2))^2/b^4/g+1/4*(g*x+f)^4*(A+B*ln(e*(b*x+a)^2
/(d*x+c)^2))^2/g-B*(-a*d+b*c)*(-a*d*g-b*c*g+2*b*d*f)*(2*a*b*d^2*f*g-a^2*d^2
*g^2-b^2*(c^2*g^2-2*c*d*f*g+2*d^2*f^2))*(A+B*ln(e*(b*x+a)^2/(d*x+c)^2))*ln(
(-a*d+b*c)/b/(d*x+c))/b^4/d^4+2/3*B^2*(-a*d+b*c)^4*g^3*ln((b*x+a)/(d*x+c))/
b^4/d^4+B^2*(-a*d+b*c)^3*g^2*(-a*d*g-3*b*c*g+4*b*d*f)*ln((b*x+a)/(d*x+c))/b
^4/d^4+2/3*B^2*(-a*d+b*c)^4*g^3*ln(d*x+c)/b^4/d^4+B^2*(-a*d+b*c)^3*g^2*(-a*
d*g-3*b*c*g+4*b*d*f)*ln(d*x+c)/b^4/d^4+2*B^2*(-a*d+b*c)^2*g*(a^2*d^2*g^2-2*
a*b*d*g*(-c*g+2*d*f)+b^2*(3*c^2*g^2-8*c*d*f*g+6*d^2*f^2))*ln(d*x+c)/b^4/d^4
-2*B^2*(-a*d+b*c)*(-a*d*g-b*c*g+2*b*d*f)*(2*a*b*d^2*f*g-a^2*d^2*g^2-b^2*(c^
2*g^2-2*c*d*f*g+2*d^2*f^2))*polylog(2,d*(b*x+a)/b/(d*x+c))/b^4/d^4
```

**Rubi [A]** time = 1.81, antiderivative size = 973, normalized size of antiderivative = 1.12, number of steps used = 33, number of rules used = 13, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.419$, Rules used = {2525, 12, 2528, 2486, 31, 72, 2524, 2418, 2390, 2301, 2394, 2393, 2391}

$$\frac{B^2\log^2(a+bx)(bf-ag)^4}{b^4g}-\frac{B\log(a+bx)\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)(bf-ag)^4}{b^4g}-\frac{2B^2\log(a+bx)\log\left(\frac{b(c+dx)}{bc-ad}\right)(bf-}{b^4g}$$

Antiderivative was successfully verified.

```
[In] Int[(f + g*x)^3*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])^2,x]
```

```
[Out] (-2*B^2*(b*c - a*d)^2*(b*c + a*d)*g^3*x)/(3*b^3*d^3) + (B^2*(b*c - a*d)^2*g
^2*(4*b*d*f - b*c*g - a*d*g)*x)/(b^3*d^3) - (A*B*(b*c - a*d)*g*(a^2*d^2*g^2
 - a*b*d*g*(4*d*f - c*g) + b^2*(6*d^2*f^2 - 4*c*d*f*g + c^2*g^2))*x)/(b^3*d
^3) + (B^2*(b*c - a*d)^2*g^3*x^2)/(3*b^2*d^2) - (2*a^3*B^2*(b*c - a*d)*g^3*
Log[a + b*x])/(3*b^4*d) + (a^2*B^2*(b*c - a*d)*g^2*(4*b*d*f - b*c*g - a*d*g
)*Log[a + b*x])/(b^4*d^2) + (B^2*(b*f - a*g)^4*Log[a + b*x]^2)/(b^4*g) - (B
```

```
^2*(b*c - a*d)*g*(a^2*d^2*g^2 - a*b*d*g*(4*d*f - c*g) + b^2*(6*d^2*f^2 - 4*
c*d*f*g + c^2*g^2))*(a + b*x)*Log[(e*(a + b*x)^2)/(c + d*x)^2])/(b^4*d^3) -
 (B*(b*c - a*d)*g^2*(4*b*d*f - b*c*g - a*d*g)*x^2*(A + B*Log[(e*(a + b*x)^2
)/(c + d*x)^2]))/(2*b^2*d^2) - (B*(b*c - a*d)*g^3*x^3*(A + B*Log[(e*(a + b*
x)^2)/(c + d*x)^2]))/(3*b*d) - (B*(b*f - a*g)^4*Log[a + b*x]*(A + B*Log[(e*
(a + b*x)^2)/(c + d*x)^2]))/(b^4*g) + ((f + g*x)^4*(A + B*Log[(e*(a + b*x)^
2)/(c + d*x)^2])^2)/(4*g) + (2*B^2*c^3*(b*c - a*d)*g^3*Log[c + d*x])/(3*b*d
^4) - (B^2*c^2*(b*c - a*d)*g^2*(4*b*d*f - b*c*g - a*d*g)*Log[c + d*x])/(b^2
*d^4) + (2*B^2*(b*c - a*d)^2*g*(a^2*d^2*g^2 - a*b*d*g*(4*d*f - c*g) + b^2*(
6*d^2*f^2 - 4*c*d*f*g + c^2*g^2))*Log[c + d*x])/(b^4*d^4) - (2*B^2*(d*f - c
*g)^4*Log[-((d*(a + b*x))/(b*c - a*d))]*Log[c + d*x])/(d^4*g) + (B*(d*f - c
*g)^4*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])*Log[c + d*x])/(d^4*g) + (B^2
*(d*f - c*g)^4*Log[c + d*x]^2)/(d^4*g) - (2*B^2*(b*f - a*g)^4*Log[a + b*x]*
Log[(b*(c + d*x))/(b*c - a*d)])/(b^4*g) - (2*B^2*(b*f - a*g)^4*PolyLog[2, -
((d*(a + b*x))/(b*c - a*d))])/(b^4*g) - (2*B^2*(d*f - c*g)^4*PolyLog[2, (b*
(c + d*x))/(b*c - a*d)])/(d^4*g)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 72

```
Int[((e_.) + (f_.)*(x_))^(p_.)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
 x_Symbol] :> Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x]
 /; FreeQ[{a, b, c, d, e, f}, x] && IntegerQ[p]
```

Rule 2301

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

Rule 2390

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_))^(q_.), x_Symbol] :> Dist[1/e, Subst[Int[((f*x)/d)^q*(a + b*Log[c*x^
n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && E
qQ[e*f - d*g, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2393

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + (c*e*x)/g])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2394

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d + e*x
)^n]))/g, x] - Dist[(b*e*n)/g, Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2418

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*(RFx_), x_Sy
mbol] :> With[{u = ExpandIntegrand[(a + b*Log[c*(d + e*x)^n])^p, RFx, x]},
Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d, e, n}, x] && RationalFunctionQ[
RFx, x] && IntegerQ[p]
```

Rule 2486

```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))
^(r_.)]^(s_.), x_Symbol] :> Simp[((a + b*x)*Log[e*(f*(a + b*x)^p*(c + d*x)^
q)^r]^s)/b, x] + Dist[(q*r*s*(b*c - a*d))/b, Int[Log[e*(f*(a + b*x)^p*(c +
d*x)^q)^r]^(s - 1)/(c + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, p, q, r, s
}, x] && NeQ[b*c - a*d, 0] && EqQ[p + q, 0] && IGtQ[s, 0]
```

Rule 2524

```
Int[((a_.) + Log[(c_.)*(RFx_)^(p_.)]*(b_.))^(n_.)/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[(Log[d + e*x]*(a + b*Log[c*RFx^p])^n)/e, x] - Dist[(b*n*p)/e
, Int[(Log[d + e*x]*(a + b*Log[c*RFx^p])^(n - 1)*D[RFx, x])/RFx, x], x] /;
FreeQ[{a, b, c, d, e, p}, x] && RationalFunctionQ[RFx, x] && IGtQ[n, 0]
```

Rule 2525

```
Int[((a_.) + Log[(c_.)*(RFx_)^(p_.)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_))^(m_.
), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(a + b*Log[c*RFx^p])^n)/(e*(m + 1))
```

```
, x] - Dist[(b*n*p)/(e*(m + 1)), Int[SimplifyIntegrand[((d + e*x)^(m + 1)*(
a + b*Log[c*RFx^p])^(n - 1)*D[RFx, x])/RFx, x], x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && RationalFunctionQ[RFx, x] && IGtQ[n, 0] && (EqQ[n, 1] ||
IntegerQ[m]) && NeQ[m, -1]
```

Rule 2528

```
Int[((a_.) + Log[(c_.)*(RFx_)^(p_.)]*(b_.))^(n_.)*(RGx_), x_Symbol] :> With
[{u = ExpandIntegrand[(a + b*Log[c*RFx^p])^n, RGx, x]}, Int[u, x] /; SumQ[u
]] /; FreeQ[{a, b, c, p}, x] && RationalFunctionQ[RFx, x] && RationalFuncti
onQ[RGx, x] && IGtQ[n, 0]
```

Rubi steps

$$\int (f+gx)^3 \left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2 dx = \frac{(f+gx)^4\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}{4g} - \frac{B\int \frac{2(bc-ad)(f+gx)^4\left(A+B\log\left(\frac{e(a+bx)}{(c+dx)^2}\right.\right.}{(a+bx)(c+dx)}}{2g}$$

$$= \frac{(f+gx)^4\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}{4g} - \frac{(B(bc-ad))\int \frac{(f+gx)^4\left(A+B\log\left(\frac{e(}{(}\right.\right.}{(a+bx)(c+dx}}{g}$$

$$= \frac{(f+gx)^4\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}{4g} - \frac{(B(bc-ad))\int\left(\frac{g^2\left(a^2d^2g^2-abdg(4a\right.}{}\right.}{}$$

$$= \frac{(f+gx)^4\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}{4g} - \frac{\left(B(bc-ad)g^3\right)\int x^2\left(A+B\log\right.}{bd}$$

$$= -\frac{AB(bc-ad)g\left(a^2d^2g^2-abdg(4df-cg)+b^2\left(6d^2f^2-4cdfg+c\right.\right.}{b^3d^3}$$

$$= -\frac{AB(bc-ad)g\left(a^2d^2g^2-abdg(4df-cg)+b^2\left(6d^2f^2-4cdfg+c\right.\right.}{b^3d^3}$$

$$= -\frac{AB(bc-ad)g\left(a^2d^2g^2-abdg(4df-cg)+b^2\left(6d^2f^2-4cdfg+c\right.\right.}{b^3d^3}$$

$$= -\frac{2B^2(bc-ad)^2(bc+ad)g^3x}{3b^3d^3} + \frac{B^2(bc-ad)^2g^2(4bdf-bcg-adg)x}{b^3d^3}$$

$$= -\frac{2B^2(bc-ad)^2(bc+ad)g^3x}{3b^3d^3} + \frac{B^2(bc-ad)^2g^2(4bdf-bcg-adg)x}{b^3d^3}$$

$$= -\frac{2B^2(bc-ad)^2(bc+ad)g^3x}{3b^3d^3} + \frac{B^2(bc-ad)^2g^2(4bdf-bcg-adg)x}{b^3d^3}$$

$$= -\frac{2B^2(bc-ad)^2(bc+ad)g^3x}{3b^3d^3} + \frac{B^2(bc-ad)^2g^2(4bdf-bcg-adg)x}{b^3d^3}$$

**Mathematica [A]** time = 1.00, size = 746, normalized size = 0.86

$$\frac{(f+gx)^4\left(B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+A\right)^2-\frac{2B\left(2Bg^4(bc-ad)\left(2a^3d^3\log(a+bx)+bdx(bc-ad)(2ad+2bc-bdx)-2b^3c^3\log(c+dx)\right)+6Abdg^2x(bc-ad)\left(a^2d^2\right.\right.}{}}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(f + g*x)^3*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])^2,x]
```

```
[Out] ((f + g*x)^4*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])^2 - (2*B*(6*A*b*d*(b*
c - a*d)*g^2*(a^2*d^2*g^2 + a*b*d*g*(-4*d*f + c*g) + b^2*(6*d^2*f^2 - 4*c*d
*f*g + c^2*g^2))*x + 6*B*d*(b*c - a*d)*g^2*(a^2*d^2*g^2 + a*b*d*g*(-4*d*f +
 c*g) + b^2*(6*d^2*f^2 - 4*c*d*f*g + c^2*g^2))*(a + b*x)*Log[(e*(a + b*x)^2
)/(c + d*x)^2] + 3*b^2*d^2*(b*c - a*d)*g^3*(4*b*d*f - b*c*g - a*d*g)*x^2*(A
 + B*Log[(e*(a + b*x)^2)/(c + d*x)^2]) + 2*b^3*d^3*(b*c - a*d)*g^4*x^3*(A +
 B*Log[(e*(a + b*x)^2)/(c + d*x)^2]) + 6*d^4*(b*f - a*g)^4*Log[a + b*x]*(A
+ B*Log[(e*(a + b*x)^2)/(c + d*x)^2]) - 12*B*(b*c - a*d)^2*g^2*(a^2*d^2*g^2
 + a*b*d*g*(-4*d*f + c*g) + b^2*(6*d^2*f^2 - 4*c*d*f*g + c^2*g^2))*Log[c +
d*x] - 6*b^4*(d*f - c*g)^4*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])*Log[c +
 d*x] + 2*B*(b*c - a*d)*g^4*(b*d*(b*c - a*d)*x*(2*b*c + 2*a*d - b*d*x) + 2*
a^3*d^3*Log[a + b*x] - 2*b^3*c^3*Log[c + d*x]) - 6*B*(b*c - a*d)*g^3*(-4*b*
d*f + b*c*g + a*d*g)*(-(a^2*d^2*Log[a + b*x]) + b*(d*(-(b*c) + a*d)*x + b*c
^2*Log[c + d*x])) - 6*B*d^4*(b*f - a*g)^4*(Log[a + b*x]*(Log[a + b*x] - 2*L
og[(b*(c + d*x))/(b*c - a*d)]) - 2*PolyLog[2, (d*(a + b*x))/(-(b*c) + a*d)]
) + 6*b^4*B*(d*f - c*g)^4*((2*Log[(d*(a + b*x))/(-(b*c) + a*d)] - Log[c + d
*x])*Log[c + d*x] + 2*PolyLog[2, (b*(c + d*x))/(b*c - a*d)])))/(3*b^4*d^4))
/(4*g)
```

**fricas [F]** time = 1.08, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(A^2g^3x^3+3\,A^2fg^2x^2+3\,A^2f^2gx+A^2f^3+\left(B^2g^3x^3+3\,B^2fg^2x^2+3\,B^2f^2gx+B^2f^3\right)\log\left(\frac{b^2ex^2+2\,ab}{d^2x^2+2\,c}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^3*(A+B*log(e*(b*x+a)^2/(d*x+c)^2))^2,x, algorithm="fricas
")
```

```
[Out] integral(A^2*g^3*x^3 + 3*A^2*f*g^2*x^2 + 3*A^2*f^2*g*x + A^2*f^3 + (B^2*g^3
*x^3 + 3*B^2*f*g^2*x^2 + 3*B^2*f^2*g*x + B^2*f^3)*log((b^2*e*x^2 + 2*a*b*e*
x + a^2*e)/(d^2*x^2 + 2*c*d*x + c^2))^2 + 2*(A*B*g^3*x^3 + 3*A*B*f*g^2*x^2
+ 3*A*B*f^2*g*x + A*B*f^3)*log((b^2*e*x^2 + 2*a*b*e*x + a^2*e)/(d^2*x^2 + 2
*c*d*x + c^2)), x)
```

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \left(gx+f\right)^{3}\left(B\log\left(\frac{\left(bx+a\right)^{2}e}{\left(dx+c\right)^{2}}\right)+A\right)^{2}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^3*(A+B*log(e*(b*x+a)^2/(d*x+c)^2))^2,x, algorithm="giac")
[Out] integrate((g*x + f)^3*(B*log((b*x + a)^2*e/(d*x + c)^2) + A)^2, x)
```

**maple [F]** time = 1.35, size = 0, normalized size = 0.00

$$\int \left(gx+f\right)^{3}\left(B\ln\left(\frac{\left(bx+a\right)^{2}e}{\left(dx+c\right)^{2}}\right)+A\right)^{2}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x+f)^3*(B*ln((b*x+a)^2/(d*x+c)^2*e)+A)^2,x)
[Out] int((g*x+f)^3*(B*ln((b*x+a)^2/(d*x+c)^2*e)+A)^2,x)
```

**maxima [B]** time = 1.96, size = 2351, normalized size = 2.71

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^3*(A+B*log(e*(b*x+a)^2/(d*x+c)^2))^2,x, algorithm="maxima
")
[Out] 1/4*A^2*g^3*x^4 + A^2*f*g^2*x^3 + 3/2*A^2*f^2*g*x^2 + 2*(x*log(b^2*e*x^2/(d
^2*x^2 + 2*c*d*x + c^2) + 2*a*b*e*x/(d^2*x^2 + 2*c*d*x + c^2) + a^2*e/(d^2*
x^2 + 2*c*d*x + c^2)) + 2*a*log(b*x + a)/b - 2*c*log(d*x + c)/d)*A*B*f^3 +
3*(x^2*log(b^2*e*x^2/(d^2*x^2 + 2*c*d*x + c^2) + 2*a*b*e*x/(d^2*x^2 + 2*c*d
*x + c^2) + a^2*e/(d^2*x^2 + 2*c*d*x + c^2)) - 2*a^2*log(b*x + a)/b^2 + 2*c
^2*log(d*x + c)/d^2 - 2*(b*c - a*d)*x/(b*d))*A*B*f^2*g + 2*(x^3*log(b^2*e*x
^2/(d^2*x^2 + 2*c*d*x + c^2) + 2*a*b*e*x/(d^2*x^2 + 2*c*d*x + c^2) + a^2*e/
(d^2*x^2 + 2*c*d*x + c^2)) + 2*a^3*log(b*x + a)/b^3 - 2*c^3*log(d*x + c)/d^
3 - ((b^2*c*d - a*b*d^2)*x^2 - 2*(b^2*c^2 - a^2*d^2)*x)/(b^2*d^2))*A*B*f*g^
2 + 1/6*(3*x^4*log(b^2*e*x^2/(d^2*x^2 + 2*c*d*x + c^2) + 2*a*b*e*x/(d^2*x^2
 + 2*c*d*x + c^2) + a^2*e/(d^2*x^2 + 2*c*d*x + c^2)) - 6*a^4*log(b*x + a)/b
^4 + 6*c^4*log(d*x + c)/d^4 - (2*(b^3*c*d^2 - a*b^2*d^3)*x^3 - 3*(b^3*c^2*d
 - a^2*b*d^3)*x^2 + 6*(b^3*c^3 - a^3*d^3)*x)/(b^3*d^3))*A*B*g^3 + A^2*f^3*x
 - 1/3*(6*a^3*c*d^3*g^3 - 3*(8*c*d^3*f*g^2 - c^2*d^2*g^3)*a^2*b + 2*(18*c*d
^3*f^2*g - 6*c^2*d^2*f*g^2 + c^3*d*g^3)*a*b^2 + (12*c*d^3*f^3*log(e) - (3*g
```

```
^3*log(e) + 11*g^3)*c^4 + 12*(f*g^2*log(e) + 3*f*g^2)*c^3*d - 18*(f^2*g*log
(e) + 2*f^2*g)*c^2*d^2)*b^3)*B^2*log(d*x + c)/(b^3*d^4) + 2*(4*a*b^3*d^4*f^
3 - 6*a^2*b^2*d^4*f^2*g + 4*a^3*b*d^4*f*g^2 - a^4*d^4*g^3 - (4*c*d^3*f^3 -
6*c^2*d^2*f^2*g + 4*c^3*d*f*g^2 - c^4*g^3)*b^4)*(log(b*x + a)*log((b*d*x +
a*d)/(b*c - a*d) + 1) + dilog(-(b*d*x + a*d)/(b*c - a*d)))*B^2/(b^4*d^4) +
1/12*(3*B^2*b^4*d^4*g^3*x^4*log(e)^2 + 4*(a*b^3*d^4*g^3*log(e) + (3*d^4*f*g
^2*log(e)^2 - c*d^3*g^3*log(e))*b^4)*B^2*x^3 - 2*((3*g^3*log(e) - 2*g^3)*a^
2*b^2*d^4 - 4*(3*d^4*f*g^2*log(e) - c*d^3*g^3)*a*b^3 - (9*d^4*f^2*g*log(e)^
2 - 12*c*d^3*f*g^2*log(e) + (3*g^3*log(e) + 2*g^3)*c^2*d^2)*b^4)*B^2*x^2 +
4*((3*g^3*log(e) - 5*g^3)*a^3*b*d^4 + (5*c*d^3*g^3 - 12*(f*g^2*log(e) - f*g
^2)*d^4)*a^2*b^2 + (18*d^4*f^2*g*log(e) - 24*c*d^3*f*g^2 + 5*c^2*d^2*g^3)*a
*b^3 + (3*d^4*f^3*log(e)^2 - 18*c*d^3*f^2*g*log(e) - (3*g^3*log(e) + 5*g^3)
*c^3*d + 12*(f*g^2*log(e) + f*g^2)*c^2*d^2)*b^4)*B^2*x + 12*(B^2*b^4*d^4*g^
3*x^4 + 4*B^2*b^4*d^4*f*g^2*x^3 + 6*B^2*b^4*d^4*f^2*g*x^2 + 4*B^2*b^4*d^4*f
^3*x + (4*a*b^3*d^4*f^3 - 6*a^2*b^2*d^4*f^2*g + 4*a^3*b*d^4*f*g^2 - a^4*d^4
*g^3)*B^2)*log(b*x + a)^2 + 12*(B^2*b^4*d^4*g^3*x^4 + 4*B^2*b^4*d^4*f*g^2*x
^3 + 6*B^2*b^4*d^4*f^2*g*x^2 + 4*B^2*b^4*d^4*f^3*x + (4*c*d^3*f^3 - 6*c^2*d
^2*f^2*g + 4*c^3*d*f*g^2 - c^4*g^3)*B^2*b^4)*log(d*x + c)^2 + 4*(3*B^2*b^4*
d^4*g^3*x^4*log(e) + 2*(a*b^3*d^4*g^3 + (6*d^4*f*g^2*log(e) - c*d^3*g^3)*b^
4)*B^2*x^3 + 3*(4*a*b^3*d^4*f*g^2 - a^2*b^2*d^4*g^3 + (6*d^4*f^2*g*log(e) -
 4*c*d^3*f*g^2 + c^2*d^2*g^3)*b^4)*B^2*x^2 + 6*(6*a*b^3*d^4*f^2*g - 4*a^2*b
^2*d^4*f*g^2 + a^3*b*d^4*g^3 + (2*d^4*f^3*log(e) - 6*c*d^3*f^2*g + 4*c^2*d^
2*f*g^2 - c^3*d*g^3)*b^4)*B^2*x - ((3*g^3*log(e) - 11*g^3)*a^4*d^4 + 2*(c*d
^3*g^3 - 6*(f*g^2*log(e) - 3*f*g^2)*d^4)*a^3*b - 3*(4*c*d^3*f*g^2 - c^2*d^2
*g^3 - 6*(f^2*g*log(e) - 2*f^2*g)*d^4)*a^2*b^2 - 6*(2*d^4*f^3*log(e) - 6*c*
d^3*f^2*g + 4*c^2*d^2*f*g^2 - c^3*d*g^3)*a*b^3)*B^2)*log(b*x + a) - 4*(3*B^
2*b^4*d^4*g^3*x^4*log(e) + 2*(a*b^3*d^4*g^3 + (6*d^4*f*g^2*log(e) - c*d^3*g
^3)*b^4)*B^2*x^3 + 3*(4*a*b^3*d^4*f*g^2 - a^2*b^2*d^4*g^3 + (6*d^4*f^2*g*lo
g(e) - 4*c*d^3*f*g^2 + c^2*d^2*g^3)*b^4)*B^2*x^2 + 6*(6*a*b^3*d^4*f^2*g - 4
*a^2*b^2*d^4*f*g^2 + a^3*b*d^4*g^3 + (2*d^4*f^3*log(e) - 6*c*d^3*f^2*g + 4*
c^2*d^2*f*g^2 - c^3*d*g^3)*b^4)*B^2*x + 6*(B^2*b^4*d^4*g^3*x^4 + 4*B^2*b^4*
d^4*f*g^2*x^3 + 6*B^2*b^4*d^4*f^2*g*x^2 + 4*B^2*b^4*d^4*f^3*x + (4*a*b^3*d^
4*f^3 - 6*a^2*b^2*d^4*f^2*g + 4*a^3*b*d^4*f*g^2 - a^4*d^4*g^3)*B^2)*log(b*x
 + a))*log(d*x + c))/(b^4*d^4)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.00

$$\int \left(f+g\,x\right)^3\,{\left(A+B\,\ln\left(\frac{e\,{\left(a+b\,x\right)}^2}{{\left(c+d\,x\right)}^2}\right)\right)}^2 \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f + g*x)^3*(A + B*log((e*(a + b*x)^2)/(c + d*x)^2))^2,x)
```

```
[Out] int((f + g*x)^3*(A + B*log((e*(a + b*x)^2)/(c + d*x)^2))^2, x)
```

**sympy [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)**3*(A+B*ln(e*(b*x+a)**2/(d*x+c)**2))**2,x)
```

```
[Out] Timed out
```

# 3.273 $\int (f+gx)^2 \left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2 dx$

**Optimal**. Leaf size=542

$$\frac{4B(bc-ad)\left(a^2d^2g^2-abdg(3df-cg)+b^2\left(c^2g^2-3cdfg+3d^2f^2\right)\right)\log\left(\frac{bc-ad}{b(c+dx)}\right)\left(B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+A\right)}{3b^3d^3}+\frac{8B^2(bc$$

```
[Out] 4/3*B^2*(-a*d+b*c)^2*g^2*x/b^2/d^2-4/3*B*(-a*d+b*c)*g*(-a*d*g-2*b*c*g+3*b*d
*f)*(b*x+a)*(A+B*ln(e*(b*x+a)^2/(d*x+c)^2))/b^3/d^2-2/3*B*(-a*d+b*c)*g^2*(d
*x+c)^2*(A+B*ln(e*(b*x+a)^2/(d*x+c)^2))/b/d^3-1/3*(-a*g+b*f)^3*(A+B*ln(e*(b
*x+a)^2/(d*x+c)^2))^2/b^3/g+1/3*(g*x+f)^3*(A+B*ln(e*(b*x+a)^2/(d*x+c)^2))^2
/g+4/3*B*(-a*d+b*c)*(a^2*d^2*g^2-a*b*d*g*(-c*g+3*d*f)+b^2*(c^2*g^2-3*c*d*f*
g+3*d^2*f^2))*(A+B*ln(e*(b*x+a)^2/(d*x+c)^2))*ln((-a*d+b*c)/b/(d*x+c))/b^3/
d^3+4/3*B^2*(-a*d+b*c)^3*g^2*ln((b*x+a)/(d*x+c))/b^3/d^3+4/3*B^2*(-a*d+b*c)
^3*g^2*ln(d*x+c)/b^3/d^3+8/3*B^2*(-a*d+b*c)^2*g*(-a*d*g-2*b*c*g+3*b*d*f)*ln
(d*x+c)/b^3/d^3+8/3*B^2*(-a*d+b*c)*(a^2*d^2*g^2-a*b*d*g*(-c*g+3*d*f)+b^2*(c
^2*g^2-3*c*d*f*g+3*d^2*f^2))*polylog(2,d*(b*x+a)/b/(d*x+c))/b^3/d^3
```

**Rubi [A]** time = 1.20, antiderivative size = 659, normalized size of antiderivative = 1.22, number of steps used = 29, number of rules used = 13, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.419, Rules used = {2525, 12, 2528, 2486, 31, 72, 2524, 2418, 2390, 2301, 2394, 2393, 2391}

$$-\frac{8B^2(bf-ag)^3\text{PolyLog}\left(2,-\frac{d(a+bx)}{bc-ad}\right)}{3b^3g}-\frac{8B^2(df-cg)^3\text{PolyLog}\left(2,\frac{b(c+dx)}{bc-ad}\right)}{3d^3g}+\frac{4a^2B^2g^2(bc-ad)\log(a+bx)}{3b^3d}-\frac{4ABg$$

Antiderivative was successfully verified.

```
[In] Int[(f + g*x)^2*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])^2,x]
```

```
[Out] (4*B^2*(b*c - a*d)^2*g^2*x)/(3*b^2*d^2) - (4*A*B*(b*c - a*d)*g*(3*b*d*f - b
*c*g - a*d*g)*x)/(3*b^2*d^2) + (4*a^2*B^2*(b*c - a*d)*g^2*Log[a + b*x])/(3*
b^3*d) + (4*B^2*(b*f - a*g)^3*Log[a + b*x]^2)/(3*b^3*g) - (4*B^2*(b*c - a*d
)*g*(3*b*d*f - b*c*g - a*d*g)*(a + b*x)*Log[(e*(a + b*x)^2)/(c + d*x)^2])/(
3*b^3*d^2) - (2*B*(b*c - a*d)*g^2*x^2*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^
2]))/(3*b*d) - (4*B*(b*f - a*g)^3*Log[a + b*x]*(A + B*Log[(e*(a + b*x)^2)/(
c + d*x)^2]))/(3*b^3*g) + ((f + g*x)^3*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)
^2])^2)/(3*g) - (4*B^2*c^2*(b*c - a*d)*g^2*Log[c + d*x])/(3*b*d^3) + (8*B^2
*(b*c - a*d)^2*g*(3*b*d*f - b*c*g - a*d*g)*Log[c + d*x])/(3*b^3*d^3) - (8*B
^2*(d*f - c*g)^3*Log[-((d*(a + b*x))/(b*c - a*d))]*Log[c + d*x])/(3*d^3*g)
+ (4*B*(d*f - c*g)^3*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])*Log[c + d*x])
/(3*d^3*g) + (4*B^2*(d*f - c*g)^3*Log[c + d*x]^2)/(3*d^3*g) - (8*B^2*(b*f -
```

```
a*g)^3*Log[a + b*x]*Log[(b*(c + d*x))/(b*c - a*d)])/(3*b^3*g) - (8*B^2*(b*
f - a*g)^3*PolyLog[2, -((d*(a + b*x))/(b*c - a*d))])/(3*b^3*g) - (8*B^2*(d*
f - c*g)^3*PolyLog[2, (b*(c + d*x))/(b*c - a*d)])/(3*d^3*g)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 72

```
Int[((e_.) + (f_.)*(x_))^(p_.)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
 x_Symbol] :> Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x]
 /; FreeQ[{a, b, c, d, e, f}, x] && IntegerQ[p]
```

Rule 2301

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

Rule 2390

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_))^(q_.), x_Symbol] :> Dist[1/e, Subst[Int[((f*x)/d)^q*(a + b*Log[c*x^
n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && E
qQ[e*f - d*g, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2393

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + (c*e*x)/g])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2394

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d + e*x
)^n]))/g, x] - Dist[(b*e*n)/g, Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2418

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*(RFx_), x_Sy
mbol] :> With[{u = ExpandIntegrand[(a + b*Log[c*(d + e*x)^n])^p, RFx, x]},
Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d, e, n}, x] && RationalFunctionQ[
RFx, x] && IntegerQ[p]
```

Rule 2486

```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))
^(r_.)]^(s_.), x_Symbol] :> Simp[((a + b*x)*Log[e*(f*(a + b*x)^p*(c + d*x)^
q)^r]^s)/b, x] + Dist[(q*r*s*(b*c - a*d))/b, Int[Log[e*(f*(a + b*x)^p*(c +
d*x)^q)^r]^(s - 1)/(c + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, p, q, r, s
}, x] && NeQ[b*c - a*d, 0] && EqQ[p + q, 0] && IGtQ[s, 0]
```

Rule 2524

```
Int[((a_.) + Log[(c_.)*(RFx_)^(p_.)]*(b_.))^(n_.)/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[(Log[d + e*x]*(a + b*Log[c*RFx^p])^n)/e, x] - Dist[(b*n*p)/e
, Int[(Log[d + e*x]*(a + b*Log[c*RFx^p])^(n - 1)*D[RFx, x])/RFx, x], x] /;
FreeQ[{a, b, c, d, e, p}, x] && RationalFunctionQ[RFx, x] && IGtQ[n, 0]
```

Rule 2525

```
Int[((a_.) + Log[(c_.)*(RFx_)^(p_.)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_))^(m_.
), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(a + b*Log[c*RFx^p])^n)/(e*(m + 1))
, x] - Dist[(b*n*p)/(e*(m + 1)), Int[SimplifyIntegrand[((d + e*x)^(m + 1)*(
a + b*Log[c*RFx^p])^(n - 1)*D[RFx, x])/RFx, x], x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && RationalFunctionQ[RFx, x] && IGtQ[n, 0] && (EqQ[n, 1] ||
IntegerQ[m]) && NeQ[m, -1]
```

Rule 2528

```
Int[((a_.) + Log[(c_.)*(RFx_)^(p_.)]*(b_.))^(n_.)*(RGx_), x_Symbol] :> With
[{u = ExpandIntegrand[(a + b*Log[c*RFx^p])^n, RGx, x]}, Int[u, x] /; SumQ[u
]] /; FreeQ[{a, b, c, p}, x] && RationalFunctionQ[RFx, x] && RationalFuncti
onQ[RGx, x] && IGtQ[n, 0]
```

Rubi steps

$$\int (f+gx)^2 \left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2 dx = \frac{(f+gx)^3\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}{3g} - \frac{(2B)\int \frac{2(bc-ad)(f+gx)^3\left(A+B\log\left(\frac{e(a}{(c-}\right.\right.}{(a+bx)(c+dx)}}{3g}$$

$$= \frac{(f+gx)^3\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}{3g} - \frac{(4B(bc-ad))\int \frac{(f+gx)^3\left(A+B\log\left(\right.\right.}{(a+bx)(c+}}{3g}$$

$$= \frac{(f+gx)^3\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}{3g} - \frac{(4B(bc-ad))\int\left(\frac{g^2(3bdf-bcg-adg}{}\right.}{}$$

$$= \frac{(f+gx)^3\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}{3g} - \frac{\left(4B(bc-ad)g^2\right)\int x\left(A+B\log\right.}{3bd}$$

$$= -\frac{4AB(bc-ad)g(3bdf-bcg-adg)x}{3b^2d^2} - \frac{2B(bc-ad)g^2x^2\left(A+B\log\right.}{3bd}$$

$$= -\frac{4AB(bc-ad)g(3bdf-bcg-adg)x}{3b^2d^2} - \frac{4B^2(bc-ad)g(3bdf-bcg}{3b}$$

$$= -\frac{4AB(bc-ad)g(3bdf-bcg-adg)x}{3b^2d^2} - \frac{4B^2(bc-ad)g(3bdf-bcg}{3b}$$

$$= \frac{4B^2(bc-ad)^2g^2x}{3b^2d^2} - \frac{4AB(bc-ad)g(3bdf-bcg-adg)x}{3b^2d^2} + \frac{4a^2B^2(b}{}$$

$$= \frac{4B^2(bc-ad)^2g^2x}{3b^2d^2} - \frac{4AB(bc-ad)g(3bdf-bcg-adg)x}{3b^2d^2} + \frac{4a^2B^2(b}{}$$

$$= \frac{4B^2(bc-ad)^2g^2x}{3b^2d^2} - \frac{4AB(bc-ad)g(3bdf-bcg-adg)x}{3b^2d^2} + \frac{4a^2B^2(b}{}$$

$$= \frac{4B^2(bc-ad)^2g^2x}{3b^2d^2} - \frac{4AB(bc-ad)g(3bdf-bcg-adg)x}{3b^2d^2} + \frac{4a^2B^2(b}{}$$

**Mathematica [A]** time = 0.55, size = 497, normalized size = 0.92

$$\frac{(f+gx)^3\left(B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+A\right)^2-\frac{2B\left(-2Bg^3(bc-ad)\left(a^2d^2\log(a+bx)-b\left(dx(ad-bc)+bc^2\log(c+dx)\right)\right)-2b^3(df-cg)^3\log(c+dx)\left(B\log\left(\frac{e(a+bx)}{(c+dx)}\right.\right.}{}}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(f + g*x)^2*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])^2,x]
```

```
[Out] ((f + g*x)^3*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])^2 - (2*B*(2*A*b*d*(b*
c - a*d)*g^2*(3*b*d*f - b*c*g - a*d*g)*x + 2*B*d*(b*c - a*d)*g^2*(3*b*d*f -
 b*c*g - a*d*g)*(a + b*x)*Log[(e*(a + b*x)^2)/(c + d*x)^2] + b^2*d^2*(b*c -
 a*d)*g^3*x^2*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2]) + 2*d^3*(b*f - a*g)^
3*Log[a + b*x]*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2]) + 4*B*(b*c - a*d)^2
*g^2*(-3*b*d*f + b*c*g + a*d*g)*Log[c + d*x] - 2*b^3*(d*f - c*g)^3*(A + B*L
og[(e*(a + b*x)^2)/(c + d*x)^2])*Log[c + d*x] - 2*B*(b*c - a*d)*g^3*(a^2*d^
2*Log[a + b*x] - b*(d*(-(b*c) + a*d)*x + b*c^2*Log[c + d*x])) - 2*B*d^3*(b*
f - a*g)^3*(Log[a + b*x]*(Log[a + b*x] - 2*Log[(b*(c + d*x))/(b*c - a*d)])
- 2*PolyLog[2, (d*(a + b*x))/(-(b*c) + a*d)]) + 2*b^3*B*(d*f - c*g)^3*((2*L
og[(d*(a + b*x))/(-(b*c) + a*d)] - Log[c + d*x])*Log[c + d*x] + 2*PolyLog[2
, (b*(c + d*x))/(b*c - a*d)])))/(b^3*d^3))/(3*g)
```

**fricas [F]** time = 1.10, size = 0, normalized size = 0.00

$$\text{integral}\left(A^2g^2x^2+2\,A^2fgx+A^2f^2+\left(B^2g^2x^2+2\,B^2fgx+B^2f^2\right)\log\left(\frac{b^2ex^2+2\,abex+a^2e}{d^2x^2+2\,cdx+c^2}\right)^2+2\left(ABg^2x^2+2\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^2*(A+B*log(e*(b*x+a)^2/(d*x+c)^2))^2,x, algorithm="fricas
")
```

```
[Out] integral(A^2*g^2*x^2 + 2*A^2*f*g*x + A^2*f^2 + (B^2*g^2*x^2 + 2*B^2*f*g*x +
 B^2*f^2)*log((b^2*e*x^2 + 2*a*b*e*x + a^2*e)/(d^2*x^2 + 2*c*d*x + c^2))^2
+ 2*(A*B*g^2*x^2 + 2*A*B*f*g*x + A*B*f^2)*log((b^2*e*x^2 + 2*a*b*e*x + a^2*
e)/(d^2*x^2 + 2*c*d*x + c^2)), x)
```

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int\left(gx+f\right)^2\left(B\log\left(\frac{(bx+a)^2e}{(dx+c)^2}\right)+A\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^2*(A+B*log(e*(b*x+a)^2/(d*x+c)^2))^2,x, algorithm="giac")
```

```
[Out] integrate((g*x + f)^2*(B*log((b*x + a)^2*e/(d*x + c)^2) + A)^2, x)
```

**maple [F]** time = 1.24, size = 0, normalized size = 0.00

$$\int \left(gx+f\right)^{2} \left(B\ln\left(\frac{\left(bx+a\right)^{2} e}{\left(dx+c\right)^{2}}\right)+A\right)^{2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x+f)^2*(B*ln((b*x+a)^2/(d*x+c)^2*e)+A)^2,x)
```

```
[Out] int((g*x+f)^2*(B*ln((b*x+a)^2/(d*x+c)^2*e)+A)^2,x)
```

**maxima [B]** time = 1.77, size = 1458, normalized size = 2.69

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^2*(A+B*log(e*(b*x+a)^2/(d*x+c)^2))^2,x, algorithm="maxima
")
```

```
[Out] 1/3*A^2*g^2*x^3 + A^2*f*g*x^2 + 2*(x*log(b^2*e*x^2/(d^2*x^2 + 2*c*d*x + c^2
) + 2*a*b*e*x/(d^2*x^2 + 2*c*d*x + c^2) + a^2*e/(d^2*x^2 + 2*c*d*x + c^2))
+ 2*a*log(b*x + a)/b - 2*c*log(d*x + c)/d)*A*B*f^2 + 2*(x^2*log(b^2*e*x^2/(
d^2*x^2 + 2*c*d*x + c^2) + 2*a*b*e*x/(d^2*x^2 + 2*c*d*x + c^2) + a^2*e/(d^2
*x^2 + 2*c*d*x + c^2)) - 2*a^2*log(b*x + a)/b^2 + 2*c^2*log(d*x + c)/d^2 -
2*(b*c - a*d)*x/(b*d))*A*B*f*g + 2/3*(x^3*log(b^2*e*x^2/(d^2*x^2 + 2*c*d*x
+ c^2) + 2*a*b*e*x/(d^2*x^2 + 2*c*d*x + c^2) + a^2*e/(d^2*x^2 + 2*c*d*x + c
^2)) + 2*a^3*log(b*x + a)/b^3 - 2*c^3*log(d*x + c)/d^3 - ((b^2*c*d - a*b*d^
2)*x^2 - 2*(b^2*c^2 - a^2*d^2)*x)/(b^2*d^2))*A*B*g^2 + A^2*f^2*x + 4/3*(2*a
^2*c*d^2*g^2 - (6*c*d^2*f*g - c^2*d*g^2)*a*b - (3*c*d^2*f^2*log(e) + (g^2*l
og(e) + 3*g^2)*c^3 - 3*(f*g*log(e) + 2*f*g)*c^2*d)*b^2)*B^2*log(d*x + c)/(b
^2*d^3) + 8/3*(3*a*b^2*d^3*f^2 - 3*a^2*b*d^3*f*g + a^3*d^3*g^2 - (3*c*d^2*f
^2 - 3*c^2*d*f*g + c^3*g^2)*b^3)*(log(b*x + a)*log((b*d*x + a*d)/(b*c - a*d
) + 1) + dilog(-(b*d*x + a*d)/(b*c - a*d)))*B^2/(b^3*d^3) + 1/3*(B^2*b^3*d^
3*g^2*x^3*log(e)^2 + (2*a*b^2*d^3*g^2*log(e) + (3*d^3*f*g*log(e)^2 - 2*c*d^
2*g^2*log(e))*b^3)*B^2*x^2 - (4*(g^2*log(e) - g^2)*a^2*b*d^3 - 4*(3*d^3*f*g
*log(e) - 2*c*d^2*g^2)*a*b^2 - (3*d^3*f^2*log(e)^2 - 12*c*d^2*f*g*log(e) +
4*(g^2*log(e) + g^2)*c^2*d)*b^3)*B^2*x + 4*(B^2*b^3*d^3*g^2*x^3 + 3*B^2*b^3
*d^3*f*g*x^2 + 3*B^2*b^3*d^3*f^2*x + (3*a*b^2*d^3*f^2 - 3*a^2*b*d^3*f*g + a
^3*d^3*g^2)*B^2)*log(b*x + a)^2 + 4*(B^2*b^3*d^3*g^2*x^3 + 3*B^2*b^3*d^3*f*
g*x^2 + 3*B^2*b^3*d^3*f^2*x + (3*c*d^2*f^2 - 3*c^2*d*f*g + c^3*g^2)*B^2*b^3
)*log(d*x + c)^2 + 4*(B^2*b^3*d^3*g^2*x^3*log(e) + (a*b^2*d^3*g^2 + (3*d^3*
```

```
f*g*log(e) - c*d^2*g^2)*b^3)*B^2*x^2 + (6*a*b^2*d^3*f*g - 2*a^2*b*d^3*g^2 +
 (3*d^3*f^2*log(e) - 6*c*d^2*f*g + 2*c^2*d*g^2)*b^3)*B^2*x + ((g^2*log(e) -
 3*g^2)*a^3*d^3 + (c*d^2*g^2 - 3*(f*g*log(e) - 2*f*g)*d^3)*a^2*b + (3*d^3*f
^2*log(e) - 6*c*d^2*f*g + 2*c^2*d*g^2)*a*b^2)*B^2)*log(b*x + a) - 4*(B^2*b^
3*d^3*g^2*x^3*log(e) + (a*b^2*d^3*g^2 + (3*d^3*f*g*log(e) - c*d^2*g^2)*b^3)
*B^2*x^2 + (6*a*b^2*d^3*f*g - 2*a^2*b*d^3*g^2 + (3*d^3*f^2*log(e) - 6*c*d^2
*f*g + 2*c^2*d*g^2)*b^3)*B^2*x + 2*(B^2*b^3*d^3*g^2*x^3 + 3*B^2*b^3*d^3*f*g
*x^2 + 3*B^2*b^3*d^3*f^2*x + (3*a*b^2*d^3*f^2 - 3*a^2*b*d^3*f*g + a^3*d^3*g
^2)*B^2)*log(b*x + a))*log(d*x + c))/(b^3*d^3)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.00

$$\int (f+gx)^2\left(A+B\ln\left(\frac{e\,(a+b\,x)^2}{(c+d\,x)^2}\right)\right)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f + g*x)^2*(A + B*log((e*(a + b*x)^2)/(c + d*x)^2))^2,x)
[Out] int((f + g*x)^2*(A + B*log((e*(a + b*x)^2)/(c + d*x)^2))^2, x)
```

**sympy [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)**2*(A+B*ln(e*(b*x+a)**2/(d*x+c)**2))**2,x)
[Out] Timed out
```

# 3.274 $\int (f+gx)\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2\,dx$

**Optimal**. Leaf size=281

$$\frac{2B(bc-ad)(-adg-bcg+2bdf)\log\left(\frac{bc-ad}{b(c+dx)}\right)\left(B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+A\right)}{b^2d^2}-\frac{(bf-ag)^2\left(B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+A\right)^2}{2b^2g}-\frac{2Bg(a}{}$$

```
[Out] -2*B*(-a*d+b*c)*g*(b*x+a)*(A+B*ln(e*(b*x+a)^2/(d*x+c)^2))/b^2/d-1/2*(-a*g+b
*f)^2*(A+B*ln(e*(b*x+a)^2/(d*x+c)^2))^2/b^2/g+1/2*(g*x+f)^2*(A+B*ln(e*(b*x+
a)^2/(d*x+c)^2))^2/g+2*B*(-a*d+b*c)*(-a*d*g-b*c*g+2*b*d*f)*(A+B*ln(e*(b*x+a
)^2/(d*x+c)^2))*ln((-a*d+b*c)/b/(d*x+c))/b^2/d^2+4*B^2*(-a*d+b*c)^2*g*ln(d*
x+c)/b^2/d^2+4*B^2*(-a*d+b*c)*(-a*d*g-b*c*g+2*b*d*f)*polylog(2,d*(b*x+a)/b/
(d*x+c))/b^2/d^2
```

**Rubi [A]** time = 0.96, antiderivative size = 450, normalized size of antiderivative = 1.60, number of steps used = 25, number of rules used = 12, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.414, Rules used = {2525, 12, 2528, 2486, 31, 2524, 2418, 2390, 2301, 2394, 2393, 2391}

$$-\frac{4B^2(bf-ag)^2\text{PolyLog}\left(2,-\frac{d(a+bx)}{bc-ad}\right)}{b^2g}-\frac{4B^2(df-cg)^2\text{PolyLog}\left(2,\frac{b(c+dx)}{bc-ad}\right)}{d^2g}-\frac{2B(bf-ag)^2\log(a+bx)\left(B\log\left(\frac{e(a}{(c}\right.\right.}{b^2g}$$

Antiderivative was successfully verified.

```
[In] Int[(f + g*x)*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])^2,x]
```

```
[Out] (-2*A*B*(b*c - a*d)*g*x)/(b*d) + (2*B^2*(b*f - a*g)^2*Log[a + b*x]^2)/(b^2*
g) - (2*B^2*(b*c - a*d)*g*(a + b*x)*Log[(e*(a + b*x)^2)/(c + d*x)^2])/(b^2*
d) - (2*B*(b*f - a*g)^2*Log[a + b*x]*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2
]))/(b^2*g) + ((f + g*x)^2*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])^2)/(2*g
) + (4*B^2*(b*c - a*d)^2*g*Log[c + d*x])/(b^2*d^2) - (4*B^2*(d*f - c*g)^2*L
og[-((d*(a + b*x))/(b*c - a*d))]*Log[c + d*x])/(d^2*g) + (2*B*(d*f - c*g)^2
*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])*Log[c + d*x])/(d^2*g) + (2*B^2*(d
*f - c*g)^2*Log[c + d*x]^2)/(d^2*g) - (4*B^2*(b*f - a*g)^2*Log[a + b*x]*Log
[(b*(c + d*x))/(b*c - a*d)])/(b^2*g) - (4*B^2*(b*f - a*g)^2*PolyLog[2, -((d
*(a + b*x))/(b*c - a*d))])/(b^2*g) - (4*B^2*(d*f - c*g)^2*PolyLog[2, (b*(c
+ d*x))/(b*c - a*d)])/(d^2*g)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 2301

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

Rule 2390

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_))^(q_.), x_Symbol] :> Dist[1/e, Subst[Int[((f*x)/d)^q*(a + b*Log[c*x^
n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && E
qQ[e*f - d*g, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2393

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + (c*e*x)/g])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2394

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d + e*x
)^n]))/g, x] - Dist[(b*e*n)/g, Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2418

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*(RFx_), x_Sy
mbol] :> With[{u = ExpandIntegrand[(a + b*Log[c*(d + e*x)^n])^p, RFx, x]},
Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d, e, n}, x] && RationalFunctionQ[
RFx, x] && IntegerQ[p]
```

Rule 2486

```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))
^(r_.)]^(s_.), x_Symbol] :> Simp[((a + b*x)*Log[e*(f*(a + b*x)^p*(c + d*x)^
q)^r]^s)/b, x] + Dist[(q*r*s*(b*c - a*d))/b, Int[Log[e*(f*(a + b*x)^p*(c +
d*x)^q)^r]^(s - 1)/(c + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, p, q, r, s
}, x] && NeQ[b*c - a*d, 0] && EqQ[p + q, 0] && IGtQ[s, 0]
```

Rule 2524

```
Int[((a_.) + Log[(c_.)*(RFx_)^(p_.)]*(b_.))^(n_.)/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[(Log[d + e*x]*(a + b*Log[c*RFx^p])^n)/e, x] - Dist[(b*n*p)/e
, Int[(Log[d + e*x]*(a + b*Log[c*RFx^p])^(n - 1)*D[RFx, x])/RFx, x], x] /;
FreeQ[{a, b, c, d, e, p}, x] && RationalFunctionQ[RFx, x] && IGtQ[n, 0]
```

Rule 2525

```
Int[((a_.) + Log[(c_.)*(RFx_)^(p_.)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_))^(m_.
), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(a + b*Log[c*RFx^p])^n)/(e*(m + 1))
, x] - Dist[(b*n*p)/(e*(m + 1)), Int[SimplifyIntegrand[((d + e*x)^(m + 1)*(
a + b*Log[c*RFx^p])^(n - 1)*D[RFx, x])/RFx, x], x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && RationalFunctionQ[RFx, x] && IGtQ[n, 0] && (EqQ[n, 1] ||
IntegerQ[m]) && NeQ[m, -1]
```

Rule 2528

```
Int[((a_.) + Log[(c_.)*(RFx_)^(p_.)]*(b_.))^(n_.)*(RGx_), x_Symbol] :> With
[{u = ExpandIntegrand[(a + b*Log[c*RFx^p])^n, RGx, x]}, Int[u, x] /; SumQ[u
]] /; FreeQ[{a, b, c, p}, x] && RationalFunctionQ[RFx, x] && RationalFuncti
onQ[RGx, x] && IGtQ[n, 0]
```

Rubi steps

$$\int (f+gx)\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2 dx = \frac{(f+gx)^2\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}{2g} - \frac{B\int \frac{2(bc-ad)(f+gx)^2\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)\cdots}{(a+bx)(c+dx)}}{g}\cdots$$

$$= \frac{(f+gx)^2\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}{2g} - \frac{(2B(bc-ad))\int \frac{(f+gx)^2\left(A+B\log\left(\frac{e(a\cdots}{(c\cdots}\right.\right.}{(a+bx)(c+dx)}}{g}\cdots$$

$$= \frac{(f+gx)^2\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}{2g} - \frac{(2B(bc-ad))\int\left(\frac{g^2\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right.\right.\cdots}{bd}\right.\cdots}{\cdots}$$

$$= \frac{(f+gx)^2\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}{2g} - \frac{(2B(bc-ad)g)\int\left(A+B\log\left(\frac{e(a\cdots}{(c+\cdots}\right.\right.}{bd}\cdots$$

$$= -\frac{2AB(bc-ad)gx}{bd} - \frac{2B(bf-ag)^2\log(a+bx)\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}{b^2g}\cdots$$

$$= -\frac{2AB(bc-ad)gx}{bd} - \frac{2B^2(bc-ad)g(a+bx)\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}{b^2d} - \frac{2B(bf-\cdots}{\cdots}$$

$$= -\frac{2AB(bc-ad)gx}{bd} - \frac{2B^2(bc-ad)g(a+bx)\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}{b^2d} - \frac{2B(bf-\cdots}{\cdots}$$

$$= -\frac{2AB(bc-ad)gx}{bd} - \frac{2B^2(bc-ad)g(a+bx)\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}{b^2d} - \frac{2B(bf-\cdots}{\cdots}$$

$$= -\frac{2AB(bc-ad)gx}{bd} - \frac{2B^2(bc-ad)g(a+bx)\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}{b^2d} - \frac{2B(bf-\cdots}{\cdots}$$

$$= -\frac{2AB(bc-ad)gx}{bd} + \frac{2B^2(bf-ag)^2\log^2(a+bx)}{b^2g} - \frac{2B^2(bc-ad)g(a+\cdots}{b^2\cdots}$$

$$= -\frac{2AB(bc-ad)gx}{bd} + \frac{2B^2(bf-ag)^2\log^2(a+bx)}{b^2g} - \frac{2B^2(bc-ad)g(a+\cdots}{b^2\cdots}$$

**Mathematica [A]** time = 0.31, size = 351, normalized size = 1.25

$$(f+gx)^2\left(B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+A\right)^2-\frac{4B\left(-b^2(df-cg)^2\log(c+dx)\left(B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+A\right)+d^2(bf-ag)^2\log(a+bx)\left(B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+A\right)+Abdg^2 \cdots}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Integrate[(f + g*x)*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])^2,x]
```

```
[Out] ((f + g*x)^2*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])^2 - (4*B*(A*b*d*(b*c
- a*d)*g^2*x + B*d*(b*c - a*d)*g^2*(a + b*x)*Log[(e*(a + b*x)^2)/(c + d*x)^
2] + d^2*(b*f - a*g)^2*Log[a + b*x]*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2]
) - 2*B*(b*c - a*d)^2*g^2*Log[c + d*x] - b^2*(d*f - c*g)^2*(A + B*Log[(e*(a
 + b*x)^2)/(c + d*x)^2])*Log[c + d*x] - B*d^2*(b*f - a*g)^2*(Log[a + b*x]*(
Log[a + b*x] - 2*Log[(b*(c + d*x))/(b*c - a*d)]) - 2*PolyLog[2, (d*(a + b*x
))/(-(b*c) + a*d)]) + b^2*B*(d*f - c*g)^2*((2*Log[(d*(a + b*x))/(-(b*c) + a
*d)] - Log[c + d*x])*Log[c + d*x] + 2*PolyLog[2, (b*(c + d*x))/(b*c - a*d)]
)))/(b^2*d^2))/(2*g)
```

**fricas [F]** time = 0.74, size = 0, normalized size = 0.00

$$\text{integral}\left(A^2gx+A^2f+\left(B^2gx+B^2f\right)\log\left(\frac{b^2ex^2+2\,abex+a^2e}{d^2x^2+2\,cdx+c^2}\right)^2+2\left(ABgx+ABf\right)\log\left(\frac{b^2ex^2+2\,abex+a \cdots}{d^2x^2+2\,cdx+c^2}\right) \cdots\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)*(A+B*log(e*(b*x+a)^2/(d*x+c)^2))^2,x, algorithm="fricas")
```

```
[Out] integral(A^2*g*x + A^2*f + (B^2*g*x + B^2*f)*log((b^2*e*x^2 + 2*a*b*e*x + a
^2*e)/(d^2*x^2 + 2*c*d*x + c^2))^2 + 2*(A*B*g*x + A*B*f)*log((b^2*e*x^2 + 2
*a*b*e*x + a^2*e)/(d^2*x^2 + 2*c*d*x + c^2)), x)
```

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int\left(gx+f\right)\left(B\log\left(\frac{(bx+a)^2e}{(dx+c)^2}\right)+A\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)*(A+B*log(e*(b*x+a)^2/(d*x+c)^2))^2,x, algorithm="giac")
```

```
[Out] integrate((g*x + f)*(B*log((b*x + a)^2*e/(d*x + c)^2) + A)^2, x)
```

**maple [F]** time = 1.01, size = 0, normalized size = 0.00

$$\int \left(gx + f\right)\left(B\ln\left(\frac{(bx+a)^2 e}{(dx+c)^2}\right) + A\right)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x+f)*(B*ln((b*x+a)^2/(d*x+c)^2*e)+A)^2,x)
```

```
[Out] int((g*x+f)*(B*ln((b*x+a)^2/(d*x+c)^2*e)+A)^2,x)
```

**maxima [B]** time = 1.55, size = 786, normalized size = 2.80

$$\frac{1}{2}A^2gx^2+2\left(x\log\left(\frac{b^2ex^2}{d^2x^2+2\,cdx+c^2}+\frac{2\,abex}{d^2x^2+2\,cdx+c^2}+\frac{a^2e}{d^2x^2+2\,cdx+c^2}\right)+\frac{2\,a\log\left(bx+a\right)}{b}-\frac{2\,c\log\left(dx+\right.}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)*(A+B*log(e*(b*x+a)^2/(d*x+c)^2))^2,x, algorithm="maxima")
```

```
[Out] 1/2*A^2*g*x^2 + 2*(x*log(b^2*e*x^2/(d^2*x^2 + 2*c*d*x + c^2) + 2*a*b*e*x/(d
^2*x^2 + 2*c*d*x + c^2) + a^2*e/(d^2*x^2 + 2*c*d*x + c^2)) + 2*a*log(b*x +
a)/b - 2*c*log(d*x + c)/d)*A*B*f + (x^2*log(b^2*e*x^2/(d^2*x^2 + 2*c*d*x +
c^2) + 2*a*b*e*x/(d^2*x^2 + 2*c*d*x + c^2) + a^2*e/(d^2*x^2 + 2*c*d*x + c^2
)) - 2*a^2*log(b*x + a)/b^2 + 2*c^2*log(d*x + c)/d^2 - 2*(b*c - a*d)*x/(b*d
))*A*B*g + A^2*f*x - 2*(2*a*c*d*g + (2*c*d*f*log(e) - (g*log(e) + 2*g)*c^2)
*b)*B^2*log(d*x + c)/(b*d^2) + 4*(2*a*b*d^2*f - a^2*d^2*g - (2*c*d*f - c^2*
g)*b^2)*(log(b*x + a)*log((b*d*x + a*d)/(b*c - a*d) + 1) + dilog(-(b*d*x +
a*d)/(b*c - a*d)))*B^2/(b^2*d^2) + 1/2*(B^2*b^2*d^2*g*x^2*log(e)^2 + 2*(2*a
*b*d^2*g*log(e) + (d^2*f*log(e)^2 - 2*c*d*g*log(e))*b^2)*B^2*x + 4*(B^2*b^2
*d^2*g*x^2 + 2*B^2*b^2*d^2*f*x + (2*a*b*d^2*f - a^2*d^2*g)*B^2)*log(b*x + a
)^2 + 4*(B^2*b^2*d^2*g*x^2 + 2*B^2*b^2*d^2*f*x + (2*c*d*f - c^2*g)*B^2*b^2)
*log(d*x + c)^2 + 4*(B^2*b^2*d^2*g*x^2*log(e) + 2*(a*b*d^2*g + (d^2*f*log(e
) - c*d*g)*b^2)*B^2*x - ((g*log(e) - 2*g)*a^2*d^2 - 2*(d^2*f*log(e) - c*d*g
)*a*b)*B^2)*log(b*x + a) - 4*(B^2*b^2*d^2*g*x^2*log(e) + 2*(a*b*d^2*g + (d^
2*f*log(e) - c*d*g)*b^2)*B^2*x + 2*(B^2*b^2*d^2*g*x^2 + 2*B^2*b^2*d^2*f*x +
 (2*a*b*d^2*f - a^2*d^2*g)*B^2)*log(b*x + a))*log(d*x + c))/(b^2*d^2)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.00

$$\int \left(f + g\,x\right)\,{\left(A + B\,\ln\left(\frac{e\,{\left(a + b\,x\right)}^2}{{\left(c + d\,x\right)}^2}\right)\right)}^2 \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f + g*x)*(A + B*log((e*(a + b*x)^2)/(c + d*x)^2))^2,x)
[Out] int((f + g*x)*(A + B*log((e*(a + b*x)^2)/(c + d*x)^2))^2, x)
```

**sympy** **[F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)*(A+B*ln(e*(b*x+a)**2/(d*x+c)**2))**2,x)
[Out] Timed out
```

# 3.275 $\int \left(A+B \log \left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2 dx$

**Optimal**. Leaf size=129

$$\frac{4B(bc-ad)\log\left(\frac{bc-ad}{b(c+dx)}\right)\left(B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+A\right)}{bd}+\frac{(a+bx)\left(B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+A\right)^2}{b}+\frac{8B^2(bc-ad)\text{Li}_2\left(\frac{d(a+bx)}{b(c+dx)}\right)}{bd}$$

```
[Out] (b*x+a)*(A+B*ln(e*(b*x+a)^2/(d*x+c)^2))^2/b+4*B*(-a*d+b*c)*(A+B*ln(e*(b*x+a
)^2/(d*x+c)^2))*ln((-a*d+b*c)/b/(d*x+c))/b/d+8*B^2*(-a*d+b*c)*polylog(2,d*(
b*x+a)/b/(d*x+c))/b/d
```

**Rubi [A]** time = 0.77, antiderivative size = 252, normalized size of antiderivative = 1.95, number of steps used = 22, number of rules used = 10, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.435, Rules used = {2523, 12, 2528, 2524, 2418, 2390, 2301, 2394, 2393, 2391}

$$\frac{8aB^2\text{PolyLog}\left(2,-\frac{d(a+bx)}{bc-ad}\right)}{b}+\frac{8B^2c\text{PolyLog}\left(2,\frac{b(c+dx)}{bc-ad}\right)}{d}+\frac{4aB\log(a+bx)\left(B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+A\right)}{b}-\frac{4Bc\log(c+dx)}{}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])^2,x]
```

```
[Out] (-4*a*B^2*Log[a + b*x]^2)/b + (4*a*B*Log[a + b*x]*(A + B*Log[(e*(a + b*x)^2
)/(c + d*x)^2]))/b + x*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])^2 + (8*B^2*
c*Log[-((d*(a + b*x))/(b*c - a*d))]*Log[c + d*x])/d - (4*B*c*(A + B*Log[(e*
(a + b*x)^2)/(c + d*x)^2])*Log[c + d*x])/d - (4*B^2*c*Log[c + d*x]^2)/d + (
8*a*B^2*Log[a + b*x]*Log[(b*(c + d*x))/(b*c - a*d)])/b + (8*a*B^2*PolyLog[2
, -((d*(a + b*x))/(b*c - a*d))])/b + (8*B^2*c*PolyLog[2, (b*(c + d*x))/(b*c
 - a*d)])/d
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2301

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

Rule 2390

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_))^(q_.), x_Symbol] :> Dist[1/e, Subst[Int[((f*x)/d)^q*(a + b*Log[c*x^
n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && E
qQ[e*f - d*g, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2393

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + (c*e*x)/g])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2394

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d + e*x
)^n]))/g, x] - Dist[(b*e*n)/g, Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2418

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*(RFx_), x_Sy
mbol] :> With[{u = ExpandIntegrand[(a + b*Log[c*(d + e*x)^n])^p, RFx, x]},
Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d, e, n}, x] && RationalFunctionQ[
RFx, x] && IntegerQ[p]
```

Rule 2523

```
Int[((a_.) + Log[(c_.)*(RFx_)^(p_.)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a +
 b*Log[c*RFx^p])^n, x] - Dist[b*n*p, Int[SimplifyIntegrand[(x*(a + b*Log[c*
RFx^p])^(n - 1)*D[RFx, x])/RFx, x], x], x] /; FreeQ[{a, b, c, p}, x] && Rat
ionalFunctionQ[RFx, x] && IGtQ[n, 0]
```

Rule 2524

```
Int[((a_.) + Log[(c_.)*(RFx_)^(p_.)]*(b_.))^(n_.)/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[(Log[d + e*x]*(a + b*Log[c*RFx^p])^n)/e, x] - Dist[(b*n*p)/e
, Int[(Log[d + e*x]*(a + b*Log[c*RFx^p])^(n - 1)*D[RFx, x])/RFx, x], x] /;
FreeQ[{a, b, c, d, e, p}, x] && RationalFunctionQ[RFx, x] && IGtQ[n, 0]
```

Rule 2528

```
Int[((a_.) + Log[(c_.)*(RFx_)^(p_.)]*(b_.))^(n_.)*(RGx_), x_Symbol] :> With
[{u = ExpandIntegrand[(a + b*Log[c*RFx^p])^n, RGx, x]}, Int[u, x] /; SumQ[u
]] /; FreeQ[{a, b, c, p}, x] && RationalFunctionQ[RFx, x] && RationalFuncti
onQ[RGx, x] && IGtQ[n, 0]
```

Rubi steps

$$\int \left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2 dx = x\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2 - (2B)\int \frac{2(bc-ad)x\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}{(a+bx)(c+dx)}dx$$

$$= x\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2 - (4B(bc-ad))\int \frac{x\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}{(a+bx)(c+dx)}dx$$

$$= x\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2 - (4B(bc-ad))\int \left(-\frac{a\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}{(bc-ad)(a+bx)} + \right.$$

$$= x\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2 + (4aB)\int \frac{A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}{a+bx}dx - (4Bc)\int$$

$$= \frac{4aB\log(a+bx)\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}{b} + x\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2 - \frac{4Bc}{}$$

$$= \frac{4aB\log(a+bx)\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}{b} + x\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2 - \frac{4Bc}{}$$

$$= \frac{4aB\log(a+bx)\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}{b} + x\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2 - \frac{4Bc}{}$$

$$= \frac{4aB\log(a+bx)\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}{b} + x\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2 - \frac{4Bc}{}$$

$$= \frac{4aB\log(a+bx)\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}{b} + x\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2 + \frac{8B^2}{}$$

$$= -\frac{4aB^2\log^2(a+bx)}{b} + \frac{4aB\log(a+bx)\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}{b} + x\left(A+B\log\right.$$

$$= -\frac{4aB^2\log^2(a+bx)}{b} + \frac{4aB\log(a+bx)\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}{b} + x\left(A+B\log\right.$$

**Mathematica [A]** time = 0.17, size = 220, normalized size = 1.71

$$\frac{4B\left(ad\log(a+bx)\left(B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+A\right) - bc\log(c+dx)\left(B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+A\right) - aBd\left(\log(a+bx)\left(\log(a+bx\right.\right.\right.}{bd}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])^2,x]
```

```
[Out] x*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])^2 + (4*B*(a*d*Log[a + b*x]*(A +
B*Log[(e*(a + b*x)^2)/(c + d*x)^2]) - b*c*(A + B*Log[(e*(a + b*x)^2)/(c + d
*x)^2])*Log[c + d*x] - a*B*d*(Log[a + b*x]*(Log[a + b*x] - 2*Log[(b*(c + d*
x))/(b*c - a*d)]) - 2*PolyLog[2, (d*(a + b*x))/(-(b*c) + a*d)]) + b*B*c*((2
*Log[(d*(a + b*x))/(-(b*c) + a*d)] - Log[c + d*x])*Log[c + d*x] + 2*PolyLog
[2, (b*(c + d*x))/(b*c - a*d)])))/(b*d)
```

**fricas [F]** time = 0.76, size = 0, normalized size = 0.00

$$\text{integral}\left(B^2\log\left(\frac{b^2ex^2+2\,abex+a^2e}{d^2x^2+2\,cdx+c^2}\right)^2+2\,AB\log\left(\frac{b^2ex^2+2\,abex+a^2e}{d^2x^2+2\,cdx+c^2}\right)+A^2,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*(b*x+a)^2/(d*x+c)^2))^2,x, algorithm="fricas")
```

```
[Out] integral(B^2*log((b^2*e*x^2 + 2*a*b*e*x + a^2*e)/(d^2*x^2 + 2*c*d*x + c^2))
^2 + 2*A*B*log((b^2*e*x^2 + 2*a*b*e*x + a^2*e)/(d^2*x^2 + 2*c*d*x + c^2)) +
 A^2, x)
```

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int\left(B\log\left(\frac{(bx+a)^2e}{(dx+c)^2}\right)+A\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*(b*x+a)^2/(d*x+c)^2))^2,x, algorithm="giac")
```

```
[Out] integrate((B*log((b*x + a)^2*e/(d*x + c)^2) + A)^2, x)
```

**maple [F]** time = 0.88, size = 0, normalized size = 0.00

$$\int\left(B\ln\left(\frac{(bx+a)^2\,e}{(dx+c)^2}\right)+A\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*ln((b*x+a)^2/(d*x+c)^2*e)+A)^2,x)
```

```
[Out] int((B*ln((b*x+a)^2/(d*x+c)^2*e)+A)^2,x)
```

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$2\left(x\log\left(\frac{(bx+a)^2e}{(dx+c)^2}\right)+\frac{2\left(\frac{ae\log(bx+a)}{b}-\frac{ce\log(dx+c)}{d}\right)}{e}\right)AB+A^2x+B^2\left(\frac{4\left(bdx\log\left(bx+a\right)^2+\left(bdx+bc\right)\log\left(dx+c\right)^2\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*(b*x+a)^2/(d*x+c)^2))^2,x, algorithm="maxima")
```

```
[Out] 2*(x*log((b*x + a)^2*e/(d*x + c)^2) + 2*(a*e*log(b*x + a)/b - c*e*log(d*x +
 c)/d)/e)*A*B + A^2*x + B^2*(4*(b*d*x*log(b*x + a)^2 + (b*d*x + b*c)*log(d*
x + c)^2 - (b*d*x*log(e) + 2*(b*d*x + a*d)*log(b*x + a))*log(d*x + c))/(b*d
) + integrate(((log(e)^2 + 4*log(e))*b^2*d*x^2 + a*b*c*log(e)^2 + (b^2*c*lo
g(e)^2 + (log(e)^2 + 4*log(e))*a*b*d)*x + 4*(b^2*d*x^2*log(e) + a*b*c*log(e
) + 2*a^2*d + (a*b*d*(log(e) + 4) + b^2*c*(log(e) - 2))*x)*log(b*x + a))/(b
^2*d*x^2 + a*b*c + (b^2*c + a*b*d)*x), x))
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.01

$$\int {\left(A+B\,\ln\left(\frac{e\,{\left(a+b\,x\right)}^2}{{\left(c+d\,x\right)}^2}\right)\right)}^2 \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*log((e*(a + b*x)^2)/(c + d*x)^2))^2,x)
```

```
[Out] int((A + B*log((e*(a + b*x)^2)/(c + d*x)^2))^2, x)
```

**sympy [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*ln(e*(b*x+a)**2/(d*x+c)**2))**2,x)
```

```
[Out] Timed out
```

## 3.276 $\int \frac{\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}{f+gx}\,dx$

**Optimal**. Leaf size=285

$$\frac{4B\text{Li}_2\left(\frac{(df-cg)(a+bx)}{(bf-ag)(c+dx)}\right)\left(B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+A\right)}{g}+\frac{\log\left(1-\frac{(a+bx)(df-cg)}{(c+dx)(bf-ag)}\right)\left(B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+A\right)^2}{g}-\frac{4B\text{Li}_2\left(\frac{d(a+bx)}{b(c+dx)}\right)\left(B\log\right.}{g}$$

```
[Out] -(A+B*ln(e*(b*x+a)^2/(d*x+c)^2))^2*ln((-a*d+b*c)/b/(d*x+c))/g+(A+B*ln(e*(b*
x+a)^2/(d*x+c)^2))^2*ln(1-(-c*g+d*f)*(b*x+a)/(-a*g+b*f)/(d*x+c))/g-4*B*(A+B
*ln(e*(b*x+a)^2/(d*x+c)^2))*polylog(2,d*(b*x+a)/b/(d*x+c))/g+4*B*(A+B*ln(e*
(b*x+a)^2/(d*x+c)^2))*polylog(2,(-c*g+d*f)*(b*x+a)/(-a*g+b*f)/(d*x+c))/g+8*
B^2*polylog(3,d*(b*x+a)/b/(d*x+c))/g-8*B^2*polylog(3,(-c*g+d*f)*(b*x+a)/(-a
*g+b*f)/(d*x+c))/g
```

**Rubi [B]** time = 5.85, antiderivative size = 2126, normalized size of antiderivative = 7.46, number of steps used = 44, number of rules used = 21, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.677, Rules used = {2524, 12, 2528, 2418, 2390, 2301, 2394, 2393, 2391, 6688, 6742, 2500, 2433, 2375, 2317, 2374, 6589, 2440, 2437, 2435, 2315}

result too large to display

Antiderivative was successfully verified.

```
[In] Int[(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])^2/(f + g*x),x]
```

```
[Out] (-4*A*B*Log[-((g*(a + b*x))/(b*f - a*g))]*Log[f + g*x])/g - (B^2*Log[(a + b
*x)^2]^2*Log[f + g*x])/g - (B^2*Log[(c + d*x)^(-2)]^2*Log[f + g*x])/g + (4*
B^2*Log[-((g*(a + b*x))/(b*f - a*g))]*(Log[(a + b*x)^2] + Log[(c + d*x)^(-2
)] - Log[(e*(a + b*x)^2)/(c + d*x)^2])*Log[f + g*x])/g + ((A + B*Log[(e*(a
+ b*x)^2)/(c + d*x)^2])^2*Log[f + g*x])/g + (8*B^2*Log[-((d*(a + b*x))/(b*c
 - a*d))]*Log[c + d*x]*Log[f + g*x])/g - (4*B^2*Log[-((g*(a + b*x))/(b*f -
a*g))]*(Log[(c + d*x)^(-2)] + 2*Log[c + d*x])*Log[f + g*x])/g + (8*B^2*Log[
a + b*x]*Log[(b*(c + d*x))/(b*c - a*d)]*Log[f + g*x])/g + (4*A*B*Log[-((g*(
c + d*x))/(d*f - c*g))]*Log[f + g*x])/g - (4*B^2*(2*Log[a + b*x] - Log[(a +
 b*x)^2])*Log[-((g*(c + d*x))/(d*f - c*g))]*Log[f + g*x])/g - (4*B^2*(Log[(
a + b*x)^2] + Log[(c + d*x)^(-2)] - Log[(e*(a + b*x)^2)/(c + d*x)^2])*Log[-
((g*(c + d*x))/(d*f - c*g))]*Log[f + g*x])/g + (B^2*Log[(a + b*x)^2]^2*Log[
(b*(f + g*x))/(b*f - a*g)])/g + (B^2*Log[(c + d*x)^(-2)]^2*Log[(d*(f + g*x)
)/(d*f - c*g)])/g + (4*B^2*(Log[(b*(c + d*x))/(b*c - a*d)] + Log[(b*f - a*g
)/(b*(f + g*x))] - Log[((b*f - a*g)*(c + d*x))/((b*c - a*d)*(f + g*x))])*Lo
g[-(((b*c - a*d)*(f + g*x))/((d*f - c*g)*(a + b*x)))]^2)/g - (4*B^2*(Log[(b
```

```
*(c + d*x))/(b*c - a*d)] - Log[-((g*(c + d*x))/(d*f - c*g))])*(Log[a + b*x]
 + Log[-(((b*c - a*d)*(f + g*x))/((d*f - c*g)*(a + b*x)))])^2)/g + (4*B^2*(
Log[-((d*(a + b*x))/(b*c - a*d))] + Log[(d*f - c*g)/(d*(f + g*x))] - Log[-(
((d*f - c*g)*(a + b*x))/((b*c - a*d)*(f + g*x)))])*Log[((b*c - a*d)*(f + g*
x))/((b*f - a*g)*(c + d*x))]^2)/g - (4*B^2*(Log[-((d*(a + b*x))/(b*c - a*d)
)] - Log[-((g*(a + b*x))/(b*f - a*g))])*(Log[c + d*x] + Log[((b*c - a*d)*(f
 + g*x))/((b*f - a*g)*(c + d*x))])^2)/g + (8*B^2*(Log[f + g*x] - Log[-(((b*
c - a*d)*(f + g*x))/((d*f - c*g)*(a + b*x)))])*PolyLog[2, -((d*(a + b*x))/(
b*c - a*d))])/g + (4*B^2*Log[(a + b*x)^2]*PolyLog[2, -((g*(a + b*x))/(b*f -
 a*g))])/g + (8*B^2*(Log[f + g*x] - Log[((b*c - a*d)*(f + g*x))/((b*f - a*g
)*(c + d*x))])*PolyLog[2, (b*(c + d*x))/(b*c - a*d)])/g - (4*B^2*Log[(c + d
*x)^(-2)]*PolyLog[2, -((g*(c + d*x))/(d*f - c*g))])/g - (8*B^2*Log[-(((b*c
- a*d)*(f + g*x))/((d*f - c*g)*(a + b*x)))]*PolyLog[2, (g*(a + b*x))/(b*(f
+ g*x))])/g + (8*B^2*Log[-(((b*c - a*d)*(f + g*x))/((d*f - c*g)*(a + b*x)))
]*PolyLog[2, -(((d*f - c*g)*(a + b*x))/((b*c - a*d)*(f + g*x)))])/g - (8*B^
2*Log[((b*c - a*d)*(f + g*x))/((b*f - a*g)*(c + d*x))]*PolyLog[2, (g*(c + d
*x))/(d*(f + g*x))])/g + (8*B^2*Log[((b*c - a*d)*(f + g*x))/((b*f - a*g)*(c
 + d*x))]*PolyLog[2, ((b*f - a*g)*(c + d*x))/((b*c - a*d)*(f + g*x))])/g -
(4*A*B*PolyLog[2, (b*(f + g*x))/(b*f - a*g)])/g + (4*B^2*(Log[(a + b*x)^2]
+ Log[(c + d*x)^(-2)] - Log[(e*(a + b*x)^2)/(c + d*x)^2])*PolyLog[2, (b*(f
+ g*x))/(b*f - a*g)])/g - (4*B^2*(Log[(c + d*x)^(-2)] + 2*Log[c + d*x])*Pol
yLog[2, (b*(f + g*x))/(b*f - a*g)])/g + (8*B^2*(Log[c + d*x] + Log[((b*c -
a*d)*(f + g*x))/((b*f - a*g)*(c + d*x))])*PolyLog[2, (b*(f + g*x))/(b*f - a
*g)])/g + (4*A*B*PolyLog[2, (d*(f + g*x))/(d*f - c*g)])/g - (4*B^2*(2*Log[a
 + b*x] - Log[(a + b*x)^2])*PolyLog[2, (d*(f + g*x))/(d*f - c*g)])/g - (4*B
^2*(Log[(a + b*x)^2] + Log[(c + d*x)^(-2)] - Log[(e*(a + b*x)^2)/(c + d*x)^
2])*PolyLog[2, (d*(f + g*x))/(d*f - c*g)])/g + (8*B^2*(Log[a + b*x] + Log[-
(((b*c - a*d)*(f + g*x))/((d*f - c*g)*(a + b*x)))])*PolyLog[2, (d*(f + g*x)
)/(d*f - c*g)])/g - (8*B^2*PolyLog[3, -((d*(a + b*x))/(b*c - a*d))])/g - (8
*B^2*PolyLog[3, -((g*(a + b*x))/(b*f - a*g))])/g - (8*B^2*PolyLog[3, (b*(c
+ d*x))/(b*c - a*d)])/g - (8*B^2*PolyLog[3, -((g*(c + d*x))/(d*f - c*g))])/
g - (8*B^2*PolyLog[3, (g*(a + b*x))/(b*(f + g*x))])/g + (8*B^2*PolyLog[3, -
(((d*f - c*g)*(a + b*x))/((b*c - a*d)*(f + g*x)))])/g - (8*B^2*PolyLog[3, (
g*(c + d*x))/(d*(f + g*x))])/g + (8*B^2*PolyLog[3, ((b*f - a*g)*(c + d*x))/
((b*c - a*d)*(f + g*x))])/g - (8*B^2*PolyLog[3, (b*(f + g*x))/(b*f - a*g)])
/g - (8*B^2*PolyLog[3, (d*(f + g*x))/(d*f - c*g)])/g
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2301

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2317

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symb
ol] :> Simp[(Log[1 + (e*x)/d]*(a + b*Log[c*x^n])^p)/e, x] - Dist[(b*n*p)/e,
 Int[(Log[1 + (e*x)/d]*(a + b*Log[c*x^n])^(p - 1))/x, x], x] /; FreeQ[{a, b
, c, d, e, n}, x] && IGtQ[p, 0]
```

Rule 2374

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> -Simp[(PolyLog[2, -(d*f*x^m)]*(a + b*Log[c*x
^n])^p)/m, x] + Dist[(b*n*p)/m, Int[(PolyLog[2, -(d*f*x^m)]*(a + b*Log[c*x^
n])^(p - 1))/x, x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p, 0]
&& EqQ[d*e, 1]
```

Rule 2375

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))^(r_.)]*((a_.) + Log[(c_.)*(x_)^(n_
.)]*(b_.))^(p_.))/(x_), x_Symbol] :> Simp[(Log[d*(e + f*x^m)^r]*(a + b*Log[
c*x^n])^(p + 1))/(b*n*(p + 1)), x] - Dist[(f*m*r)/(b*n*(p + 1)), Int[(x^(m
- 1)*(a + b*Log[c*x^n])^(p + 1))/(e + f*x^m), x], x] /; FreeQ[{a, b, c, d,
e, f, r, m, n}, x] && IGtQ[p, 0] && NeQ[d*e, 1]
```

Rule 2390

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_))^(q_.), x_Symbol] :> Dist[1/e, Subst[Int[((f*x)/d)^q*(a + b*Log[c*x^
n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && E
qQ[e*f - d*g, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2393

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + (c*e*x)/g])/x, x], x, f + g*x
```

], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*(e*f - d*g), 0]

Rule 2394

Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_)), x_Symbol] :> Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d + e*x)^n]))/g, x] - Dist[(b*e*n)/g, Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]

Rule 2418

Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*(RFx_), x_Symbol] :> With[{u = ExpandIntegrand[(a + b*Log[c*(d + e*x)^n])^p, RFx, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d, e, n}, x] && RationalFunctionQ[RFx, x] && IntegerQ[p]

Rule 2433

Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + Log[(h_.)*((i_.) + (j_.)*(x_))^(m_.)]*(g_.))*((k_.) + (l_.)*(x_))^(r_.), x_Symbol] :> Dist[1/e, Subst[Int[((k*x)/d)^r*(a + b*Log[c*x^n])^p*(f + g*Log[h*((e*i - d*j)/e + (j*x)/e)^m]), x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, j, k, l, n, p, r}, x] && EqQ[e*k - d*l, 0]

Rule 2435

Int[(Log[(a_) + (b_.)*(x_)]*Log[(c_) + (d_.)*(x_)])/(x_), x_Symbol] :> Simp[Log[-((b*x)/a)]*Log[a + b*x]*Log[c + d*x], x] + (Simp[(1*(Log[-((b*x)/a)] - Log[-(((b*c - a*d)*x)/(a*(c + d*x)))] + Log[(b*c - a*d)/(b*(c + d*x))])*Log[(a*(c + d*x))/(c*(a + b*x))]^2)/2, x] - Simp[(1*(Log[-((b*x)/a)] - Log[-((d*x)/c)])*(Log[a + b*x] + Log[(a*(c + d*x))/(c*(a + b*x))])^2)/2, x] + Simp[(Log[c + d*x] - Log[(a*(c + d*x))/(c*(a + b*x))])*PolyLog[2, 1 + (b*x)/a], x] + Simp[(Log[a + b*x] + Log[(a*(c + d*x))/(c*(a + b*x))])*PolyLog[2, 1 + (d*x)/c], x] + Simp[Log[(a*(c + d*x))/(c*(a + b*x))]*PolyLog[2, (c*(a + b*x))/(a*(c + d*x))], x] - Simp[Log[(a*(c + d*x))/(c*(a + b*x))]*PolyLog[2, (d*(a + b*x))/(b*(c + d*x))], x] - Simp[PolyLog[3, 1 + (b*x)/a], x] - Simp[PolyLog[3, 1 + (d*x)/c], x] + Simp[PolyLog[3, (c*(a + b*x))/(a*(c + d*x))], x] - Simp[PolyLog[3, (d*(a + b*x))/(b*(c + d*x))], x]) /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]

Rule 2437

Int[(Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*Log[(h_.)*((i_.) + (j_.)*(x_))^(m_.)])/(x_), x_Symbol] :> Dist[m, Int[(Log[i + j*x]*Log[c*(d + e*x)^n])/x, x

```
], x] - Dist[m*Log[i + j*x] - Log[h*(i + j*x)^m], Int[Log[c*(d + e*x)^n]/x,
 x], x] /; FreeQ[{c, d, e, h, i, j, m, n}, x] && NeQ[e*i - d*j, 0] && NeQ[i
 + j*x, h*(i + j*x)^m]
```

Rule 2440

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + Log[(h_.)
*((i_.) + (j_.)*(x_))^(m_.)]*(g_.))*((k_) + (l_.)*(x_))^(r_.), x_Symbol] :>
 Dist[1/l, Subst[Int[x^r*(a + b*Log[c*(-((e*k - d*l)/l) + (e*x)/l)^n])*(f +
 g*Log[h*(-((j*k - i*l)/l) + (j*x)/l)^m]), x], x, k + l*x], x] /; FreeQ[{a,
 b, c, d, e, f, g, h, i, j, k, l, m, n}, x] && IntegerQ[r]
```

Rule 2500

```
Int[(Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.)
)^(r_.)]*((s_.) + Log[(i_.)*((g_.) + (h_.)*(x_))^(n_.)]*(t_.)))/((j_.) + (k
_.)*(x_)), x_Symbol] :> Dist[Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r] - Log[(a
+ b*x)^(p*r)] - Log[(c + d*x)^(q*r)], Int[(s + t*Log[i*(g + h*x)^n])/(j + k
*x), x], x] + (Int[(Log[(a + b*x)^(p*r)]*(s + t*Log[i*(g + h*x)^n]))/(j + k
*x), x] + Int[(Log[(c + d*x)^(q*r)]*(s + t*Log[i*(g + h*x)^n]))/(j + k*x),
x]) /; FreeQ[{a, b, c, d, e, f, g, h, i, j, k, s, t, n, p, q, r}, x] && NeQ
[b*c - a*d, 0]
```

Rule 2524

```
Int[((a_.) + Log[(c_.)*(RFx_)^(p_.)]*(b_.))^(n_.)/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[(Log[d + e*x]*(a + b*Log[c*RFx^p])^n)/e, x] - Dist[(b*n*p)/e
, Int[(Log[d + e*x]*(a + b*Log[c*RFx^p])^(n - 1)*D[RFx, x])/RFx, x], x] /;
FreeQ[{a, b, c, d, e, p}, x] && RationalFunctionQ[RFx, x] && IGtQ[n, 0]
```

Rule 2528

```
Int[((a_.) + Log[(c_.)*(RFx_)^(p_.)]*(b_.))^(n_.)*(RGx_), x_Symbol] :> With
[{u = ExpandIntegrand[(a + b*Log[c*RFx^p])^n, RGx, x]}, Int[u, x] /; SumQ[u
]] /; FreeQ[{a, b, c, p}, x] && RationalFunctionQ[RFx, x] && RationalFuncti
onQ[RGx, x] && IGtQ[n, 0]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6688

```
Int[u_, x_Symbol] :> With[{v = SimplifyIntegrand[u, x]}, Int[v, x] /; Simpl
erIntegrandQ[v, u, x]]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\int \frac{\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}{f+gx}\,dx = \frac{\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2\log(f+gx)}{g} - \frac{(2B)\int \frac{(c+dx)^2\left(-\frac{2de(a+bx)^2}{(c+dx)^3}+\frac{2be(a+bx)}{(c+dx)^2}\right)\left(A+B\log\left(\frac{e(a+}{(c+}\right.\right.}{e(a+bx)^2}}{g}$$

$$= \frac{\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2\log(f+gx)}{g} - \frac{(2B)\int \frac{(c+dx)^2\left(-\frac{2de(a+bx)^2}{(c+dx)^3}+\frac{2be(a+bx)}{(c+dx)^2}\right)\left(A+B\log\left(\frac{e(a+}{(c+}\right.\right.}{(a+bx)^2}}{eg}$$

$$= \frac{\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2\log(f+gx)}{g} - \frac{(2B)\int \frac{2(bc-ad)e\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)\log(f+gx)}{(a+bx)(c+dx)}\,dx}{eg}$$

$$= \frac{\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2\log(f+gx)}{g} - \frac{(4B(bc-ad))\int \frac{\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)\log(f+gx)}{(a+bx)(c+dx)}\,d}{g}$$

$$= \frac{\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2\log(f+gx)}{g} - \frac{(4B(bc-ad))\int\left(\frac{b\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)\log(f+gx)}{(bc-ad)(a+bx)}\right.}{g}$$

$$= \frac{\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2\log(f+gx)}{g} - \frac{(4bB)\int \frac{\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)\log(f+gx)}{a+bx}\,dx}{g} + \frac{(4B}{}$$

$$= \frac{\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2\log(f+gx)}{g} - \frac{(4bB)\int\left(\frac{A\log(f+gx)}{a+bx}+\frac{B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\log(f+gx)}{a+bx}\right.}{g}$$

$$= \frac{\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2\log(f+gx)}{g} - \frac{(4AbB)\int\frac{\log(f+gx)}{a+bx}\,dx}{g} - \frac{\left(4bB^2\right)\int\frac{\log\left(\frac{e(a+bx}{(c+dx)}\right.}{a}}{g}$$

$$= -\frac{4AB\log\left(-\frac{g(a+bx)}{bf-ag}\right)\log(f+gx)}{g} + \frac{\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2\log(f+gx)}{g} + \frac{4AB\,\mathrm{l}}{}$$

$$= -\frac{4AB\log\left(-\frac{g(a+bx)}{bf-ag}\right)\log(f+gx)}{g} + \frac{4B^2\log\left(-\frac{g(a+bx)}{bf-ag}\right)\left(\log\left((a+bx)^2\right)+\log\left(\frac{}{(c}\right.\right.}{g}$$

$$= -\frac{4AB\log\left(-\frac{g(a+bx)}{bf-ag}\right)\log(f+gx)}{g} - \frac{B^2\log^2\left((a+bx)^2\right)\log(f+gx)}{g} - \frac{B^2\log^2\left(\frac{}{(}\right.}{}$$

$$-\frac{4AB\log\left(-\frac{g(a+bx)}{}\right)\log(f+gx)}{} \quad B^2\log^2\left((a+bx)^2\right)\ldots \quad B^2\log^2\left(\right.$$

**Mathematica [F]** time = 2.30, size = 0, normalized size = 0.00

$$\int \frac{\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}{f+gx}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])^2/(f + g*x),x]
[Out] Integrate[(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])^2/(f + g*x), x]
```

**fricas [F]** time = 0.80, size = 0, normalized size = 0.00

$$\operatorname{integral}\left(\frac{B^2\log\left(\frac{b^2ex^2+2\,abex+a^2e}{d^2x^2+2\,cdx+c^2}\right)^2+2\,AB\log\left(\frac{b^2ex^2+2\,abex+a^2e}{d^2x^2+2\,cdx+c^2}\right)+A^2}{gx+f},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*(b*x+a)^2/(d*x+c)^2))^2/(g*x+f),x, algorithm="fricas")
[Out] integral((B^2*log((b^2*e*x^2 + 2*a*b*e*x + a^2*e)/(d^2*x^2 + 2*c*d*x + c^2)
)^2 + 2*A*B*log((b^2*e*x^2 + 2*a*b*e*x + a^2*e)/(d^2*x^2 + 2*c*d*x + c^2))
+ A^2)/(g*x + f), x)
```

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(B\log\left(\frac{(bx+a)^2e}{(dx+c)^2}\right)+A\right)^2}{gx+f}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*(b*x+a)^2/(d*x+c)^2))^2/(g*x+f),x, algorithm="giac")
[Out] integrate((B*log((b*x + a)^2*e/(d*x + c)^2) + A)^2/(g*x + f), x)
```

**maple [F]** time = 1.06, size = 0, normalized size = 0.00

$$\int \frac{\left(B\ln\left(\frac{(bx+a)^2e}{(dx+c)^2}\right)+A\right)^2}{gx+f}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*ln((b*x+a)^2/(d*x+c)^2*e)+A)^2/(g*x+f),x)
```

```
[Out] int((B*ln((b*x+a)^2/(d*x+c)^2*e)+A)^2/(g*x+f),x)
```

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$\frac{A^2\log\left(gx+f\right)}{g}-\int -\frac{4\,B^2\log\left(bx+a\right)^2+B^2\log(e)^2+2\,AB\log(e)+4\left(B^2\log(e)+AB\right)\log\left(bx+a\right)-4\left(2\,B^2\,\mathrm{lo}\right.}{gx+f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*(b*x+a)^2/(d*x+c)^2))^2/(g*x+f),x, algorithm="maxima")
```

```
[Out] A^2*log(g*x + f)/g - integrate(-(4*B^2*log(b*x + a)^2 + B^2*log(e)^2 + 2*A*
B*log(e) + 4*(B^2*log(e) + A*B)*log(b*x + a) - 4*(2*B^2*log(b*x + a) + B^2*
log(e) + A*B)*log(d*x + c))/(g*x + f), x)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{{\left(A+B\,\ln\left(\frac{e\,{\left(a+b\,x\right)}^2}{{\left(c+d\,x\right)}^2}\right)\right)}^2}{f+g\,x} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*log((e*(a + b*x)^2)/(c + d*x)^2))^2/(f + g*x),x)
```

```
[Out] int((A + B*log((e*(a + b*x)^2)/(c + d*x)^2))^2/(f + g*x), x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(A + B \log{\left(\frac{a^{2} e}{c^{2} + 2 c d x + d^{2} x^{2}} + \frac{2 a b e x}{c^{2} + 2 c d x + d^{2} x^{2}} + \frac{b^{2} e x^{2}}{c^{2} + 2 c d x + d^{2} x^{2}} \right)}\right)^{2}}{f + g x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*ln(e*(b*x+a)**2/(d*x+c)**2))**2/(g*x+f),x)
```

```
[Out] Integral((A + B*log(a**2*e/(c**2 + 2*c*d*x + d**2*x**2) + 2*a*b*e*x/(c**2 +
 2*c*d*x + d**2*x**2) + b**2*e*x**2/(c**2 + 2*c*d*x + d**2*x**2)))**2/(f +
g*x), x)
```

# 3.277 $\int \frac{\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}{(f+gx)^2}\,dx$

**Optimal**. Leaf size=200

$$\frac{4B(bc-ad)\log\left(1-\frac{(a+bx)(df-cg)}{(c+dx)(bf-ag)}\right)\left(B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+A\right)}{(bf-ag)(df-cg)}+\frac{(a+bx)\left(B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+A\right)^2}{(f+gx)(bf-ag)}+\frac{8B^2(bc-ad)\mathrm{Li}_2\left(\frac{(df}{(bf}\right.}{(bf-ag)(df-}$$

```
[Out] (b*x+a)*(A+B*ln(e*(b*x+a)^2/(d*x+c)^2))^2/(-a*g+b*f)/(g*x+f)+4*B*(-a*d+b*c)
*(A+B*ln(e*(b*x+a)^2/(d*x+c)^2))*ln(1-(-c*g+d*f)*(b*x+a)/(-a*g+b*f)/(d*x+c)
)/(-a*g+b*f)/(-c*g+d*f)+8*B^2*(-a*d+b*c)*polylog(2,(-c*g+d*f)*(b*x+a)/(-a*g
+b*f)/(d*x+c))/(-a*g+b*f)/(-c*g+d*f)
```

**Rubi [B]** time = 1.30, antiderivative size = 620, normalized size of antiderivative = 3.10, number of steps used = 32, number of rules used = 10, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.323, Rules used = {2525, 12, 2528, 2524, 2418, 2390, 2301, 2394, 2393, 2391}

$$\frac{8bB^2\text{PolyLog}\left(2,-\frac{d(a+bx)}{bc-ad}\right)}{g(bf-ag)}+\frac{8B^2d\text{PolyLog}\left(2,\frac{b(c+dx)}{bc-ad}\right)}{g(df-cg)}-\frac{8B^2(bc-ad)\text{PolyLog}\left(2,\frac{b(f+gx)}{bf-ag}\right)}{(bf-ag)(df-cg)}+\frac{8B^2(bc-ad)\text{PolyLo}}{(bf-ag)(df}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])^2/(f + g*x)^2,x]
```

```
[Out] (-4*b*B^2*Log[a + b*x]^2)/(g*(b*f - a*g)) + (4*b*B*Log[a + b*x]*(A + B*Log[
(e*(a + b*x)^2)/(c + d*x)^2]))/(g*(b*f - a*g)) - (A + B*Log[(e*(a + b*x)^2)
/(c + d*x)^2])^2/(g*(f + g*x)) + (8*B^2*d*Log[-((d*(a + b*x))/(b*c - a*d))]
*Log[c + d*x])/(g*(d*f - c*g)) - (4*B*d*(A + B*Log[(e*(a + b*x)^2)/(c + d*x
)^2])*Log[c + d*x])/(g*(d*f - c*g)) - (4*B^2*d*Log[c + d*x]^2)/(g*(d*f - c*
g)) + (8*b*B^2*Log[a + b*x]*Log[(b*(c + d*x))/(b*c - a*d)])/(g*(b*f - a*g))
 - (8*B^2*(b*c - a*d)*Log[-((g*(a + b*x))/(b*f - a*g))]*Log[f + g*x])/((b*f
 - a*g)*(d*f - c*g)) + (4*B*(b*c - a*d)*(A + B*Log[(e*(a + b*x)^2)/(c + d*x
)^2])*Log[f + g*x])/((b*f - a*g)*(d*f - c*g)) + (8*B^2*(b*c - a*d)*Log[-((g
*(c + d*x))/(d*f - c*g))]*Log[f + g*x])/((b*f - a*g)*(d*f - c*g)) + (8*b*B^
2*PolyLog[2, -((d*(a + b*x))/(b*c - a*d))])/(g*(b*f - a*g)) + (8*B^2*d*Poly
Log[2, (b*(c + d*x))/(b*c - a*d)])/(g*(d*f - c*g)) - (8*B^2*(b*c - a*d)*Pol
yLog[2, (b*(f + g*x))/(b*f - a*g)])/((b*f - a*g)*(d*f - c*g)) + (8*B^2*(b*c
 - a*d)*PolyLog[2, (d*(f + g*x))/(d*f - c*g)])/((b*f - a*g)*(d*f - c*g))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2301

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

Rule 2390

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_))^(q_.), x_Symbol] :> Dist[1/e, Subst[Int[((f*x)/d)^q*(a + b*Log[c*x^
n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && E
qQ[e*f - d*g, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2393

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + (c*e*x)/g])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2394

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d + e*x
)^n]))/g, x] - Dist[(b*e*n)/g, Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2418

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*(RFx_), x_Sy
mbol] :> With[{u = ExpandIntegrand[(a + b*Log[c*(d + e*x)^n])^p, RFx, x]},
Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d, e, n}, x] && RationalFunctionQ[
RFx, x] && IntegerQ[p]
```

Rule 2524

```
Int[((a_.) + Log[(c_.)*(RFx_)^(p_.)]*(b_.))^(n_.)/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[(Log[d + e*x]*(a + b*Log[c*RFx^p])^n)/e, x] - Dist[(b*n*p)/e
, Int[(Log[d + e*x]*(a + b*Log[c*RFx^p])^(n - 1)*D[RFx, x])/RFx, x], x] /;
FreeQ[{a, b, c, d, e, p}, x] && RationalFunctionQ[RFx, x] && IGtQ[n, 0]
```

Rule 2525

```
Int[((a_.) + Log[(c_.)*(RFx_)^(p_.)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_))^(m_.
), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(a + b*Log[c*RFx^p])^n)/(e*(m + 1))
, x] - Dist[(b*n*p)/(e*(m + 1)), Int[SimplifyIntegrand[((d + e*x)^(m + 1)*(
a + b*Log[c*RFx^p])^(n - 1)*D[RFx, x])/RFx, x], x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && RationalFunctionQ[RFx, x] && IGtQ[n, 0] && (EqQ[n, 1] ||
IntegerQ[m]) && NeQ[m, -1]
```

Rule 2528

```
Int[((a_.) + Log[(c_.)*(RFx_)^(p_.)]*(b_.))^(n_.)*(RGx_), x_Symbol] :> With
[{u = ExpandIntegrand[(a + b*Log[c*RFx^p])^n, RGx, x]}, Int[u, x] /; SumQ[u
]] /; FreeQ[{a, b, c, p}, x] && RationalFunctionQ[RFx, x] && RationalFuncti
onQ[RGx, x] && IGtQ[n, 0]
```

Rubi steps

$$\int \frac{\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}{(f+gx)^2}\,dx = -\frac{\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}{g(f+gx)} + \frac{(2B)\int \frac{2(bc-ad)\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}{(a+bx)(c+dx)(f+gx)}\,dx}{g}$$

$$= -\frac{\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}{g(f+gx)} + \frac{(4B(bc-ad))\int \frac{A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}{(a+bx)(c+dx)(f+gx)}\,dx}{g}$$

$$= -\frac{\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}{g(f+gx)} + \frac{(4B(bc-ad))\int \left(\frac{b^2\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}{(bc-ad)(bf-ag)(a+bx)} + \frac{d^2\left(A+B\log\left(\frac{e(a+bx)}{(c+dx)}\right.\right.}{(bc-ad)(-df+cg)(c+}\right.}{g}$$

$$= -\frac{\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}{g(f+gx)} + \frac{\left(4b^2B\right)\int \frac{A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}{a+bx}\,dx}{g(bf-ag)} - \frac{\left(4Bd^2\right)\int \frac{A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right.}{c+dx}}{g(df-cg)}$$

$$= \frac{4bB\log(a+bx)\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}{g(bf-ag)} - \frac{\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}{g(f+gx)} - \frac{4Bd\left(A+B\,\mathrm{l}\right.}{}$$

$$= \frac{4bB\log(a+bx)\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}{g(bf-ag)} - \frac{\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}{g(f+gx)} - \frac{4Bd\left(A+B\,\mathrm{l}\right.}{}$$

$$= \frac{4bB\log(a+bx)\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}{g(bf-ag)} - \frac{\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}{g(f+gx)} - \frac{4Bd\left(A+B\,\mathrm{l}\right.}{}$$

$$= \frac{4bB\log(a+bx)\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}{g(bf-ag)} - \frac{\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}{g(f+gx)} - \frac{4Bd\left(A+B\,\mathrm{l}\right.}{}$$

$$= \frac{4bB\log(a+bx)\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}{g(bf-ag)} - \frac{\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}{g(f+gx)} + \frac{8B^2d\log\left(-\frac{d}{}\right.}{g(}$$

$$= -\frac{4bB^2\log^2(a+bx)}{g(bf-ag)} + \frac{4bB\log(a+bx)\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}{g(bf-ag)} - \frac{\left(A+B\log\left(\frac{e(a+}{(c+}\right.\right.}{g(f+gx)}$$

$$= -\frac{4bB^2\log^2(a+bx)}{g(bf-ag)} + \frac{4bB\log(a+bx)\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}{g(bf-ag)} - \frac{\left(A+B\log\left(\frac{e(a+}{(c+}\right.\right.}{g(f+gx)}$$

**Mathematica [B]** time = 0.59, size = 409, normalized size = 2.04

$$\frac{4B\left(b\log(a+bx)(df-cg)\left(B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+A\right)-d(bf-ag)\log(c+dx)\left(B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+A\right)+g(bc-ad)\log(f+gx)\left(B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+A\right)-bB(df-cg)\left(\log(a\cdots\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])^2/(f + g*x)^2,x]
```

```
[Out] (-((A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])^2/(f + g*x)) + (4*B*(b*(d*f - c
*g)*Log[a + b*x]*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2]) - d*(b*f - a*g)*(
A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])*Log[c + d*x] + (b*c - a*d)*g*(A + B
*Log[(e*(a + b*x)^2)/(c + d*x)^2])*Log[f + g*x] - b*B*(d*f - c*g)*(Log[a +
b*x]*(Log[a + b*x] - 2*Log[(b*(c + d*x))/(b*c - a*d)]) - 2*PolyLog[2, (d*(a
 + b*x))/(-(b*c) + a*d)]) + B*d*(b*f - a*g)*((2*Log[(d*(a + b*x))/(-(b*c) +
 a*d)] - Log[c + d*x])*Log[c + d*x] + 2*PolyLog[2, (b*(c + d*x))/(b*c - a*d
)]) - 2*B*(b*c - a*d)*g*((Log[(g*(a + b*x))/(-(b*f) + a*g)] - Log[(g*(c + d
*x))/(-(d*f) + c*g)])*Log[f + g*x] + PolyLog[2, (b*(f + g*x))/(b*f - a*g)]
- PolyLog[2, (d*(f + g*x))/(d*f - c*g)])))/((b*f - a*g)*(d*f - c*g)))/g
```

**fricas [F]** time = 1.46, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{B^2\log\left(\frac{b^2ex^2+2\,abex+a^2e}{d^2x^2+2\,cdx+c^2}\right)^2+2\,AB\log\left(\frac{b^2ex^2+2\,abex+a^2e}{d^2x^2+2\,cdx+c^2}\right)+A^2}{g^2x^2+2\,fgx+f^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*(b*x+a)^2/(d*x+c)^2))^2/(g*x+f)^2,x, algorithm="fricas
")
```

```
[Out] integral((B^2*log((b^2*e*x^2 + 2*a*b*e*x + a^2*e)/(d^2*x^2 + 2*c*d*x + c^2)
)^2 + 2*A*B*log((b^2*e*x^2 + 2*a*b*e*x + a^2*e)/(d^2*x^2 + 2*c*d*x + c^2))
+ A^2)/(g^2*x^2 + 2*f*g*x + f^2), x)
```

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\left(B\log\left(\frac{(bx+a)^2e}{(dx+c)^2}\right)+A\right)^2}{(gx+f)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*(b*x+a)^2/(d*x+c)^2))^2/(g*x+f)^2,x, algorithm="giac")
```

```
[Out] integrate((B*log((b*x + a)^2*e/(d*x + c)^2) + A)^2/(g*x + f)^2, x)
```

**maple [F]** time = 1.26, size = 0, normalized size = 0.00

$$\int \frac{\left(B \ln \left(\frac{(bx+a)^2 e}{(dx+c)^2}\right) + A\right)^2}{\left(gx + f\right)^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*ln((b*x+a)^2/(d*x+c)^2*e)+A)^2/(g*x+f)^2,x)
```

```
[Out] int((B*ln((b*x+a)^2/(d*x+c)^2*e)+A)^2/(g*x+f)^2,x)
```

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$2\,AB\left(\frac{2\,b\log\left(bx+a\right)}{bfg-ag^2} - \frac{2\,d\log\left(dx+c\right)}{dfg-cg^2} + \frac{2\,(bc-ad)\log\left(gx+f\right)}{bdf^2+acg^2-(bc+ad)fg} - \frac{\log\left(\frac{b^2ex^2}{d^2x^2+2\,cdx+c^2} + \frac{2\,abex}{d^2x^2+2\,cdx+c^2} + \frac{a^2}{d^2x^2+2}\right.}{g^2x+fg}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*(b*x+a)^2/(d*x+c)^2))^2/(g*x+f)^2,x, algorithm="maxima
")
```

```
[Out] 2*A*B*(2*b*log(b*x + a)/(b*f*g - a*g^2) - 2*d*log(d*x + c)/(d*f*g - c*g^2)
+ 2*(b*c - a*d)*log(g*x + f)/(b*d*f^2 + a*c*g^2 - (b*c + a*d)*f*g) - log(b^
2*e*x^2/(d^2*x^2 + 2*c*d*x + c^2) + 2*a*b*e*x/(d^2*x^2 + 2*c*d*x + c^2) + a
^2*e/(d^2*x^2 + 2*c*d*x + c^2))/(g^2*x + f*g)) - B^2*(4*log(d*x + c)^2/(g^2
*x + f*g) + integrate(-(d*g*x*log(e)^2 + c*g*log(e)^2 + 4*(d*g*x + c*g)*log
(b*x + a)^2 + 4*(d*g*x*log(e) + c*g*log(e))*log(b*x + a) - 4*((g*log(e) - 2
*g)*d*x + c*g*log(e) - 2*d*f + 2*(d*g*x + c*g)*log(b*x + a))*log(d*x + c))/
(d*g^3*x^3 + c*f^2*g + (2*d*f*g^2 + c*g^3)*x^2 + (d*f^2*g + 2*c*f*g^2)*x),
x)) - A^2/(g^2*x + f*g)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{{\left(A+B\,\ln\left(\frac{e\,{\left(a+b\,x\right)}^2}{{\left(c+d\,x\right)}^2}\right)\right)}^2}{{\left(f+g\,x\right)}^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*log((e*(a + b*x)^2)/(c + d*x)^2))^2/(f + g*x)^2,x)
```

```
[Out] int((A + B*log((e*(a + b*x)^2)/(c + d*x)^2))^2/(f + g*x)^2, x)
```

**sympy [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*ln(e*(b*x+a)**2/(d*x+c)**2))**2/(g*x+f)**2,x)
[Out] Timed out
```

# 3.278 $\int \frac{\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}{(f+gx)^3}\,dx$

**Optimal**. Leaf size=381

$$\frac{b^2\left(B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+A\right)^2}{2g(bf-ag)^2}+\frac{2Bg(a+bx)(bc-ad)\left(B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+A\right)}{(f+gx)(bf-ag)^2(df-cg)}+\frac{2B(bc-ad)(-adg-bcg+2bdf)\log\left(1-\right.}{(bf-ag)^2(d}$$

```
[Out] 2*B*(-a*d+b*c)*g*(b*x+a)*(A+B*ln(e*(b*x+a)^2/(d*x+c)^2))/(-a*g+b*f)^2/(-c*g
+d*f)/(g*x+f)+1/2*b^2*(A+B*ln(e*(b*x+a)^2/(d*x+c)^2))^2/g/(-a*g+b*f)^2-1/2*
(A+B*ln(e*(b*x+a)^2/(d*x+c)^2))^2/g/(g*x+f)^2+4*B^2*(-a*d+b*c)^2*g*ln((g*x+
f)/(d*x+c))/(-a*g+b*f)^2/(-c*g+d*f)^2+2*B*(-a*d+b*c)*(-a*d*g-b*c*g+2*b*d*f)
*(A+B*ln(e*(b*x+a)^2/(d*x+c)^2))*ln(1-(-c*g+d*f)*(b*x+a)/(-a*g+b*f)/(d*x+c)
)/(-a*g+b*f)^2/(-c*g+d*f)^2+4*B^2*(-a*d+b*c)*(-a*d*g-b*c*g+2*b*d*f)*polylog
(2,(-c*g+d*f)*(b*x+a)/(-a*g+b*f)/(d*x+c))/(-a*g+b*f)^2/(-c*g+d*f)^2
```

**Rubi [B]** time = 1.64, antiderivative size = 899, normalized size of antiderivative = 2.36, number of steps used = 36, number of rules used = 11, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.355, Rules used = {2525, 12, 2528, 2524, 2418, 2390, 2301, 2394, 2393, 2391, 72}

$$-\frac{2B^2\log^2(a+bx)b^2}{g(bf-ag)^2}+\frac{2B\log(a+bx)\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)b^2}{g(bf-ag)^2}+\frac{4B^2\log(a+bx)\log\left(\frac{b(c+dx)}{bc-ad}\right)b^2}{g(bf-ag)^2}+\frac{4B^2\text{PolyLog}\left(2\right.}{g(bf-}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])^2/(f + g*x)^3,x]
```

```
[Out] (4*b*B^2*(b*c - a*d)*Log[a + b*x])/((b*f - a*g)^2*(d*f - c*g)) - (2*b^2*B^2
*Log[a + b*x]^2)/(g*(b*f - a*g)^2) - (2*B*(b*c - a*d)*(A + B*Log[(e*(a + b*
x)^2)/(c + d*x)^2]))/((b*f - a*g)*(d*f - c*g)*(f + g*x)) + (2*b^2*B*Log[a +
 b*x]*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2]))/(g*(b*f - a*g)^2) - (A + B*
Log[(e*(a + b*x)^2)/(c + d*x)^2])^2/(2*g*(f + g*x)^2) - (4*B^2*d*(b*c - a*d
)*Log[c + d*x])/((b*f - a*g)*(d*f - c*g)^2) + (4*B^2*d^2*Log[-((d*(a + b*x)
)/(b*c - a*d))]*Log[c + d*x])/(g*(d*f - c*g)^2) - (2*B*d^2*(A + B*Log[(e*(a
 + b*x)^2)/(c + d*x)^2])*Log[c + d*x])/(g*(d*f - c*g)^2) - (2*B^2*d^2*Log[c
 + d*x]^2)/(g*(d*f - c*g)^2) + (4*b^2*B^2*Log[a + b*x]*Log[(b*(c + d*x))/(b
*c - a*d)])/(g*(b*f - a*g)^2) + (4*B^2*(b*c - a*d)^2*g*Log[f + g*x])/((b*f
- a*g)^2*(d*f - c*g)^2) - (4*B^2*(b*c - a*d)*(2*b*d*f - b*c*g - a*d*g)*Log[
-((g*(a + b*x))/(b*f - a*g))]*Log[f + g*x])/((b*f - a*g)^2*(d*f - c*g)^2) +
 (2*B*(b*c - a*d)*(2*b*d*f - b*c*g - a*d*g)*(A + B*Log[(e*(a + b*x)^2)/(c +
 d*x)^2])*Log[f + g*x])/((b*f - a*g)^2*(d*f - c*g)^2) + (4*B^2*(b*c - a*d)*
```

```
(2*b*d*f - b*c*g - a*d*g)*Log[-((g*(c + d*x))/(d*f - c*g))]*Log[f + g*x])/(
(b*f - a*g)^2*(d*f - c*g)^2) + (4*b^2*B^2*PolyLog[2, -((d*(a + b*x))/(b*c -
 a*d))])/(g*(b*f - a*g)^2) + (4*B^2*d^2*PolyLog[2, (b*(c + d*x))/(b*c - a*d
)])/(g*(d*f - c*g)^2) - (4*B^2*(b*c - a*d)*(2*b*d*f - b*c*g - a*d*g)*PolyLo
g[2, (b*(f + g*x))/(b*f - a*g)])/((b*f - a*g)^2*(d*f - c*g)^2) + (4*B^2*(b*
c - a*d)*(2*b*d*f - b*c*g - a*d*g)*PolyLog[2, (d*(f + g*x))/(d*f - c*g)])/(
(b*f - a*g)^2*(d*f - c*g)^2)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 72

```
Int[((e_.) + (f_.)*(x_))^(p_.)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
 x_Symbol] :> Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x]
 /; FreeQ[{a, b, c, d, e, f}, x] && IntegerQ[p]
```

Rule 2301

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

Rule 2390

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_))^(q_.), x_Symbol] :> Dist[1/e, Subst[Int[((f*x)/d)^q*(a + b*Log[c*x^
n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && E
qQ[e*f - d*g, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2393

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + (c*e*x)/g])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2394

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d + e*x
)^n]))/g, x] - Dist[(b*e*n)/g, Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2418

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*(RFx_), x_Sy
mbol] :> With[{u = ExpandIntegrand[(a + b*Log[c*(d + e*x)^n])^p, RFx, x]},
Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d, e, n}, x] && RationalFunctionQ[
RFx, x] && IntegerQ[p]
```

Rule 2524

```
Int[((a_.) + Log[(c_.)*(RFx_)^(p_.)]*(b_.))^(n_.)/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[(Log[d + e*x]*(a + b*Log[c*RFx^p])^n)/e, x] - Dist[(b*n*p)/e
, Int[(Log[d + e*x]*(a + b*Log[c*RFx^p])^(n - 1)*D[RFx, x])/RFx, x], x] /;
FreeQ[{a, b, c, d, e, p}, x] && RationalFunctionQ[RFx, x] && IGtQ[n, 0]
```

Rule 2525

```
Int[((a_.) + Log[(c_.)*(RFx_)^(p_.)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_))^(m_.
), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(a + b*Log[c*RFx^p])^n)/(e*(m + 1))
, x] - Dist[(b*n*p)/(e*(m + 1)), Int[SimplifyIntegrand[((d + e*x)^(m + 1)*(
a + b*Log[c*RFx^p])^(n - 1)*D[RFx, x])/RFx, x], x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && RationalFunctionQ[RFx, x] && IGtQ[n, 0] && (EqQ[n, 1] ||
IntegerQ[m]) && NeQ[m, -1]
```

Rule 2528

```
Int[((a_.) + Log[(c_.)*(RFx_)^(p_.)]*(b_.))^(n_.)*(RGx_), x_Symbol] :> With
[{u = ExpandIntegrand[(a + b*Log[c*RFx^p])^n, RGx, x]}, Int[u, x] /; SumQ[u
]] /; FreeQ[{a, b, c, p}, x] && RationalFunctionQ[RFx, x] && RationalFuncti
onQ[RGx, x] && IGtQ[n, 0]
```

Rubi steps

$$\int \frac{\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}{(f+gx)^3}\,dx = -\frac{\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}{2g(f+gx)^2} + \frac{B\int \frac{2(bc-ad)\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}{(a+bx)(c+dx)(f+gx)^2}\,dx}{g}$$

$$= -\frac{\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}{2g(f+gx)^2} + \frac{(2B(bc-ad))\int \frac{A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}{(a+bx)(c+dx)(f+gx)^2}\,dx}{g}$$

$$= -\frac{\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}{2g(f+gx)^2} + \frac{(2B(bc-ad))\int \left(\frac{b^3\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}{(bc-ad)(bf-ag)^2(a+bx)} - \frac{d^3\left(A+B\log\left(\frac{e(a+}{(c+}\right.\right.}{(bc-ad)(-df+cg)^2}\right.}{}$$

$$= -\frac{\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}{2g(f+gx)^2} + \frac{\left(2b^3B\right)\int \frac{A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}{a+bx}\,dx}{g(bf-ag)^2} - \frac{\left(2Bd^3\right)\int \frac{A+B\log\left(\frac{e(a+b}{(c+dx}\right.}{c+dx}}{g(df-cg)^2}$$

$$= -\frac{2B(bc-ad)\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}{(bf-ag)(df-cg)(f+gx)} + \frac{2b^2B\log(a+bx)\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}{g(bf-ag)^2}$$

$$= -\frac{2B(bc-ad)\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}{(bf-ag)(df-cg)(f+gx)} + \frac{2b^2B\log(a+bx)\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}{g(bf-ag)^2}$$

$$= -\frac{2B(bc-ad)\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}{(bf-ag)(df-cg)(f+gx)} + \frac{2b^2B\log(a+bx)\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}{g(bf-ag)^2}$$

$$= \frac{4bB^2(bc-ad)\log(a+bx)}{(bf-ag)^2(df-cg)} - \frac{2B(bc-ad)\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}{(bf-ag)(df-cg)(f+gx)} + \frac{2b^2B\log(a+}{}$$

$$= \frac{4bB^2(bc-ad)\log(a+bx)}{(bf-ag)^2(df-cg)} - \frac{2B(bc-ad)\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}{(bf-ag)(df-cg)(f+gx)} + \frac{2b^2B\log(a+}{}$$

$$= \frac{4bB^2(bc-ad)\log(a+bx)}{(bf-ag)^2(df-cg)} - \frac{2b^2B^2\log^2(a+bx)}{g(bf-ag)^2} - \frac{2B(bc-ad)\left(A+B\log\left(\frac{e(a+}{(c+}\right.\right.}{(bf-ag)(df-cg)(f+g}$$

$$= \frac{4bB^2(bc-ad)\log(a+bx)}{(bf-ag)^2(df-cg)} - \frac{2b^2B^2\log^2(a+bx)}{g(bf-ag)^2} - \frac{2B(bc-ad)\left(A+B\log\left(\frac{e(a+}{(c+}\right.\right.}{(bf-ag)(df-cg)(f+g}$$

**Mathematica [A]** time = 1.58, size = 603, normalized size = 1.58

$$-\frac{\frac{4B(f+gx)\left(-b^2(f+gx)\log(a+bx)(df-cg)^2\left(B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+A\right)+d^2(f+gx)(bf-ag)^2\log(c+dx)\left(B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+A\right)+g(bc-ad)(bf-ag)(df-cg)\left(B\log\left(\right.\right.\right.}{}}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])^2/(f + g*x)^3,x]
```

```
[Out] -1/2*((A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])^2 + (4*B*(f + g*x)*((b*c - a
*d)*g*(b*f - a*g)*(d*f - c*g)*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2]) - b^
2*(d*f - c*g)^2*(f + g*x)*Log[a + b*x]*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)
^2]) + d^2*(b*f - a*g)^2*(f + g*x)*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])
*Log[c + d*x] + (b*c - a*d)*g*(-2*b*d*f + b*c*g + a*d*g)*(f + g*x)*(A + B*L
og[(e*(a + b*x)^2)/(c + d*x)^2])*Log[f + g*x] - 2*B*(b*c - a*d)*g*(f + g*x)
*(b*(d*f - c*g)*Log[a + b*x] + (-(b*d*f) + a*d*g)*Log[c + d*x] + (b*c - a*d
)*g*Log[f + g*x]) + b^2*B*(d*f - c*g)^2*(f + g*x)*(Log[a + b*x]*(Log[a + b*
x] - 2*Log[(b*(c + d*x))/(b*c - a*d)]) - 2*PolyLog[2, (d*(a + b*x))/(-(b*c)
 + a*d)]) - B*d^2*(b*f - a*g)^2*(f + g*x)*((2*Log[(d*(a + b*x))/(-(b*c) + a
*d)] - Log[c + d*x])*Log[c + d*x] + 2*PolyLog[2, (b*(c + d*x))/(b*c - a*d)]
) - 2*B*(b*c - a*d)*g*(-2*b*d*f + b*c*g + a*d*g)*(f + g*x)*((Log[(g*(a + b*
x))/(-(b*f) + a*g)] - Log[(g*(c + d*x))/(-(d*f) + c*g)])*Log[f + g*x] + Pol
yLog[2, (b*(f + g*x))/(b*f - a*g)] - PolyLog[2, (d*(f + g*x))/(d*f - c*g)])
))/((b*f - a*g)^2*(d*f - c*g)^2))/(g*(f + g*x)^2)
```

**fricas [F]** time = 0.87, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{B^2\log\left(\frac{b^2ex^2+2\,abex+a^2e}{d^2x^2+2\,cdx+c^2}\right)^2+2\,AB\log\left(\frac{b^2ex^2+2\,abex+a^2e}{d^2x^2+2\,cdx+c^2}\right)+A^2}{g^3x^3+3\,fg^2x^2+3\,f^2gx+f^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*(b*x+a)^2/(d*x+c)^2))^2/(g*x+f)^3,x, algorithm="fricas
")
```

```
[Out] integral((B^2*log((b^2*e*x^2 + 2*a*b*e*x + a^2*e)/(d^2*x^2 + 2*c*d*x + c^2)
)^2 + 2*A*B*log((b^2*e*x^2 + 2*a*b*e*x + a^2*e)/(d^2*x^2 + 2*c*d*x + c^2))
+ A^2)/(g^3*x^3 + 3*f*g^2*x^2 + 3*f^2*g*x + f^3), x)
```

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(B\log\left(\frac{(bx+a)^2e}{(dx+c)^2}\right)+A\right)^2}{(gx+f)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*(b*x+a)^2/(d*x+c)^2))^2/(g*x+f)^3,x, algorithm="giac")
[Out] integrate((B*log((b*x + a)^2*e/(d*x + c)^2) + A)^2/(g*x + f)^3, x)
```

**maple [F]** time = 1.75, size = 0, normalized size = 0.00

$$\int \frac{\left(B \ln \left(\frac{(bx+a)^{2} e}{(dx+c)^{2}}\right)+A\right)^{2}}{\left(gx+f\right)^{3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*ln((b*x+a)^2/(d*x+c)^2*e)+A)^2/(g*x+f)^3,x)
[Out] int((B*ln((b*x+a)^2/(d*x+c)^2*e)+A)^2/(g*x+f)^3,x)
```

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$\left(\frac{2\, b^{2} \log \left(bx+a\right)}{b^{2} f^{2} g-2\, abfg^{2}+a^{2} g^{3}}-\frac{2\, d^{2} \log \left(dx+c\right)}{d^{2} f^{2} g-2\, cdfg^{2}+c^{2} g^{3}}+\frac{2\left(2\left(b^{2} cd-abd^{2}\right)f-\left(b^{2} c^{2}-a^{2} d^{2}\right)g\right)}{b^{2} d^{2} f^{4}+a^{2} c^{2} g^{4}-2\left(b^{2} cd+abd^{2}\right)f^{3} g+\left(b^{2} c^{2}+4\, abcd+\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*(b*x+a)^2/(d*x+c)^2))^2/(g*x+f)^3,x, algorithm="maxima
")
[Out] (2*b^2*log(b*x + a)/(b^2*f^2*g - 2*a*b*f*g^2 + a^2*g^3) - 2*d^2*log(d*x + c
)/(d^2*f^2*g - 2*c*d*f*g^2 + c^2*g^3) + 2*(2*(b^2*c*d - a*b*d^2)*f - (b^2*c
^2 - a^2*d^2)*g)*log(g*x + f)/(b^2*d^2*f^4 + a^2*c^2*g^4 - 2*(b^2*c*d + a*b
*d^2)*f^3*g + (b^2*c^2 + 4*a*b*c*d + a^2*d^2)*f^2*g^2 - 2*(a*b*c^2 + a^2*c*
d)*f*g^3) - 2*(b*c - a*d)/(b*d*f^3 + a*c*f*g^2 - (b*c + a*d)*f^2*g + (b*d*f
^2*g + a*c*g^3 - (b*c + a*d)*f*g^2)*x) - log(b^2*e*x^2/(d^2*x^2 + 2*c*d*x +
 c^2) + 2*a*b*e*x/(d^2*x^2 + 2*c*d*x + c^2) + a^2*e/(d^2*x^2 + 2*c*d*x + c^
2))/(g^3*x^2 + 2*f*g^2*x + f^2*g))*A*B - B^2*(2*log(d*x + c)^2/(g^3*x^2 + 2
*f*g^2*x + f^2*g) + integrate(-(d*g*x*log(e)^2 + c*g*log(e)^2 + 4*(d*g*x +
c*g)*log(b*x + a)^2 + 4*(d*g*x*log(e) + c*g*log(e))*log(b*x + a) - 4*((g*lo
g(e) - g)*d*x + c*g*log(e) - d*f + 2*(d*g*x + c*g)*log(b*x + a))*log(d*x +
c))/(d*g^4*x^4 + c*f^3*g + (3*d*f*g^3 + c*g^4)*x^3 + 3*(d*f^2*g^2 + c*f*g^3
)*x^2 + (d*f^3*g + 3*c*f^2*g^2)*x), x)) - 1/2*A^2/(g^3*x^2 + 2*f*g^2*x + f^
2*g)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(A + B \ln\left(\frac{e\,(a+b\,x)^2}{(c+d\,x)^2}\right)\right)^2}{\left(f + g\,x\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*log((e*(a + b*x)^2)/(c + d*x)^2))^2/(f + g*x)^3,x)
[Out] int((A + B*log((e*(a + b*x)^2)/(c + d*x)^2))^2/(f + g*x)^3, x)
```

**sympy [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*ln(e*(b*x+a)**2/(d*x+c)**2))**2/(g*x+f)**3,x)
[Out] Timed out
```

# 3.279 $\int \frac{\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}{(f+gx)^4}\,dx$

**Optimal**. Leaf size=724

$$\frac{4B(bc-ad)\left(a^2d^2g^2-abdg(3df-cg)+b^2\left(c^2g^2-3cdfg+3d^2f^2\right)\right)\log\left(1-\frac{(a+bx)(df-cg)}{(c+dx)(bf-ag)}\right)\left(B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+A\right.}{3(bf-ag)^3(df-cg)^3}$$

```
[Out] 4/3*B^2*(-a*d+b*c)^2*g^2*(d*x+c)/(-a*g+b*f)^2/(-c*g+d*f)^3/(g*x+f)-2/3*B*(-
a*d+b*c)*g^2*(d*x+c)^2*(A+B*ln(e*(b*x+a)^2/(d*x+c)^2))/(-a*g+b*f)/(-c*g+d*f
)^3/(g*x+f)^2+4/3*B*(-a*d+b*c)*g*(-2*a*d*g-b*c*g+3*b*d*f)*(b*x+a)*(A+B*ln(e
*(b*x+a)^2/(d*x+c)^2))/(-a*g+b*f)^3/(-c*g+d*f)^2/(g*x+f)+1/3*b^3*(A+B*ln(e*
(b*x+a)^2/(d*x+c)^2))^2/g/(-a*g+b*f)^3-1/3*(A+B*ln(e*(b*x+a)^2/(d*x+c)^2))^
2/g/(g*x+f)^3+4/3*B^2*(-a*d+b*c)^3*g^2*ln((b*x+a)/(d*x+c))/(-a*g+b*f)^3/(-c
*g+d*f)^3-4/3*B^2*(-a*d+b*c)^3*g^2*ln((g*x+f)/(d*x+c))/(-a*g+b*f)^3/(-c*g+d
*f)^3+8/3*B^2*(-a*d+b*c)^2*g*(-2*a*d*g-b*c*g+3*b*d*f)*ln((g*x+f)/(d*x+c))/(
-a*g+b*f)^3/(-c*g+d*f)^3+4/3*B*(-a*d+b*c)*(a^2*d^2*g^2-a*b*d*g*(-c*g+3*d*f)
+b^2*(c^2*g^2-3*c*d*f*g+3*d^2*f^2))*(A+B*ln(e*(b*x+a)^2/(d*x+c)^2))*ln(1-(-
c*g+d*f)*(b*x+a)/(-a*g+b*f)/(d*x+c))/(-a*g+b*f)^3/(-c*g+d*f)^3+8/3*B^2*(-a*
d+b*c)*(a^2*d^2*g^2-a*b*d*g*(-c*g+3*d*f)+b^2*(c^2*g^2-3*c*d*f*g+3*d^2*f^2))
*polylog(2,(-c*g+d*f)*(b*x+a)/(-a*g+b*f)/(d*x+c))/(-a*g+b*f)^3/(-c*g+d*f)^3
```

**Rubi [A]** time = 2.54, antiderivative size = 1369, normalized size of antiderivative = 1.89, number of steps used = 40, number of rules used = 11, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.355, Rules used = {2525, 12, 2528, 2524, 2418, 2390, 2301, 2394, 2393, 2391, 72}

$$-\frac{4B^2\log^2(a+bx)b^3}{3g(bf-ag)^3}+\frac{4B\log(a+bx)\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)b^3}{3g(bf-ag)^3}+\frac{8B^2\log(a+bx)\log\left(\frac{b(c+dx)}{bc-ad}\right)b^3}{3g(bf-ag)^3}+\frac{8B^2\text{PolyLog}(}{3g(bf}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])^2/(f + g*x)^4,x]
```

```
[Out] (-4*B^2*(b*c - a*d)^2*g)/(3*(b*f - a*g)^2*(d*f - c*g)^2*(f + g*x)) + (4*b^2
*B^2*(b*c - a*d)*Log[a + b*x])/(3*(b*f - a*g)^3*(d*f - c*g)) + (8*b*B^2*(b*
c - a*d)*(2*b*d*f - b*c*g - a*d*g)*Log[a + b*x])/(3*(b*f - a*g)^3*(d*f - c*
g)^2) - (4*b^3*B^2*Log[a + b*x]^2)/(3*g*(b*f - a*g)^3) - (2*B*(b*c - a*d)*(
A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2]))/(3*(b*f - a*g)*(d*f - c*g)*(f + g*
x)^2) - (4*B*(b*c - a*d)*(2*b*d*f - b*c*g - a*d*g)*(A + B*Log[(e*(a + b*x)^
2)/(c + d*x)^2]))/(3*(b*f - a*g)^2*(d*f - c*g)^2*(f + g*x)) + (4*b^3*B*Log[
```

```
a + b*x]*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2]))/(3*g*(b*f - a*g)^3) - (A
 + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])^2/(3*g*(f + g*x)^3) - (4*B^2*d^2*(b*
c - a*d)*Log[c + d*x])/(3*(b*f - a*g)*(d*f - c*g)^3) - (8*B^2*d*(b*c - a*d)
*(2*b*d*f - b*c*g - a*d*g)*Log[c + d*x])/(3*(b*f - a*g)^2*(d*f - c*g)^3) +
(8*B^2*d^3*Log[-((d*(a + b*x))/(b*c - a*d))]*Log[c + d*x])/(3*g*(d*f - c*g)
^3) - (4*B*d^3*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])*Log[c + d*x])/(3*g*
(d*f - c*g)^3) - (4*B^2*d^3*Log[c + d*x]^2)/(3*g*(d*f - c*g)^3) + (8*b^3*B^
2*Log[a + b*x]*Log[(b*(c + d*x))/(b*c - a*d)])/(3*g*(b*f - a*g)^3) + (4*B^2
*(b*c - a*d)^2*g*(2*b*d*f - b*c*g - a*d*g)*Log[f + g*x])/((b*f - a*g)^3*(d*
f - c*g)^3) - (8*B^2*(b*c - a*d)*(a^2*d^2*g^2 - a*b*d*g*(3*d*f - c*g) + b^2
*(3*d^2*f^2 - 3*c*d*f*g + c^2*g^2))*Log[-((g*(a + b*x))/(b*f - a*g))]*Log[f
 + g*x])/(3*(b*f - a*g)^3*(d*f - c*g)^3) + (4*B*(b*c - a*d)*(a^2*d^2*g^2 -
a*b*d*g*(3*d*f - c*g) + b^2*(3*d^2*f^2 - 3*c*d*f*g + c^2*g^2))*(A + B*Log[(
e*(a + b*x)^2)/(c + d*x)^2])*Log[f + g*x])/(3*(b*f - a*g)^3*(d*f - c*g)^3)
+ (8*B^2*(b*c - a*d)*(a^2*d^2*g^2 - a*b*d*g*(3*d*f - c*g) + b^2*(3*d^2*f^2
- 3*c*d*f*g + c^2*g^2))*Log[-((g*(c + d*x))/(d*f - c*g))]*Log[f + g*x])/(3*
(b*f - a*g)^3*(d*f - c*g)^3) + (8*b^3*B^2*PolyLog[2, -((d*(a + b*x))/(b*c -
 a*d))])/(3*g*(b*f - a*g)^3) + (8*B^2*d^3*PolyLog[2, (b*(c + d*x))/(b*c - a
*d)])/(3*g*(d*f - c*g)^3) - (8*B^2*(b*c - a*d)*(a^2*d^2*g^2 - a*b*d*g*(3*d*
f - c*g) + b^2*(3*d^2*f^2 - 3*c*d*f*g + c^2*g^2))*PolyLog[2, (b*(f + g*x))/
(b*f - a*g)])/(3*(b*f - a*g)^3*(d*f - c*g)^3) + (8*B^2*(b*c - a*d)*(a^2*d^2
*g^2 - a*b*d*g*(3*d*f - c*g) + b^2*(3*d^2*f^2 - 3*c*d*f*g + c^2*g^2))*PolyL
og[2, (d*(f + g*x))/(d*f - c*g)])/(3*(b*f - a*g)^3*(d*f - c*g)^3)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 72

```
Int[((e_.) + (f_.)*(x_))^(p_.)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
 x_Symbol] :> Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x]
 /; FreeQ[{a, b, c, d, e, f}, x] && IntegerQ[p]
```

Rule 2301

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

Rule 2390

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_))^(q_.), x_Symbol] :> Dist[1/e, Subst[Int[((f*x)/d)^q*(a + b*Log[c*x^
n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && E
```

```
qQ[e*f - d*g, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2393

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + (c*e*x)/g])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2394

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d + e*x
)^n]))/g, x] - Dist[(b*e*n)/g, Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2418

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*(RFx_), x_Sy
mbol] :> With[{u = ExpandIntegrand[(a + b*Log[c*(d + e*x)^n])^p, RFx, x]},
Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d, e, n}, x] && RationalFunctionQ[
RFx, x] && IntegerQ[p]
```

Rule 2524

```
Int[((a_.) + Log[(c_.)*(RFx_)^(p_.)]*(b_.))^(n_.)/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[(Log[d + e*x]*(a + b*Log[c*RFx^p])^n)/e, x] - Dist[(b*n*p)/e
, Int[(Log[d + e*x]*(a + b*Log[c*RFx^p])^(n - 1)*D[RFx, x])/RFx, x], x] /;
FreeQ[{a, b, c, d, e, p}, x] && RationalFunctionQ[RFx, x] && IGtQ[n, 0]
```

Rule 2525

```
Int[((a_.) + Log[(c_.)*(RFx_)^(p_.)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_))^(m_.
), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(a + b*Log[c*RFx^p])^n)/(e*(m + 1))
, x] - Dist[(b*n*p)/(e*(m + 1)), Int[SimplifyIntegrand[((d + e*x)^(m + 1)*(
a + b*Log[c*RFx^p])^(n - 1)*D[RFx, x])/RFx, x], x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && RationalFunctionQ[RFx, x] && IGtQ[n, 0] && (EqQ[n, 1] ||
IntegerQ[m]) && NeQ[m, -1]
```

Rule 2528

```
Int[((a_.) + Log[(c_.)*(RFx_)^(p_.)]*(b_.))^(n_.)*(RGx_), x_Symbol] :> With
[{u = ExpandIntegrand[(a + b*Log[c*RFx^p])^n, RGx, x]}, Int[u, x] /; SumQ[u
]] /; FreeQ[{a, b, c, p}, x] && RationalFunctionQ[RFx, x] && RationalFuncti
onQ[RGx, x] && IGtQ[n, 0]
```

Rubi steps

$$\int \frac{\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}{(f+gx)^4}\,dx = -\frac{\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}{3g(f+gx)^3} + \frac{(2B)\int \frac{2(bc-ad)\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}{(a+bx)(c+dx)(f+gx)^3}\,dx}{3g}$$

$$= -\frac{\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}{3g(f+gx)^3} + \frac{(4B(bc-ad))\int \frac{A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}{(a+bx)(c+dx)(f+gx)^3}\,dx}{3g}$$

$$= -\frac{\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}{3g(f+gx)^3} + \frac{(4B(bc-ad))\int \left(\frac{b^4\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}{(bc-ad)(bf-ag)^3(a+bx)} + \frac{d^4\left(A+B\log\left(\frac{e(a+\cdots}{(c+\cdots}\right.\right.}{(bc-ad)(-df+cg)^3\cdots}\right.}{\cdots}$$

$$= -\frac{\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}{3g(f+gx)^3} + \frac{\left(4b^4B\right)\int \frac{A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}{a+bx}\,dx}{3g(bf-ag)^3} - \frac{\left(4Bd^4\right)\int \frac{A+B\log\left(\frac{e(a+b\cdots}{(c+dx\cdots}\right.}{c+dx}\cdots}{3g(df-cg)^3}$$

$$= -\frac{2B(bc-ad)\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}{3(bf-ag)(df-cg)(f+gx)^2} - \frac{4B(bc-ad)(2bdf-bcg-adg)\left(A+B\,\mathrm{lo}\cdots\right.}{3(bf-ag)^2(df-cg)^2(f+gx\cdots}$$

$$= -\frac{2B(bc-ad)\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}{3(bf-ag)(df-cg)(f+gx)^2} - \frac{4B(bc-ad)(2bdf-bcg-adg)\left(A+B\,\mathrm{lo}\cdots\right.}{3(bf-ag)^2(df-cg)^2(f+gx\cdots}$$

$$= -\frac{2B(bc-ad)\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}{3(bf-ag)(df-cg)(f+gx)^2} - \frac{4B(bc-ad)(2bdf-bcg-adg)\left(A+B\,\mathrm{lo}\cdots\right.}{3(bf-ag)^2(df-cg)^2(f+gx\cdots}$$

$$= -\frac{4B^2(bc-ad)^2g}{3(bf-ag)^2(df-cg)^2(f+gx)} + \frac{4b^2B^2(bc-ad)\log(a+bx)}{3(bf-ag)^3(df-cg)} + \frac{8bB^2(bc-ad)\cdots}{3(\cdots}$$

$$= -\frac{4B^2(bc-ad)^2g}{3(bf-ag)^2(df-cg)^2(f+gx)} + \frac{4b^2B^2(bc-ad)\log(a+bx)}{3(bf-ag)^3(df-cg)} + \frac{8bB^2(bc-ad)\cdots}{3(\cdots}$$

$$= -\frac{4B^2(bc-ad)^2g}{3(bf-ag)^2(df-cg)^2(f+gx)} + \frac{4b^2B^2(bc-ad)\log(a+bx)}{3(bf-ag)^3(df-cg)} + \frac{8bB^2(bc-ad)\cdots}{3(\cdots}$$

$$= -\frac{4B^2(bc-ad)^2g}{3(bf-ag)^2(df-cg)^2(f+gx)} + \frac{4b^2B^2(bc-ad)\log(a+bx)}{3(bf-ag)^3(df-cg)} + \frac{8bB^2(bc-ad)\cdots}{3(\cdots}$$

**Mathematica [A]** time = 3.37, size = 909, normalized size = 1.26

$$-\frac{\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2+\frac{2B(f+gx)\left(2d^3(f+gx)^2\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)\log(c+dx)(bf-ag)^3-2Bd^3(f+gx)^2\left(\left(2\log\left(\frac{d(a+bx)}{ad-bc}\right)-\log(c+dx)\right)\log(c+\cdots}{\cdots}}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])^2/(f + g*x)^4,x]
```

```
[Out] -1/3*((A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])^2 + (2*B*(f + g*x)*((b*c - a
*d)*g*(b*f - a*g)^2*(d*f - c*g)^2*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])
+ 2*(b*c - a*d)*g*(b*f - a*g)*(-(d*f) + c*g)*(-2*b*d*f + b*c*g + a*d*g)*(f
+ g*x)*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2]) - 2*b^3*(d*f - c*g)^3*(f +
g*x)^2*Log[a + b*x]*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2]) + 2*d^3*(b*f -
 a*g)^3*(f + g*x)^2*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])*Log[c + d*x] -
 2*(b*c - a*d)*g*(a^2*d^2*g^2 + a*b*d*g*(-3*d*f + c*g) + b^2*(3*d^2*f^2 - 3
*c*d*f*g + c^2*g^2))*(f + g*x)^2*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])*L
og[f + g*x] - 4*B*(b*c - a*d)*g*(2*b*d*f - b*c*g - a*d*g)*(f + g*x)^2*(b*(d
*f - c*g)*Log[a + b*x] + (-(b*d*f) + a*d*g)*Log[c + d*x] + (b*c - a*d)*g*Lo
g[f + g*x]) + 2*B*(b*c - a*d)*g*(f + g*x)*((b*c - a*d)*g*(b*f - a*g)*(d*f -
 c*g) - b^2*(d*f - c*g)^2*(f + g*x)*Log[a + b*x] + d^2*(b*f - a*g)^2*(f + g
*x)*Log[c + d*x] + (b*c - a*d)*g*(-2*b*d*f + b*c*g + a*d*g)*(f + g*x)*Log[f
 + g*x]) + 2*b^3*B*(d*f - c*g)^3*(f + g*x)^2*(Log[a + b*x]*(Log[a + b*x] -
2*Log[(b*(c + d*x))/(b*c - a*d)]) - 2*PolyLog[2, (d*(a + b*x))/(-(b*c) + a*
d)]) - 2*B*d^3*(b*f - a*g)^3*(f + g*x)^2*((2*Log[(d*(a + b*x))/(-(b*c) + a*
d)] - Log[c + d*x])*Log[c + d*x] + 2*PolyLog[2, (b*(c + d*x))/(b*c - a*d)])
 + 4*B*(b*c - a*d)*g*(a^2*d^2*g^2 + a*b*d*g*(-3*d*f + c*g) + b^2*(3*d^2*f^2
 - 3*c*d*f*g + c^2*g^2))*(f + g*x)^2*((Log[(g*(a + b*x))/(-(b*f) + a*g)] -
Log[(g*(c + d*x))/(-(d*f) + c*g)])*Log[f + g*x] + PolyLog[2, (b*(f + g*x))/
(b*f - a*g)] - PolyLog[2, (d*(f + g*x))/(d*f - c*g)])))/((b*f - a*g)^3*(d*f
 - c*g)^3))/(g*(f + g*x)^3)
```

**fricas [F]** time = 0.98, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{B^2\log\left(\frac{b^2ex^2+2\,abex+a^2e}{d^2x^2+2\,cdx+c^2}\right)^2+2\,AB\log\left(\frac{b^2ex^2+2\,abex+a^2e}{d^2x^2+2\,cdx+c^2}\right)+A^2}{g^4x^4+4\,fg^3x^3+6\,f^2g^2x^2+4\,f^3gx+f^4},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*(b*x+a)^2/(d*x+c)^2))^2/(g*x+f)^4,x, algorithm="fricas
")
```

```
[Out] integral((B^2*log((b^2*e*x^2 + 2*a*b*e*x + a^2*e)/(d^2*x^2 + 2*c*d*x + c^2)
)^2 + 2*A*B*log((b^2*e*x^2 + 2*a*b*e*x + a^2*e)/(d^2*x^2 + 2*c*d*x + c^2))
+ A^2)/(g^4*x^4 + 4*f*g^3*x^3 + 6*f^2*g^2*x^2 + 4*f^3*g*x + f^4), x)
```

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(B\log\left(\frac{(bx+a)^2e}{(dx+c)^2}\right)+A\right)^2}{(gx+f)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*(b*x+a)^2/(d*x+c)^2))^2/(g*x+f)^4,x, algorithm="giac")
```

```
[Out] integrate((B*log((b*x + a)^2*e/(d*x + c)^2) + A)^2/(g*x + f)^4, x)
```

**maple [F]** time = 2.71, size = 0, normalized size = 0.00

$$\int \frac{\left(B\ln\left(\frac{(bx+a)^2e}{(dx+c)^2}\right)+A\right)^2}{(gx+f)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*ln((b*x+a)^2/(d*x+c)^2*e)+A)^2/(g*x+f)^4,x)
```

```
[Out] int((B*ln((b*x+a)^2/(d*x+c)^2*e)+A)^2/(g*x+f)^4,x)
```

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*(b*x+a)^2/(d*x+c)^2))^2/(g*x+f)^4,x, algorithm="maxima
")
```

```
[Out] 2/3*(2*b^3*log(b*x + a)/(b^3*f^3*g - 3*a*b^2*f^2*g^2 + 3*a^2*b*f*g^3 - a^3*
g^4) - 2*d^3*log(d*x + c)/(d^3*f^3*g - 3*c*d^2*f^2*g^2 + 3*c^2*d*f*g^3 - c^
3*g^4) + 2*(3*(b^3*c*d^2 - a*b^2*d^3)*f^2 - 3*(b^3*c^2*d - a^2*b*d^3)*f*g +
 (b^3*c^3 - a^3*d^3)*g^2)*log(g*x + f)/(b^3*d^3*f^6 + a^3*c^3*g^6 - 3*(b^3*
c*d^2 + a*b^2*d^3)*f^5*g + 3*(b^3*c^2*d + 3*a*b^2*c*d^2 + a^2*b*d^3)*f^4*g^
2 - (b^3*c^3 + 9*a*b^2*c^2*d + 9*a^2*b*c*d^2 + a^3*d^3)*f^3*g^3 + 3*(a*b^2*
c^3 + 3*a^2*b*c^2*d + a^3*c*d^2)*f^2*g^4 - 3*(a^2*b*c^3 + a^3*c^2*d)*f*g^5)
 - (5*(b^2*c*d - a*b*d^2)*f^2 - 3*(b^2*c^2 - a^2*d^2)*f*g + (a*b*c^2 - a^2*
c*d)*g^2 + 2*(2*(b^2*c*d - a*b*d^2)*f*g - (b^2*c^2 - a^2*d^2)*g^2)*x)/(b^2*
```

```
d^2*f^6 + a^2*c^2*f^2*g^4 - 2*(b^2*c*d + a*b*d^2)*f^5*g + (b^2*c^2 + 4*a*b*
c*d + a^2*d^2)*f^4*g^2 - 2*(a*b*c^2 + a^2*c*d)*f^3*g^3 + (b^2*d^2*f^4*g^2 +
 a^2*c^2*g^6 - 2*(b^2*c*d + a*b*d^2)*f^3*g^3 + (b^2*c^2 + 4*a*b*c*d + a^2*d
^2)*f^2*g^4 - 2*(a*b*c^2 + a^2*c*d)*f*g^5)*x^2 + 2*(b^2*d^2*f^5*g + a^2*c^2
*f*g^5 - 2*(b^2*c*d + a*b*d^2)*f^4*g^2 + (b^2*c^2 + 4*a*b*c*d + a^2*d^2)*f^
3*g^3 - 2*(a*b*c^2 + a^2*c*d)*f^2*g^4)*x) - log(b^2*e*x^2/(d^2*x^2 + 2*c*d*
x + c^2) + 2*a*b*e*x/(d^2*x^2 + 2*c*d*x + c^2) + a^2*e/(d^2*x^2 + 2*c*d*x +
 c^2))/(g^4*x^3 + 3*f*g^3*x^2 + 3*f^2*g^2*x + f^3*g))*A*B - 1/3*B^2*(4*log(
d*x + c)^2/(g^4*x^3 + 3*f*g^3*x^2 + 3*f^2*g^2*x + f^3*g) + 3*integrate(-1/3
*(3*d*g*x*log(e)^2 + 3*c*g*log(e)^2 + 12*(d*g*x + c*g)*log(b*x + a)^2 + 12*
(d*g*x*log(e) + c*g*log(e))*log(b*x + a) - 4*((3*g*log(e) - 2*g)*d*x + 3*c*
g*log(e) - 2*d*f + 6*(d*g*x + c*g)*log(b*x + a))*log(d*x + c))/(d*g^5*x^5 +
 c*f^4*g + (4*d*f*g^4 + c*g^5)*x^4 + 2*(3*d*f^2*g^3 + 2*c*f*g^4)*x^3 + 2*(2
*d*f^3*g^2 + 3*c*f^2*g^3)*x^2 + (d*f^4*g + 4*c*f^3*g^2)*x), x)) - 1/3*A^2/(
g^4*x^3 + 3*f*g^3*x^2 + 3*f^2*g^2*x + f^3*g)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(A+B\,\ln\left(\frac{e\,{\left(a+b\,x\right)}^2}{{\left(c+d\,x\right)}^2}\right)\right)^2}{{\left(f+g\,x\right)}^4} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*log((e*(a + b*x)^2)/(c + d*x)^2))^2/(f + g*x)^4,x)
[Out] int((A + B*log((e*(a + b*x)^2)/(c + d*x)^2))^2/(f + g*x)^4, x)
```

**sympy [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*ln(e*(b*x+a)**2/(d*x+c)**2))**2/(g*x+f)**4,x)
[Out] Timed out
```

# 3.280 $\int \frac{\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}{(f+gx)^5}\,dx$

**Optimal**. Leaf size=1154

$$\frac{\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2 b^4}{4g(bf-ag)^4}-\frac{\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}{4g(f+gx)^4}+\frac{B(bc-ad)g\left(\left(6d^2f^2-4cdgf+c^2g^2\right)b^2-2adg(4df-cg)b}{(bf-ag)^4(df-cg)^3(f}$$

```
[Out] -1/3*B^2*(-a*d+b*c)^2*g^3*(d*x+c)^2/(-a*g+b*f)^2/(-c*g+d*f)^4/(g*x+f)^2-2/3
*B^2*(-a*d+b*c)^3*g^3*(d*x+c)/(-a*g+b*f)^3/(-c*g+d*f)^4/(g*x+f)+B^2*(-a*d+b
*c)^2*g^2*(-3*a*d*g-b*c*g+4*b*d*f)*(d*x+c)/(-a*g+b*f)^3/(-c*g+d*f)^4/(g*x+f
)+1/3*B*(-a*d+b*c)*g^3*(d*x+c)^3*(A+B*ln(e*(b*x+a)^2/(d*x+c)^2))/(-a*g+b*f)
/(-c*g+d*f)^4/(g*x+f)^3-1/2*B*(-a*d+b*c)*g^2*(-3*a*d*g-b*c*g+4*b*d*f)*(d*x+
c)^2*(A+B*ln(e*(b*x+a)^2/(d*x+c)^2))/(-a*g+b*f)^2/(-c*g+d*f)^4/(g*x+f)^2+B*
(-a*d+b*c)*g*(3*a^2*d^2*g^2-2*a*b*d*g*(-c*g+4*d*f)+b^2*(c^2*g^2-4*c*d*f*g+6
*d^2*f^2))*(b*x+a)*(A+B*ln(e*(b*x+a)^2/(d*x+c)^2))/(-a*g+b*f)^4/(-c*g+d*f)^
3/(g*x+f)+1/4*b^4*(A+B*ln(e*(b*x+a)^2/(d*x+c)^2))^2/g/(-a*g+b*f)^4-1/4*(A+B
*ln(e*(b*x+a)^2/(d*x+c)^2))^2/g/(g*x+f)^4-2/3*B^2*(-a*d+b*c)^4*g^3*ln((b*x+
a)/(d*x+c))/(-a*g+b*f)^4/(-c*g+d*f)^4+B^2*(-a*d+b*c)^3*g^2*(-3*a*d*g-b*c*g+
4*b*d*f)*ln((b*x+a)/(d*x+c))/(-a*g+b*f)^4/(-c*g+d*f)^4+2/3*B^2*(-a*d+b*c)^4
*g^3*ln((g*x+f)/(d*x+c))/(-a*g+b*f)^4/(-c*g+d*f)^4-B^2*(-a*d+b*c)^3*g^2*(-3
*a*d*g-b*c*g+4*b*d*f)*ln((g*x+f)/(d*x+c))/(-a*g+b*f)^4/(-c*g+d*f)^4+2*B^2*(
-a*d+b*c)^2*g*(3*a^2*d^2*g^2-2*a*b*d*g*(-c*g+4*d*f)+b^2*(c^2*g^2-4*c*d*f*g+
6*d^2*f^2))*ln((g*x+f)/(d*x+c))/(-a*g+b*f)^4/(-c*g+d*f)^4-B*(-a*d+b*c)*(-a*
d*g-b*c*g+2*b*d*f)*(2*a*b*d^2*f*g-a^2*d^2*g^2-b^2*(c^2*g^2-2*c*d*f*g+2*d^2*
f^2))*(A+B*ln(e*(b*x+a)^2/(d*x+c)^2))*ln(1-(-c*g+d*f)*(b*x+a)/(-a*g+b*f)/(d
*x+c))/(-a*g+b*f)^4/(-c*g+d*f)^4-2*B^2*(-a*d+b*c)*(-a*d*g-b*c*g+2*b*d*f)*(2
*a*b*d^2*f*g-a^2*d^2*g^2-b^2*(c^2*g^2-2*c*d*f*g+2*d^2*f^2))*polylog(2,(-c*g
+d*f)*(b*x+a)/(-a*g+b*f)/(d*x+c))/(-a*g+b*f)^4/(-c*g+d*f)^4
```

**Rubi [A]** time = 3.54, antiderivative size = 1854, normalized size of antiderivative = 1.61, number of steps used = 44, number of rules used = 11, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.355, Rules used = {2525, 12, 2528, 2524, 2418, 2390, 2301, 2394, 2393, 2391, 72}

result too large to display

Antiderivative was successfully verified.

```
[In] Int[(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])^2/(f + g*x)^5,x]
```

```
[Out] -(B^2*(b*c - a*d)^2*g)/(3*(b*f - a*g)^2*(d*f - c*g)^2*(f + g*x)^2) - (5*B^2
*(b*c - a*d)^2*g*(2*b*d*f - b*c*g - a*d*g))/(3*(b*f - a*g)^3*(d*f - c*g)^3*
```

```
(f + g*x)) + (2*b^3*B^2*(b*c - a*d)*Log[a + b*x])/(3*(b*f - a*g)^4*(d*f - c
*g)) + (b^2*B^2*(b*c - a*d)*(2*b*d*f - b*c*g - a*d*g)*Log[a + b*x])/((b*f -
 a*g)^4*(d*f - c*g)^2) + (2*b*B^2*(b*c - a*d)*(a^2*d^2*g^2 - a*b*d*g*(3*d*f
 - c*g) + b^2*(3*d^2*f^2 - 3*c*d*f*g + c^2*g^2))*Log[a + b*x])/((b*f - a*g)
^4*(d*f - c*g)^3) - (b^4*B^2*Log[a + b*x]^2)/(g*(b*f - a*g)^4) - (B*(b*c -
a*d)*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2]))/(3*(b*f - a*g)*(d*f - c*g)*(
f + g*x)^3) - (B*(b*c - a*d)*(2*b*d*f - b*c*g - a*d*g)*(A + B*Log[(e*(a + b
*x)^2)/(c + d*x)^2]))/(2*(b*f - a*g)^2*(d*f - c*g)^2*(f + g*x)^2) - (B*(b*c
 - a*d)*(a^2*d^2*g^2 - a*b*d*g*(3*d*f - c*g) + b^2*(3*d^2*f^2 - 3*c*d*f*g +
 c^2*g^2))*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2]))/((b*f - a*g)^3*(d*f -
c*g)^3*(f + g*x)) + (b^4*B*Log[a + b*x]*(A + B*Log[(e*(a + b*x)^2)/(c + d*x
)^2]))/(g*(b*f - a*g)^4) - (A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])^2/(4*g*
(f + g*x)^4) - (2*B^2*d^3*(b*c - a*d)*Log[c + d*x])/(3*(b*f - a*g)*(d*f - c
*g)^4) - (B^2*d^2*(b*c - a*d)*(2*b*d*f - b*c*g - a*d*g)*Log[c + d*x])/((b*f
 - a*g)^2*(d*f - c*g)^4) - (2*B^2*d*(b*c - a*d)*(a^2*d^2*g^2 - a*b*d*g*(3*d
*f - c*g) + b^2*(3*d^2*f^2 - 3*c*d*f*g + c^2*g^2))*Log[c + d*x])/((b*f - a*
g)^3*(d*f - c*g)^4) + (2*B^2*d^4*Log[-((d*(a + b*x))/(b*c - a*d))]*Log[c +
d*x])/(g*(d*f - c*g)^4) - (B*d^4*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])*L
og[c + d*x])/(g*(d*f - c*g)^4) - (B^2*d^4*Log[c + d*x]^2)/(g*(d*f - c*g)^4)
 + (2*b^4*B^2*Log[a + b*x]*Log[(b*(c + d*x))/(b*c - a*d)])/(g*(b*f - a*g)^4
) + (B^2*(b*c - a*d)^2*g*(2*b*d*f - b*c*g - a*d*g)^2*Log[f + g*x])/((b*f -
a*g)^4*(d*f - c*g)^4) + (8*B^2*(b*c - a*d)^2*g*(a^2*d^2*g^2 - a*b*d*g*(3*d*
f - c*g) + b^2*(3*d^2*f^2 - 3*c*d*f*g + c^2*g^2))*Log[f + g*x])/(3*(b*f - a
*g)^4*(d*f - c*g)^4) + (2*B^2*(b*c - a*d)*(2*b*d*f - b*c*g - a*d*g)*(2*a*b*
d^2*f*g - a^2*d^2*g^2 - b^2*(2*d^2*f^2 - 2*c*d*f*g + c^2*g^2))*Log[-((g*(a
+ b*x))/(b*f - a*g))]*Log[f + g*x])/((b*f - a*g)^4*(d*f - c*g)^4) - (B*(b*c
 - a*d)*(2*b*d*f - b*c*g - a*d*g)*(2*a*b*d^2*f*g - a^2*d^2*g^2 - b^2*(2*d^2
*f^2 - 2*c*d*f*g + c^2*g^2))*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])*Log[f
 + g*x])/((b*f - a*g)^4*(d*f - c*g)^4) - (2*B^2*(b*c - a*d)*(2*b*d*f - b*c*
g - a*d*g)*(2*a*b*d^2*f*g - a^2*d^2*g^2 - b^2*(2*d^2*f^2 - 2*c*d*f*g + c^2*
g^2))*Log[-((g*(c + d*x))/(d*f - c*g))]*Log[f + g*x])/((b*f - a*g)^4*(d*f -
 c*g)^4) + (2*b^4*B^2*PolyLog[2, -((d*(a + b*x))/(b*c - a*d))])/(g*(b*f - a
*g)^4) + (2*B^2*d^4*PolyLog[2, (b*(c + d*x))/(b*c - a*d)])/(g*(d*f - c*g)^4
) + (2*B^2*(b*c - a*d)*(2*b*d*f - b*c*g - a*d*g)*(2*a*b*d^2*f*g - a^2*d^2*g
^2 - b^2*(2*d^2*f^2 - 2*c*d*f*g + c^2*g^2))*PolyLog[2, (b*(f + g*x))/(b*f -
 a*g)])/((b*f - a*g)^4*(d*f - c*g)^4) - (2*B^2*(b*c - a*d)*(2*b*d*f - b*c*g
 - a*d*g)*(2*a*b*d^2*f*g - a^2*d^2*g^2 - b^2*(2*d^2*f^2 - 2*c*d*f*g + c^2*g
^2))*PolyLog[2, (d*(f + g*x))/(d*f - c*g)])/((b*f - a*g)^4*(d*f - c*g)^4)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 72

```
Int[((e_.) + (f_.)*(x_))^(p_.)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
 x_Symbol] :> Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x]
 /; FreeQ[{a, b, c, d, e, f}, x] && IntegerQ[p]
```

Rule 2301

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

Rule 2390

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_))^(q_.), x_Symbol] :> Dist[1/e, Subst[Int[((f*x)/d)^q*(a + b*Log[c*x^
n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && E
qQ[e*f - d*g, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2393

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + (c*e*x)/g])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2394

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d + e*x
)^n]))/g, x] - Dist[(b*e*n)/g, Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2418

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*(RFx_), x_Sy
mbol] :> With[{u = ExpandIntegrand[(a + b*Log[c*(d + e*x)^n])^p, RFx, x]},
Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d, e, n}, x] && RationalFunctionQ[
RFx, x] && IntegerQ[p]
```

Rule 2524

```
Int[((a_.) + Log[(c_.)*(RFx_)^(p_.)]*(b_.))^(n_.)/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[(Log[d + e*x]*(a + b*Log[c*RFx^p])^n)/e, x] - Dist[(b*n*p)/e
, Int[(Log[d + e*x]*(a + b*Log[c*RFx^p])^(n - 1)*D[RFx, x])/RFx, x], x] /;
FreeQ[{a, b, c, d, e, p}, x] && RationalFunctionQ[RFx, x] && IGtQ[n, 0]
```

Rule 2525

```
Int[((a_.) + Log[(c_.)*(RFx_)^(p_.)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_))^(m_.
), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(a + b*Log[c*RFx^p])^n)/(e*(m + 1))
, x] - Dist[(b*n*p)/(e*(m + 1)), Int[SimplifyIntegrand[((d + e*x)^(m + 1)*(
a + b*Log[c*RFx^p])^(n - 1)*D[RFx, x])/RFx, x], x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && RationalFunctionQ[RFx, x] && IGtQ[n, 0] && (EqQ[n, 1] ||
IntegerQ[m]) && NeQ[m, -1]
```

Rule 2528

```
Int[((a_.) + Log[(c_.)*(RFx_)^(p_.)]*(b_.))^(n_.)*(RGx_), x_Symbol] :> With
[{u = ExpandIntegrand[(a + b*Log[c*RFx^p])^n, RGx, x]}, Int[u, x] /; SumQ[u
]] /; FreeQ[{a, b, c, p}, x] && RationalFunctionQ[RFx, x] && RationalFuncti
onQ[RGx, x] && IGtQ[n, 0]
```

Rubi steps

$$\int \frac{\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}{(f+gx)^5}\,dx = -\frac{\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}{4g(f+gx)^4} + \frac{B\int \frac{2(bc-ad)\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}{(a+bx)(c+dx)(f+gx)^4}\,dx}{2g}$$

$$= -\frac{\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}{4g(f+gx)^4} + \frac{(B(bc-ad))\int \frac{A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}{(a+bx)(c+dx)(f+gx)^4}\,dx}{g}$$

$$= -\frac{\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}{4g(f+gx)^4} + \frac{(B(bc-ad))\int \left(\frac{b^5\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}{(bc-ad)(bf-ag)^4(a+bx)} - \frac{d^5\left(A+B\log\left(\frac{e(a+bx}{(c+dx}\right.\right.}{(bc-ad)(-df+cg)^4(c}\right.}{}$$

$$= -\frac{\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}{4g(f+gx)^4} + \frac{\left(b^5B\right)\int \frac{A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}{a+bx}\,dx}{g(bf-ag)^4} - \frac{\left(Bd^5\right)\int \frac{A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right.}{c+dx}}{g(df-cg)^4}$$

$$= -\frac{B(bc-ad)\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}{3(bf-ag)(df-cg)(f+gx)^3} - \frac{B(bc-ad)(2bdf-bcg-adg)\left(A+B\log\right.}{2(bf-ag)^2(df-cg)^2(f+gx)^2}$$

$$= -\frac{B(bc-ad)\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}{3(bf-ag)(df-cg)(f+gx)^3} - \frac{B(bc-ad)(2bdf-bcg-adg)\left(A+B\log\right.}{2(bf-ag)^2(df-cg)^2(f+gx)^2}$$

$$= -\frac{B(bc-ad)\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}{3(bf-ag)(df-cg)(f+gx)^3} - \frac{B(bc-ad)(2bdf-bcg-adg)\left(A+B\log\right.}{2(bf-ag)^2(df-cg)^2(f+gx)^2}$$

$$= -\frac{B^2(bc-ad)^2g}{3(bf-ag)^2(df-cg)^2(f+gx)^2} - \frac{5B^2(bc-ad)^2g(2bdf-bcg-adg)}{3(bf-ag)^3(df-cg)^3(f+gx)} + \frac{2b^3B^2}{3(b}$$

$$= -\frac{B^2(bc-ad)^2g}{3(bf-ag)^2(df-cg)^2(f+gx)^2} - \frac{5B^2(bc-ad)^2g(2bdf-bcg-adg)}{3(bf-ag)^3(df-cg)^3(f+gx)} + \frac{2b^3B^2}{3(b}$$

$$= -\frac{B^2(bc-ad)^2g}{3(bf-ag)^2(df-cg)^2(f+gx)^2} - \frac{5B^2(bc-ad)^2g(2bdf-bcg-adg)}{3(bf-ag)^3(df-cg)^3(f+gx)} + \frac{2b^3B^2}{3(b}$$

$$= -\frac{B^2(bc-ad)^2g}{3(bf-ag)^2(df-cg)^2(f+gx)^2} - \frac{5B^2(bc-ad)^2g(2bdf-bcg-adg)}{3(bf-ag)^3(df-cg)^3(f+gx)} + \frac{2b^3B^2}{3(b}$$

**Mathematica [A]** time = 7.34, size = 1453, normalized size = 1.26

$$\frac{B(bc-ad)\left(\frac{\log(a+bx)\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)b^4}{(bc-ad)(bf-ag)^4}-\frac{B\left(\log^2(a+bx)-2\log\left(\frac{b(c+dx)}{bc-ad}\right)\log(a+bx)-2\text{Li}_2\left(-\frac{d(a+bx)}{bc-ad}\right)\right)b^4}{(bc-ad)(bf-ag)^4}-\frac{g\left(\left(3d^2f^2-3cdgf+c^2g^2\right)b^2-adg(3\cdots\right.}{(bf-ag)^3\cdots}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])^2/(f + g*x)^5,x]
```

```
[Out] -1/4*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])^2/(g*(f + g*x)^4) + (B*(b*c -
 a*d)*(-1/3*(g*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2]))/((b*f - a*g)*(d*f
- c*g)*(f + g*x)^3) - (g*(2*b*d*f - b*c*g - a*d*g)*(A + B*Log[(e*(a + b*x)^
2)/(c + d*x)^2]))/(2*(b*f - a*g)^2*(d*f - c*g)^2*(f + g*x)^2) - (g*(a^2*d^2
*g^2 - a*b*d*g*(3*d*f - c*g) + b^2*(3*d^2*f^2 - 3*c*d*f*g + c^2*g^2))*(A +
B*Log[(e*(a + b*x)^2)/(c + d*x)^2]))/((b*f - a*g)^3*(d*f - c*g)^3*(f + g*x)
) + (b^4*Log[a + b*x]*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2]))/((b*c - a*d
)*(b*f - a*g)^4) - (d^4*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])*Log[c + d*
x])/((b*c - a*d)*(d*f - c*g)^4) + (g*(2*b*d*f - b*c*g - a*d*g)*(2*b^2*d^2*f
^2 - 2*b^2*c*d*f*g - 2*a*b*d^2*f*g + b^2*c^2*g^2 + a^2*d^2*g^2)*(A + B*Log[
(e*(a + b*x)^2)/(c + d*x)^2])*Log[f + g*x])/((b*f - a*g)^4*(d*f - c*g)^4) +
 (2*B*(b*c - a*d)*g*(a^2*d^2*g^2 - a*b*d*g*(3*d*f - c*g) + b^2*(3*d^2*f^2 -
 3*c*d*f*g + c^2*g^2))*((b*Log[a + b*x])/((b*c - a*d)*(b*f - a*g)) - (d*Log
[c + d*x])/((b*c - a*d)*(d*f - c*g)) + (g*Log[f + g*x])/((b*f - a*g)*(d*f -
 c*g))))/((b*f - a*g)^3*(d*f - c*g)^3) - (B*(b*c - a*d)*g*(2*b*d*f - b*c*g
- a*d*g)*(g/((b*f - a*g)*(d*f - c*g)*(f + g*x)) - (b^2*Log[a + b*x])/((b*c
- a*d)*(b*f - a*g)^2) + (d^2*Log[c + d*x])/((b*c - a*d)*(d*f - c*g)^2) - (g
*(2*b*d*f - b*c*g - a*d*g)*Log[f + g*x])/((b*f - a*g)^2*(d*f - c*g)^2)))/((
b*f - a*g)^2*(d*f - c*g)^2) - (B*(b*c - a*d)*g*(g/((b*f - a*g)*(d*f - c*g)*
(f + g*x)^2) + (2*g*(2*b*d*f - b*c*g - a*d*g))/((b*f - a*g)^2*(d*f - c*g)^2
*(f + g*x)) - (2*b^3*Log[a + b*x])/((b*c - a*d)*(b*f - a*g)^3) + (2*d^3*Log
[c + d*x])/((b*c - a*d)*(d*f - c*g)^3) - (2*g*(a^2*d^2*g^2 - a*b*d*g*(3*d*f
 - c*g) + b^2*(3*d^2*f^2 - 3*c*d*f*g + c^2*g^2))*Log[f + g*x])/((b*f - a*g)
^3*(d*f - c*g)^3)))/(3*(b*f - a*g)*(d*f - c*g)) - (b^4*B*(Log[a + b*x]^2 -
2*Log[a + b*x]*Log[(b*(c + d*x))/(b*c - a*d)] - 2*PolyLog[2, -((d*(a + b*x)
)/(b*c - a*d))]))/((b*c - a*d)*(b*f - a*g)^4) + (B*d^4*(2*Log[-((d*(a + b*x
))/(b*c - a*d))]*Log[c + d*x] - Log[c + d*x]^2 + 2*PolyLog[2, (b*(c + d*x))
/(b*c - a*d)]))/((b*c - a*d)*(d*f - c*g)^4) - (2*B*g*(2*b*d*f - b*c*g - a*d
*g)*(2*b^2*d^2*f^2 - 2*b^2*c*d*f*g - 2*a*b*d^2*f*g + b^2*c^2*g^2 + a^2*d^2*
g^2)*(Log[-((g*(a + b*x))/(b*f - a*g))]*Log[f + g*x] - Log[-((g*(c + d*x))/
(d*f - c*g))]*Log[f + g*x] + PolyLog[2, (b*(f + g*x))/(b*f - a*g)] - PolyLo
g[2, (d*(f + g*x))/(d*f - c*g)]))/((b*f - a*g)^4*(d*f - c*g)^4)))/g
```

**fricas [F]** time = 2.08, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{B^2\log\left(\frac{b^2ex^2+2\,abex+a^2e}{d^2x^2+2\,cdx+c^2}\right)^2+2\,AB\log\left(\frac{b^2ex^2+2\,abex+a^2e}{d^2x^2+2\,cdx+c^2}\right)+A^2}{g^5x^5+5\,fg^4x^4+10\,f^2g^3x^3+10\,f^3g^2x^2+5\,f^4gx+f^5},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*(b*x+a)^2/(d*x+c)^2))^2/(g*x+f)^5,x, algorithm="fricas
")
```

```
[Out] integral((B^2*log((b^2*e*x^2 + 2*a*b*e*x + a^2*e)/(d^2*x^2 + 2*c*d*x + c^2)
)^2 + 2*A*B*log((b^2*e*x^2 + 2*a*b*e*x + a^2*e)/(d^2*x^2 + 2*c*d*x + c^2))
+ A^2)/(g^5*x^5 + 5*f*g^4*x^4 + 10*f^2*g^3*x^3 + 10*f^3*g^2*x^2 + 5*f^4*g*x
 + f^5), x)
```

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\left(B\log\left(\frac{(bx+a)^2e}{(dx+c)^2}\right)+A\right)^2}{(gx+f)^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*(b*x+a)^2/(d*x+c)^2))^2/(g*x+f)^5,x, algorithm="giac")
```

```
[Out] integrate((B*log((b*x + a)^2*e/(d*x + c)^2) + A)^2/(g*x + f)^5, x)
```

**maple [F]** time = 4.09, size = 0, normalized size = 0.00

$$\int\frac{\left(B\ln\left(\frac{(bx+a)^2e}{(dx+c)^2}\right)+A\right)^2}{(gx+f)^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*ln((b*x+a)^2/(d*x+c)^2*e)+A)^2/(g*x+f)^5,x)
```

```
[Out] int((B*ln((b*x+a)^2/(d*x+c)^2*e)+A)^2/(g*x+f)^5,x)
```

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*(b*x+a)^2/(d*x+c)^2))^2/(g*x+f)^5,x, algorithm="maxima
")

[Out] 1/6*(6*b^4*log(b*x + a)/(b^4*f^4*g - 4*a*b^3*f^3*g^2 + 6*a^2*b^2*f^2*g^3 -
4*a^3*b*f*g^4 + a^4*g^5) - 6*d^4*log(d*x + c)/(d^4*f^4*g - 4*c*d^3*f^3*g^2
+ 6*c^2*d^2*f^2*g^3 - 4*c^3*d*f*g^4 + c^4*g^5) + 6*(4*(b^4*c*d^3 - a*b^3*d^
4)*f^3 - 6*(b^4*c^2*d^2 - a^2*b^2*d^4)*f^2*g + 4*(b^4*c^3*d - a^3*b*d^4)*f*
g^2 - (b^4*c^4 - a^4*d^4)*g^3)*log(g*x + f)/(b^4*d^4*f^8 + a^4*c^4*g^8 - 4*
(b^4*c*d^3 + a*b^3*d^4)*f^7*g + 2*(3*b^4*c^2*d^2 + 8*a*b^3*c*d^3 + 3*a^2*b^
2*d^4)*f^6*g^2 - 4*(b^4*c^3*d + 6*a*b^3*c^2*d^2 + 6*a^2*b^2*c*d^3 + a^3*b*d
^4)*f^5*g^3 + (b^4*c^4 + 16*a*b^3*c^3*d + 36*a^2*b^2*c^2*d^2 + 16*a^3*b*c*d
^3 + a^4*d^4)*f^4*g^4 - 4*(a*b^3*c^4 + 6*a^2*b^2*c^3*d + 6*a^3*b*c^2*d^2 +
a^4*c*d^3)*f^3*g^5 + 2*(3*a^2*b^2*c^4 + 8*a^3*b*c^3*d + 3*a^4*c^2*d^2)*f^2*
g^6 - 4*(a^3*b*c^4 + a^4*c^3*d)*f*g^7) - (26*(b^3*c*d^2 - a*b^2*d^3)*f^4 -
31*(b^3*c^2*d - a^2*b*d^3)*f^3*g + (11*b^3*c^3 + 15*a*b^2*c^2*d - 15*a^2*b*
c*d^2 - 11*a^3*d^3)*f^2*g^2 - 7*(a*b^2*c^3 - a^3*c*d^2)*f*g^3 + 2*(a^2*b*c^
3 - a^3*c^2*d)*g^4 + 6*(3*(b^3*c*d^2 - a*b^2*d^3)*f^2*g^2 - 3*(b^3*c^2*d -
a^2*b*d^3)*f*g^3 + (b^3*c^3 - a^3*d^3)*g^4)*x^2 + 3*(14*(b^3*c*d^2 - a*b^2*
d^3)*f^3*g - 15*(b^3*c^2*d - a^2*b*d^3)*f^2*g^2 + (5*b^3*c^3 + 3*a*b^2*c^2*
d - 3*a^2*b*c*d^2 - 5*a^3*d^3)*f*g^3 - (a*b^2*c^3 - a^3*c*d^2)*g^4)*x)/(b^3
*d^3*f^9 + a^3*c^3*f^3*g^6 - 3*(b^3*c*d^2 + a*b^2*d^3)*f^8*g + 3*(b^3*c^2*d
 + 3*a*b^2*c*d^2 + a^2*b*d^3)*f^7*g^2 - (b^3*c^3 + 9*a*b^2*c^2*d + 9*a^2*b*
c*d^2 + a^3*d^3)*f^6*g^3 + 3*(a*b^2*c^3 + 3*a^2*b*c^2*d + a^3*c*d^2)*f^5*g^
4 - 3*(a^2*b*c^3 + a^3*c^2*d)*f^4*g^5 + (b^3*d^3*f^6*g^3 + a^3*c^3*g^9 - 3*
(b^3*c*d^2 + a*b^2*d^3)*f^5*g^4 + 3*(b^3*c^2*d + 3*a*b^2*c*d^2 + a^2*b*d^3)
*f^4*g^5 - (b^3*c^3 + 9*a*b^2*c^2*d + 9*a^2*b*c*d^2 + a^3*d^3)*f^3*g^6 + 3*
(a*b^2*c^3 + 3*a^2*b*c^2*d + a^3*c*d^2)*f^2*g^7 - 3*(a^2*b*c^3 + a^3*c^2*d)
*f*g^8)*x^3 + 3*(b^3*d^3*f^7*g^2 + a^3*c^3*f*g^8 - 3*(b^3*c*d^2 + a*b^2*d^3
)*f^6*g^3 + 3*(b^3*c^2*d + 3*a*b^2*c*d^2 + a^2*b*d^3)*f^5*g^4 - (b^3*c^3 +
9*a*b^2*c^2*d + 9*a^2*b*c*d^2 + a^3*d^3)*f^4*g^5 + 3*(a*b^2*c^3 + 3*a^2*b*c
^2*d + a^3*c*d^2)*f^3*g^6 - 3*(a^2*b*c^3 + a^3*c^2*d)*f^2*g^7)*x^2 + 3*(b^3
*d^3*f^8*g + a^3*c^3*f^2*g^7 - 3*(b^3*c*d^2 + a*b^2*d^3)*f^7*g^2 + 3*(b^3*c
^2*d + 3*a*b^2*c*d^2 + a^2*b*d^3)*f^6*g^3 - (b^3*c^3 + 9*a*b^2*c^2*d + 9*a^
2*b*c*d^2 + a^3*d^3)*f^5*g^4 + 3*(a*b^2*c^3 + 3*a^2*b*c^2*d + a^3*c*d^2)*f^
4*g^5 - 3*(a^2*b*c^3 + a^3*c^2*d)*f^3*g^6)*x) - 3*log(b^2*e*x^2/(d^2*x^2 +
2*c*d*x + c^2) + 2*a*b*e*x/(d^2*x^2 + 2*c*d*x + c^2) + a^2*e/(d^2*x^2 + 2*c
*d*x + c^2))/(g^5*x^4 + 4*f*g^4*x^3 + 6*f^2*g^3*x^2 + 4*f^3*g^2*x + f^4*g))
*A*B - B^2*(log(d*x + c)^2/(g^5*x^4 + 4*f*g^4*x^3 + 6*f^2*g^3*x^2 + 4*f^3*g
^2*x + f^4*g) + integrate(-(d*g*x*log(e)^2 + c*g*log(e)^2 + 4*(d*g*x + c*g)
*log(b*x + a)^2 + 4*(d*g*x*log(e) + c*g*log(e))*log(b*x + a) - 2*((2*g*log(
e) - g)*d*x + 2*c*g*log(e) - d*f + 4*(d*g*x + c*g)*log(b*x + a))*log(d*x +
c))/(d*g^6*x^6 + c*f^5*g + (5*d*f*g^5 + c*g^6)*x^5 + 5*(2*d*f^2*g^4 + c*f*g
^5)*x^4 + 10*(d*f^3*g^3 + c*f^2*g^4)*x^3 + 5*(d*f^4*g^2 + 2*c*f^3*g^3)*x^2
+ (d*f^5*g + 5*c*f^4*g^2)*x), x)) - 1/4*A^2/(g^5*x^4 + 4*f*g^4*x^3 + 6*f^2*
g^3*x^2 + 4*f^3*g^2*x + f^4*g)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(A + B \ln\left(\frac{e\,(a+b\,x)^2}{(c+d\,x)^2}\right)\right)^2}{\left(f + g\,x\right)^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*log((e*(a + b*x)^2)/(c + d*x)^2))^2/(f + g*x)^5,x)
[Out] int((A + B*log((e*(a + b*x)^2)/(c + d*x)^2))^2/(f + g*x)^5, x)
```

**sympy [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*ln(e*(b*x+a)**2/(d*x+c)**2))**2/(g*x+f)**5,x)
[Out] Timed out
```

# 3.281 $\int \frac{(f+gx)^2}{A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}\,dx$

**Optimal**. Leaf size=34

$$\text{Int}\left(\frac{(f+gx)^2}{B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+A},x\right)$$

```
[Out] Unintegrable((g*x+f)^2/(A+B*ln(e*(b*x+a)^2/(d*x+c)^2)),x)
```

**Rubi [A]** time = 0.17, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{(f+gx)^2}{A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(f + g*x)^2/(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2]),x]
```

```
[Out] f^2*Defer[Int][(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])^(-1), x] + 2*f*g*De
fer[Int][x/(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2]), x] + g^2*Defer[Int][x^
2/(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2]), x]
```

Rubi steps

$$\begin{aligned}\int \frac{(f+gx)^2}{A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}\,dx &= \int \left(\frac{f^2}{A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}+\frac{2fgx}{A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}+\frac{g^2x^2}{A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}\right)dx\\ &= f^2\int \frac{1}{A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}\,dx+(2fg)\int \frac{x}{A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}\,dx+g^2\int \frac{x^2}{A+B\log\left(\right.}\end{aligned}$$

**Mathematica [A]** time = 0.18, size = 0, normalized size = 0.00

$$\int \frac{(f+gx)^2}{A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(f + g*x)^2/(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2]),x]
```

```
[Out] Integrate[(f + g*x)^2/(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2]), x]
```

**fricas [A]** time = 0.85, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{g^2x^2 + 2\,fgx + f^2}{B\log\left(\frac{b^2ex^2+2\,abex+a^2e}{d^2x^2+2\,cdx+c^2}\right) + A}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^2/(A+B*log(e*(b*x+a)^2/(d*x+c)^2)),x, algorithm="fricas")
```

```
[Out] integral((g^2*x^2 + 2*f*g*x + f^2)/(B*log((b^2*e*x^2 + 2*a*b*e*x + a^2*e)/(
d^2*x^2 + 2*c*d*x + c^2)) + A), x)
```

**giac [A]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(gx + f\right)^2}{B\log\left(\frac{(bx+a)^2e}{(dx+c)^2}\right) + A}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^2/(A+B*log(e*(b*x+a)^2/(d*x+c)^2)),x, algorithm="giac")
```

```
[Out] integrate((g*x + f)^2/(B*log((b*x + a)^2*e/(d*x + c)^2) + A), x)
```

**maple [A]** time = 0.82, size = 0, normalized size = 0.00

$$\int \frac{\left(gx + f\right)^2}{B\ln\left(\frac{(bx+a)^2e}{(dx+c)^2}\right) + A}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x+f)^2/(B*ln((b*x+a)^2/(d*x+c)^2*e)+A),x)
```

```
[Out] int((g*x+f)^2/(B*ln((b*x+a)^2/(d*x+c)^2*e)+A),x)
```

**maxima [A]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(gx + f\right)^2}{B\log\left(\frac{(bx+a)^2e}{(dx+c)^2}\right) + A}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^2/(A+B*log(e*(b*x+a)^2/(d*x+c)^2)),x, algorithm="maxima")
[Out] integrate((g*x + f)^2/(B*log((b*x + a)^2*e/(d*x + c)^2) + A), x)
```

**mupad [A]** time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{\left(f+g\,x\right)^2}{A+B\,\ln\left(\frac{e\,{\left(a+b\,x\right)}^2}{{\left(c+d\,x\right)}^2}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f + g*x)^2/(A + B*log((e*(a + b*x)^2)/(c + d*x)^2)),x)
[Out] int((f + g*x)^2/(A + B*log((e*(a + b*x)^2)/(c + d*x)^2)), x)
```

**sympy [A]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(f+gx\right)^{2}}{A+B\log\left(\frac{a^{2}e}{c^{2}+2cdx+d^{2}x^{2}}+\frac{2abex}{c^{2}+2cdx+d^{2}x^{2}}+\frac{b^{2}ex^{2}}{c^{2}+2cdx+d^{2}x^{2}}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)**2/(A+B*ln(e*(b*x+a)**2/(d*x+c)**2)),x)
[Out] Integral((f + g*x)**2/(A + B*log(a**2*e/(c**2 + 2*c*d*x + d**2*x**2) + 2*a*
b*e*x/(c**2 + 2*c*d*x + d**2*x**2) + b**2*e*x**2/(c**2 + 2*c*d*x + d**2*x**
2))), x)
```

# 3.282 $\int \frac{f+gx}{A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}\,dx$

**Optimal**. Leaf size=32

$$\text{Int}\left(\frac{f+gx}{B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+A},x\right)$$

```
[Out] Unintegrable((g*x+f)/(A+B*ln(e*(b*x+a)^2/(d*x+c)^2)),x)
```

**Rubi [A]** time = 0.09, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{f+gx}{A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(f + g*x)/(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2]),x]
```

```
[Out] f*Defer[Int][(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])^(-1), x] + g*Defer[In
t][x/(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2]), x]
```

Rubi steps

$$\begin{aligned}\int \frac{f+gx}{A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}\,dx &= \int \left(\frac{f}{A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}+\frac{gx}{A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}\right)dx\\ &= f\int \frac{1}{A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}\,dx+g\int \frac{x}{A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}\,dx\end{aligned}$$

**Mathematica [A]** time = 0.15, size = 0, normalized size = 0.00

$$\int \frac{f+gx}{A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(f + g*x)/(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2]),x]
```

```
[Out] Integrate[(f + g*x)/(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2]), x]
```

**fricas** [A] time = 0.69, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{gx+f}{B\log\left(\frac{b^2ex^2+2\,abex+a^2e}{d^2x^2+2\,cdx+c^2}\right)+A},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)/(A+B*log(e*(b*x+a)^2/(d*x+c)^2)),x, algorithm="fricas")
```

```
[Out] integral((g*x + f)/(B*log((b^2*e*x^2 + 2*a*b*e*x + a^2*e)/(d^2*x^2 + 2*c*d*
x + c^2)) + A), x)
```

**giac** [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{gx+f}{B\log\left(\frac{(bx+a)^2e}{(dx+c)^2}\right)+A}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)/(A+B*log(e*(b*x+a)^2/(d*x+c)^2)),x, algorithm="giac")
```

```
[Out] integrate((g*x + f)/(B*log((b*x + a)^2*e/(d*x + c)^2) + A), x)
```

**maple** [A] time = 0.69, size = 0, normalized size = 0.00

$$\int \frac{gx+f}{B\ln\left(\frac{(bx+a)^2e}{(dx+c)^2}\right)+A}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x+f)/(B*ln((b*x+a)^2/(d*x+c)^2*e)+A),x)
```

```
[Out] int((g*x+f)/(B*ln((b*x+a)^2/(d*x+c)^2*e)+A),x)
```

**maxima** [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{gx+f}{B\log\left(\frac{(bx+a)^2e}{(dx+c)^2}\right)+A}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)/(A+B*log(e*(b*x+a)^2/(d*x+c)^2)),x, algorithm="maxima")
```

```
[Out] integrate((g*x + f)/(B*log((b*x + a)^2*e/(d*x + c)^2) + A), x)
```

**mupad [A]** time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{f+g\,x}{A+B\,\ln\left(\frac{e\,{\left(a+b\,x\right)}^2}{{\left(c+d\,x\right)}^2}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f + g*x)/(A + B*log((e*(a + b*x)^2)/(c + d*x)^2)),x)
[Out] int((f + g*x)/(A + B*log((e*(a + b*x)^2)/(c + d*x)^2)), x)
```

**sympy [A]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{f+gx}{A+B\log\left(\frac{a^2e}{c^2+2cdx+d^2x^2}+\frac{2abex}{c^2+2cdx+d^2x^2}+\frac{b^2ex^2}{c^2+2cdx+d^2x^2}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)/(A+B*ln(e*(b*x+a)**2/(d*x+c)**2)),x)
[Out] Integral((f + g*x)/(A + B*log(a**2*e/(c**2 + 2*c*d*x + d**2*x**2) + 2*a*b*e
*x/(c**2 + 2*c*d*x + d**2*x**2) + b**2*e*x**2/(c**2 + 2*c*d*x + d**2*x**2))
), x)
```

# 3.283 $\int \frac{1}{A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}\,dx$

**Optimal**. Leaf size=26

$$\text{Int}\left(\frac{1}{B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+A},x\right)$$

```
[Out] Unintegrable(1/(A+B*ln(e*(b*x+a)^2/(d*x+c)^2)),x)
```

**Rubi [A]** time = 0.01, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{1}{A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])^(-1),x]
[Out] Defer[Int][(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])^(-1), x]
```

Rubi steps

$$\int \frac{1}{A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}\,dx=\int \frac{1}{A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}\,dx$$

**Mathematica [A]** time = 0.04, size = 0, normalized size = 0.00

$$\int \frac{1}{A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])^(-1),x]
[Out] Integrate[(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])^(-1), x]
```

**fricas** [A] time = 0.87, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{1}{B\log\left(\frac{b^2ex^2+2\,abex+a^2e}{d^2x^2+2\,cdx+c^2}\right)+A},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(A+B*log(e*(b*x+a)^2/(d*x+c)^2)),x, algorithm="fricas")
```

```
[Out] integral(1/(B*log((b^2*e*x^2 + 2*a*b*e*x + a^2*e)/(d^2*x^2 + 2*c*d*x + c^2)
) + A), x)
```

**giac** [A] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{1}{B\log\left(\frac{(bx+a)^2e}{(dx+c)^2}\right)+A}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(A+B*log(e*(b*x+a)^2/(d*x+c)^2)),x, algorithm="giac")
```

```
[Out] integrate(1/(B*log((b*x + a)^2*e/(d*x + c)^2) + A), x)
```

**maple** [A] time = 0.66, size = 0, normalized size = 0.00

$$\int\frac{1}{B\ln\left(\frac{(bx+a)^2e}{(dx+c)^2}\right)+A}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(B*ln((b*x+a)^2/(d*x+c)^2*e)+A),x)
```

```
[Out] int(1/(B*ln((b*x+a)^2/(d*x+c)^2*e)+A),x)
```

**maxima** [A] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{1}{B\log\left(\frac{(bx+a)^2e}{(dx+c)^2}\right)+A}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(A+B*log(e*(b*x+a)^2/(d*x+c)^2)),x, algorithm="maxima")
```

```
[Out] integrate(1/(B*log((b*x + a)^2*e/(d*x + c)^2) + A), x)
```

**mupad [A]** time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{1}{A+B\,\ln\left(\frac{e\,{\left(a+b\,x\right)}^2}{{\left(c+d\,x\right)}^2}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(A + B*log((e*(a + b*x)^2)/(c + d*x)^2)),x)
[Out] int(1/(A + B*log((e*(a + b*x)^2)/(c + d*x)^2)), x)
```

**sympy [A]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(A+B*ln(e*(b*x+a)**2/(d*x+c)**2)),x)
[Out] Integral(1/(A + B*log(e*(a + b*x)**2/(c + d*x)**2)), x)
```

# 3.284 $\int \frac{1}{(f+gx)\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}\,dx$

**Optimal**. Leaf size=34

$$\text{Int}\left(\frac{1}{(f+gx)\left(B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+A\right)},x\right)$$

```
[Out] Unintegrable(1/(g*x+f)/(A+B*ln(e*(b*x+a)^2/(d*x+c)^2)),x)
```

**Rubi [A]** time = 0.06, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{1}{(f+gx)\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[1/((f + g*x)*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])),x]
```

```
[Out] Defer[Int][1/((f + g*x)*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])), x]
```

Rubi steps

$$\int \frac{1}{(f+gx)\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}\,dx=\int \frac{1}{(f+gx)\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}\,dx$$

**Mathematica [A]** time = 0.09, size = 0, normalized size = 0.00

$$\int \frac{1}{(f+gx)\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[1/((f + g*x)*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])),x]
```

```
[Out] Integrate[1/((f + g*x)*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])), x]
```

**fricas** [A] time = 0.93, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{1}{Agx + Af + \left(Bgx + Bf\right)\log\left(\frac{b^2ex^2+2\,abex+a^2e}{d^2x^2+2\,cdx+c^2}\right)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(g*x+f)/(A+B*log(e*(b*x+a)^2/(d*x+c)^2)),x, algorithm="fricas")
```

```
[Out] integral(1/(A*g*x + A*f + (B*g*x + B*f)*log((b^2*e*x^2 + 2*a*b*e*x + a^2*e)
/(d^2*x^2 + 2*c*d*x + c^2))), x)
```

**giac** [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(gx+f\right)\left(B\log\left(\frac{(bx+a)^2e}{(dx+c)^2}\right)+A\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(g*x+f)/(A+B*log(e*(b*x+a)^2/(d*x+c)^2)),x, algorithm="giac")
```

```
[Out] integrate(1/((g*x + f)*(B*log((b*x + a)^2*e/(d*x + c)^2) + A)), x)
```

**maple** [A] time = 0.84, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(gx+f\right)\left(B\ln\left(\frac{(bx+a)^2e}{(dx+c)^2}\right)+A\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(g*x+f)/(B*ln((b*x+a)^2/(d*x+c)^2*e)+A),x)
```

```
[Out] int(1/(g*x+f)/(B*ln((b*x+a)^2/(d*x+c)^2*e)+A),x)
```

**maxima** [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(gx+f\right)\left(B\log\left(\frac{(bx+a)^2e}{(dx+c)^2}\right)+A\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(g*x+f)/(A+B*log(e*(b*x+a)^2/(d*x+c)^2)),x, algorithm="maxima")
```

```
[Out] integrate(1/((g*x + f)*(B*log((b*x + a)^2*e/(d*x + c)^2) + A)), x)
```

**mupad [A]** time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{1}{\left(f+g\,x\right)\,\left(A+B\,\ln\left(\frac{e\,{\left(a+b\,x\right)}^2}{{\left(c+d\,x\right)}^2}\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((f + g*x)*(A + B*log((e*(a + b*x)^2)/(c + d*x)^2))),x)
```

```
[Out] int(1/((f + g*x)*(A + B*log((e*(a + b*x)^2)/(c + d*x)^2))), x)
```

**sympy [A]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(A+B\log\left(\frac{a^2e}{c^2+2cdx+d^2x^2}+\frac{2abex}{c^2+2cdx+d^2x^2}+\frac{b^2ex^2}{c^2+2cdx+d^2x^2}\right)\right)\left(f+gx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(g*x+f)/(A+B*ln(e*(b*x+a)**2/(d*x+c)**2)),x)
```

```
[Out] Integral(1/((A + B*log(a**2*e/(c**2 + 2*c*d*x + d**2*x**2) + 2*a*b*e*x/(c**
2 + 2*c*d*x + d**2*x**2) + b**2*e*x**2/(c**2 + 2*c*d*x + d**2*x**2)))*(f +
g*x)), x)
```

# 3.285 $\int \frac{1}{(f+gx)^2\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}\,dx$

**Optimal**. Leaf size=34

$$\text{Int}\left(\frac{1}{(f+gx)^2\left(B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+A\right)},x\right)$$

```
[Out] Unintegrable(1/(g*x+f)^2/(A+B*ln(e*(b*x+a)^2/(d*x+c)^2)),x)
```

**Rubi [A]** time = 0.07, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{1}{(f+gx)^2\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[1/((f + g*x)^2*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])),x]
[Out] Defer[Int][1/((f + g*x)^2*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])), x]
```

Rubi steps

$$\int \frac{1}{(f+gx)^2\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}\,dx=\int \frac{1}{(f+gx)^2\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}\,dx$$

**Mathematica [A]** time = 0.09, size = 0, normalized size = 0.00

$$\int \frac{1}{(f+gx)^2\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[1/((f + g*x)^2*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])),x]
[Out] Integrate[1/((f + g*x)^2*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])), x]
```

**fricas [A]** time = 1.84, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{1}{Ag^2x^2 + 2\,Afgx + Af^2 + \left(Bg^2x^2 + 2\,Bfgx + Bf^2\right)\log\left(\frac{b^2ex^2+2\,abex+a^2e}{d^2x^2+2\,cdx+c^2}\right)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(g*x+f)^2/(A+B*log(e*(b*x+a)^2/(d*x+c)^2)),x, algorithm="fricas
")
```

```
[Out] integral(1/(A*g^2*x^2 + 2*A*f*g*x + A*f^2 + (B*g^2*x^2 + 2*B*f*g*x + B*f^2)
*log((b^2*e*x^2 + 2*a*b*e*x + a^2*e)/(d^2*x^2 + 2*c*d*x + c^2))), x)
```

**giac [A]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(gx+f\right)^2\left(B\log\left(\frac{(bx+a)^2e}{(dx+c)^2}\right)+A\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(g*x+f)^2/(A+B*log(e*(b*x+a)^2/(d*x+c)^2)),x, algorithm="giac")
```

```
[Out] integrate(1/((g*x + f)^2*(B*log((b*x + a)^2*e/(d*x + c)^2) + A)), x)
```

**maple [A]** time = 0.90, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(gx+f\right)^2\left(B\ln\left(\frac{(bx+a)^2e}{(dx+c)^2}\right)+A\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(g*x+f)^2/(B*ln((b*x+a)^2/(d*x+c)^2*e)+A),x)
```

```
[Out] int(1/(g*x+f)^2/(B*ln((b*x+a)^2/(d*x+c)^2*e)+A),x)
```

**maxima [A]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(gx+f\right)^2\left(B\log\left(\frac{(bx+a)^2e}{(dx+c)^2}\right)+A\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(g*x+f)^2/(A+B*log(e*(b*x+a)^2/(d*x+c)^2)),x, algorithm="maxima
")
```

```
[Out] integrate(1/((g*x + f)^2*(B*log((b*x + a)^2*e/(d*x + c)^2) + A)), x)
```

**mupad [A]** time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{1}{{\left(f+g\,x\right)}^2\,\left(A+B\,\ln\left(\frac{e\,{\left(a+b\,x\right)}^2}{{\left(c+d\,x\right)}^2}\right)\right)} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((f + g*x)^2*(A + B*log((e*(a + b*x)^2)/(c + d*x)^2))),x)
[Out] int(1/((f + g*x)^2*(A + B*log((e*(a + b*x)^2)/(c + d*x)^2))), x)
```

**sympy [A]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(A + B \log{\left(\frac{a^{2} e}{c^{2} + 2 c d x + d^{2} x^{2}} + \frac{2 a b e x}{c^{2} + 2 c d x + d^{2} x^{2}} + \frac{b^{2} e x^{2}}{c^{2} + 2 c d x + d^{2} x^{2}} \right)}\right) \left(f + g x\right)^{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(g*x+f)**2/(A+B*ln(e*(b*x+a)**2/(d*x+c)**2)),x)
[Out] Integral(1/((A + B*log(a**2*e/(c**2 + 2*c*d*x + d**2*x**2) + 2*a*b*e*x/(c**
2 + 2*c*d*x + d**2*x**2) + b**2*e*x**2/(c**2 + 2*c*d*x + d**2*x**2)))*(f +
g*x)**2), x)
```

# 3.286 $\int \frac{1}{(f+gx)^3\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}\,dx$

**Optimal**. Leaf size=34

$$\text{Int}\left(\frac{1}{(f+gx)^3\left(B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+A\right)},x\right)$$

```
[Out] Unintegrable(1/(g*x+f)^3/(A+B*ln(e*(b*x+a)^2/(d*x+c)^2)),x)
```

**Rubi [A]** time = 0.06, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{1}{(f+gx)^3\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[1/((f + g*x)^3*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])),x]
```

```
[Out] Defer[Int][1/((f + g*x)^3*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])), x]
```

Rubi steps

$$\int \frac{1}{(f+gx)^3\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}\,dx=\int \frac{1}{(f+gx)^3\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}\,dx$$

**Mathematica [A]** time = 0.10, size = 0, normalized size = 0.00

$$\int \frac{1}{(f+gx)^3\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[1/((f + g*x)^3*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])),x]
```

```
[Out] Integrate[1/((f + g*x)^3*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])), x]
```

**fricas** [A] time = 1.54, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{1}{Ag^3x^3 + 3\,Afg^2x^2 + 3\,Af^2gx + Af^3 + \left(Bg^3x^3 + 3\,Bfg^2x^2 + 3\,Bf^2gx + Bf^3\right)\log\left(\frac{b^2ex^2+2\,abex+a^2e}{d^2x^2+2\,cdx+c^2}\right)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(g*x+f)^3/(A+B*log(e*(b*x+a)^2/(d*x+c)^2)),x, algorithm="fricas
")
```

```
[Out] integral(1/(A*g^3*x^3 + 3*A*f*g^2*x^2 + 3*A*f^2*g*x + A*f^3 + (B*g^3*x^3 +
3*B*f*g^2*x^2 + 3*B*f^2*g*x + B*f^3)*log((b^2*e*x^2 + 2*a*b*e*x + a^2*e)/(d
^2*x^2 + 2*c*d*x + c^2))), x)
```

**giac** [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(gx+f)^3\left(B\log\left(\frac{(bx+a)^2e}{(dx+c)^2}\right)+A\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(g*x+f)^3/(A+B*log(e*(b*x+a)^2/(d*x+c)^2)),x, algorithm="giac")
```

```
[Out] integrate(1/((g*x + f)^3*(B*log((b*x + a)^2*e/(d*x + c)^2) + A)), x)
```

**maple** [A] time = 1.05, size = 0, normalized size = 0.00

$$\int \frac{1}{(gx+f)^3\left(B\ln\left(\frac{(bx+a)^2e}{(dx+c)^2}\right)+A\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(g*x+f)^3/(B*ln((b*x+a)^2/(d*x+c)^2*e)+A),x)
```

```
[Out] int(1/(g*x+f)^3/(B*ln((b*x+a)^2/(d*x+c)^2*e)+A),x)
```

**maxima** [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(gx+f)^3\left(B\log\left(\frac{(bx+a)^2e}{(dx+c)^2}\right)+A\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(g*x+f)^3/(A+B*log(e*(b*x+a)^2/(d*x+c)^2)),x, algorithm="maxima
")
```

```
[Out] integrate(1/((g*x + f)^3*(B*log((b*x + a)^2*e/(d*x + c)^2) + A)), x)
```

**mupad** [**A**] time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{1}{\left(f+g\,x\right)^3\,\left(A+B\,\ln\left(\frac{e\,{\left(a+b\,x\right)}^2}{{\left(c+d\,x\right)}^2}\right)\right)} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((f + g*x)^3*(A + B*log((e*(a + b*x)^2)/(c + d*x)^2))),x)
```

```
[Out] int(1/((f + g*x)^3*(A + B*log((e*(a + b*x)^2)/(c + d*x)^2))), x)
```

**sympy** [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(g*x+f)**3/(A+B*ln(e*(b*x+a)**2/(d*x+c)**2)),x)
```

```
[Out] Timed out
```

# 3.287 $\int \frac{(f+gx)^2}{\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}\,dx$

**Optimal**. Leaf size=34

$$\text{Int}\left(\frac{(f+gx)^2}{\left(B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+A\right)^2},x\right)$$

```
[Out] Unintegrable((g*x+f)^2/(A+B*ln(e*(b*x+a)^2/(d*x+c)^2))^2,x)
```

**Rubi [A]** time = 0.19, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{(f+gx)^2}{\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(f + g*x)^2/(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])^2,x]
```

```
[Out] f^2*Defer[Int][(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])^(-2), x] + 2*f*g*De
fer[Int][x/(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])^2, x] + g^2*Defer[Int][
x^2/(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])^2, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{(f+gx)^2}{\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}\,dx &= \int \left(\frac{f^2}{\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}+\frac{2fgx}{\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}+\frac{g^2x^2}{\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}\right) \\
&= f^2\int \frac{1}{\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}\,dx+(2fg)\int \frac{x}{\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}\,dx+g^2\int \frac{}{\left(A\right.}
\end{aligned}$$

**Mathematica [A]** time = 0.72, size = 0, normalized size = 0.00

$$\int \frac{(f+gx)^2}{\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(f + g*x)^2/(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])^2,x]
[Out] Integrate[(f + g*x)^2/(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])^2, x]
```

**fricas [A]** time = 1.69, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{g^2x^2+2\,fgx+f^2}{B^2\log\left(\frac{b^2ex^2+2\,abex+a^2e}{d^2x^2+2\,cdx+c^2}\right)^2+2\,AB\log\left(\frac{b^2ex^2+2\,abex+a^2e}{d^2x^2+2\,cdx+c^2}\right)+A^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^2/(A+B*log(e*(b*x+a)^2/(d*x+c)^2))^2,x, algorithm="fricas
")
[Out] integral((g^2*x^2 + 2*f*g*x + f^2)/(B^2*log((b^2*e*x^2 + 2*a*b*e*x + a^2*e)
/(d^2*x^2 + 2*c*d*x + c^2))^2 + 2*A*B*log((b^2*e*x^2 + 2*a*b*e*x + a^2*e)/(
d^2*x^2 + 2*c*d*x + c^2)) + A^2), x)
```

**giac [A]** time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{(gx+f)^2}{\left(B\log\left(\frac{(bx+a)^2e}{(dx+c)^2}\right)+A\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^2/(A+B*log(e*(b*x+a)^2/(d*x+c)^2))^2,x, algorithm="giac")
[Out] integrate((g*x + f)^2/(B*log((b*x + a)^2*e/(d*x + c)^2) + A)^2, x)
```

**maple [A]** time = 0.78, size = 0, normalized size = 0.00

$$\int\frac{(gx+f)^2}{\left(B\ln\left(\frac{(bx+a)^2e}{(dx+c)^2}\right)+A\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x+f)^2/(B*ln((b*x+a)^2/(d*x+c)^2*e)+A)^2,x)
[Out] int((g*x+f)^2/(B*ln((b*x+a)^2/(d*x+c)^2*e)+A)^2,x)
```

**maxima [A]** time = 0.00, size = 0, normalized size = 0.00

$$-\frac{bdg^2x^4 + acf^2 + \left(adg^2 + \left(2\,dfg + cg^2\right)b\right)x^3 + \left(\left(2\,dfg + cg^2\right)a + \left(df^2 + 2\,cfg\right)b\right)x^2 + \left(bcf^2 + \left(df^2 + 2\,cfg\right)a\right)x}{2\left(2\,(bc - ad)B^2\log\left(bx + a\right) - 2\,(bc - ad)B^2\log\left(dx + c\right) + (bc - ad)AB + \left(bc\log(e) - ad\log(e)\right)B^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^2/(A+B*log(e*(b*x+a)^2/(d*x+c)^2))^2,x, algorithm="maxima
")
```

```
[Out] -1/2*(b*d*g^2*x^4 + a*c*f^2 + (a*d*g^2 + (2*d*f*g + c*g^2)*b)*x^3 + ((2*d*f
*g + c*g^2)*a + (d*f^2 + 2*c*f*g)*b)*x^2 + (b*c*f^2 + (d*f^2 + 2*c*f*g)*a)*
x)/(2*(b*c - a*d)*B^2*log(b*x + a) - 2*(b*c - a*d)*B^2*log(d*x + c) + (b*c
- a*d)*A*B + (b*c*log(e) - a*d*log(e))*B^2) + integrate(1/2*(4*b*d*g^2*x^3
+ b*c*f^2 + 3*(a*d*g^2 + (2*d*f*g + c*g^2)*b)*x^2 + (d*f^2 + 2*c*f*g)*a + 2
*((2*d*f*g + c*g^2)*a + (d*f^2 + 2*c*f*g)*b)*x)/(2*(b*c - a*d)*B^2*log(b*x
+ a) - 2*(b*c - a*d)*B^2*log(d*x + c) + (b*c - a*d)*A*B + (b*c*log(e) - a*d
*log(e))*B^2), x)
```

**mupad [A]** time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{\left(f + g\,x\right)^2}{\left(A + B\,\ln\left(\frac{e\,\left(a+b\,x\right)^2}{\left(c+d\,x\right)^2}\right)\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f + g*x)^2/(A + B*log((e*(a + b*x)^2)/(c + d*x)^2))^2,x)
```

```
[Out] int((f + g*x)^2/(A + B*log((e*(a + b*x)^2)/(c + d*x)^2))^2, x)
```

**sympy [A]** time = 0.00, size = 0, normalized size = 0.00

$$\frac{acf^2 + 2acfgx + acg^2x^2 + adf^2x + 2adfgx^2 + adg^2x^3 + bcf^2x + 2bcfgx^2 + bcg^2x^3 + bdf^2x^2 + 2bdfgx^3 + bdg^2x^4}{2ABad - 2ABbc + \left(2B^2ad - 2B^2bc\right)\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)**2/(A+B*ln(e*(b*x+a)**2/(d*x+c)**2))**2,x)
```

```
[Out] (a*c*f**2 + 2*a*c*f*g*x + a*c*g**2*x**2 + a*d*f**2*x + 2*a*d*f*g*x**2 + a*d
*g**2*x**3 + b*c*f**2*x + 2*b*c*f*g*x**2 + b*c*g**2*x**3 + b*d*f**2*x**2 +
2*b*d*f*g*x**3 + b*d*g**2*x**4)/(2*A*B*a*d - 2*A*B*b*c + (2*B**2*a*d - 2*B*
```

```
*2*b*c)*log(e*(a + b*x)**2/(c + d*x)**2)) - (Integral(a*d*f**2/(A + B*log(a
**2*e/(c**2 + 2*c*d*x + d**2*x**2) + 2*a*b*e*x/(c**2 + 2*c*d*x + d**2*x**2)
 + b**2*e*x**2/(c**2 + 2*c*d*x + d**2*x**2))), x) + Integral(b*c*f**2/(A +
B*log(a**2*e/(c**2 + 2*c*d*x + d**2*x**2) + 2*a*b*e*x/(c**2 + 2*c*d*x + d**
2*x**2) + b**2*e*x**2/(c**2 + 2*c*d*x + d**2*x**2))), x) + Integral(2*a*c*f
*g/(A + B*log(a**2*e/(c**2 + 2*c*d*x + d**2*x**2) + 2*a*b*e*x/(c**2 + 2*c*d
*x + d**2*x**2) + b**2*e*x**2/(c**2 + 2*c*d*x + d**2*x**2))), x) + Integral
(2*a*c*g**2*x/(A + B*log(a**2*e/(c**2 + 2*c*d*x + d**2*x**2) + 2*a*b*e*x/(c
**2 + 2*c*d*x + d**2*x**2) + b**2*e*x**2/(c**2 + 2*c*d*x + d**2*x**2))), x)
 + Integral(3*a*d*g**2*x**2/(A + B*log(a**2*e/(c**2 + 2*c*d*x + d**2*x**2)
+ 2*a*b*e*x/(c**2 + 2*c*d*x + d**2*x**2) + b**2*e*x**2/(c**2 + 2*c*d*x + d*
*2*x**2))), x) + Integral(3*b*c*g**2*x**2/(A + B*log(a**2*e/(c**2 + 2*c*d*x
 + d**2*x**2) + 2*a*b*e*x/(c**2 + 2*c*d*x + d**2*x**2) + b**2*e*x**2/(c**2
+ 2*c*d*x + d**2*x**2))), x) + Integral(2*b*d*f**2*x/(A + B*log(a**2*e/(c**
2 + 2*c*d*x + d**2*x**2) + 2*a*b*e*x/(c**2 + 2*c*d*x + d**2*x**2) + b**2*e*
x**2/(c**2 + 2*c*d*x + d**2*x**2))), x) + Integral(4*b*d*g**2*x**3/(A + B*l
og(a**2*e/(c**2 + 2*c*d*x + d**2*x**2) + 2*a*b*e*x/(c**2 + 2*c*d*x + d**2*x
**2) + b**2*e*x**2/(c**2 + 2*c*d*x + d**2*x**2))), x) + Integral(4*a*d*f*g*
x/(A + B*log(a**2*e/(c**2 + 2*c*d*x + d**2*x**2) + 2*a*b*e*x/(c**2 + 2*c*d*
x + d**2*x**2) + b**2*e*x**2/(c**2 + 2*c*d*x + d**2*x**2))), x) + Integral(
4*b*c*f*g*x/(A + B*log(a**2*e/(c**2 + 2*c*d*x + d**2*x**2) + 2*a*b*e*x/(c**
2 + 2*c*d*x + d**2*x**2) + b**2*e*x**2/(c**2 + 2*c*d*x + d**2*x**2))), x) +
 Integral(6*b*d*f*g*x**2/(A + B*log(a**2*e/(c**2 + 2*c*d*x + d**2*x**2) + 2
*a*b*e*x/(c**2 + 2*c*d*x + d**2*x**2) + b**2*e*x**2/(c**2 + 2*c*d*x + d**2*
x**2))), x))/(2*B*(a*d - b*c))
```

# 3.288 $\int \frac{f+gx}{\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}\,dx$

**Optimal**. Leaf size=32

$$\mathrm{Int}\left(\frac{f+gx}{\left(B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+A\right)^2},x\right)$$

```
[Out] Unintegrable((g*x+f)/(A+B*ln(e*(b*x+a)^2/(d*x+c)^2))^2,x)
```

**Rubi [A]** time = 0.10, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{f+gx}{\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(f + g*x)/(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])^2,x]
```

```
[Out] f*Defer[Int][(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])^(-2), x] + g*Defer[In
t][x/(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])^2, x]
```

Rubi steps

$$\begin{aligned}\int \frac{f+gx}{\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}\,dx &= \int \left(\frac{f}{\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}+\frac{gx}{\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}\right)dx\\ &= f\int \frac{1}{\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}\,dx+g\int \frac{x}{\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}\,dx\end{aligned}$$

**Mathematica [A]** time = 0.43, size = 0, normalized size = 0.00

$$\int \frac{f+gx}{\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(f + g*x)/(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])^2,x]
```

```
[Out] Integrate[(f + g*x)/(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])^2, x]
```

**fricas [A]** time = 2.12, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{gx+f}{B^2\log\left(\frac{b^2ex^2+2\,abex+a^2e}{d^2x^2+2\,cdx+c^2}\right)^2+2\,AB\log\left(\frac{b^2ex^2+2\,abex+a^2e}{d^2x^2+2\,cdx+c^2}\right)+A^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)/(A+B*log(e*(b*x+a)^2/(d*x+c)^2))^2,x, algorithm="fricas")
```

```
[Out] integral((g*x + f)/(B^2*log((b^2*e*x^2 + 2*a*b*e*x + a^2*e)/(d^2*x^2 + 2*c*
d*x + c^2))^2 + 2*A*B*log((b^2*e*x^2 + 2*a*b*e*x + a^2*e)/(d^2*x^2 + 2*c*d*
x + c^2)) + A^2), x)
```

**giac [A]** time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{gx+f}{\left(B\log\left(\frac{(bx+a)^2e}{(dx+c)^2}\right)+A\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)/(A+B*log(e*(b*x+a)^2/(d*x+c)^2))^2,x, algorithm="giac")
```

```
[Out] integrate((g*x + f)/(B*log((b*x + a)^2*e/(d*x + c)^2) + A)^2, x)
```

**maple [A]** time = 0.77, size = 0, normalized size = 0.00

$$\int\frac{gx+f}{\left(B\ln\left(\frac{(bx+a)^2e}{(dx+c)^2}\right)+A\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x+f)/(B*ln((b*x+a)^2/(d*x+c)^2*e)+A)^2,x)
```

```
[Out] int((g*x+f)/(B*ln((b*x+a)^2/(d*x+c)^2*e)+A)^2,x)
```

**maxima [A]** time = 0.00, size = 0, normalized size = 0.00

$$-\frac{bdgx^3+acf+\left(adg+\left(df+cg\right)b\right)x^2+\left(bcf+\left(df+cg\right)a\right)x}{2\left(2\,(bc-ad)B^2\log\left(bx+a\right)-2\,(bc-ad)B^2\log\left(dx+c\right)+(bc-ad)AB+\left(bc\log(e)-ad\log(e)\right)B^2\right)}+\int\frac{}{2\left(2\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)/(A+B*log(e*(b*x+a)^2/(d*x+c)^2))^2,x, algorithm="maxima")
```

```
[Out] -1/2*(b*d*g*x^3 + a*c*f + (a*d*g + (d*f + c*g)*b)*x^2 + (b*c*f + (d*f + c*g
)*a)*x)/(2*(b*c - a*d)*B^2*log(b*x + a) - 2*(b*c - a*d)*B^2*log(d*x + c) +
(b*c - a*d)*A*B + (b*c*log(e) - a*d*log(e))*B^2) + integrate(1/2*(3*b*d*g*x
^2 + b*c*f + (d*f + c*g)*a + 2*(a*d*g + (d*f + c*g)*b)*x)/(2*(b*c - a*d)*B^
2*log(b*x + a) - 2*(b*c - a*d)*B^2*log(d*x + c) + (b*c - a*d)*A*B + (b*c*lo
g(e) - a*d*log(e))*B^2), x)
```

**mupad [A]** time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{f+g\,x}{{\left(A+B\,\ln\left(\frac{e\,{\left(a+b\,x\right)}^2}{{\left(c+d\,x\right)}^2}\right)\right)}^2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f + g*x)/(A + B*log((e*(a + b*x)^2)/(c + d*x)^2))^2,x)
```

```
[Out] int((f + g*x)/(A + B*log((e*(a + b*x)^2)/(c + d*x)^2))^2, x)
```

**sympy [A]** time = 0.00, size = 0, normalized size = 0.00

$$\frac{acf + acgx + adfx + adgx^2 + bcfx + bcgx^2 + bdfx^2 + bdgx^3}{2ABad - 2ABbc + \left(2B^2ad - 2B^2bc\right)\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)} - \frac{\int \frac{acg}{A+B\log\left(\frac{a^2e}{c^2+2cdx+d^2x^2} + \frac{2abex}{c^2+2cdx+d^2x^2} + \frac{b^2ex^2}{c^2+2cdx+d^2x^2}\right)}\,dx + \dots}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)/(A+B*ln(e*(b*x+a)**2/(d*x+c)**2))**2,x)
```

```
[Out] (a*c*f + a*c*g*x + a*d*f*x + a*d*g*x**2 + b*c*f*x + b*c*g*x**2 + b*d*f*x**2
 + b*d*g*x**3)/(2*A*B*a*d - 2*A*B*b*c + (2*B**2*a*d - 2*B**2*b*c)*log(e*(a
+ b*x)**2/(c + d*x)**2)) - (Integral(a*c*g/(A + B*log(a**2*e/(c**2 + 2*c*d*
x + d**2*x**2) + 2*a*b*e*x/(c**2 + 2*c*d*x + d**2*x**2) + b**2*e*x**2/(c**2
 + 2*c*d*x + d**2*x**2))), x) + Integral(a*d*f/(A + B*log(a**2*e/(c**2 + 2*
c*d*x + d**2*x**2) + 2*a*b*e*x/(c**2 + 2*c*d*x + d**2*x**2) + b**2*e*x**2/(
c**2 + 2*c*d*x + d**2*x**2))), x) + Integral(b*c*f/(A + B*log(a**2*e/(c**2
+ 2*c*d*x + d**2*x**2) + 2*a*b*e*x/(c**2 + 2*c*d*x + d**2*x**2) + b**2*e*x*
*2/(c**2 + 2*c*d*x + d**2*x**2))), x) + Integral(2*a*d*g*x/(A + B*log(a**2*
e/(c**2 + 2*c*d*x + d**2*x**2) + 2*a*b*e*x/(c**2 + 2*c*d*x + d**2*x**2) + b
**2*e*x**2/(c**2 + 2*c*d*x + d**2*x**2))), x) + Integral(2*b*c*g*x/(A + B*l
og(a**2*e/(c**2 + 2*c*d*x + d**2*x**2) + 2*a*b*e*x/(c**2 + 2*c*d*x + d**2*x
```

```
**2) + b**2*e*x**2/(c**2 + 2*c*d*x + d**2*x**2))), x) + Integral(2*b*d*f*x/
(A + B*log(a**2*e/(c**2 + 2*c*d*x + d**2*x**2) + 2*a*b*e*x/(c**2 + 2*c*d*x
+ d**2*x**2) + b**2*e*x**2/(c**2 + 2*c*d*x + d**2*x**2))), x) + Integral(3*
b*d*g*x**2/(A + B*log(a**2*e/(c**2 + 2*c*d*x + d**2*x**2) + 2*a*b*e*x/(c**2
 + 2*c*d*x + d**2*x**2) + b**2*e*x**2/(c**2 + 2*c*d*x + d**2*x**2))), x))/(
2*B*(a*d - b*c))
```

## 3.289 $\int \frac{1}{\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}\,dx$

**Optimal**. Leaf size=26

$$\text{Int}\left(\frac{1}{\left(B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+A\right)^2},x\right)$$

```
[Out] Unintegrable(1/(A+B*ln(e*(b*x+a)^2/(d*x+c)^2))^2,x)
```

**Rubi** [A] time = 0.01, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{1}{\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])^(-2),x]
```

```
[Out] Defer[Int][(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])^(-2), x]
```

Rubi steps

$$\int \frac{1}{\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}\,dx=\int \frac{1}{\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}\,dx$$

**Mathematica** [A] time = 0.23, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])^(-2),x]
```

```
[Out] Integrate[(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])^(-2), x]
```

**fricas** [**A**] time = 1.04, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{1}{B^{2}\log\left(\frac{b^{2}ex^{2}+2\,abex+a^{2}e}{d^{2}x^{2}+2\,cdx+c^{2}}\right)^{2}+2\,AB\log\left(\frac{b^{2}ex^{2}+2\,abex+a^{2}e}{d^{2}x^{2}+2\,cdx+c^{2}}\right)+A^{2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(A+B*log(e*(b*x+a)^2/(d*x+c)^2))^2,x, algorithm="fricas")
```

```
[Out] integral(1/(B^2*log((b^2*e*x^2 + 2*a*b*e*x + a^2*e)/(d^2*x^2 + 2*c*d*x + c^
2))^2 + 2*A*B*log((b^2*e*x^2 + 2*a*b*e*x + a^2*e)/(d^2*x^2 + 2*c*d*x + c^2)
) + A^2), x)
```

**giac** [**A**] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{1}{\left(B\log\left(\frac{(bx+a)^{2}e}{(dx+c)^{2}}\right)+A\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(A+B*log(e*(b*x+a)^2/(d*x+c)^2))^2,x, algorithm="giac")
```

```
[Out] integrate((B*log((b*x + a)^2*e/(d*x + c)^2) + A)^(-2), x)
```

**maple** [**A**] time = 0.76, size = 0, normalized size = 0.00

$$\int\frac{1}{\left(B\ln\left(\frac{(bx+a)^{2}e}{(dx+c)^{2}}\right)+A\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(B*ln((b*x+a)^2/(d*x+c)^2*e)+A)^2,x)
```

```
[Out] int(1/(B*ln((b*x+a)^2/(d*x+c)^2*e)+A)^2,x)
```

**maxima** [**A**] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{bdx^{2}+ac+(bc+ad)x}{2\left(2\,(bc-ad)B^{2}\log(bx+a)-2\,(bc-ad)B^{2}\log(dx+c)+(bc-ad)AB+\left(bc\log(e)-ad\log(e)\right)B^{2}\right)}+\int\frac{}{2\left(2\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(A+B*log(e*(b*x+a)^2/(d*x+c)^2))^2,x, algorithm="maxima")
```

```
[Out] -1/2*(b*d*x^2 + a*c + (b*c + a*d)*x)/(2*(b*c - a*d)*B^2*log(b*x + a) - 2*(b
*c - a*d)*B^2*log(d*x + c) + (b*c - a*d)*A*B + (b*c*log(e) - a*d*log(e))*B^
2) + integrate(1/2*(2*b*d*x + b*c + a*d)/(2*(b*c - a*d)*B^2*log(b*x + a) -
2*(b*c - a*d)*B^2*log(d*x + c) + (b*c - a*d)*A*B + (b*c*log(e) - a*d*log(e)
)*B^2), x)
```

**mupad [A]** time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{1}{\left(A+B\,\ln\left(\frac{e\,{\left(a+b\,x\right)}^2}{{\left(c+d\,x\right)}^2}\right)\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(A + B*log((e*(a + b*x)^2)/(c + d*x)^2))^2,x)
```

```
[Out] int(1/(A + B*log((e*(a + b*x)^2)/(c + d*x)^2))^2, x)
```

**sympy [A]** time = 0.00, size = 0, normalized size = 0.00

$$\frac{ac+adx+bcx+bdx^2}{2ABad-2ABbc+\left(2B^2ad-2B^2bc\right)\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}-\frac{\int\frac{ad}{A+B\log\left(\frac{a^2e}{c^2+2cdx+d^2x^2}+\frac{2abex}{c^2+2cdx+d^2x^2}+\frac{b^2ex^2}{c^2+2cdx+d^2x^2}\right)}\,dx+\int\frac{}{A+B\log\left(\frac{}{c^2\ldots}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(A+B*ln(e*(b*x+a)**2/(d*x+c)**2))**2,x)
```

```
[Out] (a*c + a*d*x + b*c*x + b*d*x**2)/(2*A*B*a*d - 2*A*B*b*c + (2*B**2*a*d - 2*B
**2*b*c)*log(e*(a + b*x)**2/(c + d*x)**2)) - (Integral(a*d/(A + B*log(a**2*
e/(c**2 + 2*c*d*x + d**2*x**2) + 2*a*b*e*x/(c**2 + 2*c*d*x + d**2*x**2) + b
**2*e*x**2/(c**2 + 2*c*d*x + d**2*x**2))), x) + Integral(b*c/(A + B*log(a**
2*e/(c**2 + 2*c*d*x + d**2*x**2) + 2*a*b*e*x/(c**2 + 2*c*d*x + d**2*x**2) +
 b**2*e*x**2/(c**2 + 2*c*d*x + d**2*x**2))), x) + Integral(2*b*d*x/(A + B*l
og(a**2*e/(c**2 + 2*c*d*x + d**2*x**2) + 2*a*b*e*x/(c**2 + 2*c*d*x + d**2*x
**2) + b**2*e*x**2/(c**2 + 2*c*d*x + d**2*x**2))), x))/(2*B*(a*d - b*c))
```

# 3.290 $\int \frac{1}{(f+gx)\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}\,dx$

**Optimal**. Leaf size=34

$$\operatorname{Int}\left(\frac{1}{(f+gx)\left(B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+A\right)^2},x\right)$$

```
[Out] Unintegrable(1/(g*x+f)/(A+B*ln(e*(b*x+a)^2/(d*x+c)^2))^2,x)
```

**Rubi [A]** time = 0.07, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{1}{(f+gx)\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[1/((f + g*x)*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])^2),x]
[Out] Defer[Int][1/((f + g*x)*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])^2), x]
```

Rubi steps

$$\int \frac{1}{(f+gx)\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}\,dx=\int \frac{1}{(f+gx)\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}\,dx$$

**Mathematica [A]** time = 0.50, size = 0, normalized size = 0.00

$$\int \frac{1}{(f+gx)\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[1/((f + g*x)*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])^2),x]
```

```
[Out] Integrate[1/((f + g*x)*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])^2), x]
```

**fricas [A]** time = 2.10, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{1}{A^2gx + A^2f + \left(B^2gx + B^2f\right)\log\left(\frac{b^2ex^2+2\,abex+a^2e}{d^2x^2+2\,cdx+c^2}\right)^2 + 2\left(ABgx + ABf\right)\log\left(\frac{b^2ex^2+2\,abex+a^2e}{d^2x^2+2\,cdx+c^2}\right)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(g*x+f)/(A+B*log(e*(b*x+a)^2/(d*x+c)^2))^2,x, algorithm="fricas
")
```

```
[Out] integral(1/(A^2*g*x + A^2*f + (B^2*g*x + B^2*f)*log((b^2*e*x^2 + 2*a*b*e*x
+ a^2*e)/(d^2*x^2 + 2*c*d*x + c^2))^2 + 2*(A*B*g*x + A*B*f)*log((b^2*e*x^2
+ 2*a*b*e*x + a^2*e)/(d^2*x^2 + 2*c*d*x + c^2))), x)
```

**giac [A]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(gx + f\right)\left(B\log\left(\frac{(bx+a)^2e}{(dx+c)^2}\right) + A\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(g*x+f)/(A+B*log(e*(b*x+a)^2/(d*x+c)^2))^2,x, algorithm="giac")
```

```
[Out] integrate(1/((g*x + f)*(B*log((b*x + a)^2*e/(d*x + c)^2) + A)^2), x)
```

**maple [A]** time = 1.05, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(gx + f\right)\left(B\ln\left(\frac{(bx+a)^2e}{(dx+c)^2}\right) + A\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(g*x+f)/(B*ln((b*x+a)^2/(d*x+c)^2*e)+A)^2,x)
```

```
[Out] int(1/(g*x+f)/(B*ln((b*x+a)^2/(d*x+c)^2*e)+A)^2,x)
```

**maxima [A]** time = 0.00, size = 0, normalized size = 0.00

$$-\frac{bdx^2 + ac + (bc + ad}{2\left(\left(bcf - adf\right)AB + \left(bcf\log(e) - adf\log(e)\right)B^2 + \left(\left(bcg - adg\right)AB + \left(bcg\log(e) - adg\log(e)\right)B^2\right)x + 2\left(\left(bcg - \right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(g*x+f)/(A+B*log(e*(b*x+a)^2/(d*x+c)^2))^2,x, algorithm="maxima
")
```

```
[Out] -1/2*(b*d*x^2 + a*c + (b*c + a*d)*x)/((b*c*f - a*d*f)*A*B + (b*c*f*log(e) -
 a*d*f*log(e))*B^2 + ((b*c*g - a*d*g)*A*B + (b*c*g*log(e) - a*d*g*log(e))*B
^2)*x + 2*((b*c*g - a*d*g)*B^2*x + (b*c*f - a*d*f)*B^2)*log(b*x + a) - 2*((
b*c*g - a*d*g)*B^2*x + (b*c*f - a*d*f)*B^2)*log(d*x + c)) + integrate(1/2*(
b*d*g*x^2 + 2*b*d*f*x + b*c*f + (d*f - c*g)*a)/((b*c*f^2 - a*d*f^2)*A*B + (
b*c*f^2*log(e) - a*d*f^2*log(e))*B^2 + ((b*c*g^2 - a*d*g^2)*A*B + (b*c*g^2*
log(e) - a*d*g^2*log(e))*B^2)*x^2 + 2*((b*c*f*g - a*d*f*g)*A*B + (b*c*f*g*l
og(e) - a*d*f*g*log(e))*B^2)*x + 2*((b*c*g^2 - a*d*g^2)*B^2*x^2 + 2*(b*c*f*
g - a*d*f*g)*B^2*x + (b*c*f^2 - a*d*f^2)*B^2)*log(b*x + a) - 2*((b*c*g^2 -
a*d*g^2)*B^2*x^2 + 2*(b*c*f*g - a*d*f*g)*B^2*x + (b*c*f^2 - a*d*f^2)*B^2)*l
og(d*x + c)), x)
```

**mupad** [A] time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{1}{\left(f+g\,x\right)\,{\left(A+B\,\ln\left(\frac{e\,{\left(a+b\,x\right)}^2}{{\left(c+d\,x\right)}^2}\right)\right)}^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((f + g*x)*(A + B*log((e*(a + b*x)^2)/(c + d*x)^2))^2),x)
```

```
[Out] int(1/((f + g*x)*(A + B*log((e*(a + b*x)^2)/(c + d*x)^2))^2), x)
```

**sympy** [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(g*x+f)/(A+B*ln(e*(b*x+a)**2/(d*x+c)**2))**2,x)
```

```
[Out] Timed out
```

# 3.291 $\int \frac{1}{(f+gx)^2\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}\,dx$

**Optimal**. Leaf size=34

$$\mathrm{Int}\left(\frac{1}{(f+gx)^2\left(B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+A\right)^2},x\right)$$

```
[Out] Unintegrable(1/(g*x+f)^2/(A+B*ln(e*(b*x+a)^2/(d*x+c)^2))^2,x)
```

**Rubi [A]** time = 0.08, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{1}{(f+gx)^2\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[1/((f + g*x)^2*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])^2),x]
[Out] Defer[Int][1/((f + g*x)^2*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])^2), x]
```

Rubi steps

$$\int \frac{1}{(f+gx)^2\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}\,dx = \int \frac{1}{(f+gx)^2\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}\,dx$$

**Mathematica [A]** time = 0.61, size = 0, normalized size = 0.00

$$\int \frac{1}{(f+gx)^2\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[1/((f + g*x)^2*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])^2),x]
```

```
[Out] Integrate[1/((f + g*x)^2*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])^2), x]
```

**fricas [A]** time = 0.80, size = 0, normalized size = 0.00

$$\operatorname{integral}\left(\frac{1}{A^2g^2x^2 + 2\,A^2fgx + A^2f^2 + \left(B^2g^2x^2 + 2\,B^2fgx + B^2f^2\right)\log\left(\frac{b^2ex^2+2\,abex+a^2e}{d^2x^2+2\,cdx+c^2}\right)^2 + 2\left(ABg^2x^2 + 2\,ABf\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(g*x+f)^2/(A+B*log(e*(b*x+a)^2/(d*x+c)^2))^2,x, algorithm="fric
as")
```

```
[Out] integral(1/(A^2*g^2*x^2 + 2*A^2*f*g*x + A^2*f^2 + (B^2*g^2*x^2 + 2*B^2*f*g*
x + B^2*f^2)*log((b^2*e*x^2 + 2*a*b*e*x + a^2*e)/(d^2*x^2 + 2*c*d*x + c^2))
^2 + 2*(A*B*g^2*x^2 + 2*A*B*f*g*x + A*B*f^2)*log((b^2*e*x^2 + 2*a*b*e*x + a
^2*e)/(d^2*x^2 + 2*c*d*x + c^2))), x)
```

**giac [A]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(gx+f)^2\left(B\log\left(\frac{(bx+a)^2e}{(dx+c)^2}\right)+A\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(g*x+f)^2/(A+B*log(e*(b*x+a)^2/(d*x+c)^2))^2,x, algorithm="giac
")
```

```
[Out] integrate(1/((g*x + f)^2*(B*log((b*x + a)^2*e/(d*x + c)^2) + A)^2), x)
```

**maple [A]** time = 1.33, size = 0, normalized size = 0.00

$$\int \frac{1}{(gx+f)^2\left(B\ln\left(\frac{(bx+a)^2e}{(dx+c)^2}\right)+A\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(g*x+f)^2/(B*ln((b*x+a)^2/(d*x+c)^2*e)+A)^2,x)
```

```
[Out] int(1/(g*x+f)^2/(B*ln((b*x+a)^2/(d*x+c)^2*e)+A)^2,x)
```

**maxima [A]** time = 0.00, size = 0, normalized size = 0.00

$$-\frac{}{2\left(\left(bcf^2-adf^2\right)AB+\left(bcf^2\log(e)-adf^2\log(e)\right)B^2+\left(\left(bcg^2-adg^2\right)AB+\left(bcg^2\log(e)-adg^2\log(e)\right)B^2\right)x^2\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(g*x+f)^2/(A+B*log(e*(b*x+a)^2/(d*x+c)^2))^2,x, algorithm="maxi
ma")
```

```
[Out] -1/2*(b*d*x^2 + a*c + (b*c + a*d)*x)/((b*c*f^2 - a*d*f^2)*A*B + (b*c*f^2*lo
g(e) - a*d*f^2*log(e))*B^2 + ((b*c*g^2 - a*d*g^2)*A*B + (b*c*g^2*log(e) - a
*d*g^2*log(e))*B^2)*x^2 + 2*((b*c*f*g - a*d*f*g)*A*B + (b*c*f*g*log(e) - a*
d*f*g*log(e))*B^2)*x + 2*((b*c*g^2 - a*d*g^2)*B^2*x^2 + 2*(b*c*f*g - a*d*f*
g)*B^2*x + (b*c*f^2 - a*d*f^2)*B^2)*log(b*x + a) - 2*((b*c*g^2 - a*d*g^2)*B
^2*x^2 + 2*(b*c*f*g - a*d*f*g)*B^2*x + (b*c*f^2 - a*d*f^2)*B^2)*log(d*x + c
)) - integrate(-1/2*(b*c*f + (d*f - 2*c*g)*a - (a*d*g - (2*d*f - c*g)*b)*x)
/(((b*c*g^3 - a*d*g^3)*A*B + (b*c*g^3*log(e) - a*d*g^3*log(e))*B^2)*x^3 + (
b*c*f^3 - a*d*f^3)*A*B + (b*c*f^3*log(e) - a*d*f^3*log(e))*B^2 + 3*((b*c*f*
g^2 - a*d*f*g^2)*A*B + (b*c*f*g^2*log(e) - a*d*f*g^2*log(e))*B^2)*x^2 + 3*(
(b*c*f^2*g - a*d*f^2*g)*A*B + (b*c*f^2*g*log(e) - a*d*f^2*g*log(e))*B^2)*x
+ 2*((b*c*g^3 - a*d*g^3)*B^2*x^3 + 3*(b*c*f*g^2 - a*d*f*g^2)*B^2*x^2 + 3*(b
*c*f^2*g - a*d*f^2*g)*B^2*x + (b*c*f^3 - a*d*f^3)*B^2)*log(b*x + a) - 2*((b
*c*g^3 - a*d*g^3)*B^2*x^3 + 3*(b*c*f*g^2 - a*d*f*g^2)*B^2*x^2 + 3*(b*c*f^2*
g - a*d*f^2*g)*B^2*x + (b*c*f^3 - a*d*f^3)*B^2)*log(d*x + c)), x)
```

**mupad** [A] time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{1}{(f+gx)^2\left(A+B\ln\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((f + g*x)^2*(A + B*log((e*(a + b*x)^2)/(c + d*x)^2))^2),x)
```

```
[Out] int(1/((f + g*x)^2*(A + B*log((e*(a + b*x)^2)/(c + d*x)^2))^2), x)
```

**sympy** [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(g*x+f)**2/(A+B*ln(e*(b*x+a)**2/(d*x+c)**2))**2,x)
```

```
[Out] Timed out
```

# 3.292 $\int \frac{1}{(f+gx)^3\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}\,dx$

**Optimal**. Leaf size=34

$$\mathrm{Int}\left(\frac{1}{(f+gx)^3\left(B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+A\right)^2},x\right)$$

```
[Out] Unintegrable(1/(g*x+f)^3/(A+B*ln(e*(b*x+a)^2/(d*x+c)^2))^2,x)
```

**Rubi [A]** time = 0.07, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{1}{(f+gx)^3\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[1/((f + g*x)^3*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])^2),x]
```

```
[Out] Defer[Int][1/((f + g*x)^3*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])^2), x]
```

Rubi steps

$$\int \frac{1}{(f+gx)^3\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}\,dx=\int \frac{1}{(f+gx)^3\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}\,dx$$

**Mathematica [A]** time = 0.64, size = 0, normalized size = 0.00

$$\int \frac{1}{(f+gx)^3\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[1/((f + g*x)^3*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])^2),x]
```

[Out] Integrate[1/((f + g*x)^3*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])^2), x]

**fricas** [A] time = 1.02, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{1}{A^2g^3x^3 + 3\,A^2fg^2x^2 + 3\,A^2f^2gx + A^2f^3 + \left(B^2g^3x^3 + 3\,B^2fg^2x^2 + 3\,B^2f^2gx + B^2f^3\right)\log\left(\frac{b^2ex^2+2\,abex+a}{d^2x^2+2\,cdx+c}\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(g*x+f)^3/(A+B*log(e*(b*x+a)^2/(d*x+c)^2))^2,x, algorithm="fric
as")
```

```
[Out] integral(1/(A^2*g^3*x^3 + 3*A^2*f*g^2*x^2 + 3*A^2*f^2*g*x + A^2*f^3 + (B^2*
g^3*x^3 + 3*B^2*f*g^2*x^2 + 3*B^2*f^2*g*x + B^2*f^3)*log((b^2*e*x^2 + 2*a*b
*e*x + a^2*e)/(d^2*x^2 + 2*c*d*x + c^2))^2 + 2*(A*B*g^3*x^3 + 3*A*B*f*g^2*x
^2 + 3*A*B*f^2*g*x + A*B*f^3)*log((b^2*e*x^2 + 2*a*b*e*x + a^2*e)/(d^2*x^2
+ 2*c*d*x + c^2))), x)
```

**giac** [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{{\left(gx + f\right)}^{3} {\left(B \log\left(\frac{{\left(bx + a\right)}^{2} e}{{\left(dx + c\right)}^{2}}\right) + A\right)}^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(g*x+f)^3/(A+B*log(e*(b*x+a)^2/(d*x+c)^2))^2,x, algorithm="giac
")
```

```
[Out] integrate(1/((g*x + f)^3*(B*log((b*x + a)^2*e/(d*x + c)^2) + A)^2), x)
```

**maple** [A] time = 1.92, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(gx + f\right)^{3} \left(B \ln\left(\frac{\left(bx + a\right)^{2} e}{\left(dx + c\right)^{2}}\right) + A\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(g*x+f)^3/(B*ln((b*x+a)^2/(d*x+c)^2*e)+A)^2,x)
```

```
[Out] int(1/(g*x+f)^3/(B*ln((b*x+a)^2/(d*x+c)^2*e)+A)^2,x)
```

**maxima** [A] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{}{2\left(\left(\left(bcg^3 - adg^3\right)AB + \left(bcg^3\log(e) - adg^3\log(e)\right)B^2\right)x^3 + \left(bcf^3 - adf^3\right)AB + \left(bcf^3\log(e) - adf^3\log(e)\right)B^2 + \right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(g*x+f)^3/(A+B*log(e*(b*x+a)^2/(d*x+c)^2))^2,x, algorithm="maxi
ma")
```

```
[Out] -1/2*(b*d*x^2 + a*c + (b*c + a*d)*x)/(((b*c*g^3 - a*d*g^3)*A*B + (b*c*g^3*l
og(e) - a*d*g^3*log(e))*B^2)*x^3 + (b*c*f^3 - a*d*f^3)*A*B + (b*c*f^3*log(e
) - a*d*f^3*log(e))*B^2 + 3*((b*c*f*g^2 - a*d*f*g^2)*A*B + (b*c*f*g^2*log(e
) - a*d*f*g^2*log(e))*B^2)*x^2 + 3*((b*c*f^2*g - a*d*f^2*g)*A*B + (b*c*f^2*
g*log(e) - a*d*f^2*g*log(e))*B^2)*x + 2*((b*c*g^3 - a*d*g^3)*B^2*x^3 + 3*(b
*c*f*g^2 - a*d*f*g^2)*B^2*x^2 + 3*(b*c*f^2*g - a*d*f^2*g)*B^2*x + (b*c*f^3
- a*d*f^3)*B^2)*log(b*x + a) - 2*((b*c*g^3 - a*d*g^3)*B^2*x^3 + 3*(b*c*f*g^
2 - a*d*f*g^2)*B^2*x^2 + 3*(b*c*f^2*g - a*d*f^2*g)*B^2*x + (b*c*f^3 - a*d*f
^3)*B^2)*log(d*x + c)) - integrate(1/2*(b*d*g*x^2 - b*c*f - (d*f - 3*c*g)*a
 + 2*(a*d*g - (d*f - c*g)*b)*x)/(((b*c*g^4 - a*d*g^4)*A*B + (b*c*g^4*log(e)
 - a*d*g^4*log(e))*B^2)*x^4 + 4*((b*c*f*g^3 - a*d*f*g^3)*A*B + (b*c*f*g^3*l
og(e) - a*d*f*g^3*log(e))*B^2)*x^3 + (b*c*f^4 - a*d*f^4)*A*B + (b*c*f^4*log
(e) - a*d*f^4*log(e))*B^2 + 6*((b*c*f^2*g^2 - a*d*f^2*g^2)*A*B + (b*c*f^2*g
^2*log(e) - a*d*f^2*g^2*log(e))*B^2)*x^2 + 4*((b*c*f^3*g - a*d*f^3*g)*A*B +
 (b*c*f^3*g*log(e) - a*d*f^3*g*log(e))*B^2)*x + 2*((b*c*g^4 - a*d*g^4)*B^2*
x^4 + 4*(b*c*f*g^3 - a*d*f*g^3)*B^2*x^3 + 6*(b*c*f^2*g^2 - a*d*f^2*g^2)*B^2
*x^2 + 4*(b*c*f^3*g - a*d*f^3*g)*B^2*x + (b*c*f^4 - a*d*f^4)*B^2)*log(b*x +
 a) - 2*((b*c*g^4 - a*d*g^4)*B^2*x^4 + 4*(b*c*f*g^3 - a*d*f*g^3)*B^2*x^3 +
6*(b*c*f^2*g^2 - a*d*f^2*g^2)*B^2*x^2 + 4*(b*c*f^3*g - a*d*f^3*g)*B^2*x + (
b*c*f^4 - a*d*f^4)*B^2)*log(d*x + c)), x)
```

**mupad [A]** time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{1}{{\left(f+g\,x\right)}^3\,{\left(A+B\,\ln\left(\frac{e\,{\left(a+b\,x\right)}^2}{{\left(c+d\,x\right)}^2}\right)\right)}^2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((f + g*x)^3*(A + B*log((e*(a + b*x)^2)/(c + d*x)^2))^2),x)
```

```
[Out] int(1/((f + g*x)^3*(A + B*log((e*(a + b*x)^2)/(c + d*x)^2))^2), x)
```

**sympy [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(g*x+f)**3/(A+B*ln(e*(b*x+a)**2/(d*x+c)**2))**2,x)
```

```
[Out] Timed out
```

# 3.293 $\int (g+hx)^4 \left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)\,dx$

**Optimal**. Leaf size=365

$$-\frac{Bh^2nx^2(bc-ad)\left(a^2d^2h^2-abdh(5dg-ch)+b^2\left(c^2h^2-5cdgh+10d^2g^2\right)\right)}{10b^3d^3}+\frac{Bhnx(bc-ad)\left(a^3d^3h^3-a^2bd^2h^2(5a\right.}{}$$

```
[Out] 1/5*B*(-a*d+b*c)*h*(a^3*d^3*h^3-a^2*b*d^2*h^2*(-c*h+5*d*g)+a*b^2*d*h*(c^2*h
^2-5*c*d*g*h+10*d^2*g^2)-b^3*(-c^3*h^3+5*c^2*d*g*h^2-10*c*d^2*g^2*h+10*d^3*
g^3))*n*x/b^4/d^4-1/10*B*(-a*d+b*c)*h^2*(a^2*d^2*h^2-a*b*d*h*(-c*h+5*d*g)+b
^2*(c^2*h^2-5*c*d*g*h+10*d^2*g^2))*n*x^2/b^3/d^3-1/15*B*(-a*d+b*c)*h^3*(-a*
d*h-b*c*h+5*b*d*g)*n*x^3/b^2/d^2-1/20*B*(-a*d+b*c)*h^4*n*x^4/b/d-1/5*B*(-a*
h+b*g)^5*n*ln(b*x+a)/b^5/h+1/5*B*(-c*h+d*g)^5*n*ln(d*x+c)/d^5/h+1/5*(h*x+g)
^5*(A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))/h
```

**Rubi [A]** time = 0.71, antiderivative size = 377, normalized size of antiderivative = 1.03, number of steps used = 5, number of rules used = 3, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.097, Rules used = {6742, 2492, 72}

$$-\frac{Bh^2nx^2(bc-ad)\left(a^2d^2h^2-abdh(5dg-ch)+b^2\left(c^2h^2-5cdgh+10d^2g^2\right)\right)}{10b^3d^3}+\frac{Bhnx(bc-ad)\left(-a^2bd^2h^2(5dg-ch)\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(g + h*x)^4*(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n]),x]
```

```
[Out] (B*(b*c - a*d)*h*(a^3*d^3*h^3 - a^2*b*d^2*h^2*(5*d*g - c*h) + a*b^2*d*h*(10
*d^2*g^2 - 5*c*d*g*h + c^2*h^2) - b^3*(10*d^3*g^3 - 10*c*d^2*g^2*h + 5*c^2*
d*g*h^2 - c^3*h^3))*n*x)/(5*b^4*d^4) - (B*(b*c - a*d)*h^2*(a^2*d^2*h^2 - a*
b*d*h*(5*d*g - c*h) + b^2*(10*d^2*g^2 - 5*c*d*g*h + c^2*h^2))*n*x^2)/(10*b^
3*d^3) - (B*(b*c - a*d)*h^3*(5*b*d*g - b*c*h - a*d*h)*n*x^3)/(15*b^2*d^2) -
 (B*(b*c - a*d)*h^4*n*x^4)/(20*b*d) + (A*(g + h*x)^5)/(5*h) - (B*(b*g - a*h
)^5*n*Log[a + b*x])/(5*b^5*h) + (B*(d*g - c*h)^5*n*Log[c + d*x])/(5*d^5*h)
+ (B*(g + h*x)^5*Log[(e*(a + b*x)^n)/(c + d*x)^n])/(5*h)
```

Rule 72

```
Int[((e_.) + (f_.)*(x_))^(p_.)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
 x_Symbol] :> Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x]
 /; FreeQ[{a, b, c, d, e, f}, x] && IntegerQ[p]
```

Rule 2492

```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))
^(r_.)]^(s_.)*((g_.) + (h_.)*(x_))^(m_.), x_Symbol] :> Simp[((g + h*x)^(m +
```

```
 1)*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^s)/(h*(m + 1)), x] - Dist[(p*r*s*(
b*c - a*d))/(h*(m + 1)), Int[((g + h*x)^(m + 1)*Log[e*(f*(a + b*x)^p*(c + d
*x)^q)^r]^(s - 1))/((a + b*x)*(c + d*x)), x], x] /; FreeQ[{a, b, c, d, e, f
, g, h, m, p, q, r, s}, x] && NeQ[b*c - a*d, 0] && EqQ[p + q, 0] && IGtQ[s,
 0] && NeQ[m, -1]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\int (g+hx)^4 \left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)\,dx = \int \left(A(g+hx)^4+B(g+hx)^4\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)\,dx$$

$$= \frac{A(g+hx)^5}{5h} + B\int (g+hx)^4\log\left(e(a+bx)^n(c+dx)^{-n}\right)\,dx$$

$$= \frac{A(g+hx)^5}{5h} + \frac{B(g+hx)^5\log\left(e(a+bx)^n(c+dx)^{-n}\right)}{5h} - \frac{(B(b}{}$$

$$= \frac{A(g+hx)^5}{5h} + \frac{B(g+hx)^5\log\left(e(a+bx)^n(c+dx)^{-n}\right)}{5h} - \frac{(B(b}{}$$

$$= \frac{B(bc-ad)h\left(a^3d^3h^3-a^2bd^2h^2(5dg-ch)+ab^2dh\left(10d^2g^2\right.\right.}{}$$

**Mathematica [A]** time = 1.01, size = 463, normalized size = 1.27

$$\frac{bdx\left(Bhn(bc-ad)\left(12a^3d^3h^3-6a^2bd^2h^2(-2ch+10dg+dhx)+2ab^2dh\left(6c^2h^2-3cdh(10g+hx)+d^2\left(60g^2+1\right.\right.\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(g + h*x)^4*(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n]),x]
```

```
[Out] (b*d*x*(12*A*b^4*d^4*(5*g^4 + 10*g^3*h*x + 10*g^2*h^2*x^2 + 5*g*h^3*x^3 + h
^4*x^4) + B*(b*c - a*d)*h*n*(12*a^3*d^3*h^3 - 6*a^2*b*d^2*h^2*(10*d*g - 2*c
*h + d*h*x) + 2*a*b^2*d*h*(6*c^2*h^2 - 3*c*d*h*(10*g + h*x) + d^2*(60*g^2 +
 15*g*h*x + 2*h^2*x^2)) - b^3*(-12*c^3*h^3 + 6*c^2*d*h^2*(10*g + h*x) - 2*c
*d^2*h*(60*g^2 + 15*g*h*x + 2*h^2*x^2) + d^3*(120*g^3 + 60*g^2*h*x + 20*g*h
^2*x^2 + 3*h^3*x^3)))) + 12*a^2*B*d^5*h*(-10*b^3*g^3 + 10*a*b^2*g^2*h - 5*a
^2*b*g*h^2 + a^3*h^3)*n*Log[a + b*x] - 12*b^4*B*(-5*a*d^5*g^4 + b*c*(5*d^4*
```

```
g^4 - 10*c*d^3*g^3*h + 10*c^2*d^2*g^2*h^2 - 5*c^3*d*g*h^3 + c^4*h^4))*n*Log
[c + d*x] + 12*b^4*B*d^5*(5*a*g^4 + b*x*(5*g^4 + 10*g^3*h*x + 10*g^2*h^2*x^
2 + 5*g*h^3*x^3 + h^4*x^4))*Log[(e*(a + b*x)^n)/(c + d*x)^n])/(60*b^5*d^5)
```

**fricas [B]** time = 0.86, size = 805, normalized size = 2.21

$$\frac{12\,Ab^5d^5h^4x^5 + 3\left(20\,Ab^5d^5gh^3 - \left(Bb^5cd^4 - Bab^4d^5\right)h^4n\right)x^4 + 4\left(30\,Ab^5d^5g^2h^2 - \left(5\left(Bb^5cd^4 - Bab^4d^5\right)gh^3 - \left(Bb\right.\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x+g)^4*(A+B*log(e*(b*x+a)^n/((d*x+c)^n))),x, algorithm="fricas
")
```

```
[Out] 1/60*(12*A*b^5*d^5*h^4*x^5 + 3*(20*A*b^5*d^5*g*h^3 - (B*b^5*c*d^4 - B*a*b^4
*d^5)*h^4*n)*x^4 + 4*(30*A*b^5*d^5*g^2*h^2 - (5*(B*b^5*c*d^4 - B*a*b^4*d^5)
*g*h^3 - (B*b^5*c^2*d^3 - B*a^2*b^3*d^5)*h^4)*n)*x^3 + 6*(20*A*b^5*d^5*g^3*
h - (10*(B*b^5*c*d^4 - B*a*b^4*d^5)*g^2*h^2 - 5*(B*b^5*c^2*d^3 - B*a^2*b^3*
d^5)*g*h^3 + (B*b^5*c^3*d^2 - B*a^3*b^2*d^5)*h^4)*n)*x^2 + 12*(5*A*b^5*d^5*
g^4 - (10*(B*b^5*c*d^4 - B*a*b^4*d^5)*g^3*h - 10*(B*b^5*c^2*d^3 - B*a^2*b^3
*d^5)*g^2*h^2 + 5*(B*b^5*c^3*d^2 - B*a^3*b^2*d^5)*g*h^3 - (B*b^5*c^4*d - B*
a^4*b*d^5)*h^4)*n)*x + 12*(B*b^5*d^5*h^4*n*x^5 + 5*B*b^5*d^5*g*h^3*n*x^4 +
10*B*b^5*d^5*g^2*h^2*n*x^3 + 10*B*b^5*d^5*g^3*h*n*x^2 + 5*B*b^5*d^5*g^4*n*x
 + (5*B*a*b^4*d^5*g^4 - 10*B*a^2*b^3*d^5*g^3*h + 10*B*a^3*b^2*d^5*g^2*h^2 -
 5*B*a^4*b*d^5*g*h^3 + B*a^5*d^5*h^4)*n)*log(b*x + a) - 12*(B*b^5*d^5*h^4*n
*x^5 + 5*B*b^5*d^5*g*h^3*n*x^4 + 10*B*b^5*d^5*g^2*h^2*n*x^3 + 10*B*b^5*d^5*
g^3*h*n*x^2 + 5*B*b^5*d^5*g^4*n*x + (5*B*b^5*c*d^4*g^4 - 10*B*b^5*c^2*d^3*g
^3*h + 10*B*b^5*c^3*d^2*g^2*h^2 - 5*B*b^5*c^4*d*g*h^3 + B*b^5*c^5*h^4)*n)*l
og(d*x + c) + 12*(B*b^5*d^5*h^4*x^5 + 5*B*b^5*d^5*g*h^3*x^4 + 10*B*b^5*d^5*
g^2*h^2*x^3 + 10*B*b^5*d^5*g^3*h*x^2 + 5*B*b^5*d^5*g^4*x)*log(e))/(b^5*d^5)
```

**giac [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x+g)^4*(A+B*log(e*(b*x+a)^n/((d*x+c)^n))),x, algorithm="giac")
```

```
[Out] Timed out
```

**maple [C]** time = 0.66, size = 2576, normalized size = 7.06

Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((h*x+g)^4*(A+B*ln(e*(b*x+a)^n/((d*x+c)^n))),x)

[Out] 1/5*h^4*B*ln(e)*x^5+B*ln(e)*g^4*x+h^3*A*g*x^4+2*h^2*A*g^2*x^3+2*h*A*g^3*x^2
+A*g^4*x+1/5*h^4*B*x^5*ln((b*x+a)^n)+ln((b*x+a)^n)*x*B*g^4-1/5*(h*x+g)^5*B/
h*ln((d*x+c)^n)+1/2/d^2*h^3*B*c^2*g*n*x^2-1/d*h^2*B*c*g^2*n*x^2+h^3/b^3*B*a
^3*g*n*x-2*h^2/b^2*B*a^2*g^2*n*x+2*h/b*B*a*g^3*n*x-1/d^3*h^3*B*c^3*g*n*x+2/
d^2*h^2*B*c^2*g^2*n*x-2/d*h*B*c*g^3*n*x+I*h^2*B*Pi*g^2*x^3*csgn(I*(b*x+a)^n
)*csgn(I*(b*x+a)^n/((d*x+c)^n))^2+I*h*B*Pi*g^3*x^2*csgn(I*(b*x+a)^n)*csgn(I
*(b*x+a)^n/((d*x+c)^n))^2+I*h*B*Pi*g^3*x^2*csgn(I/((d*x+c)^n))*csgn(I*(b*x+
a)^n/((d*x+c)^n))^2+I*h*B*Pi*g^3*x^2*csgn(I*(b*x+a)^n/((d*x+c)^n))*csgn(I*e
/((d*x+c)^n)*(b*x+a)^n)^2+I*h*B*Pi*g^3*x^2*csgn(I*e)*csgn(I*e/((d*x+c)^n)*(
b*x+a)^n)^2+I*h^2*B*Pi*g^2*x^3*csgn(I*e)*csgn(I*e/((d*x+c)^n)*(b*x+a)^n)^2+
I*h^2*B*Pi*g^2*x^3*csgn(I/((d*x+c)^n))*csgn(I*(b*x+a)^n/((d*x+c)^n))^2+I*h^
2*B*Pi*g^2*x^3*csgn(I*(b*x+a)^n/((d*x+c)^n))*csgn(I*e/((d*x+c)^n)*(b*x+a)^n
)^2-1/2*I*B*Pi*g^4*x*csgn(I*(b*x+a)^n)*csgn(I/((d*x+c)^n))*csgn(I*(b*x+a)^n
/((d*x+c)^n))-I*h*B*Pi*g^3*x^2*csgn(I*(b*x+a)^n)*csgn(I/((d*x+c)^n))*csgn(I
*(b*x+a)^n/((d*x+c)^n))-I*h^2*B*Pi*g^2*x^3*csgn(I*(b*x+a)^n)*csgn(I/((d*x+c
)^n))*csgn(I*(b*x+a)^n/((d*x+c)^n))-I*h*B*Pi*g^3*x^2*csgn(I*e)*csgn(I*(b*x+
a)^n/((d*x+c)^n))*csgn(I*e/((d*x+c)^n)*(b*x+a)^n)-1/2*I*h^3*B*Pi*g*x^4*csgn
(I*e)*csgn(I*(b*x+a)^n/((d*x+c)^n))*csgn(I*e/((d*x+c)^n)*(b*x+a)^n)-1/2*I*h
^3*B*Pi*g*x^4*csgn(I*(b*x+a)^n)*csgn(I/((d*x+c)^n))*csgn(I*(b*x+a)^n/((d*x+
c)^n))-I*h^2*B*Pi*g^2*x^3*csgn(I*e)*csgn(I*(b*x+a)^n/((d*x+c)^n))*csgn(I*e/
((d*x+c)^n)*(b*x+a)^n)+1/5*h^4*A*x^5-2/d^3*h^2*B*ln(d*x+c)*c^3*g^2*n+2/d^2*
h*B*ln(d*x+c)*c^2*g^3*n-h^3/b^4*B*ln(-b*x-a)*a^4*g*n+2*h^2/b^3*B*ln(-b*x-a)
*a^3*g^2*n-2*h/b^2*B*ln(-b*x-a)*a^2*g^3*n+1/2*I*B*Pi*g^4*x*csgn(I*(b*x+a)^n
)*csgn(I*(b*x+a)^n/((d*x+c)^n))^2+1/2*I*B*Pi*g^4*x*csgn(I*e)*csgn(I*e/((d*x
+c)^n)*(b*x+a)^n)^2+1/2*I*B*Pi*g^4*x*csgn(I*(b*x+a)^n/((d*x+c)^n))*csgn(I*e
/((d*x+c)^n)*(b*x+a)^n)^2+1/2*I*B*Pi*g^4*x*csgn(I/((d*x+c)^n))*csgn(I*(b*x+
a)^n/((d*x+c)^n))^2-I*h*B*Pi*g^3*x^2*csgn(I*e/((d*x+c)^n)*(b*x+a)^n)^3+1/10
*I*h^4*B*Pi*x^5*csgn(I*e)*csgn(I*e/((d*x+c)^n)*(b*x+a)^n)^2+1/10*I*h^4*B*Pi
*x^5*csgn(I*(b*x+a)^n)*csgn(I*(b*x+a)^n/((d*x+c)^n))^2+1/10*I*h^4*B*Pi*x^5*
csgn(I/((d*x+c)^n))*csgn(I*(b*x+a)^n/((d*x+c)^n))^2+1/10*I*h^4*B*Pi*x^5*csg
n(I*(b*x+a)^n/((d*x+c)^n))*csgn(I*e/((d*x+c)^n)*(b*x+a)^n)^2-1/2*I*h^3*B*Pi
*g*x^4*csgn(I*(b*x+a)^n/((d*x+c)^n))^3-1/2*I*h^3*B*Pi*g*x^4*csgn(I*e/((d*x+
c)^n)*(b*x+a)^n)^3-I*h^2*B*Pi*g^2*x^3*csgn(I*(b*x+a)^n/((d*x+c)^n))^3-I*h^2
*B*Pi*g^2*x^3*csgn(I*e/((d*x+c)^n)*(b*x+a)^n)^3-I*h*B*Pi*g^3*x^2*csgn(I*(b*
x+a)^n/((d*x+c)^n))^3+h^3*B*g*x^4*ln((b*x+a)^n)+2*h^2*B*g^2*x^3*ln((b*x+a)^
n)+2*h*B*g^3*x^2*ln((b*x+a)^n)+1/5/h*B*ln(d*x+c)*g^5*n+h^3*B*ln(e)*g*x^4+2*
h^2*B*ln(e)*g^2*x^3+2*h*B*ln(e)*g^3*x^2-1/d*B*ln(d*x+c)*c*g^4*n+1/b*B*ln(-b
*x-a)*a*g^4*n-1/5/d^5*h^4*B*ln(d*x+c)*c^5*n+1/5*h^4/b^5*B*ln(-b*x-a)*a^5*n-
1/10*I*h^4*B*Pi*x^5*csgn(I*(b*x+a)^n/((d*x+c)^n))^3-1/10*I*h^4*B*Pi*x^5*csg
n(I*e/((d*x+c)^n)*(b*x+a)^n)^3-1/2*I*B*Pi*g^4*x*csgn(I*(b*x+a)^n/((d*x+c)^n
))^3-1/2*I*B*Pi*g^4*x*csgn(I*e/((d*x+c)^n)*(b*x+a)^n)^3+1/3*h^3/b*B*a*g*n*x
^3-1/3/d*h^3*B*c*g*n*x^3-1/2*h^3/b^2*B*a^2*g*n*x^2+h^2/b*B*a*g^2*n*x^2+1/10
*h^4/b^3*B*a^3*n*x^2-1/10/d^3*h^4*B*c^3*n*x^2-1/5*h^4/b^4*B*a^4*n*x+1/5/d^4
```

```
*h^4*B*c^4*n*x+1/d^4*h^3*B*ln(d*x+c)*c^4*g*n-1/2*I*B*Pi*g^4*x*csgn(I*e)*csg
n(I*(b*x+a)^n/((d*x+c)^n))*csgn(I*e/((d*x+c)^n)*(b*x+a)^n)+1/2*I*h^3*B*Pi*g
*x^4*csgn(I/((d*x+c)^n))*csgn(I*(b*x+a)^n/((d*x+c)^n))^2+1/2*I*h^3*B*Pi*g*x
^4*csgn(I*(b*x+a)^n/((d*x+c)^n))*csgn(I*e/((d*x+c)^n)*(b*x+a)^n)^2-1/10*I*h
^4*B*Pi*x^5*csgn(I*(b*x+a)^n)*csgn(I/((d*x+c)^n))*csgn(I*(b*x+a)^n/((d*x+c)
^n))-1/10*I*h^4*B*Pi*x^5*csgn(I*e)*csgn(I*(b*x+a)^n/((d*x+c)^n))*csgn(I*e/(
(d*x+c)^n)*(b*x+a)^n)+1/2*I*h^3*B*Pi*g*x^4*csgn(I*e)*csgn(I*e/((d*x+c)^n)*(
b*x+a)^n)^2+1/2*I*h^3*B*Pi*g*x^4*csgn(I*(b*x+a)^n)*csgn(I*(b*x+a)^n/((d*x+c
)^n))^2+1/20*h^4/b*B*a*n*x^4-1/20/d*h^4*B*c*n*x^4-1/15*h^4/b^2*B*a^2*n*x^3+
1/15/d^2*h^4*B*c^2*n*x^3
```

**maxima [A]** time = 0.80, size = 671, normalized size = 1.84

$$\frac{1}{5}\,Bh^4x^5\log\left(\frac{(bx+a)^ne}{(dx+c)^n}\right)+\frac{1}{5}\,Ah^4x^5+Bgh^3x^4\log\left(\frac{(bx+a)^ne}{(dx+c)^n}\right)+Agh^3x^4+2\,Bg^2h^2x^3\log\left(\frac{(bx+a)^ne}{(dx+c)^n}\right)+2\,Ag^2h^2x^3+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x+g)^4*(A+B*log(e*(b*x+a)^n/((d*x+c)^n))),x, algorithm="maxima
")
```

```
[Out] 1/5*B*h^4*x^5*log((b*x + a)^n*e/(d*x + c)^n) + 1/5*A*h^4*x^5 + B*g*h^3*x^4*
log((b*x + a)^n*e/(d*x + c)^n) + A*g*h^3*x^4 + 2*B*g^2*h^2*x^3*log((b*x + a
)^n*e/(d*x + c)^n) + 2*A*g^2*h^2*x^3 + 2*B*g^3*h*x^2*log((b*x + a)^n*e/(d*x
 + c)^n) + 2*A*g^3*h*x^2 + B*g^4*x*log((b*x + a)^n*e/(d*x + c)^n) + A*g^4*x
 + (a*e*n*log(b*x + a)/b - c*e*n*log(d*x + c)/d)*B*g^4/e - 2*(a^2*e*n*log(b
*x + a)/b^2 - c^2*e*n*log(d*x + c)/d^2 + (b*c*e*n - a*d*e*n)*x/(b*d))*B*g^3
*h/e + (2*a^3*e*n*log(b*x + a)/b^3 - 2*c^3*e*n*log(d*x + c)/d^3 - ((b^2*c*d
*e*n - a*b*d^2*e*n)*x^2 - 2*(b^2*c^2*e*n - a^2*d^2*e*n)*x)/(b^2*d^2))*B*g^2
*h^2/e - 1/6*(6*a^4*e*n*log(b*x + a)/b^4 - 6*c^4*e*n*log(d*x + c)/d^4 + (2*
(b^3*c*d^2*e*n - a*b^2*d^3*e*n)*x^3 - 3*(b^3*c^2*d*e*n - a^2*b*d^3*e*n)*x^2
 + 6*(b^3*c^3*e*n - a^3*d^3*e*n)*x)/(b^3*d^3))*B*g*h^3/e + 1/60*(12*a^5*e*n
*log(b*x + a)/b^5 - 12*c^5*e*n*log(d*x + c)/d^5 - (3*(b^4*c*d^3*e*n - a*b^3
*d^4*e*n)*x^4 - 4*(b^4*c^2*d^2*e*n - a^2*b^2*d^4*e*n)*x^3 + 6*(b^4*c^3*d*e*
n - a^3*b*d^4*e*n)*x^2 - 12*(b^4*c^4*e*n - a^4*d^4*e*n)*x)/(b^4*d^4))*B*h^4
/e
```

**mupad [B]** time = 5.13, size = 1434, normalized size = 3.93

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g + h*x)^4*(A + B*log((e*(a + b*x)^n)/(c + d*x)^n)),x)
```

```
[Out] x*((5*A*b*d*g^4 + 20*A*a*d*g^3*h + 20*A*b*c*g^3*h + 30*A*a*c*g^2*h^2 + 10*B
*a*d*g^3*h*n - 10*B*b*c*g^3*h*n)/(5*b*d) - ((5*a*d + 5*b*c)*((20*A*a*c*g*h^
3 + 20*A*b*d*g^3*h + 30*A*a*d*g^2*h^2 + 30*A*b*c*g^2*h^2 + 10*B*a*d*g^2*h^2
*n - 10*B*b*c*g^2*h^2*n)/(5*b*d) + ((5*a*d + 5*b*c)*((((5*A*a*d*h^4 + 5*A*b
*c*h^4 + 20*A*b*d*g*h^3 + B*a*d*h^4*n - B*b*c*h^4*n)/(5*b*d) - (A*h^4*(5*a*
d + 5*b*c))/(5*b*d))*(5*a*d + 5*b*c))/(5*b*d) - (5*A*a*c*h^4 + 20*A*a*d*g*h
^3 + 20*A*b*c*g*h^3 + 30*A*b*d*g^2*h^2 + 5*B*a*d*g*h^3*n - 5*B*b*c*g*h^3*n)
/(5*b*d) + (A*a*c*h^4)/(b*d)))/(5*b*d) - (a*c*((5*A*a*d*h^4 + 5*A*b*c*h^4 +
 20*A*b*d*g*h^3 + B*a*d*h^4*n - B*b*c*h^4*n)/(5*b*d) - (A*h^4*(5*a*d + 5*b*
c))/(5*b*d)))/(b*d)))/(5*b*d) + (a*c*((((5*A*a*d*h^4 + 5*A*b*c*h^4 + 20*A*b
*d*g*h^3 + B*a*d*h^4*n - B*b*c*h^4*n)/(5*b*d) - (A*h^4*(5*a*d + 5*b*c))/(5*
b*d))*(5*a*d + 5*b*c))/(5*b*d) - (5*A*a*c*h^4 + 20*A*a*d*g*h^3 + 20*A*b*c*g
*h^3 + 30*A*b*d*g^2*h^2 + 5*B*a*d*g*h^3*n - 5*B*b*c*g*h^3*n)/(5*b*d) + (A*a
*c*h^4)/(b*d)))/(b*d)) + log((e*(a + b*x)^n)/(c + d*x)^n)*((B*h^4*x^5)/5 +
B*g^4*x + 2*B*g^2*h^2*x^3 + 2*B*g^3*h*x^2 + B*g*h^3*x^4) + x^4*((5*A*a*d*h^
4 + 5*A*b*c*h^4 + 20*A*b*d*g*h^3 + B*a*d*h^4*n - B*b*c*h^4*n)/(20*b*d) - (A
*h^4*(5*a*d + 5*b*c))/(20*b*d)) - x^3*((((5*A*a*d*h^4 + 5*A*b*c*h^4 + 20*A*
b*d*g*h^3 + B*a*d*h^4*n - B*b*c*h^4*n)/(5*b*d) - (A*h^4*(5*a*d + 5*b*c))/(5
*b*d))*(5*a*d + 5*b*c))/(15*b*d) - (5*A*a*c*h^4 + 20*A*a*d*g*h^3 + 20*A*b*c
*g*h^3 + 30*A*b*d*g^2*h^2 + 5*B*a*d*g*h^3*n - 5*B*b*c*g*h^3*n)/(15*b*d) + (
A*a*c*h^4)/(3*b*d)) + x^2*((20*A*a*c*g*h^3 + 20*A*b*d*g^3*h + 30*A*a*d*g^2*
h^2 + 30*A*b*c*g^2*h^2 + 10*B*a*d*g^2*h^2*n - 10*B*b*c*g^2*h^2*n)/(10*b*d)
+ ((5*a*d + 5*b*c)*((((5*A*a*d*h^4 + 5*A*b*c*h^4 + 20*A*b*d*g*h^3 + B*a*d*h
^4*n - B*b*c*h^4*n)/(5*b*d) - (A*h^4*(5*a*d + 5*b*c))/(5*b*d))*(5*a*d + 5*b
*c))/(5*b*d) - (5*A*a*c*h^4 + 20*A*a*d*g*h^3 + 20*A*b*c*g*h^3 + 30*A*b*d*g^
2*h^2 + 5*B*a*d*g*h^3*n - 5*B*b*c*g*h^3*n)/(5*b*d) + (A*a*c*h^4)/(b*d)))/(1
0*b*d) - (a*c*((5*A*a*d*h^4 + 5*A*b*c*h^4 + 20*A*b*d*g*h^3 + B*a*d*h^4*n -
B*b*c*h^4*n)/(5*b*d) - (A*h^4*(5*a*d + 5*b*c))/(5*b*d)))/(2*b*d)) + (A*h^4*
x^5)/5 + (log(a + b*x)*((B*a^5*h^4*n)/5 + B*a*b^4*g^4*n + 2*B*a^3*b^2*g^2*h
^2*n - B*a^4*b*g*h^3*n - 2*B*a^2*b^3*g^3*h*n))/b^5 - (log(c + d*x)*(B*c^5*h
^4*n + 5*B*c*d^4*g^4*n + 10*B*c^3*d^2*g^2*h^2*n - 5*B*c^4*d*g*h^3*n - 10*B*
c^2*d^3*g^3*h*n))/(5*d^5)
```

**sympy [F(-2)]** time = 0.00, size = 0, normalized size = 0.00

Exception raised: HeuristicGCDFailed

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x+g)**4*(A+B*ln(e*(b*x+a)**n/((d*x+c)**n))),x)
```

```
[Out] Exception raised: HeuristicGCDFailed
```

# 3.294 $\int (g+hx)^3 \left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right) dx$

**Optimal**. Leaf size=236

$$-\frac{Bhnx(bc-ad)\left(a^2d^2h^2-abdh(4dg-ch)+b^2\left(c^2h^2-4cdgh+6d^2g^2\right)\right)}{4b^3d^3}+\frac{(g+hx)^4\left(B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right.}{4h}$$

```
[Out] -1/4*B*(-a*d+b*c)*h*(a^2*d^2*h^2-a*b*d*h*(-c*h+4*d*g)+b^2*(c^2*h^2-4*c*d*g*
h+6*d^2*g^2))*n*x/b^3/d^3-1/8*B*(-a*d+b*c)*h^2*(-a*d*h-b*c*h+4*b*d*g)*n*x^2
/b^2/d^2-1/12*B*(-a*d+b*c)*h^3*n*x^3/b/d-1/4*B*(-a*h+b*g)^4*n*ln(b*x+a)/b^4
/h+1/4*B*(-c*h+d*g)^4*n*ln(d*x+c)/d^4/h+1/4*(h*x+g)^4*(A+B*ln(e*(b*x+a)^n/(
(d*x+c)^n)))/h
```

**Rubi [A]** time = 0.46, antiderivative size = 248, normalized size of antiderivative = 1.05, number of steps used = 5, number of rules used = 3, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.097, Rules used = {6742, 2492, 72}

$$-\frac{Bhnx(bc-ad)\left(a^2d^2h^2-abdh(4dg-ch)+b^2\left(c^2h^2-4cdgh+6d^2g^2\right)\right)}{4b^3d^3}-\frac{Bh^2nx^2(bc-ad)(-adh-bch+4bdg)}{8b^2d^2}-$$

Antiderivative was successfully verified.

```
[In] Int[(g + h*x)^3*(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n]),x]
```

```
[Out] -(B*(b*c - a*d)*h*(a^2*d^2*h^2 - a*b*d*h*(4*d*g - c*h) + b^2*(6*d^2*g^2 - 4
*c*d*g*h + c^2*h^2))*n*x)/(4*b^3*d^3) - (B*(b*c - a*d)*h^2*(4*b*d*g - b*c*h
 - a*d*h)*n*x^2)/(8*b^2*d^2) - (B*(b*c - a*d)*h^3*n*x^3)/(12*b*d) + (A*(g +
 h*x)^4)/(4*h) - (B*(b*g - a*h)^4*n*Log[a + b*x])/(4*b^4*h) + (B*(d*g - c*h
)^4*n*Log[c + d*x])/(4*d^4*h) + (B*(g + h*x)^4*Log[(e*(a + b*x)^n)/(c + d*x
)^n])/(4*h)
```

Rule 72

```
Int[((e_.) + (f_.)*(x_))^(p_.)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
 x_Symbol] :> Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x]
 /; FreeQ[{a, b, c, d, e, f}, x] && IntegerQ[p]
```

Rule 2492

```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))
^(r_.)]^(s_.)*((g_.) + (h_.)*(x_))^(m_.), x_Symbol] :> Simp[((g + h*x)^(m +
 1)*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^s)/(h*(m + 1)), x] - Dist[(p*r*s*(
b*c - a*d))/(h*(m + 1)), Int[((g + h*x)^(m + 1)*Log[e*(f*(a + b*x)^p*(c + d
*x)^q)^r]^(s - 1))/((a + b*x)*(c + d*x)), x], x] /; FreeQ[{a, b, c, d, e, f
, g, h, m, p, q, r, s}, x] && NeQ[b*c - a*d, 0] && EqQ[p + q, 0] && IGtQ[s,
```

```
0] && NeQ[m, -1]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\begin{aligned}
\int (g+hx)^3 \left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right) dx &= \int \left(A(g+hx)^3+B(g+hx)^3\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right) dx \\
&= \frac{A(g+hx)^4}{4h}+B\int (g+hx)^3\log\left(e(a+bx)^n(c+dx)^{-n}\right) dx \\
&= \frac{A(g+hx)^4}{4h}+\frac{B(g+hx)^4\log\left(e(a+bx)^n(c+dx)^{-n}\right)}{4h}-\frac{(B(b
\\
&= \frac{A(g+hx)^4}{4h}+\frac{B(g+hx)^4\log\left(e(a+bx)^n(c+dx)^{-n}\right)}{4h}-\frac{(B(b
\\
&= -\frac{B(bc-ad)h\left(a^2d^2h^2-abdh(4dg-ch)+b^2\left(6d^2g^2-4cdg
}{4b^3d^3}
\end{aligned}$$

**Mathematica [A]** time = 0.61, size = 314, normalized size = 1.33

$$\frac{bdx\left(6Ab^3d^3\left(4g^3+6g^2hx+4gh^2x^2+h^3x^3\right)-Bhn(bc-ad)\left(6a^2d^2h^2-3abdh(-2ch+8dg+dhx)+b^2\left(6c^2h^2-
}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(g + h*x)^3*(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n]),x]
```

```
[Out] (b*d*x*(6*A*b^3*d^3*(4*g^3 + 6*g^2*h*x + 4*g*h^2*x^2 + h^3*x^3) - B*(b*c -
a*d)*h*n*(6*a^2*d^2*h^2 - 3*a*b*d*h*(8*d*g - 2*c*h + d*h*x) + b^2*(6*c^2*h^
2 - 3*c*d*h*(8*g + h*x) + 2*d^2*(18*g^2 + 6*g*h*x + h^2*x^2)))) - 6*a^2*B*d
^4*h*(6*b^2*g^2 - 4*a*b*g*h + a^2*h^2)*n*Log[a + b*x] + 6*b^3*B*(4*a*d^4*g^
3 + b*c*(-4*d^3*g^3 + 6*c*d^2*g^2*h - 4*c^2*d*g*h^2 + c^3*h^3))*n*Log[c + d
*x] + 6*b^3*B*d^4*(4*a*g^3 + b*x*(4*g^3 + 6*g^2*h*x + 4*g*h^2*x^2 + h^3*x^3
))*Log[(e*(a + b*x)^n)/(c + d*x)^n])/(24*b^4*d^4)
```

**fricas [B]** time = 0.86, size = 571, normalized size = 2.42

$$\frac{6\,Ab^4d^4h^3x^4+2\left(12\,Ab^4d^4gh^2-\left(Bb^4cd^3-Bab^3d^4\right)h^3n\right)x^3+3\left(12\,Ab^4d^4g^2h-\left(4\left(Bb^4cd^3-Bab^3d^4\right)gh^2-\left(Bb^4
}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x+g)^3*(A+B*log(e*(b*x+a)^n/((d*x+c)^n))),x, algorithm="fricas
")
```

```
[Out] 1/24*(6*A*b^4*d^4*h^3*x^4 + 2*(12*A*b^4*d^4*g*h^2 - (B*b^4*c*d^3 - B*a*b^3*
d^4)*h^3*n)*x^3 + 3*(12*A*b^4*d^4*g^2*h - (4*(B*b^4*c*d^3 - B*a*b^3*d^4)*g*
h^2 - (B*b^4*c^2*d^2 - B*a^2*b^2*d^4)*h^3)*n)*x^2 + 6*(4*A*b^4*d^4*g^3 - (6
*(B*b^4*c*d^3 - B*a*b^3*d^4)*g^2*h - 4*(B*b^4*c^2*d^2 - B*a^2*b^2*d^4)*g*h^
2 + (B*b^4*c^3*d - B*a^3*b*d^4)*h^3)*n)*x + 6*(B*b^4*d^4*h^3*n*x^4 + 4*B*b^
4*d^4*g*h^2*n*x^3 + 6*B*b^4*d^4*g^2*h*n*x^2 + 4*B*b^4*d^4*g^3*n*x + (4*B*a*
b^3*d^4*g^3 - 6*B*a^2*b^2*d^4*g^2*h + 4*B*a^3*b*d^4*g*h^2 - B*a^4*d^4*h^3)*
n)*log(b*x + a) - 6*(B*b^4*d^4*h^3*n*x^4 + 4*B*b^4*d^4*g*h^2*n*x^3 + 6*B*b^
4*d^4*g^2*h*n*x^2 + 4*B*b^4*d^4*g^3*n*x + (4*B*b^4*c*d^3*g^3 - 6*B*b^4*c^2*
d^2*g^2*h + 4*B*b^4*c^3*d*g*h^2 - B*b^4*c^4*h^3)*n)*log(d*x + c) + 6*(B*b^4
*d^4*h^3*x^4 + 4*B*b^4*d^4*g*h^2*x^3 + 6*B*b^4*d^4*g^2*h*x^2 + 4*B*b^4*d^4*
g^3*x)*log(e))/(b^4*d^4)
```

**giac [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x+g)^3*(A+B*log(e*(b*x+a)^n/((d*x+c)^n))),x, algorithm="giac")
```

```
[Out] Timed out
```

**maple [C]** time = 0.57, size = 1967, normalized size = 8.33

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((h*x+g)^3*(A+B*ln(e*(b*x+a)^n/((d*x+c)^n))),x)
```

```
[Out] ln((b*x+a)^n)*x*B*g^3-1/4*(h*x+g)^4*B/h*ln((d*x+c)^n)+1/4/h*B*ln(-d*x-c)*g^
4*n+h^2*B*g*x^3*ln((b*x+a)^n)+3/2*h*B*g^2*x^2*ln((b*x+a)^n)+h^2*B*ln(e)*g*x
^3+3/2*h*B*ln(e)*g^2*x^2+B*ln(e)*g^3*x+1/4*h^3*B*x^4*ln((b*x+a)^n)+1/4*h^3*
B*ln(e)*x^4+3/4*I*h*B*Pi*g^2*x^2*csgn(I*e)*csgn(I*e/((d*x+c)^n)*(b*x+a)^n)^
2+3/4*I*h*B*Pi*g^2*x^2*csgn(I*(b*x+a)^n)*csgn(I*(b*x+a)^n/((d*x+c)^n))^2+1/
2*I*h^2*B*Pi*g*x^3*csgn(I*e)*csgn(I*e/((d*x+c)^n)*(b*x+a)^n)^2+3/4*I*h*B*Pi
*g^2*x^2*csgn(I/((d*x+c)^n))*csgn(I*(b*x+a)^n/((d*x+c)^n))^2+3/4*I*h*B*Pi*g
^2*x^2*csgn(I*(b*x+a)^n/((d*x+c)^n))*csgn(I*e/((d*x+c)^n)*(b*x+a)^n)^2-1/8*
I*h^3*B*Pi*x^4*csgn(I*e)*csgn(I*(b*x+a)^n/((d*x+c)^n))*csgn(I*e/((d*x+c)^n)
*(b*x+a)^n)-1/8*I*h^3*B*Pi*x^4*csgn(I*(b*x+a)^n)*csgn(I/((d*x+c)^n))*csgn(I
*(b*x+a)^n/((d*x+c)^n))+1/2*I*h^2*B*Pi*g*x^3*csgn(I*(b*x+a)^n/((d*x+c)^n))*
```

```
csgn(I*e/((d*x+c)^n)*(b*x+a)^n)^2+1/2*I*h^2*B*Pi*g*x^3*csgn(I*(b*x+a)^n)*cs
gn(I*(b*x+a)^n/((d*x+c)^n))^2+1/2*I*h^2*B*Pi*g*x^3*csgn(I/((d*x+c)^n))*csgn
(I*(b*x+a)^n/((d*x+c)^n))^2-1/2*I*B*Pi*g^3*x*csgn(I*e)*csgn(I*(b*x+a)^n/((d
*x+c)^n))*csgn(I*e/((d*x+c)^n)*(b*x+a)^n)-1/2*I*B*Pi*g^3*x*csgn(I*(b*x+a)^n
)*csgn(I/((d*x+c)^n))*csgn(I*(b*x+a)^n/((d*x+c)^n))+1/12*h^3/b*B*a*n*x^3-1/
12/d*h^3*B*c*n*x^3-1/8*h^3/b^2*B*a^2*n*x^2+1/8/d^2*h^3*B*c^2*n*x^2+1/4*h^3/
b^3*B*a^3*n*x+h^2*A*g*x^3+3/2*h*A*g^2*x^2+A*g^3*x+1/4/d^4*h^3*B*ln(-d*x-c)*
c^4*n-1/4*h^3/b^4*B*ln(b*x+a)*a^4*n-1/d*B*ln(-d*x-c)*c*g^3*n+1/b*B*ln(b*x+a
)*a*g^3*n-1/2*I*B*Pi*g^3*x*csgn(I*(b*x+a)^n/((d*x+c)^n))^3-1/2*I*B*Pi*g^3*x
*csgn(I*e/((d*x+c)^n)*(b*x+a)^n)^3-1/8*I*h^3*B*Pi*x^4*csgn(I*(b*x+a)^n/((d*
x+c)^n))^3-1/8*I*h^3*B*Pi*x^4*csgn(I*e/((d*x+c)^n)*(b*x+a)^n)^3+1/4*h^3*A*x
^4-3/4*I*h*B*Pi*g^2*x^2*csgn(I*e)*csgn(I*(b*x+a)^n/((d*x+c)^n))*csgn(I*e/((
d*x+c)^n)*(b*x+a)^n)-1/2*I*h^2*B*Pi*g*x^3*csgn(I*e)*csgn(I*(b*x+a)^n/((d*x+
c)^n))*csgn(I*e/((d*x+c)^n)*(b*x+a)^n)-3/4*I*h*B*Pi*g^2*x^2*csgn(I*(b*x+a)^
n)*csgn(I/((d*x+c)^n))*csgn(I*(b*x+a)^n/((d*x+c)^n))-1/2*I*h^2*B*Pi*g*x^3*c
sgn(I*(b*x+a)^n)*csgn(I/((d*x+c)^n))*csgn(I*(b*x+a)^n/((d*x+c)^n))-1/4/d^3*
h^3*B*c^3*n*x-1/d^3*h^2*B*ln(-d*x-c)*c^3*g*n+3/2/d^2*h*B*ln(-d*x-c)*c^2*g^2
*n+h^2/b^3*B*ln(b*x+a)*a^3*g*n-3/2*h/b^2*B*ln(b*x+a)*a^2*g^2*n-3/4*I*h*B*Pi
*g^2*x^2*csgn(I*(b*x+a)^n/((d*x+c)^n))^3-3/4*I*h*B*Pi*g^2*x^2*csgn(I*e/((d*
x+c)^n)*(b*x+a)^n)^3+1/8*I*h^3*B*Pi*x^4*csgn(I/((d*x+c)^n))*csgn(I*(b*x+a)^
n/((d*x+c)^n))^2+1/8*I*h^3*B*Pi*x^4*csgn(I*(b*x+a)^n/((d*x+c)^n))*csgn(I*e/
((d*x+c)^n)*(b*x+a)^n)^2-1/2*I*h^2*B*Pi*g*x^3*csgn(I*(b*x+a)^n/((d*x+c)^n))
^3-1/2*I*h^2*B*Pi*g*x^3*csgn(I*e/((d*x+c)^n)*(b*x+a)^n)^3+1/8*I*h^3*B*Pi*x^
4*csgn(I*(b*x+a)^n)*csgn(I*(b*x+a)^n/((d*x+c)^n))^2+1/2*I*B*Pi*g^3*x*csgn(I
*(b*x+a)^n)*csgn(I*(b*x+a)^n/((d*x+c)^n))^2+1/2*I*B*Pi*g^3*x*csgn(I*e)*csgn
(I*e/((d*x+c)^n)*(b*x+a)^n)^2+1/2*I*B*Pi*g^3*x*csgn(I*(b*x+a)^n/((d*x+c)^n)
)*csgn(I*e/((d*x+c)^n)*(b*x+a)^n)^2+1/2*I*B*Pi*g^3*x*csgn(I/((d*x+c)^n))*cs
gn(I*(b*x+a)^n/((d*x+c)^n))^2+1/8*I*h^3*B*Pi*x^4*csgn(I*e)*csgn(I*e/((d*x+c
)^n)*(b*x+a)^n)^2+1/2*h^2/b*B*a*g*n*x^2-1/2/d*h^2*B*c*g*n*x^2-h^2/b^2*B*a^2
*g*n*x+3/2*h/b*B*a*g^2*n*x+1/d^2*h^2*B*c^2*g*n*x-3/2/d*h*B*c*g^2*n*x
```

**maxima [B]** time = 0.66, size = 467, normalized size = 1.98

$$\frac{1}{4}Bh^3x^4\log\left(\frac{(bx+a)^ne}{(dx+c)^n}\right)+\frac{1}{4}Ah^3x^4+Bgh^2x^3\log\left(\frac{(bx+a)^ne}{(dx+c)^n}\right)+Agh^2x^3+\frac{3}{2}Bg^2hx^2\log\left(\frac{(bx+a)^ne}{(dx+c)^n}\right)+\frac{3}{2}Ag^2hx^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x+g)^3*(A+B*log(e*(b*x+a)^n/((d*x+c)^n))),x, algorithm="maxima
")
```

```
[Out] 1/4*B*h^3*x^4*log((b*x + a)^n*e/(d*x + c)^n) + 1/4*A*h^3*x^4 + B*g*h^2*x^3*
log((b*x + a)^n*e/(d*x + c)^n) + A*g*h^2*x^3 + 3/2*B*g^2*h*x^2*log((b*x + a
)^n*e/(d*x + c)^n) + 3/2*A*g^2*h*x^2 + B*g^3*x*log((b*x + a)^n*e/(d*x + c)^
n) + A*g^3*x + (a*e*n*log(b*x + a)/b - c*e*n*log(d*x + c)/d)*B*g^3/e - 3/2*
```

```
(a^2*e*n*log(b*x + a)/b^2 - c^2*e*n*log(d*x + c)/d^2 + (b*c*e*n - a*d*e*n)*
x/(b*d))*B*g^2*h/e + 1/2*(2*a^3*e*n*log(b*x + a)/b^3 - 2*c^3*e*n*log(d*x +
c)/d^3 - ((b^2*c*d*e*n - a*b*d^2*e*n)*x^2 - 2*(b^2*c^2*e*n - a^2*d^2*e*n)*x
)/(b^2*d^2))*B*g*h^2/e - 1/24*(6*a^4*e*n*log(b*x + a)/b^4 - 6*c^4*e*n*log(d
*x + c)/d^4 + (2*(b^3*c*d^2*e*n - a*b^2*d^3*e*n)*x^3 - 3*(b^3*c^2*d*e*n - a
^2*b*d^3*e*n)*x^2 + 6*(b^3*c^3*e*n - a^3*d^3*e*n)*x)/(b^3*d^3))*B*h^3/e
```

**mupad [B]** time = 4.76, size = 767, normalized size = 3.25

$$x\left(\frac{4\,A\,b\,d\,g^3+12\,A\,a\,c\,g\,h^2+12\,A\,a\,d\,g^2\,h+12\,A\,b\,c\,g^2\,h+6\,B\,a\,d\,g^2\,h\,n-6\,B\,b\,c\,g^2\,h\,n}{4\,b\,d}+\frac{(4\,a\,d+4\,b\,c)\left(\left(\frac{4\,A\,a\,d\,h^3}{}\right.\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g + h*x)^3*(A + B*log((e*(a + b*x)^n)/(c + d*x)^n)),x)
```

```
[Out] x*((4*A*b*d*g^3 + 12*A*a*c*g*h^2 + 12*A*a*d*g^2*h + 12*A*b*c*g^2*h + 6*B*a*
d*g^2*h*n - 6*B*b*c*g^2*h*n)/(4*b*d) + ((4*a*d + 4*b*c)*((((4*A*a*d*h^3 + 4
*A*b*c*h^3 + 12*A*b*d*g*h^2 + B*a*d*h^3*n - B*b*c*h^3*n)/(4*b*d) - (A*h^3*(
4*a*d + 4*b*c))/(4*b*d))*(4*a*d + 4*b*c))/(4*b*d) - (4*A*a*c*h^3 + 12*A*a*d
*g*h^2 + 12*A*b*c*g*h^2 + 12*A*b*d*g^2*h + 4*B*a*d*g*h^2*n - 4*B*b*c*g*h^2*
n)/(4*b*d) + (A*a*c*h^3)/(b*d)))/(4*b*d) - (a*c*((4*A*a*d*h^3 + 4*A*b*c*h^3
 + 12*A*b*d*g*h^2 + B*a*d*h^3*n - B*b*c*h^3*n)/(4*b*d) - (A*h^3*(4*a*d + 4*
b*c))/(4*b*d)))/(b*d)) + log((e*(a + b*x)^n)/(c + d*x)^n)*((B*h^3*x^4)/4 +
B*g^3*x + (3*B*g^2*h*x^2)/2 + B*g*h^2*x^3) - x^2*((((4*A*a*d*h^3 + 4*A*b*c*
h^3 + 12*A*b*d*g*h^2 + B*a*d*h^3*n - B*b*c*h^3*n)/(4*b*d) - (A*h^3*(4*a*d +
 4*b*c))/(4*b*d))*(4*a*d + 4*b*c))/(8*b*d) - (4*A*a*c*h^3 + 12*A*a*d*g*h^2
+ 12*A*b*c*g*h^2 + 12*A*b*d*g^2*h + 4*B*a*d*g*h^2*n - 4*B*b*c*g*h^2*n)/(8*b
*d) + (A*a*c*h^3)/(2*b*d)) + x^3*((4*A*a*d*h^3 + 4*A*b*c*h^3 + 12*A*b*d*g*h
^2 + B*a*d*h^3*n - B*b*c*h^3*n)/(12*b*d) - (A*h^3*(4*a*d + 4*b*c))/(12*b*d)
) + (A*h^3*x^4)/4 - (log(a + b*x)*(B*a^4*h^3*n - 4*B*a*b^3*g^3*n - 4*B*a^3*
b*g*h^2*n + 6*B*a^2*b^2*g^2*h*n))/(4*b^4) + (log(c + d*x)*(B*c^4*h^3*n - 4*
B*c*d^3*g^3*n - 4*B*c^3*d*g*h^2*n + 6*B*c^2*d^2*g^2*h*n))/(4*d^4)
```

**sympy [F(-2)]** time = 0.00, size = 0, normalized size = 0.00

Exception raised: HeuristicGCDFailed

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x+g)**3*(A+B*ln(e*(b*x+a)**n/((d*x+c)**n))),x)
```

```
[Out] Exception raised: HeuristicGCDFailed
```

# 3.295 $\int (g+hx)^2 \left(A + B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right) dx$

**Optimal**. Leaf size=158

$$\frac{(g+hx)^3\left(B\log\left(e(a+bx)^n(c+dx)^{-n}\right)+A\right)}{3h}-\frac{Bn(bg-ah)^3\log(a+bx)}{3b^3h}-\frac{Bhnx(bc-ad)(-adh-bch+3bdg)}{3b^2d^2}-B$$

```
[Out] -1/3*B*(-a*d+b*c)*h*(-a*d*h-b*c*h+3*b*d*g)*n*x/b^2/d^2-1/6*B*(-a*d+b*c)*h^2
*n*x^2/b/d-1/3*B*(-a*h+b*g)^3*n*ln(b*x+a)/b^3/h+1/3*B*(-c*h+d*g)^3*n*ln(d*x
+c)/d^3/h+1/3*(h*x+g)^3*(A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))/h
```

**Rubi [A]** time = 0.24, antiderivative size = 170, normalized size of antiderivative = 1.08, number of steps used = 5, number of rules used = 3, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.097, Rules used = {6742, 2492, 72}

$$-\frac{Bhnx(bc-ad)(-adh-bch+3bdg)}{3b^2d^2}-\frac{Bn(bg-ah)^3\log(a+bx)}{3b^3h}+\frac{B(g+hx)^3\log\left(e(a+bx)^n(c+dx)^{-n}\right)}{3h}-\frac{Bh^2nx^2}{6}$$

Antiderivative was successfully verified.

```
[In] Int[(g + h*x)^2*(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n]),x]
```

```
[Out] -(B*(b*c - a*d)*h*(3*b*d*g - b*c*h - a*d*h)*n*x)/(3*b^2*d^2) - (B*(b*c - a*
d)*h^2*n*x^2)/(6*b*d) + (A*(g + h*x)^3)/(3*h) - (B*(b*g - a*h)^3*n*Log[a +
b*x])/(3*b^3*h) + (B*(d*g - c*h)^3*n*Log[c + d*x])/(3*d^3*h) + (B*(g + h*x)
^3*Log[(e*(a + b*x)^n)/(c + d*x)^n])/(3*h)
```

Rule 72

```
Int[((e_.) + (f_.)*(x_))^(p_.)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
 x_Symbol] :> Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x]
 /; FreeQ[{a, b, c, d, e, f}, x] && IntegerQ[p]
```

Rule 2492

```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))
^(r_.)]^(s_.)*((g_.) + (h_.)*(x_))^(m_.), x_Symbol] :> Simp[((g + h*x)^(m +
 1)*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^s)/(h*(m + 1)), x] - Dist[(p*r*s*(
b*c - a*d))/(h*(m + 1)), Int[((g + h*x)^(m + 1)*Log[e*(f*(a + b*x)^p*(c + d
*x)^q)^r]^(s - 1))/((a + b*x)*(c + d*x)), x], x] /; FreeQ[{a, b, c, d, e, f
, g, h, m, p, q, r, s}, x] && NeQ[b*c - a*d, 0] && EqQ[p + q, 0] && IGtQ[s,
 0] && NeQ[m, -1]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\int (g+hx)^2 \left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right) dx = \int \left(A(g+hx)^2+B(g+hx)^2\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right) dx$$

$$= \frac{A(g+hx)^3}{3h} + B\int (g+hx)^2 \log\left(e(a+bx)^n(c+dx)^{-n}\right) dx$$

$$= \frac{A(g+hx)^3}{3h} + \frac{B(g+hx)^3\log\left(e(a+bx)^n(c+dx)^{-n}\right)}{3h} - \frac{(B(bc}{}$$

$$= \frac{A(g+hx)^3}{3h} + \frac{B(g+hx)^3\log\left(e(a+bx)^n(c+dx)^{-n}\right)}{3h} - \frac{(B(bc}{}$$

$$= -\frac{B(bc-ad)h(3bdg-bch-adh)nx}{3b^2d^2} - \frac{B(bc-ad)h^2nx^2}{6bd} + \frac{A(}{}$$

**Mathematica [A]** time = 0.39, size = 204, normalized size = 1.29

$$\frac{2a^2Bd^3hn(ah-3bg)\log(a+bx)+b\left(dx\left(Bhn(bc-ad)(2adh+2bch-6bdg-bdhx)+2Ab^2d^2\left(3g^2+3ghx+h^2x\right.\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(g + h*x)^2*(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n]),x]
```

```
[Out] (2*a^2*B*d^3*h*(-3*b*g + a*h)*n*Log[a + b*x] + b*(d*x*(B*(b*c - a*d)*h*n*(-
6*b*d*g + 2*b*c*h + 2*a*d*h - b*d*h*x) + 2*A*b^2*d^2*(3*g^2 + 3*g*h*x + h^2
*x^2)) - 2*b*B*(-3*a*d^3*g^2 + b*c*(3*d^2*g^2 - 3*c*d*g*h + c^2*h^2))*n*Log
[c + d*x] + 2*b*B*d^3*(3*a*g^2 + b*x*(3*g^2 + 3*g*h*x + h^2*x^2))*Log[(e*(a
 + b*x)^n)/(c + d*x)^n]))/(6*b^3*d^3)
```

**fricas [B]** time = 0.93, size = 365, normalized size = 2.31

$$\frac{2\,Ab^3d^3h^2x^3+\left(6\,Ab^3d^3gh-\left(Bb^3cd^2-Bab^2d^3\right)h^2n\right)x^2+2\left(3\,Ab^3d^3g^2-\left(3\left(Bb^3cd^2-Bab^2d^3\right)gh-\left(Bb^3c^2d-Ba\right.\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x+g)^2*(A+B*log(e*(b*x+a)^n/((d*x+c)^n))),x, algorithm="fricas
")
```

```
[Out] 1/6*(2*A*b^3*d^3*h^2*x^3 + (6*A*b^3*d^3*g*h - (B*b^3*c*d^2 - B*a*b^2*d^3)*h
^2*n)*x^2 + 2*(3*A*b^3*d^3*g^2 - (3*(B*b^3*c*d^2 - B*a*b^2*d^3)*g*h - (B*b^
3*c^2*d - B*a^2*b*d^3)*h^2)*n)*x + 2*(B*b^3*d^3*h^2*n*x^3 + 3*B*b^3*d^3*g*h
*n*x^2 + 3*B*b^3*d^3*g^2*n*x + (3*B*a*b^2*d^3*g^2 - 3*B*a^2*b*d^3*g*h + B*a
^3*d^3*h^2)*n)*log(b*x + a) - 2*(B*b^3*d^3*h^2*n*x^3 + 3*B*b^3*d^3*g*h*n*x^
2 + 3*B*b^3*d^3*g^2*n*x + (3*B*b^3*c*d^2*g^2 - 3*B*b^3*c^2*d*g*h + B*b^3*c^
3*h^2)*n)*log(d*x + c) + 2*(B*b^3*d^3*h^2*x^3 + 3*B*b^3*d^3*g*h*x^2 + 3*B*b
^3*d^3*g^2*x)*log(e))/(b^3*d^3)
```

**giac [B]** time = 97.81, size = 298, normalized size = 1.89

$$\frac{1}{3}\left(Ah^2+Bh^2\right)x^3+\frac{1}{3}\left(Bh^2nx^3+3\,Bghnx^2+3\,Bg^2nx\right)\log\left(bx+a\right)-\frac{1}{3}\left(Bh^2nx^3+3\,Bghnx^2+3\,Bg^2nx\right)\log\left(dx+\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x+g)^2*(A+B*log(e*(b*x+a)^n/((d*x+c)^n))),x, algorithm="giac")
```

```
[Out] 1/3*(A*h^2 + B*h^2)*x^3 + 1/3*(B*h^2*n*x^3 + 3*B*g*h*n*x^2 + 3*B*g^2*n*x)*l
og(b*x + a) - 1/3*(B*h^2*n*x^3 + 3*B*g*h*n*x^2 + 3*B*g^2*n*x)*log(d*x + c)
- 1/6*(B*b*c*h^2*n - B*a*d*h^2*n - 6*A*b*d*g*h - 6*B*b*d*g*h)*x^2/(b*d) + 1
/3*(3*B*a*b^2*g^2*n - 3*B*a^2*b*g*h*n + B*a^3*h^2*n)*log(b*x + a)/b^3 - 1/3
*(3*B*c*d^2*g^2*n - 3*B*c^2*d*g*h*n + B*c^3*h^2*n)*log(-d*x - c)/d^3 - 1/3*
(3*B*b^2*c*d*g*h*n - 3*B*a*b*d^2*g*h*n - B*b^2*c^2*h^2*n + B*a^2*d^2*h^2*n
- 3*A*b^2*d^2*g^2 - 3*B*b^2*d^2*g^2)*x/(b^2*d^2)
```

**maple [C]** time = 0.51, size = 1389, normalized size = 8.79

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((h*x+g)^2*(A+B*ln(e*(b*x+a)^n/((d*x+c)^n))),x)
```

```
[Out] -1/3*(h*x+g)^3*B/h*ln((d*x+c)^n)+h/b*B*a*g*n*x-1/d*h*B*c*g*n*x+ln((b*x+a)^n
)*x*B*g^2+1/3*h^2*B*ln(e)*x^3+1/3*h^2*B*x^3*ln((b*x+a)^n)+B*ln(e)*g^2*x-1/3
*h^2/b^2*B*a^2*n*x+1/3/d^2*h^2*B*c^2*n*x+1/d^2*h*B*ln(d*x+c)*c^2*g*n-h/b^2*
B*ln(-b*x-a)*a^2*g*n+1/2*I*B*Pi*g^2*x*csgn(I*(b*x+a)^n)*csgn(I*(b*x+a)^n/((
d*x+c)^n))^2+1/2*I*B*Pi*g^2*x*csgn(I/((d*x+c)^n))*csgn(I*(b*x+a)^n/((d*x+c)
^n))^2+1/2*I*B*Pi*g^2*x*csgn(I*(b*x+a)^n/((d*x+c)^n))*csgn(I*e/((d*x+c)^n)*
(b*x+a)^n)^2+1/2*I*B*Pi*g^2*x*csgn(I*e)*csgn(I*e/((d*x+c)^n)*(b*x+a)^n)^2+1
/6*I*h^2*B*Pi*x^3*csgn(I*(b*x+a)^n)*csgn(I*(b*x+a)^n/((d*x+c)^n))^2+1/6*I*h
^2*B*Pi*x^3*csgn(I/((d*x+c)^n))*csgn(I*(b*x+a)^n/((d*x+c)^n))^2-1/6*I*h^2*B
*Pi*x^3*csgn(I*(b*x+a)^n)*csgn(I/((d*x+c)^n))*csgn(I*(b*x+a)^n/((d*x+c)^n))
-1/6*I*h^2*B*Pi*x^3*csgn(I*e)*csgn(I*(b*x+a)^n/((d*x+c)^n))*csgn(I*e/((d*x+
c)^n)*(b*x+a)^n)-1/2*I*B*Pi*g^2*x*csgn(I*(b*x+a)^n)*csgn(I/((d*x+c)^n))*csg
```

```
n(I*(b*x+a)^n/((d*x+c)^n))-1/2*I*B*Pi*g^2*x*csgn(I*e)*csgn(I*(b*x+a)^n/((d*
x+c)^n))*csgn(I*e/((d*x+c)^n)*(b*x+a)^n)+1/2*I*h*B*Pi*g*x^2*csgn(I*(b*x+a)^
n)*csgn(I*(b*x+a)^n/((d*x+c)^n))^2+1/2*I*h*B*Pi*g*x^2*csgn(I/((d*x+c)^n))*c
sgn(I*(b*x+a)^n/((d*x+c)^n))^2+1/2*I*h*B*Pi*g*x^2*csgn(I*(b*x+a)^n/((d*x+c)
^n))*csgn(I*e/((d*x+c)^n)*(b*x+a)^n)^2+1/2*I*h*B*Pi*g*x^2*csgn(I*e)*csgn(I*
e/((d*x+c)^n)*(b*x+a)^n)^2+1/3*h^2*A*x^3+1/3/h*B*ln(d*x+c)*g^3*n+h*B*ln(e)*
g*x^2+h*B*g*x^2*ln((b*x+a)^n)+1/6*I*h^2*B*Pi*x^3*csgn(I*(b*x+a)^n/((d*x+c)^
n))*csgn(I*e/((d*x+c)^n)*(b*x+a)^n)^2+1/6*I*h^2*B*Pi*x^3*csgn(I*e)*csgn(I*e
/((d*x+c)^n)*(b*x+a)^n)^2-1/2*I*h*B*Pi*g*x^2*csgn(I*(b*x+a)^n/((d*x+c)^n))^
3-1/2*I*h*B*Pi*g*x^2*csgn(I*e/((d*x+c)^n)*(b*x+a)^n)^3+h*A*g*x^2+A*g^2*x-1/
6/d*h^2*B*c*n*x^2-1/d*B*ln(d*x+c)*c*g^2*n+1/b*B*ln(-b*x-a)*a*g^2*n-1/3/d^3*
h^2*B*ln(d*x+c)*c^3*n+1/3*h^2/b^3*B*ln(-b*x-a)*a^3*n-1/6*I*h^2*B*Pi*x^3*csg
n(I*(b*x+a)^n/((d*x+c)^n))^3-1/6*I*h^2*B*Pi*x^3*csgn(I*e/((d*x+c)^n)*(b*x+a
)^n)^3-1/2*I*B*Pi*g^2*x*csgn(I*(b*x+a)^n/((d*x+c)^n))^3-1/2*I*B*Pi*g^2*x*cs
gn(I*e/((d*x+c)^n)*(b*x+a)^n)^3-1/2*I*h*B*Pi*g*x^2*csgn(I*(b*x+a)^n)*csgn(I
/((d*x+c)^n))*csgn(I*(b*x+a)^n/((d*x+c)^n))-1/2*I*h*B*Pi*g*x^2*csgn(I*e)*cs
gn(I*(b*x+a)^n/((d*x+c)^n))*csgn(I*e/((d*x+c)^n)*(b*x+a)^n)+1/6*h^2/b*B*a*n
*x^2
```

**maxima [A]** time = 0.94, size = 294, normalized size = 1.86

$$\frac{1}{3}Bh^2x^3\log\left(\frac{(bx+a)^n e}{(dx+c)^n}\right)+\frac{1}{3}Ah^2x^3+Bghx^2\log\left(\frac{(bx+a)^n e}{(dx+c)^n}\right)+Aghx^2+Bg^2x\log\left(\frac{(bx+a)^n e}{(dx+c)^n}\right)+Ag^2x+\frac{\left(\frac{aen\log(bx+a}{b}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x+g)^2*(A+B*log(e*(b*x+a)^n/((d*x+c)^n))),x, algorithm="maxima
")
```

```
[Out] 1/3*B*h^2*x^3*log((b*x + a)^n*e/(d*x + c)^n) + 1/3*A*h^2*x^3 + B*g*h*x^2*lo
g((b*x + a)^n*e/(d*x + c)^n) + A*g*h*x^2 + B*g^2*x*log((b*x + a)^n*e/(d*x +
 c)^n) + A*g^2*x + (a*e*n*log(b*x + a)/b - c*e*n*log(d*x + c)/d)*B*g^2/e -
(a^2*e*n*log(b*x + a)/b^2 - c^2*e*n*log(d*x + c)/d^2 + (b*c*e*n - a*d*e*n)*
x/(b*d))*B*g*h/e + 1/6*(2*a^3*e*n*log(b*x + a)/b^3 - 2*c^3*e*n*log(d*x + c)
/d^3 - ((b^2*c*d*e*n - a*b*d^2*e*n)*x^2 - 2*(b^2*c^2*e*n - a^2*d^2*e*n)*x)/
(b^2*d^2))*B*h^2/e
```

**mupad [B]** time = 4.47, size = 372, normalized size = 2.35

$$x^2\left(\frac{3\,A\,a\,d\,h^2+3\,A\,b\,c\,h^2+6\,A\,b\,d\,g\,h+B\,a\,d\,h^2\,n-B\,b\,c\,h^2\,n}{6\,b\,d}-\frac{A\,h^2\,\left(3\,a\,d+3\,b\,c\right)}{6\,b\,d}\right)+\ln\left(\frac{e\,{\left(a+b\,x\right)}^n}{{\left(c+d\,x\right)}^n}\right)\,\left(B\,g^2\,x+\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g + h*x)^2*(A + B*log((e*(a + b*x)^n)/(c + d*x)^n)),x)
```

```
[Out] x^2*((3*A*a*d*h^2 + 3*A*b*c*h^2 + 6*A*b*d*g*h + B*a*d*h^2*n - B*b*c*h^2*n)/
(6*b*d) - (A*h^2*(3*a*d + 3*b*c))/(6*b*d)) + log((e*(a + b*x)^n)/(c + d*x)^
n)*((B*h^2*x^3)/3 + B*g^2*x + B*g*h*x^2) - x*(((3*a*d + 3*b*c)*((3*A*a*d*h^
2 + 3*A*b*c*h^2 + 6*A*b*d*g*h + B*a*d*h^2*n - B*b*c*h^2*n)/(3*b*d) - (A*h^2
*(3*a*d + 3*b*c))/(3*b*d)))/(3*b*d) - (3*A*a*c*h^2 + 3*A*b*d*g^2 + 6*A*a*d*
g*h + 6*A*b*c*g*h + 3*B*a*d*g*h*n - 3*B*b*c*g*h*n)/(3*b*d) + (A*a*c*h^2)/(b
*d)) + (A*h^2*x^3)/3 + (log(a + b*x)*(B*a^3*h^2*n + 3*B*a*b^2*g^2*n - 3*B*a
^2*b*g*h*n))/(3*b^3) - (log(c + d*x)*(B*c^3*h^2*n + 3*B*c*d^2*g^2*n - 3*B*c
^2*d*g*h*n))/(3*d^3)
```

**sympy [F(-2)]** time = 0.00, size = 0, normalized size = 0.00

Exception raised: HeuristicGCDFailed

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x+g)**2*(A+B*ln(e*(b*x+a)**n/((d*x+c)**n))),x)
```

```
[Out] Exception raised: HeuristicGCDFailed
```

# 3.296 $\int (g+hx)\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)dx$

**Optimal**. Leaf size=116

$$\frac{(g+hx)^2\left(B\log\left(e(a+bx)^n(c+dx)^{-n}\right)+A\right)}{2h}-\frac{Bn(bg-ah)^2\log(a+bx)}{2b^2h}-\frac{Bhnx(bc-ad)}{2bd}+\frac{Bn(dg-ch)^2\log(c+d}{2d^2h}$$

```
[Out] -1/2*B*(-a*d+b*c)*h*n*x/b/d-1/2*B*(-a*h+b*g)^2*n*ln(b*x+a)/b^2/h+1/2*B*(-c*
h+d*g)^2*n*ln(d*x+c)/d^2/h+1/2*(h*x+g)^2*(A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))/
h
```

**Rubi [A]** time = 0.15, antiderivative size = 128, normalized size of antiderivative = 1.10, number of steps used = 5, number of rules used = 3, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.103, Rules used = {6742, 2492, 72}

$$-\frac{Bn(bg-ah)^2\log(a+bx)}{2b^2h}+\frac{B(g+hx)^2\log\left(e(a+bx)^n(c+dx)^{-n}\right)}{2h}-\frac{Bhnx(bc-ad)}{2bd}+\frac{A(g+hx)^2}{2h}+\frac{Bn(dg-ch)^2\,\text{lo}}{2d^2h}$$

Antiderivative was successfully verified.

```
[In] Int[(g + h*x)*(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n]),x]
```

```
[Out] -(B*(b*c - a*d)*h*n*x)/(2*b*d) + (A*(g + h*x)^2)/(2*h) - (B*(b*g - a*h)^2*n
*Log[a + b*x])/(2*b^2*h) + (B*(d*g - c*h)^2*n*Log[c + d*x])/(2*d^2*h) + (B*
(g + h*x)^2*Log[(e*(a + b*x)^n)/(c + d*x)^n])/(2*h)
```

Rule 72

```
Int[((e_.) + (f_.)*(x_))^(p_.)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
 x_Symbol] :> Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x]
 /; FreeQ[{a, b, c, d, e, f}, x] && IntegerQ[p]
```

Rule 2492

```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))
^(r_.)]^(s_.)*((g_.) + (h_.)*(x_))^(m_.), x_Symbol] :> Simp[((g + h*x)^(m +
 1)*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^s)/(h*(m + 1)), x] - Dist[(p*r*s*(
b*c - a*d))/(h*(m + 1)), Int[((g + h*x)^(m + 1)*Log[e*(f*(a + b*x)^p*(c + d
*x)^q)^r]^(s - 1))/((a + b*x)*(c + d*x)), x], x] /; FreeQ[{a, b, c, d, e, f
, g, h, m, p, q, r, s}, x] && NeQ[b*c - a*d, 0] && EqQ[p + q, 0] && IGtQ[s,
 0] && NeQ[m, -1]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\int (g+hx)\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)\,dx=\int \left(A(g+hx)+B(g+hx)\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)\,dx$$
$$=\frac{A(g+hx)^2}{2h}+B\int (g+hx)\log\left(e(a+bx)^n(c+dx)^{-n}\right)\,dx$$
$$=\frac{A(g+hx)^2}{2h}+\frac{B(g+hx)^2\log\left(e(a+bx)^n(c+dx)^{-n}\right)}{2h}-\frac{(B(bc$$
$$=\frac{A(g+hx)^2}{2h}+\frac{B(g+hx)^2\log\left(e(a+bx)^n(c+dx)^{-n}\right)}{2h}-\frac{(B(bc$$
$$=-\frac{B(bc-ad)hnx}{2bd}+\frac{A(g+hx)^2}{2h}-\frac{B(bg-ah)^2n\log(a+bx)}{2b^2h}$$

**Mathematica [A]** time = 0.19, size = 124, normalized size = 1.07

$$\frac{-a^2Bd^2hn\log(a+bx)+bd\left(x(Bhn(ad-bc)+Abd(2g+hx))+Bd(2ag+bx(2g+hx))\log\left(e(a+bx)^n(c+dx)^{-n}\right.\right.}{2b^2d^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(g + h*x)*(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n]),x]
```

```
[Out] (-(a^2*B*d^2*h*n*Log[a + b*x]) + b*B*(2*a*d^2*g + b*c*(-2*d*g + c*h))*n*Log
[c + d*x] + b*d*(x*(B*(-(b*c) + a*d)*h*n + A*b*d*(2*g + h*x)) + B*d*(2*a*g
+ b*x*(2*g + h*x))*Log[(e*(a + b*x)^n)/(c + d*x)^n]))/(2*b^2*d^2)
```

**fricas [A]** time = 0.81, size = 192, normalized size = 1.66

$$\frac{Ab^2d^2hx^2+\left(2\,Ab^2d^2g-\left(Bb^2cd-Babd^2\right)hn\right)x+\left(Bb^2d^2hnx^2+2\,Bb^2d^2gnx+\left(2\,Babd^2g-Ba^2d^2h\right)n\right)\log\left(bx+\right.}{2\,b^2d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x+g)*(A+B*log(e*(b*x+a)^n/((d*x+c)^n))),x, algorithm="fricas")
```

```
[Out] 1/2*(A*b^2*d^2*h*x^2 + (2*A*b^2*d^2*g - (B*b^2*c*d - B*a*b*d^2)*h*n)*x + (B
*b^2*d^2*h*n*x^2 + 2*B*b^2*d^2*g*n*x + (2*B*a*b*d^2*g - B*a^2*d^2*h)*n)*log
(b*x + a) - (B*b^2*d^2*h*n*x^2 + 2*B*b^2*d^2*g*n*x + (2*B*b^2*c*d*g - B*b^2
*c^2*h)*n)*log(d*x + c) + (B*b^2*d^2*h*x^2 + 2*B*b^2*d^2*g*x)*log(e))/(b^2*
d^2)
```

**giac [A]** time = 7.10, size = 149, normalized size = 1.28

$$\frac{1}{2}(Ah+Bh)x^2+\frac{1}{2}\left(Bhnx^2+2\,Bgnx\right)\log\left(bx+a\right)-\frac{1}{2}\left(Bhnx^2+2\,Bgnx\right)\log\left(dx+c\right)-\frac{\left(Bbchn-Badhn-2\,Abdg-\right.}{2\,bd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x+g)*(A+B*log(e*(b*x+a)^n/((d*x+c)^n))),x, algorithm="giac")
```

```
[Out] 1/2*(A*h + B*h)*x^2 + 1/2*(B*h*n*x^2 + 2*B*g*n*x)*log(b*x + a) - 1/2*(B*h*n
*x^2 + 2*B*g*n*x)*log(d*x + c) - 1/2*(B*b*c*h*n - B*a*d*h*n - 2*A*b*d*g - 2
*B*b*d*g)*x/(b*d) + 1/2*(2*B*a*b*g*n - B*a^2*h*n)*log(b*x + a)/b^2 - 1/2*(2
*B*c*d*g*n - B*c^2*h*n)*log(-d*x - c)/d^2
```

**maple [C]** time = 0.45, size = 839, normalized size = 7.23

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((h*x+g)*(A+B*ln(e*(b*x+a)^n/((d*x+c)^n))),x)
```

```
[Out] 1/2*A*h*x^2+A*g*x-1/2*B*x*(h*x+2*g)*ln((d*x+c)^n)+1/2*ln((b*x+a)^n)*x^2*B*h
+ln((b*x+a)^n)*x*B*g+1/2*B*ln(e)*h*x^2+B*ln(e)*g*x+1/4*I*B*Pi*h*x^2*csgn(I*
(b*x+a)^n)*csgn(I*(b*x+a)^n/((d*x+c)^n))^2+1/4*I*B*Pi*h*x^2*csgn(I/((d*x+c)
^n))*csgn(I*(b*x+a)^n/((d*x+c)^n))^2-1/2/b^2*B*ln(b*x+a)*a^2*h*n+1/b*B*ln(b
*x+a)*a*g*n+1/2/d^2*B*ln(-d*x-c)*c^2*h*n-1/d*B*ln(-d*x-c)*c*g*n-1/4*I*B*Pi*
h*x^2*csgn(I*e/((d*x+c)^n)*(b*x+a)^n)^3-1/4*I*B*Pi*h*x^2*csgn(I*(b*x+a)^n/(
(d*x+c)^n))^3-1/2*I*B*Pi*g*x*csgn(I*e/((d*x+c)^n)*(b*x+a)^n)^3-1/2*I*B*Pi*g
*x*csgn(I*(b*x+a)^n/((d*x+c)^n))^3-1/2/d*B*c*h*n*x+1/2*I*B*Pi*g*x*csgn(I*e)
*csgn(I*e/((d*x+c)^n)*(b*x+a)^n)^2+1/2*I*B*Pi*g*x*csgn(I*(b*x+a)^n/((d*x+c)
^n))*csgn(I*e/((d*x+c)^n)*(b*x+a)^n)^2+1/2*I*B*Pi*g*x*csgn(I*(b*x+a)^n)*csg
n(I*(b*x+a)^n/((d*x+c)^n))^2+1/2*I*B*Pi*g*x*csgn(I/((d*x+c)^n))*csgn(I*(b*x
+a)^n/((d*x+c)^n))^2+1/4*I*B*Pi*h*x^2*csgn(I*e)*csgn(I*e/((d*x+c)^n)*(b*x+a
)^n)^2+1/4*I*B*Pi*h*x^2*csgn(I*(b*x+a)^n/((d*x+c)^n))*csgn(I*e/((d*x+c)^n)*
(b*x+a)^n)^2-1/2*I*B*Pi*g*x*csgn(I*e)*csgn(I*(b*x+a)^n/((d*x+c)^n))*csgn(I*
e/((d*x+c)^n)*(b*x+a)^n)-1/2*I*B*Pi*g*x*csgn(I*(b*x+a)^n)*csgn(I/((d*x+c)^n
))*csgn(I*(b*x+a)^n/((d*x+c)^n))-1/4*I*B*Pi*h*x^2*csgn(I*e)*csgn(I*(b*x+a)^
n/((d*x+c)^n))*csgn(I*e/((d*x+c)^n)*(b*x+a)^n)-1/4*I*B*Pi*h*x^2*csgn(I*(b*x
+a)^n)*csgn(I/((d*x+c)^n))*csgn(I*(b*x+a)^n/((d*x+c)^n))+1/2/b*B*a*h*n*x
```

**maxima [A]** time = 0.62, size = 154, normalized size = 1.33

$$\frac{1}{2}\,Bhx^2\log\left(\frac{(bx+a)^n e}{(dx+c)^n}\right)+\frac{1}{2}\,Ahx^2+Bgx\log\left(\frac{(bx+a)^n e}{(dx+c)^n}\right)+Agx+\frac{\left(\frac{aen\log(bx+a)}{b}-\frac{cen\log(dx+c)}{d}\right)Bg}{e}-\frac{\left(\frac{a^2en\log(bx+a)}{b^2}-\frac{c^2}{}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x+g)*(A+B*log(e*(b*x+a)^n/((d*x+c)^n))),x, algorithm="maxima")
```

```
[Out] 1/2*B*h*x^2*log((b*x + a)^n*e/(d*x + c)^n) + 1/2*A*h*x^2 + B*g*x*log((b*x +
 a)^n*e/(d*x + c)^n) + A*g*x + (a*e*n*log(b*x + a)/b - c*e*n*log(d*x + c)/d
)*B*g/e - 1/2*(a^2*e*n*log(b*x + a)/b^2 - c^2*e*n*log(d*x + c)/d^2 + (b*c*e
*n - a*d*e*n)*x/(b*d))*B*h/e
```

**mupad [B]** time = 4.39, size = 154, normalized size = 1.33

$$\ln\left(\frac{e\,{\left(a+b\,x\right)}^n}{{\left(c+d\,x\right)}^n}\right)\,\left(\frac{B\,h\,x^2}{2}+B\,g\,x\right)+x\,\left(\frac{2\,A\,a\,d\,h+2\,A\,b\,c\,h+2\,A\,b\,d\,g+B\,a\,d\,h\,n-B\,b\,c\,h\,n}{2\,b\,d}-\frac{A\,h\,\left(2\,a\,d+2\,b\,c\right)}{2\,b\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g + h*x)*(A + B*log((e*(a + b*x)^n)/(c + d*x)^n)),x)
```

```
[Out] log((e*(a + b*x)^n)/(c + d*x)^n)*(B*g*x + (B*h*x^2)/2) + x*((2*A*a*d*h + 2*
A*b*c*h + 2*A*b*d*g + B*a*d*h*n - B*b*c*h*n)/(2*b*d) - (A*h*(2*a*d + 2*b*c)
)/(2*b*d)) - (log(a + b*x)*(B*a^2*h*n - 2*B*a*b*g*n))/(2*b^2) + (log(c + d*
x)*(B*c^2*h*n - 2*B*c*d*g*n))/(2*d^2) + (A*h*x^2)/2
```

**sympy [F(-2)]** time = 0.00, size = 0, normalized size = 0.00

Exception raised: HeuristicGCDFailed

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x+g)*(A+B*ln(e*(b*x+a)**n/((d*x+c)**n))),x)
```

```
[Out] Exception raised: HeuristicGCDFailed
```

# 3.297 $\int \left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)\,dx$

**Optimal**. Leaf size=57

$$\frac{B(a+bx)\log\left(e(a+bx)^n(c+dx)^{-n}\right)}{b}-\frac{Bn(bc-ad)\log(c+dx)}{bd}+Ax$$

```
[Out] A*x-B*(-a*d+b*c)*n*ln(d*x+c)/b/d+B*(b*x+a)*ln(e*(b*x+a)^n/((d*x+c)^n))/b
```

**Rubi [A]** time = 0.03, antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {2486, 31}

$$\frac{B(a+bx)\log\left(e(a+bx)^n(c+dx)^{-n}\right)}{b}-\frac{Bn(bc-ad)\log(c+dx)}{bd}+Ax$$

Antiderivative was successfully verified.

```
[In] Int[A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n],x]
```

```
[Out] A*x - (B*(b*c - a*d)*n*Log[c + d*x])/(b*d) + (B*(a + b*x)*Log[(e*(a + b*x)^
n)/(c + d*x)^n])/b
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 2486

```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))
^(r_.)]^(s_.), x_Symbol] :> Simp[((a + b*x)*Log[e*(f*(a + b*x)^p*(c + d*x)^
q)^r]^s)/b, x] + Dist[(q*r*s*(b*c - a*d))/b, Int[Log[e*(f*(a + b*x)^p*(c +
d*x)^q)^r]^(s - 1)/(c + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, p, q, r, s
}, x] && NeQ[b*c - a*d, 0] && EqQ[p + q, 0] && IGtQ[s, 0]
```

Rubi steps

$$\begin{aligned}\int \left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)\,dx &= Ax+B\int \log\left(e(a+bx)^n(c+dx)^{-n}\right)\,dx\\ &= Ax+\frac{B(a+bx)\log\left(e(a+bx)^n(c+dx)^{-n}\right)}{b}-\frac{(B(bc-ad)n)\int \frac{1}{c+dx}\,dx}{b}\\ &= Ax-\frac{B(bc-ad)n\log(c+dx)}{bd}+\frac{B(a+bx)\log\left(e(a+bx)^n(c+dx)^{-n}\right)}{b}\end{aligned}$$

**Mathematica** [A] time = 0.01, size = 57, normalized size = 1.00

$$\frac{B(a+bx)\log\left(e(a+bx)^n(c+dx)^{-n}\right)}{b}-\frac{Bn(bc-ad)\log(c+dx)}{bd}+Ax$$

Antiderivative was successfully verified.

```
[In] Integrate[A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n],x]
```

```
[Out] A*x - (B*(b*c - a*d)*n*Log[c + d*x])/(b*d) + (B*(a + b*x)*Log[(e*(a + b*x)^
n)/(c + d*x)^n])/b
```

**fricas** [A] time = 0.90, size = 59, normalized size = 1.04

$$\frac{Bbdx\log(e)+Abdx+(Bbdnx+Badn)\log(bx+a)-(Bbdnx+Bbcn)\log(dx+c)}{bd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(A+B*log(e*(b*x+a)^n/((d*x+c)^n)),x, algorithm="fricas")
```

```
[Out] (B*b*d*x*log(e) + A*b*d*x + (B*b*d*n*x + B*a*d*n)*log(b*x + a) - (B*b*d*n*x
 + B*b*c*n)*log(d*x + c))/(b*d)
```

**giac** [A] time = 0.20, size = 55, normalized size = 0.96

$$\left(nx\log(bx+a)-nx\log(dx+c)+\frac{an\log(bx+a)}{b}-\frac{cn\log(-dx-c)}{d}+x\right)B+Ax$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(A+B*log(e*(b*x+a)^n/((d*x+c)^n)),x, algorithm="giac")
```

```
[Out] (n*x*log(b*x + a) - n*x*log(d*x + c) + a*n*log(b*x + a)/b - c*n*log(-d*x -
c)/d + x)*B + A*x
```

**maple** [B] time = 0.05, size = 123, normalized size = 2.16

$$\frac{B\,a^2dn\ln(bx+a)}{(ad-bc)\,b}-\frac{Bacn\ln(bx+a)}{ad-bc}-\frac{Bacn\ln(dx+c)}{ad-bc}+\frac{Bb\,c^2n\ln(dx+c)}{(ad-bc)\,d}+Bx\ln\left(e\,(bx+a)^n\,(dx+c)^{-n}\right)+Ax$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(A+B*ln(e*(b*x+a)^n/((d*x+c)^n)),x)
```

```
[Out] A*x+B*x*ln(e*(b*x+a)^n/((d*x+c)^n))-1/(a*d-b*c)*B*a*c*n*ln(d*x+c)+1/(a*d-b*
c)*B*b*c^2/d*n*ln(d*x+c)+1/(a*d-b*c)*B*a^2/b*d*n*ln(b*x+a)-1/(a*d-b*c)*B*a*
c*n*ln(b*x+a)
```

**maxima [A]** time = 0.82, size = 59, normalized size = 1.04

$$Bx\log\left(\frac{(bx+a)^n e}{(dx+c)^n}\right)+Ax+\frac{\left(\frac{aen\log(bx+a)}{b}-\frac{cen\log(dx+c)}{d}\right)B}{e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(A+B*log(e*(b*x+a)^n/((d*x+c)^n)),x, algorithm="maxima")
```

```
[Out] B*x*log((b*x + a)^n*e/(d*x + c)^n) + A*x + (a*e*n*log(b*x + a)/b - c*e*n*lo
g(d*x + c)/d)*B/e
```

**mupad [B]** time = 4.11, size = 53, normalized size = 0.93

$$Ax+Bx\ln\left(\frac{e\,(a+b\,x)^n}{(c+d\,x)^n}\right)+\frac{B\,a\,n\,\ln(a+b\,x)}{b}-\frac{B\,c\,n\,\ln(c+d\,x)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(A + B*log((e*(a + b*x)^n)/(c + d*x)^n),x)
```

```
[Out] A*x + B*x*log((e*(a + b*x)^n)/(c + d*x)^n) + (B*a*n*log(a + b*x))/b - (B*c*
n*log(c + d*x))/d
```

**sympy [F(-2)]** time = 0.00, size = 0, normalized size = 0.00

Exception raised: HeuristicGCDFailed

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(A+B*ln(e*(b*x+a)**n/((d*x+c)**n)),x)
```

```
[Out] Exception raised: HeuristicGCDFailed
```

# 3.298 $\int \frac{A+B\log(e(a+bx)^n(c+dx)^{-n})}{g+hx}\,dx$

**Optimal**. Leaf size=148

$$\frac{\log(g+hx)\left(B\log\left(e(a+bx)^n(c+dx)^{-n}\right)+A\right)}{h}-\frac{Bn\text{Li}_2\left(\frac{b(g+hx)}{bg-ah}\right)}{h}-\frac{Bn\log(g+hx)\log\left(-\frac{h(a+bx)}{bg-ah}\right)}{h}+\frac{Bn\text{Li}_2\left(\frac{d(g+hx)}{dg-ch}\right)}{h}$$

```
[Out] -B*n*ln(-h*(b*x+a)/(-a*h+b*g))*ln(h*x+g)/h+B*n*ln(-h*(d*x+c)/(-c*h+d*g))*ln
(h*x+g)/h+(A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))*ln(h*x+g)/h-B*n*polylog(2,b*(h*
x+g)/(-a*h+b*g))/h+B*n*polylog(2,d*(h*x+g)/(-c*h+d*g))/h
```

**Rubi [A]** time = 0.19, antiderivative size = 156, normalized size of antiderivative = 1.05, number of steps used = 9, number of rules used = 5, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.161, Rules used = {6742, 2494, 2394, 2393, 2391}

$$-\frac{Bn\text{PolyLog}\left(2,\frac{b(g+hx)}{bg-ah}\right)}{h}+\frac{Bn\text{PolyLog}\left(2,\frac{d(g+hx)}{dg-ch}\right)}{h}+\frac{B\log(g+hx)\log\left(e(a+bx)^n(c+dx)^{-n}\right)}{h}-\frac{Bn\log(g+hx)\log}{h}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])/(g + h*x),x]
```

```
[Out] (A*Log[g + h*x])/h - (B*n*Log[-((h*(a + b*x))/(b*g - a*h))]*Log[g + h*x])/h
 + (B*n*Log[-((h*(c + d*x))/(d*g - c*h))]*Log[g + h*x])/h + (B*Log[(e*(a +
b*x)^n)/(c + d*x)^n]*Log[g + h*x])/h - (B*n*PolyLog[2, (b*(g + h*x))/(b*g -
 a*h)])/h + (B*n*PolyLog[2, (d*(g + h*x))/(d*g - c*h)])/h
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2393

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + (c*e*x)/g])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2394

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d + e*x
)^n]))/g, x] - Dist[(b*e*n)/g, Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
```

```
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2494

```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))
^(r_.)]/((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(Log[g + h*x]*Log[e*(f*(a +
 b*x)^p*(c + d*x)^q)^r])/h, x] + (-Dist[(b*p*r)/h, Int[Log[g + h*x]/(a + b*
x), x], x] - Dist[(d*q*r)/h, Int[Log[g + h*x]/(c + d*x), x], x]) /; FreeQ[{
a, b, c, d, e, f, g, h, p, q, r}, x] && NeQ[b*c - a*d, 0]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\int \frac{A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)}{g+hx}\,dx = \int \left(\frac{A}{g+hx}+\frac{B\log\left(e(a+bx)^n(c+dx)^{-n}\right)}{g+hx}\right)dx$$

$$= \frac{A\log(g+hx)}{h}+B\int \frac{\log\left(e(a+bx)^n(c+dx)^{-n}\right)}{g+hx}\,dx$$

$$= \frac{A\log(g+hx)}{h}+\frac{B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\log(g+hx)}{h}-\frac{(bBn)\int \frac{\log}{}}{h}$$

$$= \frac{A\log(g+hx)}{h}-\frac{Bn\log\left(-\frac{h(a+bx)}{bg-ah}\right)\log(g+hx)}{h}+\frac{Bn\log\left(-\frac{h(c+dx)}{dg-ch}\right)\log}{h}$$

$$= \frac{A\log(g+hx)}{h}-\frac{Bn\log\left(-\frac{h(a+bx)}{bg-ah}\right)\log(g+hx)}{h}+\frac{Bn\log\left(-\frac{h(c+dx)}{dg-ch}\right)\log}{h}$$

$$= \frac{A\log(g+hx)}{h}-\frac{Bn\log\left(-\frac{h(a+bx)}{bg-ah}\right)\log(g+hx)}{h}+\frac{Bn\log\left(-\frac{h(c+dx)}{dg-ch}\right)\log}{h}$$

**Mathematica [A]** time = 0.11, size = 150, normalized size = 1.01

$$\frac{\log(g+hx)\left(B\left(\log\left(e(a+bx)^n(c+dx)^{-n}\right)-n\log(a+bx)+n\log(c+dx)\right)+A\right)+Bn\left(\text{Li}_2\left(\frac{h(a+bx)}{ah-bg}\right)+\log(a+bx\right.}{h}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])/(g + h*x),x]
```

```
[Out] ((A + B*(-(n*Log[a + b*x]) + n*Log[c + d*x] + Log[(e*(a + b*x)^n)/(c + d*x)
^n]))*Log[g + h*x] + B*n*(Log[a + b*x]*Log[(b*(g + h*x))/(b*g - a*h)] + Pol
yLog[2, (h*(a + b*x))/(-(b*g) + a*h)]) - B*n*(Log[c + d*x]*Log[(d*(g + h*x)
)/(d*g - c*h)] + PolyLog[2, (h*(c + d*x))/(-(d*g) + c*h)]))/h
```

**fricas [F]** time = 0.97, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{B\log\left(\frac{(bx+a)^n e}{(dx+c)^n}\right)+A}{hx+g}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*(b*x+a)^n/((d*x+c)^n)))/(h*x+g),x, algorithm="fricas")
```

```
[Out] integral((B*log((b*x + a)^n*e/(d*x + c)^n) + A)/(h*x + g), x)
```

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{B\log\left(\frac{(bx+a)^n e}{(dx+c)^n}\right)+A}{hx+g}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*(b*x+a)^n/((d*x+c)^n)))/(h*x+g),x, algorithm="giac")
```

```
[Out] integrate((B*log((b*x + a)^n*e/(d*x + c)^n) + A)/(h*x + g), x)
```

**maple [C]** time = 0.45, size = 597, normalized size = 4.03

$$-\frac{i\pi B\,\mathrm{csgn}\left(ie\right)\mathrm{csgn}\left(i\left(bx+a\right)^{n}\left(dx+c\right)^{-n}\right)\mathrm{csgn}\left(ie\left(bx+a\right)^{n}\left(dx+c\right)^{-n}\right)\ln\left(hx+g\right)}{2h}+\frac{i\pi B\,\mathrm{csgn}\left(ie\right)\mathrm{csgn}\left(ie\left(bx\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))/(h*x+g),x)
```

```
[Out] -1/2*I*ln(h*x+g)/h*B*Pi*csgn(I*e/((d*x+c)^n)*(b*x+a)^n)^3-1/2*I*ln(h*x+g)/h
*B*Pi*csgn(I*e)*csgn(I*(b*x+a)^n/((d*x+c)^n))*csgn(I*e/((d*x+c)^n)*(b*x+a)^
n)+1/2*I*ln(h*x+g)/h*B*Pi*csgn(I*(b*x+a)^n/((d*x+c)^n))*csgn(I*e/((d*x+c)^n
)*(b*x+a)^n)^2+1/2*I*ln(h*x+g)/h*B*Pi*csgn(I/((d*x+c)^n))*csgn(I*(b*x+a)^n/
((d*x+c)^n))^2-1/2*I*ln(h*x+g)/h*B*Pi*csgn(I*(b*x+a)^n/((d*x+c)^n))^3+1/2*I
*ln(h*x+g)/h*B*Pi*csgn(I*e)*csgn(I*e/((d*x+c)^n)*(b*x+a)^n)^2+1/2*I*ln(h*x+
```

```
g)/h*B*Pi*csgn(I*(b*x+a)^n)*csgn(I*(b*x+a)^n/((d*x+c)^n))^2-1/2*I*ln(h*x+g)
/h*B*Pi*csgn(I*(b*x+a)^n)*csgn(I/((d*x+c)^n))*csgn(I*(b*x+a)^n/((d*x+c)^n))
+ln(h*x+g)/h*B*ln(e)+A*ln(h*x+g)/h+B*ln(h*x+g)/h*ln((b*x+a)^n)-B/h*n*dilog(
(b*(h*x+g)+a*h-b*g)/(a*h-b*g))-B/h*n*ln(h*x+g)*ln((b*(h*x+g)+a*h-b*g)/(a*h-
b*g))-B*ln(h*x+g)/h*ln((d*x+c)^n)+B/h*n*dilog(((h*x+g)*d+c*h-d*g)/(c*h-d*g)
)+B/h*n*ln(h*x+g)*ln(((h*x+g)*d+c*h-d*g)/(c*h-d*g))
```

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$-B\int -\frac{\log\left((bx+a)^n\right)-\log\left((dx+c)^n\right)+\log(e)}{hx+g}\,dx+\frac{A\log\left(hx+g\right)}{h}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*(b*x+a)^n/((d*x+c)^n)))/(h*x+g),x, algorithm="maxima")
```

```
[Out] -B*integrate(-(log((b*x + a)^n) - log((d*x + c)^n) + log(e))/(h*x + g), x)
+ A*log(h*x + g)/h
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{A+B\,\ln\left(\frac{e\,{\left(a+b\,x\right)}^n}{{\left(c+d\,x\right)}^n}\right)}{g+h\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*log((e*(a + b*x)^n)/(c + d*x)^n))/(g + h*x),x)
```

```
[Out] int((A + B*log((e*(a + b*x)^n)/(c + d*x)^n))/(g + h*x), x)
```

**sympy [F(-2)]** time = 0.00, size = 0, normalized size = 0.00

Exception raised: HeuristicGCDFailed

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*ln(e*(b*x+a)**n/((d*x+c)**n)))/(h*x+g),x)
```

```
[Out] Exception raised: HeuristicGCDFailed
```

# 3.299 $\int \frac{A+B\log(e(a+bx)^n(c+dx)^{-n})}{(g+hx)^2}\,dx$

**Optimal**. Leaf size=120

$$-\frac{B\log\left(e(a+bx)^n(c+dx)^{-n}\right)+A}{h(g+hx)}+\frac{Bn(bc-ad)\log(g+hx)}{(bg-ah)(dg-ch)}+\frac{bBn\log(a+bx)}{h(bg-ah)}-\frac{Bdn\log(c+dx)}{h(dg-ch)}$$

```
[Out] b*B*n*ln(b*x+a)/h/(-a*h+b*g)-B*d*n*ln(d*x+c)/h/(-c*h+d*g)+(-A-B*ln(e*(b*x+a
)^n/((d*x+c)^n)))/h/(h*x+g)+B*(-a*d+b*c)*n*ln(h*x+g)/(-a*h+b*g)/(-c*h+d*g)
```

**Rubi [A]** time = 0.12, antiderivative size = 132, normalized size of antiderivative = 1.10, number of steps used = 6, number of rules used = 4, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.129, Rules used = {6742, 2490, 36, 31}

$$\frac{B(a+bx)\log\left(e(a+bx)^n(c+dx)^{-n}\right)}{(g+hx)(bg-ah)}-\frac{Bn(bc-ad)\log(c+dx)}{(bg-ah)(dg-ch)}+\frac{Bn(bc-ad)\log(g+hx)}{(bg-ah)(dg-ch)}-\frac{A}{h(g+hx)}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])/(g + h*x)^2,x]
```

```
[Out] -(A/(h*(g + h*x))) - (B*(b*c - a*d)*n*Log[c + d*x])/((b*g - a*h)*(d*g - c*h
)) + (B*(a + b*x)*Log[(e*(a + b*x)^n)/(c + d*x)^n])/((b*g - a*h)*(g + h*x))
 + (B*(b*c - a*d)*n*Log[g + h*x])/((b*g - a*h)*(d*g - c*h))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 2490

```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))
^(r_.)]^(s_.)/((g_.) + (h_.)*(x_))^2, x_Symbol] :> Simp[((a + b*x)*Log[e*(f
*(a + b*x)^p*(c + d*x)^q)^r]^s)/((b*g - a*h)*(g + h*x)), x] - Dist[(p*r*s*(
b*c - a*d))/(b*g - a*h), Int[Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^(s - 1)/(
(c + d*x)*(g + h*x)), x], x] /; FreeQ[{a, b, c, d, e, f, g, h, p, q, r, s},
 x] && NeQ[b*c - a*d, 0] && EqQ[p + q, 0] && NeQ[b*g - a*h, 0] && IGtQ[s, 0
]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\int \frac{A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)}{(g+hx)^2}\,dx = \int \left(\frac{A}{(g+hx)^2}+\frac{B\log\left(e(a+bx)^n(c+dx)^{-n}\right)}{(g+hx)^2}\right)dx$$

$$= -\frac{A}{h(g+hx)}+B\int \frac{\log\left(e(a+bx)^n(c+dx)^{-n}\right)}{(g+hx)^2}\,dx$$

$$= -\frac{A}{h(g+hx)}+\frac{B(a+bx)\log\left(e(a+bx)^n(c+dx)^{-n}\right)}{(bg-ah)(g+hx)}-\frac{(B(bc-ad)n)\int \ldots}{bg-a\ldots}$$

$$= -\frac{A}{h(g+hx)}+\frac{B(a+bx)\log\left(e(a+bx)^n(c+dx)^{-n}\right)}{(bg-ah)(g+hx)}-\frac{(Bd(bc-ad)n)\int \ldots}{(bg-ah)(dg-\ldots}$$

$$= -\frac{A}{h(g+hx)}-\frac{B(bc-ad)n\log(c+dx)}{(bg-ah)(dg-ch)}+\frac{B(a+bx)\log\left(e(a+bx)^n(c+\ldots\right.}{(bg-ah)(g+hx)}$$

**Mathematica [A]** time = 0.20, size = 117, normalized size = 0.98

$$\frac{-\frac{B\log(e(a+bx)^n(c+dx)^{-n})}{g+hx}+\frac{Bn(b\log(a+bx)(dg-ch)+\log(c+dx)(adh-bdg)+h(bc-ad)\log(g+hx))}{(bg-ah)(dg-ch)}-\frac{A}{g+hx}}{h}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])/(g + h*x)^2,x]
```

```
[Out] (-(A/(g + h*x)) - (B*Log[(e*(a + b*x)^n)/(c + d*x)^n])/(g + h*x) + (B*n*(b*
(d*g - c*h)*Log[a + b*x] + (-(b*d*g) + a*d*h)*Log[c + d*x] + (b*c - a*d)*h*
Log[g + h*x]))/((b*g - a*h)*(d*g - c*h)))/h
```

**fricas [B]** time = 11.67, size = 250, normalized size = 2.08

$$-\frac{Abdg^2+Aach^2-(Abc+Aad)gh-\left(\left(Bbdgh-Bbch^2\right)nx+\left(Badgh-Bach^2\right)n\right)\log(bx+a)+\left(\left(Bbdgh-Badh^2\ldots\right.\right.}{bdg^3h+acgh^3-(bc\ldots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*(b*x+a)^n/((d*x+c)^n)))/(h*x+g)^2,x, algorithm="fricas
")
```

```
[Out] -(A*b*d*g^2 + A*a*c*h^2 - (A*b*c + A*a*d)*g*h - ((B*b*d*g*h - B*b*c*h^2)*n*
x + (B*a*d*g*h - B*a*c*h^2)*n)*log(b*x + a) + ((B*b*d*g*h - B*a*d*h^2)*n*x
+ (B*b*c*g*h - B*a*c*h^2)*n)*log(d*x + c) - ((B*b*c - B*a*d)*h^2*n*x + (B*b
*c - B*a*d)*g*h*n)*log(h*x + g) + (B*b*d*g^2 + B*a*c*h^2 - (B*b*c + B*a*d)*
g*h)*log(e))/(b*d*g^3*h + a*c*g*h^3 - (b*c + a*d)*g^2*h^2 + (b*d*g^2*h^2 +
a*c*h^4 - (b*c + a*d)*g*h^3)*x)
```

**giac [A]** time = 0.38, size = 166, normalized size = 1.38

$$\frac{Bb^2n\log(|-bx-a|)}{b^2gh-abh^2}-\frac{Bd^2n\log(|dx+c|)}{d^2gh-cdh^2}-\frac{Bn\log(bx+a)}{h^2x+gh}+\frac{Bn\log(dx+c)}{h^2x+gh}+\frac{(Bbcn-Badn)\log(hx+g)}{bdg^2-bcgh-adgh+ach^2}-\frac{A-}{h^2x-}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*(b*x+a)^n/((d*x+c)^n)))/(h*x+g)^2,x, algorithm="giac")
```

```
[Out] B*b^2*n*log(abs(-b*x - a))/(b^2*g*h - a*b*h^2) - B*d^2*n*log(abs(d*x + c))/
(d^2*g*h - c*d*h^2) - B*n*log(b*x + a)/(h^2*x + g*h) + B*n*log(d*x + c)/(h^
2*x + g*h) + (B*b*c*n - B*a*d*n)*log(h*x + g)/(b*d*g^2 - b*c*g*h - a*d*g*h
+ a*c*h^2) - (A + B)/(h^2*x + g*h)
```

**maple [C]** time = 0.53, size = 1796, normalized size = 14.97

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))/(h*x+g)^2,x)
```

```
[Out] B/h/(h*x+g)*ln((d*x+c)^n)-1/2*(2*A*a*c*h^2+2*A*b*d*g^2-I*B*Pi*b*c*h*csgn(I/
((d*x+c)^n))*csgn(I*(b*x+a)^n/((d*x+c)^n))^2*g-I*B*Pi*b*c*h*csgn(I*(b*x+a)^
n/((d*x+c)^n))*csgn(I*e/((d*x+c)^n)*(b*x+a)^n)^2*g-I*B*Pi*a*d*h*csgn(I/((d*
x+c)^n))*csgn(I*(b*x+a)^n/((d*x+c)^n))^2*g-I*B*Pi*b*d*g^2*csgn(I*(b*x+a)^n)
*csgn(I/((d*x+c)^n))*csgn(I*(b*x+a)^n/((d*x+c)^n))-I*B*Pi*a*d*h*csgn(I*(b*x
+a)^n/((d*x+c)^n))*csgn(I*e/((d*x+c)^n)*(b*x+a)^n)^2*g-I*B*Pi*a*c*h^2*csgn(
I*e)*csgn(I*(b*x+a)^n/((d*x+c)^n))*csgn(I*e/((d*x+c)^n)*(b*x+a)^n)-I*B*Pi*b
*d*g^2*csgn(I*e)*csgn(I*(b*x+a)^n/((d*x+c)^n))*csgn(I*e/((d*x+c)^n)*(b*x+a)
^n)+I*B*Pi*a*d*h*csgn(I*(b*x+a)^n)*csgn(I/((d*x+c)^n))*csgn(I*(b*x+a)^n/((d
*x+c)^n))*g+I*B*Pi*b*c*h*csgn(I*e)*csgn(I*(b*x+a)^n/((d*x+c)^n))*csgn(I*e/(
(d*x+c)^n)*(b*x+a)^n)*g-I*B*Pi*a*d*h*csgn(I*e)*csgn(I*e/((d*x+c)^n)*(b*x+a)
^n)^2*g-I*B*Pi*a*d*h*csgn(I*(b*x+a)^n)*csgn(I*(b*x+a)^n/((d*x+c)^n))^2*g+I*
B*Pi*a*c*h^2*csgn(I*(b*x+a)^n)*csgn(I*(b*x+a)^n/((d*x+c)^n))^2+I*B*Pi*a*c*h
^2*csgn(I/((d*x+c)^n))*csgn(I*(b*x+a)^n/((d*x+c)^n))^2+I*B*Pi*a*c*h^2*csgn(
I*(b*x+a)^n/((d*x+c)^n))*csgn(I*e/((d*x+c)^n)*(b*x+a)^n)^2+I*B*Pi*a*d*h*csg
n(I*(b*x+a)^n/((d*x+c)^n))^3*g-2*A*a*d*h*g-2*A*b*c*h*g+2*B*ln(h*x+g)*a*d*h^
2*n*x-2*B*ln(h*x+g)*b*c*h^2*n*x-2*B*ln(-d*x-c)*a*d*h^2*n*x+2*B*ln(-b*x-a)*b
```

```
*c*h^2*n*x+2*B*ln(h*x+g)*a*d*g*h*n-2*B*ln(h*x+g)*b*c*g*h*n-2*B*ln(-d*x-c)*a
*d*g*h*n+2*B*ln(-b*x-a)*b*c*g*h*n+2*B*a*c*h^2*ln((b*x+a)^n)+2*B*b*d*g^2*ln(
(b*x+a)^n)+I*B*Pi*b*d*g^2*csgn(I*(b*x+a)^n/((d*x+c)^n))*csgn(I*e/((d*x+c)^n
)*(b*x+a)^n)^2+I*B*Pi*a*c*h^2*csgn(I*e)*csgn(I*e/((d*x+c)^n)*(b*x+a)^n)^2-I
*B*Pi*b*c*h*csgn(I*(b*x+a)^n)*csgn(I*(b*x+a)^n/((d*x+c)^n))^2*g-I*B*Pi*a*c*
h^2*csgn(I*(b*x+a)^n)*csgn(I/((d*x+c)^n))*csgn(I*(b*x+a)^n/((d*x+c)^n))-I*B
*Pi*b*c*h*csgn(I*e)*csgn(I*e/((d*x+c)^n)*(b*x+a)^n)^2*g+2*B*ln(-d*x-c)*b*d*
g^2*n-2*B*ln(-b*x-a)*b*d*g^2*n+I*B*Pi*b*c*h*csgn(I*(b*x+a)^n)*csgn(I/((d*x+
c)^n))*csgn(I*(b*x+a)^n/((d*x+c)^n))*g+I*B*Pi*a*d*h*csgn(I*e)*csgn(I*(b*x+a
)^n/((d*x+c)^n))*csgn(I*e/((d*x+c)^n)*(b*x+a)^n)*g-I*B*Pi*b*d*g^2*csgn(I*(b
*x+a)^n/((d*x+c)^n))^3-I*B*Pi*b*d*g^2*csgn(I*e/((d*x+c)^n)*(b*x+a)^n)^3+I*B
*Pi*b*d*g^2*csgn(I*(b*x+a)^n)*csgn(I*(b*x+a)^n/((d*x+c)^n))^2+I*B*Pi*b*d*g^
2*csgn(I/((d*x+c)^n))*csgn(I*(b*x+a)^n/((d*x+c)^n))^2+I*B*Pi*b*c*h*csgn(I*e
/((d*x+c)^n)*(b*x+a)^n)^3*g+I*B*Pi*b*d*g^2*csgn(I*e)*csgn(I*e/((d*x+c)^n)*(
b*x+a)^n)^2-2*B*a*d*g*h*ln((b*x+a)^n)-2*B*b*c*g*h*ln((b*x+a)^n)+2*B*ln(e)*a
*c*h^2+2*B*ln(e)*b*d*g^2-I*B*Pi*a*c*h^2*csgn(I*(b*x+a)^n/((d*x+c)^n))^3-I*B
*Pi*a*c*h^2*csgn(I*e/((d*x+c)^n)*(b*x+a)^n)^3+I*B*Pi*a*d*h*csgn(I*e/((d*x+c
)^n)*(b*x+a)^n)^3*g+I*B*Pi*b*c*h*csgn(I*(b*x+a)^n/((d*x+c)^n))^3*g+2*B*ln(-
d*x-c)*b*d*g*h*n*x-2*B*ln(-b*x-a)*b*d*g*h*n*x-2*B*ln(e)*b*c*h*g-2*B*ln(e)*a
*d*h*g)/(h*x+g)/(a*c*h^2-a*d*g*h-b*c*g*h+b*d*g^2)/h
```

**maxima [A]** time = 0.76, size = 151, normalized size = 1.26

$$\frac{\left(\frac{ben\log(bx+a)}{bgh-ah^2}-\frac{den\log(dx+c)}{dgh-ch^2}-\frac{(bcen-aden)\log(hx+g)}{(dgh-ch^2)a-(dg^2-cgh)b}\right)B}{e}-\frac{B\log\left(\frac{(bx+a)^n e}{(dx+c)^n}\right)}{h^2x+gh}-\frac{A}{h^2x+gh}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*(b*x+a)^n/((d*x+c)^n)))/(h*x+g)^2,x, algorithm="maxima
")
```

```
[Out] (b*e*n*log(b*x + a)/(b*g*h - a*h^2) - d*e*n*log(d*x + c)/(d*g*h - c*h^2) -
(b*c*e*n - a*d*e*n)*log(h*x + g)/((d*g*h - c*h^2)*a - (d*g^2 - c*g*h)*b))*B
/e - B*log((b*x + a)^n*e/(d*x + c)^n)/(h^2*x + g*h) - A/(h^2*x + g*h)
```

**mupad [B]** time = 4.72, size = 141, normalized size = 1.18

$$\frac{B\,d\,n\,\ln(c+d\,x)}{c\,h^2-d\,g\,h}-\frac{\ln\left(g+h\,x\right)\,(B\,a\,d\,n-B\,b\,c\,n)}{a\,c\,h^2+b\,d\,g^2-a\,d\,g\,h-b\,c\,g\,h}-\frac{B\,\ln\left(\frac{e\,(a+b\,x)^n}{(c+d\,x)^n}\right)}{h\,\left(g+h\,x\right)}-\frac{B\,b\,n\,\ln(a+b\,x)}{a\,h^2-b\,g\,h}-\frac{A}{x\,h^2+g\,h}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*log((e*(a + b*x)^n)/(c + d*x)^n))/(g + h*x)^2,x)
```

```
[Out] (B*d*n*log(c + d*x))/(c*h^2 - d*g*h) - (log(g + h*x)*(B*a*d*n - B*b*c*n))/(
a*c*h^2 + b*d*g^2 - a*d*g*h - b*c*g*h) - (B*log((e*(a + b*x)^n)/(c + d*x)^n
))/(h*(g + h*x)) - (B*b*n*log(a + b*x))/(a*h^2 - b*g*h) - A/(g*h + h^2*x)
```

**sympy [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*ln(e*(b*x+a)**n/((d*x+c)**n)))/(h*x+g)**2,x)
```

```
[Out] Timed out
```

# 3.300 $\int \frac{A+B\log(e(a+bx)^n(c+dx)^{-n})}{(g+hx)^3}\,dx$

**Optimal**. Leaf size=191

$$-\frac{B\log\left(e(a+bx)^n(c+dx)^{-n}\right)+A}{2h(g+hx)^2}+\frac{b^2Bn\log(a+bx)}{2h(bg-ah)^2}-\frac{Bn(bc-ad)}{2(g+hx)(bg-ah)(dg-ch)}+\frac{Bn(bc-ad)\log(g+hx)(-a}{2(bg-ah)^2(dg-}$$

```
[Out] -1/2*B*(-a*d+b*c)*n/(-a*h+b*g)/(-c*h+d*g)/(h*x+g)+1/2*b^2*B*n*ln(b*x+a)/h/(
-a*h+b*g)^2-1/2*B*d^2*n*ln(d*x+c)/h/(-c*h+d*g)^2+1/2*(-A-B*ln(e*(b*x+a)^n/(
(d*x+c)^n)))/h/(h*x+g)^2+1/2*B*(-a*d+b*c)*(-a*d*h-b*c*h+2*b*d*g)*n*ln(h*x+g
)/(-a*h+b*g)^2/(-c*h+d*g)^2
```

**Rubi [A]** time = 0.30, antiderivative size = 203, normalized size of antiderivative = 1.06, number of steps used = 5, number of rules used = 3, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.097, Rules used = {6742, 2492, 72}

$$\frac{b^2Bn\log(a+bx)}{2h(bg-ah)^2}-\frac{B\log\left(e(a+bx)^n(c+dx)^{-n}\right)}{2h(g+hx)^2}-\frac{Bn(bc-ad)}{2(g+hx)(bg-ah)(dg-ch)}+\frac{Bn(bc-ad)\log(g+hx)(-adh-bc}{2(bg-ah)^2(dg-ch)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])/(g + h*x)^3,x]
```

```
[Out] -A/(2*h*(g + h*x)^2) - (B*(b*c - a*d)*n)/(2*(b*g - a*h)*(d*g - c*h)*(g + h*
x)) + (b^2*B*n*Log[a + b*x])/(2*h*(b*g - a*h)^2) - (B*d^2*n*Log[c + d*x])/(
2*h*(d*g - c*h)^2) - (B*Log[(e*(a + b*x)^n)/(c + d*x)^n])/(2*h*(g + h*x)^2)
 + (B*(b*c - a*d)*(2*b*d*g - b*c*h - a*d*h)*n*Log[g + h*x])/(2*(b*g - a*h)^
2*(d*g - c*h)^2)
```

Rule 72

```
Int[((e_.) + (f_.)*(x_))^(p_.)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
 x_Symbol] :> Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x]
 /; FreeQ[{a, b, c, d, e, f}, x] && IntegerQ[p]
```

Rule 2492

```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))
^(r_.)]^(s_.)*((g_.) + (h_.)*(x_))^(m_.), x_Symbol] :> Simp[((g + h*x)^(m +
 1)*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^s)/(h*(m + 1)), x] - Dist[(p*r*s*(
b*c - a*d))/(h*(m + 1)), Int[((g + h*x)^(m + 1)*Log[e*(f*(a + b*x)^p*(c + d
*x)^q)^r]^(s - 1))/((a + b*x)*(c + d*x)), x], x] /; FreeQ[{a, b, c, d, e, f
, g, h, m, p, q, r, s}, x] && NeQ[b*c - a*d, 0] && EqQ[p + q, 0] && IGtQ[s,
 0] && NeQ[m, -1]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\int \frac{A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)}{(g+hx)^3}\,dx = \int\left(\frac{A}{(g+hx)^3}+\frac{B\log\left(e(a+bx)^n(c+dx)^{-n}\right)}{(g+hx)^3}\right)dx$$

$$= -\frac{A}{2h(g+hx)^2}+B\int\frac{\log\left(e(a+bx)^n(c+dx)^{-n}\right)}{(g+hx)^3}\,dx$$

$$= -\frac{A}{2h(g+hx)^2}-\frac{B\log\left(e(a+bx)^n(c+dx)^{-n}\right)}{2h(g+hx)^2}+\frac{(B(bc-ad)n)\int\frac{}{(a+bx)(}}{2h}$$

$$= -\frac{A}{2h(g+hx)^2}-\frac{B\log\left(e(a+bx)^n(c+dx)^{-n}\right)}{2h(g+hx)^2}+\frac{(B(bc-ad)n)\int\left(\frac{}{(bc-ad}\right.}{}$$

$$= -\frac{A}{2h(g+hx)^2}-\frac{B(bc-ad)n}{2(bg-ah)(dg-ch)(g+hx)}+\frac{b^2Bn\log(a+bx)}{2h(bg-ah)^2}-\frac{Ba}{}$$

**Mathematica [A]** time = 0.62, size = 178, normalized size = 0.93

$$-\frac{Bn(bc-ad)\left(\frac{\frac{d^2\log(c+dx)}{bc-ad}+\frac{h\left(\frac{(bg-ah)(dg-ch)}{g+hx}+\log(g+hx)(adh+bch-2bdg)\right)}{(bg-ah)^2}}{(dg-ch)^2}-\frac{b^2\log(a+bx)}{(bc-ad)(bg-ah)^2}\right)+\frac{B\log\left(e(a+bx)^n(c+dx)^{-n}\right)}{(g+hx)^2}+\frac{A}{(g+hx)^2}}{2h}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])/(g + h*x)^3,x]
```

```
[Out] -1/2*(A/(g + h*x)^2 + (B*Log[(e*(a + b*x)^n)/(c + d*x)^n])/(g + h*x)^2 + B*
(b*c - a*d)*n*(-((b^2*Log[a + b*x])/((b*c - a*d)*(b*g - a*h)^2)) + ((d^2*Lo
g[c + d*x])/(b*c - a*d) + (h*(((b*g - a*h)*(d*g - c*h))/(g + h*x) + (-2*b*d
*g + b*c*h + a*d*h)*Log[g + h*x]))/(b*g - a*h)^2)/(d*g - c*h)^2))/h
```

**fricas [B]** time = 149.49, size = 1127, normalized size = 5.90

$$-\frac{Ab^2d^2g^4+Aa^2c^2h^4-2\left(Ab^2cd+Aabd^2\right)g^3h+\left(Ab^2c^2+4\,Aabcd+Aa^2d^2\right)g^2h^2-2\left(Aabc^2+Aa^2cd\right)gh^3+\left(\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*(b*x+a)^n/((d*x+c)^n)))/(h*x+g)^3,x, algorithm="fricas
")
```

```
[Out] -1/2*(A*b^2*d^2*g^4 + A*a^2*c^2*h^4 - 2*(A*b^2*c*d + A*a*b*d^2)*g^3*h + (A*
b^2*c^2 + 4*A*a*b*c*d + A*a^2*d^2)*g^2*h^2 - 2*(A*a*b*c^2 + A*a^2*c*d)*g*h^
3 + ((B*b^2*c*d - B*a*b*d^2)*g^2*h^2 - (B*b^2*c^2 - B*a^2*d^2)*g*h^3 + (B*a
*b*c^2 - B*a^2*c*d)*h^4)*n*x + ((B*b^2*c*d - B*a*b*d^2)*g^3*h - (B*b^2*c^2
- B*a^2*d^2)*g^2*h^2 + (B*a*b*c^2 - B*a^2*c*d)*g*h^3)*n - ((B*b^2*d^2*g^2*h
^2 - 2*B*b^2*c*d*g*h^3 + B*b^2*c^2*h^4)*n*x^2 + 2*(B*b^2*d^2*g^3*h - 2*B*b^
2*c*d*g^2*h^2 + B*b^2*c^2*g*h^3)*n*x + (2*B*a*b*d^2*g^3*h - B*a^2*c^2*h^4 -
 (4*B*a*b*c*d + B*a^2*d^2)*g^2*h^2 + 2*(B*a*b*c^2 + B*a^2*c*d)*g*h^3)*n)*lo
g(b*x + a) + ((B*b^2*d^2*g^2*h^2 - 2*B*a*b*d^2*g*h^3 + B*a^2*d^2*h^4)*n*x^2
 + 2*(B*b^2*d^2*g^3*h - 2*B*a*b*d^2*g^2*h^2 + B*a^2*d^2*g*h^3)*n*x + (2*B*b
^2*c*d*g^3*h - B*a^2*c^2*h^4 - (B*b^2*c^2 + 4*B*a*b*c*d)*g^2*h^2 + 2*(B*a*b
*c^2 + B*a^2*c*d)*g*h^3)*n)*log(d*x + c) - ((2*(B*b^2*c*d - B*a*b*d^2)*g*h^
3 - (B*b^2*c^2 - B*a^2*d^2)*h^4)*n*x^2 + 2*(2*(B*b^2*c*d - B*a*b*d^2)*g^2*h
^2 - (B*b^2*c^2 - B*a^2*d^2)*g*h^3)*n*x + (2*(B*b^2*c*d - B*a*b*d^2)*g^3*h
- (B*b^2*c^2 - B*a^2*d^2)*g^2*h^2)*n)*log(h*x + g) + (B*b^2*d^2*g^4 + B*a^2
*c^2*h^4 - 2*(B*b^2*c*d + B*a*b*d^2)*g^3*h + (B*b^2*c^2 + 4*B*a*b*c*d + B*a
^2*d^2)*g^2*h^2 - 2*(B*a*b*c^2 + B*a^2*c*d)*g*h^3)*log(e))/(b^2*d^2*g^6*h +
 a^2*c^2*g^2*h^5 - 2*(b^2*c*d + a*b*d^2)*g^5*h^2 + (b^2*c^2 + 4*a*b*c*d + a
^2*d^2)*g^4*h^3 - 2*(a*b*c^2 + a^2*c*d)*g^3*h^4 + (b^2*d^2*g^4*h^3 + a^2*c^
2*h^7 - 2*(b^2*c*d + a*b*d^2)*g^3*h^4 + (b^2*c^2 + 4*a*b*c*d + a^2*d^2)*g^2
*h^5 - 2*(a*b*c^2 + a^2*c*d)*g*h^6)*x^2 + 2*(b^2*d^2*g^5*h^2 + a^2*c^2*g*h^
6 - 2*(b^2*c*d + a*b*d^2)*g^4*h^3 + (b^2*c^2 + 4*a*b*c*d + a^2*d^2)*g^3*h^4
 - 2*(a*b*c^2 + a^2*c*d)*g^2*h^5)*x)
```

**giac [B]** time = 0.80, size = 523, normalized size = 2.74

$$\frac{Bb^3n\log(|bx+a|)}{2\left(b^3g^2h-2\,ab^2gh^2+a^2bh^3\right)}-\frac{Bd^3n\log(|dx+c|)}{2\left(d^3g^2h-2\,cd^2gh^2+c^2dh^3\right)}-\frac{Bn\log(bx+a)}{2\left(h^3x^2+2\,gh^2x+g^2h\right)}+\frac{Bn\log(dx+c)}{2\left(h^3x^2+2\,gh^2x+g^2h\right)}+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*(b*x+a)^n/((d*x+c)^n)))/(h*x+g)^3,x, algorithm="giac")
```

```
[Out] 1/2*B*b^3*n*log(abs(b*x + a))/(b^3*g^2*h - 2*a*b^2*g*h^2 + a^2*b*h^3) - 1/2
*B*d^3*n*log(abs(d*x + c))/(d^3*g^2*h - 2*c*d^2*g*h^2 + c^2*d*h^3) - 1/2*B*
n*log(b*x + a)/(h^3*x^2 + 2*g*h^2*x + g^2*h) + 1/2*B*n*log(d*x + c)/(h^3*x^
2 + 2*g*h^2*x + g^2*h) + 1/2*(2*B*b^2*c*d*g*n - 2*B*a*b*d^2*g*n - B*b^2*c^2
*h*n + B*a^2*d^2*h*n)*log(h*x + g)/(b^2*d^2*g^4 - 2*b^2*c*d*g^3*h - 2*a*b*d
^2*g^3*h + b^2*c^2*g^2*h^2 + 4*a*b*c*d*g^2*h^2 + a^2*d^2*g^2*h^2 - 2*a*b*c^
2*g*h^3 - 2*a^2*c*d*g*h^3 + a^2*c^2*h^4) - 1/2*(B*b*c*h^2*n*x - B*a*d*h^2*n
```

```
*x + B*b*c*g*h*n - B*a*d*g*h*n + A*b*d*g^2 + B*b*d*g^2 - A*b*c*g*h - B*b*c*
g*h - A*a*d*g*h - B*a*d*g*h + A*a*c*h^2 + B*a*c*h^2)/(b*d*g^2*h^3*x^2 - b*c
*g*h^4*x^2 - a*d*g*h^4*x^2 + a*c*h^5*x^2 + 2*b*d*g^3*h^2*x - 2*b*c*g^2*h^3*
x - 2*a*d*g^2*h^3*x + 2*a*c*g*h^4*x + b*d*g^4*h - b*c*g^3*h^2 - a*d*g^3*h^2
 + a*c*g^2*h^3)
```

**maple [C]** time = 0.84, size = 4925, normalized size = 25.79

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))/(h*x+g)^3,x)
```

```
[Out] 1/2*B/h/(h*x+g)^2*ln((d*x+c)^n)-1/4*(-2*I*B*Pi*b^2*c*d*g^3*h*csgn(I/((d*x+c
)^n))*csgn(I*(b*x+a)^n/((d*x+c)^n))^2+2*I*B*Pi*a*b*c^2*g*h^3*csgn(I*e)*csgn
(I*(b*x+a)^n/((d*x+c)^n))*csgn(I*e/((d*x+c)^n)*(b*x+a)^n)+2*I*B*Pi*b^2*c*d*
g^3*h*csgn(I*(b*x+a)^n)*csgn(I/((d*x+c)^n))*csgn(I*(b*x+a)^n/((d*x+c)^n))-2
*B*a^2*c*d*h^4*n*x+2*B*a^2*d^2*g*h^3*n*x+2*B*a*b*c^2*h^4*n*x-2*B*b^2*c^2*g*
h^3*n*x+2*A*a^2*c^2*h^4+2*A*b^2*d^2*g^4+2*B*a^2*d^2*g^2*h^2*n-2*B*b^2*c^2*g
^2*h^2*n+2*I*B*Pi*a*b*d^2*g^3*h*csgn(I*(b*x+a)^n/((d*x+c)^n))^3-I*B*Pi*a^2*
c^2*h^4*csgn(I*e)*csgn(I*(b*x+a)^n/((d*x+c)^n))*csgn(I*e/((d*x+c)^n)*(b*x+a
)^n)+2*I*B*Pi*a*b*d^2*g^3*h*csgn(I*e/((d*x+c)^n)*(b*x+a)^n)^3+2*A*b^2*c^2*g
^2*h^2-2*B*ln(-h*x-g)*a^2*d^2*h^4*n*x^2+2*B*ln(-h*x-g)*b^2*c^2*h^4*n*x^2+2*
B*ln(-d*x-c)*a^2*d^2*h^4*n*x^2-2*B*ln(b*x+a)*b^2*c^2*h^4*n*x^2-2*B*ln(-h*x-
g)*a^2*d^2*g^2*h^2*n+2*B*ln(-h*x-g)*b^2*c^2*g^2*h^2*n+2*B*ln(-d*x-c)*a^2*d^
2*g^2*h^2*n-2*B*ln(b*x+a)*b^2*c^2*g^2*h^2*n+I*B*Pi*a^2*d^2*g^2*h^2*csgn(I*(
b*x+a)^n)*csgn(I*(b*x+a)^n/((d*x+c)^n))^2+I*B*Pi*a^2*d^2*g^2*h^2*csgn(I/((d
*x+c)^n))*csgn(I*(b*x+a)^n/((d*x+c)^n))^2-I*B*Pi*b^2*d^2*g^4*csgn(I*e/((d*x
+c)^n)*(b*x+a)^n)^3-4*B*ln(-h*x-g)*a^2*d^2*g*h^3*n*x+4*B*ln(-h*x-g)*b^2*c^2
*g*h^3*n*x+4*B*ln(-d*x-c)*a^2*d^2*g*h^3*n*x+4*B*ln(-d*x-c)*b^2*d^2*g^3*h*n*
x-4*B*ln(b*x+a)*b^2*c^2*g*h^3*n*x-4*B*ln(b*x+a)*b^2*d^2*g^3*h*n*x+4*B*ln(-h
*x-g)*a*b*d^2*g^3*h*n-4*B*ln(-h*x-g)*b^2*c*d*g^3*h*n-4*B*ln(-d*x-c)*a*b*d^2
*g^3*h*n+4*B*ln(b*x+a)*b^2*c*d*g^3*h*n+2*B*ln(-d*x-c)*b^2*d^2*g^2*h^2*n*x^2
-2*B*ln(b*x+a)*b^2*d^2*g^2*h^2*n*x^2-2*I*B*Pi*a*b*d^2*g^3*h*csgn(I*(b*x+a)^
n/((d*x+c)^n))*csgn(I*e/((d*x+c)^n)*(b*x+a)^n)^2+2*A*a^2*d^2*g^2*h^2-2*I*B*
Pi*a^2*c*d*g*h^3*csgn(I*(b*x+a)^n)*csgn(I*(b*x+a)^n/((d*x+c)^n))^2-2*I*B*Pi
*a^2*c*d*g*h^3*csgn(I/((d*x+c)^n))*csgn(I*(b*x+a)^n/((d*x+c)^n))^2-2*I*B*Pi
*a^2*c*d*g*h^3*csgn(I*(b*x+a)^n/((d*x+c)^n))*csgn(I*e/((d*x+c)^n)*(b*x+a)^n
)^2+8*A*a*b*c*d*g^2*h^2-I*B*Pi*a^2*d^2*g^2*h^2*csgn(I*(b*x+a)^n)*csgn(I/((d
*x+c)^n))*csgn(I*(b*x+a)^n/((d*x+c)^n))-4*I*B*Pi*a*b*c*d*g^2*h^2*csgn(I*(b*
x+a)^n/((d*x+c)^n))^3-4*I*B*Pi*a*b*c*d*g^2*h^2*csgn(I*e/((d*x+c)^n)*(b*x+a)
^n)^3-I*B*Pi*a^2*c^2*h^4*csgn(I*(b*x+a)^n)*csgn(I/((d*x+c)^n))*csgn(I*(b*x+
a)^n/((d*x+c)^n))+4*I*B*Pi*a*b*c*d*g^2*h^2*csgn(I*e)*csgn(I*e/((d*x+c)^n)*(
b*x+a)^n)^2-I*B*Pi*b^2*c^2*g^2*h^2*csgn(I*(b*x+a)^n/((d*x+c)^n))^3+I*B*Pi*a
^2*c^2*h^4*csgn(I*(b*x+a)^n/((d*x+c)^n))*csgn(I*e/((d*x+c)^n)*(b*x+a)^n)^2+
```

```
I*B*Pi*b^2*d^2*g^4*csgn(I*e)*csgn(I*e/((d*x+c)^n)*(b*x+a)^n)^2+I*B*Pi*b^2*d
^2*g^4*csgn(I*(b*x+a)^n)*csgn(I*(b*x+a)^n/((d*x+c)^n))^2+I*B*Pi*b^2*d^2*g^4
*csgn(I/((d*x+c)^n))*csgn(I*(b*x+a)^n/((d*x+c)^n))^2+8*B*ln(e)*a*b*c*d*g^2*
h^2+2*I*B*Pi*a*b*c^2*g*h^3*csgn(I*(b*x+a)^n/((d*x+c)^n))^3+2*I*B*Pi*a*b*c^2
*g*h^3*csgn(I*e/((d*x+c)^n)*(b*x+a)^n)^3-2*I*B*Pi*a*b*d^2*g^3*h*csgn(I*e)*c
sgn(I*e/((d*x+c)^n)*(b*x+a)^n)^2-2*I*B*Pi*a*b*d^2*g^3*h*csgn(I*(b*x+a)^n)*c
sgn(I*(b*x+a)^n/((d*x+c)^n))^2-I*B*Pi*a^2*d^2*g^2*h^2*csgn(I*e)*csgn(I*(b*x
+a)^n/((d*x+c)^n))*csgn(I*e/((d*x+c)^n)*(b*x+a)^n)+2*B*ln(e)*a^2*d^2*g^2*h^
2+2*B*ln(e)*b^2*c^2*g^2*h^2+2*B*a^2*d^2*g^2*h^2*ln((b*x+a)^n)+2*I*B*Pi*b^2*
c*d*g^3*h*csgn(I*(b*x+a)^n/((d*x+c)^n))^3+2*I*B*Pi*b^2*c*d*g^3*h*csgn(I*e/(
(d*x+c)^n)*(b*x+a)^n)^3-I*B*Pi*b^2*d^2*g^4*csgn(I*e)*csgn(I*(b*x+a)^n/((d*x
+c)^n))*csgn(I*e/((d*x+c)^n)*(b*x+a)^n)+2*B*ln(e)*a^2*c^2*h^4+2*B*ln(e)*b^2
*d^2*g^4+2*I*B*Pi*a*b*d^2*g^3*h*csgn(I*(b*x+a)^n)*csgn(I/((d*x+c)^n))*csgn(
I*(b*x+a)^n/((d*x+c)^n))+4*I*B*Pi*a*b*c*d*g^2*h^2*csgn(I*(b*x+a)^n)*csgn(I*
(b*x+a)^n/((d*x+c)^n))^2+4*I*B*Pi*a*b*c*d*g^2*h^2*csgn(I/((d*x+c)^n))*csgn(
I*(b*x+a)^n/((d*x+c)^n))^2+2*B*ln(-d*x-c)*b^2*d^2*g^4*n-2*B*ln(b*x+a)*b^2*d
^2*g^4*n+2*I*B*Pi*a*b*c^2*g*h^3*csgn(I*(b*x+a)^n)*csgn(I/((d*x+c)^n))*csgn(
I*(b*x+a)^n/((d*x+c)^n))+2*I*B*Pi*a*b*d^2*g^3*h*csgn(I*e)*csgn(I*(b*x+a)^n/
((d*x+c)^n))*csgn(I*e/((d*x+c)^n)*(b*x+a)^n)+2*I*B*Pi*a^2*c*d*g*h^3*csgn(I*
e)*csgn(I*(b*x+a)^n/((d*x+c)^n))*csgn(I*e/((d*x+c)^n)*(b*x+a)^n)-2*I*B*Pi*b
^2*c*d*g^3*h*csgn(I*e)*csgn(I*e/((d*x+c)^n)*(b*x+a)^n)^2-2*I*B*Pi*b^2*c*d*g
^3*h*csgn(I*(b*x+a)^n)*csgn(I*(b*x+a)^n/((d*x+c)^n))^2-4*B*a*b*d^2*g^3*h*ln
((b*x+a)^n)-4*B*b^2*c*d*g^3*h*ln((b*x+a)^n)-4*B*a^2*c*d*g*h^3*ln((b*x+a)^n)
-4*B*a*b*c^2*g*h^3*ln((b*x+a)^n)-I*B*Pi*a^2*d^2*g^2*h^2*csgn(I*(b*x+a)^n/((
d*x+c)^n))^3-I*B*Pi*a^2*d^2*g^2*h^2*csgn(I*e/((d*x+c)^n)*(b*x+a)^n)^3+I*B*P
i*a^2*d^2*g^2*h^2*csgn(I*(b*x+a)^n/((d*x+c)^n))*csgn(I*e/((d*x+c)^n)*(b*x+a
)^n)^2+I*B*Pi*b^2*c^2*g^2*h^2*csgn(I*e)*csgn(I*e/((d*x+c)^n)*(b*x+a)^n)^2+I
*B*Pi*b^2*c^2*g^2*h^2*csgn(I*(b*x+a)^n)*csgn(I*(b*x+a)^n/((d*x+c)^n))^2+I*B
*Pi*b^2*c^2*g^2*h^2*csgn(I/((d*x+c)^n))*csgn(I*(b*x+a)^n/((d*x+c)^n))^2-I*B
*Pi*b^2*c^2*g^2*h^2*csgn(I*e)*csgn(I*(b*x+a)^n/((d*x+c)^n))*csgn(I*e/((d*x+
c)^n)*(b*x+a)^n)-I*B*Pi*b^2*c^2*g^2*h^2*csgn(I*(b*x+a)^n)*csgn(I/((d*x+c)^n
))*csgn(I*(b*x+a)^n/((d*x+c)^n))-2*I*B*Pi*a^2*c*d*g*h^3*csgn(I*e)*csgn(I*e/
((d*x+c)^n)*(b*x+a)^n)^2+2*B*b^2*c^2*g^2*h^2*ln((b*x+a)^n)-I*B*Pi*b^2*d^2*g
^4*csgn(I*(b*x+a)^n)*csgn(I/((d*x+c)^n))*csgn(I*(b*x+a)^n/((d*x+c)^n))+I*B*
Pi*b^2*c^2*g^2*h^2*csgn(I*(b*x+a)^n/((d*x+c)^n))*csgn(I*e/((d*x+c)^n)*(b*x+
a)^n)^2+I*B*Pi*a^2*d^2*g^2*h^2*csgn(I*e)*csgn(I*e/((d*x+c)^n)*(b*x+a)^n)^2+
2*I*B*Pi*a^2*c*d*g*h^3*csgn(I*(b*x+a)^n/((d*x+c)^n))^3-4*B*ln(e)*a^2*c*d*g*
h^3-4*B*ln(e)*a*b*c^2*g*h^3-4*B*ln(e)*a*b*d^2*g^3*h-4*B*ln(e)*b^2*c*d*g^3*h
-2*I*B*Pi*a*b*d^2*g^3*h*csgn(I/((d*x+c)^n))*csgn(I*(b*x+a)^n/((d*x+c)^n))^2
-4*A*a^2*c*d*g*h^3-4*A*a*b*c^2*g*h^3-4*A*a*b*d^2*g^3*h+2*I*B*Pi*a^2*c*d*g*h
^3*csgn(I*e/((d*x+c)^n)*(b*x+a)^n)^3-4*A*b^2*c*d*g^3*h-2*I*B*Pi*b^2*c*d*g^3
*h*csgn(I*(b*x+a)^n/((d*x+c)^n))*csgn(I*e/((d*x+c)^n)*(b*x+a)^n)^2-2*I*B*Pi
*a*b*c^2*g*h^3*csgn(I*e)*csgn(I*e/((d*x+c)^n)*(b*x+a)^n)^2+8*B*a*b*c*d*g^2*
h^2*ln((b*x+a)^n)-2*B*a*b*d^2*g^2*h^2*n*x+2*B*b^2*c*d*g^2*h^2*n*x-2*I*B*Pi*
a*b*c^2*g*h^3*csgn(I*(b*x+a)^n)*csgn(I*(b*x+a)^n/((d*x+c)^n))^2-2*I*B*Pi*a*
```

```
b*c^2*g*h^3*csgn(I/((d*x+c)^n))*csgn(I*(b*x+a)^n/((d*x+c)^n))^2-2*I*B*Pi*a*
b*c^2*g*h^3*csgn(I*(b*x+a)^n/((d*x+c)^n))*csgn(I*e/((d*x+c)^n)*(b*x+a)^n)^2
+I*B*Pi*b^2*d^2*g^4*csgn(I*(b*x+a)^n/((d*x+c)^n))*csgn(I*e/((d*x+c)^n)*(b*x
+a)^n)^2+I*B*Pi*a^2*c^2*h^4*csgn(I*e)*csgn(I*e/((d*x+c)^n)*(b*x+a)^n)^2+I*B
*Pi*a^2*c^2*h^4*csgn(I*(b*x+a)^n)*csgn(I*(b*x+a)^n/((d*x+c)^n))^2+I*B*Pi*a^
2*c^2*h^4*csgn(I/((d*x+c)^n))*csgn(I*(b*x+a)^n/((d*x+c)^n))^2-I*B*Pi*b^2*c^
2*g^2*h^2*csgn(I*e/((d*x+c)^n)*(b*x+a)^n)^3-I*B*Pi*a^2*c^2*h^4*csgn(I*(b*x+
a)^n/((d*x+c)^n))^3-I*B*Pi*a^2*c^2*h^4*csgn(I*e/((d*x+c)^n)*(b*x+a)^n)^3-I*
B*Pi*b^2*d^2*g^4*csgn(I*(b*x+a)^n/((d*x+c)^n))^3-4*B*ln(-d*x-c)*a*b*d^2*g*h
^3*n*x^2+4*B*ln(b*x+a)*b^2*c*d*g*h^3*n*x^2+8*B*ln(-h*x-g)*a*b*d^2*g^2*h^2*n
*x-8*B*ln(-h*x-g)*b^2*c*d*g^2*h^2*n*x-8*B*ln(-d*x-c)*a*b*d^2*g^2*h^2*n*x+8*
B*ln(b*x+a)*b^2*c*d*g^2*h^2*n*x+4*B*ln(-h*x-g)*a*b*d^2*g*h^3*n*x^2-4*B*ln(-
h*x-g)*b^2*c*d*g*h^3*n*x^2+2*I*B*Pi*a^2*c*d*g*h^3*csgn(I*(b*x+a)^n)*csgn(I/
((d*x+c)^n))*csgn(I*(b*x+a)^n/((d*x+c)^n))+2*I*B*Pi*b^2*c*d*g^3*h*csgn(I*e)
*csgn(I*(b*x+a)^n/((d*x+c)^n))*csgn(I*e/((d*x+c)^n)*(b*x+a)^n)+2*B*a^2*c^2*
h^4*ln((b*x+a)^n)+2*B*b^2*d^2*g^4*ln((b*x+a)^n)-4*I*B*Pi*a*b*c*d*g^2*h^2*cs
gn(I*e)*csgn(I*(b*x+a)^n/((d*x+c)^n))*csgn(I*e/((d*x+c)^n)*(b*x+a)^n)-4*I*B
*Pi*a*b*c*d*g^2*h^2*csgn(I*(b*x+a)^n)*csgn(I/((d*x+c)^n))*csgn(I*(b*x+a)^n/
((d*x+c)^n))+4*I*B*Pi*a*b*c*d*g^2*h^2*csgn(I*(b*x+a)^n/((d*x+c)^n))*csgn(I*
e/((d*x+c)^n)*(b*x+a)^n)^2-2*B*a^2*c*d*g*h^3*n+2*B*a*b*c^2*g*h^3*n-2*B*a*b*
d^2*g^3*h*n+2*B*b^2*c*d*g^3*h*n)/(h*x+g)^2/(a*c*h^2-a*d*g*h-b*c*g*h+b*d*g^2
)/(-c*h+d*g)/h/(-a*h+b*g)
```

**maxima [B]** time = 0.91, size = 382, normalized size = 2.00

$$\frac{\left(\frac{b^2en\log(bx+a)}{b^2g^2h-2\,abgh^2+a^2h^3} - \frac{d^2en\log(dx+c)}{d^2g^2h-2\,cdgh^2+c^2h^3} - \frac{\left(2\,abd^2egn-a^2d^2ehn-\left(2\,cdegn-c^2ehn\right)b^2\right)\log\left(hx+g\right)}{\left(d^2g^2h^2-2\,cdgh^3+c^2h^4\right)a^2-2\left(d^2g^3h-2\,cdg^2h^2+c^2gh^3\right)ab+\left(d^2g^4-2\,cdg^3h+c^2g^2h^2\right)b^2} + \frac{\cdots}{\left(dg^2h-c\cdots\right.}\right.}{2\,e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*(b*x+a)^n/((d*x+c)^n)))/(h*x+g)^3,x, algorithm="maxima
")
```

```
[Out] 1/2*(b^2*e*n*log(b*x + a)/(b^2*g^2*h - 2*a*b*g*h^2 + a^2*h^3) - d^2*e*n*log
(d*x + c)/(d^2*g^2*h - 2*c*d*g*h^2 + c^2*h^3) - (2*a*b*d^2*e*g*n - a^2*d^2*
e*h*n - (2*c*d*e*g*n - c^2*e*h*n)*b^2)*log(h*x + g)/((d^2*g^2*h^2 - 2*c*d*g
*h^3 + c^2*h^4)*a^2 - 2*(d^2*g^3*h - 2*c*d*g^2*h^2 + c^2*g*h^3)*a*b + (d^2*
g^4 - 2*c*d*g^3*h + c^2*g^2*h^2)*b^2) + (b*c*e*n - a*d*e*n)/((d*g^2*h - c*g
*h^2)*a - (d*g^3 - c*g^2*h)*b + ((d*g*h^2 - c*h^3)*a - (d*g^2*h - c*g*h^2)*
b)*x))*B/e - 1/2*B*log((b*x + a)^n*e/(d*x + c)^n)/(h^3*x^2 + 2*g*h^2*x + g^
2*h) - 1/2*A/(h^3*x^2 + 2*g*h^2*x + g^2*h)
```

**mupad [B]** time = 6.35, size = 431, normalized size = 2.26

$$\frac{\ln\left(g+h\,x\right)\,\left(h\,\left(B\,a^2\,d^2\,n-B\,b^2\,c^2\,n\right)-2\,B\,a\,b\,d^2\,g\,n+2\,B\,b^2\,c\,d\,g\,n\right)}{2\,a^2\,c^2\,h^4-4\,a^2\,c\,d\,g\,h^3+2\,a^2\,d^2\,g^2\,h^2-4\,a\,b\,c^2\,g\,h^3+8\,a\,b\,c\,d\,g^2\,h^2-4\,a\,b\,d^2\,g^3\,h+2\,b^2\,c^2\,g^2\,h^2-4\,b^2\,c\,d\,g^3\,h+2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*log((e*(a + b*x)^n)/(c + d*x)^n))/(g + h*x)^3,x)
```

```
[Out] (log(g + h*x)*(h*(B*a^2*d^2*n - B*b^2*c^2*n) - 2*B*a*b*d^2*g*n + 2*B*b^2*c*
d*g*n))/(2*a^2*c^2*h^4 + 2*b^2*d^2*g^4 + 2*a^2*d^2*g^2*h^2 + 2*b^2*c^2*g^2*
h^2 - 4*a*b*c^2*g*h^3 - 4*a*b*d^2*g^3*h - 4*a^2*c*d*g*h^3 - 4*b^2*c*d*g^3*h
 + 8*a*b*c*d*g^2*h^2) - ((A*a*c*h^2 + A*b*d*g^2 - A*a*d*g*h - A*b*c*g*h - B
*a*d*g*h*n + B*b*c*g*h*n)/(a*c*h^2 + b*d*g^2 - a*d*g*h - b*c*g*h) - (x*(B*a
*d*h^2*n - B*b*c*h^2*n))/(a*c*h^2 + b*d*g^2 - a*d*g*h - b*c*g*h))/(2*g^2*h
+ 2*h^3*x^2 + 4*g*h^2*x) - (B*log((e*(a + b*x)^n)/(c + d*x)^n))/(2*h*(g^2 +
 h^2*x^2 + 2*g*h*x)) + (B*b^2*n*log(a + b*x))/(2*a^2*h^3 + 2*b^2*g^2*h - 4*
a*b*g*h^2) - (B*d^2*n*log(c + d*x))/(2*c^2*h^3 + 2*d^2*g^2*h - 4*c*d*g*h^2)
```

**sympy [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*ln(e*(b*x+a)**n/((d*x+c)**n)))/(h*x+g)**3,x)
```

```
[Out] Timed out
```

# 3.301 $\int \frac{A+B\log(e(a+bx)^n(c+dx)^{-n})}{(g+hx)^4}\,dx$

**Optimal**. Leaf size=284

$$\frac{Bn(bc-ad)\log(g+hx)\left(a^2d^2h^2-abdh(3dg-ch)+b^2\left(c^2h^2-3cdgh+3d^2g^2\right)\right)}{3(bg-ah)^3(dg-ch)^3}-\frac{B\log\left(e(a+bx)^n(c+dx)^{-n}\right)+}{3h(g+hx)^3}$$

```
[Out] -1/6*B*(-a*d+b*c)*n/(-a*h+b*g)/(-c*h+d*g)/(h*x+g)^2-1/3*B*(-a*d+b*c)*(-a*d*
h-b*c*h+2*b*d*g)*n/(-a*h+b*g)^2/(-c*h+d*g)^2/(h*x+g)+1/3*b^3*B*n*ln(b*x+a)/
h/(-a*h+b*g)^3-1/3*B*d^3*n*ln(d*x+c)/h/(-c*h+d*g)^3+1/3*(-A-B*ln(e*(b*x+a)^
n/((d*x+c)^n)))/h/(h*x+g)^3+1/3*B*(-a*d+b*c)*(a^2*d^2*h^2-a*b*d*h*(-c*h+3*d
*g)+b^2*(c^2*h^2-3*c*d*g*h+3*d^2*g^2))*n*ln(h*x+g)/(-a*h+b*g)^3/(-c*h+d*g)^
3
```

**Rubi [A]** time = 0.54, antiderivative size = 296, normalized size of antiderivative = 1.04, number of steps used = 5, number of rules used = 3, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.097, Rules used = {6742, 2492, 72}

$$\frac{Bn(bc-ad)\log(g+hx)\left(a^2d^2h^2-abdh(3dg-ch)+b^2\left(c^2h^2-3cdgh+3d^2g^2\right)\right)}{3(bg-ah)^3(dg-ch)^3}+\frac{b^3Bn\log(a+bx)}{3h(bg-ah)^3}-\frac{B\log\left(e(a\right.}{3h}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])/(g + h*x)^4,x]
```

```
[Out] -A/(3*h*(g + h*x)^3) - (B*(b*c - a*d)*n)/(6*(b*g - a*h)*(d*g - c*h)*(g + h*
x)^2) - (B*(b*c - a*d)*(2*b*d*g - b*c*h - a*d*h)*n)/(3*(b*g - a*h)^2*(d*g -
 c*h)^2*(g + h*x)) + (b^3*B*n*Log[a + b*x])/(3*h*(b*g - a*h)^3) - (B*d^3*n*
Log[c + d*x])/(3*h*(d*g - c*h)^3) - (B*Log[(e*(a + b*x)^n)/(c + d*x)^n])/(3
*h*(g + h*x)^3) + (B*(b*c - a*d)*(a^2*d^2*h^2 - a*b*d*h*(3*d*g - c*h) + b^2
*(3*d^2*g^2 - 3*c*d*g*h + c^2*h^2))*n*Log[g + h*x])/(3*(b*g - a*h)^3*(d*g -
 c*h)^3)
```

Rule 72

```
Int[((e_.) + (f_.)*(x_))^(p_.)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
 x_Symbol] :> Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x]
 /; FreeQ[{a, b, c, d, e, f}, x] && IntegerQ[p]
```

Rule 2492

```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))
^(r_.)]^(s_.)*((g_.) + (h_.)*(x_))^(m_.), x_Symbol] :> Simp[((g + h*x)^(m +
 1)*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^s)/(h*(m + 1)), x] - Dist[(p*r*s*(
```

```
b*c - a*d))/(h*(m + 1)), Int[((g + h*x)^(m + 1)*Log[e*(f*(a + b*x)^p*(c + d
*x)^q)^r]^(s - 1))/((a + b*x)*(c + d*x)), x], x] /; FreeQ[{a, b, c, d, e, f
, g, h, m, p, q, r, s}, x] && NeQ[b*c - a*d, 0] && EqQ[p + q, 0] && IGtQ[s,
 0] && NeQ[m, -1]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\begin{aligned}\int \frac{A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)}{(g+hx)^4}\,dx &= \int \left(\frac{A}{(g+hx)^4}+\frac{B\log\left(e(a+bx)^n(c+dx)^{-n}\right)}{(g+hx)^4}\right)dx\\ &= -\frac{A}{3h(g+hx)^3}+B\int \frac{\log\left(e(a+bx)^n(c+dx)^{-n}\right)}{(g+hx)^4}\,dx\\ &= -\frac{A}{3h(g+hx)^3}-\frac{B\log\left(e(a+bx)^n(c+dx)^{-n}\right)}{3h(g+hx)^3}+\frac{(B(bc-ad)n)\int \frac{}{(a+bx)(c+}}{3h}\\ &= -\frac{A}{3h(g+hx)^3}-\frac{B\log\left(e(a+bx)^n(c+dx)^{-n}\right)}{3h(g+hx)^3}+\frac{(B(bc-ad)n)\int \left(\frac{}{(bc-ad)(}\right.}{}\\ &= -\frac{A}{3h(g+hx)^3}-\frac{B(bc-ad)n}{6(bg-ah)(dg-ch)(g+hx)^2}-\frac{B(bc-ad)(2bdg-bch}{3(bg-ah)^2(dg-ch)^2(}\end{aligned}$$

**Mathematica [A]** time = 1.14, size = 273, normalized size = 0.96

$$-\frac{Bn(bc-ad)\left(-\frac{2h\log(g+hx)\left(a^2d^2h^2+abdh(ch-3dg)+b^2\left(c^2h^2-3cdgh+3d^2g^2\right)\right)}{(bg-ah)^3(dg-ch)^3}-\frac{2b^3\log(a+bx)}{(bc-ad)(bg-ah)^3}+\frac{2d^3\log(c+dx)}{(bc-ad)(dg-ch)^3}-\frac{2h(adh+bch-2bdg}{(g+hx)(bg-ah)^2(dg-}\right.}{6h}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])/(g + h*x)^4,x]
```

```
[Out] -1/6*((2*A)/(g + h*x)^3 + (2*B*Log[(e*(a + b*x)^n)/(c + d*x)^n])/(g + h*x)^
3 + B*(b*c - a*d)*n*(h/((b*g - a*h)*(d*g - c*h)*(g + h*x)^2) - (2*h*(-2*b*d
*g + b*c*h + a*d*h))/((b*g - a*h)^2*(d*g - c*h)^2*(g + h*x)) - (2*b^3*Log[a
 + b*x])/((b*c - a*d)*(b*g - a*h)^3) + (2*d^3*Log[c + d*x])/((b*c - a*d)*(d
*g - c*h)^3) - (2*h*(a^2*d^2*h^2 + a*b*d*h*(-3*d*g + c*h) + b^2*(3*d^2*g^2
- 3*c*d*g*h + c^2*h^2))*Log[g + h*x])/((b*g - a*h)^3*(d*g - c*h)^3)))/h
```

**fricas [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*(b*x+a)^n/((d*x+c)^n)))/(h*x+g)^4,x, algorithm="fricas
")
```

```
[Out] Timed out
```

**giac [B]** time = 3.18, size = 1512, normalized size = 5.32

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*(b*x+a)^n/((d*x+c)^n)))/(h*x+g)^4,x, algorithm="giac")
```

```
[Out] 1/3*B*b^4*n*log(abs(b*x + a))/(b^4*g^3*h - 3*a*b^3*g^2*h^2 + 3*a^2*b^2*g*h^
3 - a^3*b*h^4) - 1/3*B*d^4*n*log(abs(d*x + c))/(d^4*g^3*h - 3*c*d^3*g^2*h^2
 + 3*c^2*d^2*g*h^3 - c^3*d*h^4) - 1/3*B*n*log(b*x + a)/(h^4*x^3 + 3*g*h^3*x
^2 + 3*g^2*h^2*x + g^3*h) + 1/3*B*n*log(d*x + c)/(h^4*x^3 + 3*g*h^3*x^2 + 3
*g^2*h^2*x + g^3*h) + 1/3*(3*B*b^3*c*d^2*g^2*n - 3*B*a*b^2*d^3*g^2*n - 3*B*
b^3*c^2*d*g*h*n + 3*B*a^2*b*d^3*g*h*n + B*b^3*c^3*h^2*n - B*a^3*d^3*h^2*n)*
log(h*x + g)/(b^3*d^3*g^6 - 3*b^3*c*d^2*g^5*h - 3*a*b^2*d^3*g^5*h + 3*b^3*c
^2*d*g^4*h^2 + 9*a*b^2*c*d^2*g^4*h^2 + 3*a^2*b*d^3*g^4*h^2 - b^3*c^3*g^3*h^
3 - 9*a*b^2*c^2*d*g^3*h^3 - 9*a^2*b*c*d^2*g^3*h^3 - a^3*d^3*g^3*h^3 + 3*a*b
^2*c^3*g^2*h^4 + 9*a^2*b*c^2*d*g^2*h^4 + 3*a^3*c*d^2*g^2*h^4 - 3*a^2*b*c^3*
g*h^5 - 3*a^3*c^2*d*g*h^5 + a^3*c^3*h^6) - 1/6*(4*B*b^2*c*d*g*h^3*n*x^2 - 4
*B*a*b*d^2*g*h^3*n*x^2 - 2*B*b^2*c^2*h^4*n*x^2 + 2*B*a^2*d^2*h^4*n*x^2 + 9*
B*b^2*c*d*g^2*h^2*n*x - 9*B*a*b*d^2*g^2*h^2*n*x - 5*B*b^2*c^2*g*h^3*n*x + 5
*B*a^2*d^2*g*h^3*n*x + B*a*b*c^2*h^4*n*x - B*a^2*c*d*h^4*n*x + 5*B*b^2*c*d*
g^3*h*n - 5*B*a*b*d^2*g^3*h*n - 3*B*b^2*c^2*g^2*h^2*n + 3*B*a^2*d^2*g^2*h^2
*n + B*a*b*c^2*g*h^3*n - B*a^2*c*d*g*h^3*n + 2*A*b^2*d^2*g^4 + 2*B*b^2*d^2*
g^4 - 4*A*b^2*c*d*g^3*h - 4*B*b^2*c*d*g^3*h - 4*A*a*b*d^2*g^3*h - 4*B*a*b*d
^2*g^3*h + 2*A*b^2*c^2*g^2*h^2 + 2*B*b^2*c^2*g^2*h^2 + 8*A*a*b*c*d*g^2*h^2
+ 8*B*a*b*c*d*g^2*h^2 + 2*A*a^2*d^2*g^2*h^2 + 2*B*a^2*d^2*g^2*h^2 - 4*A*a*b
*c^2*g*h^3 - 4*B*a*b*c^2*g*h^3 - 4*A*a^2*c*d*g*h^3 - 4*B*a^2*c*d*g*h^3 + 2*
A*a^2*c^2*h^4 + 2*B*a^2*c^2*h^4)/(b^2*d^2*g^4*h^4*x^3 - 2*b^2*c*d*g^3*h^5*x
^3 - 2*a*b*d^2*g^3*h^5*x^3 + b^2*c^2*g^2*h^6*x^3 + 4*a*b*c*d*g^2*h^6*x^3 +
a^2*d^2*g^2*h^6*x^3 - 2*a*b*c^2*g*h^7*x^3 - 2*a^2*c*d*g*h^7*x^3 + a^2*c^2*h
^8*x^3 + 3*b^2*d^2*g^5*h^3*x^2 - 6*b^2*c*d*g^4*h^4*x^2 - 6*a*b*d^2*g^4*h^4*
x^2 + 3*b^2*c^2*g^3*h^5*x^2 + 12*a*b*c*d*g^3*h^5*x^2 + 3*a^2*d^2*g^3*h^5*x^
2 - 6*a*b*c^2*g^2*h^6*x^2 - 6*a^2*c*d*g^2*h^6*x^2 + 3*a^2*c^2*g*h^7*x^2 + 3
*b^2*d^2*g^6*h^2*x - 6*b^2*c*d*g^5*h^3*x - 6*a*b*d^2*g^5*h^3*x + 3*b^2*c^2*
```

```
g^4*h^4*x + 12*a*b*c*d*g^4*h^4*x + 3*a^2*d^2*g^4*h^4*x - 6*a*b*c^2*g^3*h^5*
x - 6*a^2*c*d*g^3*h^5*x + 3*a^2*c^2*g^2*h^6*x + b^2*d^2*g^7*h - 2*b^2*c*d*g
^6*h^2 - 2*a*b*d^2*g^6*h^2 + b^2*c^2*g^5*h^3 + 4*a*b*c*d*g^5*h^3 + a^2*d^2*
g^5*h^3 - 2*a*b*c^2*g^4*h^4 - 2*a^2*c*d*g^4*h^4 + a^2*c^2*g^3*h^5)
```

**maple [C]** time = 1.19, size = 9645, normalized size = 33.96

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))/(h*x+g)^4,x)
```

```
[Out] result too large to display
```

**maxima [B]** time = 1.89, size = 920, normalized size = 3.24

$$\left(\frac{2\,b^3en\log(bx+a)}{b^3g^3h-3\,ab^2g^2h^2+3\,a^2bgh^3-a^3h^4}-\frac{2\,d^3en\log(dx+c)}{d^3g^3h-3\,cd^2g^2h^2+3\,c^2dgh^3-c^3h^4}+\frac{2\left(3\,ab^2d^3eg^2n-3\,a^2bd^3eghn+a^3d\right.}{\left(d^3g^3h^3-3\,cd^2g^2h^4+3\,c^2dgh^5-c^3h^6\right)a^3-3\left(d^3g^4h^2-3\,cd^2g^3h^3+3\,c^2dg^2h^4-c^3\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*(b*x+a)^n/((d*x+c)^n)))/(h*x+g)^4,x, algorithm="maxima
")
```

```
[Out] 1/6*(2*b^3*e*n*log(b*x + a)/(b^3*g^3*h - 3*a*b^2*g^2*h^2 + 3*a^2*b*g*h^3 -
a^3*h^4) - 2*d^3*e*n*log(d*x + c)/(d^3*g^3*h - 3*c*d^2*g^2*h^2 + 3*c^2*d*g*
h^3 - c^3*h^4) + 2*(3*a*b^2*d^3*e*g^2*n - 3*a^2*b*d^3*e*g*h*n + a^3*d^3*e*h
^2*n - (3*c*d^2*e*g^2*n - 3*c^2*d*e*g*h*n + c^3*e*h^2*n)*b^3)*log(h*x + g)/
((d^3*g^3*h^3 - 3*c*d^2*g^2*h^4 + 3*c^2*d*g*h^5 - c^3*h^6)*a^3 - 3*(d^3*g^4
*h^2 - 3*c*d^2*g^3*h^3 + 3*c^2*d*g^2*h^4 - c^3*g*h^5)*a^2*b + 3*(d^3*g^5*h
- 3*c*d^2*g^4*h^2 + 3*c^2*d*g^3*h^3 - c^3*g^2*h^4)*a*b^2 - (d^3*g^6 - 3*c*d
^2*g^5*h + 3*c^2*d*g^4*h^2 - c^3*g^3*h^3)*b^3) - ((3*d^2*e*g*h*n - c*d*e*h^
2*n)*a^2 - (5*d^2*e*g^2*n - c^2*e*h^2*n)*a*b + (5*c*d*e*g^2*n - 3*c^2*e*g*h
*n)*b^2 - 2*(2*a*b*d^2*e*g*h*n - a^2*d^2*e*h^2*n - (2*c*d*e*g*h*n - c^2*e*h
^2*n)*b^2)*x)/((d^2*g^4*h^2 - 2*c*d*g^3*h^3 + c^2*g^2*h^4)*a^2 - 2*(d^2*g^5
*h - 2*c*d*g^4*h^2 + c^2*g^3*h^3)*a*b + (d^2*g^6 - 2*c*d*g^5*h + c^2*g^4*h^
2)*b^2 + ((d^2*g^2*h^4 - 2*c*d*g*h^5 + c^2*h^6)*a^2 - 2*(d^2*g^3*h^3 - 2*c*
d*g^2*h^4 + c^2*g*h^5)*a*b + (d^2*g^4*h^2 - 2*c*d*g^3*h^3 + c^2*g^2*h^4)*b^
2)*x^2 + 2*((d^2*g^3*h^3 - 2*c*d*g^2*h^4 + c^2*g*h^5)*a^2 - 2*(d^2*g^4*h^2
- 2*c*d*g^3*h^3 + c^2*g^2*h^4)*a*b + (d^2*g^5*h - 2*c*d*g^4*h^2 + c^2*g^3*h
^3)*b^2)*x))*B/e - 1/3*B*log((b*x + a)^n*e/(d*x + c)^n)/(h^4*x^3 + 3*g*h^3*
x^2 + 3*g^2*h^2*x + g^3*h) - 1/3*A/(h^4*x^3 + 3*g*h^3*x^2 + 3*g^2*h^2*x + g
^3*h)
```

**mupad [B]** time = 9.26, size = 1183, normalized size = 4.17

$$\frac{B d^3 n \ln(c+dx)}{3c^3h^4 - 9c^2dgh^3 + 9cd^2g^2h^2 - 3d^3g^3h} - \frac{}{3a^3c^3h^6 - 9a^3c^2dgh^5 + 9a^3cd^2g^2h^4 - 3a^3d^3g^3h^3 - 9a^2bc^3g}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*log((e*(a + b*x)^n)/(c + d*x)^n))/(g + h*x)^4,x)
```

```
[Out] (B*d^3*n*log(c + d*x))/(3*c^3*h^4 - 3*d^3*g^3*h + 9*c*d^2*g^2*h^2 - 9*c^2*d
*g*h^3) - (log(g + h*x)*(h^2*(B*a^3*d^3*n - B*b^3*c^3*n) - h*(3*B*a^2*b*d^3
*g*n - 3*B*b^3*c^2*d*g*n) + 3*B*a*b^2*d^3*g^2*n - 3*B*b^3*c*d^2*g^2*n))/(3*
a^3*c^3*h^6 + 3*b^3*d^3*g^6 - 3*a^3*d^3*g^3*h^3 - 3*b^3*c^3*g^3*h^3 - 9*a^2
*b*c^3*g*h^5 - 9*a*b^2*d^3*g^5*h - 9*a^3*c^2*d*g*h^5 - 9*b^3*c*d^2*g^5*h +
9*a*b^2*c^3*g^2*h^4 + 9*a^2*b*d^3*g^4*h^2 + 9*a^3*c*d^2*g^2*h^4 + 9*b^3*c^2
*d*g^4*h^2 + 27*a*b^2*c*d^2*g^4*h^2 - 27*a*b^2*c^2*d*g^3*h^3 - 27*a^2*b*c*d
^2*g^3*h^3 + 27*a^2*b*c^2*d*g^2*h^4) - (B*log((e*(a + b*x)^n)/(c + d*x)^n))
/(3*h*(g^3 + h^3*x^3 + 3*g^2*h*x + 3*g*h^2*x^2)) - (B*b^3*n*log(a + b*x))/(
3*a^3*h^4 - 3*b^3*g^3*h + 9*a*b^2*g^2*h^2 - 9*a^2*b*g*h^3) - ((2*A*a^2*c^2*
h^4 + 2*A*b^2*d^2*g^4 + 2*A*a^2*d^2*g^2*h^2 + 2*A*b^2*c^2*g^2*h^2 + 3*B*a^2
*d^2*g^2*h^2*n - 3*B*b^2*c^2*g^2*h^2*n - 4*A*a*b*c^2*g*h^3 - 4*A*a*b*d^2*g^
3*h - 4*A*a^2*c*d*g*h^3 - 4*A*b^2*c*d*g^3*h + 8*A*a*b*c*d*g^2*h^2 + B*a*b*c
^2*g*h^3*n - 5*B*a*b*d^2*g^3*h*n - B*a^2*c*d*g*h^3*n + 5*B*b^2*c*d*g^3*h*n)
/(2*(a^2*c^2*h^4 + b^2*d^2*g^4 + a^2*d^2*g^2*h^2 + b^2*c^2*g^2*h^2 - 2*a*b*
c^2*g*h^3 - 2*a*b*d^2*g^3*h - 2*a^2*c*d*g*h^3 - 2*b^2*c*d*g^3*h + 4*a*b*c*d
*g^2*h^2)) + (x*(B*a*b*c^2*h^4*n - B*a^2*c*d*h^4*n + 5*B*a^2*d^2*g*h^3*n -
5*B*b^2*c^2*g*h^3*n - 9*B*a*b*d^2*g^2*h^2*n + 9*B*b^2*c*d*g^2*h^2*n))/(2*(a
^2*c^2*h^4 + b^2*d^2*g^4 + a^2*d^2*g^2*h^2 + b^2*c^2*g^2*h^2 - 2*a*b*c^2*g*
h^3 - 2*a*b*d^2*g^3*h - 2*a^2*c*d*g*h^3 - 2*b^2*c*d*g^3*h + 4*a*b*c*d*g^2*h
^2)) + (x^2*(B*a^2*d^2*h^4*n - B*b^2*c^2*h^4*n - 2*B*a*b*d^2*g*h^3*n + 2*B*
b^2*c*d*g*h^3*n))/(a^2*c^2*h^4 + b^2*d^2*g^4 + a^2*d^2*g^2*h^2 + b^2*c^2*g^
2*h^2 - 2*a*b*c^2*g*h^3 - 2*a*b*d^2*g^3*h - 2*a^2*c*d*g*h^3 - 2*b^2*c*d*g^3
*h + 4*a*b*c*d*g^2*h^2))/(3*g^3*h + 3*h^4*x^3 + 9*g^2*h^2*x + 9*g*h^3*x^2)
```

**sympy [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*ln(e*(b*x+a)**n/((d*x+c)**n)))/(h*x+g)**4,x)
```

```
[Out] Timed out
```

# 3.302 $\int \frac{A+B\log(e(a+bx)^n(c+dx)^{-n})}{(g+hx)^5}\,dx$

**Optimal**. Leaf size=389

$$-\frac{Bn(bc-ad)\left(a^2d^2h^2-abdh(3dg-ch)+b^2\left(c^2h^2-3cdgh+3d^2g^2\right)\right)}{4(g+hx)(bg-ah)^3(dg-ch)^3}-\frac{Bn(bc-ad)\log(g+hx)(-adh-bch+2bd}{4(bg}$$

```
[Out] -1/12*B*(-a*d+b*c)*n/(-a*h+b*g)/(-c*h+d*g)/(h*x+g)^3-1/8*B*(-a*d+b*c)*(-a*d
*h-b*c*h+2*b*d*g)*n/(-a*h+b*g)^2/(-c*h+d*g)^2/(h*x+g)^2-1/4*B*(-a*d+b*c)*(a
^2*d^2*h^2-a*b*d*h*(-c*h+3*d*g)+b^2*(c^2*h^2-3*c*d*g*h+3*d^2*g^2))*n/(-a*h+
b*g)^3/(-c*h+d*g)^3/(h*x+g)+1/4*b^4*B*n*ln(b*x+a)/h/(-a*h+b*g)^4-1/4*B*d^4*
n*ln(d*x+c)/h/(-c*h+d*g)^4+1/4*(-A-B*ln(e*(b*x+a)^n/((d*x+c)^n)))/h/(h*x+g)
^4-1/4*B*(-a*d+b*c)*(-a*d*h-b*c*h+2*b*d*g)*(2*a*b*d^2*g*h-a^2*d^2*h^2-b^2*(
c^2*h^2-2*c*d*g*h+2*d^2*g^2))*n*ln(h*x+g)/(-a*h+b*g)^4/(-c*h+d*g)^4
```

**Rubi [A]** time = 0.82, antiderivative size = 401, normalized size of antiderivative = 1.03, number of steps used = 5, number of rules used = 3, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.097, Rules used = {6742, 2492, 72}

$$-\frac{Bn(bc-ad)\left(a^2d^2h^2-abdh(3dg-ch)+b^2\left(c^2h^2-3cdgh+3d^2g^2\right)\right)}{4(g+hx)(bg-ah)^3(dg-ch)^3}-\frac{Bn(bc-ad)\log(g+hx)(-adh-bch+2bd}{4(b}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])/(g + h*x)^5,x]
```

```
[Out] -A/(4*h*(g + h*x)^4) - (B*(b*c - a*d)*n)/(12*(b*g - a*h)*(d*g - c*h)*(g + h
*x)^3) - (B*(b*c - a*d)*(2*b*d*g - b*c*h - a*d*h)*n)/(8*(b*g - a*h)^2*(d*g
- c*h)^2*(g + h*x)^2) - (B*(b*c - a*d)*(a^2*d^2*h^2 - a*b*d*h*(3*d*g - c*h)
 + b^2*(3*d^2*g^2 - 3*c*d*g*h + c^2*h^2))*n)/(4*(b*g - a*h)^3*(d*g - c*h)^3
*(g + h*x)) + (b^4*B*n*Log[a + b*x])/(4*h*(b*g - a*h)^4) - (B*d^4*n*Log[c +
 d*x])/(4*h*(d*g - c*h)^4) - (B*Log[(e*(a + b*x)^n)/(c + d*x)^n])/(4*h*(g +
 h*x)^4) - (B*(b*c - a*d)*(2*b*d*g - b*c*h - a*d*h)*(2*a*b*d^2*g*h - a^2*d^
2*h^2 - b^2*(2*d^2*g^2 - 2*c*d*g*h + c^2*h^2))*n*Log[g + h*x])/(4*(b*g - a*
h)^4*(d*g - c*h)^4)
```

Rule 72

```
Int[((e_.) + (f_.)*(x_))^(p_.)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
 x_Symbol] :> Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x]
 /; FreeQ[{a, b, c, d, e, f}, x] && IntegerQ[p]
```

Rule 2492

```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))
^(r_.)]^(s_.)*((g_.) + (h_.)*(x_))^(m_.), x_Symbol] :> Simp[((g + h*x)^(m +
 1)*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^s)/(h*(m + 1)), x] - Dist[(p*r*s*(
b*c - a*d))/(h*(m + 1)), Int[((g + h*x)^(m + 1)*Log[e*(f*(a + b*x)^p*(c + d
*x)^q)^r]^(s - 1))/((a + b*x)*(c + d*x)), x], x] /; FreeQ[{a, b, c, d, e, f
, g, h, m, p, q, r, s}, x] && NeQ[b*c - a*d, 0] && EqQ[p + q, 0] && IGtQ[s,
 0] && NeQ[m, -1]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\int \frac{A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)}{(g+hx)^5}\,dx = \int\left(\frac{A}{(g+hx)^5}+\frac{B\log\left(e(a+bx)^n(c+dx)^{-n}\right)}{(g+hx)^5}\right)dx$$

$$= -\frac{A}{4h(g+hx)^4}+B\int\frac{\log\left(e(a+bx)^n(c+dx)^{-n}\right)}{(g+hx)^5}\,dx$$

$$= -\frac{A}{4h(g+hx)^4}-\frac{B\log\left(e(a+bx)^n(c+dx)^{-n}\right)}{4h(g+hx)^4}+\frac{(B(bc-ad)n)\int\frac{\cdots}{(a+bx)(\cdots}}{4h}$$

$$= -\frac{A}{4h(g+hx)^4}-\frac{B\log\left(e(a+bx)^n(c+dx)^{-n}\right)}{4h(g+hx)^4}+\frac{(B(bc-ad)n)\int\left(\frac{\cdots}{(bc-a\cdots}\right.}{\cdots}$$

$$= -\frac{A}{4h(g+hx)^4}-\frac{B(bc-ad)n}{12(bg-ah)(dg-ch)(g+hx)^3}-\frac{B(bc-ad)(2bdg-b\cdots}{8(bg-ah)^2(dg-ch)\cdots}$$

**Mathematica [A]** time = 1.21, size = 366, normalized size = 0.94

$$-\frac{-Bn(bc-ad)\left(-\frac{h\left(a^2d^2h^2+abdh(ch-3dg)+b^2\left(c^2h^2-3cdgh+3d^2g^2\right)\right)}{(g+hx)(bg-ah)^3(dg-ch)^3}-\frac{h\log(g+hx)(adh+bch-2bdg)\left(a^2d^2h^2-2abd^2gh+b^2\left(c^2h^2-2cdgh+2d^2g^2\cdots\right.\right.}{(bg-ah)^4(dg-ch)^4}\cdots}{4\cdots}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])/(g + h*x)^5,x]
```

```
[Out] -1/4*(A/(g + h*x)^4 + (B*Log[(e*(a + b*x)^n)/(c + d*x)^n])/(g + h*x)^4 - B*
(b*c - a*d)*n*(-1/3*h/((b*g - a*h)*(d*g - c*h)*(g + h*x)^3) + (h*(-2*b*d*g
+ b*c*h + a*d*h))/(2*(b*g - a*h)^2*(d*g - c*h)^2*(g + h*x)^2) - (h*(a^2*d^2
```

```
*h^2 + a*b*d*h*(-3*d*g + c*h) + b^2*(3*d^2*g^2 - 3*c*d*g*h + c^2*h^2)))/((b
*g - a*h)^3*(d*g - c*h)^3*(g + h*x)) + (b^4*Log[a + b*x])/((b*c - a*d)*(b*g
 - a*h)^4) - (d^4*Log[c + d*x])/((b*c - a*d)*(d*g - c*h)^4) - (h*(-2*b*d*g
+ b*c*h + a*d*h)*(-2*a*b*d^2*g*h + a^2*d^2*h^2 + b^2*(2*d^2*g^2 - 2*c*d*g*h
 + c^2*h^2))*Log[g + h*x])/((b*g - a*h)^4*(d*g - c*h)^4)))/h
```

**fricas [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*(b*x+a)^n/((d*x+c)^n)))/(h*x+g)^5,x, algorithm="fricas
")
```

```
[Out] Timed out
```

**giac [B]** time = 15.66, size = 3293, normalized size = 8.47

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*(b*x+a)^n/((d*x+c)^n)))/(h*x+g)^5,x, algorithm="giac")
```

```
[Out] 1/4*B*b^5*n*log(abs(b*x + a))/(b^5*g^4*h - 4*a*b^4*g^3*h^2 + 6*a^2*b^3*g^2*
h^3 - 4*a^3*b^2*g*h^4 + a^4*b*h^5) - 1/4*B*d^5*n*log(abs(d*x + c))/(d^5*g^4
*h - 4*c*d^4*g^3*h^2 + 6*c^2*d^3*g^2*h^3 - 4*c^3*d^2*g*h^4 + c^4*d*h^5) - 1
/4*B*n*log(b*x + a)/(h^5*x^4 + 4*g*h^4*x^3 + 6*g^2*h^3*x^2 + 4*g^3*h^2*x +
g^4*h) + 1/4*B*n*log(d*x + c)/(h^5*x^4 + 4*g*h^4*x^3 + 6*g^2*h^3*x^2 + 4*g^
3*h^2*x + g^4*h) + 1/4*(4*B*b^4*c*d^3*g^3*n - 4*B*a*b^3*d^4*g^3*n - 6*B*b^4
*c^2*d^2*g^2*h*n + 6*B*a^2*b^2*d^4*g^2*h*n + 4*B*b^4*c^3*d*g*h^2*n - 4*B*a^
3*b*d^4*g*h^2*n - B*b^4*c^4*h^3*n + B*a^4*d^4*h^3*n)*log(h*x + g)/(b^4*d^4*
g^8 - 4*b^4*c*d^3*g^7*h - 4*a*b^3*d^4*g^7*h + 6*b^4*c^2*d^2*g^6*h^2 + 16*a*
b^3*c*d^3*g^6*h^2 + 6*a^2*b^2*d^4*g^6*h^2 - 4*b^4*c^3*d*g^5*h^3 - 24*a*b^3*
c^2*d^2*g^5*h^3 - 24*a^2*b^2*c*d^3*g^5*h^3 - 4*a^3*b*d^4*g^5*h^3 + b^4*c^4*
g^4*h^4 + 16*a*b^3*c^3*d*g^4*h^4 + 36*a^2*b^2*c^2*d^2*g^4*h^4 + 16*a^3*b*c*
d^3*g^4*h^4 + a^4*d^4*g^4*h^4 - 4*a*b^3*c^4*g^3*h^5 - 24*a^2*b^2*c^3*d*g^3*
h^5 - 24*a^3*b*c^2*d^2*g^3*h^5 - 4*a^4*c*d^3*g^3*h^5 + 6*a^2*b^2*c^4*g^2*h^
6 + 16*a^3*b*c^3*d*g^2*h^6 + 6*a^4*c^2*d^2*g^2*h^6 - 4*a^3*b*c^4*g*h^7 - 4*
a^4*c^3*d*g*h^7 + a^4*c^4*h^8) - 1/24*(18*B*b^3*c*d^2*g^2*h^4*n*x^3 - 18*B*
a*b^2*d^3*g^2*h^4*n*x^3 - 18*B*b^3*c^2*d*g*h^5*n*x^3 + 18*B*a^2*b*d^3*g*h^5
*n*x^3 + 6*B*b^3*c^3*h^6*n*x^3 - 6*B*a^3*d^3*h^6*n*x^3 + 60*B*b^3*c*d^2*g^3
*h^3*n*x^2 - 60*B*a*b^2*d^3*g^3*h^3*n*x^2 - 63*B*b^3*c^2*d*g^2*h^4*n*x^2 +
63*B*a^2*b*d^3*g^2*h^4*n*x^2 + 21*B*b^3*c^3*g*h^5*n*x^2 + 9*B*a*b^2*c^2*d*g
*h^5*n*x^2 - 9*B*a^2*b*c*d^2*g*h^5*n*x^2 - 21*B*a^3*d^3*g*h^5*n*x^2 - 3*B*a
*b^2*c^3*h^6*n*x^2 + 3*B*a^3*c*d^2*h^6*n*x^2 + 68*B*b^3*c*d^2*g^4*h^2*n*x -
```

68*B*a*b^2*d^3*g^4*h^2*n*x - 76*B*b^3*c^2*d*g^3*h^3*n*x + 76*B*a^2*b*d^3*g
^3*h^3*n*x + 26*B*b^3*c^3*g^2*h^4*n*x + 24*B*a*b^2*c^2*d*g^2*h^4*n*x - 24*B
*a^2*b*c*d^2*g^2*h^4*n*x - 26*B*a^3*d^3*g^2*h^4*n*x - 10*B*a*b^2*c^3*g*h^5*
n*x + 10*B*a^3*c*d^2*g*h^5*n*x + 2*B*a^2*b*c^3*h^6*n*x - 2*B*a^3*c^2*d*h^6*
n*x + 26*B*b^3*c*d^2*g^5*h*n - 26*B*a*b^2*d^3*g^5*h*n - 31*B*b^3*c^2*d*g^4*
h^2*n + 31*B*a^2*b*d^3*g^4*h^2*n + 11*B*b^3*c^3*g^3*h^3*n + 15*B*a*b^2*c^2*
d*g^3*h^3*n - 15*B*a^2*b*c*d^2*g^3*h^3*n - 11*B*a^3*d^3*g^3*h^3*n - 7*B*a*b
^2*c^3*g^2*h^4*n + 7*B*a^3*c*d^2*g^2*h^4*n + 2*B*a^2*b*c^3*g*h^5*n - 2*B*a^
3*c^2*d*g*h^5*n + 6*A*b^3*d^3*g^6 + 6*B*b^3*d^3*g^6 - 18*A*b^3*c*d^2*g^5*h
- 18*B*b^3*c*d^2*g^5*h - 18*A*a*b^2*d^3*g^5*h - 18*B*a*b^2*d^3*g^5*h + 18*A
*b^3*c^2*d*g^4*h^2 + 18*B*b^3*c^2*d*g^4*h^2 + 54*A*a*b^2*c*d^2*g^4*h^2 + 54
*B*a*b^2*c*d^2*g^4*h^2 + 18*A*a^2*b*d^3*g^4*h^2 + 18*B*a^2*b*d^3*g^4*h^2 -
6*A*b^3*c^3*g^3*h^3 - 6*B*b^3*c^3*g^3*h^3 - 54*A*a*b^2*c^2*d*g^3*h^3 - 54*B
*a*b^2*c^2*d*g^3*h^3 - 54*A*a^2*b*c*d^2*g^3*h^3 - 54*B*a^2*b*c*d^2*g^3*h^3
- 6*A*a^3*d^3*g^3*h^3 - 6*B*a^3*d^3*g^3*h^3 + 18*A*a*b^2*c^3*g^2*h^4 + 18*B
*a*b^2*c^3*g^2*h^4 + 54*A*a^2*b*c^2*d*g^2*h^4 + 54*B*a^2*b*c^2*d*g^2*h^4 +
18*A*a^3*c*d^2*g^2*h^4 + 18*B*a^3*c*d^2*g^2*h^4 - 18*A*a^2*b*c^3*g*h^5 - 18
*B*a^2*b*c^3*g*h^5 - 18*A*a^3*c^2*d*g*h^5 - 18*B*a^3*c^2*d*g*h^5 + 6*A*a^3*
c^3*h^6 + 6*B*a^3*c^3*h^6)/(b^3*d^3*g^6*h^5*x^4 - 3*b^3*c*d^2*g^5*h^6*x^4 -
3*a*b^2*d^3*g^5*h^6*x^4 + 3*b^3*c^2*d*g^4*h^7*x^4 + 9*a*b^2*c*d^2*g^4*h^7*
x^4 + 3*a^2*b*d^3*g^4*h^7*x^4 - b^3*c^3*g^3*h^8*x^4 - 9*a*b^2*c^2*d*g^3*h^8
*x^4 - 9*a^2*b*c*d^2*g^3*h^8*x^4 - a^3*d^3*g^3*h^8*x^4 + 3*a*b^2*c^3*g^2*h^
9*x^4 + 9*a^2*b*c^2*d*g^2*h^9*x^4 + 3*a^3*c*d^2*g^2*h^9*x^4 - 3*a^2*b*c^3*g
*h^10*x^4 - 3*a^3*c^2*d*g*h^10*x^4 + a^3*c^3*h^11*x^4 + 4*b^3*d^3*g^7*h^4*x
^3 - 12*b^3*c*d^2*g^6*h^5*x^3 - 12*a*b^2*d^3*g^6*h^5*x^3 + 12*b^3*c^2*d*g^5
*h^6*x^3 + 36*a*b^2*c*d^2*g^5*h^6*x^3 + 12*a^2*b*d^3*g^5*h^6*x^3 - 4*b^3*c^
3*g^4*h^7*x^3 - 36*a*b^2*c^2*d*g^4*h^7*x^3 - 36*a^2*b*c*d^2*g^4*h^7*x^3 - 4
*a^3*d^3*g^4*h^7*x^3 + 12*a*b^2*c^3*g^3*h^8*x^3 + 36*a^2*b*c^2*d*g^3*h^8*x^
3 + 12*a^3*c*d^2*g^3*h^8*x^3 - 12*a^2*b*c^3*g^2*h^9*x^3 - 12*a^3*c^2*d*g^2*
h^9*x^3 + 4*a^3*c^3*g*h^10*x^3 + 6*b^3*d^3*g^8*h^3*x^2 - 18*b^3*c*d^2*g^7*h
^4*x^2 - 18*a*b^2*d^3*g^7*h^4*x^2 + 18*b^3*c^2*d*g^6*h^5*x^2 + 54*a*b^2*c*d
^2*g^6*h^5*x^2 + 18*a^2*b*d^3*g^6*h^5*x^2 - 6*b^3*c^3*g^5*h^6*x^2 - 54*a*b^
2*c^2*d*g^5*h^6*x^2 - 54*a^2*b*c*d^2*g^5*h^6*x^2 - 6*a^3*d^3*g^5*h^6*x^2 +
18*a*b^2*c^3*g^4*h^7*x^2 + 54*a^2*b*c^2*d*g^4*h^7*x^2 + 18*a^3*c*d^2*g^4*h^
7*x^2 - 18*a^2*b*c^3*g^3*h^8*x^2 - 18*a^3*c^2*d*g^3*h^8*x^2 + 6*a^3*c^3*g^2
*h^9*x^2 + 4*b^3*d^3*g^9*h^2*x - 12*b^3*c*d^2*g^8*h^3*x - 12*a*b^2*d^3*g^8*
h^3*x + 12*b^3*c^2*d*g^7*h^4*x + 36*a*b^2*c*d^2*g^7*h^4*x + 12*a^2*b*d^3*g^
7*h^4*x - 4*b^3*c^3*g^6*h^5*x - 36*a*b^2*c^2*d*g^6*h^5*x - 36*a^2*b*c*d^2*g
^6*h^5*x - 4*a^3*d^3*g^6*h^5*x + 12*a*b^2*c^3*g^5*h^6*x + 36*a^2*b*c^2*d*g^
5*h^6*x + 12*a^3*c*d^2*g^5*h^6*x - 12*a^2*b*c^3*g^4*h^7*x - 12*a^3*c^2*d*g^
4*h^7*x + 4*a^3*c^3*g^3*h^8*x + b^3*d^3*g^10*h - 3*b^3*c*d^2*g^9*h^2 - 3*a*
b^2*d^3*g^9*h^2 + 3*b^3*c^2*d*g^8*h^3 + 9*a*b^2*c*d^2*g^8*h^3 + 3*a^2*b*d^3
*g^8*h^3 - b^3*c^3*g^7*h^4 - 9*a*b^2*c^2*d*g^7*h^4 - 9*a^2*b*c*d^2*g^7*h^4
- a^3*d^3*g^7*h^4 + 3*a*b^2*c^3*g^6*h^5 + 9*a^2*b*c^2*d*g^6*h^5 + 3*a^3*c*d
^2*g^6*h^5 - 3*a^2*b*c^3*g^5*h^6 - 3*a^3*c^2*d*g^5*h^6 + a^3*c^3*g^4*h^7)

**maple [C]** time = 1.73, size = 16077, normalized size = 41.33

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))/(h*x+g)^5,x)
```

```
[Out] result too large to display
```

**maxima [B]** time = 2.97, size = 1912, normalized size = 4.92

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*(b*x+a)^n/((d*x+c)^n)))/(h*x+g)^5,x, algorithm="maxima
")
```

```
[Out] 1/24*(6*b^4*e*n*log(b*x + a)/(b^4*g^4*h - 4*a*b^3*g^3*h^2 + 6*a^2*b^2*g^2*h
^3 - 4*a^3*b*g*h^4 + a^4*h^5) - 6*d^4*e*n*log(d*x + c)/(d^4*g^4*h - 4*c*d^3
*g^3*h^2 + 6*c^2*d^2*g^2*h^3 - 4*c^3*d*g*h^4 + c^4*h^5) - 6*(4*a*b^3*d^4*e*
g^3*n - 6*a^2*b^2*d^4*e*g^2*h*n + 4*a^3*b*d^4*e*g*h^2*n - a^4*d^4*e*h^3*n -
 (4*c*d^3*e*g^3*n - 6*c^2*d^2*e*g^2*h*n + 4*c^3*d*e*g*h^2*n - c^4*e*h^3*n)*
b^4)*log(h*x + g)/((d^4*g^4*h^4 - 4*c*d^3*g^3*h^5 + 6*c^2*d^2*g^2*h^6 - 4*c
^3*d*g*h^7 + c^4*h^8)*a^4 - 4*(d^4*g^5*h^3 - 4*c*d^3*g^4*h^4 + 6*c^2*d^2*g^
3*h^5 - 4*c^3*d*g^2*h^6 + c^4*g*h^7)*a^3*b + 6*(d^4*g^6*h^2 - 4*c*d^3*g^5*h
^3 + 6*c^2*d^2*g^4*h^4 - 4*c^3*d*g^3*h^5 + c^4*g^2*h^6)*a^2*b^2 - 4*(d^4*g^
7*h - 4*c*d^3*g^6*h^2 + 6*c^2*d^2*g^5*h^3 - 4*c^3*d*g^4*h^4 + c^4*g^3*h^5)*
a*b^3 + (d^4*g^8 - 4*c*d^3*g^7*h + 6*c^2*d^2*g^6*h^2 - 4*c^3*d*g^5*h^3 + c^
4*g^4*h^4)*b^4) - ((11*d^3*e*g^2*h^2*n - 7*c*d^2*e*g*h^3*n + 2*c^2*d*e*h^4*
n)*a^3 - (31*d^3*e*g^3*h*n - 15*c*d^2*e*g^2*h^2*n + 2*c^3*e*h^4*n)*a^2*b +
(26*d^3*e*g^4*n - 15*c^2*d*e*g^2*h^2*n + 7*c^3*e*g*h^3*n)*a*b^2 - (26*c*d^2
*e*g^4*n - 31*c^2*d*e*g^3*h*n + 11*c^3*e*g^2*h^2*n)*b^3 + 6*(3*a*b^2*d^3*e*
g^2*h^2*n - 3*a^2*b*d^3*e*g*h^3*n + a^3*d^3*e*h^4*n - (3*c*d^2*e*g^2*h^2*n
- 3*c^2*d*e*g*h^3*n + c^3*e*h^4*n)*b^3)*x^2 + 3*((5*d^3*e*g*h^3*n - c*d^2*e
*h^4*n)*a^3 - 3*(5*d^3*e*g^2*h^2*n - c*d^2*e*g*h^3*n)*a^2*b + (14*d^3*e*g^3
*h*n - 3*c^2*d*e*g*h^3*n + c^3*e*h^4*n)*a*b^2 - (14*c*d^2*e*g^3*h*n - 15*c^
2*d*e*g^2*h^2*n + 5*c^3*e*g*h^3*n)*b^3)*x)/((d^3*g^6*h^3 - 3*c*d^2*g^5*h^4
+ 3*c^2*d*g^4*h^5 - c^3*g^3*h^6)*a^3 - 3*(d^3*g^7*h^2 - 3*c*d^2*g^6*h^3 + 3
*c^2*d*g^5*h^4 - c^3*g^4*h^5)*a^2*b + 3*(d^3*g^8*h - 3*c*d^2*g^7*h^2 + 3*c^
2*d*g^6*h^3 - c^3*g^5*h^4)*a*b^2 - (d^3*g^9 - 3*c*d^2*g^8*h + 3*c^2*d*g^7*h
^2 - c^3*g^6*h^3)*b^3 + ((d^3*g^3*h^6 - 3*c*d^2*g^2*h^7 + 3*c^2*d*g*h^8 - c
^3*h^9)*a^3 - 3*(d^3*g^4*h^5 - 3*c*d^2*g^3*h^6 + 3*c^2*d*g^2*h^7 - c^3*g*h^
8)*a^2*b + 3*(d^3*g^5*h^4 - 3*c*d^2*g^4*h^5 + 3*c^2*d*g^3*h^6 - c^3*g^2*h^7
)*a*b^2 - (d^3*g^6*h^3 - 3*c*d^2*g^5*h^4 + 3*c^2*d*g^4*h^5 - c^3*g^3*h^6)*b
```

```
^3)*x^3 + 3*((d^3*g^4*h^5 - 3*c*d^2*g^3*h^6 + 3*c^2*d*g^2*h^7 - c^3*g*h^8)*
a^3 - 3*(d^3*g^5*h^4 - 3*c*d^2*g^4*h^5 + 3*c^2*d*g^3*h^6 - c^3*g^2*h^7)*a^2
*b + 3*(d^3*g^6*h^3 - 3*c*d^2*g^5*h^4 + 3*c^2*d*g^4*h^5 - c^3*g^3*h^6)*a*b^
2 - (d^3*g^7*h^2 - 3*c*d^2*g^6*h^3 + 3*c^2*d*g^5*h^4 - c^3*g^4*h^5)*b^3)*x^
2 + 3*((d^3*g^5*h^4 - 3*c*d^2*g^4*h^5 + 3*c^2*d*g^3*h^6 - c^3*g^2*h^7)*a^3
- 3*(d^3*g^6*h^3 - 3*c*d^2*g^5*h^4 + 3*c^2*d*g^4*h^5 - c^3*g^3*h^6)*a^2*b +
 3*(d^3*g^7*h^2 - 3*c*d^2*g^6*h^3 + 3*c^2*d*g^5*h^4 - c^3*g^4*h^5)*a*b^2 -
(d^3*g^8*h - 3*c*d^2*g^7*h^2 + 3*c^2*d*g^6*h^3 - c^3*g^5*h^4)*b^3)*x))*B/e
- 1/4*B*log((b*x + a)^n*e/(d*x + c)^n)/(h^5*x^4 + 4*g*h^4*x^3 + 6*g^2*h^3*x
^2 + 4*g^3*h^2*x + g^4*h) - 1/4*A/(h^5*x^4 + 4*g*h^4*x^3 + 6*g^2*h^3*x^2 +
4*g^3*h^2*x + g^4*h)
```

**mupad [B]** time = 14.28, size = 2570, normalized size = 6.61

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*log((e*(a + b*x)^n)/(c + d*x)^n))/(g + h*x)^5,x)
```

```
[Out] ((x*(13*B*a^3*d^3*g^2*h^4*n - 13*B*b^3*c^3*g^2*h^4*n - B*a^2*b*c^3*h^6*n +
B*a^3*c^2*d*h^6*n + 5*B*a*b^2*c^3*g*h^5*n - 5*B*a^3*c*d^2*g*h^5*n + 34*B*a*
b^2*d^3*g^4*h^2*n - 38*B*a^2*b*d^3*g^3*h^3*n - 34*B*b^3*c*d^2*g^4*h^2*n + 3
8*B*b^3*c^2*d*g^3*h^3*n - 12*B*a*b^2*c^2*d*g^2*h^4*n + 12*B*a^2*b*c*d^2*g^2
*h^4*n))/(3*(a^3*c^3*h^6 + b^3*d^3*g^6 - a^3*d^3*g^3*h^3 - b^3*c^3*g^3*h^3
- 3*a^2*b*c^3*g*h^5 - 3*a*b^2*d^3*g^5*h - 3*a^3*c^2*d*g*h^5 - 3*b^3*c*d^2*g
^5*h + 3*a*b^2*c^3*g^2*h^4 + 3*a^2*b*d^3*g^4*h^2 + 3*a^3*c*d^2*g^2*h^4 + 3*
b^3*c^2*d*g^4*h^2 + 9*a*b^2*c*d^2*g^4*h^2 - 9*a*b^2*c^2*d*g^3*h^3 - 9*a^2*b
*c*d^2*g^3*h^3 + 9*a^2*b*c^2*d*g^2*h^4)) - (6*A*a^3*c^3*h^6 + 6*A*b^3*d^3*g
^6 - 6*A*a^3*d^3*g^3*h^3 - 6*A*b^3*c^3*g^3*h^3 + 18*A*a*b^2*c^3*g^2*h^4 + 1
8*A*a^2*b*d^3*g^4*h^2 + 18*A*a^3*c*d^2*g^2*h^4 + 18*A*b^3*c^2*d*g^4*h^2 - 1
1*B*a^3*d^3*g^3*h^3*n + 11*B*b^3*c^3*g^3*h^3*n - 18*A*a^2*b*c^3*g*h^5 - 18*
A*a*b^2*d^3*g^5*h - 18*A*a^3*c^2*d*g*h^5 - 18*A*b^3*c*d^2*g^5*h + 2*B*a^2*b
*c^3*g*h^5*n - 26*B*a*b^2*d^3*g^5*h*n - 2*B*a^3*c^2*d*g*h^5*n + 26*B*b^3*c*
d^2*g^5*h*n + 54*A*a*b^2*c*d^2*g^4*h^2 - 54*A*a*b^2*c^2*d*g^3*h^3 - 54*A*a^
2*b*c*d^2*g^3*h^3 + 54*A*a^2*b*c^2*d*g^2*h^4 - 7*B*a*b^2*c^3*g^2*h^4*n + 31
*B*a^2*b*d^3*g^4*h^2*n + 7*B*a^3*c*d^2*g^2*h^4*n - 31*B*b^3*c^2*d*g^4*h^2*n
 + 15*B*a*b^2*c^2*d*g^3*h^3*n - 15*B*a^2*b*c*d^2*g^3*h^3*n)/(6*(a^3*c^3*h^6
 + b^3*d^3*g^6 - a^3*d^3*g^3*h^3 - b^3*c^3*g^3*h^3 - 3*a^2*b*c^3*g*h^5 - 3*
a*b^2*d^3*g^5*h - 3*a^3*c^2*d*g*h^5 - 3*b^3*c*d^2*g^5*h + 3*a*b^2*c^3*g^2*h
^4 + 3*a^2*b*d^3*g^4*h^2 + 3*a^3*c*d^2*g^2*h^4 + 3*b^3*c^2*d*g^4*h^2 + 9*a*
b^2*c*d^2*g^4*h^2 - 9*a*b^2*c^2*d*g^3*h^3 - 9*a^2*b*c*d^2*g^3*h^3 + 9*a^2*b
*c^2*d*g^2*h^4)) + (x^3*(B*a^3*d^3*h^6*n - B*b^3*c^3*h^6*n - 3*B*a^2*b*d^3*
g*h^5*n + 3*B*b^3*c^2*d*g*h^5*n + 3*B*a*b^2*d^3*g^2*h^4*n - 3*B*b^3*c*d^2*g
^2*h^4*n))/(a^3*c^3*h^6 + b^3*d^3*g^6 - a^3*d^3*g^3*h^3 - b^3*c^3*g^3*h^3 -
 3*a^2*b*c^3*g*h^5 - 3*a*b^2*d^3*g^5*h - 3*a^3*c^2*d*g*h^5 - 3*b^3*c*d^2*g^
```

```
5*h + 3*a*b^2*c^3*g^2*h^4 + 3*a^2*b*d^3*g^4*h^2 + 3*a^3*c*d^2*g^2*h^4 + 3*b
^3*c^2*d*g^4*h^2 + 9*a*b^2*c*d^2*g^4*h^2 - 9*a*b^2*c^2*d*g^3*h^3 - 9*a^2*b*
c*d^2*g^3*h^3 + 9*a^2*b*c^2*d*g^2*h^4) + (x^2*(B*a*b^2*c^3*h^6*n - B*a^3*c*
d^2*h^6*n + 7*B*a^3*d^3*g*h^5*n - 7*B*b^3*c^3*g*h^5*n + 20*B*a*b^2*d^3*g^3*
h^3*n - 21*B*a^2*b*d^3*g^2*h^4*n - 20*B*b^3*c*d^2*g^3*h^3*n + 21*B*b^3*c^2*
d*g^2*h^4*n - 3*B*a*b^2*c^2*d*g*h^5*n + 3*B*a^2*b*c*d^2*g*h^5*n))/(2*(a^3*c
^3*h^6 + b^3*d^3*g^6 - a^3*d^3*g^3*h^3 - b^3*c^3*g^3*h^3 - 3*a^2*b*c^3*g*h^
5 - 3*a*b^2*d^3*g^5*h - 3*a^3*c^2*d*g*h^5 - 3*b^3*c*d^2*g^5*h + 3*a*b^2*c^3
*g^2*h^4 + 3*a^2*b*d^3*g^4*h^2 + 3*a^3*c*d^2*g^2*h^4 + 3*b^3*c^2*d*g^4*h^2
+ 9*a*b^2*c*d^2*g^4*h^2 - 9*a*b^2*c^2*d*g^3*h^3 - 9*a^2*b*c*d^2*g^3*h^3 + 9
*a^2*b*c^2*d*g^2*h^4)))/(4*g^4*h + 4*h^5*x^4 + 16*g^3*h^2*x + 16*g*h^4*x^3
+ 24*g^2*h^3*x^2) + (log(g + h*x)*(h*(6*B*a^2*b^2*d^4*g^2*n - 6*B*b^4*c^2*d
^2*g^2*n) - h^2*(4*B*a^3*b*d^4*g*n - 4*B*b^4*c^3*d*g*n) + h^3*(B*a^4*d^4*n
- B*b^4*c^4*n) - 4*B*a*b^3*d^4*g^3*n + 4*B*b^4*c*d^3*g^3*n))/(4*a^4*c^4*h^8
 + 4*b^4*d^4*g^8 + 4*a^4*d^4*g^4*h^4 + 4*b^4*c^4*g^4*h^4 + 24*a^2*b^2*c^4*g
^2*h^6 + 24*a^2*b^2*d^4*g^6*h^2 + 24*a^4*c^2*d^2*g^2*h^6 + 24*b^4*c^2*d^2*g
^6*h^2 - 16*a^3*b*c^4*g*h^7 - 16*a*b^3*d^4*g^7*h - 16*a^4*c^3*d*g*h^7 - 16*
b^4*c*d^3*g^7*h - 16*a*b^3*c^4*g^3*h^5 - 16*a^3*b*d^4*g^5*h^3 - 16*a^4*c*d^
3*g^3*h^5 - 16*b^4*c^3*d*g^5*h^3 + 64*a*b^3*c*d^3*g^6*h^2 + 64*a*b^3*c^3*d*
g^4*h^4 + 64*a^3*b*c*d^3*g^4*h^4 + 64*a^3*b*c^3*d*g^2*h^6 - 96*a*b^3*c^2*d^
2*g^5*h^3 - 96*a^2*b^2*c*d^3*g^5*h^3 - 96*a^2*b^2*c^3*d*g^3*h^5 - 96*a^3*b*
c^2*d^2*g^3*h^5 + 144*a^2*b^2*c^2*d^2*g^4*h^4) - (B*log((e*(a + b*x)^n)/(c
+ d*x)^n))/(4*h*(g^4 + h^4*x^4 + 4*g^3*h*x + 4*g*h^3*x^3 + 6*g^2*h^2*x^2))
+ (B*b^4*n*log(a + b*x))/(4*a^4*h^5 + 4*b^4*g^4*h - 16*a*b^3*g^3*h^2 + 24*a
^2*b^2*g^2*h^3 - 16*a^3*b*g*h^4) - (B*d^4*n*log(c + d*x))/(4*c^4*h^5 + 4*d^
4*g^4*h - 16*c*d^3*g^3*h^2 + 24*c^2*d^2*g^2*h^3 - 16*c^3*d*g*h^4)
```

**sympy [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*ln(e*(b*x+a)**n/((d*x+c)**n)))/(h*x+g)**5,x)
```

```
[Out] Timed out
```

# 3.303 $\int (g+hx)^2 \left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^2 dx$

**Optimal**. Leaf size=570

$$\frac{2Bn(bc-ad)\left(a^2d^2h^2-abdh(3dg-ch)+b^2\left(c^2h^2-3cdgh+3d^2g^2\right)\right)\log\left(\frac{bc-ad}{b(c+dx)}\right)\left(B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right.}{3b^3d^3}$$

```
[Out] 1/3*B^2*(-a*d+b*c)^2*h^2*n^2*x/b^2/d^2+1/3*B^2*(-a*d+b*c)^3*h^2*n^2*ln((b*x
+a)/(d*x+c))/b^3/d^3+1/3*B^2*(-a*d+b*c)^3*h^2*n^2*ln(d*x+c)/b^3/d^3+2/3*B^2
*(-a*d+b*c)^2*h*(-a*d*h-2*b*c*h+3*b*d*g)*n^2*ln(d*x+c)/b^3/d^3-2/3*B*(-a*d+
b*c)*h*(-a*d*h-2*b*c*h+3*b*d*g)*n*(b*x+a)*(A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))
/b^3/d^2-1/3*B*(-a*d+b*c)*h^2*n*(d*x+c)^2*(A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))
/b/d^3+2/3*B*(-a*d+b*c)*(a^2*d^2*h^2-a*b*d*h*(-c*h+3*d*g)+b^2*(c^2*h^2-3*c*
d*g*h+3*d^2*g^2))*n*ln((-a*d+b*c)/b/(d*x+c))*(A+B*ln(e*(b*x+a)^n/((d*x+c)^n
)))/b^3/d^3-1/3*(-a*h+b*g)^3*(A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))^2/b^3/h+1/3*
(h*x+g)^3*(A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))^2/h+2/3*B^2*(-a*d+b*c)*(a^2*d^2
*h^2-a*b*d*h*(-c*h+3*d*g)+b^2*(c^2*h^2-3*c*d*g*h+3*d^2*g^2))*n^2*polylog(2,
d*(b*x+a)/b/(d*x+c))/b^3/d^3
```

**Rubi [A]** time = 1.29, antiderivative size = 697, normalized size of antiderivative = 1.22, number of steps used = 23, number of rules used = 11, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {6742, 2492, 72, 2514, 2486, 31, 2488, 2411, 2343, 2333, 2315}

$$-\frac{2B^2n^2(bg-ah)^3\text{PolyLog}\left(2,\frac{bc-ad}{d(a+bx)}+1\right)}{3b^3h}-\frac{2B^2n^2(dg-ch)^3\text{PolyLog}\left(2,\frac{d(a+bx)}{b(c+dx)}\right)}{3d^3h}+\frac{a^2B^2h^2n^2(bc-ad)\log(a+b}{3b^3d}$$

Antiderivative was successfully verified.

```
[In] Int[(g + h*x)^2*(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])^2,x]
```

```
[Out] (-2*A*B*(b*c - a*d)*h*(3*b*d*g - b*c*h - a*d*h)*n*x)/(3*b^2*d^2) + (B^2*(b*
c - a*d)^2*h^2*n^2*x)/(3*b^2*d^2) - (A*B*(b*c - a*d)*h^2*n*x^2)/(3*b*d) + (
A^2*(g + h*x)^3)/(3*h) - (2*A*B*(b*g - a*h)^3*n*Log[a + b*x])/(3*b^3*h) + (
a^2*B^2*(b*c - a*d)*h^2*n^2*Log[a + b*x])/(3*b^3*d) + (2*A*B*(d*g - c*h)^3*
n*Log[c + d*x])/(3*d^3*h) - (B^2*c^2*(b*c - a*d)*h^2*n^2*Log[c + d*x])/(3*b
*d^3) + (2*B^2*(b*c - a*d)^2*h*(3*b*d*g - b*c*h - a*d*h)*n^2*Log[c + d*x])/
(3*b^3*d^3) - (B^2*(b*c - a*d)*h^2*n*x^2*Log[(e*(a + b*x)^n)/(c + d*x)^n])/
(3*b*d) - (2*B^2*(b*c - a*d)*h*(3*b*d*g - b*c*h - a*d*h)*n*(a + b*x)*Log[(e
*(a + b*x)^n)/(c + d*x)^n])/(3*b^3*d^2) + (2*A*B*(g + h*x)^3*Log[(e*(a + b*
x)^n)/(c + d*x)^n])/(3*h) + (2*B^2*(b*g - a*h)^3*n*Log[-((b*c - a*d)/(d*(a
+ b*x)))]*Log[(e*(a + b*x)^n)/(c + d*x)^n])/(3*b^3*h) - (2*B^2*(d*g - c*h)^
3*n*Log[(b*c - a*d)/(b*(c + d*x))]*Log[(e*(a + b*x)^n)/(c + d*x)^n])/(3*d^3
```

```
*h) + (B^2*(g + h*x)^3*Log[(e*(a + b*x)^n)/(c + d*x)^n]^2)/(3*h) - (2*B^2*(
d*g - c*h)^3*n^2*PolyLog[2, (d*(a + b*x))/(b*(c + d*x))])/(3*d^3*h) - (2*B^
2*(b*g - a*h)^3*n^2*PolyLog[2, 1 + (b*c - a*d)/(d*(a + b*x))])/(3*b^3*h)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 72

```
Int[((e_.) + (f_.)*(x_))^(p_.)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
 x_Symbol] :> Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x]
 /; FreeQ[{a, b, c, d, e, f}, x] && IntegerQ[p]
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2333

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)/(x_))^(q_.)*(
x_)^(m_.), x_Symbol] :> Int[(e + d*x)^q*(a + b*Log[c*x^n])^p, x] /; FreeQ[{
a, b, c, d, e, m, n, p}, x] && EqQ[m, q] && IntegerQ[q]
```

Rule 2343

```
Int[((a_.) + Log[(c_.)*(x_)^(n_)]*(b_.))/((x_)*((d_) + (e_.)*(x_)^(r_.))),
x_Symbol] :> Dist[1/n, Subst[Int[(a + b*Log[c*x])/(x*(d + e*x^(r/n))), x],
x, x^n], x] /; FreeQ[{a, b, c, d, e, n, r}, x] && IntegerQ[r/n]
```

Rule 2411

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + (g_
.)*(x_))^(q_.)*((h_.) + (i_.)*(x_))^(r_.), x_Symbol] :> Dist[1/e, Subst[Int
[((g*x)/e)^q*((e*h - d*i)/e + (i*x)/e)^r*(a + b*Log[c*x^n])^p, x], x, d + e
*x], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, n, p, q, r}, x] && EqQ[e*f - d
*g, 0] && (IGtQ[p, 0] || IGtQ[r, 0]) && IntegerQ[2*r]
```

Rule 2486

```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))
^(r_.)]^(s_.), x_Symbol] :> Simp[((a + b*x)*Log[e*(f*(a + b*x)^p*(c + d*x)^
q)^r]^s)/b, x] + Dist[(q*r*s*(b*c - a*d))/b, Int[Log[e*(f*(a + b*x)^p*(c +
d*x)^q)^r]^(s - 1)/(c + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, p, q, r, s
```

```
}, x] && NeQ[b*c - a*d, 0] && EqQ[p + q, 0] && IGtQ[s, 0]
```

Rule 2488

```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))
^(r_.)]^(s_.)/((g_.) + (h_.)*(x_)), x_Symbol] :> -Simp[(Log[-((b*c - a*d)/(
d*(a + b*x)))]*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^s)/h, x] + Dist[(p*r*s*
(b*c - a*d))/h, Int[(Log[-((b*c - a*d)/(d*(a + b*x)))]*Log[e*(f*(a + b*x)^p
*(c + d*x)^q)^r]^(s - 1))/((a + b*x)*(c + d*x)), x], x] /; FreeQ[{a, b, c,
d, e, f, g, h, p, q, r, s}, x] && NeQ[b*c - a*d, 0] && EqQ[p + q, 0] && EqQ
[b*g - a*h, 0] && IGtQ[s, 0]
```

Rule 2492

```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))
^(r_.)]^(s_.)*((g_.) + (h_.)*(x_))^(m_.), x_Symbol] :> Simp[((g + h*x)^(m +
 1)*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^s)/(h*(m + 1)), x] - Dist[(p*r*s*(
b*c - a*d))/(h*(m + 1)), Int[((g + h*x)^(m + 1)*Log[e*(f*(a + b*x)^p*(c + d
*x)^q)^r]^(s - 1))/((a + b*x)*(c + d*x)), x], x] /; FreeQ[{a, b, c, d, e, f
, g, h, m, p, q, r, s}, x] && NeQ[b*c - a*d, 0] && EqQ[p + q, 0] && IGtQ[s,
 0] && NeQ[m, -1]
```

Rule 2514

```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))
^(r_.)]^(s_.)*(RFx_), x_Symbol] :> With[{u = ExpandIntegrand[Log[e*(f*(a +
b*x)^p*(c + d*x)^q)^r]^s, RFx, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c
, d, e, f, p, q, r, s}, x] && RationalFunctionQ[RFx, x] && IGtQ[s, 0]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\int (g+hx)^2 \left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^2 dx = \int \left(A^2(g+hx)^2 + 2AB(g+hx)^2\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right.$$

$$= \frac{A^2(g+hx)^3}{3h} + (2AB)\int (g+hx)^2\log\left(e(a+bx)^n(c+dx)^{-n}\right.$$

$$= \frac{A^2(g+hx)^3}{3h} + \frac{2AB(g+hx)^3\log\left(e(a+bx)^n(c+dx)^{-n}\right)}{3h} +$$

$$= \frac{A^2(g+hx)^3}{3h} + \frac{2AB(g+hx)^3\log\left(e(a+bx)^n(c+dx)^{-n}\right)}{3h} +$$

$$= -\frac{2AB(bc-ad)h(3bdg-bch-adh)nx}{3b^2d^2} - \frac{AB(bc-ad)h^2nx^2}{3bd}$$

$$= -\frac{2AB(bc-ad)h(3bdg-bch-adh)nx}{3b^2d^2} - \frac{AB(bc-ad)h^2nx^2}{3bd}$$

$$= -\frac{2AB(bc-ad)h(3bdg-bch-adh)nx}{3b^2d^2} - \frac{AB(bc-ad)h^2nx^2}{3bd}$$

$$= -\frac{2AB(bc-ad)h(3bdg-bch-adh)nx}{3b^2d^2} + \frac{B^2(bc-ad)^2h^2n^2x}{3b^2d^2}$$

$$= -\frac{2AB(bc-ad)h(3bdg-bch-adh)nx}{3b^2d^2} + \frac{B^2(bc-ad)^2h^2n^2x}{3b^2d^2}$$

$$= -\frac{2AB(bc-ad)h(3bdg-bch-adh)nx}{3b^2d^2} + \frac{B^2(bc-ad)^2h^2n^2x}{3b^2d^2}$$

**Mathematica [A]** time = 1.84, size = 906, normalized size = 1.59

$$\frac{-aB^2\left(3b^2g^2-3abhg+a^2h^2\right)n^2\log^2(a+bx)d^3 + Bn\log(a+bx)\left(2Bc\left(3d^2g^2-3cdhg+c^2h^2\right)n\log(c+dx)b^3 + \right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(g + h*x)^2*(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])^2,x]
```

```
[Out] (-(a*B^2*d^3*(3*b^2*g^2 - 3*a*b*g*h + a^2*h^2)*n^2*Log[a + b*x]^2) + B*n*Lo
g[a + b*x]*(2*b^3*B*c*(3*d^2*g^2 - 3*c*d*g*h + c^2*h^2)*n*Log[c + d*x] + 2*
```

```
B*(3*a*b^2*d^3*g^2 - 3*a^2*b*d^3*g*h + a^3*d^3*h^2 - b^3*c*(3*d^2*g^2 - 3*c
*d*g*h + c^2*h^2))*n*Log[(b*(c + d*x))/(b*c - a*d)] + a*d*(2*A*d^2*(3*b^2*g
^2 - 3*a*b*g*h + a^2*h^2) + B*(-3*a^2*d^2*h^2 + a*b*d*h*(6*d*g + c*h) + 2*b
^2*(3*d^2*g^2 - 3*c*d*g*h + c^2*h^2))*n + 2*B*d^2*(3*b^2*g^2 - 3*a*b*g*h +
a^2*h^2)*Log[(e*(a + b*x)^n)/(c + d*x)^n])) + b*(-(b^2*B^2*c*(3*d^2*g^2 - 3
*c*d*g*h + c^2*h^2)*n^2*Log[c + d*x]^2) + B*n*Log[c + d*x]*(-2*A*b^2*c*(3*d
^2*g^2 - 3*c*d*g*h + c^2*h^2) + B*(2*a^2*c*d^2*h^2 - 3*b^2*c^2*h*(-2*d*g +
c*h) + a*b*d*(-6*d^2*g^2 - 6*c*d*g*h + c^2*h^2))*n - 2*b^2*B*c*(3*d^2*g^2 -
 3*c*d*g*h + c^2*h^2)*Log[(e*(a + b*x)^n)/(c + d*x)^n]) + d*(a^2*B*d^2*h^2*
n*(-2*A + B*n)*x + a*b*B*n*(A*d^2*(-6*g^2 + 6*g*h*x + h^2*x^2) - 2*B*n*(3*d
^2*g^2 + c^2*h^2 + c*d*h*(-3*g + h*x))) + b^2*x*(B^2*c^2*h^2*n^2 + A^2*d^2*
(3*g^2 + 3*g*h*x + h^2*x^2) + A*B*c*h*n*(2*c*h - d*(6*g + h*x))) + B*(-2*a^
2*B*d^2*h^2*n*x + a*b*B*d^2*n*(-6*g^2 + 6*g*h*x + h^2*x^2) + b^2*x*(B*c*h*n
*(-6*d*g + 2*c*h - d*h*x) + 2*A*d^2*(3*g^2 + 3*g*h*x + h^2*x^2)))*Log[(e*(a
 + b*x)^n)/(c + d*x)^n] + b^2*B^2*d^2*x*(3*g^2 + 3*g*h*x + h^2*x^2)*Log[(e*
(a + b*x)^n)/(c + d*x)^n]^2)) + 2*B^2*(3*a*b^2*d^3*g^2 - 3*a^2*b*d^3*g*h +
a^3*d^3*h^2 - b^3*c*(3*d^2*g^2 - 3*c*d*g*h + c^2*h^2))*n^2*PolyLog[2, (d*(a
 + b*x))/(-(b*c) + a*d)])/(3*b^3*d^3)
```

**fricas [F]** time = 1.14, size = 0, normalized size = 0.00

$$\text{integral}\left(A^2h^2x^2 + 2\,A^2ghx + A^2g^2 + \left(B^2h^2x^2 + 2\,B^2ghx + B^2g^2\right)\log\left(\frac{(bx+a)^n e}{(dx+c)^n}\right)^2 + 2\left(ABh^2x^2 + 2\,ABghx + \right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x+g)^2*(A+B*log(e*(b*x+a)^n/((d*x+c)^n)))^2,x, algorithm="fric
as")
```

```
[Out] integral(A^2*h^2*x^2 + 2*A^2*g*h*x + A^2*g^2 + (B^2*h^2*x^2 + 2*B^2*g*h*x +
 B^2*g^2)*log((b*x + a)^n*e/(d*x + c)^n)^2 + 2*(A*B*h^2*x^2 + 2*A*B*g*h*x +
 A*B*g^2)*log((b*x + a)^n*e/(d*x + c)^n), x)
```

**giac [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x+g)^2*(A+B*log(e*(b*x+a)^n/((d*x+c)^n)))^2,x, algorithm="giac
")
```

```
[Out] Timed out
```

**maple [C]** time = 4.82, size = 22955, normalized size = 40.27

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((h*x+g)^2*(A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))^2,x)
```

```
[Out] result too large to display
```

**maxima [B]** time = 6.79, size = 1671, normalized size = 2.93

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x+g)^2*(A+B*log(e*(b*x+a)^n/((d*x+c)^n)))^2,x, algorithm="maxi
ma")
```

```
[Out] 2/3*A*B*h^2*x^3*log((b*x + a)^n*e/(d*x + c)^n) + 1/3*A^2*h^2*x^3 + 2*A*B*g*
h*x^2*log((b*x + a)^n*e/(d*x + c)^n) + A^2*g*h*x^2 + 2*A*B*g^2*x*log((b*x +
 a)^n*e/(d*x + c)^n) + A^2*g^2*x + 2*(a*e*n*log(b*x + a)/b - c*e*n*log(d*x
+ c)/d)*A*B*g^2/e - 2*(a^2*e*n*log(b*x + a)/b^2 - c^2*e*n*log(d*x + c)/d^2
+ (b*c*e*n - a*d*e*n)*x/(b*d))*A*B*g*h/e + 1/3*(2*a^3*e*n*log(b*x + a)/b^3
- 2*c^3*e*n*log(d*x + c)/d^3 - ((b^2*c*d*e*n - a*b*d^2*e*n)*x^2 - 2*(b^2*c^
2*e*n - a^2*d^2*e*n)*x)/(b^2*d^2))*A*B*h^2/e + 1/3*(2*a^2*c*d^2*h^2*n^2 - (
6*c*d^2*g*h*n^2 - c^2*d*h^2*n^2)*a*b - (6*c*d^2*g^2*n*log(e) + (3*h^2*n^2 +
 2*h^2*n*log(e))*c^3 - 6*(g*h*n^2 + g*h*n*log(e))*c^2*d)*b^2)*B^2*log(d*x +
 c)/(b^2*d^3) + 2/3*(3*a*b^2*d^3*g^2*n^2 - 3*a^2*b*d^3*g*h*n^2 + a^3*d^3*h^
2*n^2 - (3*c*d^2*g^2*n^2 - 3*c^2*d*g*h*n^2 + c^3*h^2*n^2)*b^3)*(log(b*x + a
)*log((b*d*x + a*d)/(b*c - a*d) + 1) + dilog(-(b*d*x + a*d)/(b*c - a*d)))*B
^2/(b^3*d^3) + 1/3*(B^2*b^3*d^3*h^2*x^3*log(e)^2 + 2*(3*c*d^2*g^2*n^2 - 3*c
^2*d*g*h*n^2 + c^3*h^2*n^2)*B^2*b^3*log(b*x + a)*log(d*x + c) - (3*c*d^2*g^
2*n^2 - 3*c^2*d*g*h*n^2 + c^3*h^2*n^2)*B^2*b^3*log(d*x + c)^2 + (a*b^2*d^3*
h^2*n*log(e) - (c*d^2*h^2*n*log(e) - 3*d^3*g*h*log(e)^2)*b^3)*B^2*x^2 - (3*
a*b^2*d^3*g^2*n^2 - 3*a^2*b*d^3*g*h*n^2 + a^3*d^3*h^2*n^2)*B^2*log(b*x + a)
^2 + ((h^2*n^2 - 2*h^2*n*log(e))*a^2*b*d^3 - 2*(c*d^2*h^2*n^2 - 3*d^3*g*h*n
*log(e))*a*b^2 - (6*c*d^2*g*h*n*log(e) - 3*d^3*g^2*log(e)^2 - (h^2*n^2 + 2*
h^2*n*log(e))*c^2*d)*b^3)*B^2*x - ((3*h^2*n^2 - 2*h^2*n*log(e))*a^3*d^3 - (
c*d^2*h^2*n^2 + 6*(g*h*n^2 - g*h*n*log(e))*d^3)*a^2*b + 2*(3*c*d^2*g*h*n^2
- c^2*d*h^2*n^2 - 3*d^3*g^2*n*log(e))*a*b^2)*B^2*log(b*x + a) + (B^2*b^3*d^
3*h^2*x^3 + 3*B^2*b^3*d^3*g*h*x^2 + 3*B^2*b^3*d^3*g^2*x)*log((b*x + a)^n)^2
 + (B^2*b^3*d^3*h^2*x^3 + 3*B^2*b^3*d^3*g*h*x^2 + 3*B^2*b^3*d^3*g^2*x)*log(
(d*x + c)^n)^2 + (2*B^2*b^3*d^3*h^2*x^3*log(e) - 2*(3*c*d^2*g^2*n - 3*c^2*d
*g*h*n + c^3*h^2*n)*B^2*b^3*log(d*x + c) + (a*b^2*d^3*h^2*n - (c*d^2*h^2*n
- 6*d^3*g*h*log(e))*b^3)*B^2*x^2 + 2*(3*a*b^2*d^3*g*h*n - a^2*b*d^3*h^2*n -
 (3*c*d^2*g*h*n - c^2*d*h^2*n - 3*d^3*g^2*log(e))*b^3)*B^2*x + 2*(3*a*b^2*d
^3*g^2*n - 3*a^2*b*d^3*g*h*n + a^3*d^3*h^2*n)*B^2*log(b*x + a))*log((b*x +
a)^n) - (2*B^2*b^3*d^3*h^2*x^3*log(e) - 2*(3*c*d^2*g^2*n - 3*c^2*d*g*h*n +
c^3*h^2*n)*B^2*b^3*log(d*x + c) + (a*b^2*d^3*h^2*n - (c*d^2*h^2*n - 6*d^3*g
```

```
*h*log(e))*b^3)*B^2*x^2 + 2*(3*a*b^2*d^3*g*h*n - a^2*b*d^3*h^2*n - (3*c*d^2
*g*h*n - c^2*d*h^2*n - 3*d^3*g^2*log(e))*b^3)*B^2*x + 2*(3*a*b^2*d^3*g^2*n
- 3*a^2*b*d^3*g*h*n + a^3*d^3*h^2*n)*B^2*log(b*x + a) + 2*(B^2*b^3*d^3*h^2*
x^3 + 3*B^2*b^3*d^3*g*h*x^2 + 3*B^2*b^3*d^3*g^2*x)*log((b*x + a)^n))*log((d
*x + c)^n))/(b^3*d^3)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.00

$$\int (g+h\,x)^2\,\left(A+B\,\ln\left(\frac{e\,(a+b\,x)^n}{(c+d\,x)^n}\right)\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g + h*x)^2*(A + B*log((e*(a + b*x)^n)/(c + d*x)^n))^2,x)
```

```
[Out] int((g + h*x)^2*(A + B*log((e*(a + b*x)^n)/(c + d*x)^n))^2, x)
```

**sympy [F(-2)]** time = 0.00, size = 0, normalized size = 0.00

Exception raised: HeuristicGCDFailed

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x+g)**2*(A+B*ln(e*(b*x+a)**n/((d*x+c)**n)))**2,x)
```

```
[Out] Exception raised: HeuristicGCDFailed
```

# 3.304 $\int (g+hx) \left(A + B \log \left(e(a + bx)^n (c + dx)^{-n}\right)\right)^2 dx$

**Optimal**. Leaf size=294

$$\frac{Bn(bc-ad)(-adh-bch+2bdg)\log\left(\frac{bc-ad}{b(c+dx)}\right)\left(B\log\left(e(a+bx)^n(c+dx)^{-n}\right)+A\right)}{b^2d^2}-\frac{(bg-ah)^2\left(B\log\left(e(a+bx)^n(c\right.\right.}{2b^2h}$$

```
[Out] B^2*(-a*d+b*c)^2*h*n^2*ln(d*x+c)/b^2/d^2-B*(-a*d+b*c)*h*n*(b*x+a)*(A+B*ln(e
*(b*x+a)^n/((d*x+c)^n)))/b^2/d+B*(-a*d+b*c)*(-a*d*h-b*c*h+2*b*d*g)*n*ln((-a
*d+b*c)/b/(d*x+c))*(A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))/b^2/d^2-1/2*(-a*h+b*g)
^2*(A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))^2/b^2/h+1/2*(h*x+g)^2*(A+B*ln(e*(b*x+a
)^n/((d*x+c)^n)))^2/h+B^2*(-a*d+b*c)*(-a*d*h-b*c*h+2*b*d*g)*n^2*polylog(2,d
*(b*x+a)/b/(d*x+c))/b^2/d^2
```

**Rubi [A]** time = 0.96, antiderivative size = 449, normalized size of antiderivative = 1.53, number of steps used = 20, number of rules used = 11, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.355, Rules used = {6742, 2492, 72, 2514, 2486, 31, 2488, 2411, 2343, 2333, 2315}

$$-\frac{B^2n^2(bg-ah)^2\text{PolyLog}\left(2,\frac{bc-ad}{d(a+bx)}+1\right)}{b^2h}-\frac{B^2n^2(dg-ch)^2\text{PolyLog}\left(2,\frac{d(a+bx)}{b(c+dx)}\right)}{d^2h}-\frac{ABn(bg-ah)^2\log(a+bx)}{b^2h}+\frac{AB}{}$$

Antiderivative was successfully verified.

```
[In] Int[(g + h*x)*(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])^2,x]
```

```
[Out] -((A*B*(b*c - a*d)*h*n*x)/(b*d)) + (A^2*(g + h*x)^2)/(2*h) - (A*B*(b*g - a*
h)^2*n*Log[a + b*x])/(b^2*h) + (A*B*(d*g - c*h)^2*n*Log[c + d*x])/(d^2*h) +
 (B^2*(b*c - a*d)^2*h*n^2*Log[c + d*x])/(b^2*d^2) - (B^2*(b*c - a*d)*h*n*(a
 + b*x)*Log[(e*(a + b*x)^n)/(c + d*x)^n])/(b^2*d) + (A*B*(g + h*x)^2*Log[(e
*(a + b*x)^n)/(c + d*x)^n])/h + (B^2*(b*g - a*h)^2*n*Log[-((b*c - a*d)/(d*(
a + b*x)))]*Log[(e*(a + b*x)^n)/(c + d*x)^n])/(b^2*h) - (B^2*(d*g - c*h)^2*
n*Log[(b*c - a*d)/(b*(c + d*x))]*Log[(e*(a + b*x)^n)/(c + d*x)^n])/(d^2*h)
+ (B^2*(g + h*x)^2*Log[(e*(a + b*x)^n)/(c + d*x)^n]^2)/(2*h) - (B^2*(d*g -
c*h)^2*n^2*PolyLog[2, (d*(a + b*x))/(b*(c + d*x))])/(d^2*h) - (B^2*(b*g - a
*h)^2*n^2*PolyLog[2, 1 + (b*c - a*d)/(d*(a + b*x))])/(b^2*h)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 72

```
Int[((e_.) + (f_.)*(x_))^(p_.)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
 x_Symbol] :> Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x]
 /; FreeQ[{a, b, c, d, e, f}, x] && IntegerQ[p]
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2333

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)/(x_))^(q_.)*(
x_)^(m_.), x_Symbol] :> Int[(e + d*x)^q*(a + b*Log[c*x^n])^p, x] /; FreeQ[{
a, b, c, d, e, m, n, p}, x] && EqQ[m, q] && IntegerQ[q]
```

Rule 2343

```
Int[((a_.) + Log[(c_.)*(x_)^(n_)]*(b_.))/((x_)*((d_) + (e_.)*(x_)^(r_.))),
x_Symbol] :> Dist[1/n, Subst[Int[(a + b*Log[c*x])/(x*(d + e*x^(r/n))), x],
x, x^n], x] /; FreeQ[{a, b, c, d, e, n, r}, x] && IntegerQ[r/n]
```

Rule 2411

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + (g_
.)*(x_))^(q_.)*((h_.) + (i_.)*(x_))^(r_.), x_Symbol] :> Dist[1/e, Subst[Int
[((g*x)/e)^q*((e*h - d*i)/e + (i*x)/e)^r*(a + b*Log[c*x^n])^p, x], x, d + e
*x], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, n, p, q, r}, x] && EqQ[e*f - d
*g, 0] && (IGtQ[p, 0] || IGtQ[r, 0]) && IntegerQ[2*r]
```

Rule 2486

```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))
^(r_.)]^(s_.), x_Symbol] :> Simp[((a + b*x)*Log[e*(f*(a + b*x)^p*(c + d*x)^
q)^r]^s)/b, x] + Dist[(q*r*s*(b*c - a*d))/b, Int[Log[e*(f*(a + b*x)^p*(c +
d*x)^q)^r]^(s - 1)/(c + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, p, q, r, s
}, x] && NeQ[b*c - a*d, 0] && EqQ[p + q, 0] && IGtQ[s, 0]
```

Rule 2488

```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))
^(r_.)]^(s_.)/((g_.) + (h_.)*(x_)), x_Symbol] :> -Simp[(Log[-((b*c - a*d)/(
d*(a + b*x)))]*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^s)/h, x] + Dist[(p*r*s*
(b*c - a*d))/h, Int[(Log[-((b*c - a*d)/(d*(a + b*x)))]*Log[e*(f*(a + b*x)^p
*(c + d*x)^q)^r]^(s - 1))/((a + b*x)*(c + d*x)), x], x] /; FreeQ[{a, b, c,
d, e, f, g, h, p, q, r, s}, x] && NeQ[b*c - a*d, 0] && EqQ[p + q, 0] && EqQ
```

```
[b*g - a*h, 0] && IGtQ[s, 0]
```

Rule 2492

```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))
^(r_.)]^(s_.)*((g_.) + (h_.)*(x_))^(m_.), x_Symbol] :> Simp[((g + h*x)^(m +
 1)*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^s)/(h*(m + 1)), x] - Dist[(p*r*s*(
b*c - a*d))/(h*(m + 1)), Int[((g + h*x)^(m + 1)*Log[e*(f*(a + b*x)^p*(c + d
*x)^q)^r]^(s - 1))/((a + b*x)*(c + d*x)), x], x] /; FreeQ[{a, b, c, d, e, f
, g, h, m, p, q, r, s}, x] && NeQ[b*c - a*d, 0] && EqQ[p + q, 0] && IGtQ[s,
 0] && NeQ[m, -1]
```

Rule 2514

```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))
^(r_.)]^(s_.)*(RFx_), x_Symbol] :> With[{u = ExpandIntegrand[Log[e*(f*(a +
b*x)^p*(c + d*x)^q)^r]^s, RFx, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c
, d, e, f, p, q, r, s}, x] && RationalFunctionQ[RFx, x] && IGtQ[s, 0]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\int (g+hx)\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^2\,dx = \int \left(A^2(g+hx)+2AB(g+hx)\log\left(e(a+bx)^n(c+dx)^{-n}\right)+\right.$$

$$= \frac{A^2(g+hx)^2}{2h}+(2AB)\int (g+hx)\log\left(e(a+bx)^n(c+dx)^{-n}\right.$$

$$= \frac{A^2(g+hx)^2}{2h}+\frac{AB(g+hx)^2\log\left(e(a+bx)^n(c+dx)^{-n}\right)}{h}+$$

$$= \frac{A^2(g+hx)^2}{2h}+\frac{AB(g+hx)^2\log\left(e(a+bx)^n(c+dx)^{-n}\right)}{h}+$$

$$= -\frac{AB(bc-ad)hnx}{bd}+\frac{A^2(g+hx)^2}{2h}-\frac{AB(bg-ah)^2n\log(a}{b^2h}$$

$$= -\frac{AB(bc-ad)hnx}{bd}+\frac{A^2(g+hx)^2}{2h}-\frac{AB(bg-ah)^2n\log(a}{b^2h}$$

$$= -\frac{AB(bc-ad)hnx}{bd}+\frac{A^2(g+hx)^2}{2h}-\frac{AB(bg-ah)^2n\log(a}{b^2h}$$

$$= -\frac{AB(bc-ad)hnx}{bd}+\frac{A^2(g+hx)^2}{2h}-\frac{AB(bg-ah)^2n\log(a}{b^2h}$$

$$= -\frac{AB(bc-ad)hnx}{bd}+\frac{A^2(g+hx)^2}{2h}-\frac{AB(bg-ah)^2n\log(a}{b^2h}$$

$$= -\frac{AB(bc-ad)hnx}{bd}+\frac{A^2(g+hx)^2}{2h}-\frac{AB(bg-ah)^2n\log(a}{b^2h}$$

**Mathematica [A]** time = 0.97, size = 472, normalized size = 1.61

$$\frac{-2Bn\log(a+bx)\left(ad\left(A(adh-2bdg)+Bd(ah-2bg)\log\left(e(a+bx)^n(c+dx)^{-n}\right)+Bn(-adh+bch-2bdg)\right)-B\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(g + h*x)*(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])^2,x]
```

```
[Out] (a*B^2*d^2*(-2*b*g + a*h)*n^2*Log[a + b*x]^2 - 2*B*n*Log[a + b*x]*(b^2*B*c*
(-2*d*g + c*h)*n*Log[c + d*x] - B*(b*c - a*d)*(-2*b*d*g + b*c*h + a*d*h)*n*
```

```
Log[(b*(c + d*x))/(b*c - a*d)] + a*d*(A*(-2*b*d*g + a*d*h) + B*(-2*b*d*g +
b*c*h - a*d*h)*n + B*d*(-2*b*g + a*h)*Log[(e*(a + b*x)^n)/(c + d*x)^n])) +
b*(b*B^2*c*(-2*d*g + c*h)*n^2*Log[c + d*x]^2 + 2*B*n*Log[c + d*x]*(A*b*c*(-
2*d*g + c*h) + B*(b*c^2*h - a*d*(2*d*g + c*h))*n + b*B*c*(-2*d*g + c*h)*Log
[(e*(a + b*x)^n)/(c + d*x)^n]) + d*(A*b*x*(2*A*d*g - 2*B*c*h*n + A*d*h*x) +
 2*a*B*n*(-2*A*d*g - 2*B*d*g*n + B*c*h*n + A*d*h*x) + 2*B*(a*B*d*n*(-2*g +
h*x) + b*x*(2*A*d*g - B*c*h*n + A*d*h*x))*Log[(e*(a + b*x)^n)/(c + d*x)^n]
+ b*B^2*d*x*(2*g + h*x)*Log[(e*(a + b*x)^n)/(c + d*x)^n]^2)) + 2*B^2*(b*c -
 a*d)*(-2*b*d*g + b*c*h + a*d*h)*n^2*PolyLog[2, (d*(a + b*x))/(-(b*c) + a*d
)])/(2*b^2*d^2)
```

**fricas [F]** time = 0.94, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(A^2hx + A^2g + \left(B^2hx + B^2g\right)\log\left(\frac{(bx+a)^n e}{(dx+c)^n}\right)^2 + 2\left(ABhx + ABg\right)\log\left(\frac{(bx+a)^n e}{(dx+c)^n}\right), x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x+g)*(A+B*log(e*(b*x+a)^n/((d*x+c)^n)))^2,x, algorithm="fricas
")
```

```
[Out] integral(A^2*h*x + A^2*g + (B^2*h*x + B^2*g)*log((b*x + a)^n*e/(d*x + c)^n)
^2 + 2*(A*B*h*x + A*B*g)*log((b*x + a)^n*e/(d*x + c)^n), x)
```

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int (hx+g)\left(B\log\left(\frac{(bx+a)^n e}{(dx+c)^n}\right) + A\right)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x+g)*(A+B*log(e*(b*x+a)^n/((d*x+c)^n)))^2,x, algorithm="giac")
```

```
[Out] integrate((h*x + g)*(B*log((b*x + a)^n*e/(d*x + c)^n) + A)^2, x)
```

**maple [C]** time = 2.71, size = 11007, normalized size = 37.44

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((h*x+g)*(A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))^2,x)
```

```
[Out] result too large to display
```

**maxima [B]** time = 6.53, size = 903, normalized size = 3.07

$$ABhx^2\log\left(\frac{(bx+a)^n e}{(dx+c)^n}\right)+\frac{1}{2}A^2hx^2+2ABgx\log\left(\frac{(bx+a)^n e}{(dx+c)^n}\right)+A^2gx+\frac{2\left(\frac{aen\log(bx+a)}{b}-\frac{cen\log(dx+c)}{d}\right)ABg}{e}-\frac{\left(\frac{a^2en\log}{b}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x+g)*(A+B*log(e*(b*x+a)^n/((d*x+c)^n)))^2,x, algorithm="maxima
")
```

```
[Out] A*B*h*x^2*log((b*x + a)^n*e/(d*x + c)^n) + 1/2*A^2*h*x^2 + 2*A*B*g*x*log((b
*x + a)^n*e/(d*x + c)^n) + A^2*g*x + 2*(a*e*n*log(b*x + a)/b - c*e*n*log(d*
x + c)/d)*A*B*g/e - (a^2*e*n*log(b*x + a)/b^2 - c^2*e*n*log(d*x + c)/d^2 +
(b*c*e*n - a*d*e*n)*x/(b*d))*A*B*h/e - (a*c*d*h*n^2 + (2*c*d*g*n*log(e) - (
h*n^2 + h*n*log(e))*c^2)*b)*B^2*log(d*x + c)/(b*d^2) + (2*a*b*d^2*g*n^2 - a
^2*d^2*h*n^2 - (2*c*d*g*n^2 - c^2*h*n^2)*b^2)*(log(b*x + a)*log((b*d*x + a*
d)/(b*c - a*d) + 1) + dilog(-(b*d*x + a*d)/(b*c - a*d)))*B^2/(b^2*d^2) + 1/
2*(B^2*b^2*d^2*h*x^2*log(e)^2 + 2*(2*c*d*g*n^2 - c^2*h*n^2)*B^2*b^2*log(b*x
 + a)*log(d*x + c) - (2*c*d*g*n^2 - c^2*h*n^2)*B^2*b^2*log(d*x + c)^2 - (2*
a*b*d^2*g*n^2 - a^2*d^2*h*n^2)*B^2*log(b*x + a)^2 + 2*(a*b*d^2*h*n*log(e) -
 (c*d*h*n*log(e) - d^2*g*log(e)^2)*b^2)*B^2*x + 2*((h*n^2 - h*n*log(e))*a^2
*d^2 - (c*d*h*n^2 - 2*d^2*g*n*log(e))*a*b)*B^2*log(b*x + a) + (B^2*b^2*d^2*
h*x^2 + 2*B^2*b^2*d^2*g*x)*log((b*x + a)^n)^2 + (B^2*b^2*d^2*h*x^2 + 2*B^2*
b^2*d^2*g*x)*log((d*x + c)^n)^2 + 2*(B^2*b^2*d^2*h*x^2*log(e) - (2*c*d*g*n
- c^2*h*n)*B^2*b^2*log(d*x + c) + (a*b*d^2*h*n - (c*d*h*n - 2*d^2*g*log(e))
*b^2)*B^2*x + (2*a*b*d^2*g*n - a^2*d^2*h*n)*B^2*log(b*x + a))*log((b*x + a)
^n) - 2*(B^2*b^2*d^2*h*x^2*log(e) - (2*c*d*g*n - c^2*h*n)*B^2*b^2*log(d*x +
 c) + (a*b*d^2*h*n - (c*d*h*n - 2*d^2*g*log(e))*b^2)*B^2*x + (2*a*b*d^2*g*n
 - a^2*d^2*h*n)*B^2*log(b*x + a) + (B^2*b^2*d^2*h*x^2 + 2*B^2*b^2*d^2*g*x)*
log((b*x + a)^n))*log((d*x + c)^n))/(b^2*d^2)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.00

$$\int (g+h\,x)\,\left(A+B\,\ln\left(\frac{e\,(a+b\,x)^n}{(c+d\,x)^n}\right)\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g + h*x)*(A + B*log((e*(a + b*x)^n)/(c + d*x)^n))^2,x)
```

```
[Out] int((g + h*x)*(A + B*log((e*(a + b*x)^n)/(c + d*x)^n))^2, x)
```

**sympy [F(-2)]** time = 0.00, size = 0, normalized size = 0.00

Exception raised: HeuristicGCDFailed

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x+g)*(A+B*ln(e*(b*x+a)**n/((d*x+c)**n)))**2,x)
```

```
[Out] Exception raised: HeuristicGCDFailed
```

# 3.305 $\int \left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^2\,dx$

**Optimal**. Leaf size=137

$$\frac{2Bn(bc-ad)\log\left(\frac{bc-ad}{b(c+dx)}\right)\left(B\log\left(e(a+bx)^n(c+dx)^{-n}\right)+A\right)}{bd}+\frac{(a+bx)\left(B\log\left(e(a+bx)^n(c+dx)^{-n}\right)+A\right)^2}{b}+\frac{2B}{}$$

```
[Out] 2*B*(-a*d+b*c)*n*ln((-a*d+b*c)/b/(d*x+c))*(A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))
/b/d+(b*x+a)*(A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))^2/b+2*B^2*(-a*d+b*c)*n^2*pol
ylog(2,d*(b*x+a)/b/(d*x+c))/b/d
```

**Rubi [A]** time = 0.32, antiderivative size = 195, normalized size of antiderivative = 1.42, number of steps used = 10, number of rules used = 8, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.320, Rules used = {6742, 2486, 31, 2488, 2411, 2343, 2333, 2315}

$$\frac{2B^2n^2(bc-ad)\text{PolyLog}\left(2,\frac{d(a+bx)}{b(c+dx)}\right)}{bd}+\frac{2AB(a+bx)\log\left(e(a+bx)^n(c+dx)^{-n}\right)}{b}-\frac{2ABn(bc-ad)\log(c+dx)}{bd}+\frac{B^2(}{}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])^2,x]
```

```
[Out] A^2*x - (2*A*B*(b*c - a*d)*n*Log[c + d*x])/(b*d) + (2*A*B*(a + b*x)*Log[(e*
(a + b*x)^n)/(c + d*x)^n])/b + (2*B^2*(b*c - a*d)*n*Log[(b*c - a*d)/(b*(c +
 d*x))]*Log[(e*(a + b*x)^n)/(c + d*x)^n])/(b*d) + (B^2*(a + b*x)*Log[(e*(a
+ b*x)^n)/(c + d*x)^n]^2)/b + (2*B^2*(b*c - a*d)*n^2*PolyLog[2, (d*(a + b*x
))/(b*(c + d*x))])/(b*d)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2333

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)/(x_))^(q_.)*(
x_)^(m_.), x_Symbol] :> Int[(e + d*x)^q*(a + b*Log[c*x^n])^p, x] /; FreeQ[{
a, b, c, d, e, m, n, p}, x] && EqQ[m, q] && IntegerQ[q]
```

Rule 2343

```
Int[((a_.) + Log[(c_.)*(x_)^(n_)]*(b_.))/((x_)*((d_) + (e_.)*(x_)^(r_.))),
x_Symbol] :> Dist[1/n, Subst[Int[(a + b*Log[c*x])/(x*(d + e*x^(r/n))), x],
x, x^n], x] /; FreeQ[{a, b, c, d, e, n, r}, x] && IntegerQ[r/n]
```

Rule 2411

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + (g_
.)*(x_))^(q_.)*((h_.) + (i_.)*(x_))^(r_.), x_Symbol] :> Dist[1/e, Subst[Int
[((g*x)/e)^q*((e*h - d*i)/e + (i*x)/e)^r*(a + b*Log[c*x^n])^p, x], x, d + e
*x], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, n, p, q, r}, x] && EqQ[e*f - d
*g, 0] && (IGtQ[p, 0] || IGtQ[r, 0]) && IntegerQ[2*r]
```

Rule 2486

```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))
^(r_.)]^(s_.), x_Symbol] :> Simp[((a + b*x)*Log[e*(f*(a + b*x)^p*(c + d*x)^
q)^r]^s)/b, x] + Dist[(q*r*s*(b*c - a*d))/b, Int[Log[e*(f*(a + b*x)^p*(c +
d*x)^q)^r]^(s - 1)/(c + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, p, q, r, s
}, x] && NeQ[b*c - a*d, 0] && EqQ[p + q, 0] && IGtQ[s, 0]
```

Rule 2488

```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))
^(r_.)]^(s_.)/((g_.) + (h_.)*(x_)), x_Symbol] :> -Simp[(Log[-((b*c - a*d)/(
d*(a + b*x)))]*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^s)/h, x] + Dist[(p*r*s*
(b*c - a*d))/h, Int[(Log[-((b*c - a*d)/(d*(a + b*x)))]*Log[e*(f*(a + b*x)^p
*(c + d*x)^q)^r]^(s - 1))/((a + b*x)*(c + d*x)), x], x] /; FreeQ[{a, b, c,
d, e, f, g, h, p, q, r, s}, x] && NeQ[b*c - a*d, 0] && EqQ[p + q, 0] && EqQ
[b*g - a*h, 0] && IGtQ[s, 0]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\int \left(A + B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^2 dx = \int \left(A^2 + 2AB\log\left(e(a+bx)^n(c+dx)^{-n}\right) + B^2\log^2\left(e(a+bx)^n(c+\right.\right.$$

$$= A^2x + (2AB)\int \log\left(e(a+bx)^n(c+dx)^{-n}\right)dx + B^2\int \log^2\left(e(a+b\right.$$

$$= A^2x + \frac{2AB(a+bx)\log\left(e(a+bx)^n(c+dx)^{-n}\right)}{b} + \frac{B^2(a+bx)\log^2\left(e\right.}{}$$

$$= A^2x - \frac{2AB(bc-ad)n\log(c+dx)}{bd} + \frac{2AB(a+bx)\log\left(e(a+bx)^n(c\right.}{b}$$

$$= A^2x - \frac{2AB(bc-ad)n\log(c+dx)}{bd} + \frac{2AB(a+bx)\log\left(e(a+bx)^n(c\right.}{b}$$

$$= A^2x - \frac{2AB(bc-ad)n\log(c+dx)}{bd} + \frac{2AB(a+bx)\log\left(e(a+bx)^n(c\right.}{b}$$

$$= A^2x - \frac{2AB(bc-ad)n\log(c+dx)}{bd} + \frac{2AB(a+bx)\log\left(e(a+bx)^n(c\right.}{b}$$

$$= A^2x - \frac{2AB(bc-ad)n\log(c+dx)}{bd} + \frac{2AB(a+bx)\log\left(e(a+bx)^n(c\right.}{b}$$

**Mathematica [A]** time = 0.16, size = 217, normalized size = 1.58

$$\frac{2ABd(a+bx)\log\left(e(a+bx)^n(c+dx)^{-n}\right) - 2ABn(bc-ad)\log(c+dx) + B^2n(bc-ad)\left(2n\mathrm{Li}_2\left(\frac{b(c+dx)}{bc-ad}\right) - \log\left(\frac{bc}{bc}\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])^2,x]
```

```
[Out] (A^2*b*d*x - 2*A*B*(b*c - a*d)*n*Log[c + d*x] + 2*A*B*d*(a + b*x)*Log[(e*(a
 + b*x)^n)/(c + d*x)^n] + B^2*d*(a + b*x)*Log[(e*(a + b*x)^n)/(c + d*x)^n]^
2 + B^2*(b*c - a*d)*n*(-(Log[(b*c - a*d)/(b*c + b*d*x)]*(2*n*Log[(d*(a + b*
x))/(-(b*c) + a*d)] - 2*Log[(e*(a + b*x)^n)/(c + d*x)^n] + n*Log[(b*c - a*d
)/(b*c + b*d*x)])) + 2*n*PolyLog[2, (b*(c + d*x))/(b*c - a*d)]))/(b*d)
```

**fricas [F]** time = 1.31, size = 0, normalized size = 0.00

$$\text{integral}\left(B^2\log\left(\frac{(bx+a)^n e}{(dx+c)^n}\right)^2 + 2\,AB\log\left(\frac{(bx+a)^n e}{(dx+c)^n}\right) + A^2, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*(b*x+a)^n/((d*x+c)^n)))^2,x, algorithm="fricas")
```

```
[Out] integral(B^2*log((b*x + a)^n*e/(d*x + c)^n)^2 + 2*A*B*log((b*x + a)^n*e/(d*
x + c)^n) + A^2, x)
```

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \left(B\log\left(\frac{(bx+a)^n e}{(dx+c)^n}\right) + A\right)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*(b*x+a)^n/((d*x+c)^n)))^2,x, algorithm="giac")
```

```
[Out] integrate((B*log((b*x + a)^n*e/(d*x + c)^n) + A)^2, x)
```

**maple [C]** time = 1.32, size = 4749, normalized size = 34.66

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))^2,x)
```

```
[Out] I*A*B*Pi*csgn(I*(b*x+a)^n)*csgn(I*(b*x+a)^n/((d*x+c)^n))^2*x+I*x*ln((b*x+a)
^n)*B^2*Pi*csgn(I/((d*x+c)^n))*csgn(I*(b*x+a)^n/((d*x+c)^n))^2+1/2*B^2*Pi^2
*x*csgn(I*(b*x+a)^n/((d*x+c)^n))^4*csgn(I*e/((d*x+c)^n)*(b*x+a)^n)^2-1/2*B^
2*Pi^2*x*csgn(I*(b*x+a)^n/((d*x+c)^n))^3*csgn(I*e/((d*x+c)^n)*(b*x+a)^n)^3-
1/4*B^2*Pi^2*x*csgn(I*(b*x+a)^n/((d*x+c)^n))^2*csgn(I*e/((d*x+c)^n)*(b*x+a)
^n)^4+1/2*B^2*Pi^2*x*csgn(I*(b*x+a)^n/((d*x+c)^n))*csgn(I*e/((d*x+c)^n)*(b*
x+a)^n)^5-1/4*B^2*Pi^2*x*csgn(I*e)^2*csgn(I*e/((d*x+c)^n)*(b*x+a)^n)^4-1/4*
B^2*Pi^2*x*csgn(I*e/((d*x+c)^n)*(b*x+a)^n)^6+B^2*a/b*ln((b*x+a)^n)^2+2*B*x*
ln((b*x+a)^n)*A-1/2*B^2*Pi^2*x*csgn(I*e)*csgn(I*(b*x+a)^n)*csgn(I*(b*x+a)^n
/((d*x+c)^n))^2*csgn(I*e/((d*x+c)^n)*(b*x+a)^n)^2+I*B^2*ln(e)*Pi*csgn(I*e)*
csgn(I*e/((d*x+c)^n)*(b*x+a)^n)^2*x-I*B^2*ln(e)*Pi*csgn(I*(b*x+a)^n)*csgn(I
/((d*x+c)^n))*csgn(I*(b*x+a)^n/((d*x+c)^n))*x-I*A*B*Pi*csgn(I*e)*csgn(I*(b*
x+a)^n/((d*x+c)^n))*csgn(I*e/((d*x+c)^n)*(b*x+a)^n)*x+2*A*B*ln(e)*x-2*n^2*B
^2*c/d+2*x*ln((b*x+a)^n)*B^2*ln(e)+B^2*ln(e)^2*x-I*B^2*c*n/d*ln(d*x+c)*Pi*c
sgn(I*(b*x+a)^n/((d*x+c)^n))*csgn(I*e/((d*x+c)^n)*(b*x+a)^n)^2+I/b*B^2*ln(b
```

*x+a)*Pi*a*n*csgn(I*e)*csgn(I*e/((d*x+c)^n)*(b*x+a)^n)^2+I/b*B^2*ln(b*x+a)*
Pi*a*n*csgn(I*(b*x+a)^n)*csgn(I*(b*x+a)^n/((d*x+c)^n))^2+B^2*x*ln((d*x+c)^n
)^2+B^2*x*ln((b*x+a)^n)^2-1/4*B^2*Pi^2*x*csgn(I*(b*x+a)^n/((d*x+c)^n))^6-I*
x*ln((b*x+a)^n)*B^2*Pi*csgn(I*(b*x+a)^n/((d*x+c)^n))^3-I*x*ln((b*x+a)^n)*B^
2*Pi*csgn(I*e/((d*x+c)^n)*(b*x+a)^n)^3-I*B^2*ln(e)*Pi*csgn(I*(b*x+a)^n/((d*
x+c)^n))^3*x-I*B^2*ln(e)*Pi*csgn(I*e/((d*x+c)^n)*(b*x+a)^n)^3*x-I*A*B*Pi*cs
gn(I*(b*x+a)^n)*csgn(I/((d*x+c)^n))*csgn(I*(b*x+a)^n/((d*x+c)^n))*x-I*x*ln(
(b*x+a)^n)*B^2*Pi*csgn(I*e)*csgn(I*(b*x+a)^n/((d*x+c)^n))*csgn(I*e/((d*x+c)
^n)*(b*x+a)^n)-I*x*ln((b*x+a)^n)*B^2*Pi*csgn(I*(b*x+a)^n)*csgn(I/((d*x+c)^n
))*csgn(I*(b*x+a)^n/((d*x+c)^n))+(-2*B^2*ln((b*x+a)^n)*x-B*(-I*B*Pi*b*d*x*c
sgn(I*e)*csgn(I*(b*x+a)^n/((d*x+c)^n))*csgn(I*e/((d*x+c)^n)*(b*x+a)^n)+I*B*
Pi*b*d*x*csgn(I*e)*csgn(I*e/((d*x+c)^n)*(b*x+a)^n)^2-I*B*Pi*b*d*x*csgn(I*(b
*x+a)^n)*csgn(I/((d*x+c)^n))*csgn(I*(b*x+a)^n/((d*x+c)^n))+I*B*Pi*b*d*x*csg
n(I*(b*x+a)^n)*csgn(I*(b*x+a)^n/((d*x+c)^n))^2+I*B*Pi*b*d*x*csgn(I/((d*x+c)
^n))*csgn(I*(b*x+a)^n/((d*x+c)^n))^2-I*B*Pi*b*d*x*csgn(I*(b*x+a)^n/((d*x+c)
^n))^3+I*B*Pi*b*d*x*csgn(I*(b*x+a)^n/((d*x+c)^n))*csgn(I*e/((d*x+c)^n)*(b*x
+a)^n)^2-I*B*Pi*b*d*x*csgn(I*e/((d*x+c)^n)*(b*x+a)^n)^3+2*B*ln(e)*b*d*x+2*B
*a*d*n*ln(b*x+a)-2*B*ln(d*x+c)*b*c*n+2*A*b*d*x)/b/d)*ln((d*x+c)^n)-1/2*B^2*
Pi^2*x*csgn(I*(b*x+a)^n)*csgn(I/((d*x+c)^n))*csgn(I*(b*x+a)^n/((d*x+c)^n))*
csgn(I*e/((d*x+c)^n)*(b*x+a)^n)^3+1/2*B^2*Pi^2*x*csgn(I*e)*csgn(I*(b*x+a)^n
)*csgn(I*(b*x+a)^n/((d*x+c)^n))^3*csgn(I*e/((d*x+c)^n)*(b*x+a)^n)+1/2*B^2*P
i^2*x*csgn(I*e)*csgn(I*(b*x+a)^n/((d*x+c)^n))^3*csgn(I*e/((d*x+c)^n)*(b*x+a
)^n)^2+1/2*B^2*Pi^2*x*csgn(I*e)*csgn(I*(b*x+a)^n/((d*x+c)^n))^2*csgn(I*e/((
d*x+c)^n)*(b*x+a)^n)^3-B^2*Pi^2*x*csgn(I*e)*csgn(I*(b*x+a)^n/((d*x+c)^n))*c
sgn(I*e/((d*x+c)^n)*(b*x+a)^n)^4-1/4*B^2*Pi^2*x*csgn(I*(b*x+a)^n)^2*csgn(I/
((d*x+c)^n))^2*csgn(I*(b*x+a)^n/((d*x+c)^n))^2+1/2*B^2*Pi^2*x*csgn(I*(b*x+a
)^n)^2*csgn(I/((d*x+c)^n))*csgn(I*(b*x+a)^n/((d*x+c)^n))^3+1/2*B^2*Pi^2*x*c
sgn(I*(b*x+a)^n)*csgn(I/((d*x+c)^n))^2*csgn(I*(b*x+a)^n/((d*x+c)^n))^3-B^2*
Pi^2*x*csgn(I*(b*x+a)^n)*csgn(I/((d*x+c)^n))*csgn(I*(b*x+a)^n/((d*x+c)^n))^
4+2/b*A*B*ln(b*x+a)*a*n-I*A*B*Pi*csgn(I*(b*x+a)^n/((d*x+c)^n))^3*x-I*A*B*Pi
*csgn(I*e/((d*x+c)^n)*(b*x+a)^n)^3*x-2/d*n*B^2*ln((b*x+a)^n)*c*ln(d*x+c)+I*
x*ln((b*x+a)^n)*B^2*Pi*csgn(I*(b*x+a)^n/((d*x+c)^n))*csgn(I*e/((d*x+c)^n)*(
b*x+a)^n)^2+I*x*ln((b*x+a)^n)*B^2*Pi*csgn(I*e)*csgn(I*e/((d*x+c)^n)*(b*x+a)
^n)^2+I*x*ln((b*x+a)^n)*B^2*Pi*csgn(I*(b*x+a)^n)*csgn(I*(b*x+a)^n/((d*x+c)^
n))^2+A^2*x-2*B*c*n/d*ln(d*x+c)*A+2/b*B^2*a*n^2*ln(b*x+a)*ln((-a*d+b*c+(b*x
+a)*d)/(-a*d+b*c))+2/b*B^2*ln(b*x+a)*ln(e)*a*n-2*B^2*c*n/d*ln(d*x+c)*ln(e)+
I*B^2*c*n/d*ln(d*x+c)*Pi*csgn(I*(b*x+a)^n)*csgn(I/((d*x+c)^n))*csgn(I*(b*x+
a)^n/((d*x+c)^n))+I*A*B*Pi*csgn(I/((d*x+c)^n))*csgn(I*(b*x+a)^n/((d*x+c)^n)
)^2*x+I*A*B*Pi*csgn(I*(b*x+a)^n/((d*x+c)^n))*csgn(I*e/((d*x+c)^n)*(b*x+a)^n
)^2*x-I/b*B^2*ln(b*x+a)*Pi*a*n*csgn(I*e)*csgn(I*(b*x+a)^n/((d*x+c)^n))*csgn
(I*e/((d*x+c)^n)*(b*x+a)^n)-I*B^2*c*n/d*ln(d*x+c)*Pi*csgn(I*e)*csgn(I*e/((d
*x+c)^n)*(b*x+a)^n)^2-I*B^2*c*n/d*ln(d*x+c)*Pi*csgn(I*(b*x+a)^n)*csgn(I*(b*
x+a)^n/((d*x+c)^n))^2-I*B^2*c*n/d*ln(d*x+c)*Pi*csgn(I/((d*x+c)^n))*csgn(I*(
b*x+a)^n/((d*x+c)^n))^2+1/2*B^2*Pi^2*x*csgn(I*e)*csgn(I/((d*x+c)^n))*csgn(I
*(b*x+a)^n/((d*x+c)^n))^3*csgn(I*e/((d*x+c)^n)*(b*x+a)^n)-1/2*B^2*Pi^2*x*cs

```
gn(I*e)*csgn(I/((d*x+c)^n))*csgn(I*(b*x+a)^n/((d*x+c)^n))^2*csgn(I*e/((d*x+
c)^n)*(b*x+a)^n)^2+1/2*B^2*Pi^2*x*csgn(I*(b*x+a)^n)*csgn(I/((d*x+c)^n))*csg
n(I*(b*x+a)^n/((d*x+c)^n))^2*csgn(I*e/((d*x+c)^n)*(b*x+a)^n)^2-1/4*B^2*Pi^2
*x*csgn(I/((d*x+c)^n))^2*csgn(I*(b*x+a)^n/((d*x+c)^n))^4+1/2*B^2*Pi^2*x*csg
n(I/((d*x+c)^n))*csgn(I*(b*x+a)^n/((d*x+c)^n))^5+2*B^2*a*n^2/b-I*B^2*ln(e)*
Pi*csgn(I*e)*csgn(I*(b*x+a)^n/((d*x+c)^n))*csgn(I*e/((d*x+c)^n)*(b*x+a)^n)*
x-I/b*B^2*ln(b*x+a)*Pi*a*n*csgn(I*e/((d*x+c)^n)*(b*x+a)^n)^3-I/b*B^2*ln(b*x
+a)*Pi*a*n*csgn(I*(b*x+a)^n/((d*x+c)^n))^3+2*n^2*B^2*c/d*ln(d*x+c)*ln((b*(d
*x+c)+a*d-b*c)/(a*d-b*c))+1/2*B^2*Pi^2*x*csgn(I*e)*csgn(I*e/((d*x+c)^n)*(b*
x+a)^n)^5-1/4*B^2*Pi^2*x*csgn(I*(b*x+a)^n)^2*csgn(I*(b*x+a)^n/((d*x+c)^n))^
4+1/2*B^2*Pi^2*x*csgn(I*(b*x+a)^n)*csgn(I*(b*x+a)^n/((d*x+c)^n))^5-n^2*B^2*
c/d*ln(d*x+c)^2-1/2*B^2*Pi^2*x*csgn(I*(b*x+a)^n)*csgn(I*(b*x+a)^n/((d*x+c)^
n))^3*csgn(I*e/((d*x+c)^n)*(b*x+a)^n)^2+1/2*B^2*Pi^2*x*csgn(I*(b*x+a)^n)*cs
gn(I*(b*x+a)^n/((d*x+c)^n))^2*csgn(I*e/((d*x+c)^n)*(b*x+a)^n)^3-1/2*B^2*Pi^
2*x*csgn(I/((d*x+c)^n))*csgn(I*(b*x+a)^n/((d*x+c)^n))^3*csgn(I*e/((d*x+c)^n
)*(b*x+a)^n)^2+1/2*B^2*Pi^2*x*csgn(I/((d*x+c)^n))*csgn(I*(b*x+a)^n/((d*x+c)
^n))^2*csgn(I*e/((d*x+c)^n)*(b*x+a)^n)^3+I/b*B^2*ln(b*x+a)*Pi*a*n*csgn(I/((
d*x+c)^n))*csgn(I*(b*x+a)^n/((d*x+c)^n))^2+I/b*B^2*ln(b*x+a)*Pi*a*n*csgn(I*
(b*x+a)^n/((d*x+c)^n))*csgn(I*e/((d*x+c)^n)*(b*x+a)^n)^2+2*n^2*B^2*c/d*dilo
g((b*(d*x+c)+a*d-b*c)/(a*d-b*c))+2/b*B^2*a*n^2*dilog((-a*d+b*c+(b*x+a)*d)/(
-a*d+b*c))+2*n^2*B^2/b*a*ln(b*(d*x+c)+a*d-b*c)-2*B^2/b*a*n*ln((b*x+a)^n)+I*
B^2*ln(e)*Pi*csgn(I*(b*x+a)^n)*csgn(I*(b*x+a)^n/((d*x+c)^n))^2*x+I*B^2*ln(e
)*Pi*csgn(I/((d*x+c)^n))*csgn(I*(b*x+a)^n/((d*x+c)^n))^2*x+I*B^2*ln(e)*Pi*c
sgn(I*(b*x+a)^n/((d*x+c)^n))*csgn(I*e/((d*x+c)^n)*(b*x+a)^n)^2*x+I*A*B*Pi*c
sgn(I*e)*csgn(I*e/((d*x+c)^n)*(b*x+a)^n)^2*x-1/4*B^2*Pi^2*x*csgn(I*e)^2*csg
n(I*(b*x+a)^n/((d*x+c)^n))^2*csgn(I*e/((d*x+c)^n)*(b*x+a)^n)^2+1/2*B^2*Pi^2
*x*csgn(I*e)^2*csgn(I*(b*x+a)^n/((d*x+c)^n))*csgn(I*e/((d*x+c)^n)*(b*x+a)^n
)^3-1/2*B^2*Pi^2*x*csgn(I*e)*csgn(I*(b*x+a)^n/((d*x+c)^n))^4*csgn(I*e/((d*x
+c)^n)*(b*x+a)^n)+I*B^2*c*n/d*ln(d*x+c)*Pi*csgn(I*(b*x+a)^n/((d*x+c)^n))^3+
I*B^2*c*n/d*ln(d*x+c)*Pi*csgn(I*e/((d*x+c)^n)*(b*x+a)^n)^3-1/2*B^2*Pi^2*x*c
sgn(I*e)*csgn(I*(b*x+a)^n)*csgn(I/((d*x+c)^n))*csgn(I*(b*x+a)^n/((d*x+c)^n)
)^2*csgn(I*e/((d*x+c)^n)*(b*x+a)^n)+1/2*B^2*Pi^2*x*csgn(I*e)*csgn(I*(b*x+a)
^n)*csgn(I/((d*x+c)^n))*csgn(I*(b*x+a)^n/((d*x+c)^n))*csgn(I*e/((d*x+c)^n)*
(b*x+a)^n)^2-I/b*B^2*ln(b*x+a)*Pi*a*n*csgn(I*(b*x+a)^n)*csgn(I/((d*x+c)^n))
*csgn(I*(b*x+a)^n/((d*x+c)^n))+I*B^2*c*n/d*ln(d*x+c)*Pi*csgn(I*e)*csgn(I*(b
*x+a)^n/((d*x+c)^n))*csgn(I*e/((d*x+c)^n)*(b*x+a)^n)
```

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$2\,ABx\log\left(\frac{(bx+a)^n e}{(dx+c)^n}\right)+A^2x+B^2\left(\frac{2\,bcn^2\log(bx+a)\log(dx+c)-bcn^2\log(dx+c)^2+bdx\log\left((bx+a)^n\right)^2+b\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*(b*x+a)^n/((d*x+c)^n)))^2,x, algorithm="maxima")
```

```
[Out] 2*A*B*x*log((b*x + a)^n*e/(d*x + c)^n) + A^2*x + B^2*((2*b*c*n^2*log(b*x +
a)*log(d*x + c) - b*c*n^2*log(d*x + c)^2 + b*d*x*log((b*x + a)^n)^2 + b*d*x
*log((d*x + c)^n)^2 + 2*(a*d*n*log(b*x + a) - b*c*n*log(d*x + c) + b*d*x*lo
g(e))*log((b*x + a)^n) - 2*(a*d*n*log(b*x + a) - b*c*n*log(d*x + c) + b*d*x
*log((b*x + a)^n) + b*d*x*log(e))*log((d*x + c)^n))/(b*d) - integrate(-(b^2
*d*x^2*log(e)^2 + a*b*c*log(e)^2 - ((2*n*log(e) - log(e)^2)*b^2*c - (2*n*lo
g(e) + log(e)^2)*a*b*d)*x - 2*(b^2*c*n^2*x + 2*a*b*c*n^2 - a^2*d*n^2)*log(b
*x + a))/(b^2*d*x^2 + a*b*c + (b^2*c + a*b*d)*x), x)) + 2*(a*e*n*log(b*x +
a)/b - c*e*n*log(d*x + c)/d)*A*B/e
```

**mupad** **[F]** time = 0.00, size = -1, normalized size = -0.01

$$\int \left(A+B\,\ln\left(\frac{e\,{\left(a+b\,x\right)}^n}{{\left(c+d\,x\right)}^n}\right)\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*log((e*(a + b*x)^n)/(c + d*x)^n))^2,x)
```

```
[Out] int((A + B*log((e*(a + b*x)^n)/(c + d*x)^n))^2, x)
```

**sympy** **[F(-2)]** time = 0.00, size = 0, normalized size = 0.00

Exception raised: HeuristicGCDFailed

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*ln(e*(b*x+a)**n/((d*x+c)**n)))**2,x)
```

```
[Out] Exception raised: HeuristicGCDFailed
```

# 3.306 $\int \frac{\left(A+B\log(e(a+bx)^n(c+dx)^{-n})\right)^2}{g+hx}\,dx$

**Optimal**. Leaf size=301

$$\frac{2Bn\text{Li}_2\left(\frac{(dg-ch)(a+bx)}{(bg-ah)(c+dx)}\right)\left(B\log\left(e(a+bx)^n(c+dx)^{-n}\right)+A\right)}{h}+\frac{\log\left(1-\frac{(a+bx)(dg-ch)}{(c+dx)(bg-ah)}\right)\left(B\log\left(e(a+bx)^n(c+dx)^{-n}\right)+A\right.}{h}$$

```
[Out] -ln((-a*d+b*c)/b/(d*x+c))*(A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))^2/h+(A+B*ln(e*(
b*x+a)^n/((d*x+c)^n)))^2*ln(1-(-c*h+d*g)*(b*x+a)/(-a*h+b*g)/(d*x+c))/h-2*B*
n*(A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))*polylog(2,d*(b*x+a)/b/(d*x+c))/h+2*B*n*
(A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))*polylog(2,(-c*h+d*g)*(b*x+a)/(-a*h+b*g)/(
d*x+c))/h+2*B^2*n^2*polylog(3,d*(b*x+a)/b/(d*x+c))/h-2*B^2*n^2*polylog(3,(-
c*h+d*g)*(b*x+a)/(-a*h+b*g)/(d*x+c))/h
```

**Rubi [A]** time = 0.82, antiderivative size = 473, normalized size of antiderivative = 1.57, number of steps used = 16, number of rules used = 10, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.303, Rules used = {6742, 2494, 2394, 2393, 2391, 2489, 2488, 2506, 6610, 2503}

$$-\frac{2ABn\text{PolyLog}\left(2,\frac{b(g+hx)}{bg-ah}\right)}{h}+\frac{2B^2n\log\left(e(a+bx)^n(c+dx)^{-n}\right)\text{PolyLog}\left(2,1-\frac{(g+hx)(bc-ad)}{(c+dx)(bg-ah)}\right)}{h}-\frac{2B^2n\text{PolyLog}\left(2,1\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])^2/(g + h*x),x]
```

```
[Out] -((B^2*Log[(b*c - a*d)/(b*(c + d*x))]*Log[(e*(a + b*x)^n)/(c + d*x)^n]^2)/h
) + (A^2*Log[g + h*x])/h - (2*A*B*n*Log[-((h*(a + b*x))/(b*g - a*h))]*Log[g
 + h*x])/h + (2*A*B*n*Log[-((h*(c + d*x))/(d*g - c*h))]*Log[g + h*x])/h + (
2*A*B*Log[(e*(a + b*x)^n)/(c + d*x)^n]*Log[g + h*x])/h + (B^2*Log[(e*(a + b
*x)^n)/(c + d*x)^n]^2*Log[((b*c - a*d)*(g + h*x))/((b*g - a*h)*(c + d*x))])
/h - (2*A*B*n*PolyLog[2, (b*(g + h*x))/(b*g - a*h)])/h + (2*A*B*n*PolyLog[2
, (d*(g + h*x))/(d*g - c*h)])/h - (2*B^2*n*Log[(e*(a + b*x)^n)/(c + d*x)^n]
*PolyLog[2, 1 - (b*c - a*d)/(b*(c + d*x))])/h + (2*B^2*n*Log[(e*(a + b*x)^n
)/(c + d*x)^n]*PolyLog[2, 1 - ((b*c - a*d)*(g + h*x))/((b*g - a*h)*(c + d*x
))])/h + (2*B^2*n^2*PolyLog[3, 1 - (b*c - a*d)/(b*(c + d*x))])/h - (2*B^2*n
^2*PolyLog[3, 1 - ((b*c - a*d)*(g + h*x))/((b*g - a*h)*(c + d*x))])/h
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2393

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + (c*e*x)/g])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2394

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d + e*x
)^n]))/g, x] - Dist[(b*e*n)/g, Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2488

```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))
^(r_.)]^(s_.)/((g_.) + (h_.)*(x_)), x_Symbol] :> -Simp[(Log[-((b*c - a*d)/(
d*(a + b*x)))]*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^s)/h, x] + Dist[(p*r*s*
(b*c - a*d))/h, Int[(Log[-((b*c - a*d)/(d*(a + b*x)))]*Log[e*(f*(a + b*x)^p
*(c + d*x)^q)^r]^(s - 1))/((a + b*x)*(c + d*x)), x], x] /; FreeQ[{a, b, c,
d, e, f, g, h, p, q, r, s}, x] && NeQ[b*c - a*d, 0] && EqQ[p + q, 0] && EqQ
[b*g - a*h, 0] && IGtQ[s, 0]
```

Rule 2489

```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))
^(r_.)]^(s_)/((g_.) + (h_.)*(x_)), x_Symbol] :> Dist[d/h, Int[Log[e*(f*(a +
 b*x)^p*(c + d*x)^q)^r]^s/(c + d*x), x], x] - Dist[(d*g - c*h)/h, Int[Log[e
*(f*(a + b*x)^p*(c + d*x)^q)^r]^s/((c + d*x)*(g + h*x)), x], x] /; FreeQ[{a
, b, c, d, e, f, g, h, p, q, r, s}, x] && NeQ[b*c - a*d, 0] && EqQ[p + q, 0
] && NeQ[b*g - a*h, 0] && NeQ[d*g - c*h, 0] && IGtQ[s, 1]
```

Rule 2494

```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))
^(r_.)]/((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(Log[g + h*x]*Log[e*(f*(a +
 b*x)^p*(c + d*x)^q)^r])/h, x] + (-Dist[(b*p*r)/h, Int[Log[g + h*x]/(a + b*
x), x], x] - Dist[(d*q*r)/h, Int[Log[g + h*x]/(c + d*x), x], x]) /; FreeQ[{
a, b, c, d, e, f, g, h, p, q, r}, x] && NeQ[b*c - a*d, 0]
```

Rule 2503

```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))
^(r_.)]^(s_.)*(u_), x_Symbol] :> With[{g = Coeff[Simplify[1/(u*(a + b*x))],
```

```
 x, 0], h = Coeff[Simplify[1/(u*(a + b*x))], x, 1]}, -Simp[(Log[e*(f*(a + b
*x)^p*(c + d*x)^q)^r]^s*Log[-(((b*c - a*d)*(g + h*x))/((d*g - c*h)*(a + b*x
)))])/(b*g - a*h), x] + Dist[(p*r*s*(b*c - a*d))/(b*g - a*h), Int[(Log[e*(f
*(a + b*x)^p*(c + d*x)^q)^r]^(s - 1)*Log[-(((b*c - a*d)*(g + h*x))/((d*g -
c*h)*(a + b*x)))])/((a + b*x)*(c + d*x)), x], x] /; NeQ[b*g - a*h, 0] && Ne
Q[d*g - c*h, 0]] /; FreeQ[{a, b, c, d, e, f, p, q, r, s}, x] && NeQ[b*c - a
*d, 0] && IGtQ[s, 0] && EqQ[p + q, 0] && LinearQ[Simplify[1/(u*(a + b*x))],
 x]
```

Rule 2506

```
Int[Log[v_]*Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_)
)^(q_.))^(r_.)]^(s_.)*(u_), x_Symbol] :> With[{g = Simplify[((v - 1)*(c + d
*x))/(a + b*x)], h = Simplify[u*(a + b*x)*(c + d*x)]}, -Simp[(h*PolyLog[2,
1 - v]*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^s)/(b*c - a*d), x] + Dist[h*p*r
*s, Int[(PolyLog[2, 1 - v]*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^(s - 1))/((
a + b*x)*(c + d*x)), x], x] /; FreeQ[{g, h}, x]] /; FreeQ[{a, b, c, d, e, f
, p, q, r, s}, x] && NeQ[b*c - a*d, 0] && IGtQ[s, 0] && EqQ[p + q, 0]
```

Rule 6610

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\int \frac{\left(A + B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^2}{g+hx}\,dx = \int\left(\frac{A^2}{g+hx} + \frac{2AB\log\left(e(a+bx)^n(c+dx)^{-n}\right)}{g+hx} + \frac{B^2\log^2\left(e(a+bx)^n\right.}{g+hx}\right.$$

$$= \frac{A^2\log(g+hx)}{h} + (2AB)\int\frac{\log\left(e(a+bx)^n(c+dx)^{-n}\right)}{g+hx}\,dx + B^2\int\frac{\mathrm{l}}{}$$

$$= \frac{A^2\log(g+hx)}{h} + \frac{2AB\log\left(e(a+bx)^n(c+dx)^{-n}\right)\log(g+hx)}{h} + \frac{(B}{}$$

$$= -\frac{B^2\log\left(\frac{bc-ad}{b(c+dx)}\right)\log^2\left(e(a+bx)^n(c+dx)^{-n}\right)}{h} + \frac{A^2\log(g+hx)}{h} - \frac{2}{}$$

$$= -\frac{B^2\log\left(\frac{bc-ad}{b(c+dx)}\right)\log^2\left(e(a+bx)^n(c+dx)^{-n}\right)}{h} + \frac{A^2\log(g+hx)}{h} - \frac{2}{}$$

$$= -\frac{B^2\log\left(\frac{bc-ad}{b(c+dx)}\right)\log^2\left(e(a+bx)^n(c+dx)^{-n}\right)}{h} + \frac{A^2\log(g+hx)}{h} - \frac{2}{}$$

**Mathematica [B]** time = 0.49, size = 1082, normalized size = 3.59

$$\frac{-2n\left(-n\log(a+bx) + n\log(c+dx) + \log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)\left(\log(c+dx)\log\left(\frac{d(g+hx)}{dg-ch}\right) + \mathrm{Li}_2\left(\frac{h(c+dx)}{ch-dg}\right)\right)B^2}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])^2/(g + h*x),x]
```

```
[Out] ((A + B*(-(n*Log[a + b*x]) + n*Log[c + d*x] + Log[(e*(a + b*x)^n)/(c + d*x)
^n]))^2*Log[g + h*x] + 2*B*n*(A + B*(-(n*Log[a + b*x]) + n*Log[c + d*x] + L
og[(e*(a + b*x)^n)/(c + d*x)^n]))*(Log[a + b*x]*Log[(b*(g + h*x))/(b*g - a*
h)] + PolyLog[2, (h*(a + b*x))/(-(b*g) + a*h)]) - 2*A*B*n*(Log[c + d*x]*Log
[(d*(g + h*x))/(d*g - c*h)] + PolyLog[2, (h*(c + d*x))/(-(d*g) + c*h)]) - 2
*B^2*n*(-(n*Log[a + b*x]) + n*Log[c + d*x] + Log[(e*(a + b*x)^n)/(c + d*x)^
n])*(Log[c + d*x]*Log[(d*(g + h*x))/(d*g - c*h)] + PolyLog[2, (h*(c + d*x))
/(-(d*g) + c*h)]) + B^2*n^2*(Log[a + b*x]^2*Log[(b*(g + h*x))/(b*g - a*h)]
+ 2*Log[a + b*x]*PolyLog[2, (h*(a + b*x))/(-(b*g) + a*h)] - 2*PolyLog[3, (h
*(a + b*x))/(-(b*g) + a*h)]) + B^2*n^2*(Log[c + d*x]^2*Log[(d*(g + h*x))/(d
*g - c*h)] + 2*Log[c + d*x]*PolyLog[2, (h*(c + d*x))/(-(d*g) + c*h)] - 2*Po
lyLog[3, (h*(c + d*x))/(-(d*g) + c*h)]) - 2*B^2*n^2*(Log[a + b*x]*Log[c + d
*x]*Log[(b*(g + h*x))/(b*g - a*h)] + (Log[(h*(c + d*x))/(-(d*g) + c*h)]*(-2
```

```
*Log[a + b*x] + Log[(h*(c + d*x))/(-(d*g) + c*h)])*(Log[(b*(g + h*x))/(b*g
- a*h)] - Log[(d*(g + h*x))/(d*g - c*h)]))/2 + Log[(h*(c + d*x))/(-(d*g) +
c*h)]*Log[((b*g - a*h)*(c + d*x))/((d*g - c*h)*(a + b*x))]*(-Log[(b*(g + h*
x))/(b*g - a*h)] + Log[(d*(g + h*x))/(d*g - c*h)]) + (Log[((b*g - a*h)*(c +
 d*x))/((d*g - c*h)*(a + b*x))]^2*(Log[(-(b*c) + a*d)/(d*(a + b*x))] + Log[
(b*(g + h*x))/(b*g - a*h)] - Log[((-(b*c) + a*d)*(g + h*x))/((d*g - c*h)*(a
 + b*x))]))/2 + (Log[c + d*x] - Log[((b*g - a*h)*(c + d*x))/((d*g - c*h)*(a
 + b*x))])*PolyLog[2, (h*(a + b*x))/(-(b*g) + a*h)] + (Log[a + b*x] + Log[(
(b*g - a*h)*(c + d*x))/((d*g - c*h)*(a + b*x))])*PolyLog[2, (h*(c + d*x))/(
-(d*g) + c*h)] + Log[((b*g - a*h)*(c + d*x))/((d*g - c*h)*(a + b*x))]*(Poly
Log[2, (b*(c + d*x))/(d*(a + b*x))] - PolyLog[2, ((b*g - a*h)*(c + d*x))/((
d*g - c*h)*(a + b*x))]) - PolyLog[3, (h*(a + b*x))/(-(b*g) + a*h)] - PolyLo
g[3, (h*(c + d*x))/(-(d*g) + c*h)] - PolyLog[3, (b*(c + d*x))/(d*(a + b*x))
] + PolyLog[3, ((b*g - a*h)*(c + d*x))/((d*g - c*h)*(a + b*x))]))/h
```

**fricas [F]** time = 0.87, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{B^2 \log\left(\frac{(bx+a)^n e}{(dx+c)^n}\right)^2 + 2\,AB \log\left(\frac{(bx+a)^n e}{(dx+c)^n}\right) + A^2}{hx+g}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*(b*x+a)^n/((d*x+c)^n)))^2/(h*x+g),x, algorithm="fricas
")
```

```
[Out] integral((B^2*log((b*x + a)^n*e/(d*x + c)^n)^2 + 2*A*B*log((b*x + a)^n*e/(d
*x + c)^n) + A^2)/(h*x + g), x)
```

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(B \log\left(\frac{(bx+a)^n e}{(dx+c)^n}\right) + A\right)^2}{hx+g}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*(b*x+a)^n/((d*x+c)^n)))^2/(h*x+g),x, algorithm="giac")
```

```
[Out] integrate((B*log((b*x + a)^n*e/(d*x + c)^n) + A)^2/(h*x + g), x)
```

**maple [F]** time = 2.63, size = 0, normalized size = 0.00

$$\int \frac{\left(B \ln\left(e\,(bx+a)^n\,(dx+c)^{-n}\right) + A\right)^2}{hx+g}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))^2/(h*x+g),x)
```

```
[Out] int((A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))^2/(h*x+g),x)
```

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$\frac{A^2 \log\left(hx+g\right)}{h} + \int \frac{B^2 \log\left((bx+a)^n\right)^2 + B^2 \log\left((dx+c)^n\right)^2 + B^2 \log(e)^2 + 2\,AB \log(e) + 2\left(B^2 \log(e) + AB\right)}{hx+g}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*(b*x+a)^n/((d*x+c)^n)))^2/(h*x+g),x, algorithm="maxima
")
```

```
[Out] A^2*log(h*x + g)/h + integrate((B^2*log((b*x + a)^n)^2 + B^2*log((d*x + c)^
n)^2 + B^2*log(e)^2 + 2*A*B*log(e) + 2*(B^2*log(e) + A*B)*log((b*x + a)^n)
- 2*(B^2*log((b*x + a)^n) + B^2*log(e) + A*B)*log((d*x + c)^n))/(h*x + g),
x)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(A + B\,\ln\left(\frac{e\,(a+b\,x)^n}{(c+d\,x)^n}\right)\right)^2}{g + h\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*log((e*(a + b*x)^n)/(c + d*x)^n))^2/(g + h*x),x)
```

```
[Out] int((A + B*log((e*(a + b*x)^n)/(c + d*x)^n))^2/(g + h*x), x)
```

**sympy [F(-2)]** time = 0.00, size = 0, normalized size = 0.00

Exception raised: HeuristicGCDFailed

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*ln(e*(b*x+a)**n/((d*x+c)**n)))**2/(h*x+g),x)
```

```
[Out] Exception raised: HeuristicGCDFailed
```

# 3.307 $\int \frac{\left(A+B\log(e(a+bx)^n(c+dx)^{-n})\right)^2}{(g+hx)^2}\,dx$

**Optimal**. Leaf size=208

$$\frac{2Bn(bc-ad)\log\left(1-\frac{(a+bx)(dg-ch)}{(c+dx)(bg-ah)}\right)\left(B\log\left(e(a+bx)^n(c+dx)^{-n}\right)+A\right)}{(bg-ah)(dg-ch)}+\frac{(a+bx)\left(B\log\left(e(a+bx)^n(c+dx)^{-n}\right)+A\right.}{(g+hx)(bg-ah)}$$

```
[Out] (b*x+a)*(A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))^2/(-a*h+b*g)/(h*x+g)+2*B*(-a*d+b*
c)*n*(A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))*ln(1-(-c*h+d*g)*(b*x+a)/(-a*h+b*g)/(
d*x+c))/(-a*h+b*g)/(-c*h+d*g)+2*B^2*(-a*d+b*c)*n^2*polylog(2,(-c*h+d*g)*(b*
x+a)/(-a*h+b*g)/(d*x+c))/(-a*h+b*g)/(-c*h+d*g)
```

**Rubi [A]** time = 0.41, antiderivative size = 343, normalized size of antiderivative = 1.65, number of steps used = 10, number of rules used = 7, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.212, Rules used = {6742, 2490, 36, 31, 2503, 2502, 2315}

$$\frac{2B^2n^2(bc-ad)\text{PolyLog}\left(2,1-\frac{(g+hx)(bc-ad)}{(c+dx)(bg-ah)}\right)}{(bg-ah)(dg-ch)}+\frac{2AB(a+bx)\log\left(e(a+bx)^n(c+dx)^{-n}\right)}{(g+hx)(bg-ah)}-\frac{2ABn(bc-ad)\log(c+dx}{(bg-ah)(dg-ch)}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])^2/(g + h*x)^2,x]
```

```
[Out] -(A^2/(h*(g + h*x))) - (2*A*B*(b*c - a*d)*n*Log[c + d*x])/((b*g - a*h)*(d*g
 - c*h)) + (2*A*B*(a + b*x)*Log[(e*(a + b*x)^n)/(c + d*x)^n])/((b*g - a*h)*
(g + h*x)) + (B^2*(a + b*x)*Log[(e*(a + b*x)^n)/(c + d*x)^n]^2)/((b*g - a*h
)*(g + h*x)) + (2*A*B*(b*c - a*d)*n*Log[g + h*x])/((b*g - a*h)*(d*g - c*h))
 + (2*B^2*(b*c - a*d)*n*Log[(e*(a + b*x)^n)/(c + d*x)^n]*Log[((b*c - a*d)*(
g + h*x))/((b*g - a*h)*(c + d*x))])/((b*g - a*h)*(d*g - c*h)) + (2*B^2*(b*c
 - a*d)*n^2*PolyLog[2, 1 - ((b*c - a*d)*(g + h*x))/((b*g - a*h)*(c + d*x))]
)/((b*g - a*h)*(d*g - c*h))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2490

```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))
^(r_.)]^(s_.)/((g_.) + (h_.)*(x_))^2, x_Symbol] :> Simp[((a + b*x)*Log[e*(f
*(a + b*x)^p*(c + d*x)^q)^r]^s)/((b*g - a*h)*(g + h*x)), x] - Dist[(p*r*s*(
b*c - a*d))/(b*g - a*h), Int[Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^(s - 1)/(
(c + d*x)*(g + h*x)), x], x] /; FreeQ[{a, b, c, d, e, f, g, h, p, q, r, s},
 x] && NeQ[b*c - a*d, 0] && EqQ[p + q, 0] && NeQ[b*g - a*h, 0] && IGtQ[s, 0
]
```

Rule 2502

```
Int[Log[((e_.)*((c_.) + (d_.)*(x_)))/((a_.) + (b_.)*(x_))]*(u_), x_Symbol]
:> With[{g = Coeff[Simplify[1/(u*(a + b*x))], x, 0], h = Coeff[Simplify[1/(
u*(a + b*x))], x, 1]}, -Dist[(b - d*e)/(h*(b*c - a*d)), Subst[Int[Log[e*x]/
(1 - e*x), x], x, (c + d*x)/(a + b*x)], x] /; EqQ[g*(b - d*e) - h*(a - c*e)
, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b*c - a*d, 0] && LinearQ[Simplify
[1/(u*(a + b*x))], x]
```

Rule 2503

```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))
^(r_.)]^(s_.)*(u_), x_Symbol] :> With[{g = Coeff[Simplify[1/(u*(a + b*x))],
 x, 0], h = Coeff[Simplify[1/(u*(a + b*x))], x, 1]}, -Simp[(Log[e*(f*(a + b
*x)^p*(c + d*x)^q)^r]^s*Log[-(((b*c - a*d)*(g + h*x))/((d*g - c*h)*(a + b*x
)))])/(b*g - a*h), x] + Dist[(p*r*s*(b*c - a*d))/(b*g - a*h), Int[(Log[e*(f
*(a + b*x)^p*(c + d*x)^q)^r]^(s - 1)*Log[-(((b*c - a*d)*(g + h*x))/((d*g -
c*h)*(a + b*x)))])/((a + b*x)*(c + d*x)), x], x] /; NeQ[b*g - a*h, 0] && Ne
Q[d*g - c*h, 0]] /; FreeQ[{a, b, c, d, e, f, p, q, r, s}, x] && NeQ[b*c - a
*d, 0] && IGtQ[s, 0] && EqQ[p + q, 0] && LinearQ[Simplify[1/(u*(a + b*x))],
 x]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\int \frac{\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^2}{(g+hx)^2}\,dx = \int\left(\frac{A^2}{(g+hx)^2}+\frac{2AB\log\left(e(a+bx)^n(c+dx)^{-n}\right)}{(g+hx)^2}+\frac{B^2\log^2\left(e(a+bx\right.}{(g+hx}\right.$$

$$= -\frac{A^2}{h(g+hx)}+(2AB)\int\frac{\log\left(e(a+bx)^n(c+dx)^{-n}\right)}{(g+hx)^2}\,dx+B^2\int\frac{\log^2\left(\right.}{}$$

$$= -\frac{A^2}{h(g+hx)}+\frac{2AB(a+bx)\log\left(e(a+bx)^n(c+dx)^{-n}\right)}{(bg-ah)(g+hx)}+\frac{B^2(a+bx)\,\mathrm{l}}{(}$$

$$= -\frac{A^2}{h(g+hx)}+\frac{2AB(a+bx)\log\left(e(a+bx)^n(c+dx)^{-n}\right)}{(bg-ah)(g+hx)}+\frac{B^2(a+bx)\,\mathrm{l}}{(}$$

$$= -\frac{A^2}{h(g+hx)}-\frac{2AB(bc-ad)n\log(c+dx)}{(bg-ah)(dg-ch)}+\frac{2AB(a+bx)\log\left(e(a+\right.}{(bg-ah)(g+}$$

$$= -\frac{A^2}{h(g+hx)}-\frac{2AB(bc-ad)n\log(c+dx)}{(bg-ah)(dg-ch)}+\frac{2AB(a+bx)\log\left(e(a+\right.}{(bg-ah)(g+}$$

**Mathematica [B]**   time = 1.30, size = 3460, normalized size = 16.63

Result too large to show

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])^2/(g + h*x)^2,x]
```

```
[Out] (-(A^2*b*d*g^2) + A^2*b*c*g*h + a*A^2*d*g*h - a*A^2*c*h^2 + 2*A*b*B*d*g^2*n
*Log[a + b*x] - 2*A*b*B*c*g*h*n*Log[a + b*x] + 2*A*b*B*d*g*h*n*x*Log[a + b*
x] - 2*A*b*B*c*h^2*n*x*Log[a + b*x] - b*B^2*d*g^2*n^2*Log[a + b*x]^2 + b*B^
2*c*g*h*n^2*Log[a + b*x]^2 - b*B^2*d*g*h*n^2*x*Log[a + b*x]^2 + b*B^2*c*h^2
*n^2*x*Log[a + b*x]^2 - 2*A*b*B*d*g^2*n*Log[c + d*x] + 2*a*A*B*d*g*h*n*Log[
c + d*x] - 2*A*b*B*d*g*h*n*x*Log[c + d*x] + 2*a*A*B*d*h^2*n*x*Log[c + d*x]
+ 2*b*B^2*d*g^2*n^2*Log[a + b*x]*Log[c + d*x] - 2*a*B^2*d*g*h*n^2*Log[a + b
*x]*Log[c + d*x] + 2*b*B^2*d*g*h*n^2*x*Log[a + b*x]*Log[c + d*x] - 2*a*B^2*
d*h^2*n^2*x*Log[a + b*x]*Log[c + d*x] - b*B^2*d*g^2*n^2*Log[c + d*x]^2 + a*
B^2*d*g*h*n^2*Log[c + d*x]^2 - b*B^2*d*g*h*n^2*x*Log[c + d*x]^2 + a*B^2*d*h
^2*n^2*x*Log[c + d*x]^2 - 2*b*B^2*c*g*h*n^2*Log[a + b*x]*Log[(h*(c + d*x))/
(-(d*g) + c*h)] + 2*a*B^2*d*g*h*n^2*Log[a + b*x]*Log[(h*(c + d*x))/(-(d*g)
+ c*h)] - 2*b*B^2*c*h^2*n^2*x*Log[a + b*x]*Log[(h*(c + d*x))/(-(d*g) + c*h)
] + 2*a*B^2*d*h^2*n^2*x*Log[a + b*x]*Log[(h*(c + d*x))/(-(d*g) + c*h)] + b*
```

```
B^2*c*g*h*n^2*Log[(h*(c + d*x))/(-(d*g) + c*h)]^2 - a*B^2*d*g*h*n^2*Log[(h*
(c + d*x))/(-(d*g) + c*h)]^2 + b*B^2*c*h^2*n^2*x*Log[(h*(c + d*x))/(-(d*g)
+ c*h)]^2 - a*B^2*d*h^2*n^2*x*Log[(h*(c + d*x))/(-(d*g) + c*h)]^2 - 2*b*B^2
*c*g*h*n^2*Log[(-(b*c) + a*d)/(d*(a + b*x))]*Log[((b*g - a*h)*(c + d*x))/((
d*g - c*h)*(a + b*x))] + 2*a*B^2*d*g*h*n^2*Log[(-(b*c) + a*d)/(d*(a + b*x))
]*Log[((b*g - a*h)*(c + d*x))/((d*g - c*h)*(a + b*x))] - 2*b*B^2*c*h^2*n^2*
x*Log[(-(b*c) + a*d)/(d*(a + b*x))]*Log[((b*g - a*h)*(c + d*x))/((d*g - c*h
)*(a + b*x))] + 2*a*B^2*d*h^2*n^2*x*Log[(-(b*c) + a*d)/(d*(a + b*x))]*Log[(
(b*g - a*h)*(c + d*x))/((d*g - c*h)*(a + b*x))] - 2*b*B^2*c*g*h*n^2*Log[(h*
(c + d*x))/(-(d*g) + c*h)]*Log[((b*g - a*h)*(c + d*x))/((d*g - c*h)*(a + b*
x))] + 2*a*B^2*d*g*h*n^2*Log[(h*(c + d*x))/(-(d*g) + c*h)]*Log[((b*g - a*h)
*(c + d*x))/((d*g - c*h)*(a + b*x))] - 2*b*B^2*c*h^2*n^2*x*Log[(h*(c + d*x)
)/(-(d*g) + c*h)]*Log[((b*g - a*h)*(c + d*x))/((d*g - c*h)*(a + b*x))] + 2*
a*B^2*d*h^2*n^2*x*Log[(h*(c + d*x))/(-(d*g) + c*h)]*Log[((b*g - a*h)*(c + d
*x))/((d*g - c*h)*(a + b*x))] + b*B^2*c*g*h*n^2*Log[((b*g - a*h)*(c + d*x))
/((d*g - c*h)*(a + b*x))]^2 - a*B^2*d*g*h*n^2*Log[((b*g - a*h)*(c + d*x))/(
(d*g - c*h)*(a + b*x))]^2 + b*B^2*c*h^2*n^2*x*Log[((b*g - a*h)*(c + d*x))/(
(d*g - c*h)*(a + b*x))]^2 - a*B^2*d*h^2*n^2*x*Log[((b*g - a*h)*(c + d*x))/(
(d*g - c*h)*(a + b*x))]^2 - 2*A*b*B*d*g^2*Log[(e*(a + b*x)^n)/(c + d*x)^n]
+ 2*A*b*B*c*g*h*Log[(e*(a + b*x)^n)/(c + d*x)^n] + 2*a*A*B*d*g*h*Log[(e*(a
+ b*x)^n)/(c + d*x)^n] - 2*a*A*B*c*h^2*Log[(e*(a + b*x)^n)/(c + d*x)^n] + 2
*b*B^2*d*g^2*n*Log[a + b*x]*Log[(e*(a + b*x)^n)/(c + d*x)^n] - 2*b*B^2*c*g*
h*n*Log[a + b*x]*Log[(e*(a + b*x)^n)/(c + d*x)^n] + 2*b*B^2*d*g*h*n*x*Log[a
 + b*x]*Log[(e*(a + b*x)^n)/(c + d*x)^n] - 2*b*B^2*c*h^2*n*x*Log[a + b*x]*L
og[(e*(a + b*x)^n)/(c + d*x)^n] - 2*b*B^2*d*g^2*n*Log[c + d*x]*Log[(e*(a +
b*x)^n)/(c + d*x)^n] + 2*a*B^2*d*g*h*n*Log[c + d*x]*Log[(e*(a + b*x)^n)/(c
+ d*x)^n] - 2*b*B^2*d*g*h*n*x*Log[c + d*x]*Log[(e*(a + b*x)^n)/(c + d*x)^n]
 + 2*a*B^2*d*h^2*n*x*Log[c + d*x]*Log[(e*(a + b*x)^n)/(c + d*x)^n] - b*B^2*
d*g^2*Log[(e*(a + b*x)^n)/(c + d*x)^n]^2 + b*B^2*c*g*h*Log[(e*(a + b*x)^n)/
(c + d*x)^n]^2 + a*B^2*d*g*h*Log[(e*(a + b*x)^n)/(c + d*x)^n]^2 - a*B^2*c*h
^2*Log[(e*(a + b*x)^n)/(c + d*x)^n]^2 - 2*A*b*B*d*g^2*n*Log[(b*(g + h*x))/(
b*g - a*h)] + 2*A*b*B*c*g*h*n*Log[(b*(g + h*x))/(b*g - a*h)] - 2*A*b*B*d*g*
h*n*x*Log[(b*(g + h*x))/(b*g - a*h)] + 2*A*b*B*c*h^2*n*x*Log[(b*(g + h*x))/
(b*g - a*h)] + 2*b*B^2*d*g^2*n^2*Log[a + b*x]*Log[(b*(g + h*x))/(b*g - a*h)
] - 2*a*B^2*d*g*h*n^2*Log[a + b*x]*Log[(b*(g + h*x))/(b*g - a*h)] + 2*b*B^2
*d*g*h*n^2*x*Log[a + b*x]*Log[(b*(g + h*x))/(b*g - a*h)] - 2*a*B^2*d*h^2*n^
2*x*Log[a + b*x]*Log[(b*(g + h*x))/(b*g - a*h)] - 2*b*B^2*d*g^2*n^2*Log[(h*
(c + d*x))/(-(d*g) + c*h)]*Log[(b*(g + h*x))/(b*g - a*h)] + 2*b*B^2*c*g*h*n
^2*Log[(h*(c + d*x))/(-(d*g) + c*h)]*Log[(b*(g + h*x))/(b*g - a*h)] - 2*b*B
^2*d*g*h*n^2*x*Log[(h*(c + d*x))/(-(d*g) + c*h)]*Log[(b*(g + h*x))/(b*g - a
*h)] + 2*b*B^2*c*h^2*n^2*x*Log[(h*(c + d*x))/(-(d*g) + c*h)]*Log[(b*(g + h*
x))/(b*g - a*h)] - 2*b*B^2*d*g^2*n*Log[(e*(a + b*x)^n)/(c + d*x)^n]*Log[(b*
(g + h*x))/(b*g - a*h)] + 2*b*B^2*c*g*h*n*Log[(e*(a + b*x)^n)/(c + d*x)^n]*
Log[(b*(g + h*x))/(b*g - a*h)] - 2*b*B^2*d*g*h*n*x*Log[(e*(a + b*x)^n)/(c +
 d*x)^n]*Log[(b*(g + h*x))/(b*g - a*h)] + 2*b*B^2*c*h^2*n*x*Log[(e*(a + b*x
```

```
)^n)/(c + d*x)^n]*Log[(b*(g + h*x))/(b*g - a*h)] + 2*A*b*B*d*g^2*n*Log[(d*(
g + h*x))/(d*g - c*h)] - 2*a*A*B*d*g*h*n*Log[(d*(g + h*x))/(d*g - c*h)] + 2
*A*b*B*d*g*h*n*x*Log[(d*(g + h*x))/(d*g - c*h)] - 2*a*A*B*d*h^2*n*x*Log[(d*
(g + h*x))/(d*g - c*h)] - 2*b*B^2*d*g^2*n^2*Log[a + b*x]*Log[(d*(g + h*x))/
(d*g - c*h)] + 2*a*B^2*d*g*h*n^2*Log[a + b*x]*Log[(d*(g + h*x))/(d*g - c*h)
] - 2*b*B^2*d*g*h*n^2*x*Log[a + b*x]*Log[(d*(g + h*x))/(d*g - c*h)] + 2*a*B
^2*d*h^2*n^2*x*Log[a + b*x]*Log[(d*(g + h*x))/(d*g - c*h)] + 2*b*B^2*d*g^2*
n^2*Log[(h*(c + d*x))/(-(d*g) + c*h)]*Log[(d*(g + h*x))/(d*g - c*h)] - 2*b*
B^2*c*g*h*n^2*Log[(h*(c + d*x))/(-(d*g) + c*h)]*Log[(d*(g + h*x))/(d*g - c*
h)] + 2*b*B^2*d*g*h*n^2*x*Log[(h*(c + d*x))/(-(d*g) + c*h)]*Log[(d*(g + h*x
))/(d*g - c*h)] - 2*b*B^2*c*h^2*n^2*x*Log[(h*(c + d*x))/(-(d*g) + c*h)]*Log
[(d*(g + h*x))/(d*g - c*h)] + 2*b*B^2*d*g^2*n*Log[(e*(a + b*x)^n)/(c + d*x)
^n]*Log[(d*(g + h*x))/(d*g - c*h)] - 2*a*B^2*d*g*h*n*Log[(e*(a + b*x)^n)/(c
 + d*x)^n]*Log[(d*(g + h*x))/(d*g - c*h)] + 2*b*B^2*d*g*h*n*x*Log[(e*(a + b
*x)^n)/(c + d*x)^n]*Log[(d*(g + h*x))/(d*g - c*h)] - 2*a*B^2*d*h^2*n*x*Log[
(e*(a + b*x)^n)/(c + d*x)^n]*Log[(d*(g + h*x))/(d*g - c*h)] + 2*B^2*(b*c -
a*d)*h*n^2*(g + h*x)*PolyLog[2, (h*(a + b*x))/(-(b*g) + a*h)] - 2*B^2*(b*c
- a*d)*h*n^2*(g + h*x)*PolyLog[2, (h*(c + d*x))/(-(d*g) + c*h)] - 2*b*B^2*c
*g*h*n^2*PolyLog[2, (b*(c + d*x))/(d*(a + b*x))] + 2*a*B^2*d*g*h*n^2*PolyLo
g[2, (b*(c + d*x))/(d*(a + b*x))] - 2*b*B^2*c*h^2*n^2*x*PolyLog[2, (b*(c +
d*x))/(d*(a + b*x))] + 2*a*B^2*d*h^2*n^2*x*PolyLog[2, (b*(c + d*x))/(d*(a +
 b*x))])/(h*(-(b*g) + a*h)*(-(d*g) + c*h)*(g + h*x))
```

**fricas [F]** time = 1.03, size = 0, normalized size = 0.00

$$\operatorname{integral}\left(\frac{B^2\log\left(\frac{(bx+a)^n e}{(dx+c)^n}\right)^2+2\,AB\log\left(\frac{(bx+a)^n e}{(dx+c)^n}\right)+A^2}{h^2x^2+2\,ghx+g^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*(b*x+a)^n/((d*x+c)^n)))^2/(h*x+g)^2,x, algorithm="fric
as")
```

```
[Out] integral((B^2*log((b*x + a)^n*e/(d*x + c)^n)^2 + 2*A*B*log((b*x + a)^n*e/(d
*x + c)^n) + A^2)/(h^2*x^2 + 2*g*h*x + g^2), x)
```

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\left(B\log\left(\frac{(bx+a)^n e}{(dx+c)^n}\right)+A\right)^2}{\left(hx+g\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*(b*x+a)^n/((d*x+c)^n)))^2/(h*x+g)^2,x, algorithm="giac
")
```

```
[Out] integrate((B*log((b*x + a)^n*e/(d*x + c)^n) + A)^2/(h*x + g)^2, x)
```

**maple [F]** time = 4.45, size = 0, normalized size = 0.00

$$\int \frac{\left(B \ln \left(e \left(bx+a\right)^{n} \left(dx+c\right)^{-n}\right)+A\right)^{2}}{\left(hx+g\right)^{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))^2/(h*x+g)^2,x)
```

```
[Out] int((A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))^2/(h*x+g)^2,x)
```

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$-B^{2}\left(\frac{\log\left(\left(dx+c\right)^{n}\right)^{2}}{h^{2}x+gh}+\int -\frac{dhx\log(e)^{2}+ch\log(e)^{2}+(dhx+ch)\log\left(\left(bx+a\right)^{n}\right)^{2}+2\left(dhx\log(e)+ch\log(e)\right)}{dh^{3}x^{3}+cg^{2}h+(}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*(b*x+a)^n/((d*x+c)^n)))^2/(h*x+g)^2,x, algorithm="maxi
ma")
```

```
[Out] -B^2*(log((d*x + c)^n)^2/(h^2*x + g*h) + integrate(-(d*h*x*log(e)^2 + c*h*l
og(e)^2 + (d*h*x + c*h)*log((b*x + a)^n)^2 + 2*(d*h*x*log(e) + c*h*log(e))*
log((b*x + a)^n) + 2*(d*g*n + (h*n - h*log(e))*d*x - c*h*log(e) - (d*h*x +
c*h)*log((b*x + a)^n))*log((d*x + c)^n))/(d*h^3*x^3 + c*g^2*h + (2*d*g*h^2
+ c*h^3)*x^2 + (d*g^2*h + 2*c*g*h^2)*x), x)) + 2*(b*e*n*log(b*x + a)/(b*g*h
 - a*h^2) - d*e*n*log(d*x + c)/(d*g*h - c*h^2) - (b*c*e*n - a*d*e*n)*log(h*
x + g)/((d*g*h - c*h^2)*a - (d*g^2 - c*g*h)*b))*A*B/e - 2*A*B*log((b*x + a)
^n*e/(d*x + c)^n)/(h^2*x + g*h) - A^2/(h^2*x + g*h)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{{\left(A+B\,\ln\left(\frac{e\,{\left(a+b\,x\right)}^n}{{\left(c+d\,x\right)}^n}\right)\right)}^2}{{\left(g+h\,x\right)}^2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*log((e*(a + b*x)^n)/(c + d*x)^n))^2/(g + h*x)^2,x)
```

```
[Out] int((A + B*log((e*(a + b*x)^n)/(c + d*x)^n))^2/(g + h*x)^2, x)
```

**sympy [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*ln(e*(b*x+a)**n/((d*x+c)**n)))**2/(h*x+g)**2,x)
[Out] Timed out
```

# 3.308 $\int \frac{\left(A+B\log(e(a+bx)^n(c+dx)^{-n})\right)^2}{(g+hx)^3}\,dx$

**Optimal**. Leaf size=393

$$\frac{b^2\left(B\log\left(e(a+bx)^n(c+dx)^{-n}\right)+A\right)^2}{2h(bg-ah)^2}+\frac{Bhn(a+bx)(bc-ad)\left(B\log\left(e(a+bx)^n(c+dx)^{-n}\right)+A\right)}{(g+hx)(bg-ah)^2(dg-ch)}+\frac{Bn(bc-ad)(\cdots}{\cdots}$$

```
[Out] B*(-a*d+b*c)*h*n*(b*x+a)*(A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))/(-a*h+b*g)^2/(-c
*h+d*g)/(h*x+g)+1/2*b^2*(A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))^2/h/(-a*h+b*g)^2-
1/2*(A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))^2/h/(h*x+g)^2+B^2*(-a*d+b*c)^2*h*n^2*
ln((h*x+g)/(d*x+c))/(-a*h+b*g)^2/(-c*h+d*g)^2+B*(-a*d+b*c)*(-a*d*h-b*c*h+2*
b*d*g)*n*(A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))*ln(1-(-c*h+d*g)*(b*x+a)/(-a*h+b*
g)/(d*x+c))/(-a*h+b*g)^2/(-c*h+d*g)^2+B^2*(-a*d+b*c)*(-a*d*h-b*c*h+2*b*d*g)
*n^2*polylog(2,(-c*h+d*g)*(b*x+a)/(-a*h+b*g)/(d*x+c))/(-a*h+b*g)^2/(-c*h+d*
g)^2
```

**Rubi [B]** time = 1.63, antiderivative size = 968, normalized size of antiderivative = 2.46, number of steps used = 29, number of rules used = 16, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.485, Rules used = {6742, 2492, 72, 2514, 2488, 2411, 2343, 2333, 2315, 2490, 36, 31, 2494, 2394, 2393, 2391}

$$-\frac{A^2}{2h(g+hx)^2}+\frac{b^2Bn\log(a+bx)A}{h(bg-ah)^2}-\frac{Bd^2n\log(c+dx)A}{h(dg-ch)^2}-\frac{B\log\left(e(a+bx)^n(c+dx)^{-n}\right)A}{h(g+hx)^2}+\frac{B(bc-ad)(2bdg-bch\cdots}{(bg-ah)^2\cdots}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])^2/(g + h*x)^3,x]
```

```
[Out] -A^2/(2*h*(g + h*x)^2) - (A*B*(b*c - a*d)*n)/((b*g - a*h)*(d*g - c*h)*(g +
h*x)) + (A*b^2*B*n*Log[a + b*x])/(h*(b*g - a*h)^2) - (A*B*d^2*n*Log[c + d*x
])/(h*(d*g - c*h)^2) - (B^2*(b*c - a*d)^2*h*n^2*Log[c + d*x])/((b*g - a*h)^
2*(d*g - c*h)^2) - (A*B*Log[(e*(a + b*x)^n)/(c + d*x)^n])/(h*(g + h*x)^2) +
 (B^2*(b*c - a*d)*h*n*(a + b*x)*Log[(e*(a + b*x)^n)/(c + d*x)^n])/((b*g - a
*h)^2*(d*g - c*h)*(g + h*x)) - (b^2*B^2*n*Log[-((b*c - a*d)/(d*(a + b*x)))]
*Log[(e*(a + b*x)^n)/(c + d*x)^n])/(h*(b*g - a*h)^2) + (B^2*d^2*n*Log[(b*c
- a*d)/(b*(c + d*x))]*Log[(e*(a + b*x)^n)/(c + d*x)^n])/(h*(d*g - c*h)^2) -
 (B^2*Log[(e*(a + b*x)^n)/(c + d*x)^n]^2)/(2*h*(g + h*x)^2) + (A*B*(b*c - a
*d)*(2*b*d*g - b*c*h - a*d*h)*n*Log[g + h*x])/((b*g - a*h)^2*(d*g - c*h)^2)
 + (B^2*(b*c - a*d)^2*h*n^2*Log[g + h*x])/((b*g - a*h)^2*(d*g - c*h)^2) - (
B^2*(b*c - a*d)*(2*b*d*g - b*c*h - a*d*h)*n^2*Log[-((h*(a + b*x))/(b*g - a*
h))]*Log[g + h*x])/((b*g - a*h)^2*(d*g - c*h)^2) + (B^2*(b*c - a*d)*(2*b*d*
g - b*c*h - a*d*h)*n^2*Log[-((h*(c + d*x))/(d*g - c*h))]*Log[g + h*x])/((b*
```

```
g - a*h)^2*(d*g - c*h)^2) + (B^2*(b*c - a*d)*(2*b*d*g - b*c*h - a*d*h)*n*Lo
g[(e*(a + b*x)^n)/(c + d*x)^n]*Log[g + h*x])/((b*g - a*h)^2*(d*g - c*h)^2)
+ (B^2*d^2*n^2*PolyLog[2, (d*(a + b*x))/(b*(c + d*x))])/(h*(d*g - c*h)^2) -
 (B^2*(b*c - a*d)*(2*b*d*g - b*c*h - a*d*h)*n^2*PolyLog[2, (b*(g + h*x))/(b
*g - a*h)])/((b*g - a*h)^2*(d*g - c*h)^2) + (B^2*(b*c - a*d)*(2*b*d*g - b*c
*h - a*d*h)*n^2*PolyLog[2, (d*(g + h*x))/(d*g - c*h)])/((b*g - a*h)^2*(d*g
- c*h)^2) + (b^2*B^2*n^2*PolyLog[2, 1 + (b*c - a*d)/(d*(a + b*x))])/(h*(b*g
 - a*h)^2)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 72

```
Int[((e_.) + (f_.)*(x_))^(p_.)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
 x_Symbol] :> Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x]
 /; FreeQ[{a, b, c, d, e, f}, x] && IntegerQ[p]
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2333

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)/(x_))^(q_.)*(
x_)^(m_.), x_Symbol] :> Int[(e + d*x)^q*(a + b*Log[c*x^n])^p, x] /; FreeQ[{
a, b, c, d, e, m, n, p}, x] && EqQ[m, q] && IntegerQ[q]
```

Rule 2343

```
Int[((a_.) + Log[(c_.)*(x_)^(n_)]*(b_.))/((x_)*((d_) + (e_.)*(x_)^(r_.))),
x_Symbol] :> Dist[1/n, Subst[Int[(a + b*Log[c*x])/(x*(d + e*x^(r/n))), x],
x, x^n], x] /; FreeQ[{a, b, c, d, e, n, r}, x] && IntegerQ[r/n]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2393

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + (c*e*x)/g])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2394

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d + e*x
)^n]))/g, x] - Dist[(b*e*n)/g, Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2411

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + (g_
.)*(x_))^(q_.)*((h_.) + (i_.)*(x_))^(r_.), x_Symbol] :> Dist[1/e, Subst[Int
[((g*x)/e)^q*((e*h - d*i)/e + (i*x)/e)^r*(a + b*Log[c*x^n])^p, x], x, d + e
*x], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, n, p, q, r}, x] && EqQ[e*f - d
*g, 0] && (IGtQ[p, 0] || IGtQ[r, 0]) && IntegerQ[2*r]
```

Rule 2488

```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))
^(r_.)]^(s_.)/((g_.) + (h_.)*(x_)), x_Symbol] :> -Simp[(Log[-((b*c - a*d)/(
d*(a + b*x)))]*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^s)/h, x] + Dist[(p*r*s*
(b*c - a*d))/h, Int[(Log[-((b*c - a*d)/(d*(a + b*x)))]*Log[e*(f*(a + b*x)^p
*(c + d*x)^q)^r]^(s - 1))/((a + b*x)*(c + d*x)), x], x] /; FreeQ[{a, b, c,
d, e, f, g, h, p, q, r, s}, x] && NeQ[b*c - a*d, 0] && EqQ[p + q, 0] && EqQ
[b*g - a*h, 0] && IGtQ[s, 0]
```

Rule 2490

```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))
^(r_.)]^(s_.)/((g_.) + (h_.)*(x_))^2, x_Symbol] :> Simp[((a + b*x)*Log[e*(f
*(a + b*x)^p*(c + d*x)^q)^r]^s)/((b*g - a*h)*(g + h*x)), x] - Dist[(p*r*s*(
b*c - a*d))/(b*g - a*h), Int[Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^(s - 1)/(
(c + d*x)*(g + h*x)), x], x] /; FreeQ[{a, b, c, d, e, f, g, h, p, q, r, s},
 x] && NeQ[b*c - a*d, 0] && EqQ[p + q, 0] && NeQ[b*g - a*h, 0] && IGtQ[s, 0
]
```

Rule 2492

```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))
^(r_.)]^(s_.)*((g_.) + (h_.)*(x_))^(m_.), x_Symbol] :> Simp[((g + h*x)^(m +
 1)*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^s)/(h*(m + 1)), x] - Dist[(p*r*s*(
b*c - a*d))/(h*(m + 1)), Int[((g + h*x)^(m + 1)*Log[e*(f*(a + b*x)^p*(c + d
*x)^q)^r]^(s - 1))/((a + b*x)*(c + d*x)), x], x] /; FreeQ[{a, b, c, d, e, f
, g, h, m, p, q, r, s}, x] && NeQ[b*c - a*d, 0] && EqQ[p + q, 0] && IGtQ[s,
 0] && NeQ[m, -1]
```

Rule 2494

```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))
^(r_.)]/((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(Log[g + h*x]*Log[e*(f*(a +
 b*x)^p*(c + d*x)^q)^r])/h, x] + (-Dist[(b*p*r)/h, Int[Log[g + h*x]/(a + b*
x), x], x] - Dist[(d*q*r)/h, Int[Log[g + h*x]/(c + d*x), x], x]) /; FreeQ[{
a, b, c, d, e, f, g, h, p, q, r}, x] && NeQ[b*c - a*d, 0]
```

Rule 2514

```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))
^(r_.)]^(s_.)*(RFx_), x_Symbol] :> With[{u = ExpandIntegrand[Log[e*(f*(a +
b*x)^p*(c + d*x)^q)^r]^s, RFx, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c
, d, e, f, p, q, r, s}, x] && RationalFunctionQ[RFx, x] && IGtQ[s, 0]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\int \frac{\left(A + B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^2}{(g+hx)^3}\,dx = \int\left(\frac{A^2}{(g+hx)^3} + \frac{2AB\log\left(e(a+bx)^n(c+dx)^{-n}\right)}{(g+hx)^3} + \frac{B^2\log^2\left(e(a+b\right.}{(g+h}\right.$$

$$= -\frac{A^2}{2h(g+hx)^2} + (2AB)\int \frac{\log\left(e(a+bx)^n(c+dx)^{-n}\right)}{(g+hx)^3}\,dx + B^2\int \frac{\log}{}$$

$$= -\frac{A^2}{2h(g+hx)^2} - \frac{AB\log\left(e(a+bx)^n(c+dx)^{-n}\right)}{h(g+hx)^2} - \frac{B^2\log^2\left(e(a+bx\right.}{2h(g+h}$$

$$= -\frac{A^2}{2h(g+hx)^2} - \frac{AB\log\left(e(a+bx)^n(c+dx)^{-n}\right)}{h(g+hx)^2} - \frac{B^2\log^2\left(e(a+bx\right.}{2h(g+h}$$

$$= -\frac{A^2}{2h(g+hx)^2} - \frac{AB(bc-ad)n}{(bg-ah)(dg-ch)(g+hx)} + \frac{Ab^2Bn\log(a+bx)}{h(bg-ah)^2}$$

$$= -\frac{A^2}{2h(g+hx)^2} - \frac{AB(bc-ad)n}{(bg-ah)(dg-ch)(g+hx)} + \frac{Ab^2Bn\log(a+bx)}{h(bg-ah)^2}$$

$$= -\frac{A^2}{2h(g+hx)^2} - \frac{AB(bc-ad)n}{(bg-ah)(dg-ch)(g+hx)} + \frac{Ab^2Bn\log(a+bx)}{h(bg-ah)^2}$$

$$= -\frac{A^2}{2h(g+hx)^2} - \frac{AB(bc-ad)n}{(bg-ah)(dg-ch)(g+hx)} + \frac{Ab^2Bn\log(a+bx)}{h(bg-ah)^2}$$

$$= -\frac{A^2}{2h(g+hx)^2} - \frac{AB(bc-ad)n}{(bg-ah)(dg-ch)(g+hx)} + \frac{Ab^2Bn\log(a+bx)}{h(bg-ah)^2}$$

$$= -\frac{A^2}{2h(g+hx)^2} - \frac{AB(bc-ad)n}{(bg-ah)(dg-ch)(g+hx)} + \frac{Ab^2Bn\log(a+bx)}{h(bg-ah)^2}$$

**Mathematica [B]** time = 6.46, size = 15406, normalized size = 39.20

Result too large to show

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])^2/(g + h*x)^3,x]
```

```
[Out] Result too large to show
```

**fricas [F]** time = 0.72, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{B^2\log\left(\frac{(bx+a)^n e}{(dx+c)^n}\right)^2 + 2\,AB\log\left(\frac{(bx+a)^n e}{(dx+c)^n}\right) + A^2}{h^3x^3 + 3\,gh^2x^2 + 3\,g^2hx + g^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*(b*x+a)^n/((d*x+c)^n)))^2/(h*x+g)^3,x, algorithm="fric
as")
```

```
[Out] integral((B^2*log((b*x + a)^n*e/(d*x + c)^n)^2 + 2*A*B*log((b*x + a)^n*e/(d
*x + c)^n) + A^2)/(h^3*x^3 + 3*g*h^2*x^2 + 3*g^2*h*x + g^3), x)
```

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(B\log\left(\frac{(bx+a)^n e}{(dx+c)^n}\right) + A\right)^2}{\left(hx+g\right)^3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*(b*x+a)^n/((d*x+c)^n)))^2/(h*x+g)^3,x, algorithm="giac
")
```

```
[Out] integrate((B*log((b*x + a)^n*e/(d*x + c)^n) + A)^2/(h*x + g)^3, x)
```

**maple [F]** time = 1.96, size = 0, normalized size = 0.00

$$\int \frac{\left(B\ln\left(e\,(bx+a)^n\,(dx+c)^{-n}\right) + A\right)^2}{\left(hx+g\right)^3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))^2/(h*x+g)^3,x)
```

```
[Out] int((A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))^2/(h*x+g)^3,x)
```

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$-\frac{1}{2}\,B^2\left(\frac{\log\left((dx+c)^n\right)^2}{h^3x^2 + 2\,gh^2x + g^2h} + 2\int -\frac{dhx\log(e)^2 + ch\log(e)^2 + (dhx+ch)\log\left((bx+a)^n\right)^2 + 2\left(dhx\log(e) + ch\,l\right.}{dh^4x^4 + cg^3h + \left(3\,d\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*(b*x+a)^n/((d*x+c)^n)))^2/(h*x+g)^3,x, algorithm="maxi
ma")
```

```
[Out] -1/2*B^2*(log((d*x + c)^n)^2/(h^3*x^2 + 2*g*h^2*x + g^2*h) + 2*integrate(-(
d*h*x*log(e)^2 + c*h*log(e)^2 + (d*h*x + c*h)*log((b*x + a)^n)^2 + 2*(d*h*x
*log(e) + c*h*log(e))*log((b*x + a)^n) + (d*g*n + (h*n - 2*h*log(e))*d*x -
2*c*h*log(e) - 2*(d*h*x + c*h)*log((b*x + a)^n))*log((d*x + c)^n))/(d*h^4*x
^4 + c*g^3*h + (3*d*g*h^3 + c*h^4)*x^3 + 3*(d*g^2*h^2 + c*g*h^3)*x^2 + (d*g
^3*h + 3*c*g^2*h^2)*x), x)) + (b^2*e*n*log(b*x + a)/(b^2*g^2*h - 2*a*b*g*h^
2 + a^2*h^3) - d^2*e*n*log(d*x + c)/(d^2*g^2*h - 2*c*d*g*h^2 + c^2*h^3) - (
2*a*b*d^2*e*g*n - a^2*d^2*e*h*n - (2*c*d*e*g*n - c^2*e*h*n)*b^2)*log(h*x +
g)/((d^2*g^2*h^2 - 2*c*d*g*h^3 + c^2*h^4)*a^2 - 2*(d^2*g^3*h - 2*c*d*g^2*h^
2 + c^2*g*h^3)*a*b + (d^2*g^4 - 2*c*d*g^3*h + c^2*g^2*h^2)*b^2) + (b*c*e*n
- a*d*e*n)/((d*g^2*h - c*g*h^2)*a - (d*g^3 - c*g^2*h)*b + ((d*g*h^2 - c*h^3
)*a - (d*g^2*h - c*g*h^2)*b)*x))*A*B/e - A*B*log((b*x + a)^n*e/(d*x + c)^n)
/(h^3*x^2 + 2*g*h^2*x + g^2*h) - 1/2*A^2/(h^3*x^2 + 2*g*h^2*x + g^2*h)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(A+B\,\ln\left(\frac{e\,(a+b\,x)^n}{(c+d\,x)^n}\right)\right)^2}{\left(g+h\,x\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*log((e*(a + b*x)^n)/(c + d*x)^n))^2/(g + h*x)^3,x)
```

```
[Out] int((A + B*log((e*(a + b*x)^n)/(c + d*x)^n))^2/(g + h*x)^3, x)
```

**sympy [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*ln(e*(b*x+a)**n/((d*x+c)**n)))**2/(h*x+g)**3,x)
```

```
[Out] Timed out
```

## 3.309 $\int (g+hx)^2 \left(A + B\log \left(e(a+bx)^n(c+dx)^{-n}\right)\right)^3 dx$

**Optimal**. Leaf size=875

$$-\frac{B^3h^2n^3\log(c+dx)(bc-ad)^3}{b^3d^3}-\frac{B^2h^2n^2\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)\log\left(1-\frac{b(c+dx)}{d(a+bx)}\right)(bc-ad)^3}{b^3d^3}+\frac{B^3h^2n^3\mathrm{Li}}{}$$

```
[Out] -B^3*(-a*d+b*c)^3*h^2*n^3*ln(d*x+c)/b^3/d^3+B^2*(-a*d+b*c)^2*h^2*n^2*(b*x+a
)*(A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))/b^3/d^2-2*B^2*(-a*d+b*c)^2*h*(-a*d*h-2*
b*c*h+3*b*d*g)*n^2*ln((-a*d+b*c)/b/(d*x+c))*(A+B*ln(e*(b*x+a)^n/((d*x+c)^n)
))/b^3/d^3-B*(-a*d+b*c)*h*(-a*d*h-2*b*c*h+3*b*d*g)*n*(b*x+a)*(A+B*ln(e*(b*x
+a)^n/((d*x+c)^n)))^2/b^3/d^2-1/2*B*(-a*d+b*c)*h^2*n*(d*x+c)^2*(A+B*ln(e*(b
*x+a)^n/((d*x+c)^n)))^2/b/d^3+B*(-a*d+b*c)*(a^2*d^2*h^2-a*b*d*h*(-c*h+3*d*g
)+b^2*(c^2*h^2-3*c*d*g*h+3*d^2*g^2))*n*ln((-a*d+b*c)/b/(d*x+c))*(A+B*ln(e*(
b*x+a)^n/((d*x+c)^n)))^2/b^3/d^3-1/3*(-a*h+b*g)^3*(A+B*ln(e*(b*x+a)^n/((d*x
+c)^n)))^3/b^3/h+1/3*(h*x+g)^3*(A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))^3/h-B^2*(-
a*d+b*c)^3*h^2*n^2*(A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))*ln(1-b*(d*x+c)/d/(b*x+
a))/b^3/d^3-2*B^3*(-a*d+b*c)^2*h*(-a*d*h-2*b*c*h+3*b*d*g)*n^3*polylog(2,d*(
b*x+a)/b/(d*x+c))/b^3/d^3+2*B^2*(-a*d+b*c)*(a^2*d^2*h^2-a*b*d*h*(-c*h+3*d*g
)+b^2*(c^2*h^2-3*c*d*g*h+3*d^2*g^2))*n^2*(A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))*
polylog(2,d*(b*x+a)/b/(d*x+c))/b^3/d^3+B^3*(-a*d+b*c)^3*h^2*n^3*polylog(2,b
*(d*x+c)/d/(b*x+a))/b^3/d^3-2*B^3*(-a*d+b*c)*(a^2*d^2*h^2-a*b*d*h*(-c*h+3*d
*g)+b^2*(c^2*h^2-3*c*d*g*h+3*d^2*g^2))*n^3*polylog(3,d*(b*x+a)/b/(d*x+c))/b
^3/d^3
```

**Rubi** [A] time = 3.48, antiderivative size = 1640, normalized size of antiderivative = 1.87, number of steps used = 53, number of rules used = 13, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.394, Rules used = {6742, 2492, 72, 2514, 2486, 31, 2488, 2411, 2343, 2333, 2315, 2506, 6610}

result too large to display

Antiderivative was successfully verified.

```
[In] Int[(g + h*x)^2*(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])^3,x]
```

```
[Out] -((A^2*B*(b*c - a*d)*h*(3*b*d*g - b*c*h - a*d*h)*n*x)/(b^2*d^2)) + (A*B^2*(
b*c - a*d)^2*h^2*n^2*x)/(b^2*d^2) - (A^2*B*(b*c - a*d)*h^2*n*x^2)/(2*b*d) +
 (A^3*(g + h*x)^3)/(3*h) - (A^2*B*(b*g - a*h)^3*n*Log[a + b*x])/(b^3*h) + (
a^2*A*B^2*(b*c - a*d)*h^2*n^2*Log[a + b*x])/(b^3*d) + (A^2*B*(d*g - c*h)^3*
n*Log[c + d*x])/(d^3*h) - (A*B^2*c^2*(b*c - a*d)*h^2*n^2*Log[c + d*x])/(b*d
^3) + (2*A*B^2*(b*c - a*d)^2*h*(3*b*d*g - b*c*h - a*d*h)*n^2*Log[c + d*x])/
(b^3*d^3) - (B^3*(b*c - a*d)^3*h^2*n^3*Log[c + d*x])/(b^3*d^3) - (A*B^2*(b*
c - a*d)*h^2*n*x^2*Log[(e*(a + b*x)^n)/(c + d*x)^n])/(b*d) - (2*A*B^2*(b*c
- a*d)*h*(3*b*d*g - b*c*h - a*d*h)*n*(a + b*x)*Log[(e*(a + b*x)^n)/(c + d*x
```

```
)^n])/(b^3*d^2) + (B^3*(b*c - a*d)^2*h^2*n^2*(a + b*x)*Log[(e*(a + b*x)^n)/
(c + d*x)^n])/(b^3*d^2) + (A^2*B*(g + h*x)^3*Log[(e*(a + b*x)^n)/(c + d*x)^
n])/h + (2*A*B^2*(b*g - a*h)^3*n*Log[-((b*c - a*d)/(d*(a + b*x)))]*Log[(e*(
a + b*x)^n)/(c + d*x)^n])/(b^3*h) - (a^2*B^3*(b*c - a*d)*h^2*n^2*Log[-((b*c
 - a*d)/(d*(a + b*x)))]*Log[(e*(a + b*x)^n)/(c + d*x)^n])/(b^3*d) - (2*A*B^
2*(d*g - c*h)^3*n*Log[(b*c - a*d)/(b*(c + d*x))]*Log[(e*(a + b*x)^n)/(c + d
*x)^n])/(d^3*h) + (B^3*c^2*(b*c - a*d)*h^2*n^2*Log[(b*c - a*d)/(b*(c + d*x)
)]*Log[(e*(a + b*x)^n)/(c + d*x)^n])/(b*d^3) - (2*B^3*(b*c - a*d)^2*h*(3*b*
d*g - b*c*h - a*d*h)*n^2*Log[(b*c - a*d)/(b*(c + d*x))]*Log[(e*(a + b*x)^n)
/(c + d*x)^n])/(b^3*d^3) - (B^3*(b*c - a*d)*h^2*n*x^2*Log[(e*(a + b*x)^n)/(
c + d*x)^n]^2)/(2*b*d) - (B^3*(b*c - a*d)*h*(3*b*d*g - b*c*h - a*d*h)*n*(a
+ b*x)*Log[(e*(a + b*x)^n)/(c + d*x)^n]^2)/(b^3*d^2) + (A*B^2*(g + h*x)^3*L
og[(e*(a + b*x)^n)/(c + d*x)^n]^2)/h + (B^3*(b*g - a*h)^3*n*Log[-((b*c - a*
d)/(d*(a + b*x)))]*Log[(e*(a + b*x)^n)/(c + d*x)^n]^2)/(b^3*h) - (B^3*(d*g
- c*h)^3*n*Log[(b*c - a*d)/(b*(c + d*x))]*Log[(e*(a + b*x)^n)/(c + d*x)^n]^
2)/(d^3*h) + (B^3*(g + h*x)^3*Log[(e*(a + b*x)^n)/(c + d*x)^n]^3)/(3*h) - (
2*A*B^2*(d*g - c*h)^3*n^2*PolyLog[2, (d*(a + b*x))/(b*(c + d*x))])/(d^3*h)
+ (B^3*c^2*(b*c - a*d)*h^2*n^3*PolyLog[2, (d*(a + b*x))/(b*(c + d*x))])/(b*
d^3) - (2*B^3*(b*c - a*d)^2*h*(3*b*d*g - b*c*h - a*d*h)*n^3*PolyLog[2, (d*(
a + b*x))/(b*(c + d*x))])/(b^3*d^3) - (2*A*B^2*(b*g - a*h)^3*n^2*PolyLog[2,
 1 + (b*c - a*d)/(d*(a + b*x))])/(b^3*h) + (a^2*B^3*(b*c - a*d)*h^2*n^3*Pol
yLog[2, 1 + (b*c - a*d)/(d*(a + b*x))])/(b^3*d) - (2*B^3*(b*g - a*h)^3*n^2*
Log[(e*(a + b*x)^n)/(c + d*x)^n]*PolyLog[2, 1 + (b*c - a*d)/(d*(a + b*x))])
/(b^3*h) - (2*B^3*(d*g - c*h)^3*n^2*Log[(e*(a + b*x)^n)/(c + d*x)^n]*PolyLo
g[2, 1 - (b*c - a*d)/(b*(c + d*x))])/(d^3*h) - (2*B^3*(b*g - a*h)^3*n^3*Pol
yLog[3, 1 + (b*c - a*d)/(d*(a + b*x))])/(b^3*h) + (2*B^3*(d*g - c*h)^3*n^3*
PolyLog[3, 1 - (b*c - a*d)/(b*(c + d*x))])/(d^3*h)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 72

```
Int[((e_.) + (f_.)*(x_))^(p_.)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
 x_Symbol] :> Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x]
 /; FreeQ[{a, b, c, d, e, f}, x] && IntegerQ[p]
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2333

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)/(x_))^(q_.)*(
x_)^(m_.), x_Symbol] :> Int[(e + d*x)^q*(a + b*Log[c*x^n])^p, x] /; FreeQ[{
a, b, c, d, e, m, n, p}, x] && EqQ[m, q] && IntegerQ[q]
```

Rule 2343

```
Int[((a_.) + Log[(c_.)*(x_)^(n_)]*(b_.))/((x_)*((d_) + (e_.)*(x_)^(r_.))),
x_Symbol] :> Dist[1/n, Subst[Int[(a + b*Log[c*x])/(x*(d + e*x^(r/n))), x],
x, x^n], x] /; FreeQ[{a, b, c, d, e, n, r}, x] && IntegerQ[r/n]
```

Rule 2411

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + (g_
.)*(x_))^(q_.)*((h_.) + (i_.)*(x_))^(r_.), x_Symbol] :> Dist[1/e, Subst[Int
[((g*x)/e)^q*((e*h - d*i)/e + (i*x)/e)^r*(a + b*Log[c*x^n])^p, x], x, d + e
*x], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, n, p, q, r}, x] && EqQ[e*f - d
*g, 0] && (IGtQ[p, 0] || IGtQ[r, 0]) && IntegerQ[2*r]
```

Rule 2486

```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))
^(r_.)]^(s_.), x_Symbol] :> Simp[((a + b*x)*Log[e*(f*(a + b*x)^p*(c + d*x)^
q)^r]^s)/b, x] + Dist[(q*r*s*(b*c - a*d))/b, Int[Log[e*(f*(a + b*x)^p*(c +
d*x)^q)^r]^(s - 1)/(c + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, p, q, r, s
}, x] && NeQ[b*c - a*d, 0] && EqQ[p + q, 0] && IGtQ[s, 0]
```

Rule 2488

```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))
^(r_.)]^(s_.)/((g_.) + (h_.)*(x_)), x_Symbol] :> -Simp[(Log[-((b*c - a*d)/(
d*(a + b*x)))]*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^s)/h, x] + Dist[(p*r*s*
(b*c - a*d))/h, Int[(Log[-((b*c - a*d)/(d*(a + b*x)))]*Log[e*(f*(a + b*x)^p
*(c + d*x)^q)^r]^(s - 1))/((a + b*x)*(c + d*x)), x], x] /; FreeQ[{a, b, c,
d, e, f, g, h, p, q, r, s}, x] && NeQ[b*c - a*d, 0] && EqQ[p + q, 0] && EqQ
[b*g - a*h, 0] && IGtQ[s, 0]
```

Rule 2492

```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))
^(r_.)]^(s_.)*((g_.) + (h_.)*(x_))^(m_.), x_Symbol] :> Simp[((g + h*x)^(m +
 1)*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^s)/(h*(m + 1)), x] - Dist[(p*r*s*(
b*c - a*d))/(h*(m + 1)), Int[((g + h*x)^(m + 1)*Log[e*(f*(a + b*x)^p*(c + d
*x)^q)^r]^(s - 1))/((a + b*x)*(c + d*x)), x], x] /; FreeQ[{a, b, c, d, e, f
, g, h, m, p, q, r, s}, x] && NeQ[b*c - a*d, 0] && EqQ[p + q, 0] && IGtQ[s,
```

```
 0] && NeQ[m, -1]
```

Rule 2506

```
Int[Log[v_]*Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_)
)^(q_.))^(r_.)]^(s_.)*(u_), x_Symbol] :> With[{g = Simplify[((v - 1)*(c + d
*x))/(a + b*x)], h = Simplify[u*(a + b*x)*(c + d*x)]}, -Simp[(h*PolyLog[2,
1 - v]*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^s)/(b*c - a*d), x] + Dist[h*p*r
*s, Int[(PolyLog[2, 1 - v]*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^(s - 1))/((
a + b*x)*(c + d*x)), x], x] /; FreeQ[{g, h}, x]] /; FreeQ[{a, b, c, d, e, f
, p, q, r, s}, x] && NeQ[b*c - a*d, 0] && IGtQ[s, 0] && EqQ[p + q, 0]
```

Rule 2514

```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))
^(r_.)]^(s_.)*(RFx_), x_Symbol] :> With[{u = ExpandIntegrand[Log[e*(f*(a +
b*x)^p*(c + d*x)^q)^r]^s, RFx, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c
, d, e, f, p, q, r, s}, x] && RationalFunctionQ[RFx, x] && IGtQ[s, 0]
```

Rule 6610

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\int (g+hx)^2 \left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^3 dx = \int \left(A^3(g+hx)^2+3A^2B(g+hx)^2\log\left(e(a+bx)^n(c+dx)^{-n}\right.\right.$$

$$= \frac{A^3(g+hx)^3}{3h} + \left(3A^2B\right)\int (g+hx)^2\log\left(e(a+bx)^n(c+dx)\right.$$

$$= \frac{A^3(g+hx)^3}{3h} + \frac{A^2B(g+hx)^3\log\left(e(a+bx)^n(c+dx)^{-n}\right)}{h} +$$

$$= \frac{A^3(g+hx)^3}{3h} + \frac{A^2B(g+hx)^3\log\left(e(a+bx)^n(c+dx)^{-n}\right)}{h} +$$

$$= -\frac{A^2B(bc-ad)h(3bdg-bch-adh)nx}{b^2d^2} - \frac{A^2B(bc-ad)h^2nx}{2bd}$$

$$= -\frac{A^2B(bc-ad)h(3bdg-bch-adh)nx}{b^2d^2} - \frac{A^2B(bc-ad)h^2nx}{2bd}$$

$$= -\frac{A^2B(bc-ad)h(3bdg-bch-adh)nx}{b^2d^2} - \frac{A^2B(bc-ad)h^2nx}{2bd}$$

$$= -\frac{A^2B(bc-ad)h(3bdg-bch-adh)nx}{b^2d^2} + \frac{AB^2(bc-ad)^2h^2n}{b^2d^2}$$

$$= -\frac{A^2B(bc-ad)h(3bdg-bch-adh)nx}{b^2d^2} + \frac{AB^2(bc-ad)^2h^2n}{b^2d^2}$$

$$= -\frac{A^2B(bc-ad)h(3bdg-bch-adh)nx}{b^2d^2} + \frac{AB^2(bc-ad)^2h^2n}{b^2d^2}$$

$$= -\frac{A^2B(bc-ad)h(3bdg-bch-adh)nx}{b^2d^2} + \frac{AB^2(bc-ad)^2h^2n}{b^2d^2}$$

$$= -\frac{A^2B(bc-ad)h(3bdg-bch-adh)nx}{b^2d^2} + \frac{AB^2(bc-ad)^2h^2n}{b^2d^2}$$

$$= -\frac{A^2B(bc-ad)h(3bdg-bch-adh)nx}{b^2d^2} + \frac{AB^2(bc-ad)^2h^2n}{b^2d^2}$$

**Mathematica** [F] time = 6.02, size = 0, normalized size = 0.00

$$\int (g+hx)^2 \left(A+B\log \left(e(a+bx)^n(c+dx)^{-n}\right)\right)^3 \, dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(g + h*x)^2*(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])^3,x]
```

```
[Out] Integrate[(g + h*x)^2*(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])^3, x]
```

**fricas** [F] time = 0.79, size = 0, normalized size = 0.00

$$\text{integral}\left(A^3h^2x^2 + 2\,A^3ghx + A^3g^2 + \left(B^3h^2x^2 + 2\,B^3ghx + B^3g^2\right)\log\left(\frac{(bx+a)^n e}{(dx+c)^n}\right)^3 + 3\left(AB^2h^2x^2 + 2\,AB^2ghx\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x+g)^2*(A+B*log(e*(b*x+a)^n/((d*x+c)^n)))^3,x, algorithm="fric
as")
```

```
[Out] integral(A^3*h^2*x^2 + 2*A^3*g*h*x + A^3*g^2 + (B^3*h^2*x^2 + 2*B^3*g*h*x +
 B^3*g^2)*log((b*x + a)^n*e/(d*x + c)^n)^3 + 3*(A*B^2*h^2*x^2 + 2*A*B^2*g*h
*x + A*B^2*g^2)*log((b*x + a)^n*e/(d*x + c)^n)^2 + 3*(A^2*B*h^2*x^2 + 2*A^2
*B*g*h*x + A^2*B*g^2)*log((b*x + a)^n*e/(d*x + c)^n), x)
```

**giac** [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x+g)^2*(A+B*log(e*(b*x+a)^n/((d*x+c)^n)))^3,x, algorithm="giac
")
```

```
[Out] Timed out
```

**maple** [F] time = 8.29, size = 0, normalized size = 0.00

$$\int \left(hx+g\right)^{2} \left(B\ln\left(e\left(bx+a\right)^{n}\left(dx+c\right)^{-n}\right)+A\right)^{3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((h*x+g)^2*(A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))^3,x)
```

```
[Out] int((h*x+g)^2*(A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))^3,x)
```

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x+g)^2*(A+B*log(e*(b*x+a)^n/((d*x+c)^n)))^3,x, algorithm="maxi
ma")
```

```
[Out] A^2*B*h^2*x^3*log((b*x + a)^n*e/(d*x + c)^n) + 1/3*A^3*h^2*x^3 + 3*A^2*B*g*
h*x^2*log((b*x + a)^n*e/(d*x + c)^n) + A^3*g*h*x^2 + 3*A^2*B*g^2*x*log((b*x
 + a)^n*e/(d*x + c)^n) + A^3*g^2*x + 3*(a*e*n*log(b*x + a)/b - c*e*n*log(d*
x + c)/d)*A^2*B*g^2/e - 3*(a^2*e*n*log(b*x + a)/b^2 - c^2*e*n*log(d*x + c)/
d^2 + (b*c*e*n - a*d*e*n)*x/(b*d))*A^2*B*g*h/e + 1/2*(2*a^3*e*n*log(b*x + a
)/b^3 - 2*c^3*e*n*log(d*x + c)/d^3 - ((b^2*c*d*e*n - a*b*d^2*e*n)*x^2 - 2*(
b^2*c^2*e*n - a^2*d^2*e*n)*x)/(b^2*d^2))*A^2*B*h^2/e - 1/6*(2*(B^3*b^3*d^3*
h^2*x^3 + 3*B^3*b^3*d^3*g*h*x^2 + 3*B^3*b^3*d^3*g^2*x)*log((d*x + c)^n)^3 +
 3*(2*(3*c*d^2*g^2*n - 3*c^2*d*g*h*n + c^3*h^2*n)*B^3*b^3*log(d*x + c) - 2*
(3*a*b^2*d^3*g^2*n - 3*a^2*b*d^3*g*h*n + a^3*d^3*h^2*n)*B^3*log(b*x + a) -
2*(B^3*b^3*d^3*h^2*log(e) + A*B^2*b^3*d^3*h^2)*x^3 - (6*A*B^2*b^3*d^3*g*h +
 (a*b^2*d^3*h^2*n - (c*d^2*h^2*n - 6*d^3*g*h*log(e))*b^3)*B^3)*x^2 - 2*(3*A
*B^2*b^3*d^3*g^2 + (3*a*b^2*d^3*g*h*n - a^2*b*d^3*h^2*n - (3*c*d^2*g*h*n -
c^2*d*h^2*n - 3*d^3*g^2*log(e))*b^3)*B^3)*x - 2*(B^3*b^3*d^3*h^2*x^3 + 3*B^
3*b^3*d^3*g*h*x^2 + 3*B^3*b^3*d^3*g^2*x)*log((b*x + a)^n))*log((d*x + c)^n)
^2)/(b^3*d^3) - integrate(-(B^3*b^3*c*d^2*g^2*log(e)^3 + 3*A*B^2*b^3*c*d^2*
g^2*log(e)^2 + (B^3*b^3*d^3*h^2*log(e)^3 + 3*A*B^2*b^3*d^3*h^2*log(e)^2)*x^
3 + (B^3*b^3*d^3*h^2*x^3 + B^3*b^3*c*d^2*g^2 + (2*d^3*g*h + c*d^2*h^2)*B^3*
b^3*x^2 + (d^3*g^2 + 2*c*d^2*g*h)*B^3*b^3*x)*log((b*x + a)^n)^3 + (3*(2*d^3
*g*h*log(e)^2 + c*d^2*h^2*log(e)^2)*A*B^2*b^3 + (2*d^3*g*h*log(e)^3 + c*d^2
*h^2*log(e)^3)*B^3*b^3)*x^2 + 3*(B^3*b^3*c*d^2*g^2*log(e) + A*B^2*b^3*c*d^2
*g^2 + (B^3*b^3*d^3*h^2*log(e) + A*B^2*b^3*d^3*h^2)*x^3 + ((2*d^3*g*h + c*d
^2*h^2)*A*B^2*b^3 + (2*d^3*g*h*log(e) + c*d^2*h^2*log(e))*B^3*b^3)*x^2 + ((
d^3*g^2 + 2*c*d^2*g*h)*A*B^2*b^3 + (d^3*g^2*log(e) + 2*c*d^2*g*h*log(e))*B^
3*b^3)*x)*log((b*x + a)^n)^2 + (3*(d^3*g^2*log(e)^2 + 2*c*d^2*g*h*log(e)^2)
*A*B^2*b^3 + (d^3*g^2*log(e)^3 + 2*c*d^2*g*h*log(e)^3)*B^3*b^3)*x + 3*(B^3*
b^3*c*d^2*g^2*log(e)^2 + 2*A*B^2*b^3*c*d^2*g^2*log(e) + (B^3*b^3*d^3*h^2*lo
g(e)^2 + 2*A*B^2*b^3*d^3*h^2*log(e))*x^3 + (2*(2*d^3*g*h*log(e) + c*d^2*h^2
*log(e))*A*B^2*b^3 + (2*d^3*g*h*log(e)^2 + c*d^2*h^2*log(e)^2)*B^3*b^3)*x^2
 + (2*(d^3*g^2*log(e) + 2*c*d^2*g*h*log(e))*A*B^2*b^3 + (d^3*g^2*log(e)^2 +
 2*c*d^2*g*h*log(e)^2)*B^3*b^3)*x)*log((b*x + a)^n) - (3*B^3*b^3*c*d^2*g^2*
log(e)^2 + 6*A*B^2*b^3*c*d^2*g^2*log(e) - 2*(3*c*d^2*g^2*n^2 - 3*c^2*d*g*h*
n^2 + c^3*h^2*n^2)*B^3*b^3*log(d*x + c) + 2*(3*a*b^2*d^3*g^2*n^2 - 3*a^2*b*
d^3*g*h*n^2 + a^3*d^3*h^2*n^2)*B^3*log(b*x + a) + (2*(h^2*n + 3*h^2*log(e))
*A*B^2*b^3*d^3 + (2*h^2*n*log(e) + 3*h^2*log(e)^2)*B^3*b^3*d^3)*x^3 + (6*(c
*d^2*h^2*log(e) + (g*h*n + 2*g*h*log(e))*d^3)*A*B^2*b^3 + (a*b^2*d^3*h^2*n^
```

```
2 - ((h^2*n^2 - 3*h^2*log(e)^2)*c*d^2 - 6*(g*h*n*log(e) + g*h*log(e)^2)*d^3
)*b^3)*B^3)*x^2 + 3*(B^3*b^3*d^3*h^2*x^3 + B^3*b^3*c*d^2*g^2 + (2*d^3*g*h +
 c*d^2*h^2)*B^3*b^3*x^2 + (d^3*g^2 + 2*c*d^2*g*h)*B^3*b^3*x)*log((b*x + a)^
n)^2 + (6*(2*c*d^2*g*h*log(e) + (g^2*n + g^2*log(e))*d^3)*A*B^2*b^3 + (6*a*
b^2*d^3*g*h*n^2 - 2*a^2*b*d^3*h^2*n^2 + (2*c^2*d*h^2*n^2 - 6*(g*h*n^2 - g*h
*log(e)^2)*c*d^2 + 3*(2*g^2*n*log(e) + g^2*log(e)^2)*d^3)*b^3)*B^3)*x + 2*(
3*B^3*b^3*c*d^2*g^2*log(e) + 3*A*B^2*b^3*c*d^2*g^2 + (3*A*B^2*b^3*d^3*h^2 +
 (h^2*n + 3*h^2*log(e))*B^3*b^3*d^3)*x^3 + 3*((2*d^3*g*h + c*d^2*h^2)*A*B^2
*b^3 + (c*d^2*h^2*log(e) + (g*h*n + 2*g*h*log(e))*d^3)*B^3*b^3)*x^2 + 3*((d
^3*g^2 + 2*c*d^2*g*h)*A*B^2*b^3 + (2*c*d^2*g*h*log(e) + (g^2*n + g^2*log(e)
)*d^3)*B^3*b^3)*x)*log((b*x + a)^n))*log((d*x + c)^n))/(b^3*d^3*x + b^3*c*d
^2), x)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.00

$$\int {\left (g+h\,x\right )}^2\,{\left (A+B\,\ln \left (\frac {e\,{\left (a+b\,x\right )}^n}{{\left (c+d\,x\right )}^n}\right )\right )}^3 \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g + h*x)^2*(A + B*log((e*(a + b*x)^n)/(c + d*x)^n))^3,x)
[Out] int((g + h*x)^2*(A + B*log((e*(a + b*x)^n)/(c + d*x)^n))^3, x)
```

**sympy [F(-2)]** time = 0.00, size = 0, normalized size = 0.00

Exception raised: HeuristicGCDFailed

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x+g)**2*(A+B*ln(e*(b*x+a)**n/((d*x+c)**n)))**3,x)
[Out] Exception raised: HeuristicGCDFailed
```

# 3.310 $\int (g+hx)\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^3\,dx$

**Optimal**. Leaf size=466

$$\frac{3B^2n^2(bc-ad)(-adh-bch+2bdg)\mathrm{Li}_2\left(\frac{d(a+bx)}{b(c+dx)}\right)\left(B\log\left(e(a+bx)^n(c+dx)^{-n}\right)+A\right)}{b^2d^2}-\frac{3B^2hn^2(bc-ad)^2\log\left(\frac{bc-ad}{b(c+dx)}\right.}{}$$

```
[Out] -3*B^2*(-a*d+b*c)^2*h*n^2*ln((-a*d+b*c)/b/(d*x+c))*(A+B*ln(e*(b*x+a)^n/((d*
x+c)^n)))/b^2/d^2-3/2*B*(-a*d+b*c)*h*n*(b*x+a)*(A+B*ln(e*(b*x+a)^n/((d*x+c)
^n)))^2/b^2/d+3/2*B*(-a*d+b*c)*(-a*d*h-b*c*h+2*b*d*g)*n*ln((-a*d+b*c)/b/(d*
x+c))*(A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))^2/b^2/d^2-1/2*(-a*h+b*g)^2*(A+B*ln(
e*(b*x+a)^n/((d*x+c)^n)))^3/b^2/h+1/2*(h*x+g)^2*(A+B*ln(e*(b*x+a)^n/((d*x+c
)^n)))^3/h-3*B^3*(-a*d+b*c)^2*h*n^3*polylog(2,d*(b*x+a)/b/(d*x+c))/b^2/d^2+
3*B^2*(-a*d+b*c)*(-a*d*h-b*c*h+2*b*d*g)*n^2*(A+B*ln(e*(b*x+a)^n/((d*x+c)^n)
))*polylog(2,d*(b*x+a)/b/(d*x+c))/b^2/d^2-3*B^3*(-a*d+b*c)*(-a*d*h-b*c*h+2*
b*d*g)*n^3*polylog(3,d*(b*x+a)/b/(d*x+c))/b^2/d^2
```

**Rubi [B]** time = 2.11, antiderivative size = 1030, normalized size of antiderivative = 2.21, number of steps used = 35, number of rules used = 13, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.419$, Rules used = {6742, 2492, 72, 2514, 2486, 31, 2488, 2411, 2343, 2333, 2315, 2506, 6610}

$$\frac{(g+hx)^2A^3}{2h}-\frac{3B(bc-ad)hnxA^2}{2bd}-\frac{3B(bg-ah)^2n\log(a+bx)A^2}{2b^2h}+\frac{3B(dg-ch)^2n\log(c+dx)A^2}{2d^2h}+\frac{3B(g+hx)^2\log}{}$$

Antiderivative was successfully verified.

```
[In] Int[(g + h*x)*(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])^3,x]
```

```
[Out] (-3*A^2*B*(b*c - a*d)*h*n*x)/(2*b*d) + (A^3*(g + h*x)^2)/(2*h) - (3*A^2*B*(
b*g - a*h)^2*n*Log[a + b*x])/(2*b^2*h) + (3*A^2*B*(d*g - c*h)^2*n*Log[c + d
*x])/(2*d^2*h) + (3*A*B^2*(b*c - a*d)^2*h*n^2*Log[c + d*x])/(b^2*d^2) - (3*
A*B^2*(b*c - a*d)*h*n*(a + b*x)*Log[(e*(a + b*x)^n)/(c + d*x)^n])/(b^2*d) +
 (3*A^2*B*(g + h*x)^2*Log[(e*(a + b*x)^n)/(c + d*x)^n])/(2*h) + (3*A*B^2*(b
*g - a*h)^2*n*Log[-((b*c - a*d)/(d*(a + b*x)))]*Log[(e*(a + b*x)^n)/(c + d*
x)^n])/(b^2*h) - (3*A*B^2*(d*g - c*h)^2*n*Log[(b*c - a*d)/(b*(c + d*x))]*Lo
g[(e*(a + b*x)^n)/(c + d*x)^n])/(d^2*h) - (3*B^3*(b*c - a*d)^2*h*n^2*Log[(b
*c - a*d)/(b*(c + d*x))]*Log[(e*(a + b*x)^n)/(c + d*x)^n])/(b^2*d^2) - (3*B
^3*(b*c - a*d)*h*n*(a + b*x)*Log[(e*(a + b*x)^n)/(c + d*x)^n]^2)/(2*b^2*d)
+ (3*A*B^2*(g + h*x)^2*Log[(e*(a + b*x)^n)/(c + d*x)^n]^2)/(2*h) + (3*B^3*(
b*g - a*h)^2*n*Log[-((b*c - a*d)/(d*(a + b*x)))]*Log[(e*(a + b*x)^n)/(c + d
*x)^n]^2)/(2*b^2*h) - (3*B^3*(d*g - c*h)^2*n*Log[(b*c - a*d)/(b*(c + d*x))]
*Log[(e*(a + b*x)^n)/(c + d*x)^n]^2)/(2*d^2*h) + (B^3*(g + h*x)^2*Log[(e*(a
```

```
 + b*x)^n)/(c + d*x)^n]^3)/(2*h) - (3*A*B^2*(d*g - c*h)^2*n^2*PolyLog[2, (d
*(a + b*x))/(b*(c + d*x))])/(d^2*h) - (3*B^3*(b*c - a*d)^2*h*n^3*PolyLog[2,
 (d*(a + b*x))/(b*(c + d*x))])/(b^2*d^2) - (3*A*B^2*(b*g - a*h)^2*n^2*PolyL
og[2, 1 + (b*c - a*d)/(d*(a + b*x))])/(b^2*h) - (3*B^3*(b*g - a*h)^2*n^2*Lo
g[(e*(a + b*x)^n)/(c + d*x)^n]*PolyLog[2, 1 + (b*c - a*d)/(d*(a + b*x))])/(
b^2*h) - (3*B^3*(d*g - c*h)^2*n^2*Log[(e*(a + b*x)^n)/(c + d*x)^n]*PolyLog[
2, 1 - (b*c - a*d)/(b*(c + d*x))])/(d^2*h) - (3*B^3*(b*g - a*h)^2*n^3*PolyL
og[3, 1 + (b*c - a*d)/(d*(a + b*x))])/(b^2*h) + (3*B^3*(d*g - c*h)^2*n^3*Po
lyLog[3, 1 - (b*c - a*d)/(b*(c + d*x))])/(d^2*h)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 72

```
Int[((e_.) + (f_.)*(x_))^(p_.)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
 x_Symbol] :> Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x]
 /; FreeQ[{a, b, c, d, e, f}, x] && IntegerQ[p]
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2333

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)/(x_))^(q_.)*(
x_)^(m_.), x_Symbol] :> Int[(e + d*x)^q*(a + b*Log[c*x^n])^p, x] /; FreeQ[{
a, b, c, d, e, m, n, p}, x] && EqQ[m, q] && IntegerQ[q]
```

Rule 2343

```
Int[((a_.) + Log[(c_.)*(x_)^(n_)]*(b_.))/((x_)*((d_) + (e_.)*(x_)^(r_.))),
x_Symbol] :> Dist[1/n, Subst[Int[(a + b*Log[c*x])/(x*(d + e*x^(r/n))), x],
x, x^n], x] /; FreeQ[{a, b, c, d, e, n, r}, x] && IntegerQ[r/n]
```

Rule 2411

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + (g_
.)*(x_))^(q_.)*((h_.) + (i_.)*(x_))^(r_.), x_Symbol] :> Dist[1/e, Subst[Int
[((g*x)/e)^q*((e*h - d*i)/e + (i*x)/e)^r*(a + b*Log[c*x^n])^p, x], x, d + e
*x], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, n, p, q, r}, x] && EqQ[e*f - d
*g, 0] && (IGtQ[p, 0] || IGtQ[r, 0]) && IntegerQ[2*r]
```

Rule 2486

```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))
^(r_.)]^(s_.), x_Symbol] :> Simp[((a + b*x)*Log[e*(f*(a + b*x)^p*(c + d*x)^
q)^r]^s)/b, x] + Dist[(q*r*s*(b*c - a*d))/b, Int[Log[e*(f*(a + b*x)^p*(c +
d*x)^q)^r]^(s - 1)/(c + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, p, q, r, s
}, x] && NeQ[b*c - a*d, 0] && EqQ[p + q, 0] && IGtQ[s, 0]
```

Rule 2488

```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))
^(r_.)]^(s_.)/((g_.) + (h_.)*(x_)), x_Symbol] :> -Simp[(Log[-((b*c - a*d)/(
d*(a + b*x)))]*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^s)/h, x] + Dist[(p*r*s*
(b*c - a*d))/h, Int[(Log[-((b*c - a*d)/(d*(a + b*x)))]*Log[e*(f*(a + b*x)^p
*(c + d*x)^q)^r]^(s - 1))/((a + b*x)*(c + d*x)), x], x] /; FreeQ[{a, b, c,
d, e, f, g, h, p, q, r, s}, x] && NeQ[b*c - a*d, 0] && EqQ[p + q, 0] && EqQ
[b*g - a*h, 0] && IGtQ[s, 0]
```

Rule 2492

```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))
^(r_.)]^(s_.)*((g_.) + (h_.)*(x_))^(m_.), x_Symbol] :> Simp[((g + h*x)^(m +
 1)*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^s)/(h*(m + 1)), x] - Dist[(p*r*s*(
b*c - a*d))/(h*(m + 1)), Int[((g + h*x)^(m + 1)*Log[e*(f*(a + b*x)^p*(c + d
*x)^q)^r]^(s - 1))/((a + b*x)*(c + d*x)), x], x] /; FreeQ[{a, b, c, d, e, f
, g, h, m, p, q, r, s}, x] && NeQ[b*c - a*d, 0] && EqQ[p + q, 0] && IGtQ[s,
 0] && NeQ[m, -1]
```

Rule 2506

```
Int[Log[v_]*Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_)
)^(q_.))^(r_.)]^(s_.)*(u_), x_Symbol] :> With[{g = Simplify[((v - 1)*(c + d
*x))/(a + b*x)], h = Simplify[u*(a + b*x)*(c + d*x)]}, -Simp[(h*PolyLog[2,
1 - v]*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^s)/(b*c - a*d), x] + Dist[h*p*r
*s, Int[(PolyLog[2, 1 - v]*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^(s - 1))/((
a + b*x)*(c + d*x)), x], x] /; FreeQ[{g, h}, x]] /; FreeQ[{a, b, c, d, e, f
, p, q, r, s}, x] && NeQ[b*c - a*d, 0] && IGtQ[s, 0] && EqQ[p + q, 0]
```

Rule 2514

```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))
^(r_.)]^(s_.)*(RFx_), x_Symbol] :> With[{u = ExpandIntegrand[Log[e*(f*(a +
b*x)^p*(c + d*x)^q)^r]^s, RFx, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c
, d, e, f, p, q, r, s}, x] && RationalFunctionQ[RFx, x] && IGtQ[s, 0]
```

Rule 6610

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\int (g+hx)\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^3 dx = \int \left(A^3(g+hx)+3A^2B(g+hx)\log\left(e(a+bx)^n(c+dx)^{-n}\right)+\right.$$

$$= \frac{A^3(g+hx)^2}{2h} + \left(3A^2B\right)\int (g+hx)\log\left(e(a+bx)^n(c+dx)^{-n}\right.$$

$$= \frac{A^3(g+hx)^2}{2h} + \frac{3A^2B(g+hx)^2\log\left(e(a+bx)^n(c+dx)^{-n}\right)}{2h} +$$

$$= \frac{A^3(g+hx)^2}{2h} + \frac{3A^2B(g+hx)^2\log\left(e(a+bx)^n(c+dx)^{-n}\right)}{2h} +$$

$$= -\frac{3A^2B(bc-ad)hnx}{2bd} + \frac{A^3(g+hx)^2}{2h} - \frac{3A^2B(bg-ah)^2n\log}{2b^2h}$$

$$= -\frac{3A^2B(bc-ad)hnx}{2bd} + \frac{A^3(g+hx)^2}{2h} - \frac{3A^2B(bg-ah)^2n\log}{2b^2h}$$

$$= -\frac{3A^2B(bc-ad)hnx}{2bd} + \frac{A^3(g+hx)^2}{2h} - \frac{3A^2B(bg-ah)^2n\log}{2b^2h}$$

$$= -\frac{3A^2B(bc-ad)hnx}{2bd} + \frac{A^3(g+hx)^2}{2h} - \frac{3A^2B(bg-ah)^2n\log}{2b^2h}$$

$$= -\frac{3A^2B(bc-ad)hnx}{2bd} + \frac{A^3(g+hx)^2}{2h} - \frac{3A^2B(bg-ah)^2n\log}{2b^2h}$$

$$= -\frac{3A^2B(bc-ad)hnx}{2bd} + \frac{A^3(g+hx)^2}{2h} - \frac{3A^2B(bg-ah)^2n\log}{2b^2h}$$

$$= -\frac{3A^2B(bc-ad)hnx}{2bd} + \frac{A^3(g+hx)^2}{2h} - \frac{3A^2B(bg-ah)^2n\log}{2b^2h}$$

**Mathematica [F]** time = 3.29, size = 0, normalized size = 0.00

$$\int (g+hx)\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^3 dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(g + h*x)*(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])^3,x]
```

```
[Out] Integrate[(g + h*x)*(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])^3, x]
```

**fricas [F]** time = 0.94, size = 0, normalized size = 0.00

$$\text{integral}\left(A^3hx + A^3g + \left(B^3hx + B^3g\right)\log\left(\frac{(bx+a)^n e}{(dx+c)^n}\right)^3 + 3\left(AB^2hx + AB^2g\right)\log\left(\frac{(bx+a)^n e}{(dx+c)^n}\right)^2 + 3\left(A^2Bhx + \right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x+g)*(A+B*log(e*(b*x+a)^n/((d*x+c)^n)))^3,x, algorithm="fricas
")
```

```
[Out] integral(A^3*h*x + A^3*g + (B^3*h*x + B^3*g)*log((b*x + a)^n*e/(d*x + c)^n)
^3 + 3*(A*B^2*h*x + A*B^2*g)*log((b*x + a)^n*e/(d*x + c)^n)^2 + 3*(A^2*B*h*
x + A^2*B*g)*log((b*x + a)^n*e/(d*x + c)^n), x)
```

**giac [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x+g)*(A+B*log(e*(b*x+a)^n/((d*x+c)^n)))^3,x, algorithm="giac")
```

```
[Out] Timed out
```

**maple [F]** time = 8.96, size = 0, normalized size = 0.00

$$\int \left(hx+g\right)\left(B\ln\left(e\left(bx+a\right)^{n}\left(dx+c\right)^{-n}\right)+A\right)^{3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((h*x+g)*(A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))^3,x)
```

```
[Out] int((h*x+g)*(A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))^3,x)
```

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x+g)*(A+B*log(e*(b*x+a)^n/((d*x+c)^n)))^3,x, algorithm="maxima
")
```

```
[Out] 3/2*A^2*B*h*x^2*log((b*x + a)^n*e/(d*x + c)^n) + 1/2*A^3*h*x^2 + 3*A^2*B*g*
x*log((b*x + a)^n*e/(d*x + c)^n) + A^3*g*x + 3*(a*e*n*log(b*x + a)/b - c*e*
n*log(d*x + c)/d)*A^2*B*g/e - 3/2*(a^2*e*n*log(b*x + a)/b^2 - c^2*e*n*log(d
*x + c)/d^2 + (b*c*e*n - a*d*e*n)*x/(b*d))*A^2*B*h/e - 1/2*((B^3*b^2*d^2*h*
x^2 + 2*B^3*b^2*d^2*g*x)*log((d*x + c)^n)^3 + 3*((2*c*d*g*n - c^2*h*n)*B^3*
b^2*log(d*x + c) - (2*a*b*d^2*g*n - a^2*d^2*h*n)*B^3*log(b*x + a) - (B^3*b^
2*d^2*h*log(e) + A*B^2*b^2*d^2*h)*x^2 - (2*A*B^2*b^2*d^2*g + (a*b*d^2*h*n -
 (c*d*h*n - 2*d^2*g*log(e))*b^2)*B^3)*x - (B^3*b^2*d^2*h*x^2 + 2*B^3*b^2*d^
2*g*x)*log((b*x + a)^n))*log((d*x + c)^n)^2)/(b^2*d^2) - integrate(-(B^3*b^
2*c*d*g*log(e)^3 + 3*A*B^2*b^2*c*d*g*log(e)^2 + (B^3*b^2*d^2*h*x^2 + B^3*b^
2*c*d*g + (d^2*g + c*d*h)*B^3*b^2*x)*log((b*x + a)^n)^3 + (B^3*b^2*d^2*h*lo
g(e)^3 + 3*A*B^2*b^2*d^2*h*log(e)^2)*x^2 + 3*(B^3*b^2*c*d*g*log(e) + A*B^2*
b^2*c*d*g + (B^3*b^2*d^2*h*log(e) + A*B^2*b^2*d^2*h)*x^2 + ((d^2*g + c*d*h)
*A*B^2*b^2 + (d^2*g*log(e) + c*d*h*log(e))*B^3*b^2)*x)*log((b*x + a)^n)^2 +
 (3*(d^2*g*log(e)^2 + c*d*h*log(e)^2)*A*B^2*b^2 + (d^2*g*log(e)^3 + c*d*h*l
og(e)^3)*B^3*b^2)*x + 3*(B^3*b^2*c*d*g*log(e)^2 + 2*A*B^2*b^2*c*d*g*log(e)
+ (B^3*b^2*d^2*h*log(e)^2 + 2*A*B^2*b^2*d^2*h*log(e))*x^2 + (2*(d^2*g*log(e
) + c*d*h*log(e))*A*B^2*b^2 + (d^2*g*log(e)^2 + c*d*h*log(e)^2)*B^3*b^2)*x)
*log((b*x + a)^n) - 3*(B^3*b^2*c*d*g*log(e)^2 + 2*A*B^2*b^2*c*d*g*log(e) -
(2*c*d*g*n^2 - c^2*h*n^2)*B^3*b^2*log(d*x + c) + (2*a*b*d^2*g*n^2 - a^2*d^2
*h*n^2)*B^3*log(b*x + a) + ((h*n + 2*h*log(e))*A*B^2*b^2*d^2 + (h*n*log(e)
+ h*log(e)^2)*B^3*b^2*d^2)*x^2 + (B^3*b^2*d^2*h*x^2 + B^3*b^2*c*d*g + (d^2*
g + c*d*h)*B^3*b^2*x)*log((b*x + a)^n)^2 + (2*(c*d*h*log(e) + (g*n + g*log(
e))*d^2)*A*B^2*b^2 + (a*b*d^2*h*n^2 - ((h*n^2 - h*log(e)^2)*c*d - (2*g*n*lo
g(e) + g*log(e)^2)*d^2)*b^2)*B^3)*x + (2*B^3*b^2*c*d*g*log(e) + 2*A*B^2*b^2
*c*d*g + ((h*n + 2*h*log(e))*B^3*b^2*d^2 + 2*A*B^2*b^2*d^2*h)*x^2 + 2*((d^2
*g + c*d*h)*A*B^2*b^2 + (c*d*h*log(e) + (g*n + g*log(e))*d^2)*B^3*b^2)*x)*l
og((b*x + a)^n))*log((d*x + c)^n))/(b^2*d^2*x + b^2*c*d), x)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.00

$$\int \left(g+h\,x\right)\,{\left(A+B\,\ln\left(\frac{e\,{\left(a+b\,x\right)}^n}{{\left(c+d\,x\right)}^n}\right)\right)}^3 \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g + h*x)*(A + B*log((e*(a + b*x)^n)/(c + d*x)^n))^3,x)
```

```
[Out] int((g + h*x)*(A + B*log((e*(a + b*x)^n)/(c + d*x)^n))^3, x)
```

**sympy [F(-2)]** time = 0.00, size = 0, normalized size = 0.00

Exception raised: HeuristicGCDFailed

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x+g)*(A+B*ln(e*(b*x+a)**n/((d*x+c)**n)))**3,x)
[Out] Exception raised: HeuristicGCDFailed
```

## 3.311 $\int \left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^3\,dx$

**Optimal**. Leaf size=203

$$\frac{6B^2n^2(bc-ad)\text{Li}_2\left(\frac{d(a+bx)}{b(c+dx)}\right)\left(B\log\left(e(a+bx)^n(c+dx)^{-n}\right)+A\right)}{bd}+\frac{3Bn(bc-ad)\log\left(\frac{bc-ad}{b(c+dx)}\right)\left(B\log\left(e(a+bx)^n(c+\right.\right.}{bd}$$

```
[Out] 3*B*(-a*d+b*c)*n*ln((-a*d+b*c)/b/(d*x+c))*(A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))
^2/b/d+(b*x+a)*(A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))^3/b+6*B^2*(-a*d+b*c)*n^2*(
A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))*polylog(2,d*(b*x+a)/b/(d*x+c))/b/d-6*B^3*(
-a*d+b*c)*n^3*polylog(3,d*(b*x+a)/b/(d*x+c))/b/d
```

**Rubi [B]** time = 0.59, antiderivative size = 408, normalized size of antiderivative = 2.01, number of steps used = 14, number of rules used = 10, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {6742, 2486, 31, 2488, 2411, 2343, 2333, 2315, 2506, 6610}

$$\frac{6AB^2n^2(bc-ad)\text{PolyLog}\left(2,\frac{d(a+bx)}{b(c+dx)}\right)}{bd}+\frac{6B^3n^2(bc-ad)\text{PolyLog}\left(2,1-\frac{bc-ad}{b(c+dx)}\right)\log\left(e(a+bx)^n(c+dx)^{-n}\right)}{bd}-\frac{6B^3n^3}{}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])^3,x]
```

```
[Out] A^3*x - (3*A^2*B*(b*c - a*d)*n*Log[c + d*x])/(b*d) + (3*A^2*B*(a + b*x)*Log
[(e*(a + b*x)^n)/(c + d*x)^n])/b + (6*A*B^2*(b*c - a*d)*n*Log[(b*c - a*d)/(
b*(c + d*x))]*Log[(e*(a + b*x)^n)/(c + d*x)^n])/(b*d) + (3*A*B^2*(a + b*x)*
Log[(e*(a + b*x)^n)/(c + d*x)^n]^2)/b + (3*B^3*(b*c - a*d)*n*Log[(b*c - a*d
)/(b*(c + d*x))]*Log[(e*(a + b*x)^n)/(c + d*x)^n]^2)/(b*d) + (B^3*(a + b*x)
*Log[(e*(a + b*x)^n)/(c + d*x)^n]^3)/b + (6*A*B^2*(b*c - a*d)*n^2*PolyLog[2
, (d*(a + b*x))/(b*(c + d*x))])/(b*d) + (6*B^3*(b*c - a*d)*n^2*Log[(e*(a +
b*x)^n)/(c + d*x)^n]*PolyLog[2, 1 - (b*c - a*d)/(b*(c + d*x))])/(b*d) - (6*
B^3*(b*c - a*d)*n^3*PolyLog[3, 1 - (b*c - a*d)/(b*(c + d*x))])/(b*d)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2333

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)/(x_))^(q_.)*(
x_)^(m_.), x_Symbol] :> Int[(e + d*x)^q*(a + b*Log[c*x^n])^p, x] /; FreeQ[{
a, b, c, d, e, m, n, p}, x] && EqQ[m, q] && IntegerQ[q]
```

Rule 2343

```
Int[((a_.) + Log[(c_.)*(x_)^(n_)]*(b_.))/((x_)*((d_) + (e_.)*(x_)^(r_.))),
x_Symbol] :> Dist[1/n, Subst[Int[(a + b*Log[c*x])/(x*(d + e*x^(r/n))), x],
x, x^n], x] /; FreeQ[{a, b, c, d, e, n, r}, x] && IntegerQ[r/n]
```

Rule 2411

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + (g_
.)*(x_))^(q_.)*((h_.) + (i_.)*(x_))^(r_.), x_Symbol] :> Dist[1/e, Subst[Int
[((g*x)/e)^q*((e*h - d*i)/e + (i*x)/e)^r*(a + b*Log[c*x^n])^p, x], x, d + e
*x], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, n, p, q, r}, x] && EqQ[e*f - d
*g, 0] && (IGtQ[p, 0] || IGtQ[r, 0]) && IntegerQ[2*r]
```

Rule 2486

```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))
^(r_.)]^(s_.), x_Symbol] :> Simp[((a + b*x)*Log[e*(f*(a + b*x)^p*(c + d*x)^
q)^r]^s)/b, x] + Dist[(q*r*s*(b*c - a*d))/b, Int[Log[e*(f*(a + b*x)^p*(c +
d*x)^q)^r]^(s - 1)/(c + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, p, q, r, s
}, x] && NeQ[b*c - a*d, 0] && EqQ[p + q, 0] && IGtQ[s, 0]
```

Rule 2488

```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))
^(r_.)]^(s_.)/((g_.) + (h_.)*(x_)), x_Symbol] :> -Simp[(Log[-((b*c - a*d)/(
d*(a + b*x)))]*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^s)/h, x] + Dist[(p*r*s*
(b*c - a*d))/h, Int[(Log[-((b*c - a*d)/(d*(a + b*x)))]*Log[e*(f*(a + b*x)^p
*(c + d*x)^q)^r]^(s - 1))/((a + b*x)*(c + d*x)), x], x] /; FreeQ[{a, b, c,
d, e, f, g, h, p, q, r, s}, x] && NeQ[b*c - a*d, 0] && EqQ[p + q, 0] && EqQ
[b*g - a*h, 0] && IGtQ[s, 0]
```

Rule 2506

```
Int[Log[v_]*Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_)
)^(q_.))^(r_.)]^(s_.)*(u_), x_Symbol] :> With[{g = Simplify[((v - 1)*(c + d
*x))/(a + b*x)], h = Simplify[u*(a + b*x)*(c + d*x)]}, -Simp[(h*PolyLog[2,
1 - v]*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^s)/(b*c - a*d), x] + Dist[h*p*r
*s, Int[(PolyLog[2, 1 - v]*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^(s - 1))/((
a + b*x)*(c + d*x)), x], x] /; FreeQ[{g, h}, x]] /; FreeQ[{a, b, c, d, e, f
```

```
, p, q, r, s}, x] && NeQ[b*c - a*d, 0] && IGtQ[s, 0] && EqQ[p + q, 0]
```

Rule 6610

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\int \left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^3\,dx=\int \left(A^3+3A^2B\log\left(e(a+bx)^n(c+dx)^{-n}\right)+3AB^2\log^2\left(e(a+bx)^n(c\right.\right.$$

$$=A^3x+\left(3A^2B\right)\int\log\left(e(a+bx)^n(c+dx)^{-n}\right)\,dx+\left(3AB^2\right)\int\log^2\left(e(\right.$$

$$=A^3x+\frac{3A^2B(a+bx)\log\left(e(a+bx)^n(c+dx)^{-n}\right)}{b}+\frac{3AB^2(a+bx)\log}{}$$

$$=A^3x-\frac{3A^2B(bc-ad)n\log(c+dx)}{bd}+\frac{3A^2B(a+bx)\log\left(e(a+bx)^n(\right.}{b}$$

$$=A^3x-\frac{3A^2B(bc-ad)n\log(c+dx)}{bd}+\frac{3A^2B(a+bx)\log\left(e(a+bx)^n(\right.}{b}$$

$$=A^3x-\frac{3A^2B(bc-ad)n\log(c+dx)}{bd}+\frac{3A^2B(a+bx)\log\left(e(a+bx)^n(\right.}{b}$$

$$=A^3x-\frac{3A^2B(bc-ad)n\log(c+dx)}{bd}+\frac{3A^2B(a+bx)\log\left(e(a+bx)^n(\right.}{b}$$

$$=A^3x-\frac{3A^2B(bc-ad)n\log(c+dx)}{bd}+\frac{3A^2B(a+bx)\log\left(e(a+bx)^n(\right.}{b}$$

**Mathematica [A]** time = 0.31, size = 378, normalized size = 1.86

$$\frac{3A^2Bd(a+bx)\log\left(e(a+bx)^n(c+dx)^{-n}\right)-3A^2Bn(bc-ad)\log(c+dx)+3AB^2n(bc-ad)\left(2n\text{Li}_2\left(\frac{b(c+dx)}{bc-ad}\right)-\text{lo}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])^3,x]
```

```
[Out] (A^3*b*d*x - 3*A^2*B*(b*c - a*d)*n*Log[c + d*x] + 3*A^2*B*d*(a + b*x)*Log[(
e*(a + b*x)^n)/(c + d*x)^n] + 3*A*B^2*d*(a + b*x)*Log[(e*(a + b*x)^n)/(c +
d*x)^n]^2 + B^3*d*(a + b*x)*Log[(e*(a + b*x)^n)/(c + d*x)^n]^3 + 3*A*B^2*(b
*c - a*d)*n*(-(Log[(b*c - a*d)/(b*c + b*d*x)]*(2*n*Log[(d*(a + b*x))/(-(b*c
) + a*d)] - 2*Log[(e*(a + b*x)^n)/(c + d*x)^n] + n*Log[(b*c - a*d)/(b*c + b
*d*x)])) + 2*n*PolyLog[2, (b*(c + d*x))/(b*c - a*d)]) + 3*B^3*(b*c - a*d)*n
*(Log[(e*(a + b*x)^n)/(c + d*x)^n]^2*Log[(b*c - a*d)/(b*c + b*d*x)] + 2*n*L
og[(e*(a + b*x)^n)/(c + d*x)^n]*PolyLog[2, (d*(a + b*x))/(b*(c + d*x))] - 2
*n^2*PolyLog[3, (d*(a + b*x))/(b*(c + d*x))]))/(b*d)
```

**fricas [F]** time = 2.94, size = 0, normalized size = 0.00

$$\text{integral}\left(B^3\log\left(\frac{(bx+a)^n e}{(dx+c)^n}\right)^3+3\,AB^2\log\left(\frac{(bx+a)^n e}{(dx+c)^n}\right)^2+3\,A^2B\log\left(\frac{(bx+a)^n e}{(dx+c)^n}\right)+A^3,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*(b*x+a)^n/((d*x+c)^n)))^3,x, algorithm="fricas")
```

```
[Out] integral(B^3*log((b*x + a)^n*e/(d*x + c)^n)^3 + 3*A*B^2*log((b*x + a)^n*e/(
d*x + c)^n)^2 + 3*A^2*B*log((b*x + a)^n*e/(d*x + c)^n) + A^3, x)
```

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int\left(B\log\left(\frac{(bx+a)^n e}{(dx+c)^n}\right)+A\right)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*(b*x+a)^n/((d*x+c)^n)))^3,x, algorithm="giac")
```

```
[Out] integrate((B*log((b*x + a)^n*e/(d*x + c)^n) + A)^3, x)
```

**maple [F]** time = 4.52, size = 0, normalized size = 0.00

$$\int\left(B\ln\left(e\,(bx+a)^n\,(dx+c)^{-n}\right)+A\right)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))^3,x)

[Out] int((A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))^3,x)

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$3\,A^2Bx\log\left(\frac{(bx+a)^n e}{(dx+c)^n}\right)+A^3x+\frac{3\left(\frac{aen\log(bx+a)}{b}-\frac{cen\log(dx+c)}{d}\right)A^2B}{e}-\frac{B^3bdx\log\left((dx+c)^n\right)^3-3\left(B^3adn\log\left(bx+a\right)\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*log(e*(b*x+a)^n/((d*x+c)^n)))^3,x, algorithm="maxima")

[Out] 3*A^2*B*x*log((b*x + a)^n*e/(d*x + c)^n) + A^3*x + 3*(a*e*n*log(b*x + a)/b - c*e*n*log(d*x + c)/d)*A^2*B/e - (B^3*b*d*x*log((d*x + c)^n)^3 - 3*(B^3*a*d*n*log(b*x + a) - B^3*b*c*n*log(d*x + c) + B^3*b*d*x*log((b*x + a)^n) + (B^3*b*d*log(e) + A*B^2*b*d)*x)*log((d*x + c)^n)^2)/(b*d) - integrate(-(B^3*b*c*log(e)^3 + 3*A*B^2*b*c*log(e)^2 + (B^3*b*d*x + B^3*b*c)*log((b*x + a)^n)^3 + 3*(B^3*b*c*log(e) + A*B^2*b*c + (B^3*b*d*log(e) + A*B^2*b*d)*x)*log((b*x + a)^n)^2 + (B^3*b*d*log(e)^3 + 3*A*B^2*b*d*log(e)^2)*x + 3*(B^3*b*c*log(e)^2 + 2*A*B^2*b*c*log(e) + (B^3*b*d*log(e)^2 + 2*A*B^2*b*d*log(e))*x)*log((b*x + a)^n) - 3*(2*B^3*a*d*n^2*log(b*x + a) - 2*B^3*b*c*n^2*log(d*x + c) + B^3*b*c*log(e)^2 + 2*A*B^2*b*c*log(e) + (B^3*b*d*x + B^3*b*c)*log((b*x + a)^n)^2 + ((2*n*log(e) + log(e)^2)*B^3*b*d + 2*A*B^2*b*d*(n + log(e)))*x + 2*(B^3*b*c*log(e) + A*B^2*b*c + (B^3*b*d*(n + log(e)) + A*B^2*b*d)*x)*log((b*x + a)^n))*log((d*x + c)^n))/(b*d*x + b*c), x)

**mupad [F]** time = 0.00, size = -1, normalized size = -0.00

$$\int {\left(A+B\,\ln\left(\frac{e\,{\left(a+b\,x\right)}^n}{{\left(c+d\,x\right)}^n}\right)\right)}^3 \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A + B*log((e*(a + b*x)^n)/(c + d*x)^n))^3,x)

[Out] int((A + B*log((e*(a + b*x)^n)/(c + d*x)^n))^3, x)

**sympy [F(-2)]** time = 0.00, size = 0, normalized size = 0.00

Exception raised: HeuristicGCDFailed

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*ln(e*(b*x+a)**n/((d*x+c)**n)))**3,x)

[Out] Exception raised: HeuristicGCDFailed

# 3.312 $\int \frac{\left(A+B\log(e(a+bx)^n(c+dx)^{-n})\right)^3}{g+hx}\,dx$

**Optimal**. Leaf size=425

$$-\frac{6B^2n^2\text{Li}_3\left(\frac{(dg-ch)(a+bx)}{(bg-ah)(c+dx)}\right)\left(B\log\left(e(a+bx)^n(c+dx)^{-n}\right)+A\right)}{h}+\frac{6B^2n^2\text{Li}_3\left(\frac{d(a+bx)}{b(c+dx)}\right)\left(B\log\left(e(a+bx)^n(c+dx)^{-n}\right)+\right.}{h}$$

```
[Out] -ln((-a*d+b*c)/b/(d*x+c))*(A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))^3/h+(A+B*ln(e*(
b*x+a)^n/((d*x+c)^n)))^3*ln(1-(-c*h+d*g)*(b*x+a)/(-a*h+b*g)/(d*x+c))/h-3*B*
n*(A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))^2*polylog(2,d*(b*x+a)/b/(d*x+c))/h+3*B*
n*(A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))^2*polylog(2,(-c*h+d*g)*(b*x+a)/(-a*h+b*
g)/(d*x+c))/h+6*B^2*n^2*(A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))*polylog(3,d*(b*x+
a)/b/(d*x+c))/h-6*B^2*n^2*(A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))*polylog(3,(-c*h
+d*g)*(b*x+a)/(-a*h+b*g)/(d*x+c))/h-6*B^3*n^3*polylog(4,d*(b*x+a)/b/(d*x+c)
)/h+6*B^3*n^3*polylog(4,(-c*h+d*g)*(b*x+a)/(-a*h+b*g)/(d*x+c))/h
```

**Rubi [B]** time = 1.64, antiderivative size = 921, normalized size of antiderivative = 2.17, number of steps used = 25, number of rules used = 11, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {6742, 2494, 2394, 2393, 2391, 2489, 2488, 2506, 6610, 2503, 2508}

$$\frac{\log(g+hx)A^3}{h}-\frac{3Bn\log\left(-\frac{h(a+bx)}{bg-ah}\right)\log(g+hx)A^2}{h}+\frac{3Bn\log\left(-\frac{h(c+dx)}{dg-ch}\right)\log(g+hx)A^2}{h}+\frac{3B\log\left(e(a+bx)^n(c+\right.}{h}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])^3/(g + h*x),x]
```

```
[Out] (-3*A*B^2*Log[(b*c - a*d)/(b*(c + d*x))]*Log[(e*(a + b*x)^n)/(c + d*x)^n]^2
)/h - (B^3*Log[(b*c - a*d)/(b*(c + d*x))]*Log[(e*(a + b*x)^n)/(c + d*x)^n]^
3)/h + (A^3*Log[g + h*x])/h - (3*A^2*B*n*Log[-((h*(a + b*x))/(b*g - a*h))]*
Log[g + h*x])/h + (3*A^2*B*n*Log[-((h*(c + d*x))/(d*g - c*h))]*Log[g + h*x]
)/h + (3*A^2*B*Log[(e*(a + b*x)^n)/(c + d*x)^n]*Log[g + h*x])/h + (3*A*B^2*
Log[(e*(a + b*x)^n)/(c + d*x)^n]^2*Log[((b*c - a*d)*(g + h*x))/((b*g - a*h)
*(c + d*x))])/h + (B^3*Log[(e*(a + b*x)^n)/(c + d*x)^n]^3*Log[((b*c - a*d)*
(g + h*x))/((b*g - a*h)*(c + d*x))])/h - (3*A^2*B*n*PolyLog[2, (b*(g + h*x)
)/(b*g - a*h)])/h + (3*A^2*B*n*PolyLog[2, (d*(g + h*x))/(d*g - c*h)])/h - (
6*A*B^2*n*Log[(e*(a + b*x)^n)/(c + d*x)^n]*PolyLog[2, 1 - (b*c - a*d)/(b*(c
 + d*x))])/h - (3*B^3*n*Log[(e*(a + b*x)^n)/(c + d*x)^n]^2*PolyLog[2, 1 - (
b*c - a*d)/(b*(c + d*x))])/h + (6*A*B^2*n*Log[(e*(a + b*x)^n)/(c + d*x)^n]*
PolyLog[2, 1 - ((b*c - a*d)*(g + h*x))/((b*g - a*h)*(c + d*x))])/h + (3*B^3
*n*Log[(e*(a + b*x)^n)/(c + d*x)^n]^2*PolyLog[2, 1 - ((b*c - a*d)*(g + h*x)
)/((b*g - a*h)*(c + d*x))])/h + (6*A*B^2*n^2*PolyLog[3, 1 - (b*c - a*d)/(b*
```

```
(c + d*x))])/h + (6*B^3*n^2*Log[(e*(a + b*x)^n)/(c + d*x)^n]*PolyLog[3, 1 -
 (b*c - a*d)/(b*(c + d*x))])/h - (6*A*B^2*n^2*PolyLog[3, 1 - ((b*c - a*d)*(
g + h*x))/((b*g - a*h)*(c + d*x))])/h - (6*B^3*n^2*Log[(e*(a + b*x)^n)/(c +
 d*x)^n]*PolyLog[3, 1 - ((b*c - a*d)*(g + h*x))/((b*g - a*h)*(c + d*x))])/h
 - (6*B^3*n^3*PolyLog[4, 1 - (b*c - a*d)/(b*(c + d*x))])/h + (6*B^3*n^3*Pol
yLog[4, 1 - ((b*c - a*d)*(g + h*x))/((b*g - a*h)*(c + d*x))])/h
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2393

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + (c*e*x)/g])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2394

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d + e*x
)^n]))/g, x] - Dist[(b*e*n)/g, Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2488

```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))
^(r_.)]^(s_.)/((g_.) + (h_.)*(x_)), x_Symbol] :> -Simp[(Log[-((b*c - a*d)/(
d*(a + b*x)))]*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^s)/h, x] + Dist[(p*r*s*
(b*c - a*d))/h, Int[(Log[-((b*c - a*d)/(d*(a + b*x)))]*Log[e*(f*(a + b*x)^p
*(c + d*x)^q)^r]^(s - 1))/((a + b*x)*(c + d*x)), x], x] /; FreeQ[{a, b, c,
d, e, f, g, h, p, q, r, s}, x] && NeQ[b*c - a*d, 0] && EqQ[p + q, 0] && EqQ
[b*g - a*h, 0] && IGtQ[s, 0]
```

Rule 2489

```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))
^(r_.)]^(s_)/((g_.) + (h_.)*(x_)), x_Symbol] :> Dist[d/h, Int[Log[e*(f*(a +
 b*x)^p*(c + d*x)^q)^r]^s/(c + d*x), x], x] - Dist[(d*g - c*h)/h, Int[Log[e
*(f*(a + b*x)^p*(c + d*x)^q)^r]^s/((c + d*x)*(g + h*x)), x], x] /; FreeQ[{a
, b, c, d, e, f, g, h, p, q, r, s}, x] && NeQ[b*c - a*d, 0] && EqQ[p + q, 0
] && NeQ[b*g - a*h, 0] && NeQ[d*g - c*h, 0] && IGtQ[s, 1]
```

Rule 2494

```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))
^(r_.)]/((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(Log[g + h*x]*Log[e*(f*(a +
 b*x)^p*(c + d*x)^q)^r])/h, x] + (-Dist[(b*p*r)/h, Int[Log[g + h*x]/(a + b*
x), x], x] - Dist[(d*q*r)/h, Int[Log[g + h*x]/(c + d*x), x], x]) /; FreeQ[{
a, b, c, d, e, f, g, h, p, q, r}, x] && NeQ[b*c - a*d, 0]
```

Rule 2503

```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))
^(r_.)]^(s_.)*(u_), x_Symbol] :> With[{g = Coeff[Simplify[1/(u*(a + b*x))],
 x, 0], h = Coeff[Simplify[1/(u*(a + b*x))], x, 1]}, -Simp[(Log[e*(f*(a + b
*x)^p*(c + d*x)^q)^r]^s*Log[-(((b*c - a*d)*(g + h*x))/((d*g - c*h)*(a + b*x
)))])/(b*g - a*h), x] + Dist[(p*r*s*(b*c - a*d))/(b*g - a*h), Int[(Log[e*(f
*(a + b*x)^p*(c + d*x)^q)^r]^(s - 1)*Log[-(((b*c - a*d)*(g + h*x))/((d*g -
c*h)*(a + b*x)))])/((a + b*x)*(c + d*x)), x], x] /; NeQ[b*g - a*h, 0] && Ne
Q[d*g - c*h, 0]] /; FreeQ[{a, b, c, d, e, f, p, q, r, s}, x] && NeQ[b*c - a
*d, 0] && IGtQ[s, 0] && EqQ[p + q, 0] && LinearQ[Simplify[1/(u*(a + b*x))],
 x]
```

Rule 2506

```
Int[Log[v_]*Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_)
)^(q_.))^(r_.)]^(s_.)*(u_), x_Symbol] :> With[{g = Simplify[((v - 1)*(c + d
*x))/(a + b*x)], h = Simplify[u*(a + b*x)*(c + d*x)]}, -Simp[(h*PolyLog[2,
1 - v]*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^s)/(b*c - a*d), x] + Dist[h*p*r
*s, Int[(PolyLog[2, 1 - v]*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^(s - 1))/((
a + b*x)*(c + d*x)), x], x] /; FreeQ[{g, h}, x]] /; FreeQ[{a, b, c, d, e, f
, p, q, r, s}, x] && NeQ[b*c - a*d, 0] && IGtQ[s, 0] && EqQ[p + q, 0]
```

Rule 2508

```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))
^(r_.)]^(s_.)*(u_)*PolyLog[n_, v_], x_Symbol] :> With[{g = Simplify[(v*(c +
 d*x))/(a + b*x)], h = Simplify[u*(a + b*x)*(c + d*x)]}, Simp[(h*PolyLog[n
+ 1, v]*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^s)/(b*c - a*d), x] - Dist[h*p*
r*s, Int[(PolyLog[n + 1, v]*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^(s - 1))/(
(a + b*x)*(c + d*x)), x], x] /; FreeQ[{g, h}, x]] /; FreeQ[{a, b, c, d, e,
f, n, p, q, r, s}, x] && NeQ[b*c - a*d, 0] && IGtQ[s, 0] && EqQ[p + q, 0]
```

Rule 6610

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\int \frac{\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^3}{g+hx}\,dx = \int \left(\frac{A^3}{g+hx}+\frac{3A^2B\log\left(e(a+bx)^n(c+dx)^{-n}\right)}{g+hx}+\frac{3AB^2\log^2\left(e(a+b\right.}{g+h}\right.$$

$$=\frac{A^3\log(g+hx)}{h}+\left(3A^2B\right)\int \frac{\log\left(e(a+bx)^n(c+dx)^{-n}\right)}{g+hx}\,dx+\left(3AB^2\right.$$

$$=\frac{A^3\log(g+hx)}{h}+\frac{3A^2B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\log(g+hx)}{h}+\frac{(3}{}$$

$$=-\frac{3AB^2\log\left(\frac{bc-ad}{b(c+dx)}\right)\log^2\left(e(a+bx)^n(c+dx)^{-n}\right)}{h}-\frac{B^3\log\left(\frac{bc-ad}{b(c+dx)}\right)\mathrm{lc}}{}$$

$$=-\frac{3AB^2\log\left(\frac{bc-ad}{b(c+dx)}\right)\log^2\left(e(a+bx)^n(c+dx)^{-n}\right)}{h}-\frac{B^3\log\left(\frac{bc-ad}{b(c+dx)}\right)\mathrm{lc}}{}$$

$$=-\frac{3AB^2\log\left(\frac{bc-ad}{b(c+dx)}\right)\log^2\left(e(a+bx)^n(c+dx)^{-n}\right)}{h}-\frac{B^3\log\left(\frac{bc-ad}{b(c+dx)}\right)\mathrm{lc}}{}$$

$$=-\frac{3AB^2\log\left(\frac{bc-ad}{b(c+dx)}\right)\log^2\left(e(a+bx)^n(c+dx)^{-n}\right)}{h}-\frac{B^3\log\left(\frac{bc-ad}{b(c+dx)}\right)\mathrm{lc}}{}$$

**Mathematica [F]** time = 1.53, size = 0, normalized size = 0.00

$$\int \frac{\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^3}{g+hx}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])^3/(g + h*x),x]
```

```
[Out] Integrate[(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])^3/(g + h*x), x]
```

**fricas [F]** time = 1.13, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{B^3\log\left(\frac{(bx+a)^n e}{(dx+c)^n}\right)^3+3\,AB^2\log\left(\frac{(bx+a)^n e}{(dx+c)^n}\right)^2+3\,A^2B\log\left(\frac{(bx+a)^n e}{(dx+c)^n}\right)+A^3}{hx+g},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*(b*x+a)^n/((d*x+c)^n)))^3/(h*x+g),x, algorithm="fricas
")
```

```
[Out] integral((B^3*log((b*x + a)^n*e/(d*x + c)^n)^3 + 3*A*B^2*log((b*x + a)^n*e/
(d*x + c)^n)^2 + 3*A^2*B*log((b*x + a)^n*e/(d*x + c)^n) + A^3)/(h*x + g), x
)
```

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(B\log\left(\frac{(bx+a)^n e}{(dx+c)^n}\right)+A\right)^3}{hx+g}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*(b*x+a)^n/((d*x+c)^n)))^3/(h*x+g),x, algorithm="giac")
```

```
[Out] integrate((B*log((b*x + a)^n*e/(d*x + c)^n) + A)^3/(h*x + g), x)
```

**maple [F]** time = 3.76, size = 0, normalized size = 0.00

$$\int \frac{\left(B\ln\left(e\,(bx+a)^n\,(dx+c)^{-n}\right)+A\right)^3}{hx+g}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))^3/(h*x+g),x)
```

```
[Out] int((A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))^3/(h*x+g),x)
```

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$\frac{A^3\log\left(hx+g\right)}{h}-\int -\frac{B^3\log\left((bx+a)^n\right)^3-B^3\log\left((dx+c)^n\right)^3+B^3\log(e)^3+3\,AB^2\log(e)^2+3\,A^2B\log(e)+3}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*(b*x+a)^n/((d*x+c)^n)))^3/(h*x+g),x, algorithm="maxima
")
```

```
[Out] A^3*log(h*x + g)/h - integrate(-(B^3*log((b*x + a)^n)^3 - B^3*log((d*x + c)
^n)^3 + B^3*log(e)^3 + 3*A*B^2*log(e)^2 + 3*A^2*B*log(e) + 3*(B^3*log(e) +
A*B^2)*log((b*x + a)^n)^2 + 3*(B^3*log((b*x + a)^n) + B^3*log(e) + A*B^2)*l
og((d*x + c)^n)^2 + 3*(B^3*log(e)^2 + 2*A*B^2*log(e) + A^2*B)*log((b*x + a)
^n) - 3*(B^3*log((b*x + a)^n)^2 + B^3*log(e)^2 + 2*A*B^2*log(e) + A^2*B + 2
*(B^3*log(e) + A*B^2)*log((b*x + a)^n))*log((d*x + c)^n))/(h*x + g), x)
```

**mupad** [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(A + B\,\ln\left(\frac{e\,(a+b\,x)^n}{(c+d\,x)^n}\right)\right)^3}{g + h\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*log((e*(a + b*x)^n)/(c + d*x)^n))^3/(g + h*x),x)
```

```
[Out] int((A + B*log((e*(a + b*x)^n)/(c + d*x)^n))^3/(g + h*x), x)
```

**sympy** [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: HeuristicGCDFailed

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*ln(e*(b*x+a)**n/((d*x+c)**n)))**3/(h*x+g),x)
```

```
[Out] Exception raised: HeuristicGCDFailed
```

# 3.313 $\int \frac{\left(A+B\log(e(a+bx)^n(c+dx)^{-n})\right)^3}{(g+hx)^2}\,dx$

**Optimal**. Leaf size=302

$$\frac{6B^2n^2(bc-ad)\text{Li}_2\left(\frac{(dg-ch)(a+bx)}{(bg-ah)(c+dx)}\right)\left(B\log\left(e(a+bx)^n(c+dx)^{-n}\right)+A\right)}{(bg-ah)(dg-ch)}+\frac{3Bn(bc-ad)\log\left(1-\frac{(a+bx)(dg-ch)}{(c+dx)(bg-ah)}\right)\left(B\log\right.}{(bg-ah)(dg-c}$$

```
[Out] (b*x+a)*(A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))^3/(-a*h+b*g)/(h*x+g)+3*B*(-a*d+b*
c)*n*(A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))^2*ln(1-(-c*h+d*g)*(b*x+a)/(-a*h+b*g)
/(d*x+c))/(-a*h+b*g)/(-c*h+d*g)+6*B^2*(-a*d+b*c)*n^2*(A+B*ln(e*(b*x+a)^n/((
d*x+c)^n)))*polylog(2,(-c*h+d*g)*(b*x+a)/(-a*h+b*g)/(d*x+c))/(-a*h+b*g)/(-c
*h+d*g)-6*B^3*(-a*d+b*c)*n^3*polylog(3,(-c*h+d*g)*(b*x+a)/(-a*h+b*g)/(d*x+c
))/(-a*h+b*g)/(-c*h+d*g)
```

**Rubi [B]** time = 0.81, antiderivative size = 650, normalized size of antiderivative = 2.15, number of steps used = 14, number of rules used = 9, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {6742, 2490, 36, 31, 2503, 2502, 2315, 2506, 6610}

$$\frac{6AB^2n^2(bc-ad)\text{PolyLog}\left(2,1-\frac{(g+hx)(bc-ad)}{(c+dx)(bg-ah)}\right)}{(bg-ah)(dg-ch)}+\frac{6B^3n^2(bc-ad)\log\left(e(a+bx)^n(c+dx)^{-n}\right)\text{PolyLog}\left(2,1-\frac{(g+hx}{(c+dx}\right.}{(bg-ah)(dg-ch)}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])^3/(g + h*x)^2,x]
```

```
[Out] -(A^3/(h*(g + h*x))) - (3*A^2*B*(b*c - a*d)*n*Log[c + d*x])/((b*g - a*h)*(d
*g - c*h)) + (3*A^2*B*(a + b*x)*Log[(e*(a + b*x)^n)/(c + d*x)^n])/((b*g - a
*h)*(g + h*x)) + (3*A*B^2*(a + b*x)*Log[(e*(a + b*x)^n)/(c + d*x)^n]^2)/((b
*g - a*h)*(g + h*x)) + (B^3*(a + b*x)*Log[(e*(a + b*x)^n)/(c + d*x)^n]^3)/(
(b*g - a*h)*(g + h*x)) + (3*A^2*B*(b*c - a*d)*n*Log[g + h*x])/((b*g - a*h)*
(d*g - c*h)) + (6*A*B^2*(b*c - a*d)*n*Log[(e*(a + b*x)^n)/(c + d*x)^n]*Log[
((b*c - a*d)*(g + h*x))/((b*g - a*h)*(c + d*x))])/((b*g - a*h)*(d*g - c*h))
 + (3*B^3*(b*c - a*d)*n*Log[(e*(a + b*x)^n)/(c + d*x)^n]^2*Log[((b*c - a*d)
*(g + h*x))/((b*g - a*h)*(c + d*x))])/((b*g - a*h)*(d*g - c*h)) + (6*A*B^2*
(b*c - a*d)*n^2*PolyLog[2, 1 - ((b*c - a*d)*(g + h*x))/((b*g - a*h)*(c + d*
x))])/((b*g - a*h)*(d*g - c*h)) + (6*B^3*(b*c - a*d)*n^2*Log[(e*(a + b*x)^n
)/(c + d*x)^n]*PolyLog[2, 1 - ((b*c - a*d)*(g + h*x))/((b*g - a*h)*(c + d*x
))])/((b*g - a*h)*(d*g - c*h)) - (6*B^3*(b*c - a*d)*n^3*PolyLog[3, 1 - ((b*
c - a*d)*(g + h*x))/((b*g - a*h)*(c + d*x))])/((b*g - a*h)*(d*g - c*h))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2490

```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))
^(r_.)]^(s_.)/((g_.) + (h_.)*(x_))^2, x_Symbol] :> Simp[((a + b*x)*Log[e*(f
*(a + b*x)^p*(c + d*x)^q)^r]^s)/((b*g - a*h)*(g + h*x)), x] - Dist[(p*r*s*(
b*c - a*d))/(b*g - a*h), Int[Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^(s - 1)/(
(c + d*x)*(g + h*x)), x], x] /; FreeQ[{a, b, c, d, e, f, g, h, p, q, r, s},
 x] && NeQ[b*c - a*d, 0] && EqQ[p + q, 0] && NeQ[b*g - a*h, 0] && IGtQ[s, 0
]
```

Rule 2502

```
Int[Log[((e_.)*((c_.) + (d_.)*(x_)))/((a_.) + (b_.)*(x_))]*(u_), x_Symbol]
:> With[{g = Coeff[Simplify[1/(u*(a + b*x))], x, 0], h = Coeff[Simplify[1/(
u*(a + b*x))], x, 1]}, -Dist[(b - d*e)/(h*(b*c - a*d)), Subst[Int[Log[e*x]/
(1 - e*x), x], x, (c + d*x)/(a + b*x)], x] /; EqQ[g*(b - d*e) - h*(a - c*e)
, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b*c - a*d, 0] && LinearQ[Simplify
[1/(u*(a + b*x))], x]
```

Rule 2503

```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))
^(r_.)]^(s_.)*(u_), x_Symbol] :> With[{g = Coeff[Simplify[1/(u*(a + b*x))],
 x, 0], h = Coeff[Simplify[1/(u*(a + b*x))], x, 1]}, -Simp[(Log[e*(f*(a + b
*x)^p*(c + d*x)^q)^r]^s*Log[-(((b*c - a*d)*(g + h*x))/((d*g - c*h)*(a + b*x
)))])/(b*g - a*h), x] + Dist[(p*r*s*(b*c - a*d))/(b*g - a*h), Int[(Log[e*(f
*(a + b*x)^p*(c + d*x)^q)^r]^(s - 1)*Log[-(((b*c - a*d)*(g + h*x))/((d*g -
c*h)*(a + b*x)))])/((a + b*x)*(c + d*x)), x], x] /; NeQ[b*g - a*h, 0] && Ne
Q[d*g - c*h, 0]] /; FreeQ[{a, b, c, d, e, f, p, q, r, s}, x] && NeQ[b*c - a
*d, 0] && IGtQ[s, 0] && EqQ[p + q, 0] && LinearQ[Simplify[1/(u*(a + b*x))],
```

```
x]
```

Rule 2506

```
Int[Log[v_]*Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_)
)^(q_.))^(r_.)]^(s_.)*(u_), x_Symbol] :> With[{g = Simplify[((v - 1)*(c + d
*x))/(a + b*x)], h = Simplify[u*(a + b*x)*(c + d*x)]}, -Simp[(h*PolyLog[2,
1 - v]*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^s)/(b*c - a*d), x] + Dist[h*p*r
*s, Int[(PolyLog[2, 1 - v]*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^(s - 1))/((
a + b*x)*(c + d*x)), x], x] /; FreeQ[{g, h}, x]] /; FreeQ[{a, b, c, d, e, f
, p, q, r, s}, x] && NeQ[b*c - a*d, 0] && IGtQ[s, 0] && EqQ[p + q, 0]
```

Rule 6610

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\int \frac{\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^3}{(g+hx)^2}\,dx = \int\left(\frac{A^3}{(g+hx)^2}+\frac{3A^2B\log\left(e(a+bx)^n(c+dx)^{-n}\right)}{(g+hx)^2}+\frac{3AB^2\log^2\left(e(a+\right.}{(g+}\right.$$

$$= -\frac{A^3}{h(g+hx)}+\left(3A^2B\right)\int\frac{\log\left(e(a+bx)^n(c+dx)^{-n}\right)}{(g+hx)^2}\,dx+\left(3AB^2\right)\int$$

$$= -\frac{A^3}{h(g+hx)}+\frac{3A^2B(a+bx)\log\left(e(a+bx)^n(c+dx)^{-n}\right)}{(bg-ah)(g+hx)}+\frac{3AB^2(a+}{}$$

$$= -\frac{A^3}{h(g+hx)}+\frac{3A^2B(a+bx)\log\left(e(a+bx)^n(c+dx)^{-n}\right)}{(bg-ah)(g+hx)}+\frac{3AB^2(a+}{}$$

$$= -\frac{A^3}{h(g+hx)}-\frac{3A^2B(bc-ad)n\log(c+dx)}{(bg-ah)(dg-ch)}+\frac{3A^2B(a+bx)\log\left(e(a\right.}{(bg-ah)(g}$$

$$= -\frac{A^3}{h(g+hx)}-\frac{3A^2B(bc-ad)n\log(c+dx)}{(bg-ah)(dg-ch)}+\frac{3A^2B(a+bx)\log\left(e(a\right.}{(bg-ah)(g}$$

**Mathematica [F]** time = 3.13, size = 0, normalized size = 0.00

$$\int \frac{\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^3}{(g+hx)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])^3/(g + h*x)^2,x]
[Out] Integrate[(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])^3/(g + h*x)^2, x]
```

**fricas [F]** time = 0.82, size = 0, normalized size = 0.00

$$\operatorname{integral}\left(\frac{B^3\log\left(\frac{(bx+a)^n e}{(dx+c)^n}\right)^3+3\,AB^2\log\left(\frac{(bx+a)^n e}{(dx+c)^n}\right)^2+3\,A^2B\log\left(\frac{(bx+a)^n e}{(dx+c)^n}\right)+A^3}{h^2x^2+2\,ghx+g^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*(b*x+a)^n/((d*x+c)^n)))^3/(h*x+g)^2,x, algorithm="fric
as")
```

```
[Out] integral((B^3*log((b*x + a)^n*e/(d*x + c)^n)^3 + 3*A*B^2*log((b*x + a)^n*e/
(d*x + c)^n)^2 + 3*A^2*B*log((b*x + a)^n*e/(d*x + c)^n) + A^3)/(h^2*x^2 + 2
*g*h*x + g^2), x)
```

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(B\log\left(\frac{(bx+a)^n e}{(dx+c)^n}\right)+A\right)^3}{(hx+g)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*(b*x+a)^n/((d*x+c)^n)))^3/(h*x+g)^2,x, algorithm="giac
")
```

```
[Out] integrate((B*log((b*x + a)^n*e/(d*x + c)^n) + A)^3/(h*x + g)^2, x)
```

**maple [F]** time = 3.36, size = 0, normalized size = 0.00

$$\int \frac{\left(B\ln\left(e\,(bx+a)^n\,(dx+c)^{-n}\right)+A\right)^3}{(hx+g)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))^3/(h*x+g)^2,x)
```

```
[Out] int((A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))^3/(h*x+g)^2,x)
```

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$\frac{B^3\log\left((dx+c)^n\right)^3}{h^2x+gh}+\frac{3\left(\frac{ben\log(bx+a)}{bgh-ah^2}-\frac{den\log(dx+c)}{dgh-ch^2}-\frac{(bcen-aden)\log(hx+g)}{(dgh-ch^2)a-(dg^2-cgh)b}\right)A^2B}{e}-\frac{3A^2B\log\left(\frac{(bx+a)^n e}{(dx+c)^n}\right)}{h^2x+gh}-\frac{A^3}{h^2x+gh}+\int$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*(b*x+a)^n/((d*x+c)^n)))^3/(h*x+g)^2,x, algorithm="maxi
ma")
```

```
[Out] B^3*log((d*x + c)^n)^3/(h^2*x + g*h) + 3*(b*e*n*log(b*x + a)/(b*g*h - a*h^2
) - d*e*n*log(d*x + c)/(d*g*h - c*h^2) - (b*c*e*n - a*d*e*n)*log(h*x + g)/(
(d*g*h - c*h^2)*a - (d*g^2 - c*g*h)*b))*A^2*B/e - 3*A^2*B*log((b*x + a)^n*e
/(d*x + c)^n)/(h^2*x + g*h) - A^3/(h^2*x + g*h) + integrate((B^3*c*h*log(e)
^3 + 3*A*B^2*c*h*log(e)^2 + (B^3*d*h*x + B^3*c*h)*log((b*x + a)^n)^3 + 3*(B
^3*c*h*log(e) + A*B^2*c*h + (B^3*d*h*log(e) + A*B^2*d*h)*x)*log((b*x + a)^n
```

```
)^2 + 3*(A*B^2*c*h - (d*g*n - c*h*log(e))*B^3 - ((h*n - h*log(e))*B^3*d - A
*B^2*d*h)*x + (B^3*d*h*x + B^3*c*h)*log((b*x + a)^n))*log((d*x + c)^n)^2 +
(B^3*d*h*log(e)^3 + 3*A*B^2*d*h*log(e)^2)*x + 3*(B^3*c*h*log(e)^2 + 2*A*B^2
*c*h*log(e) + (B^3*d*h*log(e)^2 + 2*A*B^2*d*h*log(e))*x)*log((b*x + a)^n) -
 3*(B^3*c*h*log(e)^2 + 2*A*B^2*c*h*log(e) + (B^3*d*h*x + B^3*c*h)*log((b*x
+ a)^n)^2 + (B^3*d*h*log(e)^2 + 2*A*B^2*d*h*log(e))*x + 2*(B^3*c*h*log(e) +
 A*B^2*c*h + (B^3*d*h*log(e) + A*B^2*d*h)*x)*log((b*x + a)^n))*log((d*x + c
)^n))/(d*h^3*x^3 + c*g^2*h + (2*d*g*h^2 + c*h^3)*x^2 + (d*g^2*h + 2*c*g*h^2
)*x), x)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(A+B\,\ln\left(\frac{e\,(a+b\,x)^n}{(c+d\,x)^n}\right)\right)^3}{\left(g+h\,x\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*log((e*(a + b*x)^n)/(c + d*x)^n))^3/(g + h*x)^2,x)
```

```
[Out] int((A + B*log((e*(a + b*x)^n)/(c + d*x)^n))^3/(g + h*x)^2, x)
```

**sympy [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*ln(e*(b*x+a)**n/((d*x+c)**n)))**3/(h*x+g)**2,x)
```

```
[Out] Timed out
```

# 3.314 $\int \frac{\left(A+B\log(e(a+bx)^n(c+dx)^{-n})\right)^3}{(g+hx)^3}\,dx$

**Optimal**. Leaf size=629

$$\frac{b^2\left(B\log\left(e(a+bx)^n(c+dx)^{-n}\right)+A\right)^3}{2h(bg-ah)^2}+\frac{3B^2n^2(bc-ad)(-adh-bch+2bdg)\text{Li}_2\left(\frac{(dg-ch)(a+bx)}{(bg-ah)(c+dx)}\right)\left(B\log\left(e(a+bx)^n\right.\right.}{(bg-ah)^2(dg-ch)^2}$$

```
[Out] 3/2*B*(-a*d+b*c)*h*n*(b*x+a)*(A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))^2/(-a*h+b*g)
^2/(-c*h+d*g)/(h*x+g)+1/2*b^2*(A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))^3/h/(-a*h+b
*g)^2-1/2*(A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))^3/h/(h*x+g)^2+3*B^2*(-a*d+b*c)^
2*h*n^2*(A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))*ln(1-(-c*h+d*g)*(b*x+a)/(-a*h+b*g
)/(d*x+c))/(-a*h+b*g)^2/(-c*h+d*g)^2+3/2*B*(-a*d+b*c)*(-a*d*h-b*c*h+2*b*d*g
)*n*(A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))^2*ln(1-(-c*h+d*g)*(b*x+a)/(-a*h+b*g)/
(d*x+c))/(-a*h+b*g)^2/(-c*h+d*g)^2+3*B^3*(-a*d+b*c)^2*h*n^3*polylog(2,(-c*h
+d*g)*(b*x+a)/(-a*h+b*g)/(d*x+c))/(-a*h+b*g)^2/(-c*h+d*g)^2+3*B^2*(-a*d+b*c
)*(-a*d*h-b*c*h+2*b*d*g)*n^2*(A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))*polylog(2,(-
c*h+d*g)*(b*x+a)/(-a*h+b*g)/(d*x+c))/(-a*h+b*g)^2/(-c*h+d*g)^2-3*B^3*(-a*d+
b*c)*(-a*d*h-b*c*h+2*b*d*g)*n^3*polylog(3,(-c*h+d*g)*(b*x+a)/(-a*h+b*g)/(d*
x+c))/(-a*h+b*g)^2/(-c*h+d*g)^2
```

**Rubi [B]** time = 3.75, antiderivative size = 2207, normalized size of antiderivative = 3.51, number of steps used = 49, number of rules used = 21, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.636, Rules used = {6742, 2492, 72, 2514, 2488, 2411, 2343, 2333, 2315, 2490, 36, 31, 2494, 2394, 2393, 2391, 2506, 6610, 2503, 2502, 2489}

result too large to display

Antiderivative was successfully verified.

```
[In] Int[(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])^3/(g + h*x)^3,x]
```

```
[Out] -A^3/(2*h*(g + h*x)^2) - (3*A^2*B*(b*c - a*d)*n)/(2*(b*g - a*h)*(d*g - c*h)
*(g + h*x)) + (3*A^2*b^2*B*n*Log[a + b*x])/(2*h*(b*g - a*h)^2) - (3*A^2*B*d
^2*n*Log[c + d*x])/(2*h*(d*g - c*h)^2) - (3*A*B^2*(b*c - a*d)^2*h*n^2*Log[c
 + d*x])/((b*g - a*h)^2*(d*g - c*h)^2) - (3*A^2*B*Log[(e*(a + b*x)^n)/(c +
d*x)^n])/(2*h*(g + h*x)^2) + (3*A*B^2*(b*c - a*d)*h*n*(a + b*x)*Log[(e*(a +
 b*x)^n)/(c + d*x)^n])/((b*g - a*h)^2*(d*g - c*h)*(g + h*x)) - (3*A*b^2*B^2
*n*Log[-((b*c - a*d)/(d*(a + b*x)))]*Log[(e*(a + b*x)^n)/(c + d*x)^n])/(h*(
b*g - a*h)^2) + (3*A*B^2*d^2*n*Log[(b*c - a*d)/(b*(c + d*x))]*Log[(e*(a + b
*x)^n)/(c + d*x)^n])/(h*(d*g - c*h)^2) - (3*A*B^2*Log[(e*(a + b*x)^n)/(c +
d*x)^n]^2)/(2*h*(g + h*x)^2) + (3*B^3*(b*c - a*d)*h*n*(a + b*x)*Log[(e*(a +
 b*x)^n)/(c + d*x)^n]^2)/(2*(b*g - a*h)^2*(d*g - c*h)*(g + h*x)) - (3*b^2*B
^3*n*Log[-((b*c - a*d)/(d*(a + b*x)))]*Log[(e*(a + b*x)^n)/(c + d*x)^n]^2)/
```

```
(2*h*(b*g - a*h)^2) + (3*B^3*d^2*n*Log[(b*c - a*d)/(b*(c + d*x))]*Log[(e*(a
 + b*x)^n)/(c + d*x)^n]^2)/(2*h*(d*g - c*h)^2) - (3*B^3*(b*c - a*d)*(2*b*d*
g - b*c*h - a*d*h)*n*Log[(b*c - a*d)/(b*(c + d*x))]*Log[(e*(a + b*x)^n)/(c
+ d*x)^n]^2)/(2*(b*g - a*h)^2*(d*g - c*h)^2) - (B^3*Log[(e*(a + b*x)^n)/(c
+ d*x)^n]^3)/(2*h*(g + h*x)^2) + (3*A^2*B*(b*c - a*d)*(2*b*d*g - b*c*h - a*
d*h)*n*Log[g + h*x])/(2*(b*g - a*h)^2*(d*g - c*h)^2) + (3*A*B^2*(b*c - a*d)
^2*h*n^2*Log[g + h*x])/((b*g - a*h)^2*(d*g - c*h)^2) - (3*A*B^2*(b*c - a*d)
*(2*b*d*g - b*c*h - a*d*h)*n^2*Log[-((h*(a + b*x))/(b*g - a*h))]*Log[g + h*
x])/((b*g - a*h)^2*(d*g - c*h)^2) + (3*A*B^2*(b*c - a*d)*(2*b*d*g - b*c*h -
 a*d*h)*n^2*Log[-((h*(c + d*x))/(d*g - c*h))]*Log[g + h*x])/((b*g - a*h)^2*
(d*g - c*h)^2) + (3*A*B^2*(b*c - a*d)*(2*b*d*g - b*c*h - a*d*h)*n*Log[(e*(a
 + b*x)^n)/(c + d*x)^n]*Log[g + h*x])/((b*g - a*h)^2*(d*g - c*h)^2) + (3*B^
3*(b*c - a*d)^2*h*n^2*Log[(e*(a + b*x)^n)/(c + d*x)^n]*Log[((b*c - a*d)*(g
+ h*x))/((b*g - a*h)*(c + d*x))])/((b*g - a*h)^2*(d*g - c*h)^2) + (3*B^3*(b
*c - a*d)*(2*b*d*g - b*c*h - a*d*h)*n*Log[(e*(a + b*x)^n)/(c + d*x)^n]^2*Lo
g[((b*c - a*d)*(g + h*x))/((b*g - a*h)*(c + d*x))])/(2*(b*g - a*h)^2*(d*g -
 c*h)^2) + (3*A*B^2*d^2*n^2*PolyLog[2, (d*(a + b*x))/(b*(c + d*x))])/(h*(d*
g - c*h)^2) - (3*A*B^2*(b*c - a*d)*(2*b*d*g - b*c*h - a*d*h)*n^2*PolyLog[2,
 (b*(g + h*x))/(b*g - a*h)])/((b*g - a*h)^2*(d*g - c*h)^2) + (3*A*B^2*(b*c
- a*d)*(2*b*d*g - b*c*h - a*d*h)*n^2*PolyLog[2, (d*(g + h*x))/(d*g - c*h)])
/((b*g - a*h)^2*(d*g - c*h)^2) + (3*A*b^2*B^2*n^2*PolyLog[2, 1 + (b*c - a*d
)/(d*(a + b*x))])/(h*(b*g - a*h)^2) + (3*b^2*B^3*n^2*Log[(e*(a + b*x)^n)/(c
 + d*x)^n]*PolyLog[2, 1 + (b*c - a*d)/(d*(a + b*x))])/(h*(b*g - a*h)^2) + (
3*B^3*d^2*n^2*Log[(e*(a + b*x)^n)/(c + d*x)^n]*PolyLog[2, 1 - (b*c - a*d)/(
b*(c + d*x))])/(h*(d*g - c*h)^2) - (3*B^3*(b*c - a*d)*(2*b*d*g - b*c*h - a*
d*h)*n^2*Log[(e*(a + b*x)^n)/(c + d*x)^n]*PolyLog[2, 1 - (b*c - a*d)/(b*(c
+ d*x))])/((b*g - a*h)^2*(d*g - c*h)^2) + (3*B^3*(b*c - a*d)^2*h*n^3*PolyLo
g[2, 1 - ((b*c - a*d)*(g + h*x))/((b*g - a*h)*(c + d*x))])/((b*g - a*h)^2*(
d*g - c*h)^2) + (3*B^3*(b*c - a*d)*(2*b*d*g - b*c*h - a*d*h)*n^2*Log[(e*(a
+ b*x)^n)/(c + d*x)^n]*PolyLog[2, 1 - ((b*c - a*d)*(g + h*x))/((b*g - a*h)*
(c + d*x))])/((b*g - a*h)^2*(d*g - c*h)^2) + (3*b^2*B^3*n^3*PolyLog[3, 1 +
(b*c - a*d)/(d*(a + b*x))])/(h*(b*g - a*h)^2) - (3*B^3*d^2*n^3*PolyLog[3, 1
 - (b*c - a*d)/(b*(c + d*x))])/(h*(d*g - c*h)^2) + (3*B^3*(b*c - a*d)*(2*b*
d*g - b*c*h - a*d*h)*n^3*PolyLog[3, 1 - (b*c - a*d)/(b*(c + d*x))])/((b*g -
 a*h)^2*(d*g - c*h)^2) - (3*B^3*(b*c - a*d)*(2*b*d*g - b*c*h - a*d*h)*n^3*P
olyLog[3, 1 - ((b*c - a*d)*(g + h*x))/((b*g - a*h)*(c + d*x))])/((b*g - a*h
)^2*(d*g - c*h)^2)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
```

```
- a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 72

```
Int[((e_.) + (f_.)*(x_))^(p_.)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
 x_Symbol] :> Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x]
 /; FreeQ[{a, b, c, d, e, f}, x] && IntegerQ[p]
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2333

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)/(x_))^(q_.)*(
x_)^(m_.), x_Symbol] :> Int[(e + d*x)^q*(a + b*Log[c*x^n])^p, x] /; FreeQ[{
a, b, c, d, e, m, n, p}, x] && EqQ[m, q] && IntegerQ[q]
```

Rule 2343

```
Int[((a_.) + Log[(c_.)*(x_)^(n_)]*(b_.))/((x_)*((d_) + (e_.)*(x_)^(r_.))),
x_Symbol] :> Dist[1/n, Subst[Int[(a + b*Log[c*x])/(x*(d + e*x^(r/n))), x],
x, x^n], x] /; FreeQ[{a, b, c, d, e, n, r}, x] && IntegerQ[r/n]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2393

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + (c*e*x)/g])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2394

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d + e*x
)^n]))/g, x] - Dist[(b*e*n)/g, Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2411

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + (g_
.)*(x_))^(q_.)*((h_.) + (i_.)*(x_))^(r_.), x_Symbol] :> Dist[1/e, Subst[Int
[((g*x)/e)^q*((e*h - d*i)/e + (i*x)/e)^r*(a + b*Log[c*x^n])^p, x], x, d + e
*x], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, n, p, q, r}, x] && EqQ[e*f - d
*g, 0] && (IGtQ[p, 0] || IGtQ[r, 0]) && IntegerQ[2*r]
```

Rule 2488

```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))
^(r_.)]^(s_.)/((g_.) + (h_.)*(x_)), x_Symbol] :> -Simp[(Log[-((b*c - a*d)/(
d*(a + b*x)))]*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^s)/h, x] + Dist[(p*r*s*
(b*c - a*d))/h, Int[(Log[-((b*c - a*d)/(d*(a + b*x)))]*Log[e*(f*(a + b*x)^p
*(c + d*x)^q)^r]^(s - 1))/((a + b*x)*(c + d*x)), x], x] /; FreeQ[{a, b, c,
d, e, f, g, h, p, q, r, s}, x] && NeQ[b*c - a*d, 0] && EqQ[p + q, 0] && EqQ
[b*g - a*h, 0] && IGtQ[s, 0]
```

Rule 2489

```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))
^(r_.)]^(s_)/((g_.) + (h_.)*(x_)), x_Symbol] :> Dist[d/h, Int[Log[e*(f*(a +
 b*x)^p*(c + d*x)^q)^r]^s/(c + d*x), x], x] - Dist[(d*g - c*h)/h, Int[Log[e
*(f*(a + b*x)^p*(c + d*x)^q)^r]^s/((c + d*x)*(g + h*x)), x], x] /; FreeQ[{a
, b, c, d, e, f, g, h, p, q, r, s}, x] && NeQ[b*c - a*d, 0] && EqQ[p + q, 0
] && NeQ[b*g - a*h, 0] && NeQ[d*g - c*h, 0] && IGtQ[s, 1]
```

Rule 2490

```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))
^(r_.)]^(s_.)/((g_.) + (h_.)*(x_))^2, x_Symbol] :> Simp[((a + b*x)*Log[e*(f
*(a + b*x)^p*(c + d*x)^q)^r]^s)/((b*g - a*h)*(g + h*x)), x] - Dist[(p*r*s*(
b*c - a*d))/(b*g - a*h), Int[Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^(s - 1)/(
(c + d*x)*(g + h*x)), x], x] /; FreeQ[{a, b, c, d, e, f, g, h, p, q, r, s},
 x] && NeQ[b*c - a*d, 0] && EqQ[p + q, 0] && NeQ[b*g - a*h, 0] && IGtQ[s, 0
]
```

Rule 2492

```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))
^(r_.)]^(s_.)*((g_.) + (h_.)*(x_))^(m_.), x_Symbol] :> Simp[((g + h*x)^(m +
 1)*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^s)/(h*(m + 1)), x] - Dist[(p*r*s*(
b*c - a*d))/(h*(m + 1)), Int[((g + h*x)^(m + 1)*Log[e*(f*(a + b*x)^p*(c + d
*x)^q)^r]^(s - 1))/((a + b*x)*(c + d*x)), x], x] /; FreeQ[{a, b, c, d, e, f
, g, h, m, p, q, r, s}, x] && NeQ[b*c - a*d, 0] && EqQ[p + q, 0] && IGtQ[s,
```

```
0] && NeQ[m, -1]
```

Rule 2494

```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))
^(r_.)]/((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(Log[g + h*x]*Log[e*(f*(a +
 b*x)^p*(c + d*x)^q)^r])/h, x] + (-Dist[(b*p*r)/h, Int[Log[g + h*x]/(a + b*
x), x], x] - Dist[(d*q*r)/h, Int[Log[g + h*x]/(c + d*x), x], x]) /; FreeQ[{
a, b, c, d, e, f, g, h, p, q, r}, x] && NeQ[b*c - a*d, 0]
```

Rule 2502

```
Int[Log[((e_.)*((c_.) + (d_.)*(x_)))/((a_.) + (b_.)*(x_))]*(u_), x_Symbol]
:> With[{g = Coeff[Simplify[1/(u*(a + b*x))], x, 0], h = Coeff[Simplify[1/(
u*(a + b*x))], x, 1]}, -Dist[(b - d*e)/(h*(b*c - a*d)), Subst[Int[Log[e*x]/
(1 - e*x), x], x, (c + d*x)/(a + b*x)], x] /; EqQ[g*(b - d*e) - h*(a - c*e)
, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b*c - a*d, 0] && LinearQ[Simplify
[1/(u*(a + b*x))], x]
```

Rule 2503

```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))
^(r_.)]^(s_.)*(u_), x_Symbol] :> With[{g = Coeff[Simplify[1/(u*(a + b*x))],
 x, 0], h = Coeff[Simplify[1/(u*(a + b*x))], x, 1]}, -Simp[(Log[e*(f*(a + b
*x)^p*(c + d*x)^q)^r]^s*Log[-(((b*c - a*d)*(g + h*x))/((d*g - c*h)*(a + b*x
)))])/(b*g - a*h), x] + Dist[(p*r*s*(b*c - a*d))/(b*g - a*h), Int[(Log[e*(f
*(a + b*x)^p*(c + d*x)^q)^r]^(s - 1)*Log[-(((b*c - a*d)*(g + h*x))/((d*g -
c*h)*(a + b*x)))])/((a + b*x)*(c + d*x)), x], x] /; NeQ[b*g - a*h, 0] && Ne
Q[d*g - c*h, 0]] /; FreeQ[{a, b, c, d, e, f, p, q, r, s}, x] && NeQ[b*c - a
*d, 0] && IGtQ[s, 0] && EqQ[p + q, 0] && LinearQ[Simplify[1/(u*(a + b*x))],
 x]
```

Rule 2506

```
Int[Log[v_]*Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_)
)^(q_.))^(r_.)]^(s_.)*(u_), x_Symbol] :> With[{g = Simplify[((v - 1)*(c + d
*x))/(a + b*x)], h = Simplify[u*(a + b*x)*(c + d*x)]}, -Simp[(h*PolyLog[2,
1 - v]*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^s)/(b*c - a*d), x] + Dist[h*p*r
*s, Int[(PolyLog[2, 1 - v]*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^(s - 1))/((
a + b*x)*(c + d*x)), x], x] /; FreeQ[{g, h}, x]] /; FreeQ[{a, b, c, d, e, f
, p, q, r, s}, x] && NeQ[b*c - a*d, 0] && IGtQ[s, 0] && EqQ[p + q, 0]
```

Rule 2514

```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))
```

```
^(r_.)]^(s_.)*(RFx_), x_Symbol] :> With[{u = ExpandIntegrand[Log[e*(f*(a +
b*x)^p*(c + d*x)^q)^r]^s, RFx, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c
, d, e, f, p, q, r, s}, x] && RationalFunctionQ[RFx, x] && IGtQ[s, 0]
```

Rule 6610

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\int \frac{\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^3}{(g+hx)^3}\,dx = \int \left(\frac{A^3}{(g+hx)^3} + \frac{3A^2B\log\left(e(a+bx)^n(c+dx)^{-n}\right)}{(g+hx)^3} + \frac{3AB^2\log^2\left(e(a\right.}{(g}\right.$$

$$= -\frac{A^3}{2h(g+hx)^2} + \left(3A^2B\right)\int \frac{\log\left(e(a+bx)^n(c+dx)^{-n}\right)}{(g+hx)^3}\,dx + \left(3AB^2\right.$$

$$= -\frac{A^3}{2h(g+hx)^2} - \frac{3A^2B\log\left(e(a+bx)^n(c+dx)^{-n}\right)}{2h(g+hx)^2} - \frac{3AB^2\log^2\left(e(a\right.}{2h(g}$$

$$= -\frac{A^3}{2h(g+hx)^2} - \frac{3A^2B\log\left(e(a+bx)^n(c+dx)^{-n}\right)}{2h(g+hx)^2} - \frac{3AB^2\log^2\left(e(a\right.}{2h(g}$$

$$= -\frac{A^3}{2h(g+hx)^2} - \frac{3A^2B(bc-ad)n}{2(bg-ah)(dg-ch)(g+hx)} + \frac{3A^2b^2Bn\log(a+b}{2h(bg-ah)^2}$$

$$= -\frac{A^3}{2h(g+hx)^2} - \frac{3A^2B(bc-ad)n}{2(bg-ah)(dg-ch)(g+hx)} + \frac{3A^2b^2Bn\log(a+b}{2h(bg-ah)^2}$$

$$= -\frac{A^3}{2h(g+hx)^2} - \frac{3A^2B(bc-ad)n}{2(bg-ah)(dg-ch)(g+hx)} + \frac{3A^2b^2Bn\log(a+b}{2h(bg-ah)^2}$$

$$= -\frac{A^3}{2h(g+hx)^2} - \frac{3A^2B(bc-ad)n}{2(bg-ah)(dg-ch)(g+hx)} + \frac{3A^2b^2Bn\log(a+b}{2h(bg-ah)^2}$$

$$= -\frac{A^3}{2h(g+hx)^2} - \frac{3A^2B(bc-ad)n}{2(bg-ah)(dg-ch)(g+hx)} + \frac{3A^2b^2Bn\log(a+b}{2h(bg-ah)^2}$$

$$= -\frac{A^3}{2h(g+hx)^2} - \frac{3A^2B(bc-ad)n}{2(bg-ah)(dg-ch)(g+hx)} + \frac{3A^2b^2Bn\log(a+b}{2h(bg-ah)^2}$$

**Mathematica [F]** time = 6.37, size = 0, normalized size = 0.00

$$\int \frac{\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^3}{(g+hx)^3}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])^3/(g + h*x)^3,x]
```

```
[Out] Integrate[(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])^3/(g + h*x)^3, x]
```

**fricas [F]** time = 1.00, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{B^3 \log\left(\frac{(bx+a)^n e}{(dx+c)^n}\right)^3 + 3\,AB^2 \log\left(\frac{(bx+a)^n e}{(dx+c)^n}\right)^2 + 3\,A^2 B \log\left(\frac{(bx+a)^n e}{(dx+c)^n}\right) + A^3}{h^3x^3 + 3\,gh^2x^2 + 3\,g^2hx + g^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*(b*x+a)^n/((d*x+c)^n)))^3/(h*x+g)^3,x, algorithm="fric
as")
```

```
[Out] integral((B^3*log((b*x + a)^n*e/(d*x + c)^n)^3 + 3*A*B^2*log((b*x + a)^n*e/
(d*x + c)^n)^2 + 3*A^2*B*log((b*x + a)^n*e/(d*x + c)^n) + A^3)/(h^3*x^3 + 3
*g*h^2*x^2 + 3*g^2*h*x + g^3), x)
```

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(B \log\left(\frac{(bx+a)^n e}{(dx+c)^n}\right) + A\right)^3}{(hx+g)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*(b*x+a)^n/((d*x+c)^n)))^3/(h*x+g)^3,x, algorithm="giac
")
```

```
[Out] integrate((B*log((b*x + a)^n*e/(d*x + c)^n) + A)^3/(h*x + g)^3, x)
```

**maple [F]** time = 5.69, size = 0, normalized size = 0.00

$$\int \frac{\left(B \ln\left(e\,(bx+a)^n\,(dx+c)^{-n}\right) + A\right)^3}{(hx+g)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))^3/(h*x+g)^3,x)
```

```
[Out] int((A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))^3/(h*x+g)^3,x)
```

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$\frac{B^3 \log\left((dx+c)^n\right)^3}{2\left(h^3x^2 + 2\,gh^2x + g^2h\right)} + \frac{3\left(\frac{b^2en\log(bx+a)}{b^2g^2h - 2\,abgh^2 + a^2h^3} - \frac{d^2en\log(dx+c)}{d^2g^2h - 2\,cdgh^2 + c^2h^3} - \frac{\left(2\,abd^2egn - a^2d^2ehn - \left(2\,cdegn - c^2ehn\right)b^2\right)\log(}{\left(d^2g^2h^2 - 2\,cdgh^3 + c^2h^4\right)a^2 - 2\left(d^2g^3h - 2\,cdg^2h^2 + c^2gh^3\right)ab + \left(d^2g\right.}}{2\,e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*(b*x+a)^n/((d*x+c)^n)))^3/(h*x+g)^3,x, algorithm="maxi
ma")
```

```
[Out] 1/2*B^3*log((d*x + c)^n)^3/(h^3*x^2 + 2*g*h^2*x + g^2*h) + 3/2*(b^2*e*n*log
(b*x + a)/(b^2*g^2*h - 2*a*b*g*h^2 + a^2*h^3) - d^2*e*n*log(d*x + c)/(d^2*g
^2*h - 2*c*d*g*h^2 + c^2*h^3) - (2*a*b*d^2*e*g*n - a^2*d^2*e*h*n - (2*c*d*e
*g*n - c^2*e*h*n)*b^2)*log(h*x + g)/((d^2*g^2*h^2 - 2*c*d*g*h^3 + c^2*h^4)*
a^2 - 2*(d^2*g^3*h - 2*c*d*g^2*h^2 + c^2*g*h^3)*a*b + (d^2*g^4 - 2*c*d*g^3*
h + c^2*g^2*h^2)*b^2) + (b*c*e*n - a*d*e*n)/((d*g^2*h - c*g*h^2)*a - (d*g^3
 - c*g^2*h)*b + ((d*g*h^2 - c*h^3)*a - (d*g^2*h - c*g*h^2)*b)*x))*A^2*B/e -
 3/2*A^2*B*log((b*x + a)^n*e/(d*x + c)^n)/(h^3*x^2 + 2*g*h^2*x + g^2*h) - 1
/2*A^3/(h^3*x^2 + 2*g*h^2*x + g^2*h) + integrate(1/2*(2*B^3*c*h*log(e)^3 +
6*A*B^2*c*h*log(e)^2 + 2*(B^3*d*h*x + B^3*c*h)*log((b*x + a)^n)^3 + 6*(B^3*
c*h*log(e) + A*B^2*c*h + (B^3*d*h*log(e) + A*B^2*d*h)*x)*log((b*x + a)^n)^2
 + 3*(2*A*B^2*c*h - (d*g*n - 2*c*h*log(e))*B^3 - ((h*n - 2*h*log(e))*B^3*d
- 2*A*B^2*d*h)*x + 2*(B^3*d*h*x + B^3*c*h)*log((b*x + a)^n))*log((d*x + c)^
n)^2 + 2*(B^3*d*h*log(e)^3 + 3*A*B^2*d*h*log(e)^2)*x + 6*(B^3*c*h*log(e)^2
+ 2*A*B^2*c*h*log(e) + (B^3*d*h*log(e)^2 + 2*A*B^2*d*h*log(e))*x)*log((b*x
+ a)^n) - 6*(B^3*c*h*log(e)^2 + 2*A*B^2*c*h*log(e) + (B^3*d*h*x + B^3*c*h)*
log((b*x + a)^n)^2 + (B^3*d*h*log(e)^2 + 2*A*B^2*d*h*log(e))*x + 2*(B^3*c*h
*log(e) + A*B^2*c*h + (B^3*d*h*log(e) + A*B^2*d*h)*x)*log((b*x + a)^n))*log
((d*x + c)^n))/(d*h^4*x^4 + c*g^3*h + (3*d*g*h^3 + c*h^4)*x^3 + 3*(d*g^2*h^
2 + c*g*h^3)*x^2 + (d*g^3*h + 3*c*g^2*h^2)*x), x)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(A+B\,\ln\left(\frac{e\,(a+b\,x)^n}{(c+d\,x)^n}\right)\right)^3}{\left(g+h\,x\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*log((e*(a + b*x)^n)/(c + d*x)^n))^3/(g + h*x)^3,x)
```

```
[Out] int((A + B*log((e*(a + b*x)^n)/(c + d*x)^n))^3/(g + h*x)^3, x)
```

**sympy [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*ln(e*(b*x+a)**n/((d*x+c)**n)))**3/(h*x+g)**3,x)
```

```
[Out] Timed out
```

# Chapter 4

# Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Maple and for Mathematica/Rubi. There is a version for grading Sympy and version for use with Sagemath.

The following are links to the current source code.

The following are the listings of source code of the grading functions.

### 4.0.1 Mathematica and Rubi grading function

```
(* Original version thanks to Albert Rich emailed on 03/21/2017 *)
(* ::Package:: *)

(* ::Subsection:: *)
(*GradeAntiderivative[result,optimal]*)


(* ::Text:: *)
(*If result and optimal are mathematical expressions, *)
(*          GradeAntiderivative[result,optimal] returns*)
(*  "F" if the result fails to integrate an expression that*)
(*      is integrable*)
(*  "C" if result involves higher level functions than necessary*)
(*  "B" if result is more than twice the size of the optimal*)
(*      antiderivative*)
(*  "A" if result can be considered optimal*)


GradeAntiderivative[result_,optimal_] :=
  If[ExpnType[result]<=ExpnType[optimal],
```

```
    If[FreeQ[result,Complex] || Not[FreeQ[optimal,Complex]],
      If[LeafCount[result]<=2*LeafCount[optimal],
        "A",
      "B"],
    "C"],
  If[FreeQ[result,Integrate] && FreeQ[result,Int],
    "C",
  "F"]]


(* ::Text:: *)
(*The following summarizes the type number assigned an *)
(*expression based on the functions it involves*)
(*1 = rational function*)
(*2 = algebraic function*)
(*3 = elementary function*)
(*4 = special function*)
(*5 = hyperpergeometric function*)
(*6 = appell function*)
(*7 = rootsum function*)
(*8 = integrate function*)
(*9 = unknown function*)


ExpnType[expn_] :=
  If[AtomQ[expn],
    1,
  If[ListQ[expn],
    Max[Map[ExpnType,expn]],
  If[Head[expn]===Power,
    If[IntegerQ[expn[[2]]],
      ExpnType[expn[[1]]],
    If[Head[expn[[2]]]===Rational,
      If[IntegerQ[expn[[1]]] || Head[expn[[1]]]===Rational,
        1,
      Max[ExpnType[expn[[1]]],2]],
    Max[ExpnType[expn[[1]]],ExpnType[expn[[2]]],3]]],
  If[Head[expn]===Plus || Head[expn]===Times,
    Max[ExpnType[First[expn]],ExpnType[Rest[expn]]],
  If[ElementaryFunctionQ[Head[expn]],
    Max[3,ExpnType[expn[[1]]]]],
  If[SpecialFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],4]],
  If[HypergeometricFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],5]],
  If[AppellFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],6]],
```

```
  If[Head[expn]===RootSum,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
  If[Head[expn]===Integrate || Head[expn]===Int,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],8]],
  9]]]]]]]]]]


ElementaryFunctionQ[func_] :=
  MemberQ[{
        Exp,Log,
        Sin,Cos,Tan,Cot,Sec,Csc,
        ArcSin,ArcCos,ArcTan,ArcCot,ArcSec,ArcCsc,
        Sinh,Cosh,Tanh,Coth,Sech,Csch,
        ArcSinh,ArcCosh,ArcTanh,ArcCoth,ArcSech,ArcCsch
},func]


SpecialFunctionQ[func_] :=
  MemberQ[{
        Erf, Erfc, Erfi,
        FresnelS, FresnelC,
        ExpIntegralE, ExpIntegralEi, LogIntegral,
        SinIntegral, CosIntegral, SinhIntegral, CoshIntegral,
        Gamma, LogGamma, PolyGamma,
        Zeta, PolyLog, ProductLog,
        EllipticF, EllipticE, EllipticPi
},func]


HypergeometricFunctionQ[func_] :=
  MemberQ[{Hypergeometric1F1,Hypergeometric2F1,HypergeometricPFQ},func]


AppellFunctionQ[func_] :=
  MemberQ[{AppellF1},func]
```

### 4.0.2 Maple grading function

```
# File: GradeAntiderivative.mpl
# Original version thanks to Albert Rich emailed on 03/21/2017

#Nasser 03/22/2017  Use Maple leaf count instead since buildin
#Nasser 03/23/2017  missing 'ln' for ElementaryFunctionQ added
#Nasser 03/24/2017  corrected the check for complex result
#Nasser 10/27/2017  check for leafsize and do not call ExpnType()
#                   if leaf size is "too large". Set at 500,000
```

```
#Nasser 12/22/2019  Added debug flag, added 'dilog' to special functions
#                   see problem 156, file Apostol_Problems

GradeAntiderivative := proc(result,optimal)
local leaf_count_result, leaf_count_optimal,ExpnType_result,ExpnType_optimal,
   debug:=false;

      leaf_count_result:=leafcount(result);
      #do NOT call ExpnType() if leaf size is too large. Recursion problem
      if leaf_count_result > 500000 then
          return "B";
      fi;

      leaf_count_optimal:=leafcount(optimal);

      ExpnType_result:=ExpnType(result);
      ExpnType_optimal:=ExpnType(optimal);

      if debug then
         print("ExpnType_result",ExpnType_result," ExpnType_optimal=",
   ExpnType_optimal);
      fi;

# If result and optimal are mathematical expressions,
#  GradeAntiderivative[result,optimal] returns
#   "F" if the result fails to integrate an expression that
#       is integrable
#   "C" if result involves higher level functions than necessary
#   "B" if result is more than twice the size of the optimal
#       antiderivative
#   "A" if result can be considered optimal

  #This check below actually is not needed, since I only
  #call this grading only for passed integrals. i.e. I check
  #for "F" before calling this. But no harm of keeping it here.
  #just in case.


  if not type(result,freeof('int')) then
     return "F";
  end if;


  if ExpnType_result<=ExpnType_optimal then
      if debug then
           print("ExpnType_result<=ExpnType_optimal");
      fi;
```

```
      if is_contains_complex(result) then
         if is_contains_complex(optimal) then
            if debug then
               print("both result and optimal complex");
            fi;
            #both result and optimal complex
            if leaf_count_result<=2*leaf_count_optimal then
               return "A";
            else
               return "B";
            end if
         else #result contains complex but optimal is not
            if debug then
               print("result contains complex but optimal is not");
            fi;
            return "C";
         end if
      else # result do not contain complex
           # this assumes optimal do not as well
           if debug then
               print("result do not contain complex, this assumes optimal do not
     as well");
           fi;
           if leaf_count_result<=2*leaf_count_optimal then
              if debug then
                 print("leaf_count_result<=2*leaf_count_optimal");
              fi;
              return "A";
           else
              if debug then
                 print("leaf_count_result>2*leaf_count_optimal");
              fi;
              return "B";
           end if
      end if
  else #ExpnType(result) > ExpnType(optimal)
      if debug then
         print("ExpnType(result) > ExpnType(optimal)");
      fi;
      return "C";
  end if

end proc:

#
# is_contains_complex(result)
# takes expressions and returns true if it contains "I" else false
```

```
#
#Nasser 032417
is_contains_complex:= proc(expression)
  return (has(expression,I));
end proc:

# The following summarizes the type number assigned an expression
# based on the functions it involves
# 1 = rational function
# 2 = algebraic function
# 3 = elementary function
# 4 = special function
# 5 = hyperpergeometric function
# 6 = appell function
# 7 = rootsum function
# 8 = integrate function
# 9 = unknown function

ExpnType := proc(expn)
  if type(expn,'atomic') then
    1
  elif type(expn,'list') then
    apply(max,map(ExpnType,expn))
  elif type(expn,'sqrt') then
    if type(op(1,expn),'rational') then
       1
    else
       max(2,ExpnType(op(1,expn)))
    end if
  elif type(expn,'`^`') then
    if type(op(2,expn),'integer') then
      ExpnType(op(1,expn))
    elif type(op(2,expn),'rational') then
      if type(op(1,expn),'rational') then
         1
      else
         max(2,ExpnType(op(1,expn)))
      end if
    else
         max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    end if
  elif type(expn,'`+`') or type(expn,'`*`') then
    max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif ElementaryFunctionQ(op(0,expn)) then
    max(3,ExpnType(op(1,expn)))
  elif SpecialFunctionQ(op(0,expn)) then
    max(4,apply(max,map(ExpnType,[op(expn)])))
```

```
  elif HypergeometricFunctionQ(op(0,expn)) then
    max(5,apply(max,map(ExpnType,[op(expn)])))
  elif AppellFunctionQ(op(0,expn)) then
    max(6,apply(max,map(ExpnType,[op(expn)])))
  elif op(0,expn)='int' then
    max(8,apply(max,map(ExpnType,[op(expn)]))) else
  9
  end if
end proc:


ElementaryFunctionQ := proc(func)
  member(func,[
        exp,log,ln,
        sin,cos,tan,cot,sec,csc,
        arcsin,arccos,arctan,arccot,arcsec,arccsc,
        sinh,cosh,tanh,coth,sech,csch,
        arcsinh,arccosh,arctanh,arccoth,arcsech,arccsch])
end proc:

SpecialFunctionQ := proc(func)
  member(func,[
        erf,erfc,erfi,
        FresnelS,FresnelC,
        Ei,Ei,Li,Si,Ci,Shi,Chi,
        GAMMA,lnGAMMA,Psi,Zeta,polylog,dilog,LambertW,
        EllipticF,EllipticE,EllipticPi])
end proc:

HypergeometricFunctionQ := proc(func)
  member(func,[Hypergeometric1F1,hypergeom,HypergeometricPFQ])
end proc:

AppellFunctionQ := proc(func)
  member(func,[AppellF1])
end proc:

# u is a sum or product.  rest(u) returns all but the
# first term or factor of u.
rest := proc(u) local v;
  if nops(u)=2 then
     op(2,u)
  else
     apply(op(0,u),op(2..nops(u),u))
  end if
end proc:
```

```
#leafcount(u) returns the number of nodes in u.
#Nasser 3/23/17  Replaced by build-in leafCount from package in Maple
leafcount := proc(u)
   MmaTranslator[Mma][LeafCount](u);
end proc:
```

### 4.0.3 Sympy grading function

```
#Dec 24, 2019. Nasser M. Abbasi:
#              Port of original Maple grading function by
#              Albert Rich to use with Sympy/Python
#Dec 27, 2019  Nasser. Added `RootSum`. See problem 177, Timofeev file
#              added 'exp_polar'
from sympy import *

def leaf_count(expr):
    #sympy do not have leaf count function. This is approximation
    return round(1.7*count_ops(expr))

def is_sqrt(expr):
    if isinstance(expr,Pow):
        if expr.args[1] == Rational(1,2):
            return True
        else:
            return False
    else:
        return False

def is_elementary_function(func):
    return func in [exp,log,ln,sin,cos,tan,cot,sec,csc,
            asin,acos,atan,acot,asec,acsc,sinh,cosh,tanh,coth,sech,csch,
            asinh,acosh,atanh,acoth,asech,acsch
        ]

def is_special_function(func):
    return func in [ erf,erfc,erfi,
             fresnels,fresnelc,Ei,Ei,Li,Si,Ci,Shi,Chi,
             gamma,loggamma,digamma,zeta,polylog,LambertW,
             elliptic_f,elliptic_e,elliptic_pi,exp_polar
         ]

def is_hypergeometric_function(func):
    return func in [hyper]

def is_appell_function(func):
    return func in [appellf1]
```

```
def is_atom(expn):
    try:
        if expn.isAtom or isinstance(expn,int) or isinstance(expn,float):
           return True
        else:
           return False

    except AttributeError as error:
        return False

def expnType(expn):
    debug=False
    if debug:
        print("expn=",expn,"type(expn)=",type(expn))

    if is_atom(expn):
       return 1
    elif isinstance(expn,list):
        return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
            return 1
        else:
            return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)))
    elif isinstance(expn,Pow):   #type(expn,'`^`')
        if isinstance(expn.args[1],Integer):  #type(op(2,expn),'integer')
            return expnType(expn.args[0])   #ExpnType(op(1,expn))
        elif isinstance(expn.args[1],Rational):  #type(op(2,expn),'rational')
            if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
                return 1
            else:
                return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)
   ))
        else:
            return max(3,expnType(expn.args[0]),expnType(expn.args[1])) #max(3,
   ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    elif isinstance(expn,Add) or isinstance(expn,Mul): #type(expn,'`+`') or type
   (expn,'`*`')
        m1 = expnType(expn.args[0])
        m2 = expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.func):  #ElementaryFunctionQ(op(0,expn))
        return max(3,expnType(expn.args[0]))  #max(3,ExpnType(op(1,expn)))
    elif is_special_function(expn.func): #SpecialFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(4,m1)   #max(4,apply(max,map(ExpnType,[op(expn)])))
```

```
    elif is_hypergeometric_function(expn.func): #HypergeometricFunctionQ(op(0,
   expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(5,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif is_appell_function(expn.func):
        m1 = max(map(expnType, list(expn.args)))
        return max(6,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif isinstance(expn,RootSum):
        m1 = max(map(expnType, list(expn.args))) #Apply[Max,Append[Map[ExpnType,
   Apply[List,expn]],7]],
        return max(7,m1)
    elif str(expn).find("Integral") != -1:
        m1 = max(map(expnType, list(expn.args)))
        return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):

    leaf_count_result  = leaf_count(result)
    leaf_count_optimal = leaf_count(optimal)

    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if str(result).find("Integral") != -1:
        return "F"

    if expnType_result <= expnType_optimal:
        if result.has(I):
            if optimal.has(I): #both result and optimal complex
                if leaf_count_result <= 2*leaf_count_optimal:
                    return "A"
                else:
                    return "B"
            else: #result contains complex but optimal is not
                return "C"
        else: # result do not contain complex, this assumes optimal do not as
   well
            if leaf_count_result <= 2*leaf_count_optimal:
                return "A"
            else:
                return "B"
    else:
        return "C"
```

### 4.0.4 SageMath grading function

```
#Dec 24, 2019. Nasser: Ported original Maple grading function by
#              Albert Rich to use with Sagemath. This is used to
#              grade Fricas, Giac and Maxima results.
#Dec 24, 2019. Nasser: Added 'exp_integral_e' and 'sng', 'sin_integral'
#              'arctan2','floor','abs','log_integral'

from sage.all import *
from sage.symbolic.operators import add_vararg, mul_vararg

debug=False;

def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)

def is_sqrt(expr):
    if expr.operator() == operator.pow:   #isinstance(expr,Pow):
        if expr.operands()[1]==1/2: #expr.args[1] == Rational(1,2):
            if debug: print ("expr is sqrt")
            return True
        else:
            return False
    else:
        return False

def is_elementary_function(func):
    debug=False
    m = func.name() in ['exp','log','ln',
            'sin','cos','tan','cot','sec','csc',
            'arcsin','arccos','arctan','arccot','arcsec','arccsc',
            'sinh','cosh','tanh','coth','sech','csch',
            'arcsinh','arccosh','arctanh','arccoth','arcsech','arccsch','sgn',
        'arctan2','floor','abs'
        ]
    if debug:
        if m:
```

```
            print ("func ", func , " is elementary_function")
        else:
            print ("func ", func , " is NOT elementary_function")


    return m

def is_special_function(func):
    debug=False
    if debug: print ("type(func)=", type(func))

    m= func.name() in ['erf','erfc','erfi','fresnel_sin','fresnel_cos','Ei',
           'Ei','Li','Si','sin_integral','Ci','cos_integral','Shi','
    sinh_integral'
           'Chi','cosh_integral','gamma','log_gamma','psi,zeta',
           'polylog','lambert_w','elliptic_f','elliptic_e',
           'elliptic_pi','exp_integral_e','log_integral']

    if debug:
        print ("m=",m)
        if m:
            print ("func ", func ," is special_function")
        else:
            print ("func ", func ," is NOT special_function")


    return m


def is_hypergeometric_function(func):
    return func.name() in ['hypergeometric','hypergeometric_M','hypergeometric_U
    ']

def is_appell_function(func):
    return func.name() in ['hypergeometric']   #[appellf1] can't find this in
    sagemath

def is_atom(expn):

    debug=False
    if debug: print ("Enter is_atom")


    #thanks to answer at https://ask.sagemath.org/question/49179/what-is-
    sagemath-equivalent-to-atomic-type-in-maple/
    try:
        if expn.parent() is SR:
```

```
            return expn.operator() is None
        if expn.parent() in (ZZ, QQ, AA, QQbar):
            return expn in expn.parent() # Should always return True
        if hasattr(expn.parent(),"base_ring") and hasattr(expn.parent(),"gens"):
            return expn in expn.parent().base_ring() or expn in expn.parent().
    gens()
        return False

    except AttributeError as error:
        return False


def expnType(expn):

    if debug:
        print (">>>>>Enter expnType, expn=", expn)
        print (">>>>>is_atom(expn)=", is_atom(expn))

    if is_atom(expn):
        return 1
    elif type(expn)==list:   #isinstance(expn,list):
        return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if  type(expn.operands()[0])==Rational: #type(isinstance(expn.args[0],
    Rational):
            return 1
        else:
            return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.
    args[0]))
    elif expn.operator() == operator.pow:   #isinstance(expn,Pow)
        if type(expn.operands()[1])==Integer:  #isinstance(expn.args[1],Integer)
            return expnType(expn.operands()[0])   #expnType(expn.args[0])
        elif type(expn.operands()[1])==Rational:  #isinstance(expn.args[1],
    Rational)
            if type(expn.operands()[0])==Rational: #isinstance(expn.args[0],
    Rational)
                return 1
            else:
                return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.
    args[0]))
        else:
            return max(3,expnType(expn.operands()[0]),expnType(expn.operands()
    [1])) #max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1]))
    elif expn.operator() == add_vararg or expn.operator() == mul_vararg: #
    isinstance(expn,Add) or isinstance(expn,Mul)
        m1 = expnType(expn.operands()[0]) #expnType(expn.args[0])
        m2 = expnType(expn.operands()[1:]) #expnType(list(expn.args[1:]))
```

```
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.operator()):  #is_elementary_function(expn.
   func)
        return max(3,expnType(expn.operands()[0]))
    elif is_special_function(expn.operator()): #is_special_function(expn.func)
        m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(
   expn.args)))
        return max(4,m1)   #max(4,m1)
    elif is_hypergeometric_function(expn.operator()): #
   is_hypergeometric_function(expn.func)
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(
   expn.args)))
        return max(5,m1)   #max(5,m1)
    elif is_appell_function(expn.operator()):
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(
   expn.args)))
        return max(6,m1)   #max(6,m1)
    elif str(expn).find("Integral") != -1: #this will never happen, since it
                #is checked before calling the grading function that is passed.
                #but kept it here.
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(
   expn.args)))
        return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):

    if debug: print ("Enter grade_antiderivative for sagemath")

    leaf_count_result  = tree_size(result) #leaf_count(result)
    leaf_count_optimal = tree_size(optimal) #leaf_count(optimal)

    if debug: print ("leaf_count_result=", leaf_count_result, "
   leaf_count_optimal=",leaf_count_optimal)


    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if debug: print ("expnType_result=", expnType_result, "expnType_optimal=",
   expnType_optimal)

    if expnType_result <= expnType_optimal:
        if result.has(I):
            if optimal.has(I): #both result and optimal complex
```

```
            if leaf_count_result <= 2*leaf_count_optimal:
                return "A"
            else:
                return "B"
        else: #result contains complex but optimal is not
            return "C"
    else: # result do not contain complex, this assumes optimal do not as
well
        if leaf_count_result <= 2*leaf_count_optimal:
            return "A"
        else:
            return "B"
else:
    return "C"
```